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# EVALUATION OF SOME CONTROL-VOLUME TECHNIQUES FOR ANALYSIS OF SHOCK - BOUNDARY-LAYER INTERACTIONS IN SUPERSONIC INLETS

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# EVALUATION OF SOME CONTROL-VOLUME TECHNIQUES FOR ANALYSIS OF SHOCK - BOUNDARY-LAYER INTERACTIONS IN SUPERSONIC INLETS

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## SUMMARY

A number of control-volume models used to analyze the shock - boundary-layer-bleed interaction were investigated. The bleed assumptions of the various models and their influence on the analytical solutions are discussed. The results of the analysis using these models are compared with experimental boundary-layer data taken in a supersonic inlet. The experimental Mach number upstream of the interaction was 1.66, and the oblique-shock pressure ratio was 1.33. The boundary-layer data included bleed flow rates up to approximately 0.6 of the upstream boundary-layer mass flow rate.

Two models proved significantly better when compared with the experimental data. The first model assumed the bleed was removed from the control volume with a momentum that was characterized by a pressure intermediate between the upstream and downstream pressures. The second model assumed the control volume was bounded by a streamline dividing the bleed and residual flows. This model eliminated the need to specify the momentum of the bleed flow. Comparison of the results using the two models showed that specifying the bleed pressure in one model was equivalent to specifying the pressure along the dividing streamline in the other.

## INTRODUCTION

A large number of methods for analyzing the shock - boundary-layer interaction problem can be termed control-volume techniques. While these techniques may differ in detail, they are all characterized by the fact that no attempt is made to calculate the local boundary-layer properties in the interaction region. Instead a real or implied control volume is assumed to exist around the interaction region, and the downstream properties of the flow are determined by conservation of mass and momentum through the control volume. Typical of the control-volume techniques are those of Hammitt (ref. 1) and Seebaugh, Paynter, and Childs (ref. 2). In the latter technique the analysis is extended to include the effects of boundary-layer bleed.

In this study five control-volume models used to analyze an oblique-shock interaction with a turbulent boundary layer with bleed are evaluated. The physical significance of the assumptions with regard to bleed is discussed for the various bleed models. The evaluation is made by comparing the results of the various models with boundary-layer measurements taken in a supersonic inlet. These measurements are reported in reference 3. The models include those presented in reference 2, the dividing-streamline model of reference 3, and a model similar to the slot model of reference 2 but with the capability of specifying the removal of the bleed at a pressure other than the upstream pressure. The modified-slot model is an attempt to describe more accurately the momentum of the bleed flow leaving the control volume.

The shock - boundary-layer interactions studied had an upstream Mach number of 1.66 and an incident oblique-shock pressure ratio of 1.33. The bleed flow rates ranged up to approximately 0.6 of the upstream boundary-layer mass flow rate. The Reynolds number based on momentum thickness at the upstream boundary-layer measurement location was 3528.1.

## SYMBOLS

| | |
|---|---|
| C | Crocco number |
| H | shape factor, $\delta^*/\theta$ |
| K | function of C and $N_i$ (see eq. (2)) |
| $M_e$ | edge Mach number |
| $m_b$ | bleed rate |
| $m_{bl}$ | upstream boundary-layer flow rate |
| N | power-law exponent |
| p | static pressure |
| t | static temperature |
| U | velocity |
| $U^*$ | $U/U_e$ |
| y | linear coordinate perpendicular to wall |
| y' | y measured from scoop height |
| $y_s$ | scoop height |

| | |
|---|---|
| $y_u$ | initial dividing streamline height |
| $\gamma$ | ratio of specific heats |
| $\Delta$ | transformed boundary-layer thickness |
| $\delta$ | boundary-layer thickness |
| $\delta^*$ | displacement thickness |
| $\theta$ | momentum thickness |
| $\rho$ | density |
| $\rho^*$ | $\rho/\rho_e$ |

Subscripts:

| | |
|---|---|
| b | bleed location |
| e | local boundary-layer edge |
| i | incompressible |
| t | stagnation |
| u | streamline between bleed and residual flow regions |
| 1, 2, 3, 4 | flow stations defined in diagrams of flow models (fig. 1) |

## DESCRIPTION OF ANALYSIS AND DATA

In this study the results of an analysis using five different control-volume models are compared with measurements taken for a shock - boundary-layer interaction with bleed in the interaction region. Figure 1 illustrates schematically the various bleed models. Figures 1(a) to (c) show three models from reference 2, which are denoted as the porous-wall, flush-slot, and scoop bleed models, respectively. Figure 1(d) shows a modified-slot model and figure 1(e) the dividing-streamline model of reference 3. The models differ primarily in how they treat the removal of the bleed flow. The assumptions that the static-pressure distribution outside the control volume may be determined from oblique-shock theory and that mass entrainment and friction forces on the control volume are negligible are common to all models.

The results for the various models may be anticipated by analyzing the assumptions of each model with regard to bleed. For the porous-wall model of figure 1(a) the bleed flow is assumed to leave the control volume with no streamwise momentum. This in effect attributes the momentum associated with the bleed flow to the residual flow in the control volume. This assumption leads to overestimation of the effectiveness of the bleed by predicting smaller than actual downstream boundary-layer thicknesses. One explanation of why this occurs is that, while the bleed flow has no streamwise momentum when in the porous wall, the momentum has not been given to the residual flow but has been dissipated in the high transverse shear stress at the wall that is associated with the bleed (ref. 4). Thus, the assumption of negligible friction forces on the control volume leads to an error.

The second model, the flush-slot model (fig. 1(b)), assumes that the bleed flow leaves the control volume with the same streamwise momentum that it had entering the control volume. This assumption is most correct when the bleed flow is removed immediately upstream of the interaction region. However, the model tends to overestimate the downstream boundary-layer thicknesses when bleed is removed at the higher pressures of the interaction region. This overestimation occurs because the increasing pressure force is acting on the bleed flow as well as the residual flow, while the model assumes it acts only on the residual flow.

The third model, the scoop model (fig. 1(c)), assumes that the bleed flow is removed with the momentum it had entering the control volume. This model differs from the flush-slot model in that the pressure acting on the bleed flow at the entrance to the control volume is not included in the momentum balance. Green in reference 4 states that, of the three models from reference 2, the scoop model most nearly describes the intended physical situation and should be the most applicable to other bleed configurations as well.

The penultimate model, the modified-slot model, allows the bleed flow to be removed at a pressure other than the upstream pressure. When bleed is removed in the interaction region, this model more accurately accounts for the bleed momentum removed from the control volume. For the control volume shown in figure 1(d), the continuity equation of the modified-slot model is

$$\rho_e U_{e,1}\left(1 - \frac{m_b}{m_{bl}}\right)\int_0^1 \rho^* U^* d\left(\frac{y}{\delta}\right)\Bigg|_1 = \rho_{e,3} U_{e,3}\int_0^1 \rho^* U^* d\left(\frac{y}{\delta}\right)\Bigg|_3 \qquad (1)$$

and the momentum equation is

$$\delta_1(p_1 - p_2) - \delta_3(p_3 - p_2) = \delta_3 \gamma p_3 M_{e,3}^2 \int_0^1 \rho^* U^{*2} d\left(\frac{y}{\delta}\right)\Bigg|_3$$

$$- \delta_1 \gamma p_1 M_{e,1}^2 \int_0^1 \rho^* U^{*2} d\left(\frac{y}{\delta}\right)\Bigg|_1 + \delta_b \gamma p_b M_{e,b}^2 \int_0^{y_u/\delta_b} \rho^* U^{*2} d\left(\frac{y}{\delta}\right)\Bigg|_b \quad (2)$$

Using the Dorodnitsyn-Howarth transformation and rearranging as in reference 2 give the continuity equation in the form

$$\frac{\Delta_1}{\Delta_3} = \frac{p_3 M_{e,3} \sqrt{\frac{t_{e,1}}{t_t}} (1 + N_{i,1}) N_{i,3}}{p_1 M_{e,1} \sqrt{\frac{t_{e,3}}{t_t}} N_{i,1} (1 + N_{i,3}) \left(1 - \frac{m_b}{m_{bl}}\right)} \quad (3)$$

and the momentum equation in the form

$$\frac{\Delta_1}{\Delta_3} = \frac{K_3(p_3 - p_2) + \gamma p_3 M_{e,3}^2 \frac{N_{i,3}}{2 + N_{i,3}}}{K_1(p_1 - p_2) + \gamma p_1 M_{e,1}^2 \frac{N_{i,1}}{2 + N_{i,1}} - \frac{\Delta_b}{\Delta_1} \gamma p_b M_{e,b}^2 \frac{N_{i,b}}{2 + N_{i,b}} \left(\frac{m_b}{m_{bl}}\right)^{(2+N_{i,b}/1+N_{i,b})}} \quad (4)$$

where

$$K = \frac{1}{1 - C_e^2}\left(1 - \frac{C_e^2 N_i}{2 + N_i}\right) \quad (5)$$

The value of K is evaluated at the appropriate station, and the parameters with the subscript b are evaluated at the bleed location. The right sides of the transformed continuity and momentum equations can be equated and the expression solved for $N_{i,3}$. In this expression, the bleed pressure and bleed edge Mach number can be estimated from the location of the bleed, but the values of $N_{i,b}$ and $\Delta_b/\Delta_1$ cannot be estimated initially. The method used in this report to evaluate these parameters involves a two-step approach. First, the equations are solved for zero bleed with $p_2$ and $p_3$ equal to $p_b$ and $M_{e,3}$ taken as $M_{e,b}$. That is, the equations are assumed to be valid for an interaction that proceeds without bleed to some intermediate values of pressure and Mach number. The values obtained for $N_{i,3}$ and $\Delta_1/\Delta_3$ are then taken as $N_{i,b}$ and $\Delta_1/\Delta_b$, respectively. The assumptions implicit in this technique are that the values of $N_i$ and $\Delta$ vary continuously through the interaction region as a function of $p_b$ and $M_{e,b}$ and that these values do characterize the bleed flow leaving the control volume. The equations are then solved again with bleed for $N_{i,3}$ by using the calculated values of $N_{i,b}$ and $\Delta_b/\Delta_1$. Some problems might arise in using this method for strong shocks and $p_b$ approaching $p_3$, where a solution for zero bleed might not exist. However, this problem was not encountered for the test conditions of reference 3.

The final model, shown in figure 1(e) is the dividing-streamline model of reference 3. In this model the lower boundary of the control volume is taken as the streamline defined by the fluid particle entering the most downstream location of the bleed system. Therefore, the method of removing the bleed mass flow does not enter into the analysis. However, the average pressure along the dividing streamline $p_4$ must be specified. As pointed out in reference 3, the dividing-streamline model is numerically equivalent to the scoop model of figure 1(c) if the pressure $p_4$ is assumed to be equal to $p_2$ and the boundary-layer thicknesses at station 3 are defined from the original wall location rather than the displaced position as in the scoop model. When $p_4$ is not equal to $p_2$, the model does not have an equivalent physical interpretation in the scoop bleed model.

The data used in evaluating the various models were those presented in reference 3. Boundary-layer measurements were made both upstream and downstream of a shock - boundary-layer interaction on the cowl of an inlet. The data used were for three bleed configurations having normal bleed holes located at and immediately downstream of the shock impingement point. The upstream Mach number and Reynolds number based on momentum thickness were 1.66 and 3528, respectively. The incident shock had a pressure ratio of 1.33. The upstream momentum thickness was 0.03129 centimeter, and the bleed flow rates ranged up to approximately 0.6 of the upstream boundary-layer mass flow rate. Details of the boundary-layer measurements are given in reference 3.

## RESULTS AND DISCUSSION

To determine the effect of the bleed assumptions on the control-volume models, the results of three analyses that differ only in how they treat the bleed flow are shown in figure 2. These are results for the porous-wall, flush-slot, and modified-slot models. It was not possible to make a direct comparison with the data of reference 3 for the flush-slot model since an analytical solution for this model could not be obtained for an upstream experimental incompressible shape factor $H_{i,1}$ of 1.373. Therefore, in this figure a value of $H_{i,1} = 1.3$ was used for all three models of this report, while the data of reference 3 were taken at $H_{i,1} = 1.373$. The experimental external conditions were used in the calculations. Although there is a slight difference between the shape factors of the calculations and the data, the comparison should be useful from a qualitative if not an exact quantitative standpoint.

The upper plot in figure 2 shows the displacement thickness ratio across the interaction region for the three models as a function of bleed rate. The result for the porous-wall model shows a very steep decrease in displacement thickness with increasing bleed. This result could be anticipated from the discussion of this model in the previous section. The assumption in this model of no momentum leaving with the bleed flow results in an excess amount of momentum attributed to the residual flow. This excess causes the displacement thickness to decrease rapidly and become zero for a bleed flow ratio of 0.15.

The displacement thickness ratio for the flush-slot model decreases with increasing bleed, reaches a minimum, and then increases rapidly until the analysis breaks down at a bleed flow ratio of 0.35. This behavior at the higher bleed rates results from the assumption that the bleed is removed at the upstream pressure. This assumption tends to overestimate the amount of momentum leaving the control volume and results in an overprediction of the downstream displacement thicknesses.

The modified-slot model, which takes $p_b = p_2$, gives results that fall between those for the other two models. Compared with the data from reference 3, the results show the correct qualitative behavior, although the displacement thickness is underestimated. Part of this discrepancy is caused by the difference between the upstream shape factor used in the calculation and that of the data. The modification of the bleed assumptions for this model substantially improves the results, especially for high bleed rates.

In the lower plot in figure 2 the momentum thickness ratios for various bleed rates are shown for the same three models in addition to the data from reference 3. The results for the momentum thicknesses are essentially the same as for the displacement thicknesses. As expected, the results for the modified-slot model fall between those for the flush-slot and porous-wall models.

In figure 3 the displacement and momentum thicknesses for various bleed rates are shown for an analysis using the modified-slot model. The data from reference 3 are also shown. The calculations are for $H_{i,1} = 1.373$, which corresponds to the upstream shape factor of the data. The results shown are for two values of bleed pressure ratio $p_b/p_1$, 1.2 and 1.328, the latter corresponding to $p_b = p_2$. The results for $p_b/p_1 = 1.2$ give the better fit of the data, especially at the higher bleed rates.

Similar plots are shown for the dividing-streamline model in figure 4. The average pressure along the dividing streamline is taken equal to the bleed pressure for the modified-slot model in figure 3. The value of $p_2/p_1 = 1.328$ results in $p_2 = p_4$, which also corresponds numerically to the scoop model. The results for this model are similar to those obtained for the modified slot. The value of $p_4/p_1 = 1.2$ gives the better fit of the data. This fit indicates that the pressure assumed along the dividing streamline in the dividing-streamline model and the bleed pressure in the modified-slot model are essentially equivalent. However, since the dividing-streamline model is the easier one to use, it would be the better choice, and its use would avoid the two-step procedure involved in the modified-slot model. Although both methods give the best results for a pressure ratio of 1.2, the limited amount of experimental data used in this correlation suggests that caution should be used in applying this value to situations that differ greatly from the experimental conditions of reference 3.

## SUMMARY OF RESULTS

A number of control-volume models used to describe the oblique-shock - boundary-layer-bleed interaction problem were investigated. The predictions of the various models were compared with the results of boundary-layer measurements made in a supersonic inlet. The experimental conditions of the interactions studied were characterized by an upstream Mach number of 1.66, an incident oblique-shock pressure ratio of 1.33, and boundary-layer bleed rates up to approximately 0.6 of the total boundary-layer flow. From these calculations the following results were obtained:

1. At the high bleed rates and upstream values of shape factors represented by the data, the flush-slot and porous-wall models did not give satisfactory results.
2. The results for the modified-slot model compared well with data for a ratio of bleed to upstream pressure of 1.2.
3. When compared with the data, the dividing-streamline model gave the best results for a ratio of average dividing-streamline to upstream pressure of 1.2.
4. The specification of the bleed pressure in the modified-slot model and the streamline pressure in the dividing-streamline model were equivalent.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 26, 1974,
505-04.

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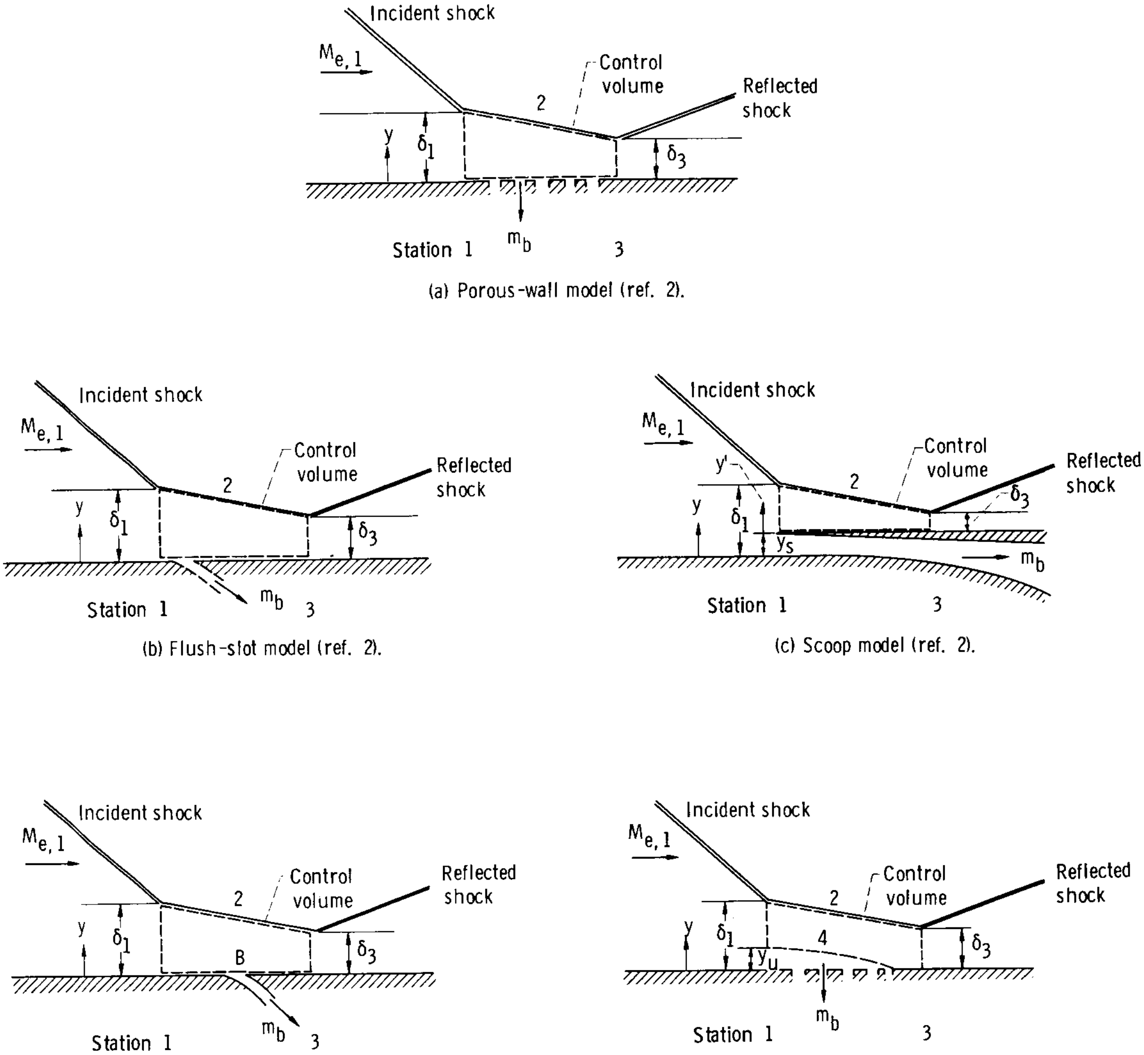


(a) Porous-wall model (ref. 2).

(b) Flush-slot model (ref. 2).

(c) Scoop model (ref. 2).

(d) Modified-slot model.

(e) Dividing-streamline model.

Figure 1. - Shock - boundary-layer interaction bleed models.

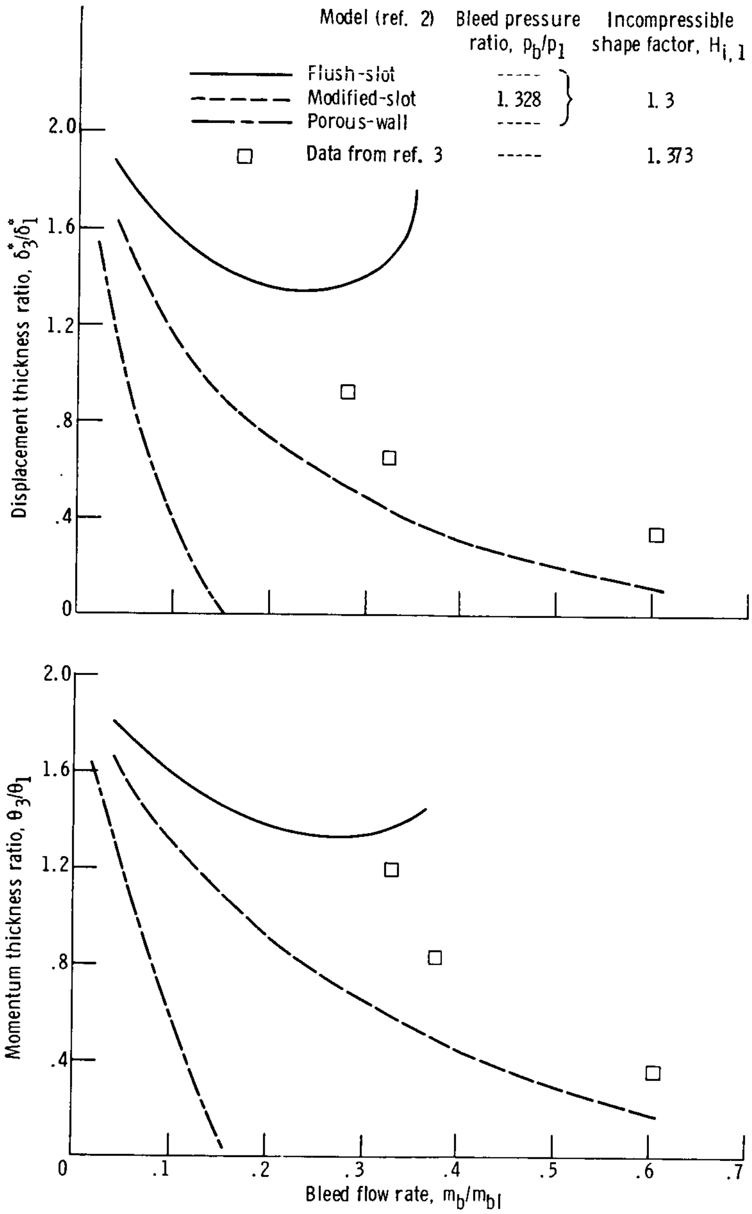


Figure 2. - Displacement and momentum thickness ratios as function of bleed flow rate for three models of reference 2.

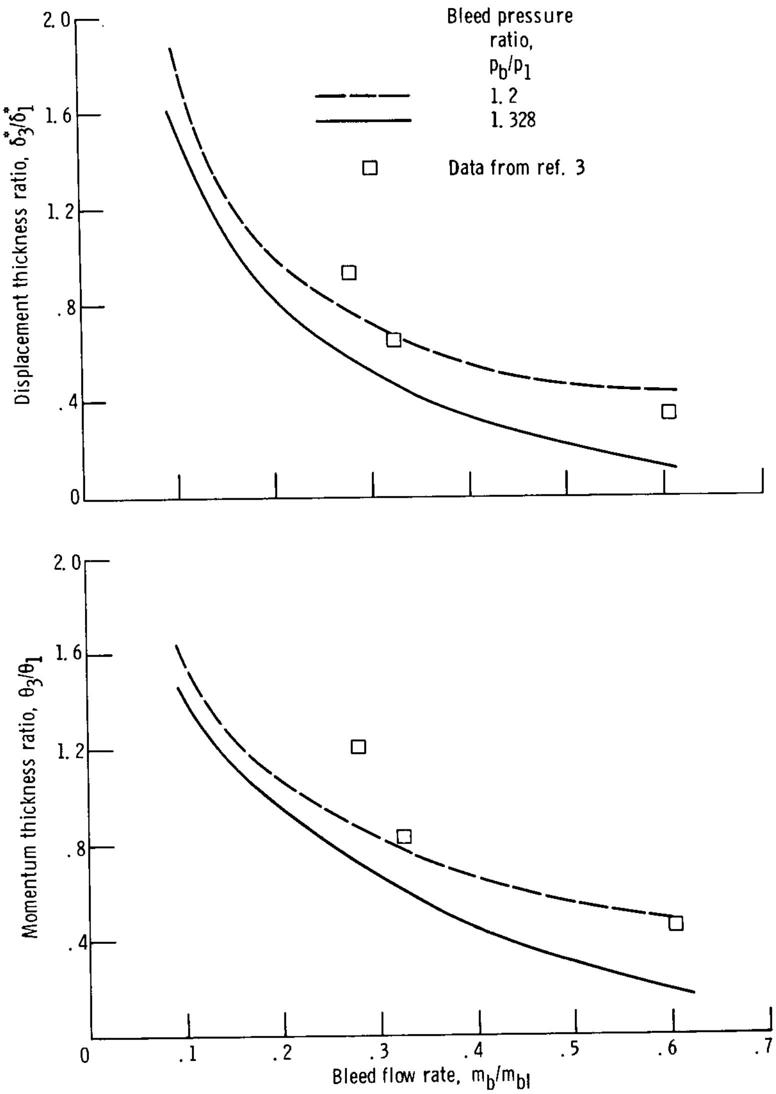


Figure 3. - Displacement and momentum thickness ratios as function of bleed flow rate for modified-slot model.

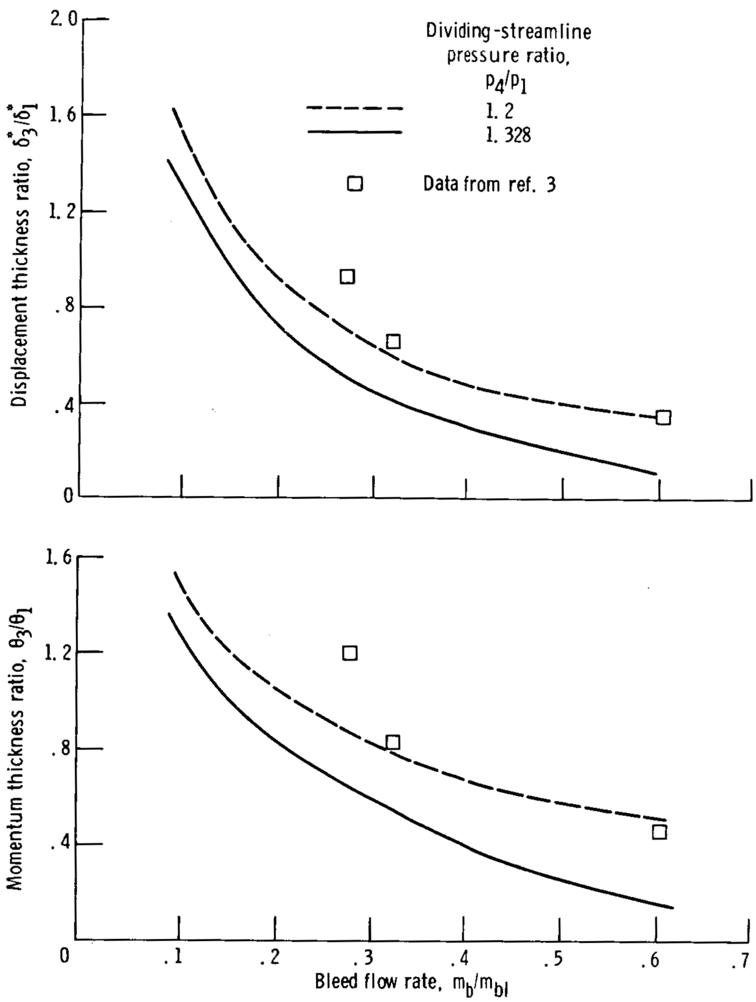


Figure 4. - Displacement and momentum thickness ratios as function of bleed flow rate for dividing-streamline model.