GEOTHERMAL RESERVOIR SIMULATION

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The prediction of long-term geothermal reservoir performance and the environmental impact of exploiting this resource are two important problems associated with the utilization of geothermal energy for power production. Our research effort addresses these problems through numerical simulation. Computer codes based on the solution of partial-differential equations using finite-element techniques are being prepared to simulate multiphase energy transport, energy transport in fractured porous reservoirs, well bore phenomena, and subsidence.

I. INTRODUCTION

Geothermal reservoir simulation, in general, refers to reproducing and predicting the behavior of a geothermal reservoir by use of a model. In this discussion we consider only mathematical models which we define as: (1) a set of equations that describe the physical processes active in a geothermal reservoir, and (2) the solution of these equations subject to boundary and initial conditions. Because of the complex nature of the equations and boundary conditions, the solution generally involves numerical techniques in conjunction with the digital computer.

One simulates a geothermal reservoir to estimate the quantity of recoverable energy and the rate at which mass and energy may be extracted. To achieve these objectives, known geological information obtained from both surface techniques and drilling is utilized to determine equation parameters, boundary, and initial conditions. During the earlier stages of field development, this information is limited, and the simulation model will undoubtedly be crude. Through the simulation of a variety of producing schemes, however, the energy available and its rate of extraction may be roughly estimated. If the decision is made to develop the field further, the reservoir model may be used to help answer such engineering questions as optimal well location and spacing, or whether or not to reinject condensate. As more geological information becomes available through continued drilling, the simulation model may be updated to give a more accurate analysis of the geothermal reservoir.

A. Previous Investigation

The literature on the simulation of both heat and fluid flow in porous media is extensive. Much of the literature is concerned with the extraction of petroleum by in situ combustion or hot-fluid injection (e.g., Refs. 1 and 2). Another area of interest is that associated with free convection models (see, for example, Refs. 3 and 4). If, however, only geothermal reservoir simulation is considered, the literature is relatively limited.

Whiting and Ramey (Ref. 5) were the first to apply a reservoir model to a geothermal system. Their model allowed for two-phase (steam-water) flow, but did not take into consideration the spatial dependence of the solution (it was a lumped parameter model). Harlow and Pracht (Ref. 6) developed a single phase (hot-water) model coupled with a rock-fracture model. They attempted to demonstrate that geothermal energy could be extracted from hot dry rocks. Mercer (Ref. 7) applied a hot-water model to the Wairakei hydrothermal system. The model was limited spatially to the two horizontal dimensions and was unable to reproduce historical data past 1962 due to the formation of steam in the reservoir. Brigham and Morrow (Ref. 8), in an attempt to allow some spatial variation in their solution, developed three lumped-parameter models based on vapor liquid distribution. Toronyi (Ref. 9) developed a two-dimensional (areal or cross-section) reservoir model coupled with a wellbore model. His reservoir model allowed only two-phase flow, being restricted to the saturated-vapor pressure curve.

B. A General Simulator

The above models could be thought of as contributions toward a general model capable of completely describing the response of a reservoir to exploitation. The development of such a model constitutes the principal objective of our current research effort.

The physical properties of the fluid in a geothermal reservoir vary spatially and temporally to the extent that vertical velocities are often of the same order of magnitude as horizontal velocities. Consequently, a general model must be a distributed parameter model with the capability of three-dimensional simulation.

To answer important engineering questions, a general model must not only accommodate the transient flow of compressed hot water, steam-water mixtures, and super-heated steam, but also must allow for phase changes (see Fig. 1). Presently we are neglecting species transport and chemical reactions that may occur in the geothermal system.

Subsidence of the land surface is often encountered when large quantities of fluid are removed from subsurface reservoirs. Consequently, a general model should have the capability of forecasting the spatial and temporal distribution of subsidence resulting from geothermal development. Such a simulator must solve not only the fluid flow and energy transport equations but also the equations describing the elastic behavior of the reservoir skeleton.

The governing equations employed in reservoir simulation are generally based on the physics of flow through porous media. While this approach may hold in selected cases, there are many reservoirs that are productive only because large fractures are encountered during drilling. To simulate such a fractured reservoir, the classical continuum approach to porous media flow must be modified and extended to accommodate the influence of fracture flow.

While we are currently developing models which will include each of the important capabilities described above, we cannot herein describe all aspects of our research program. Consequently, we have elected to focus our attention on multi-phase energy transport, a topic which should be of interest to anyone contemplating the development of a geothermal reservoir simulation capability.

II.   THEORETICAL DEVELOPMENT

The general governing equations consist of mass, momentum, and energy balances for each phase present in a geothermal system.

A.   Mass Balance

The mass balances for steam, s, and water, w, may be written as:

$$- \frac{\partial}{\partial x_i} (\bar{v}_s \rho_s) + q_s + d_v = \frac{\partial (\phi S_s \rho_s)}{\partial t} \tag{1}$$

and

$$- \frac{\partial}{\partial x_i} (\bar{v}_w \rho_w) + q_w - d_v = \frac{\partial (\phi S_w \rho_w)}{\partial t} \tag{2}$$

where

$\bar{v}$ = phase average velocity, $[Lt^{-1}]$

ρ = average density, $[ML^{-3}]$

q = source term, $[ML^{-3}t^{-1}]$

d_v = rate of vaporization, $[ML^{-3}t^{-1}]$

ϕ = porosity, dimensionless

S = saturation, dimensionless, where $S_w + S_s = 1$

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B. Momentum Balance

For velocity, we assume that Darcy's equation for multiphase flow may be used (see Ref. 10):

$$\bar{v}_s = - \frac{\bar{\bar{k}} k_{rs}}{\mu_s} \left(\frac{\partial p_s}{\partial x_j} - \rho_s \bar{g} \right) \tag{3}$$

and

$$\bar{v}_w = - \frac{\bar{\bar{k}} k_{rw}}{\mu_w} \left(\frac{\partial p_w}{\partial x_j} - \rho_w \bar{g} \right) \tag{4}$$

where

$\bar{\bar{k}}$ = local intrinsic permeability tensor, $[L^2]$

k_r = relative permeability, dimensionless

μ = dynamic viscosity, $[ML^{-1}t^{-1}]$

p = pressure, $[ML^{-1}t^{-2}]$

$\bar{g}$ = gravitational acceleration, $[Lt^{-2}]$

Combining Eqs. (1) with (3) and (2) with (4) gives

$$\frac{\partial}{\partial x_i} \left\{ \frac{\bar{\bar{k}} k_{rs} \rho_s}{\mu_s} \left(\frac{\partial p_s}{\partial x_j} - \rho_s \bar{g} \right) \right\} + q_s + d_v = \frac{\partial}{\partial t} (\phi S_s \rho_s) \tag{5}$$

and

$$\frac{\partial}{\partial x_i} \left\{ \frac{\bar{\bar{k}} k_{rw} \rho_w}{\mu_w} \left(\frac{\partial p_w}{\partial x_j} - \rho_w \bar{g} \right) \right\} + q_w - d_v = \frac{\partial}{\partial t} (\phi S_w \rho_w) \tag{6}$$

These equations describe the flow of steam and water in a porous medium where a pressure difference exists between the two phases. This pressure difference is generally defined as capillary pressure, p_c, where

$$p_c = p_s - p_w \tag{7}$$

Probably the most important effect that capillary pressure has in a geothermal reservoir is to lower the vapor-pressure curve. Ramey, et al. (Ref. 11) point out that the reason for the lowering of the vapor-pressure curve is that vapor-pressure data found in steam tables (Refs. 12 and 13) are based on flat steam-water interfaces, whereas capillary pressure causes the interface in porous media to be curved. The amount the vapor-pressure curve is lowered in a geothermal reservoir is not completely understood. The work of Calhoun, et al. (Ref. 14), on consolidated rock does show a lowering of the vapor-pressure curve with decreased fluid saturation. The efforts of Cady (Ref. 15) and Bilhartz (Ref. 16), however, indicate no significant vapor pressure lowering in their experiments using unconsolidated sands. An important difference in the two results is that the experiments of Calhoun, et al., were made at a temperature of 36°C, and those conducted by Cady and Bilhartz ranged from approximately 121 to 240°C. Further work on the importance of capillary pressure in geothermal reservoirs is required. In this paper, capillary pressure is assumed to be negligible. Under this assumption Eqs. (5) and (6) combine to reduce the number of equations and unknown parameters by one:

$$\frac{\partial}{\partial x_i} \left\{ \frac{\bar{\bar{k}} k_{rs} \rho_s}{\mu_s} \left(\frac{\partial p}{\partial x_j} - \rho_s \bar{g} \right) \right\} + \frac{\partial}{\partial x_i} \left\{ \frac{\bar{\bar{k}} k_{rw} \rho_w}{\mu_w} \left(\frac{\partial p}{\partial x_j} - \rho_w \bar{g} \right) \right\} + q_s + q_w = \frac{\partial(\phi \rho)}{\partial t} \tag{8}$$

where ρ is the density of the total steam water mixture, defined as:

$$\rho = \rho_s S_s + \rho_w S_w \tag{9}$$

Note that in combining Eqs. (5) and (6) the vaporization term, d_v, has also been eliminated.

C. Energy Balance

Three energy balances, analogous to the mass balances written for steam and water, can be written for steam, water, and rock. If it is assumed that the movement of fluid (water and steam) through porous media is sufficiently slow, and the surface areas of all phases concerned are sufficiently large, then thermal equilibrium exists between the rock, steam, and water. This assumption reduces the three energy-balance equations to the following one:

$$-\frac{\partial}{\partial x_i}(\bar{v}_s \rho_s h_s + \bar{v}_w \rho_w h_w) + q_w h_w + q_s h_s + \frac{\partial}{\partial x_i}\left(\bar{\bar{K}}_m \frac{\partial T}{\partial x_j}\right)$$

$$= \frac{\partial}{\partial t}[\phi \rho h + (1 - \phi)\rho_r h_r] - \frac{Dp^*}{Dt} \tag{10}$$

where

$$\frac{Dp^*}{Dt} = \frac{\partial \phi p}{\partial t} + (\bar{v}_s + \bar{v}_w)\frac{\partial p}{\partial x_i} \tag{11}$$

The term described by Eq. (11) represents the compressible work term, and is often assumed negligible and omitted from the energy balance. Other definitions of terms in Eq. (10) include:

h_s = enthalpy of saturated steam, $[L^2 t^{-2}]$

h_w = enthalpy of saturated water, $[L^2 t^{-2}]$

T = temperature, $[T]$

$\bar{\bar{K}}_m$ = thermal dispersion tensor for the medium $[MLt^{-3}T^{-1}]$

ρ_r = average rock density, $[ML^{-3}]$

h_r = rock enthalpy, $[L^2 t^{-2}]$

h = total enthalpy of the mixture, as defined by

$$h = \frac{S_s \rho_s h_s + S_w \rho_w h_w}{\rho} \tag{12}$$

D.   Assumptions

By making the assumptions of thermal equilibrium and negligible capillary pressure, we are left with two equations. We must obtain additional relationships for the remaining dependent variables as functions of these two.

The two dependent variables specified as unknowns must uniquely define the thermodynamic state of the system. Further, it is desirable that these variables be commonly measured in a field situation. For these reasons, we

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choose to put Eqs. (8) and (10) in terms of the dependent variables h, the total enthalpy of the mixture, and p, the fluid pressure. To do this we make the following assumptions:

(1) Porosity, ϕ, is a function of pressure alone.

(2) Rock density, ρ_r, may be treated as constant.

(3) Total density, ρ, steam density, ρ_s, and water density, ρ_w, are considered functions of pressure and enthalpy.

(4) Steam enthalpy, h_s, and water enthalpy, h_w, can be treated as functions of pressure.

(5) Temperature, T, can be treated as a function of pressure in the two-phase region, and can be treated as a function of pressure and enthalpy in the single-phase region.

(6) Rock enthalpy, h_r, can be considered as a function of temperature.

(7) Viscosities, μ_s and μ_w, are considered functions of temperature.

(8) Saturations, S_w and S_s, are functions of enthalpy and pressure.

(9) Relative permeability can be treated as a function of saturation alone and relationships similar to those in Brooks and Corey (Ref. 17) may be used.

(10) Thermal dispersion tensor can be generalized and treated as a property of the medium.

Ramey, et al. (Ref. 18), point out that relative permeability can also be a function of temperature. Using unconsolidated sand and working with oil and water, Poston, et al. (Ref. 19), observed that for increased temperatures, the relative permeability curves shift to the left on the saturation axis. Regarding the thermal dispersion tensor it should be noted that Mercer (Ref. 7) separates the medium thermal dispersion tensor into three parts: conduction in the solid phase, diffusion in the liquid phase, and a velocity-dependent dispersion in the liquid phase. Further, Somerton, et al. (Ref. 20), point out that the thermal conductivity of the medium is a function of temperature, porosity, and water saturation.

E. Final Equations

Making the above assumptions, Eqs. (8) and (10) become:

$$\frac{\partial}{\partial x_i}\left\{\frac{\bar{\bar{k}}k_{rs}\rho_s}{\mu_s}\left(\frac{\partial p}{\partial x_j} - \rho_s\bar{g}\right)\right\}$$

$$+ \frac{\partial}{\partial x_i}\left\{\frac{\bar{\bar{k}}k_{rw}\rho_w}{\mu_w}\left(\frac{\partial p}{\partial x_j} - \rho_w\bar{g}\right)\right\}$$

$$+ q_s + q_w = \left\{\rho\frac{d\phi}{dp} + \phi\left(\frac{\partial\rho}{\partial p}\right)_h\right\}\frac{\partial p}{\partial t} + \phi\left(\frac{\partial\rho}{\partial h}\right)_p\frac{\partial h}{\partial t} \qquad (13)$$

and

$$h_s\frac{\partial}{\partial x_i}\left\{\frac{\bar{\bar{k}}k_{rs}\rho_s}{\mu_s}\left(\frac{\partial p}{\partial x_j} - \rho_s\bar{g}\right)\right\}$$

$$+ h_w\frac{\partial}{\partial x_i}\left\{\frac{\bar{\bar{k}}k_{rw}\rho_w}{\mu_w}\left(\frac{\partial p}{\partial x_j} - \rho_w\bar{g}\right)\right\}$$

$$- \left(\bar{v}_s\rho_s\frac{dh_s}{dp} + \bar{v}_w\rho_w\frac{dh_w}{dp}\right)\frac{\partial p}{\partial x_i}$$

$$+ q_w h_w + q_s h_s$$

$$+ \frac{\partial}{\partial x_i}\left\{\bar{\bar{K}}_m\left[\left(\frac{\partial T}{\partial p}\right)_h\frac{\partial p}{\partial x_j} + \left(\frac{\partial T}{\partial h}\right)_p\frac{\partial h}{\partial x_j}\right]\right\} = \left\{\left(\rho h - \rho_r h_r\right)\frac{d\phi}{dp} + \phi h\left(\frac{\partial\rho}{\partial p}\right)_h\right.$$

$$\left. + (1-\phi)\rho_r\left(\frac{\partial h_r}{\partial p}\right)_h\right\}\frac{\partial p}{\partial t} + \left\{\phi h\left(\frac{\partial\rho}{\partial h}\right)_p\right.$$

$$\left. + (1-\phi)\rho_r\left(\frac{\partial h_r}{\partial h}\right)_p + \phi\rho\right\}\frac{\partial h}{\partial t} - \frac{Dp}{Dt}$$

$$(14)$$

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Equations (13) and (14) are in terms of the dependent variables enthalpy and pressure. As can be seen from Fig. 1, a pressure and enthalpy value for a given point in space will determine whether compressed hot water, a steam-water mixture or super-heated steam exists at that point. Thus, Eqs. (13) and (14) may be used to describe most types of geothermal reservoirs. To simulate the behavior of such a reservoir, however, a solution for Eqs. (13) and (14) is required.

Equations (13) and (14) are nonlinear, and their solution requires the use of numerical techniques. Such techniques are beyond the scope of this paper and will be mentioned only briefly. They include finite difference techniques, the finite element method, or some combination of the two. The nonlinear coefficients are generally handled by iteration, using, for example, the Newton-Raphson method (Refs. 2 and 9). Finally, some form of weighting of the spatially dependent variables is commonly used, upstream weighting of permeability, for example (Ref. 2).

III. CONCLUSIONS

We have presented the governing equations for energy transport and the flow of water and steam in porous media in order to provide a basis for the mathematical modeling of geothermal reservoirs. The primitive form of these equations is straightforward and available in various forms in the literature. Our objective here was to take the general mass and energy balances and reduce them to workable equations. It should be emphasized that the method described represents only one of many possible approaches. The development of a method for obtaining solutions to these equations is a challenging problem and beyond the scope of this paper.

This particular approach to geothermal modeling has been guided by several objectives: (1) the model must provide efficient and accurate solutions consistent with the available geological and hydrological data, (2) it should be applicable to hot-water, steam-water, and super-heated steam geothermal systems, and (3) the method must be easily applied to natural geothermal reservoirs. Two assumptions, peculiar to this development, were made to achieve these objectives: (1) thermal equilibrium exists between all phases, in steam, water, and rock; and (2) capillary pressure between the steam and water phases is negligible. Invoking these assumptions the basic relationships reduce to two partial differential equations written in terms of convenient field variables, fluid pressure and enthalpy. These final two equations in conjunction with well-known thermodynamic relationships provide the basis for a numerical model which we feel fulfills our initial objectives.

The suitability of this model for simulation of natural geothermal reservoirs is limited in two general ways: (1) by the validity of the basic assumptions (negligible capillary pressure, porous media, etc.), and (2) by the availability of reliable geological and hydrological and thermodynamical data for the reservoir to be simulated. Within these constraints, the suggested approach should prove useful in geothermal reservoir development and management.

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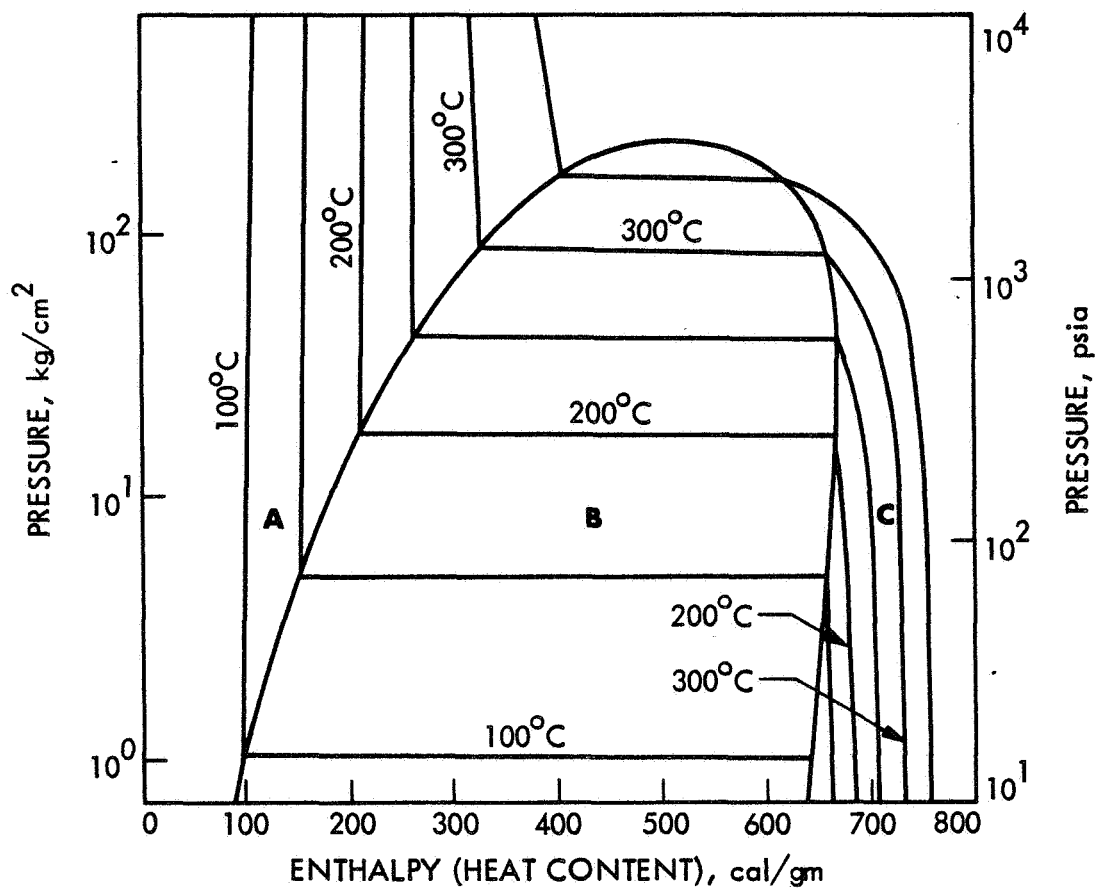
Fig. 1. Pressure-enthalpy diagram for pure water and vapor showing
temperature contours (after Ref. 21). Regions are for: (A) compressed
hot water, (B) steam-water mixture, and (C) super-heated steam.