N75 19149

PARAMETER ESTIMATION IN LINEAR MODELS OF THE HUMAN OPERATOR IN A CLOSED LOOP WITH APPLICATION OF DETERMINISTIC TEST SIGNALS.

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Abstract

The human operator can be described by a linear model and a remnant added to the output of this model. In practical cases this remnant is always non-zero. Therefore the presence of this remnant has to be taken into account in any identification method applied in human operator research.

Parameter estimation techniques are discussed with emphasis on unbiased estimates in the presence of noise. A distinction between open and closed loop systems is made. A method is given based on the application of external forcing functions consisting of a sum of sinusoids; this method is thus based on the estimation of Fourier coefficients and is applicable for models with poles and zeros in open and closed loop systems.

1 Introduction

To describe the behavior of the human operator controlling time invariant linear system with time constants up to the order of seconds, the Describing Function Method is a powerful too [1]. This DFM says that the system to be considered, in this case the human operator's behavior, can be assumed to consist of a linear time invariant system given by a transfer function and a remnant uncorrelated with the system input and added to the ouput of the system. The methods of estimating the parameters of these quasi-linear models can be devided into two main groups [2].

- General Methods: From the observation of input-ouput of a system, the impulse response or the transfer function is determined. These methods deal with classical identification techniques, such as the determination of Bode or Nyquist plots from sinusoidaltest signals. More recent additions are correlation techniques, power density analyses, FFT's and averaged response techniques.
- More Specific Methods: By observing the input-output of a system, the parameters of a defined mathematical model can be estimated. Therefore both the system to be analyzed and the mathematical model are given the same input. From the differences between the outputs of system and model the parameters can be determined according to a certain performance criterion and by means of a well considered strategy.

The fundamental difference between both the methods is that in a General Method only knowledge of the bandwidth of the input of the system to be analyzed is required (to choose a proper test signal), while for a More Specific Method the structure of the model itself must be known as well. This paper will deal with the discussion of a More Specific Method applied to the description of the human operator in open and closed loops.

2 Identification in an open loop

As pointed out before, a system in an open loop can be described by a linear transfer function $H(p)$ and a remnant $n(t)$ uncorrelated with the system input. For a known transfer function $H(p)$ the parameters of a model based on this transfer function can be calculated from a set of equations which results from minimising a function $E(\varepsilon(t);t)$ [3], where the quantity $\varepsilon(t)$ is the difference between system output $y(t)$ and model output $y^*(t)$ (see Fig. 1).

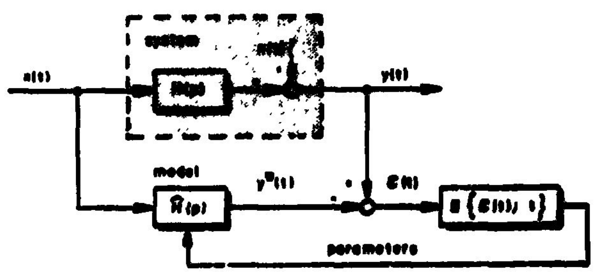

FIGURE 1:

Block diagram of system identification by means of parameter estimation; p denotes the Heaviside operator d/dt.

If the function $E(\varepsilon(t);t)$ is equal to the mean squared error $\frac{1}{T}\int_0^T[\varepsilon(t)]^2\,dt$ then also the error $\varepsilon(t)$ will be uncorrelated with the system input $x(t)$. In the special case that the model output can be written as:

$$y^*(t) = \sum_{i=1}^{n} a_i f_i(x(t)), \qquad (1)$$

the parameters a_i can be calculated from a set of linear equations; models having this property are called "linear in the parameters". A system having a transfer function with zeros only is an example of such a model; systems described by a transfer function with poles and zeros are not linear in the parameters. However, for these systems a so-called "generalised model" [2] can be defined (see Fig. 2). Here, not the difference $\varepsilon(t)$ between system output $y(t)$ and model output $y^*(t)$ is minimised according to a given criterion $E(\varepsilon(t);t)$,

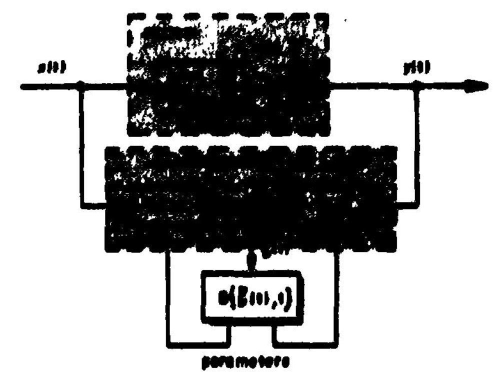

FIGURE 2:

Parameter estimation by using a generalised model.

but the difference $\xi(t)$ is minimised according to the criterion $E(\xi(t);t)$. The quantity $\xi(t)$ is defined as the difference between two signals, which are obtained by modifying the system input $x(t)$ with the transfer function $N(p)$ and the output $y(t)$ with the transfer function $D(p)$; the transfer functions $N(p)$ and $D(p)$ possess only zeros. However, it can be shown that the estimates of the parameters obtained in this way will be biased if the error signal $\xi(t)$ is non-white [3]. Therefore, this method can be applied only if the remnant n(t) is zero or very small in relation to the system output y(t). If an unbiased estimate is required in most practical cases either the parameters of a set of filters for the signals x(t) and y(t) have to be estimated which whiten the signal ε(t), and this has to be accomplished iteratively [3], or the model parameters have to be estimated from the original criterion E(ε(t);t) which leads to a set of nonlinear equations. In practice these equations have to be solved by hill-climbing techniques, either analog or digital, which is also an iterative procedure.

3 Identification in a closed loop

For a system in a closed loop the remnant n(t) is no longer uncorrelated with the system input x(t), because it circles around by way of the feedback. This means that the application of open loop methods in a closed loop will lead to a bias in the estimation of the parameters. There are two possibilities to solve this problem.

3.1 Transformation of the closed loop system into an equivalent open loop system (see Fig. 3)

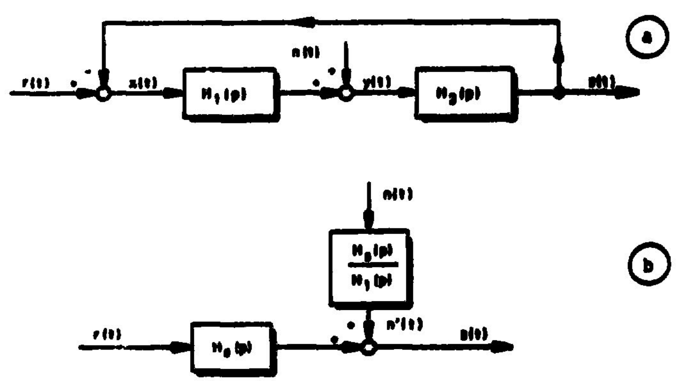

FIGURE 3:

System in a closed loop (a) and the equivalent open loop system (b).

If $H_1(p)$ is the unknown transfer function, and $H_2(p)$ is a known transfer function, then the closed loop transfer function $H_g(p)$ is:

$$H_g(p) = \frac{H_1(p)H_2(p)}{1+H_1(p)H_2(p)} \quad (2)$$

this transfer function can be estimated by means of an open loop method as illustrated in Fig. 1. The transfer function $H_1(p)$ then follows from:

$$H_1(p) = \frac{H_g(p)}{H_2(p)(1-H_g(p))} \quad (3)$$

Based on Eq. (3) a model of $H_g(p)$ can be built as a closed loop system with a known transfer function $H_2(p)$ and an unknown transfer function $H_1(p)$. In such a system a parameter estimation technique can be accomplished according to Fig. 4 or according to Fig. 5.

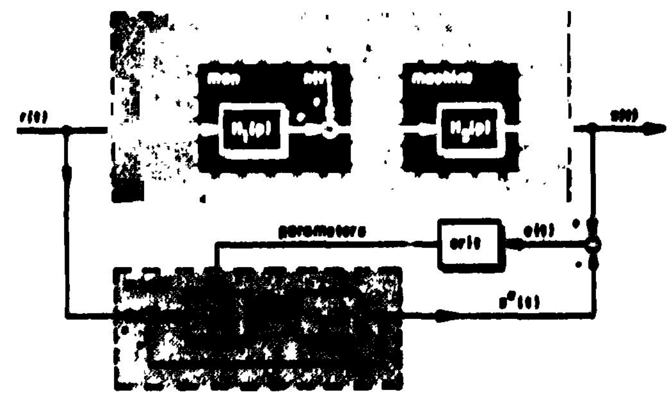

FIGURE 4:

Parameter estimation in a closed loop where $H_2(p)$ is a known system.

The latter method, for instance, is applied by Johannsen [4]. The method mentioned before implies that knowledge of the transfer function $H_2(p)$ of the machine is required. However, in many practic-

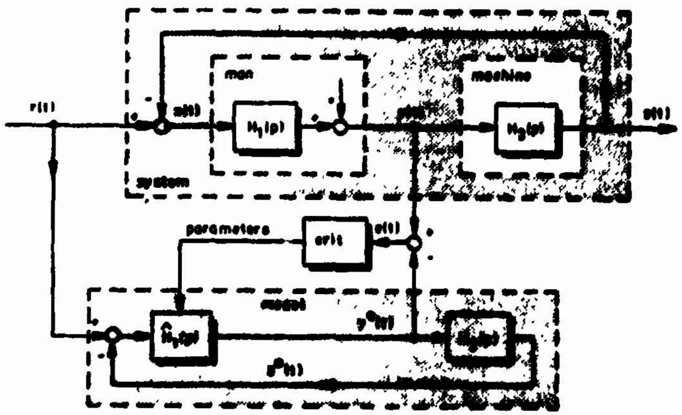

FIGURE 5:
Modified version of the method of Fig. 4.

al situations this knowledge is not available, at least not to the degree of accuracy required to get a reliable estimate of the transfer function $H_1(p)$.

3.2 Application of filtering techniques to closed loop systems

In a closed loop an unbiased estimate can be obtained only if the remnant is zero or if the remnant can be separated from the signals used for the identification; this can be achieved by applying filters. Fig. 6 shows a block diagram of a method to obtain unbiased estimates. The input x(t) and the output y(t) of the unknown system $H_1(p)$ are filtered in such a way that only those components of the signals x(t) and y(t) which originate in the external forcing function r(t) contribute in the parameter estimation. If the forcing function r(t) is a stochastic signal, the filter operation consists of the computation of the cross-covariance functions of the forcing function r(t) with the input x(t) and with the output y(t) respectiv-

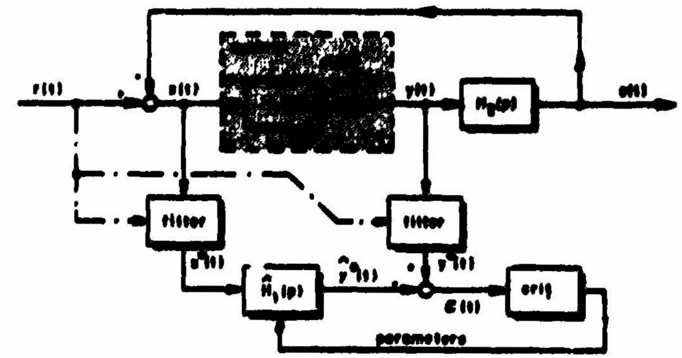

FIGURE 6:
Unbiased parameter estimation for a system in a closed loop where both the transfer functions $H_1(p)$ and $H_2(p)$ are unknown.

ely. A more attractive way of filtering can be achieved if the external forcing function r(t) consists of a sum of sinusoids. Now the application of the filters is equivalent to the estimation of a set of Fourier coefficients. Therefore, many investigators [5; 6; 7] apply a sum of sinusoids as a forcing function in experiments to identify the transfer function of the human controller. In this class of investigations it is important to choose the number of sinusoids in the forcing function sufficiently high, so that the input appears as a random signal to the human controller.

3.3 A closed loop method using a deterministic test signal

Starting from a different background, viz. the application of binary multi-frequency test signals for system identification [8], van den Bos arrived independently at the method just indicated. Moreover, he also showed that it could be applied to the generalised model in an open loop system as well as in a closed loop one without getting biased estimates [9]. Hence the method just-mentioned can be treated as a method for a system in an open loop with zero remnant. This outline will be followed in the next derivation.

Consider a system having a transfer function H(p):

$$H(p) = \frac{\alpha_0+\alpha_1 p+\ldots+\alpha_l p^l}{1+\beta_1 p+\ldots+\beta_m p^m} e^{-\tau_v p} \quad (4)$$

in which p is the Heaviside operator p=d/dt. The input x(t) to this system is described by

$$x(t) = \sum_{k=1}^{n} (a_k \cos\omega_k t + b_k \sin\omega_k t) + n_1(t) = x^*(t) + n_1(t); \quad (5)$$

the output y(t) is described by:

$$y(t) = \sum_{k=1}^{n} (c_k \cos\omega_k t + d_k \sin\omega_k t) + n_2(t) = y^*(t) + n_2(t). \quad (6)$$

The estimates $\hat{a}_k$ and $\hat{b}_k$ of the Fourier coëfficients a_k and b_k can be obtained from:

$$\hat{a}_k = \frac{2}{T}\int_0^T x(t)\cos\omega_k t\, dt = a_k + \frac{2}{T}\int_0^T n_1(t)\cos\omega_k t\, dt; \quad (7)$$

$$\hat{b}_k = \frac{2}{T}\int_0^T x(t)\sin\omega_k t\, dt = b_k + \frac{2}{T}\int_0^T n_1(t)\sin\omega_k t\, dt \quad (8)$$

In these formulas the observation time T is the period of the fundamental frequency of the signals x(t) and y(t). Similar expressions are valid for the estimates $\hat{c}_k$ and $\hat{d}_k$ of the coëfficients c_k and d_k. The filtered signals $\hat{x}^*(t)$ and $\hat{y}^*(t)$ can be defined as:

$$\hat{x}^*(t) = \sum_{k=1}^{n} (\hat{a}_k \cos\omega_k t + \hat{b}_k \sin\omega_k t); \quad (9)$$

$$\hat{y}^*(t) = \sum_{k=1}^{n} (\hat{c}_k \cos\omega_k t + \hat{d}_k \sin\omega_k t). \quad (10)$$

Hence the larger part of the disturbances $n_1(t)$ and $n_2(t)$ is filtered out from the signals x(t) and y(t); only the small part around the circular frequencies ω_k remains. The relation between the deterministic parts $x^*(t)$ and $y^*(t)$ of the input x(t) and output y(t) respectively can be described by:

$$(1+\beta_1 p+\ldots+\beta_m p^m)\, y^*(t) = (\alpha_0+\alpha_1 p+\ldots+\alpha_l p^l)\, x^*(t-\tau_v). \quad (11)$$

In a model with parameters $\hat{\alpha}_0, \hat{\alpha}_1, \ldots \hat{\alpha}_l, \hat{\beta}_1 \ldots \hat{\beta}_m, \hat{\tau}_v$ and with an input $\hat{x}^*(t)$ a similar relation is given by

$$(1+\hat{\beta}_1 p+\ldots+\hat{\beta}_m p^m)\hat{y}^*(t) = (\hat{\alpha}_0+\hat{\alpha}_1 p+\ldots+\hat{\alpha}_l p^l)\hat{x}^*(t-\hat{\tau}_v) + \xi(t). \quad (12)$$

A more general way of writing this equation is:

$$\xi(t) = y(t) - \underline{\alpha}^T \underline{z}(t,\tau) = y(t) - \underline{z}^T(t,\tau)\underline{\alpha}. \quad (13)$$

In Eq. (13) the function y(t) corresponds to $\hat{y}^*(t)$ in Eq. (12). The vector $\underline{\alpha}$ consists of the estimates $\hat{\alpha}_i$ and $\hat{\beta}_j$ of the unknown parameters, and the vector $\underline{z}(t,\tau)$ consists of the negatives of all sensitivity functions

$\frac{\partial\xi(t)}{\partial\hat{\alpha}_i}$ and $\frac{\partial\xi(t)}{\partial\hat{\beta}_j}$, which in this case are equal to $-p^i(\hat{x}(t-\tau))$ and $p^j(\hat{y}^*(t))$ respectively. The optimal value of τ corresponds to the estimated time delay $\hat{\tau}_v$. Now, consider the criterion function

$$E(\underline{\alpha},\tau) = \frac{2}{T}\int_0^T \xi^2(\underline{\alpha},\tau,t)\, dt. \quad (14)$$

Minimisation of this criterion function with respect to the parameters to be estimated yields a set of equations from which these parameters can be solved. Figure 7 which is in fact a combination of the Figs 2 and 6 shows the general idea of this method. The method is elaborated in more detail as follows. Define the scalar

$$\eta = \frac{2}{T}\int_0^T y^2(t)\, dt; \quad (15)$$

the vector

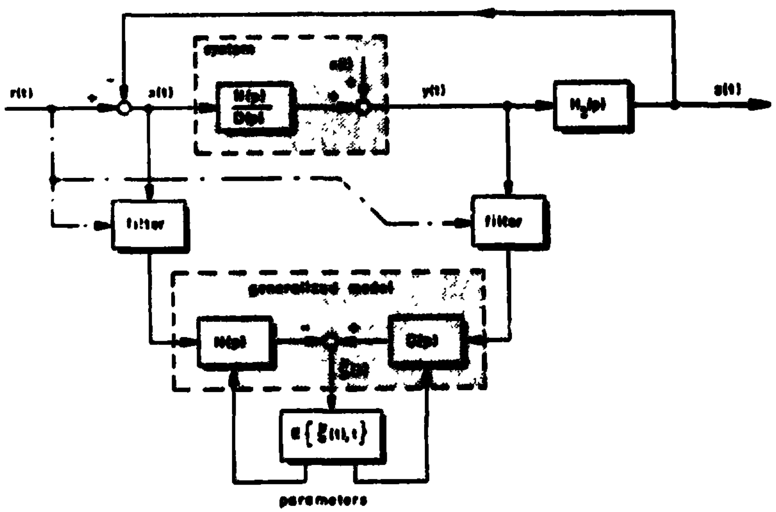

FIGURE 7:
Unbiased parameter estimation in a closed loop with a generalised model.

$$\underline{u}(\tau) = \frac{2}{T}\int_0^T \underline{z}(t,\tau)\, y(t)\, dt; \tag{16}$$

and the matrix

$$Z(\tau) = \frac{2}{T}\int_0^T \underline{z}(t,\tau)\, \underline{z}^T(t,\tau)\, dt. \tag{17}$$

Then

$$Z(\tau) = Z^T(\tau), \tag{18}$$

and

$$E(\underline{a},\tau) = n - \underline{a}^T\underline{u}(\tau) - \underline{u}^T(\tau)\underline{a} + \underline{a}^T Z(\tau)\, \underline{a}, \tag{19}$$

where

$$\underline{a}^T\underline{u}(\tau) = \underline{u}^T(\tau)\underline{a}, \tag{20}$$

so that $E(\underline{a},\tau)$ can also be written as:

$$E(\underline{a},\tau) = n - 2\underline{u}^T(\tau)\underline{a} + \underline{a}^T Z(\tau)\underline{a}. \tag{21}$$

Minimisation of $E(\underline{a},\tau)$ with respect to the parameter vector $\underline{a}$ yields:

$$\text{grad}_{\underline{a}}\, E(\underline{a},\tau) = -2\underline{u}^T(\tau) + 2\underline{a}^T Z(\tau) = \underline{0}^T, \tag{22}$$

from which follows:

$$\underline{a} = Z^{-1}(\tau)\, \underline{u}(\tau). \tag{23}$$

Denote the criterion $E(\underline{a},\tau)$ which is minimized with respect to $\underline{a}$, as $E_0(\tau)$, then insertion of Eq. (23) into Eq. (21) leads to:

$$E_0(\tau) = n - \underline{u}^T(\tau)\, Z^{-1}(\tau)\, \underline{u}(\tau). \tag{24}$$

This means, that the parameter τ can be solved even before the parameter vector $\underline{a}$ is known. The solution follows from:

$$\frac{\partial E_0(\tau)}{\partial\tau} = -f_1(\tau) = 0, \tag{25}$$

which can be written more explicitly as:

$$f_1(\tau) = \frac{\partial}{\partial\tau}(\underline{u}^T(\tau))\, Z^{-1}(\tau)\, \underline{u}(\tau) + \underline{u}^T(\tau)\frac{\partial}{\partial\tau}(Z^{-1}(\tau))\, \underline{u}(\tau)$$
$$+ \underline{u}^T(\tau)\, Z^{-1}(\tau)\frac{\partial}{\partial\tau}\, \underline{u}(\tau) = 0, \tag{26}$$

where

$$\frac{\partial}{\partial\tau}(\underline{u}^T(\tau))Z^{-1}(\tau)\underline{u}(\tau) = \underline{u}^T(\tau)Z^{-1}(\tau)\, \frac{\partial}{\partial\tau}(\underline{u}(\tau)). \tag{27}$$

By writting $Z^{-1}(\tau)$ as:

$$Z^{-1}(\tau) = \frac{\text{Adj } Z(\tau)}{|Z(\tau)|}, \tag{28}$$

and by multiplying both sides of Eq. (26) with $|Z(\tau)|^2$ this equation can be transformed into:

$$f_2(\tau) = |Z(\tau)|^2\, f_1(\tau) = |Z(\tau)|\Big[2\underline{u}^T(\tau)\text{ Adj } Z(\tau)\frac{\partial}{\partial\tau}(\underline{u}(\tau))$$
$$+ \underline{u}^T(\tau)\frac{\partial}{\partial\tau}(\text{Adj } Z(\tau))\underline{u}(\tau)\Big] - \frac{\partial}{\partial\tau}(|Z(\tau)|)\underline{u}^T(\tau)\text{ Adj } Z(\tau)\, \underline{u}(\tau) = 0 \tag{29}$$

Eq. (29) can be solved by an iteration procedure such as the Newton-Raphson algorithm. In order to apply this algorithm, the derivative of $f_2(\tau)$ has to be known. The derivative of $f_2(\tau)$ is given by:

$$\frac{\partial f_2(\tau)}{\partial \tau} = |Z(\tau)|\Big[2\frac{\partial}{\partial \tau}\{\underline{u}^T(\tau)\}\ \mathrm{Adj}\ Z(\tau)\frac{\partial}{\partial \tau}\{\underline{u}(\tau)\}$$
$$+ 4\underline{u}^T(\tau)\frac{\partial}{\partial \tau}\ \{\mathrm{Adj}\ Z(\tau)\}\frac{\partial}{\partial \tau}\{\underline{u}(\tau)\} + 2\underline{u}^T(\tau)\mathrm{Adj}\ Z(\tau)\frac{\partial^2}{\partial \tau^2}\ \underline{u}(\tau)\}$$
$$+ \underline{u}^T(\tau)\frac{\partial^2}{\partial \tau^2}\ \{\mathrm{Adj}\ Z(\tau)\}\underline{u}(\tau)\Big] - \frac{\partial^2}{\partial \tau^2}\{|Z(\tau)|\}\underline{u}^T(\tau)\ \mathrm{adj}\ Z(\tau)\underline{u}(\tau). \qquad (30)$$

Eq. (29) will have an infinite number of solutions, but owing to the fact that in practice the range of possible solutions for the value of τ can be given, normally an unambiguous solution can be found. The starting point for the iteration procedure can be chosen by solving Eq. (24) for a small number of values of τ in the range of interest, so that Eq. (29) can be solved. Finally, the parameter vector $\underline{a}$ can be computed from Eq. (23).

For a given model and for known estimates of the Fourier coefficients of the signals $x(t)$ and $y(t)$, all elements of the vectors and matrices mentioned in Eqs (29) and (30) can be computed. For example, if:

$$H(p) = \frac{a_0+a_1p}{1+a_2p}\, e^{-\tau_v p} . \qquad (31)$$

then Eq. (12) can be written as:

$$\xi(t) = \hat{y}^*(t) - \left[\hat{a}_0\hat{x}^*(t-\hat{\tau}_v) + \hat{a}_1p(\hat{x}^*(t-\hat{\tau}_v)) - \hat{a}_2p(\hat{y}^*(t))\right]; \qquad (32)$$

which means that in this case the elements $\zeta_i(t,\tau)$ of the vector $\underline{z}(t,\tau)$ in Eq. (13) are:

$$\zeta_0(t,\tau) = \hat{x}^*(t-\tau) = \sum_{k=1}^{n}\{\hat{a}_k\ \cos\omega_k(t-\tau) + \hat{b}_k\ \sin\omega_k(t-\tau)\}; \qquad (33)$$

$$\zeta_1(t,\tau) = p(\hat{x}^*(t-\tau)) = \sum_{k=1}^{n}\omega_k\{\hat{b}_k\ \cos\omega_k(t-\tau) - \hat{a}_k\ \sin\omega_k(\tau-t)\}; \qquad (34)$$

$$\zeta_3(t,\tau) = -p(\hat{y}^*(t)) = \sum_{k=1}^{n}\omega_k\{\hat{c}_k\ \sin\omega_k t - \hat{d}_k\ \cos\omega_k t\}. \qquad (35)$$

The elements of the vector $\underline{u}(\tau)$ and the matrix $Z(\tau)$ can be calculated in a way similar to the one given below:

$$z_{12}(\tau) = \frac{2}{T}\int_0^T \zeta_1(t,\tau)\ \zeta_2(t,\tau)\ dt =$$
$$= -\frac{2}{T}\int_0^T p(\hat{x}^*(t-\tau))\ p(\hat{y}^*(t)).$$

$$z_{12}(\tau) = -\sum_{k=1}^{n}\omega_k^{\,2}\{(\hat{a}_k\hat{c}_k+\hat{b}_k\hat{d}_k)\ \cos\omega_k\tau + (\hat{a}_k\hat{d}_k-\hat{b}_k\hat{c}_k)\ \sin\omega_k\tau\}. \qquad (36)$$

From the foregoing it follows that all information necessary for the estimation of the parameters is available if the estimates of the Fourier coefficients of the signals involved are known.

4 Multiloop systems

For systems having more than one inp… and more than one output, the identification of unknown transfer functions is less straightforward. Consider, for instance, the system of Fig. 8 which represents a man-bicycle system where the rider has to perform two tasks viz. the stabilisation of the bicycle and the following of a given track. In this system inputs and outputs of the unknowr transfer functions $H_1(\nu)$, $H_2(\nu)$, $H_3(\nu)$ and $H_4(\nu)$ are coupled, not only within the human operator, but also within the bicycle simulator. The signals $R_1(\nu)$ and $R_2(\nu)$ are introduced externally as forcing functions. If the relations between inputs and outputs of the bicycle simulator are linear, then all signals in the system can be described as linear functions of the forcing functions $R_1(\nu)$, $R_2(\nu)$ and the remnants $N_1(\nu)$ and $N_2(\nu)$. For instance:

$$W(\nu) = F_1(\nu)R_1(\nu) + F_2(\nu)R_2(\nu) + F_3(\nu)N_1(\nu) + F_4(\nu)N_2(\nu); \qquad (37)$$

$$X(\nu) = G_1(\nu)R_1(\nu) + G_2(\nu)R_2(\nu) + G_3(\nu)N_1(\nu) + G_4(\nu)N_2(\nu), \qquad (38)$$

where the transfer functio $F_1(\nu)$, $F_2(\nu)$, $F_3(\nu)$, $F_4(\nu)$, $G_1(\nu)$, $G_2(\nu)$,

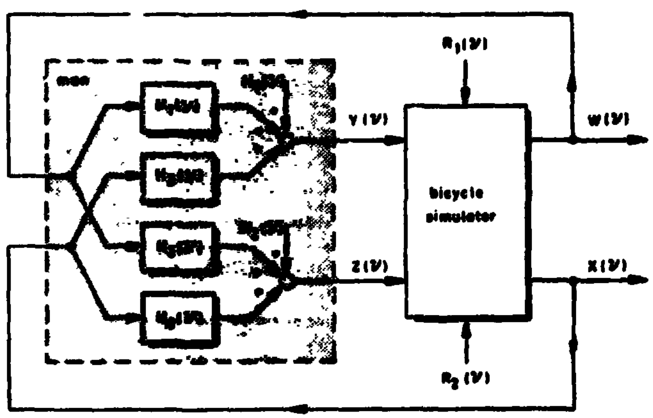

FIGURE 8:
Example of a multiloop system.

$G_3(\nu)$ and $G_4(\nu)$ describe the relations between the external inputs $R_1(\nu)$ and $R_2(\nu)$ on the one hand and the signals $W(\nu)$ and $X(\nu)$ on the other. From Fig. 8 and Eqs (37) and (38) it follows that:

$$Y(\nu) = \{H_1(\nu)F_1(\nu)+H_2(\nu)G_1(\nu)\}R_1(\nu) + \{H_1(\nu)F_2(\nu)+H_2(\nu)G_2(\nu)\}R_2(\nu)$$
$$+ \{1+H_1(\nu)F_3(\nu)+H_2(\nu)G_3(\nu)\}N_1(\nu) + \{H_1(\nu)F_4(\nu)+H_2(\nu)G_4(\nu)\}N_2(\nu); \quad (39)$$

$$Z(\nu) = \{H_3(\nu)F_1(\nu)+H_4(\nu)G_1(\nu)\}R_1(\nu) + \{H_3(\nu)F_2(\nu)+H_4(\nu)G_2(\nu)\}R_2(\nu)$$
$$+ \{H_3(\nu)F_3(\nu)+H_4(\nu)G_3(\nu)\}N_1(\nu) + \{1+H_3(\nu)F_4(\nu)+H_4(\nu)G_4(\nu)\}N_2(\nu). \quad (40)$$

Suppose that for each of the signals $W(\nu)$, $X(\nu)$, $Y(\nu)$ and $Z(\nu)$ the components $W_1(\nu)$, $W_2(\nu)$, $X_1(\nu)$, $X_2(\nu)$, $Y_1(\nu)$, $Y_2(\nu)$, $Z_1(\nu)$ and $Z_2(\nu)$, originating in the external test signals $R_1(\nu)$ and $R_2(\nu)$ can be

separated. Then a set of filtered signals can be distinguished, viz.:

$$\left.\begin{aligned}
W_1(\nu) &= F_1(\nu)R_1(\nu);\\
W_2(\nu) &= F_2(\nu)R_2(\nu);\\
X_1(\nu) &= G_1(\nu)R_1(\nu);\\
X_2(\nu) &= G_2(\nu)R_2(\nu);\\
Y_1(\nu) &= \{H_1(\nu)F_1(\nu) + H_2(\nu)G_1(\nu)\}R_1(\nu);\\
Y_2(\nu) &= \{H_1(\nu)F_2(\nu) + H_2(\nu)G_2(\nu)\}R_2(\nu);\\
Z_1(\nu) &= \{H_3(\nu)F_1(\nu) + H_4(\nu)G_1(\nu)\}R_1(\nu);\\
Z_2(\nu) &= \{H_3(\nu)F_2(\nu) + H_4(\nu)G_2(\nu)\}R_2(\nu).
\end{aligned}\right\} \quad (41)$$

By eliminating $F_1(\nu)$, $F_2(\nu)$, $G_1(\nu)$ and $G_2(\nu)$ the set of Eqs (41) can be reduced to a set of 4 equations from which the 4 unknown transfer functions $H_1(\nu)$, $H_2(\nu)$, $H_3(\nu)$ and $H_4(\nu)$ can be solved. As an example:

$$H_1(\nu) = \frac{X_2(\nu)Y_1(\nu)-X_1(\nu)Y_2(\nu)}{X_2(\nu)W_1(\nu)-X_1(\nu)W_2(\nu)}, \quad (42)$$

or

$$H_1(\nu) = \frac{Y_1(\nu)-\frac{Y_2(\nu)}{X_2(\nu)}X_1(\nu)}{W_1(\nu)-\frac{W_2(\nu)}{X_2(\nu)}X_1(\nu)} = \frac{Y_1'(\nu)}{W_1'(\nu)}. \quad (43)$$

Separation of the components of the signals originating in the two forcing functions is possible when both these test signals are composed of a number of sinusoids. In order to distinguish between the components originating in each of the two test signals it is necessary that no common frequencies occur in both test signals. However, now the problem arises that application of Eqs (42) or (43) is not possible because for a given frequency either the signals with the index 1 or those with the index 2 or both are zero. In general the transfer functions considered here are sufficiently smooth, i.e. the transfer functions can be considered to be constant within a frequency range $\Delta\nu = 1/T$. Now the problem can

be solved as indicated below.
Choose a test signal $R_1(\nu)$ consisting of a set of sinusoids with frequencies ν_k $(k=1,2,\dots n)$ and choose the signal $R_2(\nu)$ which has the same number of sinusoids with frequencies $\nu_k+\Delta\nu(k=1,2,\dots n)$.
Then if it is assumed that:

$$\frac{Y_2(\nu_k+\Delta\nu)}{X_2(\nu_k+\Delta\nu)} \approx \frac{Y_2(\nu_k)}{X_2(\nu_k)}, \qquad (44)$$

and if the same approximation can be applied to all quotients in the equations for the transfer functions $H_1(\nu)$, $H_2(\nu)$, $H_3(\nu)$ and $H_4(\nu)$, it i .ow possible to compute the decoupled inputs and outputs like $W_1'(\nu)$ and $Y_1'(\nu)$ in Eq. (43). These decoupled inputs and outputs can then be used in the parameter estimation method described under Par. 3. Another possibility to obtain the decoupled inputs and outputs at the frequencies desired is to apply an interpolation procedure. This method, for instance, is used by Stapleford et al [10].

Finally, it should be mentioned, that the number of parameters to be estimated should be chosen as small as possible, i.e. the structure of the model should be as simple as possible. A redundancy in the number of parameters means an increase in the number of near optimal solutions in the parameter space. Small disturbances due to noise may have the effect that for the same unknown system different sets of solutions can be found which lie far apart.

5 References

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