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MODELS OF MAN AS A SUBOPTIMAL PREDICTOR

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ABSTRACT

Models of man making predictions of future states of discrete linear dynamic systems are considered. The task is forced-pace, but the pace is slow enough to eliminate the effects of reaction time and neuromuscular lag. The best of the several models considered includes the constraints of limited memory and observation noise.

INTRODUCTION

Many human activities depend on the ability to make predictions. We usually are fairly accurate when making the simple predictions required for such tasks as walking or opening a door which depend on a (perhaps unconscious) prediction of future positions of one's legs, arms, etc. However, as we try to predict further into the future and/or one's understanding of or experience with the process decreases, our predictive abilities degrade. For example, our abilities are somewhat limited when trying to predict the effect today's technology will have in the next century.

The improvement of man's ability to predict has paralleled the development of civilization. Economic, political, and technological progress is dependent on the confidence in the future that comes with the ability to make predictions. The further into the future that can be acceptably predicted, the better that investments, policies, and strategies can be planned.

This paper considers man making predictions of future states of discrete linear dynamic systems. Several models of the human in this task are presented. Two of the models, which assume simple extrapolation strategies, do not predict as well as the human. Two other models, which utilize two methods of system identification, predict much better than the human. A fifth model, which is basically a linear regression system identifier with a limited memory and noisy observations, matches human performance quite well. These models and others are used to discuss the limitations imposed on human predictive abilities by physiological and cognitive constraints. The effect of these constraints is related to the distance into the future that the human attempts to predict.

THE TASK

The following prediction task has been considered. The subject sat in a darkened booth and viewed the computer-generated display shown in Figure 1. The display represents the output of a discrete linear dynamic system given by

$$x_{N+1} = c_o y_{N+1} + \underline{C}^T \underline{X}_N \qquad (1)$$

where

$$\underline{X}_N = \begin{bmatrix} x_N \\ \dot{x}_N \\ \ddot{x}_N \end{bmatrix} = \text{system state at time } N,$$

$$\underline{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \text{a vector of constants},$$

y_{N+1} = input at time N+1 sampled from a zero-mean Gaussian process,

c_o = a constant,

and the derivatives noted in $\underline{X}$ were approximated using one-sided difference equations. Amplitude x is on the horizontal scale and time increases downward on the vertical scale. The points displayed are Δt = 1.0 units apart.

The subject viewed the last ten points of system output. His task was to predict the eleventh point, the horizontal position of which he controlled with a potentiometer. The horizontal line at the top of the display represented the time in which the prediction had to be completed. The length of the line decreased as the time remaining decreased. When the length of the bar went to zero, the subject's prediction was read by the computer and output on paper tape along with the optimal prediction and the actual next point. All the points on the display (with the exception of the subject's prediction) then shifted one unit up the vertical scale and a new point appeared below these points representing the actual point which the subject has just tried to predict. Thus, assuming that the subject had not moved his potentiometer during the updating interval, he saw his prediction error as the difference in horizontal position between the new tenth point and the eleventh point which he had predicted. The process was then repeated. The time per prediction was fifteen seconds. Each trial lasted at least twenty minutes making for at least eighty predictions per trial.

Eight subjects were used, four of which were well acquainted with system dynamics and optimal control while the nearest acquaintance with the task of the other four was freshman calculus. They were instructed to minimize RMS prediction error.

Each subject performed a randomly chosen sequence of eight trials. Each trial was characterized by an approximation to the normalized integrated absolute autocorrelation function I given by

$$I = \frac{1}{\sigma_x^2} \sum_{i=0}^{\infty} |\phi_{xx}(i\Delta t)| \qquad (2)$$

where

$\phi_{xx}(i\Delta t)$ = autocorrelation function,

σ_x^2 = variance of displayed signal.

Equation (2) is a measure of the "memory" of the system or, in other words, it is a measure of the confidence with which the next point can be predicted based solely on the information contained in a limited number of past points. This measure is independent of input variance. The input variance was chosen so as to yield a constant output standard deviation σ_x of 100.0 (with a range of 1000.0).

The results of the experiment are shown in Figure 2. It was assumed that the last sixty of the eighty-plus predictions represented a more or less steady-state (not necessarily non-tim[illegible] behavior. Since there was no statistically significant difference bet[illegible] formance of those subjects familiar with system dynamics and those unfa[illegible] data shown is across all eight subjects. Each value of the RMS predict[illegible] with respect to optimal e_{os} represents an average across twenty predictions for a particular subject. Thus, the data includes three values of e_{os} per subject per trial. Hence, $\sigma_{e_{os}}$ is a measure of the dispersion of average prediction errors. Since the performance of the models was measured in exactly the same way, this averaging of averages presents no problem.

MODELS

The following models represent several approaches to describing human performance in the above task. They will only be discussed in general. The reader is referred to Rouse [5] for the derivations.

While all five models vary to some degree, they all include an observation noise parameter to account for the fact that a human observer cannot perfectly estimate the magnitudes of physical stimuli. This psychophysical phenomenon was formalized by the well-known Weber-Fechner law and its application to the observation of continuous time series has been experimentally investigated by Levison, Baron, and Kleinman [3] and discussed by Crossman [1]. Specifically, the idea is applied here by assuming that the standard deviation σ_x of the human's estimate of a quantity x is given by

$$\frac{\sigma_x}{|x|} = F = \text{constant.} \tag{3}$$

A very simple model is that of linear extrapolation from the previous two points

$$\hat{x}_{N+1} = x_N + (x_N - x_{N-1}) \tag{4}$$

where $\hat{x}$ is the human's prediction. Since one-sided difference equations have been used to approximate derivatives, (4) can be written as

$$\hat{x}_{N+1} = x_N + \dot{x}_N \tag{5}$$

A slightly more sophisticated model assumes that the human fits a second-order curve through the last three points and then extrapolates that curve to make his prediction. This can easily be shown to yield

$$\hat{x}_{N+1} = x_N + \dot{x}_N + \ddot{x}_N \tag{6}$$

The RMS error between each of these models and the optimal was determined analytically and is shown in Figure 3. As can be seen, these models do not perform as well as the subjects. Increasing F above zero would only make matters worse.

The simple extrapolater models do not learn about the system from observing it. Their strategy is fixed. We will now consider a model that collects data and then performs a linear regression on that data to identify the system. This model is termed a learning model because the quality of the identification improves in time. The model learns more and more about the system as it collects more data. This definition of learning is different from classical operant conditioning. The model gains nothing from its mistakes. Learning is considered here only in the sense of gaining information from its environment.

The learning model makes predictions using

$$\hat{x}_{N+1} = \hat{\underline{C}}\, \underline{X}_N \tag{7}$$

where

$$\hat{\underline{C}} = (\underline{A}^T\underline{A})^{-1}\underline{A}^T\underline{H} \tag{8a}$$

and

$$\underline{A} = \begin{bmatrix} x_{N-1} & \dot{x}_{N-1} & \ddot{x}_{N-1} \\ x_{N-2} & \dot{x}_{N-2} & \ddot{x}_{N-2} \\ & \vdots & \end{bmatrix} \tag{8b}$$

$$\underline{H} = \begin{bmatrix} x_N \\ x_{N-1} \\ \vdots \end{bmatrix} \tag{8c}$$

Figure 4 compares the learning model with the experimental data. The data for the model is averaged over the last sixty of one hundred trials. This model does much better than the subjects. (In the limit of infinite trials, the model would be perfectly optimal for F = 0.0) Increasing F was attempted but an F or 0.10 or 0.20 made very little difference and it would not be consistent with psychophysical data to use an F of 1.0, 2.0, or higher.

The following model is a discrete equivalent of Ware's model [7]. It will be termed the autocorrelation model and makes predictions using

$$\hat{x}_{N+1} = \hat{c}_1 x_N + \hat{c}_2 \dot{x}_N \tag{9}$$

where

$$\hat{c}_1 = \frac{\phi_{xx}(\Delta t) + \phi_{xx}(2\Delta t)}{\phi_{xx}(o) + \phi_{xx}(\Delta t)}, \tag{10a}$$

$$\hat{c}_2 = \frac{\phi_{xx}^2(\Delta t) - \phi_{xx}(o)\phi_{xx}(2\Delta t)}{\phi_{xx}^2(o) - \phi_{xx}^2(\Delta t)}, \tag{10b}$$

and $\phi_{xx}(i\Delta t)$ is the autocorrelation function. This model is compared with the experimental data in Figure 5. It does much better than the subjects did. This is not surprising since the expansion of a few terms in the learning model (8a) shows terms such as $x_N x_{N+1}$, $x_N x_{N+2}$, etc.

The last model to be considered is a limited memory model. The mathematics involved are similar to that of the learning model except that there is a double-exponential memory weighting function that effectively forgets old data. This is accomplished by changing (8a) to

$$\hat{\underline{C}} = (\underline{A}^T \underline{W} \underline{A})^{-1} \underline{A}^T \underline{W} \underline{H} \tag{11}$$

where $\underline{W}$ is a diagonal square matrix with diagonal elements w_i^2 given by

$$w_i^2 = K_1 e^{-b(\text{[illegible]}-i)} + K_2 e^{-\text{[illegible]}(i-\text{[illegible]})} \tag{12}$$

where

$$K_1 = a/(a-b),$$

$$K_2 = -b/(a-b),$$

$$a = D + \sqrt{D^2 - 1},$$

$$b = D - \sqrt{D^2 - 1},$$

and $D > 1.0$ is a free parameter in the model. Computationally, it is necessary to truncate (12) in order for $\underline{W}$ to be of finite order. It was assumed that if $w_i^2 < 0.10$, then $w_i^2 = 0.0$.

The limited memory model is compared to the experimental data in Figure 6. The parameters of the model were found by fitting the data for the trial with lowest I. The rest of the trials were then run with the same parameters. This emphasizes the closeness of the fit of the model to data.

The D of 2.5 leads to an effective memory length of thirteen. This means that the human uses thirteen past states to compute $\underline{\hat{C}}$. Once he calculates $\underline{\hat{C}}$, he uses only the present state to make his prediction. The state contains only three past points (assuming the human used the same derivative formulas as the model). The distinction between memory length and the length of the state vector is important. The human might very well decide that the state vector $\underline{X}$ need only be of length two or three which might include perhaps three or four past points depending on derivative approximations. However, when determining $\underline{\hat{C}}$, he should use as much past history as he can. (This assumes that the system is stationary and he realizes that such is the case.) Since the statistical quality of $\hat{C}$ is a monotonically increasing function of memory length, the human has no reason to limit memory length. Thus, it is inferred that the effective memory length of thirteen found with the model is not the result of the human purposely discounting the value of less recent data.

DISCUSSION

The above models indicate that human prediction strategies are more sophisticated than simple extrapolations, but also, that humans are suboptimal predictors.

These results do not agree with those of Ware [7] or those of Kleinman, Baron, ...on [2] who assume the human to be an optimal predictor. However, their tasks in... human's motor system to a significant degree and because of this, delays ...romotor dynamics had to be included as sources of suboptimality. Also, ...system can have the positive effect of providing proprioceptive cues by the ...for example, the arm-stick combination [4]. These factors combined with ...at subjects in their tasks were making relatively short predictions, make ...of their results with those presented here a little difficult.

At the other end of the prediction length scale, Sheridan and Rouse [6] have studied the human's ability to predict several time units into the future. As might be intu...., the human's performance becomes increasingly suboptimal with prediction length. This behavior has been modeled [5] and the conclusion reached was that the human has difficulty determining the amount of signal history due to the noise input. While this supports the proposition that limited memory constrains him from collecting sufficient data, applying the limited memory model to long prediction tasks yields unsatisfactory comparisons with the experimental data of Sheridan and Rouse. This problem was further investigated by performing a linear regression on the experimental data from which the parameters of the limited memory model were determined. The results of the regression were then used to estimate what the human might have done if he had been predicting further into the future. These predictions were inconsistent with Sheridan and Rouse's data in the same way that the limited memory model was inconsistent.

This seems to indicate that some new source of suboptimality becomes significant with longer predictions. The optimal prediction trajectories for long predictions do not look very much like the actual time series. The optimal trajectory approaches the mean of the time series exponentially. Adopting an optimal-like strategy for long predictions may require a conceptualization that humans find difficult to accept.

By way of analogy, consider asking a human uninitiated in the laws of probability to predict the flip of an unbiased coin. If the person is instructed to minimize prediction error where heads = 1 and tails = 0, the optimal strategy is to predict the expected value of 1/2. However, anyone who has instructed beginning students of probability, knows the difficulty of convincing a percentage of the students that the expected value of the above coin flip is 1/2. Usually some student will wonder how 1/2 can be the expected value ... in actuality a flip can only yield the values of 0 or 1.

It seems reasonable to assign such conceptual difficulties to the cognitive category. With this assignment, Figure 7 summarizes the relative significance of physiological and cognitive constraints on the human's ability to predict. Reaction time and neuromotor dynamics are not constraints (and propioceptive cues are not aids) when making long predictions. Cognitive constraints are not very significant (relative to other constraints) for short, fast predictions but increase in significance as the prediction length increases.

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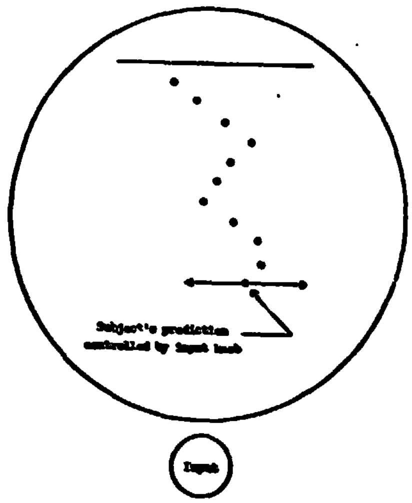
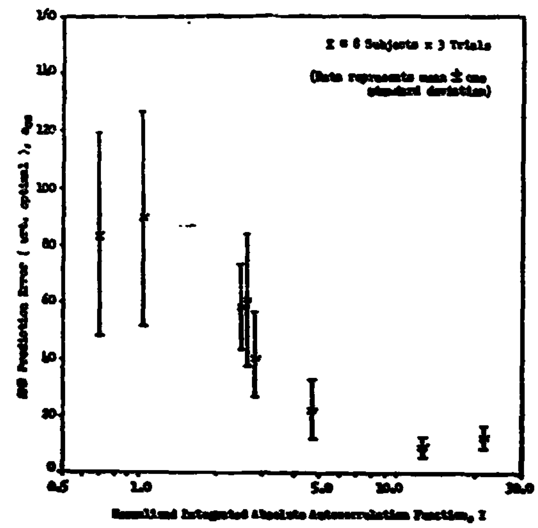


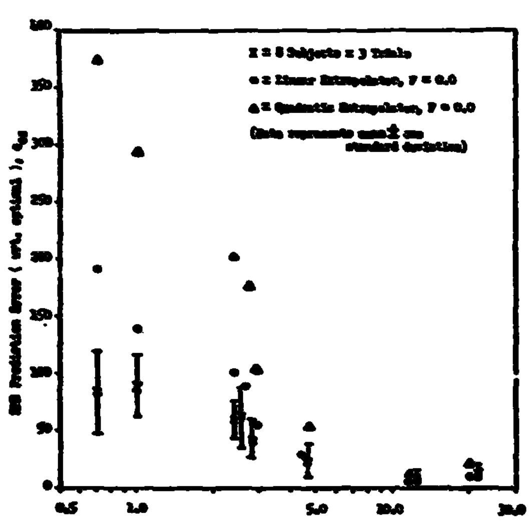
Display for Prediction Experiment

Figure 1



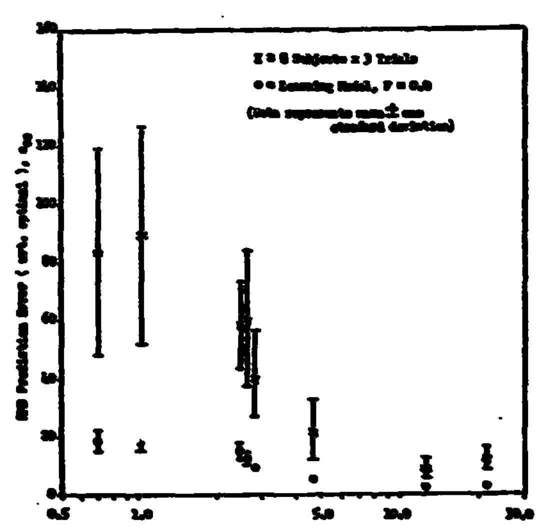
Data from Prediction Experiment

Figure 2



Comparing the Simple Extrapolator Models
with the Prediction Data

Figure 3



Comparing the Learning Model with the Prediction Data

Figure 4

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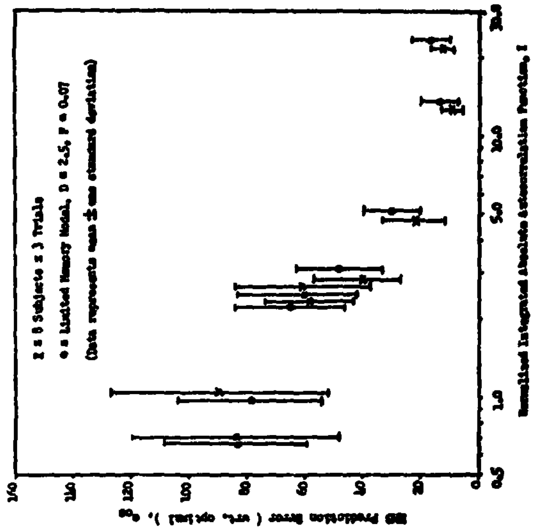
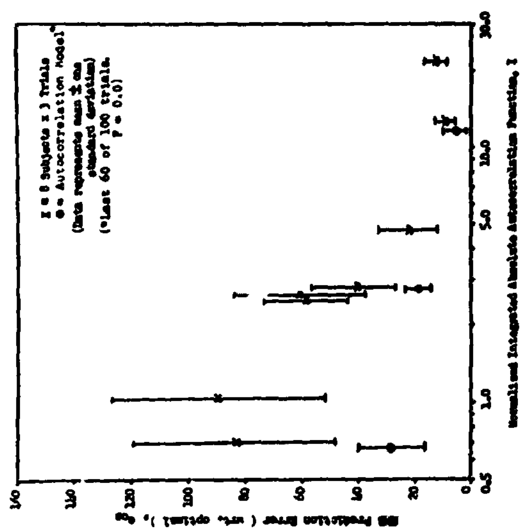


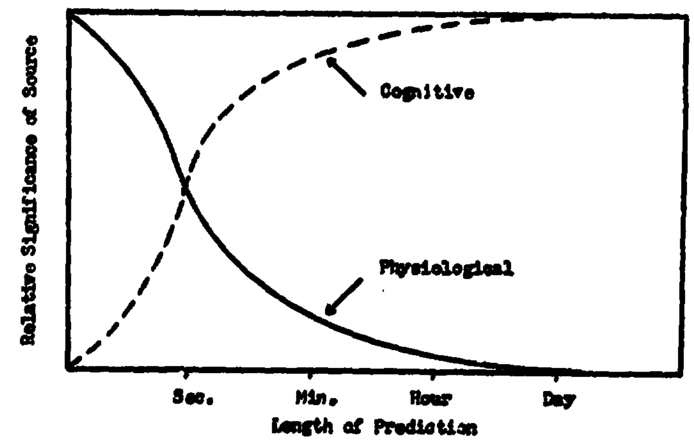
Comparing the Limited Memory Model with the Prediction Data

Figure 6



Comparing the Autocorrelation Model with the Prediction Data

Figure 5



Relative Significance of Sources of Suboptimality

Figure 7