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A LINEAR STOCHASTIC MODEL OF THE HUMAN OPERATOR

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ABSTRACT

A linear stochastic model of the human operator is developed and applied to the problem of piloted control of an aircraft. The pilot and aircraft are modeled as linear time-invariant systems containing both process and measurement noise. The loop closure by the pilot is determined by formulating the problem as an optimal stochastic control problem. The solution to the optimal control problem yields not only the pilot's optimal control output which he uses to control the vehicle, but also the optimal combination of his observations of the vehicle states upon which the pilot bases his control. In addition, a method is presented so that, using experimental pilot vehicle data, the cost functional which is minimized in the optimal control problem will be numerically equal to the Pilot Rating (PR) that the pilot would associate with the given vehicle and task.

I. INTRODUCTION

The purpose of this paper is to show how the techniques of modern control theory can be applied to the problem of defining the closed loop dynamics of a pilot controlling an aircraft in flight.

The pilot and vehicle are modeled as linear time invariant systems containing both process and measurement noise. The loop closure by the pilot is determined by formulating the problem as an optimal stochastic control problem. The solution to the optimal control problem yields not only the pilot's optimal control output which he uses to control the vehicle, but also the optimal combination of observations of the vehicle states upon which the pilot bases his control. In addition, a method is presented in this paper so that, using experimental pilot vehicle data, the cost functional which is minimized in the optimal control problem will be numerically equal to the Pilot Rating (PR) that the pilot would associate with the given vehicle task.

Because of the use of the state space notation of modern control theory, the impact of this paper is much broader than the field of pilot vehicle control; what is presented is the methodology of defining operator control of any dynamic system which can be represented as a linear time-invariant stochastic system. In fact, the theory can be readily extended to linear time-varying stochastic systems, Reference 1. Furthermore, the mathematical operator describing the pilot can also represent a nonhuman operator such as a monkey, a fixed order observer (ala Luenberger, Reference 2), or a fixed order compensator, Reference 3.

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II. VEHICLE DYNAMICS

It is assumed that the vehicle to be controlled by the pilot can be represented as a linear time invariant stochastic system as follows:

$$E\dot{x} = Fx + Gu + Mn \qquad (1)$$

where x is an $n \times 1$ column vector of the system states, u is a $m \times 1$ column vector of the controls, and η is an $\ell \times 1$ column vector of white noise inputs with autocovariance $E\,(\eta(t)\eta(\tau)) = \varkappa \cdot \delta(t-\tau)$. In this form, equation (1) can easily represent an aircraft flying in turbulence (See References 4, 5, 6, and 7 for a derivation of these equations). The pilot is assumed to "observe" or "feel" some incomplete linear combination of the system states which have been contaminated by an observation noise, μ. The pilot's observation, y, is written as

$$y = Cx + \mu \qquad (2)$$

To simplify the equation format of this analysis, we assume that the observation noise, μ, is filtered white noise. If it is desired to retain the white noise characteristics, the filter bandwidth can be made much greater than that of the controlled system of equation (1). Then, using the techniques described in Reference 7, the controlled system would be

$$\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} F & 0 \\ 0 & F_\mu \end{bmatrix} \begin{bmatrix} x \\ \mu \end{bmatrix} + \begin{bmatrix} G \\ 0 \end{bmatrix} u + \begin{bmatrix} M & 0 \\ 0 & M_\mu \end{bmatrix} \begin{bmatrix} \eta \\ \eta_\mu \end{bmatrix} \qquad (3)$$

where the observation is now expressed as

$$y = \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} x \\ \mu \end{bmatrix} \qquad (4)$$

The matrices F_μ and M_μ are used to define the bandwidth and intensity of the observation noise.

The final step in the simplification is to redefine the matrices in Equations . and (4) by

$$x = \begin{bmatrix} x \\ \hline u \end{bmatrix},\quad E = \left[\begin{array}{c|c} E & 0 \\ \hline 0 & I \end{array}\right],\quad F = \left[\begin{array}{c|c} F & 0 \\ \hline 0 & F_\mu \end{array}\right],\quad G = \begin{bmatrix} G \\ \hline 0 \end{bmatrix}$$

$$M = \left[\begin{array}{c|c} M & 0 \\ \hline 0 & M_\mu \end{array}\right],\quad \eta = \begin{bmatrix} \eta \\ \hline \eta_\mu \end{bmatrix},\quad C = [C \,\vdots\, I] \qquad (5)$$

Then we can write the following set of equations for the vehicle under control:

$$E\dot{x} = Fx + Gu + M\eta \qquad (6)$$

$$y = Cx \qquad (7)$$

where the observation noise is now included in the system equations. This system represented by Equations (6) and (7) is assumed to be controlled by the pilot.

III. PILOT DYNAMICS

The pilot dynamics are represented in state variable notation and are based on the pilot model developed in Reference 7. The pilot model is

$$E_p \dot{x}_p = F_p x_p + G_p u_p + M_p \eta_p \qquad (8)$$

where x_p is an rx1 column vector of the pilot's states, u_p is a qx1 column vector of the input controls to the pilot model, and η_p is a tx1 column vector of white noise inputs with autocovariance $E\{\eta_p(t)\eta_p(\tau)\} = \psi \cdot \delta(t-\tau)$. The matrix ψ is used to scale the intensity of the pilot's motor noise.

Several differences can be noted between this pilot model and that of Reference 7. The control input to the pilot model, u_p, is expressed in a more general form rather than in the pure observation vector form of Reference 7. This is to allow u_p to be expressed as an optimal control input to the pilot model; this will become evident later in the paper. The other difference is that the time rate of change of the observation vector is no longer included. The rationale for the exclusion of this term is based on the classical rule of thumb, Reference 8, that the pilot will seek to drive the closed loop dynamics to a k/s form, where s is the conventional Laplace

variable. Thus, the pilot will not generate a lead (which classically "causes" a $\dot{y}$) unless he is controlling a system whose dynamics are at least second order or greater. Thus, the $\dot{y}$ term apparently generated by the lead is really another state of the system which can readily be included in the observation vector, y, of Equation (7).

IV. PILOT VEHICLE DYNAMICS

The combined pilot vehicle dynamics are easily expressed using Equations (6), (7), and (8). The combined system is

$$\left[\begin{array}{c|c} E & 0 \\ \hline 0 & E_p \end{array}\right]\begin{bmatrix} \dot{x} \\ \hline \dot{x}_p \end{bmatrix} = \left[\begin{array}{c|c} F & 0 \\ \hline 0 & F_p \end{array}\right]\begin{bmatrix} x \\ \hline x_p \end{bmatrix} + \left[\begin{array}{c|c} G & 0 \\ \hline 0 & G_p \end{array}\right]\begin{bmatrix} u \\ \hline u_p \end{bmatrix} + \left[\begin{array}{c|c} M & 0 \\ \hline 0 & M_p \end{array}\right]\begin{bmatrix} \eta \\ \hline \eta_p \end{bmatrix} \qquad (9)$$

The observation vector for this system is

$$\begin{bmatrix} y \\ \hline x_p \end{bmatrix} = \left[\begin{array}{c|c} C & 0 \\ \hline 0 & I \end{array}\right]\begin{bmatrix} x \\ \hline x_p \end{bmatrix} \qquad (10)$$

To simplify the writing of the combined pilot vehicle dynamics, the following new pilot vehicle state vector, $\underline{x}$, and matrices are defined to be

$$\underline{x} = \begin{bmatrix} x \\ \hline x_p \end{bmatrix},\quad \underline{E} = \left[\begin{array}{c|c} E & 0 \\ \hline 0 & E_p \end{array}\right],\quad \underline{F} = \left[\begin{array}{c|c} F & 0 \\ \hline 0 & F_p \end{array}\right],\quad \underline{G} = \left[\begin{array}{c|c} G & 0 \\ \hline 0 & G_p \end{array}\right]$$

$$\underline{u} = \begin{bmatrix} u \\ \hline u_p \end{bmatrix},\quad \underline{y} = \begin{bmatrix} y \\ \hline y_p \end{bmatrix},\quad \underline{\eta} = \begin{bmatrix} \eta \\ \hline \eta_p \end{bmatrix},\quad \underline{M} = \left[\begin{array}{c|c} M & 0 \\ \hline 0 & M_p \end{array}\right],\quad \underline{C} = \left[\begin{array}{c|c} C & 0 \\ \hline 0 & I \end{array}\right] \qquad (11)$$

With these definitions, Equations (9) and (10) become

$$\underline{E}\dot{\underline{x}} = \underline{F}\underline{x} + \underline{G}\underline{u} + \underline{M}\underline{\eta} \qquad (12)$$

$$\underline{y} = \underline{C}\underline{x} \qquad (13)$$

Equations (12) and (13) represent the open loop pilot vehicle dynamics.

These equations can also be written as

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} + \underline{D}\underline{\eta} \qquad (14)$$

$$\underline{y} = \underline{C}\underline{x} \qquad (15)$$

where

$$\underset{\sim}{A} = \underset{\sim}{E}^{-1}\underset{\sim}{F}, \quad \underset{\sim}{B} = \underset{\sim}{E}^{-1}\underset{\sim}{G}, \quad \underset{\sim}{N} = \underset{\sim}{E}^{-1}\underset{\sim}{M} \tag{16}$$

The matrix $\underset{\sim}{E}$ will have an inverse whenever $I_{xz}^2 \neq I_{xx} I_{zz}$, Reference 5.

The structure of the control vector, $\underset{\sim}{u}$, which will determine the closed loop dynamics, will be a function of the pilot vehicle control task.

V. OPTIMAL PILOT VEHICLE CONTROL

The control vector, $\underset{\sim}{u}$, that is selected by the pilot will be a function of the pilot vehicle control task. The work of Kleinman (Reference 9), Anderson (Reference 10), Dillow (Reference 11), and Paskin (Reference 12) have indicated that the satisfaction of the control task can be thought of as the selection of an optimal control, $\underset{\sim}{u}^*$, which minimizes a particular cost functional, J; that cost functional being determined by the control task at hand. The optimal control, $\underset{\sim}{u}^*$, that is selected by the pilot can be seen from Equation (11) to be a combination of the optimal controls u^* and u_p^*. Recall from Equation (9) that u is the control that "operates" the vehicle and u_p is the control that "drives" the pilot. The pilot, therefore, is selecting both his optimal input, u_p^*, and his optimal output, u^*, as he attempts to achieve his pilot vehicle control task.

Two potentially viable forms for the cost functional will be given in this section of the paper. The validation of these functionals will be left to experiments. These forms are directly motivated by the independent works of Kleinman and Paskin (References 9 and 12) who obtained experimental verification of this general approach to the selection of cost functionals.

$J = f(\underset{\sim}{y}, \underset{\sim}{u})$

The first cost function is written assuming that the pilot minimizes some function of the pilot vehicle system observation vector, $\underset{\sim}{y}$, and the pilot vehicle control vector, $\underset{\sim}{u}$. Thus, J is written as

$$J = E\left\{\lim_{t \to \infty} \left[\underset{\sim}{y}'(t)\, Q_1 \underset{\sim}{y}(t) + \underset{\sim}{u}'(t)\, Q_2 \underset{\sim}{u}(t)\right]\right\} \tag{17}$$

where $E\{\cdot\}$ is the expectation operator, Reference 13.

The control task for the cost function Equation (17) is: given the pilot vehicle system from Equations (14) and (15),

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\underset{\sim}{x} + \underset{\sim}{B}\underset{\sim}{u} + \underset{\sim}{N}\underset{\sim}{\eta} \tag{18}$$

$$\underset{\sim}{y} = \underset{\sim}{C}\underset{\sim}{x} \tag{19}$$

Find the optimal control, $\underset{\sim}{u}^*$, where $\underset{\sim}{u}^*$ is restricted to be a linear combination of the pilot vehicle system observations

$$\underset{\sim}{u}^* = \underset{\sim}{H}\underset{\sim}{y} \tag{20}$$

such that the cost functional, J, of Equation (17) is minimized. Thus, for $J = f(\underset{\sim}{y}, \underset{\sim}{u})$, the optimal control will be, from Equation (20)

$$\underset{\sim}{u}^* = \begin{bmatrix} u^* \\ \hline u_p^* \end{bmatrix} = \left[\begin{array}{c|c} H & H_p \\ \hline K & K_p \end{array}\right] \begin{bmatrix} y \\ \hline x_p \end{bmatrix} \tag{21}$$

Equation (21) can be expanded to give

$$\begin{aligned} u^* &= Hy + H_p x_p \\ u_p^* &= Ky + K_p x_p \end{aligned} \tag{22}$$

Equation (22) can be interpreted in the following manner. Through the control u_p^* the pilot sets up a linear combination of his observations, Ky, and adjusts his own dynamics by feeding back some linear combination of his own states, $K_p x_p$, to aid in the optimization of J. In addition to this, he controls the vehicle through the control vector u^* with a linear combination of his observations, Hy, and his states $H_p x_p$. In other words, Equation (22) says that the pilot will adjust his input and output simultaneously to obtain optimal control over the vehicle. This is shown in Figure 1.

The solution for the matrix $\underset{\sim}{H}$ of Equation (20) is not trivial. However, methods to achieve this solution are available in References 1 and 14.

$J = f(\underset{\sim}{y}, \underset{\sim}{u}, \dot{\underset{\sim}{u}})$

A slightly different approach is necessary when the cost functional includes the control rate (Kleinman's cost functional, Reference 9, includes control rate). Kleinman has shown that the introduction of control rate in the cost functional for a multiple input-single output pilot control task will effectively introduce a

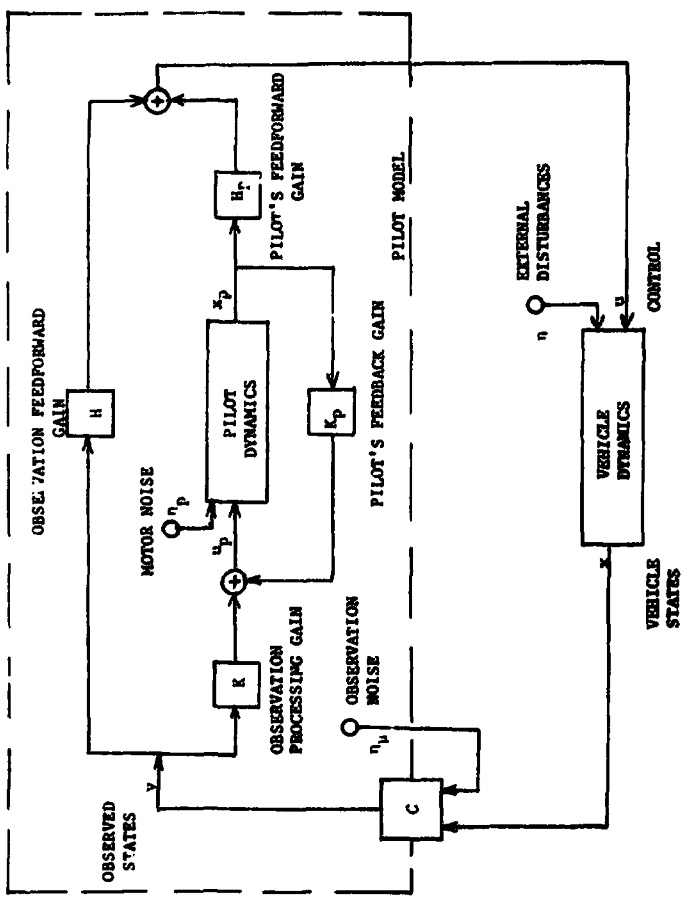

FIGURE 1 - OPTIMAL PILOT VEHICLE CONTROL FOR $J = f(y, x_p, u, u_p)$

first-order lag in the pilot dynamics. From a different viewpoint, it might be argued that the pilot's workload may be a function of his control rate as well as his control output. J is written as

$$J = E\left\{\lim_{t\to\infty}\left[y'(t)Q_1 y(t) + u'(t)Q_2 u(t) + \dot{u}'(t)Q_3\dot{u}(t)\right]\right\} \qquad (23)$$

To make this cost functional compatible with optimal stochastic control theory, we include the controls, u, as states in the system and define a new state vector, z, and a new control vector, v, such that

$$z = \begin{bmatrix} x \\ \hline u \end{bmatrix}, \quad v = \dot{u} \qquad (24)$$

Then, the pilot vehicle equations, (14) and (15), become

$$\begin{bmatrix} \dot{x} \\ \hline \dot{u} \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline 0 & 0 \end{array}\right] \begin{bmatrix} x \\ \hline u \end{bmatrix} + \begin{bmatrix} 0 \\ \hline I \end{bmatrix} \dot{u} + \begin{bmatrix} N \\ \hline 0 \end{bmatrix} \eta \qquad (25)$$

$$\begin{bmatrix} y \\ \hline u \end{bmatrix} = \left[\begin{array}{c|c} C & 0 \\ \hline 0 & I \end{array}\right] \begin{bmatrix} x \\ \hline u \end{bmatrix} \qquad (26)$$

or

$$\dot{z} = A_1 z + B_1 v + N_1\eta \qquad (27)$$

$$y_1 = C_1 z \qquad (28)$$

where z and v are from Equation (24) and

$$A_1 = \left[\begin{array}{c|c} A & B \\ \hline 0 & 0 \end{array}\right], \quad C_1 = \left[\begin{array}{c|c} C & 0 \\ \hline 0 & I \end{array}\right], \quad B_1 = \begin{bmatrix} 0 \\ \hline I \end{bmatrix}, \quad y_1 = \begin{bmatrix} y \\ \hline u \end{bmatrix}, \quad N_1 = \begin{bmatrix} N \\ \hline 0 \end{bmatrix} \qquad (29)$$

In this case, the cost functional of Equation (23) would become

$$J = E\left\{\lim_{t\to\infty}\left[y_1'(t)Q_4 y_1(t) + v'(t)Q_3 v(t)\right]\right\} \qquad (30)$$

where

$$Q_4 = \left[\begin{array}{c|c} Q_1 & 0 \\ \hline 0 & Q_2 \end{array}\right] \qquad (31)$$

Now, we can state the optimal control problem. Given the state dynamics of Equations (27) and (28), find the optimal control, v^*, which is restricted to be a linear combination of the observations, y_1.

$$v^* = H_1 y_1 \qquad (32)$$

such that the cost functional of Equation (30) is minimized. Once again, the solution for H_1 can be found using References 1 and 14.

To show the resulting closed loop dynamics, it is necessary to expand the optimal control, Equation (32). Using Equation (24) we have for the optimal control

$$v^* = \dot{\underset{\sim}{u}}^* = \underset{\sim}{H}\underset{\sim}{y} + \underset{\sim}{H}u\,\underset{\sim}{u} \qquad (33)$$

Substituting into Equation (25), the optimal closed loop pilot vehicle control dynamics become

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \dot{\underset{\sim}{u}} \end{bmatrix} = \left[\begin{array}{c|c} \underset{\sim}{A} & \underset{\sim}{B} \\ \hline 0 & 0 \end{array}\right] \begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{u} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \left[\begin{array}{c|c} \underset{\sim}{H}C & \underset{\sim}{H}u \end{array}\right] \begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{u} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{N} \\ 0 \end{bmatrix} \eta$$

or $\qquad (34)$

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \dot{\underset{\sim}{u}} \end{bmatrix} = \left[\begin{array}{c|c} \underset{\sim}{A} & \underset{\sim}{B} \\ \hline \underset{\sim}{H}C & \underset{\sim}{H}u \end{array}\right] \begin{bmatrix} \underset{\sim}{x} \\ \underset{\sim}{u} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{N} \\ 0 \end{bmatrix} \eta$$

By going back and using the equations which define the matrices in Equation (34), it can be shown that

$$\begin{bmatrix} \dot{x} \\ \dot{x}_p \\ \dot{u} \\ \dot{u}_p \end{bmatrix} = \left[\begin{array}{c|c|c|c} E^{-1}F & 0 & E^{-1}G & 0 \\ \hline 0 & E_p{}^{-1}F_p & 0 & E_p{}^{-1}G_p \\ \hline HC & H_p & \multicolumn{2}{c}{\underset{\sim}{H}u} \\ \hline KC & K_p & & \end{array}\right] \begin{bmatrix} x \\ x_p \\ u \\ u_p \end{bmatrix} + \left[\begin{array}{c|c} E^{-1}M & 0 \\ \hline 0 & E_p{}^{-1}M_p \\ \hline 0 & 0 \\ \hline 0 & 0 \end{array}\right] \begin{bmatrix} \eta \\ \eta_p \end{bmatrix} \qquad (35)$$

Equation (35) describes the optimal closed loop dynamics of the pilot vehicle system for $J = f(\underset{\sim}{y}, \underset{\sim}{u}, \dot{\underset{\sim}{u}})$. The next section will discuss the problem of obtaining the quadratic weights for the cost functionals from experimental data.

VI. QUADRATIC WEIGHTS FOR PILOT RATING

This section will address the problem of determining the elements of the quadratic weighting matrices from experimental test data. An additional goal is to show how these weighting matrices may be sized so that the numerical value of the cost functional will be equivalent to some preassigned scale of measurement such as Pilot Rating. We begin with the general cost functional

$$\begin{aligned} J &= E\left\{\lim_{t\to\infty} [x'(t)Qx(t) + u'(t)Ru(t)]\right\} \\ &= E\left\{\lim_{t\to\infty} [x'(t)Qx(t)] + \lim_{t\to\infty} [u'(t)Ru(t)]\right\} \\ &= E\left\{\lim_{t\to\infty} [x'(t)Qx(t)]\right\} + E\left\{\lim_{t\to\infty} [u'(t)Ru(t)]\right\} \\ &= \text{trace } Q\left(E\left\{\lim_{t\to\infty} [x'(t)x(t)]\right\}\right) + \text{trace } R\left(E\left\{\lim_{t\to\infty} [u'(t)u(t)]\right\}\right) \end{aligned} \qquad \ldots (36)$$

or

$$J = \text{tr}\,(QX) + \text{tr}\,(RU) \qquad (37)$$

where X and U are the steady state covariance matrices for the states and the controls, respectively. *We now restrict Q and R to be diagonal* weighting matrices

$$Q = \begin{bmatrix} q_1 & & (0) \\ & q_2 & \\ & & \ddots \\ (0) & & & q_n \end{bmatrix}, \quad R = \begin{bmatrix} r_1 & & (0) \\ & r_2 & \\ & & \ddots \\ (0) & & & r_m \end{bmatrix} \qquad (38)$$

Then the trace operator in Equation (37) will pick off the diagonal *elements of the covariance matrices to give*

$$J = \sum_{i=1}^{n} (q_i\, x_{ii}) + \sum_{j=1}^{m} (r_j\, u_{jj}) \qquad (39)$$

where x_{ii} and u_{jj} are the diagonal elements of the covariance matrices X and U, respectively. Now because X and U are covariance matrices, it is well known, Reference 4, that the steady rate root mean square (rms) values of the system states $\bar{x}_i$, and controls $\bar{u}_j$, can be expressed

$$\bar{x}_i = (x_{ii})^{1/2} \qquad \bar{u}_j = (u_{jj})^{1/2} \tag{40}$$

Thus, the performance function can be expressed in terms of the weighting coefficients and the steady state rms values of the states and controls.

$$J = \sum_{i=1}^{n} (q_i\,\bar{x}_i^{\,2}) + \sum_{j=1}^{m} (r_j\,\bar{u}_j^{\,2}) \tag{41}$$

Since the rms values of the states and controls can be measured from experimental data with a pilot in the loop, we now address the problem of choosing the weighting coefficients.

An example will now be given to show how the weighting coefficients can be selected to predict pilot ratings.

In the development of the "Paper Pilot" approach to predicting pilot acceptance of aircraft handling qualities, Anderson and Dillow (References 10 and 11) have shown that the pilot will adjust his gains and model parameters to minimize a cost functional which, for most cases, is numerically equal to Pilot Rating. The "Paper Pilot" rating functional consisted of a weighted linear combination of the root mean squared (rms) values of the vehicle states and the pilot "lead" terms. The pilot rating functional proposed in this paper is a weighted linear combination of the squar of the rms values of the vehicle states, the pilot's states, and the controls. The selection of the use of the squares of the rms values is computationally more attractive since values are merely the diagonal element of the covariance matrix solutions to the closed loop, steady state optimal pilot vehicle system. The selection of the rms squared values, however, is further motivated by the work of Schmotzer, Reference 15, where the handling qualities for F-4C and F-80 aircraft are shown to be in direct proportion to the rms squared values of the aircraft states when the aircraft are being randomly disturbed by aerodynamic turbulence.

Assume that t experiments are conducted and for each experiment, data are collected on steady state rms values for the states and controls, and the associated Pilot Rating (PR) is recorded. We would then have for the t experiments, using PR for J in equation (41),

$$\begin{aligned} PR_{(1)} &= \alpha + \sum_{i=1}^{n} q_i\,\bar{x}_{i(1)}^{\,2} + \sum_{j=1}^{m} r_j\,\bar{u}_{j(1)}^{\,2} \\ PR_{(2)} &= \alpha + \sum_{i=1}^{n} q_i\,\bar{x}_{i(2)}^{\,2} + \sum_{j=1}^{m} r_j\,\bar{u}_{j(2)}^{\,2} \\ &\;\;\vdots \qquad\qquad\qquad \vdots \\ PR_{(t)} &= \alpha + \sum_{i=1}^{n} q_i\,\bar{x}_{i(t)}^{\,2} + \sum_{j=1}^{m} r_j\,\bar{u}_{j(t)}^{\,2} \end{aligned} \tag{42}$$

where α is a constant bias term. The equation in (42) can also be expressed in matrix form by factoring the weighting coefficients such that

$$\begin{bmatrix} PR_{(1)} \\ PR_{(2)} \\ \cdot \\ \cdot \\ \cdot \\ PR_{(t)} \end{bmatrix} = \begin{bmatrix} 1 & \bar{x}_{1(1)}^{\,2} & \bar{x}_{2(1)}^{\,2} \cdots \bar{x}_{n(1)}^{\,2} & \bar{u}_{1(1)}^{\,2} & \bar{u}_{2(1)}^{\,2} \cdots \bar{u}_{m(1)}^{\,2} \\ 1 & \bar{x}_{1(2)}^{\,2} & \bar{x}_{2(2)}^{\,2} \cdots \bar{x}_{n(2)}^{\,2} & \bar{u}_{1(2)}^{\,2} & \bar{u}_{2(2)}^{\,2} \cdots \bar{u}_{m(2)}^{\,2} \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ 1 & \bar{x}_{1(t)}^{\,2} & \bar{x}_{2(t)}^{\,2} \cdots \bar{x}_{n(t)}^{\,2} & \bar{u}_{1(t)}^{\,2} & \bar{u}_{2(t)}^{\,2} \cdots \bar{u}_{m(t)}^{\,2} \end{bmatrix} \begin{bmatrix} \alpha \\ q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ q_n \\ r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_m \end{bmatrix} \tag{43}$$

or

$$[PR] = P q \qquad (44)$$

where [PR] is a txl column vector of the Pilot Ratings, q is a txl column vector of the weighting coefficients, and P is readily identified from Equation (43). It can be shown that the best fit with respect to "least squares" or "minimum norm," Reference 16, is

$$q = P^{\dagger} [PR] \qquad (45)$$

where $P^{\dagger}$ is the generalized or pseudo inverse of P.

Now, it should be evident from Equation (43) that is is not necessary to weigh or measure every state of control. The form of the cost functionals associated with Pilot Ratings and the nonzero weighting coefficients will be a function of the particular control task. Thus, if the coefficients of the weighting matrices are chosen using Equation (45), the solution of the optimal pilot vehicle control problem, as posed in this paper, should produce not only the predicted closed loop performance but an associated Pilot Rating of the vehicle dynamics as well.

VII. CLOSING COMMENTS

In this final section of the paper, I will mention some of the present shortcomings of this optimal pilot vehicle control theory and then document some of my thoughts on where we could go from here. Obviously, one primary deficiency is that the theory has not been directly validated by experiment. The formulation of the theory is, however, based on an integration of ideas from all of the listed references and should work.

The reader may have noticed by now that no guidelines have been given to determine the size or order of the pilot model; this is a present shortcoming. My guess is that the order of the pilot model will be a function of the desirability to obtain an open loop pilot vehicle system which is completely controllable and completely observable (See Reference 16 for a mathematical system definition of these terms). Rynaski and Chen, Reference 17, have suggested that the pilot model be considered as a compensator and that the order of the pilot model be determined just as Brasch and Pearson, Reference 3, determine the order of a compensator necessary to achieve system stability or pole assignment. Perhaps, the most direct way of determining the pilot model order is to use the classical pilot vehicle approach; pick a low order model from those given in Reference 7, try it, and see if it works. Part of the motivation behind the development of a new approach to Pilot Vehicle Control was the need to have a pilot model whose structure was simple enough that the individual parameters of the model could be identified through experiments. This is the primary difficulty of the approach of Kleinman and Paskin where, because of the modeling of the pilot as a Kalman filter, the dynamic order of the model is necessarily the same size as the order of the plant or vehicle under control. In these cases, the validation of the model can practically be done only on an input-output basis and not through parameter identification techniques.

Another handicap at this time is that a consistent method does not exist for determining the pilot's motor and observation noise; call the whole thing remnant if you like. However, Kleinman and Paskin, References 9 and 12, have made some progress in this area.

Scaling the cost functional to be numerically equal to the Pilot Rating must be approached with caution. Ideally, one should be able to select a set of cost functional weighting matrices which will be valid over a given class of vehicles for a particular type of control task. As an example, the weighting matrices should be invariant for cargo aircraft in a landing approach task. The future of optimal pilot vehicle control looks very promising. One immediate application of the theory is in the direct assessment of aircraft handling qualities. With this optimal pilot model it will be possible to evaluate an aircraft over preplanned "stability and control" trajectories. The trajectories can be constructed as either fixed point or time varying linearized segments of actual nonlinear maneuvers.

The use of the state space formulation in the development of this optimal pilot vehicle control theory enables the rapid coupling of this theory with the design of automatic control systems utilizing optimal control theory. A successful design effort of this type using more classical control techniques has been accomplished by Hollis, Reference 18.

One final comment is included for the engineer concerned with basic control system design. The matrix equations of Equation (8) which describe the pilot dynamics will also mathematically describe a fixed order dynamic compensator network or an observer system in the sense Luenberger, Reference 2. Thus, the optimal control developed for the pilot dynamics will also be directly applicable to the design of compensators or observers.

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