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OPTIMIZING SIGNAL-TO-ERROR RATIO IN STANDING WAVE ULTRASONIC MEASUREMENTS†

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ABSTRACT. Standing wave ultrasonic techniques are well suited to the measurement of very small changes in acoustic attenuation and phase velocity. Enhanced sensitivity to these small changes is often achieved by making the specimen part of a composite ultrasonic resonator. However, a point of maximum sensitivity on the response of such an ultrasonic resonator need not coincide with a point of maximum signal-to-error ratio. We present and analyze a model which takes into account error due to long term (i.e., low frequency) noise effects such as gain drifts and dc level shifts. This model yields a quantitative value for the signal-to-error ratio, where we define the signal as the ideal change in the monitored response and the error as the difference between the experimentally measured change and the signal. The specific frequency dependent forms for the ultrasonic response and the sensitivity enhancement factor are utilized to predict the operating point on a mechanical resonance corresponding to maximum signal-to-error ratio.

I. Introduction

Continuous (i.e., standing) wave ultrasonic techniques are well suited to the measurement of very small changes in acoustic attenuation and acoustic phase velocity.[1] The specimen under investigation typically forms part of a composite ultrasonic resonator which provides enhanced sensitivity to these small absorptive or dispersive changes.[2] In this paper we discuss certain signal-to-error ratio considerations relevant to standing wave ultrasonic measurements. Focusing our attention on the case of cw reflection experiments, we explore the implications of these signal-to-error ratio considerations. We explicitly consider phase-sensitive as well as simple diode detection schemes.

Errors resulting from certain types of instrumental drift are considered in Section II. The dependence of these errors on the relative magnitudes of the signal and the background ultrasonic response is explicitly obtained. In Section III the specific frequency dependent forms for the background responses are presented. We introduce the sensitivity enhancement factors in Section IV and use them to predict the operating point on a mechanical resonance corresponding to the maximum value of the signal-to-error ratio.

II. Signal-to-Error Ratio Considerations

In a large number of ultrasonic experiments, a small change $\Delta\alpha$ (or Δv) in the acoustic attenuation α (or phase velocity v) is measured as a function of some external variable, e.g., magnetic field, temperature, or pressure. Let us suppose that for an initial value of this external variable the measured quantity is R. We define $R+\Delta R$ as the measured quantity corresponding to the final value of this external variable. The magnitude of the small change in ultrasonic attenuation (or phase velocity) must be deduced from the measurement of ΔR. For an ideal system (i.e., one in which there is no noise), $R = Gr$ and $\Delta R = G\Delta r$, where G is the system gain and r is the background ultrasonic response presented to the input of the first amplifier. It is clearly desirable to arrange for a large change Δr. It is also desirable to measure the change Δr in the presence of a rather small background response r in cases where the signal of interest occurs on a sufficiently long time scale that the system must be dc coupled.

In order to bring out the essential features introduced by the presence of non-ideal components, we consider the following "worst case" situation. We suppose that at the same time the external variable is changed from the initial to the final value, the gain of the system abruptly drifts from G to $G(1+\varepsilon)$ and the dc level shifts from 0 to the value γ. Thus the initial measured quantity is $R = Gr$, and the final measured quantity is $(R+\Delta R) = [(r+\Delta r)G(1+\varepsilon) + \gamma]$. The resulting error is given by $[G\varepsilon(|r| + |\Delta r|) + \gamma]$, where the absolute value signs prevent possible cancellation of r and Δr, in keeping with our "worst case" treatment. The signal-to-error ratio (S/E) is

$$S/E = G|\Delta r|/[G\varepsilon(|r| + |\Delta r|) + \gamma] . \quad (1)$$

[The gain and dc level drifts of individual components of the system are related to ε and γ in Appendix A.]

In order to bring out certain features of the signal-to error ratio [Eq. (1)], it is desirable to examine cases in which the importance of the ε drift is less than, comparable to, or greater than the importance of the γ drift. In Appendix B we define a variable m such that $m > 1$ specifies γ dominance and $m < 1$ specifies ε dominance.

In Figure 1, the signal-to-error ratio is plotted versus $|\Delta r|/|r|$ for constant $|\Delta r|$ and for several values of m. Increasing the ratio $|\Delta r|/|r|$ is seen to produce dramatic improvement in the signal-to-error ratio in the region of large background response, i.e., $|\Delta r|/|r| \ll 1$, but to produce no significant effect in the region $|\Delta r|/|r| \gg 1$.

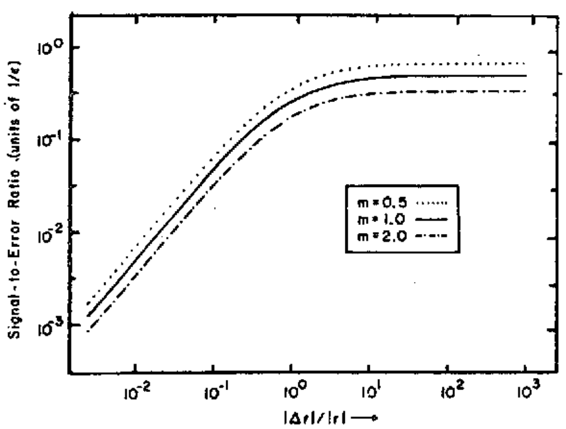

Figure 1. Signal-to-error ratio in units of $(1/\varepsilon)$ as a function of the ratio of the change in response $|\Delta r|$ to the background response $|r|$ for $m = 0.5$, 1.0, and 2.0.

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When combined with the explicit frequency dependences of $|r|$ and of $|\Delta r|$, Eq. (1) gives the frequency dependence of the signal-to-error ratio. In cases where varying the frequency to increase $|\Delta r|/|r|$ results in a decrease in $|\Delta r|$, the operating point on the mechanical resonance corresponding to the optimum trade-off is determined by the maximum value of the signal-to-error ratio.

III. Standing Wave Ultrasonic Responses

The discussion in the previous section indicates that it is often desirable to select an operating point corresponding to a relatively small value of $|r|$, provided that this can be done without unduly reducing $|\Delta r|$. Since some flexibility is permitted in the selection of the monitored response, in this section we explicitly examine the possible choices of r.

The standing wave response of a one-dimensional ultrasonic resonator driven as cos ωt and monitored in reflection is[1]

$$A_0 = A_0' \cos \omega t + A_0'' \sin \omega t \qquad (2a)$$

where

$$A_0' = \frac{e^{\alpha a} - \cos ka}{2(\cosh \alpha a - \cos ka)} \qquad (2b)$$

and

$$A_0'' = \frac{\sin ka}{2(\cosh \alpha a - \cos ka)} \quad . \qquad (2c)$$

A_0' and A_0'' are the in- and out-of-phase components of the ultrasonic response. The magnitude of the response is given by

$$|A_0| = \sqrt{A_0'^2 + A_0''^2} = \frac{e^{\alpha a/2}}{\sqrt{2}(\cosh \alpha a - \cos ka)^{1/2}} \quad . \qquad (2d)$$

[$|A_0|$, A_0', and A_0'' were called $|A|$, A_1, and A_2, respectively, in reference (1).] Here α is the acoustic attenuation, a is twice the sample length, and $k = \omega/v$ is the acoustic wavenumber. A_0 is a periodic function which exhibits mechanical resonances at frequencies such that the sample length a/2 corresponds to an integral number of half wavelengths. Using simple diode detection one obtains $|A_0|$, while with the phase-sensitive detection one may choose either the in-phase component A_0' or the out-of-phase component A_0''.

When measuring very small changes in ultrasonic attenuation or phase velocity it is sometimes possible to "tag" the signal of interest with an audio frequency modulation ω_1. Such a modulated signal can be processed by a lock-in detector which provides substantial discrimination against noise. Typically this "tagging" is accomplished by frequency modulating the rf oscillator or by the use of magnetic field modulation. Making use, for example, of some internal coupling between the magnetic and ultrasonic properties of the system, one can induce an audio frequency variation (i.e., an amplitude modulation at frequency ω_1) in the specimen's ultrasonic attenuation and/or phase velocity. Two cases of particular interest are 1) the use of an fm technique for the precise determination of ultrasonic phase velocity[1,3] and 2) the use of magnetic field modulation to measure small changes in attenuation and phase velocity associated with nuclear acoustic resonance and acoustic paramagnetic resonance.[1,4]

For clarity we limit our discussion to cases in which simple diode detection is used to monitor $|A_0|$. The "effective response" under these conditions is that part of $|A_0|$ which varies sinusoidally at the audio frequency ω_1. If the attenuation is time varying one substitutes $\alpha \to \alpha + \delta\alpha \cos \omega_1 t$ into the expression for $|A_0|$ [Eq. (2d)]. Since $k = \omega/v$, the substitution $k \to k + \delta k \cos \omega_1 t$ accounts for either oscillator frequency modulation (time varying ω) or specimen phase velocity modulation (time varying v).

The algebraic reduction of Eq. (2d) with the appropriate substitution for α or k (or possibly both) is straightforward but tedious. The "total response" is found to include terms which are time-independent and terms which occur at harmonics of the audio frequency ω_1. Ordinarily a lock-in is used to obtain the "effective response" which consists of only that part of the total response which varies as $\cos \omega_1 t$. One thus obtains $[\partial|A_0|/\partial\alpha]\delta\alpha \cos \omega_1 t$ and/or $[\partial|A_0|/\partial k]\delta k \cos \omega_1 t$ where the "effective responses" are

$$r_{eff}(\delta a) = \partial|A_0|/\partial\alpha = \frac{a[\exp(-\frac{1}{2}\alpha a) - \exp(\frac{1}{2}\alpha a)\cos ka]}{2\sqrt{2}(\cosh \alpha a - \cos ka)^{3/2}} \qquad (3a)$$

and

$$r_{eff}(\delta k) = \partial|A_0|/\partial k = \frac{-a \exp(\frac{1}{2}\alpha a)\sin ka}{2\sqrt{2}(\cosh \alpha a - \cos ka)^{3/2}} \, . \qquad (3b)$$

In practice one typically monitors one of five responses given by Eqs. (2b), (2c), (2d), (3a), or (3b). The optimum choice of response and the appropriate frequency on that response are the subjects of the next section.

IV. Optimization of Signal-to Error

In this section we incorporate the frequency dependent responses r of the previous section and the corresponding frequency dependent forms of Δr into the signal-to-error ratio formalism of Section II. Our goal is to specify the appropriate response and operating point (i.e., frequency) on that particular response at which to measure certain small changes in acoustic attenuation or phase velocity.

In the most general case, a given change in an external variable results in small changes ($\Delta\alpha$ and Δv) in both ultrasonic attenuation and phase velocity. For any choice of the monitored response r the corresponding small change Δr is given approximately by

$$\Delta r = \frac{\partial r}{\partial \alpha}\Delta\alpha + \frac{\partial r}{\partial k}\Delta k \ . \qquad (4)$$

The factors $\partial r/\partial\alpha$ and $\partial r/\partial k$ are known as the sensitivity enhancement factors for absorption and dispersion, respectively. We divide our discussion into two parts, focusing first on experiments designed to measure changes in ultrasonic attenuation and second on experiments measuring changes in ultrasonic phase velocity.

A. Measurement of Absorption ($\Delta\alpha$)

We first consider the class of experiments in which dispersion is negligible, i.e., $(\partial r/\partial\alpha)\Delta\alpha >> (\partial r/\partial k)\Delta k$. Later we discuss the more general situation described by Eq. (4), in which one must discriminate against dispersive effects Δk in order to get a correct measure of $\Delta\alpha$ from a change in the response Δr. Of interest are the particular response r, the change Δr in that response, and the resulting signal-to-error ratio. We explicitly consider each of the five responses A_0', A_0'', $|A_0|$, $r_{eff}(\delta\alpha)$ (i.e., $\partial|A_0|/\partial\alpha$), and $r_{eff}(\delta k)$ (i.e., $\partial|A_0|/\partial k$). The Δr values of interest thus are proportional to the appropriate derivatives of r,

$$A_0': \quad \partial A_0'/\partial\alpha = \frac{(a/2)(1-\cosh\alpha a \cos ka)}{(\cosh\alpha a - \cos ka)^2} , \quad (5a)$$

$$A_0'': \quad \partial A_0''/\partial\alpha = \frac{-(a/2)\sinh\alpha a \sin ka}{(\cosh\alpha a - \cos ka)^2} , \quad (5b)$$

$$A_0: \partial|A_0|/\partial\alpha = \frac{a[\exp(-\frac{1}{2}\alpha a) - \exp(\frac{1}{2}\alpha a)\cos ka]}{2\sqrt{2}(\cosh\alpha a - \cos ka)^{3/2}} , \quad (5c)$$

$$r_{eff}(\delta\alpha): \quad \partial^2|A_0|/\partial\alpha^2 =$$

$$= \frac{a^2\exp(\frac{1}{2}\alpha a)(2\cos ka \sinh\alpha a - \sin^2 ka - 2\exp(-\alpha a)\sinh\alpha a)}{4\sqrt{2}(\cosh\alpha a - \cos ka)^{5/2}} \quad (5d)$$

$$r_{eff}(\partial k): \quad \partial^2|A_0|/\partial\alpha\, \partial k =$$

$$= \frac{a^2\exp(\frac{1}{2}\alpha a)\sin ka(\cos ka - \cosh\alpha a + 3\sinh\alpha a)}{4\sqrt{2}(\cosh\alpha a - \cos ka)^{5/2}} . \quad (5e)$$

[Although we are modulating k in Eq. (5e), we note that the response $r_{eff}(\delta k)$ still exhibits a sensitivity to changes $\Delta\alpha$ in ultrasonic attenuation.]

In the a) panels of Figures 2-6 we display the frequency dependences of r and Δr. The frequency dependence of Δr is the same as that of the appropriate sensitivity enhancement factor $\partial r/\partial\alpha$ since $\Delta r \equiv (\partial r/\partial\alpha)\Delta\alpha$. The resulting signal-to-error ratio for each of the responses considered is shown in the b) panels. The functions are plotted over a range of frequencies centered about a single mechanical resonance. The signal-to-error ratios are plotted for several values of the parameter m which specifies the relative importance of ε-like and γ-like contributions to the error. All signal-to-error ratios are quantitatively specified on the left ordinate in dimensionless units of $\Delta\alpha/\alpha\varepsilon$. These are the natural units for absorptive signal-to-error ratio when γ is related to ε by the parameter m, as discussed in Appendix B.

Also shown in the a) panels of Figures 2-6 are the corresponding dispersion sensitivity enhancement factors $\partial r/\partial k$. [See Eq. (6) below.] These are of interest in the more general case [Eq. (4)], where we must determine $\Delta\alpha$ in the presence of dispersion effects [i.e., when $(\partial r/\partial k)\Delta k$ is not negligible in comparison with $(\partial r/\partial\alpha)\Delta\alpha$]. This matter is considered in what follows.

Table 1 summarizes the results for absorption presented in Figures 2-6. Particular points (i.e., frequencies) on the responses are single out because they represent optimum conditions for the measurement of $\Delta\alpha$. Points on the responses are specified in terms of angular frequency ω relative to the frequency ω_0 corresponding to the center of the response. The natural unit of frequency is $\omega_\alpha \equiv \alpha v$. [The response A_0', for example, is at one-half of its maximum value when $(\omega - \omega_0) = \omega_\alpha$.]

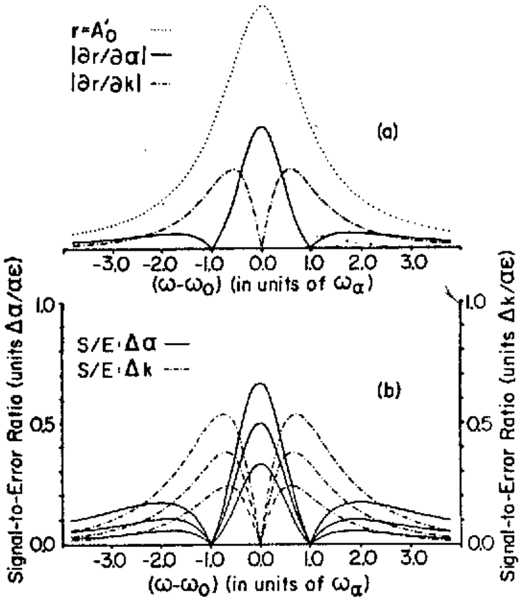

Figure 2. (a) In-phase component A_0' superimposed upon its absorption $(\partial A_0'/\partial\alpha)$ and dispersion $(\partial A_0'/\partial k)$ sensitivity enhancement factors as functions of frequency $(\omega - \omega_0)$ over the range of one mechanical resonance. (b) Signal-to-error ratio versus frequency for absorption (solid line, left ordinate, units of $\Delta\alpha/\alpha\varepsilon$) and dispersion (dashed line, right ordinate, units of $\Delta k/\alpha\varepsilon$). S/E is plotted for m = 0.5, 1.0 and 2.0.

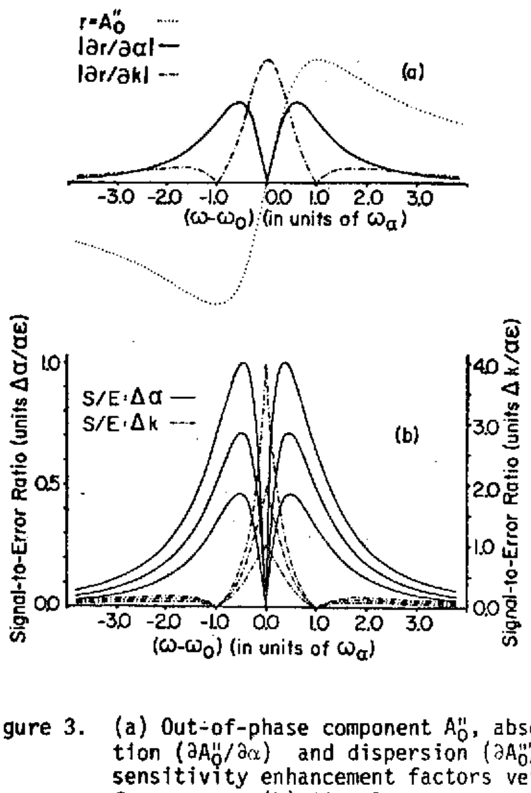

Figure 3. (a) Out-of-phase component A_0'', absorption $(\partial A_0''/\partial\alpha)$ and dispersion $(\partial A_0''/\partial k)$ sensitivity enhancement factors versus frequency. (b) Signal-to-error ratios for the measurement of absorption (solid line, left ordinate) and dispersion (dashed line, right ordinate) for the response A_0''.

When more than one point on a particular response is singled out in Table 1, the appropriate choice is to be made on the basis of details specific to a particular experiment. The maximum signal-to-error ratio for $r = A_0''$, for example, occurs at $(\omega - \omega_0) = \pm 0.4\ \omega_\alpha$. There is, however, a substantial sensitivity to dispersion at that point. Thus, in an experiment in which Δk is non-negligible, it might be desirable to operate instead at $(\omega - \omega_0) = \pm 1.0\ \omega_\alpha$. This would result in about a 30% reduction in signal-to-error ratio, which might be an acceptable price to pay to achieve a pure absorption, rather than mixed absorption-dispersion, signal. Another possible consideration is the availability of an appropriate frequency locking scheme for accurately maintaining the oscillator frequency at the chosen point on an ultrasonic response.[5] If such a scheme is not used, then an operating point corresponding to a relative maximum or minimum of the monitored response offers certain advantages. Specifically, little drift in output occurs as a result of small frequency deviations between the rf oscillator and the operating point on the mechanical resonance. (Both the mechanical resonance and the oscillator are subject to frequency drift.)

The summary in Table 1 indicates that the use of the response $r = r_{eff}(\delta k)$ (i.e., $\partial|A_0|/\partial k$) at the frequency $(\omega - \omega_0) = \pm 0.7\ \omega_\alpha$ offers several advantages for the measurement of absorption. These include 1) freedom from dispersive interference, 2) insensitivity to frequency drift, and 3) a signal-to-error ratio two times better than that of the commonly used peak of the $|A_0|$ response. Roughly comparable in quality is the choice $r = r_{eff}(\delta\alpha)$ (i.e., $\partial|A_0|/\partial\alpha$) at $(\omega - \omega_0) = 0$.

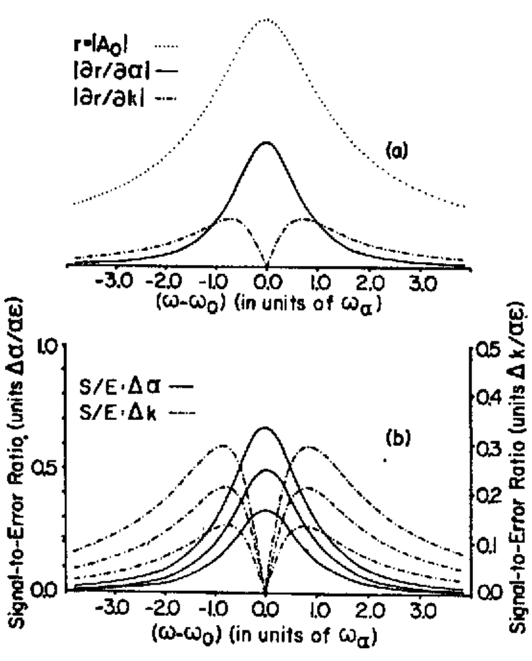

Figure 4. (a) Simple diode detected response $|A_0|$ superimposed upon plots of the corresponding absorption and dispersion sensitivities. (b) Signal-to-error ratios for the measurement of absorption (solid line, left ordinate) and dispersion (dashed line, right ordinate) using the response $|A_0|$.

Table 1. Measurement of Absorption ($\Delta\alpha$) Signal-to-error ratios for absorption at frequencies of particular interest for each of the five responses r. The values of S/E are calculated for m = 0.5.

Monitored Response	Signal/Error in units of $\Delta\alpha/\alpha\varepsilon$	Frequency $(\omega - \omega_0)$ (units ω_α)	Comments		
			$\frac{\|r\|}{\|r\|_{max}}$	$\frac{\|\Delta r\|}{\|\Delta r\|_{max}}$	$\frac{\|\partial r/\partial\alpha\|}{\|\partial r/\partial\alpha\|_{max}}$
A_0'	Max: 0.67	0.0	1.0	1.0	0.0
A_0''	Max: 1.0	±0.4	0.69	0.92	0.62
A_0''	0.67	±1.00	1.0	0.77	0.0
$\|A_0\|$	Max: 0.67	0.0	1.0	1.0	0.0
$\frac{\partial\|A_0\|}{\partial\alpha}$	Max: 1.33	0.0	1.0	1.0	0.0
$\frac{\partial\|A_0\|}{\partial k}$	Max: 1.62	±0.35	0.76	0.92	0.42
$\frac{\partial\|A_0\|}{\partial k}$	1.35	±0.70	1.0	0.90	0.0

B. Measurement of Dispersion (Δk)

Taking r equal to each of the five responses A_0', A_0'', $|A_0|$, $r_{eff}(\delta\alpha)$, and $r_{eff}(\delta k)$, and considering $\Delta r \cong (\partial r/\partial k)\Delta k$, we examine the sensitivity enhancement factors appropriate to dispersion,

$$A_0':\ \partial A_0'/\partial k = \frac{-(a/2)\sinh \alpha a \sin ka}{(\cosh \alpha a - \cos ka)^2}\ , \tag{6a}$$

$$A_0'':\ \partial A_0''/\partial k = \frac{(a/2)(\cosh \alpha a \cos ka - 1)}{(\cosh \alpha a - \cos ka)^2}\ , \tag{6b}$$

$$A_0:\ \partial|A_0|/\partial k = \frac{-a \exp(\frac{1}{2}\alpha a)\sin ka}{2\sqrt{2}(\cosh \alpha a - \cos ka)^{3/2}}\ , \tag{6c}$$

$$r_{eff}(\delta\alpha):\ \partial^2|A_0|/\partial\alpha\,\partial k = \frac{a^2\exp(\frac{1}{2}\alpha a)\sin ka(\cos ka - \cosh \alpha a + 3\sinh \alpha a)}{4\sqrt{2}(\cosh \alpha a - \cos ka)^{5/2}}\ , \tag{6d}$$

$$r_{eff}(\delta k):\ \partial^2|A_0|/\partial k^2 = \frac{a^2\exp(\frac{1}{2}\alpha a)(3 - \cos^2 ka - 2\cos ka \cosh \alpha a)}{4\sqrt{2}(\cosh \alpha a - \cos ka)^{5/2}}\ . \tag{6e}$$

These sensitivity enhancement factors are plotted for the corresponding responses r in the a) panels of Figure 2-6. The resulting signal-to-error ratios for dispersion are shown in the b) panels and are measured on the right ordinate, in dimensionless units $\Delta k/\alpha\varepsilon$.

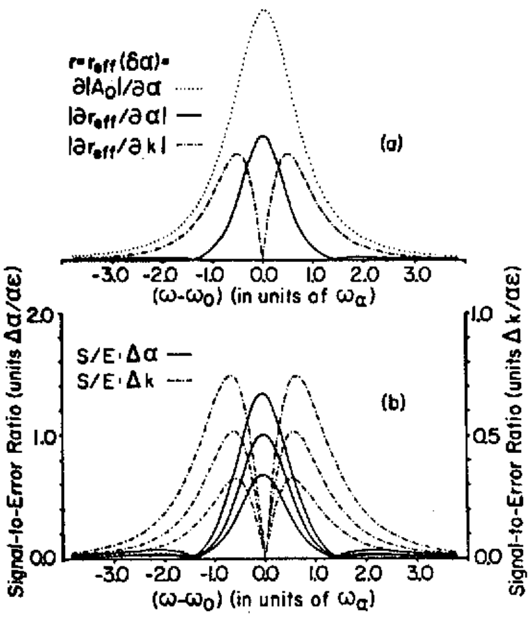

Figure 5. (a) Effective response $r_{eff}(\delta a) = \partial|A_0|/\partial\alpha$, produced by modulation scheme with corresponding absorption and dispersion sensitivity enhancement factors. (b) Signal-to-error ratios for absorption (solid line, left ordinate) and dispersion (dashed line, right ordinate).

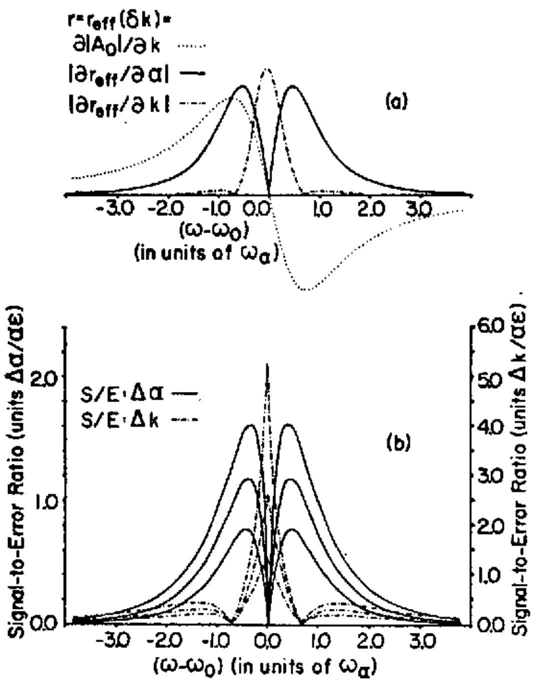

Figure 6. (a) Effective response $r_{eff}(\delta k) = \partial|A_0|/\partial k$ for frequency modulation absorption and dispersion sensitivities. (b) Signal-to-error ratios for absorption (solid line, left ordinate) and dispersion (dashed line, right ordinate).

A summary of the results for dispersion measurements is presented in Table 2. It is enlightening to compare the maximum signal-to-error ratios for three cases: 1) $(S/E)_{max} = 0.29\ \Delta k/\alpha\varepsilon$ for simple diode detection without modulation (i.e., using $|A_0|$), 2) $(S/E)_{max} = 4.0\ \Delta k/\alpha\varepsilon$ for A_0'' without modulation, and 3) $(S/E)_{max} = 5.26\ \Delta k/\alpha\varepsilon$ for $r_{eff}(\delta k)$ using frequency modulation. The roughly 15-fold improvements in signal-to-error ratios for cases 2) and 3) over case 1) arise because the signal $|\Delta r|$ is measured at maximum sensitivity against a background ultrasonic response r which is zero. We have thus reached the limit $|\Delta r|/|r| >> 1$ in the sense of Figure 1.

Table 2: Measurement of Dispersion (Δk)
Signal-to-error ratios for dispersion at frequencies of particular interest for each of the five responses r. The values of S/E are calculated for m = 0.5.

Monitored Response	Signal/Error in units of $\Delta k/\alpha\varepsilon$	Frequency $(\omega - \omega_0)$ (units ω_α)	Comments		
			$\frac{\lvert r\rvert}{\lvert r\rvert_{max}}$	$\frac{\lvert\Delta r\rvert}{\lvert\Delta r\rvert_{max}}$	$\frac{\lvert\partial r/\partial\alpha\rvert}{\lvert\partial r/\partial\alpha\rvert_{max}}$
A_0'	Max: 0.54	±0.75	0.64	0.95	0.18
A_0'	0.50	±1.00	0.50	0.77	0.0
A_0''	Max: 4.00	±0.0	0.0	1.0	0.0
$\lvert A_0\rvert$	Max: 0.29	±1.00	0.70	0.92	0.35
$\frac{\partial\lvert A_0\rvert}{\partial\alpha}$	Max: 0.74	±0.65	0.59	0.94	0.33
$\frac{\partial\lvert A_0\rvert}{\partial\alpha}$	0.40	±1.41	0.20	0.32	0.0
$\frac{\partial\lvert A_0\rvert}{\partial k}$	Max: 5.26	±0.0	0.0	1.0	0.0

Appendix A

In this appendix we discuss how the system gain drift ε and dc level drift γ are related to the gain and dc drifts of the individual components of the system. Figure A1 shows a block diagram of a typical system and labels fractional drifts associated with various components. We continue in the spirit of the "worst case" situation described in Section II. The amplitude of the oscillator is assumed to drift from a value of unity to a value of $(1+\beta)$ as the external variable goes from its initial to its final value, and similarly for the other components.

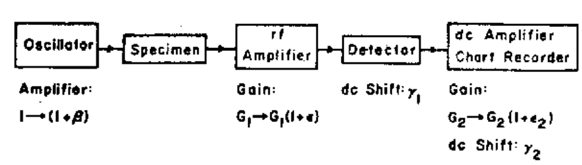

Figure A1. Block diagram of simplified spectrometer showing gain and dc level drifts.

Under these conditions,

$$R = G_1 G_2(r) \qquad \text{(A1)}$$

and

$$R + \Delta R = \{G_1(1+\varepsilon_1)(r+\Delta r)(1+\beta) + \gamma_1\} G_2(1+\varepsilon_2) + \gamma_2 \ . \qquad \text{(A2)}$$

One thus defines the effective drifts γ and ε that describe the system as a whole as

$$\gamma \equiv \gamma_1 G_2 + \gamma_1 G_2 \varepsilon_2 + \gamma_2 \cong \gamma_1 G_2 + \gamma_2 \qquad \text{(A3)}$$

and

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \beta + \varepsilon_1\beta + \varepsilon_2\beta + \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_2\beta \cong \varepsilon_1 + \varepsilon_2 + \beta \qquad \text{(A4)}$$

Appendix B

In this appendix we discuss a method to characterize the importance of the dc level shift γ relative to that of the gain drift ε in the determination of the signal-to-error ratio. One might expect the dc level drift γ to be some approximately constant fraction f of the full-scale deflection of the chart recorder. An estimate of the full-scale deflection required to display the background response R and the change ΔR is full-scale $\cong |R_{max}| + |\Delta R_{max}|$. Thus it is plausible to take $\gamma \cong f[|R_{max}| + |\Delta R_{max}|]$, or

$$\gamma \cong Gf[|r_{max}| + |\Delta r_{max}|] \ . \qquad \text{(B1)}$$

Substituting this form into the expression [Eq. (1)] for the signal-to-error ratio, one obtains

$$S/E = \frac{|\Delta r|}{[\varepsilon(|r| + |\Delta r|) + f(|r_{max}| + |\Delta r_{max}|)]} \ . \qquad \text{(B2)}$$

We are thus led to define the variable $m \equiv f/\varepsilon$ which is a measure of the relative importance of the drifts γ and ε. Hence,

$$S/E = \frac{|\Delta r|}{\varepsilon[(|r| + |\Delta r|) + m(|r_{max}| + |\Delta r_{max}|)]} \ . \qquad \text{(B3)}$$

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References

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[1]D. I. Bolef and J. G. Miller, Physical Acoustics, edited by W. P. Mason and R. N. Thurston (Academic Press, New York, 1971), Vol. 8.

[2]Joseph S. Heyman and J. G. Miller, J. Appl. Phys. 44, 3398 (1973).

[3]R. L. Melcher, D. I. Bolef, and J. B. Merry, Rev. Sci. Instrum. 39, 1618 (1968).

[4]R. G. Leisure and D. I. Bolef, Rev. Sci. Instrum. 39, 199 (1968).

[5]J. G. Miller and D. I. Bolef, Rev. Sci. Instrum. 40, 361 (1969).