FINAL REPORT

ALABAMA A&M

ROCKET ENGINE THRUST CHAMBER HEAT TRANSFER CALCULATION AND ANALYSIS

September 1974

Principal Investigator:
Hrishikesh Saha

Prepared For

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Grant No. NGR 01-001-024
Technical Monitor: Klaus Gross

ALABAMA AGRICULTURAL AND MECHANICAL UNIVERSITY
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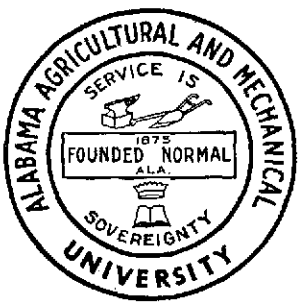

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FOREWORD

The present report describes a rocket nozzle turbulent boundary layer heat transfer rate parametric study using three different computer programs.

This investigation, entitled ROCKET ENGINE THRUST CHAMBER HEAT TRANSFER CALCULATION AND ANALYSIS, was conducted for the Propulsion and Thermodynamics Division of MSFC, National Aeronautical and Space Administration under the grant No. NGR01-001-024 with Klaus W. Gross as technical monitor. Hrishikesh Saha of Alabama Agricultural and Mechanical University was the principal investigator.

This research program contributed extensibely to improve faculty and student-research capability at the Alabama A&M University and this support by NASA-MSFC is greatly appreciated.

The author wishes to acknowledge the helpful assistance and advice received from Mr. Klaus W. Gross and Alfred Krebsbach, NASA-MSFC. The author would also like to express his appreciation to Mr. Marion I. Kent, Assistant Director, University Affairs, NASA,MSFC, for his effort in making this grant possible.

ABSTRACT

A parametric study of the heat transfer rate along the wall of a rocket nozzle is presented. The influences of different parameters; Laminar and turbulent Lewis number, mixture ratio, initial wall temperature distribution, and eddy viscosity, are considered in this study.

The numerical evaluation of these influences on heat transfer rate is done by using three different compressible, reacting Laminar and turbulent boundary layer computer programs; MABL (Mass Addition Boundary Layer Program), MABL-KE (MABL program is modified to include turbulent kinetic energy equation), and BLIMP (Boundary Layer Integral Matrix Procedure).

This study also provides an excellent opportunity to evaluate the efficiencies of these three computer programs and to suggest one of them for future computational purposes. As described in the recommendations, the suggested program needs further improvements, in its mathematical model and numerical solution method, to be used as a standard computing procedure for comparison with future experimental results.

LIST OF FIGURES

LIST OF SYMBOLS

A van Driest's Damping factor

C_f Skin friction coefficient

C_p Specific heat at constant pressure

D Diffusion coefficient

H Total enthalpy, $H = \bar{H} + H' = h + u^2/2$

h Enthalpy, $h = \bar{h} + h' = \sum_{i=1}^{n} h_i Y_i$

h_i Enthalpy of species i, $h_i = \bar{h}_i + h'_1 =$

$$\int_{T^o}^{T} C_{pi}\, dT + h_i^o$$

K Turbulent kinetic energy per unit density, $K_o = u'u' + v'v' + w'w'$, and $K = \bar{K}_o$

k Constant

L Lewis number

l Prandtl mixing length

M Mean molecular weight

$\dot{m}_w$ Mass addition rate from the wall

n Number of species

P,p Pressure

P_c Chamber stagnation pressure

P_r Prandtl number

$\dot{q}_w$ Heat Transfer rate into the wall

R Universal gas constant

r_{mi} Mass ratio of element m in species i

r_w Nozzle radius

St Stanton number

T Temperature

u velocity in x direction

v velocity in y direction

s,x Distance along the nozzle wall

C_i,Yi Mass fraction of species i

y Normal distance from the wall

z Axial distance

α Constant

α_m Element mass fraction, $\alpha_m = \sum_{i=1}^{n} r_{mi} Yi$

β Constant

γ Constant

δ Velocity thickness, $\delta = y$ at $u/u_e = 0.995$

δ_1 Velocity thickness, $\delta_1 = y$ at $u/u_e = 0.9999$

δ^* Displacement thickness, $\delta^* =$

$$\int_0^\infty (1 - pu/p_e U_e) dy$$

ε Eddy viscosity

Θ Momentum thickness, $\Theta =$

$$\int_0^\infty (pu/p_e u_e)(1 - u/u_e) dy$$

Λ Dissipation length

K,λ Thermal conductivity

μ Molecular viscosity

ρ Density

τ Shear stress

Subscripts

e Boundary layer edge

I Transpiration coolant

i Species

m Element

T Turbulent

w WAll

Superscripts

$(\quad)'$ Fluctuating term

$\overline{(\quad)}$ Time averaged quanity

SECTION I

INTRODUCTION

Boundary layer behavior along the walls of a rocket nozzle plays an important role in the performance of a rocket engine. The shear layer determines part of the thrust loss of the nozzle and the energy layer controls the heat transfer to the wall and the wall temperature. There is a need for a computer program which can calculate boundary layer properties and effects for flows with large pressure gradient, chemical reactions, and a wide variety of wall conditions.

Accurate heat transfer prediction for regeneratively cooled thrust chamber is of vital importance, since the increased energy level of the coolant has a direct impact on the engine performance. Heat transfer rate has also an effect on the contour design and material selection of the thrust chamber wall.

The primary purpose of this report is to present the results of a comparative study of boundary layer heat transfer rate at a nozzle wall, computed by the computer programs presently available at the MSFC Propulsion and Thermodynamics Division and also to select one of the programs suitable for further studies of the SSME characteristics.

The computer programs presently available at NASA-MSFC for prediction of heat transfer rate in a rocket nozzle boundary layer are

a) MABL (Mass Addition Boundary Layer, Ref. 11&12).

b) MABL-KE (MABL modification to include turbulent kinetic energy equation, Ref. 19).

c) BLIMP (Boundary Layer Integral Matrix Procedure, Ref.23-25).

Short descriptions of these programs with respect to their,

- basic assumptions made to set up the governing equations of the boundary layer flow including discussions of the conservation equations, turbulence model, coordinate transformation, and boundary conditions, and
- numerical techniques and analysis which led to the computer solutions,

are given in section 2.

For the sake of completeness, a short derivation of the steady state compressible turbulent boundary layer equations for two dimensional and axisymmetric flows of chemically reacting mixtures is given in the appendix. The turbulent kinetic energy equation to compute eddy viscosity is also included in the appendix.

Section 3 contains the various turbulence model, definitions, and presently available computational formulas of the laminar and turbulent transport coefficients and parameters; coefficients of molecular, -viscosity, μ , thermal conductivity, K, and diffusion, D_{ij} , and eddy viscosity ε, eddy conductivity K_T, eddy diffusivity D_T, and Prandtl number, P_r, Lewis number, L, and Schmidt number, S. This section also contains a derivation of the heat flux equation used in different computer programs.

A parametric study of the heat transfer rate along the wall of a rocket engine thrust chamber using different computer programs is given in section 4. The effects on the heat flux due to the variation of the parameters; Lewis number, Mixture ratio, initial wall temperature distribution, and eddy viscosity are shown in the figures.

Section 5 presents a comparative study of the available computer programs, MABL, MABL-KE, and BLIMP, and suggests one program for future computation with several modifications and improvements.

SECTION II

BOUNDARY LAYER COMPUTER PROGRAMS

The computer programs, presently used at MSFC to compute turbulent boundary layer properties in a rocket thrust chamber, are described below:

MABL (Mass Addition Boundary Layer) Computer Program.

This effort was carried out (Ref 11 & 12) for the purpose of developing a practical and immediately useful state-of-the-art engineering method for analitically predicting the boundary layer performance losses in liquid rocket engines; in particular for the space shuttle engine configurations currently under consideration by NASA. Therefore, the analysis and resulting computer program have been oriented towards the analysis of high pressure hydrogen-oxygen engines.

The boundary layer equations for compressible turbulent flow, derived from the time dependent Navier-Stokes equations using the Reynolds time-averaging procedure and the usual boundary layer order of magnitude assumptions in a curvilinear coordinate system, were considered for solution.

The analysis and program use equilibrium chemistry since the flow in the high pressure H_2-O_2 engines will most probably be very close to the equilibrium. The computer program is set up and dimensioned, at present, to handle the hydrogen-oxygen system and considers two elements (H,O) and six species (H_2, H, OH, H_2O, O_2, O). The equilibrium state calculations in the present analysis are performed by selected subroutines from the One-Dimensional Equilibrium (ODE) JANNAF Performance Program; given the element mass fractions and two thermodynamic state variables this program solves for the species mass fractions and all the other thermodynamics state variables.

As a result of the parabolic nature of the boundary layer equations, initial profiles are required to begin the solution.

The numerical method chosen for the solution of the boundary layer equations is a Crank-Nicolson implicit finite difference technique. In applying this finite difference technique the difference analogs of the equations of motion are linearized and uncoupled and solved using a tridiagonal matrix inversion algorithom.

In formulating the turbulent equations of motion it was assumed that the turbulent flux terms could be related to mean, time averaged quantities, through the use of the phenomenological mixing length-eddy viscosity concept. The turbulence eddy viscosity model, used here, was developed and extended by Cebeci (Refs. 17, 18). This model uses a two-layer representation of the eddy viscosity. In the inner region, closer to the wall, the eddy viscosity is based on Prandtl's mixing length theory, as modified by Van Driest to account for the damping effect of the wall, and as extended by Cebeci to include wall mass transfer, compresibility and pressure gradient effects. In the outer, wake-like, portion of the boundary layer, Clauser's form of the eddy viscosity, modified to include an intermittency factor, is used. Turbulent Prandtl number formulation developed by Cebeci (Ref. 17) is used here. The equations and computer program have been kept general to allow for an arbitrary variation of turbulent Lewis number. No attempt has been made to calculate or model Lewis number.

MABL-KE (MABL with Turbulent Kinetic Energy Equation) Computer Program.

The MABL program described earlier was modified to include turbulent kinetic energy equation for solving the compressible turbulent boundary

layers along the wall of a rocket thurst chamber (Ref. 19). This inclusion of the turbulent K. E. equation provides better results than the method utilizing the Van Driest-Clauser hypothesis; this also the character of chemically reactive flow with mass injection and can exhibit the past history of boundary layers. Results show the variation of eddy viscosity in flow direction including the laminarization tendency in the nozzle convergent section. The results appear to be in good agreement with available experimental data. The eddy viscosity decreases significantly in the reactive hydrogen-oxygen boundary layer when a small amount of hydrogen is injected from the wall.

The equations and boundary conditions for the compressible turbulent boundary layer are included in the appendix for completeness sake.

The numerical solution technique is similar to that of MABL as described earlier.

BLIMP (Boundary Layer Integral Matrix Procedure) Computer Program.

BLIMP version J serves as a boundary layer prediction computer program for rocket nozzle flow. The references 13, 25, 29, 30, 31, 32 describe a mathematical model and numerical solution of nonsimilar laminar or turbulent multicomponent chemically reacting boundary layer flows with arbitary equilbrium chemical systems, unequal diffusion and thermal diffusion coefficient for all species, and a variety of surface boundary conditions.

The equations of motion are developed for axisymetric or two-dimensional turbulent flow including transverse curvature effects. The usual turbulent flow technique of breaking the species, velocity,

and enthalpy fields into mean and fluctuating components, time averaging, and making appropriate order of magnitude approximations results in the governing equations. The turbulent transport terms are expressed in the Boussinesg form, i.e. eddy viscosity, eddy diffusivity, and eddy conductivity. Hence, all the terms in the equations are time-averaged quantities. In the order-of-magnitude arguments, terms of the following types have been eliminated; 1) triple correlations, 2) derivatives of turbulent correlations parallel to the wall, and 3) correlations involving turbulent components of molecular transport mechanisms. Also necessary in the mathematical formulation of the problem is the specification of the molecular transport properties, equation of state and equilibrium relations for the multicomponent gas, and a descritpion of the eddy viscosity, conductivity, and diffusivity.

The boundary layer flow is divided into a wall region and a wake region. A mixing length formulation for turbulent shear stress was chosen for the wall region. In the wake portion of the boundary layer, a constant eddy viscosity model (Clauser's equilibrium boundary layer expression (Ref. 33))is used.

A bifurcation approximation to binary diffusion coefficients introduced in (Ref. 20) utilized herein permits expilicit solution of the Stefan-Maxwell relations for the diffusion flux in terms of gradients and properties of species and of the system as a whole.

A modified Levy-Lees coordinate transformation is used to reduce the partial-differential equations of the boundary layer to total-differential equations.

The solution procedure uses splined gradratics or cubics to represent velocity, enthalpy, and species concentrations between nodes which give smooth variations of the dependent variables and their derivatives through the boundary layer, therefore allowing significantly fewer nodes for a given accuracy. This advantage is particularly valuable for turbulent boundary layers, where the very large gradients in the surface normal direction would require a great number of nodal points for an accurate solution with typical finite difference representations. In the streamwise direction, derivatives are expressed by ordinary implicit difference relations using linear or quadratic curvefits of the dependent variables. The resulting set of algebraic linear and nonlinear simultaneous equations is solved iteratively by the general Newton-Raphson technique.

TRANSPORT PROPERTIES

Historically there have been two different approaches to developing the equations of gas dynamics, i.e. equations for the conservation of species, conservation of mass, conservation of momentum, and conservation of energy along with the equation of state for the gas mixture.

One method is the phenomenological approach wherein certain relations between shear and strain, heat flux and temperature gradient, and diffusion flux and concentration gradient are postulated and the equations then developed using the laws of classical mechanics and heat flow. The method leaves the transport coefficients, that is, the constant of proportionality between shear and strain, heat flux and temperature gradient, and diffusion flux and concentration gradient, undefined with no method other than direct measurements available to determine their values. These transport coefficients are identified by the symbols μ, K, and D_{ij} for laminar flows, where, for example (Ref. 3),

$\mu = \frac{\tau_{yx}}{\partial u/\partial y}$ = Coefficient of viscosity

$K = \frac{-\dot{q}_y}{\partial T/\partial y}$ = Coefficient of thermal conductivity

and $D_{ij} = \frac{\overline{V_{iy}}\, C_i}{\partial C_i/\partial y}$ = binary diffusion coefficient

where τ_{yx} = shear stress on surface perpendicular to y axis acting in x direction in cartesian coordinate system,

$-\dot{q}_y$ = heat flux in y direction of cartesian coordinate system,

$\bar{V}_{iy}$ = component of diffusion velocity due to concentration gradients in y direction of cartesian coordinate system for a species i diffusing into a mixture of species i and j,

$C_i = \rho_i/\rho$, mass fraction of species i,

T and ρ are the temperature and density of the gas mixture, respectively.

The alternative approach is that of the mathematical theory for nonuniform gases, essentially a kinetic theory approach (Ref. 21). This method yields the fluid-dynamic equations with the transport coefficients defined in terms of certain integral relations which involve the dynamics of colliding particles. Some empiricism is involved in specifying the interparticle forces needed to evaluate the collision integrals but even here, in principle, recourse to theory will narrow the margin of uncertainty involved. The transport coefficients μ, K, and D_{ij} are found to be funtions of the gas-mixture temperature, gas-species molecular weights, and certain parameters of the interparticle force field.

The conservation equations can often be written in forms involving dimensionless ratios of various transport coefficients. These dimensionless transport parameters are defined as follows:

The PRANDTL NUMBER, which is a rough measure of the relative importance of momentum transfer and heat transfer, is defined as

$P_r = \mu C_{pf}/K$, where $C_{pf} = \sum_i C_i C_{p_i}$ = frozen Specific heat & C_{p_i} = Specific heat

It can be shown from the energy equation (Ref. 34) that

$$\frac{K(\partial T/\partial y)}{\mu u(\partial u/\partial y)} = \frac{\text{Conduction}}{\text{Shear work}} \simeq \frac{1}{P_r E} \quad \text{where}$$

$E = u_e^2/h_e$ is called Eckert's number.

For a pure polyatomic gas Prandtl number is given by the Eucken form, $P_r = 4\gamma/(9\gamma - 5)$. From this equation it is seen that P_r, which is approximately 3/4 for air ($\gamma = 1.4$), varies from 2/3 for monatomic gases ($\gamma = 5/3$) to unity as $\gamma \to 1$. In some systems (e.g., liquids) the Eucken formula is not valid and P_r can differ considerably from unity.

The SCHMIDT NUMBER, a rough measure of the relative importance of momentum transfer and mass transfer, is defined for a binary mixture by,

$S = \mu/\rho D_{12}$, from Energy equations (Ref. 34) it can be shown that

$$\frac{\rho D_{12} \sum h_i(\partial C_i/\partial y)}{\mu u(\partial u/\partial y)} = \frac{\text{diffusion}}{\text{shear work}} \simeq \frac{1}{SE}$$

If S is large, shear work predominates over diffusion. In multicomponent systems, Schmidt numbers may be defined for each pair of species. As in the case of Prandtl numbers, S is often somewhat less than unity.

The LEWIS NUMBER is the ratio of the energy transported by conduction to that transported by diffusion, and is defined for a binary mixture by

$$L = \frac{\rho D_{12} C_{pf}}{K} = \frac{Pr}{Sc} \sim \frac{\text{diffusion}}{\text{conduction}} = \frac{\rho D_{12} \sum_i h_i (\partial C_i / \partial y)}{K (\partial T / \partial y)}$$

As with the Schmidt number, Lewis numbers may be defined for each pair of species in multicomponent mixtures.

In many systems, L is very nearly unity; it is often slightly greater than unity in combustible gas mixtures. The approximation L= 1 is frequently very helpful in theoretical combustion analysis.

LAMINAR TRANSPORT PROPERTIES

The ideal gas calculations performed to date were basically for program check-out purposes and restricted to a one component gas so the Lewis number is not required. The Prandtl number is assumed to be a constant. Its value can be selected to approximate the gas being considered. Most of the ideal gas calculations have been for air and currently the viscosity calculation is based on Sutherland's law expressed as

$$\mu = 2.27 \cdot 10^{-8} \cdot T^{1/2} / (1 + 198.6/T)$$

For gases other than air a different viscosity formulation should be used.

The calculation of the laminar transport properties for a mixture of gases (for example : H_2-O_2 system) is based on the following:

The viscosity of the mixture is calculated from Wilke's semi-emperical formula (Ref. 20).

$$\mu = \sum_{i=1}^{n_s} \mu_i \left(1 + \sum_{j=1}^{n_s} \varphi_{ij} \frac{x_j}{x_i} \right)^{-1}$$

where n_s is the number of species, X_i is the mol fraction of species i; that is, $X_i = C_i / (M_i \cdot \sum_i C_i / M_i)$, μ_i is the viscosity of each species i;

$$\mu_i = 266.93 \times 10^{-7} (M_i T)^{1/2} / (\sigma^2 \Omega^{(2,2)*})$$

here M_i = molecular weight of species i

σ = collision diameter,

T = temperature

$\Omega^{(2,2)*} = \Omega^{(2,2)} / (\Omega^{(2,2)})$ rigid sphere

$\Omega^{(2,2)}$ = collision integral (Ref. 21)

and $\varphi_{ij} = \left[1 + (\mu_i/\mu_j)^{1/2} (M_j/M_i)^{1/4} \right]^2 / \left[(2)^{3/2} (1 + M_i / M_j)^{1/2} \right]$

For simpler and faster computational purposes the values of μ_i are taken from reference [40] and stored in tabular form. The more accurate predictions of Svehla for μ and P_r are given in reference (42).

THERMAL CONDUCTIVITY. For a monatomic gas of species i the Chapman-Enskog theory yields (Ref. 21)

$$K_i = \frac{15}{4} \frac{R}{M_i} \mu_i$$

where K_i = thermal conductivity

R = universal gas constant

Mason and Saxena (Ref. 41) give the thermal conductivity of a mixture of polyatomic gas by

$$K = \sum_{i=1}^{n_s} K_i \left[1 + 1.065 \sum_{\substack{j=1 \\ j \neq i}}^{n_s} \varphi_{ij} x_j / x_i \right]^{-1}$$

where $K_i = \frac{15}{4} \mu_i (.354 C_{pi} + .115 \cdot R / M_i)$

DIFFUSION COEFFICIENT. The expression for the binary diffusion coefficient is

$$D_{12} = D_{21} = 262.8 \times 10^{-5} \frac{\left[T^3 (M_1 + M_2) / (2 M_1 M_2)\right]^{\frac{1}{2}}}{p \sigma_{12}^2 \Omega^{(1,1)*}}$$

where $\sigma_{12} = \frac{1}{2} (\sigma_1 + \sigma_2)$, σ_i = Transport property collision dia.

p = pressure

T = temperature

The effects of multicomponent diffusion can be considered by using a bifurcation approximation as given in reference [26].

The frozen specific heat of the mixture is

$$C_{pf} = \sum_{i=1}^{n_s} C_i C_{p_i}$$

and the laminar Prandtl number can then be calculated as

$$P_r = \mu C_{pf} / K$$

and similarly the laminar Lewis number for a binary mixture may be computed as

$$L = \rho D_{12} C_{pf} / K$$

TURBULENT TRANSPORT PROPERTIES

The turbulent equations of a boundary layer contain terms which represent the transport of mass, momentum, and heat due to the turbulent fluctuations and are identified as follows:

$-\overline{(\rho v)' C_i}$ ~ turbulent mass transfer

$-\overline{(\rho v)' u'}$ ~ turbulent momentum transfer

$-\overline{(\rho v)' h_i'}$ ~ turbulent heat transfer

The eddy viscosity, $\mathcal{E}$, eddy conductivity, K_T, and eddy diffusivity, D_T, are defined by assuming that the turbulent flux terms can be related to mean, time averaged quantities, through the use of the phenomenological mixing length-eddy viscosity concept as follows:

$$\mathcal{E}(s,y) = -\frac{\overline{(\rho v)' u'}}{\partial \bar{u}/\partial y}$$

$$K_T(s,y) = -\frac{\overline{(\rho v)' h_i'}}{\partial \bar{T}/\partial y}$$

$$\bar{\rho} D_T(s,y) = -\frac{\overline{(\rho v)' u'}}{\partial \bar{C}_i/\partial y}$$

Furthermore, turbulent Lewis number and Prandtl number are defined, using the above, as

$$L_T = \frac{C_{pf}\,\bar{\rho}\,D_T}{K_T} \quad \text{and} \quad P_T = \frac{C_{pf}\,\mathcal{E}}{K_T}$$

EDDY VISCOSITY.

The details of Cebeci's extended eddy viscosity model is given in reference (18). The model uses a two-layer representation of the eddy viscosity. In the inner region, closer to the wall, the eddy viscosity is based on Prandtl's mixing length theory, as modified by Van Driest to account for the damping effect of the wall, and as extended by Cebeci to include wall mass transfer, compressibility and pressure gradient effects. Thus, the eddy viscosity in the inner region is given by

$$\mathcal{E}_i = \bar{\rho}\, l^2 \left|\frac{\partial \bar{u}}{\partial y}\right| \qquad \text{where the mixing length, } l, \text{ is}$$

$$l = .4\, y \left[1 - \exp\,(-y/A)\right]$$

the Van Driest's "damping factor", A, is defined as

$$A = [26\mu] / [(\tau_w \bar{\rho})^{\frac{1}{2}} N]$$

where τ_w = shear stress at the wall

and the factor, N, which accounts for pressure gradient and mass transfer effects is given by

$$N^2 = -\frac{d\bar{P}}{dx}\frac{\mu}{(\overline{\rho v})_w}\frac{1}{\tau_w}\left[1 - \exp\frac{11.8(\overline{\rho v})_w \mu_w}{(\bar{\rho}_w \tau_w)^{1/2}\mu}\right]$$

$$+ \exp\frac{11.8(\overline{\rho v})_w \mu_w}{(\bar{\rho}_w \tau_w)^{1/2}\mu}$$

If $(\overline{\rho v})_w$ equals zero the above equation becomes a degenerate form and N must be calculated as

$$N^2 = 1 + 11.8\frac{d\bar{P}}{dx}\left[\bar{\rho}_w \mu_w \left(\frac{\partial \bar{u}}{\partial y}\right)_w^3\right]^{-\frac{1}{2}}$$

In the outer, wake-like, portion of the boundary layer, Clauser's form of the eddy viscosity, modified to include an intermittency factor, is used. The outer eddy viscosity is given by

$$\varepsilon_o = 0.0168\ \bar{u}_e\ \delta^*_{inc}\left[1 + 5.5\,(y/\delta)^6\right]^{-1}$$

where the term in brackets is an approximation to Klebanoff's error function intermittency relationship and δ^*_{inc} is the incompressible displacement thickness defined as

$$\delta^*_{inc} = \int_0^\infty (1 - u/u_e)\,dy$$

The eddy viscosity for the inner region is used from the wall outward until the height at which $\varepsilon_o = \varepsilon_i$ is reached. From that point, to the boundary layer edge, the outer expression

for eddy viscosity is utilized.

For an external air flow the results using Van Driest-Clauser eddy viscosity model are in agreement with experimental measurements (Ref. 18). However, for an internal flow as in a rocket thrust chamber with a highly cooled wall, Cebeci's modified model does not show the laminarization tendency in the nozzle convergent section and the increase of eddy viscosity in the divergent section of the nozzle even for air flow (Ref. 19). In order to include the effect of the past history of flow and the strong temperature variation across the boundary layer due to chemical reaction or wall cooling, the turbulent kinetic energy equation has been introduced for the solution of the Reynolds stress (Ref. 19, also see Appendix). This method obtains eddy viscosity, ε_o, in the wake region, and uses the formula introduced earlier for the inner eddy viscosity, ε_i.

TURBULENT PRANDTL NUMBER.

The experimental data (Ref. 16) have shown that the turbulent Prandtl number varies considerably across the boundary layer. To account for this type of variation the turbulent Prandtl number is computed using Cebeci's formulation (Ref. 17) as

$$P_T = K_m \left[1 - \exp\left(-y^+/A^+\right)\right] \Big/ \left\{K_h \left[1-\exp\left(-y^+ P_r^{1/2}/B^+\right)\right]\right\}$$

where $K_m = .4$, $K_h = .44$ are the momentum and enthalpy Prandtl mixing length constants, respectively, and

$y^+ = y\, u_w^* / \nu_w$

where ν = Kinematic viscosity

$A^+ = 26 (\rho_w/\rho)^{1/2} (\mu/\mu_w)^{\frac{1}{N}}$

$$B^+ = 34 \left(\rho_w / \rho \right)^{\frac{1}{2}} \left(\mu / \mu_w \right) \frac{1}{N}$$

$$u^*_w = \left(\tau_w / \rho_w \right)^{\frac{1}{2}}$$

at the wall, y=0, the above equation reduces to

$$P_T = \left[K_m B^+ \right] / \left[K_n A^+ p_r^{\frac{1}{2}} \right]$$

Not too much is known about the turbulent Lewis number, and eddy conductivity, K_T, and eddy diffusivity, D_T, at this time. In the calculations L_T has been assumed constant close to unity.

HEAT FLUX $\vec{q}$

The local heat flow per unit time and area (heat flux) in a pure substance is given by the Fourier's law of heat conduction:

$$\vec{q} = -k \nabla T \qquad (1)$$

It states that the heat flux vector $\vec{q}$ is proportional to the temperature gradient ∇T and is oppositely directed; k is the proportionality constant or thermal conductivity. Thus in an isotropic medium heat flows by conduction in the direction of steepest temperature descent. In a moving fluid $\vec{q}$ represents the flux of thermal energy relative to the local fluid veclocity.

For mixtures there are, in addition to the conductive flux, contributions resulting from the interdiffusion of the various species present and the diffusion-thermo (Dufour) effect. The total energy flux relative to the mass average velocity is then given by

$$\vec{q} = \vec{q}^{\,(c)} + \vec{q}^{\,(d)} + \vec{q}^{\,(x)} \qquad (2)$$

(The radiant energy flux $\vec{q}^{(r)}$ may be handled separately and will be described later). Here in equation (2) $\vec{q}^{(c)} = -k \nabla T$ is the conductive energy flux, as defined in equation (1), and k is the instantaneous local thermal conductivity of the mixture. The energy flux $\vec{q}^{(d)}$ caused by interdiffusion is defined for a fluid containing n species by the expression:

$$\vec{q}^{(d)} = \sum_{i=1}^{n} \frac{\bar{H}_i}{M_i} \vec{J}_i = \sum_{i=1}^{n} \bar{H}_i \vec{J}_i \qquad (3)$$

Here $\bar{H}_i$ is the partial molal enthalpy of the ith species, and J_i is the diffusion mass flux in a multicomponent system. The Dufour energy flux $\vec{q}^{(x)}$ is quite complex in nature and is usually of minor importance. The explicit form of the Dufour-effect term in multicomponent gas mixtures has been discussed in [Ref. 21].

In order to compute heat transfer rate in a reacting turbulent boundary layer using the computer program MABL (Mass Addition Boundary Layer) [Ref. 11&12], the approximate heat flux relation given in this program is derived below; The peculiar veclocity of species i is defined as

$$\vec{V}_i = \vec{v}_i - \vec{v}_o$$

and is the veclocity of a particle of kind i with respect to a coordinate system moving with the mass average veclocity $\vec{v}_o$ of the fluid, where v_i is the random veclocity of the ith species.

The diffusion veclocity of species i with an average veclocity $\vec{\bar{v}}_i$ in a mixture flowing with a mass average velocity $\vec{v}_o$ is defined as

$$\vec{\bar{V}}_i = \vec{\bar{v}}_i - \vec{v}_o \qquad (4)$$

This veclocity is zero when only one kind of particle is present. The mass flux (per unit time and area) of species i across a surface moving with the mass average veclocity of the mixture is given by

$$\text{Mass Flux}_i = \rho_i \vec{\bar{V}}_i \text{ , where } \rho_i = \text{density of species i}$$

Then the enthalpy flux across this surface due to the mass flux of species i is

$$\text{Enthalpy flux}_i = \rho_i \vec{\bar{V}}_i \, H_i \qquad (5)$$

Where h_i = enthalpy per i species.

The total enthalpy transport (of all kinds of species) per unit time and area is

$$\sum_{i=1}^{n} \rho_i \vec{V}_i \, h_i \qquad (6)$$

since mass fraction $C_i = \rho_i / \rho$

$$\sum_{i=1}^{n} \rho_i \vec{V}_i \, h_i = \sum_{i=1}^{n} \rho \vec{V}_i \, C_i h_i \qquad (7)$$

Fick's law gives the relation (approximate formula)

$$\vec{V}_i = - D \nabla(\log C_i)$$

Where D = diffusion coefficient or,

$$\vec{V}_i C_i = - D \nabla C_i \qquad (8)$$

Substituting relation (8) in relation (7) the energy flux caused by interdiffusion of species in a gas mixture is derived, i.e.,

$$\vec{q}^{(d)} = - \rho D \sum_{i=1}^{n} h_i \nabla C_i$$

The total heat flux relative to the mass average veclocity as in equation (2) neglecting the Dufour effect is given by

$$\vec{q} = \vec{q}^{(c)} + \vec{q}^{(d)}$$

$$\vec{q} = - K \nabla T - \rho D \sum_{i=1}^{n} h_i \nabla C_i \qquad (10)$$

To express equation (10) in terms of enthalpy, ∇T may be transformed as

$$\text{Since, total enthalpy } h = \sum_{i=1}^{n} C_i h_i \qquad (11)$$

$$\text{grad } h \equiv \nabla h = (C_i \nabla h_i + h_i \nabla C_i) \qquad (12)$$

$$\text{Now, } \nabla h_i = \frac{\partial h_i}{\partial T} \nabla T = \frac{\partial (\int_0^T C_{pi} dT + h_i^\circ)}{\partial T} \nabla T$$

$$= C_{pi} \nabla T \qquad (13)$$

where the relation $h_i = \int_0^T C_{pi} dT + h_i^\circ$ was used

C_{pi} = Specific heat & h_i° = heat of formation of species i

Substituting Eqn (13) into Eqn. (12) we get

$$\nabla h = \sum_{i=1}^{n} (C_i \, C_{pi} \nabla T + h_i \, \nabla C_i)$$

$$= \bar{C}_{p_f} \nabla T + \sum_{i=1}^{n} h_i \, \nabla C_i, \text{ where } \bar{C}_{p_f} = \sum_{i=1}^{n} C_{pi} \, C_i$$

or, $$\nabla T = \frac{1}{C_{p_f}} \left(\nabla h - \sum_{i=1}^{n} h_i \nabla C_i \right) \qquad (14)$$

Substituting Eqn (14) in Eqn. (10) and rearranging we get

$$\vec{q} = - \frac{K}{\bar{C}_{p_f}} \left(\nabla h - \sum_{i=1}^{n} h_i \, \nabla C_i \right) - \rho D \sum_{i=1}^{n} h_i \nabla C_i$$

$$= - \frac{\mu K}{\mu \bar{C}_{p_f}} \left[\nabla h + \sum_{i=1}^{n} h_i \nabla C_i \left(\frac{\rho D \bar{C}_{p_f}}{K} - 1 \right) \right]$$

$$\vec{q} = - \frac{\mu}{P_r} \left[\nabla h + (L_i - 1) \sum_{i=1}^{n} h_i \, \nabla C_i \right] \qquad (15)$$

where Prandtl number $P_r = \frac{\bar{C}_p \mu}{K}$ and Lewis number $L = \frac{\rho D \bar{C}_p}{K}$

This is the heat flux equation used in the computer program MABL.

SECTION IV

RESULTS AND DISCUSSIONS OF THE PARAMETRIC STUDY

The computer programs, MABL, MABL-KE, and BLIMP are used to predict the heat transfer rate at the walls of NASA's Space Shuttle Main Engine (SSME) and RL-10 Engine thrust chambers. The results are shown in figures 1 through 7 for various parameter values.

The SSME characteristic parameters are:

Nozzle geometry - area ratio, E=77.5, throat radius, R_t=0.4294 (ft).

Stagnation conditions for normal power load-

temperature, T_o=6000^o (R)

pressure, P_o=3000 (psia)

mixture ratio, MR=6.0 for O_2/H_2 fuel system.

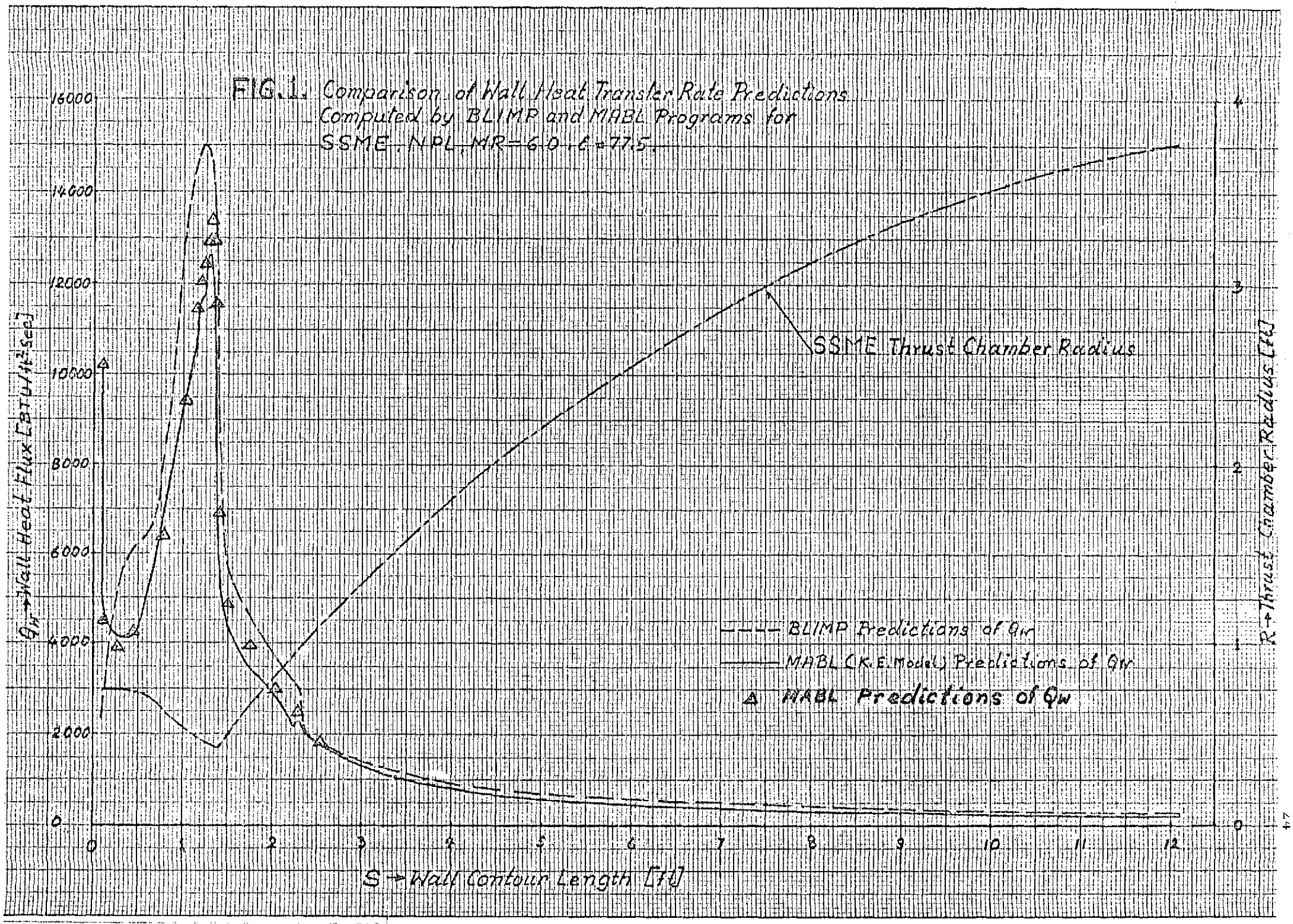
FIG. 1. Comparison of Wall Heat Transfer Rate Predictions Computed by BLIMP and MABL Programs for SSME, NPL, MR=6.0, ε=77.5.
Q_w → Wall Heat Flux [BTU/ft²sec]
S → Wall Contour Length [ft]
R → Thrust Chamber Radius [ft]
16000
14000
12000
10000
8000
6000
4000
2000
0
0
1
2
3
4
5
6
7
8
9
10
11
12
SSME Thrust Chamber Radius
---- BLIMP Predictions of Q_w
—— MABL (K.E. Model) Predictions of Q_w
Δ MABL Predictions of Q_w

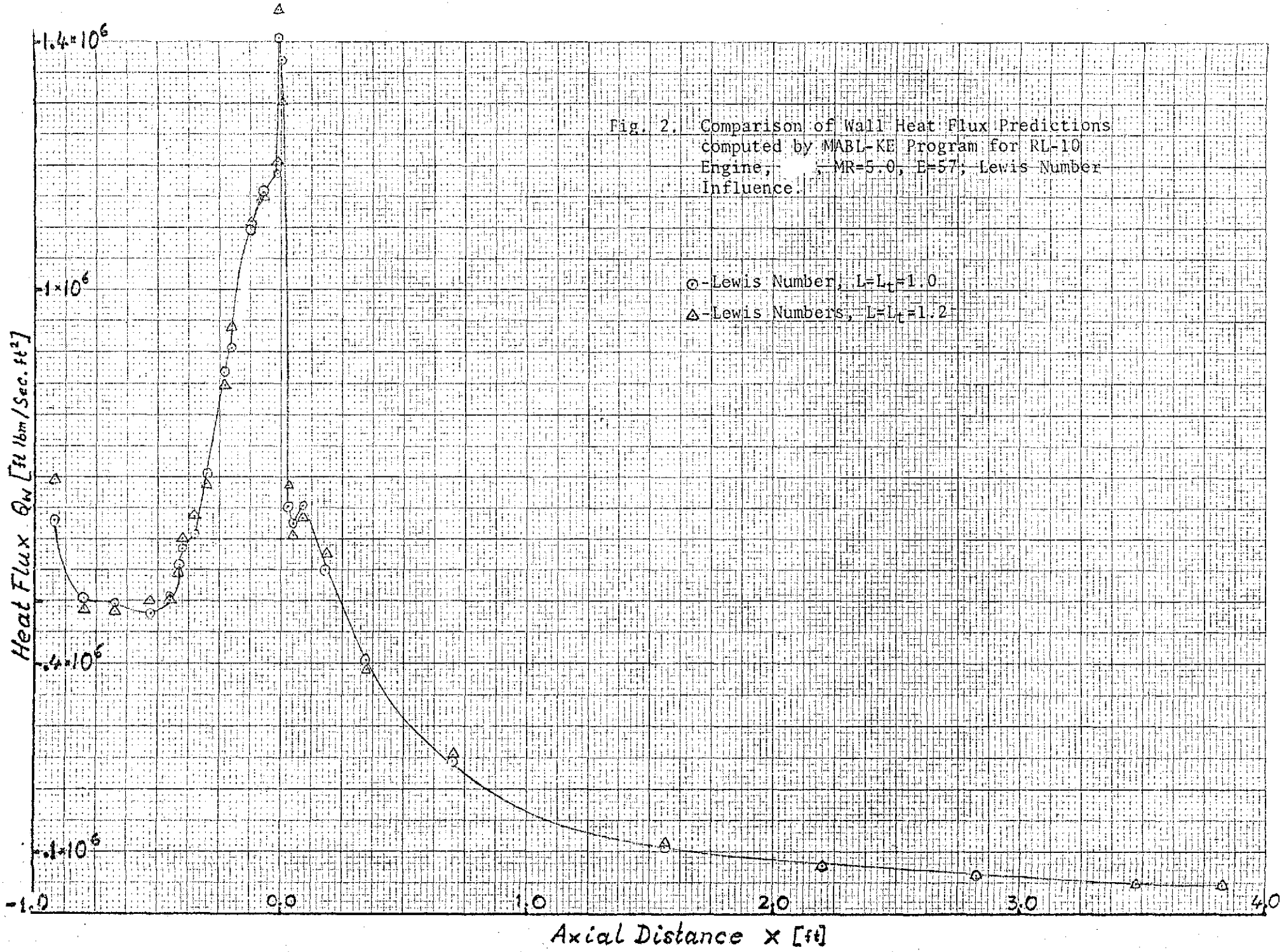

Fig. 2. Comparison of Wall Heat Flux Predictions computed by MABL-KE Program for RL-10 Engine, , MR=5.0, E=57; Lewis Number Influence.

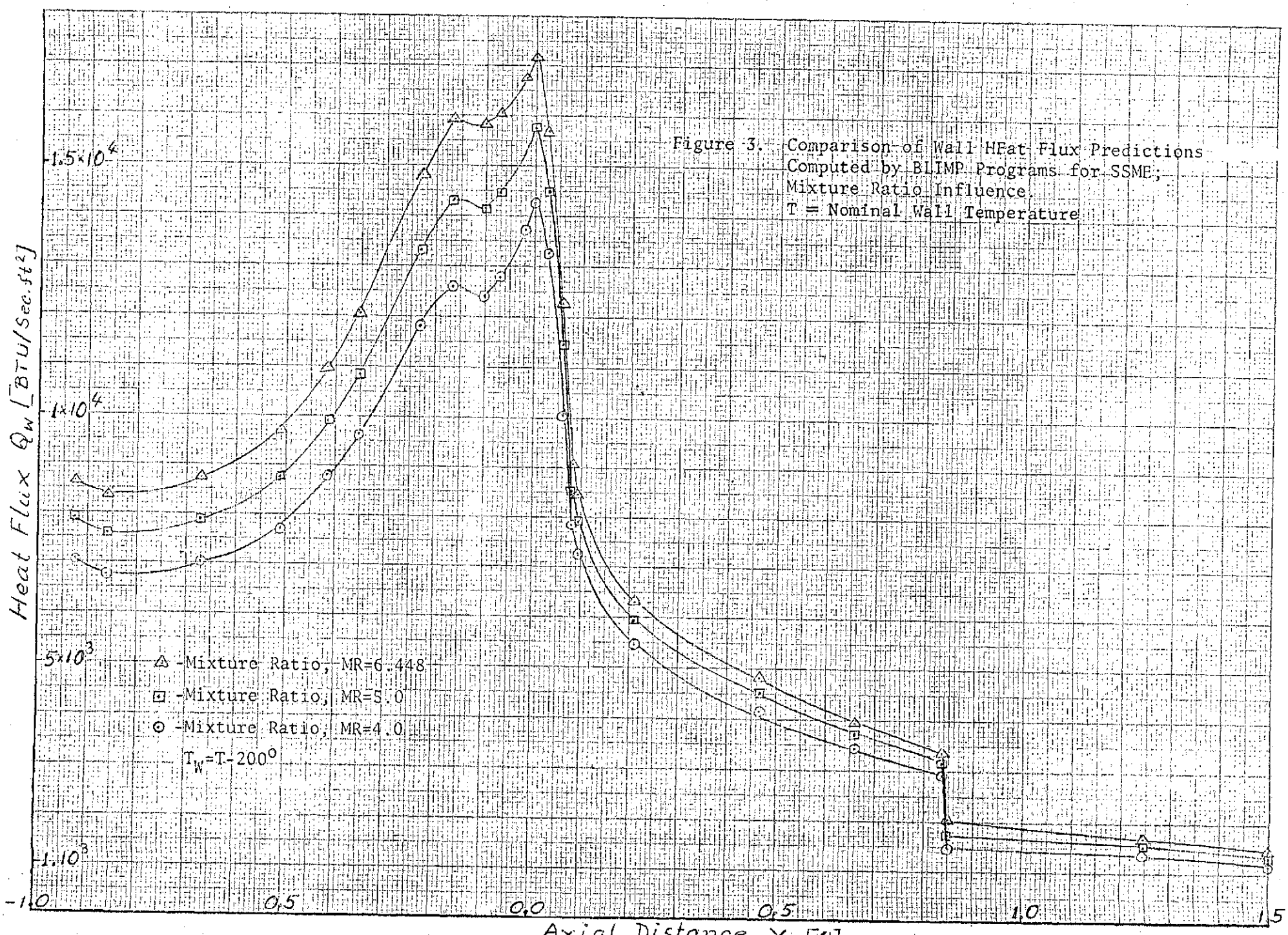

Figure 3. Comparison of Wall HEat Flux Predictions Computed by BLIMP Programs for SSME, Mixture Ratio Influence
T = Nominal Wall Temperature

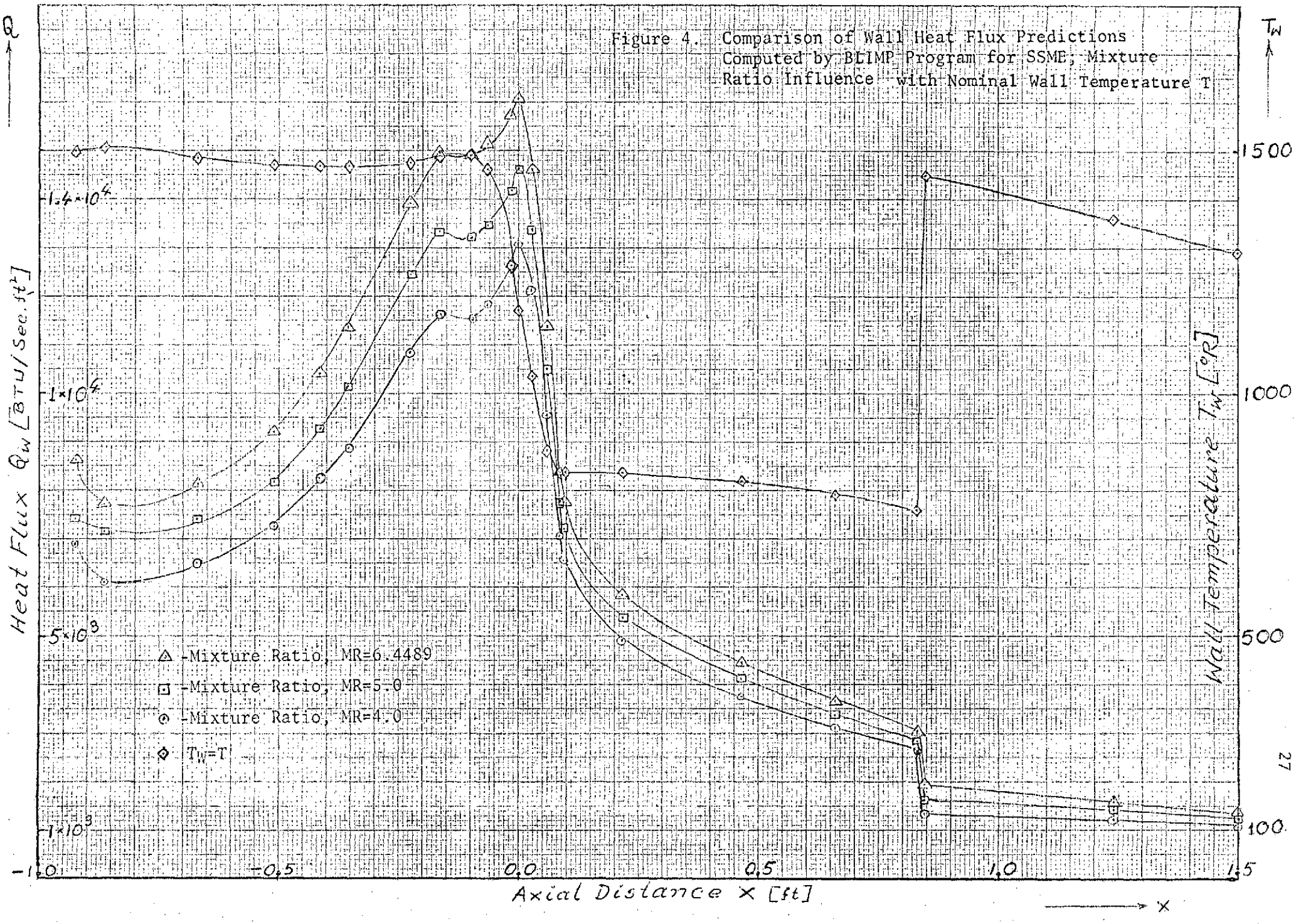

Figure 4. Comparison of Wall Heat Flux Predictions Computed by BLIMP Program for SSME; Mixture Ratio Influence with Nominal Wall Temperature T

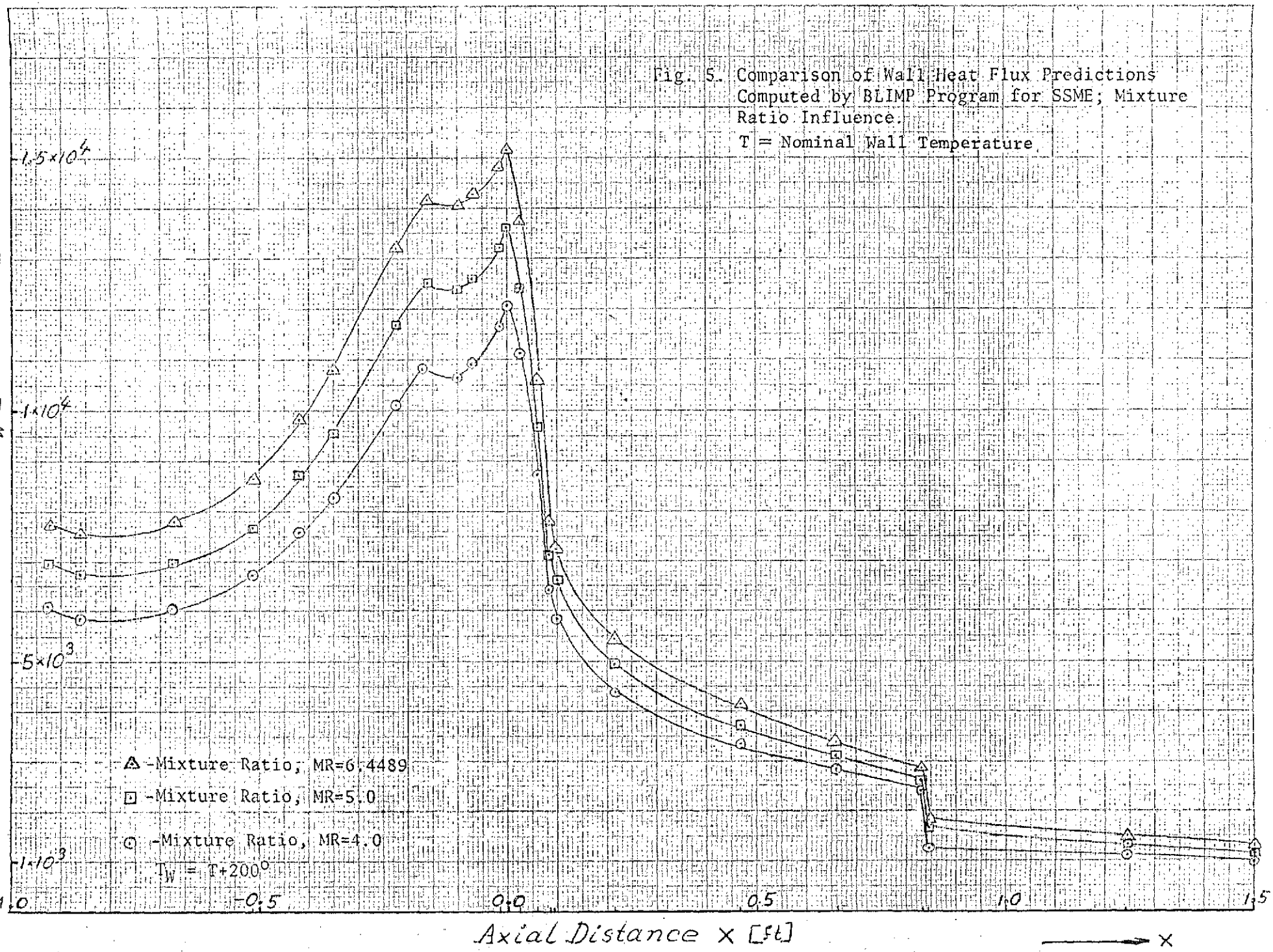

Fig. 5. Comparison of Wall Heat Flux Predictions Computed by BLIMP Program for SSME; Mixture Ratio Influence.
T = Nominal Wall Temperature

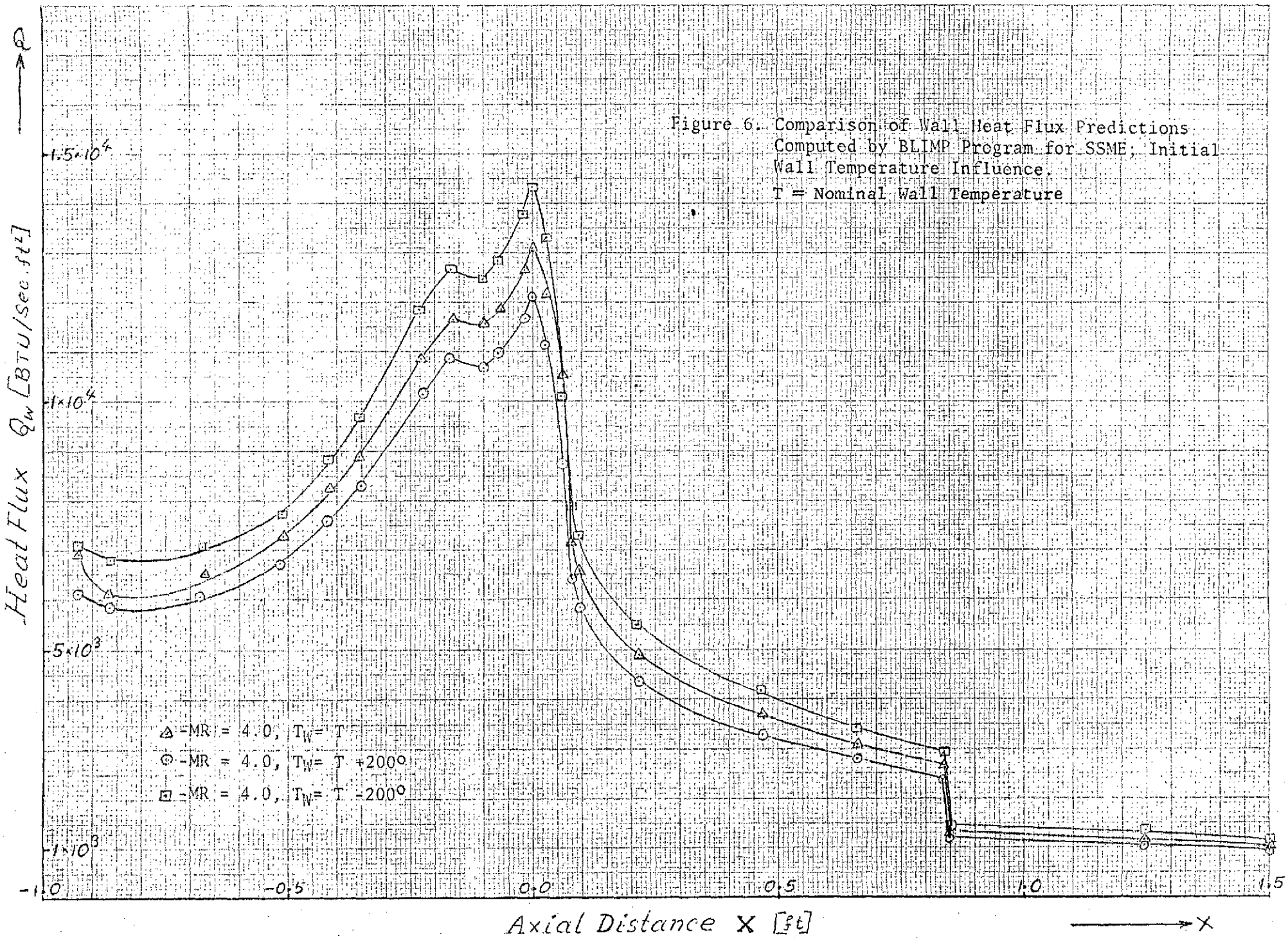

Figure 6. Comparison of Wall Heat Flux Predictions Computed by BLIMP Program for SSME; Initial Wall Temperature Influence.
T = Nominal Wall Temperature

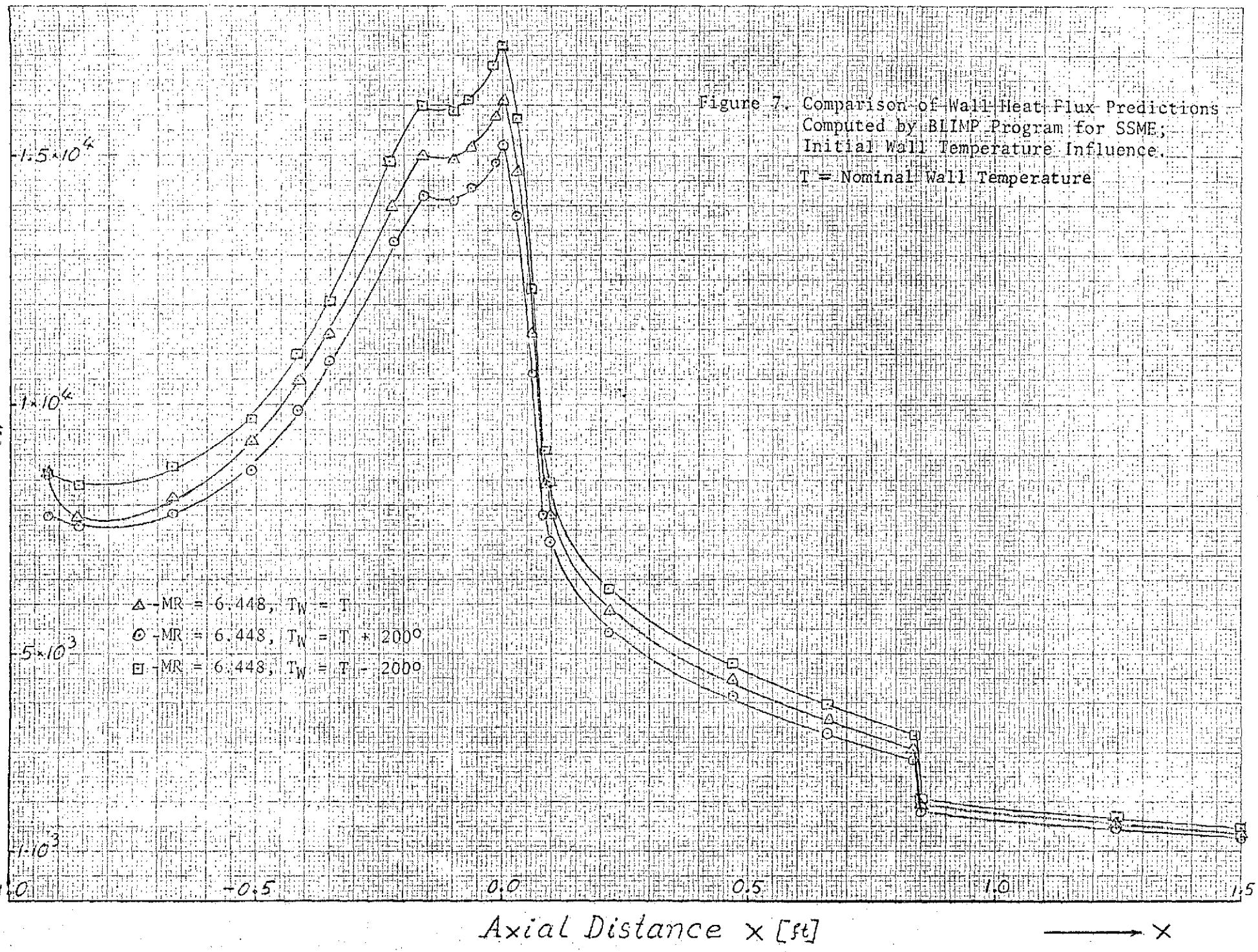

Figure 7. Comparison of Wall Heat Flux Predictions Computed by BLIMP Program for SSME; Initial Wall Temperature Influence.

T = Nominal Wall Temperature

The RL-10 Engine characteristic parameters are:

Nozzle geometry - area ratio, E-57

throat radius, Rt=0.21416(ft)

Stagnation conditions

temperature, T_o=5000^o (R)

Pressure, P_o=400 (psia)

Misture ratio, MR=5.0 for O_2/H_2 fuel system.

The heat transfer rate predictions at the wall of the SSME thrust chambers using MABL, MABL-KE and BLIMP are shown in figurel, where the abscissa represents the wall contour length. The thrust chamber radius variation is shown against the contour length to aid the visual apprehension of the heat transfer rate predictions along the wall contour: The predictions have the maximum heat transfer rate slightly upstream of the throat as was found by a few experimental results. Since the heat flux computed by the MABL program differs slightly from the results of MABL-KE, only the computed values near the throat region are shown in the figure 1.

EFFECTS OF LEWIS NUMBER ON HEAT FLUX

To determine the influence of Lewis number on heat flux the computer program MABL-KE was used for SSME & RL-10 engine configurations, since the BLIMP program does not explicitly express its governing equations in terms of lewis number. Since the MABL-KE does not compute Lewis number, fixed values about 1.0 as input to the program were chosen. Figure 2 represents the heat flux variations for L=1.0=L_t for a fixed mixture ratio of, MR=5. Even this small change in Lewis number causes visible difference in the heat flux

distributions. This influence is more prominent at the throat region, which was expected because of the sharp variations of the flow parameters there. Heat flux increases with Lewis number for a fixed mixture ratio. MABL-KE faces computational difficulties for higher Lewis numbers.

The BLIMP program considers multi-component diffusion effects by using a bifurcation approximation, which means that the Lewis number, considered but not explicitly computed by BLIMP, would be more accurate for a boundary layer flow analysis than the fixed Lewis number as assumed in MABL & MABL-KE.

EFFECTS OF MIXTURE RATIO ON HEAT FLUX

The influence of mixture ratio on heat flux was determined by using BLIMP for the SSME configuration. Figures 3,4, and 5 show the influences on heat flux due to the mixture ratios, MR=4, 5, & 6.45, each for a fixed wall temperature distribution respectively. In these figures heat flux is plotted against the axial thrust chamber distance for the SSME configuration only. The heat flux at the wall increases with the increasing mixture ratio and is more prominent near the throat region, as expected. Figure 4 shows the nominal wall temperature distribution which is a required input for computation. This temperature profile has been predicted analytically by a different computer program utilizing test results from other similar thrust chambers.

EFFECTS OF INITIAL WALL TEMPERATURE DISTRIBUTION ON HEAT FLUX

Since the initial wall temperature distribution must be given to initiate computation and since this temperature profile is a predicted quantity containing same uncentainty, it is of great importance to observe its influence on the heat flux. Figures 6 and 7 reproduces . .

the heat flux results computed by using the BLIMP program for the SSME thrust chamber. Three different wall temperature profiles, T wall, (T wall + 200 ^{o}R), and (T wall - 200^{o}R) were used to compute the heat flux for several fixed mixture ratios respectively. Heat flux increases with decreasing initial wall temperature and this influence reaches its maximum near the throat region as expected .

EFFECTS OF EDDY VISCOSITY MODEL AND PRANDTL NUMBER ON HEAT FLUX

Eddy Viscosity: Each of the computer programs, MABL, MABL-KE, and BLIMP, uses different eddy viscosity models. MABL uses the Van Driest-Clauser formula modified by Cebeci (Ref. 17). MABL-KE modifies the MABL formulation by introducing the turbulent kinetic energy equation to find eddy viscosity in the wake region (Ref. 19).

BLIMP uses the Clauser formula without any modification to compute eddy viscosity. Omori et al., in reference (19) compares eddy viscosity profiles across boundary layers, computed by using MABL-KE, experimental data with results b based on the modifeid V n Driest - Clauser formula; and his results appear to be in good agreement with available experimental data. The difference in heat flux results due to MABL & MABL-KE is too small for plotting purposes. The differences in the heat flux curves in Figure 1 are partially due to the eddy viscosity models in MABL, MABL-KE, and BLIMP programs. The formulas involved in the different models to compute eddy viscosity are given in Section II and in the Appendix.

PRANDTL Number: The computer programs MABL, MABL-KE, and BLIMP compute a Laminar Prandtl number by using the method described in Section II for a gas mixture. BLIMP uses a constant value for the turbulent Prandtl

number, P_t = 0.9; whereas, MABL and MABL-KE use the formulation of Cebeci (Ref. 17), described in Section II to compute a turbulent Prandtl number. A comparison of the heat flux results, using P_t = constant and P_t computed according to Cebeci, with experimental data is given in reference (11) which was computed by using MABL. Figure 1 shows a comparison of the heat flux results partially influenced by different turbulent Prandtl number computational models.

SECTION V

RECOMMENDATIONS

A comparative study of the three rocket nozzle turbulent boundary layer computer programs with regard to heat transfer reveals that all of them need some improvements and modifications in their mathematical models and numerical methods before any one program would be acceptable as a standard computing procedure for future theoretical investigations.

The consideration of the numerical solution methods used in these computer programs suggests that the BLIMP with its integral matrix solution method will require less nodal points and consequently less computer time than the Crank-Nicolson implicit finite difference technique used in both the MABL & MABL-KE programs to achieve compatible results and accuracy.

The BLIMP program considers multi-component diffusion by using bifurcation approximation (Ref. 26); whereas MABL and MABL-KE use a constant Lewis number, usually $L=L_t=1$, which considers the diffusion coefficient to be binary, neglecting the multi-component diffusion effect.

The MABL-KE program introduces a turbulent kinetic energy equation to calculate eddy viscosity together with a modified Van Driest-Clauser turbulent model. The BLIMP program lacks in this area and an addition of this improved model to the code would be appropriate. BLIMP should be improved with a turbulent number formula given in reference(17).

The computer programs need a wall temperature profile as a boundary condition and, at present, this prediction is made from scaled test data or industry in-house heat transfer calculations which are not accessible. It is recommended that a computer program be extended with an option to predict a thrust chamber wall temperature profile by coupling the boundary layer analysis with the cooling process.

Considering different aspects of the computer programs available, BLIMP appears to be the more suitable one to be improved, as discussed above, and used for future heat transfer analysis of rocket engine thrust chambers.

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APPENDIX A

TURBULENT BOUNDARY LAYER EQUATIONS: including the turbulent kinetic energy equation for the Reynolds stress term describing the flow properties of compressible turbulent boundary layers along the wall of a rocket thrust chamber are given below. (For detailed derivation see Ref. 35).

When the transverse-curvature effect is neglected, the compressible turbulent boundary layer equations in steady state for two-dimensional and axisymmetric flows are written in a curvilinear coordinate system as follows:

Continuity:
$$\frac{\partial}{\partial x}(\bar{\rho}\bar{u}\, r_\omega^j) + \frac{\partial}{\partial y}\left[(\bar{\rho}\bar{v} + \overline{\rho' v'})\, r_\omega^j\right] = 0 \qquad (1)$$

Momentum:
$$\bar{\rho}\bar{u}\frac{\partial \bar{u}}{\partial x} + (\bar{\rho}\bar{v} + \overline{\rho' v'})\frac{\partial \bar{u}}{\partial y} = -\frac{d\bar{P}_e}{dx} + \frac{\partial}{\partial y}\left[\mu_L \frac{\partial \bar{u}}{\partial y} - \overline{(\rho v)' u'}\right] \qquad (2)$$

Energy:
$$\bar{\rho}\bar{u}\frac{\partial \bar{H}}{\partial x} + (\bar{\rho}\bar{v} + \overline{\rho' v'})\frac{\partial \bar{H}}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{\mu_L}{P_r}\frac{\partial \bar{H}}{\partial y} - \sum_{i=1}^{n}\left[\overline{(\rho v)' h_i'}\,\bar{Y}_i + \overline{(\rho v)' Y_i'}\,\bar{h}_i\right] - \bar{u}\,\overline{(\rho v)' u'} + \mu_L\left(1 - \frac{1}{P_r}\right)\bar{u}\frac{\partial \bar{u}}{\partial y} + \frac{\mu_L}{P_r}(L-1)\sum_{i=1}^{n}\bar{h}_i\frac{\partial \bar{Y}_i}{\partial y}\right\} \qquad (3)$$

Element:
$$\bar{\rho}\bar{u}\frac{\partial \bar{\alpha}_m}{\partial x} + (\bar{\rho}\bar{v} + \overline{\rho' v'})\frac{\partial \bar{\alpha}_m}{\partial y} = \frac{\partial}{\partial y}\left[\bar{\rho} D \frac{\partial \bar{\alpha}_m}{\partial y} - \overline{(\rho v)' \alpha_m'}\right] \qquad (4)$$

where j=o for two-dimensional flow and j=1 for axisymmetric flow. Symbols are defined in the nomenclature.

The flow is assumed to be calorically perfect and obeys the equation of state,

$$\bar{P} = \frac{\bar{\rho} R \bar{T}}{M} \tag{5}$$

where the time mean of the correlation between fluctuation density and temperature, $\overline{\rho' T'}$ is neglected because of its small order of magnitude.

TURBULENT TRANSPORT PROPERTIES

In formulating the turbulent equations of motion (1-4) it was assumed that the turbulent flux terms; $\overline{(\rho v)' Y_i'}$ due to turbulent mass transfer, $\overline{(\rho v)' u'}$-turbulent momentum transfer, $\overline{(\rho v)' \sum_{i=1}^{n} \bar{Y}_i h_i'}$ - turbulent heat transfer could be related to mean , time averaged quantities, through the use of the phenomenological mixing length-eddy viscosity concept. Accordingly, the eddy viscosity, $\mathcal{E}$, eddy conductivity, λ_T and eddy deffusivity, D_T, are defined as follows:

$$-\overline{(\rho v)' u'} = \mathcal{E}(x,y)\left(\partial \bar{u}/\partial y\right) \tag{6}$$

$$-\overline{(\rho v)' \sum_{i=1}^{n} \bar{Y}_i h_i'} = \lambda_T(x,y)\left(\partial \bar{T}/\partial y\right) \tag{7}$$

$$-\overline{(\rho v)' Y_i'} = \bar{\rho} D_T(x,y)\left(\partial \bar{Y}_i/\partial y\right) \tag{8}$$

or,

$$-\overline{(\rho v)' \alpha_m'} = \bar{\rho} D_T(x,y)\left(\partial \bar{\alpha}_m/\partial y\right) \tag{9}$$

The turbulent Lewis and Prandtl numbers are defined as

$$L_T = \bar{\rho}\,\bar{C}_p\,\mathcal{D}_T/\lambda_T \tag{10}$$

and

$$P_T = \varepsilon\,\bar{C}_p/\lambda_T \tag{11}$$

Assuming

$$\overline{(\rho v)'u'} - \overline{\rho' u' v'} \simeq \overline{(\rho v)'}\,\overline{u'} \tag{12}$$

$$\overline{(\rho v)' h_i'} - \overline{\rho' v' h_i'} \simeq \overline{(\rho v)'}\,\overline{h_i'} \tag{13}$$

$$\overline{(\rho v)' Y_i'} - \overline{\rho' v' Y_i'} \simeq \overline{(\rho v)'}\,\overline{Y_i'} \tag{14}$$

$$\frac{\partial}{\partial y}\left(\bar{u}\,\frac{\partial \bar{u}}{\partial y}\right) >> \frac{1}{2}\,\frac{\partial^2 \overline{u'u'}}{\partial y^2} \tag{15}$$

and considering the definition of enthalpy

$$\frac{\partial}{\partial y}\left(\bar{h}_i \frac{\partial \bar{Y}_i}{\partial y}\right) >> \frac{\partial}{\partial y}\left(h_i' \frac{\partial Y_i'}{\partial y}\right) \tag{16}$$

$$\frac{\partial \bar{T}}{\partial y} = \frac{1}{\bar{C}_p}\left(\frac{\partial \bar{h}}{\partial y} - \sum_{i=1}^{n} \bar{h}_i \frac{\partial \bar{Y}_i}{\partial y}\right) \tag{17}$$

Eqns (2) - (4) are rewritten as

$$\bar{\rho}\,\bar{u}\,\frac{\partial \bar{u}}{\partial x} + \left(\bar{\rho}\bar{v} + \overline{\rho' v'}\right)\frac{\partial \bar{u}}{\partial y} = -\frac{d\bar{P}_e}{dx} + \frac{\partial}{\partial y}\left[(\mu + \varepsilon)\frac{\partial \bar{u}}{\partial y}\right] \tag{18}$$

$$\begin{aligned}\bar{\rho}\,\bar{u}\,\frac{\partial \bar{H}}{\partial x} + \left(\bar{\rho}\bar{v} + \overline{\rho' v'}\right)\frac{\partial \bar{H}}{\partial y} = \frac{\partial}{\partial y}\Bigg\{&\left(\frac{\mu}{P_r} + \frac{\varepsilon}{P_T}\right)\frac{\partial \bar{H}}{\partial y} \\ &+ \left[\mu\left(1 - \frac{1}{P_r}\right) + \varepsilon\left(1 - \frac{1}{P_T}\right)\right]\bar{u}\,\frac{\partial \bar{u}}{\partial y} \\ &+ \sum_{i=1}^{n}\left[\frac{\mu}{P_r}\left(L - 1\right) + \frac{\varepsilon}{P_T}\left(L_T - 1\right)\right]\bar{h}_i \frac{\partial \bar{Y}_i}{\partial y}\Bigg\}\end{aligned} \tag{19}$$

$$\bar{\rho}\,\bar{u}\,\frac{\partial \bar{\alpha}_m}{\partial x} + \left(\bar{\rho}\bar{v} + \rho' v'\right)\frac{\partial \bar{\alpha}_m}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{\mu L}{P_r} + \frac{\varepsilon L_T}{P_T}\right)\frac{\partial \bar{\alpha}_m}{\partial y}\right] \tag{20}$$

TURBULENT KINETIC ENERGY EQUATION

To compute the eddy viscosity, $\mathcal{E}$, defined in Egn (6) the following kinetic energy equation is introduced.

Considering the continuity and momentum equations for compressible turbulent boundary layers, the system of equations due to turbulent fluctuations are derived and written below in cartesian tensor notation form:

$$\overline{(\rho u_k)' u_j'} \frac{\partial \bar{u}_i}{\partial x} + \overline{(\rho u_k)' u'} \frac{\partial \bar{u}_j}{\partial x_k} + \frac{\partial \overline{(\bar{\rho} u_i' u_j' u_k')}}{\partial x_k} + \bar{\rho}\bar{u}_k \frac{\partial \overline{(u_i' u_j')}}{\partial x_k} + \overline{\left(\rho' \bar{u}_k + \rho' u_k'\right)} \frac{\partial \overline{(u_i' u_j')}}{\partial x_k} + \overline{u_i' u_j' \frac{\partial (\rho' \bar{u}_k + \rho' u_k' - \bar{\rho}' u_k')}{\partial x_k}}$$

$$= -\left[\frac{\partial \overline{(p' u_j')}}{\partial x_i} + \frac{\partial \overline{(p' u_i')}}{\partial x_j}\right] + \overline{p'\left(\frac{\partial u_j'}{\partial x_i} + \frac{\partial u_i'}{\partial x_j}\right)} + u_j' \frac{\partial \tau_{ik}'}{\partial x_k} + u_i' \frac{\partial \tau_{jk}'}{\partial x_k} \qquad (21)$$

where $$\tau_{ik}' = \mu_L \left(\frac{\partial u_i'}{\partial x_k} + \frac{\partial u_k'}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial u_l'}{\partial x_l}\right) \qquad (22)$$

Assuming: $$\overline{u_j' \frac{\partial}{\partial x_k}\left[\mu_L \left(\frac{\partial u_k'}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right)\right]} + \overline{u_i' \frac{\partial}{\partial x_k}\left[\mu_L \left(\frac{\partial u_k'}{\partial x_i} - \frac{2}{3}\delta_{jk}\frac{\partial u_l'}{\partial x_l}\right)\right]} = 0 \qquad (23)$$

and $$\frac{\partial \overline{(\rho u_k)' u_i' u_j'}}{\partial x_k} - \frac{\partial \overline{(\rho' u_k')(u_i' u_j')}}{\partial x_k} = \frac{\partial \overline{(\bar{\rho} u_i' u_k' u_k')}}{\partial x_k} \qquad (24)$$

Equation (21) is simplified as

$$\overline{(\rho u_k)' u_j'}\frac{\partial \bar{u}_i}{\partial x_k} + \overline{(\rho u_k)' u_i'}\frac{\partial \bar{u}_j}{\partial x_k} + \left(\bar{\rho}\bar{u}_k + \overline{\rho' u_k'}\right)\frac{\partial \overline{u_i' u_j'}}{\partial x_k}$$

$$+ \frac{\partial \overline{(\rho u_i' u_j' u_k')}}{\partial x_k} = -\left(\overline{\frac{\partial p' u_j'}{\partial x_i}} + \overline{\frac{\partial p' u_i'}{\partial x_j}}\right) + \overline{p'\left(\frac{\partial u_j'}{\partial x_i} + \frac{\partial u_i'}{\partial x_j}\right)}$$

$$+ \frac{\partial}{\partial x_k}\left(\mu \frac{\partial \overline{u_i' u_j'}}{\partial x_k}\right) - 2\mu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}} \qquad (25)$$

Adding the three equations for i=j=1, 2, and 3, in Eqn (25), and defining the turbulent kinetic energy, $K_o = u'u' + v'v' + \omega'\omega'$, the turbulent energy equation is obtained.

$$\bar{\rho}\bar{u}\frac{\partial K}{\partial x} + \left(\bar{\rho}\bar{v} + \overline{\rho' v'}\right)\frac{\partial K}{\partial y}$$

$$= -2\overline{(\rho v)' u'}\frac{\partial \bar{u}}{\partial y} + \frac{\partial}{\partial y}\left[\mu \frac{\partial K}{\partial y} - \left(\overline{\rho K_o + 2p'}\right)\overline{v'}\right]$$

$$- 2\mu\left(\overline{\frac{\partial u'}{\partial y}\frac{\partial u'}{\partial y}} + \overline{\frac{\partial v'}{\partial y}\frac{\partial v'}{\partial y}} + \overline{\frac{\partial \omega'}{\partial y}\frac{\partial \omega'}{\partial y}}\right) \qquad (26)$$

where $K = \bar{K}_o$ and the tendency- towards isotropy term was assumed to be negligible

$$2\overline{p'\frac{\partial v'}{\partial y}} = 0 \qquad (28)$$

The first, second, and third terms on the right hand side of equation (26) are called the turbulent production, diffusion, and dissipation terms, respectively.

According to Prandtl - Wieghart (Ref. 36) the Reynolds stress, $-\overline{(\rho v)' u'}$, the turbulent kinetic energy, K, and the gradient of time mean velocity, $\partial \bar{u}/\partial y$, are related as

$$-\overline{(\rho v)' u'} = k\,\bar{\rho}\,\Lambda\,K^{1/2}\left|\frac{\partial \bar{u}}{\partial y}\right| \qquad (29)$$

where K is a constant and Λ is a dissipation length. Two unknown terms in equation (26) are replaced by the following two relations due to Rotta

(Ref. 15, 37), assuming the compressibility effect of these terms to be small:

$$-\left(\overline{\bar{\rho} K_o + 2 p'}\right) v' = \alpha \bar{\rho} \Lambda K^{1/2} \frac{\partial K}{\partial y} \tag{30}$$

and

$$2\mu \left(\overline{\frac{\partial u'}{\partial y} \frac{\partial u'}{\partial y}} + \overline{\frac{\partial v'}{\partial y} \frac{\partial v'}{\partial y}} + \overline{\frac{\partial \omega'}{\partial y} \frac{\partial \omega'}{\partial y}} \right) = \bar{\rho} \mu \beta \frac{K}{\Lambda^2} + \gamma \frac{\bar{\rho} K^{3/2}}{\Lambda} \tag{31}$$

where α, β, and γ are constants.

Substituting equations (29-31) into equation (26) the final form of the turbulent kinetic energy equation is derived,

$$\bar{\rho} \bar{u} \frac{\partial K}{\partial x} + \left(\bar{\rho} \bar{v} + \overline{\rho' v'}\right) \frac{\partial K}{\partial y} = 2 k \bar{\rho} \Lambda K^{1/2} \left(\frac{\partial \bar{u}}{\partial y}\right) \left|\frac{\partial \bar{u}}{\partial y}\right| + \frac{\partial}{\partial y}\left[\left(\mu + q \bar{\rho} \Lambda K^{1/2}\right) \frac{\partial K}{\partial y}\right] - \bar{\rho} \mu \beta \frac{K}{\Lambda^2} - \gamma \frac{\bar{\rho} K^{3/2}}{\Lambda} \tag{32}$$

The following boundary conditions are used to solve the equations (1), (5), (18), (19), (20), and (32) simultaneously:

at the wall, y=o;

$$\begin{aligned} &\bar{u}(x,0) = 0 \\ &\bar{\rho} \bar{v} + \overline{\rho' v'} = \dot{m}_w(x) \\ &\bar{T}(x,0) = \bar{T}_w(x) \\ &\left(\frac{\partial \bar{\alpha}_m}{\partial y}\right)_{y=0} = \dot{m}_w \left(\bar{\alpha}_{m_w} - \bar{\alpha}_{m_I}\right) \frac{Pr_w}{\mu_w L_w} \end{aligned} \tag{33}$$

At the outer edge of boundary layers, $K(x,0) = 0$

$y \to \infty$;

$$\begin{aligned} &\bar{u}(x,\infty) = U_e(x) \\ &\bar{T}(x,\infty) = \bar{T}_e(x) \\ &\bar{\alpha}_m(x,\infty) = \bar{\alpha}_{m_e}(x) \\ &K(x,\infty) = 0 \end{aligned} \tag{34}$$

When the dissipation length, Λ, is given, the equations may be solved in a closed form. Assuming that the compressibility effect on the dissipation length is negligible (Ref. 38, 39), Λ is expressed as a polynomial of the distance, y, from the wall,

$$\Lambda = a\,(y/\delta_1)^4 + b\,(y/\delta_1)^3 + c\,(y/\delta)^2 + d\,(y/\delta_1) + e \qquad (35)$$

To avoid a negative value of K in the vicinity of the wall the boundary layer is separated into the laminar sublayer close to the wall and the wake region. Considering that the effect of molecular viscosity term is small compared with the remaining terms in the wake region, the terms $\frac{\partial}{\partial y}\left(\mu \frac{\partial K}{\partial y}\right) - \bar{\varepsilon}\rho\beta \frac{K}{\Lambda^2}$ are neglected. Then the eddy viscosity, ε, in the wake region is obtained from equations (6, 29) as

$$\varepsilon_o = \kappa\, \bar{\varepsilon}\, \Lambda\, K^{1/2} \left|\frac{\partial \bar{u}}{\partial y}\right| \Big/ \left(\frac{\partial \bar{u}}{\partial y}\right) \qquad (36)$$

In the region very close to the wall the modified Van Driest Model (Ref. 18) based upon the Prandtl mixing length theory is applied; that is, the inner eddy viscosity is obtained as

$$\varepsilon_i = \bar{\varepsilon}\, \ell^2 \left|\frac{\partial \bar{u}}{\partial y}\right| \qquad (37)$$

Where the mixing length, ℓ, is given

$$\ell = 0.40\, y\left[1 - \exp(-y/A)\right] \qquad (38)$$

and A = Van Driest's Damping factor includes the effect of suction or mass addition and pressure gradient as shwon by Cebecé (Ref. 18).

The inner eddy viscosity, ε_i, is used from the wall outward until the height at which $\varepsilon_o = \varepsilon_i$ is reached. From that point on to the boundary layer edge the ε_o is used.