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The Preview Control Problem with Application to

Man-Machine System Analysis

by

Masayoshi Tomizuka* and Daniel E. Whitney**
*Research Assistant, Department of Mechanical Engineering
**Associate Professor of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 U.S.A.

Abstract

The preview control problem is formulated in a general form and its solution is obtained. The analytical tool used is discrete stochastic optimal control theory. Aiming the application to manual control situations with preview, time delay, observation noise, motor noise, etc. were included in formulating the problem.

Some manual preview control experiments have been performed to qualitatively check the validity of the model, and it was found that the mechanism of the manual control problem was explained by the developed model pretty well.

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I. Introduction

In this paper, the preview control problem, in which information about future inputs and disturbances is used as well as the present and past information in deciding control, is considered. It is reasonable to expect that we can achieve better performance in some sense in the case when we know about the future compared to the case when we have no idea about the future. Such cases include:

1) Car driving
2) Airplane landing
3) Control of vehicle suspension
4) Process control problems
5) Economic or industrial problems etc.

Above 1) and 2) are related to manual control or man-machine systems. It is our objective to develop the general theory of the preview problem using optimal control theory, and to apply it to manual control problems.

Several works related to preview control have been published already. Sheridan[1] proposed three models of preview control. Bender[2] solved a class of preview control problems using Wiener filter theory and a parameter search, and applied it to the design of vehicle suspensions. Hayase and Ichikawa[3] treated the problem from the viewpoint of deterministic control theory and obtained a suboptimal control. However, the manual preview control problem is usually very complicated because of reaction time, remnant, etc., and is difficult to analyze using the above results.

The analysis of the manual control problem using optimal control theory has been done by several people[4,5,6]. Among these, the most successful work has been done by Kleinman, Baron and Levison[4], and some of their ideas are used in this paper. So far, however, this kind of analysis has been done only for the compensatory and pursuit tracking problems. In this paper, it is extended to the preview tracking problem.

In the next section, the formulation of the preview control problem is given. This is solved in Section III. Section IV describes the manual preview control experiment performed to qualitatively evaluate the validity of the proposed model, and the data from the experiment are analyzed in Section V. Conclusions and what should be done in the future work are stated in Section VI.

II. Formulation of Preview Control Problem

Let us consider the situation in Fig. 2-1. We have a plant to which we can apply control $u(t)$. The plant might meet some disturbance or noise $w(t)$, and we can measure $y(t)$ by a sensor with a measurement time delay τ and observation noise $v(t)$. We would like to control the plant so that the output $y(t)$ of the system follows the desired trajectory $y_d(t)$ (Fig. 2-2) as close as possible with a reasonable amount of control over the time interval from $t_o \le t \le t_f$.

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Depending on the amount of a priori information we have about $y_d(t)$, we can divide the problem in three cases.

Case 1. Complete knowledge about $y_d(t)$. $t_0 \leq t \leq t_f$

Case 2. Statistics or some characteristics of $y_d(t)$ is known.

Case 3. Nothing is known.

In optimal control terminology, Case 1 is called the tracking problem. In Case 3, measuring $y_d(t)$ is definitely necessary. In Case 2, measuring $y_d(t)$ is not necessary, but if it can be done the quality of control will increase. Usually only $y_d(t)$ is measured or given at time t; however, in our problem $y_d(\tau)$ is measured or given for $t \leq \tau \leq t+t_{\ell a}$ at time t (See Fig. 2-2). This is the preview problem. In our case, it is further assumed that the measurement of $y_d(\tau)$ has a measurement time delay τ_d and observation noise v_d. In this paper a priori information on $y_d(t)$ is in the form of Case 2 and it is modeled as a time correlated zero mean random process.

It is our objective to study how we can use the information about future y_d most effectively. In the following, the preview problem is formulated as an optimal control problem. What we want to decide is "?" in Fig. 2-3.

In this paper, the problem is formulated in discrete fashion and, noises are all assumed to be Gaussian and white. Before going into the mathematical formulation, basic symbols are listed. (If not familiar with discrete systems, see Bryson and Ho[8].)

$\underline{\cdot}$ means . is a vector or a matrix

Subscript i denotes time

$E[\cdot]$ means the expectation of $\cdot$

R^k denotes k- dimensional Euclidean space

($\underline{\cdot} \in R^k$ is shorthand for saying "$\underline{\cdot}$ is a k-dimensional vector")

δ_{ij} is the Dirac delta function $\left(\delta_{ij} = 1 \text{ for } i = j, = 0 \text{ otherwise}\right)$

Superscript T denotes the transpose of a vector or a matrix

A lot of control problems with preview can be formulated in the following form, although more general formulations are possible.

First, the system to be considered is given by the following difference equation.

$$\underline{x}_{i+1} = \underline{\Phi}_i \underline{x}_i + \underline{\Gamma}_i \underline{u}_i + \underline{w}_i \tag{2-1}$$

$$\underline{z}_i = \underline{C}^T_{i-da}\underline{x}_{i-da} + \underline{v}_i = \underline{y}_{i-da} + \underline{v}_i \tag{2-2}$$

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where $\underline{\Phi}_i$, $\underline{\Gamma}_i$ and $\underline{C}_{i-da}$ are (n×n), (n×m) and (n×r) matrices, respectively,

$\underline{x}_i \in R^n$, $\underline{u}_i \in R^m$, $\underline{z}_i \in R^r$, $E[\underline{w}_i] = E[\underline{v}_i] = 0$, da is a measurement time delay, da ≥ 0,

$E[\underline{x}_k] = \bar{\underline{x}}_k$ (k= -da..., 0)†, $E[\underline{w}_i\underline{w}_j^T] = \underline{W}_i\delta_{ij}$, $\underline{W}_i \geq 0$, $E[\underline{v}_i\underline{v}_j^T] = \underline{V}_i\delta_{ij}$, $\underline{V}_i > 0$,

$E[(\underline{x}_k-\bar{\underline{x}}_k)(\underline{x}_\ell-\bar{\underline{x}}_\ell)^T] = \underline{P}xx_{k,\ell}$ (k= -da, ..., 0; and ℓ = -da, ..., 0)†,

$\underline{P}xx_{k,\ell} \geq 0$, $E[\underline{w}_i\underline{v}_j^T] = E[w_i\underline{x}_k^T] = E[\underline{v}_i\underline{x}_k^T] = \underline{0}$ (k= -da, ..., 0)†.

This system has to be controlled so that its output can follow the desired trajectory, which is modelled as the output of the following shaping filter, running $N_{\ell a}$ time steps ahead ($N_{\ell a}$ denotes the amount of look ahead available).

$$\underline{x}_{d_{i+1}} = \underline{\Phi}_{s_i} \underline{x}_{d_i} + \underline{\Gamma}_{s_i} \underline{w}_{d_i} \tag{2-3}$$

$$\underline{y}_{d_i} = \underline{C}_{s_i}^T \underline{x}_{d_i} \tag{2-4}$$

where $\underline{\Phi}_{s_i}$, $\underline{\Gamma}_{s_i}$ and $\underline{C}_{s_i}$ (subscript s denotes the shaping filter) are (t×t), (t×s) and (t×r) matrices respectively, $\underline{x}_{d_i} \in R^t$, $\underline{y}_{d_i} \in R^r$, $E[\underline{w}_{d_i}$ ...

and $E[\underline{w}_{d_i}\underline{w}_{d_j}^T] = \underline{W}_{d_i}\delta_{ij}$, $\underline{W}_{d_i} \geq 0$. At time i, it is assumed that the following measurements are possible.

$$\underline{z}_{d_i}(\ell) = \underline{y}_{d_{i-db+\ell}} + \underline{v}_{d_i}(\ell), \quad 0 \leqslant \ell \leqslant N_{l.a.} \tag{2-5}$$

db is another measurement time delay.

Consequently, the following are given. $E[\underline{v}_{d_i}(\ell)] = 0$, $E[\underline{v}_{d_i}(k)\underline{v}_{d_j}^T(\ell)]$ $= \underline{V}_{d_i}(k,\ell)\delta_{ij}$ (ℓ= 0, ..., $N_{\ell a}$; k=0, ..., $N_{\ell a}$), $\underline{V}_{d_i}(k,\ell) > 0$ for k = ℓ, $\underline{V}_{d_i}(k,\ell) \geq 0$ for k $\neq \ell$, $E[\underline{x}_{d_k}] = \bar{\underline{x}}_{d_k}$ (k= -db,...,$N_{\ell a}$), $E[(\underline{x}_{d_k}-\bar{\underline{x}}_{d_k})(\underline{x}_{d_\ell}-\bar{\underline{x}}_{d_\ell})^T]$ $= P_{dd_{k,\ell}}$ (k = -db,...,$N_{\ell a}$; ℓ = -db,...,$N_{\ell a}$), $P_{dd_{k,\ell}} \geq 0$, $E[\underline{v}_{d_i}(k)\underline{w}_{d_j}^T]$

†These have to be defined because of a delay.

$$= E[\underline{v}_{d_i}(k)\underline{x}_{d_\ell}^T] = E[\underline{v}_{d_i}\underline{x}_{d_\ell}^T] = \underline{0}\ (k = 0,\dots,N_{\ell a};\ \ell = -db,\dots,N_{\ell a}).$$

Also it is assumed that $E[\underline{w}_i \underline{w}_{d_j}^T] = E[\underline{w}_i \underline{v}_{d_j}^T(m)] = E[\underline{v}_i \underline{w}_{d_j}^T] =$

$$= E[\underline{v}_i \underline{v}_{d_j}^T(m)] = E[\underline{x}_k \underline{x}_{d_\ell}^T] = E[\underline{x}_k \underline{w}_{d_j}^T] = E[\underline{x}_k \underline{v}_{d_j}^T(m)] = E[\underline{v}_i \underline{x}_{d_\ell}^T] =$$

$$E[\underline{v}_i \underline{x}_{d_\ell}^T] = \underline{0}\ (k = -da,\dots,0;\ \ell = -db,\dots,N_{\ell a};\ m = 0,\dots,N_{\ell a}).$$

The objective is to find a control policy $\underline{u}_i$, which minimizes

$$J = E\Big[\frac{1}{2}(\underline{C}_N^T\underline{x}_N - \underline{y}_{d_N})^T\underline{Q}_N(\underline{C}_N^T\underline{x}_N - \underline{y}_{d_N}) + \frac{1}{2}\sum_{i=0}^{N-1}\{(\underline{C}_i^T\underline{x}_i - \underline{y}_{d_i})^T\underline{Q}_i(\underline{C}_i^T\underline{x}_i - \underline{y}_{d_i}) + \underline{u}_i^T\underline{R}_i\underline{u}_i\}\Big] \tag{2-6}$$

where $\underline{R}_i > \underline{0}$, $\underline{Q}_i \geq \underline{0}$ and $E[\cdot]$ is the expectation taken over all underlying random quantities. Several remarks should be made at this point.

Remark 2-1: Note the difference between the preview control problem formulated here and the stochastic tracking problem. In case of the stochastic tracking problem the $\underline{y}_{d_i}$'s in (2-6) are <u>all deterministically</u> given a priori from <u>i=0 to i=N</u> while the system has the driving noise and the observation noise. In case of the preview problem, however, the information about the $\underline{y}_{d_i}$'s is given by the <u>noise-corrupted</u> measurement (2-5) and usually $N_{\ell a} < N$. Therefore, the preview control problem is more difficult than the tracking problem, and the following relation holds:

Preview Control Problem $\supset$ Stochastic Tracking Problem.

Remark 2-2: There are time delays da and db in measurement equations (2-2) and (2-5) respectively, and in case of manual control problems these respresent the time delay of the human controller.

Remark 2-3: Although the problem is formulated in discrete fashion, it can be a good approximation to the continuous problem if discretization is made small. And actually the manual preview control problem in this paper is not all discrete, and the model should be taken as the discrete approximation.

III. Solution of the Preview Control Problem

In this section, the preview control problem is transformed into the equivalent regulator problem. Then the solution can be easily obtained by dynamic programming.

It is easy to check that (2-1) and (2-2) are equivalent to

$$\underline{x}'_{i+1} = \begin{bmatrix} \underline{\xi}^1_{i+1} \\ \vdots \\ \underline{\xi}^{da}_{i+1} \\ \hline \underline{x}_{i+1} \end{bmatrix} = \left[\begin{array}{cccc|c} \underline{0} & \underline{I} & \underline{0}\dots & 0 & \\ \underline{0} & \underline{0} & \underline{I}\dots & 0 & \underline{0} \\ \vdots & & & \vdots & \\ \underline{0} & \underline{0} & \underline{0}\dots & 0 & \underline{C}_i^T \\ \hline & & \underline{0} & & \underline{\Phi}_i \end{array}\right] \begin{bmatrix} \underline{\xi}^1_i \\ \vdots \\ \underline{\xi}^{da}_i \\ \hline \underline{x}_i \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \hline \underline{\Gamma}_i \end{bmatrix} \underline{u}_i + \begin{bmatrix} \underline{0} \\ \hline \underline{w}_i \end{bmatrix}$$

$$= \underline{\Phi}'_i\,\underline{x}'_i + \underline{\Gamma}'_i\,\underline{u}_i + \underline{w}'_i, \qquad da > 0 \tag{3-1}$$

$$\underline{z}_i = [\,\underline{I}\ \underline{0}\ \dots\ \underline{0}\,]\,\underline{x}'_i + \underline{v}_i$$

$$= \underline{C}'^T\,\underline{x}'_i + \underline{v}_i, \qquad da > 0 \tag{3-2}$$

where $\underline{\xi}^j_i \in R^r (j=1,\dots,da)$, $\underline{x}'_i \in R^{r\times d_1+n}$ etc., $\underline{I}$ is an identity matrix, and all matrices have appropriate dimensions. $\underline{\xi}^j_i$'s are introduced here to handle the measurement time delay. For $da = 0$ (no time delay), just set $\underline{x}' = \underline{x}$, $\underline{\Phi}' = \underline{\Phi}$, $\underline{\Gamma}' = \underline{\Gamma}$, $\underline{w}' = \underline{w}$ and $\underline{C}' = \underline{C}$ in (3-1) and (3-2). Note that $E[\underline{w}'_i]$, $E[\underline{w}'_i\underline{w}'^T_j]$, $E[\underline{x}'_0]$ etc., can be all obtained from the quantities given in the previous section.

Also it can be checked that (2-3), (2-4) and (2-5) are equivalent to

$$\underline{Y}'_{d_{i+1}} = \begin{bmatrix} \underline{g}^1_{i+1} \\ \vdots \\ \underline{g}^{db}_{i+1} \\ \hline \underline{Y}_{d_{i+1}} \end{bmatrix} = \left[\begin{array}{cccc|cc} \underline{0} & \underline{I} & \underline{0}\dots & \underline{0} & & \\ \underline{0} & \underline{0} & \underline{I}\dots & \underline{0} & \underline{0} & \\ \vdots & & & \vdots & & \\ \underline{0} & \underline{0} & \dots\dots & \underline{0} & \underline{I} & \underline{0} \\ \hline & & \underline{0} & & & \underline{\Phi}_{d_i} \end{array}\right] \begin{bmatrix} \underline{g}^1_i \\ \vdots \\ \underline{g}^{db}_i \\ \hline \underline{Y}_{d_i} \end{bmatrix} + \begin{bmatrix} \underline{0} \\ \hline \underline{\Gamma}_{d_i} \end{bmatrix} \underline{w}_{d_{i+N_{\ell a}}}$$

$$= \underline{\Phi}'_{d_i}\,\underline{Y}'_{d_i} + \underline{\Gamma}'_{d_i}\,\underline{w}_{d_{i+N_{\ell a}}}, \quad db > 0 \tag{3-3}$$

$$\underline{Z}_{d_i} = \begin{bmatrix} \underline{z}_{d_i}(0) \\ \vdots \\ \underline{z}_{d_i}(N_{\ell a}) \end{bmatrix} = \left[\begin{array}{cccc|c} \underline{I} & & \underline{0} & & \\ & \underline{I} & & & \underline{0} \\ \underline{0} & & \ddots & & \\ & & & \underline{I} & \end{array}\right] \begin{bmatrix} \underline{Y}'_{d_i} \end{bmatrix} + \begin{bmatrix} \underline{v}_{d_i}(0) \\ \vdots \\ \underline{v}_{d_i}(N_{\ell a}) \end{bmatrix}$$

$$= \underline{C}^T_{d_i}\,\underline{Y}'_{d_i} + \underline{v}'_{d_i}, \quad N_{\ell a} > 0 \tag{3-4}$$

with

$$\underline{Y}_{d_i} = \begin{bmatrix} y_{d_i} \\ y_{d_{i+1}} \\ \vdots \\ y_{d_{i+N_{la}-1}} \\ \hline \underline{x}_{d_{i+N_{la}}} \end{bmatrix}, \quad \underline{\Phi}_{d_i} = \left[\begin{array}{cccc|c} \underline{0} & \underline{I} & \underline{0} \cdots & \underline{0} & \\ \underline{0} & \underline{0} & \underline{I} & \vdots & \underline{0} \\ \vdots & & & \underline{I} & \\ \underline{0} & \underline{0} & \cdots & \underline{0} & \underline{C}^T_{s_{i+N_{la}}} \\ \hline & & \underline{0} & & \underline{\Phi}_{s_{i+N_{la}}} \end{array}\right] \text{ and } \underline{\Gamma}_{d_i} = \begin{bmatrix} \underline{0} \\ \\ \hline \underline{\Gamma}_{s_{i+N_{la}}} \end{bmatrix} \qquad \begin{array}{c}(3\text{-}5)\\ -(3\text{-}7)\end{array}$$

where $\underline{\sigma}^j_i \in R^r$, $\underline{Y}_{d_i} \in R^{N_{la} \times r+t}$, $\underline{Z}_{d_i} \in R^{(N_{la}+1)\times r}$, $\underline{Y}_{d_i}' \in R^{(N_{la}+db)\times r+t}$ etc.

For db=0, set $\underline{Y}_{d_i}' = \underline{Y}_{d_i}$, $\underline{\Phi}_{d_i}' = \underline{\Phi}_{d_i}$ and $\underline{\Gamma}_{d_i}' = \underline{\Gamma}_{d_i}$. Furthermore, if $N_{la} = 0$, set $\underline{Y}_{d_i} = \underline{x}_{d_i}$, $\underline{\Phi}_{d_i} = \underline{\Phi}_{s_i}$, $\underline{\Gamma}_{d_i} = \underline{\Gamma}_{s_i}$ and $\underline{C}_{d_i} = \underline{C}_{s_i}$. Here again all statistics necessary to get a solution can be all defined from the quantities given in the previous section. One further assumption to be made is $\underline{V}_i' > \underline{0}$ wher $E[\underline{v}_{d_i}' \underline{v}_{d_j}'^T] = \underline{V}_i' \delta_{ij}$.

(3-1), (3-2), (3-3) and (3-4) can be combined into the following.

$$\underline{x}_{i+1} = \begin{bmatrix} \underline{x}'_{i+1} \\ \underline{Y}'_{d_{i+1}} \end{bmatrix} = \begin{bmatrix} \underline{\Phi}'_i & \underline{0} \\ \underline{0} & \underline{\Phi}'_{d_i} \end{bmatrix} \begin{bmatrix} \underline{x}'_i \\ \underline{Y}'_{d_i} \end{bmatrix} + \begin{bmatrix} \underline{\Gamma}'_i \\ \underline{0} \end{bmatrix} u_i + \begin{bmatrix} \underline{w}'_i \\ \underline{\Gamma}'_{d_i} \underline{w}_{d_{i+N_{la}}} \end{bmatrix}$$

$$= \underline{\Phi}_i \underline{x}_i + \underline{\Gamma}_i \underline{u}_i + \underline{w} \qquad (3\text{-}8)$$

$$\underline{z}_i = \begin{bmatrix} \underline{C}_i'^T & \underline{0} \\ \underline{0} & \underline{C}^T_{d_i} \end{bmatrix} \begin{bmatrix} \underline{x}'_i \\ \underline{Y}'_{d_i} \end{bmatrix} + \begin{bmatrix} \underline{v}_i \\ \underline{v}'_{d_i} \end{bmatrix}$$

$$= \underline{C}^T_i \underline{x}_i + \underline{v}_i \qquad (3\text{-}9)$$

where $\underline{x}_i \in R^{(da+N_{la}+db)\times r + n + t}$, $\underline{z}_i \in R^{(2+N_{la})\times r}$ etc., $E[\underline{x}_o] = \bar{\underline{x}}_o$,

$E[(\underline{x}_o - \bar{\underline{x}}_o)(\underline{x}_o - \bar{\underline{x}}_o)^T] = \underline{P}_o$, $E[\underline{w}_i] = E[\underline{v}_i] = 0$, $E[\underline{w}_i \underline{w}_j^T] = \begin{bmatrix} \underline{W}_i & \underline{0} \\ \underline{0} & \underline{\Gamma}'_{d_i} \underline{W}_{d_{i+N_{la}}} \underline{\Gamma}'^T_{d_i} \end{bmatrix} \delta_{ij}$

$= \underline{W}_i \delta_{ij}$, $\underline{W}_i \geq 0$, $E[\underline{v}_i \underline{v}_j^T] = \begin{bmatrix} \underline{V}_i & \underline{0} \\ \underline{0} & \underline{V}'_{d_i} \end{bmatrix} \delta_{ij} = \underline{V}_i \delta_{ij}$, $\underline{V}_i > 0$, $E[\underline{x}_o \underline{w}_j^T] = E[\underline{x}_o \underline{v}_j^T]$

$= E[\underline{w}_i \underline{v}_j^T] = 0$ are all available from the quantities given so far.

With this new variable $\underline{x}$, (2-6) can be written as

$$J = E[\ \tfrac{1}{2} \underline{x}_N^T \underline{Q}_N \underline{x}_N + \tfrac{1}{2}\sum_{i=0}^{N-1} (\ \underline{x}_i^T \underline{Q}_i \underline{x}_i + \underline{u}_i^T \underline{R}_i \underline{u}_i\ )\ ] \qquad (3\text{-}10)$$

where

$$\underline{Q}_i = \left[\begin{array}{c|c|c|c} \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \hline \underline{0} & \underline{Q}_{xx_i} & \underline{0} & -\underline{Q}_{xd_i} \\ \hline \underline{0} & \underline{0} & \underline{0} & \underline{0} \\ \hline \underline{0} & -\underline{Q}^T_{xd_i} & \underline{0} & \underline{Q}_{dd_i} \end{array}\right] \begin{array}{l} \}\ r\times da \\ \}\ n \\ \}\ r\times db \\ \}\ N_{la}\times r + t \end{array}$$

with

$$\underline{Q}_{xx_i} = \underline{C}_i \underline{Q}_i \underline{C}_i^T$$

$$\underline{Q}_{xd_i} = \begin{cases} [\ \underline{C}_i \underline{Q}_i \mid \underline{0} \mid \underline{0} \mid \ldots \mid \underline{0}\ ] & \text{for } N_{la} > 0 \\ \underline{C}_i \underline{Q}_i \underline{C}^T_{s_i} & \text{for } N_{la} = 0 \end{cases}$$

$$\underline{Q}_{dd_i} = \begin{cases} \left[\begin{array}{c|c} \underline{Q}_i & \underline{0} \\ \hline \underline{0} & \underline{0} \end{array}\right] \begin{array}{l} \}\ r \\ \}\ (N_{la}-1)\times r + t \end{array} & \text{for } N_{la} > 0 \\ \underline{C}_{s_i} \underline{Q}_i \underline{C}^T_{s_i} & \text{for } N_{la} = 0 \end{cases}$$

Then (3-8), (3-9) and (3-10) represent the usual LQG (Linear-Quadratic-Gaussian) problem, and it is well known that the feedback type solution is obtained by the dynamic programming. (For example, see Refs. 7 or 8.)

The solution is as follows. In the following, $\hat{\underline{x}}_{i|j}$ denotes the estimator of $\underline{x}_i$ with the measurement up to time j.

$$\underline{u}_i^{opt} = -\underline{G}_i \hat{\underline{x}}_{i|i} \qquad (3\text{-}11)$$

$$\hat{\underline{x}}_{i|i} = \hat{\underline{x}}_{i|i-1} + \underline{K}_i (\ \underline{z}_i - \underline{C}_i^T \hat{\underline{x}}_{i|i-1}) \qquad (3\text{-}12)$$

$$\hat{\underline{x}}_{i+1|i} = \underline{\Phi}_i \hat{\underline{x}}_{i|i} + \underline{\Gamma}_i \underline{u}_i\ , \quad \hat{\underline{x}}_{o|o} = \bar{\underline{x}}_o \qquad (3\text{-}13)$$

where

$$\underline{G}_i = (\underline{\Gamma}_i^T \underline{S}_{i+1} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{i+1} \underline{\Phi}_i \qquad (3\text{-}14)$$

$$\underline{K}_i = \underline{M}_i \underline{C}_i (\underline{C}_i^T \underline{M}_i \underline{C}_i + \underline{V}_i)^{-1} = \underline{P}_i \underline{C}_i \underline{V}_i^{-1} \qquad (3\text{-}15)$$

and

$$\underline{S}_i = \underline{\Phi}_i^T \underline{M}_{i+1} \underline{\Phi}_i + \underline{Q}_i\ , \quad \underline{S}_N = \underline{Q}_N \qquad (3\text{-}16)$$

$$\underline{M}_{i+1} = \underline{S}_{i+1} - \underline{S}_{i+1} \underline{\Gamma}_i (\underline{\Gamma}_i^T \underline{S}_{i+1} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{i+1} \quad (3\text{-}17)$$

$$\underline{N}_{i+1} = \underline{\Phi}_i \underline{P}_i \underline{\Phi}_i^T + \underline{W}_i \quad (3\text{-}18)$$

$$\underline{P}_i = \underline{N}_i - \underline{N}_i \underline{C}_i (\underline{C}_i^T \underline{N}_i \underline{C}_i + \underline{V}_i)^{-1} \underline{C}_i^T \underline{N}_i \quad (3\text{-}19)$$

$\underline{P}_0$: given

Noting the forms of $\underline{\Phi}$, $\underline{\Gamma}$, $\underline{V}$, $\underline{W}$ etc., after a lot of manupulation (3-11)-(3-19) can be shown to be equivalent to the following set of equations.

$$\underline{u}_i^{opt} = - [\underline{G}_{x_i} \vdots \underline{G}_{d_i}] \begin{bmatrix} \hat{\underline{x}}_{i|i} \\ \hat{\underline{Y}}_{d_i|i} \end{bmatrix} \quad (3\text{-}20)$$

where

$$\underline{G}_{x_i} = (\underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Phi}_i \quad (3\text{-}21)$$

$$\underline{G}_{d_i} = (\underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{xd_{i+1}} \underline{\Phi}_{d_i} \quad (3\text{-}22)$$

with

$$\underline{S}_{xx_i} = \underline{\Phi}_i^T \underline{M}_{xx_{i+1}} \underline{\Phi}_i + \underline{Q}_{xx_i} , \quad \underline{S}_{xx_N} = \underline{Q}_{xx_N} \quad (3\text{-}23)$$

$$\underline{M}_{xx_{i+1}} = \underline{S}_{xx_{i+1}} - \underline{S}_{xx_{i+1}} \underline{\Gamma}_i (\underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \quad (3\text{-}24)$$

$$\underline{S}_{xd_i} = \underline{\Phi}_i^T \underline{M}_{xd_{i+1}} \underline{\Phi}_{d_i} - \underline{Q}_{xd_i} , \quad \underline{S}_{xd_N} = -\underline{Q}_{xd_N} \quad (3\text{-}25)$$

$$\underline{M}_{xd_{i+1}} = \underline{S}_{xd_{i+1}} - \underline{S}_{xx_{i+1}} \underline{\Gamma}_i (\underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{xd_{i+1}} \quad (3\text{-}26)$$

$$\underline{S}_{dd_i} = \underline{\Phi}_{d_i}^T \underline{M}_{dd_{i+1}} \underline{\Phi}_{d_i} + \underline{Q}_{dd_i} , \quad \underline{S}_{dd_N} = \underline{Q}_{dd_N} \quad (3\text{-}27)$$

$$\underline{M}_{dd_{i+1}} = \underline{S}_{dd_{i+1}} - \underline{S}_{xd_{i+1}}^T \underline{\Gamma}_i (\underline{\Gamma}_i^T \underline{S}_{xx_{i+1}} \underline{\Gamma}_i + \underline{R}_i)^{-1} \underline{\Gamma}_i^T \underline{S}_{xd_{i+1}} \quad (3\text{-}28)$$

$\hat{\underline{x}}_{i|i}$ can be found from $\hat{\underline{x}}'_{i|i}$ which satisfies

$$\hat{\underline{x}}'_{i|i} = \hat{\underline{x}}'_{i|i-1} + \underline{K}x'_i (\underline{z}_i - \underline{C}'^T_i \hat{\underline{x}}'_{i|i-1}) , \quad \hat{\underline{x}}'_{0|0} = \bar{\underline{x}}'_0 \quad (3\text{-}29)$$

$$\hat{\underline{x}}'_{i+1|i} = \underline{\Phi}'_i \hat{\underline{x}}'_{i|i} + \underline{\Gamma}'_i \underline{u}_i \quad (3\text{-}30)$$

$$\underline{K}x'_i = \underline{P}xx'_i \underline{C}'_i \underline{V}_i^{-1} \quad (3\text{-}31)$$

$$\underline{N}xx'_{i+1} = \underline{\Phi}'_i \underline{P}xx'_i \underline{\Phi}'^T_i + \underline{W}'_i \quad (3\text{-}32)$$

$$\underline{P}xx'_i = \underline{N}xx'_i - \underline{N}xx'_i \underline{C}'_i (\underline{C}'^T_i \underline{N}xx'_i \underline{C}'_i + \underline{V}_i)^{-1} \underline{C}'^T_i \underline{N}xx'_i \quad (3\text{-}33)$$

$\underline{P}xx'_0$: given

where

$$\hat{\underline{x}}'_{i|i} = \begin{bmatrix} \hat{\xi}^1_{i|i} \\ \vdots \\ \hat{\xi}^{da}_{i|i} \\ \hat{\underline{x}}_{i|i} \end{bmatrix} \text{ etc.}$$

$\hat{\underline{Y}}_{d_{i|i}}$ can be found from $\hat{\underline{Y}}'_{d_{i|i}}$ which satisfies

$$\hat{\underline{Y}}d'_{i|i} = \hat{\underline{Y}}d'_{i|i-1} + \underline{K}d'_i (\underline{Z}d_i - \underline{C}d_i^T \hat{\underline{Y}}d'_{i|i-1}), \quad \hat{\underline{Y}}d_{0|0} = \bar{\underline{Y}}d'_0 \quad (3\text{-}34)$$

$$\hat{\underline{Y}}d'_{i+1|i} = \underline{\Phi}d'_i \hat{\underline{Y}}d'_{i|i} \quad (3\text{-}35)$$

$$\underline{K}d'_i = \underline{P}dd'_i \underline{C}d_i \underline{V}d_i^{-1} \quad (3\text{-}36)$$

$$\underline{N}dd'_{i+1} = \underline{\Phi}d'_i \underline{P}dd'_i \underline{\Phi}d_i^T + \underline{W}d'_i \quad (3\text{-}37)$$

$$\underline{P}dd'_i = \underline{N}dd'_i - \underline{N}dd'_i \underline{C}d_i (\underline{C}d_i^T \underline{N}dd'_i \underline{C}d_i + \underline{V}d_i)^{-1} \underline{C}d_i^T \underline{N}dd'_i \quad (3\text{-}38)$$

$\underline{P}dd'_0$: given

where

$$\hat{\underline{Y}}'_{d_{i|i}} = \begin{bmatrix} \hat{g}^1_{i|i} \\ \vdots \\ \hat{g}^{db}_{i|i} \\ \hat{Y}_{d_{i|i}} \end{bmatrix} \text{ etc.}$$

The following remarks are appropriate here.

Remark 3-1: (3-20) - (3-26) show that the control gains do not depend on the time delay. The estimators $\hat{x}_{i|i}$ and $\hat{y}_{d_{i|i}}$, however, are affected by the time delay.

Remark 3-2: $\underline{S}_{dd}$ and $\underline{H}_{dd}$ in (3-27) and (3-28) do not have to be calculated in deciding the closed loop structure. However they are useful in getting the average cost.

Remark 3-3: Transition from (3-15), (3-18) and (3-19) to (3-29)-(3-38) shows that the Kalman filter for (3-8) and (3-9) can be separated into two; one for the system given by (3-1) and (3-2) and the other for the desired trajectory given by (3-3) and (3-4). This "separation" property follows from the fact that the stochastic quantities in (3-1) and (3-2) are uncorrelated from those in (3-3) and (3-4).

In many cases including the manual preview control experiment in this paper, we are interested in the steady state behaviour of the preview control system. Namely, $\underline{\Phi}_i$, $\underline{\Gamma}_i$, $\underline{W}_i$, $\underline{V}_i$, $\underline{\Phi}_{d_i}$, ... are all time invariant, and $N \to \infty$. In this case, the cost function to be looked at is

$$J = E\left[\frac{1}{2}\underline{x}^T \underline{Q}\,\underline{x} + \frac{1}{2}\underline{u}^T \underline{R}\,\underline{u}\right]$$

$$= E\left[\frac{1}{2}(\underline{C}^T\underline{x} - y_d)^T \underline{Q}\,(\underline{C}^T\underline{x} - y_d) + \frac{1}{2}\underline{u}^T \underline{R}\,\underline{u}\right] \qquad (3\text{-}39)$$

For this problem, the control gains can be determined once the steady state solution of (3-23) - (3-26) are obtained. Also the Kalman filter gains are determined by the steady state solution of (3-32), (3-33), (3-37) and (3-38). Moreover, in this case the filter equations can be simplified giving a more understandable form. The filter for the state of the system becomes the series connection of the Kalman filter for the delayed state $\underline{x}_{i-da}$ and the predictor for $\underline{x}_i$ based on $\hat{\underline{x}}_{i-da|i}$.

$$\hat{\underline{x}}_{i-da|i} = \hat{\underline{x}}_{i-da|i-1} + \underline{K}_x(\underline{z}_i - \underline{C}^T\hat{\underline{x}}_{i-da|i-1}) \qquad (3\text{-}40)$$

$$\hat{\underline{x}}_{i-da+1|i} = \underline{\Phi}\,\hat{\underline{x}}_{i-da|i} + \underline{\Gamma}\,\underline{u}_{i-da} \qquad (3\text{-}41)$$

$$\underline{K}_x = \underline{P}_{xx}\underline{C}\,\underline{V}^{-1} \qquad (3\text{-}42)$$

$$\underline{N}_{xx} = \underline{\Phi}\,\underline{P}_{xx}\underline{\Phi}^T + \underline{W} \qquad (3\text{-}43)$$

$$\underline{P}_{xx} = \underline{N}_{xx} - \underline{N}_{xx}\underline{C}(\underline{C}^T\underline{N}_{xx}\underline{C} + \underline{V})^{-1}\underline{C}^T\underline{N}_{xx} \qquad (3\text{-}44)$$

$$\hat{\underline{x}}_{i|i} = \hat{\underline{x}}_{i|i-1} + (\underline{\Phi})^{da}\underline{K}_x(\underline{z}_i - \underline{C}^T\hat{\underline{x}}_{i-da|i-1})$$

$$= \hat{\underline{x}}_{i|i-1} + (\underline{\Phi})^{da}(\hat{\underline{x}}_{i-da|i} - \hat{\underline{x}}_{i-da|i-1}) \qquad (3\text{-}45)$$

$$\hat{\underline{x}}_{i+1|i} = \underline{\Phi}\,\hat{\underline{x}}_{i|i} + \underline{\Gamma}\,\underline{u}_i \qquad (3\text{-}46)$$

It can be shown that (3-40) - (3-46) represent the discrete version of the estimator for $\underline{x}$ derived by Kleinman[9] for the continuous case.

The same thing can be done for the filter for the desired trajectory.

$$\hat{\underline{y}}_{d_{i-db|i}} = \hat{\underline{y}}_{d_{i-db|i-1}} + \underline{K}_d(\underline{z}_d - \underline{C}_d^T\hat{\underline{y}}_{d_{i-db|i-1}}) \qquad (3\text{-}47)$$

$$\hat{\underline{y}}_{d_{i-db+1|i}} = \underline{\Phi}_d\,\hat{\underline{y}}_{d_{i-db|i}} \qquad (3\text{-}48)$$

$$\underline{K}_d = \underline{P}_{dd}\underline{C}_d\underline{V}_d^{-1} \qquad (3\text{-}49)$$

$$\underline{N}_{dd} = \underline{\Phi}_d\underline{P}_{dd}\underline{\Phi}_d^T + \underline{W}_d \qquad (3\text{-}50)$$

$$\underline{P}_{dd} = \underline{N}_{dd} - \underline{N}_{dd}\underline{C}_d(\underline{C}_d^T\underline{N}_{dd}\underline{C}_d + \underline{V}_d)^{-1}\underline{C}_d^T\underline{N}_{dd} \qquad (3\text{-}51)$$

$$\hat{\underline{y}}_{d_{i|i}} = \hat{\underline{y}}_{d_{i|i-1}} + (\underline{\Phi}_d)^{db}\underline{K}_d(\underline{z}_{d_i} - \underline{C}_d^T\hat{\underline{y}}_{d_{i-db|i-1}})$$

$$= \hat{\underline{y}}_{d_{i|i-1}} + (\underline{\Phi}_d)^{db}(\hat{\underline{y}}_{d_{i-db|i}} - \hat{\underline{y}}_{d_{i-db|i-1}}) \qquad (3\text{-}52)$$

$$\hat{\underline{y}}_{d_{i+1|i}} = \underline{\Phi}_d\,\hat{\underline{y}}_{d_{i|i}} \qquad (3\text{-}53)$$

The following expression can be derived, and it is useful to see the average behaviour of the total system.

$$E[(\underline{C}^T\underline{x} - y_d)^T\underline{Q}(\underline{C}^T\underline{x} - y_d)] = \mathrm{Tr}[\underline{Q}_{xx}\underline{X}_{xx}] + \mathrm{Tr}[\underline{Q}_{dd}\underline{X}_{dd}]$$

$$- 2\,\mathrm{Tr}[\underline{Q}_{xd}^T\underline{X}_{xd}] \qquad (3\text{-}54)$$

$$E[\underline{u}^T\underline{R}\,\underline{u}] = \mathrm{Tr}[\underline{G}_x^T\underline{G}_x(\underline{X}_{xx} - \tilde{\underline{P}}_{xx})] + 2\,\mathrm{Tr}[\underline{G}_d^T\underline{G}_x\underline{X}_{xd}]$$

$$+ \mathrm{Tr}[\underline{G}_d^T\underline{G}_d(\underline{X}_{dd} - \tilde{\underline{P}}_{dd})] \qquad (3\text{-}55)$$

where tr denotes the trace operation and $\tilde{P}_{xx}$, $\tilde{P}_{dd}$, X_{xx}, X_{xd}, and X_{dd} can be obtained by the following relations:

$$\tilde{P}_{xx} = (\Phi)^{da}\, \Gamma_x\, (\Phi^T)^{da} + \sum_{i=1}^{da} (\Phi)^{da-i}\, W\, (\Phi^T)^{da-i} \tag{3-56}$$

$$\tilde{P}_{dd} = (\Phi_d)^{db}\, P_{dd}\, (\Phi_d^T)^{db} + \sum_{i=1}^{db} (\Phi_d)^{db-i}\, \Gamma_d\, W_d\, \Gamma_d^T\, (\Phi_d^T)^{db-i} \tag{3-57}$$

$$X_{dd} = \Phi_d\, X_{dd}\, \Phi_d^T + \Gamma_d\, W_d\, \Gamma_d^T \tag{3-58}$$

$$X_{xd} = (\Phi - \Gamma G_x)\, X_{xd} \Phi_d^T - \Gamma G_d\, X_{dd}\, \Phi_d^T \tag{3-59}$$

$$\begin{aligned} X_{xx} = {} & (\Phi - \Gamma G_x)(X_{xx} - \tilde{P}_{xx})(\Phi - \Gamma G_x)^T - \Gamma G_d\, X_{xd}^T (\Phi - \Gamma G_x)^T \\ & - (\Phi - \Gamma G_x)\, X_{xd}\, G_d^T \Gamma^T + \Gamma G_d\, (X_{dd} - \tilde{P}_{dd})\, G_d^T \Gamma^T \\ & + \Phi \tilde{P}_{xx} \Phi^T + W \end{aligned} \tag{3-60}$$

Fig. 3-1 shows the structure of the optimal system. A more detailed illustration in case of a manual control problem will be given later.

To look at what can be predicted by the model let us consider a very simple case. A plant is a pure integrator, and the bandwidth of a desired trajectory is 4.0 rad/sec. First we assume that v, v_d and w do not exist, time delays are zero and the state variables of the plant and the shaping filter are measurable. Then optimal control is determined by using $\underline{x}$ and $\underline{Y}_d$ instead of $\underline{x}$ and $\hat{\underline{Y}}_d$ (no need to have estimators). Fig. 3-2 shows the effect of preview on $E[e^2]$ and cost J for this case. We can see that the preview beyond certain point (~10 in this example) does not have much effect on J or $E[e^2]$. To see the effect of noise, $W = 0.0007$, $V = V_d(0,0) = 0.35$, $V_d(\ell,\ell) = 0.1\ell^2 + 0.35$ and $V_d(k,\ell) = 0 (k \neq \ell)$ were assumed and $E[e^2]$ and J were calculated. We can see that the improvement by having preview is more clear when noise exists. In the graph the effect of measurement time delays (da=db=3) is also shown. We can see that the preview also helps to make the increase in $E[e^2]$ or J due to time delays small. These are all structural properties of optimal preview control systems, and similar effects can be expected in manual preview experiments.

IV. Experiment

The experiment similar to one by Reid and Drewell[10] was performed to

look at the validity of the proposed model qualitatively. It was the single degree of freedom manual preview experiment, and Fig. 5-1 shows the configuration.

In the figure, m (not u) is used to indicate the input to the controlled system. In the next section, the reason will be given. As shown in the figure, both digital and analog computers were used. The digital computer was used for three purposes: a desired trajectory (random signal) generator, a shift register for storing present and future desired outputs, and a data acquisition system. Everything was done on line, which determined the time in which the computer could finish one cycle. The input and the output of the controlled system were sampled once each cycle. In the following, each component in the figure is explained in detail.

<u>Display</u>: A CRT display with 5" diameter was used. The desired trajectory was displayed as a sequence of dots (100 pts. at the maximum) allowing subjects to have preview from 0 sec. up to about 2.6 sec. The preview length was changed by changing the number of dots which appeared on the display. The trajectory moved from right to left on the screen, and it was adjusted so that the length of the trajectory became about 2 1/4" maximum preview case (shorter for less preview) and the range in vertical direction became about 2".

The output of the controlled system was displayed by the dot on the same vertical line as the lefthand side of the desired trajectory, and it was intensified stronger than the desired trajectory so that it was easily distinguished from the desired trajectory. The distance between subject and display was about 20".

<u>Desired Trajectory</u>: The desired trajectory was a random signal generated by the digital computer. The second order digital filter was driven by Gaussian white noise which was also generated by the digital computer on line. By changing the coefficients of the digital filter, three kinds of trajectories with different bandwidth (ω_d = 1.5, 2.5, and 4.0 rad/sec.) were generated.

<u>Controlled System</u>: Three kinds of plants were implemented on the analog computer. They were a pure integrator ($\frac{1}{s}$), a damped first order system ($\frac{1}{s+2}$), and a double integrator ($\frac{1}{s^2}$).

<u>Joystick</u>: The joystick used was a Model 435 Hand Control by Measurement System INC. This stick enabled the subject to apply control just by the wrist or finger motion. The stick gain was different for each plant, but was the same for all subjects and all trajectories.

<u>Data Acquisition</u>: $\bar{e}$, $\overline{e^2}$, $\bar{m}$ and $\overline{m^2}$ were calculated by the digital computer on line where ‾ indicates the time average of ·. y, y_d, e and m were also recorded by the chart recorder.

Three graduate students at MIT served as subjects. Each subject tried all combinations of three plants and three random signals. Subjects had enough training for each combination. 0, 3, 12, 25, 50, and 100 pt. preview cases were conducted for each combination with a few exceptions. Each experimental run was three minutes following a 30 second warming-up period.

Figs. 4-2(a) and (b) show the data from experiments normalized by $\bar{v}_d^2$. Before going into the analysis by the model, several points are worth mentioning.

1) In almost all cases, preview beyond 0.7 sec ahead did not make essential improvements in either e^2 or m^2.

2) Subjects fell into one of the following two modes of looking at the display with non zero preview.

i) Mountain range mode: This was a usual mode, in which subjects felt that they were looking at a mountain range from the window of a train. In this mode, subjects did not have difficulty in tracking the trajectory.

ii) Flag mode: Once subjects fell in this mode, they took the moving wave form as a flag flapping in the air. This happened during those experimental runs with a particular combination of the preview length and the trajectory's bandwidth degrading the performance.

3) The output of the system was displayed by the dot. This confused subjects a little at zero preview in which case they were looking at two dots in the screen, although the brightness was controlled giving different intensity to two dots.

4) Learning effect still exists in the data. Subject 1 did the experiment in the order of $\frac{1}{s}$, $\frac{1}{s+2}$ and $\frac{1}{s^2}$. Subject 2 did in the order of $\frac{1}{s+2}$, $\frac{1}{s^2}$ and $\frac{1}{s}$. Subject 3 did in the order of $\frac{1}{s^2}$, $\frac{1}{s+2}$ and $\frac{1}{s}$.

V. Analysis of Data and Discussion

The result for the steady state discussed at the end of Section III can be used to analyze experiment where the plant is single input and single output. Fig. 5-1 represents the manual preview control model used in the analysis.

Note that in (2-1) $w_i = \Gamma w_{m_i}$ and that m in the experiment actually represents $u+w_m$ where w_{m_i} is the motor noise with $E[w_{m_i}] = 0$ and $E[w_{m_i} w_{m_j}] = W_m \delta_{ij}$. Then it is easy to see that

$$E[m^2] = E[u^2] + W_m \tag{5-1}$$

The analysis of the experiment using this model was so far made for the case when the plant is a pure integrator.

The difficulties in fitting the data by the present preview model are that the model has too many parameters, and that the dimension of Kalman filter equation for the desired trajectory becomes larger as the preview length increases. The following procedure was used in analysis.

First all equations including the plant were written down in the discrete form with $\Delta t = 0.0265$, which was supposed to be small enough to approximate the continuous part in the experiment and also was equal to the one computer cycle making it possible to maintain the discrete part in the experiment in the same form.

Time delays da and db were set equal to d, and it was varied from 0 to 7, or equivalently from 0 sec. to about 0.186 sec.

As for the determination of the strength of motor noise w_m and observation noise v, we followed Kleinman et al to get a rough idea. Since their formulas were for a continuous system, some modification was necessary in case of a discrete system. After necessary modification,

$$W_m = E[w_m^2] - \frac{\pi \times P_m}{0.0265} \times E[u^2] \tag{5-2}$$

$$V_y = E[v^2] - \frac{\pi \times P_y}{0.0265} \times E[e^2] \tag{5-3}$$

(P_m ~0.003, P_y ~ 0.01 were recommended by Kleinman et al.)

The determination of V_d was the most difficult task. There was no available data in the past for nonzero preview cases, so it was decided to simplify V_d as much as possible. The first assumption made was

$$E[v_{d_i}(\ell) v_{d_j}(k)] = V_d(\ell)\delta_{\ell k}\delta_{ij} \tag{5-4}$$

This choice decreased the number of parameters, but still it was not clear how $V_{d_i}(\ell)$ changes with ℓ. $V_d(\ell)$ could even have different values for different preview length. Since there was almost no doubt about setting $V_d = V$ for zero preview case, we set $B \triangleq V_d = V$ for zero preview case. Based on this, the form $V_d(\ell) = A\ell^2 + B$ was assumed for the general case and A was changed to fit data. This form of $V_d(\ell)$ is not unreasonable since as a dot is displayed further apart from the dot representing the output of the system both errors in vertical sense and horizontal ... increase and the variance of the latter is considered to be proportional to the square of the distance between two dots.

Several values of R were tried and $E[e^2]$ and $E[u^2]$ were calculated by (3-53) and (3-54). These results together with (5-1) were compared to the experimental data.

Table 5-1 shows the values of parameters which gave us a good fitting. We do not claim that Table 5-1 shows the best or unique fitting. However, it shows qualitatively that the proposed model is reasonable, and it also explains what could be happening in manual preview control.

1) One notable thing is that the time delay in the model is kept to the same value for all preview cases. This is not unrealistic. Especially, it is rather hard to believe that the time delay associated with the output of the plant becomes smaller as the preview length increases, since the new information from looking ahead is not about the plant but about the trajectory. The model demonstrates that the improvement in performance can be actually done without changing the time delay.

2) The weighting R decreases as the preview length increases. This implies that the bandwidth of the loop composed of the plant and the subject increases as the preview length increases, since a small R implies large feedback gains in general.

3) W_m, V, etc. in the table are larger than the values suggested by Kleinman et al. For zero preview case, one reason is supposed to be the nature of the display used in the experiment, which was mentioned earlier.

4) Fitting was not done for the large (25 pt. or (0.7 sec)) preview cases because of the increase of the dimension of Kalman Filters. This does not imply that the computation is impossible. But it takes a very large amount of computational time. It was mentioned before that e^2 and m^2 were not improved essentially by the preview beyond 0.7 sec. We can expect that the same kind of thing will be observed in the model if parameters are slected properly. Basically there are two reasons for this. The first reason is that human can not make the bandwidth of the loop infinite, which implies that R has some lower bound. The second reason is that in large preview cases the desired trajectory seems to be divided into two parts if we assume that human does not move his eyes left and right; one is a foveal region and the other is a peripheral region. The neighborhood of the vertical line where the output of the plant is displayed is the foveal region and the rest is the peripheral region. The observation noise for the peripheral region is supposed to be much larger than that for the foveal region and there must be some point in the peripheral region beyond which subject cannot get any essential information. This implies that there exists some limit in the quality of the estimator for the desired trajectory. If these are true, $E[e^2]$ and $E[m^2]$ will be also lower bounded in the model.

VI. Conclusion and Further Research

In this paper, a model was developed for the preview control problem using the discrete stochastic control theory. Theoretically it would be interesting to extend the present theory to continuous problems. One way to solve such problems would be to take the limit of the present solution for the discrete system.

A manual preview control experiment was performed, and it was shown that the proposed model could explain the experimental data. It would also be interesting to give the frequency domain interpretation to the developed model. Intuitively speaking, it can be expected that as the preview length increases $Y(j\omega)/Y_d(j\omega) \to 1$ for the frequency ω lower than the bandwidth of the closed loop part of the total system.

The display used in the experiment brought in some problem confusing subjects. Apart from the theory, the design of better preview display would be interesting.

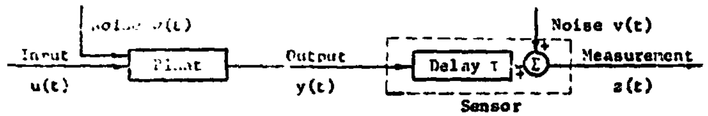


Fig. 2-1 System to be considered

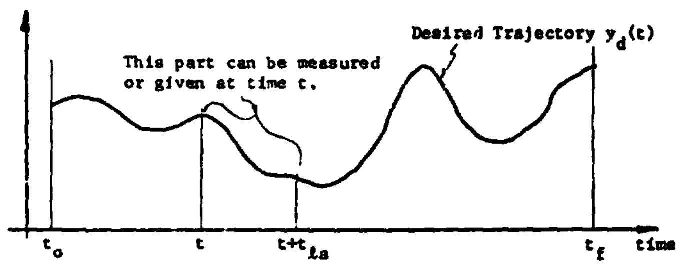


Fig. 2-2 Desired Trajectory

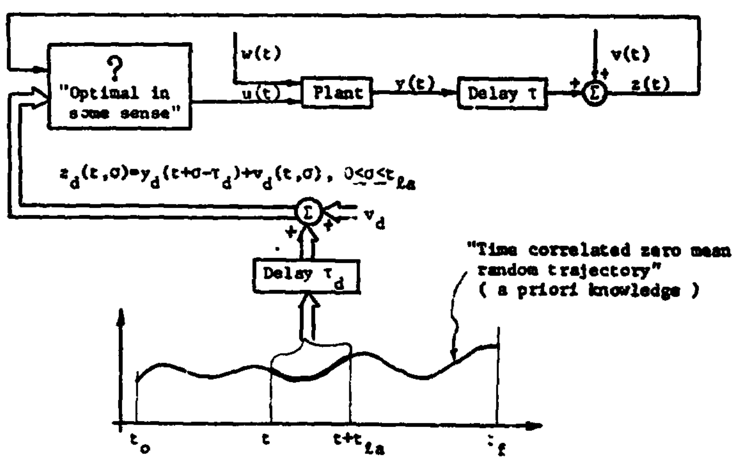


Fig. 2-3 Preview Control Problem

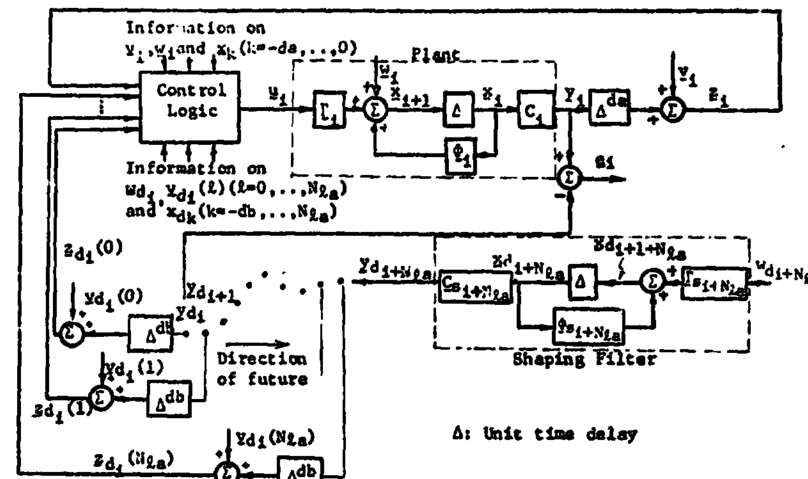


Fig. 2-4 Relation between Plant, Shaping Filter and Subscript

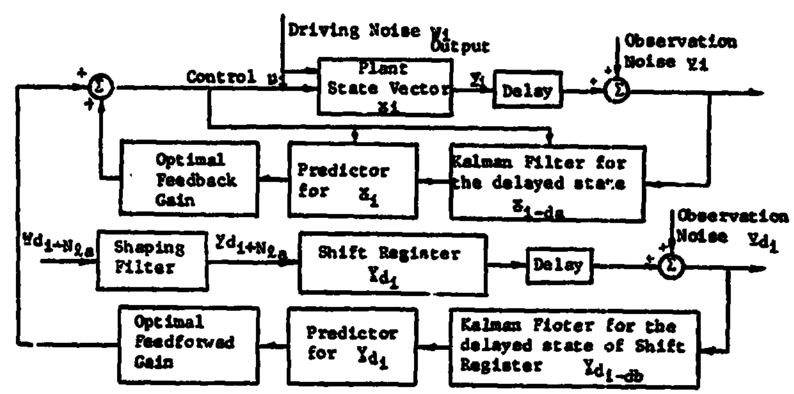


Fig. 3-1 Structure of the Optimal System

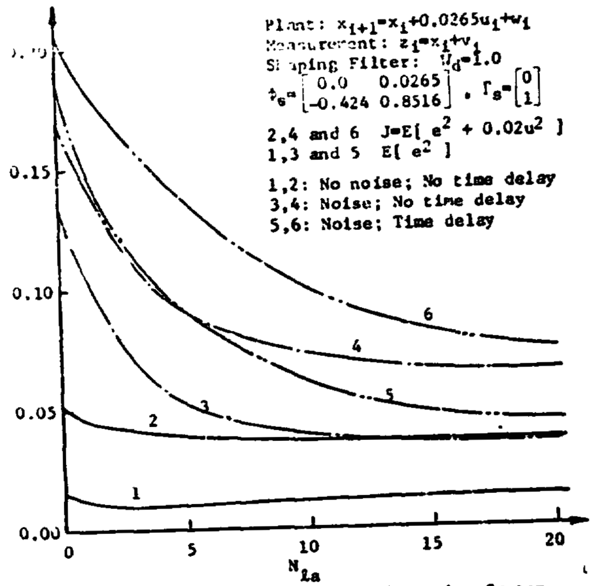


Fig. 3-2 Structual Properties of Preview System

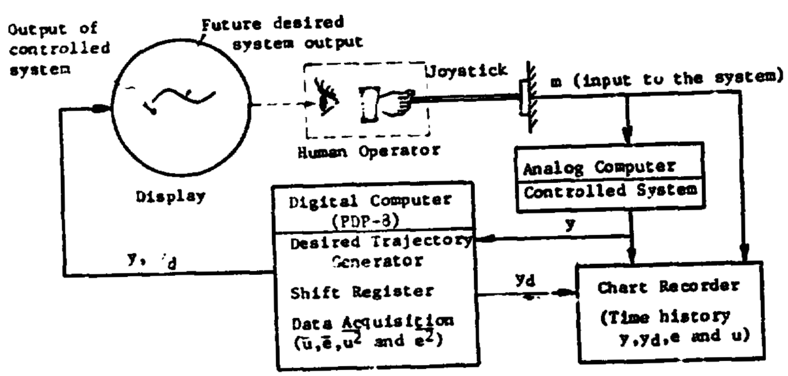


Fig. 4-1 Configuration of Manual Preview Experiment

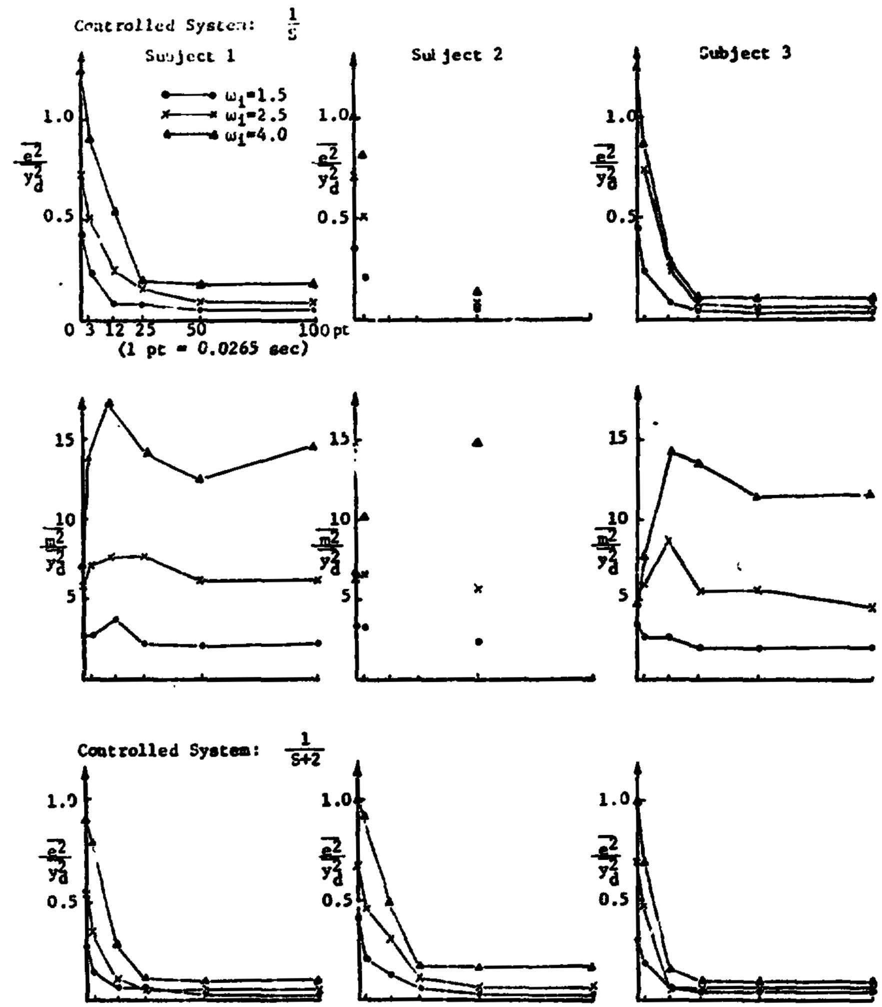


Fig. 4-2-a Experimental Results

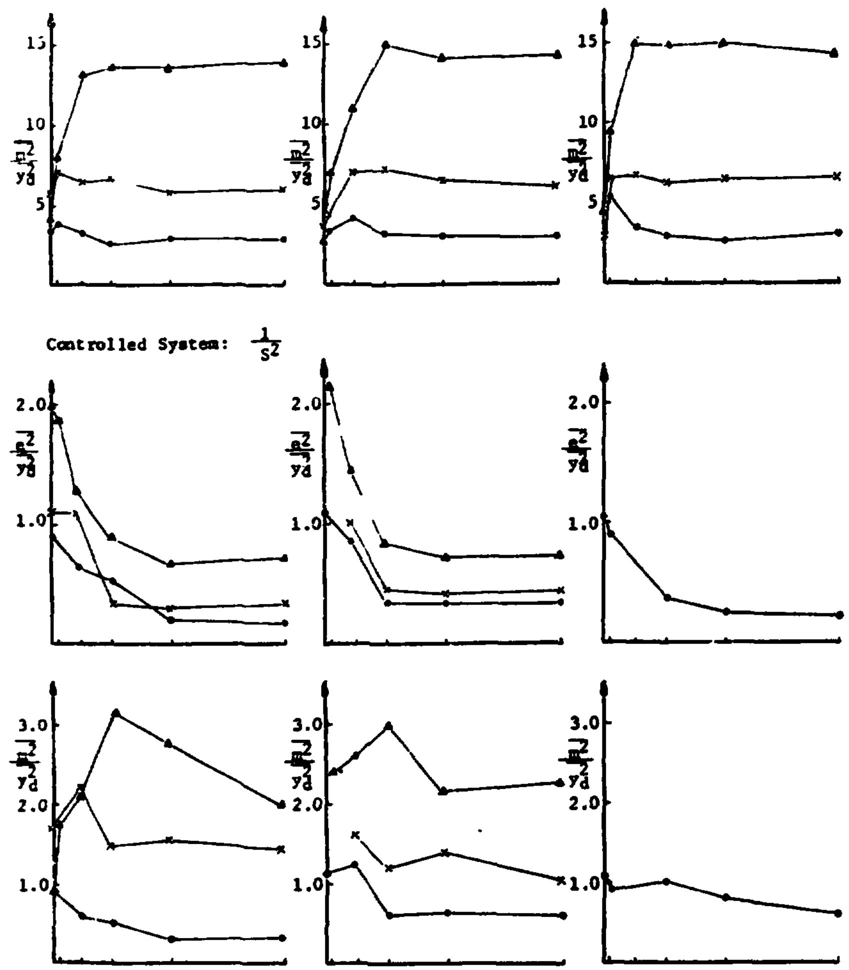
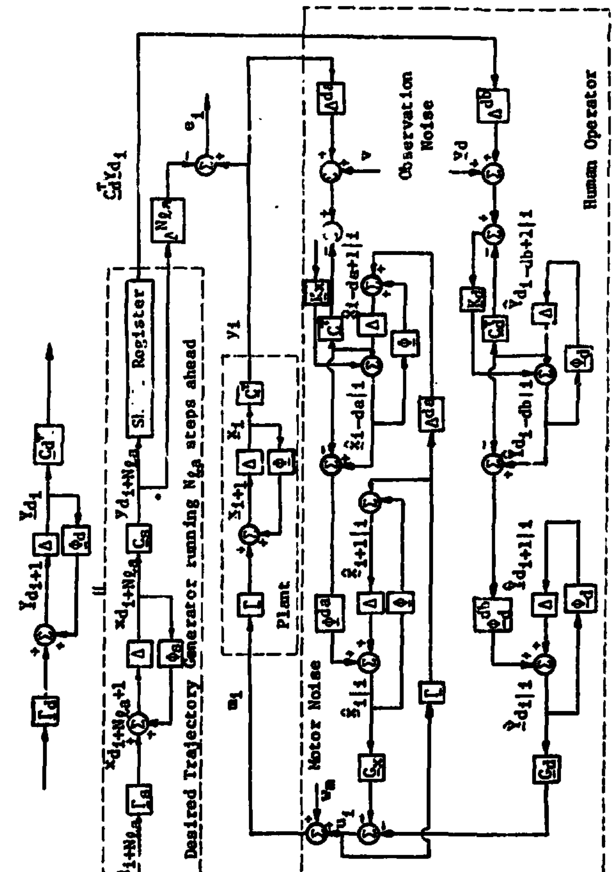


Fig. 4-2-b Experimentil Results



Δ: Unit Time Delay

Fig. 5-1 Manual Preview Control Model

ω_d	$N_{\ell a}$	Parameters					$E[e^2]/E[y_d^2]$		$E[m^2]/E[y_d^2]$	
		R†	d	$W_m/E[y_d^2]$	$V/E[v_d^2]$	$V_d(\ell)/E[y_d^2]$	Model	Experiment*	Model	Experiment*
1.5	0	0.05	6 (-0.15 sec)	0.243 (P_m~ 0.0085)	0.973 (P_y~ 0.022)	$0.0037\ell^2$ +0.973	0.366	1) 0.417 2) 0.351 3) 0.457	2.65	1) 2.[illegible]1 2) 3.30 3) 3.[illegible]
	3	0.04	"	"	"	"	0.226	1) 0.237 2) 0.207 3) 0.245	2.96	1) [illegible] 2) 3.30 3) 2.73
	12	0.03	"	"	"	"	0.099	3) 0.093	2.65	3) 2.7[illegible]
	15	0.03	"	"	"	"	0.074	1) 0.081 2) 0.041	1.98	1) 2.[illegible]1 3) 1.96
2.5	0	0.025	6	0.552 (P_m~ 0.0087)	2.43 (P_y~ 0.073)	$0.00945\ell^2$ +2.43	0.722	1) 0.721 2) 0.715	5.85	1) 5.87 2) 6.47
	3	0.020	"	"	"	"	0.515	1) 0.508 2) 0.51	.44	1) 7.14 2) 5.93
	12	0.012	"	"	"	"	0.226	1) 0.24 3) 0.24	7.85	1) 7.8 3) 8.8
	25	0.012	"	"	"	"	0.169	1) 0.16 3) 0.19	5.14	1) 7.8 3) 5.4
4.0	0	0.02	6	0.735 (P_m~ 0.0104)	3.94 (P_y~ 0.032)	$0.0175\ell^2$ +3.94	1.046	1) 1.246 2) 1.025	6.61	1) 7.04 2) 6.79
	3	0.015	"	"	"	"	0.859	1) 0.929 2) 0.836	11 3	1) 13.7 2) 10.1
	12	0.01	"	"	"	"	0.395	1) 0.548 3) 0.290	14.1	1) 17.3 3) 14.4
	25	0 002	"	"	2.63	$0.0202\ell^2$ +2.63	0.177	1) 0.196 3) 0.13	16.3	1) 14.1 3) 13.9

*The number of subject is shown before his performance.
† Q = 1

Table 5-1 Fitting of Parameters.

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