N75 17238

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MODEL OF SATURN'S RINGS THAT SATISFIES THE OBSERVED PHASE CURVE FOR OPTICAL SCATTERING

I would like to begin by stressing that this is a preliminary report, and any numbers quoted are subject to revision. If the procedure we are using is of interest to anyone, I hope they will consult with me. I wouldn't have presented this material at this time if it were not for the workshop nature of our discussion. The results are principally the work of Y. Kawata, a graduate student at the University of Massachusetts, and in essence, they are a refinement of the classical model to which Dr. Franklin has referred (see preceding contribution by Franklin).

Several workers in the classical tradition have stressed the importance of refining previous calculations to include rigorously the effect of multiple anisotropic scattering, and that is what we have done, including the effect of the solar penumbra, which becomes important in shadowing computations of this kind. Our basic idea has been to see if we could take the classical model and match the observations, including the wavelength dependence, simply by varying the particle albedo as a function of wavelength, and thus obviate the need to consider diffraction by the particles. This provides a model different from that Franklin and Cook (1965) proposed in the past.

Let me fairly briefly go over the procedure, which is well known in the literature. The basic data that we have been attempting to fit include, first of all, the phase curve, which Dr. Franklin has shown. This phase curve has three characteristic features and is shown in figure 1. We have normalized the visual and blue curves at the smallest phase angle observed by Franklin and Cook (1965). There is the opposition effect or surge in brightness near opposition, the linearly increasing portion at larger phase angles, and the very important wavelength dependence. I might stress at this point the very great desirability of obtaining phase curves that are this complete and of this quality at other wavelengths, both longer and shorter. The other critically important information is the absolute surface brightness of the rings. It is matching the shape of the phase curve over its entire range

University of Massachusetts.

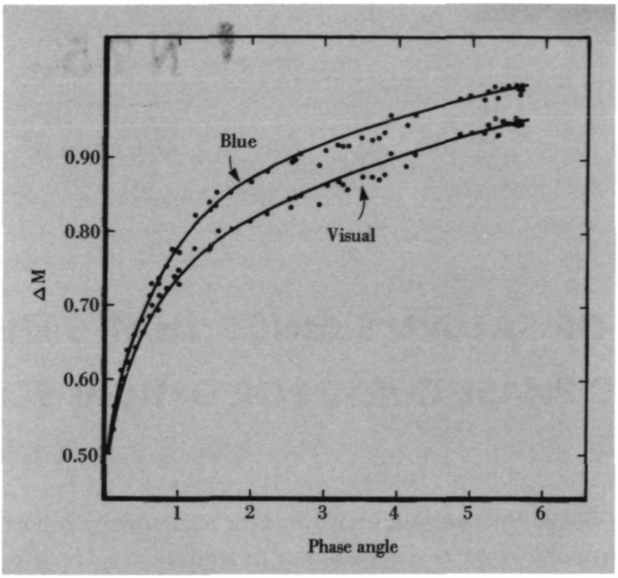

FIGURE 1.—*Phase curve for Saturn's B ring showing relative brightness as a function of phase angle in degrees.*

and matching the absolute brightness of the rings, which we have required of our model.

There are also data on the variation in the brightness of the rings as a function of the elevation angle of the Sun and Earth above the ring plane. I have found the treatment of those data in the literature somewhat confusing and have been unable to make a very good judgment as to how reliable they are. Different observers seem to get different results when they reduce the data because of seeing and various photographic effects on the plates.

As Dr. Franklin has pointed out, the principal diagnostic feature of the phase curves is the opposition effect. In the classical tradition, we have treated this as a result of mutual shadowing in a layer that is many particles thick.

We have in our model taken the rings to be homogeneous, so we have not looked at the effect of possible variations in particle properties with altitude in the ring, something proposed by some authors. Initially we took the ring particles to be monodispersed (characterized by a single particle size) and treated them as spheres for the purpose of the shadowing calculation. That doesn't, I think, significantly affect our results, because it can be shown that the magnitude of the shadowing effect at opposition is independent of the particle shape. We did not assume that the particles scattered like Mie particles. A spherical particle was used simply to give the geometry of the shadow. I will return later to the question of a distribution of particle sizes.

What we have done is to take this homogeneous layer with randomly distributed particles, solve the radiative transfer problem for radiation incident on this ring including the multiple scattered intensity for arbitrary phase functions, and include in the first-order scattering a correction for shadowing, which can be done in a straightforward manner. In this way we can account for both mutual shadowing of the particles, which Dr. Franklin has described, and anisotropic multiple scattering. We have, unlike Bobrov (1970a), for example, taken our results for parallel incident solar radiation and integrated over the solar disk to provide the range of angles of incidence that are important. If you haven't been familiar with this problem, it may seem somewhat surprising that the angular size of the Sun at Saturn can significantly influence the shape of a computed phase curve when the size of the Sun is something like 3 minutes of arc. But, in fact, that is a very important effect. It significantly reduces the magnitude of the opposition surge.

Figure 2 shows the magnitude of the shadowing effect with first-order scattering only, in stellar magnitudes between a phase angle of 0° and 6°, for models with a point Sun or models that integrate over the finite Sun. The effect is much reduced for a finite solar disk as a function of the volume density parameter, *D*, that Dr. Franklin spoke about. It is interesting to see that, when you account for the finite size of the Sun, you get a maximum phase effect at a particular volume density. If the volume density gets too large, then the rings effectively get filled up with matter and the shadowing becomes less important, essentially because the peak produced by the shadowing becomes very broad. Likewise, if the volume density becomes very small, with parallel radiation incident on the ring all your shadows would be cylindrical and would go on to infinity, so you reach an asymptotic situation. In the real world where the Sun has a finite size, you get radiation coming into the shadowed zone, so if you put the particles far enough apart you lose the shadowing effect.

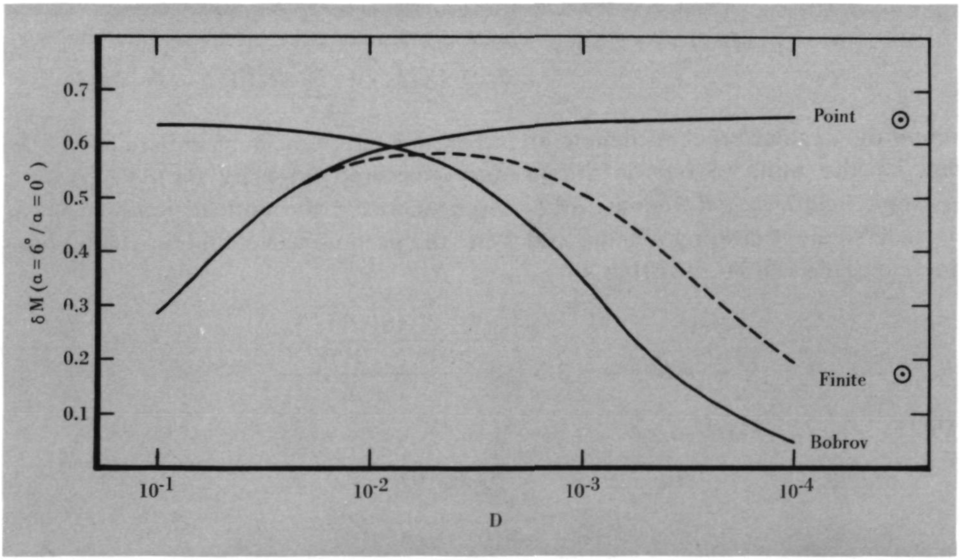

FIGURE 2.—*Magnitude of the opposition effect resulting from particle shadowing for three cases as a function of volume density* D, *considering only single scattering.*

It turns out that it is the volume density D which is the principal determinant of the shape of the opposition effect. As Dr. Franklin pointed out, we do get a pretty good handle on that and it turns out to be about 0.01 in these kinds of models.

However, we do find that, with that value of D which correctly reproduces the opposition effect, we cannot get the correct slope to the phase curve at larger phase angles by simply taking into account the primary scattering. We therefore have to go on and include the effects of multiple scattering and the effect of the phase function of the particles.

As I have said, the multiple-scattering computation is now a straightforward thing to do. The reflectivity of the rings will be given by the first-order scattering including the shadowing effect, plus the sum over higher-order scattering contributions where the shadowing effect is negligible. We find that the phase curve then depends on such parameters as the single scattering albedo of the ring particles, the optical thickness of the ring, the phase function of the ring particles, and volume density D, as well as geometric factors like the phase angle and the tilt of the ring with respect to the Sun and the Earth.

The specific intensity I of radiation reflected by the rings may be expressed as a sum of successive orders of scattering:

$$I = I_1^S + \sum_{n=2}^{\infty} I_n a^n \tag{1}$$

where I_1^S is the contribution from radiation scattered once, including the shadowing correction, and $a^n I_n$ is the contribution from radiation scattered n times. The predicted phase curve $M(\alpha)$ can be written as:

$$M(\alpha) = -2.5 \log \frac{a\Phi(\alpha)\ \langle S(\alpha)\rangle + \sum_{n=2}^{\infty} R_n(\alpha)}{a\Phi(0)\ \langle S(0)\rangle + \sum_{n=2}^{\infty} R_n(0)} \tag{2}$$

where the angular brackets denote an integration of the incident radiation over the disk of the Sun; $\langle S(\alpha)\rangle$ is the primary scattered intensity for conservative, isotropic scattering; R_n equals $a^n I_n$ for $\tau=0$ with τ the optical depth; a is the particle single scattering albedo; and $\Phi(\alpha)$ the particle phase function with phase angle α. This can be rewritten as

$$m(\alpha) = -2.5 \log \frac{\dfrac{\Phi(\alpha)\ \langle S(\alpha)\rangle}{\Phi(0)\ \langle S(0)\rangle} + X}{1+X} \tag{3}$$

where

$$X = \frac{\sum_{n=2}^{\infty} R_n(0)}{a\Phi(0)\ \langle S(0)\rangle} \tag{4}$$

This is so because the higher-order scattering component of the intensity does not change rapidly with angle. The maximum phase angle observable for Saturn is 6°, so, to a good approximation,

$$\sum_{n=2}^{\infty} R_n(0) \sim \sum_{n=2}^{\infty} R_n(\alpha) \tag{5}$$

The phase curve depends theoretically on the quantity X, which is the fraction of multiple scattering at opposition; that is, the brightness we see at opposition will include once-scattered light and multiply scattered light. Equation (3) turns out to be simple to deal with. We see that the shape of the phase curve will depend on the fraction of multiple scattering, the shape of the individual particle phase function, and the parameters that go into determining the shadowing effect in first-order scattering (principally the volume density and optical depth).

Those are the parameters which we will try to determine with this model, and, of course, there are a lot of parameters to play around with. Fortunately, we do have some other data, and particularly important in that regard are the data on the absolute brightness at opposition. I have been referring to this as the absolute brightness; in effect, it is the ratio of the brightness of the rings to the brightness of the disk.

There have been observations of the spectrum of the rings by Franklin and Cook (1965); Lebofsky et al. (1970); and Irvine and Lane (1973); and there have been some recent data by the Russian group, Kharitonova and Teifel' (1973). The spectral data are in pretty good agreement for wavelengths less than about 6000 Å, particularly if we bear in mind the color dependence of the opposition effect. But strangely enough, the data are rather discordant in the red. In the red the observations of Irvine and Lane (1973), Lebofsky et al. (1970), and Kharitonova and Teifel' (1973) are all rather different from each other. I don't know what the explanation for that is; some of it may be due to differences in tilt during the observations, or there may be actual temporal variations due to such things as difference in the insolation or differences in the brightness of the rings on the east and west side, as has been reported consistently back through the literature. I don't know what modern observations there are, but it would be interesting to see if there is any spectral difference from one side of the ring to the other.

Let me go on and show you how we can try to match both the shape of the phase curve and the observed absolute brightness with our theoretical models. We can conveniently do that with the aid of figure 3. The vertical axis represents the primary scattered radiation, including the shadowing effect. The horizontal axis in figure 3 represents the sum of the higher-order scattering, which according to the model is $\sum_{2}^{n} R_n$. The dashed curves designated R, V, and B are the loci of points that satisfy the observed absolute brightness in the red, visual, and blue, respectively. An uncertainty of ± 0.05 has been shown for B to illustrate the effect of possible uncertainty in these measurements.

For a given D and τ, the shape of the phase curve $M(\alpha)$ depends principally on the fraction of multiple scattering X. By experimenting with a wide choice of values for these parameters, for the phase function Φ, and for single scattering

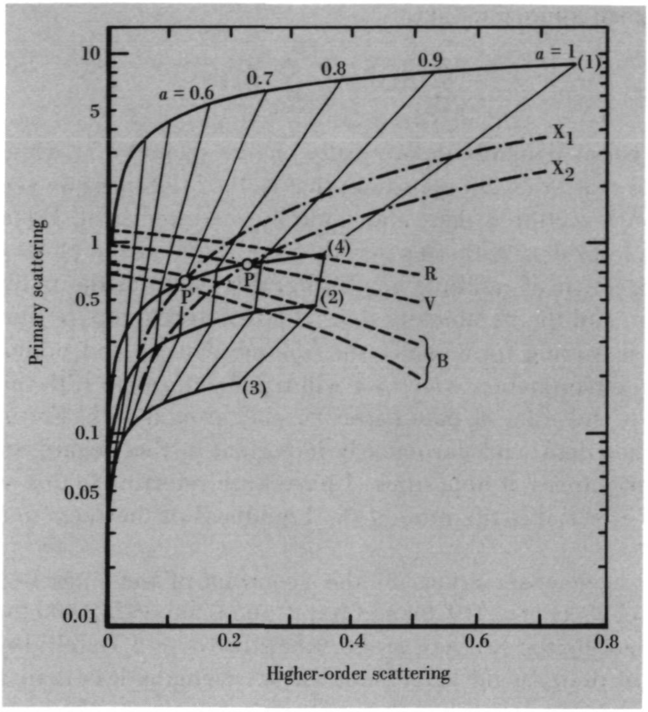

FIGURE 3.—*Scattering diagram showing the relative amount of primary and higher-order scattering necessary to match the B ring brightness and the shape of the phase curve.*

albedo a, we find that the sharp peak in the opposition effect depends primarily on the value of D, and that the observations restrict D to a narrow range around the value 0.01. Using this value of D, a τ of 1 on the basis of the observations of stellar occultations by the rings as discussed by Cook et al. (1973), and four assumed phase functions, the solid curves in figure 3 were constructed.

These phase functions, labeled (1) to (4), are shown in figure 4. They represent phase functions with the following characteristics: (1) a very strong backward peak with no forward scattering, (2) isotropic scattering, (3) a very strong forward peak with a small backward peak (reminiscent of the phase function for terrestrial clouds), and (4) a more slowly varying backward scattering phase function with a slight peak near 180°.

The straight lines in figure 3 connect points of constant single scattering albedo a and are labeled at the top of the figure. Once you have chosen an optical thickness for the rings and a volume density to more or less match the initial peak in the opposition effect, the parameter that determines the phase curve is the contribution of multiple scattering. The dot-dashed curves in figure 3 represent a match to the observed phase curves, X_1 in the blue and X_2 in the visual. The dot-dashed curves are the loci of points for which the fraction of multiple scattering is $X_1 = 0.17$ and $X_2 = 0.29$.

If we now call the intersection between curves V and X_2 in figure 3 a point P and the intersection between the middle of the range B and the curve X_1 a point P', then the theoretically computed brightness curve for the rings that passes through both P and P' will match the observed absolute brightness and also the observed phase curves.

James Pollack What is the meaning of the numbers on the vertical and horizontal axes of figure 3?

Irvine The units are arbitrary because we are ultimately talking about a ratio of ring intensity to disk intensity.

Pollack How can the primary scattering be greater than 1?

Irvine Because it isn't strictly a fraction. There is a scaling by the geometric albedo of Saturn's disk.

The power of this procedure is illustrated by the large differences between the curves in figure 3. The requirement that the model match both the shape of the

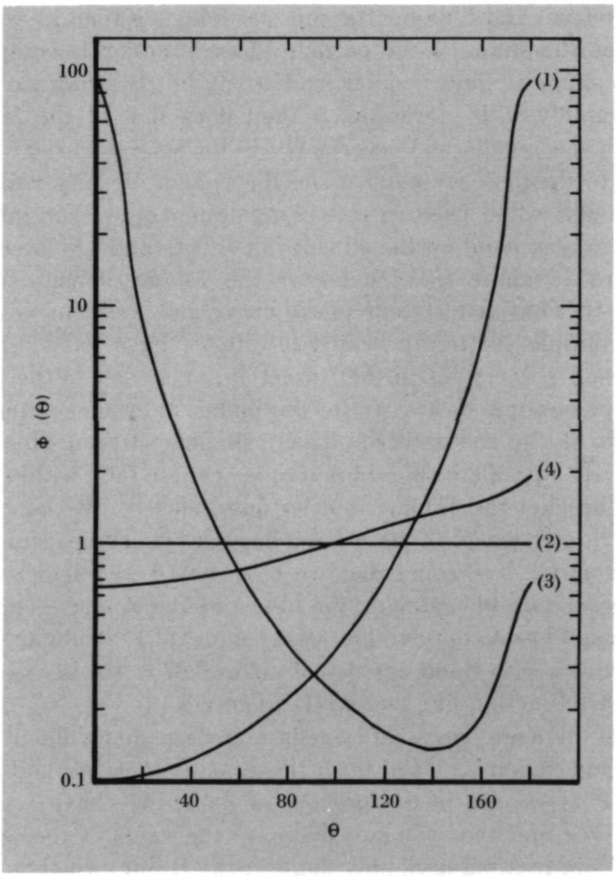

FIGURE 4.—*Particle phase functions used to generate curves labeled (1)-(4) in figure 3. $\theta=0$ is the forward direction.*

phase curve and the absolute brightness clearly puts significant restrictions on the form of the phase function. In particular, it is quite evident that neither the phase function with a very strong backward peak nor that with a very strong forward peak can match the observations. Some degree of backscatter is required to match the phase curve, so that the phase curve must be similar in shape to curve (4) in figure 3. Although the shape of the phase curve (apart from the opposition peak) depends principally on the values of Φ near 180° (corresponding to the small phase angles observable for Saturn), an appropriate phase function cannot be very different from curve (4). If it decreased much more sharply with decreasing α, it would not satisfy the normalization condition. The addition of a shallow forward peak to the phase function would be possible and would require a lower backward peak; that is, the phase function would become more isotropic.

We may now determine the single scattering albedo from the position of the points P and P' on curve (4) in figure 3. We find $a_V=0.87$ and $a_B=0.70$. By normalizing the phase function (4) to unity at $\alpha=180°$ and integrating, we may obtain the phase integral q for the ring particles as $q=2.1$. The resulting geometric albedos in the visual and blue for the ring particles are then $p_V=a_V/q=0.41$ and $p_B=0.33$. When the shape of the particle phase function is compared to that of the Moon, the shape is quite similar near $\alpha=0$, but the ring particle brightness falls off less rapidly with increasing α than does that of the Moon. This is in agreement with the results of Ververka (1973) for snow-covered objects.

Going back to figure 3, we can't make the volume density very much smaller than the particular value used in this computation or we run into the following problem: we are at a point on the shadowing effect versus volume density curve, shown in figure 2, where if we decrease the volume density we increase the shadowing effect. That distorts our phase curve and we have to modify it by increasing the multiple scattering, which means we have to increase the particle albedos. We can't go very far in that direction, or we get to the point where the new curves representing X_1 and X_2 for the higher multiple scattering don't cross the observations in the red until you have a single scattering albedo greater than 1, which is clearly a problem. We think that we can, in fact, within the parameters of this model, bracket the volume density quite closely. We can't go very far in the other direction or we don't get a large enough opposition peak.

Let me summarize by saying that we have found a satisfactory model using the classical procedure of matching the blue and visual phase curves and brightnesses. This model has an optical thickness τ equal to 1, a volume density of about 0.010, ring particles with Bond albedos of about 0.87 in the visual and 0.70 in the blue, and a phase function like curve (4) in figure 4.

None of that in itself necessarily tells you very much about Saturn's rings until you see how much you can push these parameters around and still fit the data. That is what we are in the process of doing. We have gone a little bit in this direction. We find that you can decrease the value of the optical thickness of the B ring (I am treating here only data for the B ring, which is most complete) to about a value of 0.7. That brings the volume density down a bit. It doesn't change the albedo very much, and it makes the phase function somewhat more backward-scattering. It turns out that you can increase the optical thickness by a considerable amount, but the model is not sensitive to larger values of τ. It is hard to get an upper limit on the optical thickness using a model of this nature.

Let me make a final remark about particle size. If we took the monodispersed model literally, we could, given the optical thickness and the volume density, determine the mean particle size if we knew the physical thickness of the rings. If we take a physical thickness of the rings of 2 km, we come out with a particle size of about 15 m.

That may be misleading for reasons associated with what that physical thickness means, as well as with the limitations of the monodispersed model. So we modified our computation to include a distribution of particle sizes. Of course, once you allow for that, there are an infinity of possible particle size distributions you can use. We have looked at the distributions that Bobrov (1970) considered where the number of particles with radii between ρ and $\rho + d\rho$ is proportional to $K\rho^{-s}$. This is a distribution, according to Bobrov, which is used in meteor astronomy. We have looked at distributions that increased sharply toward the small particle end and also the uniform distribution, where s equals 0. The latter case doesn't change our results very much, at least in our initial experimentation. However, as you go to distributions that have more and more small particles in them, we find that if we take a distribution as steep as ρ^{-3}, we don't seem to be able to fit the data. With ρ^{-2} we can fit the data, and the mean particle size turns out to be considerably reduced.

In the case where you have a steep distribution of particle sizes, you must differentiate between the mean size, the root mean square size, the cube root mean cube size, and so on, all of which would enter a shadowing computation of this kind. Probably the most relevant is the root mean square particle size. We have matched the data with a model that has a volume density of the order 10^{-2} (5×10^{-3} up to 2×10^{-2}) and a root mean square particle size of around 50 cm. Whether we can push that down lower as some other considerations (see contribution by Pollack) would suggest is not entirely clear at this point. I think it may be difficult to push the root mean square particle size much lower than that. For the albedos, we get a somewhat greater range, but, in any case, it is clear that the particle albedo is high and wavelength-dependent.

In agreement with what Dr. Franklin said, it would be very desirable to get additional information on the phase curve and opposition effect of big, bright particles. I would think that, as Dr. Pollack pointed out, the Jovian satellites are potentially good examples, as are, perhaps, Iapetus and Rhea. It may be possible to do something in the laboratory with ice surfaces. It is very important to try to find the effect of albedo on the shadowing effect (the effect of the complex part of the index of a refraction). Perhaps one could do some experiments with ice, putting in varying amounts of dye to reduce the transparency of the ice, and see how the opposition effect is changed.

As I said in the beginning, I also think that a critical type of observation in distinguishing between models of the kind I considered here would be complete phase curves at the extreme wavelengths available from the ground in the red or near-infrared and the ultraviolet. It is important to see whether the shape of the phase curve changes and just what the opposition effect is at those wavelengths.

Since I didn't bring it out in this discussion up to this point, perhaps I should emphasize that in the models we considered we have been assuming that the phase function for the particles was independent of wavelength; that is, as you change the albedo of the particle you change the reflectivity at all wavelengths by a constant factor. I believe, from looking at the Fresnel coefficients and from multiple scattering computations, that that will be approximately true for big, rough, bright particles as long as the absorption doesn't get too large, and it doesn't have to get too large to knock the albedo down considerably. It would be of interest as well to see if that assumption could be verified, perhaps in the laboratory.

DISCUSSION

James Pollack How much can we trust the measurements of the thickness of the rings? The reason I mention this is that in looking at the data as presented in Bobrov (1970b), you must extrapolate to the exact point of ring passage. There is no measurement per se at that exact moment. The two independent people who did this seem to get extrapolations that differ by a factor of 2, which makes me wonder whether there is any reality to what they did at all.

Hugh Kieffer I would like to complicate the issue, Jim (Pollack), a little bit. A prerequisite for this thickness measurement is the assumption that the edge-on outermost ring particles have properties that are somewhat similar to the central ring properties. That could be a terrible assumption if there is any kind of separation going on. If you throw in lunar-type material on the outside of the A ring for some type of screwy reason, you will be off by a factor of 10 in the ring thickness.

Pollack Fred (Franklin), do you think the measurements are real? Do you think we should believe them?

Fred Franklin I agree with what you say, but it seems to me it is hard to make them much thicker than the upper limit given of about 4 km. I have the impression that you could make them a good deal thinner.

Pollack Yes, that is really the point. I have no doubt that the upper limit is real. The point is, say in Bill's (Irvine) interpretation, if you make the rings a lot thinner then the particle size could be a lot smaller. That is why I asked the question. You think that is a possibility?

Franklin I certainly do.

William Irvine The parameter, which is the best determined by this approach, is the volume density. You can't push that around very much.

Robert Murphy Bill (Irvine), the albedos you are coming up with are awfully high. I will talk about that in my presentation. With those albedos there is no way to get the ring brightness temperature above 80 K which is in conflict with a number of observations in the infrared.

Irvine Including yours?

Murphy Yes.

Irvine Well, the original brightness temperature measurements gave values of about 80 K.

Murphy Allen and Murdock (1971) got about 83 K.

Irvine Okay, what do you want? Will you quote a value for the temperature at this point?

Murphy Well, I will talk about it this afternoon. In the context of this particular paper, I would like to point out that these albedos are very difficult to handle in the context of other existing data. They have to be somewhat lower.

Irvine How low do you feel they would have to be?

Murphy The values that Cook et al. (1973) have talked about, on the order of a Bond albedo of 0.6.[1]

Irvine Well, of course, that is a bolometric Bond albedo.

Murphy Right.

Irvine We would have to do the appropriate integration to get that from the values of Bond albedo I have given. But even 0.6 is fairly high in the scale of the solar system.

Dennis Matson This is in regard to Bob's (Murphy) remarks on temperature. There is an emissivity parameter that enters into these models. The temperature you get is also dependent on what you are assigned for that. I do not view the infrared data, at its face value, as being inconsistent with higher albedos. There are a number of model parameters still free at this time.

Franklin If you did have a ring model in which you had essentially a monolayer of large particles in contact, or however you would like to visualize it, do you feel that you could match the observed photometry? You have parameters to play with—surface structure, phase function, and so on. Does a monolayer have a photometric interpretation that is reasonable, or do you think that is essentially ruled out?

Irvine Well, as you know, there isn't at this point any really accurate theoretical way to treat the opposition effect in a single particle, so it is difficult to answer your question. I don't think one can answer your question theoretically. One would have to try to look at laboratory data. As you pointed out, the data seem to indicate that you don't get as large an opposition effect, as you observe in the rings, with bright samples from the laboratory. I don't think the data published so far are very complete. It would be useful to have a more systematic investigation.

Brad Smith Bill (Irvine), is there any way of using the changing ring tilt that will take place in the next half-dozen years or so to distinguish between the monolayer and your model?

Irvine Well, we can certainly make predictions for what you ought to see with changing ring tilt with our model. I think to get good observations of that effect would be very valuable. I don't know how you would predict what you ought

[1]Editors' note: Murphy's view has changed, somewhat softening his objection to Bond albedos greater than 0.6. See footnote in discussion following his contribution.

to see for the monolayer. I haven't given that enough theoretical thought and I don't know that anyone else has, either.

Franklin But you have no arguments against it?

Irvine What worries me about a monolayer is that it is difficult for me to visualize a monolayer that has an optical thickness of unity and anything like the thicknesses that we have been talking about for the rings, for then you have got great big particles that are essentially, as Dr. Franklin said, rolling over each other, at least in the thicker parts of the B ring. Maybe it doesn't appeal to me esthetically. It is a little hard to visualize. I think it might be interesting to try to look at a model like that dynamically. You will certainly have collisions which would tend to reduce the angular momentum of the individual particles, and I would think you would eventually get them spiraling inwards. It might be interesting to see if you could establish the time scale.

Smith But you eliminate those collisions which are required by the particles passing through the ring place twice every revolution.

Irvine Right.

REFERENCES

ALLEN, D. A.; AND MURDOCK, T. L.: Infrared Photometry of Saturn, Titan, and the Rings. Icarus, vol. 14, 1971, pp. 1–2.

BOBROV, M. S.: Solar System Research, vol. 4, 1970a, p. 127 (English translation).

BOBROV, M. S.: Physical Properties of Saturn's Rings. In Surfaces and Interiors of Planets and Satellites, A. Dollfus, ed., Academic Press (New York), 1970b, pp. 377–458.

COOK, A. F.; FRANKLIN, F. A.; AND PALLUCONI, F. D.: Saturn Rings—A Survey. Icarus, vol. 18, 1973, pp. 317–337.

FRANKLIN, F. A.; AND COOK, A. F.: Optical Properties of Saturn's Rings. II. Two-Color Phase Curves of the Two Bright Rings. Astron. J., vol. 70, no. 9, 1965, pp. 704–720.

IRVINE, W. M.; AND LANE, A.: Photometric Properties of Saturn's Rings. Icarus, vol. 18, 1973, pp. 171–176.

KHARITONOVA, G. A.; AND TEIFEL', V. G.: Astron. Tsirk., no. 747, 1973.

LEBOFSKY, L. A.; JOHNSON, T. V.; AND MCCORD, T. B.: Saturn's Rings: Spectral Reflectivity and Compositional Implications. Icarus, vol. 13, 1970, pp. 226–230.

VERVERKA, J.: private communication, 1973.