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PART I

A STABLE, LINEAR FREQUENCY-
MODULATED OSCILLATOR

Prepared by

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FOREWORD

This report is a technical summary presenting the results of a study by the Electrical Engineering Department, Auburn University, under the auspices of the Engineering Experiment Station toward fulfillment of the requirements in NASA Contract NAS8-26193. The report describes studies made concerning "Frequency Stabilization and Modulation Techniques for High Frequency and High Data Rate Telecommunication".

Part I of the report presented herein describes a stable, linear frequency modulated oscillator with dc response designed to accept analog, digital or television signals. Part II presented under separate cover describes a digital mixing technique investigated for its possible application in digital telemetry systems.

A STABLE, LINEAR FREQUENCY-

MODULATED OSCILLATOR

M. A. Honnell and A. W. Wales, Jr.

The development of a push-pull frequency-modulated oscillator employing field-effect transistors is described. The advantages of field-effect transistors for use in a frequency-stable oscillator are presented. Linearization of the frequency deviation was accomplished by utilizing the square-law characteristic of an FET used as a modulating amplifier. The push-pull oscillator model produced a linear frequency deviation of more than 10 MHz at a center frequency of approximately 100 MHz. Output power is within 0.6 dB of a nominal +8.5 dBm over the desired frequency range, and the modulation bandwidth is dc to 10 MHz. Frequency variation with temperature after compensation with a negative-temperature-coefficient capacitor is within ±0.05% from 0° to 60°C.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

The problem of frequency stability in S-band FM telemetry transmission can be approached using several methods, depending on the particular format of the modulating signal. When possible, a closed-loop automatic-frequency-control system provides an excellent means of minimizing frequency drift. However, in many applications, this closed-loop approach may be impractical.

In systems where the information to be transmitted is in a well-defined, synchronous form, the frequency error can be corrected at specified times during transmission. For example, in a frequency-modulated television transmitter, the frequency can be stabilized during the horizontal blanking interval of the television signal [1]. However, if the information to be transmitted is not conveniently synchronized, or if the system is to be used to transmit information in more than one format, the closed-loop system becomes impractical.

For the present application, a system was desired which could transmit information in any format, analog or digital, with a high degree of frequency stability, a frequency deviation of approximately 1% at 1 GHz, and baseband response to dc. For this reason, an inherently stable open-loop system was the only alternative.

In many applications at S-band, an oscillator is frequency-modulated at about 100 MHz and then frequency-multiplied to 1 GHz. This technique results in a ten-fold increase in frequency deviation, but it also increases the instability by a factor of ten. It was decided, therefore, that a highly stable crystal-controlled reference oscillator would be used to up-convert from 100 MHz to S-band [2]. This up-conversion technique requires a stable 100-MHz oscillator capable of being modulated with a linear frequency deviation of 10% of the carrier frequency, or 10 MHz. Such a system would result in a 1% frequency deviation at 1 GHz with a stability percentage of one-tenth that obtained at 100 MHz. It is the development of this oscillator which is presented in this study.

The requirements of good center frequency stability and wide frequency deviation led to the selection of a push-pull oscillator using field-effect transistors as the active elements. Frequency modulation was accomplished by using two varactor diodes connected in series to vary the operating frequency of the oscillator. The non-linear capacitance variation of the varactors resulted in a non-linear frequency deviation, which was corrected by utilizing the square-law characteristic of an FET. The operation of the varactor diode is explained in the next chapter, followed by a discussion of the considerations involved in designing the push-pull FET oscillator.

II. VARACTOR CHARACTERISTICS

All reverse-biased p-n junctions exhibit a voltage-dependent capacitance. This nonlinear capacitance characteristic of varactor diodes is utilized in amplifiers, harmonic generation, frequency multiplication, electronic tuning, and frequency modulation. The expression for varactor capacitance as a function of applied voltage can be written [3] as

$$C_v = C_p + C_j = C_p + \frac{C_o}{\left(1 + \frac{V_R}{V_c}\right)^n} \quad , \tag{1}$$

where the variables are

C_v = total varactor capacitance,

C_p = package capacitance,

C_j = junction capacitance,

C_o = junction capacitance with zero bias,

V_R = applied reverse-bias voltage,

V_c = contact potential (0.5 - 0.7 v for Silicon), and

n = exponent of variation (between 1/2 and 1/3, a function of junction diffusion).

A typical normalized curve of Equation (1) is shown in Figure 1.

The operation of varactors at high frequencies is best described by the small-signal equivalent circuit given in Figure 2(a). The

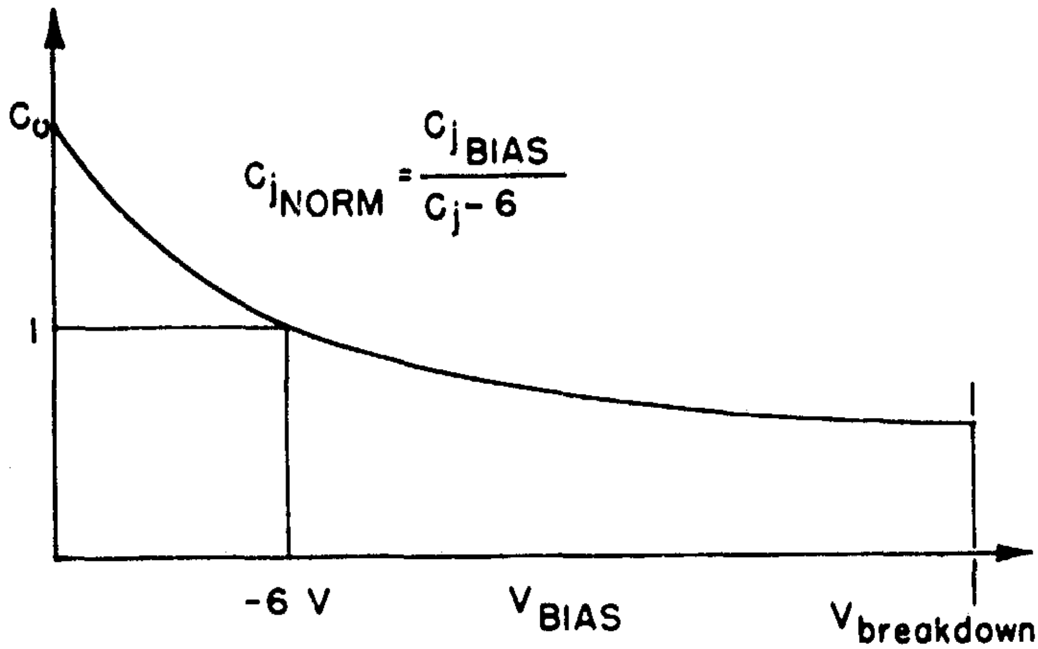

Figure 1. Normalized Varactor Capacitance as a Function of Bias Voltage.

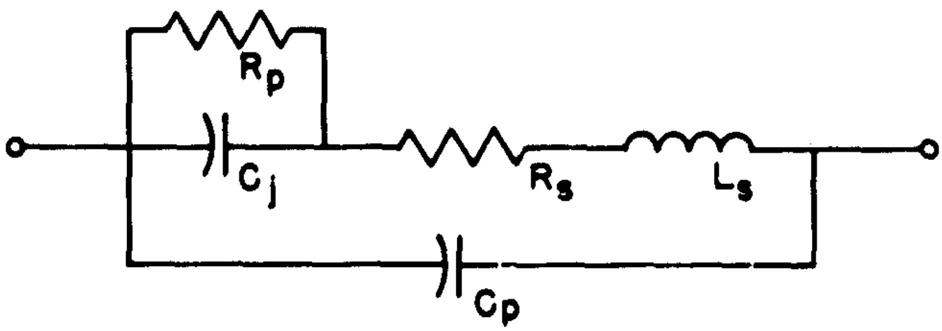

Figure 2(a) Equivalent Circuit for the Operation of a Varactor Diode at High Frequencies.

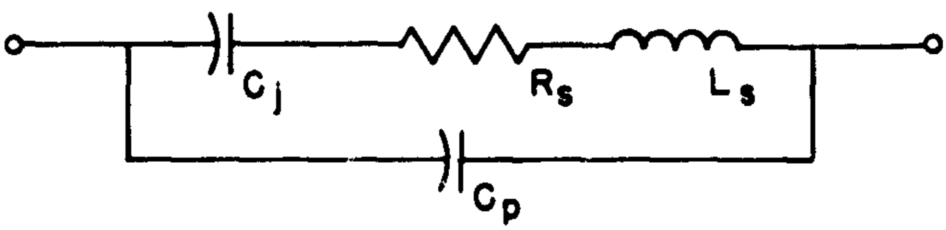

Figure 2(b). Simplified Equivalent Circuit for the Operation of a Varactor Diode at High Frequencies.

package of the varactor is modeled by the package capacitance C_p and the series inductor L_s. The junction itself is modeled by the series resistance R_s, the junction capacitance C_j, and the leakage resistance R_p. At the frequency of interest R_p is much greater than the reactance of C_j; therefore R_p can be neglected, leading to the simplified equivalent circuit given in Figure 2(b). Considering only the junction of the varactor, the quality factor Q for the device may be written [3]

$$Q = \frac{1}{2\pi\ R_s\ C_j\ f} \quad . \tag{2}$$

Since Q at any frequency f is the ratio of the varactor cut-off frequency f_c to f, or

$$Q = \frac{f_c}{f} \quad , \tag{3}$$

then

$$f_c = \frac{1}{2\pi\ R_s\ C_j} \quad . \tag{4}$$

The capacitance of a varactor biased with a fixed voltage increases with temperature. At minus one volt bias, the capacitance change is typically of the order of 0.08%/°C. At minus six volts bias, the capacitance change is of the order of 0.005%/°C. At more than six volts, the capacitance change with temperature is negligible and may be ignored [3]. Therefore, to obtain maximum frequency stability, the varactors should be biased at minus six volts or more, if possible.

III. DESIGN CONSIDERATIONS

A. Selection of the Active Element

The selection of the transistor for this oscillator involved specifications such as gain, bandwidth, and input/output characteristics. In order to attain maximum stability, it was decided that a field-effect-transistor (FET) was the most promising. It can be biased with simpler schemes than bipolar devices; its high input impedance reduces loading problems; and it can be designed so that the frequency of oscillation is relatively independent of temperature-varying bias currents [4].

In a bipolar transistor, there are two junction capacitances to deal with. These are the depletion-layer capacitance of the reverse-biased collector-base junction and the diffusion capacitance of the forward-biased base-emitter junction. This diffusion capacitance is almost directly proportional to the emitter current and can drastically affect the operating frequency as the bias current changes with temperature.

The capacitances in the FET, however, are all caused by the depletion-layer capacitance of the reverse-biased gate-channel junction. Although this capacitance changes with channel voltage variations, it is relatively independent of temperature-varying channel current. The oscillator is designed with a low drain-source resistance to minimize changes in channel voltage; thus, variations in bias current with temperature will have little effect on the output frequency. Figure 3 shows a comparison of the temperature stability of oscillators using an FET and a bipolar transistor [4]. It should be pointed out that in both FETs and bipolar transitors, junction capacitances also vary directly with temperature, regardless of any bias current effects. The fact that the FET has this single type of variation makes it the preferred device for frequency-stable oscillators.

On the basis of voltage rating, low internal capacitances, and availability, the GE-FET-2 (2N4416, or equiv.) n-channel FET was chosen for this application. The FET-2 has a maximum drain-source voltage capability of 30 volts, minimum forward transconductance (Y_{FS}) of 4500 μmho, and can be used in applications up to 400 MHz.

B. Selection of the Varactor

The varactor diode to be used in modulating the oscillator was chosen on the basis of nominal capacitance, range of capacitance variation (or tuning ratio), figure of merit Q, and local availability. The Motorola HEP R2503 proved adequate in all areas; the R2503 has the following typical characteristics:

Reverse Voltage (maximum), V_R = 30 volts

Nominal Diode Capacitance, C_T = 33 pF
(V_R = 4 Vdc, f = 1 MHz)

Figure of Merit (minimum), Q = 200

Tuning Ratio, $\frac{C_2}{C_{30}} = 3$
($C_2 = C_T$ @ 2 Vdc, $C_{30} = C_T$ @ 30 Vdc, f = 1 MHz)

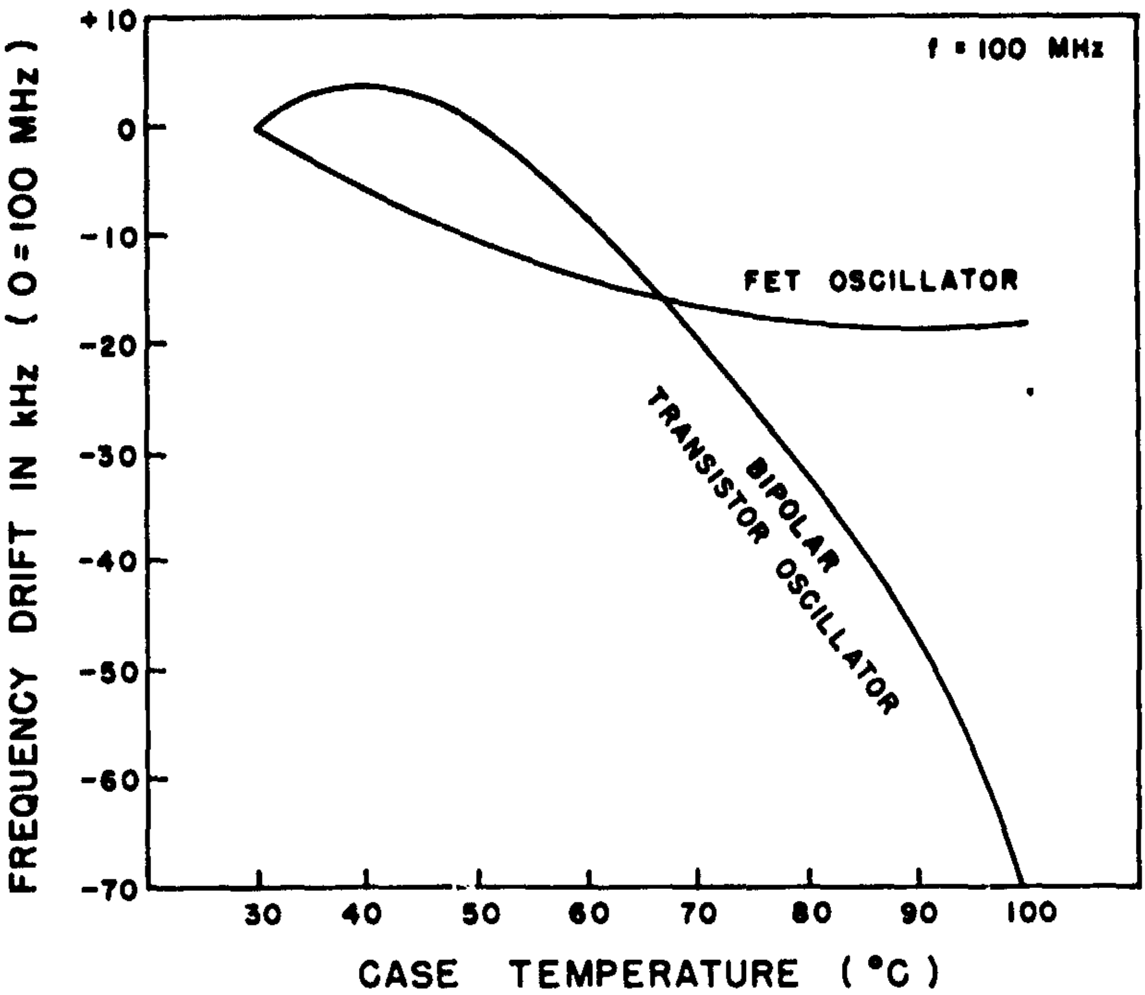

Figure 3. Comparison of Frequency Stability with Temperature for FET and Bipolar Transistor LC Oscillators.

A typical curve of capacitance versus reverse bias voltage for the R2503, as measured in the laboratory, is given in Figure 4.

C. Selection of the Basic Circuit Configuration

Several types of oscillator circuits were built and tested. Most circuits either failed to yield adequate frequency deviation or produced unacceptable output characteristics, such as poor waveform and widely-varying output power as a function of frequency.

The circuit finally chosen was a push-pull configuration of the form shown in Figure 5. Although it has not been widely publicized for oscillator use, the push-pull circuit offers many advantages. The symmetry of this circuit not only reduces all even-order harmonics, but also tends to stabilize the operating frequency of the oscillator [5,6]. Although output power was not a prime consideration in this application, the push-pull circuit can deliver more power than a comparable circuit having only one active element.

Output coupling was accomplished by winding two turns of number 22 AWG hook-up wire around the coil (one turn on each side of the center tap). This secondary coil was then connected, along with a shunt capacitance for matching purposes, to a 50-ohm load.

It is important to note that, in the push-pull configuration, the components should be closely matched to insure the symmetry of the circuit. For this reason the FETs were chosen by using a Tektronix Model 577 Curve Tracer, the capacitors were checked on a Boonton Q-Meter, and the resistors were matched with a Hewlett-Packard 3469A digital

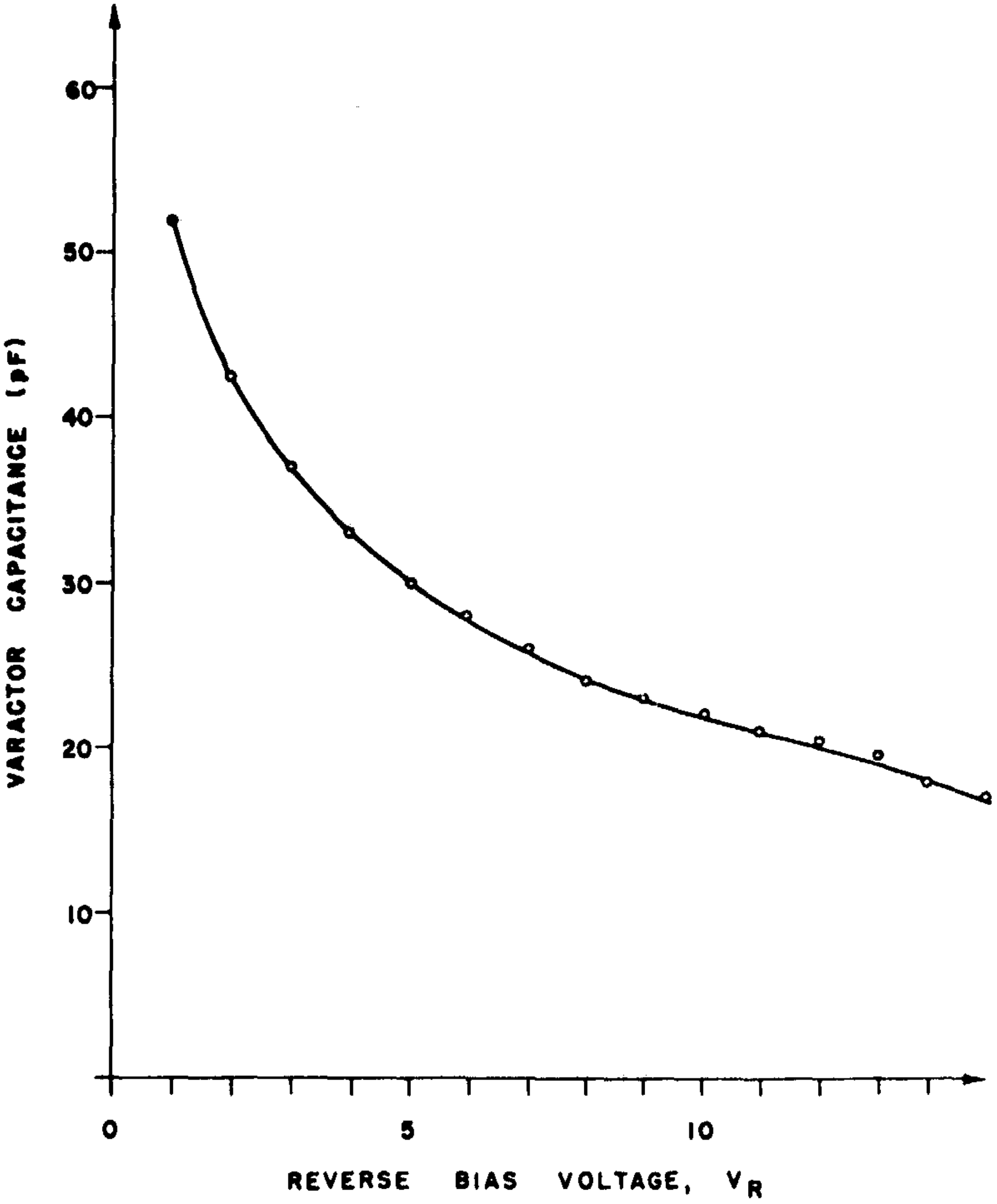

Figure 4. Capacitance as a Function of Bias Voltage for the Motorola HEP R2503 Varactor Diode.

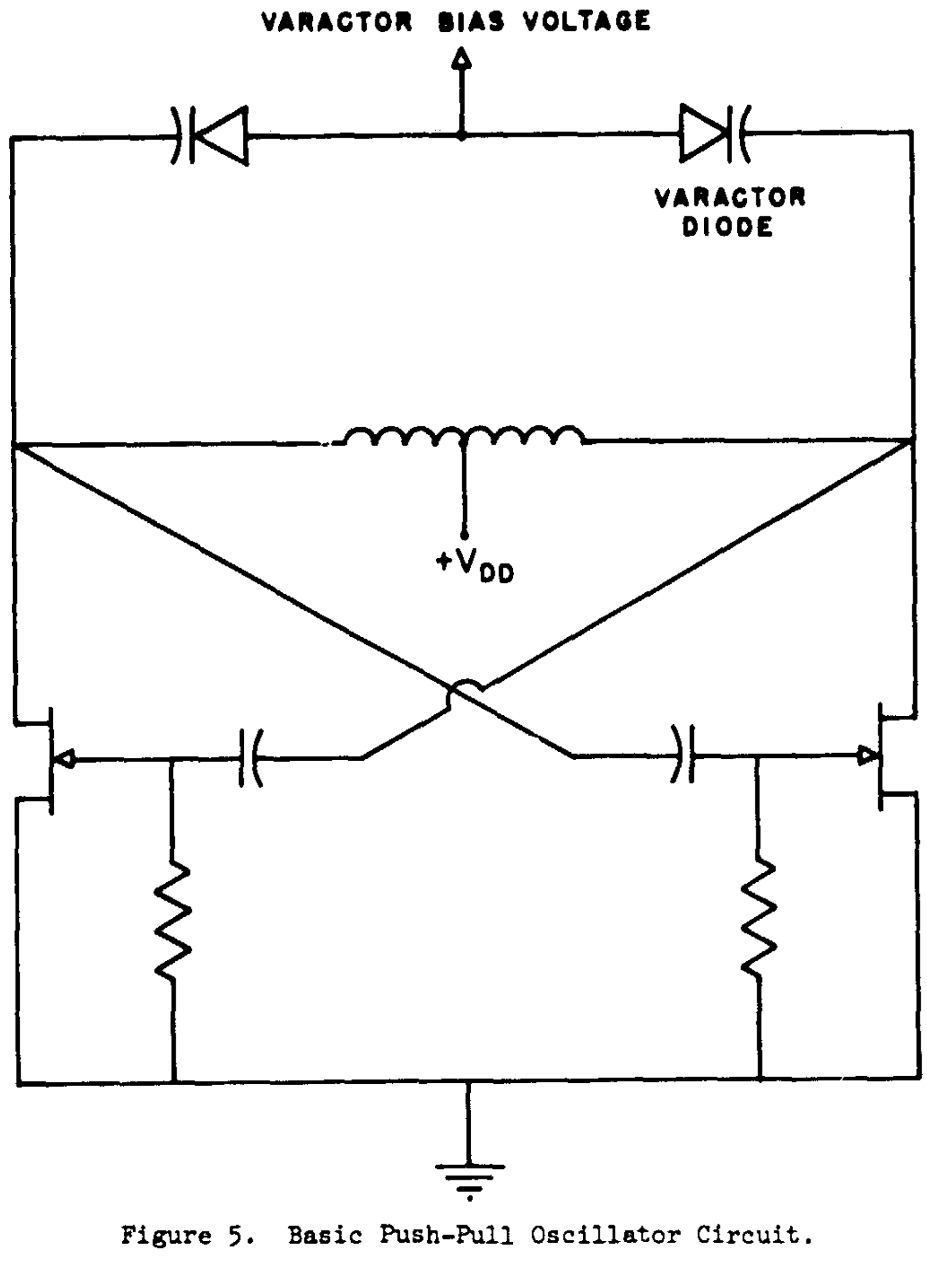

Figure 5. Basic Push-Pull Oscillator Circuit.

multimeter. The coil should be exactly center-tapped so that the symmetry of the circuit will be preserved and distortion in the output will be reduced.

The frequency of oscillation for the push-pull oscillator is

$$f_o = \frac{1}{2\pi(LC_T)^{1/2}} \; , \tag{6}$$

where C_T and L represent the equivalent capacitance and inductance of one-half of the symmetrical circuit. This equation is derived in the analysis of the push-pull oscillator included in the Appendix.

D. Linearization and Equalization

As shown in Chapter III, the capacitance variation with applied voltage for a varactor diode falls somewhere between a square-root and a cube-root relationship. This relationship leads to a non-linear frequency deviation for the basic oscillator as shown in Figure 6. In this figure, each increasing change in bias voltage results in a smaller change in frequency. In order to correct this non-linearity, it is necessary to place a linearizing circuit in the varactor input voltage path. The linearization technique employed is similar to the Gamma correction technique used in television systems.

Shaping diodes could be used to accomplish this linearization. However, it is believed that the number of diodes that would be necessary to accomplish this task would introduce too many temperature-sensitive p-n junctions into the circuit. For this reason, a linearizing circuit utilizing the square-law characteristic of an FET was employed.

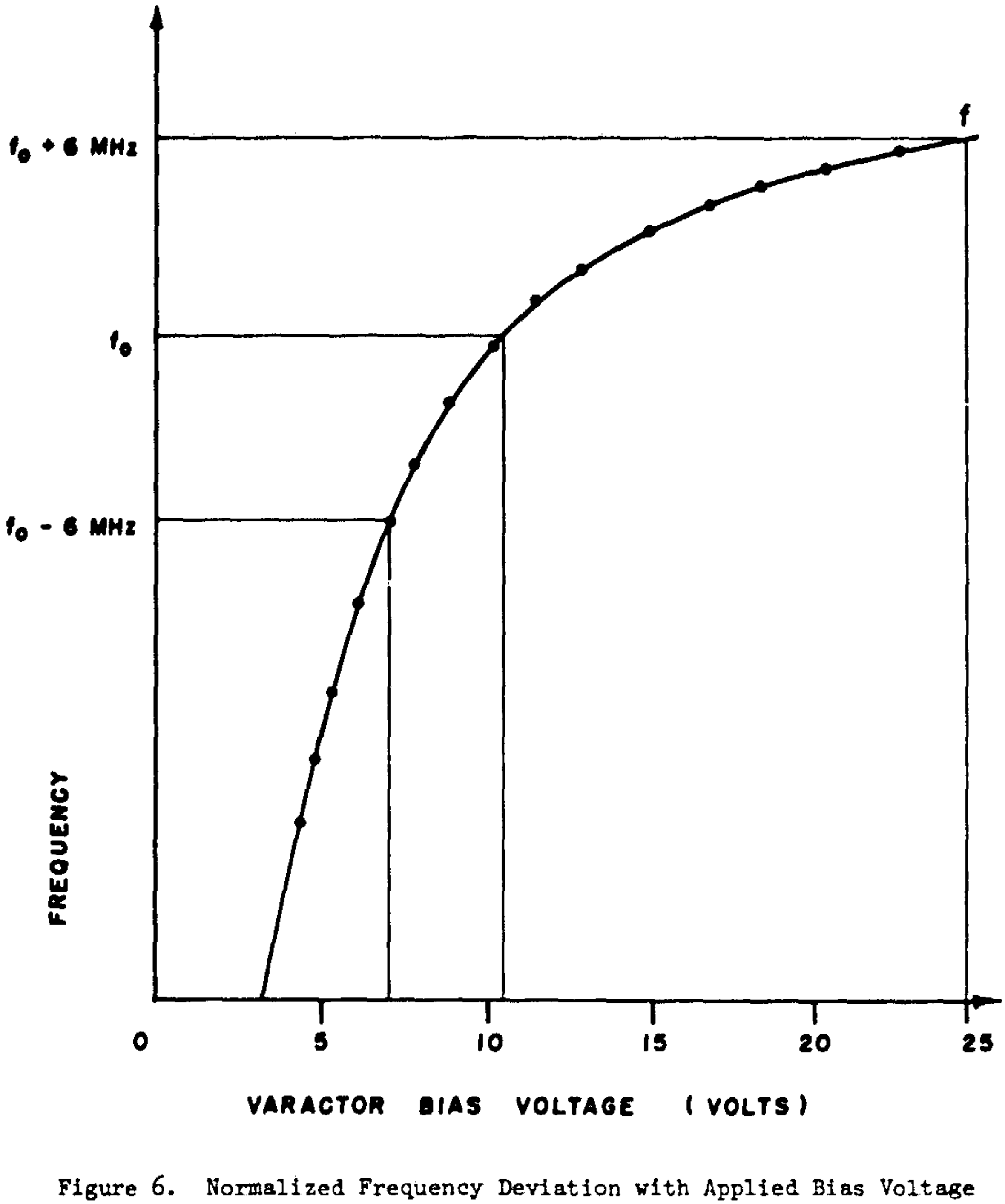

Figure 6. Normalized Frequency Deviation with Applied Bias Voltage for the Basic Push-Pull Oscillator.

A circuit containing a single FET, a drain resistance, and a small source resistance has transfer characteristics typical of those shown in Figure 7. With each incremental increase in input voltage (gate to ground), the output voltage change (drain to ground) is larger. By proper selection of the operating point and the amount of non-linearity, a single FET circuit may be used to correct much of the non-linear behavior of the basic oscillator frequency deviation. Experimental observation led to the conclusion that the maximum non-linearity with R_s equal to zero is required in order to compensate for the modulation non-linearity produced by the particular varactor selected for this application. The power supply voltage was chosen by considering the range of varactor bias necessary to achieve the desired frequency deviation shown in Figure 6, and the operating point was determined empirically by adjusting the drain resistor of the linearizing FET. Because of working-voltage requirements and availability, the GE-FET-2 (2N4416, or equivalent) was again chosen. The resulting linearized frequency deviation is shown in Figure 8.

The final oscillator circuit shown in Figure 9 includes an equalization network to extend the modulation capability to include video and high-pulse-rate digital information. This was necessary because the two varactors appeared as a parallel capacitive load to the FET linearization circuit and thereby restricted high-frequency modulation signals. The equalization includes a standard shunt-peaking inductor L_x in the drain circuit of Q_3 [7] and a bridged-T equalizer at the input of the linearizing FET circuit. Components used in the circuit of Figure 9

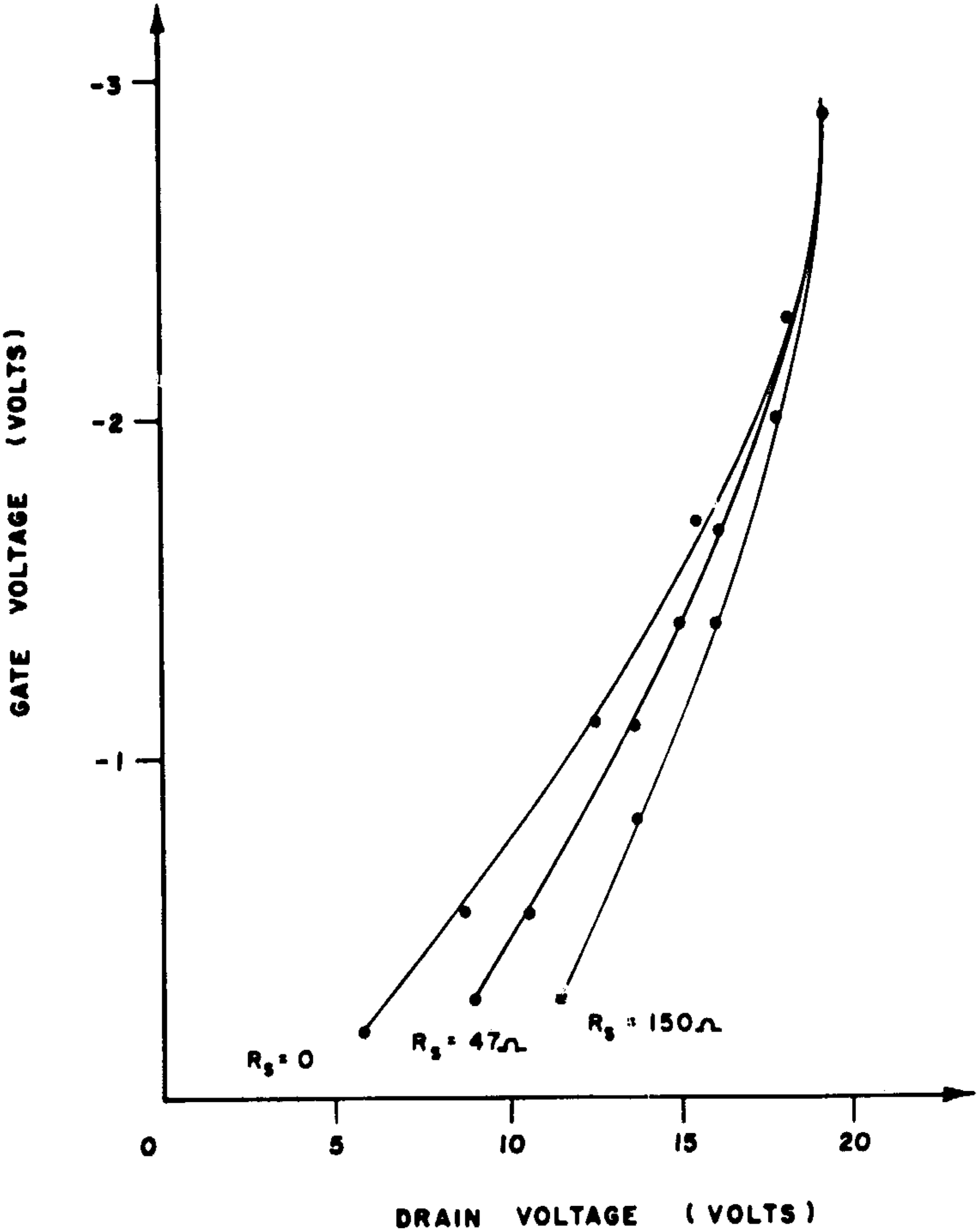

Figure 7. Typical Transfer Characteristics for an FET with Low Source Resistance R_s.

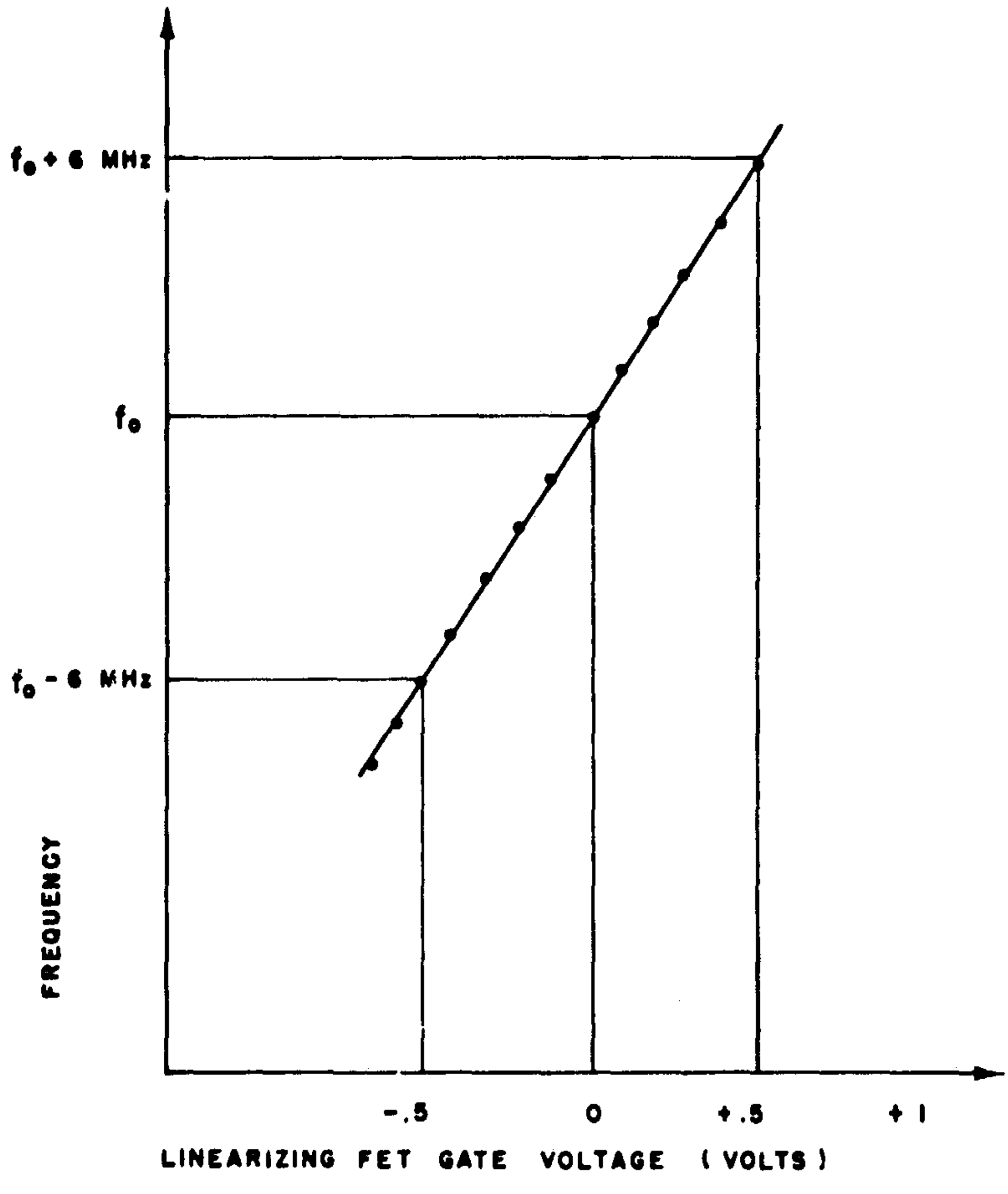

Figure 8. Normalized Frequency Deviation with Applied Voltage for the Push-Pull Oscillator after Linearization.

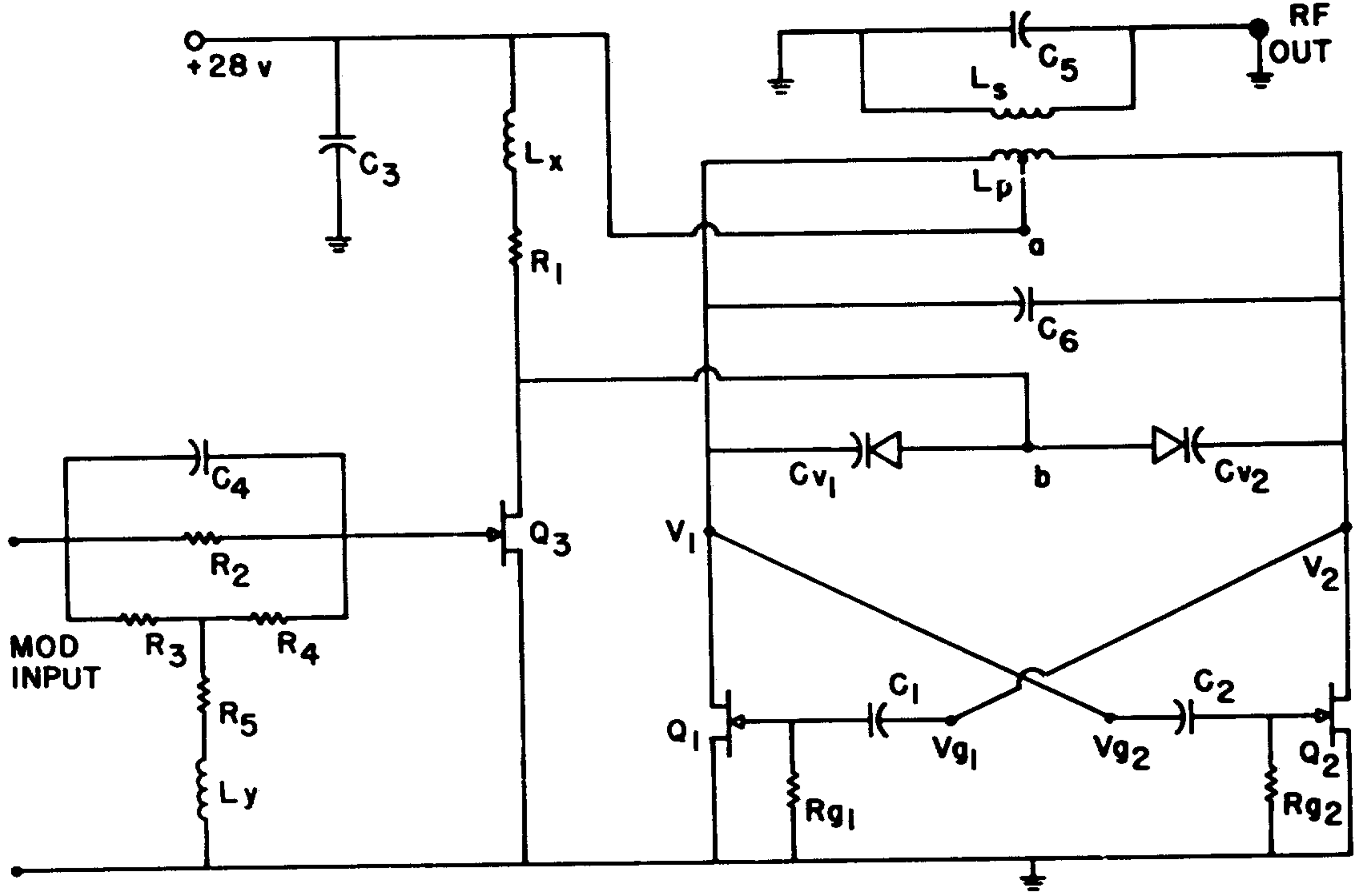

Figure 9. Final Push-Pull Oscillator Circuit, Showing Equalization and Linearization Circuits.

are identified in Table 1. With the addition of the equalization network, the oscillator has useful modulation response above 10 MHz, as demonstrated in the following chapter.

TABLE 1

LIST OF COMPONENTS FOR THE PUSH-PULL FREQUENCY-MODULATED OSCILLATOR

Item or Symbol	Value or Type Number	Manufacturer	Comments
Q_1, Q_2, Q_3	FET-2(2N4416)	General Electric	N-Channel, Junction FET
C_{V_1}, C_{V_2}	HEP-R2503(MV2109)	Motorola	Varactor Diode
C_1, C_2	10 pF	CDE	Mica
C_3	0.1 μF	TRW	Mica
C_4	150 pF	CDE	Mica
C_5	5 pF(NPO)	Erie	Ceramic
C_6	5 pF(N4200)	Erie	Ceramic
R_{g_1}, R_{g_2}	4.7 KΩ	Corning	1%, glass
R_1	1.0 KΩ	Corning	1%, glass
R_2	270 Ω	IRC	5%, ¼ watt, carbon
R_3, R_4	50 Ω	IRC	5%, ¼ watt, carbon
R_5	27 Ω	IRC	5%, ¼ watt, carbon
L_x	15 μH	J. W. Miller	
L_y	0.15 μH	J. W. Miller	
L_p	0.101 μH	JFD	Thin-film on glass
L_s	See Text		

IV. EXPERIMENTAL RESULTS

A. Frequency Stability with Temperature

The experimental model of the push-pull oscillator without modulation was subjected to temperature extremes in a Tenney environmental chamber to determine the effects of temperature on the frequency of oscillation. The plot of frequency versus temperature in Figure 10 shows an almost linear temperature characteristic for the push-pull FET oscillator. The frequency variation exhibited is largely attributed to poor heat dissipation of the epoxy-encased FETs, to excessive length of wires in the breadboarded oscillator model, and to drift in the quiescent point of the linearizing FET which supplies the varactor bias.

The near-linearity of the plot in Figure 10 made temperature compensation relatively easy. Temperature compensation was accomplished by adding a 5 pF N4200 negative-temperature-coefficient capacitor C_6 across the coil in parallel with the varactors. Also a zero-temperature-coefficient capacitor C_5 was used for the output matching capacitance to insure constant loading with varying temperature. The resulting frequency change with temperature is shown in Figure 11. The addition of the fixed compensating capacitor in parallel with the varactors lowered the operating frequency slightly and reduced the useable frequency deviation from 12 MHz to 10 MHz; but this is an insignificant change when compared to the frequency stability possible with compensation.

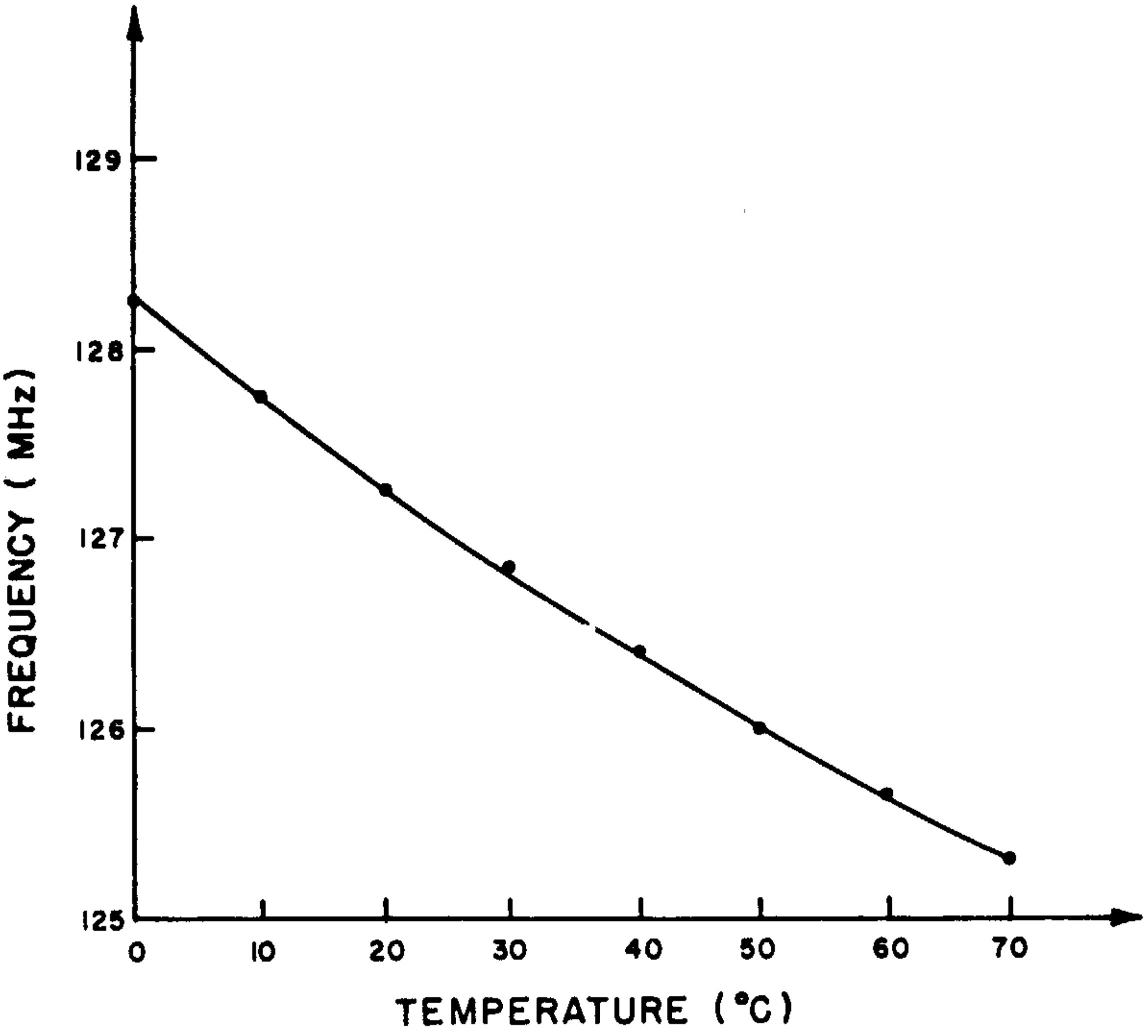

Figure 10. Frequency as a Function of Temperature for the Oscillator before Compensation.

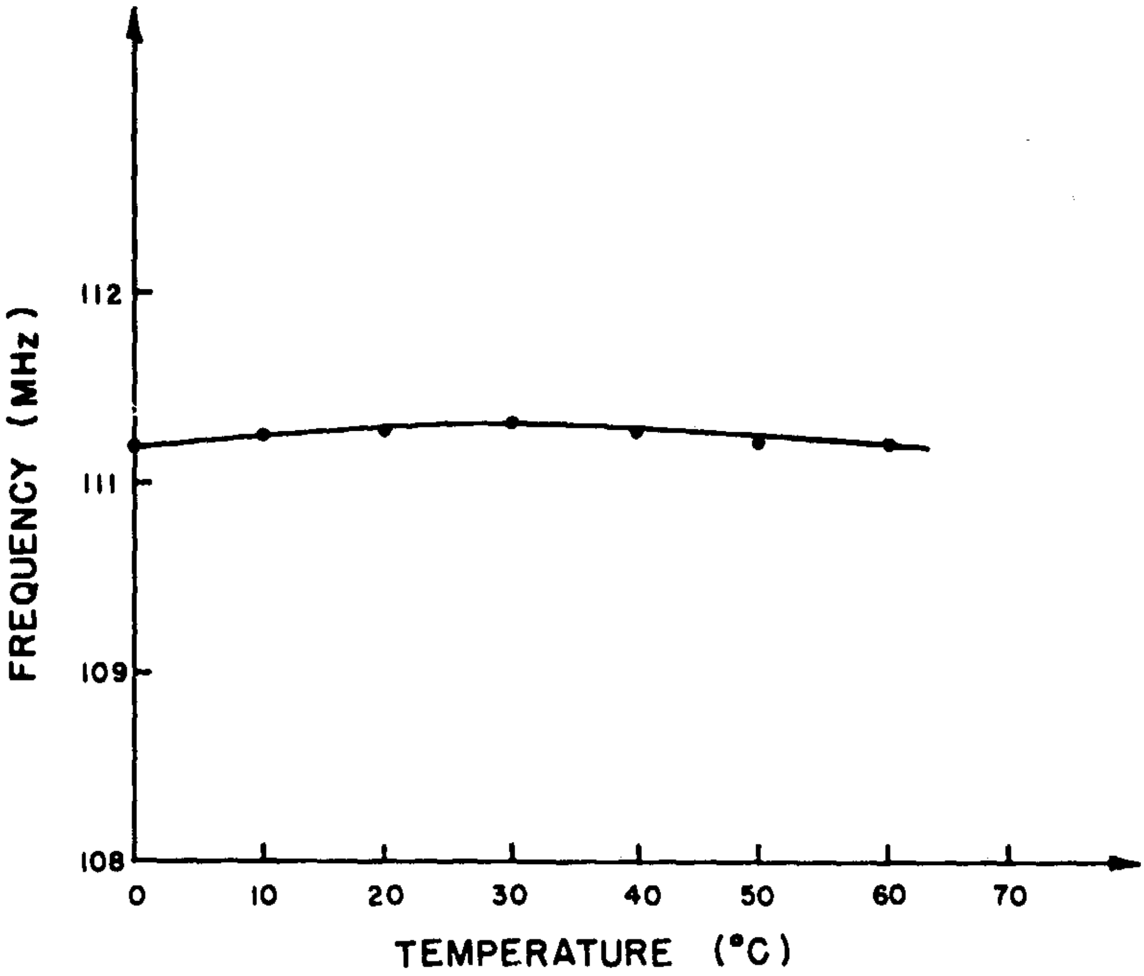

Figure 11. Frequency as a Function of Temperature for the Oscillator after Compensation.

B. Static Frequency Deviation

The static frequency deviation of the experimental push-pull oscillator was determined using the equipment configuration of Figure 12. The oscillator was linear within 0.2 MHz over a 10 MHz range centered at 113.3 MHz, as shown in Figure 13. The output power, also shown in Figure 13, was a nominal + 8.5 dBm, varying only $\pm$ 0.3 dB over the frequency range of interest. This is quite important in order for undesired amplitude modulation - and resulting distortion - to be held to a minimum. The waveform of the unmodulated 113-MHz carrier is displayed in Figure 14. This waveform shows that with careful attention to symmetry, the push-pull oscillator produces a signal with very low harmonic content.

C. Modulation Response

The experimental push-pull oscillator was subjected to various modulating signals in order to investigate its modulation bandwidth and waveform distortion. The experimental set-up used to perform these tests is shown in Figure 15. The RHG Laboratories FM receiver used to demodulate the signal has an IF frequency of 130 MHz, but its RF bandwidth capability of 60 MHz allowed it to be used successfully with the 113-MHz oscillator.

For sinusoidal modulation the oscillator proved to have a modulation bandwidth extending from dc to 10 MHz, within 0.7 dB, as shown in Figure 16. The effect of undesirable high-frequency roll-off in the video amplifier of the RHG receiver has been eliminated from the response

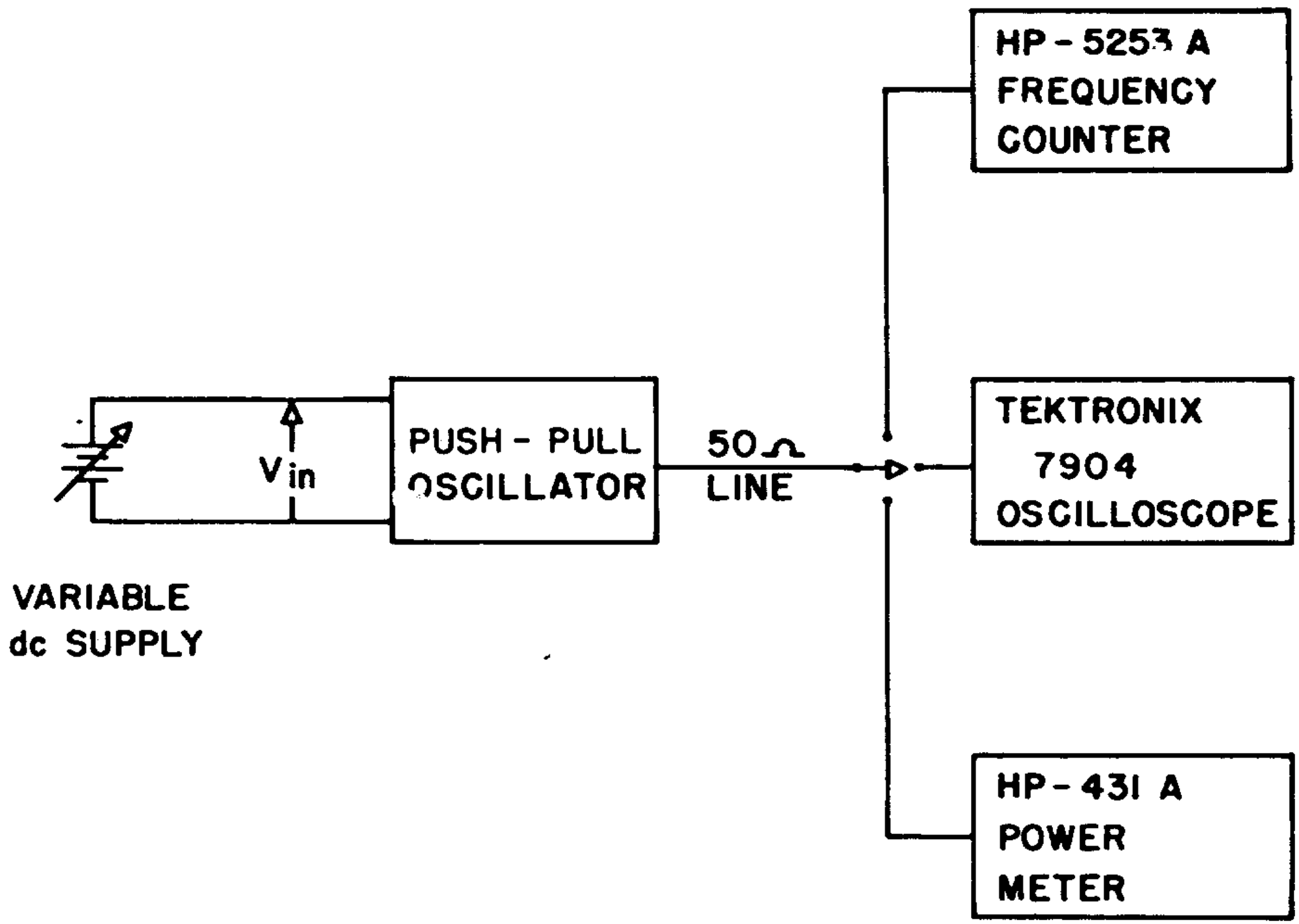

Figure 12. Equipment Configuration Used for the Static Frequency Deviation and Output Power Tests.

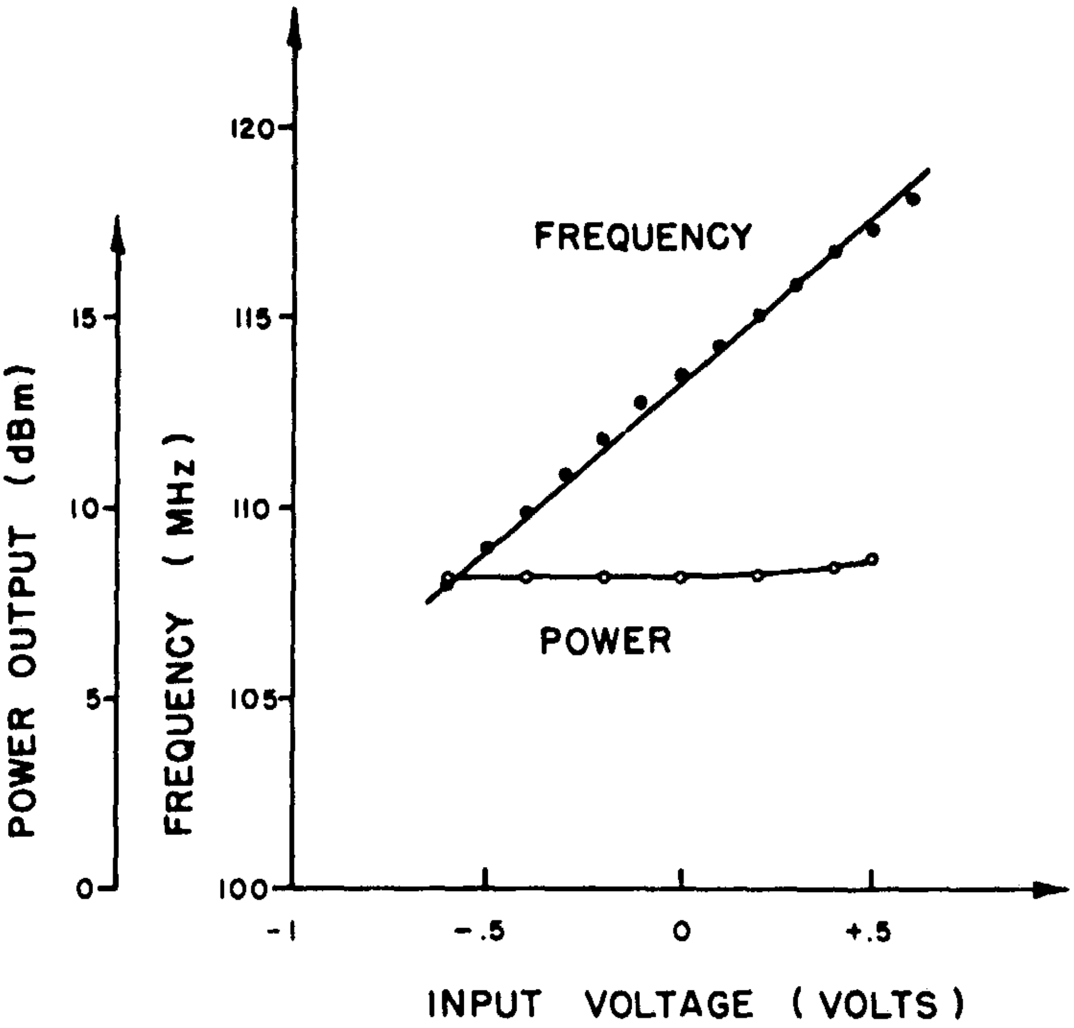

Figure 13. Static Frequency Deviation and Output Power as a Function of Input Voltage.

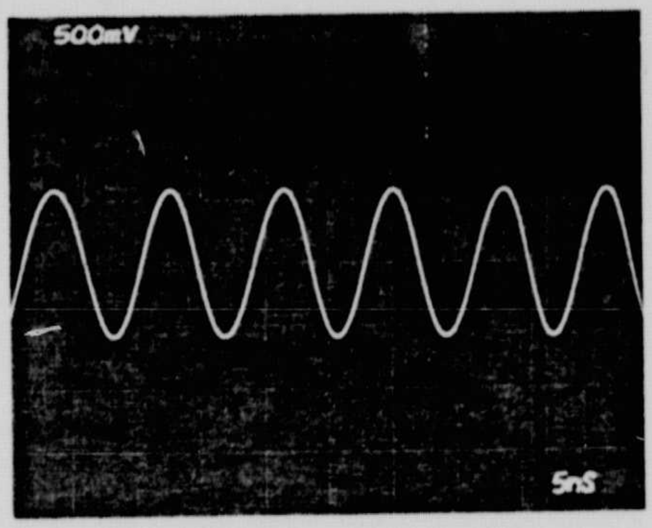

Figure 14. Output Waveform of the Oscillator without Modulation.

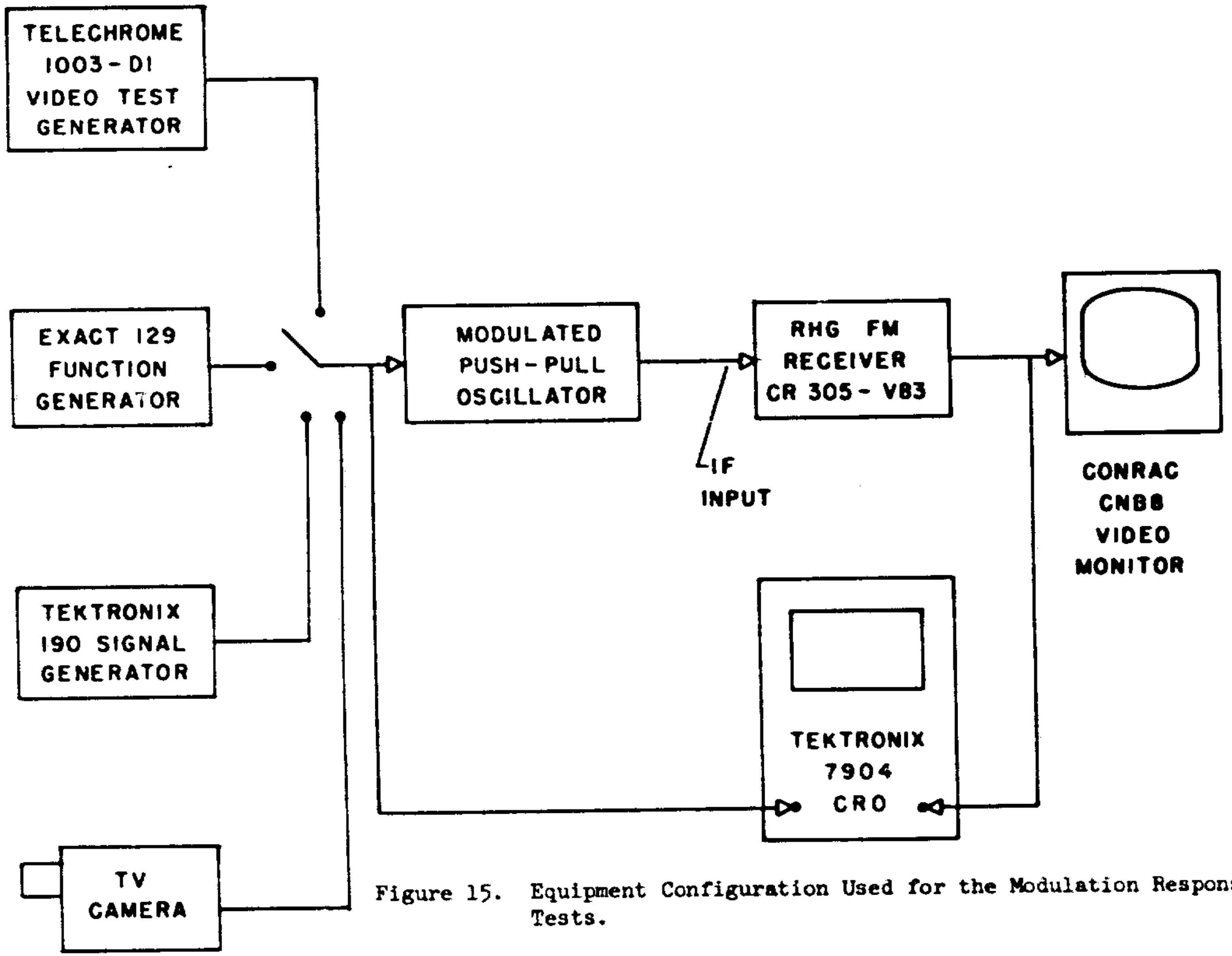

Figure 15. Equipment Configuration Used for the Modulation Response Tests.

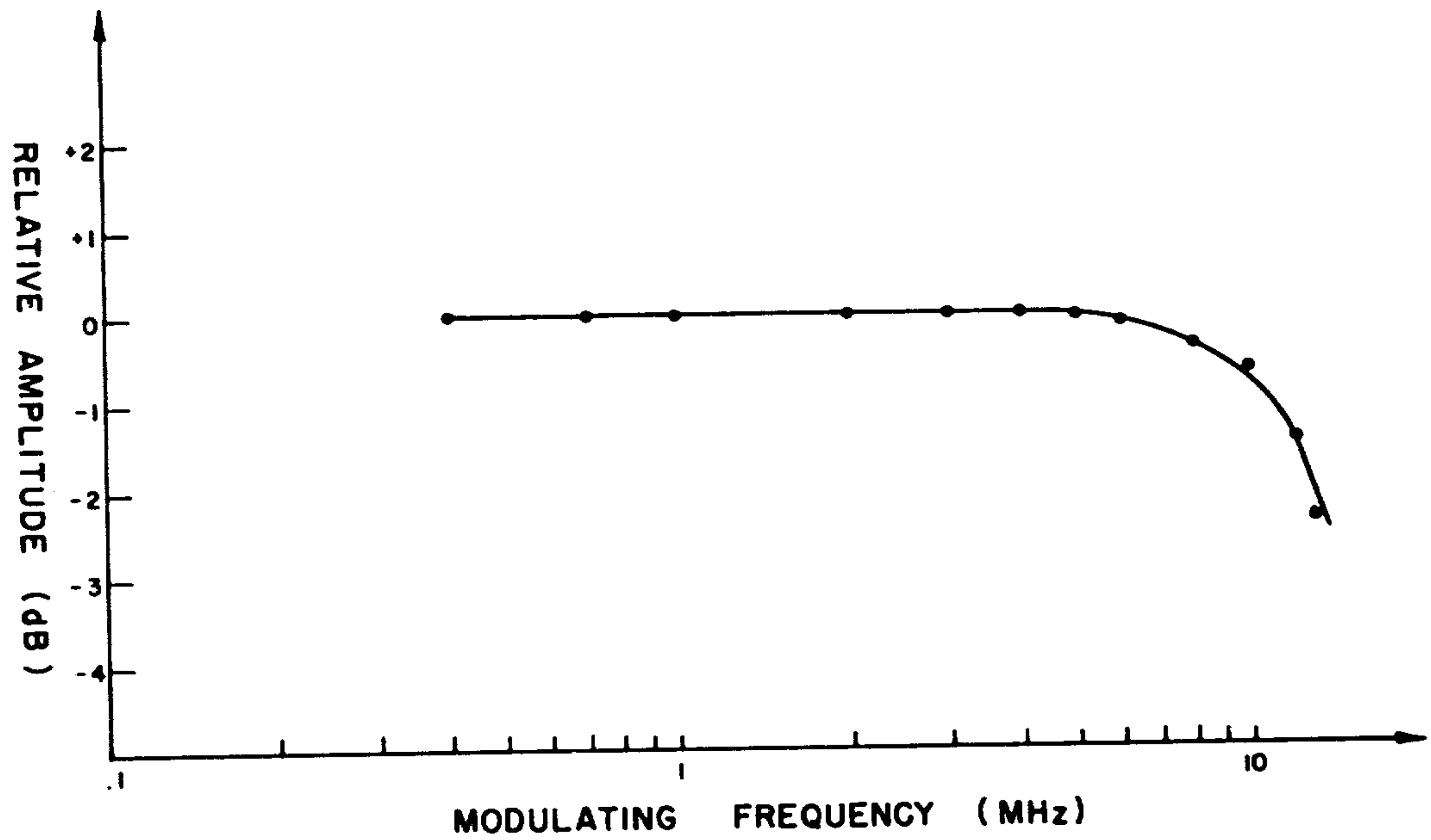

Figure 16. Modulation Response of the Push-Pull Oscillator.

plotted in Figure 16, providing a more accurate assessment of the oscillator's capabilities [8]. The modulation response of a 1-MHz square-wave is shown in Figure 17, again indicating the oscillator's good high-frequency modulation capability.

In order to determine its usefulness for television, the oscillator was subjected to various forms of video modulation, including the standard television multiburst and stairstep waveforms. The multiburst, shown in Figures 18 and 19, is a test of the system's modulation bandwidth; the stairstep pattern, in Figure 20, is a test of the linearity of the system. As a final test of video response, the oscillator was modulated by a video signal from a television camera. The results of this test are seen in Figure 21. It is evident from these photographs that the push-pull oscillator has excellent video bandwidth and linearity.

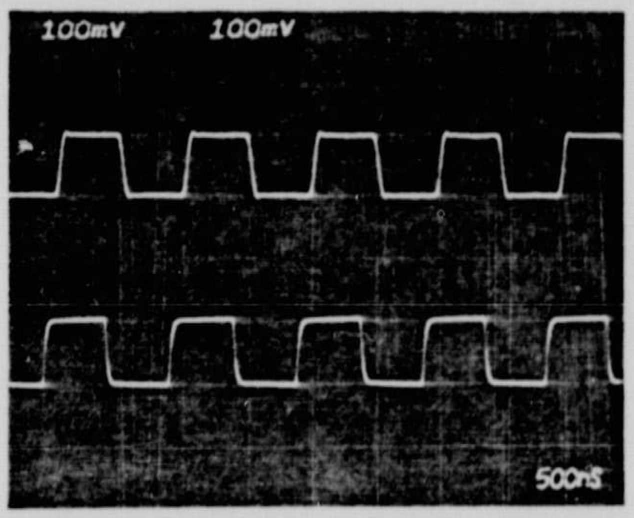

Figure 17. Modulation Response of the Oscillator Using a 1-MHz Square-Wave.

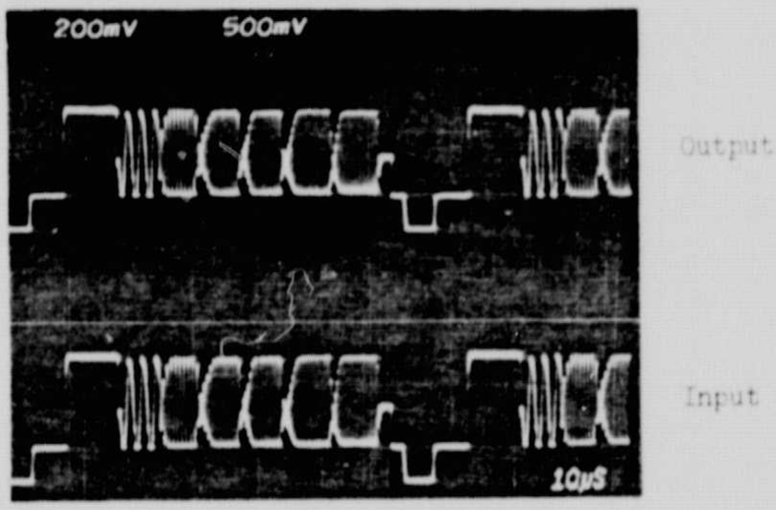

Figure 18. Modulation Response of the Oscillator Using a Television Multiburst Signal.

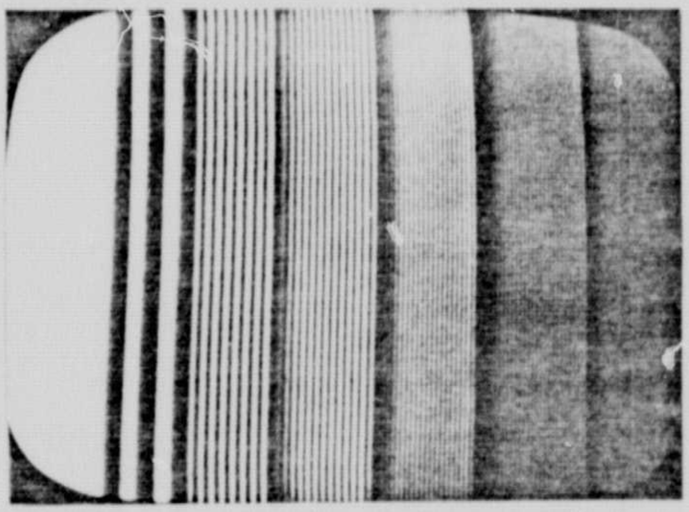

Figure 19. Video Output of the Receiver When the Oscillator is Modulated by a Multiburst Signal.

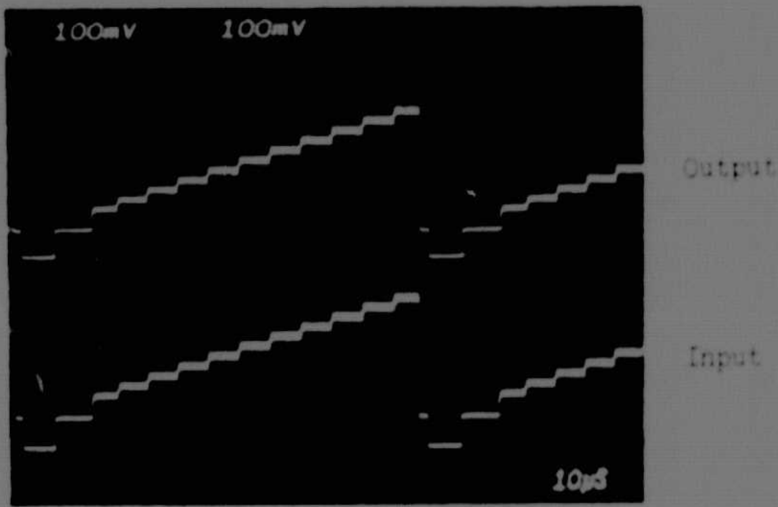

Figure 20. Video Linearity Test Using a Television Stairstep Waveform.

Figure 21. Video Output of the Receiver When the Oscillator is Modulated by a Signal from a Television Camera.

V. CONCLUSIONS

The field-effect transistor, with its high input impedance and relatively constant junction capacitances, is shown to be ideally suited for use in a frequency-stable wide-band FM oscillator with modulation response from dc to 10 MHz. The modulating amplifier, utilizing the square-law characteristic of an FET, provides linearization of the frequency deviation as a function of the modulating voltage amplitude. The temperature-varying frequency characteristic of the oscillator before compensation may be due, in part, to drift in the quiescent operating point, as a function of temperature, of the linearizing FET circuit. Since the varactors are biased directly by this FET, any change in the FET's quiescent voltage will result in an immediate change in the varactor capacitance and, therefore, a change in the center frequency of the oscillator. It is believed that further work could be done on the temperature stability of the quiescent voltage of the linearizing FET.

The push-pull configuration, with its inherently low distortion, proved to be easily implemented as a frequency-modulated oscillator. The symmetry of the circuit not only reduced the harmonic content of the output, but also simplified many of the problems associated with the use of varactor diodes. For example, the point at which the modulation is applied to the varactors is at RF ground potential, eliminating the need for complex radio-frequency filtering.

The experimental analysis of the push-pull frequency-modulated FET oscillator shows that it can be used successfully in a system designed for good frequency stability, linear operation, and low distortion. Although this work was done in the 100-MHz region because of equipment limitations, scaling techniques can be used to change the operating point of the oscillator to other frequencies, subject, of course, to device-imposed limitations. After temperature compensation, the frequency stability was excellent over a wide temperature range. Through the use of up-conversion, using a temperature-controlled oven for the modulated oscillator and a stable crystal-controlled high-frequency reference oscillator, a stable S-band, or X-band, FM transmitter with dc response can be designed.

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APPENDIX

ANALYSIS OF THE PUSH-PULL OSCILLATOR

The push-pull oscillator of Figure 9 may be analyzed by several techniques, including the negative-resistance approach [5]. In the analysis presented here, the oscillator is viewed as two symmetrically-connected LC oscillators operating in complementary fashion. In order to show more clearly the effects of symmetry, an RF equivalent circuit of the oscillator in Figure 9 is given in Figure 22. Points a and b are connected without altering the analysis since, by symmetry, they are at the same RF potential.

Resistors R_1 and R_2 account for the output resistance of each FET and all other losses which may be considered to be connected from drain to source. The equivalent input capacitances of the FETs are represented by C_{IN_1} and C_{IN_2}; while C_{OUT_1} and C_{OUT_2} include the equivalent output capacitance of each FET, the effect of the temperature-compensating capacitor C_6, and other stray capacitance from drain to source. Note that although C_6 is connected drain-to-drain, the symmetry of the circuit allows it to be considered as two drain-to-source capacitances, each having a value of twice the capacitance of C_6. C_{V_1} and C_{V_2} represent the capacitance of each varactor diode; and the inductances L_1 and L_2 are the equivalent inductances from the center tap to each end of coil L_p, including the effects of mutual inductance.

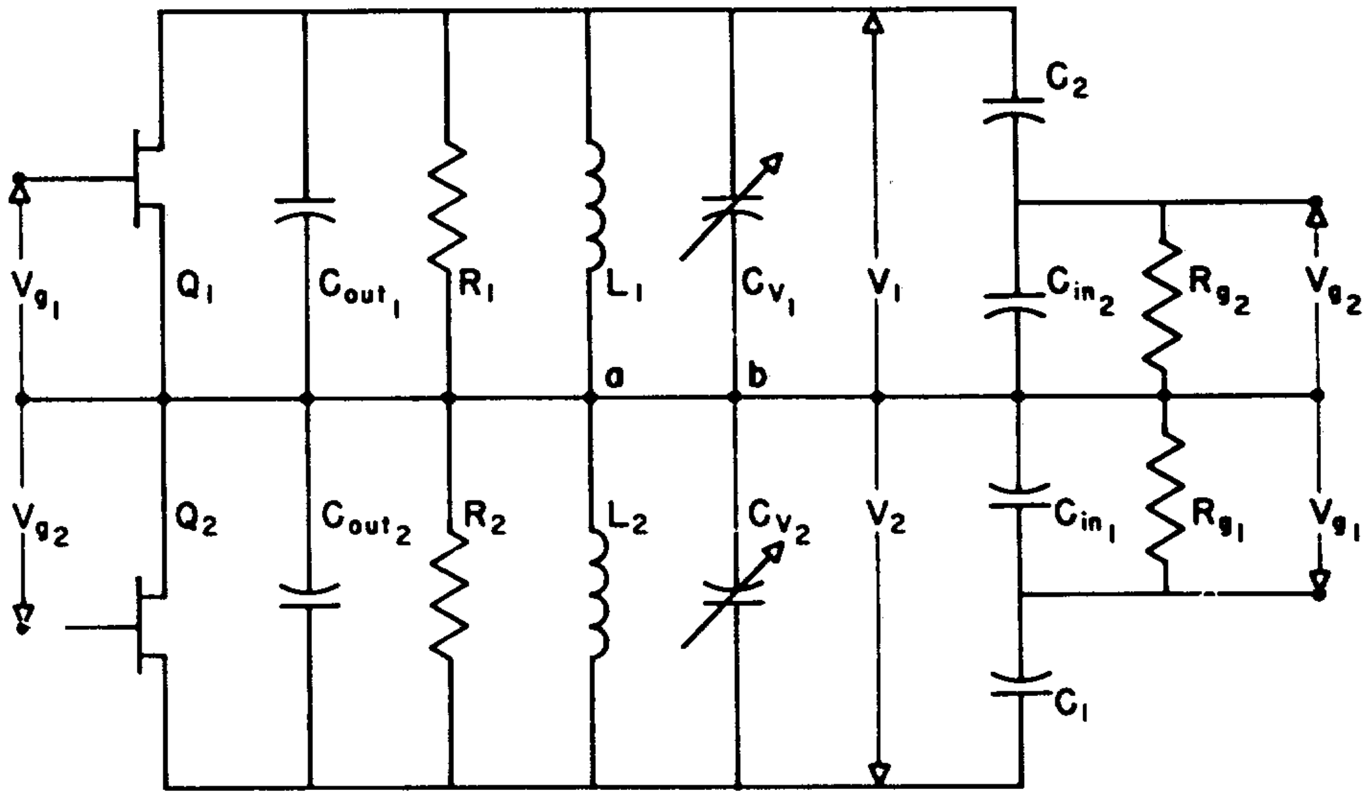

Figure 22. RF Equivalent Circuit for the Push-Pull Oscillator.

For the purpose of analysis we will consider only one of the active elements and its associated circuitry; that is, we will determine the conditions necessary for oscillation to occur in the top half of the symmetrical circuit of Figure 22. This requires that we know the loop gain AB for transistor Q_1 and its circuit. The gain A of the FET Q_1 is

$$A = -g_m Z_L \; . \qquad \text{(A-1)}$$

The load impedance Z_L seen by Q_1 is approximately

$$Z_L \doteq R_1 \,/\!/\, SL_1 \,/\!/\, \frac{1}{SC_T} \; , \qquad \text{(A-2)}$$

where

$$C_T \doteq C_{V_1} + C_{OUT_1} + \frac{C_2 \; C_{IN_2}}{C_2 + C_{IN_2}} \; .$$

This expression is approximate under the assumption that $R_{g_2} >> X_{C_{IN_2}}$. After some algebraic manipulation, the load impedance can be written as

$$Z_L \doteq \frac{R_1}{1 + \dfrac{R_1}{SL_1} + SR_1 \; C_T} \; . \qquad \text{(A-3)}$$

The gain A is then

$$A \doteq \frac{-g_m R_1}{1 - jR_1\left(\dfrac{1}{\omega L_1} - \omega C_T\right)} \; . \qquad \text{(A-4)}$$

In order to find the feedback ratio B, we must make use of the symmetry of the circuit and recognize that $V_{g_1} = -V_{g_2}$, instantaneously. Since, by voltage division,

$$V_{g_2} \doteq V_1 \frac{C_2}{C_2 + C_{IN_2}} , \tag{A-5}$$

then

$$V_{g_1} \doteq -V_1 \frac{C_2}{C_2 + C_{IN_2}} , \tag{A-6}$$

again assuming that $R_{g_2} >> X_{C_{IN_2}}$. Therefore, the feedback ratio B for Q_1 is

$$B \doteq \frac{V_{g1}}{V_1} = - \frac{C_2}{C_2 + C_{IN_2}} . \tag{A-7}$$

Assuming symmetry and dispensing with the subscripts, the loop gain for this single FET can now be written as

$$AB \doteq \frac{C}{C + C_{IN}} \; \frac{g_m R}{1 - jR\left(\frac{1}{\omega L} - \omega C_T\right)} . \tag{A-8}$$

In order for oscillation to be sustained, the Barkhausen Criterion must be satisfied [9,10]; that is,

$$AB = 1 \angle 0^{\circ} . \tag{A-9}$$

Therefore, the imaginary part of Equation (A-8) must equal zero and the real part must equal unity at the frequency of oscillation, f_o. Taking the imaginary term first,

$$- jE(1 - \omega_o^2 L\, C_T) = 0 \ . \tag{A-10}$$

This leads directly to the expression for the frequency of oscillation,

$$f_o \doteq \frac{1}{2\pi(L\, C_T)^{1/2}} \ , \tag{A-11}$$

where

$$C_T \doteq C_V + C_{OUT} + \frac{C\, C_{IN}}{C + C_{IN}} \ .$$

The expression for the minimum gain required for oscillation, $g_m R$, is obtained by equating the real part of Equation (A-8) to unity.

$$g_m R \doteq 1 + \frac{C_{IN}}{C} \tag{A-12}$$

Equation (A-11) shows that the push-pull oscillator's frequency varies approximately inversely as the square-root of varactor capacitance C_V, if

$$C_V > C_{OUT} + \frac{C\, C_{IN}}{C + C_{IN}} \ .$$

Equation (A-12) shows that the gain required for oscillation remains relatively constant over the frequency range where the previously stated assumptions are valid.

Since the push-pull oscillator can be viewed as two single-FET oscillators symmetrically connected, the conditions for oscillation of each separate oscillator apply to the push-pull configuration. Also, the analysis takes into account the existence of the other half of the oscillator by using the fact that $V_{g_1} = -V_{g_2}$ in finding the feedback ratio B.

Because the FETs in the oscillator circuit use a gate-to-source resistor to develop bias, the amplitude of oscillation is self-limited. This method of biasing results in some wave form distortion. Due to the symmetry of the push-pull circuit, all even-order harmonic distortion terms are cancelled, or minimized.