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TWO TRANSDUCER FORMULA FOR MORE PRECISE DETERMINATION OF ULTRASONIC PHASE VELOCITY FROM STANDING WAVE MEASUREMENTS†

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ABSTRACT. Ultrasonic velocity measurements in solids and liquids using standing-wave techniques cannot be accurately analyzed without the use of an appropriate transducer correction formula. We discuss the two-transducer (transmission) case. An improved transducer correction is derived which is substantially more accurate than all previous approximations over the range of parameters corresponding to velocity determinations in both solids and liquids. Previous approximations are useful only over very limited ranges. We discuss the relationship between the present result and a previously derived result for the one-transducer case. Computer simulations of velocity measurements demonstrate the accuracy of our formula under a wide variety of conditions.

I. Introduction

Standing wave ultrasonic measurements can be performed in either the reflection mode (one transducer) or the transmission mode (two transducers). For the determination of ultrasonic phase velocity it is essential in either case to incorporate an appropriate transducer correction.

An improved velocity correction for the one transducer case was discussed previously.[1] The two transducer problem is discussed only briefly in the literature.[2,3] The approximations adequate under narrow constraints in solids, for example, are found to be totally inadequate in liquid studies.[4]

In this paper we present a substantially more accurate two transducer correction formula valid for both solid and liquid specimens. Using computer simulations of velocity measurements the accuracy and range of validity of the new results are discussed and are compared with previous approximations.

II. Theory

The system we treat is a one dimensional composite resonator consisting of a specimen (with properties characterized by subscript s) and two identical transducers (subscript t). We wish to find the sound velocity v_s in the specimen having measured a set of mechanical resonance frequencies ν_c^n of the composite system. Using a transmission line analogy, Bolef and Menes[3] treat the three-element system consisting of a transducer, bond, and specimen. The transducer-specimen-transducer problem can be formulated using a similar approach.[2]

Provided that the ultrasonic attenuation is not excessive, the mechanical resonance frequencies ν_c^n of the composite system are the solutions of

$$\tan\theta_s - 2\left(\frac{r-1}{r+1}\right)\tan\theta_t - \left(\frac{r-1}{r+1}\right)^2 \tan^2\theta_t \tan\theta_s = 0 \qquad (1)$$

where

$$\theta_s = \frac{\pi\nu_c^n}{\Delta\nu_s} \quad \text{and} \quad \theta_t = \frac{\pi\nu_c^n}{\nu_t}$$

and where

$$r = (\rho_s v_s - \rho_t v_t)/(\rho_s v_s + \rho_t v_t)$$

is the reflection coefficient from specimen to transducer. The notation employed is the same as in the one transducer case.[1] For computer evaluation purposes Eq. (1) can be written in the equivalent form

$$\sin(2\theta_t + \theta_s) - r^2 \sin(2\theta_t - \theta_s) + 2r\sin\theta_s = 0 \quad .(2)$$

In the absence of transducers, the isolated specimen resonances would be equally spaced at intervals $\Delta\nu_s = v_s/2\ell_s$. However, the observed resonance spacings $\Delta\nu_c^n = \nu_c^{n+1} - \nu_c^n$ of the composite system, vary with the particular mechanical resonance pair. This leads to certain inaccuracies encountered with the so-called "uncorrected formula"

$$v_s = 2\ell_s\,\Delta\nu_c^n \quad . \qquad (3)$$

For resonances not too far from the transducer resonance frequency ν_t, an approximation[2] to Eq. (1) yields

$$v_s \simeq 2\ell_s\,\Delta\nu_c^n(1+2\delta) \qquad (4)$$

where $\delta \equiv \rho_t\ell_t/\rho_s\ell_s$. Anticipating the results of Section III, this "$1+2\delta$ formula" is found to be adequate only for experiments with solids, where δ is typically less than ~0.02.

Using Eq. (1), we develop an improved transducer correction useful over a wider range of δ. Equation (1) is quite complex in comparison with the one transducer resonance equation (Eq. (4) of Ref. 1). Using trigonometric substitutions, however, Eq. (1) is found to be factorable into two simpler equations,

$$\tan\frac{\theta_s}{2} - \left(\frac{r-1}{r+1}\right)\tan\theta_t = 0 \qquad (5)$$

$$\cot\frac{\theta_s}{2} + \left(\frac{r-1}{r+1}\right)\tan\theta_t = 0 \quad . \qquad (6)$$

(These two equations are identical to the one-transducer resonance equation of Ref. 1 provided that ℓ_s is replaced by $\ell_s/2$.) The solutions of (5) and (6) comprise the full set of two transducer resonances, symmetric and antisymmetric,[5] respectively. Taken separately, Eqs. (5) and (6) describe alternate resonances.

In order to develop a velocity approximation dependent upon single spacings $\Delta\nu_c^n = \nu_c^{n+1} - \nu_c^n$, we subtract Eq. (6) written for the n^{th} resonance from Eq. (5) written for the $(n+1)^{th}$ resonance. (The approximate expression for the velocity which is obtained is identical to that obtained if Eq. (5) for ν_c^{n+1} is subtracted from Eq. (6) for ν_c^{n+2}.) Following a procedure similar to that used for the one transducer case and employing the approximation $\Delta\nu_c^n \approx \Delta\nu_s$, we arrive at the velocity correction formula for the two transducer case,

$$v_s = 2\ell_s\,\Delta\nu_c^n\left[1 + 2\delta\left(\frac{\pi D\Delta\nu_c^n}{\nu_t} + \frac{\pi\delta^2\nu_t}{\Delta\nu_c^n}\left(DT_n^2 + T_n\right)\right)\right]^{-1} \quad , (7)$$

where

$$T_n = \tan \pi\, \nu_c^n / \nu_t$$

$$D = \left(T_{n+1} - T_n\right)^{-1} \ .$$

Although this expression differs only slightly from Eq. (6) of Ref. 1, the appropriate generalization is by no means obvious a priori. In particular, the replacement of δ by 2δ, as might be suggested by Eq. (4), is not appropriate.

In Section III we compare the accuracy of this new result with that of the $1+2\delta$ formula, the uncorrected formula, and an empirical approximation technique sometimes used in solid studies.

III. Discussion

In order to examine the behavior of the various approximate formulas for v_s, numerical iteration was used to find to an accuracy of 1 part in 10^{10} the frequencies ν_c^n satisfying Eq. (2) for an assumed set of parameters ρ_s, ρ_t, v_s, ℓ_s, ℓ_t. In these simulations, all arbitrary parameters were given values typical of those encountered in ultrasonic experiments. The resulting set of simulated mechanical resonance frequencies was used in conjunction with the expressions for v_s to compute approximate values for the phase velocity. The percent error for each approximation was computed with respect to the value of v_s assumed in the initial iteration.

In general, experiments involving solid samples feature smaller values of δ than do experiments on liquids. Accordingly, we somewhat arbitrarily choose to divide the range of δ studied into two parts. We call the region where $\delta < 0.02$ the "solids" region and the region where $\delta > 0.02$ the "liquids" region. The approximations analyzed include the uncorrected formula (Eq. (3)), the $1+2\delta$ formula (Eq. (4)), our new approximation (Eq. (7)), and an additional expression which we shall call the Bolef-Menes formula.[3,6,7] The Bolef-Menes formula involves rounding to the nearest integer m the following expression for m':

$$m' = \frac{\nu_c^n}{\Delta\nu_c^n}\,(1 - 2\delta) \quad . \tag{8a}$$

The approximate expression for the velocity is

$$v_s = \frac{2\ell_s\, \nu_c^n}{m}\left[1 + 2\delta\left(\frac{\nu_c^n - \nu_t}{\nu_c^n}\right)\right] . \tag{8b}$$

In Figure 1 we display the percent error for each of the various approximations over the range of δ typical of experiments on solid specimens ($\delta < 0.02$). The pair of mechanical resonances used in the calculations were the fourth and fifth on the high-frequency side of ν_t. The $1+2\delta$ formula (Eq. (4)) is seen to result in errors of roughly 1 part in 10^4 over most of the range. In contrast, the uncorrected formula (Eq. (3)) yields errors approximately two orders of magnitude larger than those resulting from the $1+2\delta$ formula. (The cusp-like behavior near $\delta = 0.01$ for Eqs. (4) and (8) and near $\delta = 0.004$ for Eq. (7) is due to a change in sign of the error.)

The Bolef-Menes expression (Eq. (8)) is also presented in Figure 1. We note that Eq. (8) is superior to Eq. (4) over the range of δ shown by roughly two orders of magnitude. Equation (8) rapidly deteriorates as δ nears 0.02, rendering it unusable in the "liquids" region.

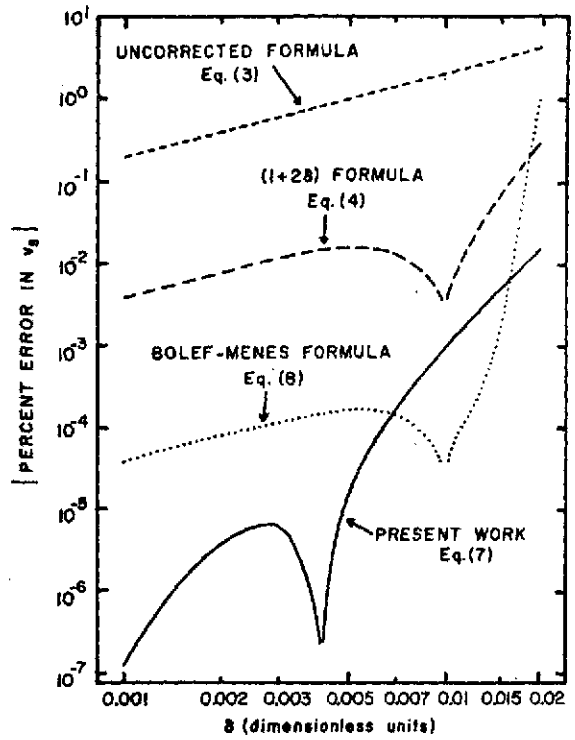

Figure 1. Absolute value of the percent error in the velocity of sound v_s versus $\delta \equiv \rho_t\ell_t/\rho_s\ell_s$ for $\delta < 0.02$ ("solids" region).

Also shown in Figure 1 are the results of the present work (Eq. (7)). The improved accuracy of Eq. (7) over the Bolef-Menes expression (Eq. (8)) may be of value only if measurements to an accuracy of 1 part in 10^7 are available.

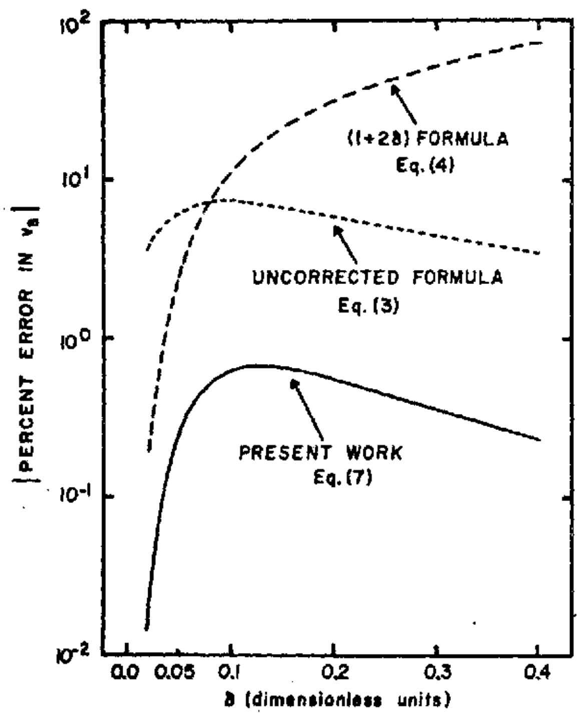

Figure 2. Absolute value of the percent error in v_s versus $\delta \equiv \rho_t\ell_t/\rho_s\ell_s$ for $\delta > 0.02$ ("liquids" region).

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Figure 2 shows the error curves in the "liquids" region ($\delta > 0.02$) for Eqs. (3), (4), and (7). (As noted above, the Bolef-Menes formula (Eq. (8)) diverges in this region.) As in Figure 1, the pair of resonances used in the calculations were the fourth and fifth on the high-frequency side of ν_t. The $1+2\delta$ formula (Eq. (4)) results in large errors for δ greater than 0.02. We note in particular that the uncorrected formula becomes more accurate than the $1+2\delta$ formula as δ increases. The accuracy of the present result (Eq. (7)) is superior to all previous results by roughly a factor of 10.

The behavior of the various approximations depends upon the distance in frequency between the resonance pair (ν_c^n, ν_c^{n+1}) and ν_t in a fashion similar to that for the one transducer case.[1] Briefly, the $1+2\delta$ formula is most accurate near ν_t, while the errors for the uncorrected formula and the present work decrease rapidly with distance from ν_t.

In situations where electromagnetic leakage complicates velocity measurements,[8] a formula involving the spacing between resonance n and n+2 may be useful. Such a double-spacing formula is given by Eq. (6) of Ref. 1 with δ replaced by 2δ and ℓ_s replaced by $\ell_s/2$. The error behavior for this double-spacing formula is very similar to that for the present result, Eq. (7).

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