Scintillations During Occultations by Planets

I. An Approximate Theory

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Abstract

Scintillations are observed during occultations of both stars and spacecraft by planetary atmospheres. Existing treatments of these scintillations ignore a major effect unique to occultations: the severe flattening of the Fresnel zone or source image by defocusing. Other large effects, due to "saturation" of the scintillation, have also been ignored. The deeper portions of atmospheric temperature and density profiles inferred from occultation data are seriously in error if other planets' atmospheres are as turbulent as our own. Thus, profiles obtained from entry probes (e.g., the Soviet Venera series) are probably more accurate than those from radio occultation (Mariner 5 and 10) data. Scintillation greatly reduces the information obtainable from occultation observations; much of the detail in published profiles is probably spurious. This paper gives an approximately - correct theoretical treatment that is a substantial improvement over published theories, and shows how a more accurate theory could be constructed. Some methods for a more accurate determination of atmospheric structure are proposed.

I. Introduction

In recent years, a large literature has grown up around the inference of atmospheric structure (e.g., temperature and pressure profiles) from observations of the intensity and/or phase of electromagnetic waves received from a point source, such as a star or a spacecraft radio transmitter, as it is occulted by a planetary atmosphere. The occultations of Regulus by Venus in 1959, of BD - 17°4388 by Neptune in 1968, of β Scorpii by Jupiter in 1971, and of Mariners 5 and 10 by Venus in 1967 and 1974 respectively, as well as numerous occultations of other Mariner spacecraft by Mars, have been used to infer planetary atmospheric profiles. Many events were observed at several stations, and some observations have been re-discussed several times, so the total number of publications is very large; I shall not attempt to review this extensive literature.

The theoretical interpretations of such data go back at least to Pannekoek (1903), who wished to explain the phenomena of the occultation of a star by Jupiter on Sept. 19 of that year. He commented that the light curves of future occultations of stars by planets should be measured accurately: "This is of importance, because it can give a determination of the temperature distribution in the outermost layers of the planetary atmosphere, or more precisely, the way in which the horizontal refraction varies with height." The problem was re-analysed (in French instead of in German) by Fabry (1929), who also considered the effects of both diffraction and an extended source, and who drew the apparent shape of the occulted object at various stages. A third derivation of the laws of occultations was published in English, with a third independent scheme of notation, by Baum and Code (1953), who had observed the occultation of σ Arietis by Jupiter in the previous year. Finally, the subject was reviewed by Link (1969), using a fourth set of symbols. Since then still other notations have been used by various authors.

This lack of uniformity is a problem, complicated here by the need to discuss scintillation as well (see Table I).

All the above authors assume a simple exponential atmosphere with a fixed scale height. More recently, it has become fashionable to reduce occultation curves by a numerical inversion process, which assumes a continuous, one-to-one mapping of rays between the successive layers of the planet's atmosphere and the successive moments of observation (or points along the plane of the telescope objective.) As will be shown below, this assumption is probably not justified, at least for the deeper stages of the occultation, and therefore the inversion results contain systematic errors.

The first hint that turbulence might play a part in the observed phenomena was a comment by Baum and Code (1953): "A sequence of photographs taken by Pettit and Richardson with the 100-inch telescope indicates that the fading is far from smooth and in fact suggests that the principal humps in [the light curve] may partly represent real features in the occultation - curve. Random variations in θ caused by turbulence in Jupiter's stratosphere would tend to jumble neighboring rays in such a way that the observed flux ... would deviate randomly from an ideal occultation-curve. Percentage variations in θ of the same order as those suffered by objects near the earth's horizon would become significant for θ greater than the angular subtense of Jupiter's scale height ... which occurs when $\phi < 0.5\ \phi_o$. Thus, large fluctuations in the tail of the observed light-curve are rather to be expected."

Osawa et al. (1968) remarked that their light-curves "have large fluctuations prevailing the main phase of occultation. Small fluctuations lasting for about two minutes after immersion and before emersion are also remarkable features of these light-curves. Random variations in refraction angle caused by turbulence in Neptune's upper atmosphere must be the cause of these fluctuations." They did not, however, take the important step of showing that realistic amounts of turbulence could account for their observations, as Baum and Code (1953) had done.

Freeman and Lyngå (1969, 1970) discussed their observations of the same event. They seem to have been the first workers to call the observed asymmetrical light fluctuations "spikes", and also to have attributed them to layers of lower-than-average refractivity gradient in the planetary atmosphere. Since then, this explanation has become widely accepted. According to this model, "spikes" in the light curve correspond to layers with a more nearly adiabatic lapse rate, and the minima between them to more isothermal or stably - stratified regions.

This view was endorsed, for example, by Kovalevsky and Link (1969), the first workers to use the numerical-inversion technique instead of curve-fitting. They claimed, without any quantitative demonstration, that the fluctuations observed at two stations several hundred kilometers apart showed "a good correlation."

Unfortunately, the human eye tends to "see" patterns in random distributions; the subjective impression of "a good correlation" may not be justified by the actual data. For example, Rages et al. (1974) concluded from the same data as Kovalevsky and Link (1969) that "the correlation is not striking." Furthermore, in a quantitative statistical discussion of spikes in the β Sco occultation, Veverka et al. (1972) concluded, "We find no evidence in our data that the pattern of fluctuations is planet-wide in the case of Jupiter, and it is therefore improper to speak of global layering."

Finally, although Veverka et al. (1974a) assume that the "spikes" are due to "layering," and claim that "the agreement among the three channels in Fig. 10 [their temperature-refractivity profiles] is excellent," closer inspection of this figure and their Fig. 13 (which shows refractivity scale height as a function of depth) shows that the fine structures of the atmospheric profiles deduced by inversion from observations at one station in different wavelengths are not at all in good agreement. While the same general features appear in these profiles, they do not occur at the same depth in the atmosphere; in many cases a local maximum on one curve occurs at the same atmospheric level as a minimum on another curve. This discrepancy is explained by the scintillation model below.

A similar problem is shown by the very disparate profiles deduced by Rages et al. (1974) from three different observations of the Neptune event. Their profiles differ by several tens of degrees at points a few hundred kilometers apart on Neptune. It is difficult to see how a planet so far from the Sun can develop such large horizontal temperature gradients, which (if real) would cause very strong thermal winds.

Other problems with temperature profiles deduced from occultations are reported by Veverka and Wasserman (1974) and Kliore et al. (1974). Both the Regulus and the Pioneer 10 occultation data lead to temperature profiles that are systematically too high, by roughly 100°K. The discrepancies are so striking that Veverka and Wasserman (1974) say, "There is no way in which the Regulus occultation light curve can be reconciled with reality," and Kilore et al. say "This temperature distribution in the upper levels of the atmosphere of Jupiter is completely different from any published model of the atmosphere."

These statements cast doubt on the validity of numerical-inversion results. The spurious results are, I believe, due to scintillation effects.

II. Scintillations: Theory

Although it has been common practice to interpret the fluctuations in signal strength at radio frequencies as scintillations, using the theory for isotropic Kolmogorov turbulence, this has not yet been applied to the optical observations. Furthermore, the theory that has been used (cf. Woo and Ishimaru, 1974) is appropriate to a medium without differential refraction, in which the Fresnel zone is circular. Finally, only weak-scintillation theory has been used, although many of the observations show very strong modulation. Thus the first need is to adopt scintillation theory to suit the special circumstances of planetary occultations.

1. Effects of Differential Refraction.

The theoretical treatments used up to now have implicitly assumed a circular Fresnel zone. But, as Fjeldbo and Eshleman (1969) point out, "Refraction has the effect of changing these zones from circular to elliptic. In the atmospheric and ionospheric regions that produce defocusing, the curvature of the wavefronts is increased in such a way as to squash the Fresnel zones into ellipses with minor axes in the vertical direction." This is strictly true only if the Fresnel zone is much smaller than a scale height; if it is comparable, the Fresnel zone (or the disk of an occulted star, whichever is larger) is more flattened on the lower side. The shapes that result are illustrated in Fabry (1929), Link (1969), or the form of the setting Sun - a most familiar example of an atmospheric occultation with differential refraction (Young, 1974). In our present examples, the free-space Fresnel zone sizes are roughly a kilometer across, and the atmospheric scale heights are typically ten times larger. Thus the elliptical approximation is fairly good, but not exact.

From the basic physical law that the power received at a point is just what flows through the first Fresnel zone, one sees that the reduction in area of this zone (or the axial ratio, in the elliptical approximation) is just the factor ϕ by which the intensity has been reduced. For an extended source (stellar disk) the corresponding result follows from the fact that a lossless optical system (the occulting atmosphere) cannot change the source's apparent surface brightness; so the defocusing produces an image-area reduction equal to the reduction in received intensity. Thus, when the received signal is cut in half, the region of the atmosphere probed is half as large vertically as horizontally (see Fig. 1).

This reduction in size has a profound effect on the scintillations, which are mainly due to the smallest elements of turbulence that are not averaged out over either the source or the Fresnel-zone area.

To estimate this effect quantitatively, an heuristic approach used previously (Young 1969, 1970) is useful. For detailed accounts of scintillation theory, see Tatarskii (1961, 1967), Lee (1969), or Lee and Harp (1969). We can regard the effect of either Fresnel-zone or source size as an averaging aperture in the turbulent medium, and then use the ray-optical model (Reiger, 1963; Young, 1967, 1969) of scintillation. Then the mean-square modulation (or "scintillation power," P) of the shadow pattern observed in the x-y plane is proportional to

$$P \sim \int_{-\infty}^{+\infty} dk_x \int_{-\infty}^{+\infty} dk_y \int_0^{\infty} dz \cdot C_n^2(z)\, k^{1/3}\, z^2\, F\,(k, z, \lambda, \ldots), \qquad (1)$$

where $k = (k_x^2 + k_y^2)^{1/2}$ is the spatial frequency of a Fourier component of the shadow pattern, z is the distance from a slab of turbulence to the observer, $C_n(z)$ is a measure of the strength of the turbulence at distance z, λ is the wavelength of the light, and F (...) is a spatial frequency filter function corresponding to source or Fresnel zone size. The effect of this filter is to cut off the infinite integrations in k-space at finite values (Young, 1970). The factor $k^{1/3}$ is due to the Kolmogorov spectrum of turbulence, and z^2 is due to a "lever-arm" effect (Young, 1969) that represents the gradual conversion of phase fluctuations in the wavefront to amplitude fluctuations.

We can readily obtain the temporal spectrum of the scintillations (Reiger, 1963; Young, 1967, 1969; Lee and Harp, 1969) by choosing the x-axis in the direction of motion of the line of sight through the turbulent medium, omitting the integration over k_x, and transforming from spatial frequency k_x in the direction of motion to temporal frequency f:

$$f = \frac{V_{\perp} k_x}{2\pi} \quad , \qquad (2)$$

where $V_\perp$ is the speed of motion. The integrals over z and k_y then give P (k_x) or P (f), the spectral power density. For a point source at infinity, F is the "diffraction filter" function (Young, 1969, 1970)

$$F_{diff} = \frac{\sin^2 (\lambda z k^2/4\pi)}{(\lambda z k^2/4\pi)^2} \quad , \tag{3}$$

which cuts off at $k \approx (\lambda z)^{-1/2}$. The high-frequency behavior of the spectral power density P(f) is then readily found to be asymptotically proportional to $f^{-8/3}$; a simple expansion for calculating the spectrum from a single slab of turbulence is given by Young (1971). For a uniform disk source of angular radius r, the averaging region at z is a circle of radius rz, and the filter is (Young, 1969)

$$F_{disk} = [2J_1(rzk)/(rzk)]^2 \quad , \tag{4}$$

where J_1 is the usual Bessel function; this cuts off at $k \approx (rz)^{-1}$. Lee (1974) has discussed the effects of other interesting source shapes.

For a region of arbitrary shape, the filter F has the analytical form of the intensity distribution in its Fraunhofer diffraction pattern. Thus, for an ellipti averaging region with semimajor axis a and semiminor axis ϕa, where $\phi \leq 1$, we have

$$F_{ell} = [2J_1(ak^*)/(ak^*)]^2 \quad , \tag{5}$$

where

$$k^* = [(k_x/\phi)^2 + k_y^2]^{1/2} \quad , \tag{6}$$

assuming the major axis in the y-direction. That is, compressing the averaging region in the x-direction by a factor $\Phi = \phi^{-1} > 1$ has effectively extended the cut-off of the filter function by this same factor along the k_x axis. In other words, when we substatute Equations (5) and (6) into Equation (1), the k_y integral cuts off at $k_y = a^{-1}$, and the k_x integral cuts off at $k_x = (a\phi)^{-1} = \Phi/a$. In the occultation light curve, ϕ is the ratio of the partially - occulted intensity to the unocculted intensity, and is typically much smaller than unity when scintillations are observed. An atmospheric averaging region that is very different in the vertical and horizontal directions results from atmospheric dispersion on Earth

(Young, 1970), and the same treatment can be used here; the chief difference is that the sense of the asymmetry is different in the occultation case. When $\Phi>>1$, we find

$$P \sim \int_0^{\Phi/a} dk_x \int_0^{1/a} dk_y \cdot k^{1/3} \sim \Phi^{4/3} a^{-7/3} \quad . \tag{7}$$

Thus the effect of flattening either the stellar disk or the Fresnel zone by a factor $\Phi>>1$ is to <u>increase</u> the mean-square scintillation by a factor of $\Phi^{4/3}$.

Now let us return to Eq. (1) and put in the integral over dz. Since the whole planetary atmosphere is at a distance $z = D$ that is large compared to the planetary radius, we can approximate

$$\int dz \cdot C_n^2(z) z^2 F(z, \ldots) \approx D^2 F(D, \ldots) \int C_n^2(z) dz, \tag{8}$$

where $\int C_n^2 \, dz$, the integrated turbulence along the line of sight, behaves like an air-mass factor. For example, for an atmosphere of radius R with an exponential distribution of turbulence with scale height H_t, we have (Young, 1969b; Fabry, 1929)

$$\int C_n^2(z) \, dz = C_n^2(0) \, (2\pi R \, H_t)^{1/2} \exp \, (-2h/H_t) \quad , \tag{9}$$

where $C_n^2(0)$ is the turbulent structure parameter at some reference level in the atmosphere where $h = 0$, and h is the minimum height of the ray above this level. For an isolated turbulent layer of thickness b and strength C_n^2, such as Woo and Ishimaru (1974) have assumed, we have

$$\int C_n^2(z) dz = 2C_n^2 \, (2bR + b^2)^{1/2} \approx 2C_n^2 \, (2bR)^{1/2}, \tag{10}$$

if the lowest point on the ray touches the bottom of the layer. The first part of Eq. (10) (which was first derived by Lambert in 1760) is equivalent to equations 2 and 3 of Woo et al. (1974). If the ray passes a depth d below the base of the layer,

$$\int C_n^2(z) dz = 2C_n^2 \left\{ [2(b + d)R + b^2 - d^2]^{1/2} - [2dR - d^2]^{1/2} \right\} \approx 2C_n^2 \left\{ [2(b + d)R]^{1/2} - (2dR)^{1/2} \right\} \quad , \tag{11}$$

which reduces to Eq. (10) as $d \to o$, and becomes $C_n^2 b(2R/d)^{1/2}$ when $d > b$. Notice that the effective path length in the atmosphere is always proportional to the geometric mean of a characteristic vertical dimension and the radius of curvature. If we call this length $L(h)$, we can summarize Equations (9-11) as

$$\int C_n^2(z)dz \sim C_{n\ max}^2 L(h), \quad (12)$$

where $C_{n\ max}^2$, the peak value of C_n^2 along the ray, is $C_n^2(0) \exp(-2h/H_t)$ for the exponential model, and

$$L(h) = (2\pi R H_t)^{1/2} \quad (13a)$$

for the exponential model, and

$$L(h) \simeq [4\ R(b - h)]^{1/2} \quad , \quad h > o \quad , \quad (13b)$$

$$L(h) \simeq [4\ Rb]^{1/2} \quad , \quad o \geqslant h \geqslant -b/2 \quad , \quad (13c)$$

and

$$L(h) \simeq [-2R\ b^2/h]^{1/2} \quad , \quad h < -b/2 \quad , \quad (13d)$$

for the one-layer model. (The approximations in Equations (13c, d) slightly overestimate $L(h)$; at $h = -b/2$, the error is on the order of 27%.)

Now let us combine these results. For simplicity, assume a simple exponential atmosphere (Pannekoek, 1903; Fabry, 1929) with refractivity scale height H_ν. In the absence of turbulence, the normalized light curve $\phi(t)$ is related to the minimum ray height h by

$$\Phi = \phi^{-1} = 1 + \exp(-h/H_\nu) \quad (14)$$

if $h = 0$ where $\phi = 1/2$; this is the conventional "occultation level." We remark that $h(\phi) = -h\ (1-\phi)$; thus, for example, the 10% and 90% intensity points are 2.2 scale heigh below and above the occultation level, the 5% and 95% points are 2.9 scale heights from this level, and the 1% and 99% points are 4.6 scale heights from it. Since typical photometric errors are a few per cent, we obviously learn nothing reliable about any part of the atmosphere more than a few scale heights from this level.

It is also easy to show that the vertical component of ray velocity through the atmosphere is directly proportional to the light remaining:

$$\frac{dh}{dt} = v_V(\phi) = \phi\, v_V(1) \quad . \tag{15}$$

If we measure time from the half-intensity point on the light curve, $v_V(t) \rightarrow H_V/t$, for large t. These simple relations provide some "feel" for the occultation phenomena.

Woo and Ishimaru (1974) have assumed that v_V is the only important component of the motion. This is true only for central occultations. In general, there is also a horizontal component v_H parallel to the planet's limb. As long as the occultation point does not move far along the limb, we can regard v_H as constant; this is generally satisfactory for stellar occultations. For radio occultations of spacecraft, a large motion along the limb occurs. As Link (1969) points out, the motion along the limb is the projection of the geometric position of the source in the sky from the center of the planet's disk to the limb. Thus v_H can be strongly accelerated, especially near the middle of a nearly-central occultation. Because of the rapid decrease in v_V and the increase in v_H, the motion is primarily horizontal for the deeper parts of every occultation. Thus, the assumption of Woo and Ishimaru (1974) is strongly violated by the data (Woo et al., 1974) to which they have attempted to apply their theory.

As long as the scintillations are weak, we can treat the observed light curve as an ideal one modulated by the scintillations. The modulation power is, from Equations 7-13,

$$P \sim \phi^{4/3}\, a^{-7/3}\, C_n^2(0)\, L(h)\, D^2 \quad , \tag{16}$$

where the major axis a of the averaging region is

$$a_{diff} = (\lambda D)^{1/2} \tag{17a}$$

for Fresnel-zone averaging, and

$$a_{\star} = rD \tag{17b}$$

if the stellar disk is larger. If they are comparable, they can be added quadratically (Young, 1969). Note that ϕ depends on h and the refractivity scale height H_v through Eq. (14), and that L depends on h and the vertical structure of turbulence through Eq. (13).

We are now in a position to estimate where, in various occultation curves, scintillation should appear. We first remark that P must generally increase rapidly with depth in the occultation, not only because of the rapid increase of $\phi^{4/3}$, but also because of the expected increase of $\int C_n^2 dz$ with depth. Even in the single-layer model of turbulence, where L decreases slowly (with about the square root of ray depth below the layer), the exponential increase in ϕ with depth must overwhelm the small decrease in L. Thus, the reason offered by Woo et al. (1974) for using a layer model of turbulence cannot be correct. This error is a direct result of omitting the dispersive flattening of the Fresnel zone in the theory by Woo and Ishimaru (1974).

Now let us estimate P, assuming that C_n is proportional to density, and adopting the following values for stellar scintillations produced in the zenith by the Earth's atmosphere (Young, 1967, 1969, 1970b): $P \sim 0.2$ for $a = 5$ cm $= 5 \times 10^{-5}$ km, $\phi \equiv 1$ (no refractive flattening), $L \simeq D \simeq 8$ km, number density $\rho_o \simeq 10^{19}$ molecules/cc. For a planet, we shall assume an exponential atmosphere with $H_v = H_t = H$, adopt Eq. (13a) for L(h), and call the number density ρ_p. We can ignore the effects of small differences in temperature and molecular refractivity between planets; these are generally only a factor of 2 or so. We then have

$$P \simeq 0.2 \left(\frac{\rho_p}{\rho_o}\right)^2 \phi^{4/3} \left(\frac{a}{5\times10^{-5}}\right)^{-7/3} \frac{L}{8} \cdot \frac{D^2}{64}$$

$$7.8\times10^9 \left(\frac{\rho_p}{\rho_o}\right)^2 \phi^{4/3} a^{-7/3} LD^2 \quad , \qquad (18)$$

for ρ_p measured at the minimum ray height h, and all dimensions in kilometers. The circumstances of several occultations, and the corresponding values of P, are given in Table II for $\phi = 0.5$, 0.2, 0.05, and 0.01 ($\Phi = 2$, 5, 20, and 100, respectively.) The relative values of P at these levels, just due to the variation of $\Phi^{4/3}\ \rho_p{}^2(h)$ with height, are 2.51, 137, $2x10^4$, and $4.5x10^6$, respectively. The very rapid increase in scintillation with depth is due to the combined effects of increasing atmospheric density and Fresnel-zone flattening; the amplitude of the fluctuations increases by a factor of over 1000 between the $\phi = 0.5$ and $\phi = 0.01$ levels.

A few special circumstances should be noted in Table II. The star β ScoA is double; this decreases P by about a factor of 2, which I neglect in the present crude treatment. The appropriate value of D for a spacecraft occultation is the spacecraft-to-planet distance instead of the Earth-to-planet distance, as pointed out by Golitsyn and Tatarski (1972). We may also note that a is on the order of a kilometer, and L on the order of 10^3 km, for all the events in Table II; the factor $a^{-7/3}$ L is similar for all. Thus, because the attenuation Φ is proportional to $\rho_p D$, the value of $\rho_p D$ (or $\rho_p{}^2\ D^2$, which appears in the scintillation equation 18) at $\phi = 0.5$ is similar for all planets. Thus, the value of P is not greatly different from one planet to another at the same point in the light curve (ϕ = constant).

Scintillations are first visible above the noise when their amplitude exceeds a few per cent, say $P \doteq 10^{-3}$. This can be expected near, or shortly after, the reference level $\phi = 0.5$, if other planets' upper atmospheres are as turbulent as ours. When $P \gtrsim 10^{-2}$, the RMS scintillation exceeds 10%, and an intensity record shows asymmetrical excursions ("spikes") because the logarithm of the intensity (rather than the intensity itself) has a normal distribution (Tatarski, 1961, 1967; Ochs and Lawrence 1969; Young, 1969, 1971); this occurs by the time the average intensity has fallen by a factor of about ten. On the extreme tail of the light curve, the theory predicts $P > 1$; I shall say more about this in subsection 4, below.

2. Temporal Power Spectrum

Now let us consider the temporal power spectrum of the fluctuations. As long as the scintillations are "weak" (i.e., $P \leq 0.1$ or so) this can be calculated by well-known methods (Reiger, 1963; Young, 1967, 1969; Lee and Harp, 1969), outlined above. But now we must allow for both the flattening of the Fresnel zone and the flattening of its trajectory through the atmosphere (see Fig. 1).

For the moment we regard the planetary limb as straight. Since the cutoff frequency in the temporal power spectrum is

$$f_c = \frac{V_\perp k_c}{2\pi} \quad , \tag{19}$$

and the cutoff wavenumber $k_c = 1/a$ when $\phi = 1$, we have

$$f_c = \frac{V_o}{2\pi a} \tag{20}$$

at the top of the atmosphere, where V_o is the initial speed of the ray relative to the planet. The horizontal components of $V_\perp$ and a are unaffected by differential refraction; the vertical compression of a to $a\phi$ is compensated by the reduction of the vertical ray velocity by the same factor, when the light is reduced to a fraction ϕ of its original intensity. Hence, to a first approximation, the scintillation frequency spectrum remains grossly similar throughout the occultation.

However, as can be seen in Fig. 2, the effect of flattening the Fresnel zone is to stretch the region of k-space that contributes substantial scintillation into an ellipse. The spatial frequency components that contribute to a temporal frequency f now lie along a line that cuts the ellipse obliquely. Thus the detailed shape of the frequency spectrum is altered; it is no longer given by the expressions derived by Woo and Ishimaru (1974), even apart from their omission of the factor $\phi^{4/3}$. Qualitatively, the effect is to smear out the fine structure (oscillations) in the curve, and to make the knee at f_c less pronounced and more gradual.

If we consider the deeper phases of an occultation, where limb curvature cannot be neglected, we see that the effects of projection enlarge the horizontal extent of the Fresnel zone in proportion to the increased horizontal velocity component. Thus the cutoff frequency, again, is unaltered.

A special remark must be made about $V_\perp$, however. Woo et al. (1974) have allowed this velocity to be a free parameter, and have adopted values of a few hundred meters per second. But, as pointed out above, the value V_o (which is fixed by orbital motion) must be used in Eq. 20. The appropriate values of V_o and f_c appear at the bottom of Table II for several events. For the Venus/M5 occultation, V_o is about 20 times larger than Woo et al. (1974) assumed. In fact, the theoretical value given in Table II (0.9Hz) is close to the spectrum turnover shown in Fig. 3(c) of Woo et al. (1974) for this event.

Below this spectral knee, the power density is on the order of 0.1 Hz^{-1} for the Mariner 5 scintillations in a region where the average power received is about 13 db below the unocculted signal (i.e., $\phi \approx 0.05$). Thus, the observed value of P is a little less than 0.1, while Table II predicts $P \approx 0.25$ for this region. The agreement is fair; however, the slight discrepancy, together with the higher refractivity of CO_2 compared to air, suggests that the stratosphere of Venus is a little <u>less</u> turbulent than our own.

For the Jupiter/β Sco event, the frequency cutoff implies a characteristic time scale ($=a/V_\perp$) of 0.05 seconds, which should correspond to the half-width (at half-maximum) of the narrowest "spikes" in the light curve. Inspection of the spike profiles published by Liller et al. (1974) confirms that this is the case. Their light curves also show that the scintillation became strong (i.e., $P \gtrsim 1$) for ϕ between 0.2 and 0.5, again in agreement with the prediction in Table II.

The published records of the Neptune/BD-17°4388 occultation are noiser. The scintillations have a large amplitude for $\phi < 0.5$, and their width appears to be on the order of 1/3 second, as well as can be estimated from recordings with a rather compressed time scale. The predicted half width is exactly 1/3 second, which is satisfactory. However, the scintillations are obviously much stronger than expected, which suggests that Neptune's upper atmosphere is considerably more turbulent than the Earth's.

On the whole, the rough calculations of Table II are in good agreement with the observations. Both the times of onset of "spikes" and, especially, their widths, are what one would expect from scintillation theory. Note that Table II merely assumes Earth-like turbulence, with no free parameters to fit the data.

3. Turbulence vs. Layers

The "spikes" have been attributed to layered structure in the occulting atmosphere by a number of investigators. Obviously, if one assumes the atmosphere is spherically symmetrical, as the numerical-inversion people do, a spike in the light curve must produce a kink in the deduced thermal profile. But the required thermal excursions in the "layers" are large--10^0 K or more.

How can random turbulent fluctuations produce such large fluctuations in the light curves, when turbulent temperature fluctuations are typically only a few tenths of a degree (RMS) at separations on the order of 1 km? One might expect that the random effects of turbulence would be much less effective than the organized effects of a uniform layer: But the layer can only act in the vertical direction, while turbulence can "focus" in two dimensions; and the ray path through a thin layer is much shorter than the effective path length L ($\sim 10^3$ km) through the turbulent atmosphere. Furthermore, a layer refracts a ray only twice (on entry and exit), but turbulence produces many random deviations along the path. For L $\sim 10^3$ km and a $\sim$ 1 km, isotropic turbulence produces about a thousand random refractions along the ray path. The net deviation is on the order of the square root of this -- say 30 -- times the deviation produced by a layer. For the same net deviation, the temperature or refractive-index fluctuations can thus be ~ 30 times smaller if turbulence is the cause, rather than layers. Thus, the roughly hundred-fold smaller temperature variations needed by the scintillation model are readily accounted for.

There is a further problem with "layers". The fine structure in the light curves always has a time scale of $a/V_\perp$. This is naturally explained by the spatial-filtering effect, if scintillation is the explanation; smaller-scale structure is smoothed out. But if layers are the explanation, we have two unsatisfactory alternatives: either the layer thickness just happens to match the shrinking Fresnel size at every depth, (an implausible coincidence); or there is still smaller detail in the layers, which is smoothed out by Fresnel filtering. But in the latter case, the finer structure must have still larger temperature excursions and gradients. However, the inferred excursions in the "layers" are already tens of degrees, and the gradients in some cases are close to the adiabatic value. The finer structure must exceed these values. If we do not accept temperature fluctuations of 100° or more, nor superadiabatic gradients, we are forced to accept the implausible coincidence cited above. As turbulence can produce the observed "spikes" with much smaller temperature excursions, the scintillation model avoids these uncomfortable problems.

Freeman and Lyngå (1970) and Rages et al. (1974) have argued that the good correlation between "spikes" seen at two telescopes 200 m apart at Mt. Stromlo is evidence for layers extending at least this far along Neptune's limb. But because the projected diameter of BD-17°4388 at Neptune was about 10 km, the atmospheric columns sampled by the two telescopes overlap almost completely; one must expect better than 95% correlation between the scintillations observed at two points so close together. Hence their argument is worthless; the tiny difference between the nearly perfect correlation expected from turbulence, and the perfect correlation expected from layers, is buried in the noise. Stations separated by at least the length a along the limb must be used to distinguish layers from turbulence.

Rages et al. conclude that no significant correlation was observed between two stations in Japan separated by 260 km, however. But this is an order of magnitude smaller than L, which requires the horizontal extent of the assumed "layers" to be longer in the line of sight than across it. Such accusing "fingers" pointing at the observer are generally a sign of some error. They are removed by assuming isotropic turbulence instead of a layered structure.

Another way to obtain two nearby ray paths is to use two different wavelengths (Young, 1969; Liller et al, 1974). As the two Fresnel zones become more flattened and their vertical separation increases, deeper in the atmosphere, the two regions sampled at the same time eventually cease to overlap. However, if the motion is nearly vertical, different wavelengths will pass through nearly the same air column in succession, and all fluctuations will be reproduced at different wavelengths after some time delay. This is seen as chromatic scintillation of stars near the terrestrial horizon; when viewed through a prism, the scintillations appear to run through the spectrum (Respighi, 1872; Zwicky, 1950). In the Jupiter/ β Sco occultation, it was seen as a delay in the appearance of "spikes" at different wavelengths (Liller et al., 1974).

Now, if the path of the star outside the atmosphere makes a 45° angle with the planet's limb, a significant drop in the peak time-lagged correlation between spikes observed at two wavelengths should be found when the dispersion separates the Fresnel zones by their (flattened) thickness, because they are then separated horizontally by their diameter as they pass a fixed depth (see Fig. 3). This occurs when the spikes in the two light curves become separated by their own widths, as at B in Fig. 3. The more obliquely the star meets the limb, the more rapidly the maximum correlation should decrease with depth, if turbulence is the cause of the spikes. On the other hand, if horizontal layers are involved, the pattern of spikes should repeat equally well in the two light curves, whether the spikes are separated by large or small time delays at the two wavelengths. The data of Liller et al. (1974) obviously agree better with turbulence than with layers: β Sco A met the limb at roughly a 30° angle, and the spikes reproduce more and more poorly as their separation in time increases. Indeed, when they are well separated in time, one light curve can show spikes that do not appear at all in another, as is obvious from Figs. 2 and 4, and Tables II, III, and IV, of Liller et al.(1974). Such observations of a still-more-grazing occultation would show this effect still more strongly.

However, there is even stronger evidence against layers in Figs. 10 and 13 of Veverka et al. (1974a). Fig. 10 shows the temperature vs. refractivity profiles for three different wavelengths; the profiles show similar wiggles (due to spike structure), but at different refractivity levels. In Fig. 13 the same data are plotted as refractivity scale height vs. depth, but have been arbitrarily "adjusted to bring the fine structure of the profiles into alignment." However, this adjustment is not legitimate. To see why, one needs only recall that the refractivity of a transparent gas can change by only a few per cent, at most, over the spectral range (3530 - 6201 Å) covered by these data. Thus, constant refractivity is (within a few per cent of a scale height, or about 2 km on Jupiter) constant height. Hence the mis-match of structure inferred from different wavelengths is real, and cannot be removed by an arbitrary "adjustment".

How does the scintillation model handle the same data? The <u>maximum</u> overlap between the two Fresnel zones shown in Fig. 3, and hence the maximum correlation between "spikes" on the corresponding light curves, occurs when the bluer zone has reached a point <u>between</u> B and B', if a red "spike" occurred at B. Hence the same atmospheric structure is traversed by the bluer Fresnel zone at a time earlier than would be expected for the layered model, i.e., <u>before</u> it reaches the depth h_1. Thus the numerical inversion of these spikes will produce a feature in the "blue" refractivity profile that occurs at a greater height in the atmosphere than the corresponding feature on the "red" profile. This is exactly what Fig. 10 of Veverka et al. (1974a) shows. Furthermore, if the profiles are "adjusted" to agree at one depth, the same discrepancy will occur, increasing with depth, at lower parts of the two profiles. This is clearly evident in Fig. 13 of Veverka et al. (1974a). These effects by themselves are enough to reject the "layer" model in favor of turbulence.

Grazing occultations offer another way to distinguish between layers and turbulence. If the star meets the limb at an angle θ, the time for the Fresnel zone to traverse its own vertical dimension is proportional to cosec θ, and the time scale for "spikes" due to layers is increased by this same factor. For the Jupiter/β Sco event, the factor was about 2.1 for the data of Veverka et al. (1974a) and Liller et al. (1974). Thus, if layers caused the "spikes", they should have been twice as wide as observed; the narrowness of the spikes <u>requires</u> turbulent fluctuations of length <u>a</u> along the limb. This can be seen also from a comparison of the observed and computed light curves shown in Fig. 9 of Veverka et al. (1974a): the calculated spikes are systematically broader and less extreme than those observed.

Fig. 1 of Veverka et al. (1974b) shows the same defect, although the Neptune event was nearly central. This is perhaps the most telling point of all: layers are unable to explain the details of the light curves, which are readily explained by isotropic turbulence.

III. Saturation of Scintillation

When the value of P calculated from weak-scintillation theory is comparable to, or larger than, unity, the theory breaks down. Observationally, the scintillation modulation "saturates" at a maximum value; beyond this, if the instrumental frequency response is limited, the measured signal variance may decrease (an effect sometimes called "supersaturation.") These effects have been observed not only with terrestrial light sources, but also in interplanetary and interstellar scintillations at radio frequencies, as well as ionospheric scintillation. The literature in each of these areas is quite large and will not be cited here; the phenomena are familiar to all scintillation workers, under such diverse names as "strong scattering", "multiple scattering", "focusing", "multipath propagation", etc.

The theoretical treatment of saturation has been difficult. In weak scintillation, we can regard the intensity fluctuations as due to weak focusing and defocusing effects, which cause the first Fresnel zone (or the solid angle subtended by a finite source) to increase and decrease in area. In strong scintillation, the Fresnel zone breaks up into a "seeing disk" (Young, 1969, 1970) containing "speckles". Thus, in place of $a = (\lambda D)^{\frac{1}{2}}$, two length scales appear in the shadow pattern: one corresponding to the overall blur (which imposes a low-pass spatial filter) and one corresponding to the speckles, which can be regarded as multiple-beam, high-order interference fringes. In broadband observations of stellar scintillation, both bandwidth (temporal coherence) and atmospheric-dispersion effects smear out the fine structure, and the overall "seeing" blur can be treated as a low-pass filter (Young, 1969, 1970). However, in the present case, the higher-order fringes or "speckles" must be included in the theory (Clifford et al., 1974; Yura, 1974; Clifford and Yura, 1974).

The effect of saturation on the frequency spectrum (Yura, 1974b) is twofold: the "knee" in the spectrum becomes more gradual and shifts to lower frequencies; and the "tail" flattens, extends to much higher frequencies, and contains an increasing fraction of the total modulation power. The appearance of an intensity record changes from a dense population of oscillations (the band-limited noise character of weak scintillation) to a sparse population of isolated large spikes, superimposed on a more slowly varying background. This progressive change in character is well illustrated by Fig. 1 of Gracheva (1967); the stronger examples look exactly like the tails of occultation curves. In particular, the isolated "spikes" in the far tails of the curves are due to the saturation predicted in Table II.

The effect of saturation on the frequency spectrum is clearly seen in Fig. 5 of Woo et al. (1974), where the "knee" has been blunted and shifted to roughly a quarter of the weak-scintillation cutoff. Comparison with Fig. 2 of Yura (1974b) shows that this corresponds to a value of σ_T^2 (or P, in our notation) of about 100. As region C of Woo et al.(1974) is some 20 db down from the unocculted signal, we have $\phi = 0.01$, at which Table II predicts $P \approx 60$. The agreement is within the uncertainties; the shape of the spectrum deep in the tail of the event seems to be adequately accounted for by saturation effects.

There is thus no evidence in the Mariner scintillation spectra for effects of a finite outer scale. There is also no evidence for "layers" of turbulence, if saturation (and "supersaturation") effects are taken into account; a simple model with roughly constant temperature structure function throughout explains all the data.

IV. Effects of Saturation on Occultation Curves

As saturated scintillation involves ray crossings, deeply saturated scintillations completely vitiate the numerical inversions of occultation curves, quite apart from the fact that scintillations produce spurious detail in the resulting profiles. Indeed, some of the numerical-invertors have already suspected ray-crossings in the deep parts of curves, where there are saturated scintillations. For example, Kliore et al. (1974) mention that "Discontinuities in the observed Doppler shift indicate the possible occurence of multipath propagati n," and Veverka et al. (1974a) admit that "we suspect that some ray crossing occurred." Veverka et al. (1974b) and Wasserman and Veverka (1973) also admit that "In seeking a solution by inversion it is implicitly assumed that ray crossing is negligible." But they then beg the question, saying, "at any rate ... there is no way in which ray crossing can be dealt with ... the assumption that it is negligible is unavoidable."

On the contrary, it is possible to estimate the statistical effects of scintillations. Weak scintillation can be considered noise, and can be filtered out by simple least-squares model-fitting, or by explicit filtering (Kovalevsky and Link, 1969). Strong scintillations are log-normally distributed, which presents a statistical problem: least-squares fits are efficient only for normal (Gaussian) noise. Strong scintillations can be rendered normal by using log ϕ instead of ϕ; but the problem then is that the logarithm of the mean intensity is not the mean logarithm of the intensity. Owing to conservation of energy, the scintillations cannot add or subtract energy from the shadow pattern, but merely move it around.* It is readily shown that

$$\langle \log I \rangle = \log \langle I \rangle - \sigma^2/2 \quad , \tag{21}$$

where $\sigma^2 = \log(1+P)$ is the variance of log I, the angle brackets denote averages, and all logarithms are Napierian. Thus, if the course of the scintillation strength P is known, the systematic correction (21) can be applied to the course of log I, and a model-fitting by least squares will be adequate.

In the case of saturated scintillations, the problem is more complicated. As mentioned above, scintillation redistributes the light on the receiving plane. As long as the scintillations are weak, the lateral scale of this redistribution is just the Fresnel zone size. But in saturation, it is the projected seeing-disk size, which can be much larger, and increases exponentially with occultation depth. On the sloping occultation curve, the effect of such a redistribution is

*Thus, the procedure of rejecting "spikes", as done by Freeman and Lyngå (1969, 1970) and suggested by Hubbard et al. (1972) as "justified if the flashes were due to random lumps in the ... atmosphere", is an unacceptable violation of energy conservation.

to make the tail of the curve brighter (and hence, longer) than it should be. This corresponds to a fictitiously high scale height or atmospheric temperature.

I have remarked above on the anomalously high temperature derived from the Venus/Regulus and Jupiter/Pioneer 10 occultations. It is also well known that the Mariner 5 temperature profile shows higher temperatures than the direct Venera probe measurements, in the region below about 55 km altitude. This is just the region of super-saturation - i.e., the onset of very strong angular scattering - according to the present interpretation (regions B and C of Woo et al., 1974). Thus the Venera profiles are probably more reliable in this region.

It may be argued that the agreement between Mariner ingress and egress profiles shows that the structure derived is correct. Actually, the agreement merely shows that the atmospheric structure (including turbulence profiles, and the onset of saturation) is similar for both ingress and egress. It does not prove that the structure derived by ignoring ray-crossings is correct.

It may be objected that the radio profiles were derived from phase rather than amplitude data. Multiple scattering makes the optical path between transmitter and receiver longer than it would be in a non-turbulent atmosphere with the same temperature profile; the "image" of the transmitter thus appears to be too far from the receiver, as if the atmosphere were less dense (warmer). Furthermore, the phase information is affected by the large-scale turbulent components, much larger than the Fresnel zone (Lee and Harp, 1969); large phase perturbations can occur before large scintillations.

Finally, the kind of fluctuations in ionospheric electron density that cause ionospheric scintillations on Earth can also produce ionospheric scintillations at other planets at radio wavelengths. This is particularly likely to be a problem at Jupiter, with an extensive and active ionosphere. Indeed, the complex structure derived in the lower Jovian ionosphere from Pioneer radio occultations may be analogous to the spurious detail derived from the β Sco occultation. A comparison with the much weaker ionosphere of Venus is instructive: Figs. 9 - 18 of Fjeldbo and Eshleman (1969) show that moderate scintillations were observed at 423.3 MHz (70 cm) and saturated ionospheric scintillations were seen at 49.8 MHz (6 meters). They add that "Because of the multipath propagation at 49.8 MHz, inversion of the Doppler data does not yield an accurate profile in the lower ionosphere." This remark applies equally well to the lower parts of the S-band occultations discussed above.

In short, the numerical inversion technique yields spurious results in the parts of occultations where strong scintillations (as indicated by "spikes" or multipath propagation) occur, i.e., the lower parts of all occultations. As Wasserman and Veverka (1973) say, "in the case of occultation curves containing spikes, this method cannot be applied meaningfully, and attempts to do so lead to deceptive results. Therefore, this approach must be avoided in such cases." (Curiously enough, they were referring to the model-fitting method; but their remarks apply equally well to the numerical-inversion method.)

I have just shown that scintillation effects vitiate the results of numerical inversion in the lower parts of occultations. But Wasserman and Veverka (1973) also showed this method was unreliable for the upper 4 scale heights of the inversion. As it is not practical to start the process until the intensity has fallen by a few per cent, their supposedly reliable part does not begin until below $\phi = 0.5$. But this is just the region in which scintillation makes the results unreliable! Thus the inversion method is unreliable everywhere.

Another way of seeing this is to regard the scintillations as noise on the curve. Because isotropic turbulence in the planet's atmosphere can deflect light sideways as well as vertically, there is no guarantee that the fluctuations observed are a conservative redistribution of the energy in an ideal light curve without scintillations; large portions of the observed curve may be systematically too bright or too faint. The very close similarity of ideal light curves produced by atmospheres with quite different temperatures and temperature gradients, shown by Wasserman and Veverka (1973) in their Figs. 5 - 7, shows that it is hopeless to extract more than an approximate scale height (or temperature) at the ϕ = 0.5 level from an occultation curve.

Wasserman and Veverka (1973) have claimed that a temperature gradient can be extracted by inversion from an occultation curve, even in the presence of noise. However, their simulated noise is wide-bandwidth white noise, which is like photon noise but unlike scintillation. The values of a in Table II are on the order of a tenth of a scale height. Thus near ϕ = 0.5 one has typically only 20 independent scintillations per scale height; if the RMS scintillation amplitude is roughly 10%, as the observations indicate, one can expect about 2% photometric error averaged over a scale height, or about twice this error in the average slope of the curve (and hence, the derived scale height) around ϕ = 0.5. Deeper in the occultation, the number of samples per scale height increase, but the scintillations increase still faster. By contrast, the 3% white noise tested by Wasserman and Veverka has about 100 independent samples per scale height near ϕ = 0.5, giving a slope uncertainty an order of magnitude lower than for real occultation curves. It is the spectral density of the noise, not its total over a wide bandwidth, that is important.

To sum up the results of this section: (1) Both the gross structure in the deeper parts of temperature profiles derived from occultation curves, and the fine structure throughout them, are spurious. (2) In general, it is only possible to derive an approximate scale height, or mean temperature, for the upper parts of occultations. (3) Contrary to the conclusions of Wasserman and Veverka (1973), a (modified) least-squares model-fitting procedure is probably more accurate than numerical inversion, which is highly susceptible to random and systematic errors due to scintillations. (4) In the region of disagreement, the Venera temperature profiles are probably more accurate than the Mariner profiles of the Venus atmosphere.

V. Discussion

The general conclusion, that the fine structure of occultation curves tells us something about turbulence, but almost nothing about the general atmospheric structure, is a dismal one indeed. To those who have a substantial professional investment in the deconvolution of such details, the suggestion that they have wasted their time in the interpretation of random noise must be especially repugnant. Can we offer them hopes of either improved observational techniques in the future, or of obtaining more reliable information from existing occultation records?

I have already suggested a means of re-analysing existing occultation curves: fit an isothermal curve by least squares to the part of the record containing only weak scintillations. This could be extended somewhat into the region of stronger scintillations by an iterative process: estimation of the RMS scintillation noise (deviation from the initial isothermal curve) on short sections - say, each half a scale height in extent - followed by estimation of the course of P or σ^2, which would be used to find the systematic corrections to the course of log I, and an improved least-squares fit. In some favorable cases, it may be possible to determine a mean temperature gradient in this way.

In the future, the largest possible telescope apertures should be used to observe stellar occultations, to reduce photon noise and terrestrial scintillations. If optical arrays several hundred meters across could be used, some aperture averaging of the planetary atmospheric shadow pattern (which has characteristic dimensions of $\underline{a}$ parallel to the planet's limb, and $\underline{\frac{1}{2}a}$ perpendicular to it, in the region of weak scintillation) may be possible. Observations at longer wavelengths would also help, because P (weak) is proportional to $\lambda^{-7/6}$ (Tatarskii, 1963, 1967; Young, 1970). A wide optical bandpass will reduce both photon noise and scintillation (through the effects of atmospheric dispersion - see Young, 1969).

Alternatively, one could combine observations made at several observatories, and solve for the single atmospheric scale height (and possibly, its gradient) that best reproduces all the observations simultaneously. This seems a promising possibilty for the Jupiter and Neptune events.

The spacecraft problem is more severe. If one goes to lower frequencies than S-band, ionospheric scintillation becomes a severe problem (Fjeldbo and Eshleman, 1969). On the other hand, at higher frequencies the Fresnel zone is smaller and the scintillations in the neutral atmosphere increase with the 7/6 power of frequency. Furthermore, projection of the Fresnel zone back to Earth gives dimensions on the order of 10^4 km for the Mariner events at Venus and the Pioneer events at Jupiter, so all parts of the Earth see essentially the same scintillation, and ground-station diversity buys no averaging of even weak scintillations. Only a receiving station on a (high) satellite can help here.

On the other hand, spacecraft scintillations are reduced as the 11/6 power of the flyby distance, so closer spacecraft encounters could help substantially. Nevertheless, when we contemplate the expected exponential increase of scintillation with depth in the atmosphere, we must expect only modest gains in depth penetration.

VI. Some Loose Ends

The whole scintillation theory used above assumes that the outer scale of turbulence is large compared to the length a, which is typically a kilometer. This assumption is directly justified by the agreement between predicted and observed "spike" widths, or frequency turnover: if the outer scale were smaller, the spikes would be narrower than observed, and the knee in the frequency spectrum would be at a higher frequency than observed. Furthermore, Woo and Yang (1975) have used the wavelength dependence of scintillation to show that the outer scale is "large"; as both wavelengths are equally affected by the factor $\phi^{4/3}$, this demonstration is not invalidated by their neglect of Fresnel-zone flattening. Furthermore, there is considerable direct evidence for a Kolmogorov-like turbulent spectrum in our own lower atmosphere up to scales of tens of kilometers (Reiter and Foltz, 1967). Of course, the spectrum becomes anisotropic at scales comparable to the height above ground. But the heights of occultation regions in planetary atmospheres are on the order of 100 km above the surface, so anisotropy should be unimportant.

One may also ask whether it is plausible to expect turbulence at the great heights of stellar occultations. In the Earth's atmosphere, turbulence is strong enough to overcome diffusive separation up to heights of 100 km, and turbulence has been reported as high as 110 km (Lloyd et al., 1973). At this altitude the air density is less than 10^{-5} of that at sea level, and comparable to the density at the Neptune occultation level. All the other events occured at higher densities. Veverka et al. (1974b) have also argued that the occultation level on Neptune is near the turbopause. Thus there are reasons, apart from the observed scintillations, to expect turbulence in the regions probed by occultations.

We may also inquire into the assumption that turbulence is exponentially distributed, with a scale height $H_t \simeq H_v$ (or approximately constant C_t). Reiger (1963) introduced this model, and showed it agreed with both airborne turbulence measurements and stellar scintillation. Later observations of planetary scintillation (Young, 1969) supported this model, as have more recent observations of stellar scintillation (Young, 1970b). Modern balloon measurements (Bufton et al., 1972) also show that although there is much patchy fine structure, the lower atmosphere follows this trend quite well.

VII. Future Needs

Obviously, a more accurate calculation of the effects of turbulence on occultation curves needs to be made. We need an accurate theory for the frequency spectra, for example. A careful investigation of the effects of multiple scattering on the Mariner phase data is needed, taking account of the phase - locking receiver characteristics. The theory of saturated scintillations should be applied to the tails of occultation curves; perhaps it will be possible to salvage some useful information from them after all. As is suggested above, the existing data should be re-analysed to separate the general atmospheric structure from the turbulence.

If we find that we know less about planetary atmospheres than we thought we did, at least we have our hands full of unfinished work; which should ultimately lead to a better understanding of atmospheric structure than we have at present.

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TABLE I

Notation used for occultations by various authors.

PHYSICAL QUANTITY	PANNEKOEK 1903	FABRY 1929	BAUM & CODE 1953	LINK 1969	THIS PAPER
Radius of planet	a	R	r_o	a	R
Distance to planet	R	D	D	1	D
Refractivity scale height	1	h	a^{-1}	H	H_ν
Fractional residual intensity	—	E/E_o	ϕ/ϕ_o	Φ^{-1}	$\phi \equiv \Phi^{-1}$
Angular refraction	η	ω	θ	ω	ω
Ray height in atmosphere	h	z	—	h_o	h
Refractivity, (n-1)	—	ε	—	$c\rho$	ν

TABLE II

Scintillation parameters of some occultations.

PLANET/SOURCE	JUPITER/βScoA	NEPTUNE/BD-17°4388	VENUS/M5	JUP./P10
Scale height, H	30	50	5	30
Distance, D	6.5×10^{8}	4.5×10^{9}	1.2×10^{4}	2×10^{5}
Radius, R	7×10^{4}	2.5×10^{4}	6.1×10^{3}	7×10^{4}
Wavelength, λ	(0.5 micron = 5×10^{-10}km)		(13 cm = 1.3×10^{-4} km)	
$a_{diff}=(\lambda D)^{\frac{1}{2}}$	0.57	1.5	1.3	5
$a_{*}=Dr$	0.55	5	0	0
a(effective)	0.8	6	1.3	5
atmospheric length, L	3.6×10^{3}	2.8×10^{3}	0.4×10^{3}	3.6×10^{3}
atmospheric density at $\phi = 0.5$, ρ_p	10^{14}	10^{13}	10^{18}	3×10^{16}
(ρ_p/ρ_o):	10^{-5}	10^{-6}	10^{-1}	3×10^{-3}
$P(\phi=0.5)$	2.4×10^{-2}	8×10^{-5}	3×10^{-5}	3×10^{-6}
$P(\phi=0.2)$	1.3	4×10^{-3}	2×10^{-3}	1.4×10^{-4}
$P(\phi=0.05)$	190	0.6	0.25	0.02
$P(\phi=0.01)$	4×10^{4}	140	60	5
(Note: All lengths are in kilometers.)				
V_o(km/sec)	16	18	7.5	–
f_c(Hz)	3.2	0.5	0.9	–

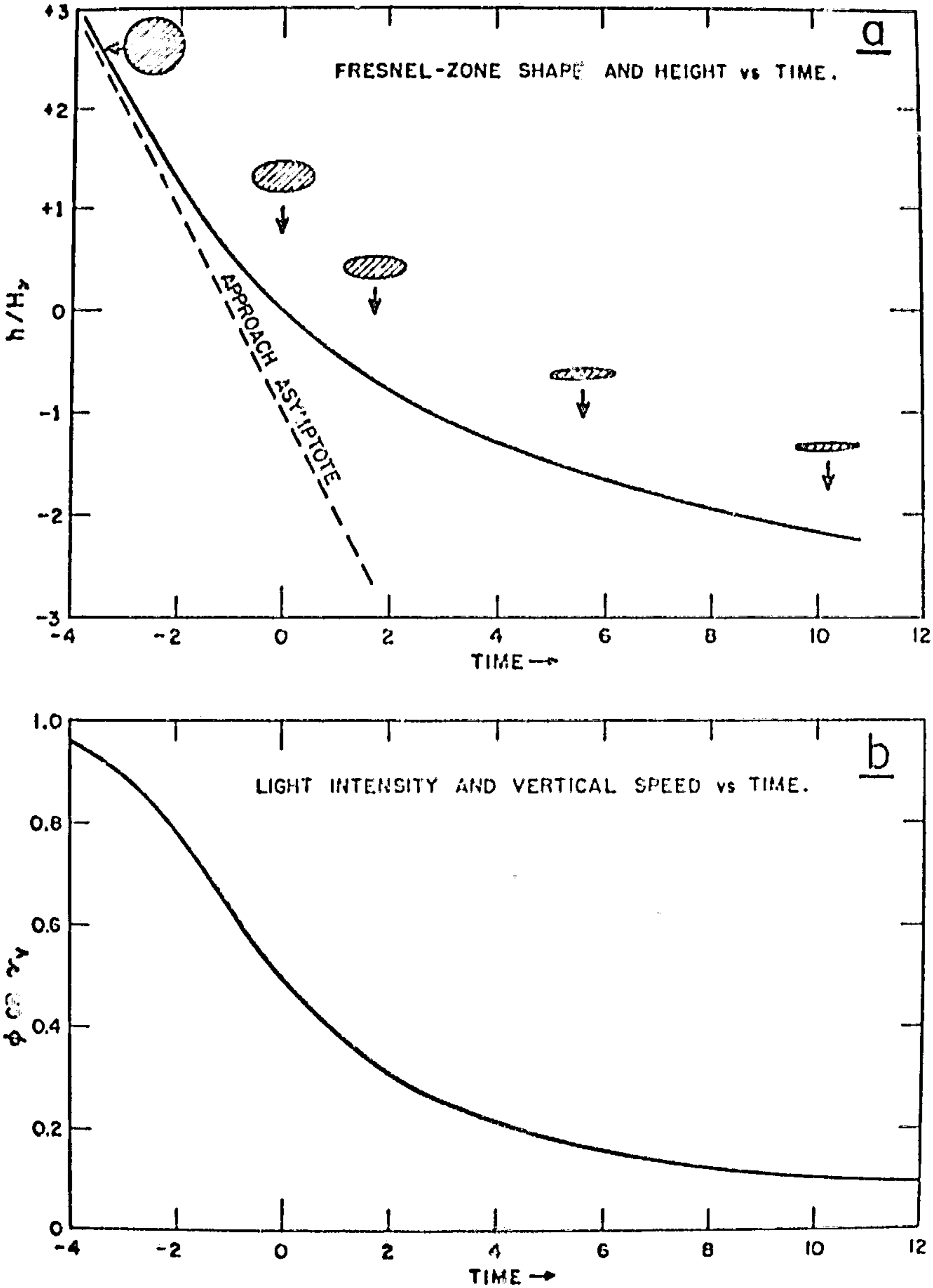
FIGURE CAPTIONS

Figure 1. (a) Path and shape of Fresnel zone, for a 45° entry angle. (b) Corresponding light intensity (or vertical velocity component). An exponential atmosphere is assumed. The horizontal velocity component is constant, so horizontal position is proportional to time.

Figure 2. Effects of Fresnel-zone and trajectory flattening on spatial and temporal frequency spectra: (a) at the start of an occultation; and (b) at the level $\phi = 0.5$. An initial entry angle of 45° is illustrated. Spatial frequencies along the dashed line all contribute to the same temporal frequency f in both cases. Note that the Fresnel zone and the vertical component of $V_{\perp}$ are both compressed by the same factor ϕ.

Figure 3. The Fresnel zones, and their paths through the atmosphere, for two wavelengths. Solid lines: red; dashed lines: blue. At the beginning of occultation (A) the two Fresnel zones overlap completely and the same scintillations appear at both wavelengths simultaneously. At some later point (B) the two Fresnel zones partly overlap but are separated by atmospheric dispersion. The overlap of the blue (dashed) Fresnel zone with the red (solid) zone remains large until B′, when it reaches the same height h_1. Thus if a spike appears in the red curve at B, it will also appear in the blue curve at B′; however, the two spikes may have different heights, as the overlap is only partial. In the tail of the curve (C) the two zones never overlap, and scintillations become uncorrelated. But a layer at height h_2 would still be encountered by the blue Fresnel zone at C′; so the correlation of

spike heights would still be as good as at A, though after a time lag. The two paths should be scaled vertically in the inverse ratio of the atmospheric refractivities at the two wavelengths.



a
FRESNEL-ZONE SHAPE AND HEIGHT vs TIME.
APPROACH ASYMPTOTE
h/H
+3
+2
+1
0
-1
-2
-3
-4
-2
0
2
4
6
8
10
12
TIME →
b
LIGHT INTENSITY AND VERTICAL SPEED vs TIME.
1.0
0.8
0.6
0.4
0.2
0
-4
-2
0
2
4
6
8
10
12
TIME →

Fig.1

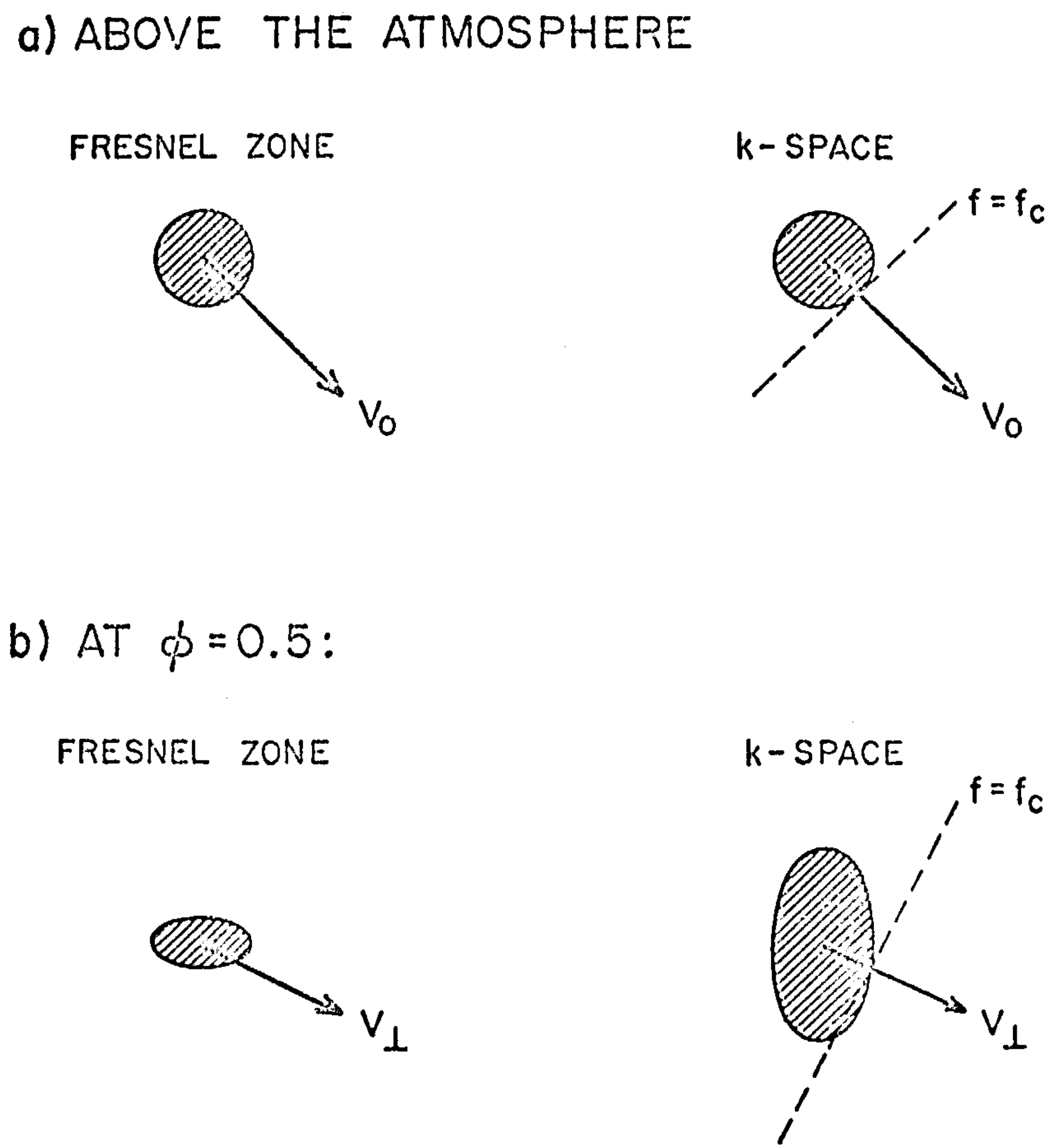

a) ABOVE THE ATMOSPHERE
FRESNEL ZONE
k - SPACE
f = f_c
V_0
V_0
b) AT φ = 0.5:
FRESNEL ZONE
k - SPACE
f = f_c
V_⊥
V_⊥

Fig.2

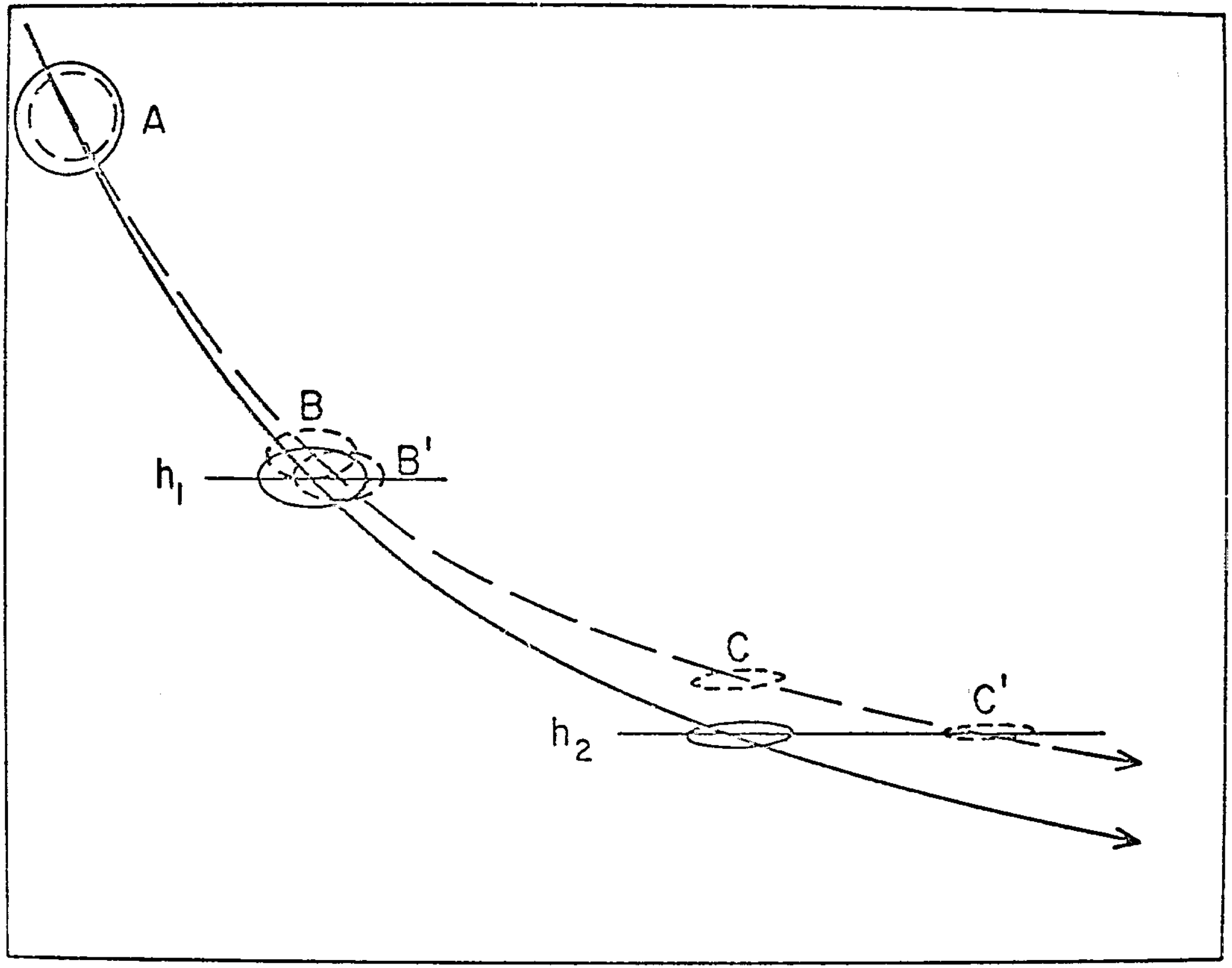


Fig. 3