(NASA-CR-142021) OPTIMIZATION OF STRUCTURES TO SATISFY AEROELASTIC REQUIREMENTS Final Technical Report (Clemson Univ.) 29 p HC $3.75 CSCL 20K

N75-15639

Unclas G3/05 07765

[redacted]

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Optimization of Structures to Satisfy Aeroelastic Requirements

by

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February 1975

Final Technical Report

The investigation described in this report was supported by the National Aeronautics and Space Administration under Grant No. NGR 41-001-027.

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Abstract

A method for the optimization of structures to satisfy flutter velocity constraints is presented along with a method for determining the flutter velocity. The material presented represents a summary of Ref. 1 through 5 which are a direct result of the research effort of this grant. A method for the optimization of structures to satisfy divergence velocity constraints is presented in Appendix B.

Acknowledgments

The author wishes to thank Dr. Kumar G. Bhatia, Mr. Jerry L. Cooper, Mr. Saifee K. Motiwalla and Mr. Yee-Yeen Chu for their diligent assistance.

The author also is indebted to Mr. Robert V. Doggett, Jr. of NASA Langley Research Center for his helpful suggestions and continuous assistance.

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1. Optimization of Structures to Satisfy a Flutter Constraint

In this section of the report a mathematical method for minimizing the structural mass of a lifting surface which is subject to a specified flutter velocity constraint will be presented. It will be assumed that structural parameters (cross-sectional areas, plate thicknesses, diameters squared, etc.) are selected in such a way that the total structural mass is a linear function of these parameters. The optimization procedure which will be presented is independent of the aerodynamic theory which might be selected.

1.1 Velocity Gradient Search

This routine is employed when it is desired to increase the flutter velocity. The flutter velocity normal derivative is calculated at a point and parameters are varied so that a step is taken in the direction of maximum increase in velocity. The desired flutter velocity is reached after several successive steps along a gradient curve in an iterative fashion as from point A to point B in Fig. 1.

The desired change in a design parameter during a velocity gradient search may be computed from the following relation which was derived in Ref. 1.:

$$\Delta P_i = V_{,i} \; (V^*-V)/\sum_{j=1}^{n} V^2_{,j} \qquad (1)$$

where V^* is the desired flutter velocity V is the current flutter velocity, $V_{,i} = \partial V/\partial P_i$ and n is the total number of design parameters.

New values of the design parameters may be determined from the relation

$$P^*_i = P_i + \Delta P_i \qquad (2)$$

A new value of the reduced frequency k may be estimated from the relation

$$k^* = k + \sum_{i=1}^{n} k_{,i} \, \Delta P_i \qquad (3)$$

This new value of k may be used as a starting value for search for the true flutter velocity in a procedure for determining the flutter velocity.

1.2 Projected Gradient Velocity Search

This routine is employed to find a relative maximum of the flutter velocity while the total mass of the structure is held constant. The parameters are varied in such a way that the search proceeds tangent to a constant mass hyperplane in the direction of the maximum rate of increase of the flutter velocity until a relative maximum is found which lies within the bounds of the parameter constraints as from B to C in Fig. 1. In Ref. 1 a procedure for searching for a maximum flutter velocity along a constant mass hyperplane was presented; however, the step size in the search was selected by trial, this resulted in a slow convergence to the maximum flutter velocity for that hyperplane. In Ref. 2 equations were presented for approximating the step size for moving from point B to C in Fig. 1. These relations are as follows:

$$S = - \sum_{j=1}^{n} V_{,j} \frac{dP_j}{ds} \Big/ \sum_{j=1}^{n} \sum_{h=1}^{n} V_{,jh} \frac{dP_j}{ds} \frac{dP_h}{ds} \qquad (4)$$

where

$$\frac{dP_j}{ds} = (V_{,j} + \theta_1 m_{,j})/2\theta_o \qquad (5)$$

$$\theta_1 = - \sum_{j=1}^{n} m_{,j} V_{,j} \Big/ \sum_{h=1}^{n} m^2_{,h} \qquad (6)$$

$$2\theta_o = \left\{ \sum_{j=1}^{n} (V^2_{,j} + V_{,j}\theta_1 m_{,j}) \right\}^{1/2} \qquad (7)$$

and where m is the total mass of the structure. New parameters may be computed from the relation

$$P^*_j = P_j + (dP_j/ds)S \qquad (8)$$

1.3 Mass Gradient Search

This routine is employed whenever it is desired to reduce the flutter velocity as from point C to point D in Fig. 1. A step is taken in the direction of the greatest rate of decrease in the structural mass and the process is repeated until the flutter velocity is less than or approximately equal to the desired value.

The changes in the design parameters are computed from the following equation which was derived in Ref. 1:

$$\Delta P_i = m_{,i} \; (V^* - V) / \sum_{j=1}^{n} V_{,j} \; m_{,j} \qquad (9)$$

The above three search procedures were applied to the design of a box beam in Ref. 1 and 2. The results of this application indicate that practical application of the above method to a real aircraft structure is feasible.

1.4 Partial Derivatives of the Flutter Velocity

Two of the search procedures which were previously described require the partial derivatives of the flutter velocity with respect to the design parameters. These partial derivatives may be found by considering the equations of oscillation of an aircraft structure which is in a state of neutral stability, i.e.

$$[K - \lambda_i (M+A)]U_i = 0 \tag{10}$$

where K, M and A are the stiffness matrix, mass matrix and aerodynamic force matrix, respectively. U_i is an eigenvector corresponding to the eigenvalue λ_i. The aerodynamic force matrix A is a function of the air density, Mach number and reduced frequency k.

Expressions for the first partial derivatives of the eigenvalues with respect to structural design parameters were derived in Ref. 1. These derivatives may be expressed in the form

$$\lambda_{i,j} = V_i^T [K_{,j} - \lambda_i (M_{,j} + A_{,j})] U_i \tag{11}$$

where the letter in the subscript which follows the comma indicates a partial derivative with respect to a design parameter, i.e.

$$\lambda_{i,j} = \partial\lambda_i/\partial P_j$$

The superscript T in equation (2) indicates the transpose of the left-hand eigenvector V where

$$V_i^T[K - \lambda_i(M+A)] = 0 \tag{12}$$

In the design process presented here it is convenient to constrain the derivatives of λ_i and k such that they are pure real. Then velocity gradient search and the projected gradient velocity search will move from one neutral stable state to another neutral stable state for the mode which determines the flutter velocity. The derivatives for this case are given in Ref. 1 and Ref. 2 and are repeated here.

$$\lambda_{i,j} = R_1 - R_2 A_1/A_2 \tag{13}$$

$$k_{,j} = - A_1/A_2 \tag{14}$$

where

$$R_1 + A_1 I_m = V_i^T [K_{,j} - \lambda_i M_{,j}] U_i \tag{15}$$

$$R_2 + A_2 I_m = - \lambda_i V_i^T \frac{\partial A}{\partial k} U_i \tag{16}$$

$$I_m = (-1)^{1/2}$$

The first partial derivative of the flutter velocity V_f is given by the relation

$$V_{f,j} = - b\omega_i k_{,j}/k^2 + b\lambda_{i,j}/2k\omega_i \tag{17}$$

where

$$\omega_i = \lambda_i^{1/2}$$

The second derivatives of the eigenvalues of equation (10) and the reduced frequency when the derivatives are constrained to be pure real are derived in Ref. 2 and may be expressed in the forms

$$\lambda_{i,jh} = R_3 - R_2 A_3/A_2 \tag{18}$$

and $$k_{,jh} = - A_3/A_2 \tag{19}$$

where
$$R_3 + A_3 I_m = V_i^T \Big\{ K_{,jh} - \lambda_{i,j} (M_{,h} + A_{,h})$$
$$- \lambda_{i,h} (M_{,j} + A_{,j}) - \lambda_i [M_{,jh} + (\partial^2 A/\partial k^2)/k_{,j} k_{,h}] \Big\} U_i \tag{20}$$
$$+ V_i^T F_{i,j} U_{i,h} + V_i^T F_{i,h} U_{i,j}$$

$$A_{,j} = \frac{\partial A}{\partial k} k_{,j} \tag{21}$$

$$A_{,jh} = \frac{\partial^2 A}{\partial k^2} k_{,j} k_{,h} + \frac{\partial A}{\partial k} k_{,jh} \tag{22}$$

$$V_i^T F_{i,j} U_{i,h} = V_i^T [K_{,j} - \lambda_i (M_{,j} + A_{,j})] \sum_{\substack{l=1 \\ l \neq i}}^{n} V_l^T [K_{,h} - \lambda_i (M_{,h} + A_{,h})] U_i U_l / (\lambda_i - \lambda_l) \tag{23}$$

The second derivatives of the flutter velocity with respect to the design parameters may be found by differentiating equation (17) with respect to a design parameter, then

$$V_{f,jh} = (b/2k\omega_i) \; [\lambda_{i,jh} - \lambda_{i,j} \lambda_{i,h}/(2\lambda_i)$$
$$-(\lambda_{i,j} k_{,h} + \lambda_{i,h} k_{,j})/k$$
$$+(2k_{,j} k_{,h}/k - k_{,jh}) 2 \lambda_i/k] \tag{24}$$

Alternate methods for finding the unconstrained derivatives of eigenvalues and eigenvectors were developed in Ref. 4 and 5. The "algebraic method" of Ref. 5 may be used to find the first derivatives of the eigenvalues and eigenvector of the non-self-adjoint system of eigenequation

$$[K - \lambda_i (M+A)] U_i = 0 \tag{10}$$

with respect to an independent variable P_j by solving the set linear equations

$$C \begin{Bmatrix} U_{i,j} \\ \lambda_{i,j} \end{Bmatrix} = DU_i \tag{25}$$

where

$$C = \left[\begin{array}{c|c} U_i^T & 0 \\ \hline K - \lambda_i (M+A) & -(M+A)U_i \end{array}\right] \tag{26}$$

and

$$D = \left[\begin{array}{c} 0 \\ \hline \lambda_i (M_{,j} + A_{,j}) - K_{,j} \end{array}\right] \tag{27}$$

and where it is required that $U_i^T U_i = 1$.

The second partial derivatives of the eigenvalues and eigenvectors may be found by differentiating equation (25) with respect to the independent parameter P_h, then

$$C \begin{Bmatrix} U_{i,jh} \\ \hline \lambda_{i,jh} \end{Bmatrix} = D_{,h} U_i + D U_{i,h} - C_{,h} \begin{Bmatrix} U_{i,j} \\ \hline \lambda_{i,j} \end{Bmatrix} \tag{28}$$

If $\lambda_{i,j}$, $U_{i,j}$ and $U_{i,h}$ are evaluated from equation (25) then $U_{i,jh}$ and $\lambda_{i,jh}$ may be found by solving equation (28).

The above process may be continued for any number of higher order derivatives.

It should be noted that the derivatives found by using the algebraic method are not constrained, and they may not be substituted into equations (17) and (24) to find the first and second derivatives of the flutter velocity; however if the reduced frequency is held constant in equations (25) through (28) and in the relations

$$V_f = b\ \omega_i/k = b\ \lambda_i^{1/2}/k \tag{29}$$

then

$$V_{f,j} = b\ \lambda_{i,j}/2k\omega_i \tag{30}$$

and

$$V_{f,jh} = b\ \lambda_{i,jh}/2k\omega_i - b\ \lambda_{i,j}\ \lambda_{i,h}/4k\omega_i^3 \tag{31}$$

The derivatives of V_f from equation (30) and (31) will (in the general case) be complex.

2. An Automated Procedure for Determining the Flutter Velocity

Efficient optimal design programs for aircraft structures which are subject to constraints on the flutter velocity require a rapid and automatic method for evaluating the flutter velocity. In Ref. 3 a computationally efficient method for finding the flutter velocity is presented. The method utilizes derivatives of the eigenvalues with respect to the reduced frequency in a curve fitting scheme for finding the critical roots of the flutter equation. The method is unaffected by the coalescence of any of the eigenvalues.

In Ref. 3 the derivatives of the eigenvalues were found by use of the methods of Ref. 1 and 2; however, the "algebraic" method of Ref. 5 could be more efficiently employed to find these derivatives.

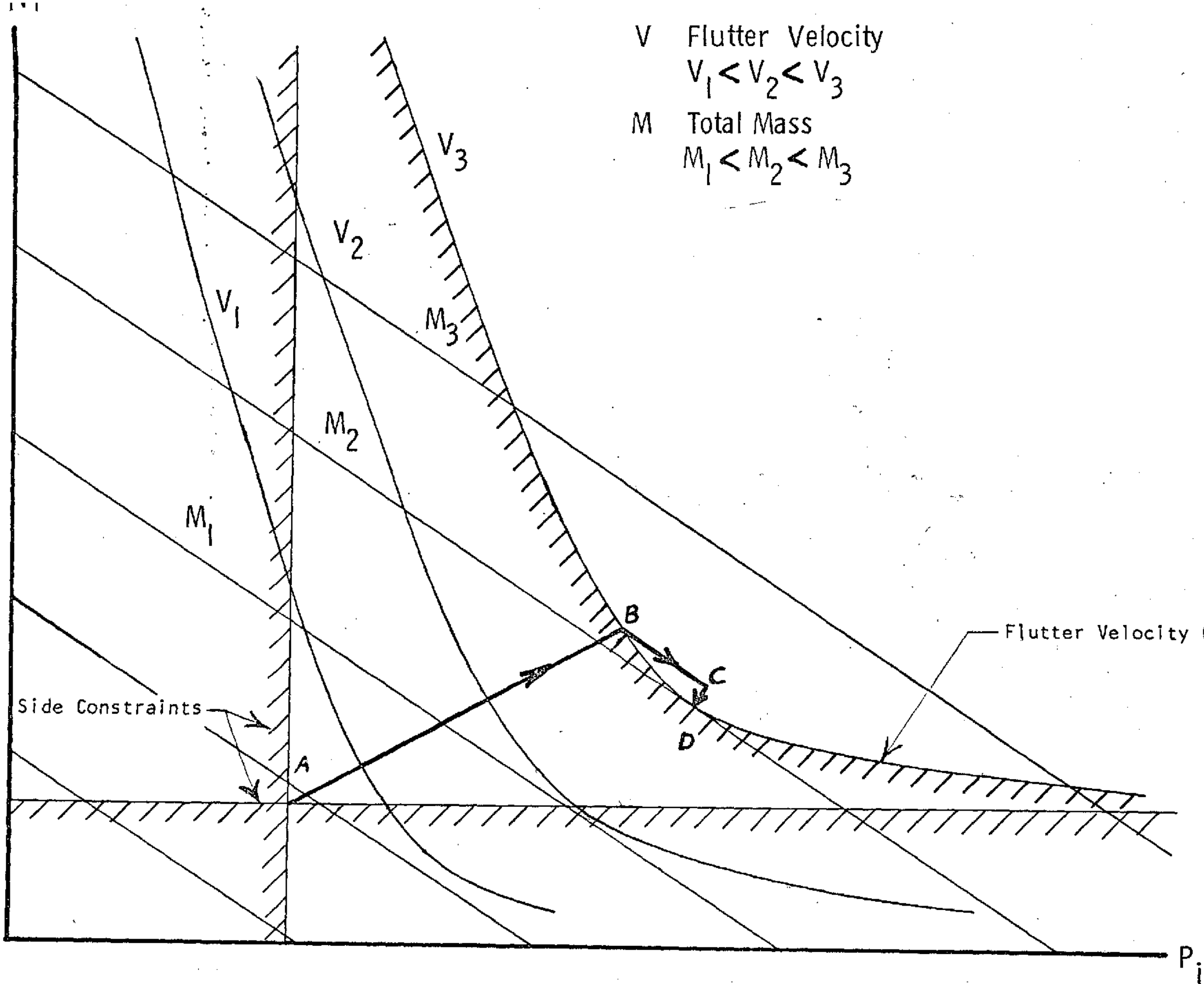
V Flutter Velocity
$V_1 < V_2 < V_3$
M Total Mass
$M_1 < M_2 < M_3$
V_1
V_2
V_3
M_1
M_2
M_3
A
B
C
D
Side Constraints
Flutter Velocity Constraint
P_i

Figure 1

Appendix A

Published References and Abstracts

As a direct result of support of this grant the following papers were published:

1. Rudisill, C.S. and Bhatia, K.G., "Optimization of Complex Structures to Satisfy Flutter Requirements," AIAA Journal, Vol. 9, No. 8, August 1971, pp. 1487-1491.

Abstract

Equations for finding the partial derivatives of the flutter velocity of an aircraft structure with respect to structural parameters are derived. A numerical procedure is developed for determining the values of the structural parameters such that a specified flutter velocity constraint is satisfied and the structural mass is a relative minimum. A search procedure is presented which utilizes two gradient search methods and a gradient projection method. The procedure is applied to the design of a box beam.

2. Rudisill, C.S. and Bhatia, K.G., "Second Derivatives of the Flutter Velocity and the Optimization of Aircraft Structures," AIAA Journal, Vol. 10, No. 12, December 1972, pp. 1569-1572.

Abstract

Equations for the second partial derivatives of the eigenvalues of the flutter equation along with the equations for finding the second partial derivatives of the flutter velocity of an aircraft structure with respect to the structural parameters are derived. These partial derivatives are used to develop expressions for the step size in a projected gradient search along a constant mass hyperlane. A projected gradient search along with a gradient mass and a gradient velocity search is used to minimize the mass of a box beam which supports a lifting surface.

3. Rudisill, C.S. and Cooper, J.L., "An Automated Procedure for Determining the Flutter Velocity," Journal of Aircraft, Vol. 10, No. 7, July 1973, pp. 442-444.

Abstract

A computationally efficient method for finding the flutter velocity of a structure is presented. The method utilizes derivatives of the eigenvalues with respect to the reduced frequency in a curve fitting scheme to find the critical roots of the flutter characteristic equation. The method is unaffected by the coalescence of any of the eigenvalues.

4. Rudisill, C.S., "Derivatives of Eigenvalues and Eigenvectors for a General Matrix," AIAA Journal, Vol. 12, No. 5, May 1974, pp. 721-722.

Abstract

A method is developed for finding all orders of the derivatives of the eigenvalues and eigenvectors of a non-self-adjoint system of algebraic eigenvalue equations. The method does not require a complete eigenanalysis of the algebraic eigenvalue problem. The method requires only the eigenvalue and eigenvector whose derivatives are sought and the method also requires the corresponding left-hand eigenvector.

5. Rudisill, C.S. and Chu, Yee-yeen, "Numerical Methods for Evaluating the Derivatives of Eigenvalues and Eigenvectors," Accepted for publication in the AIAA Journal. Publication date will probably be in the late spring of 1975.

Abstract

Two numerical methods for computing the derivatives of eigenvalues and eigenvectors are developed. The first method is an iteration method for finding the first partial derivative of the eigenvalues and eigenvectors of a self-adjoint system of algebraic eigenvalue equations. The iteration method will also find the first partial derivative of the largest eigenvalue and its corresponding eigenvector of a non-self-adjoint system, but the method cannot be used to find the derivatives of the remaining eigenvectors.

The second method will find all orders of the derivatives of the eigenvalues and eigenvectors of a non-self-adjoint system of algebraic eigenvalue equations. The method does not require a complete eigenanalysis of the algebraic eigenvalue problem. The method requires only the eigenvalue and eigenvector whose derivatives are sought.

Appendix B

OPTIMIZATION OF COMPLEX STRUCTURES TO SATISFY DIVERGENCE VELOCITY CONSTRAINT*

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ABSTRACT

Analytical expression for evaluating the partial derivatives of the torsional divergence velocity of an aircraft structure with respect to design variables is derived. An optimization procedure to satisfy a specified divergence velocity is illustrated, using gradient methods and finite-element representation, for a box beam with the lower and upper values specified for the design variables. It is shown that there is a possibility of serious designer judgement error due to inefficient performance of optimization methods against multiple constraints. A "dimension reduction technique" is proposed to help in such situations.

The authors wish to express their appreciation to Mr. R. V. Doggett, Jr. of the NASA Langley Research Center for his many valuable suggestions and advice.

*This research was supported by NASA Research Grant NGR-41-001-027

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1. INTRODUCTION

In the recent past, there have been several publications dealing with optimization with respect to a static aeroelastic constraint on the torsional divergence velocity [1,2,3,4]*. McIntosh and Eastep [2] presented a calculus of variations formulation for a tapered cantilevered wing with torsional stiffness dominated by contributions from a thin outer skin. McIntosh and Weisshhaar [3], and Armand and Vitte [4] used transition-matrix approach of optimal control theory. McIntosh and Weisshhaar [3] concluded that early optimism concerning the use of transition-matrix approach must be tempered somewhat, and it may in the long run prove most useful to adopt more sophisticated steepest ascent or gradient methods. Further, it appears that even for the gradient methods to be computationally attractive for practical problems, without excessive penalty in the computer time used to arrive at an optimum solution, closed form analytical expressions should be derived for the partial derivatives involved so that their computation does not involve individual perturbation of each design variable and corresponding evaluation of the objective function. This would be a very significant consideration for problems with a large number of design variables, and/or where evaluation of the objective function is expensive in terms of computer time. In the present paper, the authors' aim is, therefore, to develop a closed form expression for the partial derivatives of torsional divergence velocity with respect to design variables, and to present a practical method for optimum weight design of an aircraft wing structure subject to torsional divergence velocity constraint.

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*Numbers in square brackets indicate references listed at the end of paper.

II. PARTIAL DERIVATIVES OF DIVERGENCE VELOCITY

The governing characteristic equation for torsional divergence of an aircraft wing structure in a static neutral state can be expressed as, (see Ref. (5))

$$([K] - \lambda[A])\ \{U\} = 0 \tag{1}$$

where

$[K]$ = torsional stiffness matrix, function of design variables P_i, symmetric matrix.

$[A]$ = torsional air-force matrix, function of air density and wing geometry, constant and real.

λ = eigenvalue, equal to the divergence velocity squared.

$\{U\}$ = angular displacement of the wing

To define an associated row vector $\{V\}'$ of the eigenvector $\{U\}$, consider

$$([K] - \lambda[A]')\ \{V\} = 0 \tag{2a}$$

Taking the transpose of equation (2a) and using the symmetry property of $[K]$ yields

$$\{V\}'([K] - \lambda[A]) = 0 \tag{2}$$

Differentiation of the characteristic equation (1) with respect to a design variable P_i yields

$$\left(\frac{\partial[K]}{\partial P_i} - \frac{\partial\lambda}{\partial P_i}[A]\right)\{U\} + ([K] - \lambda[A])\frac{\partial\{U\}}{\partial P_i} = 0 \tag{3}$$

Premultiplying equation (3) by $\{V\}'$ and simplifying by substituting equation (2), results in the equation

$$\frac{\partial\lambda}{\partial P_i} = \frac{\{V\}'\ \dfrac{\partial[K]}{\partial P_i}\ \{U\}}{\{V\}'[A]\ \{U\}} \tag{4}$$

A similar expression can be derived using the flexibility matrix instead of the stiffness matrix, in which case the characteristic equation is given by

$$([C][A] - 1/\lambda[I])\{U\} = 0 \qquad (5)$$

where [C] is the inverse of [K].

From equation (5) it can be shown that

$$\frac{\partial\lambda}{\partial P_i} = - \frac{\lambda^2 \{V\}' (\frac{\partial [C]}{\partial P_i}) [A] \{U\}}{\{V\}'\{U\}} \qquad (6)$$

The partial derivatives of the divergence velocity can be quickly computed from either equation (4) or (6). The use of these derivatives will be discussed in section III.

III. OPTIMIZATION PROCEDURE

Two general approaches to optimization have evolved thus far: maximizing a certain eigenvalue for fixed mass, or minimizing the mass for a fixed eigenvalue [2]. The authors use the first approach, which is equivalent to projected gradient method with eigenvalue as the objective function, in conjunction with gradient mass and gradient velocity methods. This procedure is described in detail in the authors' earlier paper [6]. The optimum design with divergence velocity constraint will be illustrated for a three bay box beam (See Figure 1) with twelve design variables having upper and lower constraints.

The optimization program is independent of the formulation used for the air-force matrix. However, significant simplification is obtained when the air-force matrix [A] is symmetric, in which case equation (4) is simplified to

$$\frac{\partial \lambda}{\partial P_i} = \frac{\{U\}' \left[\frac{\partial K}{\partial P_i} \right] \{U\}}{\{U\}' [A] \{U\}} \tag{7}$$

This simplification would not be achieved using the flexibility matrix. Thus there is substantial computational advantage for the case of symmetric air-force matrix, if the stiffness approach is used. For the numerical example presented in this paper, a diagonal air-force matrix was used [5].

Table 1 lists the results obtained from the optimization program for the box-beam of Figure 1. Two sets of values for the design variables were used as initial input to the optimization program. The mass-optimization was subject to a torsional divergence velocity constraint of 600 ft./sec. with upper and lower constraints specified for all the 12 design variables. These constraints were same for both the cases of initial design values.

For the case 1, an optimum design was very quickly arrived at in two design cycles. A design cycle here defines a step taken in the multi-dimensional space of design variables during an execution of projected gradient, mass gradient or velocity gradient search; each design cycle involves computation of stiffness matrix and evaluation of the eigenvalue problem in addition to computation of derivatives. The final values for P_2, P_3 and P_{10} seem to be numerically same as the corresponding initial values. Actually the final values are slightly higher than the initial values, but due to a very small numerical difference this is not apparent from the number of digits listed in the table. Therefore, the optimum arrived at for case 1, appears to be a free optimum, i.e., the design parameters are not against any constraint.

For the case 2, the lower constraints for P_7, P_8 and P_9 slowed down the optimization procedure considerably, and it took 47 design cycles to reach a mass approximately 40% higher and a divergence velocity approximately 0.45% higher than for the previous optimum attained. This suggests the possibility of serious designer judgement error due to inefficient performance of optimization methods against multiple constraints. To circumvent this, the authors suggest the use of "dimension reduction technique". The proposed technique would monitor the partial derivatives of the relevant eigenvalue with respect to the design parameters which are against the constraints, and from these determine the design parameter which would tend to violate the constraints if a step in the desired direction were taken. Such parameters would be then held constant for the next step, thus effectively reducing the dimension of the design parameter space for this step. At the new point thus reached, a new design cycle would begin and partial derivatives of the relevant eigenvalue with respect to all the design variables would be computed, and the above process repeated. It is expected that such a technique would reduce the number of design cycles required for problems where constraints are encountered.

CONCLUSIONS

The closed form analytical expression derived for the partial derivatives of divergence velocity with respect to a design parameter is useful in the gradient type search procedures. The example solution illustrates its use. It seems that where lower constraints are specified for the design variables, a logical initial point should preferably include applicable lower constraints. This view is substantiated by authors experience that gradient velocity procedure operating in the neighborhood of lower constraints is very fast and effective in increasing the velocity to the desired value, since in increasing the velocity it would usually tend to move away from the constraints. For the case where the constraints slow down the gradient methods, the dimension reduction technique may improve their performance. The authors hope to explore the potential of this method in their future work.

REFERENCES

1. Ashley, H., and McIntosh, S.C., "Expanding the Consciousness of the Aeroelastician," *Fluid-Solid Interaction*, American Society of Mechanical Engineers, 1967, pp. 120-143.

2. McIntosh, S. C., and Eastep, F. E., "Design of Minimum-Mass Structures with Specified Stiffness Properties," *AIAA Journal*, Vol. 6, No. 5, May 1968, pp. 962-964.

3. McIntosh, S. C., Jr., Weisshhaar, T. A. and Ashley, H., "Progress in Aeroelastic Optimization--Analytical Versus Numerical Methods," SUDAAR No. 383, July 1969, Stanford University, Stanford, California.

4. Armand, J. and Vitte, W. J., "Foundations of Aeroelastic Optimization and Some Applications to Continuous Systems," SUDAAR No. 390, Jan. 1970, Stanford University, Stanford, California.

5. Scanlan, R. H., and Rosenbaum, R., *Introduction to the Study of Aircraft Vibration and Flutter*, Dover, New York, 1968, page 299.

6. Rudisill, C. S. and Bhatia, K. G., "Optimization of Complex Structures to Satisfy Flutter Requirement," To be published, *AIAA Journal*, 1971.

TABLE 1 Initial and Final Parameters with Constraints

		Area of Longitudinals (Sq. in.)			Front & Back Web Thickness ($\times 10^{-1}$ in.)			Top & Bottom Web Thickness ($\times 10^{-1}$ in.)			Rib Thickness ($\times 10^{-1}$ in.)			Total Mass (Slugs)	Divergence Velocity (ft./sec.)
		Bay 1 P_1	Bay 2 P_2	Bay 3 P_3	Bay 1 P_4	Bay 2 P_5	Bay 3 P_6	Bay 1 P_7	Bay 2 P_8	Bay 3 P_9	Bay 1 P_{10}	Bay 2 P_{11}	Bay 3 P_{12}		
Initial Values	Case 1	0.33264	0.33264	0.33264	0.13332	0.13332	0.13332	0.0666	0.0666	0.0666	0.0666	0.0666	0.0666	0.92	559.23
	Case 2	2.000	2.000	2.000	0.800	0.800	0.800	0.400	0.400	0.400	0.400	0.400	0.400	5.54	1370.41*
Constraints	Lower	0.33264	0.33264	0.33264	0.13332	0.13332	0.13332	0.0666	0.0666	0.0666	0.0666	0.0666	0.0666	0.92	559.23
	Upper	8.0064	8.0064	8.0064	3.204	3.204	3.204	1.596	1.596	1.596	1.596	1.596	1.596	22.15	2738.71*
Final Values	Case 1	0.33336	0.33264	0.33264	0.1362	0.13464	0.133368	0.081228	0.082284	0.072672	0.0666	0.066756	0.067008	0.95	599.99
	Case 2	0.3865	0.3865	0.3865	0.35184	0.35184	0.3184	0.0666	0.0666	0.0666	0.3066	0.3066	0.3066	1.30	602.65

* Beyond the range of aerodynamic theory used.

a) Number of design cycles: Case 1 - 2

Case 2 - 47

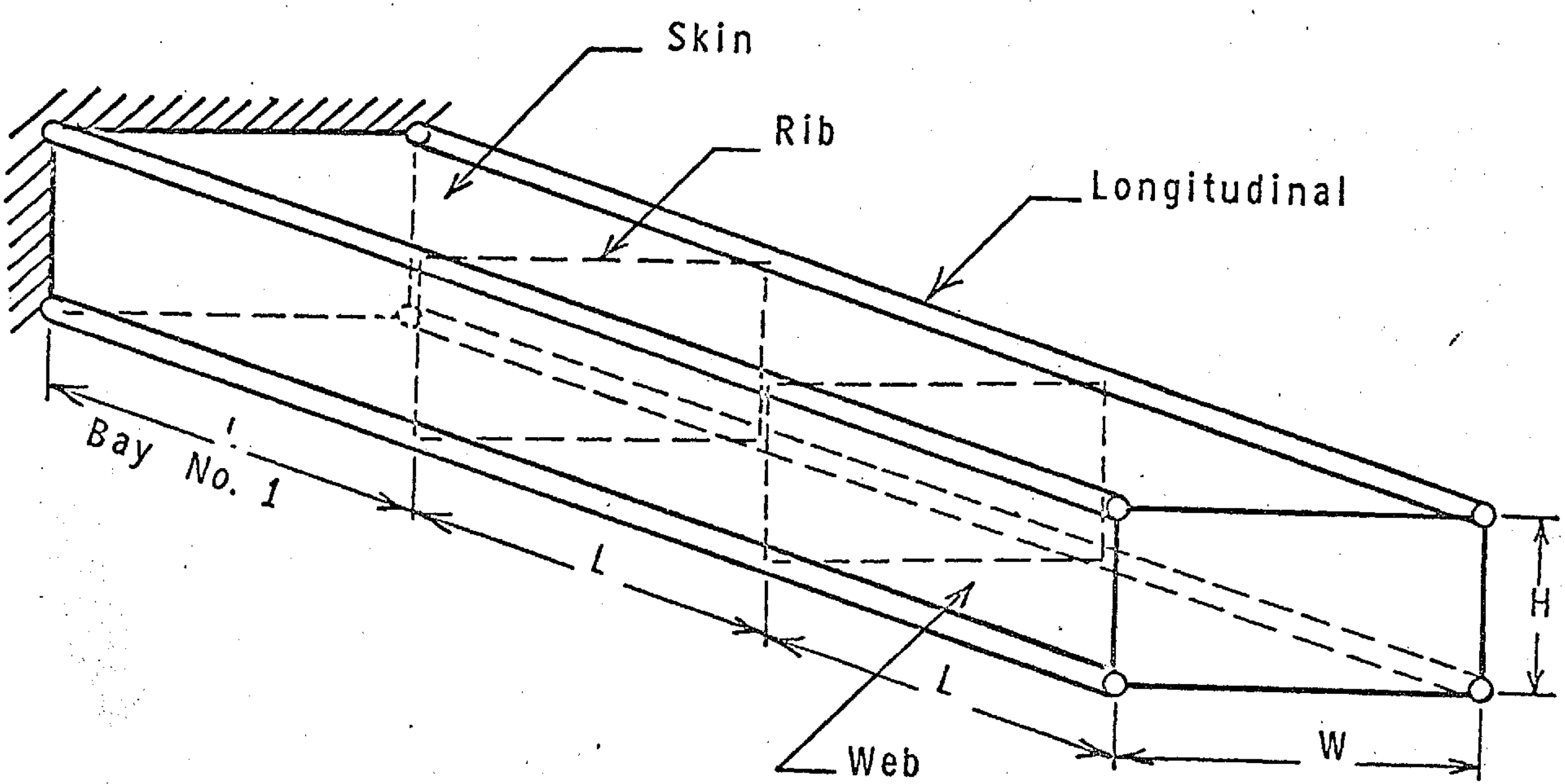

Figure 1