On Modeling Performance of Open-Loop Mechanisms

by

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Abstract

A technique is presented for modeling the performance of open-loop mechanisms or systems, where performance is measured by success or failure as a function of the selected target for movement along a constrained trajectory and without corrective feedback. A modification of signal detection theory is used, which normalizes dissimilar data and indicates two independent parameters of performance: (1) "discriminability" and (2) "optimality" of the distribution of selected moves. An illustrative application is made to experimental data.

Introduction

The control of mechanisms for performing complex sequential tasks such as materials processing, assembly and inspection of manufactured parts, aerospace vehicles and other complex processes can often be characterized as a multilevel process, as indicated in Fig. 1. At the lowest level (the inner loop) position and force variables of the mechanism may be measured continuously or at frequent intervals in time or space (a), then operated upon to drive the mechanism (a′) into continuous conformation with a reference input (b′). The controller at this level is typically a passive electronic or mechanical filter or a computer-based servocontroller.

At an intermediate level (middle loop) a stored program computer is typically used to measure position or force variables (b) of external objects with respect to the mechanism. For example these may be tactile sensors or simple optical ranging or pattern recognition devices. These data are sampled at less frequent intervals than those at (a), and serve together with commands from the human supervisor (c′), as inputs to a computer program which generates a changing reference input to the lower loop (b′).

At the highest level (outer loop) the human supervisor observes with his own eyes directly, or indirectly through aiding instruments such as television, the relation of various objects in the environment (c), including the mechanism, to each other. He acts upon this information, plus his apriori goals or criteria, to command or continually reprogram the computer controller. The man's observations (c) and commands to the computer (c′) are at less frequent intervals than the computer's measurements (b) and reference input changes (b′), which in turn are at less frequent intervals than the feedback (a) interval to the mechanism and the first level control signal (a′).

A different problem in the engineering such mechanisms or man-machine systems is to evaluate performance, especially with regard to the importance of feedback at various levels as a function of sampling frequency. In effect, a compromise must be found which most appropriately trades errors against sampling frequency of various loops, and permits modeling a variety of systems with few parameters in standardized form.

An approach is suggested which closely resembles the theory of detecting signals in noise (Green and Swets, 1966). Cohen and Ferrell (1969) applied signal detection theory to binary predictive judgement of success or failure in motor skill tasks. Here we modify the theory to allow for multi or continuous level explicit response decision which results in success or failure on each move, and we suggest other changes to make a theory designed for modeling sensory processes suitable for modeling motor processes.

Typically a complete task is specified either as a continuous trajectory or as a sequence of desired discrete states to be achieved, with constraining conditions as functions of position, time and energy. In many cases it is convenient to characterize performance at any point in time by whether it is within a tolerance range (which may be a changing function of position time or energy variables) or whether it is outside this range. Fig. 2 shows schematically two tasks, one defined continuously, the other defined as a sequence of discrete events, both with changing tolerances. "Within" this range means that no significant penalties are incurred and with relative ease it can be brought under control once the loop is closed. "Outside" this range means that the penalty is significant and that bringing the system under control means reverting to an abort mode, a change in normal plan, a return to an earlier part of th[illegible]ence, etc. For example a manipulator might accidentally drop an ob[illegible] it was carrying, or confront an unexpected obstacle to its planned path, or find itself in a kinematic singularity or "gimbal lock". A vehicle might run off the road or collide with an obstacle. Thus the sharp nonlinearity in penalty as a function of measured performance variables is characteristic or various real tasks.

We are concerned there with how far along the given continuous path or sequence of discrete stages of the task the mechanism goes in open loop fashion, i.e., before the difference between actual and desired states is measured and corrective efforts are imposed. We will assume that the longer the open loop move the greater the risk of failure, a monotonic relation. We wish to develop <u>both</u> a descriptive model (the statistical description of empirical moves, both the extent of the moves and whether they succeeded or failed) <u>and</u> a normative model (a specification of when the feedback loop should be closed to minimize expected

penalty).

<u>The Model</u>

Let x be the target (extent requested) for an open loop move at some given point (a′, b′, or c′) in the system of Fig. 1. If at a, b or c that move is observed to be (or have been during the move) outside the tolerable range, we call it a failure, f. Otherwise it is a success, s. We shall make a plot of the probability density of selecting x, p(x) and the joint probability density of selecting x <u>and</u> observing y to be a failure p(x,f).

Assumptions stated above imply that experimental plots of these quantities will be approximately of the form shown in Fig. 3; the ratio p(x,f) to p(x) increases with x, since the greater x the more the cumulative risk. The joint probability density of selecting x and succeeding is obviously the difference between the two densities,

$$p(x,s) = p(x) - p(x,f). \qquad (1)$$

By Bayes' theorem the contingent probabilities of x given f or s are

$$p(x|f) = \frac{p(x,f)}{p(f)} = \frac{p(x,f)}{\int_0^\infty p(x,f)dx} \qquad (2)$$

$$p(x|s) = \frac{p(x,s)}{p(s)} = \frac{p(x,s)}{\int_0^\infty p(x,s)dx} \qquad (3)$$

We define cumulative probability functions, i.e., the probability of a selection less than or equal to x,

$$P(x|f) = \int_0^x p(x|f)dx \qquad (4)$$

$$P(x|s) = \int_0^x p(x|s)dx \qquad (5)$$

If a cross plot is made of $P(x|f)$ and $P(x|s)$ as in Fig. 4, a rough equivalent of the receiver operating characteristic (ROC) of signal detection theory results. Though actually it is a distribution of motor responses, we shall call our curve an ROC. This crossplot serves to normalize and standardize the open loop performance characteristic so that two independent parameters of performance are readily apparent. Values of x are scaled monotonically along the crossplot curve.

The first parameter of interest is a measure of "relative discrimination" between success and failure for the particular distribution of x values selected. It is the tendency of the experimental curve toward that ROC "curve" which intersects the upper left hand corner. At this point $P(x|s) = 1$ and $P(x|f) = 0$; therefore it is a point of perfect discrimination, where lesser values of x promise the fullest possible success with no risk of failure, and greater values of x can only be failures. In practice this situation is seldom if ever present, since selecting the greatest x which can be successful usually means a significant probability of failure. This is characterized by an intermediate curve 1 or 2. Diagonal line 0 is where at each x in the distribution $P(x|s) = P(x|f)$. A curve below the diagonal implies a situation where one must experience much failure at smaller values of x to finally attain some successes at greater values. This is contrary to our monotonically increasing risk assumption and thus is a situation we assume to be irrelevant for the present.

The second parameter of interest is the central tendency of $p(x)$ relative to the optimal x. The optimal x is calculated from knowledge of the tendency to failure as a function of x (as estimated from the experimental data), plus the given rewards $R(x)$ for moving as far as x successfully and the given cost $C(x)$ for failure. Thus the expected value of return $E(\mathcal{R})$ over a distribution of moves is

$$E[\mathcal{R}(x)] = \int_0^\infty p(x,s)R(x)dx - \int_0^\infty p(x,f)C(x)dx \qquad (6)$$

However, if it were possible to select consistently a single value of x, one could maximize expected value of return by choosing

$$x_{opt} = \max_x[\mathcal{R}(x)] = \max_x [p(s|x)\ R(x) - p(f|x)C(x)] \qquad (7)$$

The tendency toward the upper left hand corner is the degree of non-overlap of the $p(x|s)$ and $p(x|f)$ distributions. With a given monotonic propensity to fail as a function of x, the percentage overlap of the two distributions can be reduced by selecting over a larger range of x values, insuring success at small x and failure at large x. During learning, such a selection over a large range of x is a good strategy to establish experimentally the dependence of $p(x|s)$ and $p(x|f)$ on x. At later stages presumably the human or computer decision maker attempts to narrow the range of x on a region of near optimal payoff. Necessarily over any narrow range of x $p(x|s)$ and $p(x|f)$ distributions will cease to be displaced relative to each other on the x axis and the ROC curve will approach the diagonal.

There are various means of scaling the tendency to the upper left hand corner. One method used in signal detection theory is to estimate from the data the distance d' between means of distributions $p(x|s)$ and $p(x|f)$ divided by the mean standard deviation. For two Gaussian distributions of equal variance this procedure results in curves for $d' = 1$ and $d' = 2$ as indicated in Fig. 4.

Since the slope of the ROC curve

$$\frac{dP(x|s)}{dP(x|f)} = \frac{d\int_0^x p(x|s)dx}{d\int_0^x p(x|f)dx} = \frac{p(x|s)}{p(x|f)} = \frac{p(s|x)}{p(f|x)}\,\frac{p(f)}{p(s)} \qquad (8)$$

and thus for any given x in a distribution

$$\frac{p(s|x)}{p(f|x)} = \frac{(\text{slope of ROC})p(s)}{p(f)} \qquad (9)$$

From this one might assert that if a single value of x were chosen consistently the return would approximate[3]

$$\mathcal{R}(x) \cong (\text{slope of ROC})p(s)R(x) - p(f)C(x). \qquad (10)$$

x_{opt} can be found by trial and error by use of equation 10. Measures of the central tendency of x are the average x or, alternatively, the median x. The difference between x_{med} and x_{opt}, if the data warrant a unique specification of the latter, can then be specified. When operating over a narrow range of x where p(x,s) and p(x,f) are constant and the ROC has unity slope, (10) reduces to the condition for x_{opt} that

$$\mathcal{R}(x) = (\text{constant})R(x) - (\text{constant})C(x) \qquad (11)$$

and the choice of x depends only on the relative magnitude and nonlinearity of R and C. The difference between x_{med} and x_{opt}, is a measure of conservatism ($x_{med} < x_{opt}$) or risk ($x_{med} > x_{opt}$) independent of the degree of discriminability.

Example of Application to Experimental Data

In order to illustrate the method, data were taken from two human subjects performing a simple open loop manipulation task which corresponds to the first task of Fig. 2, but with fixed tolerance. For convenience the "mechanism" employed was a human arm and hand holding a simple pen. In this task a subject observed the task, then with eyes closed moved the pen from a starting point toward the right, trying to keep the pen within two lines spaced 0.2 inches apart. The payoff or return function ($\mathcal{R}$) was

$$\mathcal{R} = (x)_{\text{if success}} - (3)_{\text{if fail}} \quad , x \text{ in inches} \qquad (13)$$

The subjects were explained the return function and instructed to aggregate the greatest score possible. Each subject had 41 "learning" trials, then 164 "test" trials. On all trials he could observe what happened on the last trial before going on to the next. The temporal pace was self-determined.

Experimental data for the test trials are plotted in the form of histograms, for movement x in 0.2 inch increments, in Fig. 5. Note the big difference between subjects, subject GF being conservative with small x and small p(f). Subject PS risks large x and large p(f).

Fig. 6 shows the ROC curves approximated from the test data of Fig. 5. Fig. 7 shows the ROC curve drawn for the learning trials of one subject derived by an alternative scheme, namely the point by point plot of sequential values of x. Notice that as predicted the tendency to generate discriminability (d') is greater for the learning trials than

for the test trials. This is because both absolute range and variability were greater than for the test trials. Notice especially that both subjects' data result finally in approximately the same ROC curves (Fig. 6) even though the raw histograms were quite different. Finally notice that the median responses of both subjects, lie in a region of the ROC where R(x) and slope multiply to keep R(x) relatively constant. This lack of a sharply defined optimum characterizes many tasks.

Conclusions

In designing and controling mechanisms for multidegree of freedom tasks it is important to evaluate performance of open loop moves. If each of a set of responses, intended for open loop movement of extent x, can be characterised by observation into binary performance categories of success and failure, and if a given return function specifies rewards for successful move to x and cost for failure at or before x, and if probability of failure increases monotonically with x, then a performance model analogous to the receiver operating characteristic (ROC) of signal detection theory is useful.

A cross plot of cumulative probabilities of x, given success, and of x, given failure, reveals two independent performance parameters. The first parameter is the "relative discriminability" of the set of moves, the degree to which moves of extent less than some value are surely successes and moves greater than some value are surely failures. This index is increased as the range of x of the set of moves increases, and it is decreased by random "noise" or uncontrollability in the mechanism.

The second parameter is the central tendency of selecting or targeting x, relative to the optimal, as determined from the given return function and the empirically demonstrated probabilities of success and failure as a function of x. It is directly a measure of risk vs conservatism in selecting or targeting moves.

An application to data for open loop arm movements by two human subjects illustrates how rather dissimilar distributions yield similar ROC plots. However, the data did not provide sufficiently sharp definition of the "optimal" move distance to assert that the central tendencies tended toward risk or conservatism.

Further development of the analysis and its application are in order.

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References

1. Green, David M. and John A. Swets, Signal Detection Theory and Psychophysics, John Wiley and Sons, Inc., New York, 1966.

2. Cohen, Harry S. and William R. Ferrell, "Human Operator Decision-Making in Manual Control", IEEE Transactions on Man-Machine Systems, Vol. MMS-10, No. 2, June 1969, pp. 41-47.

3. Clearly the ability to select consistly a value of x and the ability to avoid "failure" are interrelated in practical cases, so that the two situations are not directly comparable. The author is grateful to Prof. W.R. Ferrell of University of Arizona for comments on this and related points of the paper.

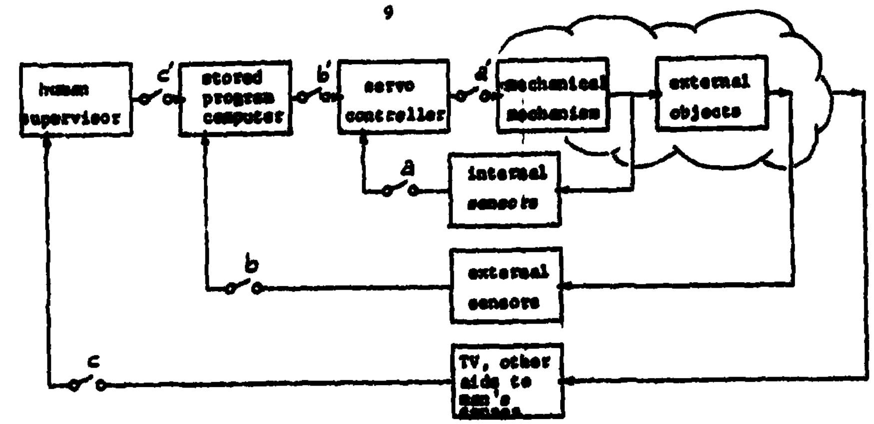

Figure 1.
Manipulator mechanism with multi-level control

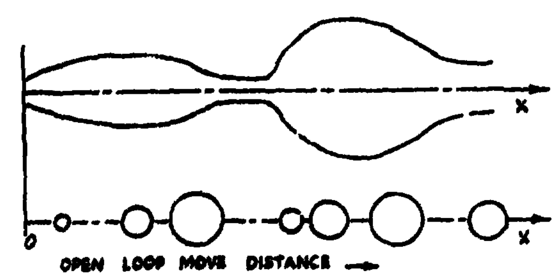

Figure 2.
Schematic representation of two open loop tasks to move along line x, yet stay within boundaries. The upper task is a continuous movement; the lower task is a sequence of discrete movements.

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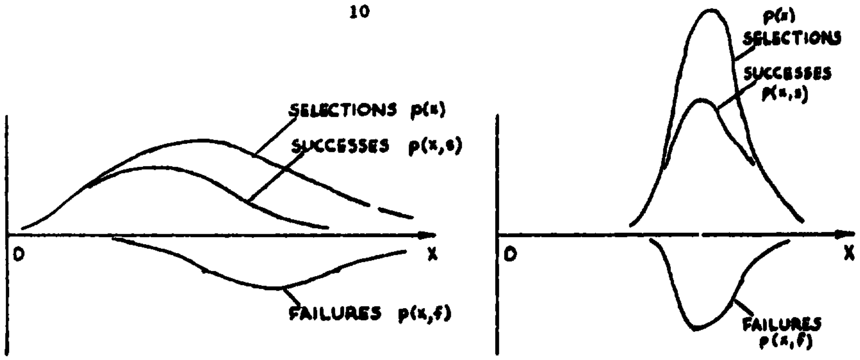

Figure 3.
Typical plots of p(x), p(x,s), p(x,f). The left hand plots represent a case of relatively high discriminability. The right hand plots represent a case of low discriminability, as might occur after a control system settled down in an operating range near the median of p(x) of the left hand distribution.

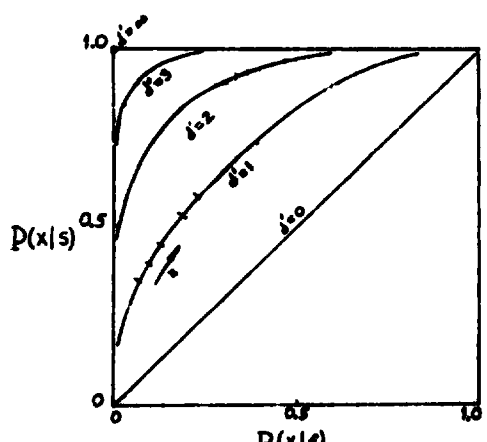

Figure 4.
Cross plots analogous to the receiver operating characteristics of signal detection theory.

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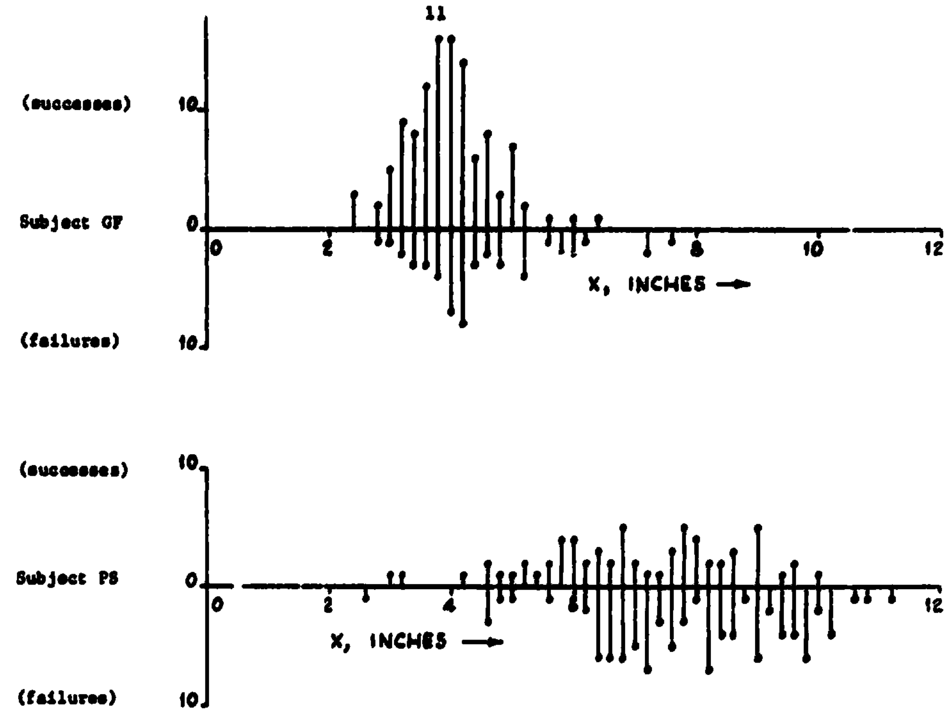

Figure 5.
Histograms of data for two human subjects performing continuous open loop arm movements within a 0.2 inch wide tolerance zone where the return function = $(x \text{ inches})_{\text{if succeed}} - (3)_{\text{if fail}}$.

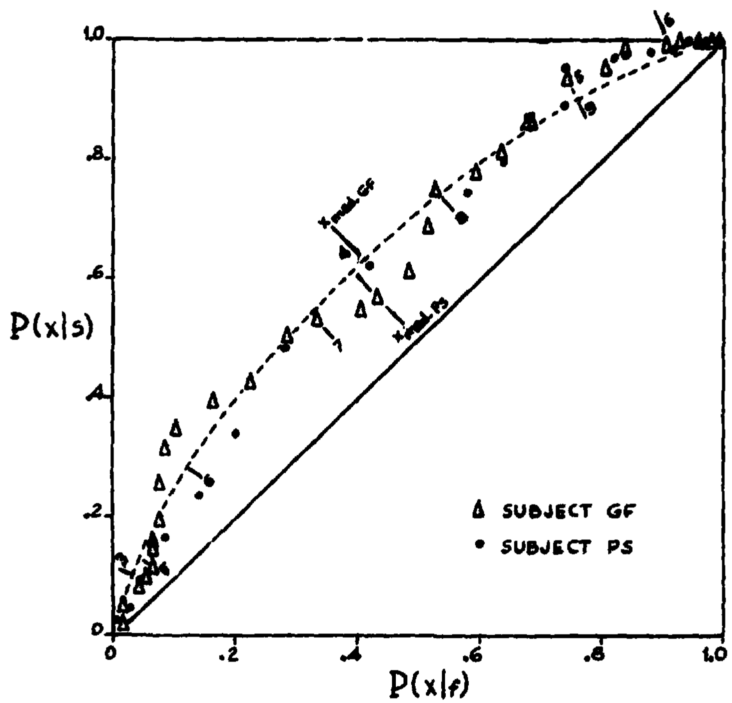

Figure 6.
ROC type crossplots of the test data of Fig.5. Median values are indicated for each subject separately. In each case the use of equation 10 proves the "optimal" to be a relatively broad region, not specifiable from empirical data as a point, but somewhere near the medians of the distributions. Numbers above line indicate inch scale for GF, numbers below line for PS.

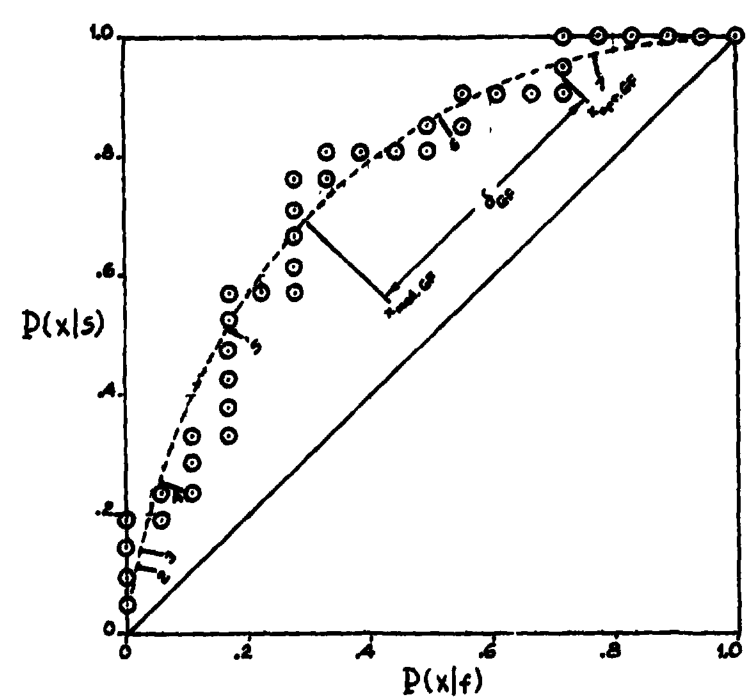

Figure 7.
ROC type crossplot of the learning trials of subject GF, illustrating the method of plotting where each successive x response is treated as a data point. Notice the relatively larger d' (higher discriminability) here than in Fig. 6. Scale along ROC curve is inches.