A STOCHASTIC MODEL OF THE ELECTROMYOGRAM

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This work was supported in part by the National Science Foundation and the Presbyterian-St.Luke's R & E grants.

Ninth Annual *Conference* on Manual Control
May 23-25, 1973
Massachusetts Institute of Technology
Cambridge, Mass. 02139

Introduction

The electrical activity of muscle has long been studied by recording from the surface of a muscle and by recording the activity of individual motor units with concentric needle electrodes. Both techniques are simple and reliable enough to be used routinely in the kinesiology and in the diagnosis of many diseases of the muscles and their motor nerves [1].

The interference pattern electromyogram (EMG) which is observed when recording from the muscle's surface is a summation of action potentials of a number of motor units. The information about these motor units latent in it is not readily apparent because of the very complex pattern. Consequently, the surface EMG has not yet seen widespread clinical application. The purpose of this study is to investigate the quantitative regularities of interference pattern formation by motor unit action potentials. To this end, we have considered the parameters of a single motor unit and how they relate to the Fourier transform analysis of an EMG. The Fourier transform of the simulated electromyogram is compared with the Fourier transform of the actual EMG recorded from various human muscles using surface electrodes.

The Parameters of the Motor Unit

The steady state discharge of single neurons is often characterized by a probability density function for the intervals between successive nerve impulses called the Inter Spike Intervals (ISI).

Buchthal and his associates [2,3], Clamann [4], Leifer [5] and several other investigators have shown that the ISI of successive motor unit spikes are normally distributed. The rates of steady state discharge in the cats are usaully between 5 to 30 pulses per second (pps) [6].

Clamann [4] has reported the firing rates between 7 to 25 pps in the human brachial biceps. The transient discharge rates may be higher, particularly in smaller muscles. A peak frequency of 150 pps has been reported in the adductor pollicis in the human hand [7]. In unfatigued human muscles, the mean and the standard deviation of the probability density function characterizing the ISI are functionally related [4], making it possible to completely specify the stochastic process by determining its mean. For the human brachial biceps muscle he arrives at the following functional relation between the mean (μ) and the variance (σ^2) of the ISI:

$$\sigma = 9.1 \times 10^{-4} \mu^2 + 4.0 \text{ msec} \quad (1)$$

When successive intervals are uncorrelated, the probability density function and its mean and variance completely specify the statistical process generating the intervals. In the normal, unfatigued muscle, under conditions of low or moderate activation, the steady discharges of motoneurons generally do not show any dependence between successive intervals [4,5,8,9]. However, at high levels of muscular activity and particularly during fatigue, the discharges tend to be grouped so that successive intervals will no longer be uncorrelated.

The EMG of a single motor unit can be looked at as a time function defined by a convolution integral:

$$e(t) = \int_0^t h(t-\tau)\, p(\tau)d\tau \quad (2)$$

where p(t) is a point process (i.e., a series of unit impulses

or Dirac delta functions) which passes through a filter whose impulse response is h(t) as in Fig. 1. The time function h(t) describes the shape of a single motor unit action potential. The width of the motor unit action potential is usually between 3 to 20 msec. [2]. The conduction process of the action potential along the muscle fiber is such that successive muscle action potentials recorded from the same motor unit cannot overlap. They remain distinct events. A typical EMG of a synthetic motor unit is shown in Figure 2. The time origin is arbitrarily fixed so that the first motor unit starts at t = ℓ msec. The Fourier transform (FT) of equation (2) is

$$E(jw) = H(jw)\, P(jw) \quad (3)$$

where H(jw) is the FT of the impulse response and P(jw) is the FT of the random pulse train.

Let $t_i = \tau_1 + \tau_2 + \cdots + \tau_i$ and $t_o = 0$. Then the i<u>th</u> pulse occurs at time $(\ell + t_{i-1})$. For a pulse train of $N + 1$ pulses, p(t) may be written as

$$p(t) = \sum_{i=0}^{N} \delta(t-\ell-t_i) \quad (4)$$

and its FT is

$$P(jw) = \sum_{i=0}^{N} e^{-jw(\ell+t_i)}$$
$$= e^{-jw\ell} \left[\sum_{i=0}^{N} e^{-jwt_i}\right]. \quad (5)$$

Without any loss of generality, we will from now on consider $\ell = 0$, i.e., the time origin is defined at the occurrence of the first pulse. Therefore,

$$P(jw) = \sum_{i=0}^{N} e^{-jwt_i} \qquad (6)$$

where t_i is a sum of i normally distributed independent ISI.

<u>Ensemble Average of P(jw)</u>

The (i+1)th pulse occurs at time t_i which is a sum of i independent random variables

$$t_i = \tau_1 + \tau_2 + \dots + \tau_i$$

The mean (μ) and variance (σ^2) of each ISI is assumed to be the same. Thus the mean and variance of t_i are given by:

$$E(t_i) = E(\tau_1 + \tau_2 + \dots + \tau_i)$$

$$= \sum_{j=1}^{i} E(\tau_j) = i\mu \qquad (7)$$

$$V(t_i) = V(\tau_1 + \tau_2 + \dots + \tau_i)$$

$$= \sum_{j=1}^{i} V(\tau_j) = i\sigma^2 \qquad (8)$$

The probability density function of t_i is given by

$$f(t_i) = \frac{1}{\sqrt{2\pi i}\,\sigma} \exp\left[\frac{-(t_i - i\mu)^2}{2i\sigma^2}\right] \qquad (9)$$

The Fourier transform P(jw) is a random function of t_i, i = 1,2, ..., N. The ensemble average is given by

$$\overline{P(jw)} = E[P(jw, t_1, t_2, \dots, t_N)]$$

$$= E\left[\sum_{i=0}^{N} e^{-jwt_i}\right]$$

$$= \sum_{i=0}^{N} E(e^{-jwt_i}) \qquad (10)$$

The expected value of e^{-jwt_i} is given by

$$E(e^{-jwt_i}) = \int_{-\infty}^{\infty} e^{-jwt_i} f(t_i)\, dt_i$$

$$= \frac{1}{\sqrt{2\pi i}\,\sigma} \int_{-\infty}^{\infty} e^{-jwt_i}\, e^{\frac{-(t_i - i\mu)^2}{2i\sigma^2}}\, dt_i$$

Let $t_i - i\mu = x$. Then

$$E[e^{-jwt_i}] = \frac{2e^{-jwi\mu}}{\sqrt{2\pi i}\,\sigma} \int_{-\infty}^{\infty} \cos wx\, e^{\frac{-x^2}{2i\sigma^2}}\, dx$$

$$= e^{-jwi\mu} \cdot e^{\frac{-iw^2\sigma^2}{2}} \qquad (11)$$

The ensemble average $\overline{P(jw)}$ from equations (10) and (11) is given by

$$\overline{P(jw)} = \sum_{i=0}^{N} e^{\frac{-iw^2\sigma^2}{2}} \cdot e^{-jwi\mu} \qquad (12)$$

From equation (2) and (12), the ensemble average of the Fourier Transform of a single motor unit is given by

$$\overline{E(jw)} = \overline{P(jw)}\, H(jw) \qquad (13)$$

and

$$|\overline{E(jw)}| = |\overline{P(jw)}|\, |H(jw)| \qquad (14)$$

In Figure 3, the FT of the EMG given by equation 14 is plotted for μ = 20 msec, σ = 4.36 msec and N = 50 (which represents about 50 pulses on the average in one second interval). H(jw) is taken for a diphasic pulse with peak to peak height 2b and width 2c as shown in Figure 2. The FT for this pulse is

$$H(jw) = \frac{4b}{cw^2}\, e^{-jwc}\, [2\sin(\tfrac{wc}{2}) - \sin(wc)] \qquad (15)$$

EMG Simulation

The EMG simulation was carried out on an IBM 1800 process computer. Our simulation is very similar to the work done by Person and Libkind [10,11]. Their studies were, however, limited to the symmetrical biphasic motor unit potentials.

In our model, M is the number of active motor units. The interspike intervals (ISI) are independent random values having a normal distribution with mean μ and standard derivation σ. Various shapes of motor unit potentials were used in this study with widths from 4 to 16 msecs. The pulse shapes were chosen to have a zero mean value. The ISI was always taken to be greater than the width of the pulse.

The first pulse for the ith motor unit was taken at a random interval uniformly distributed between zero and μ msec. All other ISI were normally distributed. The interference pattern was arrived at by summing up the action potentials of active motor units in the time interval from 0 to 1050 msec. The samples were 1 msec apart. The Fourier transform was obtained using 1024 samples and employing a fast Fourier transform algorithm.

Simulation Results

With a small number of motor units active, the pulses were frequently grouped with a "pile up effect". With an increase in the number of active motor units the time intervals free of impulses gradually diminished. A "saturation" level is reached with about 16 active motor units at mean frequency of 20 pps. Figure 4A shows that synthetic EMGs, especially the "saturated" ones, bears a close resemblance to the actual surface EMG, examples

of which appear in Figure 4B.

The theoretical analysis indicates that the average FT is the product of the FT of the motor units pulse shape and the ensemble average of the FT of a randomly occurring impulse train. Figure 5 shows the |FT| for one motor unit with a symmetric biphasic pulse of 8 msec width as shown in Figure 2 [μ = 20 msec, σ = 4.36 msec, M = 1]. Figure 6 shows the average |FT| for 15 such independent motor units. The smooth curve in Figure 6 is obtained from equation 14 for 30 spike sequence. The important point in this analysis is that average |FT| of several identical motor units approaches the FT of a single motor unit's pulse shape. Figure 7 shows some samples of the average |FT| of the surface EMG data recorded from the gastrocnemius-soleus muscle.

Discussion

In this analysis it has been assumed that mean and variance of the interspike interval remains constant and also that the shape of the various motor unit potentials in a muscle are the same. The theoretical analysis shows that the average |FT| of the synthetic EMG approaches the |FT| of the motor unit potential. This has been confirmed by simulation studies except at the very low end of the spectrum.

If one assumes that the mean and the variance of the ISI for the real EMG remain nearly stable over a short interval during constant force and also that the shape of the various motor unit potentials within the recording field of the electrodes are the same, then it is theoretically feasible to calculate the shape of the motor unit potential from the average FT. The practicality of this procedure is now being investigated.

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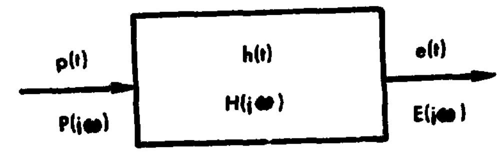

FIGURE 1: EMG Model as low pass filter

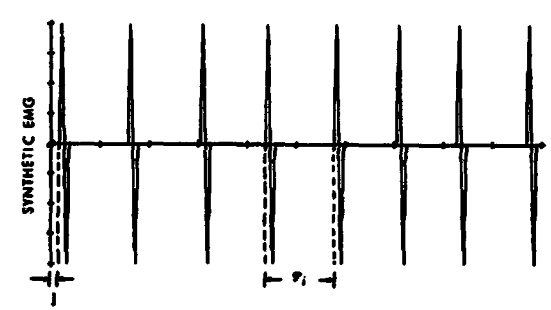

FIGURE 2: Synthetic EMG for a single motor unit with the parameters μ = 50 msec and σ = 6.27 msec. [Time = 400 msec]

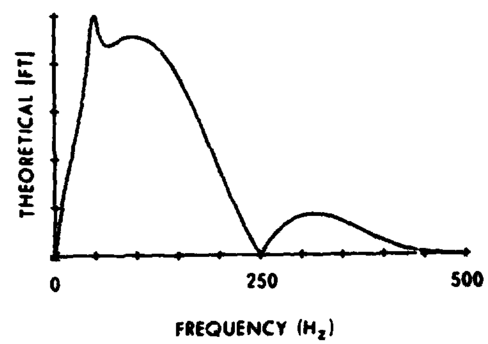

FIGURE 3: Theoretical absolute Fourier transform of the EMG for a symmetric biphasic pulse of 8 msec width, μ = 20 msec, σ = 4.36 msec and the number of pulses is 50.

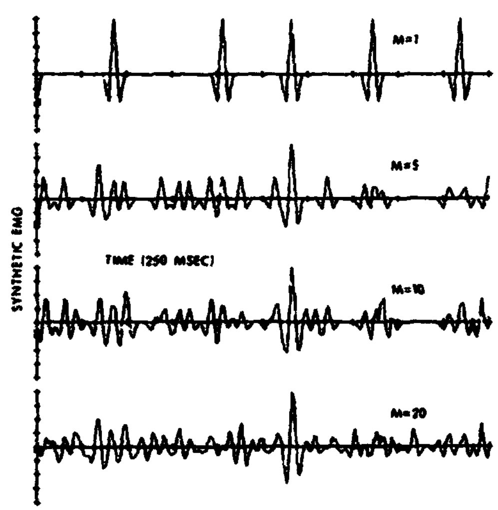

FIGURE 4A: Synthetic EMG generated for a symmetric triphasic pulse. The four traces are for 1, 5, 10 and 20 active motor units.

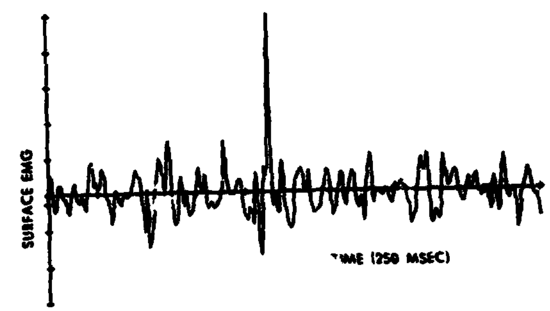

FIGURE 4B: Surface EMG from gastrocnemius-soleus muscle at low force level.

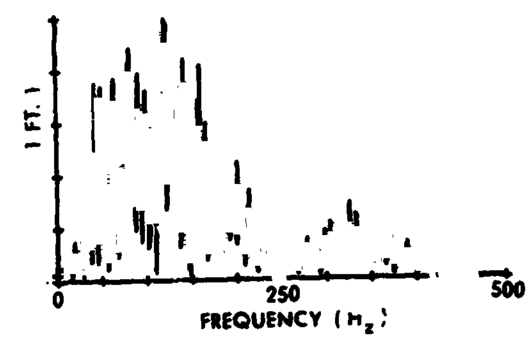

FIGURE 5: Absolute Fourier transform for a single active motor unit with biphasic pulse shape (μ = 20 msec and σ = 4.36 msec).

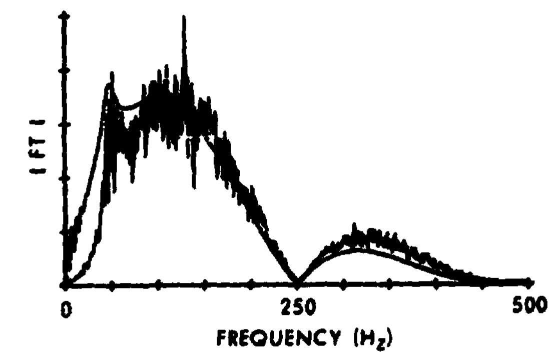

FIGURE 6: Average absolute Fourier transform for fifteen independent active motor units with bi-phasic pulse shape (μ = 20 msec and σ = 4.36 msec). The smooth curve is the theoretical absolute Fourier transform.

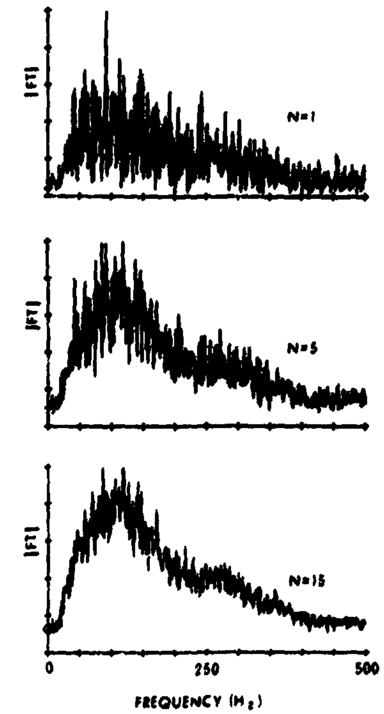

FIGURE 7: Average absolute Fourier transform of surface EMG data from gastrocnemius-soleus muscle. The three parts are average of 1, 5 and 15 samples.