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ANALYSIS OF EDGE IMPACT STRESSES IN COMPOSITE PLATES

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16. Abstract

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ABSTRACT

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*Except for Figure 6, a contact time of 35 μsec and contact length of 2 cm was used in these calculations.

INTRODUCTION

This report is part of a continuing effort by NASA to understand the basic mechanics of foreign object impact of composite materials of particular interest are damage resistant designs of jet engine fan blades under hail or bird impact. In previous reports the central or normal impact response of composite plates was examined [1]. In this report the mechanics of edge impact of composite plates are examined. This is schematically illustrated in Figure 1.

The basic approach to the study of impact of composite plates in this program has been to examine the stress waves generated by the impact forces. For central impact of plates it has been shown that in addition to wave propagation across the plate thickness, bending and extensional waves propagate away from the impact site. The stresses associated with these waves have been studied without considering the effect of boundaries such as the free or clampled edges of a fan blade. This simplification has been made on the premise that for short impact times e.g. less than 10^{-4} sec. few edge reflections have taken place, and that the highest stresses occur at the impact site. However, for edge impact, the boundary conditions greatly affect the nature of the wave mechanics.

Edge waves in solids have been studied extensively in seismology. The principal phenomenon is the entrapment of wave energy in a layer near the surface. This surface wave is known as a Rayleigh wave and travels at a velocity below the shear velocity for isotropic solids. For plates, however, two types of edge waves can occur as shown in Figure 2a. For impact transverse to the plate, flexural edge waves can occur. For in-plane, Rayleigh type edge waves are generated. In this report only in-plane edge waves will be discussed.

Wave type solutions to the equations of elastodynamics of an orthotropic plate which exponentially decay away from the edge (X_3 direction) and propagate along the edge (X_1 direction) can be found provided the edge wave velocity satisfies the equation (Reference 9).

$$\rho v^2 + \left[\frac{C_{55} - \rho v^2}{C_{33} C_{55} (C_{11} - \rho v^2)} \right]^{1/2} [C_{13}^2 - C_{33} (C_{11} - \rho v^2)] = 0$$

where ρ is the massdensity of the plate and C_{ij} are the effective plate elastic constants (denoted by $\hat{C}_{ij}$ in Reference 1). It can be shown that one real root lies in the interval

$$0 < \rho v^2 < C_{55}$$

Thus, the edge or Rayleigh wave speed is less than the shear speed in this direction $[C_{55}/\rho]^{1/2}$.

Changing the layup angle will affect the elastic constants C_{ij} and hence, change the value of the edge wave velocity. The results of this calculation are shown in Figure 2b where the C_{ij} are obtained from Reference 7 . The edge wave speed seems to obtain a maximum between $\pm$ 15 and $\pm$ 30^o layup angle which is below the extentional wave speeds labelled "dilational" and "shear" in Figure 2b and which is greater than the bending wave velocity.

In order to prevent damage to composite fan blades under foreign object impact leading edge protection has been used. (See e.g. Ref. (8)). This usually consists of a strip of metal attached to the leading edge of the fan blade. To model the effects of this impact protection strip, the in-plane edge impact of an anisotropic plate, with a beam-strip attached to the impact edge, has been studied (see Figure 1). It will be shown later that the strip will decrease the tensile stress along the edge while producing shear stress between the strip and the plate edge.

I. BASIC EQUATIONS FOR EDGE LOADING ON ANISOTROPIC PLATE

Let the plate be the half-space $x_3 > 0$. The equations of motion are given by [1] as:

$$
\begin{aligned}
C_{11}\, u_{1,11} + C_{55}\, u_{1,33} + (C_{13} + C_{55})\, u_{3,13} &= \rho\, u_{1,tt} \\
C_{33}\, u_{3,33} + C_{55}\, u_{3,11} + (C_{13} + C_{55})\, u_{1,13} &= \rho\, u_{3,tt}
\end{aligned}
\tag{1.1}
$$

here we have employed C_{11}, C_{13}, C_{33} ... to denote $\hat{C}_{11}, \hat{C}_{13}, \hat{C}_{33}$ ... of [1]. The in-plane motion is assumed to be independent of the bending deformation. With the loading condition shown in Figure 1, the boundary conditions are (without protection strip):

$$
\begin{aligned}
t_{13}(x_1,0,t) &= C_{55}(u_{1,3} + u_{3,1})\Big|_{x_3=0} = 0 \\
t_{33}(x_1,0,t) &= [C_{33}\, u_{3,3} + C_{13}\, u_{1,1}]\Big|_{x_3=0} = p\, f(x_1)\, g(t)
\end{aligned}
\tag{1.2}
$$

The following nondimensional parameters are used;

$$C^*_{11} = C_{11}/C_{66}, \quad C^*_{13} = C_{13}/C_{66}, \quad C^*_{33} = C_{33}/C_{66}, \quad C^*_{55} = C_{55}/C_{66}, \quad P^*_o = p/C_{66} \tag{1.3}$$

$$u^*_1 = u_1/\ell, \quad u^*_3 = u_3/\ell, \quad x^*_1 = x_1/\ell, \quad x^*_3 = x_3/\ell, \quad t^* = t\sqrt{C_{66}/\rho}/\ell \tag{1.4}$$

ℓ is a length parameter. C_{66} is a typical elastic constant. In what follows the equations will be assumed to be nondimensionalized using (1.3) and (1.4).

To obtain the solution, we employ transform methods. Define:

$$F(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ik_1 x_1} f(x_1)\, dx_1$$

as the Fourier transform of $f(x)$ where we assume that

$$f(x_1)\Big|_{-\infty}^{\infty} = \frac{\partial}{\partial x_1} f(x_1)\Big|_{-\infty}^{\infty} = 0$$

Then

$$F\left(\frac{\partial}{\partial x_1} f\right) = ik\, F(f), \qquad F\left(\frac{\partial^2}{\partial x_1^2} f\right) = -k_1^2\, F(f)$$

Also define

$$\mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t)\, dt$$

as the Laplace transform. The initial conditions are assumed to be,

$$u_1(x_1,x_3,0) = \frac{\partial}{\partial t} u_1(x_1,x_3,0) = 0$$

$$u_3(x_1,x_3,0) = \frac{\partial}{\partial t} u_3(x_1,x_3,0) = 0 \tag{1.5}$$

Letting $\mathcal{L}[F(u_1)] = \bar{u}_1(k,x_3,s)$, $\mathcal{L}[F(u_3)] = \bar{u}_3(k,x_3,s)$, we have the following transformed equation:

$$C_{11}(-k_1^2)\,\bar{u}_1 + C_{55}\,\bar{u}_{1,33} + (C_{13}+C_{55})\,(ik_1)\,\bar{u}_{3,3} = +s^2\,\bar{u}_1$$

$$C_{33}\,\bar{u}_{3,33} + C_{55}(-k_1^2)\,\bar{u}_3 + (C_{13}+C_{55})\,(ik_1)\,\bar{u}_{1,3} = +s^2\,\bar{u}_3 \tag{1.6}$$

and at $x_3 = 0$, the transformed boundary conditions become

$$\bar{u}_{1,3} + ik_1\,\bar{u}_3 = 0$$

$$C_{33}\,\bar{u}_{3,3} + C_{13}\,ik_1\,\bar{u}_1 = -p_o\,\overline{f(k_1)g(s)} \tag{1.7}$$

Since we are expecting surface wave propagation, we seek the solution of (1.6), (1.7) in the forms:

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} e^{-p(k,s)x_3} \qquad (\text{real }(p) \geq 0) \tag{1.8}$$

Therefore, the equations for ϕ_1, ϕ_2 are:

$$\begin{bmatrix} -s^2 - C_{11} k_1^2 + p^2 C_{55} & -ik_1 p(C_{13}+C_{55}) \\ -ik_1 p(C_{13}+C_{55}) & -s^2 - k_1^2 C_{55} + p^2 C_{33} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = 0 \tag{1.9}$$

or for a non-trivial solution,

$$\det = C_{33} C_{55} p^4 + [C_{33}(-s^2-C_{11}k_1^2) + C_{55}(-s^2-C_{55}k_1^2) + k_1^2(C_{13}+C_{55})^2] p^2$$

$$+ (s^2+C_{11}k_1^2)(s^2+C_{55}k_1^2) = 0 \tag{1.10}$$

We will choose the p's with positive real parts to ensure the decay in x_3 direction of the surface wave. Let the solutions be $p = p_1, p_2$, therefore, we have

$$p = p_1: \quad \phi_1^{(1)} \equiv C_1(k_1, s), \quad \phi_2^{(1)} = - \frac{i[-s^2-C_{11}k_1^2 + C_{55}p_1^2]}{k_1 p_1 (C_{13} + C_{55})} \phi_1^{(1)} \equiv \psi_{31} C_1 \tag{1.11}$$

$$p = p_2: \phi_1^{(2)} \equiv C_2(k_1, s), \; \phi_2^{(2)} = \psi_{32} C_2 \equiv - \frac{i[-s^2-C_{11}k_1^2 + C_{55}p_2^2]}{k_1 p_2 (C_{13}+C_{55})} \phi_1^{(2)}$$

Therefore, the displacements have the forms:

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = C_1(k_1,s) \begin{bmatrix} 1 \\ \psi_{31} \end{bmatrix} e^{-p_1 x_3} + C_2(k_1,s) \begin{bmatrix} 1 \\ \psi_{32} \end{bmatrix} e^{-p_2 x_3} \tag{1.12}$$

C_1, C_2 are determined from the boundary conditions (1.7), or

$$\begin{bmatrix} -p_1 + ik_1\psi_{31} & -p_2 + ik_1\psi_{32} \\ -p_1 C_{33}\psi_{31} + ik_1 C_{13} & -p_2 C_{33}\psi_{32} + ik_1 C_{13} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -p_o \overline{fg} \end{bmatrix} \tag{1.13}$$

Here, the determinant, $\Delta(p_1, s)$, given by,

$$\Delta = (ik_1\psi_{31} - p_1)(ik_1 C_{13} - p_2 C_{33}\psi_{32})$$

$$- (ik_1\psi_{32} - p_2)(ik_1 C_{13} - p_1 C_{33}\psi_{31}) \tag{1.14}$$

must be non-zero to ensure a solution. (Δ=0 gives the Rayleigh poles of the system, which correspond to a free surface). Therefore,

$$C_1 = \frac{1}{\Delta}[ik_1\psi_{32} - p_2]\,\overline{fg}\,p_o$$

$$C_2 = \frac{1}{\Delta}[p_1 - ik_1\psi_{31}]\,\overline{fg}\,p_o \tag{1.15}$$

and the physical displacements $u_1(x_1,x_3,t)$, $u_3(x_1,x_3,t)$ are obtained by inverting their transforms.

From the stress-strain relations

$$t_{13} = C_{55}(u_{1,3}+u_{3,1}), \quad t_{11} = C_{11}u_{1,1} + C_{13}\,u_{3,3},$$

$$t_{33} = C_{33}\,u_{3,3} + C_{13}\,u_{1,1} \tag{1.16}$$

we obtain the transforms of the stresses

$$\bar{t}_{13} = C_{55}\{(-p_1+ik_1\,\psi_{31})\,C_1\,e^{-p_1x_3}$$

$$+ (-p_2+ik_1\,\psi_{32})\,C_2\,e^{-p_2x_3}\}$$

$$\bar{t}_{11} = (C_{11}ik_1-C_{13}p_1\psi_{31})\,C_1\,e^{-p_1x_3} \tag{1.17}$$

$$+ (C_{11}ik_1-p_2\psi_{32}C_{13})\,C_2\,e^{-p_2x_3}$$

$$\bar{t}_{33} = (-p_1\psi_{31}C_{33}+ik_1C_{13})\,C_1\,e^{-p_1x_3}$$

$$+ (-p_2\psi_{32}C_{33}+C_{13}ik_1)\,C_2\,e^{-p_2x_3} \quad ,$$

and the physical stresses can be obtained by inversion.

The particular forcing function employed is

$$f(x_1)\,g(t) = [1 - (\frac{x_1}{a})^2]\,\sin\frac{\pi t}{\tau_o} \qquad 0 < t < \tau_o \qquad |x_1| \le a$$

$$= [1 - (\frac{\ell}{a})^2\, x_1^{*2}]\,\sin\,(\pi t^*/\tau_o^*) \tag{1.18}$$

where $\tau_o^* \equiv \ell/\sqrt{C_{66}/\rho}$.

Here a is a length measuring the impact area, and τ_o is the contact time [1]. The transform of this particular forcing function, in non-dimensional form becomes,

$$\overline{fg} = + \frac{4}{k_1^2(a/\ell)^2}\left[-\frac{a}{\ell}\,\cos k_1(\frac{a}{\ell}) + \sin k\,(\frac{a}{\ell})\Big/k\right]\frac{\pi\tau_o^*(1+e^{-s\tau_o^*})}{\pi^2 + \tau_o^{*2}s^2}\ . \tag{1.19}$$

II. EDGE IMPACT OF PLATE WITH EDGE PROTECTION

To prevent failure of composite fan blades under impact forces, leading edge protective strips have been employed. In practice, these strips of stainless steel are wrapped around the leading edge. To model this device, we consider a beam bonded to the edge of an anisotropic plate (Figure 1). The effect of the beam will be to thwart the force of impact, thereby decreasing the normal stresses in the composite. However, we shall show that with such a reduction in normal stress, sizeable interface shear stresses can be induced.

With the introduction of a beam of thickness b on the edge of the composite plate, the Rayleigh wave behavior will depend on the ratio of the wavelength to thickness ratio of each Fourier component in the x_1 direction. Thus one should expect the Rayleigh wave speed to vary with b/a, the thickness to impact footprint ratio. In addition the Rayleigh wave will become distorted as it propagates.

To solve the edge strip problem the solution in the composite plate follows the same procedure as the no-strip case except for the boundary conditions on the edge. In place of the zero stress conditions on the edge we relate the edge stresses t_{33}, t_{13} to the motion of the beam strip. If one considers a small element of the beam-strip along the x_1 direction, the momentum balance equations in the x_1, x_3 directions become, (for a plate of unit thickness)

$$\rho b \frac{\partial^2 U}{\partial t^2} = Eb \frac{\partial^2 U}{\partial x_1^2} + t_{13} \tag{2.1}$$

$$\rho b \frac{\partial^2 W}{\partial t^2} = - EI \frac{\partial^4 W}{\partial x_1^4} + I_\rho b \frac{\partial^4 W}{\partial x_1^2 \partial t^2} + \frac{b}{2} \frac{\partial t_{13}}{\partial x_1} + t_{33} + p f_o^g \tag{2.2}$$

In these equations U, W are the x_1, x_3 displacements of the beam element at the half thickness, and t_{33}, t_{13} are the interface stresses.

We choose the compatibility conditions between the beam and plate displacements

$$W = u_3, \text{ on } x_3 = 0. \tag{2.3}$$

$$U = u_1 + \frac{b}{2} \frac{\partial u_3}{\partial x_1}, \text{ on } x_3 = 0 \tag{2.4}$$

In the above equations b is the depth of the strip, E, I, I_ρ are respectively the Young's modulus, moment of inertia and rotary inertia. Also p f(t) g(x_1) is the edge loading now applied to the outer protective strip surface.

The equations for the plate remain as in the free edge case and a solution is obtained by taking a Laplace transform on time and a Fourier transform on the space variable x_1. With nondimensionalization the solution in the plate is assumed in the form of (1.12). The transform of the plate displacements are

$$\begin{bmatrix} \bar{u}_1 \\ \bar{u}_3 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ \psi_{31} \end{bmatrix} e^{-p_1 x_3} + C_2 \begin{bmatrix} 1 \\ \psi_{32} \end{bmatrix} e^{-p_2 x_3} \tag{2.5}$$

where p_1, p_2 are defined in (1.10) and ψ_{31}, ψ_{32} are given in (1.11). C_1, C_2 are determined from the edge boundary conditions. However, in place of the free edge conditions (1.2) we use the equations of motion for the strip (2.1), (2.2). C_1, C_2 are then solutions to the algebraic equations

$$\begin{bmatrix} G_1 & H_1 \\ G & H \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -p_o^*\overline{fg} \end{bmatrix} \tag{2.6}$$

where

$$G_1 = -k^2Ew + p_1 C_{55} - \rho ws^2 - \psi_{31}(ik^2w^2E/2 + ikC_{55} + \rho w^2s^2ik/2)$$

$$H_1 = -k^2Ew + p_2 C_{55} - \rho ws^2 - \psi_{32}(ik^3w^2E/2 + ikC_{55} + \rho w^2s^2ik/2)$$

$$G = -ikC_{13} + ikwp_1C_{55}/2 + \psi_{31}(p_1C_{33} + Ew^2k^4/12 + wk^2C_{55}/2 + k^2s^2\rho w^3/12 + \rho ws^2)$$

$$H = -ikC_{13} + ikwp_2C_{55}/2 + \psi_{32}(p_2C_{33} + Ew^2k^4/12 + wk^2C_{55}/2 + k^2s^2\rho w^3/12 + \rho ws^2) \tag{2.7}$$

where $w = b/\ell$, ρ, E, are nondimensionalized quantities and p_1, p_2, ψ_{31}, ψ_{32} are defined in (1.11).

III. NUMERICAL INVERSION

The inversions are accomplished by the Fast Fourier Transform (FFT) techniques [2], which consists of a transformation from Laplace to Fourier transforms, and a two-dimensional numerical inversion using the usual FFT alogarithm. Notice the Laplace inversion formula

$$f(t) = \frac{1}{2\pi i} \int_{\Gamma\; C-i\infty}^{C+i\infty} f(s)\, e^{st}\, ds$$

Set $s = C + i\alpha$

$$f(t) = \frac{1}{2\pi} \int_{\Gamma\; -\infty}^{\infty} f(C+i\alpha)\, e^{Ct}\, e^{i\alpha t}\, d\alpha \qquad (3.1)$$

where C and α are both real, and C is greater than the largest real part of all singularities of $f(s)$. Numerically, the double Fourier transform (or inversion) has the following form [1]:

$$f(x,t) \cong \frac{K_x K_t}{\pi^2 NM}\, e^{-i[K_x(1-\frac{1}{N})x \,+\, K_t(1-\frac{1}{M})t]} \sum_{I=1}^{N} \sum_{J=1}^{M} \bar{f}(I,J)\, e^{2\pi i[\frac{(I-1)x}{N} + \frac{(J-1)}{M} t]} \qquad (3.2)$$

where N, M are the number of points in x and t direction respectively, and K_x, K_t are the corresponding half-frequency range.

For the present problem, the determination of C is through the following considerations:

The form of inversion integrals are, in general,

$$I = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} \frac{F(k_1,s)}{\Delta(k_1,s)} e^{-px_3} e^{st} ds \qquad p = p(k_1,s),\ Re(p) \geq 0 \tag{3.3}$$

and Δ is given by (1.14). It is easy to see the singularities of the integral of (2.3) are:

a) <u>Poles at $\Delta = 0$,</u>

$$\Delta = (p_2-p_1)\ ik_1(C_{13}-\psi_{31}\psi_{32}C_{33}) + (\psi_{32}-\psi_{31})(k_1^2C_{13}+p_1p_2C_{33}) = 0 \tag{3.4}$$

implies

$$(p_2-p_1)[C_{13}k_1^2p_1p_2(C_{13}+C_{55})^2 + C_{33}(C_{55}p_1^2-s^2-C_{11}k_1^2)(C_{55}p_2^2-s^2-C_{11}k_1^2)]$$

$$-(C_{13}+C_{55})[p_1(C_{55}p_2^2-s^2-C_{11}k_1^2) - p_2(C_{55}p_1^2-s^2-C_{11}k_1^2)](k_1^2C_{13}+p_1p_2C_{33})$$

$$= 0 \tag{3.5}$$

Interpretation of this condition is best understood for the particular case of an isotropic material, i.e. for

$$C_{11} = \lambda + 2\mu = C_{33}, \quad C_{13} = \lambda, \quad C_{55} = \mu,$$

choose $C_{66} = C_{55} = \mu$

It is easy to see that from (1.10), (1.11)

$$\det = [\mu p^2 - (\mu k_1^2 + \rho s^2)]\ \{(2\mu+\lambda)\ p^2 - [(\lambda+2\mu)\ k_1^2 + \rho s^2]\}$$

$$p_1^2 = (\mu k_1^2 + \rho s^2)/\mu, \qquad p_2^2 = [(\lambda+2\mu)\ k^2 + \rho s^2]/(\lambda+2\mu)$$

$$\psi_{31} = ik_1/p_1, \qquad \psi_{32} = -p_2/ik_1,$$

Here $[\mu/\rho]^{1/2} \equiv v_s$, is the shear wave speed and $[(\lambda+2\mu)/\rho]^{1/2}$ v_p, is the longitudinal or pressure wave speed for isotropic materials.

Thus for the isotropic case that, $\Delta = 0$ implies

$$4\mu p_2 + (p_1 + \frac{k^2}{p_1}) [\lambda + \frac{p_2^2}{k^2} (\lambda + 2\mu)] = 0 \tag{3.6}$$

where p_1, p_2 are defined above. This is the equation for the Rayleigh wave speed $v_R \equiv (is/k)^{1/2}$ which is found as a real root of (3.6) (see e.g.[3]). For the case $\lambda = \mu$ (Poissons ratio = 0.25), $v_R = 0.919\, v_s$.

For the anisotropic case a computational scheme to calculate the zeroes of (3.5) has been written, (Figure 2).

b) Branch points: The branch points of the integrand in (3.3) are the same as those of the functions $p_1(k_1,s)$, $p_2(k_1,s)$, they are:

i) $p_1 = 0$, or $p_2 = 0$ which implies $(C_{11} k_1^2 + \rho s^2)(C_{55} k^2 + \rho s^2) = 0$
i.e., $k/s = \pm i\sqrt{\rho/C_{55}}$, $\pm i\sqrt{\rho/C_{11}}$ pure imaginary.

These correspond to longitudinal and shear wave speeds for an isotropic material

$$\text{ii)} \quad p_{1,2} = \pm \sqrt{-B \pm \sqrt{B^2 - 4AC} / 2A} \tag{3.7}$$

with $A = C_{33} C_{55} > 0$

$$B = -\rho s^2(C_{33}+C_{55}) + k^2[(C_{13}+C_{55})^2 - C_{11}C_{33}-C_{55}^2]$$

$$C = (C_{11}k^2+\rho s^2)\ (C_{55}k^2+\rho s^2)$$

These branch points are those values of s/k which render $B - 4AC = 0$, and are branch points of second order.[4] The distribution of these points is shown in Figure 3. It has been shown [5] that the contribution of these branch points to the value of the integrals (3.3) is important only when one considers the multi-reflected and refracted waves in layered media, or when the position of interest is very close to the impact origin. In this study we were more concerned with how the energy is propagated away from the impact point, which is mainly associated with surface waves, thus we ignored the contribution of these branch points.[5]

The contour of integration for the Laplace inversion is as shown in Figure 3. Notice the branch cuts are extended to negative infinity, in accordance with the requirement that $C >$ max real part of the singularities. The requirement that real $(p) \geq 0$ also determines the correct sheet of the Riemann surfaces. Numerically, since the branch points are located at $s/k =$ constant, as k gets large C should be large, and the factor e^{ct} in the Laplace inversion expression will rise sharply to an unmanageable size. Since, in the last paragraph, we have noticed the contribution of the branch points is unimportant, a path Γ_2 is chosen to replace Γ_1 by the Cauchy's integral theorem. Notice the advantage of integration along Γ_2 is that C_o is a positive constant independent of k. The determination of optimal C_o is discussed in [2]. Here, in order to minimize the aliasing and round off errors in numerical computations simultaneously, we choose

$$C_o = \frac{2}{3M\lambda_t} \quad \ell n\ (\hat{g}/\hat{r}) \tag{3.9}$$

where $\hat{g} = p_o/C_{66}$, $\hat{r} = 10^{-6} \times \frac{1}{\lambda_t}$. λ_t is a small time interval, less than the impact contact time. Further details are given in Appendix A.

IV. RESULTS

A computer program has been written to calculate the stresses in a plate with an elastic beam on one edge under a transient impact load distribution along the edge. A program description, flow charts, input data formats, and sample printout of the program are contained in the appendices to this report. In this section we will summarize some of the results obtained from this computer program. These results were calculated for an anisotropic plate with effective elastic constants of 55% graphite fiber/epoxy matrix composite obtained from Reference 7 and summarized in the Table.

<u>No-strip case:</u>

The Rayleigh wave can be seen in the stress t_{11} on the edge as shown in Figures 4, 5 for a graphite fiber-epoxy composite for layup angles 0, $\pm$ 15°. After the initial contact time, the stress is observed to propagate with little change at a speed near the calculated Rayleigh speed (Figure 2). This wave can be observed in the computed output in the space-time (x_1, -t) plane Figure 7, as a band of non-zero values along a diagonal from the upper left to the lower right corner of the x_1, -t plane. Caution is urged in using this program since spurious waves can enter the calculations due to the periodic nature of the finite numerical Fourier Transform. These spurious waves are data bands which lie along diagonals from upper right to lower left. In other words, only disturbances emanating from the impact source in the upper left corner of the x_1 -t plane of Figure 7 should be valid. A computer map of the space time history of the edge impact stress is shown in Figure 6. A contact time of 35 μsec and contact length 2 cm was used in these calculations.

As is characteristic of surface wave effects, the stresses due to impulsive loading on the edge decrease with distance from the edge. This is shown in Figure 8 for two different layup angles. The normal stress t_{33} appears to decrease to about 1/4 of its value on the surface at a depth equal to one half of the loading length a. The rate of decay from the edge depends on the layup angle. Another characteristic of edge impact is the development of tension in the normal stress t_{33} under the impact point. This is shown in Figure 9. Thus while the compression part decreases with distance from the edge a tension tail developes in the wave.

The effect of layup angle on the impact stresses can be seen in Figures 10, 11. The stress t_{11} at the edge is larger than the impact pressure and decreases as the layup angle goes from 0° to $\pm 45^{\circ}$ (Figure 10).

Below the surface or edge, the peak normal stress t_{33} at $x_1 = 0$ is a minimum for layup angles near $\pm 30^{\circ}$, while the shear stress t_{13} increases as the fiber angle goes from 0° to $\pm 45^{\circ}$ (Figure 10).

An unexpected result is the shift of the maximum normal stress t_{33} to points off the impact axis $x_1 = 0$ for layup angles greater than about $\pm$ 30. A pronounced peak in t_{33} versus x_1/ beyond the impact pressure foot print, can be seen in Figure 11 for $\pm 45^{\circ}$ layup angle.

Impact protection strip case:

The effect of bonding an edge impact protection strip to the half plane is shown in Figures 12-16. In Figures 12, 13, the increase in the thickness of a steel strip produces a decrease in the interface stresses

t_{11}, t_{33} but creates an interface shear stress at the strip-composite interface. This shear stress reaches a maximum for strip thickness less than the impact footprint length and decreases for greater strip thicknesses. Thus, if the strip is too thin, debonding can occur under impact due to induced interface shear.

In Figure 14 one can see that increasing the strip thickness decreases the peak normal stress t_{33} and redistributes the load over a longer length under the strip. However, while the peak compression stress is decreased by the strip, tension is created which could also produce debonding of the strip from the composite.

For the no-strip case a Rayleigh wave was seen to propagate along the edge relatively unperturbed (Figures 4, 5). With the strip present, (Fig.15) this wave becomes distorted as time increases. In fact, the beam-strip boundary conditions introduce dispersion in the edge waves which make the Rayleigh edge wave velocity dependent in the effective wave length of the disturbance.

Finally in Figure16 shear stress distributions along the strip-composite interface are shown for a thickness near the shear peak ($b/a = 0.25$) and another for $b/a = 2.0$. In the latter case the shear is distributed over a larger length resulting in a lower peak stress. Also the strip delays the time of maximum shear from 1/2 to 3/4 $T_{\hat{0}}$

Summary of Results

An analytical-numerical method has been developed to solve the response of a composite plate with a bonded edge strip to in-plane impact type forces on the edge. Results of computer simulations reveal the following:

1) Rayleigh edge waves can propagate away from the impact site with tension and compression up to values of the impact pressure, depending on layup angle.
2) Normal to the edge, the initial peak compression pulse decreases as it propagates into the plate but a tension tail develops as it propagates away from the impact site.
3) The edge stress t_{11} under impact is decreased as the fiber layup angle goes from 0° to $\pm 45^{\circ}$.
4) Protection strips of thickness less than half the impact length can develop large interface shear under impact.
5) The normal and edge stresses t_{33}, t_{11} at the edge can be decreased significantly by protection strips of thickness greater than the half impact length.

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TABLE I. - STRESS-STRAIN COEFFICIENTS FOR 55 PERCENT GRAPHITE FIBER-EPOXY MATRIX COMPOSITE

[All constants to be multiplied by 10^6 psi; data obtained from ref. 7.]

0° Layup

$$\begin{vmatrix} 27.95 & 0.3957 & 0.3957 & 0 & 0 & 0 \\ & 1.170 & 0.4601 & 0 & 0 & 0 \\ & & 1.170 & 0 & 0 & 0 \\ & & & 0.3552 & 0 & 0 \\ & & & & 0.7197 & 0 \\ & & & & & 0.3552 \end{vmatrix}$$

±15° Layup

$$\begin{vmatrix} 24.56 & 0.4000 & 1.986 & 0 & 0 & 0 \\ & 1.170 & 0.4558 & 0 & 0 & 0 \\ & & 1.374 & 0 & 0 & 0 \\ & & & 0.3552 & 0 & 0 \\ & & & & 2.310 & 0 \\ & & & & & 0.3552 \end{vmatrix}$$

±30° Layup

$$\begin{vmatrix} 16.48 & 0.4118 & 5.167 & 0 & 0 & 0 \\ & 1.170 & 0.4400 & 0 & 0 & 0 \\ & & 3.093 & 0 & 0 & 0 \\ & & & 0.3552 & 0 & 0 \\ & & & & 5.491 & 0 \\ & & & & & 0.3552 \end{vmatrix}$$

±45° Layup

$$\begin{vmatrix} 8.197 & 0.4279 & 6.758 & 0 & 0 & 0 \\ & 1.170 & 0.4279 & 0 & 0 & 0 \\ & & 8.179 & 0 & 0 & 0 \\ & & & 0.3552 & 0 & 0 \\ & & & & 7.082 & 0 \\ & & & & & 0.3552 \end{vmatrix}$$

APPENDIX A

The Determination of Parameter C_o

Consider the Laplace inversion of a function $g(t)$ as

$$g(t) = \frac{1}{2\pi i} \int_{C_o - i\infty}^{C_o + i\infty} \bar{g}(s)\, e^{st}\, ds \tag{A.1}$$

and let $s = C_o + i\omega$, C_o and ω are real. We can change (A.1) into the Fourier inversion formula

$$g(t) = \frac{1}{2\pi}\, e^{C_o t} \int_{-\infty}^{\infty} \bar{g}(C_o + i\omega)\, e^{i\omega t}\, d\omega \equiv e^{C_o t}\, \mathbf{x}(t) \tag{A.2}$$

which is then inverted by the Fast Fourier Transform technique. In the FFT scheme, a continuous function $\bar{g}(C_o + i\omega)$ is discretized and the infinite interval of integration is truncated. The error due to truncation depends on each problem but doesn't depend on C_o, thus in the determination of C_o, we will assume the truncation error is negligible. It is shown [2] that the discretizing of the transform in one domain will cause aliasing error in the other domain, e.g. sampling $\bar{g}$ at N points in a frequency interval, $0 < \infty < \Omega$, will produce a transformed function $x_p(t)$ which is periodic and which differs substantially from $x(t)$ for large enough t. For even $x(t)$ this difference can be shown [2] to be given by,

$$x_p(t) \simeq x(t) + x(t-T)$$

for $0 < t < T/2$ where $T = N/\Omega$.

It has been shown [2] also that the aliasing error is approximated by $E_a(t) = e^{-C_o(T-2t)} g(T-t)$ for the Laplace inversion (A.2).

Notice the aliasing error is a decreasing function of C_o.

The other source of error is of course the round off error in computation. Since we multiply the resulting $x(t)$ by $e^{C_o t}$ to get $g(t)$, the rounding error is of the form

$$Er(t) = e^{C_o T} r(t).$$

The error bounds are then

$$\left.\begin{aligned} \varepsilon_1 &= |\text{Max } Ea(t)| = e^{-C_o(T-2\tau)} \text{ Max}|g(T-t)| \\ \varepsilon_2 &= |\text{Max } Er(t)| = e^{C_o\tau} \text{ Max}|r(t)| \end{aligned}\right\} \quad 0 \leq t \leq \tau$$

Equating ε_1 and ε_2, the optimal C_o is then

$$C_o = \frac{\ell n\ (\text{Max } g(T-t)/\text{Max } r(t))}{T-\tau}$$

Chosing $\tau = T/4$, therefore

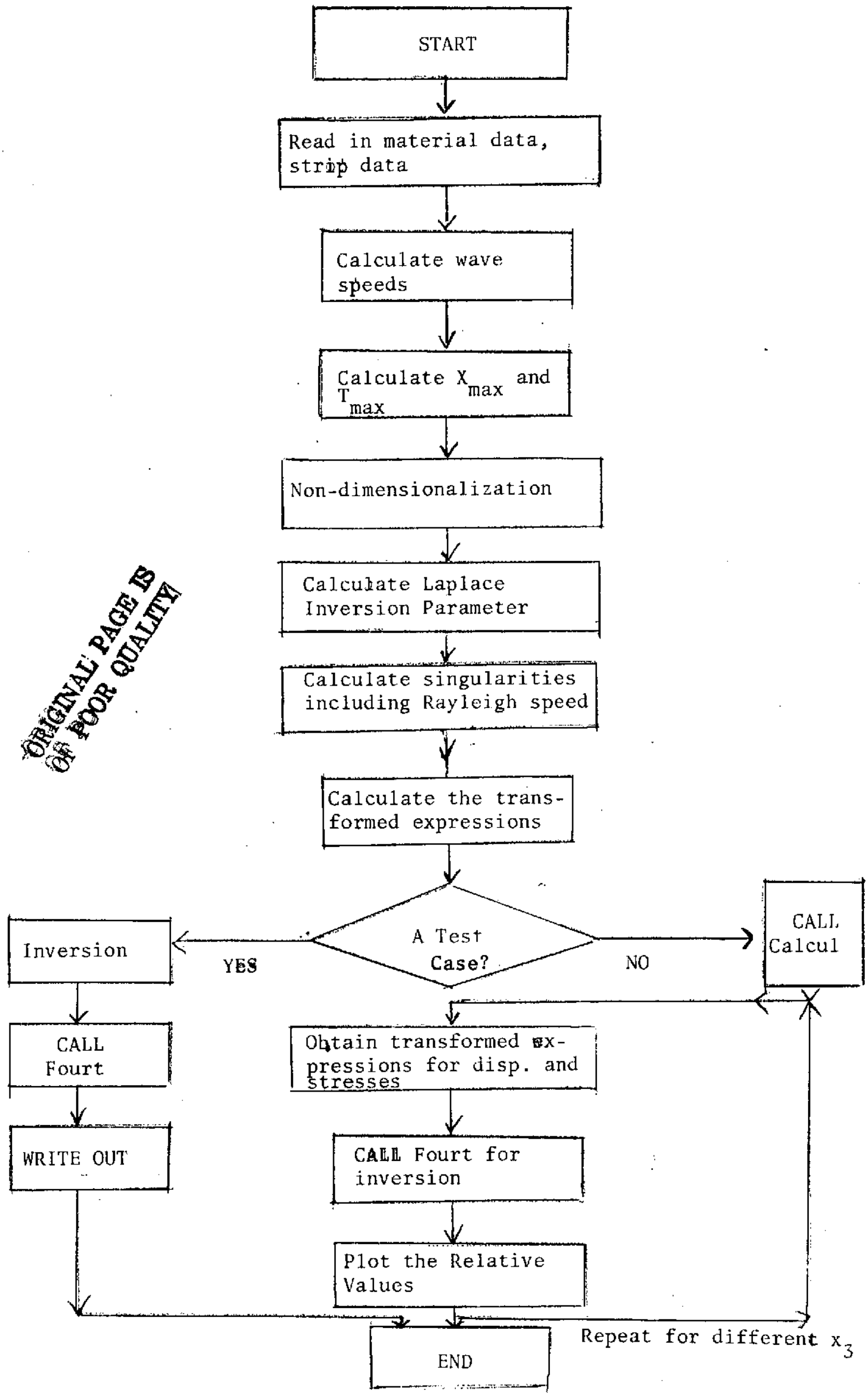
$$C_o = \frac{4}{3T} \ell n(\hat{g}/\hat{r}), \qquad \hat{g} \equiv \text{Max } g(T-t), \quad \hat{r} \equiv \text{Max } r(t)$$

Notice, empirically, $\hat{r} \approx 2 \frac{N}{T} 10^{-6}$ on single precision IBM 360 systems.

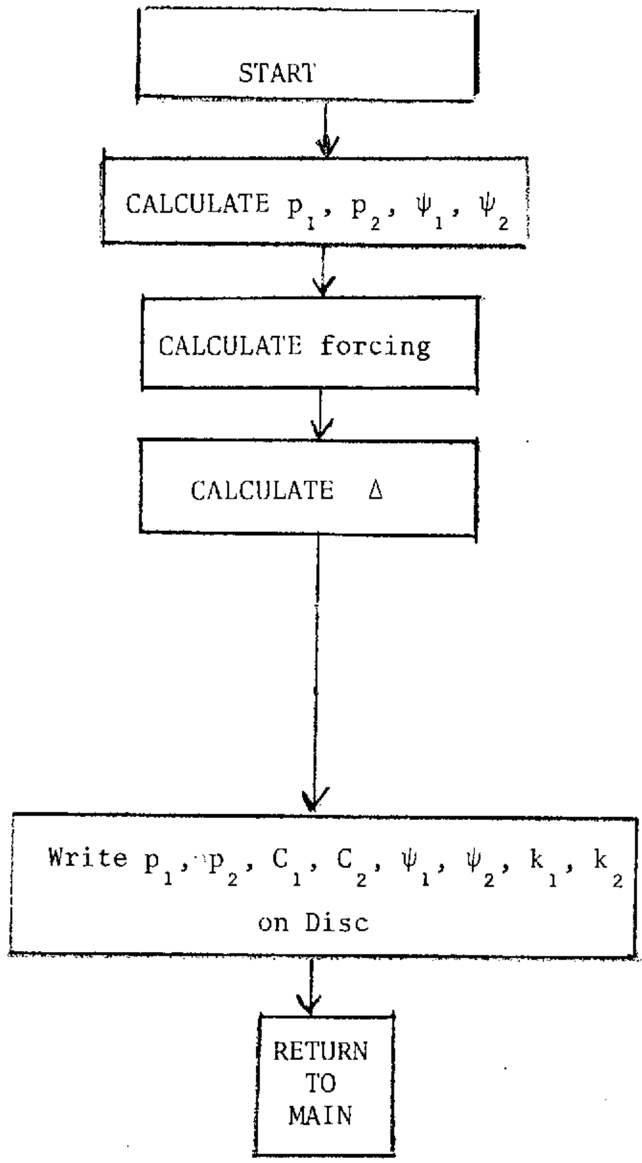
APPENDIX B

A FLOW CHART OF THE PROGRAM

1. Main Program



3. Subroutine calcul



APPENDIX C

NOTES ON COMPUTER PROGRAM

The choice of scales is very essential to the success of the present computational method. It is noticed that the accuracy depends on the number of points employed and the range of frequency spectra covered. Considering limitations of both computer storage and time, a time-space grid of 32 x 64 points was chosen for $t > 0$, $x_1 > 0$. Thus, the non-dimensionalization of all equations and quantities are both necessary and important to the obtaining of meaningful data from the limited grid size.

The numerical inaccuracies introduced have several origins:

1. Theoretically, error has been introduced by the neglecting of the outer branch points contributions. It has been shown, for isotropic cases, the contributions of these branch points behaved like r^{-2} at large distance from a delta function loading at the origin $r = 0$. Compared with the $r^{-\frac{1}{2}}$ decreases of the contribution of the residue. Thus, for small r, or near the origin, the errors might be significant. An asymptotic form of the behavior of small r has been deduced for a simple delta loading at origin on an isotropic half-space [ ]. It is shown the error thus introduced is of the order of 5% maximum response in stress.
2. The aliasing error introduced through the periodizing of the functions.
3. The round-off error in Laplace inversion along with aliasing error. have been discussed in the determination of C_o (Appendix A). It is

found that error 3 is more serious of the two. Hence in calculations the maximum non-dimensionalized time should be restricted to below 6 or 8 for reasonably good results.

4. Errors due to reflections at the boundary. Since the space grid is finite, it has been observed that whenever a wave hit the boundary of chosen space, a sizable numerical error will start propagating in, as if the wave were reflected from the boundary. The basic reason is due to the periodization of the space (x_1) domain. In computation, this error should be avoided. To correctly determine the extent of the space (x_1) domain, a priori recognization of significant wave speed (at which most of the energy travels) is important. Usually an estimation will be sufficient. Then the proper nondimensionalizing constants can be chosen.

APPENDIX D
PROGRAM INPUT AND OUTPUT

A. Method:

FFT alogarithm for numerical inversion

B. Input Data Cards:

Card 1. NTEST, NSTRIP, NP (3I5)

NTEST = 1 A test program for FFT (2-D) (beam).

0 The present program

NSTRIP = 1 With strip

0 No strip

NP = 1 Calculate: u_1 only

2 u_1, u_3

3 u_1, u_3, t_{33}

4 u_1, u_3, t_{33},, t_{11}

5 u_1, u_3, t_{33}, t_{11}, t_{13}

Card 2. CC(I) I = 1,9, RHO, ANGLE (8E10,4)

CC(I=1,9) Material constants of composite, in the order

C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{12}, C_{13}, C_{23}. (psi)

RHO Density of composite (g/cm^3)

ANGLE (degrees) lay-up angle

Card 3. VEL , DM, E1, ANU, DEN (8E10.4)

VEL - Velocity of incoming particle m/sec

DM - Diameter of impacting object cm

E1 - Youngs Modulus of impacting object (psi)

ANU - Poissons ratio of impacting object

DEN - Density of impacting object (gm/cc)

Card 4. NSTRES, NK3, DX3 (2I5, F10,3)

NSTRES = 1

NK3: Number of steps in x_3 direction

DX3: $\Delta x_3/\ell$, step size in x_3 direction

Photo elastic fringe order computer map in the x-t plane.

Card 5. RO, W, ES (3E10,4)

RO: density of strip beam (g/cm^3)

W: depth of strip beam (cm)

ES: Young's modulus of strip beam (psi)

C. Output:

1. Test problem: Appendis III

2. Values of displacements 1 : u_1

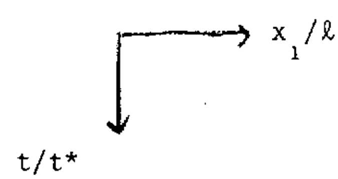
   2 : u_3

   (stress) 3 : t_{33}

   4 : t_{13}

   5 : t_{11}

3. Relative magnitude computer maps of displacements and stresses.



```
C********  PROGRAM TO CALCULATE STRESSES DUE TO EDGE IMPACT OF A PLATE*
C
C            THIS PROGRAM CALCULATES THE ELASTIC RESPONSE OF  AN
C          ANISOTROPIC PLATE TO AN IN-PLANE EDGE IMPACT FORCE ON X3=0.0
C          WHEN NSTRIP =1 , THE PROGRAM PLACES AN IMPACT PROTECTION STRIP
C          ,OR ELASTIC BEAM ON THE EDGE. THE IMPACT FORCE IS A HALF SINE
C          FUNCTION IN TIME AND IS NON ZERO FOR 0<T<T0,(MICROSEC). THE
C          FORCE IS DISTRIBUTED ALONG THE EDGE AS  P0(1-X1**2), WHERE
C          X1 IS NORMALIZED BY THE HALF WIDTH OF THE IMPACT CONTACT
C          LENGTH.
C            THE METHOD EMPLOYS A FOURIER TRANSFORM IN THE EDGE DIRECTION
C          X1,AND A LAPLACE TRANSFORM IN THE TIME DIMENSION. THE TRANSFORM
C          OF THE FORCING FUNCTION IS GIVEN IN THE SUBROUTINE CALCUL AND
C          THE SOLUTION IS OBTAINED USING A 2-DIMENSIONAL FAST FOURIER
C          INVERSION ROUTINE CALLED 'FOURT'.
C            THE OUTPUT FOR A GIVEN DEPTH X3 CONSISTS OF DISPLACEMENTS
C          U1,U3,AND STRESSES T33,T11,T13, IN THE X1-TIME PLANE .
C          THE STRESSES ARE NORMALIZED BY THE ELASTIC CONSTANT C(6,6).
C
C                                     IMPACT FORCE
C                                           I
C                                           I
C                                           I
C                                           I
C                                        V  I  V
C                                         V I V
C                                          VIV
C                                           V
C
C                   STRIP >>>  * * * * * * * * * * * * * * *  <<< STRIP
C
C                              C                             C ----> X1
C                              C                             C |
C                              C          COMPOSITE          C |
C                              C                             C |
C                              C          HALF SPACE         C |
C                              C                             C V
C                              C                             C X3
C
C          INPUT DATA
C          NN(1)--2.0* MAX X1 DISTANCE,  NN(2)--MAX NO. OF TIME UNITS
C          NSTRIP=0,NO STRIP,..NSTRIP=1,WITH STRIP
C          NP=1,--,5,  CALCULATES U1,U3,T33,T11,T13,IN THAT ORDER
C          NP=6, CALCULATES DISPL. OF A BEAM ON AN ELASTIC FOUNDATION
C          CC(9),  ELASTIC CONSTANTS OF PLATE IN THE ORDER
C                             C11,C22,C33,C44,C55,C66,C23,C13,C12,IN PSI
C          RHO,DENSITY OF PLATE IN UNITS GM/CC
C          ANGLE--LAYUP ANGLE OF COMPOSITE PLATE,DEG.,FOR INFO ONLY
C          VEL, VELOCITY OF INCOMING OBJECT METERS/SEC
C          DM,  DIAMETER OF IMPACTING OBJECT -CM.
C          E1,ANU, YOUNG'S MODULUS AND POISSON'S RATIO FOR IMPACTING BODY
C          DEN,  DENSITY OF IMPACTING BODY
C          NSTRESS=1
C          NK3,  NO.OF DEPTHS X3,(FOR NK3=1,X3=0)
C          DX3, INCREMENT IN DEPTH X3(NORMALIZED BY A0)
C          RO,  DENSITY OF PROTECTIVE STRIP GM/CC
```

* This program has two extra cards to override the Hertz contact time and contact length calculation. Remove cards #51, 52.

```
C        W, THICKNESS TO WIDTH RATIO OF BEAM
C        ES, YOUNG'S MODULUS FOR BEAM
C        WL--FOURIER WAVELENGTH (CM) OF THE ORDER OF A0 OR LESS
C        WT--FOURIER WAVE PERIOD (SEC) OF THE ORDER OF T0 OR LESS
C            CHOICE OF WL,WT DETERMINES DX,DT--DX=WL/2,DT=WT/2
C        FG,  TRANSFORM OF NORMALIZED FORCING FUNCTION F(X1)*G(T)
C             THIS IS PROVIDED IN PROGRAM BUT CAN BE CHANGED BY THE USER
C
C        OUTPUT DATA
C        T0,TC CONTACT TIME (SEC,1.E-6 SEC  ) (FROM HERTZ THEORY)
C        A0,A,  HALF THE IMPACT CONTACT LENGTH ,CM
C        F0,  MAX IMPACT FORCE FROM HERTZ THEORY ,NEWTONS
C        E,   WAVE SPEED IN PLATE SQRT(C66/RHO)
C          OR WAVE SPEED IND BEAM STRIP,SQRT(E1/DEN) UNITS CM/SEC
C        CL=SQRT(C11/RHO) LONGITUDINAL WAVE SPEED ALONG EDGE IN PLATE
C        CS=SQRT(C55/RHO),  SHEAR SPEED ALONG EDGE OF PLATE,CM/SEC
C        CR, RAYLEIGH WAVE SPEED ALONG FREE EDGE OF PLATE
C
C        DX,DT  SPACE TIME INCREMENTS IN X1-T SPACE UNITS--CM AND SEC
C        DATA(I,J) ,NORMALIZED TRANSFORM OF ONE OF DISPL. OR STRESSES
C         --BEFORE CALL FOURT,AFTER CALL FOURT DATA IS A 2 DIM MATRIX OF
C            DISPL. OR STRESSES IN X1-T SPACE,DEPENDING ON VALUE OF  K
C            IN THE LOOP 'DO 4 K=1,NP'
C        U1,U3,NORMALIZED DISPL. IN PLANE OF PLATE(E.G. U1/A0)
C        T33,T11,T13,  NORM. STRESSES (E.G. T33/C66)
C            --NOTE-- T33 ON X3=0 SHOULD REPRODUCE THE FORCING FUNCTION
C                     WHEN THERE IS NO STRIP
C            --NOTE-- AS A CHECK T13=0 ON X3=0 WHEN THERE IS NO STRIP
C        THE FRINGE ORDER MAP PLOTS THE DIFF IN PRINCIPAL STRESSES AND
C          MAY BE USED TO COMPARE WITH PHOTOELASTIC EXPERIMENTS OR TO
C            LOOK FOR POINTS OF MAX IN PLANE SHEAR STRESSES
C        ********************
C        THIS PROGRAM HAS BEEN WRITTEN BY F.MOON AND C-K KANG UNDER
C          A GRANT TO PRINCETON UNIVERSITY FROM THE NASA LEWIS RESEARCH
C          LAB.
C        ********************
C        --NOTE TO THE USER-- IN THE OUTPUT MAPS OF STRESSES OR DISPL.,
C        YOU WILL NOTICE BANDS OF SIMILAR NUMBERS RUNNING FROM THE
C        UPPER LEFT CORNER TO THE LOWER RIGHTCORNER -- THESE ARE WAVES
C        WHICH EMINATE FROM THE IMPACT POINT--HOWEVER- WAVES RUNNING
C        FROM RIGHT UPPER TO LEFT LOWER CORNER ARE SPURIOUS DUE TO THE
C        DESCRETENESS OF THE NUMERICAL FOURIER INVERSION PROGRAM
C        --ALSO DATA FOR TIMES NEAR TMAX AT THE BOTTOM OF THE MAPS ARE
C        USUALLY SPURIOUS AND SHOULD NOT BE USED
C
      COMMON /MMC/ D,DK1
      COMMON /MC/DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRIP,A0
      DIMENSION DATA(128,32),MM(40),CC(9)
      DIMENSION NN(2)
      DIMENSION RDATA(40)
      DIMENSION CIDATA(40)
      DIMENSION FRNGE(64,32)
      DIMENSION T11(64,32),T33(64,32),T13(64,32)
      COMPLEX DATA,S
      COMPLEX P1,P2,S1,S2,C1,C2,B,C,D,SI
      COMPLEX DK2,FG,SLAP,DK1
```

```
      CCMPLEX*16 BS
      NN(1)=128
      NN(2)=32
C***  READ IN AND WRITE OUT DATA AND PARAMETERS
C***  RHO,A0,T0 MUST BE IN C.G.S. UNITS, P0 MUST BE CONSISTENT WITH CC
      CALL INDUMP
      READ (5,102) NTEST,NSTRIP,NP
      READ (5,100) CC,RHO,ANGLE
      WRITE(6,490) CC
      WRITE(6,491) RHO,ANGLE
      READ(5,100) VEL,DM,E1,ANU,DEN
      P0=CC(6)
      C11= CC(1)-CC(7)**2/CC(2)
          C33= CC(3)-CC(9)**2/CC(2)
      C13= CC(8)-CC(7)*CC(9)/CC(2)
      C55= CC(5)
      PI= 3.14159265
      RHO= RHO/6.895*1.E-4
      READ (5,101) NSTRES,NK3,DX3
        WRITE (6,504) NSTRES,NK3,DX3
        SI= CMPLX(0.,1.)
C*********  CALCULATE THE IMPACT CONTACT TIME ,RADIUS,AND PRESSURE
C*********  BASED ON HERTZ CONTACT THEORY
      R=DM/2.0
      R=R*1.E-2
      E1=E1*6895.0
      E2=CC(2)*6895.0
      DEN=DEN*1.0E3
      AMASS=4./3.*PI*R**3*DEN
      AK2=4./3.*SQRT(R)*E1/((1.0-ANU*ANU)+E1/E2)
      ALF=5./4.*AMASS*VEL*VEL/AK2
      ALF=(ALF)**0.4
      TC=2.943*ALF/VEL
      F0=1.14*AMASS*VEL*VEL/ALF
      A=SQRT(ALF*R)
      A=1.0E2*A
      TC=TC*1.0E6
      WRITE(6,710) VEL,TC,A,F0
C***************************************
C***  CALCULATE THE NON-DIMENSIONAL PARAMETERS
      EE= CC(6)
      E= SQRT(EE/RHO)
      RE= RHO
C        DEFINE TRANSFORM SPACE AND DISTANCE-TIME SPACE
C***  UNIT DISTANCE   --CM.
      A0=A
      T0=TC*1.E-6
C                    TEST CASE T0== 35E-6, A0=1 CM
      A0=1.0
      T0=35.E-6
      AL=A0
C***  UNIT TIME--SEC.
      TE=AL/E
C***  SMALLEST WAVELENGTHS
      WL=A0/1.5
      WT=T0/10.0
```

```
C***  DIMENSIONS OF TRANSFORM SPACE
      XK=2.0*PI*AL/WL
      TK=2.0*PI*TE/WT
C***  NONDIMENSIONALIZE CONSTANTS
      A0=A0/AL
      T0=T0/TE
      DT=WT/2.0
      DX=WL/2.0
      WRITE(6,621) WT,WL,TK,XK
C**************************************
C        CALCULATE RAYLEIGH WAVE SPEED
      X1=0.0
      X2=C55
      N=0
      DO 90 I=1,100
      N=N+1
      X=(X1+X2)/2.0
      D1=(C55-X)/(C11-X)
      D1=SQRT(D1)
      D2=(C13)**2-C33*(C11-X)
      D3=C55*C33
      D3=SQRT(D3)
      F=D1*D2/D3
      F=X+F
      IF(F) 81,82,83
   81 X1=X
      GO TO 92
   82 GO TO 91
   83 X2=X
   92 CONTINUE
      F1=F/C55
      DIFF=ABS(F1)-1.0E-4
      IF(DIFF) 91,91,90
   90 CONTINUE
   91 CONTINUE
      WRITE(6,701) C11,C33,C55,C13
  701 FORMAT(//,10X,6H C11 =,E12.4,6H C33 =,E12.4,6H C55 =,E12.4,
     16H C13 =,E12.4,/)
      CL=C11/RHO
      CL=SQRT(CL)
      CS=C55/RHO
      CS=SQRT(CS)
      CR=X/RHO
      CR=SQRT(CR)
      RDS=CR/CS
      WRITE(6,702) CL,CS,CR,RDS,N
  702 FORMAT(//,10X,12HLONG.SPEED =,E12.4,13HSHEAR SPEED =,E12.4
     1 //,10X,16HRAYLEIGH SPEED =,E12.4,8H CR/CS =,F10.5,5X,I5)
C**************************************
C***  CONSTANTS FOR STRIP CASE
      READ (5,100) RO,W,ES
      WRITE(6,521) RO,W,ES
      RO= RO/6.895*1.E-4
      IF (RO) 51,51,50
   50 CONTINUE
      E= SQRT(ES/RO)
```

```
      WRITE (6,510) E
   51 CONTINUE
      RO= RO/RE
      ES= ES/EE
      W= W/AO/AL
C*****************************************
  205 CONTINUE
      C11= C11/EE
      C13= C13/EE
      C33= C33/EE
      C55= C55/EE
      PO= PO/EE
      RHO= RHO/RE
      FG1= PO*TO
  206 CONTINUE
      N= NN(1)
      M= NN(2)
C***  CALCULATE THE LAPLACE INVERSION PARAMETER
      WT=WT/TE
      RH=  PO*WT*1.E6
      CH= 2./3./M/WT*ALOG(RH)
      CLAP= CH
C***  CALCULATE THE SECOND ORDER BRANCH POINTS
      A= RHO*(C33-C55)**2*RHO
      E= -2.*RHO*((C33+C55)*(C13*C13+2.*C13*C55-C11*C33)+2.*C33*C55
     1*(C11+C55))
      F= (C13*C13+2.*C13*C55-C11*C33)**2-4.*C11*C33*C55*C55
      BS= 1.D0*E*E-1.D0*4.*A*F
      D= CDSQRT(BS)
      P1= .5/A*(-E+D)
      P2= F/P1/A
      P1= CSQRT(P1)
      P2= CSQRT(P2)
      WRITE (6,509)
      WRITE (6,506) P1,P2
  204 WRITE (6,508) CLAP
      N2= N/2+1
      M2= M/2+1
      IF (NTEST.EQ.1) GO TO 211
  211 IF (NSTRIP.EQ.1) WRITE (6,514)
C***  GENERATE THE  TRANSFORMED EXPRESSIONS
      REWIND 2
      DO 1 I= 1,N
      DK1= 2.*XK/N*(I-.5)-XK
      DO 1 J= 1,M
      DK2F=2.*TK/M*(J-.5)-TK
      SLAP= CMPLX(CLAP,DK2F)
      DK2= -SI*SLAP
      IF (NTEST.NE.1) GO TO 201
      FG= PI*(1.+CEXP(-SLAP))/(SLAP*SLAP+PI*PI)*
     1CSIN(DK1)/DK1*(1.+DK1*DK1/(PI*PI-DK1*DK1))
C***  TEST FOR A STRING
      DATA(I,J)=-FG/(SLAP*SLAP+DK1*DK1)
      GO TO 1
  201 CALL CALCUL(0)
    1 CONTINUE
```

```
                IF (NTEST.NE.1) GO TO 202
                WRITE (6,507)
                CALL FOURT(DATA,NN,2,1,1,0)
                DO 15 KI= 1,N2
                B= -SI*PI/N*(N-1)*(KI-1)
                DO 15 KJ= 1,M
                C= -SI*PI/M*(M-1)*(KJ-1)
                T= PI/TK*(KJ-1)
                DATA(KI,KJ)= DATA(KI,KJ)*XK*TK/PI/PI/N/M*CEXP(B)*CEXP(C)
             15 DATA(KI,KJ)= DATA(KI,KJ)*EXP(CLAP*T)
                WRITE (6,506) ((DATA(I,J), J= 1,M), I= 1,N2)
                GO TO 203
          C****************************
          C***  MAIN LOOP FOR DIFFERENT DEPTHS X3
            202 DO 6 K3= 1,NK3
                X3= (K3-1)*DX3
          C***  LOOP FOR CALCULATING DISPLACEMENTS AND STRESSES AT A GIVEN DEPTH
                DO 4 K= 1,NP
                WRITE(6,680)
                WRITE (6,501) X3
                REWIND 2
                DO 7 I= 1,N
                DO 7 J= 1,M
                READ (2) P1,P2,C1,C2,S1,S2,DK1,DK2
                B= CEXP(-X3*P1)
                C= CEXP(-X3*P2)
          C***  DISPLACEMENTS
                IF (K.EQ.1) DATA(I,J)= C1*B+C2*C
                IF (K.EQ.2) DATA(I,J)= C1*S1*B+C2*S2*C
          C***  STRESSES
                IF (K.EQ.3)
               1DATA(I,J)= (C13*SI*DK1-C33*P1*S1)*C1*B+(C13*SI*DK1-C33*P2*S2)*C2*C
                IF (K.EQ.4)
               1DATA(I,J)= (C11*SI*DK1-C13*P1*S1)*C1*B+(C11*SI*DK1-C13*P2*S2)*C2*C
                IF (K.EQ.5)
               1DATA(I,J)= C55*((-P1+SI*DK1*S1)*C1*B+(-P2+SI*DK1*S2)*C2*C)
                IF (K.LT.6) GO TO 7
          C***  DISPL. FOR A BEAM ON AN ELASTIC FOUNDATION
                EF=0.1
          C***  FORCING FUNCTION
                FG= PI*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI-DK2*DK2*T0*T0)*
               14./(DK1*A0)**2*(CSIN(DK1*A0)/DK1-A0*CCOS(DK1*A0))
                D=W*DK1*DK1/12.0*(ES*W*W*DK1*DK1-RO*W*W*DK2*DK2)-RO*W*DK2*DK2
               1 +EF
                IF (K.EQ.6)
               1DATA(I,J)=FG/D
              7 CONTINUE
                CALL FOURT(DATA,NN,2,1,1,0)
                DO 3 KI= 1,N2
                B= -SI*PI/N*(N-1)*(KI-1)
                DO 3 KJ= 1,M
                C= -SI*PI/M*(M-1)*(KJ-1)
                T= PI/TK*(KJ-1)
                DATA(KI,KJ)=    DATA(KI,KJ)*XK*TK/PI/PI/M/N*CEXP(B)*CEXP(C)
                E= CLAP*T
              3 DATA(KI,KJ)= DATA(KI,KJ)*EXP(E)
```

```
C***  CALCULATE THE PHOTOELASTIC FRINGE ORDER
      DO 290 I=1,40
      DO 290 J=1,32
      IF (K.EQ.3) T33(I,J)=REAL(DATA(I,J))
      IF (K.EQ.4) T11(I,J)=REAL(DATA(I,J))
      IF (K.EQ.5) T13(I,J)=REAL(DATA(I,J))
      IF (K.EQ.5)  GO TO 280
      GO TO 290
  280 CONTINUE
      FNGE=(T11(I,J)-T33(I,J))**2+4.0*T13(I,J)*T13(I,J)
      FRNGE(I,J)=SQRT(FNGE)
  290 CONTINUE
  214 WRITE (6,511) K
C***  FIND THE MAXIMUM VALUE
      RS= 1.E-3
      DO 14 I=1,40
      DO 14 J= 1,32
      S= DATA(I,J)
      TP= REAL(S)/RS
      IF (ABS(TP).LT.1.) GO TO 14
      RS= REAL(S)
   14 CONTINUE
      WRITE (6,516) RS
C                            PRINT REAL PART OF DISPL. AND STRESSES
      NIJ=N/4
      NIJ=10
      DO 310 I=1,NIJ
      I1=I-1
      WRITE(6,600) I1
      DO 300 J=1,32
      SR=REAL(DATA(I,J))/RS
      SIMG=AIMAG(DATA(I,J))/RS
      RDATA(J)=SR
      CIDATA(J)=SIMG
  300 CONTINUE
      WRITE(6,620)  (RDATA(L),L=1,M)
      WRITE(6,650)
      WRITE(6,620)  (CIDATA(L),L=1,11)
  310 CONTINUE
      WRITE(6,680)
      IF (K.EQ.1) WRITE(6,630)
      IF (K.EQ.2) WRITE(6,631)
      IF (K.EQ.3) WRITE(6,632)
      IF (K.EQ.4) WRITE(6,633)
      IF (K.EQ.5) WRITE(6,634)
      IF (K.EQ.6) WRITE(6,635)
      WRITE(6,640) DX,DT
C***  PLOT THE RELATIVE VALUES
      DO 12 J= 1,M
      DO 13 I=1,40
      S= DATA(I,J)
   13 MM(I)= REAL(S)/RS*100
      WRITE (6,515) MM
   12 CONTINUE
C***  PLOT A MAP OF PHOTOELASTIC FRINGE ORDER
      IF (K.LT.5) GO TO 4
```

```
          RS=1.E-3
          DO 312 I=1,20
          DO 312 J=1,20
          SS=FRNGE(I,J)
          TP=SS/RS
          IF(ABS(TP).LT.1.) GO TO 312
          RS=SS
      312 CONTINUE
          WRITE(6,680)
          WRITE(6,516) RS
          WRITE(6,690)
          DO 320 J=1,32
          DO 315 I=1,40
          SS=FRNGE(I,J)
      315 MM(I)=SS/RS*100
          WRITE(6,515) MM
      320 CONTINUE
        4 CONTINUE
        6 CONTINUE
      100 FORMAT (8E10.4)
      101 FORMAT (2I5,3F10.3)
      102 FORMAT (3I5)
      490 FORMAT(5X,55H ELASTIC CONSTANTS C11,C22,C33,C44,C55,C66,C12,C13,C3
         12,/9E12.4,/)
      491 FORMAT(5X,25H DENSITY OF PLATE GM/CC--,F10.4,5X,20H FIBER LAYUP AN
         1GLE--,F10.4)
      500 FORMAT (2F12.4,4E15.4)
      501 FORMAT (5H X3= F10.4)
      502 FORMAT (10H STRESSES   I3/(8E15.7))
      503 FORMAT (5X5H DATA, 15X10HCONSTANTS    ,20X10HPARAMETERS  /8E15.7)
      504 FORMAT (5X8HNSTRESS= I3,5X4HNX3= I4,5X4HDX3= F10.4,/)
      505 FORMAT (14H SIMPLE POLES )
      506 FORMAT (8E15.7)
      507 FORMAT (15H THIS IS A TEST )
      508 FORMAT (30H LAPLACE INVERSION PARAMETER=      F12.4)
      509 FORMAT (28H SECOND ORDER BRANCH POINTS      )
      510 FORMAT(5X,27H LONG.WAVE SPEED IN BEAM = ,F12.3,7H CM/SEC)
      511 FORMAT (14H DISPLACEMENTS  I4)
      512 FORMAT (24H RAYLEIGH SPEED CS/CR*I  )
      513 FORMAT (8F15.8)
      514 FORMAT (/50X 10HWITH STRIP)
      515 FORMAT(2X,40I3)
      516 FORMAT (16H MAXIMUM VALUE=   E15.7)
      520 FORMAT(/,5X,23H IMPACT PRESSURE(PSI)= ,E12.4,5X,25H 1/2 CONTACT LE
         1NGTH(CM)= ,F12.4,5X,23H IMPACT DURATION(SEC)= ,E12.4,/)
      521 FORMAT(/,5X,18H DENSITY OF BEAM= ,F12.4,5X,17H NORM.THICKNESS= ,
         1F12.4,5X,15H BEAM MODULUS= ,E12.4)
      600 FORMAT(/, 20X,18HNORMALIZED DIST.= ,I4)
      620 FORMAT(11F10.5)
      621 FORMAT(5X,20H WAVELEN(TIME-SEC)= ,E12.4,5X,20H WAVELEN(DIST-CM )=
         1,F12.4,/,5X,23H MAX FREQ NO(NON DIM)= ,F12.4,5X,23H MAX WAVE NO(NO
         2N DIM)= ,F12.4,/)
      630 FORMAT(30X,16H DISPLACEMENT U1,/)
      631 FORMAT(30X,16H DISPLACEMENT U3,/)
      632 FORMAT(30X,11H STRESS T33,/)
      633 FORMAT(30X,11H STRESS T11,/)
```

```
0298          634 FORMAT(30X,11H STRESS T13,/)
0299          635 FORMAT(30X,41H TEST-DISPL.OF BEAM ON ELASTIC FOUNDATION,/)
0300          640 FORMAT(20X,14H--->> X1, DX= ,F12.4,/,20X,2H |,/,20X,2H |,/,20X,
                 1 2H |,/,20X,2H V,/,20X,2H V,/,20X,9HTIME,DT= ,E12.4,//)
0301          650 FORMAT(10H IMAG PART)
0302          680 FORMAT(1H1)
0303          690 FORMAT(/,20X,17H FRINGE ORDER MAP,/,20X,31H SQRT((T11-T33)**2+4.0*
                 1T13*T13))
0304          710 FORMAT( 5X,6H VEL= ,F12.4,5X,5H T0= ,F12.4,5X,4H A= ,F12.4,5X,
                 15H F0= ,F12.4,/)
0305          203 STOP
0306              END
```

```
      C
            SUBROUTINE CALCUL(NPOLE)
            COMMON /MMC/ D,DK1
            COMMON /MC/DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRIP,A0
            COMPLEX B1,B2
            COMPLEX G0,G01,H0,H01,G10,G11,H10,H11
            COMPLEX G,H,G1,H1
            COMPLEX DK1,DK2,P1,P2,S1,S2,C1,C2,SI,B,C,D ,FG
            COMPLEX*16 BS
            PI= 3.14159265
            SI= CMPLX(0.,1.)
            A= C33*C55
            B= C33*(RHO*DK2*DK2-C11*DK1*DK1)+C55*(RHO*DK2*DK2-C55*DK1*DK1)+
           1DK1*DK1*(C13+C55)**2
            C= (RHO*DK2*DK2-C11*DK1*DK1)*(RHO*DK2*DK2-C55*DK1*DK1)
            BS= 1.D0*B*B-1.D0*4.*A*C
            D= CDSQRT(BS)
            P1= .5/A*(-B+D)
            P2= C/P1/A
            P1= CSQRT(P1)
            IF (REAL(P1).LT.0.0.AND.NPOLE.NE.2) P1= -P1
            P2= CSQRT(P2)
            IF (REAL(P2).LT.0.0.AND.NPOLE.NE.2) P2= -P2
            S1= -SI/DK1      /P1/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P1*P1)
            S2= -SI/DK1      /P2/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P2*P2)
            IF (NPOLE.EQ.2) GO TO 201
      C***  FORCING FUNCTION
            FG= PI*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI-DK2*DK2*T0*T0)*
           14./(DK1*A0)**2*(CSIN(DK1*A0)/DK1-A0*CCOS(DK1*A0))
        201 IF (NSTRIP.EQ.1) GO TO 202
            D= (SI*DK1*S1-P1)*(SI*DK1*C13-P2*C33*S2)-(SI*DK1*S2-P2)*(SI*DK1*
           1C13-P1*C33*S1)
            IF (NPOLE.EQ.2) RETURN
        206 C1= FG/D*(SI*DK1*S2-P2)
            C2= FG/D*(P1-SI*DK1*S1)
            GO TO 203
      202   CONTINUE
      C***     MATRIX FOR STRIP PROBLEM
            B1=W*W*(ES*DK1*DK1-RO*DK2*DK2)*DK1*DK1/12.0-RO*DK2*DK2
            B1=W*B1
            B2=-W*(ES*DK1*DK1-RO*DK2*DK2)
            G10= C55*(SI*DK1*S1-P1)
            H10= C55*(SI*DK1*S2-P2)
            G11=B2*(1.0+SI*DK1*S1*W/2)
            H11=B2*(1.0+SI*DK1*S2*W/2)
            G0=-(SI*DK1*C13-P1*S1*C33)
            H0=-(SI*DK1*C13-P2*S2*C33)
            G01=-SI*DK1*W*G10/2.0 + B1*S1
            H01=-SI*DK1*W*H10/2.0 + B1*S2
            H1=H10+H11
            G1=G10+G11
            G=G0+G01
            H=H0+H01
            D=G10*H0-G0*H10 + (H0*G11+H01*G10-G0*H11-G01*H10)+(H01*G11-H11*G01
           1)
            IF (NPOLE.EQ.2) RETURN
```

```
      C1= -FG/D*H1
      C2= FG/D*G1
  203 WRITE (2) P1,P2,C1,C2,S1,S2,DK1,DK2
      RETUFN
      END
```

```
   ELASTIC CONSTANTS C11,C22,C33,C44,C55,C66,C12,C13,C32,
0.2456E 08  0.1170E 07  0.1374E 07  0.3552E 06  0.2310E 07  0.3552E 06  0.4000E 06  0.1986E 07  0.4558E 06

   DENSITY OF PLATE GM/CC--     1.4400      FIBER LAYUP ANGLE--   15.0000
  NSTRESS=  1      NX3=   1      DX3=    1.0000

   VEL=      300.0000      T0=       87.3169*      A=        1.3342*      F0= 1042938.7500

   WAVELEN(TIME-SEC)=   0.3500E-05       WAVELEN(DIST-CM )=        0.6667
   MAX FREQ NO(NON DIM)=       13.7654       MAX WAVE NO(NON DIM)=       9.4248


       C11 =  0.2442E 08 C33 =  0.1196E 07 C55 =  0.2310E 07 C13 =  0.1830E 07


      LCNG.SPEED =  0.1081E 07SHEAR SPEED =  0.3326E 06

      RAYLEIGH SPEED =  0.3020E 06 CR/CS =    0.90800        12

   DENSITY OF BEAM=       2.7000      NORM.THICKNESS=       0.5000      BEAM MODULUS=   0.1124E 08
   LONG.WAVE SPEED IN BEAM =   535757.750 CM/SEC
SECOND ORDER BRANCH POINTS
 0.1997043E 02  0.0          0.0           0.2266004E 01
LAPLACE INVERSION PARAMETER=       0.5948

                                        WITH STRIP
```

* This program has test cards (#51, 52) to override the Hertz calculation T0=3510^{-6}s A=1 cm.

STRESS T33

```
--->> X1, DX=        0.3333
 |
 |
 |
 V
 V
TIME,DT=   0.1750E-05
```

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	12	7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	23	15	6	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	36	24	11	2	-1	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	49	34	17	5	0	-3	-3	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	61	44	23	8	0	-3	-4	-4	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	71	52	29	12	2	-2	-5	-6	-5	-3	-2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	80	59	34	16	5	-1	-4	-6	-6	-5	-4	-2	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
94	86	64	39	20	8	1	-3	-5	-7	-7	-6	-4	-3	-1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	90	68	42	23	10	3	-1	-4	-6	-7	-7	-6	-5	-3	-1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	91	70	44	25	13	5	0	-3	-5	-6	-7	-7	-6	-5	-3	-2	0	1	1	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
98	91	70	45	26	15	7	2	-1	-3	-5	-6	-7	-7	-7	-5	-4	-2	0	0	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0
95	87	68	44	27	16	9	4	0	-1	-3	-5	-6	-7	-7	-7	-6	-4	-3	-1	0	1	2	2	1	0	0	0	0	0	0	0	0	0	0
89	82	64	43	27	17	10	6	2	0	-1	-3	-5	-6	-7	-7	-7	-6	-5	-3	-1	0	1	2	2	2	1	0	0	0	0	0	0	0	0
80	74	59	40	26	17	11	7	4	2	0	-1	-3	-5	-6	-7	-7	-7	-6	-5	-4	-2	0	1	2	2	2	1	0	0	0	0	0	0	0
70	65	52	36	25	17	12	8	6	3	1	0	-1	-3	-4	-6	-6	-7	-7	-6	-5	-4	-2	0	0	2	2	2	1	1	0	0	0	0	0
57	53	43	31	22	16	12	9	7	5	3	1	0	-1	-3	-4	-5	-6	-7	-7	-7	-6	-4	-3	-1	0	1	2	2	2	1	0	0	0	0
43	41	34	26	19	15	12	10	8	6	5	3	1	0	-1	-2	-4	-5	-6	-7	-7	-7	-6	-5	-3	-2	0	1	2	2	2	1	0	0	0
28	27	24	19	16	13	12	10	9	7	6	5	3	2	0	0	-2	-3	-5	-6	-7	-7	-7	-6	-5	-4	-2	0	0	2	2	2	2	1	0
13	13	13	12	12	11	11	10	9	8	7	6	5	3	2	0	0	-2	-3	-4	-6	-6	-7	-7	-7	-6	-4	-3	-1	0	1	2	2	2	1
1	1	3	6	8	9	10	10	9	9	8	7	6	5	3	2	1	0	-1	-3	-4	-5	-6	-7	-7	-7	-6	-5	-3	-1	0	1	2	2	2
-4	-3	-1	0	3	6	8	9	9	9	8	8	7	6	5	4	2	1	0	-1	-2	-4	-5	-6	-7	-7	-7	-6	-5	-4	-2	0	0	2	2
-7	-6	-5	-3	0	2	5	7	9	9	9	8	8	7	6	5	4	3	1	0	-1	-2	-3	-5	-6	-7	-7	-7	-7	-6	-4	-3	-1	0	1
-7	-7	-6	-4	-3	0	2	5	7	8	9	9	8	8	7	6	5	4	3	2	0	0	-2	-3	-4	-6	-7	-7	-7	-7	-6	-5	-3	-1	0
-7	-7	-6	-6	-4	-2	0	2	4	7	8	9	9	8	8	7	6	5	4	3	2	0	0	-1	-3	-4	-5	-6	-7	-7	-7	-6	-5	-4	-2
-6	-6	-6	-5	-5	-4	-2	0	1	4	6	8	9	9	9	8	7	6	5	4	3	2	1	0	-1	-2	-4	-5	-6	-7	-7	-7	-7	-6	-4
-6	-6	-6	-5	-5	-4	-4	-2	0	1	3	5	7	9	9	9	8	7	6	5	5	4	3	1	0	0	-2	-3	-4	-6	-6	-7	-7	-7	-6
-3	-3	-4	-4	-4	-4	-4	-3	-2	-1	0	3	5	7	9	9	9	8	7	6	6	5	4	3	2	0	0	-1	-3	-4	-5	-6	-7	-7	-7
-5	-5	-4	-4	-4	-4	-4	-4	-3	-2	-1	0	2	4	6	8	9	9	9	8	7	6	5	4	3	2	1	0	-1	-2	-4	-5	-6	-7	-7
-1	-1	-2	-3	-3	-3	-4	-4	-4	-3	-3	-1	0	1	4	6	8	9	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-5	-6	-7
-5	-4	-4	-3	-3	-3	-3	-3	-3	-3	-3	-3	-2	0	1	3	5	7	9	10	9	9	7	6	5	4	4	3	2	0	0	-2	-3	-4	-5

X3= 0.0
DISPLACEMENTS 3
MAXIMUM VALUE= -0.7104896E 00

NORMALIZED DIST.= 0

0.00050	0.03979	0.14142	0.27084	0.41184	0.54907	0.67705	0.78714	0.87790	0.94377	0.98577
1.00000	0.98948	0.95134	0.89048	0.80461	0.70045	0.57593	0.43953	0.28894	0.13502	0.01142
-0.04267	-0.07263	-0.07276	-0.07738	-0.06042	-0.06357	-0.03818	-0.05201	-0.01223	-0.05377	

IMAG PART

0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00001	0.00002	0.00003

NORMALIZED DIST.= 1

0.00052	0.03504	0.12364	0.23925	0.36748	0.49373	0.61244	0.71537	0.80083	0.86362	0.90441
0.91968	0.91198	0.87880	0.82442	0.74692	0.65225	0.53870	0.41383	0.27579	0.13423	0.01894
-0.03628	-0.06768	-0.07048	-0.07543	-0.06060	-0.06261	-0.03975	-0.05059	-0.01542	-0.04968	

IMAG PART

0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00002	0.00001	0.00002	0.00003

NORMALIZED DIST.= 2

0.00046	0.02177	0.07726	0.15621	0.24997	0.34647	0.44004	0.52356	0.59467	0.64909	0.68662
0.70461	0.70446	0.68451	0.64749	0.59235	0.52308	0.43886	0.34489	0.24037	0.13188	0.03829
-0.01809	-0.05320	-0.06323	-0.06962	-0.06054	-0.06012	-0.04359	-0.04736	-0.02317	-0.04079	

IMAG PART

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002

NORMALIZED DIST.= 3

0.00028	0.00689	0.02688	0.06236	0.11326	0.17235	0.23427	0.29336	0.34651	0.39036	0.42373
0.44479	0.45370	0.44958	0.43352	0.40525	0.36662	0.31768	0.26100	0.19681	0.12822	0.06182
0.00870	-0.03042	-0.04999	-0.06009	-0.05857	-0.05702	-0.04745	-0.04482	-0.03114	-0.03385	

IMAG PART

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001

NORMALIZED DIST.= 4

0.00030	0.00089	0.00200	0.00749	0.02457	0.05280	0.08844	0.12718	0.16551	0.20047	0.23014
0.25300	0.26837	0.27564	0.27493	0.26619	0.25006	0.22685	0.19759	0.16280	0.12382	0.08191
0.03868	-0.00176	-0.03008	-0.04672	-0.05288	-0.05368	-0.04925	-0.04457	-0.03642	-0.03159	

IMAG PART

0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001

NORMALIZED DIST.= 5

0.00032	0.00039	-0.00267	-0.01052	-0.01476	-0.00909	0.00649	0.02920	0.05590	0.08350	0.10982
0.13298	0.15198	0.16592	0.17462	0.17771	0.17551	0.16799	0.15576	0.13895	0.11848	0.09467
0.06546	0.02911	-0.00428	-0.02892	-0.04250	-0.04904	-0.04829	-0.04595	-0.03865	-0.03469	

IMAG PART

0.00000	0.00000	-0.00000	-0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000

STRESS T11

--->> X1, DX= 0.3333

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V
V

TIME,DT= 0.1750E-05

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2	0	5	10	8	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-5	-1	8	18	16	9	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-4	0	12	26	24	15	7	1	-1	-2	-1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	4	19	34	30	19	9	2	-1	-4	-4	-3	0	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	11	27	43	37	23	11	2	-2	-5	-6	-6	-5	-2	0	1	2	2	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
15	20	37	53	45	27	12	2	-4	-8	-9	-9	-9	-7	-4	-1	0	2	2	2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
25	30	47	63	53	33	15	2	-6	-10	-12	-13	-12	-11	-9	-6	-3	0	1	2	2	1	1	1	2	2	1	1	1	0	0	0	0	0	0
35	40	57	73	61	39	18	3	-7	-13	-16	-17	-16	-15	-13	-11	-8	-5	-2	0	2	2	1	1	1	1	2	2	2	1	1	1	1	0	0
45	50	67	82	69	45	23	5	-6	-14	-19	-20	-20	-19	-17	-15	-13	-10	-7	-4	-1	0	2	2	1	0	0	1	1	2	2	1	2	1	1
54	59	75	89	76	51	28	9	-4	-13	-20	-23	-24	-24	-22	-20	-17	-15	-12	-9	-6	-3	0	1	1	1	0	0	0	1	1	1	1	1	1
61	66	82	95	82	57	33	14	0	-11	-19	-24	-26	-27	-27	-25	-22	-19	-16	-14	-11	-8	-5	-2	0	1	1	0	0	0	0	1	1	1	1
68	72	86	98	85	61	38	19	4	-7	-16	-23	-27	-29	-30	-29	-27	-24	-21	-18	-15	-13	-10	-7	-4	-1	0	1	0	0	0	0	0	1	1
72	76	89	100	87	64	43	24	9	-2	-12	-19	-25	-29	-31	-32	-31	-29	-26	-23	-20	-17	-14	-11	-8	-5	-3	0	0	0	0	0	-1	0	0
75	78	89	98	87	66	47	29	15	3	-6	-15	-21	-26	-30	-32	-33	-33	-31	-28	-25	-22	-19	-16	-13	-10	-7	-4	-2	0	0	0	-1	-1	-1
75	78	87	94	84	66	49	34	20	9	0	-9	-16	-22	-27	-31	-33	-34	-34	-33	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	-1	0	0	-1	-1
74	76	83	88	79	65	50	37	26	15	6	-2	-10	-16	-22	-27	-31	-34	-35	-35	-34	-32	-29	-26	-23	-19	-16	-13	-10	-8	-5	-3	-1	-1	-1
71	72	76	79	73	61	50	40	30	21	13	4	-2	-10	-16	-22	-27	-31	-34	-36	-36	-35	-33	-31	-28	-24	-21	-18	-15	-12	-9	-7	-4	-3	-2
66	66	68	68	64	56	49	41	34	26	19	12	4	-2	-9	-15	-21	-26	-31	-34	-36	-36	-36	-35	-32	-29	-26	-23	-19	-16	-14	-11	-8	-6	-4
59	59	58	56	53	50	46	42	37	31	25	19	12	5	-1	-7	-14	-20	-25	-30	-33	-36	-37	-37	-36	-34	-31	-28	-24	-21	-18	-15	-13	-10	-8
51	50	48	45	43	43	42	41	38	35	30	25	19	13	7	0	-6	-12	-18	-24	-29	-33	-35	-37	-37	-37	-35	-32	-29	-26	-23	-20	-17	-14	-12
39	39	39	38	38	38	38	39	39	38	35	31	26	21	15	8	2	-4	-11	-17	-23	-28	-32	-35	-37	-38	-38	-36	-34	-31	-28	-25	-22	-19	-16-
25	26	28	30	32	34	35	37	38	39	39	36	33	28	22	17	10	4	-2	-9	-15	-21	-26	-31	-34	-37	-38	-38	-37	-36	-33	-30	-27	-24	-21-
15	16	18	22	25	29	31	34	36	38	40	40	39	35	30	25	19	12	6	0	-7	-13	-19	-25	-30	-34	-37	-39	-39	-39	-37	-35	-32	-29	-26-
7	8	10	13	17	22	27	30	33	35	37	39	40	40	38	33	28	21	15	8	1	-5	-11	-18	-24	-29	-33	-36	-39	-40	-40	-38	-36	-34	-30-
1	2	4	7	11	15	19	24	29	32	35	37	38	39	40	39	35	30	24	18	11	4	-2	-9	-16	-22	-27	-32	-36	-38	-40	-40	-39	-37	-34-
-1	-1	0	1	4	8	13	18	22	27	31	34	36	38	39	39	39	36	32	26	20	14	7	0	-6	-12	-19	-25	-30	-34	-37	-39	-39	-39	-37-
-4	-4	-2	0	1	3	6	10	15	20	25	29	33	35	36	38	39	39	37	34	29	22	16	10	4	-2	-9	-15	-21	-27	-31	-34	-37	-38	-38-
-5	-5	-6	-6	-4	-1	1	5	8	13	17	22	27	31	34	36	37	38	39	38	36	32	26	19	13	7	1	-5	-11	-17	-23	-28	-32	-35	-36-
-8	-7	-5	-4	-3	-4	-3	0	2	6	11	16	21	25	29	33	36	38	39	39	39	37	34	29	23	16	9	3	-2	-8	-14	-20	-25	-29	-32-
-5	-6	-9	-11	-10	-7	-4	-2	-1	1	4	8	14	19	25	29	32	35	37	39	40	40	39	36	32	26	19	12	6	0	-5	-12	-18	-23	-27-

X3= 0.0
DISPLACEMENTS 4
MAXIMUM VAIUE= -0.1169061E 01

NORMALIZED DIST.= 0

0.00258	0.00591	-0.02645	-0.05231	-0.04372	-0.00204	0.06720	0.15474	0.25280	0.35369	0.45160
0.54124	0.61882	0.68126	0.72641	0.75286	0.75988	0.74715	0.71530	0.66481	0.59890	0.51992
0.39153	0.25986	0.15272	0.07511	0.01861	-0.01623	-0.04614	-0.05638	-0.08178	-0.05544	
IMAG PART										
0.00000	0.00000	0.00000	-0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00001	0.00001

NORMALIZED DIST.= 1

0.00200	0.00707	-0.00660	-0.01783	-0.00073	0.04631	0.11913	0.20892	0.30838	0.40951	0.50684
0.59461	0.66947	0.72778	0.76810	0.78838	0.78875	0.76829	0.72861	0.66949	0.59492	0.50860
0.39149	0.26578	0.16154	0.08150	0.02557	-0.01399	-0.04097	-0.05868	-0.07437	-0.06856	
IMAG PART										
0.00000	0.00000	0.00000	-0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00002

NORMALIZED DIST.= 2

0.00095	0.01745	0.05203	0.08323	0.12706	0.19085	0.27497	0.37173	0.47550	0.57739	0.67297
0.75508	0.82171	0.86766	0.89339	0.89515	0.87552	0.83186	0.76861	0.68362	0.58354	0.48166
0.39022	0.28142	0.18630	0.10048	0.04569	-0.00618	-0.02674	-0.06290	-0.05665	-0.09664	
IMAG PART										
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002

NORMALIZED DIST.= 3

0.00057	0.03187	0.10966	0.18732	0.26129	0.34028	0.43188	0.53084	0.63438	0.73316	0.82397
0.89837	0.95550	0.98870	1.00000	0.98414	0.94583	0.88079	0.79607	0.68776	0.56538	0.45448
0.38626	0.30196	0.22148	0.13213	0.07587	0.01127	-0.00827	-0.06039	-0.04181	-0.11182	
IMAG PART										
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00003

NORMALIZED DIST.= 4

0.00019	0.01996	0.08833	0.16952	0.24197	0.30671	0.37709	0.45380	0.53712	0.61997	0.69932
0.76750	0.82278	0.85892	0.87669	0.87127	0.84603	0.79712	0.73005	0.64180	0.53914	0.43862
0.38149	0.32254	0.25902	0.17532	0.11222	0.04213	0.01049	-0.04408	-0.03756	-0.10398	
IMAG PART										
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002

NORMALIZED DIST.= 5

0.00069	0.00496	0.04046	0.09727	0.15250	0.19436	0.23295	0.27765	0.33095	0.39123	0.45436
0.51583	0.57109	0.61642	0.64875	0.66607	0.66710	0.65118	0.61852	0.56999	0.50604	0.43337
0.38097	0.34064	0.29211	0.22508	0.15284	0.08553	0.03096	-0.01399	-0.04177	-0.07016	
IMAG PART										
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00001	0.00001	0.00001	0.00002

STRESS T13

--->> X1, DX= 0.3333

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V
V

TIME,DT= 0.1750E-05

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	3	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	10	15	10	0	-2	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	20	31	24	7	-2	-6	-5	-3	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	30	48	41	19	3	-6	-9	-8	-5	-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	39	63	56	31	12	-1	-9	-12	-11	-7	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	47	76	70	43	22	6	-4	-11	-14	-13	-10	-5	-1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	52	86	80	52	30	14	2	-6	-11	-15	-14	-12	-7	-2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	57	93	88	59	37	20	9	0	-6	-12	-14	-15	-13	-9	-4	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	59	98	93	64	41	25	14	5	0	-6	-11	-14	-15	-14	-11	-6	-1	1	2	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
0	61	100	95	66	44	28	18	10	4	-1	-6	-10	-13	-15	-15	-12	-8	-3	0	2	2	1	0	-1	0	0	0	0	0	0	0	0	0	0
0	60	99	95	67	45	30	21	14	8	3	0	-5	-9	-12	-15	-15	-13	-10	-5	-1	2	3	2	0	-1	-1	0	0	0	0	0	0	0	0
0	58	96	92	65	45	31	22	16	12	7	4	0	-4	-8	-11	-14	-15	-14	-11	-7	-2	1	3	3	1	0	-1	-1	0	0	0	0	0	0
0	54	90	87	62	43	30	23	17	14	10	8	4	1	-2	-6	-10	-13	-15	-14	-12	-8	-4	0	2	3	2	0	-1	-1	-1	0	1	0	0
0	50	82	80	57	40	29	22	18	15	12	10	8	5	2	-1	-5	-9	-12	-14	-14	-13	-10	-5	-1	2	3	3	1	0	-2	-1	0	0	1
0	43	72	70	51	36	27	21	17	15	14	12	10	8	6	3	0	-4	-8	-11	-13	-14	-14	-11	-7	-2	0	3	3	2	0	-1	-2	-1	0
0	36	60	59	43	31	24	19	17	15	14	13	12	11	9	7	4	0	-2	-6	-10	-12	-14	-14	-12	-8	-4	0	2	4	3	1	0	-2	-2
0	28	47	46	34	26	20	17	15	14	14	13	13	12	11	10	7	5	2	-1	-5	-8	-12	-13	-14	-13	-10	-6	-1	1	3	4	2	0	-1
0	20	33	33	25	19	16	14	13	13	13	13	13	13	12	12	10	8	6	3	0	-4	-7	-10	-13	-14	-13	-11	-7	-3	0	3	4	3	1
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0	1	2	3	5	6	7	8	9	10	11	12	12	13	13	13	13	12	11	10	8	5	2	-1	-5	-8	-11	-13	-14	-12	-10	-6	-2	1	3
0	-6	-9	-9	-5	-1	2	5	7	8	9	10	11	12	13	13	13	13	13	12	10	8	6	3	0	-4	-7	-10	-13	-14	-13	-11	-7	-3	0
0	-7	-12	-15	-14	-10	-4	0	3	6	7	9	10	11	12	12	13	13	13	13	12	11	9	7	4	0	-3	-6	-9	-12	-13	-13	-12	-9	-5
0	-5	-11	-15	-17	-16	-13	-7	-1	2	5	7	8	9	10	11	12	13	13	13	13	12	11	9	7	4	1	-1	-5	-8	-11	-13	-13	-12	-10
0	-3	-7	-11	-15	-18	-18	-15	-9	-3	2	5	7	7	8	9	10	11	12	13	13	13	12	11	10	8	5	2	0	-4	-7	-10	-12	-13	-13-
0	-2	-5	-8	-11	-14	-17	-18	-16	-12	-6	0	5	7	8	8	9	10	11	11	12	12	13	12	12	11	9	6	3	0	-2	-6	-9	-11	-13-
0	-1	-2	-4	-7	-10	-13	-16	-17	-17	-14	-8	-2	3	7	8	9	9	9	10	11	12	12	12	12	12	11	9	7	5	2	-1	-4	-8	-10-
0	0	-1	-2	-4	-6	-9	-12	-15	-17	-17	-15	-10	-4	1	6	8	9	9	9	10	11	12	12	13	13	12	11	10	8	6	3	0	-3	-6
0	0	0	0	-2	-3	-5	-7	-10	-13	-16	-17	-16	-12	-7	0	5	8	10	9	9	10	11	13	13	13	13	13	12	11	9	7	4	0	-2
0	0	0	0	0	0	-2	-4	-7	-10	-12	-15	-16	-15	-13	-9	-3	3	8	10	10	10	10	10	12	13	14	14	13	12	11	10	7	4	1
0	1	1	1	0	0	0	-1	-3	-5	-8	-11	-14	-16	-16	-14	-10	-4	1	6	10	11	11	10	10	11	13	14	14	14	13	12	10	8	5
0	-1	-1	0	1	2	1	0	-1	-2	-5	-7	-9	-12	-15	-16	-15	-11	-6	0	4	9	11	11	10	10	10	12	13	14	14	13	13	11	10

```
X3=      0.0
DISPLACEMENTS    5
MAXIMUM VALUE=  -0.4623035E 00

                          NORMALIZED DIST.=     0
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000  -0.00000  -0.00000  -0.00000  -0.00000
 -0.00000  -0.00000  -0.00000  -0.00000  -0.00000  -0.00000  -0.00000   0.00000   0.00000   0.00000   0.00000
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000  -0.00000  -0.00000  -0.00000
IMAG PART
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00000

                          NORMALIZED DIST.=     1
 -0.00043   0.02068   0.10175   0.20639   0.30728   0.39631   0.47093   0.52964   0.57264   0.59915   0.61001
  0.60465   0.58452   0.54925   0.50110   0.43993   0.36875   0.28784   0.20073   0.10784   0.01335  -0.06204
 -0.07283  -0.05896  -0.03786  -0.02592  -0.01095  -0.00946   0.00347  -0.00564   0.01473  -0.01847
IMAG PART
  0.00000  -0.00000   0.00000   0.00000   0.00000   0.00000   0.00001   0.00001   0.00001   0.00001   0.00002

                          NORMALIZED DIST.=     2
 -0.00013   0.03502   0.15215   0.31697   0.48494   0.63521   0.76207   0.86189   0.93521   0.98069   1.00000
  0.99229   0.96012   0.90298   0.82454   0.72472   0.60842   0.47603   0.33347   0.18134   0.02696  -0.09592
 -0.12971  -0.11354  -0.07743  -0.05318  -0.02567  -0.01857   0.00226  -0.00719   0.01974  -0.01810
IMAG PART
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00001   0.00001   0.00002   0.00002   0.00002   0.00003

                          NORMALIZED DIST.=     3
 -0.00001   0.02103   0.10073   0.24333   0.41035   0.56703   0.70144   0.80789   0.88641   0.93621   0.95909
  0.95511   0.92661   0.87394   0.80010   0.70587   0.59526   0.46931   0.33299   0.18781   0.03946  -0.09051
 -0.15673  -0.15637  -0.11853  -0.08225  -0.04707  -0.02788  -0.00646  -0.00410   0.01312  -0.00197
IMAG PART
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00001   0.00001   0.00002   0.00002   0.00002   0.00003

                          NORMALIZED DIST.=     4
 -0.00004  -0.00297   0.00997   0.07754   0.19345   0.31982   0.43364   0.52662   0.59598   0.64235   0.66643
  0.67025   0.65479   0.62217   0.57325   0.51049   0.43492   0.34924   0.25488   0.15515   0.05108  -0.05597
 -0.14683  -0.17773  -0.15636  -0.11319  -0.07501  -0.04060  -0.02147  -0.00133   0.00071   0.01812
IMAG PART
  0.00000   0.00000   0.00000   0.00000   0.00000   0.00000   0.00001   0.00001   0.00001   0.00002   0.00002

                          NORMALIZED DIST.=     5
  0.00030  -0.00548  -0.02595  -0.02357   0.03348   0.12663   0.22238   0.30661   0.37192   0.41857   0.44612
  0.45759   0.45334   0.43645   0.40685   0.36774   0.31856   0.26281   0.19987   0.13350   0.06310  -0.01262
 -0.10517  -0.16989  -0.18071  -0.14489  -0.10571  -0.06167  -0.03831  -0.00752  -0.00720   0.02622
IMAG PART
  0.00000   0.00000  -0.00000  -0.00000   0.00000   0.00000   0.00001   0.00001   0.00001   0.00001   0.00002
```

MAXIMUM VALUE=   0.1168567E 01

FRINGE ORDER MAP
SQRT((T11-T33)**2+4.0*T13*T13)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	2	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	11	12	12	8	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	23	25	24	17	10	6	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	33	38	37	27	16	10	8	7	4	2	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	40	50	50	37	22	12	8	10	9	7	4	1	1	2	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	45	60	62	47	28	14	6	8	11	12	10	7	3	1	2	2	2	2	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
32	47	68	73	56	35	18	5	5	10	13	14	13	10	6	2	1	2	2	2	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
28	48	74	81	64	41	22	8	2	8	13	15	16	15	12	8	4	2	2	2	2	1	1	1	2	2	1	1	1	0	0	0	0	0	0
21	48	79	89	71	47	27	12	5	9	13	16	17	18	17	14	11	7	3	2	2	2	2	1	1	1	2	2	2	1	1	1	1	0	0
14	48	83	94	76	52	31	16	9	11	14	17	18	19	19	18	16	13	9	5	3	2	2	2	1	1	0	1	1	2	2	2	2	1	1
6	47	85	98	81	56	35	19	11	12	16	19	20	21	21	21	20	18	15	11	7	4	2	2	2	1	1	0	0	1	1	1	1	1	2
1	47	85	100	83	59	38	22	13	13	17	20	22	23	23	23	22	21	20	17	13	9	6	4	2	1	1	1	1	0	0	1	1	1	1
10	47	84	99	84	62	41	24	14	13	16	20	23	25	25	25	25	24	23	21	19	15	12	8	5	3	2	1	1	1	1	0	1	1	1
18	47	82	97	84	63	43	27	16	12	15	19	23	25	27	27	27	26	25	24	23	20	17	14	10	7	4	2	1	0	1	1	1	0	1
26	48	78	92	81	62	45	30	18	12	13	17	21	24	27	28	29	28	28	27	25	24	22	19	16	12	9	6	4	2	0	1	2	2	1
33	48	73	86	77	61	45	32	21	14	11	14	18	22	25	28	29	30	30	29	28	27	25	23	21	17	14	10	8	5	3	1	0	2	2
39	49	67	78	71	58	46	34	24	16	11	11	14	18	22	26	28	30	31	31	30	29	28	26	24	22	19	16	12	9	7	4	2	0	1
44	50	61	69	64	54	44	35	27	20	14	11	11	14	18	22	26	29	30	31	31	31	30	29	27	26	23	21	17	14	11	8	6	4	1
48	50	55	58	55	49	43	36	30	24	18	13	10	11	14	18	22	26	28	30	32	32	32	31	30	28	27	25	22	19	16	13	10	7	5
51	51	50	48	46	43	40	36	32	27	22	18	13	10	11	13	17	21	25	28	30	32	32	32	32	31	29	28	26	24	21	18	14	12	9
51	49	46	42	39	37	36	35	33	30	26	22	18	14	11	10	12	16	20	24	27	30	32	33	33	32	31	30	29	27	25	23	19	16	13
41	41	41	40	37	35	33	33	34	33	30	27	23	19	15	11	10	11	15	19	23	26	29	31	33	33	33	32	31	30	29	27	24	21	18
30	31	32	34	35	34	33	33	33	34	33	32	28	25	20	16	12	10	11	14	18	22	26	29	31	33	34	34	33	33	31	30	28	26	23
19	20	23	26	30	32	33	33	32	32	34	35	34	31	27	22	18	13	10	10	12	16	21	25	28	31	33	34	35	34	34	33	31	30	28
12	12	14	18	22	26	30	33	33	32	32	33	35	35	33	29	25	20	15	11	9	11	15	19	24	27	31	33	34	35	35	35	34	32	31
5	6	8	11	15	19	24	28	31	33	33	32	32	34	35	34	32	27	22	17	12	10	10	13	18	22	26	30	32	34	35	35	35	34	33
2	2	3	5	8	12	17	22	26	29	32	32	32	32	33	34	34	33	29	24	19	15	11	10	12	16	20	24	28	31	33	34	35	34	34
2	1	0	2	4	6	9	14	19	24	28	31	32	32	31	32	33	34	34	31	26	21	16	13	11	11	13	17	22	25	29	31	32	33	33
2	2	3	3	1	1	5	8	12	16	21	25	29	31	32	32	31	32	34	34	33	29	24	18	14	12	11	12	15	19	22	26	28	30	32
7	6	4	2	1	1	1	2	5	10	14	19	24	27	30	31	32	32	33	34	35	34	32	27	21	16	13	11	11	13	17	20	23	26	28
2	4	7	9	8	5	2	0	1	4	7	12	17	22	27	29	31	32	32	33	34	35	35	33	29	24	19	14	12	11	12	15	19	22	24

JOB	0350	7/5/74	IBM 360-91
	EXECUTION TIME		34 SEC.
			25 PAGES
	CORE		150 K

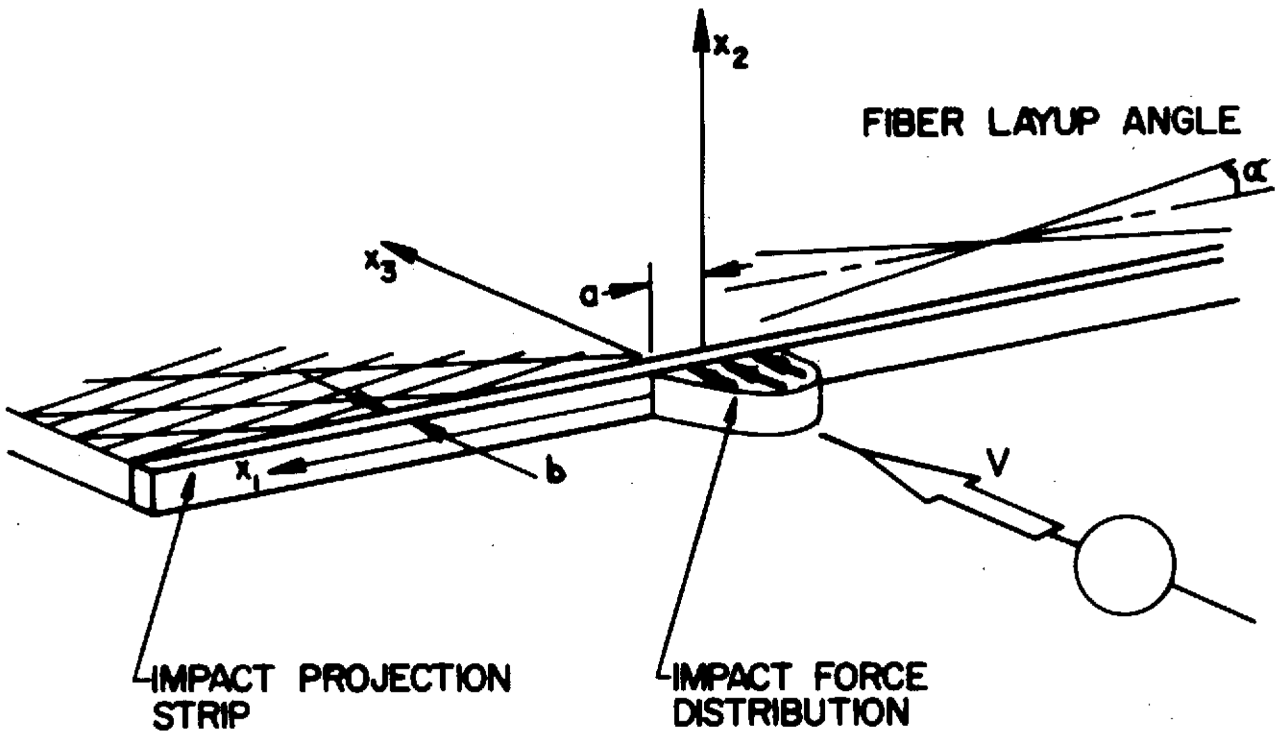
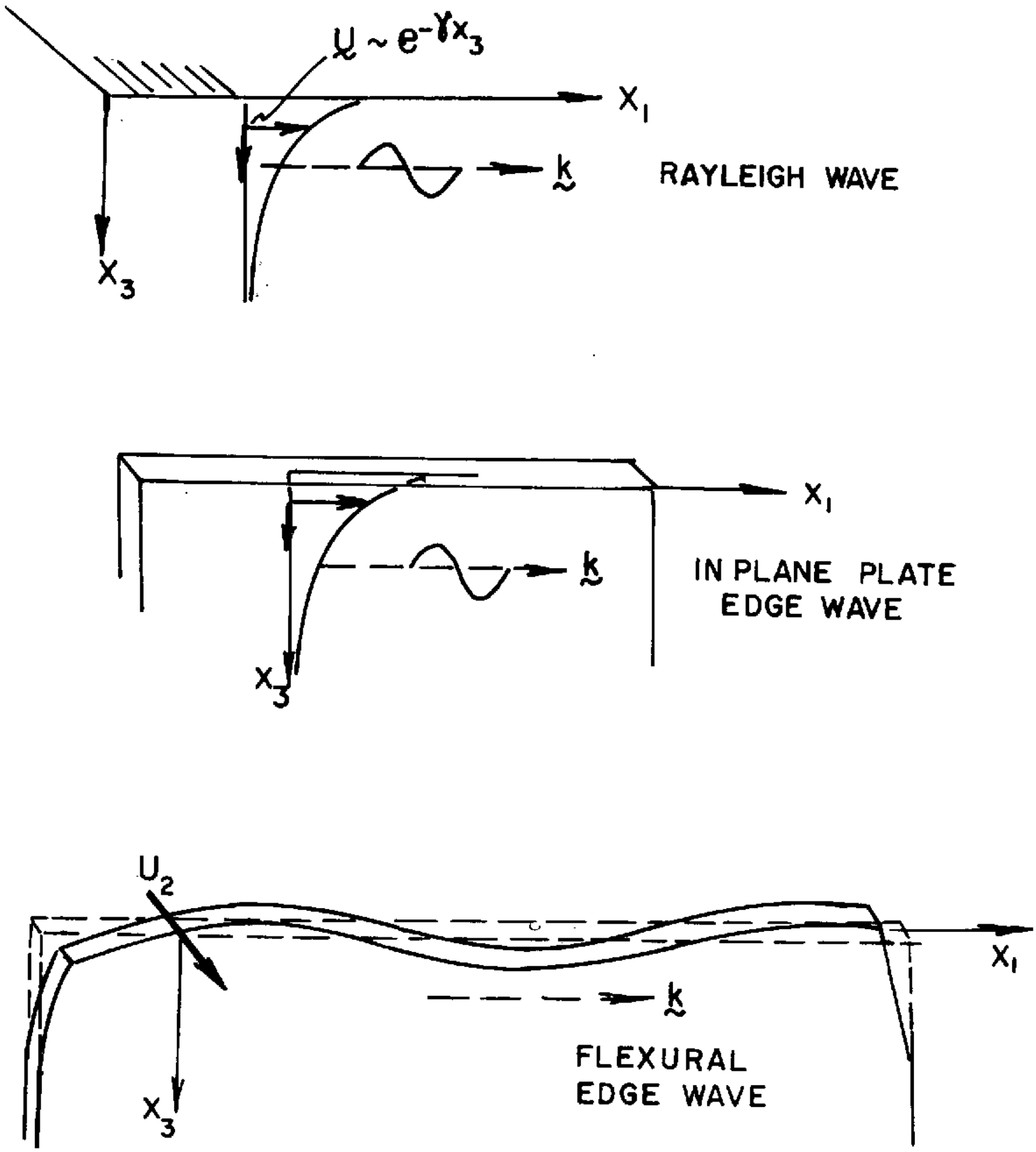


FIGURE 1

GEOMETRY OF IN-PLANE EDGE IMPACT OF A COMPOSITE PLATE WITH PROJECTION STRIP



$\underset{\sim}{U} \sim e^{-\gamma x_3}$
X_1
$\underset{\sim}{k}$
RAYLEIGH WAVE
X_3
X_1
$\underset{\sim}{k}$
IN PLANE PLATE
EDGE WAVE
X_3
U_2
X_1
$\underset{\sim}{k}$
FLEXURAL
EDGE WAVE
X_3

FIGURE 2a

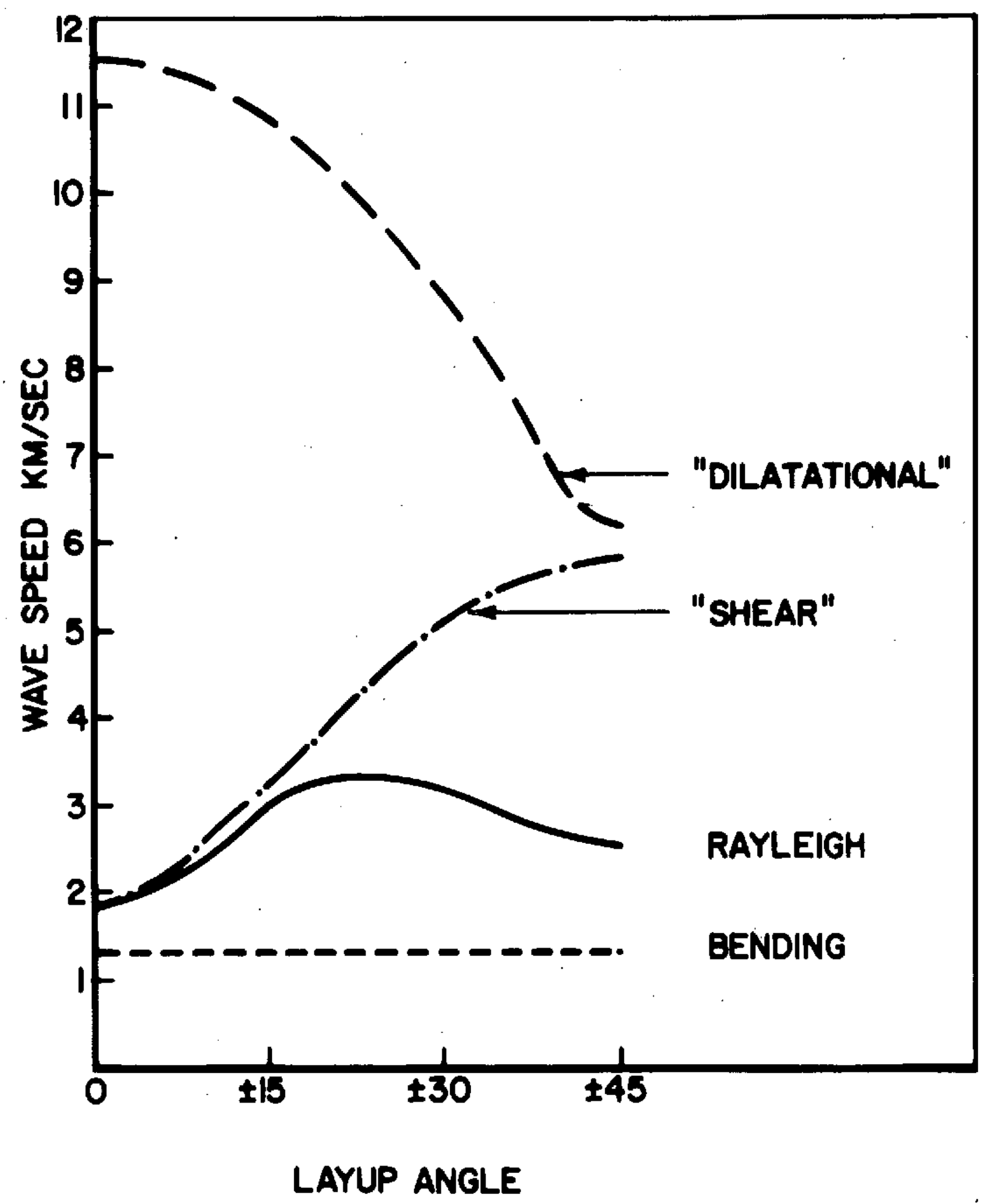


FIGURE 2b

Comparison of In-plane plate edge wave speed with body wave speeds versus fiber layup angle for 55% graphite fiber/epoxy matrix composite.

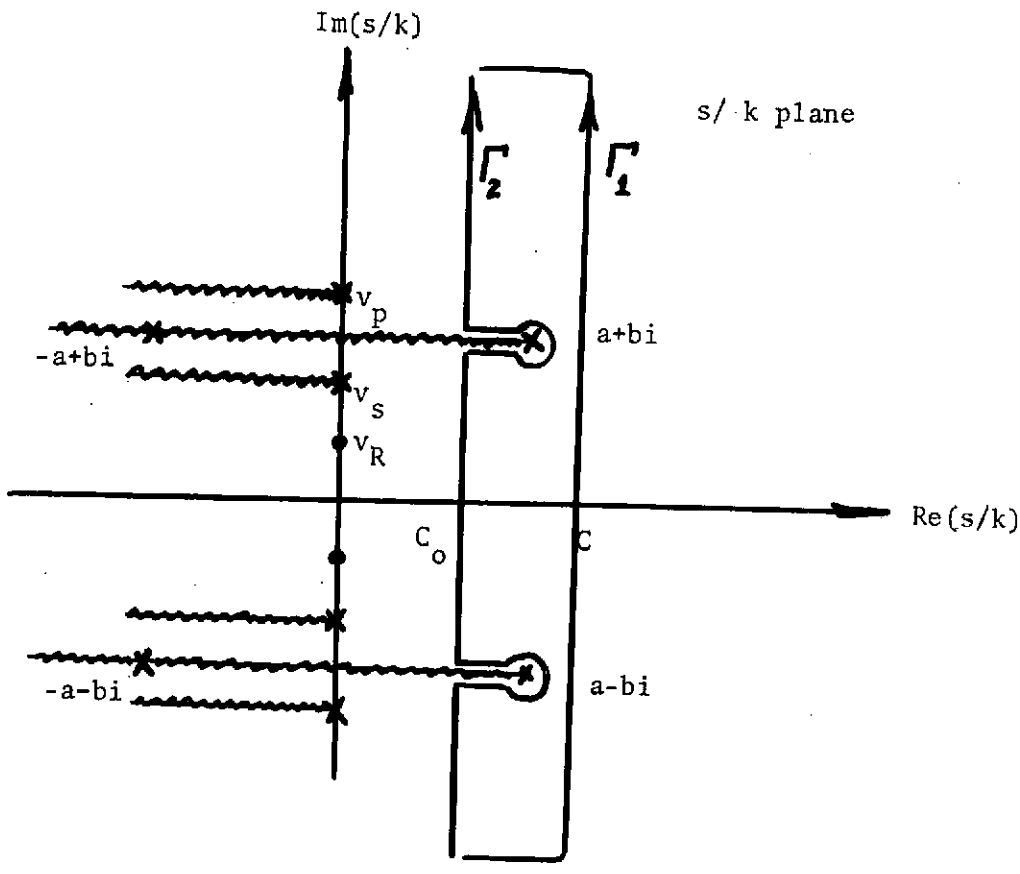


FIGURE 3.

Poles, Branch Points and Integration Contours in the complex Plane for the Numerical Solution.

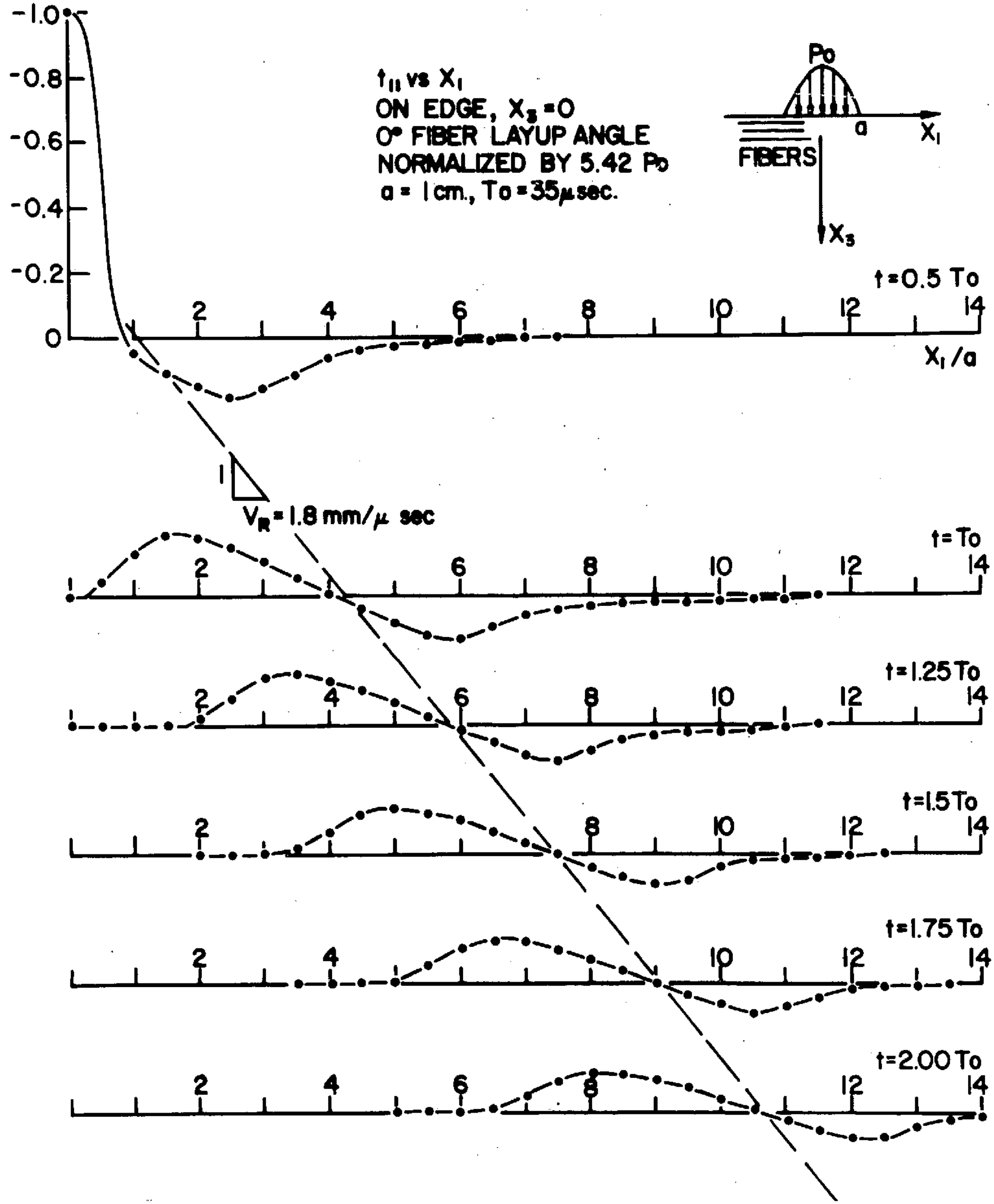


FIGURE 4

Edge Stress t_{11} Surface Wave for 0° Fiber layup Angle: 55% graphite fiber/epoxy matrix composite

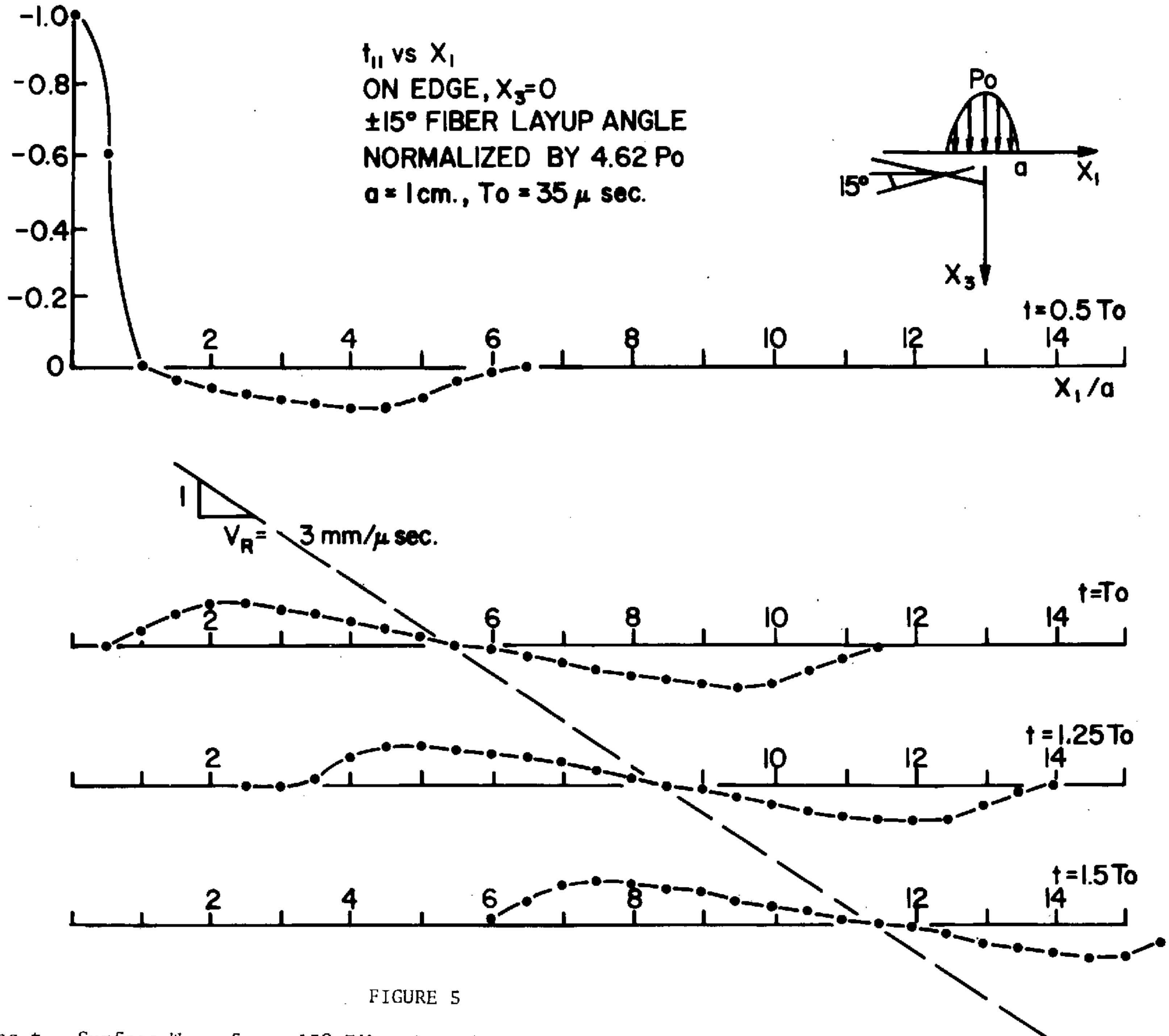


FIGURE 5

Edge Stress t_{11} Surface Wave for ± 15° Fiber Layup Angle: 55% Graphite fiber/expoxy matrix composite.

STRESS T33

---->> X1, DX= 0.4447

TIME,DT= 0.4366E-05

1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	13	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	27	17	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	40	24	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	52	32	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	63	38	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	72	44	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	79	48	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95	84	52	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	88	54	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	89	54	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	88	54	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95	84	52	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
89	79	48	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	72	44	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	63	38	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	52	32	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	40	24	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	27	16	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	13	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-5	-5	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	15	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

FIGURE 6

Computer Map of Edge Impact Pressure t_{33} Impact Time 87 μsec.

x_1

STRESS t_{11} , 0^o LAYUP

	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	3	-4	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	34	13	-14	-7	-2	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	60	28	-18	-16	-9	-4	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TIME	81	42	-15	-19	-16	-10	-5	-3	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	94	53	-10	-16	-19	-16	-10	-5	-3	-2	-2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	59	-4	-11	-15	-18	-16	-11	-6	-4	-3	-2	-2	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	96	59	2	-4	-10	-14	-17	-16	-11	-6	-4	-3	-2	-2	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0
	85	55	8	1	-3	-8	-13	-16	-15	-11	-6	-4	-3	-2	-2	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0
	66	46	14	8	2	-2	-7	-11	-15	-15	-11	-6	-4	-3	-3	-2	-2	-2	-1	-1	-1	-1	-1	0	0	0	0	0	0	0	0
	41	32	18	14	9	4	-1	-6	-10	-14	-14	-10	-6	-4	-3	-3	-2	-2	-2	-1	-1	-1	-1	-1	0	0	0	0	0	0	0
	14	17	20	18	14	10	5	0	-5	-9	-13	-13	-10	-6	-4	-3	-3	-2	-2	-2	-1	-1	-1	-1	0	0	0	0	0	0	0
END OF	0	5	13	19	18	15	11	6	1	-3	-8	-12	-13	-9	-6	-4	-3	-2	-2	-2	-1	-1	-1	-1	0	0	0	0	0	0	0
IMPACT	0	0	4	11	17	17	15	11	6	2	-2	-7	-11	-12	-9	-5	-3	-3	-2	-2	-1	-1	-1	-1	0	0	0	0	0	0	0
	0	0	0	3	10	16	16	14	11	7	2	-2	-6	-10	-11	-9	-5	-3	-2	-2	-1	-1	-1	0	0	0	0	0	0	0	0
	0	0	0	0	3	9	15	16	14	11	7	3	-1	-5	-9	-11	-8	-5	-3	-2	-1	-1	-1	0	0	0	0	0	0	0	0
	0	0	0	0	0	3	9	14	15	14	11	7	3	0	-5	-9	-10	-8	-4	-2	-2	-1	-1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	2	8	13	15	14	11	7	3	0	-4	-8	-10	-8	-4	-2	-1	-1	-1	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	2	7	13	14	13	11	7	4	0	-4	-7	-9	-8	-4	-2	-1	-1	-1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	2	7	12	14	13	11	8	4	0	-3	-7	-9	-7	-4	-2	-1	-1	-1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	2	6	11	14	13	11	8	4	0	-3	-7	-9	-7	-4	-2	-1	-1	-1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	6	11	13	13	11	8	4	0	-3	-6	-9	-7	-4	-2	-1	-1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	6	11	13	12	11	8	4	0	-2	-6	-8	-7	-4	-2	-1	-1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	5	10	13	12	10	8	4	1	-2	-6	-8	-7	-4	-2	-1	-1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	5	10	12	12	10	8	4	1	-2	-5	-8	-7	-4	-2	-1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	4	9	12	12	10	8	4	1	-2	-5	-8	-7	-4	-2	-1	0
	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	9	12	11	10	8	5	1	-1	-5	-7	-7	-4	-1	-1
	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	8	11	11	10	8	5	1	-1	-4	-7	-7	-4	-1
	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	8	11	11	10	8	5	2	-1	-4	-7	-7	-4
	-2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	8	11	11	10	8	5	2	0	-4	-6	-6
	4	1	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	7	11	11	10	8	5	2	0	-3	-5
	-7	-2	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	7	11	11	10	8	6	3	0	-1

SPURIOUS DATA

FIGURE 7

Computer Map of Edge Stress t_{11} Surface Wave in the Space-Time (x_1, t) Domain

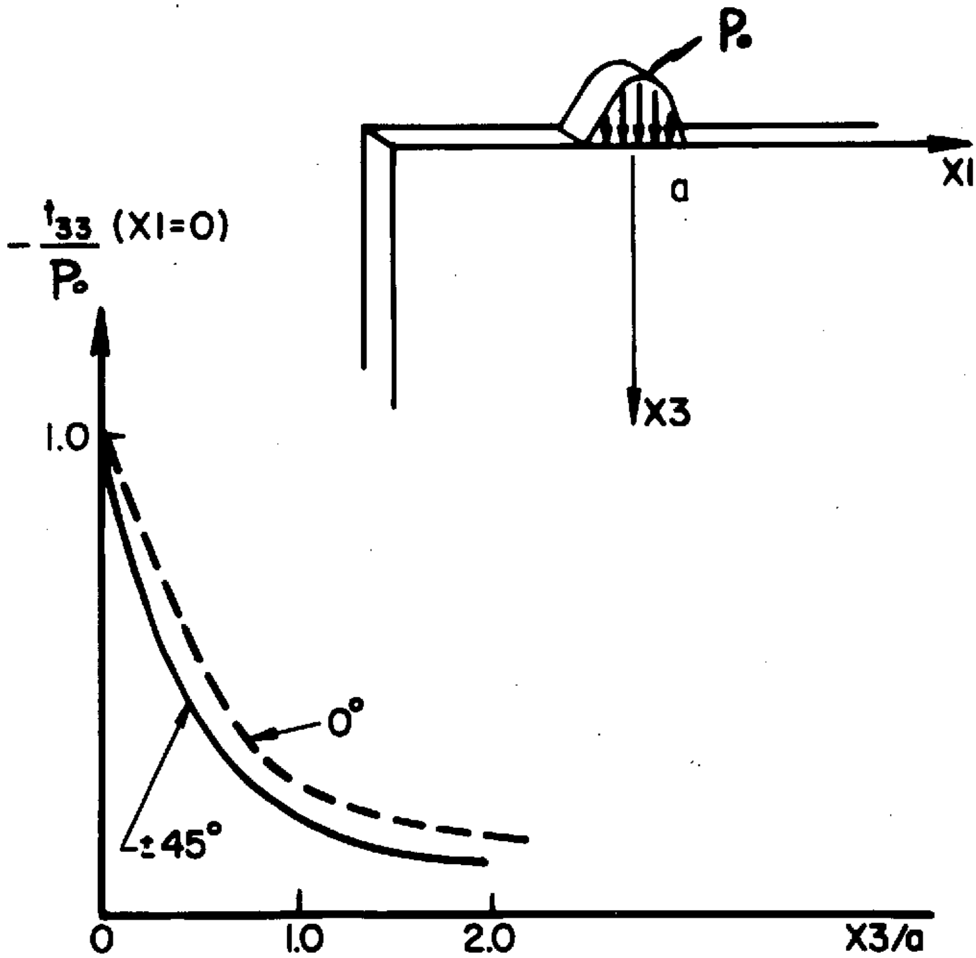


FIGURE 8

Decrease of Stress t_{33} with Distance from the Impact Edge.

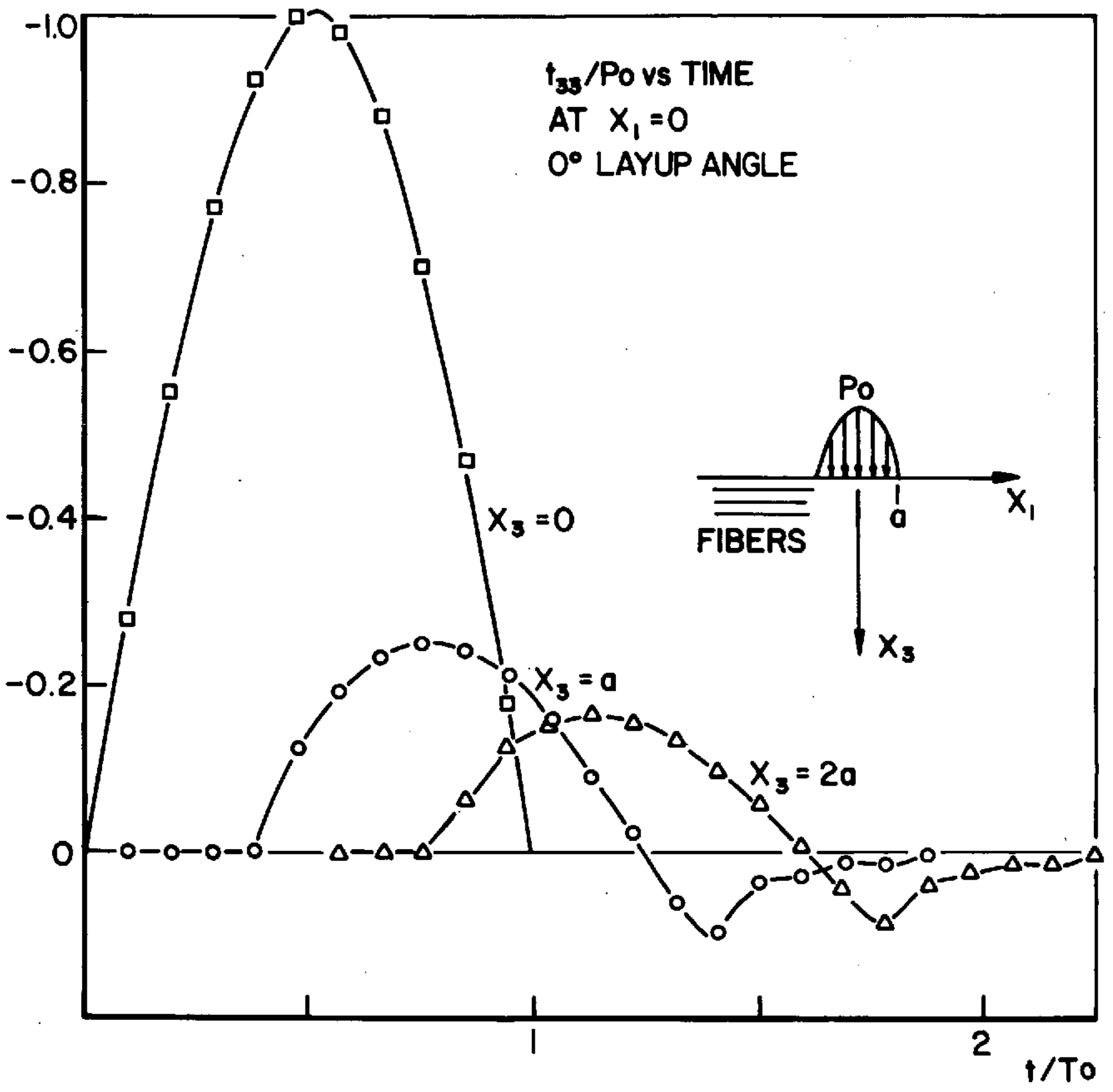


FIGURE 9

Stress t_{33} versus Time at Different Distances from the Edge under the Contact Point.

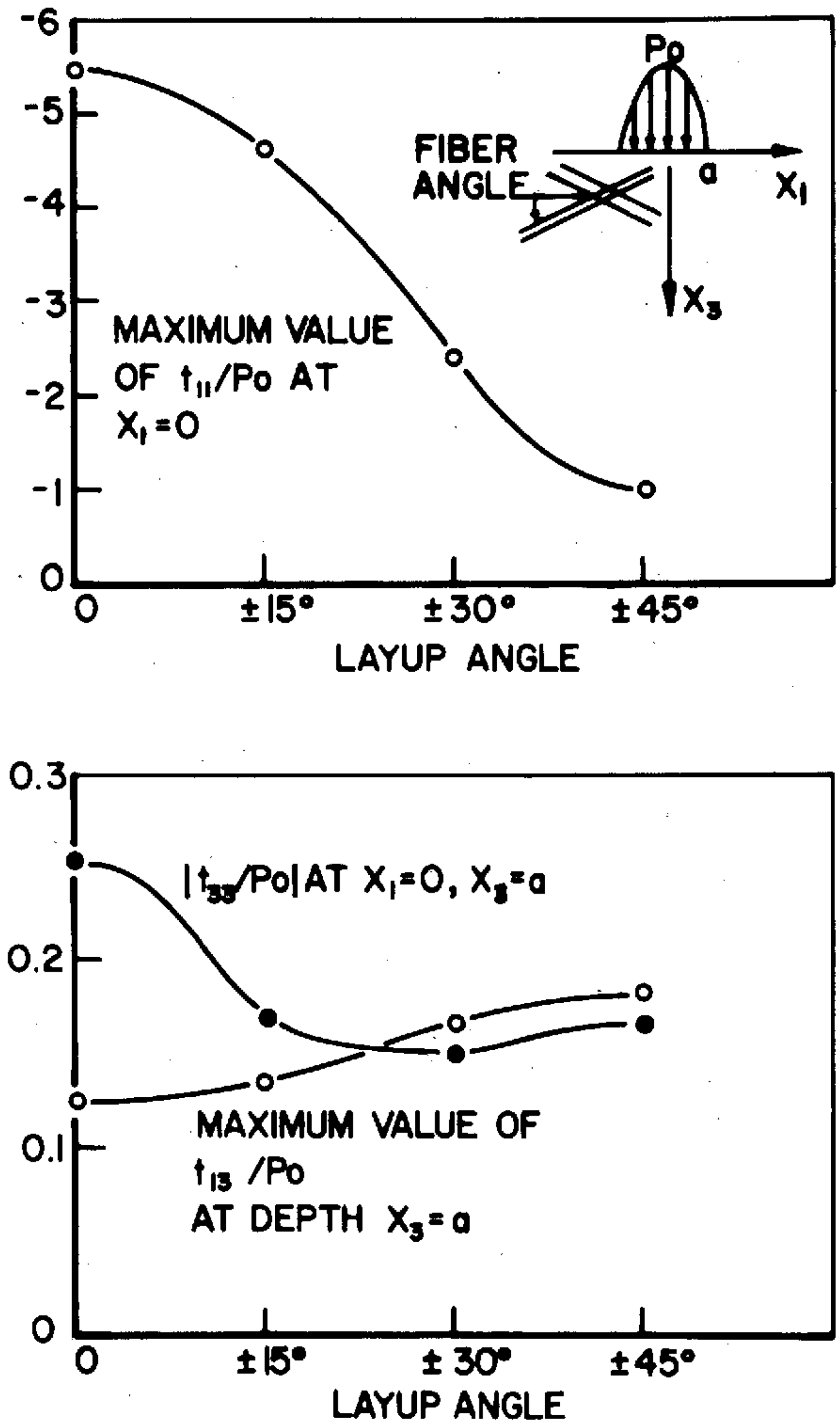


FIGURE 10

Maximum Stresses versus Layup angle for 55% Graphite Fiber/epoxy Matrix Composite.

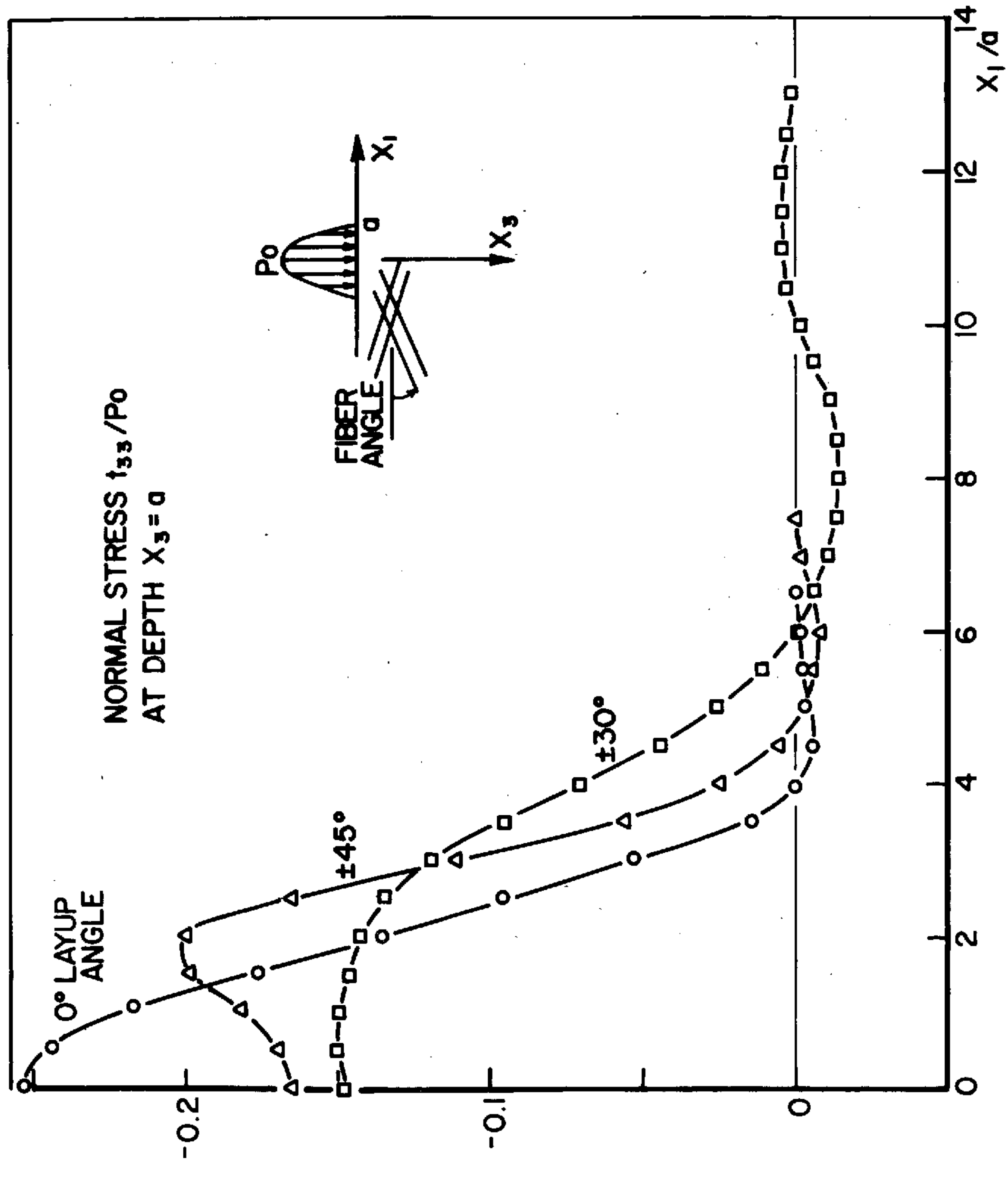
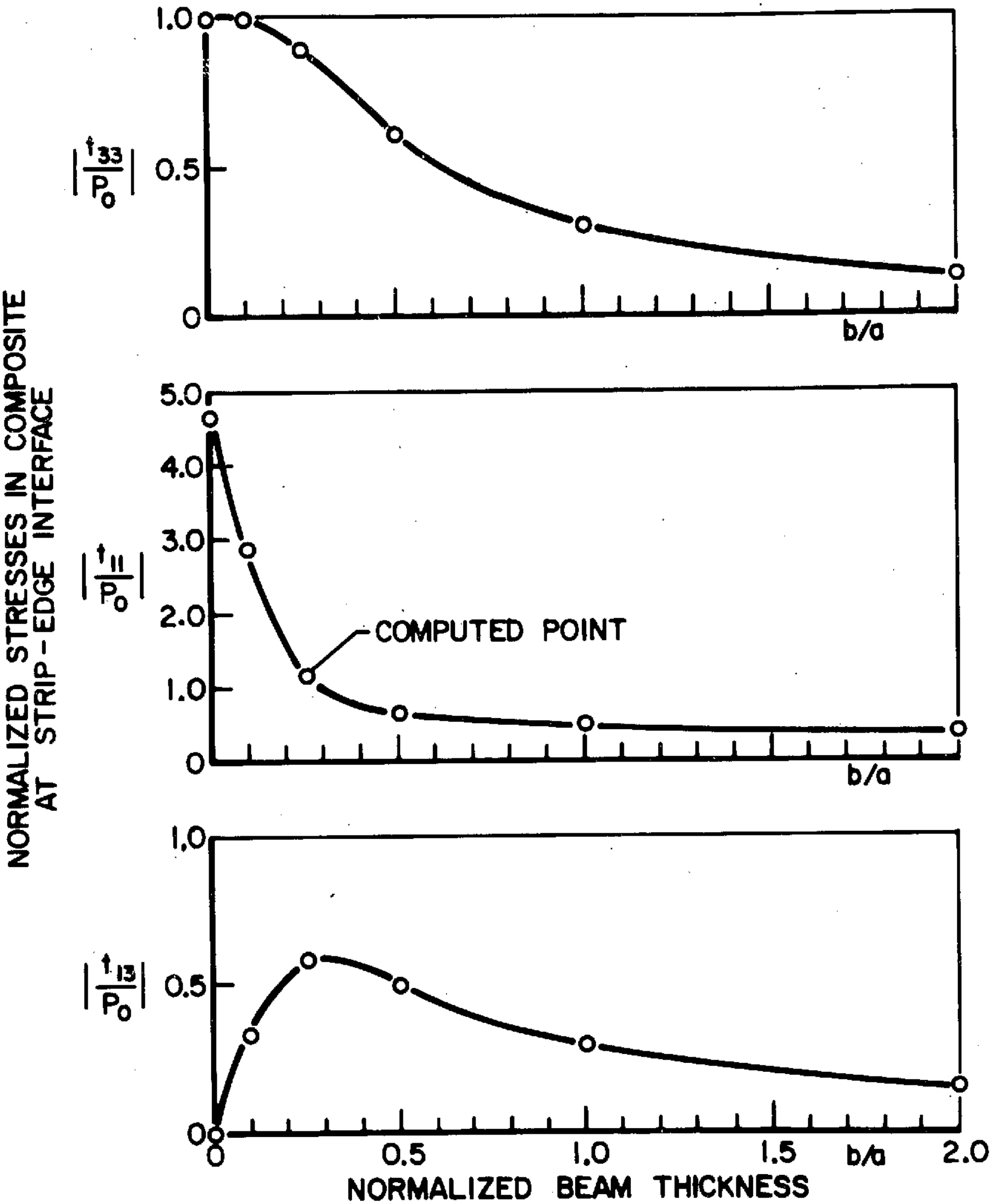


FIGURE 11

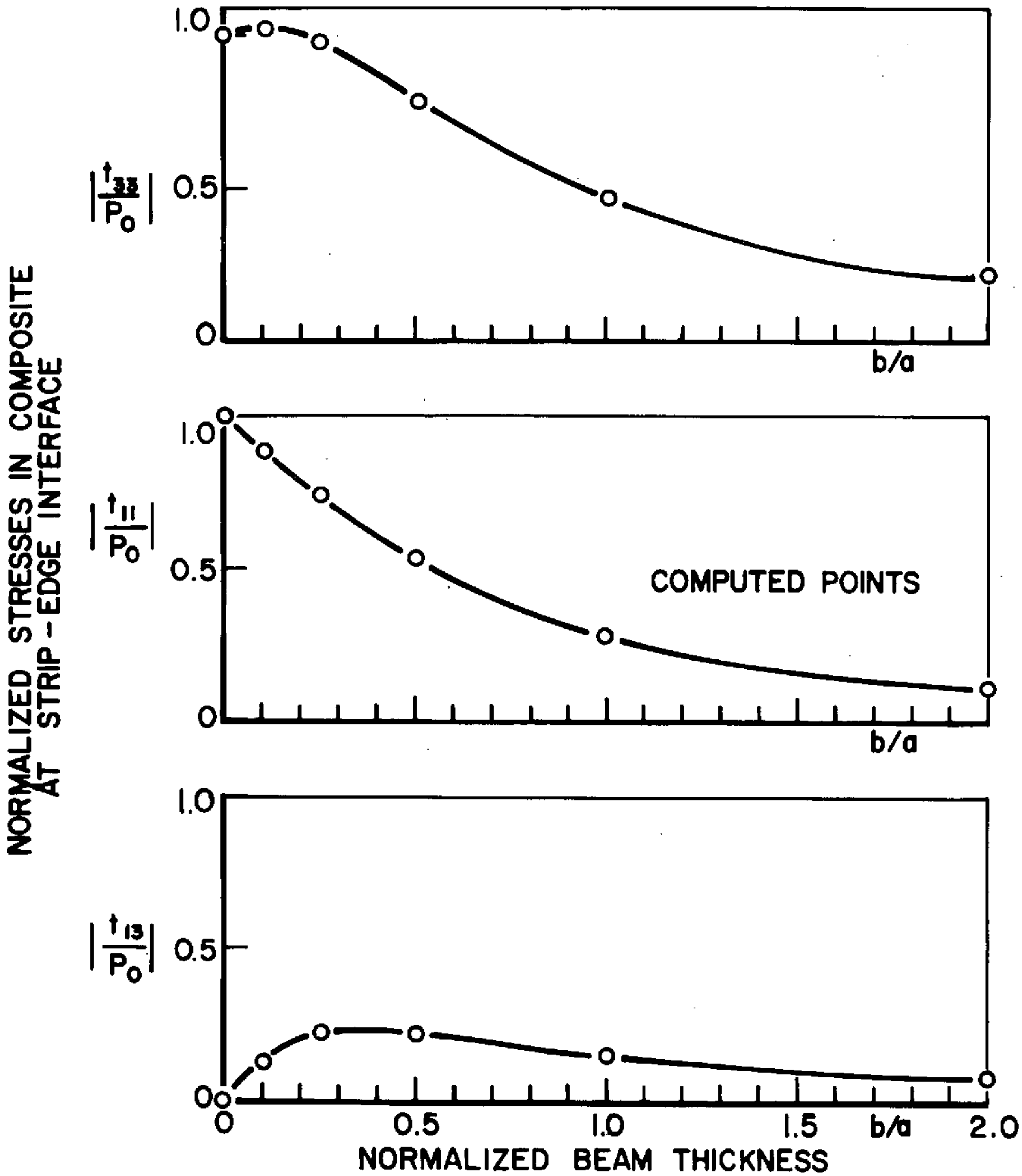
Distribution of Stress t_{33} Along the Edge for Fiber Layup angles 0^o, $\pm$ 30, $\pm$ 45^o.



DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX
± 15° LAYUP - STEEL STRIP

FIGURE 12a

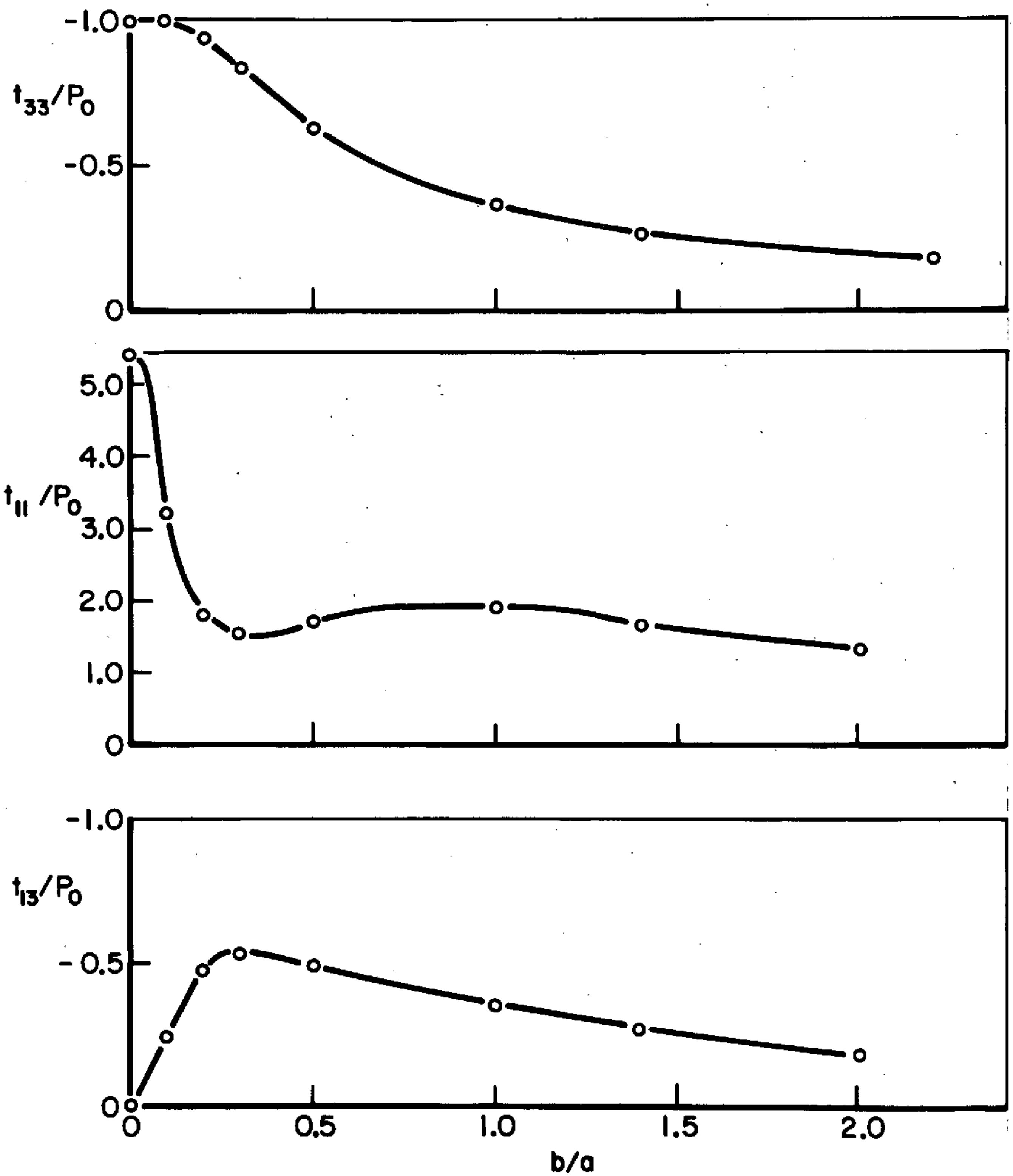
Effect of Edge Strip Thickness on Interface Stresses.



DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX
±45° LAYUP ANGLE - STEEL STRIP

FIGURE 12b

Effect of Edge Strip Thickness on Interface Stresses.



0° LAYUP ANGLE ALUMINUM STRIP

$X_3 = 0$, $a = 1\,cm$, $T_0 = 35\,\mu sec$

FIGURE 13

Effect of Edge Strip Thickness on Interface Stresses.

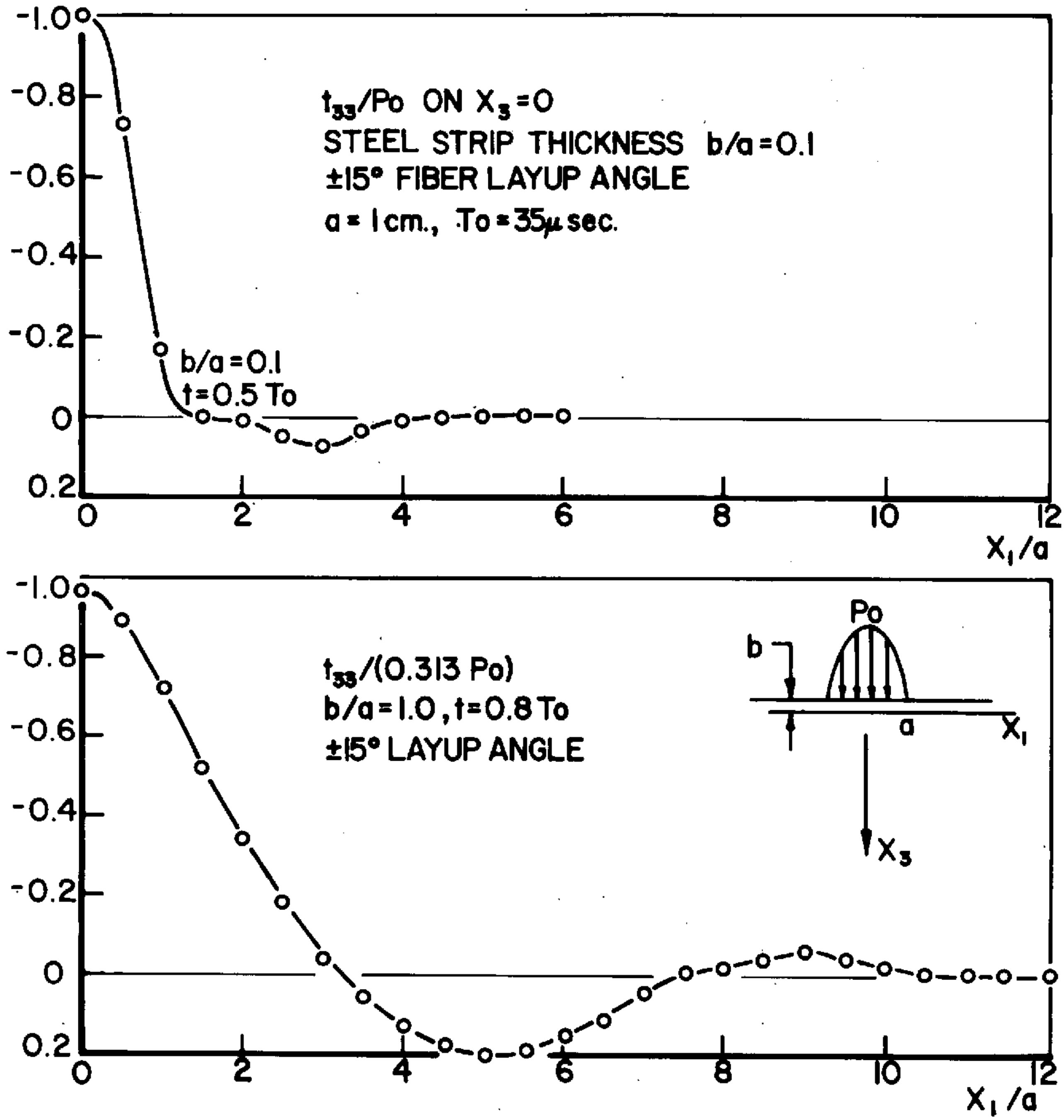


FIGURE 14

Effect of Edge Strip Thickness on Stress t_{33} Distribution Along the Edge.

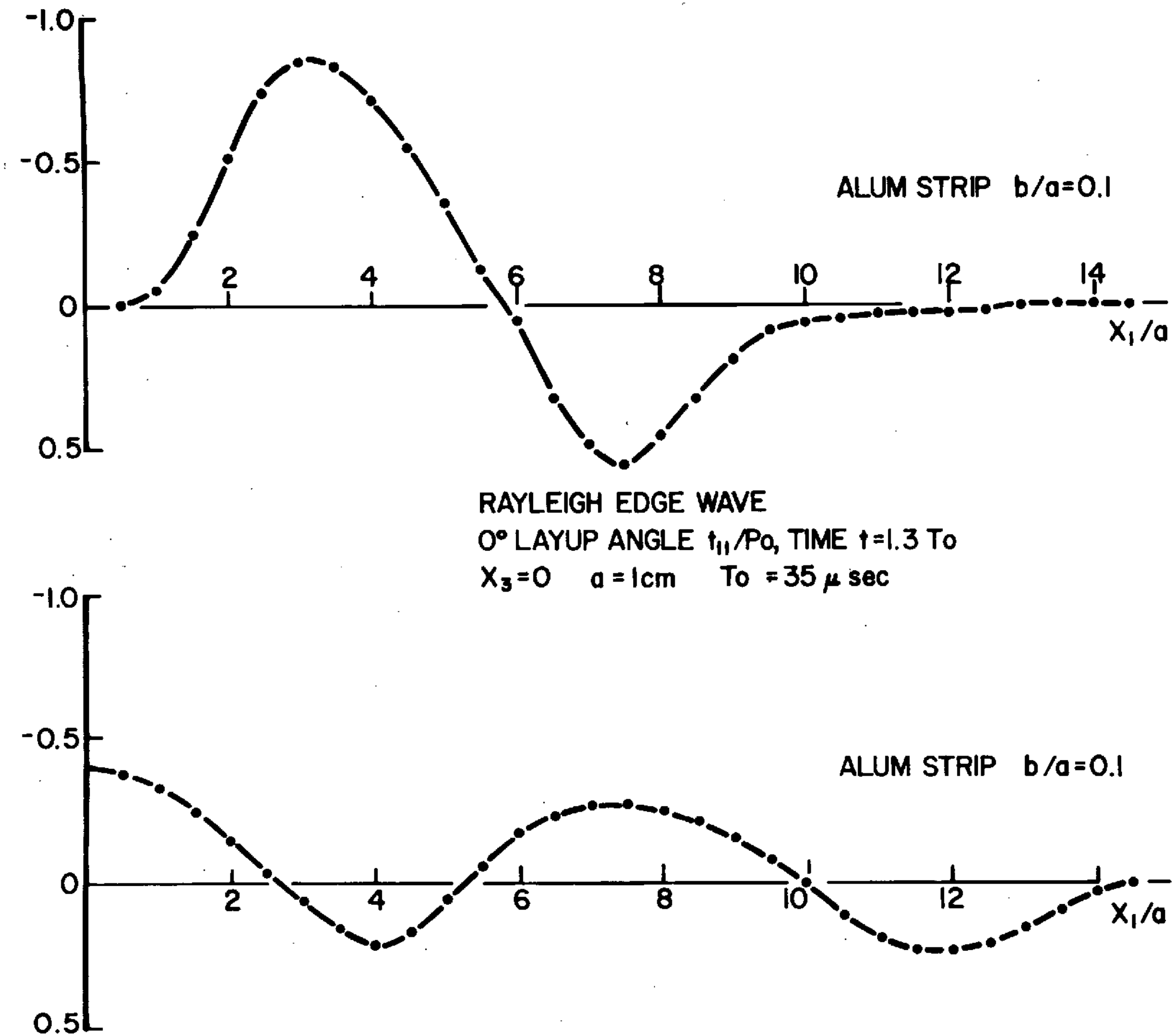


FIGURE 15

Effect of Edge Strip Thickness on the Rayleigh Edge Wave Shape.

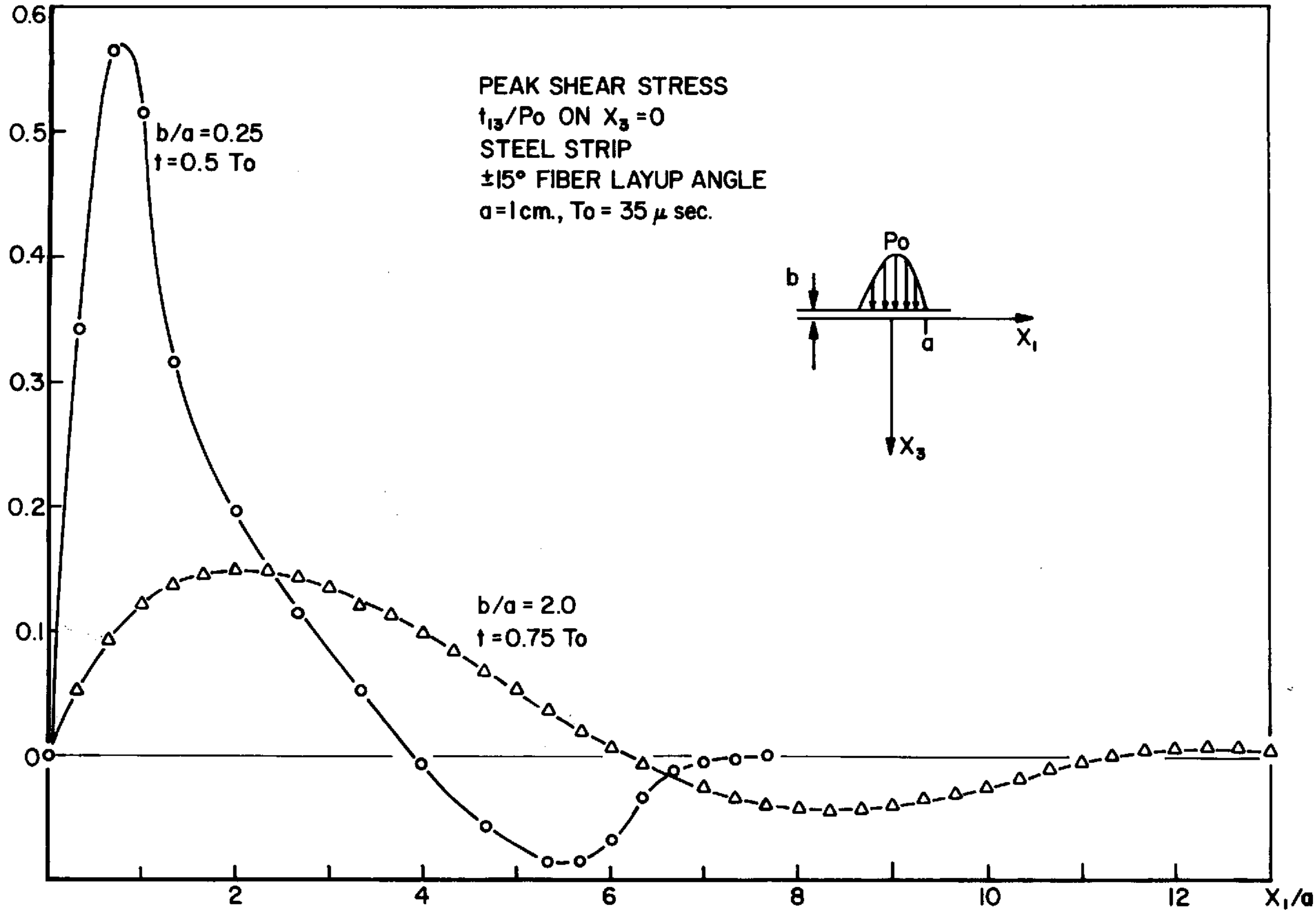


FIGURE 16

Effect of Edge Strip Thickness of Interface Shear Stress t_{13} Distribution Along the Edge.