## A STUDY OF DIGITAL GYRO COMPENSATION LOOPS

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## ABSTRACT

The primary objective of this program was to demonstrate the feasibility of replacing existing state-of-the-art analog gyro compensation loops with digital computations. This objective was realized during the course of the program.

A breadboard design was established in which one axis of a Teledyne tunedgimbal TDF gyro was caged digitally while the other was caged using conventional analog electronics. The digital loop was designed analytically to closely resemble the analog loop in performance. The breadboard was subjected* to various static and dynamic tests in order to establish the relative stability characteristics and frequency responses of the digital and analog loops. Several variations of the digital loop configuration were evaluated. The results were very favorable - it appears that digital caging is indeed a practical approach.

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## I. INTRODUCTION

## Background

In January 1974 Teledyne Systems Company completed under contract to NASA Langley Research Center an "Investigation of the Application of Two-Degree-of-Freedom Dry Tuned-Gimbal Gyroscopes to Strapdown Navigation Systems"\%. During the course of that study an entirely new approach to the control and compensation of the TDF gyro, as well as accelerometers, was suggested and studied. This technique was based upon the use of "all-digital" sensor compensation loops and control functions.

An analysis of the hardware content and costs of present day inertial navigation systems showed that a significant portion, on the order of $30 \%$, of the system, consists of analog and computer interface electronics. The advent of newly available, versatile, digital microcomputers and high-speed analog-to-digital and digital-to-analog converters now makes it practical to replace essentially all of the analog electronics by functionally equivalent digital computations.

Figure 1 shows a functional block diagram of a conventional strapdown inertial navigation system mechanization. Both the sensor compensation functions and the spin motor and pickoff excitations are generated by analog electronics which operate independently of the digital processor. The sensor torquing currents are fed through precision resistances to develop voltages which are proportional to angular rates and accelerations. These voltages are then converted into digital numbers for use in the navigation and attitude equations which must be solved by the computer.

Figure 2 shows the corresponding block diagram for a strapdown system employing digital sensor control and compensation. Here all of the compensation loop ser vo functions are mechanized as digital computations in the proc- . essor. The basic inputs to the A/D converter are the sensor pickoff signals rather than the restoring signals. The torquing signals are computed as digital numbers, converted to analog signals and, after power amplification, used to restore the sensors. These same digital torquing quantities may be used directly in the strapdown attitude and navigation computations as well. (In an optional configuration the actual torquing signals are also passed through the A/D converter for use in the attitude and navigation equations.)

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Figure l. Functional Block Diagram of Conventional Strapdown Mechanization


Figure 2. Functional Block Diagram of "All-Digital" Strapdown Mechanization

What is gained by such an "all-digital" approach is that the presently cumbersome and necessarily specialized analog and digital interface electronics can be virtually eliminated, being replaced by digital computations (software) performed in the computer hardware which is, in any case, required for the attitude and navigation computations. This results in substantial savings in cost, size, weight, and power consumption while simultaneously increasing the reliability of the system by reducing the component count.

## Objective

The objective of this program was to determine the feasibility of replacing present state-of-themart analog circuits in strapdown tuned-gimbal gyro compensation circuits with their digital counterparts. This was accomplished by designing appropriate compensation loops for the dry tuned TDF gyro, selecting appropriate data conversion and processing techniques and algorithms, and, using existing laboratory equipment, breadboarding the design for laboratory evaluation.

The principal area of engineering design involved in the program was the determination of the specific software requirements for closing the instrument loops and performing the required compensation. Specific requirements relative to processing rate, word length, computer time and memory utilization were established in order that corresponding requirements for a full complement of strapdown sensors may be readily extrapolated. Additionally, appropriate analog-to-digital and digital-to-analog conversion designs were established and techniques selected which were capable of meeting these requirements.

An available dry tuned-gimbal two-degree-of-freedom gyroscope was utilized together with an electronic circuit breadboard and existing laboratory hardware to implement the digital control loop. Selection of hardware and techniques was based on the anticipated availability of proven hardware in the late $1970^{\prime} s$. The breadboard design was evaluated using standard test and evaluation procedures.

This final report summarizes the test configuration and the program results which were obtained.

## II. ANALYSIS

The breadboard design which was established for this study, consisted of a dry-tuned two-degree-of-freedom (TDF) gyroscope with one axis being caged in a conventional manner using simplified existing "state-of-the-art" analog compensation. The second axis was caged using a digital compensation loop designed to emulate the performance of the analog compensation. Figure 3 shows a simplified block diagram of the breadboard design. An analysis of the two caging techniques employed in the design is presented in the following paragraphs.

## Analog Compensation Design

The general Teledyne gyro caging loop configuration for conventional analog compensation is shown in the simplified block diagram of Figure 4. Observe that both direct and cross axis compensation is employed in this design in order to increase the loop bandwidth and reduce the rotor hangoff during acceleration inputs. In this block diagram $\theta_{X}$ and $\theta_{Y}$ represent the gyro pickoff angles, $T_{X}$ and $T_{Y}$ the gyro torques, $P_{X}$ and $P_{Y}$ the precessional torques resulting from rate inputs $\omega_{X}$ and $\omega_{Y}$, and $M_{X}$ and $M_{Y}$ the rebalance torques provided by the compensation loops.

A more detailed block diagram of the compensation portions of the loop for the current Teledyne design is shown in Figure 5. This block diagram shows the actual transfer functions which are implemented in the analog electronics as well as the pickoff and torquer transfer characteristics. A derivation and analysis of this compensation design is included in NASA CR-132419. The pole-zero locations for the compensation electronics using this design are shown in Figure 6.

Concurrently with the digital loop demonstration program Teledyne has been involved in an extensive redesign effort on its conventional analog compensation electronics. This effort has as its goals improved loop per formance and simpler loop design. Although work is continuing in this area, it has been found that excellent performance can be obtained with relatively simple analog transfer functions. The pole-zero configuration for one such simple loop is shown in Figure 7. This configuration provides a baseline for much of the digital loop design work which was performed during the program.


Figure 3. Block Diagram of the System Used in the Digital Gyro-Caging Study


Figure 4: Block Diagram of Gyro and Caging Electronics


Figure 5. Caging Loop Mechanization


Figure 6a. Pole-Zero Locations for Cross Axis-Current Design


Figure 6b. Pole-Zero Locations for Direct Axis-Current Design


Figure 7a. Pole-Zero Locations for Cross Axis - New Design


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Figure 7b. Pole-Zero Locations for Direct Axis - New Design

## Digital Compensation Analysis

Figure 8 shows a simplified block diagram of the digital gyro caging loop. This figure should be compared with Figure 4. One of the primary objectives of the program was to design digital filters which would emulate the performance of the existing analog filter design.

The study of digital filters and digital servos has progressed rapidly in the past decade and an extensive literature now exists with regard to the se subjects. Rabiner and Rader\%, for example, provides both theoretical and practical coverage of digital filtering as well as extensive bibliographies. The large variety of digital filters and the subtleties (e.g., the aliasing effect) which arise in their implementation, however, serve to maintain digital filtering as more of an art than a science.

Some "cookbook" approaches exist for obtaining digital equivalents to analog filters. Some of these have been analyzed, aided where required by simulation, in order to determine their applicability to the digital compensation loop design. In selecting a practical approach, particular attention was paid to the impact on the computer with regard to memory and time utilization. As an example, it was desirable to synthesize digital filters whose coefficients are integer powers of two, since this allows the replacement of multiply instructions with less time consuming shift instructions.

As one simple example of using the cookbook approaches, consider the bilinear transformation approach using the transformation

$$
s \rightarrow \frac{1-z^{-1}}{1+z^{-1}}
$$

The transfer function

$$
H(s)=\frac{\frac{s}{-79 N}+1}{\left(\frac{s}{3 N}+1\right)^{2}}
$$

appearing in Figure 5 is seen to have a zero at $\omega_{1}=.79 \mathrm{~N}=496 \mathrm{rad} / \mathrm{sec}$ and a double pole at $\omega_{2}=3 N=1884 \mathrm{rad} / \mathrm{sec}$. In order to synthesize a digital filter with these characteristics we first derive an analog filter with a zero at

[^1]

Figure 8. Block Diagram of Digital Compensation Loop

$$
\omega_{1}^{\prime}=\tan \frac{\omega_{1} \tau}{2}
$$

and a double pole at

$$
\omega_{2}^{\prime}=\tan \frac{\omega_{2} \tau}{2}
$$

where $\tau$ is the computer iteration interval. Assuming, for example, an iteration rate of $200 \mathrm{~Hz}, \tau$ is 5 milliseconds and

$$
\begin{aligned}
& \omega_{1}^{\prime}=\tan \frac{(496)(.005)}{2}=2.91 \\
& \omega_{2}^{\prime}=\tan \frac{(1884)(.005)}{2}=418.59
\end{aligned}
$$

so that the "dummy" analog transfer function is

$$
\mathrm{H}^{\prime}(\mathrm{s})=\frac{\frac{\mathrm{s}}{2.91}+1}{\left(\frac{\mathrm{~s}}{418.59}+1\right)^{2}}
$$

Finally, the digital transfer function is found by introducing the transformation

$$
\begin{aligned}
& s \rightarrow \frac{1-z^{-1}}{1+z^{-1}} \text { so that } \\
& H(z)=\frac{\frac{1\left(1-z^{-1}\right)}{2.91\left(1+z^{-1}\right)}+1}{\left(\frac{1\left(1-z^{-1}\right)}{418.59\left(1+z^{-1}\right)}+1\right)^{2}} \\
& =\frac{\left[\frac{1}{2.91}\left(1-z^{-1}\right)+\left(1+z^{-1}\right)\right]\left[1+z^{-1}\right]}{\left[\frac{1}{418.59}\left(1-z^{-1}\right)+\left(1+z^{-1}\right)\right] 2}
\end{aligned}
$$

$$
\begin{aligned}
& =\frac{\left(1+\frac{1}{2.91}\right)+2 z^{-1}+\left(1-\frac{1}{2.91}\right) z^{-2}}{\left(1+\frac{2}{418.59}+\frac{1}{(418.59)^{2}}\right)+2\left(1-\frac{1}{(418.59)^{2}}\right) z^{-1}} \\
& +\left(1-\frac{2}{(418.59)}+\frac{1}{(418.59)^{2}}\right) z^{-2}
\end{aligned}
$$

The bracketed groups in this expression are fixed constants. Thus an exact mechnaization of this filter requires 6 multiplies and 4 adds per iteration of the computer. Observe, however, that one of these multiplications is by the number two. Thus for this operation a shift may replace a multiply instruction resulting in a substantial saving in computer time since a simple shift requires only a small fraction of the time needed for execution of a multiply instruction.

Further simplifications may be possible in this transfer function. For example, consider approximating the factor $(1+1 / 2.91)=1.34$ by

$$
1.375=1+\frac{1}{4}+\frac{1}{8}
$$

Then the multiplication (of the current input) by this factor may be accomplished by a double shift, a single shift, and two additions, again effecting a substantial saving in computer time. Such approximations must be carefully analyzed, however. Working 'backward" through the preceding derivation it is seen that this approximation has the effect of shifting the zero of the transfer function from $\omega_{1}=496 \mathrm{rad} / \mathrm{sec}$ to

$$
\begin{aligned}
\omega_{1} & =\frac{2}{\tau} \tan ^{-1}(2.67) \\
& =\frac{2}{.005}(1.21)=485 \frac{\mathrm{rad}}{\mathrm{sec}}
\end{aligned}
$$

In this case the effect on the filter response is probably negligible. This is not the case for all approximations which appear on the surface to be reasonable, however.

Initial efforts in designing the digital compensation filters concentrated on emulating existing "old design"analog filters as used in previous strapdown systems. The transfer functions of these analog filters are shown in the block diagram of Figure 4 and their pole-zero locations in Figure 5.

Consider first the cross axis transfer function relating $M_{Y}$ to $\theta_{X}$, i. e. the $y$-axis torquer output to the x-axis pickoff angle. The analog transfer function is

$$
\frac{M_{Y}(s)}{\theta_{X}(s)}=\frac{1.01 \mathrm{~K}_{\mathrm{sa}} \mathrm{~K}_{\mathrm{po}} \mathrm{~K}_{\mathrm{T}}\left(\frac{\mathrm{~s}}{.79 \mathrm{~N}}+1\right)\left(\frac{\mathrm{s}}{35 \mathrm{~N}}+1\right)\left(\frac{\mathrm{s}^{2}}{\mathrm{~N}^{2}}+1\right)}{\mathrm{s}\left(\frac{\mathrm{~s}}{16 \mathrm{~N}}+1\right)\left(\frac{\mathrm{s}}{3 \mathrm{~N}}+1\right)^{4}}
$$

Since the pickoff and torquer transfer functions will exist in the digital loop in any case, they do not have to be implemented digitaly. The gain factor ( $1.01 \mathrm{~K}_{\text {sa }}$ ) may also be neglected in the digital design as the gain may be provided by the power amplifier. Thus the digital transfer function to be mechanized should be equivalent to

$$
H(s)=\frac{\left(\frac{s}{.35 N}+1\right)\left(\frac{s}{.79 N}+1\right)\left(\frac{s^{2}}{N^{2}}+1\right)}{s\left(\frac{s}{3 N}+1\right)^{4}}
$$

Using the bilinear transformation technique and assuming now an 800 Hz update rate, the dummy analog breakpoints are computed

$$
\begin{aligned}
& \omega_{1}^{\prime}=\tan \frac{(2 \pi)(35)}{(2)(800)}=.13832 \\
& \omega_{2}^{\prime}=\tan \frac{(2 \pi)(79)}{(2)(800)}=.32058 \\
& \omega_{3}^{\prime}=\tan \frac{(2 \pi)(100)}{(2)(800)}=.41421 \\
& \omega_{4}^{\prime}=\tan \frac{(2 \pi)(300)}{(2)(800)}=2.41421
\end{aligned}
$$

Making the substitution

$$
s \rightarrow \frac{1-z^{-1}}{1+z^{-1}}
$$

the digital transfer function is found (after much arithmetic manipulation) to be
$H(z)=57.8787 \frac{1-1.68566 z^{-1}+.50190 z^{-2}+1.36534 z^{-3}-1.43278 z^{-4}+3.8945 z^{-5}}{1+.65685 z^{-1}-.62742 z^{-2}-.74517 z^{-3}-.25483 z^{-1}-.02944 z^{-5}}$
In order to obtain a rough estimate of computer timing requirements for implementation of the digital loop, the numerator of $H(z)$ was considered. The numerator was first approximated in order to eliminate the need for multiplications. Thus

$$
\begin{aligned}
& 1.68566 \approx 1+2^{-1}+2^{-3}+2^{-4}=1.6875 \\
& .50190 \simeq 2^{-1}=.500 \\
& 1.36534 \approx 1+2^{-2}+2^{-3}=1.375 \\
& 1.43278 \simeq 1+2^{-2}+2^{-3}+2^{-4}=1.4375 \\
& .38945 \approx 2^{-2}+2^{-3}+2^{-6}=.390625
\end{aligned}
$$

so that the numerator is implemented as

$$
\begin{gathered}
\theta_{x}(k)-\left(1+2^{-1}+2^{-3}+2^{-4}\right) \theta_{x}(k-1)+\left(2^{-1}\right) \theta_{x}(k-2)+\left(1+2^{-2}+2^{-3}\right) \theta_{x}(k-3) \\
-\left(1+2^{-2}+2^{-3}+2^{-4}\right) \theta_{x}(k-4)+\left(2^{-2}+2^{-3}+2^{-6}\right) \theta_{x}(k-5)
\end{gathered}
$$

(Working "backward" as before, these approximations resulted in analog breakpoints of $.519 \mathrm{~N}, .749 \mathrm{~N}$ and $.711 \mathrm{~N} \pm \mathrm{j} .706 \mathrm{~N}$, a reasonable agreement with the original transfer function.) Using the TDY-52B/IMP-16 instructions, a simple program was written to implement the equation above. The instruction breakdown was as follows

| Instruction | Number | Time Per <br> Instruction | Time <br> ( $\mu \mathrm{sec}$ ) |
| :--- | :---: | :---: | :---: |
| Load | 6 | 7.7 | 46.20 |
| Store | 5 | 9.1 | 45.50 |
| Reg. Copy | 1 | 8.75 | 8.75 |
| Reg. Add | 10 | 4.55 | 45.50 |
| Add/Sub | 5 | 7.7 | 38.50 |
| Shift Rt. 1 | 3 | 10.1 | 30.30 |
| Shift Rt. 2 | 5 | 14.3 | 71.50 |
| Shift Rt. 3 | 1 | 17.5 | 17.50 |

Assuming that each transfer function numerator and denominator is of roughly the same complexity, the total time required to impement the four transfer functions is
$(303.75) \times 2 \times 4=2.43 \mathrm{msec}$
which is approximately twice the available time for an 800 Hz update rate. Since this estimate does not include I/O processing, scaling, executive, control, etc. it was clear that much simpler processing would be required if the digital compensation loop were to be successfully implemented with the TDY-52B computer.

Fortunately, as described in Section II, efforts were underway concurrent to the digital loop program to simplify and improve analog compensation techniques. This work resulted in the designs described in the preceeding section, and showed that very simple analog transfer functions could be used to obtain the desired servo loop response characteristics. Furthermore, it was decided to mechanize one axis of the gyro in an analog loop in order to more accurately compare the analog loop and digital loop responses, thus reducing the digital computations which were required. These two factors resulted in a digital loop design which could be implemented with a TDY-52B even though the update rate was increased to 1200 Hz .

The "first cut" at implementing the new transfer functions started with the analog functions whose pole-zero locations are shown in Figure 7.

$$
\begin{aligned}
& F_{1}(s)=\frac{s+N}{s+3 N} \\
& F_{2}(s)=\frac{s+N}{s(s+3 N)}
\end{aligned}
$$

where $F_{I}(s)$ is the direct axis transfer function and $F_{2}(s)$ is the cross axis transfer function. Using a 1200 Hz update, the dummy breakpoint frequencies were computed to be

$$
\begin{aligned}
& \omega_{1}^{\prime}=\tan \frac{(2 \pi)(100)}{(2)(1200)}=.26795 \\
& \omega_{2}^{\prime}=\tan \frac{(2 \pi)(300)}{(2)(1200)}=1.0000
\end{aligned}
$$

so that

$$
\begin{aligned}
& F_{1}(z)=\frac{\frac{(z-1)}{(z+1)}+.26795}{\frac{(z-1)}{(z-1)}+1.0000}=\frac{1.26795 z-.73205}{2 z}=.63398\left(1-.57735 z^{-1}\right) \\
& F_{2}(z)=\frac{\frac{(z-1)}{(z+1)}+.26795}{\frac{(z-1)}{(z+1)}\left[\frac{z-1}{z+1}+1.000\right]}=.63398 \frac{\left(1+.42265 z^{-1}-.57735 z^{-2}\right)}{\left(1-z^{-1}\right)}
\end{aligned}
$$

Since the constant gain term can be readily implemented in the analog portion of the loop, it may be neglected. The transfer functions were further simplified to

$$
\begin{aligned}
& G_{1}(z)=\left(1-.5 z^{-1}\right) \\
& G_{2}(z)=\frac{\left(1+.5 z^{-1}-.5 z^{-2}\right)}{\left(1-z^{-1}\right)}
\end{aligned}
$$

for easier digital implementation. This has the effect of changing $\omega_{1}$ to .333 and $\omega_{1}$ (the analog loop zero) to 1.23 N instead of N .

The comparison of analog and digital frequency responses is shown in Figures 9a and 9b for this configuration. It is seen that the breakpoints and low - frequency asymptotics are identical, with variations occurring at intermediate points. The high frequency performance of the digital loop is, of course, different from the analog response due to the "fold-over" effect. For this reason, the filter update rate must be sufficiently high that the deleterious effects of this phenomenon are countered by attenuation in the analog portions of the loop. (Test results indicate that 1200 Hz is the minimum update frequency required to obtain the desired response and that higher frequencies are desirable.)


Figure 9a. Direct Axis Frequency Response


Figure 9b. Cross Axis Frequency Response

Several variations of parameters were tested within the constraints of the basic filter configurations described by $G_{1}(z)$ and $G_{2}(z)$ above. It was found that the transfer functions

$$
\begin{aligned}
& G_{1}(z)=\left(1-\frac{15}{16} z^{-1}\right) \\
& G_{2}(z)=\frac{1+\frac{1}{16} z^{-1}-\frac{15}{16} z^{-2}}{1-z^{-1}}
\end{aligned}
$$

provided reasonably optimal performance for the digital loop\%. Most of the results described in the report reflect this configuration. Thus the equations which were mechanized in the computer were

$$
\begin{aligned}
M_{x}(k) & =M_{x}(k-1)+\theta_{x}(k)+\left(2^{-4}\right) \theta_{x}(k-1)-\left(1-2^{-4}\right) \theta_{x}(k-2) \\
M_{y}(k) & =\theta_{y}(k)-\left(1-2^{-4}\right) \theta_{y}(k-1) \\
M(k) & =M_{x}(k)-M_{y}(k)
\end{aligned}
$$

where $M(k)$ is the computed gyro torque at the $k^{\text {th }}$ iteration.
The digital number $\mathrm{M}(\mathrm{k})$ is converted by the digital-to-analog converter and, after power amplification, used to torque the gyro. In the absense of errors, $M(k)$ represents the true torque input and is, therefore, proportional to the angular rate of the gyro case. This is precisely the information which is desired for attitude computation in a strapdown navigator. Any errors introduced by the D/A converter therefore propagate directly into attitude errors. For this reason it is necessary to use an accurate D/A converter in the digital loop.
*This corresponds to moving the zero locations in Figure 6 from 100 Hz to approximately 12.7 Hz .

The analog-to-digital conversion requirements are much less stringent. Due to the closed-loop nature of the digital compensator, errors in the $A / D$ converter are of relatively minor importance to the accuracy of rate measurement (just as pickoff measurement errors are unimportant in a conventional analog loop mechanization).

The test results indicate that a 16 bit computer word length is sufficient for mechanization of the digital loop. The D/A converter should be an accurate device with excellent (or, at least, well compensatable) bias and scale factor characteristics. The A/D converter can be less accurate without seriously compromising torque measurement accuracy. The digital loop was mechanized using a 12 bit A/D converter and a 13 bit D/A converter. These resolutions are not critical, however, for the reasons outlined above. As long as the analog torque applied accurately represents the (quantized) digital torque, no net error will result in the computation of attitude due to the D/A converter resolution.

## III. COMPUTER

The computer used in the breadboard is an IMP-16P (National Semiconductor), which is electrically and functionally interchangeable with the TSC hybrid packaged TDY-52B. Although these two machines are functionally the same, they are vastly different physically. The CPU of each consists of four 4-bit NS (National Semiconductor) GPC/P RALU slices. The microprogram of each is contained in a set of two Control Read Only Memories (CROMs) that implement the NS IMP-16 computer repertoire. Input/Output and other hardware details are also similar. The IMP-16P and TDY-52B are thus, basically, the same machine.

The most obvious difference between the IMP-16P and TDY-52B is one of size. Teledyne has packaged the heart of the IMP-16P into a small hybrid unit that is only two inches on a side and .2 "thick. The IMP-16P computer, including a convenient control panel and 4 K of 16 -bit RAM memory, occupies a box with a frontal dimension of about $101 / 2^{\prime \prime} \times 17^{\prime \prime}$ and a depth of about $24^{\prime \prime}$.

The IMP-16P was selected for the digital loop demonstration primarily because its control panel feature facilitated the operation and modification of the breadboard. The results which were obtained are, however, equally applicable to the TDY-52B.

Table 1. TDY-52B/IMP-16 Instruction Repertoire (Sheet 1 of 2)

| MNEMONIC | INSTRUCTION NAME | FUNCTION | FORMAT | EXECUTION TIME IN MICROSECONDS |
| :---: | :---: | :---: | :---: | :---: |
| LOAD AND STORE |  |  |  |  |
| LD | LOAD | $(E A)+(A C r)_{r} \mathrm{IF}$ INDIRECT $((E A)) \rightarrow(\mathrm{ACr})$ | 2 | $7.0,9.8$ <br> IF INDIRECT |
| LDB | LOAD BYTE | $(1 / 2 E A) \rightarrow(A C O L E S S$ SIGNIFICANT BYTE) | 5 | 16.8 T0 28.0 |
| LI | LOAD IMMEDIATE | $D \rightarrow(A C r)$ | 48 | 4.2 |
| ST | StORE | $(\mathrm{ACN})-(E A)$, IF INDIRECT $(\mathrm{ACr}) \rightarrow((E A))$ | 2 | 8.4.11.2 IF INDIRECT |
| STB | StORE BYTE | (ACO LESS SIGNIFICANT BYTE) $\rightarrow(1 / 2$ EA $)$ $0 \rightarrow(S E L)$ | 5 | 23.8 TO 32.2 |
| RXCH | REGISTER EXCHANGE | $(S R)-(D R), \quad(D R) \rightarrow(S R)$ | 6 | 11.2 |
| RCPY | REGISTER COPY | $(S R)-(D R)$ | 6 | 8.4 |
| LOGICAL |  |  |  |  |
| RXOR | REGISTER EXCLUSIVE OR | $(S R) \oplus(D R)-(D R)$ | 6 | 8.4 |
| RAND | REGISTER AND | $(\mathrm{SR})$ "AND" $(\mathrm{DR}) \rightarrow$ (DR) | 6 | 8.4 |
| AND | AND | (RO1) "AND" (EA) $\rightarrow$ (RO1) | 3 | 7.0 |
| OR | OR | (R01) "OR" $(E A) \rightarrow(\mathrm{RO} 1)$ | 3 | 7.0 |
| ARITHMETIC |  |  |  |  |
| RADD | REGISFER ADD | $(S R)+(D R) \rightarrow(D R) \quad O V, C Y$ | 6 | 4.2 |
| ADD | ADD | $(A C r)+(E A) \rightarrow\left(A C_{r}\right), \quad O V, C Y$ | 2 | -7.0 |
| SUB | SUBTRACT | $(A C r)-(E A)-(A C r) \quad O V, C Y$. | 2 | 7.0 |
| MPY | MULTIPLY |  | 5 | 148.410170 .8 |
| DIV | DIVIDE | $\left\{\begin{array}{l} (A C O),(A C 1)\}-(E A) \rightarrow(A C O) \text { QUOTIENT } \\ 0-(S E L) \quad O V, L \quad(A C I) \text { REMAINDER } \end{array}\right.$ | 5 | 177.870222 .6 |
| DADO | DOUBLE PRECISION ADD | $\underset{0 \rightarrow(S E L),(A C I) \mid+\{(E A),(E A+1)\} \rightarrow\{(A C O),(A C D)\}}{\{(A C O)}$ | 5 | 16.8 . |
| DSUB | DOUBLE PRECISION SUBTRACT |  | 5 | 16.8 |
| CAI | COMPLEMENT AND ADD | $\sim\left(A C_{r}\right)+D \rightarrow\left(A C_{r}\right)$ | 4B | 4.2 |
| SHIFT IMMEDIATE |  |  |  |  |
| ROL | rotate left | $\left.\begin{array}{l} 2(A C r) \rightarrow(A C r) \\ I F S E L=0,(B \mid T 15) \rightarrow(B \mid T 0) \\ I F S E L=1,(B\|T\| 5) \rightarrow(L),(L) \rightarrow(\text { BIT } 0) \end{array}\right\} D T I M E S$ | 48 | $5.6+4.20$ |
| ROR | ROTATE RIGHT | $\left.\begin{array}{l}1 / 2\left(A C_{r}\right) \rightarrow\left(A C_{r}\right) \\ \text { IF } S E L=0,(\text { BIT } 0) \rightarrow(B I T ~ 15) \\ \text { IF SEL }=1,(B I T 0) \rightarrow(L),(L) \rightarrow(B I T \quad 15)\end{array}\right\}$ D TIMES | 4B | $5.6+4.2 \mathrm{D}$ |
| SHL | Shift left | $2(A C r) \rightarrow(A C r)$ | $4 B$ | $5.6+4.2 \mathrm{D}$ |
| . |  | $0 \rightarrow$ (BIT 0) $\quad$ D TIMES |  | , |
|  |  | IF $\mathrm{SEL}=1$, (BIT 15) $-(\mathrm{L}$ ) |  |  |
| SHR | SHIFT RIGHT | $1 / 2\left(A C_{r}\right)-\left(A C_{r r}\right)$ | 4B | $5.6+4.2 \mathrm{D}$ |
|  | * | IF SEL $=0,0 \rightarrow$ (BIT 15) $\quad$ D TIMES |  |  |
|  |  | IF. SEL $=1,(\mathrm{~L}) \rightarrow$ (BIT 15), $0 \rightarrow(\mathrm{~L}) \cdot \mathrm{C}$. |  |  |
| SINGLE BIT |  |  |  |  |
| SETST | SET STATUS BIT | $1 \rightarrow$ (STATUS FLAG N) | 9 | 18.27044 .8 |
| CLRST | CLEAR STATUS Bit | $0 \rightarrow$ (STATUS FLAG N ) | 9 | 18.2 TO 44.8 |
| SETBIT | SET BIT | $\xrightarrow{\rightarrow}(\mathrm{ACO}$ BIT $N$ ) | 9 | 18.2 TO 44.8 |
| CLRBIT | CLEAR BIT | $0 \rightarrow(A C O B I T N)$ | 9 | 18.2 TO 44.8 |
| CMPBIT | COMPLEMENT BIT | $(A C O B I T N) \rightarrow(A C O B I T N)$ | 9 | 18.2 TO 44.8 |

Table 1. TDY-52B/IMP-16 Instruction Repertoire (Sheet 2 of 2 )

| MNEMONIC | INSTRUCTION NAME | FUNCTION | FORMAT | EXECUTION TIME <br> IN MICROSECONDS |
| :---: | :---: | :---: | :---: | :---: |
| JUMP |  | . |  |  |
| $J M P$ | JUMP | $\mathrm{EA} \rightarrow(\mathrm{PC})$, IF INDIRECT $(\mathrm{EA}) \rightarrow(\mathrm{ACr})$ | 4A | 4.2,7.0 IF INDIRECT |
| JMPP | JUMP THROUGH POINTER | $\left(100{ }_{16}+\mathrm{N}\right)-(\mathrm{PC})$ | 9 | 9.8 |
| 3 NTT | JUMP INDIRECT TO Level 0 | $(\mathrm{PC}) \rightarrow(\mathrm{STK}), \mathrm{O} \rightarrow(\mathrm{IEF})$ | 9 | 9.8 |
| BOC | BRANCH ON CONDITION | $\left(120_{16}+\mathrm{N}\right)-\mathrm{PC}$ <br> IF CONDITION CC IS TRUE, $(P C)+D \cdot(P C)$ | 1 | 5.6,7.0 IF BRANCH |
| JSR | JUMP TO SUBROUTINE | $(P C) \rightarrow(S T K)$ | 4A | 5.6,8.4 IF INDIRECT |
| JSRI | JUMP TO SUBROUTINE IMPLED | $\begin{aligned} & E A-(P C), \text { IF INDIRECT }(E A) \rightarrow(P C) \\ & (P C) \rightarrow(S T K) \end{aligned}$ | 8 | 5.6 |
|  |  | $\mathrm{FFBO}{ }_{16}+\mathrm{C}-(\mathrm{PC})$ $(\mathrm{PC}) \rightarrow(\mathrm{STK})$ |  |  |
| - JSRP | JUMP TO SUBROUFINE THROUGH POINTER | $(P C) \rightarrow(S T K)$ | 8 | 11.2 |
|  | POINTER | $\left(100{ }_{16}+\mathrm{C}\right) \rightarrow(\mathrm{PC})$ |  |  |
| RTS | RETURN FROM SUBROUTINE | $(\mathrm{STK})+\mathrm{C}-(\mathrm{PC})$ | 8 | 5.6 |
| RTI | RETURN FROM INTERRUPT | $(5 T K)+C-(P C)$ | 8 | 7.0 |
|  |  | 1-(IEF) |  |  |
| SKIP |  |  |  |  |
| AISZ | ADD IMMEDIATE AND SKIP IF ZERO | $\begin{aligned} & (A C r)+D=\left(A C_{r}\right) \quad O V, C Y \\ & I F(A C r)=0,(P C)+1 \rightarrow(P C) \end{aligned}$ | $4 B$ | 5.6,7.0 IF SKIP |
| SKAZ | SKIP IF "AND" 15 ZERO | IF (ROI) "AND" (EA) $-0,(\mathrm{PC})+1 \rightarrow(\mathrm{PC})$ | 3 | 8.4,9.8 IF SKIP |
| 152 | INCREMENT AND SKIP IF ZERO | $\begin{aligned} & (E A)+1-(E A) \\ & I F(E A)=0,(P C)+1-(P C) \end{aligned}$ | 4 B | 9.8, 11.2 IF SKIP |
| DSZ | DECREMENT AND SKIP IF ZERO | $(E A)-1 \rightarrow(E A)$ <br> IF $(E A)=0,(P C)+1-(P C)$ | 4 B | 11.2,12.6IF SKIP |
| SKG | SKIP IF GREATER THAN | IF $(\mathrm{ACr})>(\mathrm{EA}),(\mathrm{PC})+1 \rightarrow(\mathrm{PC})$ | 2 | 11.2 TO 14.0 |
| SKNE | SKIP IF NOT EQUAL | $1 \mathrm{~F}(\mathrm{ACr}) \neq(\mathrm{EA}),(\mathrm{PC})+1 \rightarrow(\mathrm{PC})$ | 2 | 8.4 |
| SKSTF | SKIP IF STATUS FLAG TRUE | $\begin{aligned} & \text { IF }(S T A T U S ~ F L A G ~ N)=1,(P C)+1-(P C) \\ & 0 \quad(S E L) \end{aligned}$ | 9 | 18.21044 .8 |
| SKBIT | SKIP IF BIT TRUE | $\begin{aligned} & \text { IF }(A C O \text { BIT } N)=1,(P C)+1-(P C) \\ & 0 \quad(S E L) \end{aligned}$ | 9 | 18.2 TO 44.8 |
| STACK |  |  |  |  |
| PUSHF | PUSH STATUS FLAGS ONTO STACK | (SF) - (SIK) | 8 | 5,6 |
| PULLF | PULL STATUS FLAGS FROM STACK INTO FLAG REGISTER | $(S T K)-(A C r)$ | 8 | 7.0 |
| PUSH | PUSH ONTO STACK | $(\mathrm{ACr}) \rightarrow(5 T K)$ | 4 B | 4.2 |
| PULL | . PULL FROM STACK | $(5 T K) \longrightarrow\left(A C_{r}\right)$ | 4 B | 4.2 |
| XCHRS | EXCHANGE REGISTER AND STACK | $\begin{aligned} & \left(A C_{r}\right) \rightarrow(S T K) \\ & (S T K) \rightarrow(A C r) \end{aligned}$ | 4 B | 7.0 |
| INPUT/OUTPUT |  |  |  |  |
| RIN | REGISTER INPUT | $\begin{aligned} & (A C 3)+C-(I O \text { ADDR }) \\ & (I O \text { DATA }) \rightarrow(A C O) \end{aligned}$ | 8 | 9.8 |
| ROUT | REGISTER OUTPUT | $\begin{aligned} & (A C 3)+C-(1 O A D D R) \\ & (A C O) \rightarrow(I O D A T A) \end{aligned}$ | 8 | 9.8 |
| SFLG | SET FLAG | $C \rightarrow$ (IOADDR), $1 \rightarrow(C O N T R O L$ FLAG FC) | 7 | 5.6 |
| PFLG | PULSE FLAG | $C \rightarrow(1 O A D D R), I \rightarrow$ (CONTROL FLAG FC) | 7 | 5.6 |
| - ISCAN | INTERRUPT SCAN | $1 / 2$ (ACI) $\rightarrow(A C I)$ UNTIL 1 SHIFTED OUT (AC2) + NUMBER OF SHIFTS $-(A C 2)$ | 9 | 8.4 TO 100.8 |
| HALT | HALT | PROLESSOR HALTS | 8 | - |

T92170-2A

## Notation Used in Instruction Descriptions

| Notation | Meaning |
| :---: | :---: |
| $\mathrm{ACr}^{\text {r }}$ | Denotes a specific working register ( $\mathrm{ACO}, \mathrm{AC} 1, \mathrm{AC} 2$, or AC 3 ), where $r$ is the number of the accumulator referenced in the instruction. |
| AR | Denotes the address register used for addressing memory or peripheral devices. |
| cc | Denotes the 4-bit condition code value for conditional branch instructions. |
| ctl | Denotes the 7-bit control-field value for flag, input/output, and miscellaneous instructions. |
| CY | Indicates that the Carry flag is set if there is a carry due to the instruction (either an addition or a subtraction). |
| disp | Stands for displacement value and it represents an operand in a nonmemory reference instruction or an address field in a memory reference instruction. It is an 8 -bit, signed twos-complement number except when base page is referenced; in the latter case, it is unsigned. |
| dr | Denotes the number of a destination working register that is specified in the instruction-word field. The working register is limited to one of four: $\mathrm{AC} 0, \mathrm{AC1}, \mathrm{AC} 2$, or AC 3 . |
| EA | Denotes the effective address specified by the instruction directly, indirectly, or by indexing. The contents of the effective address are used during execution of an instruction. See table 3-1. |
| fc | Denotes the number of the referenced flag (see table 3-20 under 3.6.10, Input/Output, Halt, and Flag Instructions). |
| INTEN | Denotes the Interrupt Enable control flag. |
| IOREG | Denotes an input/output register in a peripheral device. |
| L | Denotes 1-bit link (L) flag. |
| OV | Indicates that the overflow flag is set if there is an overflow due to the instruction (either an addition or a subtraction). |
| PC | Denotes the program counter. During address formation, it is incremented by 1 to contain an address 1 greater than that of the instruction being executed. |
| r | Denotes the number of a working register that is specified in the instruction-word field. The working register is limited to one of four: $\mathbf{A C 0}, \mathbf{A C 1}, \mathbf{A C 2}$, or $\mathbf{A C 3}$. |
| SEL | Denotes the Select control flag. It is used to select the carry of overflow for output on the carry and overflow (CYOV) line of the CPU, and to include the link bit (L) in shift operations. |
| sr | Denotes the number of a source working register that is specified in the instruction-word field. The working register is limited to one of four: AC0, AC1, AC2, or AC3. |
| xr | When not zero, this value designates the number of the register to be used in the indexed and relative memory-addressing modes. |

Notation Used in Instruction Descriptions (Continued)

| Notation | Meaning |
| :---: | :--- |
| () | Denotes the contents of the item within the parentheses. (ACr) is read as "the contents of ACr." <br> (EA) is read as "the contents of EA." |
| $\\| l$ | Denotes "the result of." |
| $\sim$ | Indicates the logical complement (ones complement) of the value on the right-hand side of $\sim$ |
| $\rightarrow$ | Means "replaces." |
| $\sim$ | Means "is replaced by." |
| $\square$ | Appearing in the oporand field of an instruction, denotes indirect addressing. |
| $\sim$ | Denotes an AND operation. |
| $\vee$ | Denotes an OR operation. |
| $\nabla$ | Denotes an exclusive OR operation. |

## Introduction

The intent of this study was not to demonstrate the particular processor which was used but rather to demonstrate how the program was mechanized for this particular computer in such a manner that the software could be mechanized for any given computer with equivalent characteristics. The reader should have some knowledge, however, of the computer in which the equation set was mechanized in order to fully understand the work performed. An example of how the flow charts reflect the instruction repertoire of the processor is in the handling of negative quantities. The TDY-52B/IMP-16 has a logical shift only, so that when a multiplication by $1 / 16$ is to be performed it is necessary to handle negative quantities in a different manner than positive quantities where the multiplication can be performed by a shift of 4 bit positions.

In order to give a better understanding of the digital computer used in this study a summary of instructions was included in Section III. Should the reader have further interest in this processor including the interrupt system, push-pull stack operation, logic mechanization, etc., complete descriptive manuals can be obtained from Teledyne Systems Company.

## Power On Processing

Upon recognition of Power being applied to the unit, coding is performed to disregard the initial multiplexer interrupt. This is done to allow the program to become synchronized with the timing base of the multiplexer so that after the initial interrupt the timing base of approximately 104 microseconds between interrupts is established for the remaining processing. The next task is to initialize a pointer within the processor so that whenever data is generated for storage the starting point for this stor age is established.

The interrupts are then enabled so that the processor is ready to recognize interrupts from the multiplexer. Finally a transfer is made to the Main Processing Loop. As the processor used in this study turns on with Random Access Memory (RAM) set to an all one's condition, the initial iterations of the Main Processing do not have the proper data for use as a priori iteration data so that the data generated for the first two iterations of the Main Processm ing Loop is not precise.

## POWER ON INITIALIZATION



T98824

## Main Programming Loop

The Main Programming Loop is where the majority of calculations are performed. This routine is coded to run continuously, i. e. from Power On the calculation process is repeatedly performed with the only interruptions being the $X$ peak and $Y$ peak Interrupts where data from the analog to digital converter is input. Once the data has been input within either of these interrupt routines, program control is transfered back to the Main Programing Loop at the point the program was interrupted.

Essentially the processing within this software module consists of solving the equations below:

$$
\begin{aligned}
& M_{x}(\mathrm{~K})=M_{\mathrm{x}}(\mathrm{~K}-1)+\mathrm{U}_{\mathrm{x}}(\mathrm{~K})+1 / 16 \mathrm{U}_{\mathrm{x}}(\mathrm{~K}-1)-15 / 16 \mathrm{U}_{\mathrm{x}}(\mathrm{~K}-2) \\
& \mathrm{M}_{\mathrm{y}}(\mathrm{~K})=\mathrm{U}_{\mathrm{y}}(\mathrm{~K})-15 / 16 \mathrm{U}_{\mathrm{y}}(\mathrm{~K}-1) \\
& M_{\mathrm{M}}(\mathrm{~K})=\mathrm{U}_{\mathrm{x}}(\mathrm{~K})-\mathrm{U}_{\mathrm{y}}(\mathrm{~K})
\end{aligned}
$$

where:
$M_{X} \quad=\quad$ filtered digital quantity from $X$ axis pickoff
$M_{y}=$ filtered digital quantity from $Y$ axis pickoff
M. $\quad=$ total digital torque output to $\mathrm{D} / \mathrm{A}$

Index ( $k$ ) designates quantity input or calculated on current program iterations.

Index (k-1) designates quantity input or calculated on previous iteration
Index ( $k-2$ ) designates quantity input or calculated two iterations previous
The other calculations performed in this loop and shown on the flow diagrams for the Main Processing Loop is the interrogation of two software flags, X peak flag and Y peak flag, to properly correlate the data from the storage buffers and to transform the data from its 2's complement form for the sign magnitude digital to analog convertor.

Following is the flow chart for the Main Processing Loop. (Figure 10.)

MAIN PROGRAMMING LOOP


198822
Figure 10. Main Programming Loop

## MAIN PROGRAMMING LOOP



Figure 10. Main Programming Loop (Continued)

Upon recognition by the processor that sample data from the X axis pickoff is available, the Main Processing Loop is interrupted at the end of the instruction it is performing and the program is transferred to the $X$ Peak Interrupt Servicing routine. The purpose of this routine is to accept the digitized $X$ axis pickoff data for the computation to be performed by the Main Processing Loop. In order to keep the data being computed in the Main Programming Loop correlated it was necessary to mechanize two buffer areas which are merely memory cells set aside for temporary data storage and retrieval. At any given point of time one of the buffer areas is set for reading by the Main Processing Loop and the other buffer area is set for writing or storage by the X Peak Interrupt Servicing routine. The purpose of these buffers is to ensure that the data being computed is indeed from successive iterations. A memory cell is used as a flag (X Peak Flag in flow charts) to indicate how the buffers are currently being utilized. This same flag is again interrogated in the Main Processing Loop for the same purpose. A graphic illustration of the storage areas referred to as 'swinging buffers' will perhaps clarify this coding concept. At the time the X Peak Interrupt is entered by the program assume that the two buffers are as shown below

Buffer A

| $\mathrm{U}_{\mathrm{X}}(\mathrm{K})$ | $\mathrm{U}_{\mathrm{X}}(\mathrm{K}-1)$ | $\mathrm{U}_{\mathrm{X}}(\mathrm{K}-2)$ |
| :--- | :--- | :--- |

Buffer B

| $\mathrm{U}_{\mathrm{X}}(\mathrm{K})$ | $\mathrm{U}_{\mathrm{X}}(\mathrm{K}-1)$ | $\mathrm{U}_{\mathrm{X}^{(\mathrm{K}-2)}}$ |
| :--- | :--- | :--- |

and further assume that the flag (X Peak Flag) indicates that Buffer A is the area that should be used to store data. The cell from Buffer B containing UX(K-1) must first be moved to Buffer A as it now becomes two iterations old, i. e. $U_{X}(\mathrm{~K}-2)$. In the same manner the Buffer $B$ cell containing $U_{X}(K)$ is moved to Buffer A and is now one iteration old, i. e. U $\mathrm{X}(\mathrm{K}-\mathrm{I})$. The digitized X axis pickoff data input during the current iteration is then stored in Buffer A as $U_{X}(\mathrm{~K})$. Buffer A then contains data from three successive inputs. The process is reversed the next iteration such that Buffer B contains current data. The last task of the X Peak Interrupt processing is to divide the data from the previous iteration by 16 (accomplished by a right shift of 4 places) before storing to relieve the Main Processing Loop of this computational burden. Once these tasks are performed the program is returned to the Main Process ing Loop to the point at which the interrupt occurred.

Following is the flow charts for the X Peak Interrupt Processing. (Figure 11.)


198821
Figure 11. X Peak Interrupt Servicing Flow

## Y Peak Interrupt Servicing

Upon recognition by the processor that sample data from the $Y$ axis pickoff is available the Main Processing Loop is interrupted at the end of the instruction it is performing and the program is transferred to the Y Peak Inter rupt Servicing routine. The purpose of this routine is to accept the digitized $Y$ axis pickoff data for the computation to be performed by the Main Processing Loop. This data is correlated in much the same manner as previously explained in the X Peak Interrupt Servicing in that a flag (Y Peak Flag) is used to determine which data is time correlated and buffers are used to determine read and write areas. In this process the last iteration Y axis pickoff data $\mathrm{U}_{\mathrm{Y}}(\mathrm{K}-1)$ is multiplied by $15 / 16$ as a time saving step to the Main Processing Loop. Again, as in the X Peak Interrupt Processing, a return is made to the point in the Main Processing Loop where processing was interrupted.

Following is the flow charts for the Y Peak Interrupt Processing. (Figure 12.)

*-15/16 UY $(K-1)=-1+1 / 16$ UY $(K-1)$
T98820

Figure 12. Y Peak Interrupt Servicing Flow

## V. ANALOG INTERFACE

The analog interface for the IMP 16 computer consists of an $A / D$ converter, a D/A converter, and the control logic for the two converters. A block diagram of the interface is shown in Figure 13.

The A/D converter section, shown in Figure 14, consists of an input multiplex switch, a sample and hold, a 12-bit converter, and a tri-state buffer register. The input multiplex selects an input channel which may be a peak, zero, or ground input from one of the gyros. This input depends on which channel is selected by the computer and logic decode. The input is applied to the sample and hold circuit, which will hold the input signal value until the $A / D$ performs a conversion. The $A / D$ start command and clock are supplied by the control logic. The tri-state buffer holds the A/D conversion and outputs it to the CPU until the next input enable allows the data to update.

The D/A converter accepts a 13 -bit sign-magnitude word from the CPU and converts it to a + or - DC output to drive the gyro torquer amplifier. This converter consists of a $\pm$ DC reference, a 12 -bit ladder network, plus $\pm$ output buffer amplifiers. The control logic decodes the input data word and outputs a sign bit so that the proper polarity is applied to the ladder network. The control logic also applies a 12 -bit word to the ladder network to determine amplitude. The signal from the ladder network is applied to output buffer amplifiers. The control logic enables either a + or - sample switch which applies the respective output signal to the hold capacitor located on the torquer amplifier on the platform.

The control logic of the analog interface receives the 5.72 MHz CPU clock and divides it by 10 and 12 successively to provide basic square waves of 572 KHz and approximately 48 KHz . The 48 KHz clock is used to generate periodically four sequential analog select signals, each spanning an interval of 32 clock periods. Two successive analog-to-digital conversions are made during an analog select interval, one in the first half-interval and one in the second half-interval. The 48 KHz bit times of each half-interval are designated $\mathrm{T}_{0}$ through $\mathrm{T}_{15}$. Analog sampling signals are enabled during $\mathrm{T}_{1}$ of each half-interval such that an analog peak signal is sampled in the first half-interval and an anlaog zero crossing is sampled in the second halfinterval. Analog to digital conversion is accomplished during $\mathrm{T}_{2}$ and $\mathrm{T}_{3}$ of each half cycle. An interrupt request is then sent to the CPU at $T_{4}$. The converted 12 -bit analog data may be input to the computer by the command, $\mathrm{R}_{\text {IN }}{ }^{(0070)} 16$. The analog data is identified by a three-bit code which is inputted to the computer by the command, $\mathrm{R}_{\mathrm{IN}}(0078)_{16}$.


Figure 13. $A / D-D / A$ Control Block Diagram


Figure 14. A/D Converter

The CPU outputs a 13 -bit word in sign-magnitude format for conversion to analog by the command, ROUT (0078) 16 . The data word is latched in a lst rank buffer from which it is transferred to a 2nd rank buffer at bit time $\mathrm{T}_{0}$ of a continuous 8 -bit cycle of the 48 KHz square wave. The conversion is accomplished during each $\mathrm{T}_{0}$. Also, sampling of the converted output is disabled during $T_{0}$.

## VI. TEST RESULTS

## Test Description

Figures 15 through 17 illustrate the general test setup. In this test configuration, the gyro was mounted on the rate table such that the input rates would be about the gyro x axis.

The $x$ and $y$ gyro pickoff signals were fed to both the digital servo and the analog servo. The digital servo controlled only the $y$ torquer, and the analog servo controlled only the $x$ torquer.

The rate table was driven by the oscillator portion of the wave analyzer with a constant voltage applied to the rate table.

The wave analyzer was used to measure the torquer signals and pickoff signals. The $x$ axis (direct) pickoff signal was also measured in the open loop condition. The open loop and closed loop measurements were used to calculate the gains that are plotted in Figures 18 and 19.


Figure 15. Block Diagram, Test Set Up


Figure 16. Test Set-Up for Digital Compensation Loop Feasibility Study


Figure 17. Test Set-up of Converters and Interface for Digital Compensation Loop System


DATA TAKEN 8.2.74 TMW
T97307
Figure 18. Output Signal (Torquer Voltage) vs Frequency Digital-Analog Loop $\sim 100 \mathrm{~Hz}$ Bandwidth


DATA TAKEN 8.2.74 TMW
Figure 19. Error Signal (Pickoff Angle) vs Frequency Digital-Analog Loop $\sim 100 \mathrm{~Hz}$ Bandwidth

The rate table drive frequency was varied from 10 Hz to 1 KHz with 3 to 5 measurements taken per decade depending on how rapidly the data was changing with frequency.

The integral gain time constant in the digital servo was modified (software modification) and the test repeated. There were no modifications to the analog servo.

## Discussion of Test Results

The test setup described above provided a means for one-to-one comparison of the performance of the digital and analog caging loops. In order to obtain such a comparison, however, it was necessary to match the gains of the two loops since the digital computations, as described in Section 2, assumed that the gain would be adjusted on the analog level, i.e., in the power amplifier. The criterion used to match the gain (applied torque vs pickoff angle) of the digital loop to that of the analog loop was to operate each servo 6 db below its marginal stability level. Using this criterion it was found that the digital loop gain was approximately 10 db below that of the analog loop.

Frequency response data was taken for both the digital and analog loops. Figure 20 shows the output signal (i.e., the torquer voltage) response versus frequency. Figure $2 l$ shows the error signal (i. e., pickoff angle) versus frequency. These figures are actually families of responses showing variations resulting from adjusting the "integral gain time constant" in the digital loop. This time constant is directly related to the location of the zero of the digital transfer functions described in Section 2. The range or variation corresponds to varying this zero location between 123 Hz and 12.7 Hz .

It is apparent from Figure 18 that the closed loop response of the digital loop compares very favorably with that of the analog loop in terms of the torquer output versus frequency. This quantity is, of course, of primary interest as it is the ultimate measure of gyro rate.

Figure 19 indicates that the gyro pickoff angles are substantially larger for the digital loop than for the analog loop for low frequency inputs. This is attributable to the lower gain in the digital loop which was established by the gain matching criterion described above.

The primary reason that the marginal stability gain for the digital loop is lower than for the analog loop is the effective phase shift introducted by the finite computer iteration rate. Thus as the computational frequency is increased it is expected that the digital loop gain established by the gain matching procedure would be increased and the pickoff angle versus frequency response of the digital loop would more closely approximate the corresponding analog response. Unfortunately this conclusion could not readily be verified due to computer timing limitations.

The primary ramifications of the lower loop gain in the digital loop is a lowering of the bandwidth of the caging loop which would occur in a gyro with both axes mechanized digitally. Subsequent analysis indicates that the effective bandwidth of such an all-digital gyro would be approximately 25 Hz for the configuration which was tested (i.e., 1200 Hz digital processing). This corresponds closely with the 30 Hz bandwidth used in Teledyne's current analog caging loops. This bandwidth can be further increased by increasing the computational frequency and can be expected to approach the 100 Hz of the new analog design with very high speed processing.

In summary, then, the test results were in general concert with expectations and the feasibility of using digital filtering in place of conventional analog caging loops for the TDF tuned-gimbal gyroscope was demonstrated.

## VII. SUMMARY AND CONCLUSIONS

The primary objective of this program was to demonstrate the feasibility of replacing existing state-of-the-art analog gyro compensation loops with digital computations. This objective was realized during the course of the program.

A breadboard design was established in which one axis of a Teledyne tuned-gimbal TDF gyro was caged digitally while the other was caged using conventional analog electronics. The digital loop was designed analytically to closely resemble the analog loop in performance. The breadboard was subjected to various static and dynamic tests in order to establish the relative stability characteristics and frequency responses of the digital and analog loops. Several variations of the digital loop configuration were evaluated. The results were very favorable - it appears that digital caging is indeed a practical approach.

The primary limitation to the digital compensation loop approach appears to be the requirement for high processing rates. The tests which were performed indicate that the minimum processing rate for acceptable loop performance is approximately 1200 Hz . Using the general purpose IMP-16 on TDY-52B computers essentially all of the available computer time was required to perform the digital caging loop functions for a single gyro axis at this rate. (These functions include all timing, control, sampling, etc., as well as the implementation of the actual digital filters.) For this reason it appears desirable to consider a special purpose digital processor for the digital caging loop task. Such a processor could be quite simple in concept since the only arithmetic functions required to implement the digital filters which were derived during the program are adds, subtracts, and shifts.

The primary advantages of using digital caging loops in place of conventional analog loops are reduction of cost, size, weight, and power, increased reliability, and simplified maintenance, resulting from the elimination of a substantial amount of hardware from the IMU Electronics. The electronics which are eliminated using the digital technique include pickoff demodulators, caging electronics, spin supplies, clock and timing functions, and pickoff excitation generation. New functions which must be performed with the digital loop are digital-to-analog conversion and, optionally, the special purpose digital processing. Since both of these functions may be multiplexed to handle all sensor axes, the hardware required is minimal. A preliminary design analysis has indicated the hardware requirement to perform these two
functions is of the order of 5 to $20 \%$ of that required for the analog functions which they replace depending upon the level of redundancy which is employed in the system. The resulting savings in direct acquisition cost for a fail-op/ fail. op redundant strapdown navigator is projected to be approximately $\$ 4000$ per system in quantities of 150 systems.

The performance of a system using digital caging loops is expected to be virtually identical to that using conventional analog technology. This is particularly true if a special purpose processor is employed, since the processing rate can be increased beyond the 1200 Hz which was used in the demonstration. Higher rates will provide digital loop performance which matches the analog loop performance even more closely than described in this report.

Other than the limitations imposed by computational speed, the digital loop which was breadboarded during the program appears to be suitable for use in a practical system design without any significant modifications. The general design philosophy, filtering algorithms, computer word length, $A / D$ and D/A converter accuracies and resolutions, etc., which were employed in the breadboard design are all suitable for use in a production design.

## APPENDIX A

## PROGRAM LISTING

This is the assembly language gyro program as assembled on Teledyne unspecialized assemblèr TUMPA. Assembled code was punched out in special loading format on mylar teletype tape for loading via teletype into the IMP-16P. This program will also run on the TDY-52B.

P CA. LITERAL INITIAL CHARACTER
CTHES THAN CCTAL DIGIT
CECIMAL CHARACTER IN CCTAL FIELO TJ- AA!Y CHARACTERS IN FIELU ©R GAAN RELATER PRCBLEN:
NETHIUTOLFSS :PAU LINE MARR YJT IA TASLE
NACRU TUN CEEPLY NESTED

COC1 \& C/ GYPO PRCGRAM VERSION 2.7 TAPE 907
$\operatorname{COC2}$ 1.DC,20,1/
COC3 1.PCIF,2,LDE,2/
COC4 $3, P C I F, 2,57 B, 21$
COO5 3 :PCIF,2, पACO, 21
CCC6 3.PCIF,2,OSLB,2/
COC7 \&.PCIF,2,HPY,21
COCH $5, P C i F, 7,01 \mathrm{M}, 21$
COC9 5, TRAI,3,21
CO1O 1, TRAN,4,21
cold $\$$. IRAN, 3, 1/

COI3 \$, TRAE! $5,1 /$
CO14 5, ECCL, 2,90,01
CO15 s.BnCL,2, $01,1 /$
CO16 \$, BOCL, 2,R2,2
CO17 \%, BCCL, 2, 63,3 )
COl
C019 3. $\mathrm{BOCL}, 2 \mathrm{ACl}, 1$
$C O 20$. $\mathrm{BNCL}, 2, A C 2,21$
COR1 \$.EOCL,2,ACZ, I/
CC22 $\$$ QCCL, 2,3 ASE, O/
CO23 \& BOCL, 2, PC, $1 /$
CO24 4 , $\mathrm{CCCL}, 2, \times \mathrm{CK} 2,21$
C025 \$ BCCL, 2, XR3, 21
CO?6 \$.BCLL,2,XMIT,4/
C027 \& \& QCL, 2, LiNK, $17 /$
coza igetu, $2,10 \mathrm{~V}, 1 \mathrm{Al}$
CO29 \$.BCEL, $2, C Y, 15 /$

CO31 , $\mathrm{BECL}, 2,5 \mathrm{EL}, 2 /$
CO32 S.BCCL, R, REACR, 3
CO33 3. $\mathrm{CDCL}, 2$, xHIT,4
CO 34 \& $\mathrm{BCOL} \mathrm{C}, 2, \mathrm{JC14,16/}$
CO35 \& AOCL, $2, J C 1,17$
$C 036$ 2, $4 D C L, 2, J C 2,21$
COZ7 $3 . \operatorname{HOCL}, 2, \mathrm{JC} 3,3 /$

TUMPA PAtE':1."2

COB6 \$. $\mathrm{BOCL}, 2, J C 5,31$


C042 $3 . B 0 C L, 2, J C 1 C, 121$
C043 3. BOCL,2,JC11,13/
CO4 S F FEED/

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    TUMDA PAGE 3
    CO43 $,CRC,I/ 
        0OOC2 CO48 ,PUSH,ACC%
        00003 C049 ,LISAC3,0/
        000c4 CO50 ,RIN,I7C/
        OONOS COS1 XX,RCPY,ACO,AC3/
        OONOS COS1 XX,RCPY,ACO,AC
            C053 $.[RG,1U/
            CO54 S.C/X PEAK BUFFERS
        {OO10 CO55 XFD,CCT,C/ FLAGL
        00011 CO50 XF2,LCCT,C/ CELLZ
        0DO12 CO57 XP3,ECP,E/ CELL3
        OOO13 CO58 XP4,CCT,C/ CELL4
        OOOL4 CO57 XPS,CCT,C/ FL,SG2
        OOO1S COAS XOOBCCTPO/ CELLS
        OOU1E COH1 XP7,LCPIC/ CELLO
        00117 COH2 xPg,CCT,C/ CELL7
        OOO2U CO63 C1C,CCT,40OCC/
        non2l coet C14;CCT,140000/
        i)\022 COG5 MT,CCT=O%
        COOS KT:CCT=O/
```



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        00031 C069 YN2,CCT C/ CEAL
        00032 c070 yo3,CCC,C/ CELG1
        n0032 CO70 YP3,CCT,C/ CELLZ
        00033 CO71 YP4,ECT,O/ FLAGZ
        00034 CO72 YPS,CCT,C/ CELL3
        C074 $.CRG.40/
    0040 C075 XX1,RCPY,ACO,AC3/
    00041 C076 XX2,JMO,BASE,FGD/
    0004? C077 FGC,LN,ACC,BASESXXI/
    00043 C07B ,ST,ACO,BASE,XXI
    00044 CO79 ,PLLL,AC3/
    00045 CORO ,PLLL,ACO/
    10046 COH1 , R\I,01
        COR2 $.ERG.1OC/
        COB3 S,C/ BATA STCRAGE POINTER
    COIOC COH4 CSPC,NCT,4OCC/
    00JU1 CO&5 CSP,ECT,C/
    0OSU? CO&Q SIGN,NCT,10000O/
    COR7 $.C/ GYRC MAIH
    DOT03 co8E MX, ロCT,0/
    n0104 cOB9 M,OCT.OI
        COQO S.CRG,10CN/
        CO91 s.C/ X PEAK INTERRUPT SERVIEE
    O1nOL CO92 ,LI,AC3,O1
    0l001 c093
    OLOO2 CO94 ,PUSH,ACO/
    ,LD,ACC,BASE,XP1/
```

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    TUMPA PAgE 4
    01n04 coso -'AOCC#G1,XPEAK21
```



```
    0100S COG8 :ST,ACO,8ASE,XP1/
    01007C099 OLD,ACC,BASE,XP7/
    01010 C2CO SST,ACC,GASE,XP4)
    O1O11 C1O1 XPEAK4,LD,ACO,BASE,XPO/
    01012CICZ ,BCC*;2,XPEK4/
    01013 ClO3 ,CAT,ACO,1/
    OO14 ClO4 SHZ,ACO,4
    01015 C1C5 CA!,ACC1,1)
    01015C1C6 SST,ACC,BASEOXP3
    01017 Cl07 JNF#,XPEKSI
    0102: C1O8 XPEK4,SHR,ACO,4/
    01021 CIC9 ,5T,ACC,3ASE,XP3/
    01022 C110 XPEKS,PULL,ACC/
    01022 C111 ,ST,ACC,BASE,XP2/
    01024 C112 APLLL,DC3/
    01025 C1.13 ,PLUL,ACO1
    01023 0114 ,RTI,O1
    01027 C115 XPEAKR.LI,ACO.1/
    01030 C116 ,ST,ACC,OASE,XPL/
    01031 C117 ,GC,ACC,BASE,XP3/
    01032 C118 ,5T,ACC,BASE,XP&/
    01033 C119 XPEAKO,LC,ACC,BASE,XP2/
    01034 C120 ,BCC%,2,XPEAKT/
    01035 C121 OCAI.ACD.11
    01034 C122 ,SHR,ACO,41
    01037C123 CAI,ACO,1/
    0104U C124 ,ST.ACC,BASE,XPY/
    0:041 Cl25 ,JNP#,XPEAK8/
    01042 C126 XPEAK7;SNR,ACC,41
    O1C43 C127 ,ST,ACC,BASE,XP7/
    01044 C128 XPEAKB,PLLL,ACO/
    Ol045 Cl29 ,ST,ACC,BASE,XP6/
    0104A Cl30 ,PULG&AC3)
    S1O47 C131 . PCLLL,ACEI
    OInSL CI32 ,RTI,01
    c133 s,CRCS120O/
    C134 $.C/ Y PEAK INTERRUPT SERVICE
    0120) C135 YPEAK,LS,\triangleC3,0/
    01201 C136 ,RIN,160%
    01702 C137 ,PLSH,ACC
    01203 C136 OLE,ACC,BASE,YP1
    01204 C129 ,8CC#,1,YPEAKK/
    01205C140 SL1,ACC,O1
    n1206 C141 -ST,ACC,BASE,YOI/
    01207 C142 OLE,ACC,BASE,YPS/
    01215C143 ,BCC*,2,YPEAK3/
    01?11C144 eCAI,ACO.1/
    01712 C145 ,SHR,ACD,4)
    01213 C146 ,CAI,ACO,41
```

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    Tumpa page
    O1Z14 C147 ,SLO,ACOSBASESYPS/
    01\geqslant15C14日 ,ST,ACC,BASE,YP3/
    O1广1SC149 ,JNP|,YPEAK4/
    01717C150 YPEAK3,5HR,\triangleCO,4/
    01220 C151 ,SUB,ACO,BASE,YD5/
    01221 C152 ,Si,ACC,BASE,Yp3/
    01>22 C153 YPEAK4,PLLL,ACO
    01223 C154 ,ST,ACC,BASE,YPZ/
    21724 C155 ,PLLLAAC3/
    01725 C156 sPLLL,ACO
    01226 C157 ,晈,0/
    O1727 C158 YPEAK2,LI,ACC,1/
    0123C C1S9 ST,ACC,HASF,YPI%
    N1F31 C1&O ,LC,ACC,BASE,YP2/
    0173? C161 ,BCC&;2,YPEAKSI
    01233C1E2 ,EA1,ACC,1/
    O1734 C163 ;SHR,AC',A/
    O1735 C164 ,CAI,ACO.1/
    01236 C165 - -HNA-ACO,YO24
    ,SUB,AC\varnothing,BASE,YPZ
    0124C C167 JNPG,YPEAKG/
    OL241 C168 YPEAKS,SHR,ACO,41
    O1242 C169 ,SLP,ACO,GASE,YP2/
    01743 C170 ,ST,ACC,BASE,YP6/
    01744 C171 YPEAKG,PLLL,ACO/
    OL245 CI72 ,ST,ACC,BASE,YPG/
    01245 C173 ,PLLLBAC3
    01247C174 PLLLAACO
    0125C C175 ,RTt!OI
        C176 IORG,15CO/
        C177 $,C/ DATA STCRAGE INTERRUPY SERVICE
        O1SCC C17.B OS,LC,ACC,UASE,OSD/
        015C1 C179: SHR,ACC,14/
        01502 C190
        01503 C1H1 ,HO,ACO,BASE,XPL/
        01504 C1A2 ,B0C#,1,OS2I
        01SOSCIR3 OLC,ACE,SASE,XPG/
    01306 C184 ,JNF#,LS3/
xuTVIO yood 40
    01507 C1ES DS2,LI,ACO,BASE,XP2/
    O151C C1B6 OS3,LD,AC3,AASE,OSP/
    0151! C187 ,ST,ACC,AC3,O/
    01512 C10日 ILIACC.1/
    OIS13C1A9 ,RADO,ACC,AC3/
    01514 C190 /LD,ACC,BASESYPI/
    01515C151 ,BEC#,1,054/
    01516 C192 LC,ACC,BASE,YPS/
    01511 C193 JNOH,CS5/
    0152C C194 0S4,BR,ACO,BASE,YP2/
    01521 C195 CS5,5Y,ACO,AC3,O/
    O322 C190 ,LI,ACC,1/
    01523C1&7 ,RADO,ACO,AC3/
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        pumpa page
            02051 C24% ,ADOSACO,BASE,YPS/
            O2OS2 C250 ,AODAACD,AASESYDO/
            02093 C251 ,ST,ACO,BASE,H/
            72054 C252 ,JNP*,NAINT/
    O2035 C253 MAING,LI,ACO,O1
    O2155 C254 ,AOQ,ACD,BASE;YP2I
    N2057C255 ,AED,ACD,BASE,YP3/
    O2O6C C256 ,5T,\triangleCC,BASEPAI
    02061 C257 MAINTPLDPACCPDASEPHX/
    O2062 C258 ,SLH,ACO,BASE,M,
    02063-0253 ST,ACCODASEM/
    02064 C28O MAIN1U,PCC*,2,MAIN1I/
    02063 C2a1,CA1,ACO,1/
    02n64 C202,OR,ACO,BNSE,SIGN/
    02C67 CZES NAIN11,LI,AC3,01
    02074 C204 ,RCUT,17C/
    n2071 C2G5 ,5T,ACO,BASE,HT/
    12072 C266 ,JMP*,RET/
        C2AT $,C/ FROCRAM SECTION CUMP
    0207? C2OB OUMP,LD,AC3,BASE,OSPCI
    0207% C269 CP2,RCPY,AC3,ACO/
    02075 C270 ,SHR,ACO,141
    02076 C271 ,BCC*,3,014%
    02077 C272 ,LC,ACO,AC3,O/
    O2I00 C273 ,SHR,ACO,ICI
    02101 C274 ,LC*:AC2,PUTC./
    02i0) C275 ,JSA;AC2,CI
    02103 C276 ,LE,ACC,AC3,O
    02104 C27% ,ANB*,ACO,N,SSK
    02105 C278 SlC*,AC2,PLTC.1
    02104 C279 ,JSR,AC2,O1
    12107 C29D OLI,ACC,1/
    J211C C2PI ,RAUOSACO,AC3/
    02111 C2A2 ,JNP*)CP21
    02i12 C283 PUTC.,OCT,71131/
    0211: &234 MASK,[ICT,377%
        C2日5 S.ENC/
        C2g6 $.FEEOI
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        tumida bige g
cruss rfffrence Yable listing
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline C10 & 00020 & c2c44 & & & & & & \\
\hline \(\mathrm{Cl4}\) & 00021 & C2C37 & & & & & & \\
\hline DP2 & 02074 & c2076 & 02111 & & & & & \\
\hline 0.5 & 01500 & & & & & & & \\
\hline 052 & 01509 & C1504 & & & & & & \\
\hline 053 & 0151. & C 1506 & & & & & & \\
\hline 054 & 0152 2 & C1513 & & & & & & \\
\hline 053 & 01521 & c1517 & & & & & & \\
\hline DSS & 01531 & C1502 & & & & & & \\
\hline DSP & coiol & c1509． & 01510 & 01530 & 02004 & & & \\
\hline OSPC & 00108 & 22003 & 02073 & & & & & \\
\hline OLMP & 02073 & & & & & & & \\
\hline FGI & 00042 & coc4 1 & & & & & & \\
\hline M & 00104 & c2c53 & 02060 & 02062 & 02063 & & & \\
\hline MaI＇s & 02000 & & & & & & & \\
\hline Mal：20 & 02064 & & & & & & & \\
\hline HAIAli & ． 2067 & C2064 & & & & & & \\
\hline HAI．v2 & 02023 & C2011 & & & & & & \\
\hline MAIV3 & 02033 & C2C22 & & & & & & \\
\hline Halit & 02042 & c2c33 & & & & & & \\
\hline MALS5 & 02046 & C2C36 & 02041 & 02043 & & & & \\
\hline Mas \({ }^{\text {a }}\) & 1）2059 & C2：47 & & & & & & \\
\hline MA1：7 & 02061 & C2C54 & & & & & & \\
\hline MAS＊ & ก2il？ & C2104 & & & & & & \\
\hline MT & 30022 & c1524 & 02071 & & & & & \\
\hline Mx & 00193 & C2C06 & 02020 & 02021 & 02031 & 0203202040 & 02045 & 02061 \\
\hline PLTC． & 02112 & C2101 & 021 cs & & & & & \\
\hline RET & n2010 & C2C72 & & & & & & \\
\hline 516.5 & 0010？ & c2066 & & & & & & \\
\hline XPI & 03010 & c1003 & \(010 ¢ 6\) & 01030 & C1503 & 02010 & & \\
\hline XP2 & 00011 & C1c23 & 01033 & 01507 & 02027 & & & \\
\hline XP3 & 00012 & Clci6 & c1021 & 01031 & C2030 & & & \\
\hline XP4 & 00013 & clcio & 02023 & 02026 & & & & \\
\hline XP5 & 00014 & & & & & & & \\
\hline \(\times \mathrm{XPO}\) & nonls & clcll & 01043 & 01505 & 02016 & & & \\
\hline \(\times \mathrm{P} 7\) & 00016 & c1007 & 01040 & 01043 & 02017 & & & \\
\hline \(\times \mathrm{PB}\) & nonl7 & ClC32 & 02012 & 02015 & & & & \\
\hline XPEAK2 & 01027 & Clco4 & & & & & & 1 \\
\hline XPEAK4 & 01011 & & & & & & & \\
\hline PPEAKG & 2103？ & & & & & ， & & 0 \\
\hline XPEAKT & 01042 & C1034 & & & & & & \\
\hline XPEAK日 & 01044 & C1C41 & & & & & & \\
\hline XPEX4 & 0102 J & C1012 & & & & & & 或 \\
\hline XPEVS & 01622 & c1017 & & & & & & 比 \\
\hline
\end{tabular}
xpavj 01022 clol
xx 0nnos co043 02001
xx1 0004: coc42
x>2 nounl c2000
YP1 OOn3C C120. 012CS 01230 01514 02046
YP2 00031 c1223 01231 01236 01242 01520 02056
YP3 00032 C1215 01221 02057
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## Pumpa page 10

YP4 - 00033 !
Y05 $00034 \quad C 12070121401220012450151602051$
YP6 20039612370124302052
YoEAK Olzog
YREIK2 GIJ27 C1204
YOEAK3 O1717C1210
YPEAK4 O1>22 C1216
YPEAK5 01?41 01232
YPEAK6 01244 C1240
C2B8 3.FEEO/

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tumpa page 12
C2Ag s.c/ THESE CAROS SPECIFY WORD ASSEmbly
C290 \$.IF,2,LCI
C2S1 \$SETC,6,101
C292 \$.JUNP,A/
C293 3.1F,2:LI/
C294 \$.SETC,5,23/
C205 \$. JUND, A/
C296 s.IF,2,LIN/
C297 \$.SETC,5,231
C276 \$, SETC, 6, C/
C259 8,51;8,6,4,41
C3C0 \$.JUNP, B/
C3Cl S.IF,2,LCII
C3C2 S.SETC, \(6,11 /\)
C3C3 \$.JIJNP, A,
63C4 \$.IF,2,1001
C3C5 5 , SETC,5,2300/
C3c6 \$. गJyp,k/
C3C7 3.1F,2,ST/
C30 S.SE1C, G:121
C3CP *.JUND,A/
C310 b, 1F,2,ST11
C311 5 .SEIC.6.13/
\(C 312\) 4.JU! \({ }^{4}\) P, A/
C313 3.1F,2,5191
C314 \$, SETC,5,232C/
C315 \$.JUKP,K/
C 316 S.IF,Z,RXCH/
C317 \$.SETC,5,34/
C318 5.JUNP, O/
C319 \$.IF,2:RCPY/
C320 \$.SETC.5.351
C321 5.JUND:
C322 ©IF,2,PLSH/
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C323 3.5ETC.4,201
C324 \$. JUNP, Cl
C325 5. [F,2, PLLL/
C326 3.SEFC,4,21/
C327 .JUND,C1
C328 S.IF,2:XCHRS/
C329 3.SETC,4,25/
C330 E.JUNP, C/
C331 S.IF:2,PLSNF/
C332 S.SETC,3,1/
C333 \$.JUNP, L
C334 \$.1F,2,PLLLF/
C335 SSEYC,3,51
C3き6 *. JUNロ, L/
0337 5.1F,2, ACD/
C338 \$.SETC,0,14/
C329 S. JUf, P, A/
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    Timpa päge iz
    C340 3.1F,2,00DO%
    C341 ,SETG,5,224C/
    C342 s.J!prgik
    C343 s.1F,2,CA1/
    C344 3,SETC,5,24/
    C345 S.JUND,R1
    C346 S,IF,2,RAND/
    C347 S:SETC,5,301
    C348 .JUND,D/
    C349 B.IF,?.SLB/
    C350 S.SETC,6,151
    C351 S.JUNP,A1
    C351 3.JUNP,A/
    C352 $.1F,2,N5UB/
    C333 3.SEIC,5,2260/
    C354 $.JUNP,K)
    C355 :.IF,2,HFM/
    C35% S.SETC.5:2200/
    C357 $.JUNP,K/
    C358 $.1F,2,01V/
    C359 S.SETC,5,22201
    C360 $.JUNPOK/
    C3S1 S.IF,2,ANO/
    C362 {,SETC:6,141
    C363 $.JUNP,G1
    C3C4 S.IF,2,RAND/
    C3AS S.SETC,5,37/
    C3AO $,JUNP,O\
    6367 $,1f,2,0R/
    *368 S.SETC,6,19/
    C3A9 S.JUNP,G/
    C370 s.IF,2.RXOR/
    0371 1.SFTC.5.301
    *372 **FFS
    .JUNP,N/
    2373 $.1F,2,SETEIT,3/
    2374 $,SETC,4,1621.
    375 &.JUNP,J/
    376 %.IF,2,CLRBIT,4/
    3377 3.SETC,4,103/
    378 1. LLND,J/
    :379 &.1F,2,CHPE!P,13/
    \3A0 $,SETC,4,166/
    C3Al &.JUND,J/
    S3@2 3.JF,2,SETST/
    03.3 S.SETC.4,160/
    0394 $.JurP,J/
    3A5 S,IF,2,CLRST/
    3AG $.SETC,4;161/
    C3A7 %.JUKA,J/
    :388 %.1F,R,A1521
    349 $.5FTC,5,221
    330 $.JUNP,R,
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    Tumpa Page it
        C595 $.NGRD;1/
    C596 $.RET/
    C307 &.SPEPjE/
    C59B &:NQVE;5,1,1,13/
    C558 &.NGVE;5,1,1,13/
    C5S9 {.MQVE,5,0,1,7/
    Csco s,NOVE;3,0,3,10/
    C601 5,NCVE,4,0,7,C1
    C6C2 S.hNRD,1/
    C&C3 s.HET/
    COC4 S.SPCT,F/
    CSC5 S.NDVE,5,0,4,14/
    CSC5 3,NDVE,5,0,4,141
    C6CO $.NOVE,3,0,4,101
    CoC7 S,NCVE,4,T,10,0%
    cscb s,wCRi),1/
    CBCO B.RET/
    colo $.SPCT,G/
    C611 S.MGVE,6,0,5,131
    Coll :MQVE,6,0,5,13/
    CS13 &.MCVE,4,7,2,101
    CS13 &,MEVE,4,0,2,10/
    C615 $,NCRD,1/
    C616 S.RET/
    C617 S.SPCT,H/
    C618 3.NQVE,4,0,4,71
    C619 &NMVE4,jo4,7/
    C19 :NTVE,3,0,7,01
    C2O 3.hDRO,
    C621 B.RET/
    CO22 S.5POT,J/
    6623 S.MOVE,4,0,14,41
    62,4 s,NOYE,3,0,4,0
    625 s.WTRT],1/
    Cb26 $, REY/
    &627 $.SPOT,K/
    C620 S:MOVE,5,0,20,01
    CO27 1.NOVE,3,0,2,101
    C630 s.hORT,1/
    631 3.NCVE,4,0,20,0)
    26?2 s.WERD,1/
    C633 GRET/
    6634 $.5POT,L/
    =635 $,FOVE,3.0.4,9/1
    6630 3.WORI,I/
    Ch37 $.RFT/
    C638 $.5PCT,AFC/
    C639 $.CPC,6/
    C$40 $.SFTC.7,1/
    6641 $.NOVF,5,0,4,141
    C642 s.MnvF,3,0,2,121
    6043 {.NCVE,7,0,1,10/
    8044 5.ADUP5,7,7%
    C645 3.SUB,4,7,41
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TLMPA PAGE IB
\(\operatorname{C640}\) ，MQVE， \(4,0,10,01\)
C647 \＄．hOKD．1
CO48 8．RETI
C649 S．SPCT，APC／
C6SO
Cosi \＆．SETC，\(t, 1 /\)
Cose s．MOVE，4， \(6,6,121\)
C653 3 ，पUVE，6， \(0,2,101\)
C654 \＄，ADC，5，6，61
C655 3．SU的3， \(\mathrm{C}, 31\)
COS6 3，KOVE，3，0，10，01
C6S7＊WOKi） \(1 /\)
C6S日 \＄．RET／
C059 S．SPCT，FPC／
COED ：CPC，OI
C661 S．SETC，7，1／
C6t2 2 NOVE，5，3，4，141
C663 8，NOVE，3， \(0,4,101\)
C5E4 3，APC，3，7，7／
\(\cos 5\) \＄．5Ud，4，7＞41
C6月O ，MCYE，4，3，10，01
S667 S．W02M，1／
SGB8 S．RET／
0669 \＄．SPCTAGPC／
＊670 \(\$\) ．CPC，61
－671 \＄．SETC，7，1
：672 \＄，NDVF，5，0，5，131
\(=673\) 3．MDVE， \(3,0,1\) 121
－674 \＄，NOVE，7， \(0,2,101\)
：675 \＄，AOU，6，7．71
9676 ，5以世，4，7，41
6677 3．NOVE，4，1） 10,07
C678 \＄．hURT， \(1 /\)
1.679 ．RET।
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LGAN SSDCTSHPC
LGOL \(\$ . C P C, 3 /\)
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：643 \＄，N0VE，4，3，11，7／
COR4 \＄，\(\triangle O C, 5,6,61\)
r．685 ，SUu，3，6，31
（696 3，MnVE，3，i，7，01
COR7 WWCMCII
CGRB \＄．RET／
COR9 \＆．ASN／
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figiza page lo
ASSEMGLEO NACFINE LANGLAGE LISTING

| 000001 | $00 \operatorname{cococ} 40 \cos$ |
| :---: | :---: |
| CCOOOS | yocecoc 2140 C |
| 000914 | 000 Cc 000000 |
| 000:21 | OOBCCO140coc |
| 000)33 | resecoosonoc |
| 000342 | cocesoloocto |
| 000100 | $00: \operatorname{cococ} 4 \mathrm{COO}$ |
| $091: 001$ | $00 \sim c \cos 47400$ |
| 001105 | OOCCCOC4 ${ }^{\circ} 00$ |
| 001112 | nccecocilcos |
| 001917 | nouccoc2c402 |
| 001324 | OSCCCOC434CO |
| OC1:31 | 00.ccolocc12 |
| 001935 | 00000005374 |
| $001: 43$ | notecol20cic |
| 001059 | 00cccooosano |
| 001204 | 006ccocio422 |
| 001211 | covecos50001. |
| 001215 | 00:cccou20403 |
| 001723 | OOLCCOL20032 |
| 001230 | 00iocol2ce30 |
| $001 ? 35$ | 0 OCCOO50001 |
| 0 cl 1242 |  |
| 001247 | 0 ccccos 200 c |
| 0 Cl 153 | 00cccoloocio |
| 001510 | ODCCCOLO6101 |
| 001515 | DCuCCOC10402 |
| OC152? | 00:ccocteocl |
| 001527 | 00こCこOC31400 |
| 002903 | OOECCol00041 |
| 002.03 | O2ECOOC4B200 |
| 002712 | O0.j0:0100217 |
| 002317 | OOSCU0140016 |
| 002124 | cciciocsoris |
| 002131 | $00 . \mathrm{C} 01410103$ |
| 0023.35 | 00:c 00.15 cc |
| OC2)43 | $0 \mathrm{COC}=0011002$ |
| 002:5) | n0. $0=0046000$ |
| 00235 | 00sccoutheno |
| 002:32 | Onic:015cte4 |
| 002167 | 0ucide04749C |
| 002174 |  |
| 002101 | 00:C:0104410 |
| $00^{2104}$ | 00.1c:0c23:00 |
|  |  |

## APPENDIX B

## LIST OF SYMBOLS

| A | Gyro rotor moment of inertia normal to spin axis |
| :---: | :---: |
| $\mathrm{F}_{1}(\mathrm{~s}), \mathrm{F}_{2}(\mathrm{~s})$ | Direct-axis and cross-axis analog transfer functions |
| $F_{1}(z), F_{2}(z)$ | Direct-axis and cross-axis digital transfer functions |
| $G_{1}(z), G_{2}(z)$ | Mechanized direct-axis and cross-axis digital transfer functions |
| $\mathrm{H}(\mathrm{s})$ | General analog transfer function |
| H(z) | General digital transfer function |
| $\mathrm{H}^{\prime}(\mathrm{s})$ | Dummy analog transfer function |
| k, K | Iteration index |
| $\mathrm{K}_{\mathrm{PO}}$ | Pickoff gain factor |
| $\mathrm{K}_{\text {SA }}$ | Compensation gain factor |
| $\mathrm{K}_{\mathrm{T}}$ | Torquer gain factor |
| M (k) | Total digital torque |
| $M_{x}, M_{y}$ | Components of computer digital torque resulting from X - and Y - pickoff signals |
| $M_{X}, M_{Y}$ | Analog rebalance torques |
| N | Gyro spin speed |
| $P_{X}, P_{Y}$ | Precessional torques |
| s, S | Laplace Operator |
| $\mathrm{T}_{\mathrm{X}}, \mathrm{T}_{\mathrm{Y}}$ | Total torques |
| $\mathrm{U}_{\mathrm{X}}, \mathrm{U}_{\mathrm{Y}}$ | Digitized gyro pickoff angles |
| $X_{\text {peak }}$ | Sampled X-axis pickoff angle |
| $Y_{\text {peak }}$ | Sampled Y-axis pickoff angle |

$z^{-1}$ Delay operator$\theta_{\mathrm{x}}, \theta_{\mathrm{y}}$
$\theta_{\mathrm{X}}, \theta_{\mathrm{Y}}$

## $\tau$

$\omega_{\mathrm{X}}, \omega_{\mathrm{Y}}$$\omega_{i}, i=1,2 \ldots$$\omega_{i}^{\prime}, i=1,2 \ldots$
Digitized gyro pickoff angles
Gyro pickoff angles
Iteration period
Angular rate inputs
Analog filter break frequencies
Dummy analog filter break frequencies


[^0]:    *The final report of this study was published as NASA CR-132419

[^1]:    *Rabiner, L. R. and C. M. Rader, Digital Signal Processing, IEEE Press, New York, 1972.

