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A MONTE CARLO INVESTIGATION OF THRUST IMBALANCE OF SOLID ROCKET MOTOR PAIRS

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16. ABSTRACT

A technique is described for theoretical, statistical evaluation of the thrust imbalance of pairs of solid-propellant rocket motors (SRMs) firing in parallel. Sets of the significant variables, determined as a part of the research, are selected using a random sampling technique and the imbalance calculated for a large number of motor pairs. The "simplified computer program" described in previous NASA Contractor Reports (Nos. CR-129024 and CR-129025) is used to evaluate the performance of the SRM pairs. The performance model is upgraded to include the effects of statistical variations in the ovality and alignment of the motor case and mandrel. Effects of cross-correlations of variables are minimized by selecting for the most part completely independent input variables, over forty in number. The imbalance is evaluated in terms of six time - varying parameters as well as eleven single valued ones which themselves are subject to statistical analysis. A sample study of the thrust imbalance of 50 pairs of 146 in. dia. SRMs of the type to be used on the Space Shuttle is presented. The FORTRAN IV computer program of the analysis and complete instructions for its use are included. Performance computation time for one pair of SRMs is approximately 35 seconds on the IBM 370/155 using the FORTRAN H compiler.

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and

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TABLE OF CONTENTS

Acknowledgements ii

List of Figures iv

List of Tables viii

Nomenclature ix

I. Introduction and Summary 1

II. Analysis 4

Internal Ballistics (Main Program). 4

Internal Ballistics (Area Subroutine) 7

Ovality and Malalignment (Oval Subroutine) 7

Statistical Analysis 10

III. Discussion of Input and Output 16

IV. The Computer Program 36

Input Data 36

Program Listing 44

V. Sample Study 102

VI. Concluding Remarks. 118

References 119

Appendix 120

LIST OF FIGURES

Figure II-1. Geometry for analysis of ovality and malalignment. . . 8

Figure II-2. General flowchart of setup and input subroutines . . 12

Figure II-3. Example of the two general types of CDFs 13

Figure III-1. Basic motor dimensions 21

Figure III-2. Standard star grain cross-section 25

Figure III-3 Truncated (slotted tube) star grain cross-section. . 26

Figure III-4 Wagon-wheel grain cross-section. Calculated variables are circled 27

Figure IV-1. Schematic of data deck 45

Figure IV-2. Block diagram of computer program 50

Figure V-1. Thrust versus time for one pair of SRMs of sample case (CalComp plot reduced size) 110

Figure V-2. Thrust during tailoff versus time for one pair of SRMs of sample case (CalComp plot-reduced size). . . 111

Figure V-3. Thrust imbalance versus time for one pair of SRMs of sample case (CalComp plot) 112

Figure V-4. Impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot) 113

Figure V-5. Absolute impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot). 114

Figure V-6. Histogram of absolute thrust imbalance values for fifty motor pairs from the sample study 117

Figure A-1. Thrust versus time for two SRMs with propellant density ρ difference of 3% 121

Figure A-2. Thrust versus time for two SRMs with burning rate coefficient, a_1 and a_2, difference of 3% 122

LIST OF FIGURES (CONT'D)

Figure A-3. Thrust versus time for two SRMs with burning rate exponent, n_1 and n_2, difference of 3%. 123

Figure A-4. Thrust versus time for two SRMs with characteristic velocity C* difference of 3% 124

Figure A-5. Thrust versus time for two SRMs with ratio of specific heats γ difference of 3%. 125

Figure A-6. Thrust versus time for two SRMs with nozzle exit diameter D_e difference of 3% 126

Figure A-7. Thrust versus time for two SRMs with nozzle initial throat diameter D* difference of 3%. 127

Figure A-8. Thrust versus time for two SRMs with nozzle exit divergence angle α_n difference of 0.3° (2.7%). . . . 128

Figure A-9. Thrust versus time for two SRMs with aft end grain tapered length L_{T_a} difference of 3% 129

Figure A-10. Thrust versus time for two SRMs with aft end radial displacement of tapered grain x_{Ta} difference of 0.3 inches (9.9%) 130

Figure A-11. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to bore taper z_o difference of 0.3 inches (12.4%) . . . 131

Figure A-12. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to exterior taper difference z_c of 0.3 inches (0 base value) 132

Figure A-13. Thrust versus time for two SRMs with out-of-round conditions of the grain exterior at the aft end ΔR_{cn} of 0 and 0.5 inches (α_{an} = 0) 133

Figure A-14. Thrust versus time for two SRMs with out-of-round conditions of the grain exterior at the aft end ΔR_{cn} of 0 and 0.5 inches (α_{an} = 90°) 134

Figure A-15. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the aft end ΔR_{gn} of 0 and 0.5 inches 135

LIST OF FIGURES (CONT'D)

Figure A-16. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the head end ΔR_{gh} of 0 and 0.5 inches 136

Figure A-17. Thrust versus time for two SRMs with eccentricities of grain exterior and interior at the aft end e_{xn} of 0 and 0.5 inches 137

Figure A-18. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end e_{xh} of 0 and -0.5 inches 138

Figure A-19. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end e_{xh} of 0 and -0.5 inches (e_{xn} for Figs. A-18 and A-19 is 0 and +0.5, respectively) 139

Figure A-20. Thrust versus time for two SRMs with radial nozzle throat erosion rate E_{ref} difference of 20%. . 140

Figure A-21. Thrust versus time for two SRMs with bulk propellant temperature T_{gr} difference of 3°F. 141

Figure A-22. Thrust versus time for two SRMs with ignition delay T_{igr} difference of 0.4 sec 142

Figure A-23. Thrust versus time for two SRMs with average grain outside diameter D_o difference of 0.3 inches (0.2%) 143

Figure A-24. Thrust versus time for two SRMs with average grain inside diameter D_i difference of 0.3 inches (0.5%) . 144

Figure A-25. Thrust versus time for two SRMs with angle of aft burning surface θ_G difference of 3° (28.8%) 145

Figure A-26. Thrust versus time for two SRMs with length of circular perforated grain L_{Gci} difference of 3.0 inches (0.3%) 146

Figure A-27. Thrust versus time for two SRMs with length of grain L_{Gni} associated with θ_G difference of 3% . . . 147

LIST OF FIGURES (CONT'D)

Figure A-28. Thrust versus time for two SRMs with length of star grain L_{Gsi} difference of 3.0 inches (1.6%). . . . 148

Figure A-29. Thrust versus time for two SRMs with radius of star grain fillet difference f of 0.3 inches (2.4%) 149

Figure A-30. Thrust versus time for two SRMs with average initial radius of star grain truncation R_p difference of 0.3 inches (2.4%) 150

Figure A-31. Thrust versus time for two SRMs with average web thickness of truncated star grain τ_s of 0.3 inches (3.7%) 151

LIST OF TABLES

TABLE IV-1. EXAMPLE DATA SHEETS 46

TABLE IV-2. MONTE CARLO PERFORMANCE ANALYSIS OF SRM PAIRS. . 51

TABLE V-1. FIXED INPUT VALUES OF SAMPLE CASE 103

TABLE V-2. INPUT DATA FOR STATISTICAL VARIABLES OF SAMPLE CASE 104

TABLE V-3. PRINTOUT OF STATISTICAL INPUT DATA FROM SAMPLE CASE 107

TABLE V-4. PRINTOUT OF INPUT VARIABLES FOR ONE SRM FROM SAMPLE CASE 108

TABLE V-5. PORTION OF PRINTOUT OF TRANSIENT VALUES FOR ONE SRM FROM SAMPLE CASE 109

TABLE V-6. OUTPUT VALUES FOR FINAL MOTOR PAIR AND STATISTICAL CHARACTERISTICS OF TWENTY-FIVE MOTOR PAIRS FROM SAMPLE STUDY 115

TABLE V-7. SELECTED STATISTICAL CHARACTERISTICS OF FIFTY MOTOR PAIRS FROM THE SAMPLE STUDY 116

NOMENCLATURE

An asterisk before the symbols indicates an input variable. All input subscripted and non-subscripted variables are listed separately.

English Symbol	Definition	Units Used
a_c, b_c	Major and minor semiaxis, respectively, of grain exterior used in the ovality analysis.	in.
$*a_1, *a_2$	Propellant burning rate coefficient above and below the transition pressure, respectively.	in/sec-psi^n
$*a_{c*p}, a_{\gamma p}$	Pressure sensitivity of C* and γ, respectively.	—
$*a_{c*T}$	Temperature sensitivity of C*	/°F
a_g, b_g	Major and minor semiaxis of grain interior, respectively, used in ovality analysis.	in.
A_{bn}, A_{bp}, A_{bs}	Total burning surface associated with nozzle end, port and slot surfaces, respectively.	in^2
$*A_{bnT}$	Burning surfaces (as a function of y) at nozzle end for tabular input.	in^2
$*A_{bpT}$	Burning surfaces (as a function of y) of port sides for tabular input.	in^2
$*A_{bsT}$	Burning surfaces (as a function of y) of slots for tabular input.	in^2
$*A_{phT}, *A_{pnT}$	Controlling port areas at head and aft ends, respectively (as a function of y) for purely tabular inputs.	in^2
C_v	Coefficient of variation; i.e., the ratio of the standard deviation to the mean.	—

NOMENCLATURE (Continued)

English Symbol	Definition	Units Used
$C^*, {}^*C_n^*$	Characteristic exhaust velocity at T_{gr} and P_{on} and at standard conditions, respectively.	ft/sec
$^*C_{op}$	Integer designating shape of grain ends.	—
$D^*, {}^*D_i^*$	Instantaneous and initial diameters, respectively, of the nozzle throat.	in.
$^*D_e$	Diameter of the nozzle exit.	in.
$^*D_i$	Length average initial diameter of controlling length of circular perforated grain.	in.
$^*D_o$	Length average outside diameter of circular perforated grain, excluding lengths in closures.	in.
$^*D_{t\ ref}$	Reference nozzle throat diameter used in the nozzle throat erosion equation.	in.
$^*D_{TP}$	Initial diameter of the thrust termination passageways in the grain.	in.
$^*f$	Fillet radius at star valley.	in.
$^*e_{xh}, {}^*e_{yh}$	The eccentricities of the center of the grain interior with respect to the center of the grain exterior in the x_c and y_c directions, respectively, at the head end reference plane.	in.
$^*e_{xn}, {}^*e_{yn}$	The eccentricities of the center of the grain interior with respect to the center of the grain exterior in the x_c and y_c directions, respectively, at the nozzle end reference plane.	in.
$E_n, {}^*E_{ref}$	Radial erosion rate and reference erosion rate, respectively, of the nozzle throat.	in/sec
F	Thrust	lbf.
g_o	Gravitational constant	(ft/sec^2)lbm/lbf
$^*G_{op}$	Integer designating type of grain perforation.	—
$^*h_1, {}^*h_2$	The half width of the first and second set of points, respectively, of a wagon-wheel grain.	in.
$^*h_b$	Estimated burnout altitude.	ft.
h_{cn}	Axial length of nozzle closure.	in.

English Symbol	Definition	Units Used
$*I_{eo}$	Integer designating option of ovality and malalignment calculations.	—
$*I_I$	Number of steps in the integrations of perimeters in ovality analysis subroutine.	—
$*I_{op}$	Integer designating type of program input.	—
$*I_{po}$	Integer designating options of plotting results and obtaining special outputs.	—
I_{sp}, I_T	Specific and total impulse, respectively.	lbf-sec/lbm,lbf-sec
$*I_{xi}, I_{xf}$	The initial and final seed numbers, respectively, for the random number generator.	—
K	Criterion value in Pearson's system for determining frequency curves.	—
$*\ell_1, *\ell_2$	The length of the two parallel sides of the first and second set of points, respectively, of a wagon-wheel grain.	in.
ℓ_f	Distance between center line of motor and fillet center of standard star grain.	in.
ℓ_s	Length of sides of truncated star point excluding fillets.	in.
$*\ell_{TP}$	Initial length of termination passages between centers of gravity of perimeters of bases.	in.
*L	Total initial perforated grain length including gaps at slots.	in.
$L_{Gc}, *L_{Gci}$	Instantaneous and initial total axial lengths, respectively, of circular perforated grain (not including gaps).	in.
$*L_{Gni}$	Initial slant length of grain surface at the nozzle end.	in.
$*L_{Gsi}$	Initial total length of star-shaped perforated grain.	in.

NOMENCLATURE (Continued)

English Symbol	Definition	Units Used
$*L_{Ta}$	Estimated length of grain at the aft end at start of first tailoff having an additional taper not represented by z_o or θ_G.	in.
n	Number of observations of a statistically distributed variable or burning rate exponent.	—
$*n_1, *n_2$	Burning rate exponents above and below the transition pressure, respectively.	
$*n_n$	Number of burning flat end surfaces of a star grain located at the extreme nozzle end of the chamber.	—
$*n_p$	Number of star points.	—
$*n_s$	Number of burning flat end surfaces of a star grain not located at the nozzle end of the chamber.	—
*N(j)	Integer designating whether or not a specific output plot is desired.	—
P	Pressure.	lbf/in^2
$P_{h\ max}$	Maximum head end chamber pressure calculated by the program.	psia
$*P_{ref}$	Reference average nozzle stagnation pressure used in the nozzle throat erosion equation.	psia
$*P_{tran}$	Transition pressure at which the burning rate coefficient and exponent change.	psia
$*Q_{op}$	Integer designating grain arrangement.	—
r	Burning rate.	in/sec
$*R_{is}, *R_{iws}, *R_{iww}$	Initial inside radius of the propellant web of a standard star, truncated star, and wagon-wheel grain, respectively.	in.
R_7	Distance from center of curvature of a spherical end of circular perforated grain to the burning surface associated with θ_G.	in.
$*R_c$	Outside radius of a star grain.	in.
$*R_{OA\ell}$	The propellant oxidizer to aluminum weight ratio.	—

English Symbol	Definition	Units Used
$^*R_p$	Initial radius of truncated star points.	in.
$^*S$	Number of burning flat slot sides of a circular perforated grain not including the nozzle end.	—
$^*S_{op}$	Integer designating type of star grain.	—
t	Time	sec.
t_b	Calculated total operating time of motor.	sec.
$^*t_{b1}$	Estimated time at burnout.	sec.
$^*t_{igr}$	Ignition delay at 60°F grain temperature.	sec.
$^*t_{max\ q}$	Estimated time at which maximum dynamic pressure on the vehicle occurs.	sec.
t_{ti}	Earlier time at which tailoff begins in a motor pair.	sec.
$^*T_{gr}$	Bulk temperature of the propellant grain.	°F
$^*V_{ciT}$	Initial volume of chamber gases associated with tabular input.	in^3
W_{p1}, W_{p2}, W_p	Total masses of propellant burned based on mass discharge rate, volume calculations and the arithmetic average of W_{p1} and W_{p2}.	lbm.
x	Value of general statistical variable.	Units vary
x_c, y_c	Coordinates of the grain exterior used in the ovality analysis.	in.
x_g, y_g	Coordinates of the grain interior used in the ovality analysis.	in.
$^*x_{out}$	Distance burned at which propellant breaks up.	in.
$^*x_{Ta}$	Difference in web thicknesses at ends of L_{Ta}.	in.
$^*x_{Tz}$	Difference between the initial circular perforated grain diameter and the nozzle end of L_{Gci} and the nominal value of D_i less z_o and less twice x_{Ta}.	in.
y	Distance propellant has burned from initial surface.	in.

NOMENCLATURE (Continued)

English Symbol	Definition	Units Used
$*z_o, z$	Initial and instantaneous differences, respectively, between web thicknesses due to grain bore taper at head and nozzle ends of controlling grain length, excluding any initial length associated with L_{Ta} or θ_G.	in.
$*z_c$	Initial difference between web thicknesses due to grain exterior taper at the head and aft ends of the controlling grain length.	in.
Greek Symbol		
$*\alpha_{ah}, *\alpha_{an}$	The angular orientation of the ovality of the grain interior with respect to the grain exterior at the head and nozzle end of the grain, respectively.	degrees
$*\alpha_1, *\alpha_2$	The angle between the slant sides of a wagon-wheel grain point and the center line of the point for the first and second set of points, respectively.	degrees
$*\alpha_{eb}$	Erosive burning coefficient in the Robillard-Lenoir rule.	$in^{2.8}$-$ft^{0.8}/sec^{1.8}lbf^{0.8}$
$*\alpha_n$	Nozzle exit half angle.	degrees
$*\beta$	Erosive burning pressure coefficient in the Robillard-Lenoir rule.	—
β_1, β_2	Ratio of the square of the third moment about the mean to the cube of the second moment and the ratio of the fourth moment to the square of the second moment, respectively, for a statistically distributed variable.	—
$\Delta P_c/\Delta y$	Rate of change of chamber pressure with respect to distance burned.	lbf/in^3
$*(\Delta P/\Delta y)$	Depressurization rate at which propellant is extinguished for computation control purposes.	lbf/in^3
$*\Delta R_{ch}, *\Delta R_{cn}$	One half the difference between the maximum and minimum diameter of the grain exterior at the head and nozzle end reference planes, respectively.	in

Greek Symbol	Definition	Units Used
$*\Delta R_{gh}, *\Delta R_{gn}$	One half the difference between the maximum and minimum diameter of the grain bore at the head and nozzle end reference planes, respectively.	in.
$\Delta y, *\Delta y_1$	Incremental distance burned and initial value of same, respectively.	in.
$*\zeta_F$	Thrust loss coefficient.	—
$*\theta_{cn}, *\theta_{ch}$	Approximate acute angle bonded circular perforated grain makes with motor center line at the head and nozzle closures, respectively. Also referred to as the grain contraction angles.	degrees
$*\theta_f$	Angular location of fillet center of standard star from line of symmetry.	degrees
θ_{fw}	Angular location of fillet centers with respect to radial center line of wagon-wheel grain points.	radians
$*\theta_G$	Angle burning surface element of circular grain makes with longitudinal axis of motor at the nozzle end of the chamber.	degrees
$*\theta_n$	Nozzle cant angle.	degrees
$*\theta_p$	Angle of standard star point.	degrees
θ_s	Half angle of star grain sector.	radians
θ_{sl}	Angular location of slot side of truncated star grain.	radians
$*\theta_{TP}$	Acute angle between axis of thrust termination passage and motor axis.	degrees
λ	Volumetric loading density; i.e., initial volume occupied by propellant divided by empty case volume.	—
μ_2, μ_3, μ_4	Second, third and fourth moments, respectively, of a statistically distributed variable about its mean.	Units vary

Greek Symbol	Definition	Units Used
*$(\pi_p)_K$	Temperature sensitivity coefficient of pressure at constant $K = A_b/A^*$.	/°F
*ρ_p	Solid propellant density.	$slugs/in^3$
σ	The standard deviation of a statistically distributed variable; i.e., the square root of the second moment about its mean value.	Units vary
σ_1,σ_2	The square root of the second moment of a statistically distributed variable about zero (σ_1) and $\sigma_1/\sqrt{2}$, respectively.	Units vary
τ_s	Thickness of propellant web at slot bottom of truncated star grain.	in.
*τ_{Teff}	Estimated "effective" web thickness of termination port.	in.
*τ_w	Web thickness of main propellant grain.	in.
τ_{ws}	Web thickness of standard star (same as τ_w except for some combination grains).	in.
τ_{ww}	Web thickness of wagon-wheel grain.	in.

Subscripts

a	Value during web action time.
c	Case, grain exterior or chamber value.
f	Final.
g,G	Grain interior.
h	Head end of grain.
i	Initial.
ig	Ignition.
max	Maximum value.
min	Minimum value.
n	Nozzle or nozzle end of grain.
o	Stagnation.
q	Dynamic pressure.
t	Value during tailoff.

Subscripts (Cont'd)

w	Value at web time.
100k	Value when thrust has decayed to 100,000 lbs.

Superscripts

*	Choked throat value.
–	Arithmetic average value over Δy or statistical mean.
•	Time rate of change.

I. INTRODUCTION AND SUMMARY

This report presents the results of research performed at Auburn Unviersity during the period May 31 to November 30, 1974, under modification No. 6 to the Cooperative Agreement, dated October 6, 1969, between NASA Marshall Space Flight Center and Auburn University. The principal objective of the research was to develop a technique for statistically investigating the thrust imbalance of a pair of solid rocket motors (SRMs) firing in parallel.

The study of thrust balance and imbalance is of particular interest with regard to application to the NASA Space Shuttle because two very large solid motors fire in parallel on the Shuttle. Although a similar arrangement was utilized in the Titan program with somewhat smaller motors, because of the differences in the configuration of the Titan and Shuttle, it is more imperative that the thrust on the solid motor booster stage be uniform to assure proper guidance and control for the overall vehicle.

Past analyses of SRM reproducibility have been concerned mainly with characteristics of an entire population of single motors rather than pairs. Usually a non-time varying parameter such as average thrust and total impulse has been of interest, the distributions of variables affecting the parameter are assumed normal in the statistical sense, and cross-correlation effects are neglected. Reference 1 typifies such an analysis.

For the present investigation the Monte Carlo technique (Reference 2) was selected. Sets of the significant variables are selected on a probability basis using a random sampling technique and the imbalance calculated for a large number of motor pairs using a mathematical model of the internal ballistics. This method is not limited to normal distributions of the input variables. Raw data, histograms, or cumulative distribution functions may be used for any of forty or more controlling variables to specify the statistical input.

Errors arising from neglecting the cross-correlation of variables are minimized in this study by selecting for the most part completely independent variables. However, the analysis may be readily extended to account for the cross-correlations where they are shown to exist and when they are calculable. The imbalance is evaluated in terms of six time-varying parameters as well as eleven single valued ones which themselves are subject to statistical analysis.

Application of the Monte Carlo method, which requires evaluation of a large number of motor pairs, is made practical and economical through the utilization of the simplified computer program for the internal ballistics presented in References 3, 4, and 5. Familiarity of the reader with References 3 and 4 or alternatively with 5, which is a consolidated report on 3 and 4, is assumed. Although the simplified program makes use of numerous approximations, the effects of the vast majority of the variables affecting imbalance are represented. Also, the essence of the phenomena controlling the critical tailoff portion of the thrust trace are embodied in the program which has been upgraded during the current effort to refine the analysis and to incorporate additional input variables which may now be statistically distributed. It is fundamental to the approach that, in spite of bias in the performance calculated resulting from the approximations in the ballistic analysis, the bias will reflect equally in each motor of a pair. Thus the difference in performance calculations for a pair of SRMs will be of a higher degree of accuracy than the individual motor calculations assuming that the significant parameters causing the difference are incorporated into the model and their effects precisely determined.

Owing to the complexity of the problem it was not possible to include all variables or give precise representation of the influences of all those that are included. Nevertheless, it is felt that the model developed yields a good first estimate of the difference in performance between a pair of SRMs. Early in the study, the sensitivities of performance to the variation in the various input parameters was evaluated by changing each variable one at a time a small amount between a pair of SRMs and computing the performance of each motor. Partial results are given in the Appendix in the form of pairs of thrust-time traces obtained for each variable. These traces were obtained directly from the computer output using the CalComp plotter. Although only very minor performance differences were obtained in many cases, it was decided that for the Monte Carlo investigation all but a few of the variables would be statistically determined in order to minimize cumulative errors that might result from neglect of a large number of seemingly unimportant variables.

Illustrative of the comprehensiveness of the analysis is the capability which has been added to calculate as a computer program option the approximate effects of case and mandrel ovality, eccentricity, and malalignment. On the other hand, the effects of grain temperature gradient (as opposed to bulk temperature) variation between rocket motors are not within the capability of the program to evaluate. However, the ovality analysis would permit simulation of the approximate results of one special situation for temperature gradient from which insight into the effects of this important variable may be gleaned.

The Monte Carlo computer program retains the capability of the programs of References 3, 4, and 5 to treat segmented configurations with both star and circular perforated grains present in various arrangements in the same SRM as well as monolithic grains with either a circular perforation or one of the three most common types of star grains. However, the program is more accurate if most of the grain is circular perforated as in the Space Shuttle. Also, whereas tabular input may still be used to specify portions of the burning surface, the statistical input variables influence only the burning geometry that is computed from the program equations. Basic assumptions of the program are that the propellant does not break up and is not extinguished except by being completely consumed by burning normal to the propellant surface.

Ignition transients are not calculated in the analysis. However, the variations in the initial equilibrium chamber pressures as calculated by assuming the grains are completely ignited have been incorporated along with statistically determined values of ignition delay. It is believed that this treatment embodies the significant effects of ignition upon the remainder of the trace.

Perhaps the greatest limitation of the analysis is its reliance on the availability of reliable statistical data to specify accurately the distributions of the numerous variables. For example, there simply have not been enough SRMs built in the size class of those in the Space Shuttle stage to provide direct statistical samples. For small motors, where large populations exist from which data may be directly applied or scaled to another SRM, documentation of manufacturing variations is often incomplete or unavailable. In the test cases used in this report, drawing tolerance limits were taken in a number of cases to represent the ranges corresponding to six standard deviations in assumed normally distributed dimensions. Hopefully, this research will provide a stimulus for acquisition of statistical data which will reduce reliance on assumptions of this nature. As suggested earlier and as is demonstrated in the sample case, the computer program is virtually unlimited in its ability to treat the various types of statistical distributions.

The program which is written in FORTRAN IV requires approximately 2,000 computer cards not including the data cards necessary to specify the input variables and their statistical distributions. The compilation time is approximately 1 minute and 10 seconds on the IBM 370/155 computer. Performance computation time for 1 pair of SRMs using recommended increment sizes in the several integration processes involved is approximately 35 seconds using the FORTRAN H compiler.

Section II of this report discusses the changes made to the ballistic performance analysis program and also describes how the statistically distributed variables are treated and sets of random input variables selected. The basic mathematical details of the ovality and malalignment analysis are also given. A discussion of the program input and output is presented in Section III. Although most of the input variables are defined in References 3, 4, and 5, a complete discussion is given here for a ready reference and guide to the user in specification of his problem and the interpretation of the output. Section IV gives the information required to operate the program including data card format and the program listing which contains concise printed definitions of all input variables along with identification of the statistically determined variables to serve as a checklist. In Section V a sample study is presented to demonstrate the setup procedures, format and computational capabilities of the computer program.

II. ANALYSIS

This section of the report describes the changes and additions that have been made to the ballistic analyses of References 3, 4, and 5, and presents the basic elements of the Monte Carlo analysis. Rationale to the selection of input variables is interspersed throughout the discussion. The section is divided into four parts: the first two parts give the changes to the "main" program and the "area" subroutine, the third part gives the ovality and malalignment analysis and the final part is the statistical presentation. It is assumed the reader is familiar with References 3, 4, and 5, and thus in discussing program revisions only information sufficient to identify the changes and the bases for them is given. Complete documentation of the program logic and computational changes is given in the program listing in Section IV. Although a number of completely new subroutines have been incorporated into the program, the methods used are straightforward applications of basic statistics and analytical geometry. Therefore, again concise descriptions of the analyses and their capabilities and limitations are given in lieu of detailed mathematical procedures which can be readily identified from the program listing.

Internal Ballistics (Main Program)

Inert parts and ignition. The computation of the weights of inert parts have been removed from the program since the program is now strictly a performance analysis rather than a design analysis. Also, the option of calculating the ignition transient by the methods of References 4 and 5 has been removed because the complications of the ignition transients make a detailed Monte Carlo impractical from a computer requirements standpoint. However, an ignition delay time has been incorporated as a statistical variable subject to both random variation and systematic variation due to the distribution of bulk temperature of the grain. Also, the initial equilibrium pressure is subject to variations arising from a number of sources. It is believed that this treatment of ignition embodies the significant effects of the ignition transient on the remainder of the thrust time trace.

Throat area. The throat relationship has been modified to account for Reynolds' number and Stanton's number effects of pressure and diameter on heat transfer (Ref. 6) as follows:

$$E_n = E_{n\ ref}\,(P_{on}/P_{ref})^{0.8}(D^*_{ref}/D^*)^{0.2} \tag{1}$$

$$D^* = D_i^* + 2.0\ \Sigma E_n \Delta t \tag{2}$$

$$A^* = \pi D^{*2}/4 \tag{3}$$

Mach number. To account for throat erosion effects, the Mach number at the nozzle exit from which the thrust coefficient is ultimately derived is now recalculated at each time step rather than just initially.

Characteristic velocity and ratio of specific heats. These variables, C* and γ, respectively, are calculated based on the oxidizer to aluminum ratio of the propellant $R_{OA\ell}$ which is introduced as a new independent distributed variable. In this way the cross-correlation between C* and γ has been incorporated. Data for determining separate relationships for C* and γ as a function of $R_{OA\ell}$ should be obtained from thermochemical analyses. The present program uses typical functional relationships obtained from a linear regression analysis of such data. Similar relationships must be established to fit the propellant system under consideration. More rigorously, additional composition components might be included. However, because of the relatively strong influence of the aluminum on the propellant chemistry the two-component oxidizer to aluminum system should convey the largest effects of composition on C* and γ.

Systematic variations in C* and γ have also been incorporated to give the effects of chamber pressure and propellant bulk temperature on C* and of chamber pressure on γ. The relationships are as follows:

$$C^*_{ref} = C^*_n \exp[a_{c^*T}(T_{gr}-60)] \tag{4}$$

$$C^* = C^*_{ref}(P_{on}/1000.0)^{a_{c^*p}} \tag{5}$$

$$\gamma = \gamma_n(P_{on}/1000.0)^{a_{\gamma p}} \tag{6}$$

The detailed attention given to these variables was motivated not only by the cross-correlation but also by the high sensitivity of performance to both C* and γ as revealed by the sensitivity study discussed in Section III. The effect of γ is largely due to its influence on exit Mach number and hence thrust coefficient. In the subsonic flow regime, Mach number is relatively insensitive to variation in γ and it is found convenient to neglect the variations in γ resulting from pressure variations in calculating the Mach number at the end of the grain.

The introduction of $R_{OA\ell}$ as an independent variable raises the question of possible cross-correlation with other independent propellant variables, especially the burning rate coefficient a and density ρ. When such correlations can be identified they may of course be treated in a manner similar to that given for C* and γ. Although the matter has not been investigated fully as of this writing, it appears that factors such as oxidizer particle size distributions, amount of burning rate catalyst and other composition variables will play a more important role than the oxidizer to aluminum ratio in fixing the distributions of burning rate coefficient and density; cross-correlation between $R_{OA\ell}$ and ρ are assumed negligible as have, with somewhat less justification, the correlation between a and ρ, themselves. The latter warrants further investigation as it is intuitively clear that at least a weak cross-correlation exists between a and ρ because of the effects of oxidizer particle size distribution on these parameters.

Burning rate coefficient. Bulk temperature has been introduced as an explicit variable subject to statistical variation. It modifies the burning rate coefficient:

$$a = a_{60^\circ} \exp \left[(1-n)(\pi_p)_k(T_{gr}-60)\right] \qquad (7)$$

Also, the program now makes use of two sets of burning rate coefficients and exponents in recognition of the possibility of separate values applying above and below a specified transition pressure. In conjunction with this, the level above which burning of the propellant is permitted was changed from 30 psia to 5 psia as the thrust of some SRMs may still be significant at low pressure. Choked flow is still assumed at the lower pressure levels.

Time increments. Several changes were made in the calculation of time increments for the purpose of obtaining more accurate representation of the thrust-time characteristics of the individual SRM. First, the time increment is now calculated from the burning rate and incremental distance burned using the burning rate that applies during the increment under consideration rather than the previous increment. This is done except for the purpose of computing changes in port areas which must be known before the rate is calculated. The error in the latter case is minor and a time consuming iterative solution is avoided. Also, time values and corresponding output values are now obtained at the precise y positions that the lengths of grain associated with X_{Ta} and z begin to burnout. These times are identified in the program printout by "initial tailoff begins" and "final tailoff begins," respectively. It was found desirable in general to use a y increment (Δy) of approximately 0.001 of the web thickness. This gives good precision in the calculations and also sufficient values to obtain good graphical portrayals. However, the smaller increment size introduces a difficulty in satisfying the mass flow continuity relationship during rapid pressure changes; the effect of such changes tends to be overestimated in the simplified ballistic model when small increment sizes are used. In References 3, 4, and 5, this difficulty was avoided during tailoff by calculating the nozzle end stagnation pressure directly from the pressure gradient. In the new program, the same method is also applied to the period involving the burning of the length of grain associated with X_{Ta} as similar difficulties were encountered in the region when small increment sizes were used.

Internal Ballistics (Area Subroutine)

During the present study, an error was found in the first expression for A_{bnc} in Eqs. A4c of References 4 and 5. Specifically the first value of R_3 in the equation should be replaced by

$$[(D_o/2)^2 - R_7^2]^{1/2} + [(D_o/2)^2 - (R_7 + y)^2]^{1/2}$$

and R_3 should be replaced by R_7 throughout the remainder of Eqs. A4c. The program listing contains the necessary changes to the computer program. It will be noted that additional changes applicable to Eq. A4c have been made to further refine the closure geometry effects for a circular perforated grain. Also, application of 4b and c has been modified in the computer program to avoid the possibility of negative square roots in terms such as $[D_o^2 - (D_i+2y)^2]^{\frac{1}{2}}$.

Ovality and Malalignment (Oval Subroutine)

Ovality and lack of concentricity of the grain perforation with respect to the motor case can clearly influence the ballistic performance of SRMs during the critical tailoff period. As a first approach to the problem it is assumed that the burning surface geometry embodying the effects of ovality and malalignment may be defined by specifying three radial reference planes - one near the head end of the rocket, one at the aft end of L_{Ta} and one at the aft end of z_o. The reference planes must intersect the cylindrical portion of the rocket motor case to eliminate the effect of end closures on the geometric properties to be calculated. The implied assumption here is that the central portion of the rocket dominates the influences of ovality and malalignment.

The geometry of the reference planes is illustrated by Figure II-1. To fix ideas, both the exterior and interior (bore) surfaces of the grain are assumed to be distorted into oval shaped surfaces from nominally circular ones. However, as discussed later, with some restrictions and loss in accuracy, the analysis is also applicable to grains with the star-shaped perforations treated in the program.

The exterior and interior grain perimeters are next assumed to be elliptical and the interior grain surface to remain elliptical as burning progresses. The latter assumption is not rigorous for burning normal to the surface but the error introduced is insignificant for the small degrees of ovality to be encountered in practice. Thus, the ellipse defining the burning perimeter is given by Equation (8) where y is the distance burned

$$x_g^2/(a_g + y)^2 + y_g^2/(b_g + y)^2 = 1 \qquad (8)$$

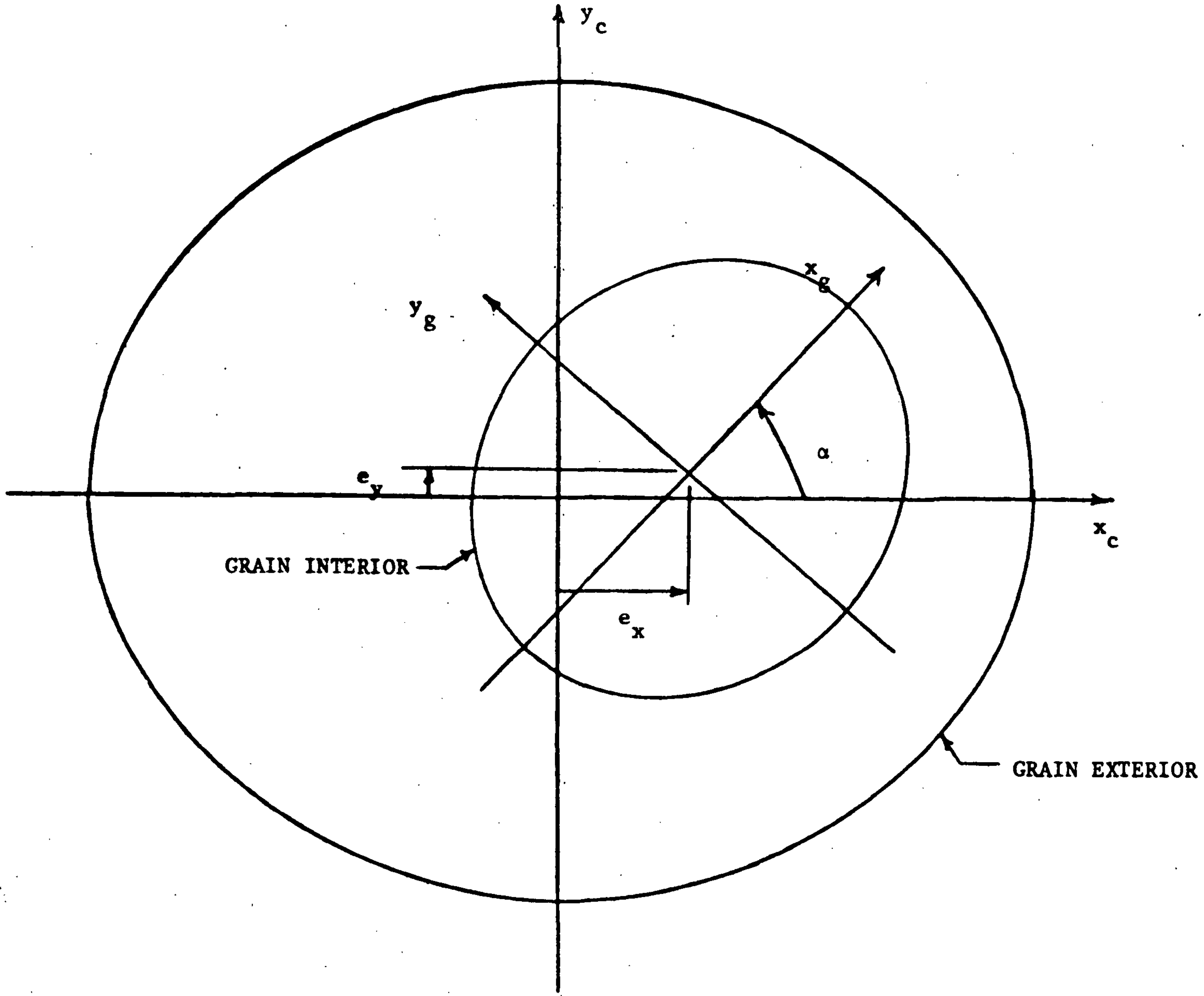

Figure II-1. Geometry for analysis of ovality and malalignment.

and a_g and b_g are semi-axes of the ellipse defining the initial burning grain perimeter. The exterior grain perimeter is expressed in the coordinates x_g and y_g of the interior perimeter by the equation

$$(x_g \cos\alpha - y_g \sin\alpha + e_x)^2 + (a_c^2/b_c^2)(x_g \sin\alpha + y_g \cos\alpha + e_y)^2 = a_c^2 \quad (9)$$

where a_c and b_c are the semi-axes of the ellipse defining the grain exterior. The burning perimeter at each reference plane is determined as follows. Equations (8) and (9) are rewritten in polar coordinates r and θ. The perimeter is determined through numerical integration of the equation

$$s \simeq \int_0^{2\pi} r d\theta \quad (10)$$

At each value of θ in the integration the radial coordinate of the grain bore and exterior is computed from (8) and (9), respectively. If the exterior value exceeds the bore value at the θ position, r is given the value calculated for the bore; otherwise, a zero value is assigned.

Burning perimeters for each reference plane thus vary from one SRM to another as a result of eccentricity of the grain bore and exterior, which is specified by the independent variables e_x and e_y, orientation of the ovality of the bore with respect to the exterior, and the out-of-roundnesses of both the bore and exterior. It is possible that some correlation may exist between these variables in particular cases. However, if the case is segmented, it would appear that the head end and aft end geometric features are independent and they are treated as such in the remainder of this report. On the other hand, in general, there should be a close correspondence between the exterior grain geometries and the locations of the bore centers at the aft ends of L_{Ta} and z_o, and it has been assumed that these are identical. The geometry of the interior grain is determined by the distributed variables D_i, z_o, and x_{Ta}; i.e., these determine the initial semi-axes and the exterior geometry is determined by D_o and z_c where z_c has been introduced to account for axial variations in the outside diameter of the grain. It is clear that the independent specifications of e_x and e_y at both ends of the rocket provides for a statistically distributed malalignment of the grain bore and exterior.

From the burning perimeter values obtained as outlined above, correction factors are next calculated and applied to the standard calculations of perimeter to account for the ovality and malalignment. For this purpose when solution of the ovality equations indicates that burnthroughs have occurred at adjacent reference planes, the burning perimeters are assumed to vary linearly between the planes. When burnthrough has occurred at only one of two adjacent reference planes, the portion of the correction factor applicable to each end is weighted in proportion to the corresponding length that has or has not experienced burnthrough to determine the overall correction factors (computer symbols: SEN, SEH, CHINAV, CHIN and CHIH).

It is recognized that the approach represents considerable idealization of the general behavior to be expected, particularly with respect to the assumed qualities of the axial distributions of the parameters between reference planes. Nevertheless, the model seems to capture the essence of the performance effects associated with ovality and malalignment. Even with highly nonlinearly distributed ovality and eccentricities between reference planes the effects on performance should be roughly the same as determined here. Also, factors such as the precise shape of the ovals would appear to play a secondary role in influencing performance variations.

Care has been taken in the above discussion to differentiate between the grain exterior and the case interior. As far as the equations go the pertinent item is the grain exterior. Its shape, however, may be influenced by the case and it is a choice of the user as to whether or not the statistical variations of ovality of the case alone should determine the variation in the grain exterior or if variation in insulation thickness should be statistically combined with the case variation to arrive at the qualities of the grain exterior.

As mentioned earlier, the analysis may also be applied to star grain configurations. In this case, the star points are in effect disregarded. The rationale for this is that by far the most important effects of malalignment and ovality occur just before and during tailoff and are dominated by the behavior of the remaining propellant web which may be approximated for this purpose by a circular perforated grain of the same web thickness. The capability to treat star grains has thus been incorporated into the computer program. However, it is applicable only when the entire grain is a star grain. When both circular perforated and star grains are present together, it is assumed that the circular perforated grains determine the performance characteristics in as far as the effects of ovality and malalignment are concerned.

An additional application for the ovality and malalignment is noteworthy. This has to do with the temperature gradient within the propellant grain which is an important variable that has not been taken directly into account in the analysis because no convenient way has yet been found to do so. Some insight into the gross effects of such gradients can be gleaned by statistically incorporating additional variations in the out-of-roundness of the grain to represent the general effects of having a biaxial burning rate which could be different in magnitude and orientation at the two main reference planes.

Statistical Analysis

This section describes the statistical analysis used in selecting values for the distributed variables. The basic computational methods as related to the logic of the statistical procedure used in the computer program are presented.

The computer routine is designed to accept several different types of data, perform the specified operations required to obtain a frequency distribution for each variable, and select a value based on its statistical frequency curve. The routine is divided into two subroutines. One subroutine, "setup," is called once and generates the required frequency curves for the statistical variables given. The other, "input," is called as required and provides specific values for each variable given. The latter subroutine utilizes the Monte Carlo technique for selection of values which will be discussed later. Figure II-2 is a general flow chart of the setup and input subroutines.

Frequency curves. The primary task of the setup subroutine is to obtain a frequency curve for each statistical variable from the data given. For each variable, the ultimate product of this subroutine, a cumulative distribution function (CDF), is obtained from its frequency curve. A CDF is a step function which jumps at regular intervals and is constant between jump points. At each jump point the magnitude of the jump is the probability that the variable will be within that interval; thus, as the name implies, the probabilities are accumulated over the range of the statistical variable. Examples of the types of CDFs produced by the setup subroutine are shown in Figure II-3. This CDF can be obtained in the present program from several types of input data, ranging from raw data points to specifying the actual CDF directly (See Section IV).

Basically, there are two classes of requests allowed for input statistical variables. The first class contains all variables which require little if any statistical analysis. This includes such requests as to hold certain variables constant or to obtain the CDF directly from a given histogram, as illustrated in Fig. II-3A. The other class requires statistical analysis to obtain the frequency curve. The analysis consists of obtaining the first four moments of the statistical variable and generating an equation which approximates the actual frequency curve for the data. The method selected for obtaining the frequency curve from the first four moments is known as Pearson's system (Ref. 7). The variables and sequence of calculations were chosen to parallel those of Pearson's system which is discussed in sufficient detail in Ref. 7 to permit direct adaptation to a computer program.

Basically, Pearson's system consists of a family of curves with a criterion value used to determine which equation of the family best describes the data. This criterion value, K, is evaluated as follows:

$$K = \beta_1(\beta_2 + 3)^2/4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6) \qquad (11)$$

where

$$\beta_1 = \mu_3{}^2/\mu_2{}^3 \qquad (12)$$

$$\beta_2 = \mu_4/\mu_2{}^2 \qquad (13)$$

Here μ_2, μ_3, μ_4 are the second, third and fourth moments of the variable about its mean, respectively.

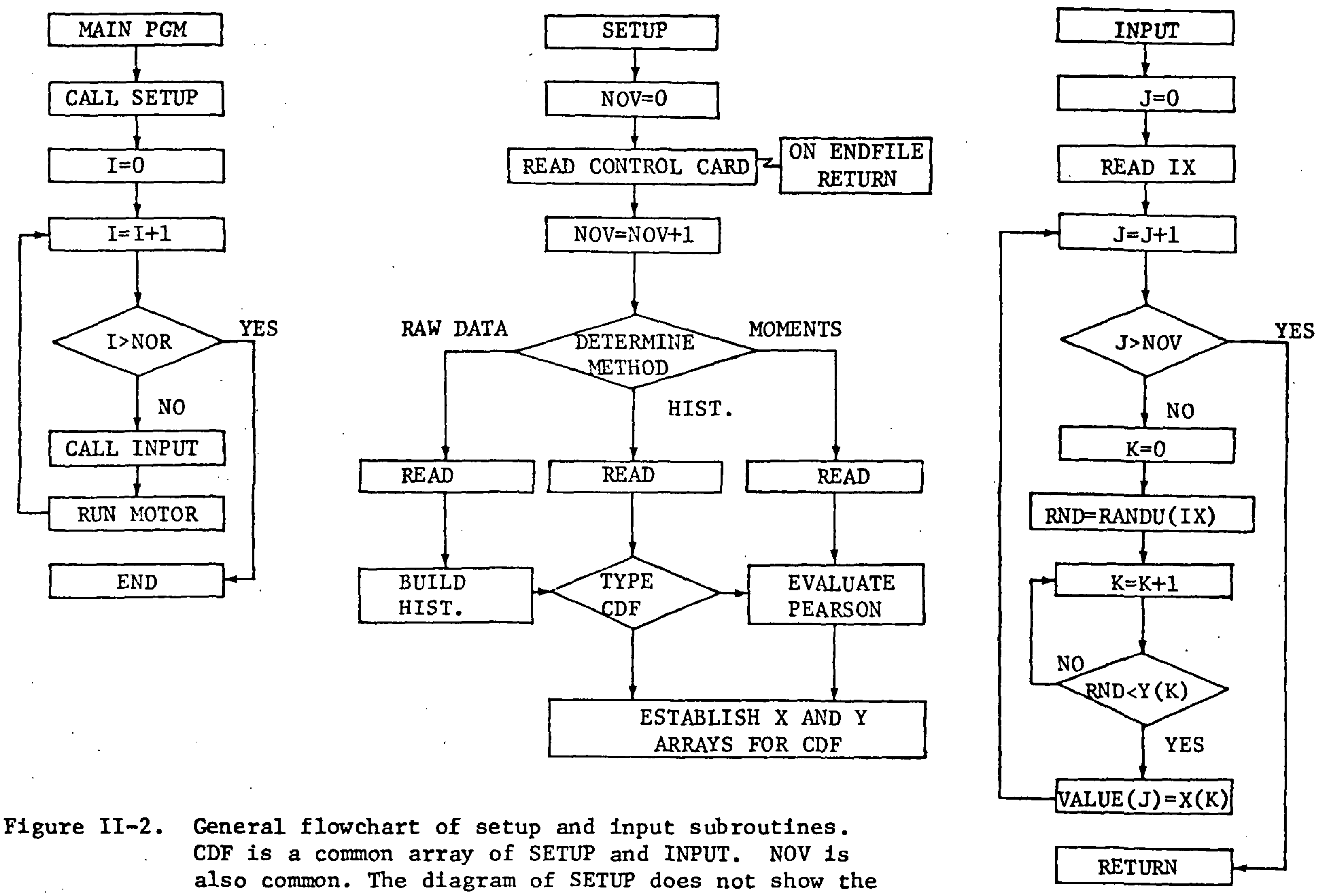

Figure II-2. General flowchart of setup and input subroutines. CDF is a common array of SETUP and INPUT. NOV is also common. The diagram of SETUP does not show the handling process of constant variables (Code 60). This was omitted due to the complicated mechanism for core savings which does not aid in the understanding of the general concept of SETUP.

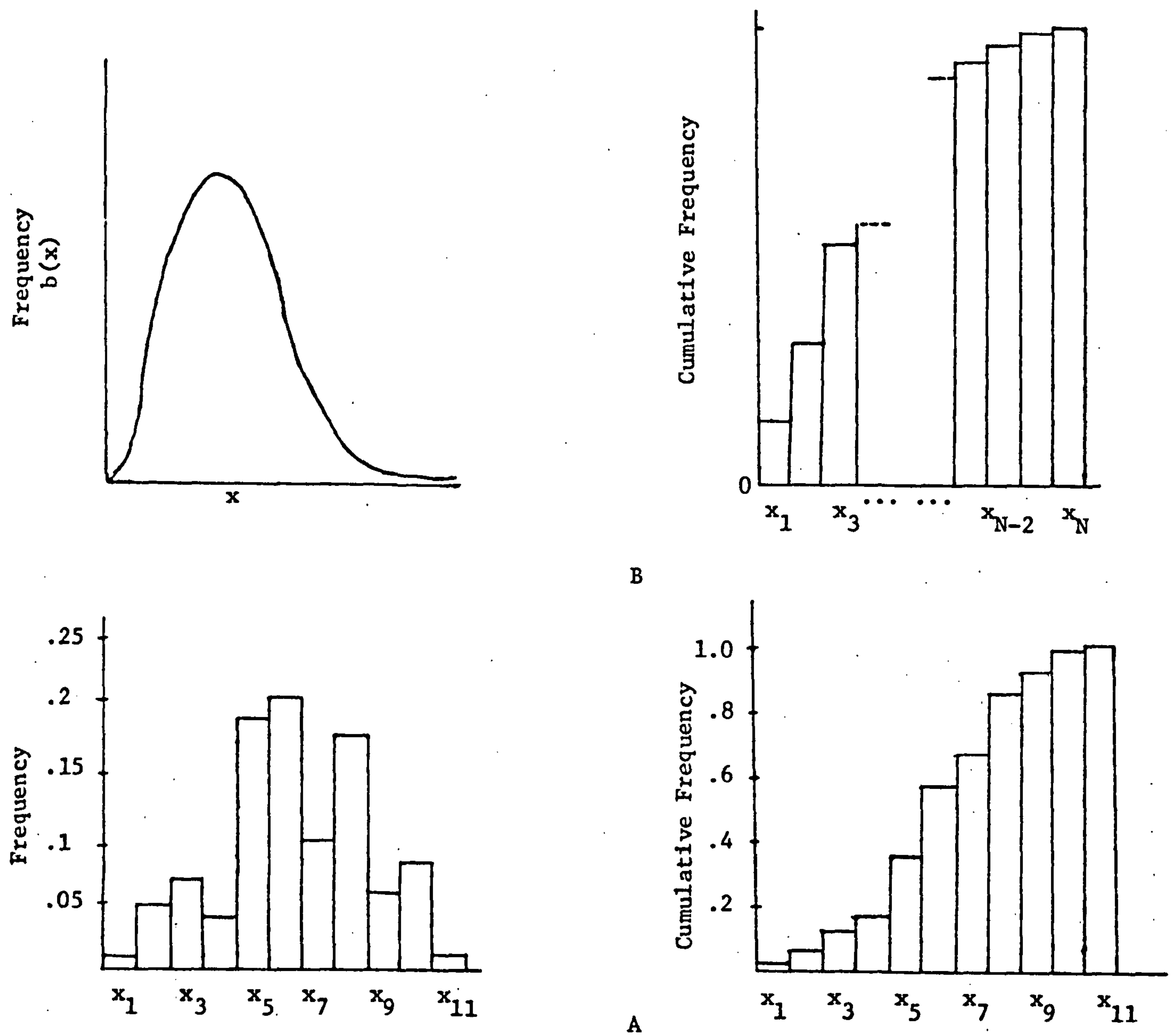

Figure II-3 Example of the two general types of CDFs.

The curve is determined to be one of the three main types as follows:

K < 0	0 < K < 1	K < 1
TYPE I	TYPE IV	TYPE VI

Transitional types exist, but are not incorporated in the present program.

The statistical analysis section of the subroutine initially obtains the first four moments. It then evaluates K and determines the appropriate type of curve to use. From these moments and the total number of data points for each statistical variable, the appropriate parameters for the proper equation are evaluated. Thus the equation representing the frequency curve is established. This equation is evaluated over the specified range and using Simpson's rule to integrate the curve, the CDF is obtained for each statistical variable, as shown in Fig. II-6B. The CDF for each variable is then stored in an array.

Monte Carlo. When called, the Monte Carlo or input subroutine generates values for all statistical variables. These values are determined based on the frequency curve for each variable. The Monte Carlo subroutine performs this function by obtaining a random number and using the CDF array produced earlier.

The CDF consists of values from 0. to 1. with corresponding variable (x) values over the appropriate range. A random number (RANDU) with a value between 0. and 1. is obtained from the random number generator discussed below. This value establishes the appropriate CDF value, and the corresponding x value is then selected from the array to fix the proper value of the variable. Each time the Monte Carlo routine is called, it determines new values for all the statistical variables; that is, it selects a new set of variables for each SRM whose performance is to be analyzed. This is accomplished as follows.

In the computer program, the CDF is stored in an array of 100 elements (a different number of elements may be specified by program changes). A second array (x) also contains 100 values corresponding to the possible values which the statistical variable may assume. For any x array value, the corresponding CDF array value is the accumulated area under the frequency curve to that point, expressed as a percentage of the total area:

$$\mathrm{CDF}(N) = \int_{x(1)}^{x(N)} f(x)\,dx \Big/ \int_{x(1)}^{x(100)} f(x)\,dx \qquad (16)$$

where f(x) is the frequency distribution. The values of the CDF array, thus, will progressively increase from an initial value of zero to a final value of one at a rate depending on the given frequency curve. To randomly select a value for a given statistical variable, the random number generator (RANDU) is invoked to obtain a random number between zero and one. This random number is compared against CDF array values until the smallest CDF value that is larger than the random number is found. The value of the corresponding element of the x array is assigned as the appropriate random value for the given statistical variable. The random numbers, obtained from a rectangular distribution from zero to one, are thus transformed by the CDF in such a way that as a series of random numbers is selected for a single variable, the corresponding variable values occur at a frequency corresponding to their probability distributions.

Random number generator. Based on its size, five statements, this subroutine would not appear to be worthy of a lengthy discussion. The subject, however, is quite critical to the statistical portion of this program. Without some understanding of the random number generator (RNG) the results may prove both surprising and less than satisfactory.

The numbers generated by this routine are not actually random numbers, but rather are referred to as pseudo-random numbers. This fact may cause a less than comfortable feeling. It turns out, however, that the characteristics of pseudo-random numbers are adequate for the present purposes. Real random numbers do not cycle. The pseudo-random numbers have a cycle, but the period is reasonably large. For IBM 370 hardware configurations, approximately 2^{30} selections are made. It is unlikely that this program will be used extensively enough for this to become a problem. The major consideration when using the RNG is that it requires a number to initiate the random generation. After the initial "seed" number, the routine will automatically reset this number for the next iteration as is done when a new set of variables is to be selected for an SRM. For any given seed number, the random number produced is always the same and the new seed number generated will also be equivalent. Thus, if one hundred random numbers are generated from a given initial seed number, the same, exact set of numbers will be generated given that same initial seed number. Often this is a useful characteristic; however, it is essential to understand that to obtain two different sets of random numbers, two different initial seed values must be given.

III. DISCUSSION OF INPUT AND OUTPUT

In this section, each input parameter is defined in the order in which it is encountered in the program. The English or Greek symbol is given first followed by the computer symbol in parentheses. The English or Greek symbols are provided mainly for convenience in consulting the basic analyses of References 3, 4 and 5 as only a few of these are used in the present report. Where appropriate, additional discussion of the variable and recommended or typical nominal numerical values are given. Sketches of geometric characteristics are presented for clarification. Although many of the variables are the same as used in References 3, 4 and 5, the discussion relative to these is repeated here for ready reference. In addition, the present outputs of the program are defined. It is the aim of this section to provide a guide to the user in the specification of his problem and the interpretation of the outputs.

Concise printed definitions of all input variables are also given with the computer program listing (See Section IV). The definitions appear in groups throughout the "main" program and the "area" subroutines. In general, each group is divided in the computer program and in this section into two subgroups: the first group containing the variables describing the SRM which are always fixed for the Monte Carlo investigation and the second group containing those that are subject to being statistically distributed.

The original basis for classification of the two types of variables was to be the analysis of the sensitivities of the performance of the SRM to the variable under consideration. Of course, each variable considered would also have to be an independent one or at least relatively so. Each variable whose variation could reasonably be expected to influence the performance calculations for the SRM was examined one at a time. A few variables whose effects were obviously of very minor significance were omitted at this point and classified without further evaluation as non-statistical (fixed value) variables. Among these were the temperature and pressure sensitivity coefficients of the propellant properties. The thrust loss coefficients were also classified as fixed, as there is no practical way of distinguishing in experimental data analysis between the variations in thrust due to the variation in the statistical variables and that due to other factors. A baseline design was then selected based on the nominal values for the sample case SRM of Section V and the performance of a pair of SRMs computed. One motor in the pair had the baseline values of the variable and the other had the baseline values except for the variable under consideration which was changed a small amount. The amount of the change was somewhat arbitrary but represents a rough estimate of the maximum range of variation for a wide variety of SRMs scaled to the size of the baseline SRM; i.e., the range reflects an estimate of the maximum variation to be expected when no special attention is given to control of the variables during manufacture. In the case of the examination of the effects of grain ovality and malalignment, several variable changes were examined together in order to reduce computational time.

Partial results are given in the Appendix in the form of the pairs of thrust-time traces obtained for each variable. These plots should be of interest to the SRM developer as they indicate possible results of deviations from manufacturing specifications and tolerances. Their utility as far as the present study goes is somewhat limited as it was decided that for the Monte Carlo investigation all but a very few of the variables would be statistically determined in order to minimize cumulative errors that might result from neglect of a large number of seemingly unimportant variables. It will be noted that there are no plots in the Appendix for a few of the distributed variables listed in the computer program and in this section of the report. This is because either the variable has no influence for the design or option selected (e.g., θ_{cn} and θ_{ch} for $C_{op} = 1$); the effects of the variable is an obvious one (θ); data is not presently available for a meaningful investigation (α and β); or the final selection of variables differed from that considered during the earlier sensitivity investigation (e.g., $R_{OA\ell}$ was substituted for γ and C^* and R_{is} and R_c were both introduced as independent variables eliminating τ_s).

The present section provides information only on nominal values and the units of the variables. Instructions for preparing the input for the description of the distribution of statistical (distributed) variables is given in Section IV based on the analysis of Section II. It should be noted here that the statistical variables may be held constant if desired by use of the proper code on the data cards for the variables. With minor program modifications it is also possible to accommodate additional distributed variables, the number of such variables being limited only by the core storage capacity of the computer. The listing of variables follows.

<u>Seed Number</u>

I_{xi} (IX1) The seed number for the random number generator. An odd eight-digit integer should be used. This number initiates the generation of random numbers forming the basis for selection of the various sets of input variables for each SRM. The seed number must be changed to change the sets of variables generated when repeating an analysis of the same motor pairs (see Section II for further discussion).

<u>User's Options - Fixed Values</u>

I_{eo}(IEO)

0 For no consideration of grain ovality or malalignment.

1 For consideration of above. Calculations of ovality and malalignment effects approximately triples computer

time requirements. These calculations should be by-passed by use of the option provided if there is a basis for assuming in a particular case that the effects are negligible.

I_{po}(IPO) 0 For no plots and no statistical or difference analysis of results.

1 For plots and tabular output of motor pair differences.

2 For tabular output of motor pair differences without plots.

3 For plots and statistical analysis of results only. Codes 1 and 2 will also yield statistical analysis of the motor pair results.

N(j)(NUMPLT(J) An integer designating whether or not a specific output plot is desired:

0 If a specific plot is desired.

1 If a specific plot is not desired.

The order of specification of NUMPLT(J) is as follows:

1 F versus T for the motor pair.

2 F versus T for the motor pair during tailoff.

3 Difference in F between the pair of motors versus T.

4 Difference in ITOT between the pair of motors versus T.

5 Time integral of absolute value of difference in F of motors versus T.

Propellant Characteristics - Distributed Values

ρ_p(RHO) Density of the solid propellant (slugs/in^3)

a_1(A1),a_2(A2) Propellant burning rate coefficients in the equation $r = aP^n$ above and below the transition pressure, respectively (in/sec-psia^n).

n_1(N1),n_2(N2) Burning rate exponents corresponding to a_1 and a_2, respectively (1).

α_{eb}(ALPHA) Erosive burning coefficient in the Robillard-Lenoir burning rate rule (Equation III-11, Ref. 3)(in$^{2.8}$-ft$^{0.8}$/sec$^{1.8}$ lbf$^{0.8}$).

β (BETA) Erosive burning pressure coefficient in the Robillard-Lenoir rule (1).

$R_{OA\ell}$ (ROAL) The oxidizer to aluminum ratio (1). Variations in this quantity determine variation in the thermochemically determined characteristic exhaust velocity CSTARN and the ratio of specific heats GAMN of the chamber gases. The latter variations are determined by a regression analysis which gives CSTARN and GAMN as functions of ROAL. The resultant functional relationship must be used in the program where indicated on the program listing (Section IV). CSTARN and GAMN are determined at 1000 psi chamber pressure and 60°F propellant bulk temperature. If no aluminum is present in the propellant CSTARN and GAMN should be set at this point to their nominal values at the reference conditions.

Basic Motor Dimensions - Fixed Values

L (L) Total initial perforated grain length including gaps at slots (in.). This is used only in the erosive burning rate equation. An estimate will suffice.

τ_w (TAU) Estimated web thickness of the controlling propellant length (in.). (See ZO below for definition of controlling length.) The actual web thickness is calculated from DI and DO (or RC and RSI, RIWS or RIWW)and includes statistical variations. If the grain is tapered, the length average value should be specified excluding lengths having additional taper not represented by ZO and segments located anywhere having a step decrease in thickness. Such step decrease must be handled by the additive tabular input option if they introduce two significantly different web thicknesses for the same grain type; e.g., two circular perforated grains. If a circular perforated grain is used, it is assumed that it will have the approximate average web thickness of the controlling propellant length.

Basic Motor Dimensions - Distributed Values

D_e (DE) Diameter of the nozzle exit (in.).

D_i* (DTI) Initial diameter of the nozzle throat (in.).

θ_n (THETA) Cant angle of the nozzle with respect to the motor (degrees).

α_n (ALFAN) Exit half angle of the nozzle (degrees).

L_{Ta} (LTAP) Estimated axial length of grain at the aft end at start of first tailoff having an additional taper not represented by ZO or THETAG (in.). See Figure III-1. This variable permits the designer to specify an additional taper at the nozzle end of a circular perforated or star grain to produce regressivity shortly before tailoff.

x_{Ta} (XT) Difference in web thickness at ends of LTAP (in.). See Figure III-1.

z_o (ZO) Initial difference between web thicknesses due to grain bore taper at the head and aft ends of controlling grain length, not including any initial length associated with LTAP or THETAG (in.). The controlling length of the grain is the axial distance between the head end of the grain and the position of expected maximum Mach number in the port. In general this length terminates whenever there is an abrupt decrease in web thickness near the aft end of the chamber.

z_c (ZC) Initial difference between web thicknesses due to grain exterior taper at the head and aft ends of the controlling grain length (in.).

Ovality and Malalignment - Distributed Values

(Not required if IEO = 1). The following variables characterize the ovality and lack of concentricity of the grain interior and exterior at two reference radial cross-sections - one near the head end of the grain and one near the nozzle end. Based on the mathematical analysis of the burning perimeters at these two planes, correction factors are calculated and applied to the burning surface calculations to account for approximate effects of ovality and malalignment. The effects of interior and exterior grain taper are taken into account through the parameters ZO, ZC, and XT. See Section II for details.

ΔR_{cn} (RONDCN), ΔR_{ch} (RONDCH) One half the difference between the maximum and minimum diameter of the grain exterior at the nozzle and head end reference planes, respectively (in.).

ΔR_{gn} (RONDGN), ΔR_{gh} (RONDGH) One half the difference between the maximum and minimum diameter of the grain interior at the nozzle and head end reference planes, respectively (in.).

e_{xn} (EXN) e_{yn} (EYN) The eccentricity of the center of the grain interior with respect to the center of the grain exterior in the x_c and y_c directions, respectively (See Fig. II-1) at the nozzle end reference plane (in.).

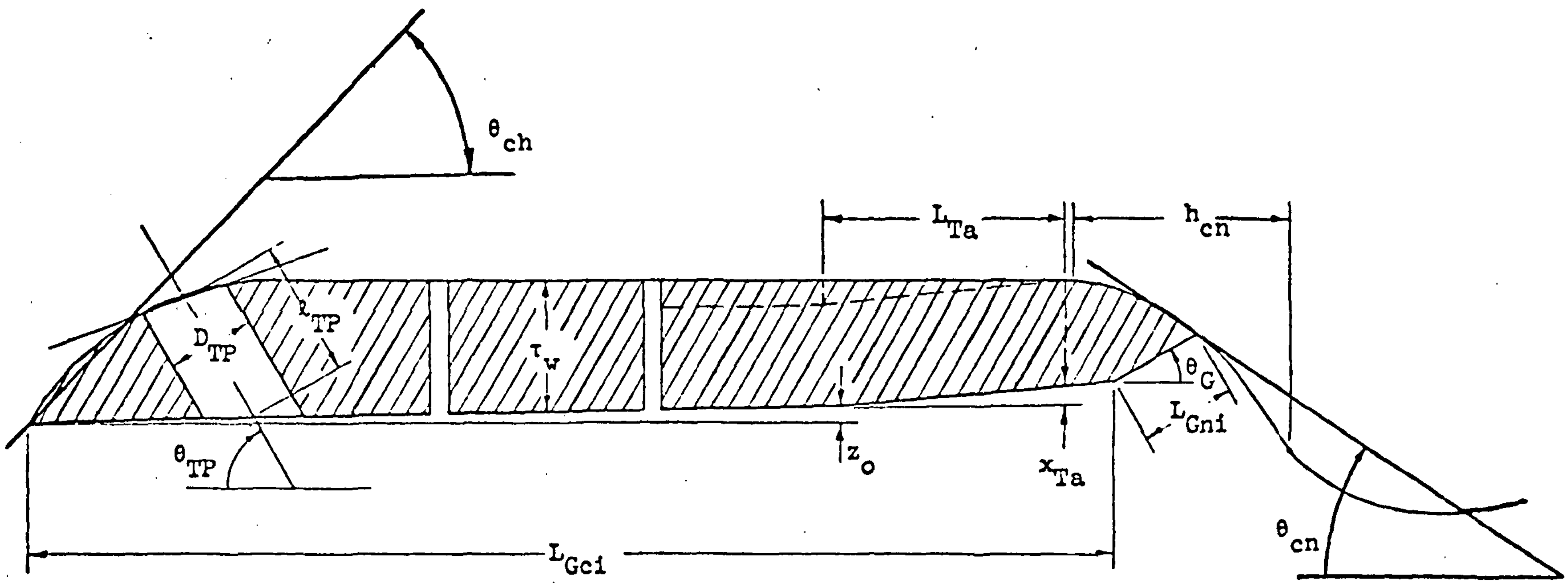

Figure III-1. Basic motor dimensions. L_{Gci}, L_{Gni}, and θ_G are used only for circular perforated grains. The controlling grain length may be L_{Gci} or some lesser value depending on the position of maximum port Mach number. τ_w is the length average web thickness over the grain length excluding the lengths associated with L_{Ta}, θ_G and the head end dome.

e_{xh} (EXH), e_{yh} (EYH) — The eccentricity of the center of the grain interior with respect to the center of the grain exterior in the x_c and y_c directions, respectively, at the head end reference plane (in.).

α_{an} (ALPHAN), α_{ah} (ALPHAH) — The angular orientation of the ovality of the grain interior with respect to the grain exterior at the nozzle and head end reference planes, respectively (degrees).

<u>Basic Performance Constants - Fixed Values</u>

Δy_1 (DELTAY) — Initial desired burn increment (in.). This increment will be used by the program for initial 5% of the web thickness burned, for the period shortly preceding and following tailoff and at such other times as the rate of pressure change is large. Larger increments will automatically be used at other times. An initial increment size of approximately $0.001\tau_w$ is recommended for the Monte Carlo program. If the increment size used is twice the recommended value, the maximum thrust imbalance calculated may be decreased by as much as 5 percent, representing a loss in program accuracy.

I_I (II) — Number of steps in the integrations of perimeters in the OVAL subroutine. Approximately 25 steps appears to offer a good compromise between accuracy and computation time, the latter being more strongly influenced than the former by the step size.

x_{out} (XOUT) — Distance burned at which the propellant breaks up (in.). This permits the option of specifying termination of burning resulting from possible structural breakup of propellant. If this option is not desired, XOUT should be set to some large value; e.g., 1000 in.

$(\Delta P/\Delta y)_{out}$ (DPOUT) — Rate of change of pressure with respect to distance burned at which the propellant is extinguished (lb/in^3). This permits the option of specifying termination of burning when it is determined that an abrupt tailoff will not permit the computer to handle the rapid change in surface area. If a gradual tailoff is expected, DPOUT may be set to some large value; e.g., 10,000 psia/in. For large motors (120 in. dia. and up) where the tailoff is expected to be abrupt, a value of 500 psia/in. is recommended. In general, larger values may be used for smaller motors.

ζ_F (ZETAF) — Thrust loss coefficient (1). In the absence of data to the contrary, a value of 0.98 is recommended.

$t_{max\ q}$(TMAXQ) Estimated time at which maximum dynamic pressure on the vehicle occurs (sec.). This permits the program to compute an estimate of the thrust differential of the motor pair at the time of maximum dynamic pressure.

t_{b1}(TB) Estimated burning time (sec.). This and HB below permit the program to calculate delivered specific impulse and thrust based on an assumed trajectory which was determined from analysis of typical large SRM applications.

h_b (HB) Estimated burnout altitude (ft.). To obtain sea level performance characteristics, HB should be set equal to zero.

P_{ref}(PREF) Reference average nozzle stagnation pressure used in the nozzle throat erosion equation (psia). See also ERREF below.

$D_{t\ ref}$(DTREF) Reference nozzle throat diameter used in the nozzle throat erosion equation (in.). See also ERREF below.

$(\pi_p)_k$(PIPK) Temperature sensitivity coefficient of pressure at constant ratio K of burning surface to throat area (/°F).

a_{c*T}(CSTART) Temperature sensitivity of CSTAR at constant K (/°F). (See Eq. 4, Section II). A typical value is 0.000038.

a_{c*p}(CSTARP) Pressure sensitivity of CSTAR (1)(See Eq. 5, Section II).

P_{tran}(PTRAN) Transition pressure at which the burning rate coefficient and exponent change (psia).

$a_{\gamma p}$(GAMP) Pressure sensitivity of the ratio of specific heats (1) (See Eq. 6, Section II). A typical value is 0.00527.

Basic Performance Constants - Distributed Values

E_{ref}(ERREF) Reference nozzle throat erosion rate (in/sec)(See Eq. 1 Section II.) A set of typical values for a carbon tape-phenolic impregnated throat is 0.00763 in/sec, 560 psia and 57.285 in. for ERREF, PREF and DTREF, respectively.

T_{gr}(TGR) Bulk temperature of the propellant grain (°F).

T_{igr}(TIGR) Ingition delay (sec.) at a TGR of 60°F. The time required to reach ninety-five percent of initial equilibrium pressure should be used.

The Program and Basic Grain Configuration and Arrangement - Fixed Values

I_{op} (INPUT) 1 For only tabular input.

2 For only equation input.

3 For a combination of 1 and 2.

G_{op} (GRAIN) 1 For an entirely circular perforated grain.

2 For star grain only.

3 For a combination of 1 and 2.

S_{op} (STAR) 0 For an entirely circular perforated grain.

1 For standard star (See Fig. III-2).

2 For truncated star (See Fig. III-3).

3 For wagon-wheel (See Fig. III-4).

NOTE: The different types of star grains may not be combined in a single configuration.

n_T (NT) Number of thrust termination passageways in grain (1) NT is zero if there are no thrust termination passageways.

Q_{op} (ORDER) 1 If a star grain is at head end and a circular perforated grain at aft controlling end.

2 If a circular perforated grain is at both ends. A star grain segment may still be present.

3 If a circular perforated grain is at head end and a star at the aft controlling end.

4 If a star grain is at both ends.

If grain = 1, value of order must be 2.

If grain = 2, value of order must be 4.

It is important to realize that ORDER establishes the controlling port area equations to be used. Thus if the nozzle end segment is not indeed the controlling one (the one that establishes the maximum Mach number in the port), ORDER should be specified to designate the actual controlling segment as the nozzle end segment. GRAIN, STAR, NT and ORDER are not used for INPUT = 1, but values must be assigned for continuity of computer operations.

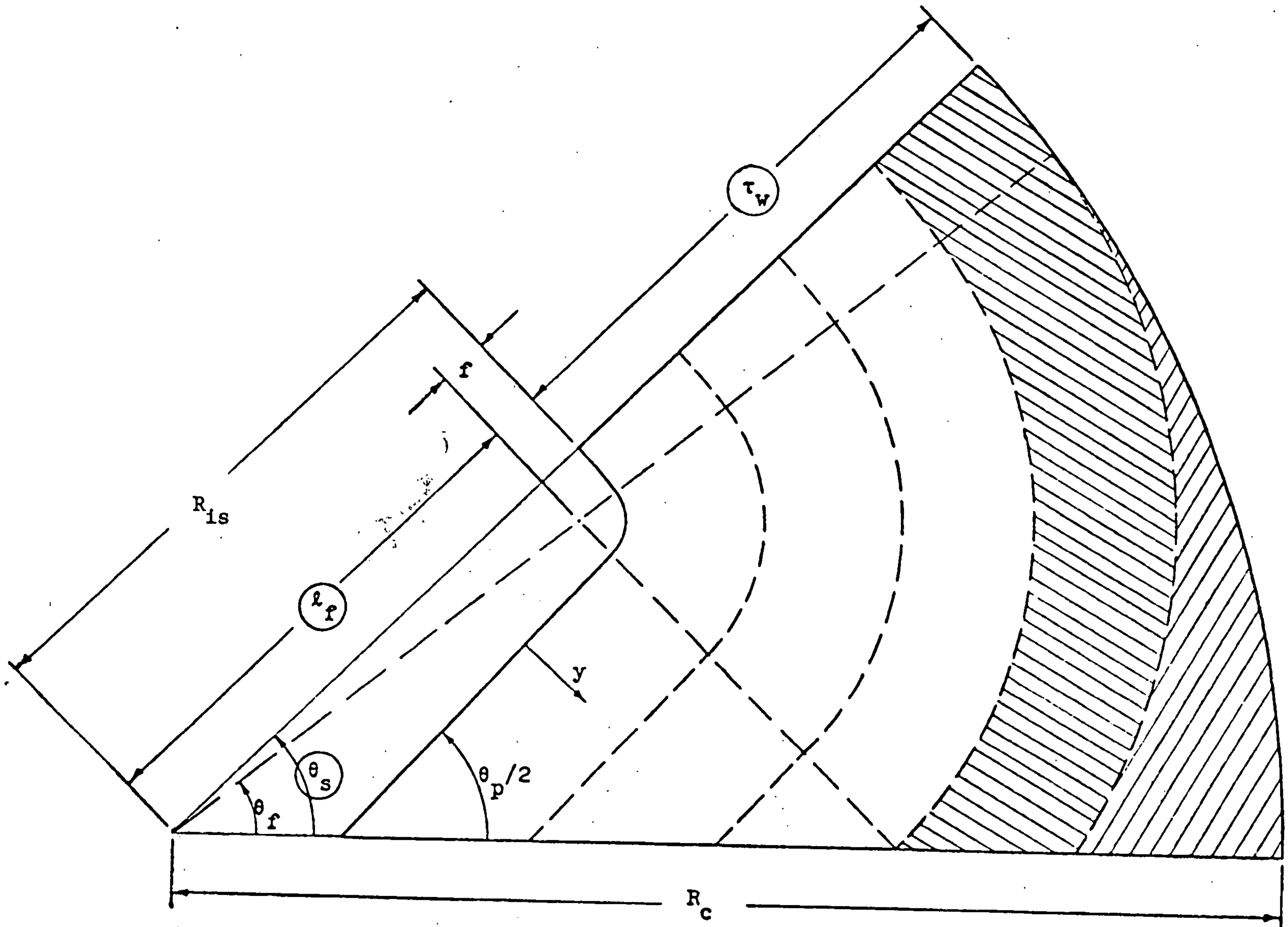

Figure III-2. Standard star grain cross-section. The evolution of the burning perimeter is shown. Calculated variables are circled. Second and third (final) zones of burning are cross-hatched.

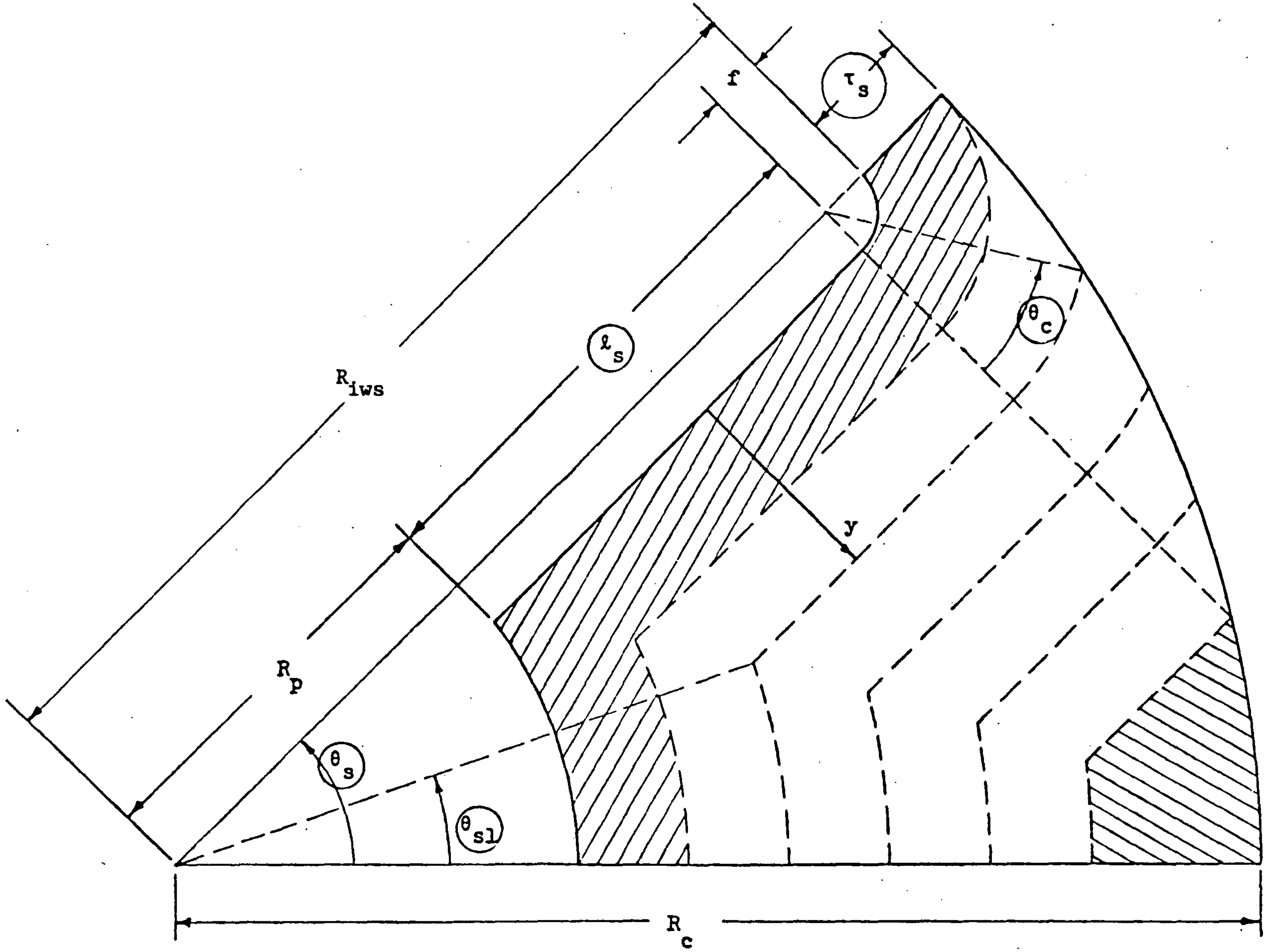

Figure III-3. Truncated (slotted tube) star grain cross-section. The evolution of the burning perimeter is shown. Calculated variables are circled. First and third (final) burning zones are cross-hatched.

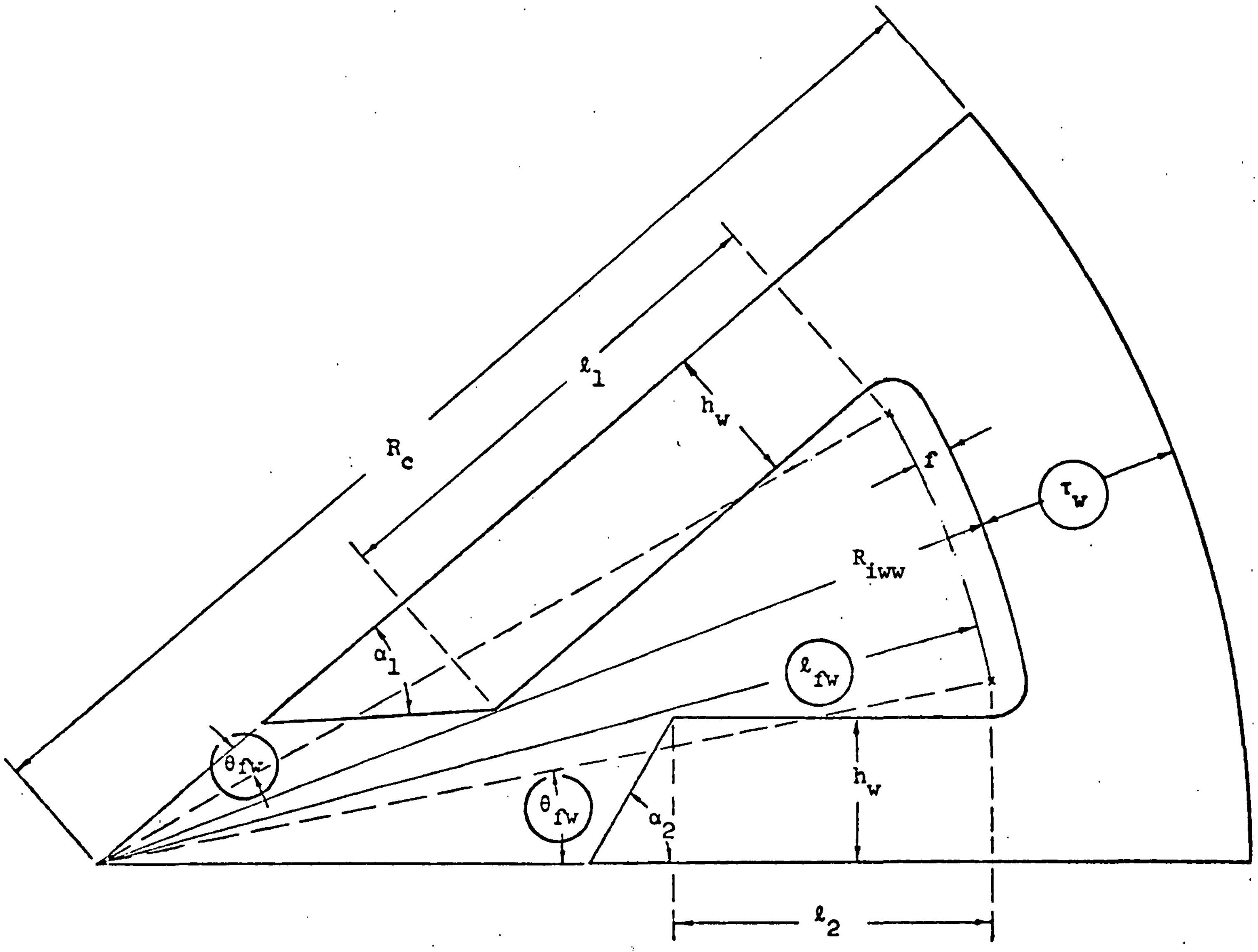

Figure III-4. Wagon-wheel grain cross-section. Calculated variables are circled.

C_{op} (COP) For extreme ends of a circular perforated grain only:

0 If both ends are conical or flat.

1 If head end is conical or flat and aft end is spherical.

2 If both ends are spherical.

3 If head end is spherical and aft end is conical or flat.

Tabular Burning Surface and Port Areas - Fixed Values

A_{bpT} (ABPK) Burning area in the port (in^2).

A_{bsT} (ABSK) Burning area in the slots (in^2).

A_{bnT} (ABNK) Burning area at the nozzle end (in^2).

A_{phT} (APHK) Port area at the nozzle end of the controlling grain length (in^2).

A_{pnT} (APNK) Port area at the head end of the grain (in^2). APHK and APNK are not required when INPUT is 2 or 3; the equation inputs must be used to provide the information in these cases. Values of all A's must be specified to completely describe the burning surface and port areas versus distance burned. The program computes intermediate values by linear interpolation. The number of values required is arbitrary and limited only by the storage capacity of the computer, but values must be specified for y = 0. Also, as an example of the procedure for specifying terminal values which must be followed, burning surfaces must be specified as zero at burnout of the tabular surfaces and at a y exceeding the highest anticipated calculated performance value. Separate input cards must be prepared for each value of y and arranged as described in Section IV of this report. The use of these tabular values in conjunction with the equation inputs (INPUT = 3) increases the flexibility of the program considerably. Frequently it is easy to estimate burning surface effects which end constraints on the equation inputs neglect. In this case a table of input values can be readily prepared from estimates of the effects. Also, the program can be run first without the tabular values and outputs used as an aid in obtaining the estimate. For example, the burning perimeters of a star grain can be determined in this way and the values used to estimate the effects of a head end closure on the star grain.

V_{ciT} (VCIT) Initial volume of chamber gases associated with tabular input (in^3).

Geometry for Circular Perforated Grain - Fixed Values

x_{Tz} (XTZO) Difference between the initial circular perforated grain diameter at the nozzle end of LGCI and the nominal value of DI less ZO and less twice XT (in.)(See Fig. III-1).

S (S) Number of burning flat ends of a circular perforated grain not including an extreme aft grain end (1).

Geometry for Circular Perforated Grain - Distributed Values

D_0 (DO) Length average outside diameter of circular perforated grain, excluding lengths extending into the closure (in.).

D_i (DI) Length average inside initial diameter of circular perforated grain (in.). Only the controlling length excluding LTAP should be considered in the averaging.

θ_G (THETAG) Angle burning surface element of circular perforated grain located at the extreme nozzle end of chamber makes with the motor axis (degrees). See Figure III-1. THETAG must be set to zero if a star grain is located at the nozzle end (GRAIN = 3, ORDER = 3) or if aft end burning surfaces are represented by tabular values. THETAG is 90° if the circular perforated grain represented by equations has a flat burning surface located at the extreme nozzle end of the chamber. If THETAG is less than or equal to 5°, a value must be assigned (zero is satisfactory), but the effect of THETAG on burning surface area is not computed (See also LGNI and LTAP). THETAG is zero if the end surface is flat and inhibited.

L_{Gci} (LGCI) Initial total axial length of circular perforated grain represented by equation inputs not including gaps (in.). LGCI excludes lengths associated with THETAG.

L_{Gni} (LGNI) Initial slant length of a burning conical circular perforated grain at the nozzle end (in.). LGNI is set equal to zero if THETAG is less than or equal to 5°. In this case the length otherwise associated with LGNI should be added to LGCI. If the error in burning surface area thus introduced is deemed significant, a correction may be introduced by making use of tabular inputs in combination with the equation inputs. Basic effects of small THETAG on tailoff may be accounted for by specification of LTAP. If a nozzle end burning surface is flat (THETAG = 90°) LGNI equals one half the difference between inside and outside local grain diameters.

θ_{cn} (THETCN) The contraction angle of a circular perforated grain bonded to the nozzle closure (degrees). See Figure III-1. Use an estimated value which yields approximately the correct volume of propellant burned. If a star shaped grain is located at the extreme nozzle end of the chamber or if tabular values are used to represent downstream burning surfaces, THETCN is 90°. THETCH is also 90° if the extreme aft end of the grain is inhibited, but only a flat-ended, inhibited grain (THETAG = 0) which does not extend into the nozzle closure may be accurately represented. THETCN is zero for a burning flat end (THETAG = 90°) which does not extend into the closure. THETCN must be assigned a value even if COP is 1 or 2, but it will not affect the numerical results.

θ_{ch} (THETCH) The contraction angle of a circular perforated grain bonded to the head end (degrees). See Figure III-1. Use an estimated value which yields approximately the correct volume of propellant burned. THETCH is 90° if the extreme forward end of the circular perforated grain (bonded or not) represented by equations is flat. A head end flat burning surface is treated by proper specification of S. THETCH must be assigned a value even if COP is 2 or 3, but it will not affect the numerical results.

Basic Geometry for Star Grains - Fixed Values (The wagon-wheel is considered a type of star grain for the purpose of this program.)

n_s (NS) Number of burning flat end surfaces of a star grain not located at extreme nozzle end of the chamber (1).

n_p (NP) The number of star points (1).

n_n (NN) Number of burning flat end surfaces (0 or 1) of a star grain located at the extreme nozzle end of the chamber (1).

Basic Geometry for Star Grains - Distributed Values

L_{Gsi} (LGSI) Initial total axial length of star-shaped perforated grain represented by equations (in.). No provision comparable to the use of LGNI for circular perforated grains is made here to treat effects resulting from THETAG greater than 5°. Adjustments may be made, however, by use of tabular input values in conjunction with the equation inputs. Also, effects of taper, including additional small taper at the nozzle end, on tailoff may be treated by use of the variables ZO, XT and LTAP.

R_c (RC) The star grain outside radius (in.).

f (FILL) The fillet radius at star valleys (in.). See Figure III-2, 3, 4.

Special Geometry for Wagon-Wheel Grain - Distributed Values

R_{iww}(RIWW)	The length average initial radius of the inside of the propellant web (in.).
ℓ_1, ℓ_2 (L1,L2)	The lengths of the pairs of parallel sides of the first and second sets of grain points, respectively (in.). See Figure III-4).
α_1, α_2 (ALPHA1,ALPHA2)	The angles between the slant sides and the center lines of the points of the first and second sets of grain points, respectively (degrees). The angles should not exceed 90 degrees.
h_w (HW)	The half-width of the star points (in.). HW must not exceed TAUWW (HW $\leq$ TAUWW).

Special Geometry for Truncated Star Grain - Distributed Values

R_p (RP)	The length average initial radius of the truncation (in.). See Figure III-3.
R_{is} (RIS)	The length average initial radius of the inside of the propellant web at the bottom of the slots (in.).

Special Geometry for Standard Star Grain - Distributed Values

θ_f (THETAF)	Angular location of the fillet center of standard star from the line of symmetry (degrees).
θ_p (THETAP)	The apex angle of the star point (degrees).
R_{iws}(RIWS)	The initial radius of the inside of the propellant web of standard star grain (in.). See Figure III-2. If the grain is tapered, the length average value should be used.

Geometry of Thrust Termination Passageways - Distributed Values

ℓ_{TP} (LTP)	Initial length of the termination passageway between the centers of gravity of perimeters of the bases (in.). See Figure III-1.
D_{TP} (DTP)	Initial diameter of the termination passage (in.).
θ_{TP} (THETTP)	The acute angle between the axis of the passage and the motor axis (degrees).
τ_{eff} (TAUEFF)	Estimated effective web thickness at the termination port (in.). The user must judge the distance burned at which the effect of the termination passage on modification of the burning surface geometry ceases to be significant. In general, this should be between two-thirds and full web thickness. The equation used to account for the burning surface is based on a passageway in a circular perforated grain terminated at the case by a flat inclined plane. Thus only a rough estimate of the effect of the termination passage is provided.

Special Equation Inputs -(Fixed Relationships required only at the option of the user. May be used when INPUT = 1, 2, or 3.)

B_{bp} (BBP) Additive burning surface input as function of y for port burning surface (in^2).

B_{bs} (BBS) Additive burning surface input as function of y for slot burning surface (in^2).

B_{bn} (BBN) Additive burning surface input as function of y for nozzle end burning surface (in^2). In order to make use of the option of specifying the B's, a minor program modification is required. The B's are all set equal to zero in the present program. If this option is to be used, the program statements assigning values to the B's are easily replaced with the desired equation inputs.

Program Outputs

The variables whose values are printed by the present program are defined below. Additional variables may also be printed with minor program modifications. In addition to the variables listed below, the present program prints out values of all input variables including those selected for each SRM from statistical distributions. The input characteristics of the statistical distributions are also printed.

Time dependent data - single motors

t (T) Operating time (sec.). This is calculated from the time of initiation of ignition.

y (Y) Average distance burned (in.).

P_{on} (PONOZ) Stagnation pressure at the nozzle end of the chamber (lb/in^2).

P_h (PHEAD) Pressure at the head end of the chamber (lb/in^2).

$A_{bp} + A_{bs} + A_{bn}$ (SUMAB) The total burning surface of the propellant (in^2).

F (F) The delivered thrust based on the assumed trajectory (lbf). Losses are included.

I_T (ITOT) Total delivered specific impulse (lbf-sec). Losses are included.

Time independent data - single motors

W_{p1} (WP1) Propellant weight calculated from mass discharge rates (lbm).

W_{p2} (WP2) Propellant weight calculated from the products of burning surfaces and incremental distances burned (lbm).

W_p (WP) Arithmetic average of WP1 and WP2 (lbm). A check on the calculation accuracy is provided by comparison of this with WP1 and WP2.

$P_{h\ max}$ (PHMAX) Maximum head end chamber pressure calculated by the program (lb/in^2).

I_{xi} (IX1) Initial seed number for the current configuration. New seed numbers are automatically selected for each variable based upon this seed number. The initial seed number may be used as an input seed number to reproduce the calculations for the configuration. See Section II.

I_{xf} (IX) Final seed number for the current configuration. This final seed number may be used as an input seed number to continue the calculations without random number cycling before 2^{30} random numbers have been selected.

Time dependent data - motor pairs. The following data are available in tabular and/or graphical form, subject to the option of the user (See IPO). To compute the imbalances between two motors the data for the motor which has the fewer computational y-steps is subtracted from the data of the other motor to determine the imbalances. No difficulty with regard to the interpretation of the results arises from this since in most instances it is the absolute value of the differences which is important. For those variables whose absolute values are not taken, changes in sign of the differences indicate points where the crossings between the two motors' traces occur.

ΔF (FDIFF) The difference in thrust between the two motors (lbf).

ΔI_T (IDIFF) The difference in total impulse between the two motors (lbf-sec).

$|\Delta I_T|$ (IADIFF) The absolute value of the difference in total impulse between the two motors (lbf-sec).

$$|\Delta I_T| = \int_0^t |F_1 - F_2|\, dt \tag{17}$$

Time independent data - motor pairs

(FMAX1,TFMX1, FMIN1,TFMN1)	The maximum and minimum algebraic thrust imbalance (lbf) and the time (secs) at which they occur, respectively, during web action time.
(FMAX2,TFMX2, FMIN2,TFMN2)	The maximum and minimum algebraic thrust imbalance (lbf) and the times (secs) at which they occur, respectively, during tailoff.
(TDFTO1,TDFTO2, DTW)	The time at which tailoff begins for the first motor of a pair to begin tailoff, for the final motor to begin tailoff and the absolute value of the difference between the two times, respectively, (secs).
(FW1,FW2,DFW)	The thrust at the beginning of tailoff for the first motor of a pair to begin tailoff, for the final motor to begin tailoff and the absolute value of the difference between the two thrusts, respectively, (lbf).
(DFTO1,DFTO2)	The absolute value of the thrust imbalance when the first motor of a pair begins tailoff and the final motor begins tailoff, respectively.
(DFMQ,TMAXQ)	The absolute value of the thrust imbalance (lbf) which exists when the maximum dynamic pressure occurs on the vehicle and the estimated time (secs) at which this event occurs, respectively.
(AFMAX,TFMAX, AFMAXT,TFMAXT)	The absolute value of the maximum thrust imbalance (lbf) which exists and the time (secs) at which it occurs during web action time and tailoff, respectively.
(FDIFIG,TDIFIG)	The absolute value of the thrust imbalance (lbf) which occurs during the initial portion of operation ($t < 0.02\ t_b$) and the time (secs) at which it occurs, respectively.
(DIT,ADIT)	The total impulse imbalance and the absolute value of the total impulse imbalance (lbf-secs) accumulated during tailoff, respectively:

$$\Delta I_t = \int_{t_{t1}}^{t_b} (F_1 - F_2)\,dt \tag{18}$$

$$|\Delta I_t| = \int_{t_{t1}}^{t_b} |F_1 - F_2|\,dt \tag{19}$$

where t_{t1}(TDFTO1) is the earlier time at which tailoff begins in the two motors and t_b is the time at which operation of both motors ends.

(DF100K,T100K) The absolute value of the thrust imbalance (lbf) which exists when the last motor reaches 100,000 lb. thrust during tailoff and the time (secs) at which it occurs, respectively.

IV. THE COMPUTER PROGRAM

This section contains the instructions for the preparation and arrangement of the data cards. Also, a complete listing of the program statements is given. The program was written for use on an IBM 370/155 computer and requires approximately 168K storage locations on that machine. The program also is designed to be used with a CALCOMP 663 drum plotter. The plotter requires one external storage device (magnetic tape or disk). In addition to the one storage device required for the plotter, three other external storage units are required. Unit 1 is used to store the output data, pertinent to the imbalance calculations, for the first motor in each pair of motors. Unit 2 is used to store the nonstatistical data which remain constant for all of the motors. Unit 4 is used to store the values of the statistical variables for use with each motor. Only minor program modifications are required to eliminate the plotting capability of the program. Also, Unit 2 can be eliminated by using repeated sets of data cards for the nonstatistical variables. Hence, it is relatively simple to modify the program to require only 2 external storage units. Elimination of the other two external storage units would require significant program modification.

Input Data

The discussion below gives the general purpose, order and FORTRAN coding information for the input data.

Card 1 Total number of individual motors to be analyzed (42X,I2)

Col. 1-42 NUMBER OF CONFIGURATIONS TO BE TESTED =

43-44 Number of rocket motors to be analyzed

Card 2 Initial seed number (I10)

Col. 1-10 Initial 8-10 digit seed number

It is necessary to describe one type of statistical analysis for each statistical input variable. The method for doing this is described below using Cards 3 through 9. Note that only one type of statistical analysis may be requested for each variable. Hence, only the card or cards necessary for that particular type of statistical analysis are input for each variable. For example, to obtain a Type II analysis only Card 5 and Cards 5A would be would be used. In addition, it is necessary that the data cards for the variables to be used in a given configuration be placed in the order in which they are input into the computer program. In some cases certain variables are not required for an analysis. In such cases, the cards for those variables should be omitted. As many Cards 5 through Cards 9A as required may be used.

Card 3 Variable name (2A4)(one card for each variable)

Col. 1-8 Name of statistical variable.

NOTE: One Card 3 immediately precedes the Card 4 thru Card 9B used for each variable. Also, END should be used as the last variable name before using Card 9B below.

Card 4 Input for Type I statistical analysis (I2, 2X, 7E10.0)

Col.	1-2	Code = 10	Raw data given; obtain CDF directly from histogram.
		Code = 11	Raw data given; obtain CDF from Pearson's equation of the frequency curve.
	5-14	X1 = Number of raw data points given.	
	15-24	X2 = Mean value of first interval of histogram.	
	25-34	X3 = Histogram interval width.	
	35-44	X4 = Number of intervals in histogram.	
	45-74	Blank	

Card 4A Subsequent Type I data cards (10E8.0)

Col. 1-8
9-16
:
72-80

Raw data points equivalent to the number specified in X1. Ten data points per card for as many cards as required (e.g., 46 data points would require 5 data cards with the last card having the final four fields blank).

Card 5 Data input for Type II statistical analysis (I2, 2X, 7E10.0)

Col.	1-2	Code = 20	Histogram given; obtain CDF directly from histogram.
		Code = 21	Histogram given; obtain CDF from Pearson's equation of the frequency curve.
	5-14	X1 = Number of intervals in histogram.	
	15-24	X2 = Mean value of first interval of histogram.	
	25-34	X3 = Interval width.	
	35-74	Blank	

Card 5A Subsequent Type II data cards (10E8.0)

Col.	1-8	The same number of data points as specified in X1, for as many data cards as necessary
	9-16	
	⋮	
	72-80	

Card 6 Input for Type III statistical analysis (I2, 2X, 7E10.0)

Col.	1-2	Code = 31	Four moments given; obtain CDF from Pearson's equation of the frequency curve.
	5-14	X1 = First moment about zero.	
	15-24	X2 = Second moment about mean.	
	25-34	X3 = Third moment about mean.	
	35-44	X4 = Fourth moment about mean.	
	45-54	X5 = Histogram interval width.	
	55-64	X6 = Mean value of first interval of histogram.	
	65-74	X7 = Total number of data points used.	

NOTE: No data cards required.

Card 7 Input for Type IV statistical analysis

Col.	1-2	Code = 40	CDF given; read in the given CDF.
	5-14	X1 = Number of intervals in CDF.	
	15-24	X2 = Mean value of first interval of CDF.	
	25-34	X3 = Interval width.	
	35-74	Blank	

Card 7A Subsequent Type IV data cards (10E8.0)

Col.	1-8	CDF values corresponding to the cumulative
	9-16	frequency up through each interval. Data
	⋮	should be provided for as many intervals as
	72-80	indicated by the value given for X1.

Card 8 Input for Type V statistical analysis (Use appropriate card below)

Card 8A Normal distribution to obtain CDF.

Col.	1-2	Code = 51
	5-14	X1 = Mean of normal distribution.
	15-24	X2 = Standard deviation.
	25-34	X3 = Beginning X value of CDF (optional).
	35-44	X4 = Ending X value of CDF (optional).
	45-74	Blank

NOTE: If either X3 or X4 is omitted, a three-sigma limit is assumed; thus, if both values are left blank, a six-sigma limit will be generated by the program. If a zero value is desired for X3 or X4, ±.0000001 should be used instead.

Card 8B Rectangular distribution to obtain CDF (I2, 2X, 7E10.0)

Col.	1-2	Code = 52
	5-14	X1 = Beginning X value.
	15-24	X2 = Ending X value
	25-74	Blank

Card 8C J-Distribution to obtain CDF

Col.	1-2	Code = 53
	5-14	X1 = Mean(beginning X value).
	15-24	X2 = Standard deviation.

Card 8C (Cont'd)

Col. 25-34 X3 = Ending X value (optional)

35-74 Blank

NOTE: The J-distribution is defined herein as the right half of a normal frequency curve. The X1 value specified should be the mean as if the full normal curve were being specified. The X3 value is optional; if not specified, a three sigma limit will be assumed. If zero is desired for the X3 value, ±.0000001 should be used instead.

Card 9 Input for Type VI statistical analysis (use appropriate card below)

Card 9A Use a constant for this value (I2, 2X, 7E10.0)

Col. 1-2 Code = 60 Use a constant value for this variable.

5-14 X1 = Desired constant value.

15-74 Blank

Card 9B Indicates end of data (I2)

Col. 1-2 Code = 90

Card 10 Initialization of variables (22F3.1)

Col. 1-66 Zero's or blank card

Card 11 Ovality and output options (5X, I1, 5X, I1, 9X, 5I)

Col. 1-5 IEO =

6 { 0 No ovality analysis
  1 Ovality analysis

7-11 IPO =

12 { 0 No plots or statistical analysis
   1 Plots, statistical analysis and tabular output
   2 Tabular output and statistical analysis
   3 Plots and statistical analysis

Card 11 (Cont'd)

Col. 13-17 NUMPLT(J) =

18	0	Plot thrust time trace
	1	Do not plot thrust time trace
19	0	Plot tailoff thrust time trace
	1	Do not plot tailoff thrust time trace
20	0	Plot thrust imbalance
	1	Do not plot thrust imbalance
21	0	Plot impulse imbalance
	1	Do not plot impulse imbalance
22	0	Plot absolute impulse imbalance
	1	Do not plot absolute impulse imbalance

Card 12 Nonstatistical motor dimensions (3X, F10.2, 5X, F10.3)

Col.	1-3	L =
	4-13	Value of L
	12-18	TAU =
	19-28	Value of TAU

Card 13 Nonstatistical performance constants (requires 4 data cards)

Card 13A (8X, F10.3, 4X, I4, 6X, F10.2, 7X, F10.2, 7X, F10.4)

Col.	1-8	DELTAY =
	8-18	Value of DELTAY
	19-22	II =
	23-26	Value of II
	27-32	XOUT =
	33-42	Value of XOUT

Card 13A (Cont'd)

Col.	43-49	DPOUT =
	50-59	Value of DPOUT
	60-66	ZETAF =
	67-76	Value of ZETAF

Card 13B (4X, F10.1, 4X, F10.1, 6X, F10.2, 7X, F10.3, 6X, F10.5)

Col.	1-4	TB =
	5-14	Value of TB
	15-18	HB =
	19-28	Value of HB
	29-34	PREF =
	35-44	Value of PREF
	45-51	DTREF =
	52-61	Value of DTREF
	62-67	PIPK =
	68-78	Value of PIPK

Card 13C (8X, F10.7, 7X, F10.2, 8X, F10.7, 6X, F10.7)

Col.	1-8	CSTART =
	9-18	Value of CSTART
	19-25	PTRAN =
	26-35	Value of PTRAN
	36-43	CSTARP =
	44-53	Value of CSTARP
	54-59	GAMP =
	60-69	Value of GAMP

Card 13D (7X, F10.3)

Col. 1-7 TMAXQ =

8-17 Value of TMAXQ

Card 14 Description of type of grain configuration (9X, I2, 9X, I2, 8X, I2, 6X, F4.0, 9X, I2, 7X, I2)

Col. 1-9 INPUT =

10-11 Value of INPUT (1, 2 or 3)

12-20 GRAIN =

21-22 Value of GRAIN (1, 2 or 3)

23-30 STAR =

31-32 Value of STAR (0, 1, 2 or 3)

33-38 NT =

39-42 Value of NT

43-51 ORDER =

52-53 Value of ORDER (1, 2, 3 or 4)

54-60 COP =

61-62 Value of COP (0, 1, 2 or 3)

Card 15 Tabular values for geometry at y = 0.0 (requires 2 data cards) (Not required if INPUT = 2)

Card 15A (6X, F6.2, 10X, F11.2, 10X, F11.2, 8X, F11.2)

Col. 1-6 YT =

7-12 0.0

13-22 ABPK =

23-33 Value of ABPK

34-43 ABSK =

44-55 Value of ABSK

Card 15A (Cont'd)

Col. 55-62 ABNK =

63-71 Value of ABNK

Card 15B (22X, F11.2, 9X, F11.2, 8X, F11.2)

Col. 1-22 APHK =

23-33 Value of APHK

34-42 APNK =

43-53 Value of APNK

54-61 VCIT =

62-72 Value of VCIT

Card 16 Tabular inputs for y greater than 0.0 (requires 2 data cards for each y value)(Not required for INPUT = 2)

Card 16A (6X, F6.2, 10X, F11.2, 10X, F11.2, 8X, F11.2)

Col. 1-6 YT =

7-12 Value of YT

13-22 ABPK =

23-33 Value of ABPK

34-43 ABSK =

44-54 Value of ABSK

55-62 ABNK =

63-73 Value of ABNK

Card 16B (22X, F11.2, 9X, F11.2)

Col. 1-22 APHK =

23-33 Value of APHK

34-42 APNK =

43-53 Value of APNK

Card 17 Non-statistical c.p. grain geometry (Not required for GRAIN = 4)(6X, F10.3, 3X, F10.0)

Col.	1-6	XTZO =
	7-16	Value of XTZO
	17-19	S =
	20-29	Value of S

Card 18 Non-statistical star grain geometry (Not required for GRAIN = 2)(4X, F10.0, 4X, F10.0, 4X, F10.0)

Col.	1-4	NS =
	5-14	Value of NS
	15-18	NP =
	19-28	Value of NP
	29-32	NN =
	33-42	Value of NN

Finally, Figure IV-1 is a schematic representation of the data deck construction, and Table IV-1 presents an example set of data. This is the same data as used in the sample case presented in Section V of this report. Note that these are all the data cards which are required for this example for any number of configurations.

Program Listing

Figure IV-2 shows a block diagram of the overall program and Table IV-2 presents the complete program listing.

As previously mentioned, the program has been designed to produce graphical presentations of the computational results. Program statements that must be removed in order to delete the plotter compilation requirements are indicated by check marks (▸) in Table IV-2. Removal of these statements is necessary if the user's computer is not equipped for CALCOMP plotting. However, if other plotters are available, generally only the plotting subroutines need be replaced.

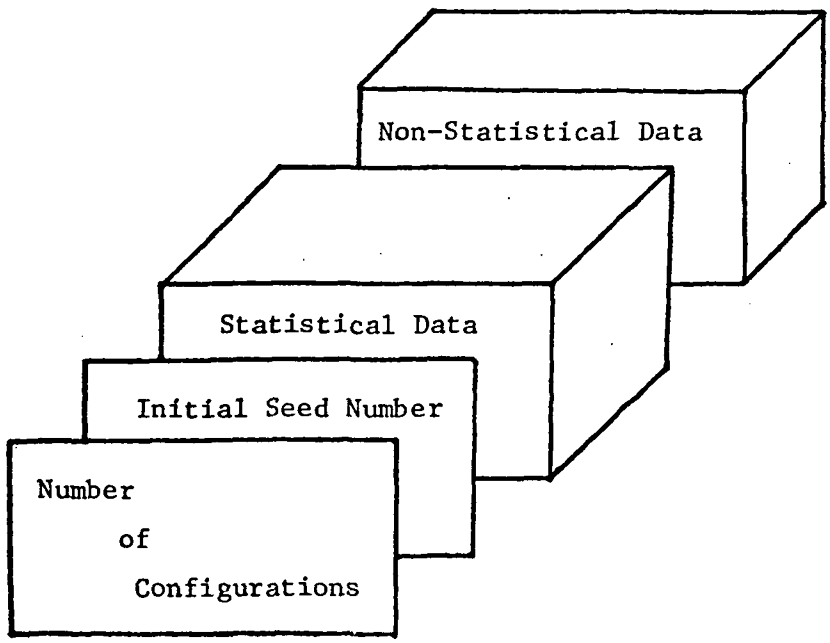

Figure IV-1. Schematic of data deck.

TABLE IV-1. EXAMPLE DATA SHEETS

```
   NUMBER ØF CØNFIGURATIØNS TØ BE TESTED = 2
1882369701
RHØ
51  0.06350   0.0000105
A1
21  11.0      0.03655   0.00001             100.0     0.03655   0.03655
1.0     3.0     5.0     2.0     13.0    16.0    10.0    12.0    4.0     7.0
1.0
N1
60  0.35
A2
21  11.0      0.03655   0.00001             100.0     0.03655   0.03655
1.0     3.0     5.0     2.0     13.0    16.0    10.0    12.0    4.0     7.0
1.0
N2
60  0.35
ALPHA
60  0.0
BETA
60  0.0
RØAL
51  4.35      0.04
DE
51  145.67    0.033333
DTI
51  54.430    0.01
THETA
60  0.0
ALFAN
60  11.25
```

TABLE IV-1. EXAMPLE DATA SHEETS (CONT'D)

LTAP		
60	176.5	
XT		
51	3.04	0.02357
ZØ		
51	2.41	0.02357
ZC		
51	0.0	0.02357
RØNDCW		
53	0.0	0.083333
RØNDCH		
53	0.0	0.083333
RØNDGW		
53	0.0	0.033333
RØNDGH		
53	0.0	0.033333
EXN		
51	0.0	0.05
EYN		
51	0.0	0.05
EXH		
51	0.0	0.05
EYH		
51	0.0	0.05
ALPHAN		
52	0.0	360.0
ALPHAH		
52	0.0	360.0
ERREF		
51	0.00763	0.00032

TABLE IV-1. EXAMPLE DATA SHEETS (CONT'D)

TGR									
51 60.0	0.2333								
TIG									
11 40.0	0.3740	0.0040	15.0	100.0	0.3740	0.4340			
0.3777	0.3811	0.4030	0.3980	0.3744	0.3795	0.4266	0.4300	0.4334	0.4300
0.3980	0.3997	0.3862	0.3845	0.3895	0.3963	0.4013	0.3929	0.4097	0.4081
0.3980	0.4030	0.3827	0.3963	0.3980	0.3996	0.3895	0.3929	0.4081	0.4132
0.4215	0.4148	0.3946	0.3946	0.3744	0.3845	0.4136	0.4148	0.4030	0.4013
DØ									
51 143.08	0.01462								
DI									
51 63.59	0.033333								
THETAG									
60 10.1990									
LGCI									
51 135.58	0.577								
LGNI									
51 51.20	0.33333								
THETCW									
60 0.0									
THETCH									
60 90.0									
LGSI									
51 189.15	0.33333								
RC									
51 71.54	0.00731								
FILL									
51 2.010	0.011111								
RP									
51 12.0	0.01667								

TABLE IV-1. EXAMPLE DATA SHEETS (CONT'D)

```
RIS
51   63.540    0.01667
END
90
0.000.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.00.0
 IEØ=1 IPØ=1
 L=1367.23     TAU=39.74
 DELTAY=0.04       II=  26 XØUT=1000.0        DPØUT=10000.0      ZETAF=0.96
 TB=122.2      HB=130000.0    PREF=560.0        DTREF=57.285       PIPK=0.00150
 CSTART=0.000038   PTRAN=0.0          CSTARP=0.0057        GAMP=0.00527
 TMAXQ=60.0
   INPUT= 2     GRAIN= 3     STAR= 2     NT=0.0      ØRDER= 1     CØP= 2
 XTZØ=0.12          S=1.0
 NS=1.0         NP=9.0         NN=0.0
```

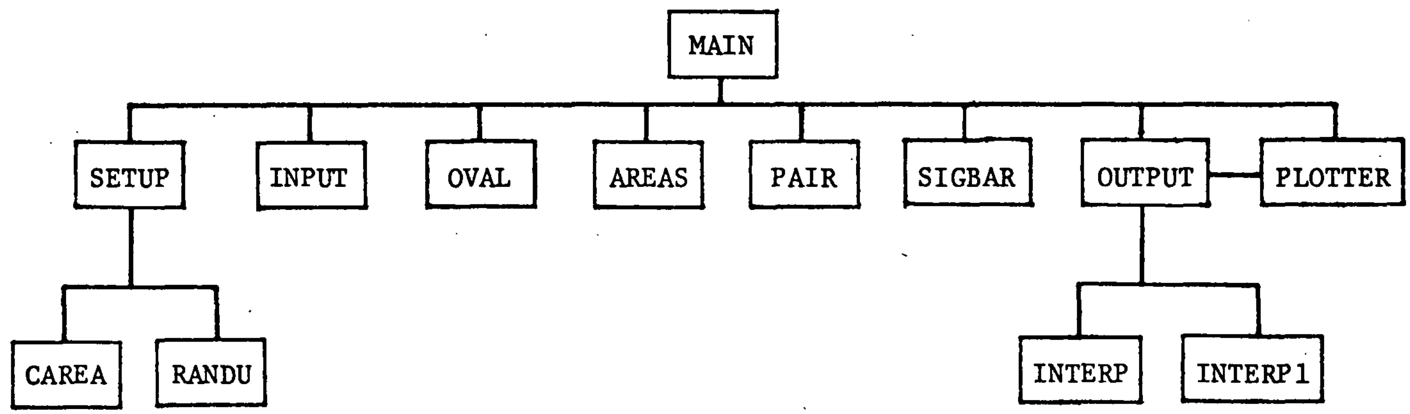

Figure IV-2. Block diagram of computer program.

TABLE IV-2

```
C     ******************************************************************
C     *            MONTE CARLO PERFORMANCE ANALYSIS OF SRM PAIRS        *
C     *                 PREPARED AT AUBURN UNIVERSITY                   *
C     *         UNDER MOD. NO. 12 TO COOPERATIVE AGREEMENT WITH         *
C     *               NASA MARSHALL SPACE FLIGHT CENTER                 *
C     *                                                                 *
C     *                              BY                                 *
C     *    R. H. SFORZINI, W. A. FOSTER, JR. AND J. S. JOHNSON, JR.     *
C     *                AEROSPACE ENGINEERING DEPARTMENT                 *
C     *                         NOVEMBER 1974                           *
C     ******************************************************************
      INTEGER GRAIN
      REAL IDIFF
      REAL MGEN,MDIS,MNOZ,MN1,JROCK,N,L,ME1,ME,ISP,ITOT,MU,MASS,ISPVAC
      REAL N1,N2,NSEG,K1,K2,KEH,KEN,NS,LCC,LTAP
      REAL M2,MOBAR,ISP2,ITVAC,KA,KB,LAMBDA
      COMMON/CONST1/ZW,AE,AT,THETA,ALFAN
      COMMON/CONST2/CAPGAM,ME,BOT,ZETAF,TB,HB,GAM
      COMMON/CONST3/S,NS,GRAIN
      COMMON/CONST4/DELDI,DO,DI,ZC,XT,ZO
      COMMON/CONST5/KPLT
      COMMON/VARIA1/T,DELY,DELTAT,PCNOZ,PHEAD,RNOZ,RHEAD,SUMAB,PHMAX
      COMMON/VARIA2/ABPORT,ABSLOT,ABNOZ,APHEAD,APNOZ,DADY,ABP2,ABN2,ABS2
      COMMON/VARIA3/ITOT,ITVAC,JROCK,ISP,ISPVAC,MDIS,MNOZ,SG,SUMMT
      COMMON/VARIA4/RNT,RHT,SUM2,R1,R2,R3,RHAVE,RNAVE,RBAR,YB,KOUNT
      COMMON/VARIA5/ABMAIN,ABTO,SUMDY,VCI,VC
      COMMON/VARIA6/YDI,TE
      COMMON/VARIA7/Y,THRUST
      COMMON/PLOTT/IPO,NDUM,IPT,IOP
      COMMON/PLOT2/NUMPLT
      COMMON/OVALA/CHIH,CHIN,SEN,SEH,CHINH,AZ,BZ
      COMMON/OVALB/CHINN,CHINAV,SENN
      COMMON/OVALC/RONDCN,RONDCH,RONDGN,RONDGH,EXN,EYN,EXH,EYH,
     2ALPHAN,ALPHAH
      COMMON/OVALM/Z,ZQ
      COMMON/OVALM2/KKI,II
      COMMON/SEED/IX
      COMMON/PAIR1/TW1,TW2,DTW,FW1,FW2,DFW1,DFW2,DFW,TMAXQ,DFMQ,
     2FDIFF,TDIFF,NX
      COMMON/PAIR2/FMAX1,TFMX1,FMIN1,TFMN1,
     2            FMAX2,TFMX2,FMIN2,TFMN2
      COMMON/PAIR3/AFMAX,TFMAX,AFMAXT,TFMAXT
      COMMON/OUT1/FDIFIG,TDIFIG,DIT,ADIT
      COMMON/OUT2/DF100K,T100K
      COMMON/DATA2/I
      COMMON/TOFF/DFTO1,DFTO2,TDFTO1,TDFTO2
      DIMENSION FDIFF(400),TDIFF(400)
      DIMENSION NUMPLT(5)
```

```
      DATA PI,G/3.14159,32.1725/
      READ(5,500) NRUNS
C   ******************************************************************
C   *      READ IN THE NUMBER OF CONFIGURATIONS TO BE TESTED          *
C   ******************************************************************
  500 FORMAT(42X,I2)
      NPAIRS=NRUNS/2
      IOP=0
      NPLOT=0
      KPLOT=0
      TW1=0.0
      FW1=0.0
      WRITE(6,11112)
11112 FORMAT(20X,'DATA FOR STATISTICAL ANALYSIS PROGRAM')
      CALL SETUP
      DO 901 I=1,NRUNS
      REWIND 2
      IX1=IX
      CALL INPUT
      WRITE(6,602) I
  602 FORMAT(1H1,42X,'CONFIGURATION NUMBER ',I2)
      IF(I-1) 5000,5000,5001
 5000 READ(5,499) SUMDY,ANS,ZW,Y,T,DELTAT,RNOZ,RHEAD,SUMAB,PHMAX,SUM2,IT
     1OT,RHT,RNT,R1,R2,R3,RHAVE,RNAVE,RBAR,ITVAC,SUMMT
      WRITE(2,499) SUMDY,ANS,ZW,Y,T,DELTAT,RNOZ,RHEAD,SUMAB,PHMAX,SUM2,I
     1TOT,RHT,RNT,R1,R2,R3,RHAVE,RNAVE,RBAR,ITVAC,SUMMT
      GO TO 5002
 5001 READ(2,499) SUMDY,ANS,ZW,Y,T,DELTAT,RNOZ,RHEAD,SUMAB,PHMAX,SUM2,IT
     1OT,RHT,RNT,R1,R2,R3,RHAVE,RNAVE,RBAR,ITVAC,SUMMT
 5002 CONTINUE
C   ******************************************************************
C   *      SET INITIAL VALUES OF SELECTED VARIABLES EQUAL TO ZERO     *
C   *      ***NOTE***  THESE VALUES MUST BE ZEROED AT THE BEGINNING OF *
C   *      EACH CONFIGURATION RUN                                     *
C   ******************************************************************
  499 FORMAT(22F3.1)
      IF(I-1) 5003,5003,5004
 5003 READ(5,491) IEO,IPO,(NUMPLT(JP),JP=1,5)
      WRITE(2,491) IEO,IPO,(NUMPLT(JP),JP=1,5)
      GO TO 5005
 5004 READ(2,491) IEO,IPO,(NUMPLT(JP),JP=1,5)
 5005 CONTINUE
  491 FORMAT(5X,I1,5X,I1,9X,5I1)
C   ******************************************************************
C   *      READ IN THE USER'S OPTIONS                                 *
C   *                                                                 *
C   *      VALUES FOR IEO ARE                                         *
C   *            0 FOR NO OVALITY                                     *
```

```
C   *             1 FOR OVALITY ANALYSIS                                        *
C   *       VALUES FOR IPO ARE                                                  *
C   *             0 FOR NO PLOTS AND NO STATISTICAL ANALYSIS                    *
C   *             1 FOR PLOTS AND TABULAR OUTPUT                                *
C   *             2 FOR TABULAR OUTPUT ONLY                                     *
C   *             3 FOR PLOTS ONLY                                              *
      CONTINUE
C   *       VALUES FOR NUMPLT(J) ARE  (NOT REQUIRED FOR IPO=0,3)                *
C   *             0 IF SPECIFIC PLOT IS DESIRED                                 *
C   *             1 IF SPECIFIC PLOT IS NOT DESIRED                             *
C   *       ORDER OF SPECIFICATION OF NUMPLT(J) IS                              *
C   *             1 THRUST VS TIME (ENTIRE TRACE)                               *
C   *             2 THRUST VS TIME (TAILOFF PORTION ONLY)                       *
C   *             3 THRUST IMBALANCE VS TIME                                    *
C   *             4 TOTAL IMPULSE IMBALANCE VS TIME                             *
C   *             5 ABSOLUTE TOTAL IMPULSE IMBALANCE VS TIME                    *
C   *****************************************************************************
      WRITE(6,492) IEO,IPO,(NUMPLT(JP),JP=1,5)
  492 FORMAT(//,20X,'OPTIONS',/,13X,'IEO= ',I1,/,13X,'IPO= ',I1,
     2/,13X,'NUMPLT(J)= ',5I2)
11111 FORMAT(E16.9)
      READ(4,11111) RHO,A1,N1,A2,N2,ALPHA,BETA,ROAL
C   *****************************************************************************
C   *       READ IN BASIC PROPELLANT CHARACTERISTICS                            *
C   *                                                                           *
C   *****************************************************************************
C   *       THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL           *
C   *            ANALYSIS PROGRAM                                               *
C   *****************************************************************************
C   *                                                                           *
C   *       RHO IS THE DENSITY OF THE PROPELLANT IN LBM/IN**3                   *
C   *       A1 IS THE BURNING RATE COEFFICIENT BELOW THE TRANSITION             *
C   *            PRESSURE                                                       *
C   *       N1 IS THE BURNING RATE EXPONENT BELOW THE TRANSITION PRESSURE       *
C   *       A2 IS THE BURNING RATE COEFFICIENT ABOVE THE TRANSITION             *
C   *            PRESSURE                                                       *
C   *       N2 IS THE BURNING RATE EXPONENT ABOVE THE TRANSITION PRESSURE       *
C   *       ALPHA AND BETA ARE THE CONSTANTS IN THE EROSIVE BURNING             *
C   *            RELATION OF ROBILLARD AND LENIOR                               *
C   *       ROAL IS THE OXIDIZER TO ALUMINUM RATIO                              *
C   *****************************************************************************
C   *                                                                           *
C   *****************************************************************************
C   *       DEFINE CSTARN AND GAMN                                              *
C   *                                                                           *
C   *       CSTARN IS THE NOMINAL THERMOCHEMICAL CHARACTERISTIC EXHAUST         *
C   *            VELOCITY IN FT/SEC AT 1000 PSI AND 60 DEG F                    *
C   *       GAMN IS THE NOMINAL RATIO OF SPECIFIC HEATS FOR THE                 *
```

```
C   *           PROPELLANT GASES                                               *
C   ****************************************************************************
C   *                                                                          *
      CSTARN=-17.8475*ROAL+5239.7
      GAMN=ROAL*5.67357E-3+1.11707
C   *                                                                          *
C   ****************************************************************************
      WRITE(6,603) RHO,A1,N1,A2,N2,ALPHA,BETA,ROAL,CSTARN,GAMN
  603 FORMAT( //,20X,'PROPELLANT CHARACTERISTICS',/,13X,'RHO= ',F8.6,/,1
     23X,'A1= ',F7.5,/,13X,'N1= ',F5.3,/,13X,'A2= ',F7.5,/,13X,'N2= ',
     3F5.3,/,13X,'ALPHA= ',F4.1,/,13X,'BETA= ',F5.1,/,13X,'ROAL= ',F7.4
     4,/,13X,'CSTARN= ',1PE11.4,/,13X,'GAMN= ',1PE11.4)
      IF(IPO)4002,4002,3999
 3999 IF(I.EQ.1) CALL GSIZE(1200.0,11.0,1121)
      IF((-1)**I) 4000,4000,4001
 4000 REWIND 1
      KPLT=1
      GO TO 4002
 4001 KPLT=2
 4002 CONTINUE
      RHO=RHO/G
      IF(I-1) 5006,5006,5007
 5006 READ(5,502) L,TAU
      WRITE(2,502) L,TAU
      GO TO 5008
 5007 READ(2,502) L,TAU
 5008 CONTINUE
      IF(IEO) 6000,6000,6001
 6000 READ(4,11111) DE,DTI,THETA,ALFAN,LTAP,XT,ZO,ZC
      GO TO 6002
 6001 READ(4,11111) DE,DTI,THETA,ALFAN,LTAP,XT,ZO,ZC,
     2RONDCN,RONDCH,RONDGN,RONDGH,EXN,EYN,EXH,EYH,ALPHAN,ALPHAH
 6002 CONTINUE
C   ****************************************************************************
C   *       READ IN BASIC MOTOR DIMENSIONS                                     *
C   *                                                                          *
C   *       L IS THE TOTAL LENGTH OF THE GRAIN IN INCHES                       *
C   *       TAU IS THE ESTIMATED AVERAGE WEB THICKNESS OF THE CONTROLLING      *
C   *            GRAIN LENGTH IN INCHES                                        *
C   *                                                                          *
C   ****************************************************************************
C   *       THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL          *
C   *            ANALYSIS PROGRAM                                              *
C   ****************************************************************************
C   *                                                                          *
C   *       DE IS THE DIAMETER OF THE NOZZLE EXIT IN INCHES                    *
C   *       DTI IS THE INITIAL DIAMETER OF THE NOZZLE THROAT IN INCHES         *
C   *       THETA IS THE CANT ANGLE OF THE NOZZLE WITH RESPECT TO THE          *
```

```
C  *            MOTOR AXIS IN DEGREES                                        *
C  *       ALFAN IS THE EXIT HALF ANGLE OF THE NOZZLE IN DEGREES             *
C  *       LTAP IS THE LENGTH OF THE GRAIN AT THE NOZZLE END HAVING          *
C  *            ADDITIONAL TAPER NOT REPRESENTED BY ZO IN INCHES             *
C  *       XT IS THE DIFFERENCE IN WEB THICKNESS ASSOCIATED WITH LTAP        *
C  *       ZO IS THE INITIAL DIFFERENCE BETWEEN WEB THICKNESSES IN           *
C  *            INCHES DUE TO GRAIN BORE TAPER AT THE HEAD AND AFT ENDS      *
C  *            OF THE CONTROLLING GRAIN LENGTH                              *
C  *       ZC IS THE INITIAL DIFFERENCE BETWEEN WEB THICKNESSES IN           *
C  *            INCHES DUE TO GRAIN EXTERIOR TAPER AT THE HEAD AND AFT       *
C  *            ENDS OF THE CONTROLLING GRAIN LENGTH                         *
      CONTINUE
C  *       RONDCN AND RONDCH ARE ONE HALF THE DIFFERENCE IN INCHES           *
C  *            BETWEEN THE MAXIMUM AND MINIMUM DIAMETER OF THE GRAIN        *
C  *            EXTERIOR AT THE NOZZLE AND HEAD END REFERENCE PLANES         *
C  *            RESPECTIVELY                                                 *
C  *       RONDGN AND RONDGH ARE ONE HALF THE DIFFERENCE IN INCHES           *
C  *            BETWEEN THE MAXIMUM AND MINIMUM DIAMETER OF THE GRAIN        *
C  *            INTERIOR AT THE NOZZLE AND HEAD END REFERENCE PLANES         *
C  *            RESPECTIVELY                                                 *
C  *       EXN,EYN,EXH AND EYH ARE THE ECCENTRICITIES IN INCHES OF THE       *
C  *            CENTER OF THE GRAIN INTERIOR WITH RESPECT TO THE GRAIN       *
C  *            EXTERIOR AT THE NOZZLE AND HEAD END REFERENCE PLANES         *
C  *            RESPECTIVELY                                                 *
C  *       ALPHAN AND ALPHAH ARE THE ANGULAR ORIENTATIONS IN DEGREES         *
C  *            OF THE OVALITY OF THE GRAIN INTERIOR WITH RESPECT TO         *
C  *            THE GRAIN EXTERIOR AT THE NOZZLE AND HEAD END REFERENCE      *
C  *            PLANES RESPECTIVELY                                          *
C  ***************************************************************************
  502 FORMAT(3X,F10.2,5X,F10.3)
      IF(IEO) 6003,6003,6004
 6003 WRITE(6,6040) L,TAU,DE,DTI,THETA,ALFAN,LTAP,XT,ZO,ZC
      GO TO 6005
 6004 WRITE(6,604) L,TAU,DE,DTI,THETA,ALFAN,LTAP,XT,ZO,ZC,
     2RONDCN,RONDCH,RONDGN,RONDGH,EXN,EYN,EXH,EYH,ALPHAN,ALPHAH
 6005 CONTINUE
  604 FORMAT(//,20X,'BASIC MOTOR DIMENSIONS',/,13X,'L= ',F8.2,/,13X,
     1'TAU= ',F6.3,/,13X,'DE= ',
     21PE11.4,/,13X,'DTI=',1PE11.4,/,13X,'THETA= ',1PE11.4,/,13X,'ALFAN=
     3 ',1PE11.4,/,13X,'LTAP= ',1PE11.4,/,13X,'XT= ',1PE11.4,/,13X,'ZO=
     4',1PE11.4,/,13X,'ZC= ',
     51PE11.4,/,13X,'RONDCN= ',1PE11.4,/,13X,'RONDCH= ',1PE11.4,/,13X,
     6'RONDGN= ',1PE11.4,/,13X,'RONDGH= ',1PE11.4,/,13X,'EXN= ',1PE11.4,
     7/,13X,'EYN= ',1PE11.4,/,13X,'EXH= ',1PE11.4,/,13X,'EYH= ',1PE11.4,
     8/,13X,'ALPHAN= ',1PE11.4,/,13X,'ALPHAH= ',1PE11.4)
 6040 FORMAT(//,20X,'BASIC MOTOR DIMENSIONS',/,13X,'L= ',F8.2,/,13X,
     1'TAU= ',F6.3,/,13X,'DE= ',
     21PE11.4,/,13X,'DTI=',1PE11.4,/,13X,'THETA= ',1PE11.4,/,13X,'ALFAN=
```

```
     3 ',1PE11.4,/,13X,'LTAP= ',1PE11.4,/,13X,'XT= ',1PE11.4,/,13X,'ZO=
     4',1PE11.4,/,13X,'ZC= ',1PE11.4)
      THETA=THETA/57.29578
      ALFAN=ALFAN/57.29578
      ALPHAN=ALPHAN/57.29578
      ALPHAH=ALPHAH/57.29578
      IF(I-1) 5009,5009,5010
 5009 READ(5,503) DELTAY,II,XOUT,DPOUT,ZETAF,TB,HB,PREF,DTREF,PIPK,
     2CSTART,PTRAN,CSTARP,GAMP,TMAXQ
      WRITE(2,503) DELTAY,II,XOUT,DPOUT,ZETAF,TB,HB,PREF,DTREF,PIPK,
     2CSTART,PTRAN,CSTARP,GAMP,TMAXQ
      GO TO 5011
 5010 READ(2,503) DELTAY,II,XOUT,DPOUT,ZETAF,TB,HB,PREF,DTREF,PIPK,
     2CSTART,PTRAN,CSTARP,GAMP,TMAXQ
 5011 CONTINUE
      READ(4,11111) ERREF,TGR,TIGR
C   *******************************************************************
C   *      READ IN BASIC PERFORMANCE CONSTANTS                        *
C   *                                                                 *
C   *      DELTAY IS THE DESIRED BURN INCREMENT DURING TAILOFF IN INCHES *
C   *      II IS THE NUMBER OF INTEGRATION STEPS USED IN OVAL         *
C   *      XOUT IS THE DISTANCE BURNED IN INCHES AT WHICH THE PROPELLANT *
C   *           BREAKS UP                                             *
C   *      DPOUT IS THE DEPRESSURIZATION RATE IN LB/IN**3 AT WHICH THE   *
C   *           PROPELLANT IS EXTINGUISHED                            *
C   *      ZETAF IS THE THRUST LOSS COEFFICIENT                       *
C   *      TMAXQ IS THE ESTIMATED TIME AT WHICH THE MAXIMUM DYNAMIC   *
C   *           PRESSURE OCCURS ON THE VEHICLE                        *
C   *      TB IS THE ESTIMATED BURN TIME IN SECONDS                   *
C   *      HB IS THE ESTIMATED BURNOUT ALTITUDE IN FEET               *
C   *      PREF IS THE REFERENCE NOZZLE STAGNATION PRESSURE           *
C   *      DTREF IS THE REFERENCE THROAT DIAMETER                     *
C   *      PIPK IS THE TEMPERATURE SENSITIVITY COEFFICIENT OF PRESSURE   *
C   *           PER DEGREE F AT CONSTANT K                            *
C   *      CSTART IS THE TEMPERATURE SENSITIVITY PER DEGREE F OF CSTAR   *
C   *           AT CONSTANT PRESSURE                                  *
C   *      CSTARP IS THE PRESSURE SENSITIVITY OF CSTAR                *
C   *      PTRAN IS THE PRESSURE IN PSIA ABOVE WHICH THE BURNING RATE    *
C   *           EXPONENT CHANGES                                      *
C   *      GAMP IS THE PRESSURE SENSITIVITY OF GAM                    *
C   *******************************************************************
      CONTINUE
C   *******************************************************************
C   *      THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL     *
C   *           ANALYSIS PROGRAM                                      *
C   *******************************************************************
C   *                                                                 *
C   *      ERREF IS THE REFERENCE THROAT EROSION RATE IN IN/SEC       *
```

```
C  *     TGR IS THE BULK TEMPERATURE OF THE GRAIN IN DEGREES F          *
C  *     TIGR IS THE IGNITION DELAY IN SECONDS AT 60 DEGREES F          *
C  *******************************************************************
  503 FORMAT(8X,F10.3,4X,I4,6X,F10.2,7X,F10.2,7X,F10.4,/,4X,F10.1,4X,
     2F10.1,6X,F10.2,7X,F10.3,6X,F10.5,/,8X,F10.7,7X,F10.2,8X,F10.7,
     36X,F10.7,/,7X,F10.3)
      WRITE(6,606) DELTAY,II,XOUT,DPOUT,ZETAF,TB,HB,ERREF,PREF,DTREF
     2,TGR,PIPK,CSTART,PTRAN,CSTARP,TIGR,GAMP,TMAXQ
  606 FORMAT(//,20X,'BASIC PERFORMANCE CONSTANTS',/,13X,'DELTAY= ',F5.3,
     1/,13X,'II= ',I4,
     1/,13X,'XOUT= ',F7.2,/,13X,'DPOUT= ',F9.2,/,13X,'ZETAF= ',F6.4,/,13
     2X,'TB= ',F5.1,/,13X,'HB= ',F7.0,/,13X,'ERREF= '
     3,F8.5,/,13X,'PREF= ',F8.2,/,13X,'DTREF= ',F7.3,/,13X,'TGR= ',F7.3,
     4/,13X,'PIPK= ',F7.5,/,13X,'CSTART= ',F10.7,/,13X,'PTRAN= ',F8.2
     5,/,13X,'CSTARP= ',F10.7,/,13X,'TIG= ',F7.4,/,13X,'GAMP= ',F10.7,
     6/,13X,'TMAXQ= ',F7.3)
      TIG=TIGR*EXP(PIPK*(60.0-TGR))
      T=TIG
      A=A1
      N=N1
      CSTARR=CSTARN*EXP(CSTART*(TGR-60.))
      GAM=GAMN
      Q=A*EXP(PIPK*(1.-N)*(TGR-60.))
      KKI=0
      CHIH=1.0
      CHIN=1.0
      CHINN=1.0
      CHINAV=1.0
      SEN=0.0
      SENN=0.0
      SEH=0.0
      CHINH=1.0
      NDUM=0
      IPT=0
      MN1=.85
      ME1=7.0
      Z=Z0+ZC
      ZQ=Z0
      S=0.0
      NS=0.0
      KOUNT=0
      KEWAT=0
      ABMAIN=0.0
      ABTO=0.0
      TW2=0.0
      DTW=0.0
      FW2=0.0
      DFW=0.0
```

```
      DELY=DELTAY
      TOP=GAM+1.
      BOT=GAM-1.
      ZAP=TOP/(2.*BOT)
      CAPGAM=SQRT(GAM)*(2./TOP)**ZAP
      AE=PI*DE*DE/4.
    1 IF(XT.LE.0.0) TE=0.0
      TCALL=(TAU-XT-ABS(Z/2.))/1.05
      IF(IEO.EQ.1.AND.Y.GT.TCALL) CALL OVAL
      IF(XT.LE.0.0) GO TO 40
      TL=(Y-TAU+XT+Z/2.)*LTAP/XT
      IF(TL.LE.0.0) TL=0.0
      IF(TL.GE.LTAP) TL=LTAP
      TE=LTAP-LTAP*CHINAV
      IF(IEO.EQ.0) TE=TL
   40 IF(T-TIG) 41,41,42
   41 DT=DTI
      CSTAR=CSTARR
      GO TO 43
   42 RADER=ERREF*((PONOZ/PREF)**0.8)*((DTREF/DT)**0.2)
      DT=DT+(2.0*RADER*DELTAT)
   43 AT=PI*DT*DT/4.
      CALL AREAS
      IF(Y.LE.0.0) VC=VCI
      IF(ABS(ZW).GT.0.0) GO TO 20
      IF(SUMAB.LE.0.0) GO TO 31
      X=(ABPORT+ABSLOT)/SUMAB
   90 MNOZ=AT*X/APNOZ*(2.*(1.+BOT/2.*MN1*MN1)/TOP)**ZAP
      IF(ABS(MNOZ-MN1).LE.0.002) GO TO 2
      MN1=MNOZ
      GO TO 90
    2 VNOZ=GAM*CSTAR*MNOZ*SQRT(((2./TOP)**(TOP/BOT))/(1.+BOT/2.*MNOZ*MNO
     1Z))
      PRAT=(1.+BOT/2.*MNOZ*MNOZ)**(-GAM/BOT)
      JROCK=AT/APNOZ
  832 SUMYA=DELY*(ABP2+ABN2+ABS2)
      IF(Y.EQ.0.0) SUMYA=0.0
      VC=VC+SUMYA
      IF(Y.GT.0.0) GO TO 11
      PONOZ=(Q*RHO*CSTAR*SUMAB/AT)**(1./(1.-N))*(1.+(CAPGAM*JROCK)**2/2.
     1)**(N/(1.-N))
      PON=PONOZ
      CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
      MDIS=AT*PONOZ/CSTAR
      P2=PONOZ
      PONOZ2=PONOZ
      PNOZ=PRAT*PONOZ
      P4=2.*MDIS*VNOZ/(APHEAD+APNOZ)+PNOZ
```

```
      IF(GRAIN.EQ.3) P4=MDIS*VNOZ/APNOZ+PNOZ
    5 PNOZ=PRAT*PONOZ
      PHEAD=2.*MDIS*VNOZ/(APHEAD+APNOZ)+PNOZ
      IF(GRAIN.EQ.3) PHEAD=MDIS*VNOZ/APNOZ+PNOZ
      IF(PHEAD.LT.PTRAN)N=N1
      IF(PHEAD.LT.PTRAN)A=A1
      IF(PHEAD.GE.PTRAN)N=N2
      IF(PHEAD.GE.PTRAN)A=A2
      RHEAD=Q*PHEAD**N
      ZIT=MDIS*X/APNOZ
      RN1=RHEAD
      PHEAD2=PHEAD
      IF(PONOZ.LT.PTRAN)N=N1
      IF(PONOZ.LT.PTRAN)A=A1
      IF(PONOZ.GE.PTRAN)N=N2
      IF(PONOZ.GE.PTRAN)A=A2
    3 RNOZ=RN1-((RN1-Q*PNOZ**N-ALPHA*ZIT**.8/(L**.2*EXP(BETA*RN1*RHO/ZIT
     1))))/(1.+ALPHA*ZIT**.8*BETA*RHO/ZIT/(L**.2*EXP(BETA*RN1*RHO/ZIT))))
      IF(ABS(RN1-RNOZ).LE.0.002) GO TO 4
      RN1=RNOZ
      GO TO 3
    4 AVE1=(RHEAD+RNOZ)/2.
      IF(Y.GT.0.0) GO TO 7
      RN2=RNOZ
      RH2=RHEAD
      PONJ=PONOZ
      DPCDY=0.0
      AVE2=AVE1
    7 RNAVE=(RNOZ+RN2)/2.
      RHAVE=(RHEAD+RH2)/2.
      MGEN=RHO/2.*((RNOZ+RHEAD)*(ABPORT+ABSLOT)+2.*Q*PONOZ**N*ABNOZ)
      DRDY=(AVE1-AVE2)/DELY
      RBAR=(AVE1+AVE2)/2.
      GMAX=1.0002*MDIS
      GMIN=0.9998*MDIS
      IF(Y.GT.0.0) GO TO 12
      GMAX=1.001*MDIS
      GMIN=0.999*MDIS
      IF(MGEN.GE.GMIN.AND.MGEN.LE.GMAX) GO TO 6
      MDIS=MGEN
      PONOZ=MDIS*CSTAR/AT
      GO TO 5
    6 PONJ=PONOZ
   17 GAM=GAMN*(PONOZ/1000.)**GAMP
      TOP=GAM+1.
      BOT=GAM-1.
      ZAP=TOP/(2.*BOT)
      CAPGAM=SQRT(GAM)*(2./TOP)**ZAP
```

```
      ME=SQRT(2./BOT*(TOP/2.*(AE*ME1/AT)**(1./ZAP)-1.))
      IF(ABS(ME-ME1).LE.0.002) GO TO 9
      ME1=ME
      GO TO 17
    9 IF(Y.LE.0.0) CALL OUTPUT
      IF(Y.LE.0.0) GO TO 10
      DELTAT=2.*DELY/(RHAVE+RNAVE)
      Z=Z+DELTAT*(RNAVE-RHAVE)
      ZQ=ZQ+DELTAT*(RNAVE-RHAVE)
      T=T+DELTAT
      CALL OUTPUT
   10 IF(Y.LE..05*TAU) GO TO 16
      SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.
      MASS=.01*MDIS
      ANS4=Y+10.0*DELTAY
      IF(KOUNT.GT.0) GO TO 16
      IF(ABS(SINK1).LE.MASS.AND.ANS4.LE.ANS-XT) GO TO 18
      GO TO 16
   18 DELY=10.*DELTAY
      GO TO 55
   16 DELY=DELTAY
   55 YLED=Y
      Y=Y+DELY
      IF(Y.GE.(TAU-XT-Z/2.).AND.KEWAT.EQ.0) DELY=TAU-XT-Z/2.-YLED
      IF(Y.GE.(TAU-XT-Z/2.).AND.KEWAT.EQ.0) Y=TAU-XT-Z/2.
      IF(Y.GE.(TAU-XT-Z/2.).AND.KEWAT.EQ.0) KEWAT=1
      ANS=TAU-ABS(Z/2)
      IF(Y.GE.ANS.AND.KOUNT.EQ.0) DELY=ANS-YLED
      IF(Y.GE.ANS.AND.KOUNT.EQ.0) Y=ANS
      DELTAT=2.*DELY/(RHAVE+RNAVE)
      SUM2=SUMAB
      RN2=RNOZ
      RH2=RHEAD
      AVE2=AVE1
      GO TO 1
   11 CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
      MDIS=AT*PONOZ/CSTAR
      GO TO 5
   12 DPCDY=(1./(1.-N))*((PHEAD2+PONOZ2)/((ABP2+ABN2+ABS2)*2.)*DADY)
      IF(ABS(DPCDY).GE.DPOUT.OR.Y.GE.XOUT) GO TO 25
      SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.+(PHEAD2+PONOZ2)/2.*(RNAV
     1E+RHAVE)/2.*(ABP2+ABN2+ABS2)/(12.*(CSTAR*CAPGAM)**2)
      STUFF=MGEN-SINK1
      MDIS=STUFF
      PONOZ=MDIS*CSTAR/AT
      IF(2.*Y+DI+DELDI.GE.DO) PONOZ=PONJ+DPCDY*DELY
      IF(STUFF.GE.GMIN.AND.STUFF.LE.GMAX) GO TO 14
      GO TO 5
```

```
   14 P1=PONOZ
      PONJ=PONOZ
      PONOZ2=(P1+P2)/2.
      P2=PONOZ
      P3=PHEAD
      PHEAD2=(P3+P4)/2.
      P4=PHEAD
      MDIS=AT*PONOZ/CSTAR
      IF(KEWAT.EQ.1) GO TO 2221
      GO TO 2222
 2221 WRITE(6,2223)
 2223 FORMAT(//,37X,'**********************************',/,
     237X,'**** INITIAL TAIL OFF BEGINS ****',/,37X,
     3'**********************************')
      KEWAT=KEWAT+1
 2222 CONTINUE
      IF(Y.LT.ANS) GO TO 17
      ZW=Z
      YW=Y
      SUMBA=SUMAB
      P1=PONOZ
      RH2=RHEAD
      RN2=RNOZ
      RAVE=AVE1
      ABMAIN=SUMAB
      ABTO=0.0
      WRITE(6,51)
   51 FORMAT(//,37X,'**********************************',/,37X,'**** FINAL
     2TAIL OFF BEGINS ****',/,37X,'**********************************')
      IF((-1)**I.LT.0) TW1=T
      IF((-1)**I.LT.0) FW1=THRUST
      IF((-1)**I.GT.0) TW2=T
      IF((-1)**I.GT.0) FW2=THRUST
      IF(TW2.NE.0.) DTW=ABS(TW2-TW1)
      IF(TW2.NE.0.) DFW=ABS(FW2-FW1)
   20 ANS2=TAU+ABS(ZW/2.)
      KOUNT=KOUNT+1
      IF(KOUNT.EQ.1) GO TO 17
      DELYW=DELTAY
      DY2=DELYW
      IF(ZW) 32,32,33
   32 IF(Y.LT.ANS2.AND.ABS(ZW).GT.DY2) GO TO 211
      SUMAB=ABMAIN
      GO TO 31
  211 SUMDY=SUMDY+DELYW
      SUMAB=(1.+SUMDY/ZW-DELYW/(2.*ZW))*ABTO-(SUMDY/ZW-DELYW/(2.*ZW))*AB
     1MAIN
      GO TO 31
```

TABLE IV-2 (CONT'D)

```
33 IF(Y.LT.ANS2.AND.ZW.GT.DY2) GO TO 21
   SUMAB=ABTO
   GO TO 31
21 SUMDY=SUMDY+DELYW
   SUMAB=(1.-SUMDY/ZW+DELYW/(2.*ZW))*ABMAIN+(SUMDY/ZW-DELYW/(2.*ZW))*
  1ABTO
31 IF(SUMAB.LE.0.0) PONOZ=PONOZ/2.
   IF(SUMAB.LE.0.0) GO TO 25
   CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
   MDIS=AT*PONOZ/CSTAR
   ABAVE=(SUMAB+SUMBA)/2.
   SUMYA=DELY*ABAVE
   VC=VC+SUMYA
   DADY=(SUMAB-SUMBA)/DELY
   PBAR=(P1+PONOZ)/2.
   SUMBA=SUMAB
22 DPCDY=PBAR/(1.-N)*1./ABAVE*DADY
   IF(PONOZ.LE.5.0) GO TO 25
   RNOZ=Q*PONOZ**N
   RHEAD=RNOZ
   RBAR=(RHEAD+RAVE)/2.
   MGEN=RHO*(RNOZ+RHEAD)/2.*SUMAB
   GMAX=1.0002*MDIS
   GMIN=0.9998*MDIS
   SINK1=VC/(CAPGAM*CSTAR)**2*RBAR*DPCDY/12.+PBAR*ABAVE/(12.*(CAPGAM
  **CSTAR)**2)*RBAR
   STUFF=MGEN-SINK1
   IF(STUFF.GE.GMIN.AND.STUFF.LE.GMAX) GO TO 23
   MDIS=STUFF
   PONOZ=PONJ+DPCDY*DELY
   IF(PONOZ.LE.0.0) PONOZ=0.0
   PBAR=(P1+PONOZ)/2.
   GO TO 22
23 RHAVE=(RH2+RHEAD)/2.
   RNAVE=(RN2+RNOZ)/2.
   RH2=RHEAD
   RN2=RNOZ
   PHEAD=PONOZ
   RAVE=RHEAD
   P1=PONOZ
   PONJ=PONOZ
   MDIS=AT*PONOZ/CSTAR
   IF(ABS(DPCDY).GE.DPOUT) GO TO 25
   IF(Y.GE.XOUT) GO TO 25
   GO TO 17
25 SUMAB=0.0
   RHEAD=0.0
   RNOZ=RHEAD
```

```
      PHEAD=PONOZ
      MDIS=AT*PONOZ/CSTAR
      WRITE(6,318)
  318 FORMAT(//,37X,'**********************************',/,37X,'**** BEGI
     1N HALF SECOND TRACE ****',/,37X,'**********************************
     1')
      DELTAT=2.0*DELY/(RHAVE+RNAVE)
      T=T+DELTAT
      CALL OUTPUT
      IF(PONOZ.LE.0.0) GO TO 100
      TIME=T
      DELTAT=.5
      TIM=TIME+5.
      PHT=PHEAD
      PONT=PONOZ
      SG=0.0
   29 T=T+DELTAT
      CSTAR=CSTARR*(PONOZ/1000.)**CSTARP
      PHEAD=PHT/EXP(CAPGAM**2*AT*CSTAR/VC*(T-TIME)*12.)
      PONOZ=PHEAD
      MDIS=PONOZ*AT/CSTAR
      Y=Y+.5*RHEAD
      CALL OUTPUT
   28 IF(T.LT.TIM.AND.PHEAD.GE.5.0) GO TO 29
  100 WP1=G*SUMMT
      WP2=RHO*(VC-VCI)*G
      WP=(WP1+WP2)/2.
      ISP=ITOT/WP
      ISPVAC=ITVAC/WP
      WRITE(6,1022)
 1022 FORMAT(//,20X,'INDIVIDUAL MOTOR DATA')
      WRITE(6,102) WP1,WP2,WP,PHMAX,IX1,IX
  102 FORMAT(13X,'WP1= ',1PE11.4,/,13X,'WP2= ',1PE11.4,/,13X,'WP= ',0000
     11PE11.4,/,13X,'PHMAX= ',1PE11.4,/,13X,'IX1= ',I10,/,13X,'IX= ',
     2I10)
      NDUM=1
      IF(IPO.NE.0) CALL OUTPUT
      IF(IPO.EQ.0) GO TO 901
      IF((-1)**I.LT.0) GO TO 901
      CALL PAIR
      CALL SIGBAR(AFMAX,S1,S2,SAFMAX,BAFMAX,I,NPAIRS,SG1,SG2)
      CALL SIGBAR(TFMAX,SA,SB,STFMAX,BTFMAX,I,NPAIRS,SG3,SG4)
      CALL SIGBAR(AFMAXT,S3,S4,SAFMXT,BAFMXT,I,NPAIRS,SG5,SG6)
      CALL SIGBAR(TFMAXT,SC,SD,STFMXT,BTFMXT,I,NPAIRS,SG7,SG8)
      CALL SIGBAR(DFT01,ST,SU,SDFT01,BDFT01,I,NPAIRS,SG19,SG20)
      CALL SIGBAR(TDFT01,SHA,SIA,STDFT1,BTDFT1,I,NPAIRS,SG9,SG10)
      CALL SIGBAR(DFT02,SV,SW,SDFT02,BDFT02,I,NPAIRS,SG21,SG22)
      CALL SIGBAR(TDFT02,SJ,SK,STDFT2,BTDFT2,I,NPAIRS,SG11,SG12)
```

```
      CALL SIGBAR(DTW,SP,SQ,SDTW,BDTW,I,NPAIRS,SG13,SG14)
      CALL SIGBAR(FW1,SLA,SMA,SFW1,BFW1,I,NPAIRS,SG15,SG16)
      CALL SIGBAR(FW2,SN,SO,SFW2,BFW2,I,NPAIRS,SG17,SG18)
      CALL SIGBAR(DFW,Z1,Z2,SDFW,BDFW,I,NPAIRS,SG37,SG38)
      CALL SIGBAR(DFMQ,SX,SY,SDFMQ,BDFMQ,I,NPAIRS,SG23,SG24)
      CALL SIGBAR(FDIFIG,SE,SF,SFDFIG,BFDFIG,I,NPAIRS,SG25,SG26)
      CALL SIGBAR(TDIFIG,SEA,SAF,STDFIG,BTDFIG,I,NPAIRS,SG27,SG28)
      CALL SIGBAR(DIT,S5,S6,SDIT,BDIT,I,NPAIRS,SG29,SG30)
      CALL SIGBAR(ADIT,S7,S8,SADIT,BADIT,I,NPAIRS,SG31,SG32)
      CALL SIGBAR(DF100K,D1,D2,SF100K,BF100K,I,NPAIRS,SG33,SG34)
      CALL SIGBAR(T100K,D3,D4,ST100K,BT100K,I,NPAIRS,SG35,SG36)
  901 CONTINUE
      IF(IPO.EQ.0) STOP
      WRITE(6,887)
  887 FORMAT(//,20X,'STANDARD DEVIATIONS AND MEANS FOR MOTOR PAIR DATA',
     2/,14X,'VAR.',6X,' STD. DEV. ',5X,'    MEAN    ')
      WRITE(6,888) SAFMAX,BAFMAX,STFMAX,BTFMAX,SAFMXT,BAFMXT,
     2STFMXT,BTFMXT,
     2SDFT01,BDFT01,STDFT1,BTDFT1,SDFT02,BDFT02,STDFT2,BTDFT2,
     2SDTW,BDTW,SFW1,BFW1,SFW2,BFW2,SDFW,BDFW,SDFMQ,BDFMQ,
     2SFDFIG,BFDFIG,STDFIG,BTDFIG,SDIT,BDIT,SADIT,BADIT,SF100K,BF100K,
     2ST100K,BT100K
  888 FORMAT(13X,'AFMAX ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'TFMAX ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'AFMAXT',5X,1PE11.4,5X,1PE11.4,/,
     213X,'TFMAXT',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DFT01 ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'TDFT01',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DFT02 ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'TDFT02',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DTW   ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'FW1   ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'FW2   ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DFW   ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DFMQ  ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'FDIFIG',5X,1PE11.4,5X,1PE11.4,/,
     213X,'TDIFIG',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DIT   ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'ADIT  ',5X,1PE11.4,5X,1PE11.4,/,
     213X,'DF100K',5X,1PE11.4,5X,1PE11.4,/,
     213X,'T100K ',5X,1PE11.4,5X,1PE11.4)
      WRITE(6,889) SG1,SG2,SG5,SG6
  889 FORMAT(//,20X,'ALTERNATE DISPERSION VALUES FOR THRUST IMBALANCE DA
     2TA',/,14X,'VAR.',6X,'  SIGMA 1  ',5X,'  SIGMA 2  ',/,
     313X,'AFMAX ',5X,1PE11.4,5X,1PE11.4,/,13X,'AFMAXT',5X,1PE11.4,
     45X,1PE11.4)
      IF(IOP.NE.0) CALL PLOT(0.0,0.0,999)
      STOP
      END
```

```
      SUBROUTINE AREAS
C   ********************************************************************
C   *      SUBROUTINE AREAS CALCULATES BURNING AREAS AND PORT AREAS FOR  *
C   *      CIRCULAR PERFORATED (C.P.) GRAINS AND STAR GRAINS OR FOR A    *
C   *      COMBINATION OF C.P. AND STAR GRAINS                           *
C   ********************************************************************
      INTEGER STAR,GRAIN,ORDER,COP
      REAL MGEN,MDIS,MNOZ,MN1,JROCK,N,L,ME1,ME,ISP,ITOT,MU,MASS,ISPVAC
      REAL LGCI,LGNI,NS,NN,NP,LGSI,NT,LTP,LGC,LS,LF
      REAL M2,MDBAR,ISP2,ITVAC,L1,L2,LFW,LFWSQD
      COMMON/CONST1/ZW,AE,AT,THETA,ALFAN
      COMMON/CONST3/S,NS,GRAIN
      COMMON/CONST4/DELDI,DO,DI,ZC,XT,ZO
      COMMON/VARIA1/T,DELY,DELTAT,PONOZ,PHEAD,RNOZ,RHEAD,SUMAB,PHMAX
      COMMON/VARIA2/ABPORT,ABSLOT,ABNOZ,APHEAD,APNOZ,DADY,ABP2,ABN2,ABS2
      COMMON/VARIA3/ITOT,ITVAC,JROCK,ISP,ISPVAC,MDIS,MNOZ,SG,SUMMT
      COMMON/VARIA4/RNT,RHT,SUM2,R1,R2,R3,RHAVE,RNAVE,RBAR,YB,KOUNT
      COMMON/VARIA5/ABMAIN,ABTO,SUMDY,VCI,VC
      COMMON/VARIA6/YDI,TE
      COMMON/VARIA7/Y,THRUST
      COMMON/OVALA/CHIH,CHIN,SEN,SEH,CHINH,AZ,BZ
      COMMON/DATA2/IDATA
      DATA PI/3.14159/
21111 FORMAT(E16.9)
      ABPC=0.0
      ABNC=0.0
      ABSC=0.0
      ABPS=0.0
      ABNS=0.0
      ABSS=0.0
      DABT=0.0
      SG=0.0
      VCIT=0.0
      ANUM=PI/4.
      PID2=PI/2.
      RNT=RNT+RNOZ*DELTAT
      RHT=RHT+RHEAD*DELTAT
      IF(Y.LE.0.0) AGS=0.0
      K=0
      IF(ABS(ZW).GT.0.0) K=1
      YB=Y
      IF(K.EQ.1) Y=YB-SUMDY/2.
    2 IF(K.EQ.2) Y=YB+ABS(ZW)/2.-SUMDY/2.
      IF(IDATA-1) 5000,5000,5001
 5000 IF(Y.LE.0.0) READ(5,500) INPUT,GRAIN,STAR,NT,ORDER,COP
      IF(Y.LE.0.0) WRITE(2,500) INPUT,GRAIN,STAR,NT,ORDER,COP
      GO TO 5002
 5001 IF(Y.LE.0.0) READ(2,500) INPUT,GRAIN,STAR,NT,ORDER,COP
```

```
 5002 CONTINUE
C   *******************************************************************
C   *      READ THE TYPE OF INPUT FOR THE PROGRAM AND THE BASIC GRAIN    *
C   *           CONFIGURATION AND ARRANGEMENT                             *
C   *      VALUES FOR INPUT ARE                                           *
C   *                1 FOR ONLY TABULAR INPUT                             *
C   *                2 FOR ONLY EQUATION INPUTS (EQUATIONS ARE BUILT      *
C   *                  INTO THE SUBROUTINE)                               *
C   *                3 FOR A COMBINATION OF 1 AND 2                       *
C   *      VALUES FOR GRAIN ARE                                           *
C   *                1 FOR STRAIGHT C.P. GRAIN                            *
C   *                2 FOR STRAIGHT STAR GRAIN                            *
C   *                3 FOR COMBINATION OF C.P. AND STAR GRAINS            *
C   *      VALUES FOR STAR ARE (WAGON WHEEL IS CONSIDERED A TYPE OF       *
C   *        STAR GRAIN IN THIS PROGRAM)                                  *
C   *                0 FOR STRAIGHT C.P. GRAIN                            *
C   *                1 FOR STANDARD STAR                                  *
C   *                2 FOR TRUNCATED STAR                                 *
C   *                3 FOR WAGON WHEEL                                    *
C   *      VALUES FOR NT ARE                                              *
C   *                0 IF THERE ARE NO TERMINATION PORTS                  *
C   *                X WHERE X IS THE NUMBER OF TERMINATION PORTS         *
C   *      VALUES OF ORDER ESTABLISH HOW A COMBINATION C.P. AND STAR      *
C   *        GRAIN IS ARRANGED                                            *
C   *                1 IF DESIGN IS STAR AT HEAD END AND C.P. AT NOZZLE   *
C   *                2 IF DESIGN IS C.P. AT HEAD END AND C.P. AT NOZZLE   *
C   *                3 IF DESIGN IS C.P. AT HEAD END AND STAR AT NOZZLE   *
C   *                4 IF DESIGN IS STAR AT HEAD END AND STAR AT NOZZLE   *
C   *             ***NOTE***  IF GRAIN=1, VALUE OF ORDER MUST BE 2        *
C   *             ***NOTE***  IF GRAIN=2, VALUE OF ORDER MUST BE 4        *
      CONTINUE
C   *      VALUES FOR COP ARE (APPLICABLE TO C.P. GRAINS ONLY)            *
C   *                0 IF BOTH ENDS ARE CONICAL OR FLAT                   *
C   *                1 IF HEAD END IS CONICAL OR FLAT AND AFT END IS      *
C   *                    HEMISPHERICAL                                    *
C   *                2 IF BOTH ENDS ARE HEMISPHERICAL                     *
C   *                3 IF HEAD END IS HEMISPHERICAL AND AFT END IS        *
C   *                    CONICAL OR FLAT                                  *
C   *******************************************************************
  500 FORMAT(9X,I2,9X,I2,8X,I2,6X,F4.0,9X,I2,7X,I2)
      IF(Y.LE.0.0) WRITE(6,607)
  607 FORMAT(//,20X,'GRAIN CONFIGURATION')
      IF(Y.LE.0.0) WRITE(6,600) INPUT,GRAIN,STAR,NT,ORDER,COP
  600 FORMAT(13X,'INPUT= ',I2,/,13X,'GRAIN= ',I2,/,13X,'STAR= ',I2,/,13X
     1,'NT= ',F4.0,/,13X,'ORDER= ',I2,/,13X,'COP= ',I2,//)
      IF(INPUT.EQ.2) GO TO 12
      IF(Y.LE.0.0) GO TO 6
      IF(YT.LE.Y.AND.K.LT.2) GO TO 8
```

```
    9 DENOM=YT-YT2
      SLOPE1=(ABPK-ABPK2)/DENOM
      SLOPE2=(ABSK-ABSK2)/DENOM
      SLOPE3=(ABNK-ABNK2)/DENOM
      SLOPE4=(APHK-APHK2)/DENOM
      SLOPE5=(APNK-APNK2)/DENOM
      B1=ABPK-SLOPE1*YT
      B2=ABSK-SLOPE2*YT
      B3=ABNK-SLOPE3*YT
      B4=APHK-SLOPE4*YT
      B5=APNK-SLOPE5*YT
      ABPT=SLOPE1*Y+B1
      ABST=SLOPE2*Y+B2
      ABNT=SLOPE3*Y+B3
      APHT=SLOPE4*Y+B4
      APNT=SLOPE5*Y+B5
      IF(INPUT.EQ.3) GO TO 3
      GO TO 52
    6 IF(IDATA-1) 5003,5003,5004
 5003 READ(5,507) YT,ABPK,ABSK,ABNK,APHK,APNK,VCIT
      WRITE(2,507) YT,ABPK,ABSK,ABNK,APHK,APNK,VCIT
      GO TO 5005
 5004 READ(2,507) YT,ABPK,ABSK,ABNK,APHK,APNK,VCIT
 5005 CONTINUE
C   *************************************************************************
C   *      READ IN TABULAR VALUES FOR Y=0.0  (NOT REQUIRED IF INPUT=2)      *
C   *                                                                       *
C   *      ABPK IS THE BURNING AREA IN THE PORT IN IN**2                    *
C   *      ABSK IS THE BURNING AREA IN THE SLOTS IN IN**2                   *
C   *      ABNK IS THE BURNING AREA IN THE NOZZLE END IN IN**2              *
C   *      APHK IS THE PORT AREA AT THE HEAD END IN IN**2                   *
C   *      APNK IS THE PORT AREA AT THE NOZZLE END IN IN**2                 *
C   *      VCIT IS THE INITIAL VOLUME OF CHAMBER GASES ASSOCIATED WITH      *
C   *          TABULAR INPUT IN IN**3                                       *
C   *************************************************************************
  507 FORMAT(6X,F6.2,10X,F11.2,10X,F11.2,8X,F11.2,/,22X,F11.2,9X,F11.2,
     1  8X,F11.2)
      WRITE(6,610)
  610 FORMAT(13X,'TABULAR VALUES FOR YT EQUAL ZERO READ IN')
      WRITE(6,583) ABPK,ABSK,ABNK,APHK,APNK
  583 FORMAT(13X,'ABPK=',1PE11.4,5X,'ABSK=',1PE11.4,5X,'ABNK=',1PE11.4,
     1  5X,'APHK=',1PE11.4,5X,'APNK=',1PE11.4)
      WRITE(6,584) VCIT
  584 FORMAT(13X,'VCIT=',1PE11.4,//)
      ABPT=ABPK
      ABST=ABSK
      ABNT=ABNK
      APHT=APHK
```

```
      APNT=APNK
      YT2=YT
      IF(INPUT.EQ.3) GO TO 3
      VCI=VCIT
      GO TO 52
    8 YT2=YT
      ABPK2=ABPK
      ABNK2=ABNK
      ABSK2=ABSK
      APHK2=APHK
      APNK2=APNK
      IF(IDATA-1) 5006,5006,5007
 5006 READ(5,505) YT,ABPK,ABSK,ABNK,APHK,APNK
      WRITE(2,505) YT,ABPK,ABSK,ABNK,APHK,APNK
      GO TO 5008
 5007 READ(2,505) YT,ABPK,ABSK,ABNK,APHK,APNK
 5008 CONTINUE
C    ********************************************************************
C    *      READ IN TABULAR VALUES FOR Y=Y     (NOT REQUIRED FOR INPUT=2)  *
C    ********************************************************************
  505 FORMAT(6X,F6.2,10X,F11.2,10X,F11.2,8X,F11.2,/,22X,F11.2,9X,F11.2)
      WRITE(6,611) YT
  611 FORMAT(///,13X,'TABULAR VALUES FOR YT= ',F7.3,'  READ IN')
      WRITE(6,583) ABPK,ABSK,ABNK,APHK,APNK
      GO TO 9
   12 ABPT=0.0
      ABNT=0.0
      ABST=0.0
    3 IF(GRAIN.NE.2) GO TO 4
      ABPC=0.0
      ABNC=0.0
      ABSC=0.0
      GO TO 7
    4 IF(IDATA-1) 5009,5009,5010
 5009 IF(Y.LE.0.0) READ(5,501) XTZO,S
      IF(Y.LE.0.0) WRITE(2,501) XTZO,S
      GO TO 5011
 5010 IF(Y.LE.0.0) READ(2,501) XTZO,S
 5011 CONTINUE
      IF(Y.LE.0.0) READ(4,21111) DO,DI,THETAG,LGCI,LGNI,THETCN,THETCH
C    ********************************************************************
C    *      READ IN BASIC GEOMETRY FOR C.P. GRAIN  (NOT REQUIRED FOR       *
C    *           STRAIGHT STAR GRAIN)                                      *
C    *      XTZO IS THE DIFFERENCE BETWEEN THE INITIAL INTERNAL GRAIN      *
C    *           DIAMETER AT THE NOZZLE END OF LGCI AND DI IN INCHES       *
C    *           LESS TWICE XT AND LESS ZO                                 *
C    *      S IS THE NUMBER OF FLAT BURNING SLOT SIDES (NOT INCLUDING      *
C    *           THE NOZZLE END)                                           *
```

```
C  *                                                                     *
C  ***********************************************************************
C  *     THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL       *
C  *          ANALYSIS PROGRAM                                           *
C  ***********************************************************************
C  *                                                                     *
C  *     DO IS THE AVERAGE OUTSIDE INITIAL GRAIN DIAMETER IN INCHES      *
C  *     DI IS THE AVERAGE INITIAL INTERNAL GRAIN DIAMETER IN INCHES     *
C  *     THETAG IS THE ANGLE THE NOZZLE END OF THE GRAIN MAKES WITH      *
C  *          THE MOTOR AXIS IN DEGREES                                  *
C  *     LGCI IS THE INITIAL TOTAL LENGTH OF THE CIRCULAR PERFORATION    *
C  *          IN INCHES                                                  *
C  *     LGNI IS THE INITIAL SLANT LENGTH OF THE BURNING CONICAL         *
C  *          GRAIN AT THE NOZZLE END IN INCHES                          *
C  *     THETCN IS THE CONTRACTION ANGLE OF THE BONDED GRAIN IN DEGREES*
C  *     THETCH IS THE CONTRACTION ANGLE AT THE HEAD END IN DEGREES      *
C  ***********************************************************************
  501 FORMAT(6X,F10.3,3X,F10.0)
      IF(Y.LE.0.0) WRITE(6,601) DO,DI,XTZO,S,THETAG,LGCI,LGNI,THETCN,TH
     1ETCH
  601 FORMAT(20X,'C.P. GRAIN GEOMETRY',/,13X,'DO= ',F7.3,/,13X,'DI= ',F7
     1.3,/,13X,'XTZO= ',F7.3,/,13X,'S= ',F4.0,/,13X,'THETAG= ',F8.5,/,13
     2X,'LGCI= ',F7.2,/,13X,'LGNI= ',F6.2,/,13X,'THETCN= ',F8.5,/,13X,
     3'THETCH= ',F8.5,//)
      IF(Y.LE.0.0) TAU=(DO-DI)/2.0
      IF(Y.LE.0.0) DELDI=2.0*XT+ZO+XTZO
      IF(Y.LE.0.0)THETAG=THETAG/57.29578
      IF(Y.LE.0.0)THETCN=THETCN/57.29578
      IF(Y.LE.0.0)THETCH=THETCH/57.29578
      DOSQD=DO*DO
      DISQD=DI*DI
      BNUM=ANUM*DOSQD
      TLL=TE
      IF(ORDER.GE.3) TLL=0.0
      YDI=2.*Y+DI
      YDISQD=YDI*YDI
      ABSC=S*ANUM*(DOSQD-YDISQD)
      IF(ABSC.LE.0.0) ABSC=0.0
      IF(YDI.GE.DO)GO TO 100
      IF(THETAG.GT.0.08727) GO TO 101
      IF(COP.EQ.0) GO TO 700
      IF(COP.EQ.1) GO TO 701
      IF(COP.EQ.2) GO TO 702
      CHCK1=DOSQD-YDISQD
      IF(CHCK1.LT.0.0) CHCK1=0.0
      LGC=LGCI-(SQRT(DOSQD-DISQD)-SQRT(CHCK1))/2.-Y*COTAN(THETCN)
      GO TO 710
  702 CHCK1=DOSQD-YDISQD
```

```
      IF(CHCK1.LT.0.0) CHCK1=0.0
      LGC=LGCI-(SQRT(DOSQD-DISQD)-SQRT(CHCK1))
      GO TO 710
  701 CHCK2=DOSQD-(YDI+DELDI)**2
      IF(CHCK2.LT.0.0) CHCK2=0.0
      LGC=LGCI-(SQRT(DOSQD-(DI+DELDI)**2)-SQRT(CHCK2))/2.
     2-Y*COTAN(THETCH)
      GO TO 710
  700 LGC=LGCI-Y*(COTAN(THETCN)+COTAN(THETCH))
  710 ABPC=PI*YDI*(LGC-TLL-S*Y)
      ABNC=0.0
      GO TO 732
  101 CONTINUE
      IF(COP.EQ.0.OR.COP.EQ.1) GO TO 720
      CHCK1=DOSQD-YDISQD
      IF(CHCK1.LT.0.0) CHCK1=0.0
      LGC=LGCI-(SQRT(DOSQD-DISQD)-SQRT(CHCK1))/2.-TLL
     2-(S+TAN(THETAG/2.))*Y
      ABPC=PI*YDI*LGC
      GO TO 730
  720 LGC=LGCI-Y*COTAN(THETCH)-TLL-(S+TAN(THETAG/2.))*Y
      ABPC=PI*YDI*LGC
  730 IF(COP.EQ.1.OR.COP.EQ.2) GO TO 731
      ABNC=PI*(LGNI-Y*COTAN(THETAG+THETCN)-Y*TAN(THETAG/2.))*(DI+
     1  DELDI+Y+LGNI*SIN(THETAG)+Y*SIN(THETCN)/SIN(THETAG+THETCN))
      GO TO 732
  731 IF(Y.LE.0.0) GO TO 7311
      GO TO 7312
 7311 R7=((DI+DELDI)/2.+LGNI*SIN(THETAG))*COS(THETAG)-SIN(THETAG)*
     1  SQRT((DO/2.)**2-((DI+DELDI)/2.+LGNI*SIN(THETAG))**2)
 7312 IF(R7+Y.LT.(DO/2.)*COS(THETAG)) GO TO 11111
      ABNC=PI*(LGNI+(1./SIN(THETAG))*((DO/2.)-LGNI*SIN(THETAG)-(DI
     2-DELDI)/2.)-Y*COTAN(THETAG)-Y*COTAN(THETAG/2.))*((DI-DELDI)/2.
     3+Y+DO/2.)
      GO TO 22222
11111 RPR=SQRT(((DO/2.)**2)-R7**2)-SQRT(((DO/2.)**2)-(R7+Y)**2)
      ABNC=PI*(LGNI-RPR-Y*TAN(THETAG/2.))*((DI+DELDI)/2.+SQRT((DO/
     1  2.)**2-(R7+Y)**2)*SIN(THETAG)+Y+(R7+Y)*COS(THETAG))
22222 CONTINUE
  732 IF(ABPC.LE.0.0) ABPC=0.0
      IF(ABNC.LE.0.0) ABNC=0.0
      GO TO 5
  100 ABNC=0.0
      ABPC=0.0
    5 APHT=ANUM*(DI+2.*RHT)**2
      IF(APHT.GE.BNUM) APHT=BNUM
      IF(K.LT.2) APHT1=APHT
      APNT=ANUM*(DI+DELDI+2.*RNT)**2
```

```
      IF(APNT.GE.BNUM) APNT=BNUM
      IF(GRAIN.NE.1) GO TO 7
      ABPS=0.0
      ABSS=0.0
      ABNS=0.0
      GO TO 50
    7 IF(IDATA-1) 5012,5012,5013
 5012 IF(Y.LE.0.0) READ(5,502) NS,NP,NN
      IF(Y.LE.0.0) WRITE(2,502) NS,NP,NN
      GO TO 5014
 5013 IF(Y.LE.0.0) READ(2,502) NS,NP,NN
 5014 CONTINUE
      IF(Y.LE.0.0) READ(4,21111) LGSI,RC,FILL
C  ******************************************************************
C  *     READ IN BASIC GEOMETRY FOR STAR GRAIN  (NOT REQUIRED FOR    *
C  *          STRAIGHT C.P. GRAIN)                                   *
C  *     NS IS THE NUMBER OF FLAT BURNING SLOT SIDES (NOT INCLUDING  *
C  *          THE NOZZLE END)                                        *
C  *     NP IS THE NUMBER OF STAR POINTS                             *
C  *     NN IS THE NUMBER OF STAR NOZZLE END BURNING SURFACES        *
C  *                                                                 *
C  ******************************************************************
C  *     THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL   *
C  *          ANALYSIS PROGRAM                                       *
C  ******************************************************************
C  *                                                                 *
C  *     LGSI IS THE INITIAL TOTAL LENGTH OF THE STAR SHAPED         *
C  *          PERFORATED GRAIN IN INCHES                             *
C  *     RC IS THE AVERAGE STAR GRAIN OUTSIDE RADIUS IN INCHES       *
C  *     FILL IS THE FILLET RADIUS IN INCHES                         *
C  ******************************************************************
  502 FORMAT(4X,F10.0,4X,F10.0,4X,F10.0)
      IF(Y.LE.0.0) WRITE(6,602) NS,LGSI,NP,RC,FILL,NN
  602 FORMAT(20X,'BASIC STAR GEOMETRY',/,13X,'NS= ',F4.0,/,13X,'LGSI= ',
     1F7.2,/,13X,'NP= ',F4.0,/,13X,'RC= ',F7.3,/,13X,'FILL= ',F7.3,/,13X
     2,'NN= ',F4.0,//)
      IF(Y.LE.0.0.AND.GRAIN.EQ.2) DO=2.0*RC
      PIDNP=PI/NP
      RCSQD=RC*RC
      FY=FILL+Y
      FYSQD=FY*FY
      IF(STAR.EQ.1) GO TO 20
      IF(STAR.EQ.2) GO TO 201
      IF(Y.GT.0.0) GO TO 179
      READ(4,21111) RIWW,L1,L2,ALPHA1,ALPHA2,HW
C  ******************************************************************
C  *     READ IN GEOMETRY FOR WAGON WHEEL (NOT REQUIRED FOR STANDARD *
C  *          OR TRUNCATED STAR GRAINS)                              *
```

```
C   *                                                                        *
C   **************************************************************************
C   *       THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL        *
C   *            ANALYSIS PROGRAM                                            *
C   **************************************************************************
C   *                                                                        *
C   *       RIWW IS THE AVERAGE RADIUS OF THE INSIDE OF THE PROPELLANT       *
C   *            WEB IN INCHES                                               *
C   *       L1 AND L2 ARE THE LENGTHS OF THE TWO PARALLEL SIDES OF THE       *
C   *            TWO SETS OF STAR POINTS IN INCHES                           *
C   *       ALPHA1 AND ALPHA2 ARE THE ANGLES BETWEEN THE SLANT SIDES OF      *
C   *            THE STAR POINTS CORRESPONDING TO L1 AND L2, RESPECTIVELY,*
C   *            AND THE CENTER LINES OF THE POINTS IN DEGREES               *
C   *       HW IS HALF THE WIDTH OF THE STAR POINTS IN INCHES                *
C   **************************************************************************
      WRITE(6,422) RIWW,L1,L2,ALPHA1,ALPHA2,HW
  422 FORMAT(20X,'WAGON WHEEL GEOMETRY',/,13X,'RIWW= ',F5.2,/,13X,
     1  'L1= ',F5.2,/,13X,'L2= ',F5.2,/,13X,'ALPHA1= ',F7.5,/,13X,
     2  'ALPHA2= ',F7.5,/,13X,'HW= ',F5.2,//)
      IF(Y.LE.0.0) TAUWW=RC-RIWW
      IF(Y.LE.0.0.AND.GRAIN.EQ.2) DI=DO-2.0*TAUWW
      ALPHA1=ALPHA1/57.29578
      ALPHA2=ALPHA2/57.29578
      ALP2=ALPHA2
      XL2=L2
      LFW=RC-TAUWW-FILL
      LFWSQD=LFW*LFW
      THETFW=ARSIN((HW+FILL)/LFW)
      SLFW=LFW*SIN(THETFW)
  179 KKK=0
      SG=0.0
      ENUM=(RCSQD-LFWSQD-FYSQD)/(2.*LFW*FY)
      ALPHA2=ALP2
      L2=XL2
  190 YTAN=Y*TAN(ALPHA2/2.)
      COSALP=COS(ALPHA2)
      SINALP=SIN(ALPHA2)
      IF(YTAN.GT.L2) GO TO 182
      IF(FY.GT.SLFW) GO TO 181
      SGW=NP*(L2-2.*YTAN+(SLFW-FILL)/SINALP-Y*COTAN(ALPHA2)+FY*
     1  (PID2+THETFW)+(LFW+FY)*(PIDNP-THETFW))
      GO TO 183
  181 IF(Y.GT.TAUWW) GO TO 184
      SGW=NP*(FY*(PIDNP+ARSIN(SLFW/FY))+(PIDNP-THETFW)*LFW)
      GO TO 183
  184 SGW=NP*FY*(THETFW+ARSIN(SLFW/FY)-ARCOS(ENUM))
      GO TO 183
  182 YPO=-SLFW
```

```
      IF(ALPHA2.GE.PID2) GO TO 222
      Q=-FILL+L2*TAN(ALPHA2)-Y/COSALP
      XPI=(-Q*TAN(ALPHA2)-SQRT(-Q*Q+FYSQD/COSALP*COSALP))*COSALP*COSALP
      YPI=XPI*TAN(ALPHA2)+Q
      XPO=(YPO-Q)*COTAN(ALPHA2)
      GO TO 223
  222 XPI=Y-L2
      YPI=-SQRT(FYSQD-XPI*XPI)
      XPO=XPI
  223 FYLS=SQRT(SLFW*SLFW+XPI*XPI)
      XPIO2=(XPI-XPO)*(XPI-XPO)
      YPIO2=(YPI-YPO)*(YPI-YPO)
      IF(FY.GT.FYLS) GO TO 186
      IF(Y.GE.TAUWW) GO TO 185
      SGW=NP*(SQRT(XPIO2+YPIO2)+FY*(PID2+THETFW-ARSIN(XPI/FY))+(LFW+FY)*
     1  (PIDNP-THETFW))
      GO TO 183
  185 SGW=NP*(SQRT(XPIO2+YPIO2)+FY*(PID2-ARSIN(XPI/FY)-ARCOS(ENUM)))
      GO TO 183
  186 IF(Y.GT.TAUWW) GO TO 187
      SGW=NP*(FY*(PIDNP+ARSIN(SLFW/FY))+(PIDNP-THETFW)*LFW)
      GO TO 183
  187 SGW=NP*FY*(THETFW+ARSIN(SLFW/FY)-ARCOS(ENUM))
  183 IF(SGW.LE.0.0) SGW=0.0
      IF(Y.GT.0.0) GO TO 188
      AGS2=.5*(PI*RCSQD-NP*LFW*SLFW*(COS(THETFW)-SIN(THETFW)*COTAN(ALPHA
     1 2)-2.*(L2+FILL*TAN(ALPHA2/2.))/LFW)-(PI-THETFW*NP)*LFWSQD-2.*NP*F
     2 ILL*(L2+SLFW/SINALP+LFW*(PIDNP-THETFW)+(PIDNP+PID2-1./SINALP)*
     1  FILL/2.))
      AGS=AGS+AGS2
  188 CONTINUE
      SG=SG+SGW
      IF(KKK.EQ.1) GO TO 24
      L2=L1
      ALPHA2=ALPHA1
      KKK=1
      GO TO 190
  201 IF(Y.LE.0.0) READ(4,21111) RP,RIS
C   ********************************************************************
C   *      READ IN GEOMETRY FOR TRUNCATED STAR  (NOT REQUIRED FOR       *
C   *             STANDARD STAR OR WAGON WHEEL)                         *
C   *                                                                   *
C   ********************************************************************
C   *      THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL    *
C   *             ANALYSIS PROGRAM                                      *
C   ********************************************************************
C   *                                                                   *
C   *      RP IS THE INITIAL RADIUS OF THE TRUNCATION IN INCHES         *
```

```
C  *      RIS IS THE AVERAGE RADIUS OF THE INSIDE OF THE PROPELLANT       *
C  *            WEB IN INCHES                                              *
C  *            OF THE SLOTS IN INCHES                                     *
C  *********************************************************************
      IF(Y.LE.0.0) WRITE(6,603) RP,RIS
  603 FORMAT(20X,'TRUNCATED STAR GEOMETRY',/,13X,'RP= ',F7.3,/,13X,'RIS=
     1 ',F7.3,//)
      IF(Y.LE.0.0) TAUS=RC-RIS
      IF(Y.LE.0.0.AND.GRAIN.EQ.2) DI=DO-2.0*TAUS
      THETAS=PIDNP
      RPY=RP+Y
      LS=RC-TAUS-FILL-RP
      RPL=RP+LS
      THETS1=THETAS-ARSIN(FY/RPY)
      IF(THETS1.LE.0.0) GO TO 110
      IF(Y.LE.TAUS) GO TO 103
      THETAC=ARSIN((RCSQD-RPL*RPL-FYSQD)/(2.*FY*RPL))
      IF(THETAC.GE.0.0) GO TO 104
      IF(Y.LT.RC-RP) GO TO 105
      SG=0.0
      GO TO 14
  103 SG=2.*NP*(RPY*THETS1+LS-(RPY*COS(THETAS-THETS1)-RP)+PID2*FY)
      GO TO 14
  104 SG=2.*NP*(RPY*THETS1+LS-(RPY*COS(THETAS-THETS1)-RP)+FY*THETAC)
      GO TO 14
  105 SG=2.*NP*(RPY*THETS1+SQRT(RCSQD-FYSQD)-SQRT(RPY*RPY-FYSQD))
   14 IF(Y.LE.0.0) AGS=PI*(RCSQD-RP*RP)-NP*(PI*FILL*FILL/2.+2.*LS*FILL)
      GO TO 31
  110 THETAF=THETAS
      THETAP=2.*THETAS
      TAUWS=TAUS
      GO TO 111
   20 IF(Y.LE.0.0) READ(4,21111) THETAF,THETAP,RIWS
C  *********************************************************************
C  *      READ IN GEOMETRY FOR STANDARD STAR  (NOT REQUIRED FOR           *
C  *            TRUNCATED STAR OR WAGON WHEEL)                             *
C  *                                                                       *
C  *********************************************************************
C  *      THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL       *
C  *            ANALYSIS PROGRAM                                           *
C  *********************************************************************
C  *                                                                       *
C  *      THETAF IS THE ANGLE LOCATION OF THE FILLET CENTER IN DEGREES    *
C  *      THETAP IS THE ANGLE OF THE STAR POINT IN DEGREES                 *
C  *      RIWS IS THE AVERAGE RADIUS OF THE INSIDE OF THE PROPELLANT      *
C  *            WEB IN INCHES                                              *
C  *********************************************************************
      IF(Y.LE.0.0) WRITE(6,604) THETAF,THETAP,RIWS
```

```
604 FORMAT(20X,'STANDARD STAR GEOMETRY',/,13X,'THETAF= ',F7.5,/,13X,'T
   1HETAP= ',F7.5,/,13X,'RIWS= ',F6.3,//)
    IF(Y.LE.0.0) TAUWS=RC-RIWS
    IF(Y.LE.0.0.AND.GRAIN.EQ.2) DI=DO-2.0*TAUWS
    THETAF=THETAF/57.29578
    THETAP=THETAP/57.29578
    THETAS=PI/NP
    THETS1=1.00
111 LF=RC-TAUWS-FILL
    CNUM=(Y+FILL)/LF
    DNUM=SIN(THETAF)/SIN(THETAP/2.)
    ENUM=(RCSQD-LF*LF-FYSQD)/(2.*LF*FY)
    FNUM=SIN(THETAF)/COS(THETAP/2.)
    IF(CNUM.LE.FNUM) GO TO 106
    IF(Y.LE.TAUWS)GO TO 107
    SG=2.*NP*FY*(THETAF+ARSIN(SIN(THETAF)/CNUM)-ARCOS(ENUM))
    GO TO 23
106 IF(Y.LE.TAUWS) SG=2.*NP*LF*(DNUM+CNUM*(PID2+THETAS-THETAP/2.
   1-COTAN(THETAP/2.))+THETAS-THETAF)
    IF(Y.LE.TAUWS) GO TO 23
    SG=2.*NP*(FY*ARSIN(ENUM-(THETAS-THETAP/2.))+LF*DNUM-FY*COTAN(THETA
   1P/2.))
    GO TO 23
107 SG=2.*NP*LF*(CNUM*(THETAS+ARSIN(SIN(THETAF)/CNUM))+THETAS-THETAF)
 23 IF(THETS1.LE.0.0) GO TO 14
    IF(Y.LE.0.0) AGS=PI*RC*RC-NP*LF*LF*(SIN(THETAF)*(COS(THETAF)-
   1SIN(THETAF)*COTAN(THETAP/2.))+THETAS-THETAF+2.*FILL/LF*(SIN(THETAF
   2)/SIN(THETAP/2.)+THETAS-THETAF+FILL/(2.*LF)*(PID2+THETAS-THE
   3TAP/2.-COTAN(THETAP/2.))))
 24 CONTINUE
 31 IF(SG.LE.0.0) SG=0.0
    IF(K.EQ.0.OR.K.EQ.2) SGN=SG
    IF(K.LE.1) SGH=SG
    IF(Y.LE.0.0) SG2=SG
    IF(K.EQ.2) GO TO 37
    RAVEDT=R1+(SG+SG2)/2.*RBAR*DELTAT
    RNDT=R2+(SG+SG2)/2.*RNAVE*DELTAT
    RHDT=R3+(SG+SG2)/2.*RHAVE*DELTAT
    R1=RAVEDT
    R2=RNDT
    R3=RHDT
    SG2=SG
    GO TO 38
 37 IF(KOUNT.NE.1) GO TO 39
    SG3=SG
    R4=R1
    R5=R2
    R6=R3
```

```
   39 RAVEDT=R4+(SG+SG3)/2.*RBAR*DELTAT
      RNDT=R5+(SG+SG3)/2.*RNAVE*DELTAT
      RHDT=R6+(SG+SG3)/2.*RHAVE*DELTAT
      R4=RAVEDT
      R5=RNDT
      R6=RHDT
      SG3=SG
   38 ABSS=(AGS-RAVEDT)*NS
      IF(ABSS.LE.0.0.OR.SG.LE.0.0) ABSS=0.0
      ABNS=(AGS-RNDT)*NN
      IF(ABNS.LE.0.0.OR.SG.LE.0.0) ABNS=0.0
      IF(ORDER.LE.2) ABPS=(LGSI-Y*(NS+NN))*SG
      IF(ORDER.LE.2) GO TO 36
      ABPS=(LGSI-TE-Y*(NS+NN))*SG
   36 PIRCRC=PI*RCSQD
      APHS=PIRCRC-AGS+RHDT
      IF(APHS.GE.PIRCRC.OR.SG.LE.0.0) APHS=PIRCRC
      APNS=PIRCRC-AGS+RNDT
      IF(K.LT.2) APHS1=APHS
      IF(APNS.GE.PIRCRC) APNS=PIRCRC
   50 IF(NT.EQ.0.0) GO TO 371
      IF(Y.LE.0.0) READ(4,21111) LTP,DTP,THETTP,TAUEFF
C    ******************************************************************
C    *     READ IN GEOMETRY ASSOCIATED WITH TERMINATION PORTS  (NOT     *
C    *           REQUIRED IF NT=0)                                      *
C    *                                                                  *
C    ******************************************************************
C    *     THE FOLLOWING VARIABLES ARE OBTAINED FROM THE STATISTICAL    *
C    *           ANALYSIS PROGRAM                                       *
C    ******************************************************************
C    *                                                                  *
C    *     LTP IS THE INITIAL LENGTH OF THE TERMINATION PASSAGES        *
C    *           IN INCHES                                              *
C    *     DTP IS THE INITIAL DIAMETER OF THE TERMINATION PASSAGE       *
C    *           IN INCHES                                              *
C    *     THETTP IS THE ACUTE ANGLE BETWEEN THE AXIS OF THE PASSAGE    *
C    *           AND THE MOTOR AXIS IN DEGREES                          *
C    *     TAUEFF IS THE ESTIMATED EFFECTIVE WEB THICKNESS AT THE       *
C    *           TERMINATION PORT IN INCHES                             *
C    ******************************************************************
      IF(Y.LE.0.0) WRITE(6,606) LTP,DTP,THETTP,TAUEFF
  606 FORMAT(20X,'TERMINATION PORT GEOMETRY',/,13X,'LTP= ',F6.2,/,13X,'D
     1TP= ',F5.2,/,13X,'THETTP= ',F7.5,/,13X,'TAUEFF= ',F6.3,//)
      THETTP=THETTP/57.29578
      DABT=NT*3.14159*((DTP+2.*Y)*(LTP-Y/SIN(THETTP))-(DTP+2.*Y)**2/4.+
     1(Y+DTP/2.)*(DTP/2.)*(1.-1./SIN(THETTP)))
      IF(Y.GE.TAUEFF) DABT=0.0
  371 IF(Y.GT.0.0) GO TO 52
```

TABLE IV-2 (CONT'D)

```
      IF(NT.NE.0.0) GO TO 45
      LTP=0.0
      DTP=0.0
   45 IF(GRAIN.NE.2) GO TO 49
      LGCI=0.0
      LGNI=0.0
      DISQD=0.0
      DOSQD=4.*RCSQD
   49 IF(GRAIN.EQ.1) LGSI=0.0
      VCI=1.1*(ANUM*DISQD*(LGCI+LGNI)+(ANUM*DOSQD-AGS)*LGSI+NT*LTP*ANUM*
     1  DTP*DTP)+VCIT
   52 BBP=0.0
      BBS=0.0
      BBN=0.0
      IF(K.NE.0) GO TO 521
      IF(KKL.EQ.0.AND.KKM.EQ.0) GO TO 521
      IF(KKL.EQ.0) ABPC=ABPC*(BZ+AZ*CHIN)
      IF(KKM.EQ.0) ABPC=ABPC*(AZ*CHIH+BZ)
      IF(KKL.EQ.0.AND.GRAIN.EQ.2) ABPS=ABPS*(BZ+AZ*CHIN)
      IF(KKM.EQ.0.AND.GRAIN.EQ.2) ABPS=ABPS*(AZ*CHIH+BZ)
  521 ABPORT=ABPT+ABPC+ABPS+DABT+BBP
      ABSLOT=ABST+ABSC+ABSS+BBS
      ABNOZ=ABNT+ABNC+ABNS+BBN
      IF(K.GE.2) GO TO 55555
      SUMAB=ABPORT+ABSLOT+ABNOZ
55555 CONTINUE
      IF(K.EQ.0) GO TO 99
      IF(ZW) 322,323,323
  322 IF(K.EQ.1) ABPORT=ABPORT*CHIN
      IF(K.EQ.2.AND.GRAIN.NE.2) ABPORT=ABPORT+SEH*LGC
      IF(K.EQ.2.AND.GRAIN.EQ.2) ABPORT=ABPORT+SEH*(LGSI-Y*(NS+NN))
      GO TO 33333
  323 IF(K.EQ.1) ABPORT=ABPORT*CHIH
      IF(K.EQ.2.AND.GRAIN.NE.2) ABPORT=ABPORT+SEN*LGC
      IF(K.EQ.2.AND.GRAIN.EQ.2) ABPORT=ABPORT+SEN*(LGSI-Y*(NS+NN))
33333 IF(K.EQ.1) ABMAIN=ABPORT+ABSLOT+ABNOZ
      K=K+1
      IF(K.GT.2) GO TO 69
      GO TO 2
   69 ABTO=ABPORT+ABSLOT+ABNOZ
   99 CONTINUE
      IF(Y.GT.0.0) GO TO 70
      ABP1=ABPORT
      ABN1=ABNOZ
      ABS1=ABSLOT
   70 ABP2=(ABP1+ABPORT)/2.
      ABN2=(ABN1+ABNOZ)/2.
      ABS2=(ABS1+ABSLOT)/2.
```

```
      IF(INPUT.EQ.1) GO TO 76
      GO TO (71,72,73,74),ORDER
   71 APHEAD=APHS1
      APNOZ=APNT
      SG=SGH
      GO TO 75
   72 APHEAD=APHT1
      APNOZ=APNT
      SG=0.0
      IF(GRAIN.EQ.3) SG=(SGH+SGN)/2.
      GO TO 75
   73 APHEAD=APHT1
      APNOZ=APNS
      SG=SGN
      GO TO 75
   74 APHEAD=APHS1
      APNOZ=APNS
      SG=SGN
      GO TO 75
   76 APHEAD=APHT
      APNOZ=APNT
   75 Y=YB
      DIFF=SUMAB-SUM2
      DADY=DIFF/DELY
      ABP1=ABPORT
      ABN1=ABNOZ
      ABS1=ABSLOT
      IF(ZW.GE.0.0) GO TO 77
      ABM1=ABMAIN
      ABMAIN=ABTO
      ABTO=ABM1
   77 RETURN
      END
```

```
      SUBROUTINE OUTPUT
C   ************************************************************************
C   *      SUBROUTINE OUTPUT CALCULATES BASIC PERFORMANCE PARAMETERS      *
C   *      AND PRINTS THEM OUT                                             *
C   *      (WEIGHT CALCULATIONS ARE PERFORMED IN THE MAIN PROGRAM)         *
C   *      T IS THE TIME IN SECS                                           *
C   *      Y IS THE DISTANCE BURNED IN INCHES                              *
C   *      SUMAB IS THE TOTAL BURNING AREA OF PROPELLANT IN IN**2          *
C   *          (IF ANY)                                                    *
C   *      F IS THE THRUST IN LBS                                          *
C   ************************************************************************
      REAL MDIS,ME,ITOT,M2,MDBAR,ITPLOT,ITPLT1,IDIFF,IADIFF
      COMMON/CONST1/ZW,AE,AT,THETA,ALFAN
      COMMON/CONST2/CAPGAM,ME,BOT,ZETAF,TB,HB,GAM
      COMMON/CONST5/KPLT
      COMMON/VARIA1/T,DELY,DELTAT,PONOZ,PHEAD,RNOZ,RHEAD,SUMAB,PHMAX
      COMMON/VARIA3/ITOT,ITVAC,JROCK,ISP,ISPVAC,MDIS,MNOZ,SG,SUMMT
      COMMON/VARIA5/ABMAIN,ABTO,SUMDY,VCI,VC
      COMMON/VARIA7/Y,F
      COMMON/PAIR1/TW1,TW2,DTW,FW1,FW2,DFW1,DFW2,DFW,TMAXQ,DFMQ,
     2FDIFF,TDIFF,NX
      COMMON/PLOTT/IPO,NDUM,NP,IOP
      COMMON/PLOT2/NUMPLT
      COMMON/OUT1/FDIFIG,TDIFIG,DIT,ADIT
      COMMON/OUT2/DF100K,T100K
      COMMON/DATA2/ICOUNT
      DIMENSION TOFPLT(400),TOTPLT(400),TOFPL1(400),TOTPL1(400)
      DIMENSION FPLOT(400),FPLOT1(400),ITPLOT(400),ITPLT1(400),
     2TPLOT(400),TPLOT1(400)
      DIMENSION FDIFF(400),IDIFF(400),TDIFF(400),IADIFF(400)
      DIMENSION NUMPLT(5)
      DATA G/32.1725/
      IF(Y.LE.0.0) NTO=0
      IF(NDUM.EQ.1) GO TO 2
      NP=NP+1
      YSFT=0.0
      YB=Y
      IF(Y.LE.0.0) M2=MDIS
      MDBAR=(M2+MDIS)/2.
      SUMMT=SUMMT+MDBAR*DELTAT
      PRES=(1.+BOT/2.*ME*ME)**(-GAM/BOT)
      ALT=HB*(T/TB)**(7./3.)
      PATM=14.696/EXP(0.43103E-04*ALT)
      IF(MDIS.LE.0.0.OR.PONOZ.LE.0.0) GO TO 45
      CF=CAPGAM*SQRT(2.*GAM/BOT*(1.-PRES**(BOT/GAM)))+AE/AT*(PRES-PATM/P
     1ONOZ)
      F=ZETAF*COS(THETA)*PONOZ*AT*((1.+COS(ALFAN))/2.*CF+(1.-COS(ALFAN))
     1/2.*AE/AT*(PRES-PATM/PONOZ))
```

```
      IF(F.LE.0.0) F=0.0
      IF(Y.LE.0.0) F2=F
      FBAR=(F+F2)/2.
      ITOT=ITOT+FBAR*DELTAT
      M2=MDIS
      F2=F
      IF(PHEAD.GT.PHMAX) PHMAX=PHEAD
      GO TO 47
   45 F=0.0
   47 WRITE(6,1) T,YB,PONOZ,PHEAD,SUMAB,F,ITOT
    1 FORMAT(//,13X,'TIME= ',F7.3,12X,'Y= ',F6.3,/,13X,
     2'PONOZ= ',1PE11.4,'  PHEAD= ',1PE11.4,'  SUMAB= ',1PE11.4,
     3'  F= ',1PE11.4,'  ITOT= ',1PE11.4)
      IF(IPO.EQ.0) RETURN
      TPLOT(NP)=T
      FPLOT(NP)=F
      ITPLOT(NP)=ITOT
      IF(TPLOT(NP).LT.100.) GO TO 50
      NTO=NTO+1
      TOTPLT(NTO)=T
      TOFPLT(NTO)=F
   50 RETURN
    2 NP=NP+2
      NTO=NTO+2
      IOP=1
      IF(KPLT-1) 4000,4000,4001
 4000 NP2=NP-2
      NTO2=NTO-2
      WRITE(1,4002) NP2
 4002 FORMAT(I4)
      WRITE(1,4003) (FPLOT(I),ITPLOT(I),TPLOT(I),I=1,NP2)
 4003 FORMAT(E12.5)
      WRITE(1,4002) NTO2
      WRITE(1,4003) (TOFPLT(I),TOTPLT(I),I=1,NTO2)
      GO TO 1004
 4001 REWIND 1
      IF(IPO.NE.3) WRITE(6,9998)
 9998 FORMAT(//,20X,'TABULATED IMBALANCE DATA',/,
     213X,'    TIME    ',10X,'   FDIFF    ',10X,'   IDIFF    ',
     210X,'   IADIFF  ')
      READ(1,4002) NP21
      READ(1,4003) (FPLOT1(I),ITPLT1(I),TPLOT1(I),I=1,NP21)
      READ(1,4002) NTO1
      READ(1,4003) (TOFPL1(I),TOTPL1(I),I=1,NTO1)
      NP1=NP21+2
  ►   IF(IPO.EQ.2) GO TO 8888
  ►   IF(ICOUNT.EQ.2) YSFT=1.5
  ►   IF(NUMPLT(1).NE.0) GO TO 7001
```

```
      CALL PLOTIT(FPLOT1,TPLOT1,NP1,FPLOT,TPLOT,NP,'THRUST (LBS)',12,
     2'TIME (SECS)',-11,0.0,400000.0,0.0,10.0,9.0,YSFT)
 7001 XSFT=18.0
      IF(NUMPLT(1).NE.0) XSFT=9.0
      NT1=NT01+2
      IF(NUMPLT(2).NE.0) GO TO 7002
      IF(NUMPLT(1).EQ.0) YSFT=0.0
      CALL PLOTIT(TOFPL1,TOTPL1,NT1,TOFPLT,TOTPLT,NTO,'THRUST (LBS)',12,
     2'TIME (SECS)',-11,0.0,400000.0,100.0,2.0,XSFT,YSFT)
 7002 XSFT=18.0
      IF(NUMPLT(1).NE.0.AND.NUMPLT(2).NE.0) XSFT=9.0
 8888 CONTINUE
      IF(NP1-NP) 2000,2000,2001
 2000 NX=NP-2
      NY=NP1-2
      CALL INTERP(TPLOT,FPLOT,NX,TPLOT1,FPLOT1,NY,FDIFF,0)
      CALL INTERP(TPLOT,ITPLOT,NX,TPLOT1,ITPLT1,NY,IDIFF,1)
      TDIFIG=TPLOT(1)
      FDIFIG=ABS(FDIFF(1))
      DO 3000 J=2,NX
      IF(TPLOT(J).GT..02*TB) GO TO 3001
      IF(ABS(FDIFF(J)).LT.ABS(FDIFF(J-1))) GO TO 3000
      FDIFIG=ABS(FDIFF(J))
      TDIFIG=TPLOT(J)
 3000 CONTINUE
 3001 CONTINUE
      DO 2004 I=1,NX
 2004 TDIFF(I)=TPLOT(I)
      DUM1=0.0
      IADIFF(1)=ABS(FDIFF(1)/2.)*TPLOT(1)
      DO 2003 I=2,NX
      FBARI=(FDIFF(I)+FDIFF(I-1))/2.
      DUM1=ABS(FBARI)*(TPLOT(I)-TPLOT(I-1))
 2003 IADIFF(I)=IADIFF(I-1)+DUM1
      IF(IPO.NE.3) WRITE(6,9999) (TPLOT(I),FDIFF(I),IDIFF(I),IADIFF(I),
     2I=1,NX)
 9999 FORMAT(13X,1PE11.4,10X,1PE11.4,10X,1PE11.4,10X,1PE11.4)
      TI=AMIN1(TW1,TW2)
      CALL INTRP1(IDIFF,TPLOT,NX,TI,DIT1,0)
      DIT=IDIFF(NX)-DIT1
      CALL INTRP1(IADIFF,TPLOT,NX,TI,ADIT1,0)
      ADIT=IADIFF(NX)-ADIT1
      CALL INTRP1(FDIFF,TPLOT,NX,TMAXQ,DFMQ,0)
      CALL INTRP1(FDIFF,TPLOT,NX,TW1,DFW1,0)
      CALL INTRP1(FDIFF,TPLOT,NX,TW2,DFW2,0)
      CALL INTRP1(TPLOT,FPLOT,NX,100000.,T100K2,1)
      CALL INTRP1(TPLOT1,FPLOT1,NY,100000.,T100K1,1)
      T100K=AMAX1(T100K1,T100K2)
```

```
      CALL INTRP1(FDIFF,TPLOT,NX,T100K,DF100K,0)
►     IF(IPO.EQ.2) GO TO 8887
►     CALL SCALE(IADIFF,8.0,NX,1)
►     YSCAL1=-ABS(8.0*IADIFF(NX+2))
►     YSCAL2=ABS(2.0*IADIFF(NX+2))
►     NX=NX+2
►     IF(NUMPLT(3).NE.0) GO TO 7003
►     IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0) YSFT=0.0
►     CALL PLOT1(TPLOT,FDIFF,NX,'THRUST IMBALANCE (LBS)',22,
►    2'TIME (SECS)',-11,-400000.,100000.,0.0,26.0,4.0,XSFT,YSFT)
►7003 XSFT=9.0
►     IF(NUMPLT(3).NE.0) XSFT=18.0
►     IF(NUMPLT(4).NE.0) GO TO 7004
►     IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0.OR.NUMPLT(3).EQ.0) YSFT=0.0
►     CALL PLOT1(TPLOT,IDIFF,NX,'IMPULSE IMBALANCE (LB-SECS)',27,
►    2'TIME (SECS)',-11,YSCAL1,YSCAL2,0.0,26.0,4.0,XSFT,YSFT)
►7004 XSFT=9.0
►     IF(NUMPLT(3).NE.0.AND.NUMPLT(4).NE.0) XSFT=18.0
►     IF(NUMPLT(5).NE.0) GO TO 7005
►     IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0.OR.NUMPLT(3).EQ.0.OR.NUMPLT(4)
►    2.EQ.0) YSFT=0.0
►     CALL PLOT1(TPLOT,IADIFF,NX,'ABS. IMPULSE IMBALANCE (LB-SECS)',32,
►    2'TIME (SECS)',-11,IADIFF(NX-1),IADIFF(NX),0.0,26.0,0.0,XSFT,YSFT)
►7005 CONTINUE
►     NX=NX-2
►8887 CONTINUE
      GO TO 1004
 2001 NX=NP1-2
      NY=NP-2
      CALL INTERP(TPLOT1,FPLOT1,NX,TPLOT,FPLOT,NY,FDIFF,0)
      CALL INTERP(TPLOT1,ITPLT1,NX,TPLOT,ITPLOT,NY,IDIFF,1)
      TDIFIG=TPLOT1(1)
      FDIFIG=ABS(FDIFF(1))
      DO 3002 J=2,NX
      IF(TPLOT(J).GT..02*TB) GO TO 3003
      IF(ABS(FDIFF(J)).LT.ABS(FDIFF(J-1))) GO TO 3002
      FDIFIG=ABS(FDIFF(J))
      FDIFIG=FDIFF(J)
      TDIFIG=TPLOT1(J)
 3002 CONTINUE
 3003 CONTINUE
      DO 2005 I=1,NX
 2005 TDIFF(I)=TPLOT1(I)
      DUM1=0.0
      IADIFF(1)=ABS(FDIFF(1)/2.)*TPLOT1(1)
      DO 2002 I=2,NX
      FBARI=(FDIFF(I)+FDIFF(I-1))/2.
      DUM1=ABS(FBARI)*(TPLOT1(I)-TPLOT1(I-1))
```

TABLE IV-2 (CONT'D)

```
2002 IADIFF(I)=IADIFF(I-1)+DUM1
      IF(IPO.NE.3) WRITE(6,9999) (TPLOT1(I),FDIFF(I),IDIFF(I),IADIFF(I),
     2I=1,NX)
      TI=AMIN1(TW1,TW2)
      CALL INTRP1(IDIFF,TPLOT1,NX,TI,DIT1,0)
      DIT=IDIFF(NX)-DIT1
      CALL INTRP1(IADIFF,TPLOT1,NX,TI,ADIT1,0)
      ADIT=IADIFF(NX)-ADIT1
      CALL INTRP1(FDIFF,TPLOT1,NX,TMAXQ,DFMQ,0)
      CALL INTRP1(FDIFF,TPLOT1,NX,TW1,DFW1,0)
      CALL INTRP1(FDIFF,TPLOT1,NX,TW2,DFW2,0)
      CALL INTRP1(TPLOT,FPLOT,NX,100000.,T100K2,1)
      CALL INTRP1(TPLOT1,FPLOT1,NY,100000.,T100K1,1)
      T100K=AMAX1(T100K1,T100K2)
      CALL INTRP1(FDIFF,TPLOT1,NX,T100K,DF100K,0)
      IF(IPO.EQ.2) GO TO 1004
      CALL SCALE(IADIFF,8.0,NX,1)
      YSCAL1=-ABS(8.0*IADIFF(NX+2))
      YSCAL2=ABS(2.0*IADIFF(NX+2))
      NX=NX+2
      IF(NUMPLT(3).NE.0) GO TO 7006
      IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0) YSFT=0.0
      CALL PLOT1(TPLOT1,FDIFF,NX,'THRUST IMBALANCE (LBS)',22,
     2'TIME (SECS)',-11,-400000.,100000.,0.0,26.0,4.0,XSFT,YSFT)
7006 XSFT=9.0
      IF(NUMPLT(3).NE.0) XSFT=18.0
      IF(NUMPLT(4).NE.0) GO TO 7007
      IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0.OR.NUMPLT(3).EQ.0) YSFT=0.0
      CALL PLOT1(TPLOT1,IDIFF,NX,'IMPULSE IMBALANCE (LB-SECS)',27,
     2'TIME (SECS)',-11,YSCAL1,YSCAL2,0.0,26.0,4.0,XSFT,YSFT)
7007 XSFT=9.0
      IF(NUMPLT(3).NE.0.AND.NUMPLT(4).NE.0) XSFT=18.0
      IF(NUMPLT(5).NE.0) GO TO 7008
      IF(NUMPLT(1).EQ.0.OR.NUMPLT(2).EQ.0.OR.NUMPLT(3).EQ.0.OR.NUMPLT(4)
     2.EQ.0) YSFT=0.0
      CALL PLOT1(TPLOT1,IADIFF,NX,'ABS. IMPULSE IMBALANCE (LB-SECS)',32,
     2'TIME (SECS)',-11,IADIFF(NX-1),IADIFF(NX),0.0,26.0,0.0,XSFT,YSFT)
7008 CONTINUE
       NX=NX-2
1004 CONTINUE
      RETURN
      END
```

```
► SUBROUTINE PLOTIT(Y1,X1,NP1,Y2,X2,NP2,YHDR,NY,XHDR,NX,SY1,SY2,
►2SX1,SX2,XSFT,YSFT)
► DIMENSION XHDR(8),YHDR(8),X1(NP1),Y1(NP1),X2(NP2),Y2(NP2)
► N1=NP1-2
► NS1=NP1-1
► N2=NP2-2
► NS2=NP2-1
► X1(NS1)=SX1
► X1(NP1)=SX2
► X2(NS2)=SX1
► X2(NP2)=SX2
► Y1(NS1)=SY1
► Y1(NP1)=SY2
► Y2(NS2)=SY1
► Y2(NP2)=SY2
► CALL PLOT(XSFT,YSFT,-3)
► CALL AXIS(0.0,0.0,YHDR,NY,8.0,90.0,SY1,SY2)
► CALL AXIS(0.0,0.0,XHDR,NX,14.0,0.0,SX1,SX2)
► CALL LINE(X1,Y1,N1,1,0,1)
► CALL LINE(X2,Y2,N2,1,0,2)
► NPLOT=NPLOT+1
► RETURN
► END
```

TABLE IV-2 (CONT'D)

```
      SUBROUTINE OVAL
      REAL M1,N1
      COMMON/CONST4/DELDI,DO,DI,ZC,XT,ZO
      COMMON/VARIA7/Y
      COMMON/OVALM/Z,ZQ
      COMMON/OVALM2/KKI,II
      COMMON/OVALA/CHIH,CHIN,SEN,SEH,CHINH,AZ,BZ
      COMMON/OVALB/CHINN,CHINAV,SENN
      COMMON/OVALC/RONDCN,RONDCH,RONDGN,RONDGH,EXN,EYN,EXH,EYH,
     2ALPHAN,ALPHAH
      DATA PI/3.14159/
      KKI=KKI+1
      IF(KKI.GT.1) GO TO 8
      AGN=(RONDGN+SQRT(RONDGN**2+DI**2))/2.
      BGN=AGN-RONDGN
      AGH=(RONDGH+SQRT(RONDGH**2+DI**2))/2.
      BGH=AGH-RONDGH
      DTH=2.*PI/II
      KKM=0
      KKL=0
      KKJ=0
      KKXT=0
      KKP=0
      AX=0.
      AZ=0.
      BZ=0.
      ACN=(RONDCN+(RONDCN**2+(DO-ZC)**2)**.5)/2.
      BCN=ACN-RONDCN
      ACH=(RONDCH+(RONDCH**2+(DO+ZC)**2)**.5)/2.
      BCH=ACH-RONDCH
      A1N=(COS(ALPHAN))**2+(ACN/BCN)**2*(SIN(ALPHAN))**2
      A1H=(COS(ALPHAH))**2+(ACH/BCH)**2*(SIN(ALPHAH))**2
      B1N=((ACN/BCN)**2-1.)*SIN(2.*ALPHAN)
      B1H=((ACH/BCH)**2-1.)*SIN(2.*ALPHAH)
      C1N=2.*(EXN*COS(ALPHAN)-(ACN/BCN)**2*EYN*SIN(ALPHAN))
      C1H=2.*(EXH*COS(ALPHAH)-(ACH/BCH)**2*EYH*SIN(ALPHAH))
      D1N=2.*((ACN/BCN)**2*EYN*COS(ALPHAN)-EXN*SIN(ALPHAN))
      D1H=2.*((ACH/BCH)**2*EYH*COS(ALPHAH)-EXH*SIN(ALPHAH))
      E1N=(SIN(ALPHAN))**2+(ACN/BCN)**2*(COS(ALPHAN))**2
      E1H=(SIN(ALPHAH))**2+(ACH/BCH)**2*(COS(ALPHAH))**2
      F1N=ACN**2-EXN**2-((ACN/BCN)*EYN)**2
      F1H=ACH**2-EXH**2-((ACH/BCH)*EYH)**2
      SENNO=PI*(DO-ZC)
      SENO=SENNO
      SEHO=PI*(DO+ZC)
    8 KK=0
      YO=Y
    3 IF(KK.EQ.1) Y=YO+ZQ/2.
```

```
      IF(KK.EQ.1) GO TO 5
    2 IF(KK.EQ.2) Y=YO-ZQ/2.
      IF(KK.EQ.2) GO TO 6
      IF(KK.EQ.0.AND.XT.GT.0.) Y=YO+XT+ZQ/2.
      IF(KK.EQ.0.AND.XT.GT.0.) GO TO 7
      KK=1
      GO TO 3
    5 THETA=0.0
      SUMO=0.
      DO 12 I=1,II
      THETA=THETA+DTH
      M1=A1N*(COS(THETA))**2+B1N*SIN(THETA)*COS(THETA)+
     2E1N*(SIN(THETA))**2
      N1=C1N*COS(THETA)+D1N*SIN(THETA)
      RC=(-N1+SQRT(N1**2+4.*M1*F1N))/(2.*M1)
      IF(RC.LT.0.) RC=(-N1-SQRT(N1**2+4.*M1*F1N))/(2.*M1)
      RG=SQRT(1./((COS(THETA)/(AGN+Y))**2+(SIN(THETA)/(BGN+Y))**2))
      IF(RG.GE.RC) KKM=1
      IF(RG.GE.RC) RG=0.
      SUMO=SUMO+RG*DTH
   12 CONTINUE
      IF(KKM.EQ.1) SEN=SUMO
      IF(SUMO.LE.0.) SEN=0.
      IF(KKM.EQ.0) GO TO 9
      CHIN=SEN/SENO
      CHINAV=(1.+CHINN)/2.
      IF(XT.LE.0.0) CHINAV=1.0
      IF(KKJ.EQ.1) CHINAV=(1.-AX)*CHIN+AX*CHINN
      CHINH=(1.+CHIN)/2.
    9 KK=2
      IF(Z.GE.0.0.AND.KKM.EQ.0) GO TO 62
      GO TO 2
    6 THETA=0.0
      SUMO=0.0
      DO 13 I=1,II
      THETA=THETA+DTH
      M1=A1H*(COS(THETA))**2+B1H*SIN(THETA)*COS(THETA)+
     2E1H*(SIN(THETA))**2
      N1=C1H*COS(THETA)+D1H*SIN(THETA)
      RC=(-N1+SQRT(N1**2+4.*M1*F1H))/(2.*M1)
      IF(RC.LT.0.) RC=(-N1-SQRT(N1**2+4.*M1*F1H))/(2.*M1)
      RG=SQRT(1./((COS(THETA)/(AGH+Y))**2+(SIN(THETA)/(BGH+Y))**2))
      IF(RG.GE.RC) KKL=1
      IF(RG.GE.RC) RG=0.
      SUMO=SUMO+RG*DTH
   13 CONTINUE
      IF(KKL.EQ.1) SEH=SUMO
      IF(SUMO.LE.0.) SEH=0.
```

```
      CHIH=SEH/SEHO
      IF(KKL.EQ.0) CHIH=1.0
      CHINH=(1.+CHIH)/2.
      IF(KKM.EQ.1) CHINH=(SEN+SEH)/(SENO+SEHO)
      GO TO 62
    7 THETA=0.0
      SUMO=0.
      DO 11 I=1,II
      THETA=THETA+DTH
      M1=A1N*(COS(THETA))**2+B1N*SIN(THETA)*COS(THETA)+
     2E1N*(SIN(THETA))**2
      N1=C1N*COS(THETA)+D1N*SIN(THETA)
      RC=(-N1+SQRT(N1**2+4.*M1*F1N))/(2.*M1)
      IF(RC.LT.0.) RC=(-N1-SQRT(N1**2+4.*M1*F1N))/(2.*M1)
      RG=SQRT(1./((COS(THETA)/(AGN+Y))**2+(SIN(THETA)/(BGN+Y))**2))
      IF(RG.GE.RC) KKJ=1
      IF(RG.GE.RC) RG=0.
      SUMO=SUMO+RG*DTH
   11 CONTINUE
      IF(KKJ.EQ.1) SENN=SUMO
      IF(SUMO.LE.0.) SENN=0.0
      IF(KKJ.EQ.0) GO TO 9
      CHINN=SENN/SENNO
      KKXT=KKXT+1
      IF(KKXT.EQ.1) YXIP=Y
      AX=(Y-YXIP)/(XT+DO/2.-DI/2.-YXIP)
      IF(AX.LE.0.) AX=0.
      IF(AX.GE.1.0) AX=1.0
      CHINAV=1.-AX+AX*CHINN
      KK=1
      IF(AX.LE.0.5.AND.XT.GE.0.02097*DO) GO TO 9
      GO TO 3
   62 Y=YO
      IF(KKL.EQ.0.AND.KKM.EQ.0) GO TO 63
      KKP=KKP+1
      IF(KKP.EQ.1) YZIP=Y
      AZ=(Y-YZIP)/(ABS(Z)/2.+DO/2.-DI/2.-YZIP)
      IF(AZ.LE.0.) AZ=0.
      BZ=1.-AZ
   63 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SETUP
      INTEGER TEMPCO,CODE
      REAL T(200)
      REAL ANS(60)
      REAL TEMPA(10),CONST(60)
      INTEGER ORDER(60),CNSTNM
      REAL FXAREA(2,100)
      REAL XX(105),YY(105)
      REAL PSEUDO(105)
      REAL X(40,105),Y(105),FX(40,105)
C     ********************************************************************
C     *      IF THE DIMENSION OF X AND FX ARE CHANGED M AND N SHOULD       *
C     *            ALSO BE RESET                                           *
C     ********************************************************************
      REAL MODE,MEAN,M1,M2,K,INC
      COMMON/SEED/IX
      INPTNM=0
      CNSTNM=0
      N=105
      NI=100
      NSI=10
      M=40
      MM=0
      NI1=NI+1
      NSI1=NSI+1
      READ(5,100)IX
   30 CONTINUE
      READ(5,106) NAM1,NAM2
  106 FORMAT(2A4)
      READ(5,102)CODE,X1,X2,X3,X4,X5,X6,X7
      WRITE(6,107) NAM1,NAM2,CODE,X1,X2,X3,X4,X5,X6,X7
  107 FORMAT(1X,2A4,5X,I2,5X,7(1PE11.4,3X))
      IF(CODE.EQ.90) GO TO 399
      INPTNM=INPTNM+1
      IF(CODE.EQ.60)GO TO 356
      MM=MM+1
      ORDER(INPTNM)=MM
      TEMPCO=CODE/10
      GO TO (31,32,33,3+,35),TEMPCD
   31 CONTINUE
      NOI=X4
      NOI1=NOI+1
      X(MM,1)=X2
      DO 311 I=2,NOI
      X(MM,I)=X(MM,I-1)+X3
  311 CONTINUE
      DO 312 I=1,NOI
      Y(I)=0.
```

```
312 CONTINUE
    H=X3
    STARTR=X2-X3/2.
    SUM=0.
    NOV=X1
    NOC=(X1+9.)/10.
    DO 313 JJ=1,NOC
    READ(5,104)(TEMPA(I),I=1,10)
    WRITE(6,109) (TEMPA(I),I=1,10)
    DO 314 J=1,10
    IF(JJ*10+J.GT.NOV)GO TO 317
    DO 315 I=1,NOI
    IF(TEMPA(J).LT.X(MM,I)+X3/2.)GO TO 316
315 CONTINUE
    GO TO 314
316 CONTINUE
    Y(I)=Y(I)+1.
    SUM=SUM+1.
314 CONTINUE
313 CONTINUE
317 CONTINUE
    IF(CODE.EQ.11)GO TO 99
    FX(MM,1)=0.
    DO 318 I=2,NOI1
    FX(MM,I)=FX(MM,I-1)+Y(I-1)/SUM
318 CONTINUE
    GO TO 30
 32 CONTINUE
    NOI=X1
    X(MM,1)=X2
    DO 220 I=2,NOI
    X(MM,I)=X(MM,I-1)+X3
220 CONTINUE
    READ(5,104)(Y(I),I=1,NOI)
    WRITE(6,109) (Y(I),I=1,NOI)
    H=X3
    STARTR=X2-X3/2.
    IF(CODE.EQ.21)GO TO 99
    SUM=0.
    DO 222 I=1,NOI
    SUM=SUM+Y(I)
222 CONTINUE
    NOI1=NOI+1
    FX(MM,1)=0.
    DO 221 I=2,NOI1
    FX(MM,I)=FX(MM,I-1)+Y(I)/SUM
221 CONTINUE
    GO TO 30
```

```
   33 CONTINUE
      MEAN=X1
      S2=X1
      U2=X2
      U3=X3
      U4=X4
      H=X5
      STARTR=X6
      SUMX=X7
      GO TO 331
   34 CONTINUE
      NOI=X1
      X(MM,1)=X2
      DO 341 I=2,NOI
      X(MM,I)=X(MM,I-1)+X3
  341 CONTINUE
      READ(5,104)(FX(MM,I),I=1,NOI)
      WRITE(6,109) (FX(MM,I),I=1,NOI)
  109 FORMAT(5X,1P10E11.4)
      GO TO 30
   35 CONTINUE
      CODE=CODE-50
      GO TO(351,352,353,354,355),CODE
  351 CONTINUE
      MEAN=X1
      SIGMA=X2
      IF(X6.EQ.0.)X6=MEAN-3.*SIGMA
      IF(X7.EQ.0.)X7=MEAN+3.*SIGMA
      X0=X6
      XN=X7
 1351 CONTINUE
      H=(XN-X0)/FLOAT(NI)
      D=H/FLOAT(NSI)
      X(MM,1)=X0
      INC=(XN-X0)/FLOAT(NI)
      DO 201 I=2,NI1
      X(MM,I)=X(MM,I-1)+H
  201 CONTINUE
      DO 202 J=2,NI1
      T(1)=X(MM,J-1)
      DO 203 KK=2,NSI1
      T(KK)=T(KK-1)+D
  203 CONTINUE
      DO 204 L=1,NSI1
      Y(L)=(1./(SQRT(6.2832)*SIGMA))*(EXP(-.5*((T(L)-MEAN)/SIGMA)**2))
  204 CONTINUE
      CALL CAREA(Y,FX,M,N,MM,NSI,J,D)
  202 CONTINUE
```

```
      DO 205 I=2,NI1
      FX(MM,I)=FX(MM,I)/FX(MM,NI1)
  205 CONTINUE
      GO TO 30
  352 CONTINUE
      INC=(X2-X1)/FLOAT(NI)
      X(MM,1)=X1
      DO 3521 I=2,NI1
      X(MM,I)=X(MM,I-1)+INC
 3521 CONTINUE
      INC=1./FLOAT(NI)
      FX(MM,1)=0.
      DO 3522 I=2,NI1
      FX(MM,I)=FX(MM,I-1)+INC
 3522 CONTINUE
      GO TO 30
  353 CONTINUE
      MEAN=X1
      SIGMA=X2
      X0=MEAN
      IF(X7.EQ.0.)X7=MEAN+3.*SIGMA
      XN=X7
      GO TO 1351
  354 CONTINUE
  355 CONTINUE
      GO TO 30
  356 CONTINUE
      CNSTNM=CNSTNM+1
      ORDER(INPTNM)=100+CNSTNM
      CONST(CNSTNM)=X1
      GO TO 30
   99 MEAN=0.
      SUMX=0.
      S1=0.
      S2=0.
      S3=0.
      S4=0.
      S5=0.
      DO 200 L=1,NOI
      I=NOI-L+1
      SUMX=SUMX+Y(L)
      S1=S1+Y(I)
      S2=S2+S1
      S3=S3+S2
      S4=S4+S3
      S5=S5+S4
  200 CONTINUE
      MEAN=S2/SUMX
```

```
      S2=S2/SUMX
      S3=S3/SUMX
      S4=S4/SUMX
      S5=S5/SUMX
      U2=2.*S3-S2*(1.+S2)
      U3=6.*S4-3.*U2*(1.+S2)-S2*(1.+S2)*(2.+S2)
      U4=24.*S5-2.*U3*(2.*(1.+S2)+1.)-U2*(6.*(1.+S2)*(2.+S2)-1.)
     9                                  -S2*(1.+S2)*(2.+S2)*(3.+S2)
  331 CONTINUE
      B1=U3**2/U2**3
      B2=U4/U2**2
      K=(B1*(B2+3.)**2)/(4.*(2.*B2-3.*B1-6.)*(4.*B2-3.*B1))
      IF(K)1,98,94
    1 R=(6.*(B2-B1-1.))/(6.+3.*B1-2.*B2)
      COM=B1*(R+2.)**2+16.*(R+1.)
      A1A2=.5*SQRT(U2)*SQRT(COM)
      COM12=R*(R+2.)*SQRT(B1/COM)
      IF(U3.LT.0.)COM12=-COM12
      M2=.5*(R-2.+COM12)
      M1=.5*(R-2.-COM12)
      Y0=(SUMX/A1A2)*(M1**M1*M2**M2)/(M1+M2)**(M1+M2)*GAMMA(M1+M2+2.)/
     9(GAMMA(M1+1.)*GAMMA(M2+1.))
      A2=A1A2/(M1/M2+1.)
      A1=A1A2-A2
      MODE=MEAN-.5*U3/U2*(R+2.)/(R-2.)
      MODE=MODE*H+STARTR
      INC=A1A2/FLOAT(N)
      X(MM,1)=MODE+(-A1)*H
      X(MM,NI1)=MODE+A2*H
      H=(X(MM,NI1)-X(MM,1))/FLOAT(NI)
      X(MM,2)=STARTR
      DO 706 I=3,NI
      X(MM,I)=X(MM,I-1)+H
  706 CONTINUE
      PSEUDO(1)=-A1
      PSEUDO(NI1)=A2
      H=A1A2/NI
      DO 701 I=2,NI
      PSEUDO(I)=PSEUDO(I-1)+H
  701 CONTINUE
      D=H/FLOAT(NSI)
      DO 702 J=2,NI1
      T(1)=PSEUDO(J-1)
      DO 703 KK=2,NSI1
      T(KK)=T(KK-1)+D
  703 CONTINUE
      DO 704 L=1,NSI1
      Y(L)=Y0*(1.+T(L)/A1)**M1*(1.-T(L)/A2)**M2
```

```
704 CONTINUE
    CALL CAREA(Y,FX,M,N,MM,NSI,J,D)
702 CONTINUE
    DO 705 I=2,NI1
    FX(MM,I)=FX(MM,I)/FX(MM,NI1)
705 CONTINUE
    GO TO 30
 94 IF(K-1)4,96,6
  4 CONTINUE
    R=(6.*(B2-B1-1.))/(2.*B2-3.*B1-6.)
    M1=.5*(R+2.)
    COM=SQRT(16.*(R-1.)-B1*(R-2.)**2)
    V=(-R*(R-2.)*SQRT(B1))/COM
    IF(U3.GE.0.)GO TO 44
    V=ABS(V)
 44 CONTINUE
    A1=SQRT(U2/16.)*COM
    MODE=MEAN-(U3*(R-2.))/((2.+U2)*(R+2.))
    THETA=ATAN(V/R)
    IF(R.LE.10.)GO TO 48
    A1=A1*H
    Y0=SUMX/A1*SQRT(R/6.2832)*(EXP(COS(THETA)**2/(3.*R)-1./
   9(12.*R)-THETA*V))/(COS(THETA))**(R+1)
 48 CONTINUE
    ORIGIN=MEAN+V*A1/R
    H=2.*ORIGIN/FLOAT(NI)
    D=H/FLOAT(NSI)
    X(MM,1)=-ORIGIN
    DO 711 I=2,NI1
    X(MM,I)=X(MM,I-1)+H
711 CONTINUE
    DO 712 J=2,NI1
    T(1)=X(MM,J-1)
    DO 713 KK=2,NSI1
    T(KK)=T(KK-1)+D
713 CONTINUE
    DO 714 L=1,NSI1
    Y(L)=Y0*(1.+T(L)**2/A1**2)**(-M1)*EXP(-V*ATAN(T(L)/A1))
714 CONTINUE
    CALL CAREA(Y,FX,M,N,MM,NSI,J,D)
712 CONTINUE
    DO 715 I=2,NI1
    FX(MM,I)=FX(MM,I)/FX(MM,NI1)
715 CONTINUE
    DO 716 I=1,NI1
    X(MM,I)=X(MM,I)+ORIGIN
716 CONTINUE
    GO TO 30
```

```
    6 CONTINUE
      IMEAN=MEAN
      MEAN=MEAN-IMEAN
      R=(6.*(B2-B1-1.))/(6.+3.*B1-2.*B2)
      COM=B1*(R+2.)**2+16.*(R+1.)
      A1=.5*SQRT(U2)*SQRT(COM)
      IF(U3.LT.0.)A1=-(ABS(A1))
      COM12=(R*(R+2.))/2.*SQRT(B1/COM)
      M1=-((R-2.)/2.-COM12)
      M2=(R-2.)/2.+COM12
      Y0=(A1**(M1-M2-1.)/GAMMA(M1-M2-1.))*(GAMMA(M1)/GAMMA(M2+1.))*SUMX
      ORIGIN=MEAN-(A1*(M1-1.))/(M1-M2-2.)
      MODE=MEAN-.5*U3/U2*(R+2.)/(R-2.)
      XN=A1+XN/H
      SAVEH=H
      H=(XN-A1)/FLOAT(NI)
      D=H/FLOAT(NSI)
      X(MM,1)=A1
      DO 721 I=2,NI1
      X(MM,I)=X(MM,I-1)+H
  721 CONTINUE
      DO 722 J=2,NI1
      T(1)=X(MM,J-1)
      DO 723 KK=2,NSI1
      T(KK)=T(KK-1)+D
  723 CONTINUE
      DO 724 L=1,NSI1
      Y(L)=Y0*(T(L)-A1)**M2*T(L)**(-M1)
  724 CONTINUE
      CALL CAREA(Y,FX,M,N,MM,NSI,J,D)
  722 CONTINUE
      DO 725 I=1,NI1
      FX(MM,I)=FX(MM,I)/FX(MM,NI1)
  725 CONTINUE
      DO 726 I=1,NI1
      X(MM,I)=(X(MM,I)-A1)*SAVEH
  726 CONTINUE
      GO TO 30
   98 WRITE(6,103)
      GO TO 399
   96 CONTINUE
      WRITE(6,105)
  399 CONTINUE
      RETURN
******************   ENTRY POINT   ***********************
      ENTRY INPUT
      REWIND 4
      DO 500 J=1,INPTNM
```

```
      IF(ORDER(J).GT.100)GO TO 501
      RND=RANDU(IX)
      DO 502 I=1,NI1
      IF(RND.LT.FX(ORDER(J),I))GO TO 503
  502 CONTINUE
  503 CONTINUE
      ANS(J)=X(ORDER(J),I)
      GO TO 500
  501 CONTINUE
      ANS(J)=CONST(ORDER(J)-100)
  500 CONTINUE
      WRITE(4,101)(ANS(I),I=1,INPTNM)
      ENDFILE 4
      REWIND 4
  100 FORMAT(I10)
  101 FORMAT(E16.9)
  102 FORMAT(I2,2X,7E10.0)
  103 FORMAT(' ','K=0')
  104 FORMAT(10E8.0)
  105 FORMAT(' ','K= 1. ')
      RETURN
      END
```

TABLE IV-2 (CONT'D)

```
      SUBROUTINE CAREA(Y,FX,M,N,MM,NSI,J,D)
      REAL FX(M,N),Y(N)
      NSI1=NSI+1
      NSI0=NSI-1
      FX(MM,1)=0.
      SUM=0.
      DO 201 I=3,NSI0,2
      SUM=SUM+4.*Y(I-1)+2.*Y(I)
  201 CONTINUE
      AREA=D/3.*(Y(1)+SUM+Y(NSI1))
      FX(MM,J)=FX(MM,J-1)+AREA
      RETURN
      END
      FUNCTION RANDU(IX)
      IX=IX*65541
      IF(IX)5,6,6
    5 IX=IX+2147483647+1
    6 RANDU=IX
      RANDU=RANDU*.4656613E-9
      RETURN
      END
```

```
      SUBROUTINE INTERP(X1,Y1,N1,X2,Y2,N2,YDIFF,ICHK)
      DIMENSION X1(N1),Y1(N1),X2(N2),Y2(N2),YDIFF(N1)
      DO 100 I=1,N1
      N3=N2-1
      DO 200 J=1,N3
      IF(I.GT.N2.AND.ICHK.EQ.0) YDIFF(I)=Y1(I)
      IF(I.GT.N2.AND.ICHK.EQ.1) YDIFF(I)=Y1(I)-Y2(N2)
      IF(I.GT.N2) GO TO 100
      IF(ABS(X1(I)-X2(J)).GT.1.E-5) GO TO 1
      YDIFF(I)=Y1(I)-Y2(J)
      GO TO 100
    1 IF(X1(I).LT.X2(J).OR.X1(I).GE.X2(J+1)) GO TO 2
      YDIFF(I)=Y1(I)-((Y2(J+1)-Y2(J))/(X2(J+1)-X2(J)))*(X1(I)-X2(J))
     2-Y2(J)
      GO TO 100
    2 IF(X1(I).GE.X2(J+1).AND.J+1.LT.N2) GO TO 200
      IF(J.EQ.1) GO TO 3
      YDIFF(I)=Y1(I)-((Y2(J)-Y2(J-1))/(X2(J)-X2(J-1)))*(X1(I)-X2(J-1))
     2-Y2(J-1)
      GO TO 100
    3 YDIFF(I)=Y1(I)-(Y2(J)/X2(J))*X1(I)-Y2(J)
  200 CONTINUE
  100 IF(ABS(YDIFF(I)).LT.ABS(Y1(I)*1.E-5)) YDIFF(I)=0.0
      IF(N1.EQ.N2.AND.ABS(X1(N1)-X2(N2)).LT.1.E-5) YDIFF(N1)=Y1(N1)
     2-Y2(N2)
      IF(ABS(YDIFF(N1)).LT.ABS(Y1(N1)*1.E-5)) YDIFF(N1)=0.0
      RETURN
      END

►     SUBROUTINE PLOT1(X,Y,N,YHDR,NY,XHDR,NX,SY1,SY2,SX1,SX2,XY,
►    2XSFT,YSFT)
►     DIMENSION X(N),Y(N)
►     DIMENSION XHDR(8),YHDR(8)
►     X(N-1)=SX1
►     X(N)=SX2
►     Y(N-1)=SY1
►     Y(N)=SY2
►     CALL PLOT(XSFT,YSFT,-3)
►     CALL AXIS(0.0,0.0,YHDR,NY,8.0,90.0,SY1,SY2)
►     CALL AXIS(0.0,XY,XHDR,NX,5.0,0.0,SX1,SX2)
►     N1=N-2
►     CALL LINE(X,Y,N1,1,0,1)
►     KPLOT=KPLOT+1
►     RETURN
►     END
```

```
      SUBROUTINE PAIR
      REAL IDIFF,IMAX1,IMIN1,IMAX2,IMIN2,IMAX,IMIN
      COMMON/PAIR1/TW1,TW2,DTW,FW1,FW2,DFW1,DFW2,DFW,TMAXO,DFMO,
     2FDIFF,TDIFF,N
      COMMON/PAIR2/FMAX1,TFMX1,FMIN1,TFMN1,
     2              FMAX2,TFMX2,FMIN2,TFMN2
      COMMON/PAIR3/AFMAX,TFMAX,AFMAXT,TFMAXT
      COMMON/OUT1/FDIFIG,TDIFIG,DIT,ADIT
      DIMENSION FDIFF(400),TDIFF(400)
      COMMON/OUT2/DF100K,T100K
      COMMON/TOFF/DFT01,DFT02,TDFT01,TDFT02
      FMAX=FDIFF(1)
      FMIN=FDIFF(1)
      FMAX1=FDIFF(1)
      FMIN1=FDIFF(1)
      TFMX1=TDIFF(1)
      TFMN1=TDIFF(1)
      T=AMIN1(TW1,TW2)
      DO 6 I=2,N
      K=I
      IF(TDIFF(I)-T) 7,7,8
    7 FMAX=AMAX1(FDIFF(I),FMAX)
      IF(FMAX.GT.FMAX1) TFMX1=TDIFF(I)
      FMAX1=FMAX
      FMIN=AMIN1(FDIFF(I),FMIN)
      IF(FMIN.LT.FMIN1) TFMN1=TDIFF(I)
      FMIN1=FMIN
    6 CONTINUE
    8 FMAX=FDIFF(K)
      FMIN=FDIFF(K)
      FMAX2=FDIFF(K)
      FMIN2=FDIFF(K)
      TFMX2=TDIFF(K)
      TFMN2=TDIFF(K)
      DO 9 I=K,N
      FMAX=AMAX1(FDIFF(I),FMAX)
      IF(FMAX.GT.FMAX2) TFMX2=TDIFF(I)
      FMAX2=FMAX
      FMIN=AMIN1(FDIFF(I),FMIN)
      IF(FMIN.LT.FMIN2) TFMN2=TDIFF(I)
      FMIN2=FMIN
    9 CONTINUE
      AFMAX1=ABS(FMAX1)
      AFMIN1=ABS(FMIN1)
      IF(AFMAX1.GE.AFMIN1) TFMAX=TFMX1
      IF(AFMIN1.GT.AFMAX1) TFMAX=TFMN1
      AFMAX=AMAX1(AFMAX1,AFMIN1)
      AFMAX2=ABS(FMAX2)
```

```
      AFMIN2=ABS(FMIN2)
      IF(AFMAX2.GE.AFMIN2) TFMAXT=TFMX2
      IF(AFMIN2.GT.AFMAX2) TFMAXT=TFMN2
      AFMAXT=AMAX1(AFMAX2,AFMIN2)
      DTW=ABS(DTW)
      DFW=ABS(DFW)
      DFW1=ABS(DFW1)
      DFW2=ABS(DFW2)
      DFMQ=ABS(DFMQ)
      FDIFIG=ABS(FDIFIG)
      DF100K=ABS(DF100K)
C   *********************************************************************
C   *                                                                   *
C   *     OUTPUT MOTOR PAIR DATA                                        *
C   *                                                                   *
C   *     FMAX1,FMIN1,TFMX1 AND TFMN1 ARE THE MAXIMUM AND MINIMUM       *
C   *          VALUES OF THRUST IMBALANCE DURING EWAT AND THE TIMES     *
C   *          AT WHICH THEY OCCUR RESPECTIVELY                         *
C   *     FMAX2,FMIN2,TFMX2 AND TFMN2 ARE THE MAXIMUM AND MINIMUM       *
C   *          VALUES OF THRUST IMBALANCE DURING TAIL-OFF AND THE TIMES *
C   *          AT WHICH THEY OCCUR RESPECTIVELY                         *
C   *     TDFTO1,TDFTO2 AND DTW ARE THE WEB TIMES FOR THE FIRST AND     *
C   *          SECOND MOTORS TO BEGIN TAILOFF AND THE ABSOLUTE VALUE    *
C   *          OF THE DIFFERENCE IN WEB TIMES RESPECTIVELY              *
C   *     FW1,FW2 AND DFW ARE THE THRUSTS AT WEB TIME FOR THE FIRST     *
C   *          AND SECOND MOTORS TO BEGIN TAILOFF AND THE ABSOLUTE      *
C   *          VALUE OF THE DIFFERENCE IN THRUSTS AT WEB TIME           *
C   *          RESPECTIVELY                                             *
C   *     DFTO1 AND DFTO2 ARE THE ABSOLUTE VALUES OF THE THRUST         *
C   *          IMBALANCES WHICH EXIST WHEN THE FIRST AND SECOND MOTORS  *
C   *          BEGIN TAILOFF RESPECTIVELY                               *
 1000 CONTINUE
C   *     DFMQ AND TMAXQ ARE THE ABSOLUTE VALUE OF THE THRUST           *
C   *          IMBALANCE WHEN THE MAXIMUM DYNAMIC PRESSURE OCCURS ON    *
C   *          THE VEHICLE AND THE TIME AT WHICH IT OCCURS RESPECTIVELY *
C   *     AFMAX AND TFMAX ARE THE ABSOLUTE VALUE OF THE MAXIMUM THRUST  *
C   *          IMBALANCE DURIN EWAT AND THE TIME AT WHICH IT OCCURS     *
C   *          RESPECTIVELY                                             *
C   *     AFMAXT AND TFMAXT ARE THE ABSOLUTE VALUE OF THE MAXIMUM       *
C   *          THRUST IMBALANCE DURING TAIL-OFF AND THE TIME AT WHICH   *
C   *          IT OCCURS RESPECTIVELY                                   *
C   *     FDIFIG AND TDIFIG ARE THE ABSOLUTE VALUE OF THE MAXIMUM       *
C   *          THRUST IMBALANCE DURING THE INITIAL PART OF OPERATION    *
C   *          AND THE TIME AT WHICH IT OCCURS RESPECTIVELY             *
C   *     DIT AND ADIT ARE THE THE TOTAL IMPULSE IMBALANCE AND THE      *
C   *          ABSOLUTE VALUE OF THE TOTAL IMPULSE IMBALANCE DURING     *
C   *          TAIL-OFF                                                 *
C   *     DF100K AND T100K ARE THE ABSOLUTE VALUE OF THE THRUST         *
```

TABLE IV-2 (CONT'D)

```
C   *           IMBALANCE WHEN THE LAST MOTOR REACHES 100K AND THE         *
C   *           TIME AT WHICH IT OCCURS RESPECTIVELY                       *
C   ************************************************************************
      IF(TW1-TW2) 700,700,701
  700 DFTO1=DFW1
      DFTO2=DFW2
      GO TO 702
  701 DFTO1=DFW2
      DFTO2=DFW1
      FW1=FW2
      FW2=FW1
  702 CONTINUE
      TDFTO1=AMIN1(TW1,TW2)
      TDFTO2=AMAX1(TW1,TW2)
      WRITE(6,1)
    1 FORMAT(//,20X,'MOTOR PAIR DATA')
      WRITE(6,2) FMAX1,TFMX1,FMIN1,TFMN1,
     2FMAX2,TFMX2,FMIN2,TFMN2,DFTO1,DFTO2,
     3TDFTO1,TDFTO2,DTW,FW1,FW2,DFW,DFMQ,TMAXQ,
     3AFMAX,TFMAX,AFMAXT,TFMAXT,FDIFIG,TDIFIG,DIT,ADIT,DF100K,T100K
    2 FORMAT(13X,'FMAX1=   ',1PE11.4,13X,'TFMX1=   ',1PE11.4,/,
     213X,'FMIN1=   ',1PE11.4,13X,'TFMN1=   ',1PE11.4,/,
     213X,'FMAX2=   ',1PE11.4,13X,'TFMX2=   ',1PE11.4,/,
     213X,'FMIN2=   ',1PE11.4,13X,'TFMN2=   ',1PE11.4,/,
     213X,'DFTO1=   ',1PE11.4,13X,'DFTO2=   ',1PE11.4,/,
     213X,'TDFTO1=  ',1PE11.4,13X,'TDFTO2=  ',1PE11.4,13X,'DTW= ',1PE11.4,
     2/,13X,'FW1=     ',1PE11.4,13X,'FW2=     ',1PE11.4,13X,'DFW= ',
     21PE11.4,/,
     213X,'DFMQ=    ',1PE11.4,13X,'TMAXQ=   ',1PE11.4,/,
     213X,'AFMAX=   ',1PE11.4,13X,'TFMAX=   ',1PE11.4,/,
     213X,'AFMAXT=  ',1PE11.4,13X,'TFMAXT=  ',1PE11.4,/,
     213X,'FDIFIG=  ',1PE11.4,13X,'TDIFIG=  ',1PE11.4,/,
     213X,'DIT=     ',1PE11.4,13X,'ADIT=    ',1PE11.4,/,
     213X,'DF100K=  ',1PE11.4,13X,'T100K=   ',1PE11.4)
      RETURN
      END
```

TABLE IV-2 (CONT'D)

```
      SUBROUTINE INTRP1(Y,T,N,TT,DY,ICHK)
      DIMENSION Y(N),T(N)
      N1=N-1
      DY=0.0
      IF(ICHK) 2,2,3
    2 DO 1 I=1,N1
      IF(TT.GE.T(I).AND.TT.LT.T(I+1)) DY=((Y(I+1)-Y(I))/(T(I+1)-T(I)))
     2*(TT-T(I))+Y(I)
      IF(DY.NE.0.0) RETURN
    1 CONTINUE
    3 DO 4 I=1,N1
      IF(TT.LE.T(I).AND.TT.GT.T(I+1)) DY=((Y(I+1)-Y(I))/(T(I+1)-T(I)))
     2*(TT-T(I))+Y(I)
      IF(DY.NE.0.0) RETURN
    4 CONTINUE
      RETURN
      END

      SUBROUTINE SIGBAR(X,XI,XI2,SIGX,BX,ICOUNT,N,SIG1,SIG2)
      XN=FLOAT(N)
      IF(ICOUNT.GT.2) GO TO 1
      XI2=0.0
      XI=0.0
    1 XI2=XI2+X**2
      XI=XI+X
      BX=XI/XN
      XIS=XI**2
      SIGX=SQRT((XI2/XN)-(XIS/XN**2))
      SIG1=SQRT(XI2/XN)
      SIG2=SQRT(XI2/(2.*XN))
      RETURN
      END
```

V. SAMPLE STUDY

In keeping with the present interest in very large SRMs the case selected for the sample study is a 146-inch diameter motor of the type being considered for use on the Space Shuttle. However, the study does not constitute a prediction of the imbalance characteristics of the Space Shuttle. No effort has been made to select the most recent design considered for the Shuttle or to minimize or maximize imbalance. Although the statistical characteristics of the input variables should be reasonable for the most part, in some cases their selection has been somewhat arbitrary since the purpose of this study is merely to demonstrate the setup procedures, format and computational capabilities of the computer program.

The SRM for the sample case has three center segments consisting of circular perforated grains, an aft segment with a circular perforated grain and a forward segment with a truncated (slotted tube) nine point star grain. The fixed values for the sample case are given in Table V-1. It will be noted that both the head end and aft end domes are represented by hemispherical closures which is seemingly inconsistent with the fact that the head end of the foremost circular perforated grain is flat. This is an attempt to artificially correct for the curvature of the star grain segment located at the extreme head end. Experience with the simplified computer program indicates that this is a satisfactory alternative procedure to specification of an effective length of star grain. Of course, the effects of the head end closure burning surface geometry could be represented more precisely by specifying tabular input values, but it should be kept in mind that only the burning surface defined by equations is subject to statistical variation. In the present case, because the entire head end segment is consumed far in advance of tailoff, only minor errors in the predicted behavior during the critical tailoff portion of the traces should be encountered as a result of the approximation used.

Table V-2 gives the input data for the statistical variables of the sample case. Included in Table V-2 are brief comments on the sources of the data and the method in which it is applied to the present study. It will be noted that in a number of cases the convention is adopted of taking the drawing tolerances as representing ±3 standard deviations in a normally distributed population of a variable. Also, where more than one dimension controls a variable input dimension, the standard deviation of the variable is taken as the square root of the sum of the squares of the standard deviations of the controlled variables (assumed to be normally distributed and uncorrelated.) An example of this is the standard deviation of the average outside diameter of the circular perforated grain which is calculated based on the standard deviations of the outside diameter of the case, and the thicknesses of the case wall, liner and insulation.

TABLE V-1. FIXED INPUT VALUES OF SAMPLE CASE

Options

IEO = 1

IPO = 1

NUMPLT(J) 0 0 0 0 0

Propellant Characteristics

L = 1367.23

TAU = 39.740

Basic Performance Constants

DELTAY = 0.040
II = 26
XOUT = 1000.00
DPOUT = 10,000.00
ZETAF = 0.9600
TMAXQ = 60.0
TB = 122.2
HB = 130,000
PREF = 560.00
DTREE = 57.285
PIPK = 0.00150
CSTART = 0.0000380
CSTARP = 0.0057000
PTRAN = 0.0
GAMP = 0.00527000

Grain Configuration

INPUT = 2

GRAIN = 3

STAR = 2

NT = 0

ORDER = 1

COP = 2

C. P. Grain Geometry

XTZO = 0.120

S = 1

Basic Star Geometry

NS = 1

NP = 9

NN = 0

TABLE V-2. INPUT DATA FOR STATISTICAL VARIABLES OF SAMPLE CASE

Variable	Code	X1	X2	Source or Comment
RHO	51	0.06350	0.0000105	Ref. 8, p. 4.5-26 (2/3 of total variation ascribed to propellant weight).
A1,A2	21	11	0.03655	X3 = 0.00001
		Data cards for A1 and A2: 1,3,5,2,13,16,10,12,4,7,1		
		Reference Values: $\bar{a}$ = 0.0366045 C_v = 0.060%		
		Source: Ref. 8, pp. 4.5-24, 4.5-25, MINUTEMAN data pair analysis scaled.		
N1,N2	60	0.035	—	
ALFA	60	0.0	—	
BETA	60	0.0	—	
ROAL	51	4.350	0.04	Ref. 1, Weighting error distribution, p. 44.
				From drawing tolerance:
DE	51	145.67	0.033333	3σ = 0.100
DTI	51	54.430	0.0100	3σ = 0.030
THETA	60	0.0	—	
ALFAN	60	11.250	—	
LTAP	60	176.5	—	
XT	51	3.0400	0.02357	3σ = 0.05 $\sqrt{2 \text{ controlling dimensions}}$
ZO	51	2.41	0.02357	3σ = 0.05 $\sqrt{2}$
ZC	51	0.0000	0.02357	3σ = 0.05 $\sqrt{2}$
RONDCN	53	0.0	0.083333	3σ = 0.250
RONDCH	53	0.0	0.083333	3σ = 0.250
RONDGN	53	0.0	0.033333	3σ = 0.100
RONDGH	53	0.0	0.033333	3σ = 0.100
EXN	51	0.0	0.05	3σ = 0.150
EYN	51	0.0	0.05	3σ = 0.150

TABLE V-2. INPUT DATA FOR STATISTICAL VARIABLES OF SAMPLE CASE (CONT'D)

Variable	Code	X1	X2	Source or Comment
EXH	51	0.0	0.05	$3\sigma = 0.150$
EYH	51	0.0	0.05	$3\sigma = 0.150$
ALPHAN	52	0.0	360	Random orientation of mandrel and case.
ALPHAH	52	0.0	360	Random orientation of mandrel case.
ERREF	51	0.00763	0.00032	POSEIDON data per Ref. 8, p. 4.5-26, $C_V = 4.2\%$.
TGR	51	60.0	0.2333	Ref. 8, p. 4.5-26, $6\sigma = 1.4°F$.
TIGR	11	40.0	0.3740	X3 = 0.0040, X4 = 15

Data card for TIGR: 0.3777, 0.3811, 0.4030, 0.3980, 0.3744, 0.3795, 0.4266, 0.4300, 0.4334, 0.4300, 0.3980, 0.3997, 0.3862, 0.3845, 0.3895, 0.3963, 0.4013, 0.3929, 0.4097, 0.4081, 0.3980, 0.4030, 0.3827, 0.3963, 0.3980, 0.3996, 0.3895, 0.3929, 0.4081, 0.4132, 0.4215, 0.4148, 0.3946, 0.3744, 0.3845, 0.4136, 0.4148, 0.4030, 0.4013.

Reference values: $\bar{T}_{igr} \simeq 0.40$, $C_V = 3.83\%$.

Source: Artificial data based on C_V of large number of motors.

Variable	Code	X1	X2	Source or Comment
DO	51	143.080	0.01462	$3\sigma = \sqrt{0.032^2 + 2 \times 0.02^2 + 0.01^2}$
DI	51	63.590	0.033333	$3\sigma = 0.10$
THETAG	60	10.1990	—	
LGCI	51	1135.58	0.577	$3\sigma = 1.0\sqrt{3\text{ segments}}$
LGNI	51	51.20	0.33333	$3\sigma = 1.0$
THETCN	60	0.0	—	
THETCH	60	90.0	—	
LGSI	51	189.15	0.3333	$3\sigma = 1.0$
RC	51	71.540	0.00731	$3\sigma = \sqrt{0.032^2 + 2\times0.02^2 + 0.01^2}/2$
FILL	51	2.010	0.011111	$3\sigma = (1\sqrt{9\text{ points}} \times 0.1)$
RP	51	12.000	0.01667	$3\sigma = 0.05$
RIS	51	63.540	0.01667	$3\sigma = 0.05$

Not only must the procedures used in manufacture and quality control of the motor production be recognized when specifying the input characteristics, but also the way a particular variable is used in the program. Thus, when a dimension (or other characteristic) of a variable is subject to random variation and the effect of the variation is averaged in the program, the standard deviation in the variable should be reduced. For example, the standard deviation in the fillet radii of the star points is reduced by the $\sqrt{9}$ because the nine star points each have equal effects on the burning surface. Similarly, the propellant average burning rate variation between pairs may be reduced substantially if propellant from the same mixer batch is divided between the pair of motors as was assumed in this sample case.

In treatment of characteristics such as burning rate, propellant density, and grain temperature, it is assumed that the primary concern is the variation between two motors of a pair and variation between pairs is not considered. Thus, for example, the standard deviation in these characteristics is only that within a pair and excludes any between pair variation as experienced from change of a lot of propellant between pairs or from different ambient temperature histories.

An actual printout of the statistical input data is shown in Table V-3 to demonstrate the computer program format. Table V-4 gives the complete set of variables (fixed and distributed) selected by the Monte Carlo program for the first motor of the first pair. Table V-5 illustrates the format for the printout of transient values for one SRM of the sample case and also shows the propellant weight and initial and final seed numbers for the configuration. The initial seed number may be used to repeat the calculations for the configuration. The tabular output of motor pair imbalance data shown at the bottom of Table V-5 may be omitted by proper specification of IPO.

Figure V-1 through V-5 illustrate the graphical data which may be obtained using the CalComp plotter. Any or all of the plots may be omitted by proper choice of the NUMPLT(J) input array (See also IPO). The figures are derived from the same initial seed number given in Table V-5.

Tables V-6 and V-7 illustrate the statistical analysis of the output data that may be obtained from the program by use of the appropriate value of IPO. Table V-6 is a facsimile of the computer output for 25 motor pairs of the sample study and Table V-7 is a compilation of selected statistical characteristics of 50 motor pairs. The latter was obtained by separate calculations from three groups of data.

A histogram (Fig. V-6) of maximum thrust imbalances during tailoff for the 50 motor pairs demonstrates that the program results are generally consistent with the type of behavior which would be expected.

TABLE V-3. PRINTOUT OF STATISTICAL INPUT DATA FROM SAMPLE CASE

```
                      DATA FOR STATISTICAL ANALYSIS PROGRAM
RHO           51      6.3500E-02    1.0500E-05    0.0           0.0           0.0           0.0           0.0
A1            21      1.1000E 01    3.6550E-02    1.0000E-05    0.0           1.0000E 02    3.6555E-02    3.6655E-02
     1.0000E 00 3.0000E 00 5.0000E 00 2.0000E 00 1.3000E 01 1.6000E 01 1.0000E 01 1.2000E 01 4.0000E 00 7.0000E 00
     1.0000E 00
N1            60      3.5000E-01    0.0           0.0           0.0           0.0           0.0           0.0
A2            21      1.1000E 01    3.6550E-02    1.0000E-05    0.0           1.0000E 02    3.6555E-02    3.6655E-02
     1.0000E 00 3.0000E 00 5.0000E 00 2.0000E 00 1.3000E 01 1.6000E 01 1.0000E 01 1.2000E 01 4.0000E 00 7.0000E 00
     1.0000E 00
N2            60      3.5000E-01    0.0           0.0           0.0           0.0           0.0           0.0
ALPHA         60      0.0           0.0           0.0           0.0           0.0           0.0           0.0
BETA          60      0.0           0.0           0.0           0.0           0.0           0.0           0.0
ROAL          51      4.3500E 00    4.0000E-02    0.0           0.0           0.0           0.0           0.0
DE            51      1.4567E 02    3.3333E-02    0.0           0.0           0.0           0.0           0.0
DTI           51      5.4430E 01    1.0000E-02    0.0           0.0           0.0           0.0           0.0
THETA         60      0.0           0.0           0.0           0.0           0.0           0.0           0.0
ALFAN         60      1.1250E 01    0.0           0.0           0.0           0.0           0.0           0.0
LTAP          60      1.7650E 02    0.0           0.0           0.0           0.0           0.0           0.0
XT            51      3.0400E 00    2.3570E-02    0.0           0.0           0.0           0.0           0.0
ZO            51      2.4100E 00    2.3570E-02    0.0           0.0           0.0           0.0           0.0
ZC            51      0.0           2.3570E-02    0.0           0.0           0.0           0.0           0.0
RONDCN        53      0.0           8.3333E-02    0.0           0.0           0.0           0.0           0.0
RONDCH        53      0.0           8.3333E-02    0.0           0.0           0.0           0.0           0.0
RONDGN        53      0.0           3.3333E-02    0.0           0.0           0.0           0.0           0.0
RONDGH        53      0.0           3.3333E-02    0.0           0.0           0.0           0.0           0.0
EXN           51      0.0           5.0000E-02    0.0           0.0           0.0           0.0           0.0
EYN           51      0.0           5.0000E-02    0.0           0.0           0.0           0.0           0.0
EXH           51      0.0           5.0000E-02    0.0           0.0           0.0           0.0           0.0
EYH           51      0.0           5.0000E-02    0.0           0.0           0.0           0.0           0.0
ALPHAN        52      0.0           3.6000E 02    0.0           0.0           0.0           0.0           0.0
ALPHAH        52      0.0           3.6000E 02    0.0           0.0           0.0           0.0           0.0
ERREF         51      7.6300E-03    3.2000E-04    0.0           0.0           0.0           0.0           0.0
TGR           51      6.0000E 01    2.3330E-01    0.0           0.0           0.0           0.0           0.0
TIGR          11      4.0000E 01    3.7400E-01    4.0000E-03    1.5000E 01    1.0000E 02    3.7400E-01    4.3400E-01
     3.7770E-01 3.8110E-01 4.0300E-01 3.9800E-01 3.7440E-01 3.7950E-01 4.2660E-01 4.3000E-01 4.3340E-01 4.3000E-01
     3.9800E-01 3.9970E-01 3.8620E-01 3.8450E-01 3.8950E-01 3.9630E-01 4.0130E-01 3.9290E-01 4.0970E-01 4.0810E-01
     3.9800E-01 4.0300E-01 3.8270E-01 3.9630E-01 3.9800E-01 3.9960E-01 3.8950E-01 3.9290E-01 4.0810E-01 4.1320E-01
     4.2150E-01 4.1480E-01 3.9460E-01 3.9460E-01 3.7440E-01 3.8450E-01 4.1360E-01 4.1480E-01 4.0300E-01 4.0130E-01
DO            51      1.4308E 02    1.4620E-02    0.0           0.0           0.0           0.0           0.0
DI            51      6.3590E 01    3.3333E-02    0.0           0.0           0.0           0.0           0.0
THETAG        60      1.0199E 01    0.0           0.0           0.0           0.0           0.0           0.0
LGCI          51      1.1356E 03    5.7700E-01    0.0           0.0           0.0           0.0           0.0
LGNI          51      5.1200E 01    3.3333E-01    0.0           0.0           0.0           0.0           0.0
THETCN        60      0.0           0.0           0.0           0.0           0.0           0.0           0.0
THETCH        60      9.0000E 01    0.0           0.0           0.0           0.0           0.0           0.0
LGSI          51      1.8915E 02    3.3330E-01    0.0           0.0           0.0           0.0           0.0
RC            51      7.1540E 01    7.3100E-03    0.0           0.0           0.0           0.0           0.0
FILL          51      2.0100E 00    1.1111E-02    0.0           0.0           0.0           0.0           0.0
RP            51      1.2000E 01    1.6667E-02    0.0           0.0           0.0           0.0           0.0
RIS           51      6.3540E 01    1.6670E-02    0.0           0.0           0.0           0.0           0.0
END           90      0.0           0.0           0.0           0.0           0.0           0.0           0.0
```

TABLE V-4. PRINTOUT OF INPUT VARIABLES FOR ONE SRM FROM SAMPLE CASE

CONFIGURATION NUMBER 1

OPTIONS
IEO= 1
IPO= 1
NUMPLT(J)= 0 0 0 0 0

PROPELLANT CHARACTERISTICS
RHO= 0.063507
A1= 0.03665
N1= 0.350
A2= 0.03667
N2= 0.350
ALPHA= 0.0
BETA= 0.0
RUAL= 4.2852
CSTARN= 5.1632E 03
GAMN= 1.1414E 00

BASIC MOTOR DIMENSIONS
L= 1367.23
TAU= 39.740
DE= 1.4575E 02
DTI= 5.4437E 01
THETA= 0.0
ALFAN= 1.1250E 01
LTAP= 1.7650E 02
XT= 3.0470E 00
ZO= 2.4142E 00
ZC= 5.6571E-03
RONDCN= 1.9500E-01
RONDCH= 2.5000E-02
RONDGN= 3.6000E-02
RONDGH= 2.0000E-02
EXN= 4.2001E-02
EYN= 2.1001E-02
EXH= 3.9001E-02
EYH= -2.0999E-02
ALPHAN= 2.5200E 02
ALPHAH= 3.3120E 02

BASIC PERFORMANCE CONSTANTS
DELTAY= 0.040
II= 26
XOUT= 1000.00
DPOUT= 10000.00
ZETAF= 0.9600
TB= 122.2
HB= 130000.
ERREF= 0.00782
PREF= 560.00
DTREF= 57.285
TGR= 59.916
PIPK= 0.00150
CSTART= 0.0000380
PTRAN= 0.0
CSTARP= 0.0057000
TIG= 0.3906
GAMP= 0.0052700
TMAXQ= 60.000

GRAIN CONFIGURATION
INPUT= 2
GRAIN= 3
STAR= 2
NT= 0.
ORDER= 1
COP= 2

C.P. GRAIN GEOMETRY
DO= 143.067
DI= 63.574
XTZO= 0.120
S= 1.
THETAG= 10.19900
LGCI= 1135.91
LGNI= 51.14
THETCN= 0.0
THETCH= 90.00000

BASIC STAR GEOMETRY
NS= 1.
LGSI= 189.21
NP= 9.
RC= 71.527
FILL= 2.006
NN= 0.

TRUNCATED STAR GEOMETRY
RP= 12.005
RIS= 63.544

TABLE V-5. PORTION OF PRINTOUT OF TRANSIENT VALUES FOR ONE SRM FROM SAMPLE CASE

```
TIME= 119.282          Y= 40.493
PONOZ=  1.5533E 01  PHEAD=  1.5533E 01  SUMAB=  3.9977E 04  F=  6.2843E 04  ITOT=  2.7288E 08

TIME= 119.726          Y= 40.533
PONOZ=  1.0838E 01  PHEAD=  1.0838E 01  SUMAB=  3.1684E 04  F=  4.3577E 04  ITOT=  2.7291E 08

TIME= 120.240          Y= 40.573
PONOZ=  6.7622E 00  PHEAD=  6.7622E 00  SUMAB=  2.3393E 04  F=  2.6849E 04  ITOT=  2.7292E 08

                 **********************************
                 **** BEGIN HALF SECOND TRACE ****
                 **********************************

TIME= 120.753          Y= 40.613
PONOZ=  0.0         PHEAD=  0.0         SUMAB=  0.0         F=  0.0         ITOT=  2.7292E 08

        INDIVIDUAL MOTOR DATA
WP1=  1.0862E 06
WP2=  1.0886E 06
WP=  1.0874E 06
PHMAX=  7.9388E 02
IX1= 1211119009
IX=   19232621
```

TABULATED IMBALANCE DATA

TIME	FDIFF	IDIFF	IADIFF
3.9060E-01	1.1408E 04	1.1316E 04	2.2280E 03
4.9929E-01	1.1269E 04	1.2617E 04	3.4604E 03
6.0808E-01	1.1952E 04	1.3879E 04	4.7235E 03
7.1686E-01	1.1992E 04	1.5178E 04	6.0258E 03
8.2562E-01	1.2033E 04	1.6492E 04	7.3323E 03
9.3435E-01	1.2083E 04	1.7780E 04	8.6433E 03
1.0431E 00	1.2056E 04	1.9023E 04	9.9559E 03
1.1518E 00	1.2094E 04	2.0298E 04	1.1268E 04
1.2604E 00	1.2040E 04	2.1736E 04	1.2579E 04
1.3691E 00	1.2119E 04	2.2926E 04	1.3892E 04
1.4777E 00	1.2146E 04	2.4277E 04	1.5210E 04
1.5863E 00	1.2091E 04	2.5634E 04	1.6526E 04
1.6949E 00	1.2157E 04	2.6897E 04	1.7842E 04
1.8034E 00	1.2171E 04	2.8320E 04	1.9162E 04
1.9119E 00	1.2205E 04	2.9748E 04	2.0484E 04
2.0205E 00	1.2175E 04	3.0826E 04	2.1808E 04
2.1289E 00	1.2192E 04	3.2321E 04	2.3129E 04
2.2374E 00	1.2209E 04	3.3567E 04	2.4453E 04
2.3458E 00	1.2273E 04	3.4972E 04	2.5780E 04

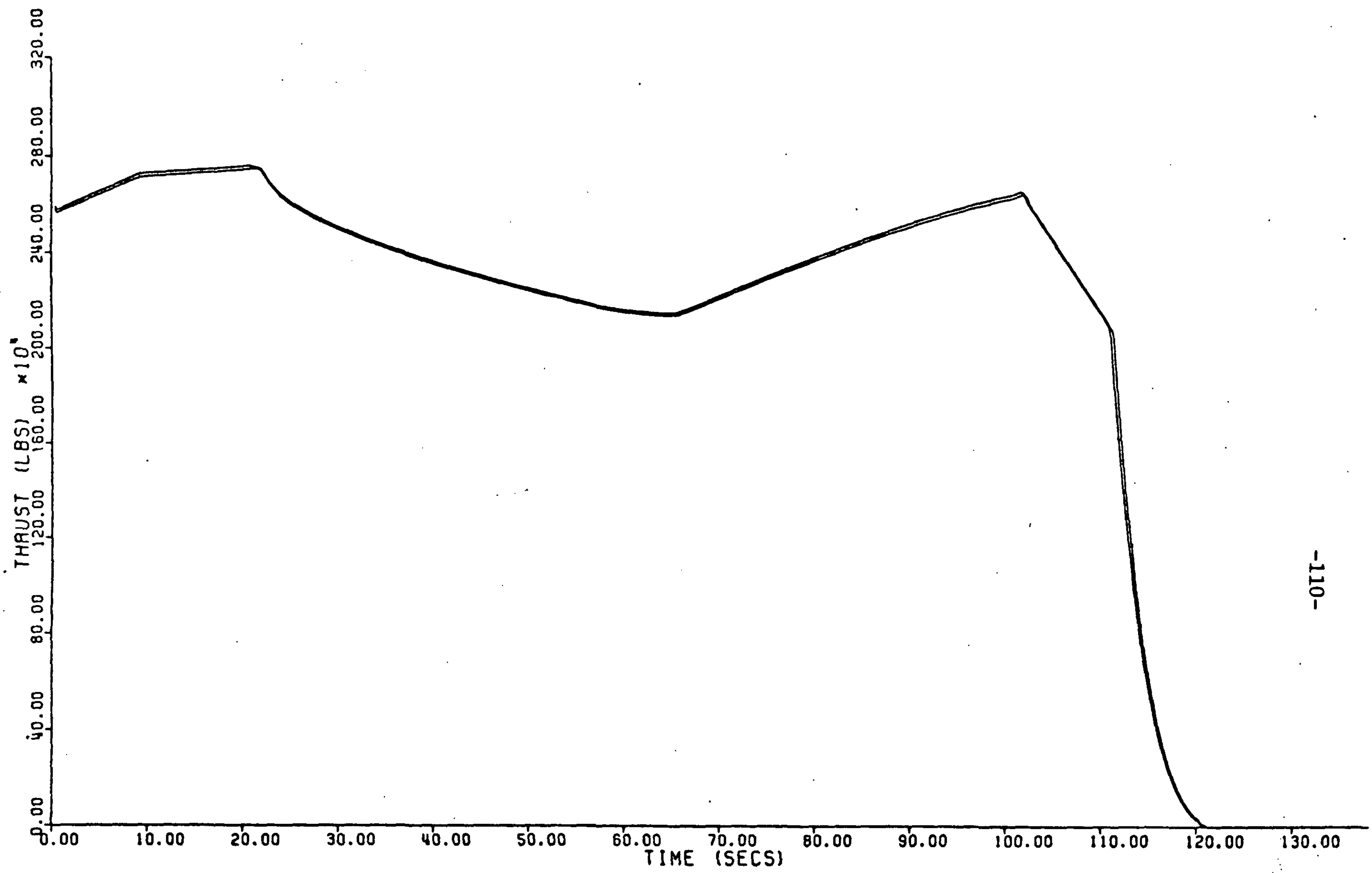

Figure V-1. Thrust versus time for one pair of SRMs of sample case (CalComp plot-reduced size).

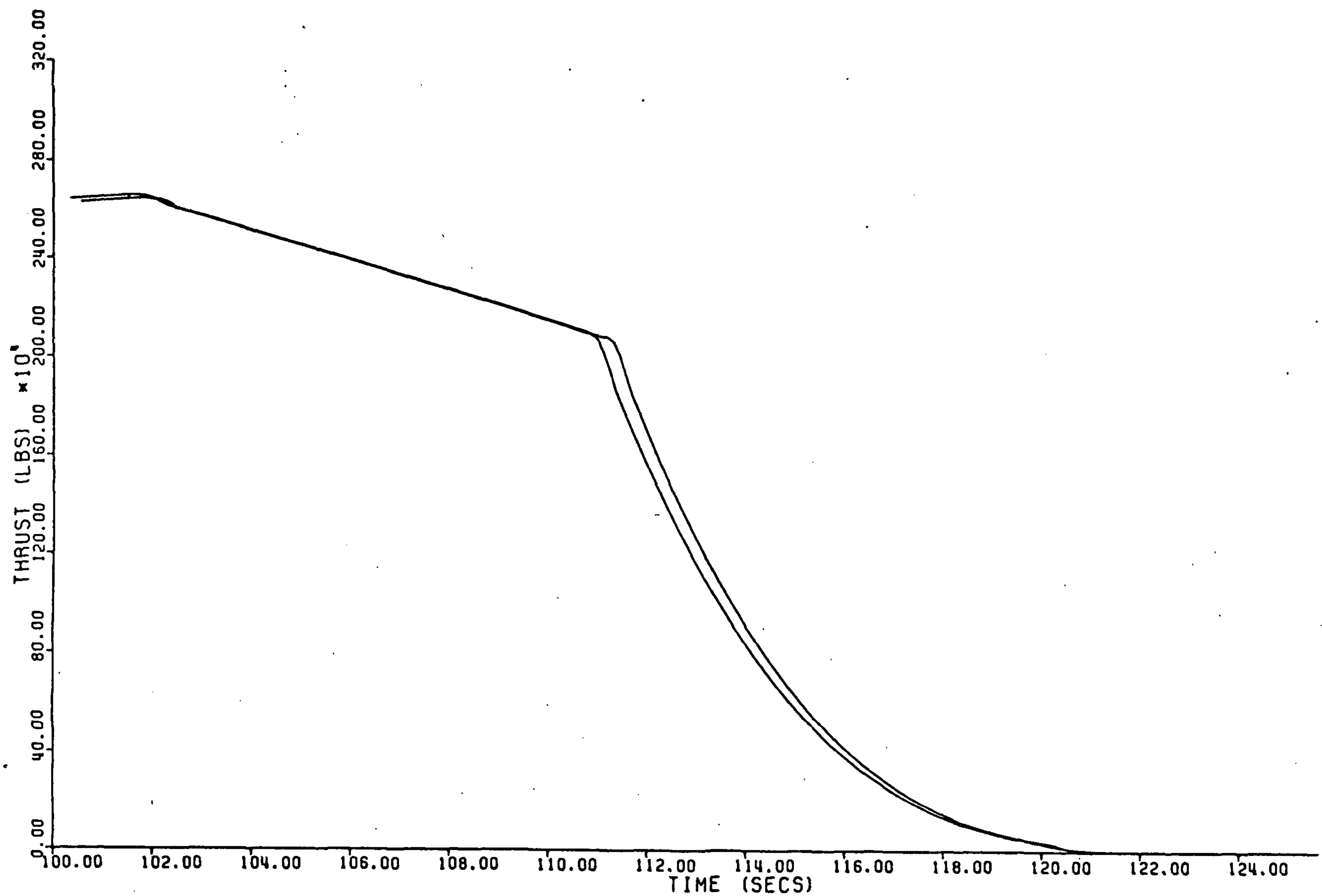

Figure V-2. Thrust during tailoff versus time for one pair of SRMs of sample case (CalComp plot-reduced size).

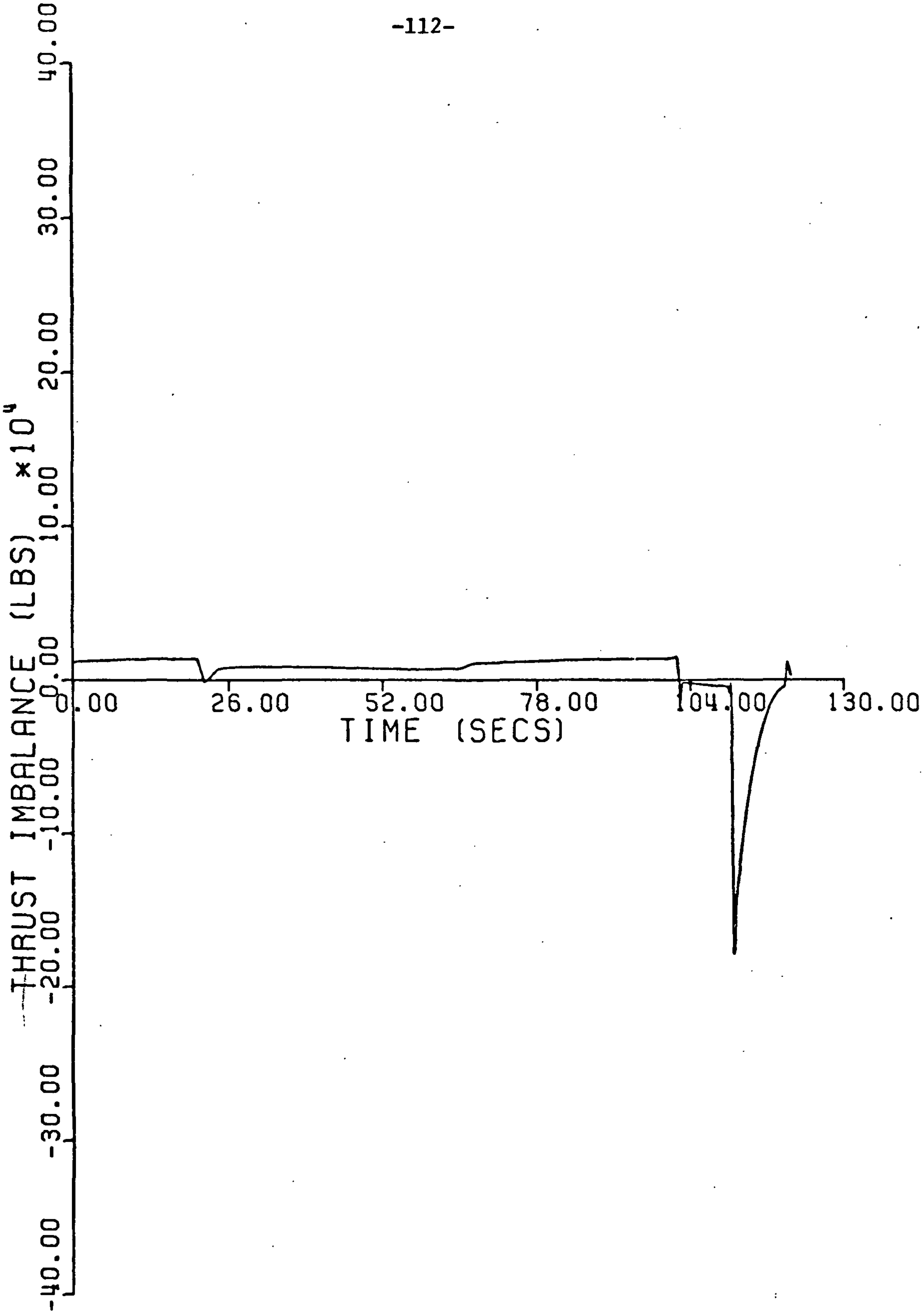

Figure V-3. Thrust imbalance versus time for one pair of SRMs of sample case (CalComp plot).

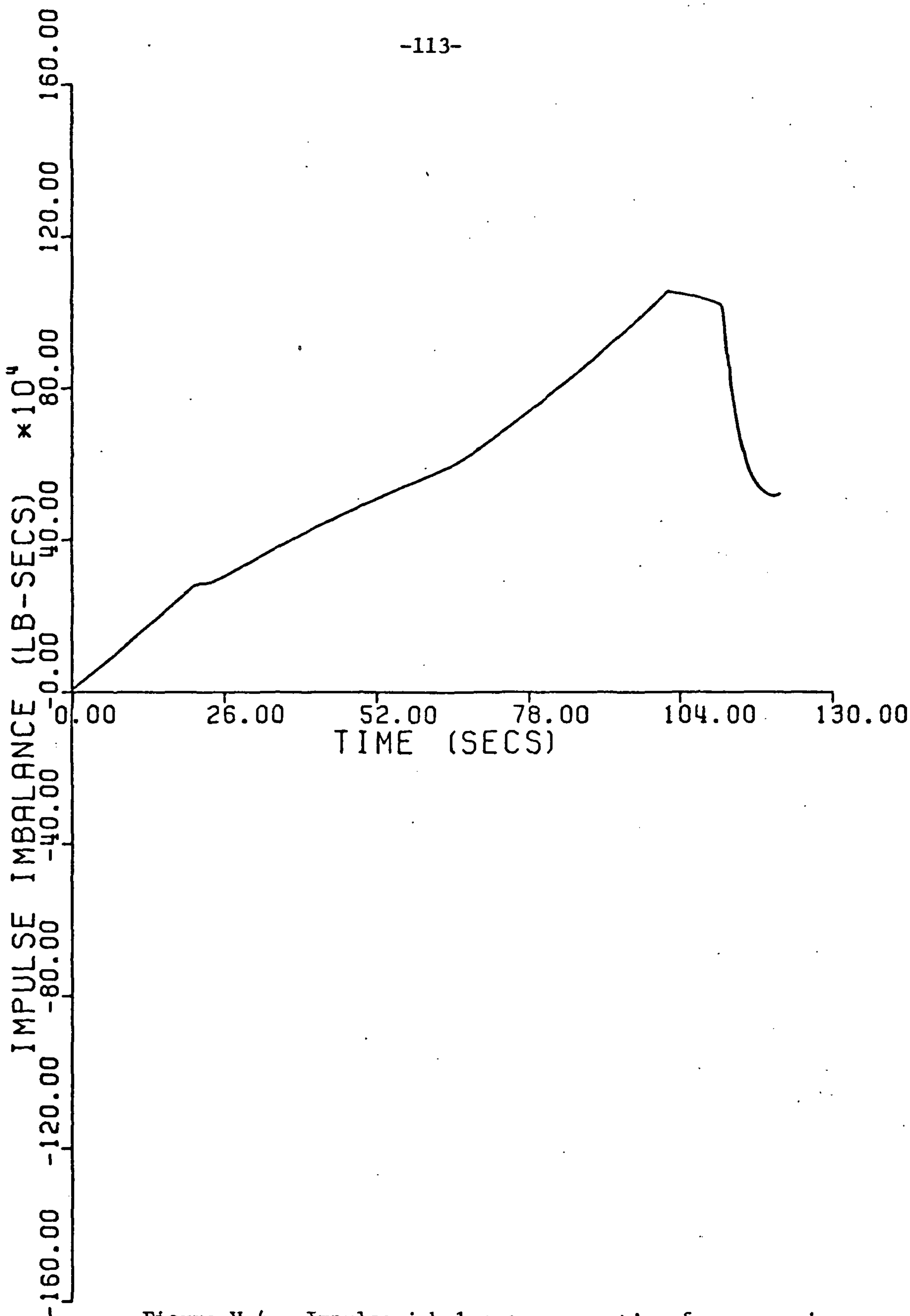

Figure V-4. Impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot).

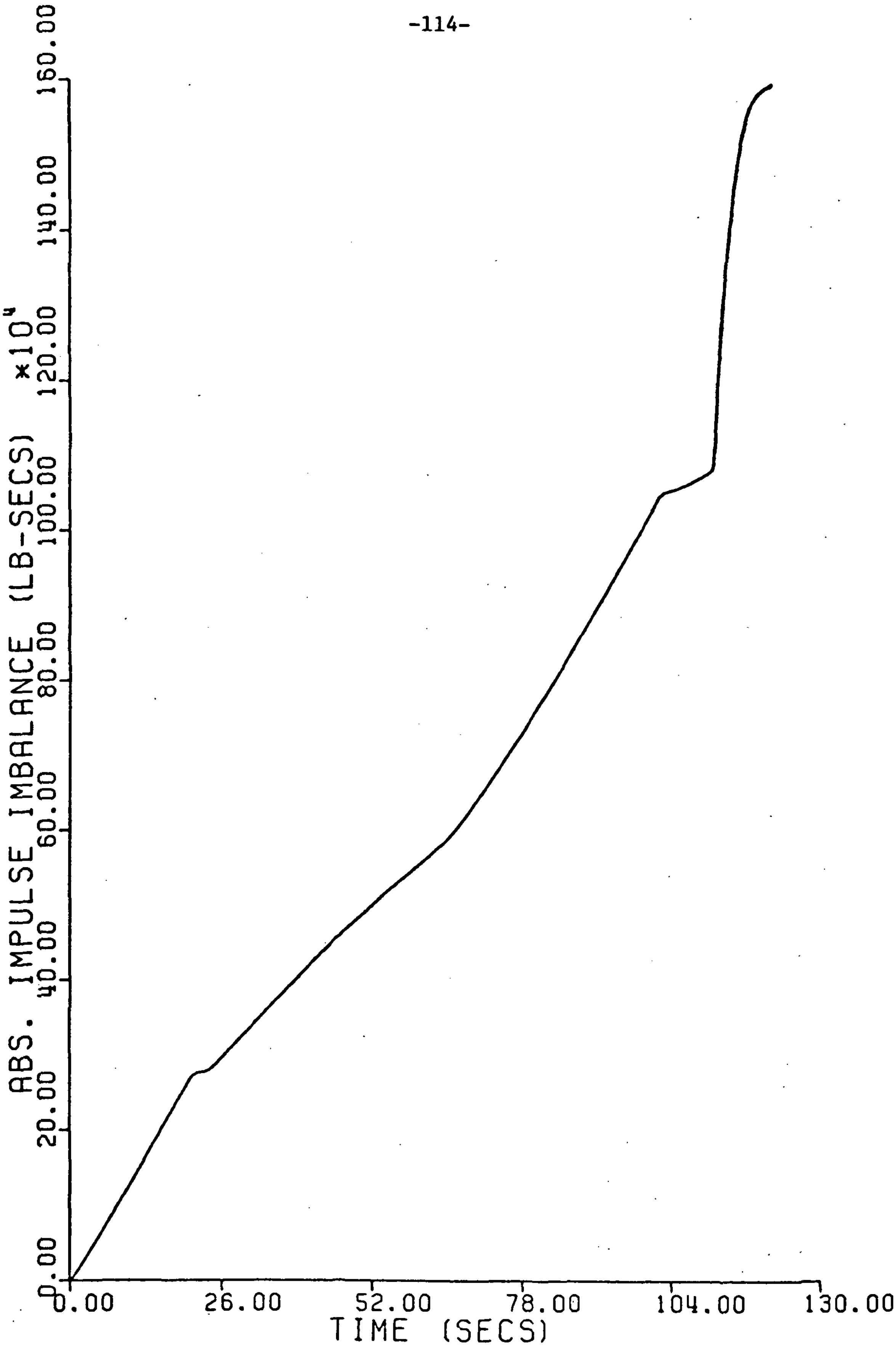

Figure V-5. Absolute impulse imbalance versus time for one pair of SRMs of sample case (CalComp plot).

TABLE V-6. OUTPUT VALUES FOR FINAL MOTOR PAIR AND STATISTICAL CHARACTERISTICS OF TWENTY-FIVE MOTOR PAIRS FROM THE SAMPLE STUDY.

MOTOR PAIR DATA

FMAX1=	3.5491E 04	TFMX1=	1.0226E 02		
FMIN1=	-1.7347E 04	TFMN1=	8.0128E 00		
FMAX2=	2.0555E 05	TFMX2=	1.1119E 02		
FMIN2=	1.5032E 03	TFMN2=	1.2134E 02		
TDFTO1=	1.1066E 02	TDFTO2=	1.1099E 02	DTW=	3.2643E-01
FW1=	2.0811E 06	FW2=	2.0809E 06	DFW=	1.7500 02
DFTO1=	2.2095E 04	DFTO2=	1.1096E 05		
DFMQ=	6.6945E 03	TMAXQ=	6.0000E 02		
AFMAX=	3.5419E 04	TFMAX=	1.0226E 02		
AFMAXT=	2.0555E 05	TFMAXT=	1.1119E 02		
FDIFIG=	1.6448E 04	TDIFIG=	2.2855E 00		
DIT=	8.6436E 05	ADIT=	8.7547E 05		
DF100K=	2.8583E 04	T100K=	1.1869E 02		

STANDARD DEVIATIONS AND MEANS FOR MOTOR PAIR DATA

	STD. DEV.	MEAN
AFMAX	0.1016E 05	0.1952E 05
TFMAX	0.3787E 02	0.8279E 02
AFMAXT	0.6072E 05	0.9811E 05
TFMAXT	0.5590E 00	0.1114E 03
DTW	0.1282E 00	0.1761E 00
FW1	0.4726E 04	0.2079E 06
FW2	0.5726E 04	0.2076E 06
DFW	0.2551E 04	0.6369E 04
DFTO1	0.6512E 04	0.6369E 04
TDFTO1	0.1291E 00	0.1107E 03
DTFO2	0.2757E 05	0.2961E 05
TDFTO2	0.1947E 00	0.1109E 03
DFMQ	0.2461E 04	0.3679E 04
FDIFIG	0.4574E 04	0.6234E 04
TDIFIG	0.1553E 00	0.2233E 01
DIT	0.4186E 06	0.5927E 05
ADIT	0.2386E 06	0.3585E 06
DF100K	0.8192E 04	0.1228E 05
T100K	0.1531E 00	0.1186E 03

ALTERNATE DISPERSION VALUES FOR THRUST IMBALANCE DATA

	SIGMA 1	SIGMA 2
AFMAX	0.2200E 05	0.1556E 05
AFMAXT	0.1154E 06	0.8159E 05

TABLE V-7. SELECTED STATISTICAL CHARACTERISTICS OF FIFTY MOTOR PAIRS FROM THE SAMPLE STUDY

PARAMETER	MEAN	STANDARD DEVIATION
Absolute value of maximum thrust imbalance during web action time (AFMAX) lbf.	19,620.40	9250.22
Time of AFMAX(TFMAX) sec.	83.89	36.59
Absolute value of maximum thrust imbalance during tailoff (AFMAXT) lbf.	110,346.00	61,130.86
Time of AFMAXT (TFMAXT) sec.	111.60	0.93
Absolute value of the difference in time at which the two motors of a pair begin tailoff (DTW) sec.	0.20	0.14
Absolute value of the thrust imbalance at input time of maximum dynamic pressure (DFMQ) lbf.	2954.46	3965.88
Algebraic value of the impulse imbalance during tailoff (DIT) lbf-sec.	-51,059.09	461,769.56
Absolute value of the area between the thrust-time traces of the pair during tailoff (ADIT) lbf-sec.	406,400.00	237,49794
Absolute value of thrust imbalance when last motor of pair reaches 100,000 lb. thrust during tailoff (DF100K) lbf-sec.	8554.64	13,469.31
Time of DF100K (T100K) sec.	118.66	0.29

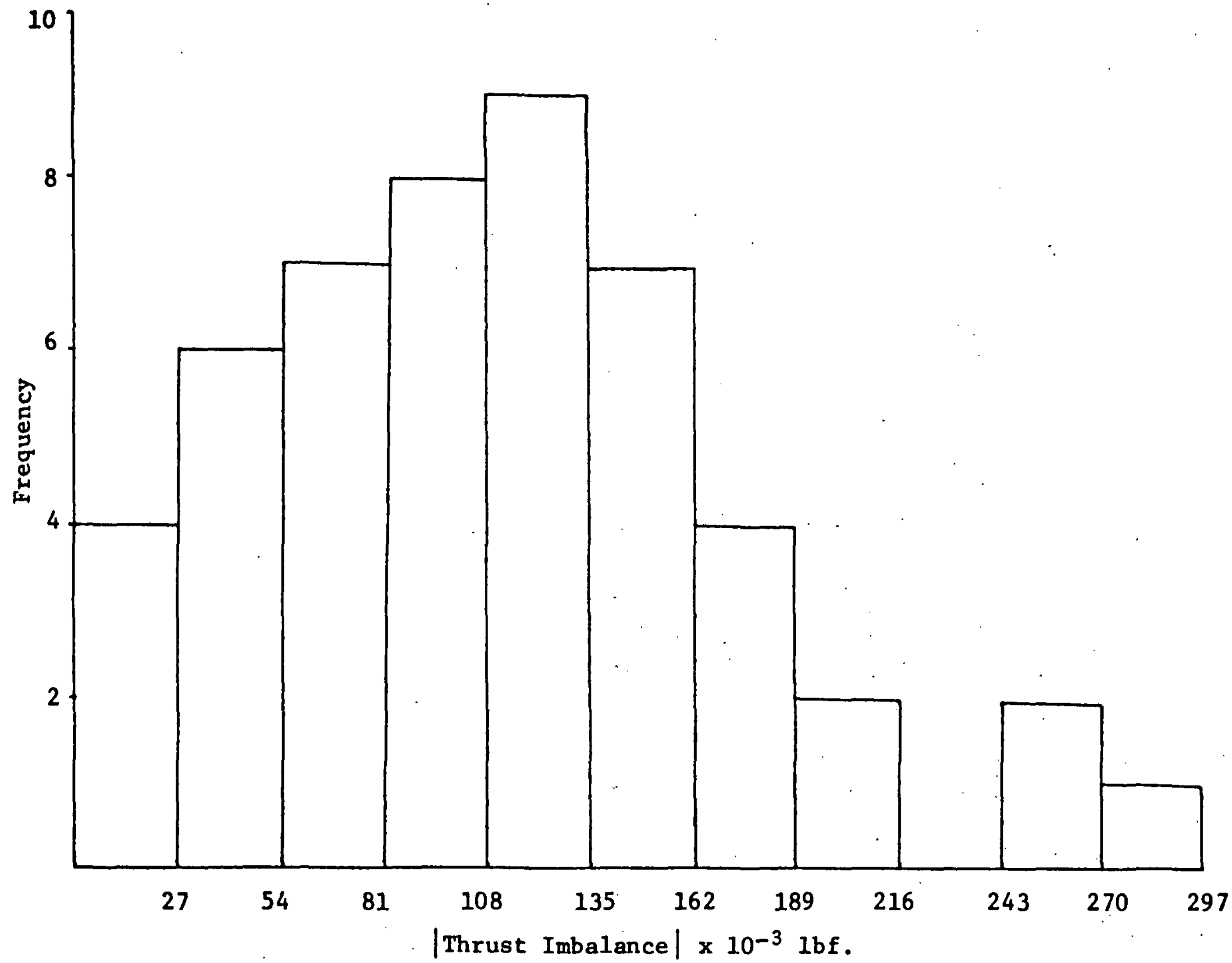

Figure V-6. Histogram of absolute thrust imbalance values for fifty motor pairs from the sample study.

VI. CONCLUDING REMARKS

A technique has been established for statistically investigating the thrust imbalance of pairs of SRMs firing in parallel. The computer program based upon the analysis permits the imbalance characteristics of a large number of SRM pairs to be evaluated in a reasonably short time.

It remains to demonstrate the accuracy of the program by comparisons of theoretical imbalance results with those from real SRM populations. Preliminary investigations of this type were conducted during the program. The results, although encouraging, are too incomplete to warrant reporting. This is largely due to the difficulty encountered in obtaining specific data to define confidently the statistical distributions of input variables for past rocket motors. The necessary data is often incomplete or not readily accessible.

Additional areas for extended effort include investigation of methods for accounting for effects of radial, axial and circumferential temperature gradient differences between motors of a pair and incorporation of between pair variations of propellant characteristics into the analysis. Ability to treat the between pair variations would improve the accuracy of the program as a device for predicting the absolute performance characteristics of SRMs.

Finally, it is noted that a large number of improvements in the basic accuracy of the simplified computer program presented in References 3, 4 and 5 have been incorporated into the program presented in this report. Those using the earlier program may wish to adopt these improvements for the purpose of design and performance analysis of single rocket motors.

REFERENCES

1. Reckmeyer, V. U., "Impulse Reproducibility," Huntsville Division, Thiokol Chemical Corporation, Huntsville, Alabama, Special Report No. 40-68, Control No. U-68-40A, October 1968,

2. Mize, J. H., and Cox, J. G., Essentials of Simulation, Prentice-Hall, Englewood Cliffs, New Jersey, 1968, pp. 73, 74, 87-92, 228.

3. Sforzini, R. H., "Design and Performance Analysis of Solid-Propellant Rocket Motors Using a Simplified Computer Program," Final Report, NASA Contractor Report NASA CR-129025, Auburn University, October 1972.

4. Sforzini, R. H., "Extension of a Simplified Computer Program for Analysis of Solid-Propellant Rocket Motors," Final Report, NASA Contractor Report NASA CR 129024, Auburn University, April 1973.

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6. "Solid Rocket Motor Performance Analysis and Prediction," NASA Space Vehicle Design Criteria (Chemical Propulsion) NASA SP-8039, National Aeronautics and Space Administration, May 1971, pp. 62-63.

7. Elderton, W. P., and Johnson, J. L., Systems of Frequency Curves, The University Press, London, 1969, pp. 35-70.

8. "Proposal for Solid Rocket Motor Project for the Space Shuttle Program," Vol. III, Design, Development and Verification Proposal, TWP 077326, Submitted to NASA George C. Marshall Space Flight Center by Wasatch Division Thiokol Chemical Corporation, 27 August, 1973, pp. 4.5-25 through 27.

APPENDIX

PERFORMANCE SENSITIVITIES

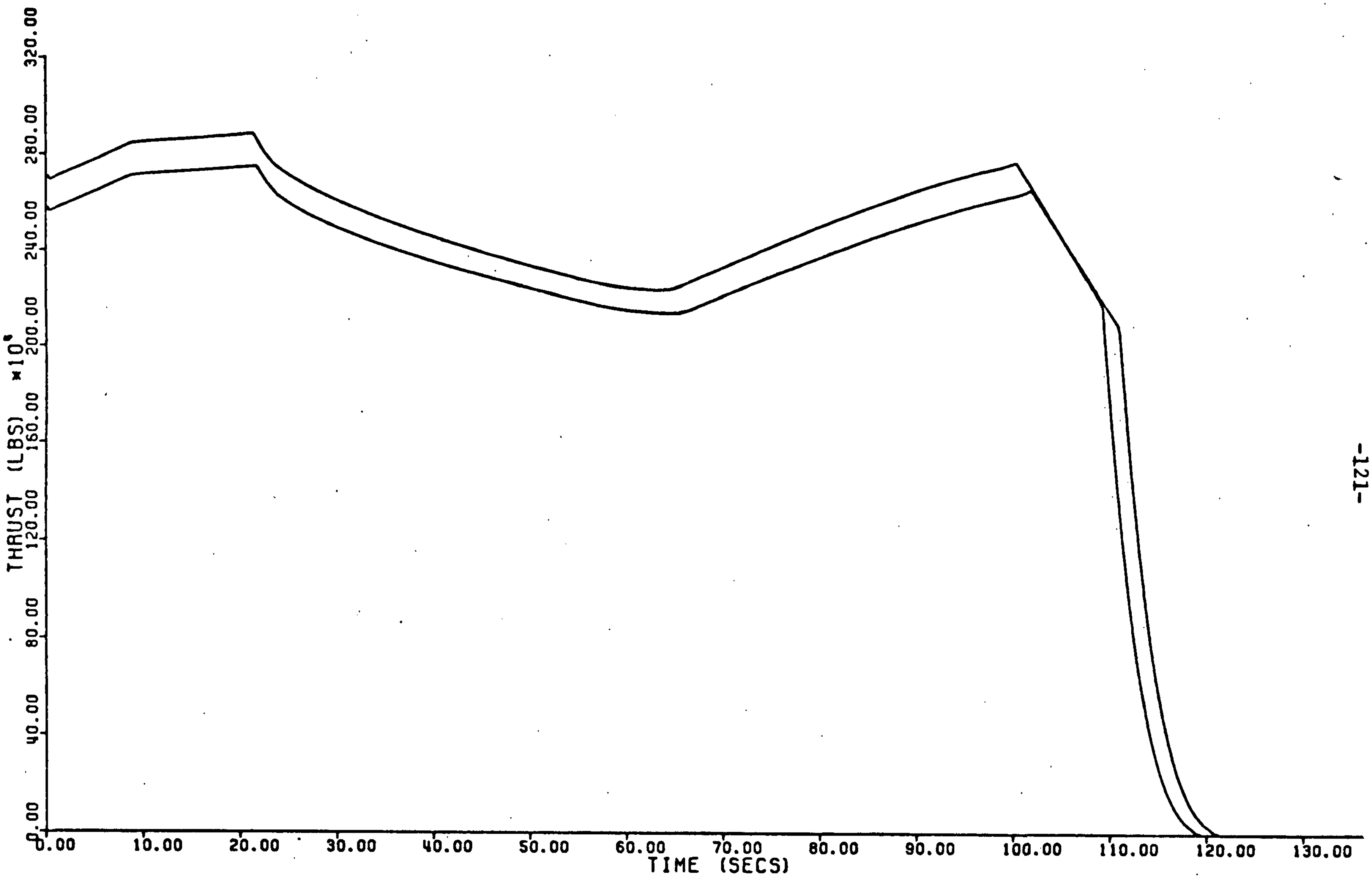

Figure A-1. Thrust versus time for two SRMs with propellant density ρ difference of 3%.

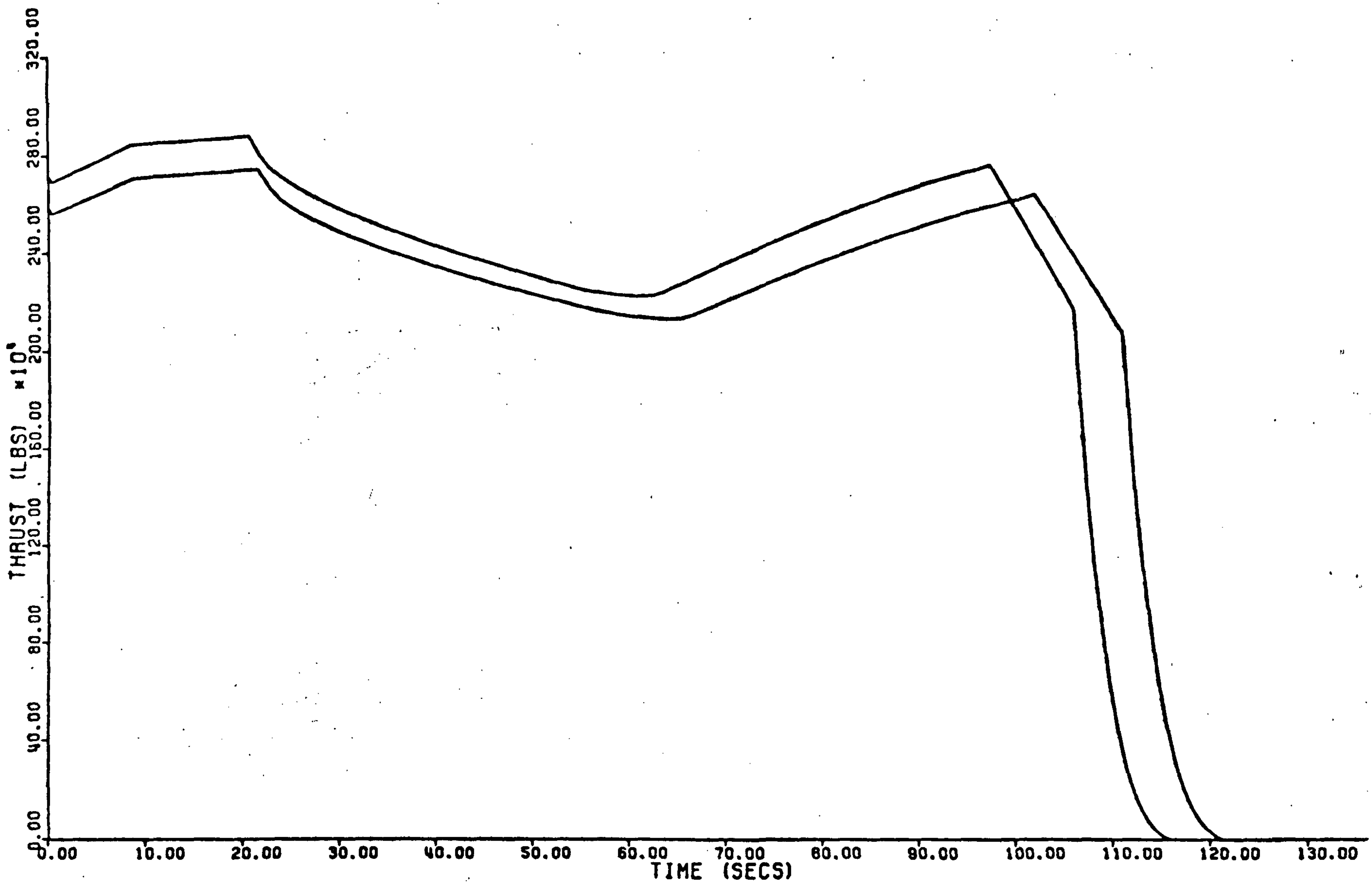

Figure A-2. Thrust versus time for two SRMs with burning rate coefficient, a_1 and a_2, difference of 3%.

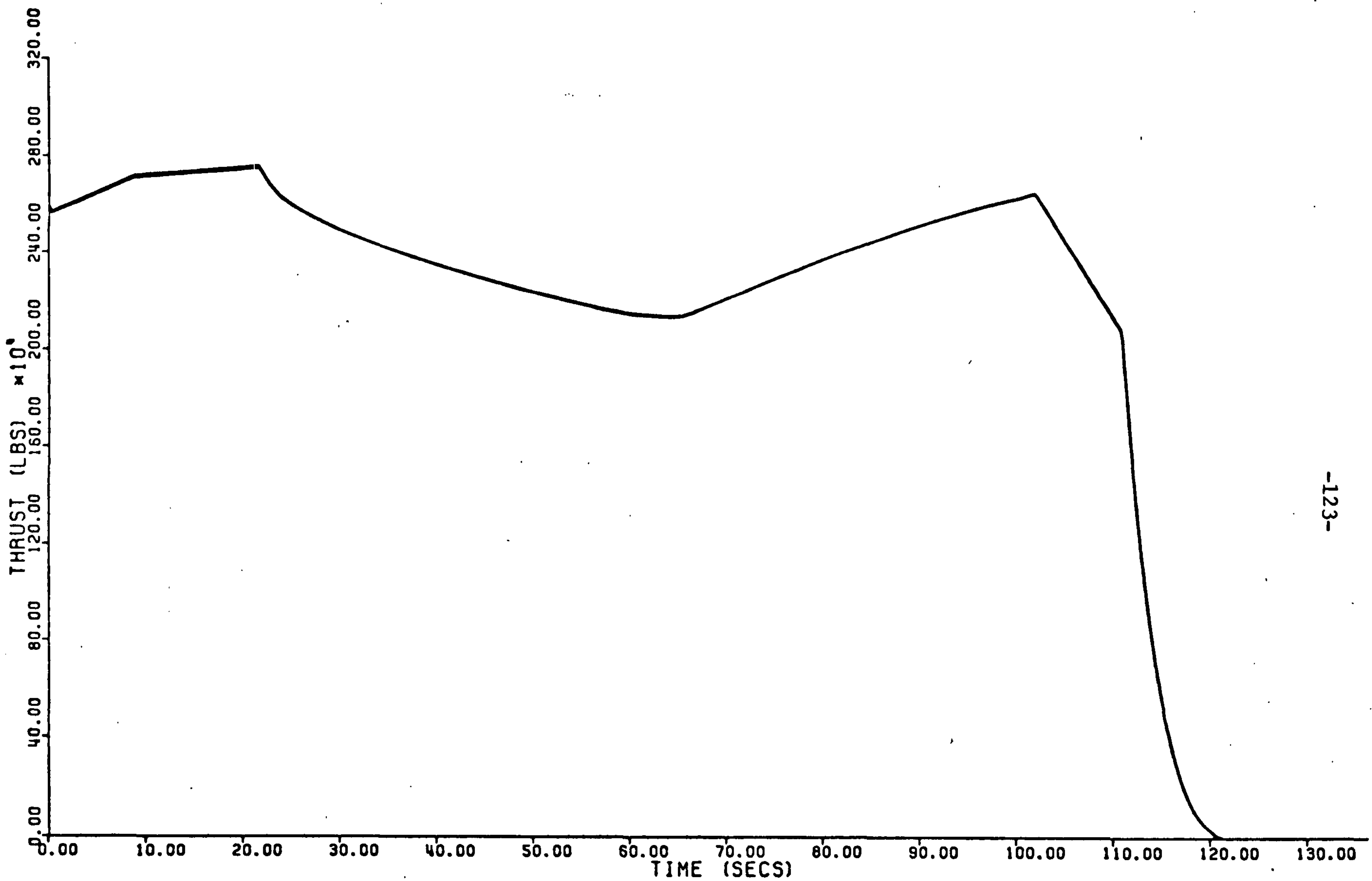

Figure A-3. Thrust versus time for two SRMs with burning rate exponent, n_1 and n_2, difference of 3%.

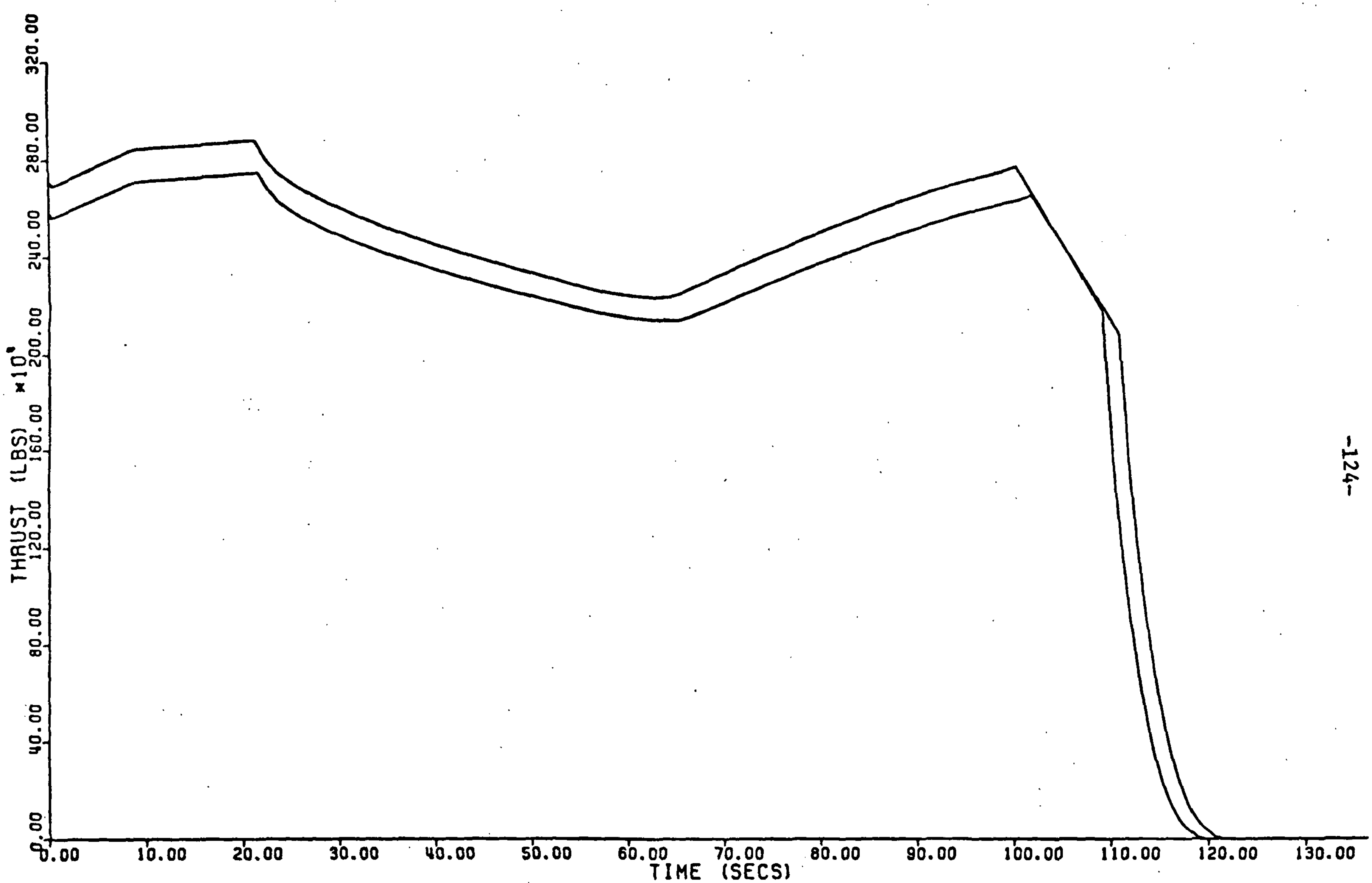

Figure A-4. Thrust versus time for two SRMs with characteristic velocity C* difference of 3%.

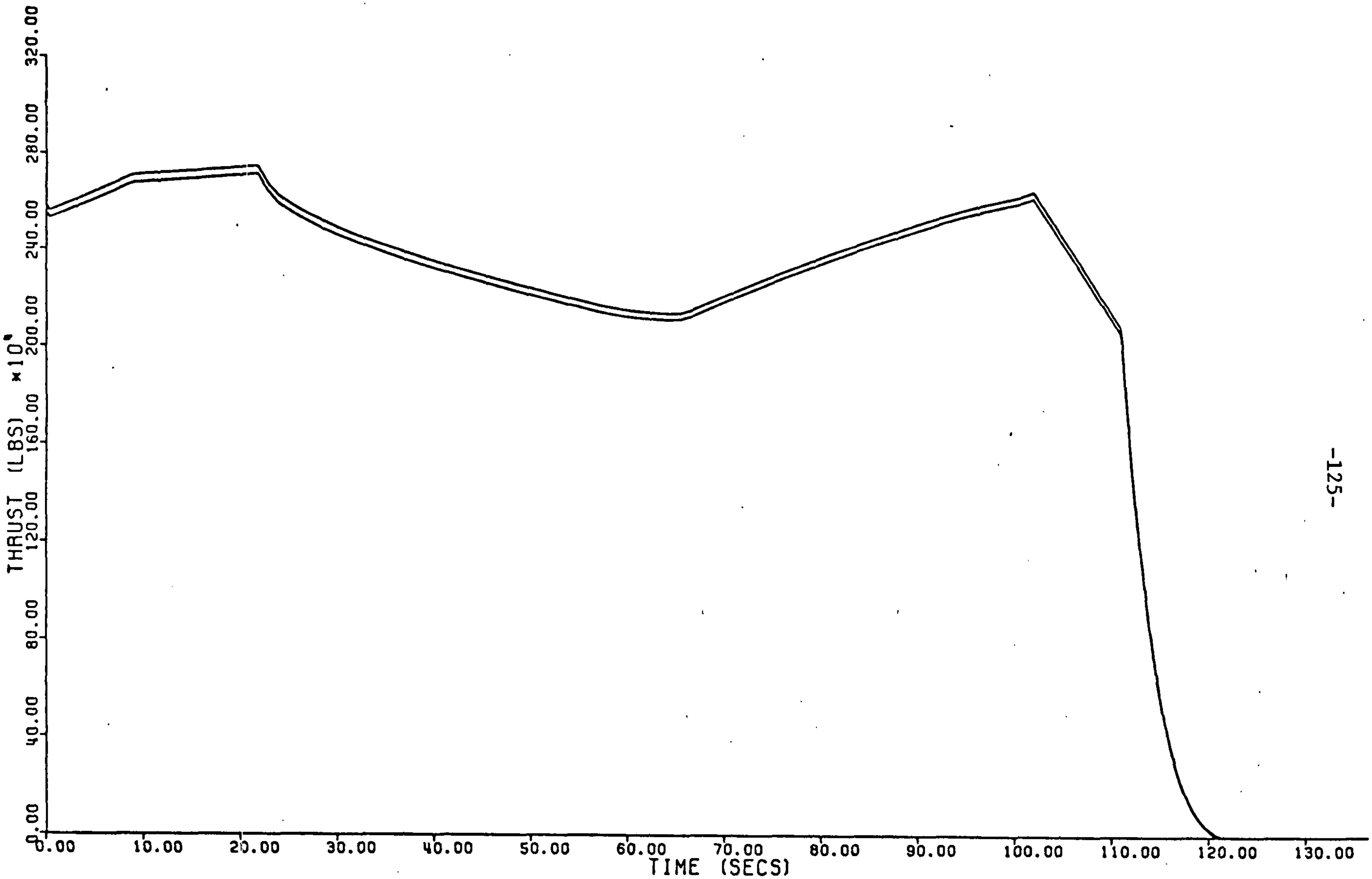

Figure A-5. Thrust versus time for two SRMs with ratio of specific heats γ difference of 3%.

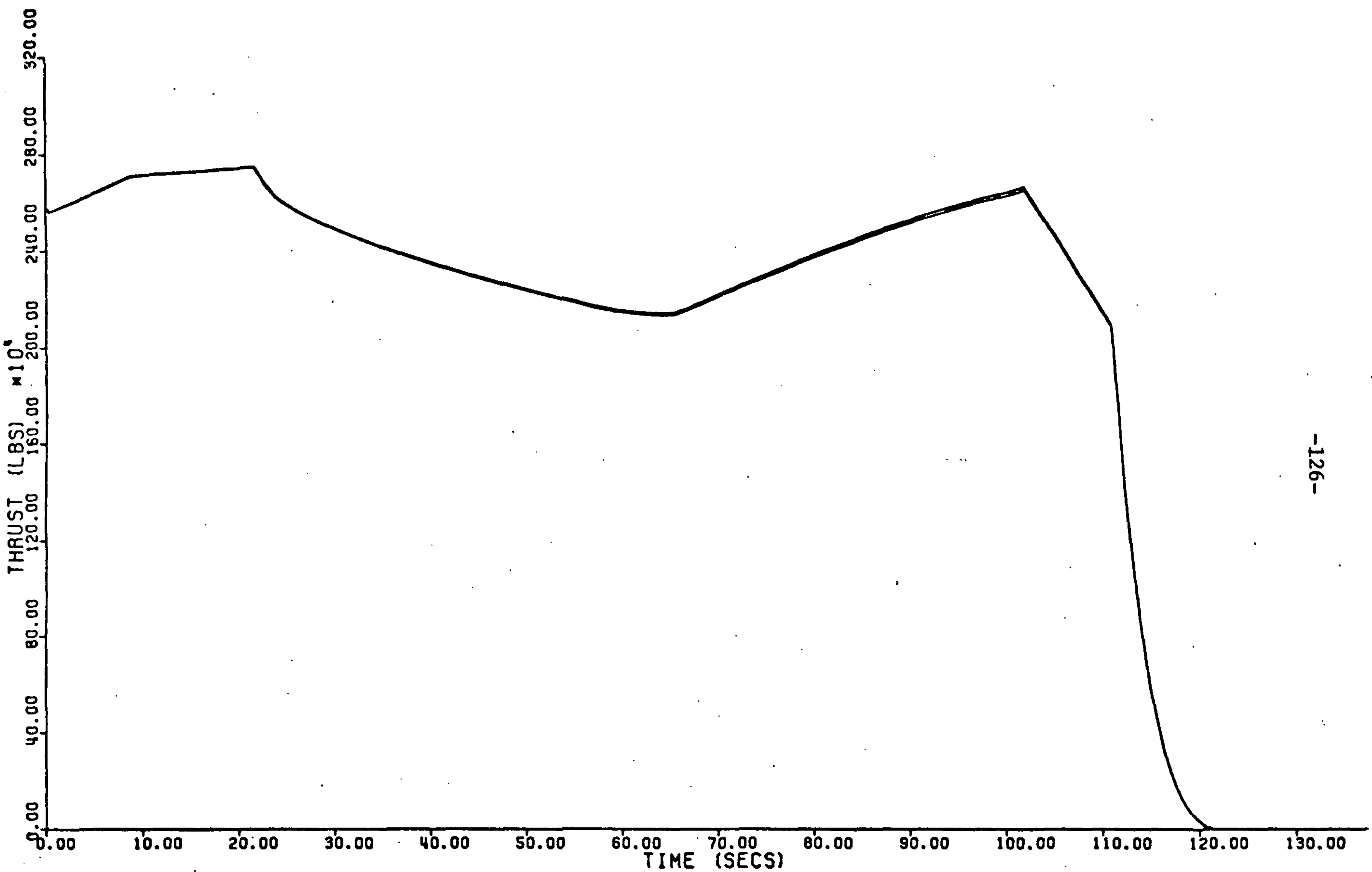

Figure A-6. Thrust versus time for two SRMs with nozzle exit diameter D_e difference of 3%.

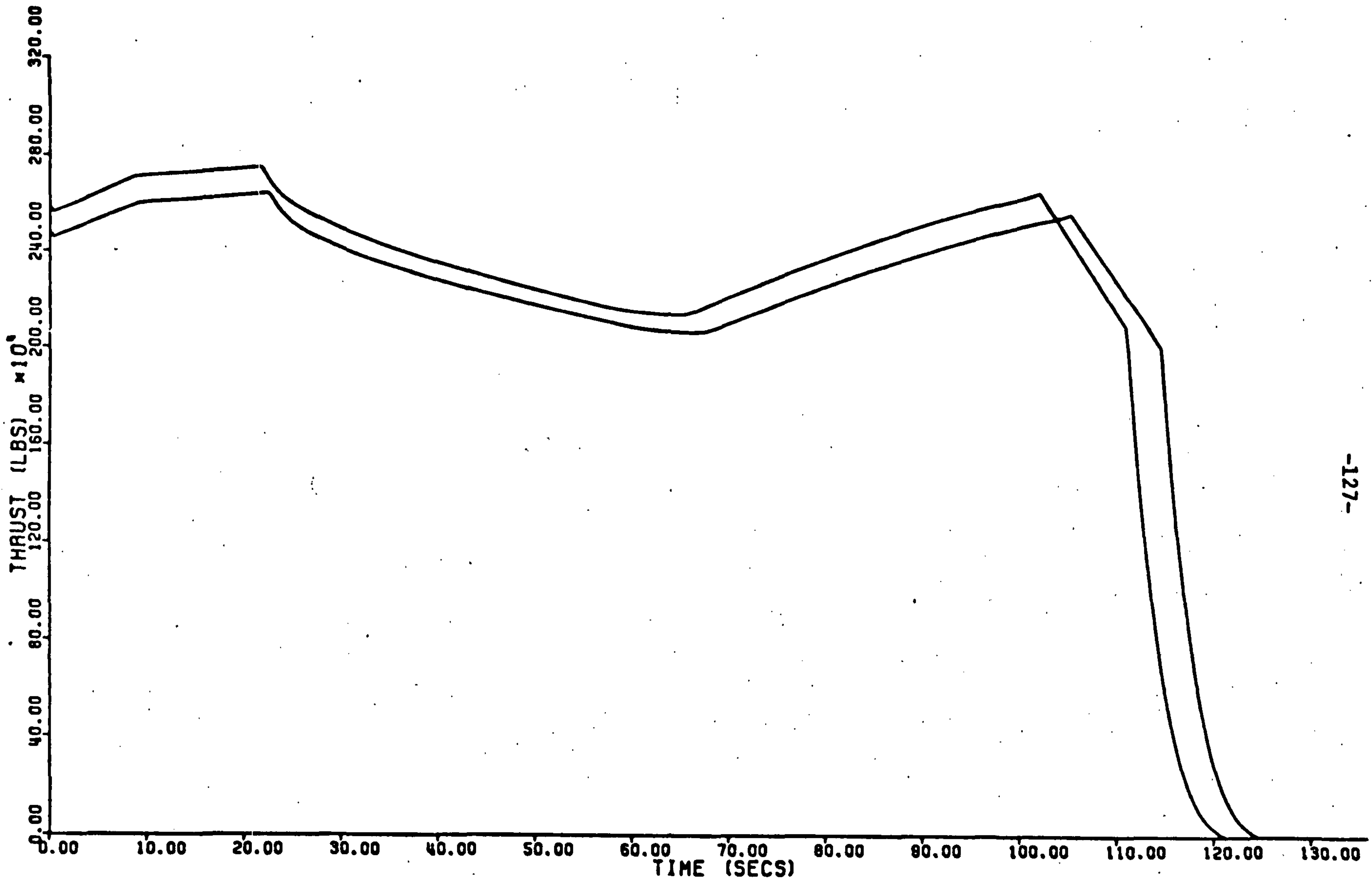

Figure A-7. Thrust versus time for two SRMs with nozzle initial throat diameter D* difference of 3%.

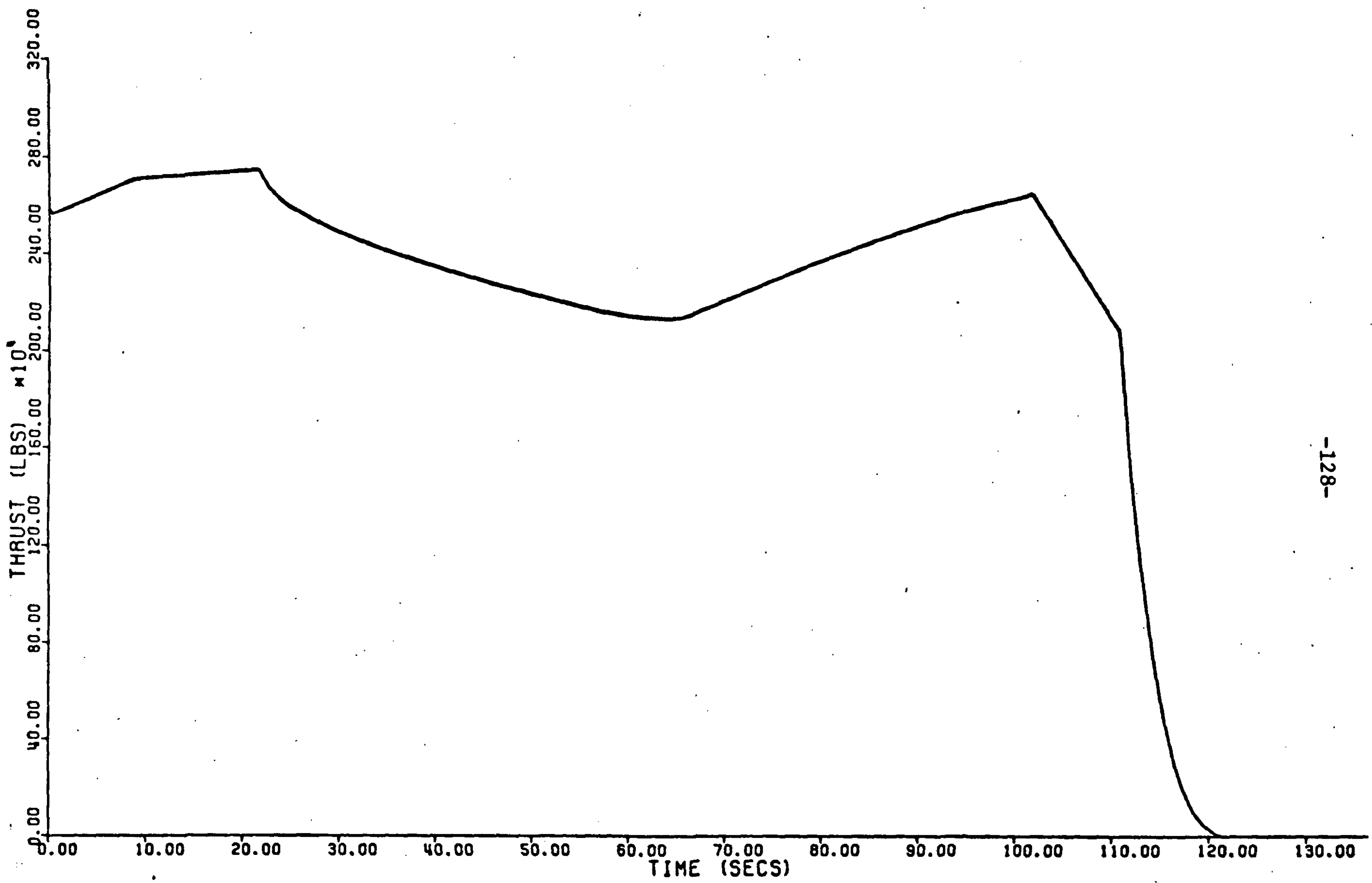

Figure A-8. Thrust versus time for two SRMs with nozzle exit divergence angle α_n difference of 0.3° (2.7%).

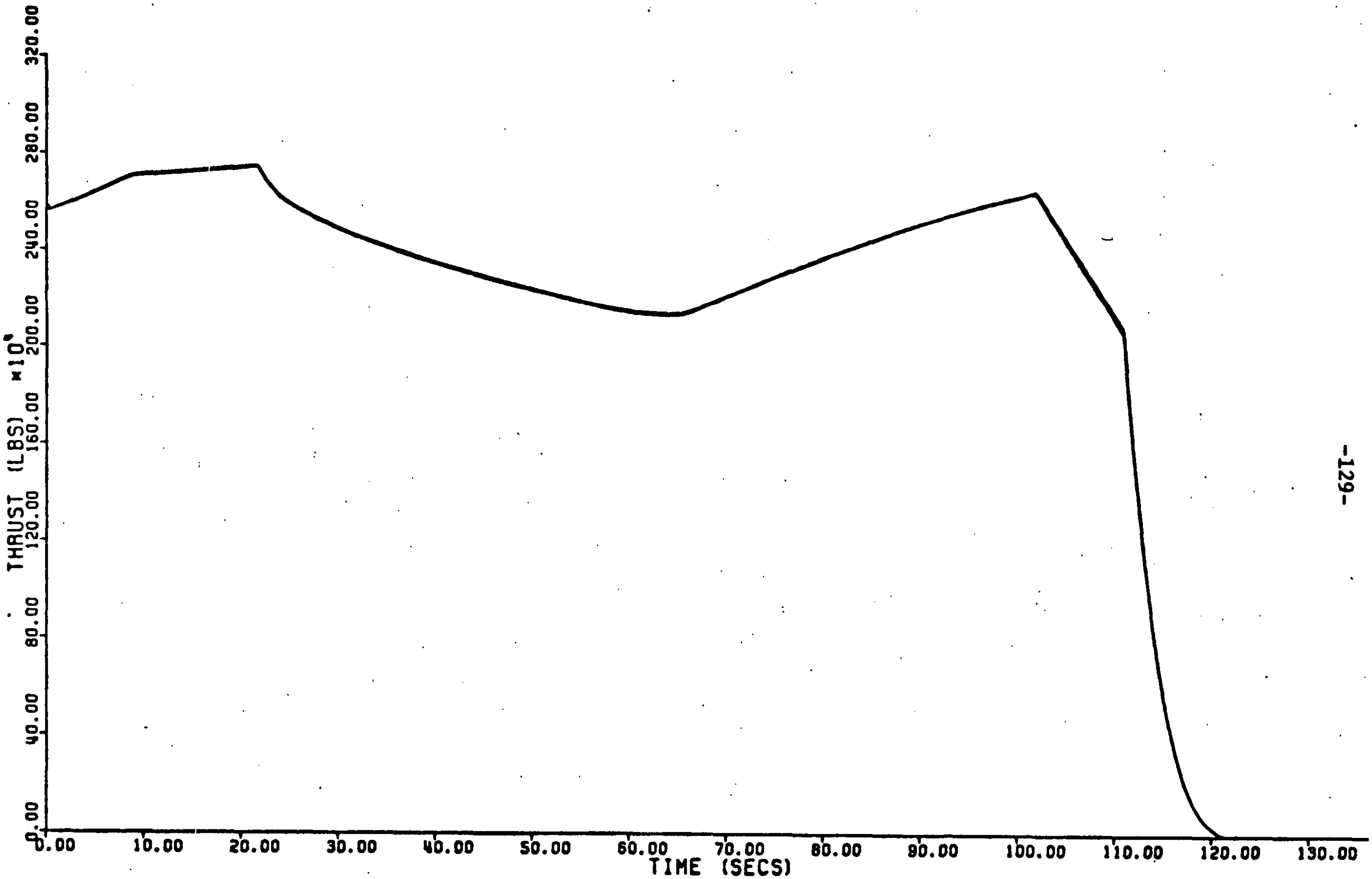

Figure A-9. Thrust versus time for two SRMs with aft end grain tapered length L_{T_a} difference of 3%.

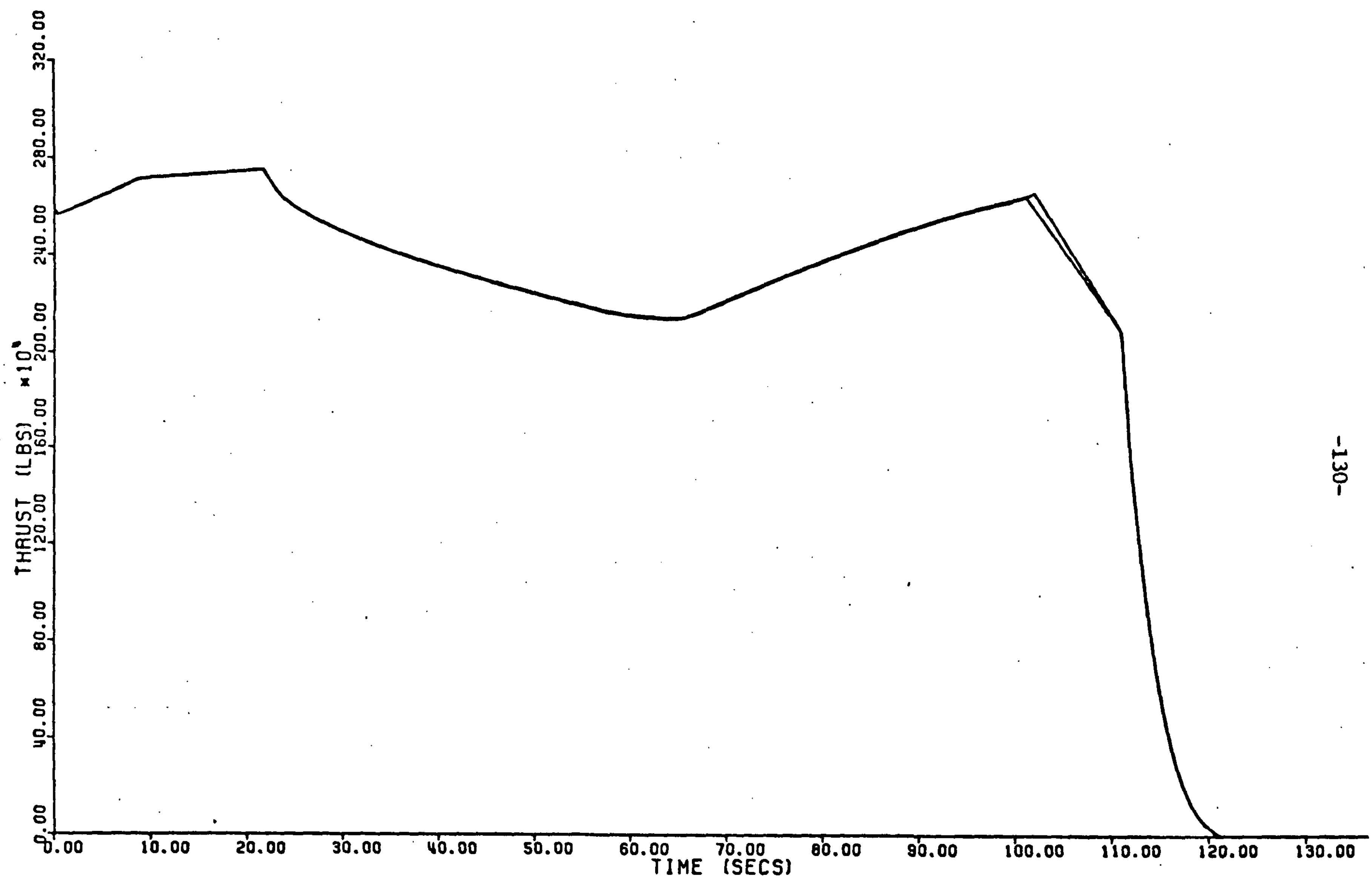

Figure A-10. Thrust versus time for two SRMs with aft end radial displacement of tapered grain x_{Ta} difference of 0.3 inches (9.9%).

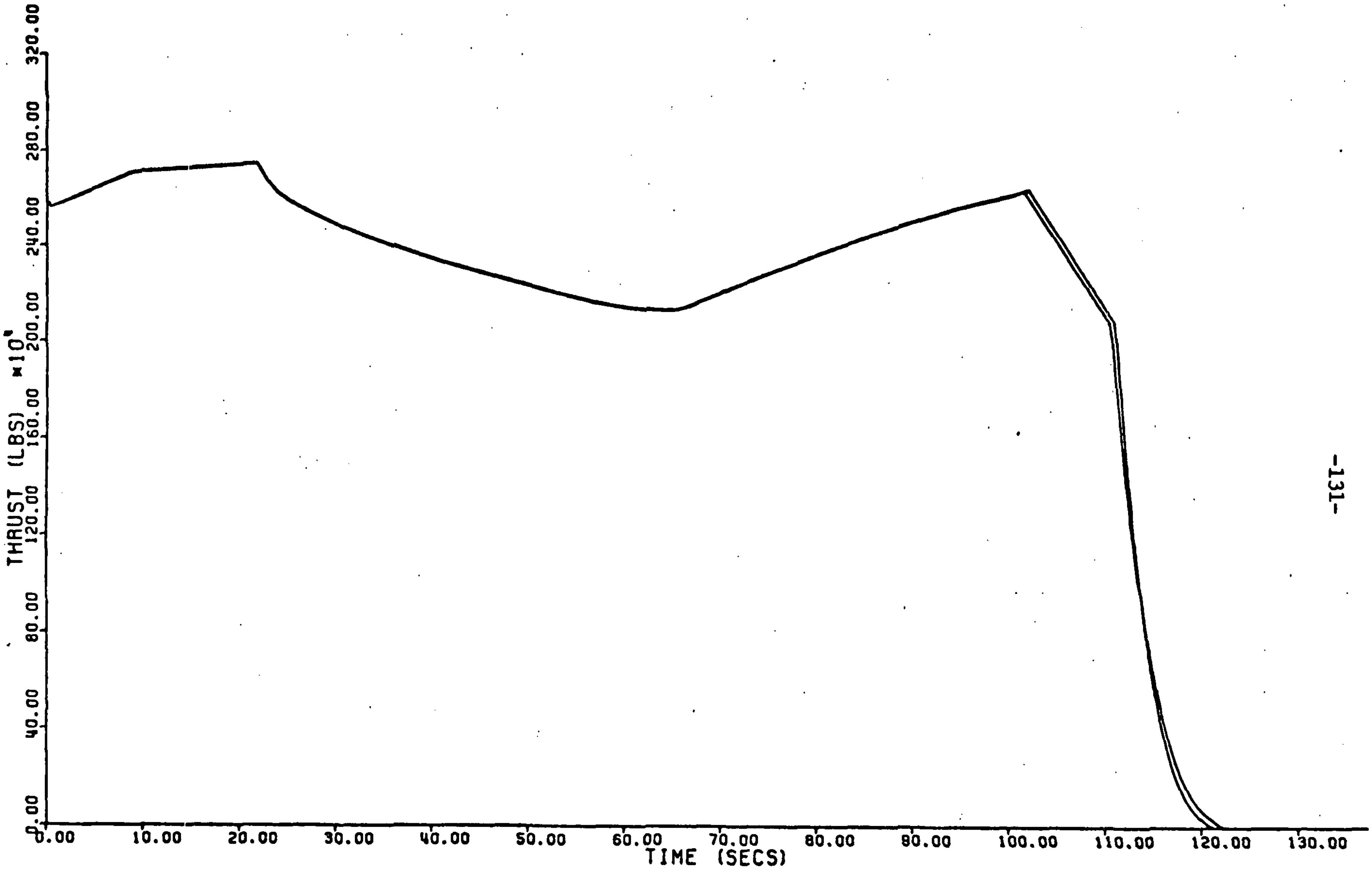

Figure A-11. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to bore taper z_o difference of 0.3 inches (12.4%).

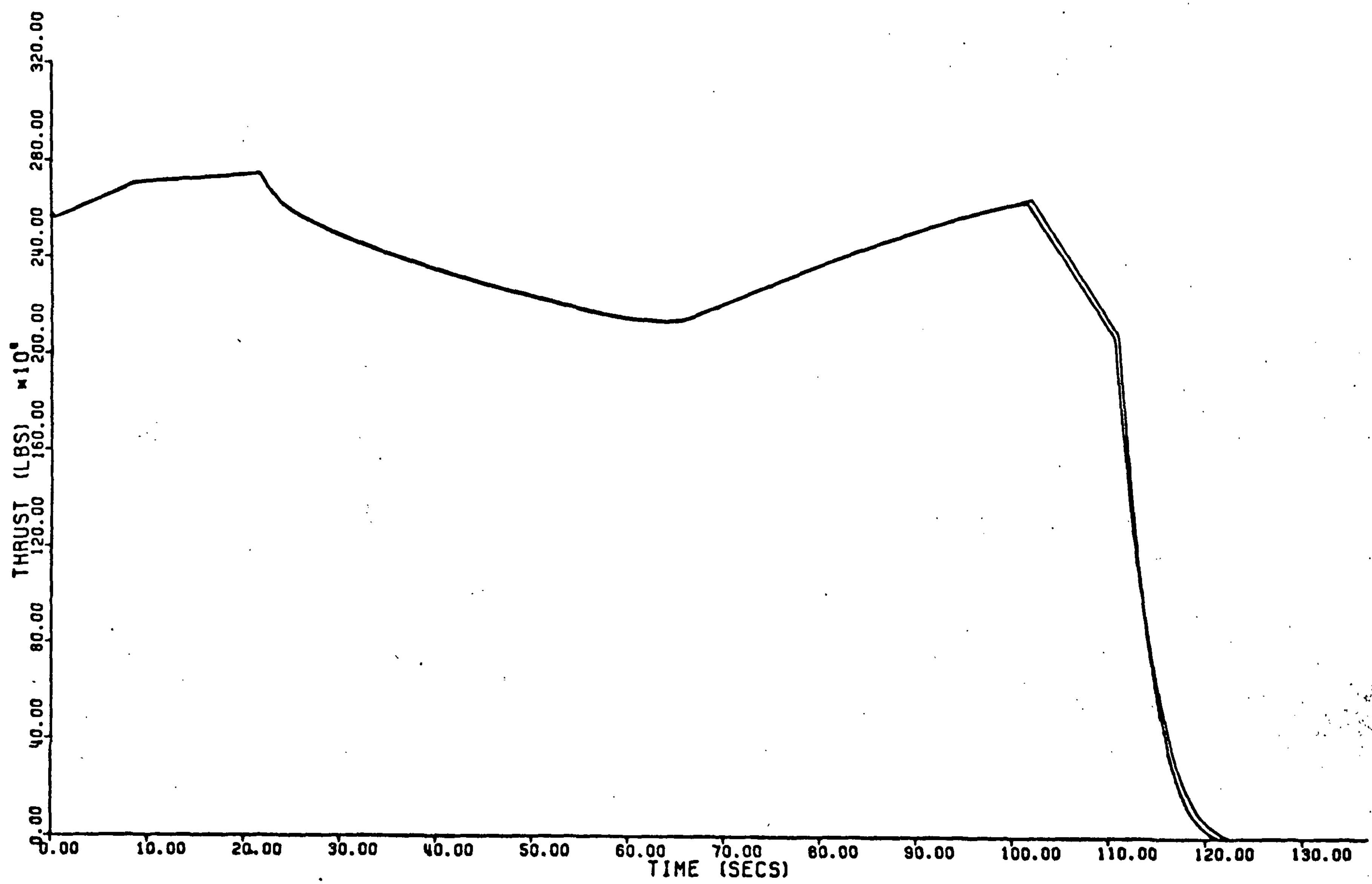

Figure A-12. Thrust versus time for two SRMs with radial displacement of the main portion of the grain due to exterior taper difference z_c of 0.3 inches (0 base value).

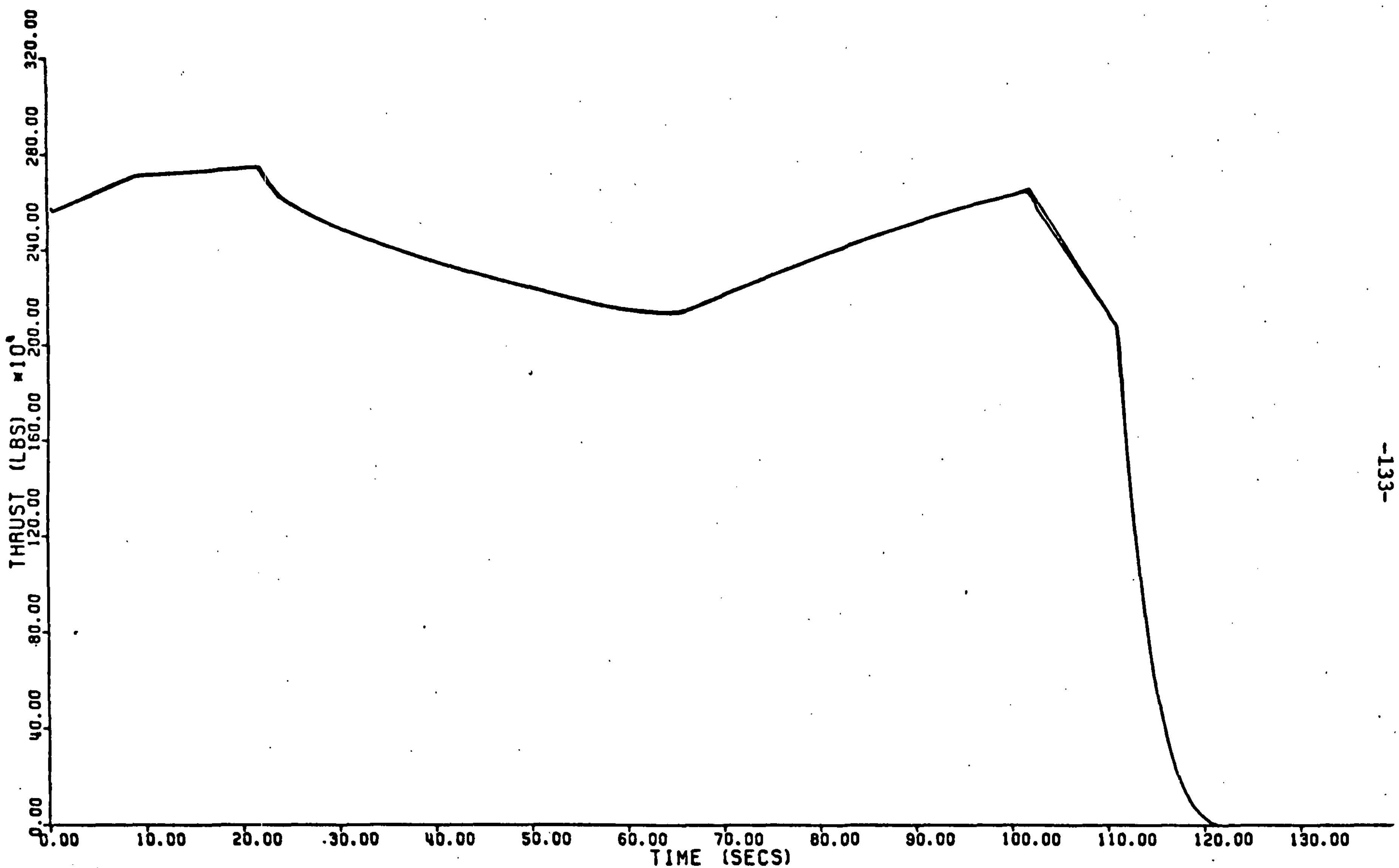

Figure A-13. Thrust versus time for two SRMs with out-of-round conditions of the grain exterior at the aft end ΔR_{cn} of 0 and 0.5 inches ($\alpha_{an} = 0$).

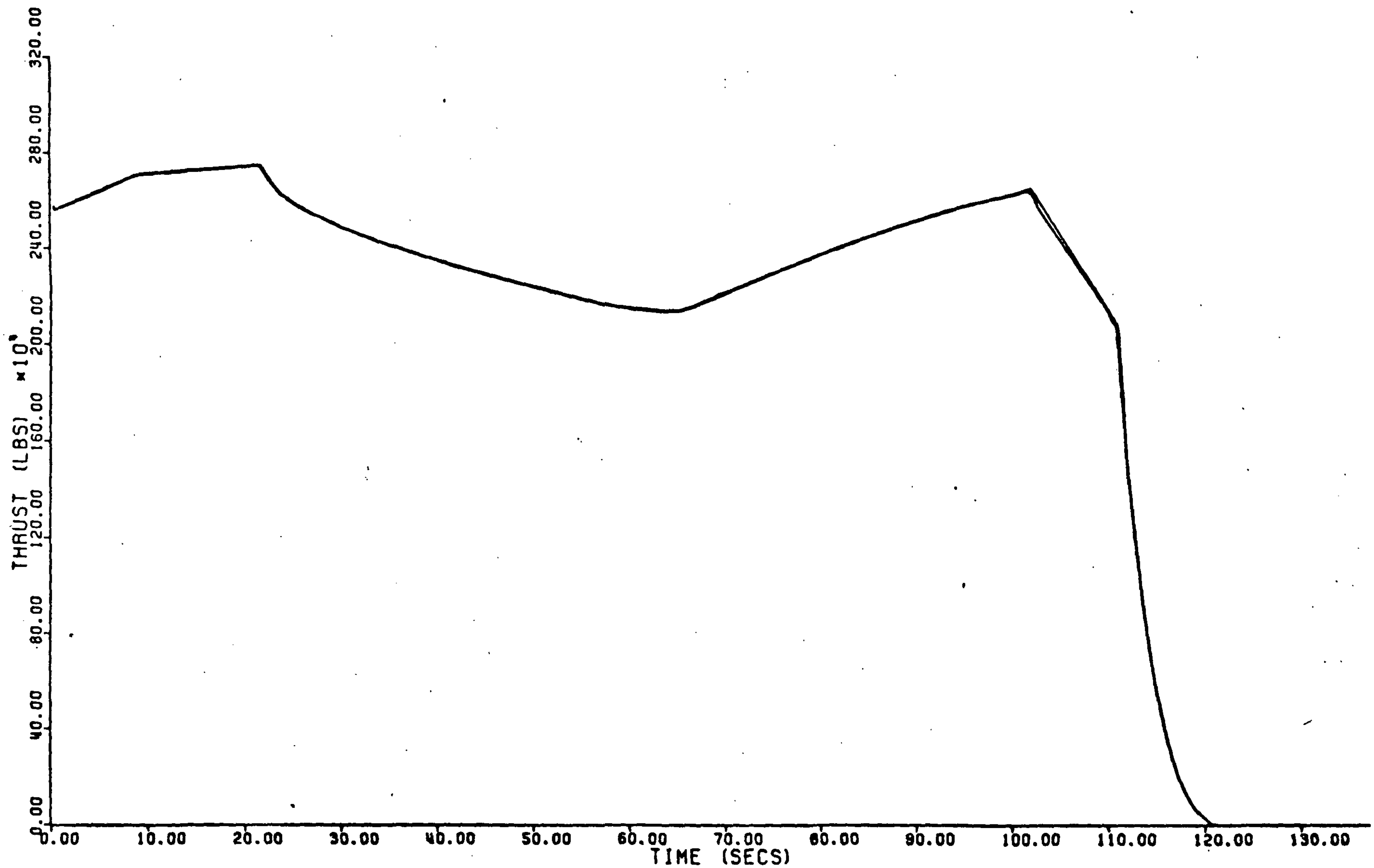

Figure A-14. Thrust versus time for two SRMs with out-of-round conditions of the grain exterior at the aft end ΔR_{cn} of 0 and 0.5 inches (α_{an}= 90°).

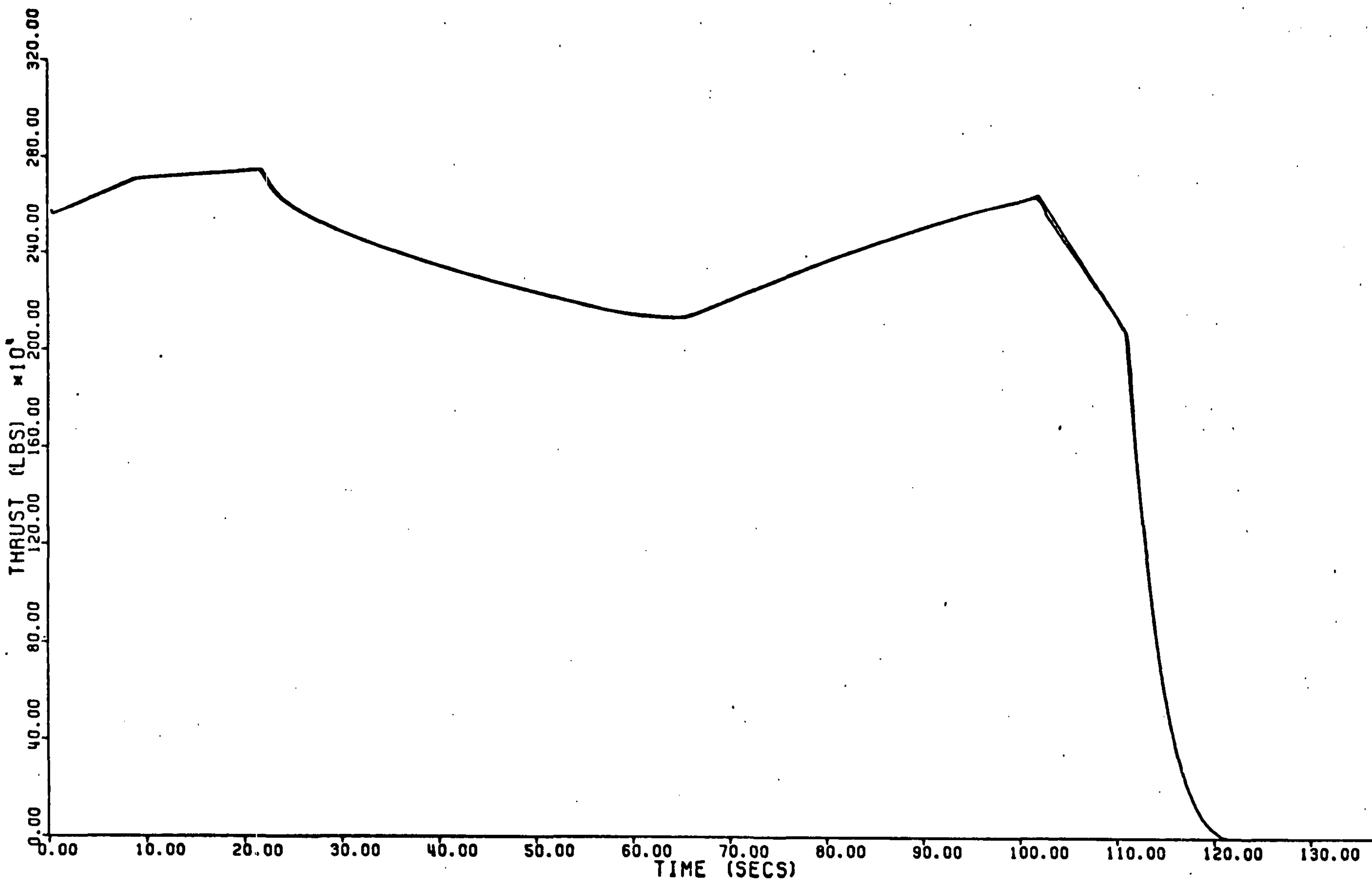

Figure A-15. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the aft end ΔR_{gn} of 0 and 0.5 inches.

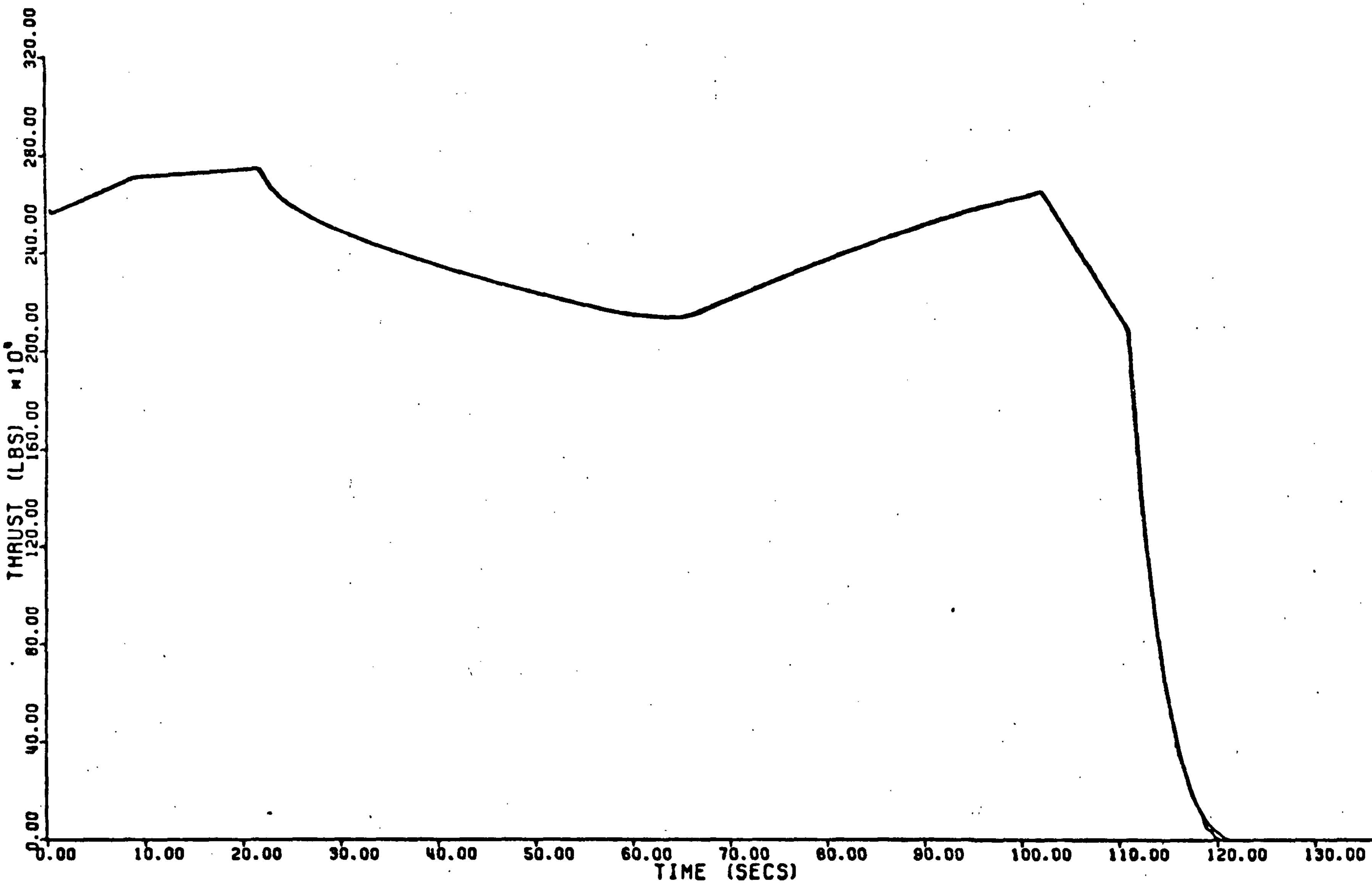

Figure A-16. Thrust versus time for two SRMs with out-of-round conditions of the grain bore at the head end ΔR_{gh} of 0 and 0.5 inches.

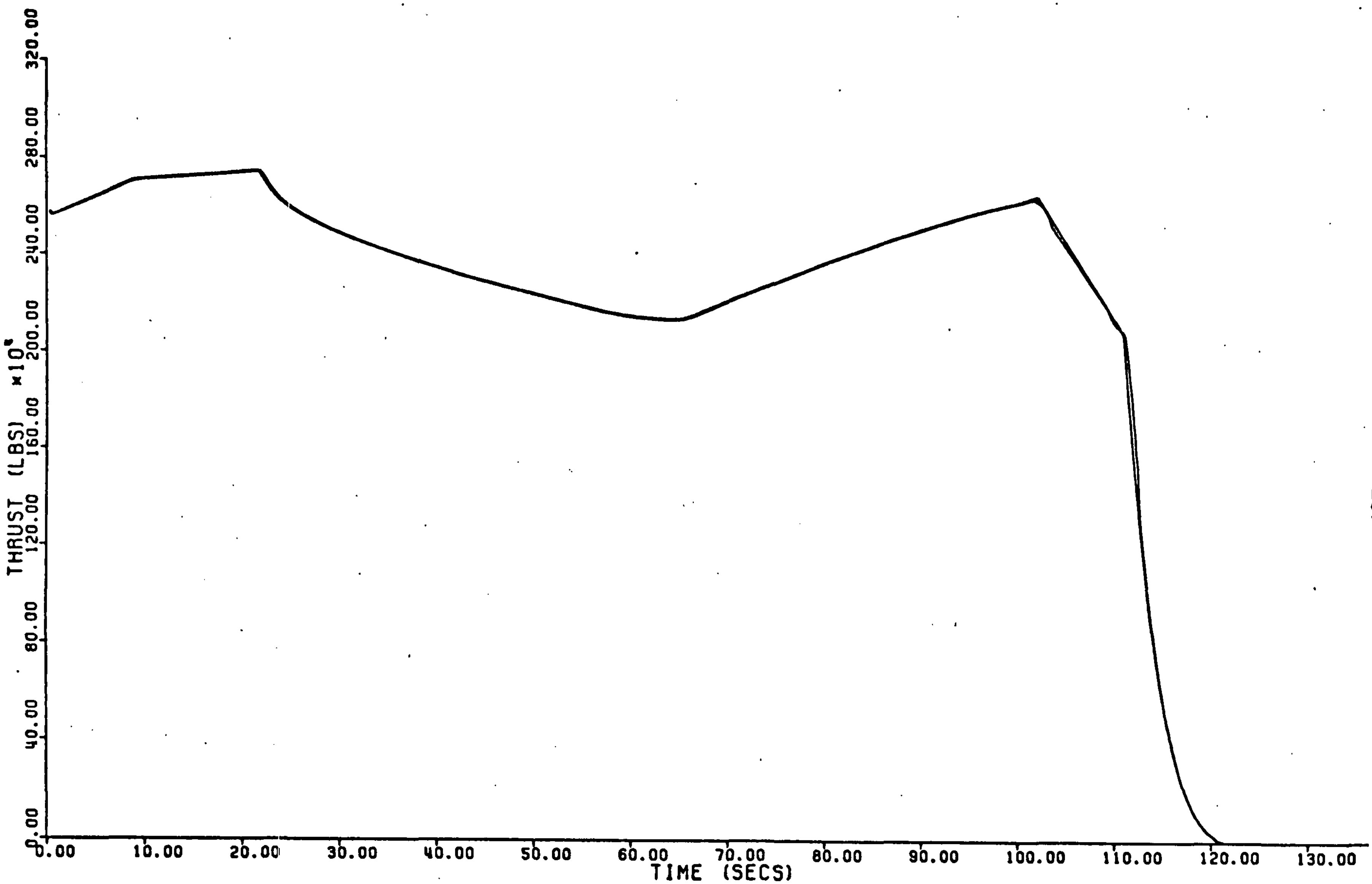

Figure A-17. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the aft end e_{xn} of 0 and 0.5 inches.

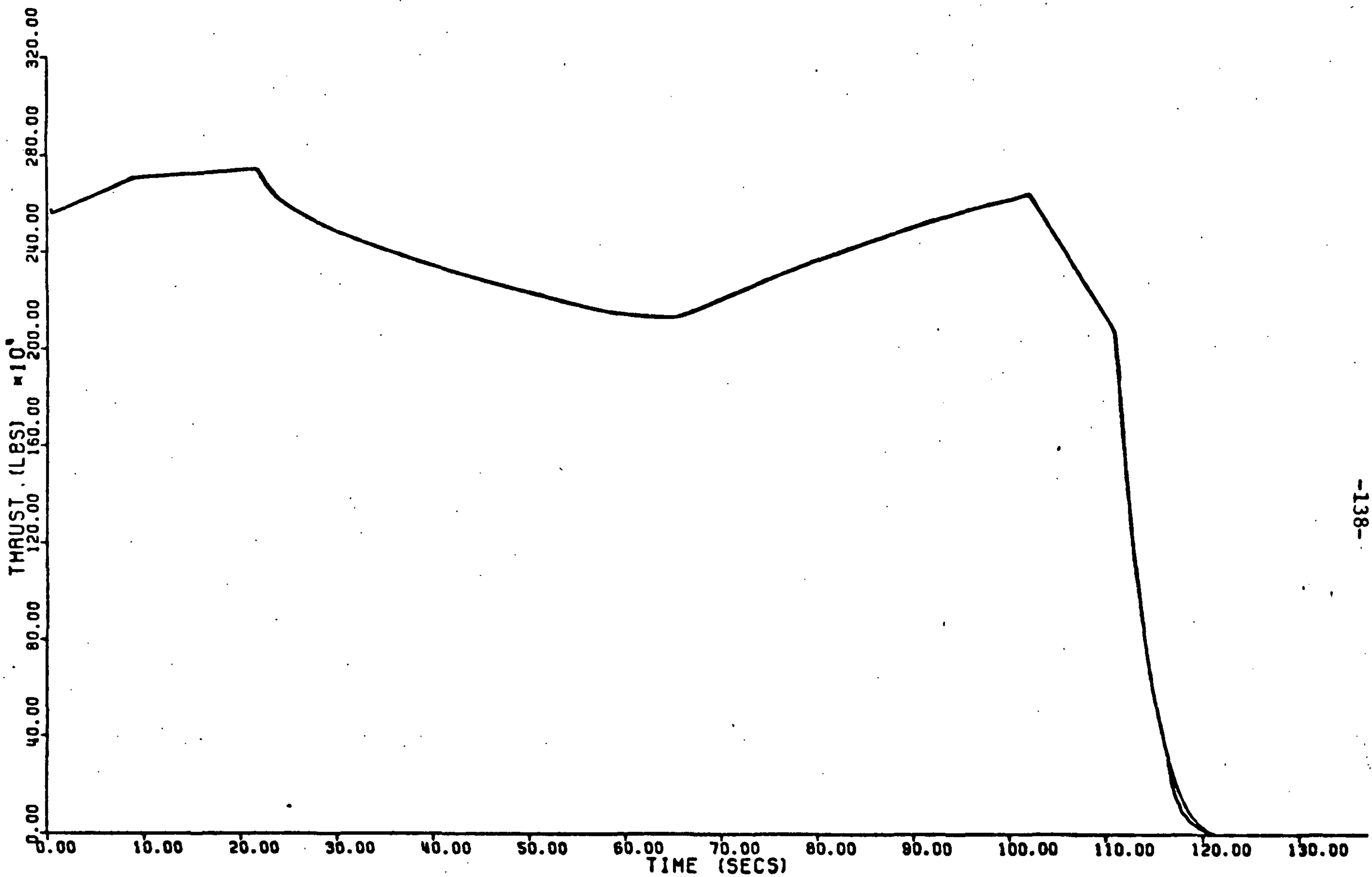

Figure A-18. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end e_{xh} of 0 and -0.5 inches.

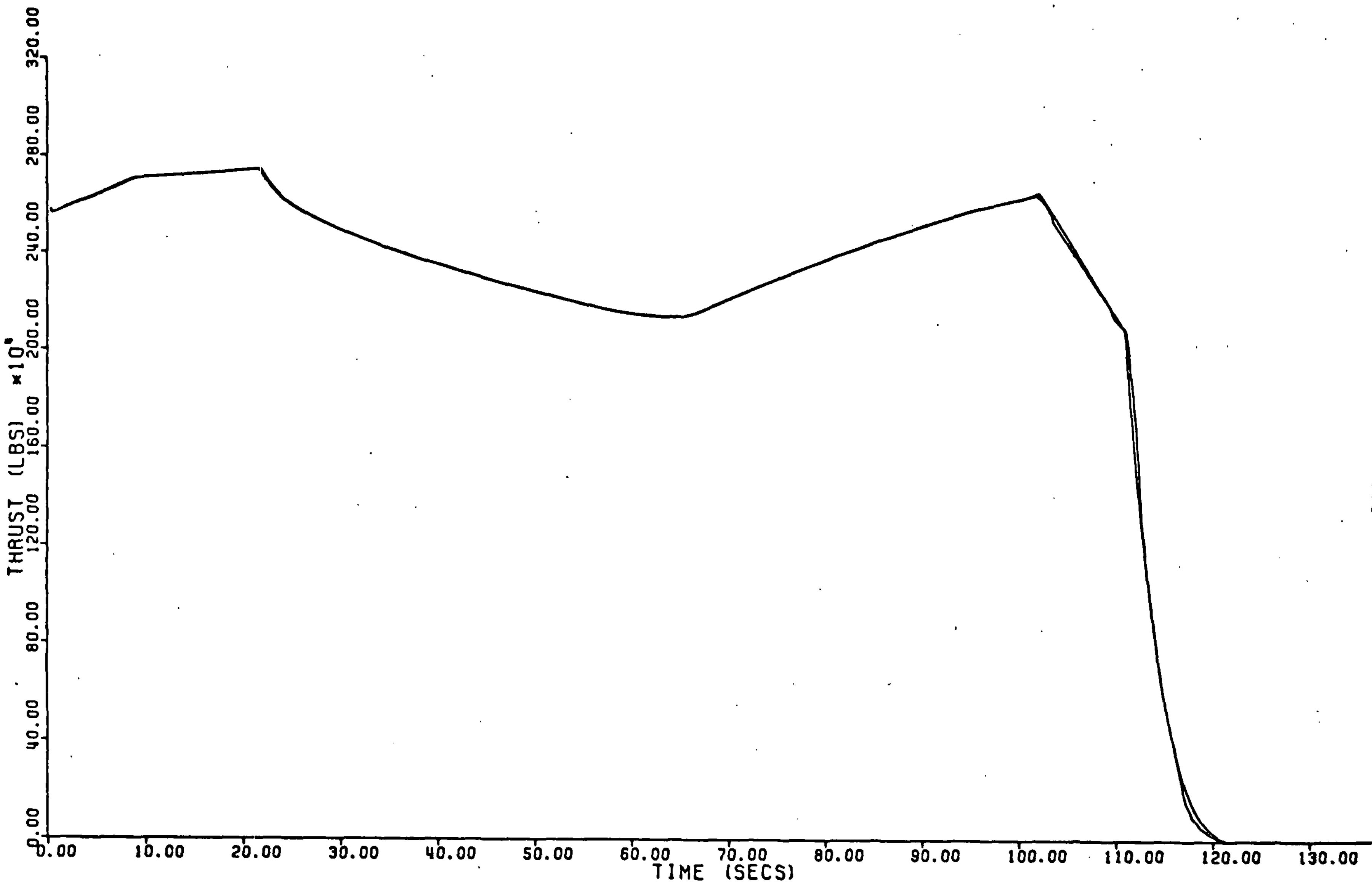

Figure A-19. Thrust versus time for two SRMs with eccentricities of centers of grain exterior and interior at the head end e_{xh} of 0 and -0.5 inches (e_{xn} for Figs. A-18 and A-19 is 0 and + 0.5, respectively).

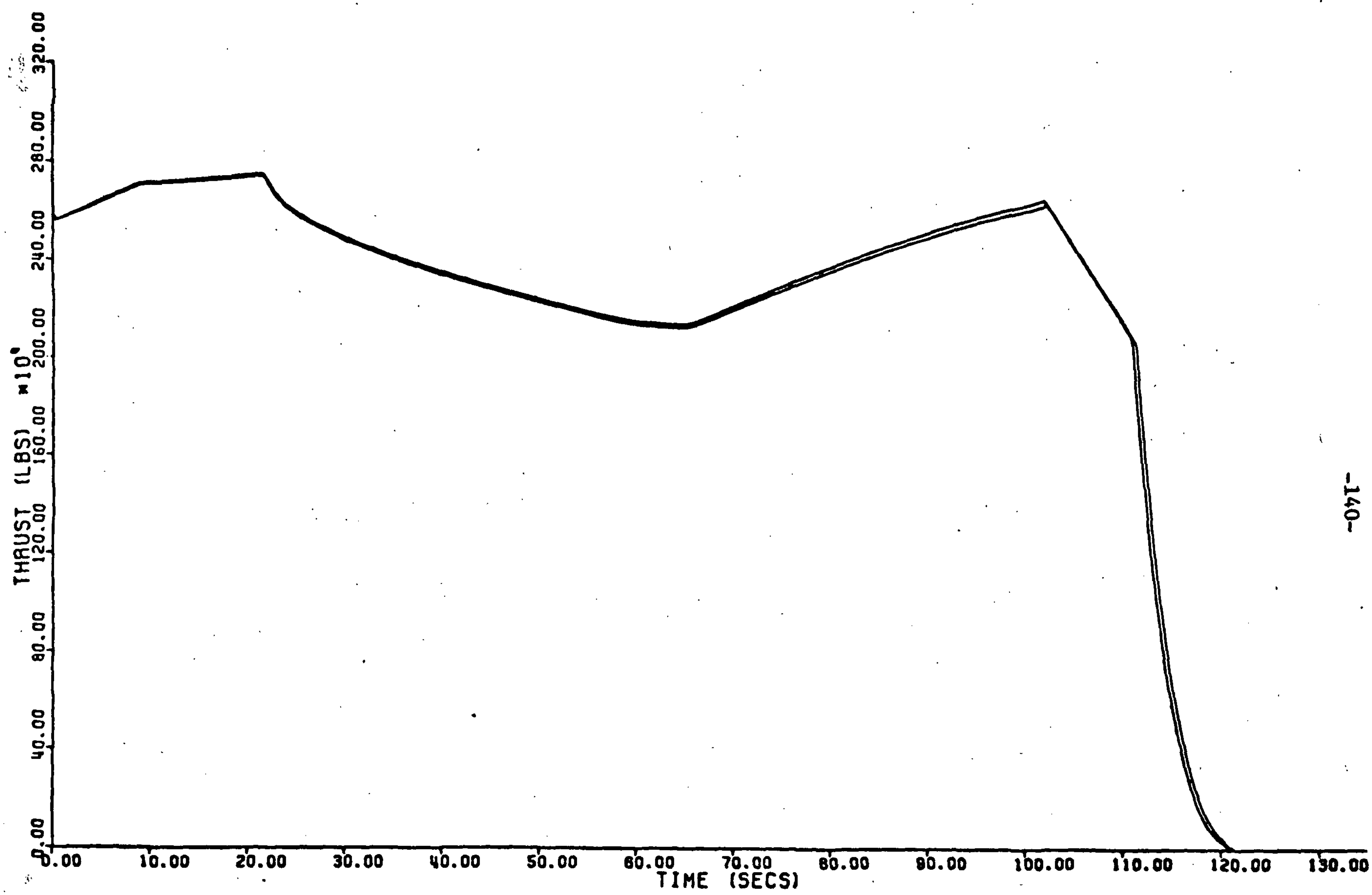

Figure A-20. Thrust versus time for two SRMs with radial nozzle throat erosion rate E_{ref} difference of 20%.

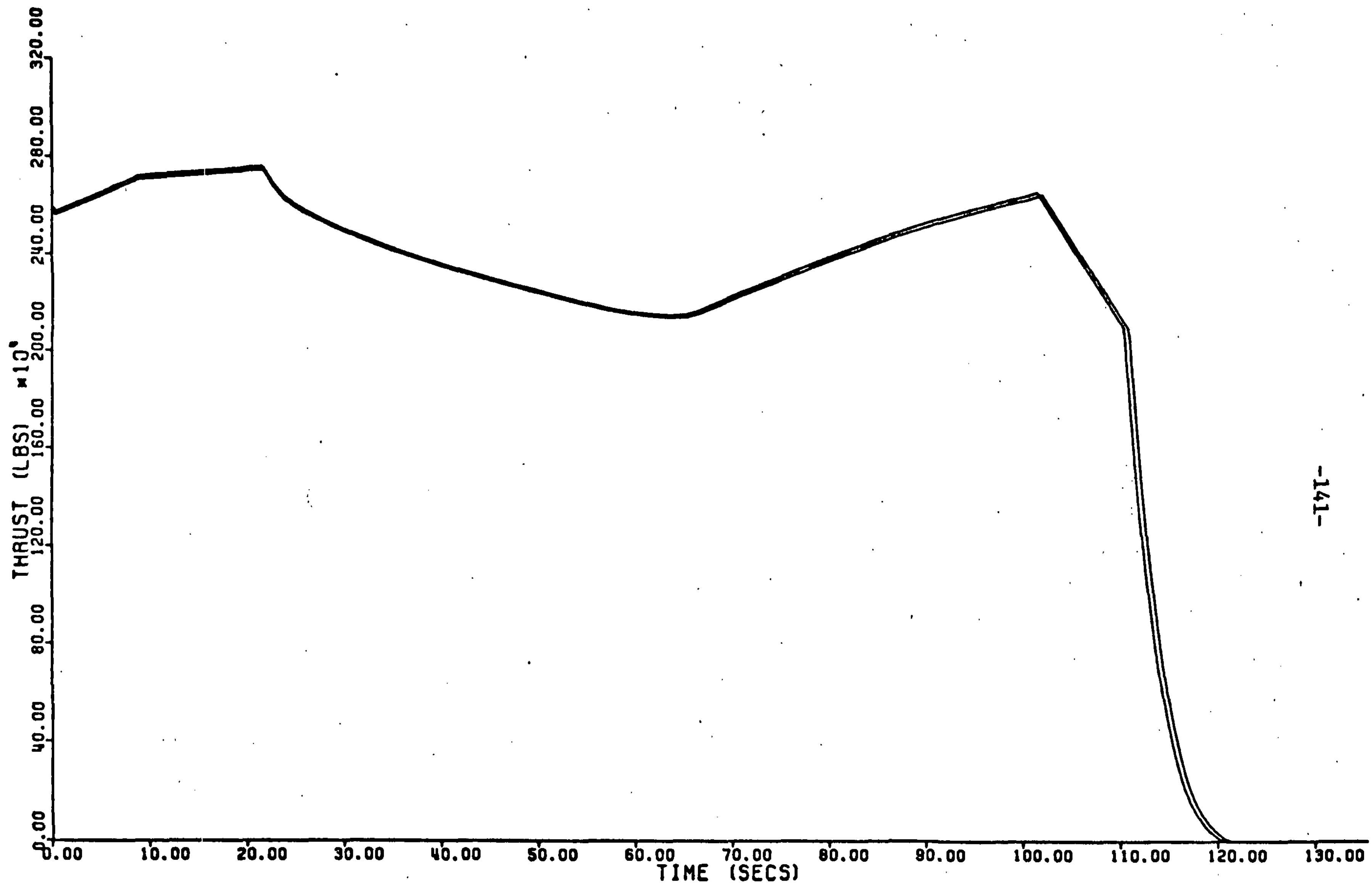

Figure A-21. Thrust versus time for two SRMs with bulk propellant temperature T_{gr} difference of 3°F.

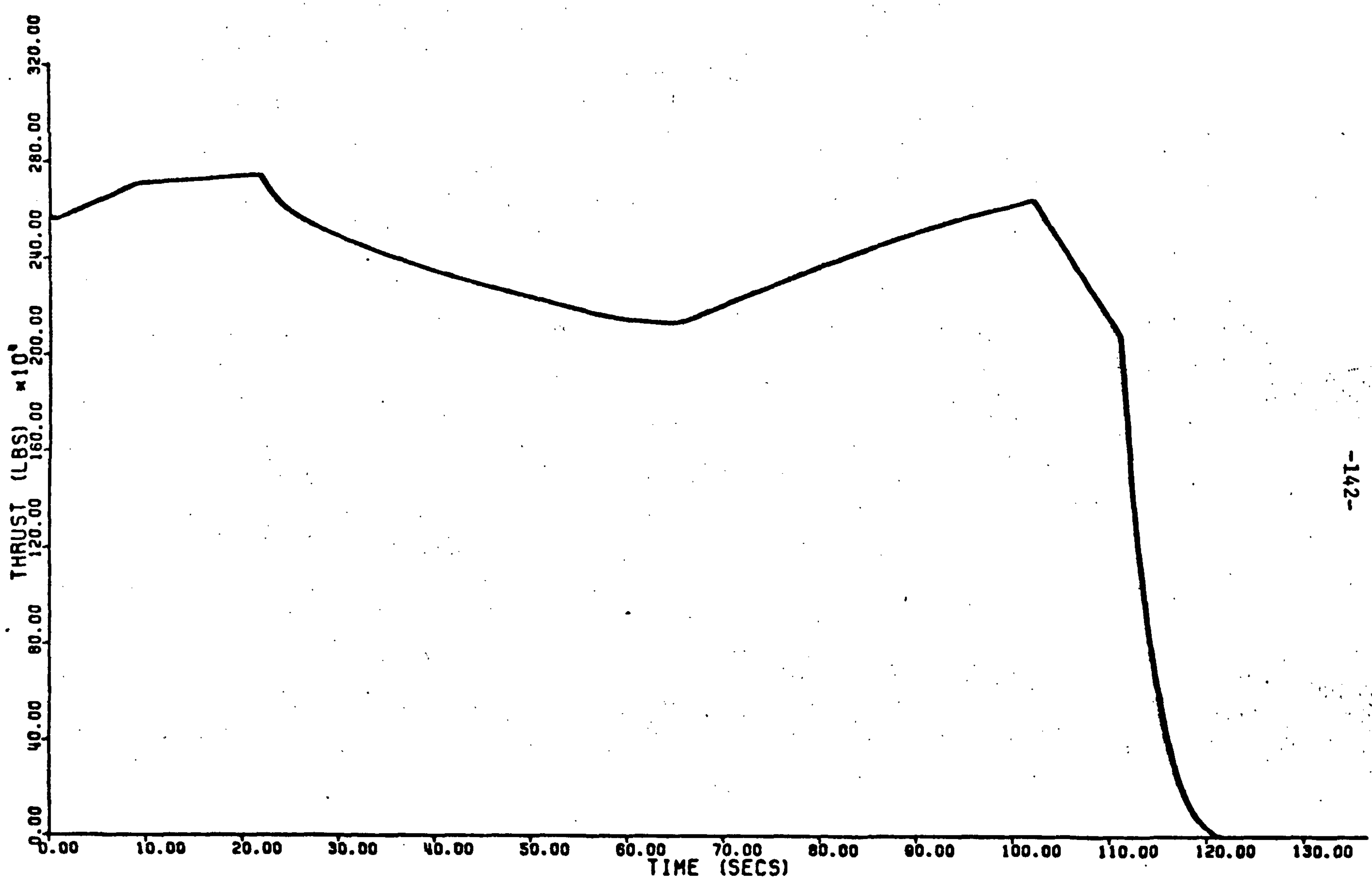

Figure A-22. Thrust versus time for two SRMs with ignition delay T_{igr} difference of 0.4 sec.

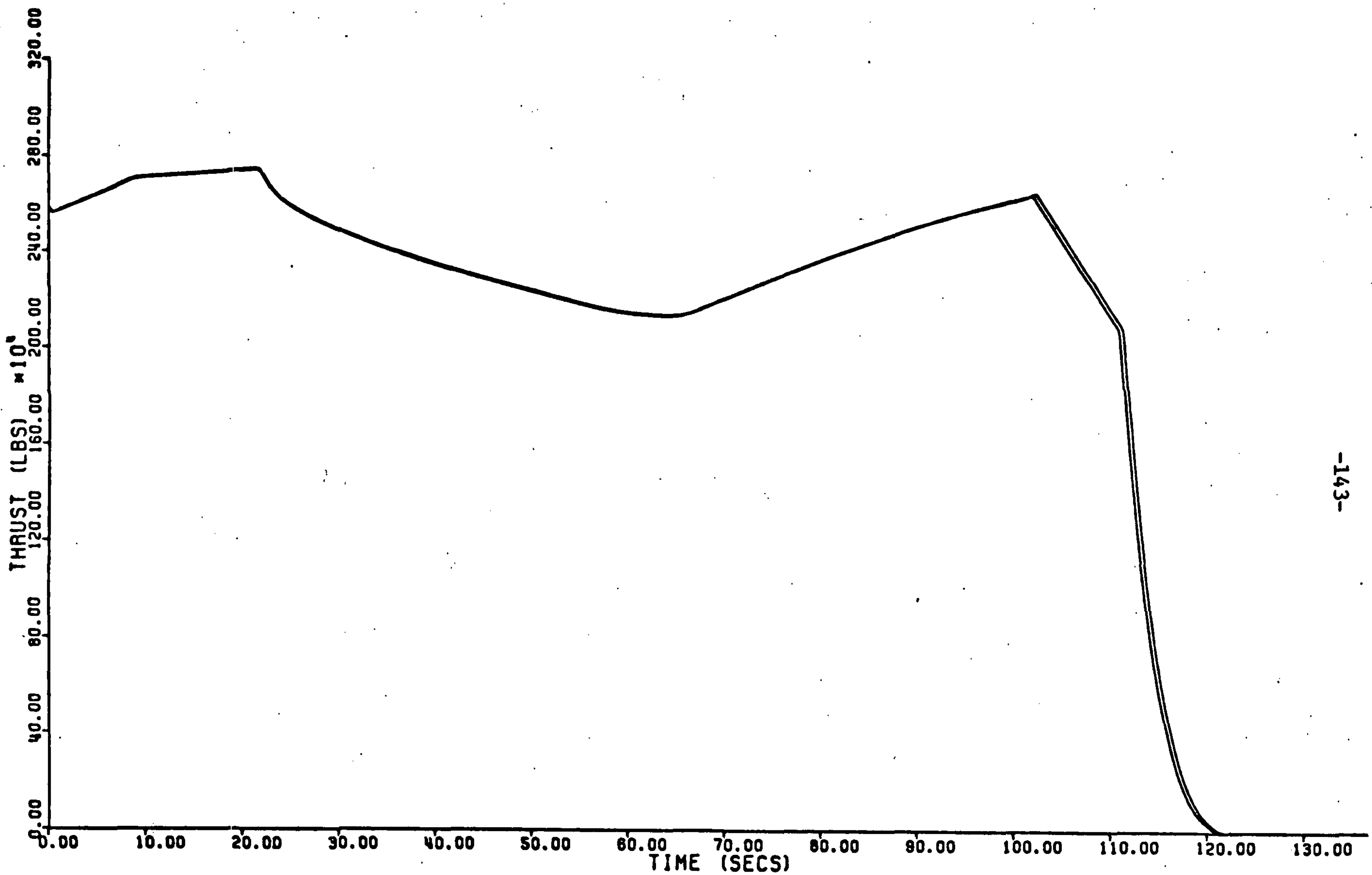

Figure A-23. Thrust versus time for two SRMs with average grain outside diameter D_o difference of 0.3 inches (0.2%).

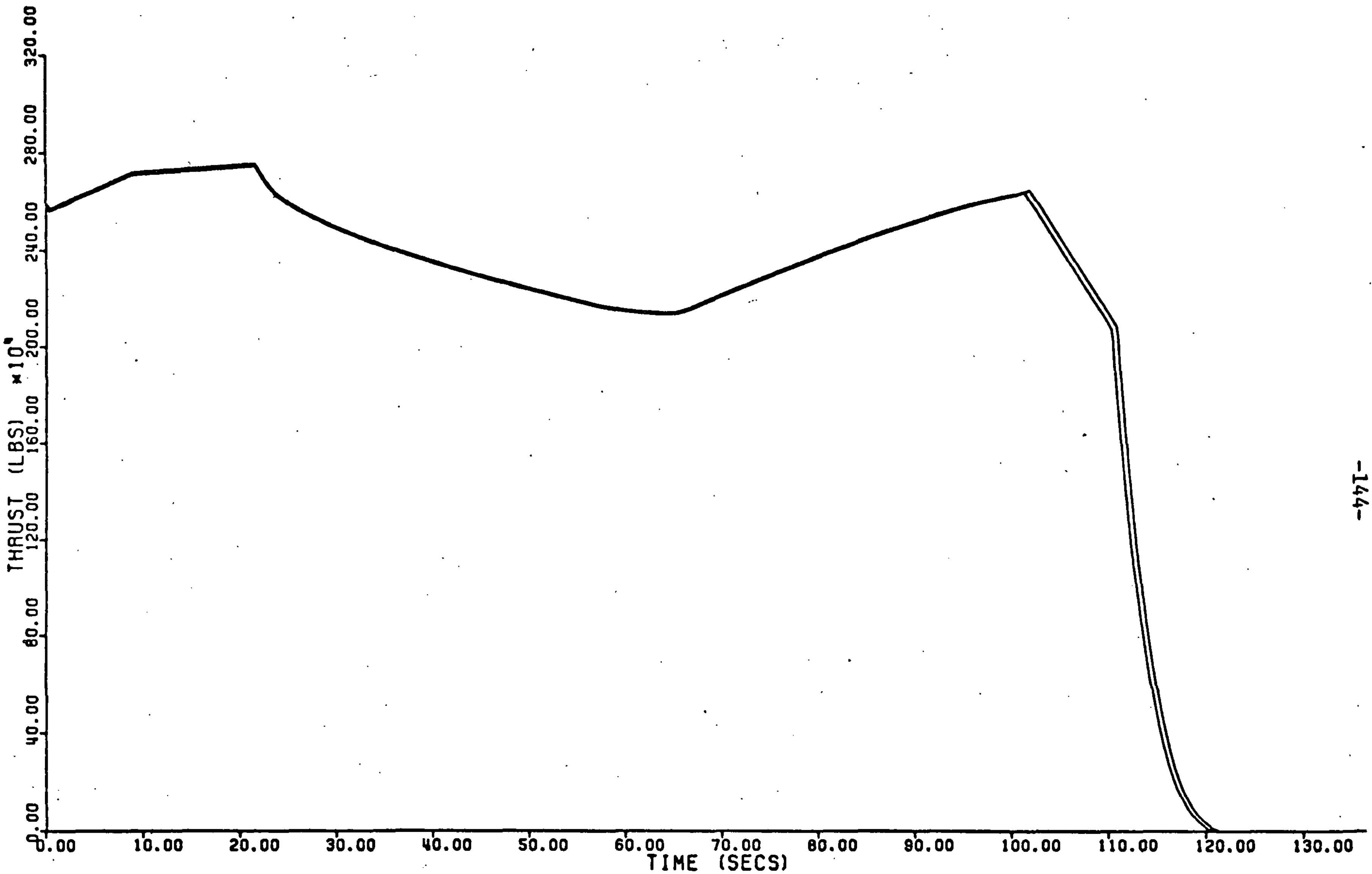

Figure A-24. Thrust versus time for two SRMs with average grain inside diameter D_i difference of 0.3 inches (0.5%).

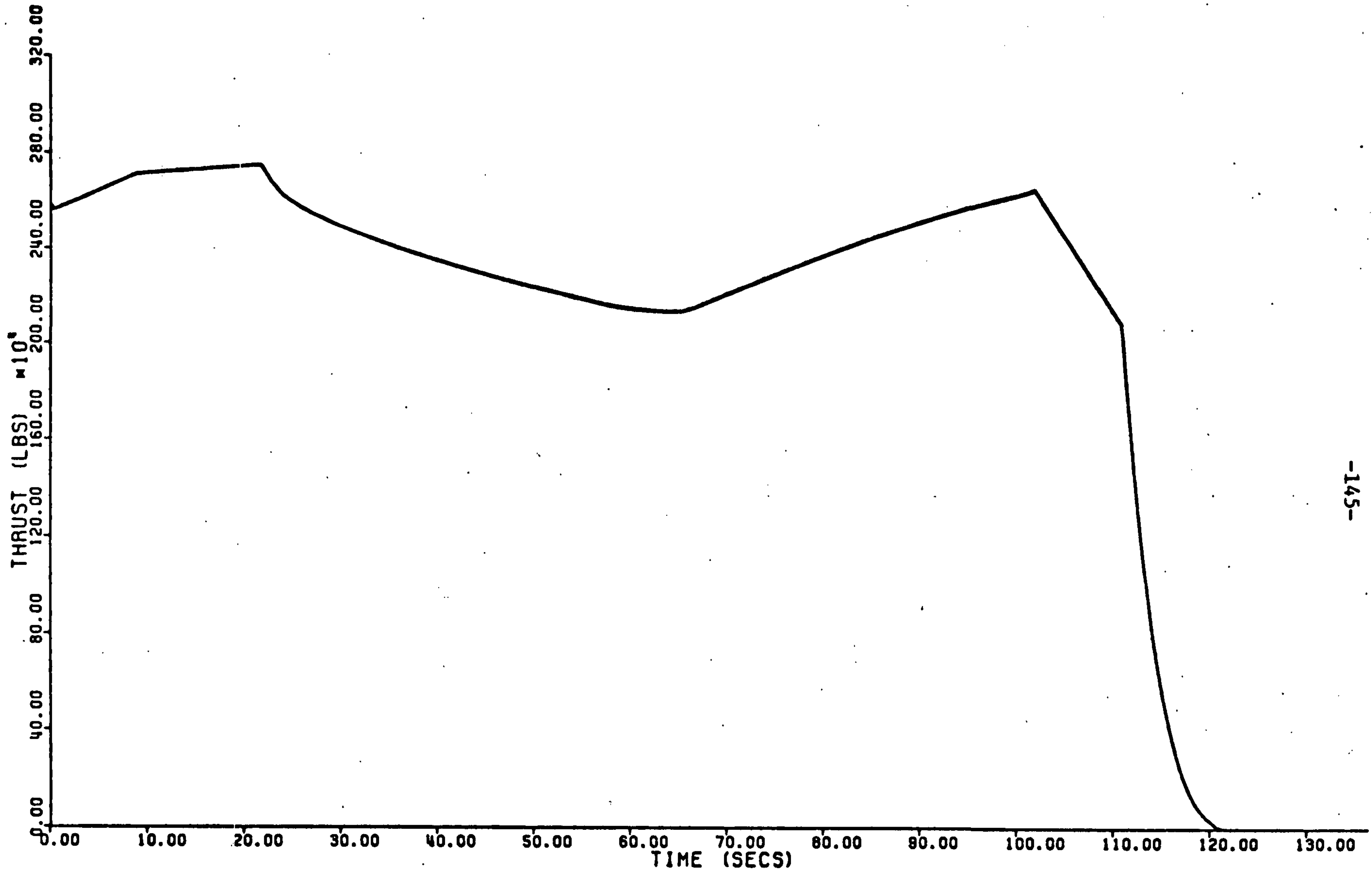

Figure A-25. Thrust versus time for two SRMs with angle of aft burning surface θ_G difference of 3° (28.8%).

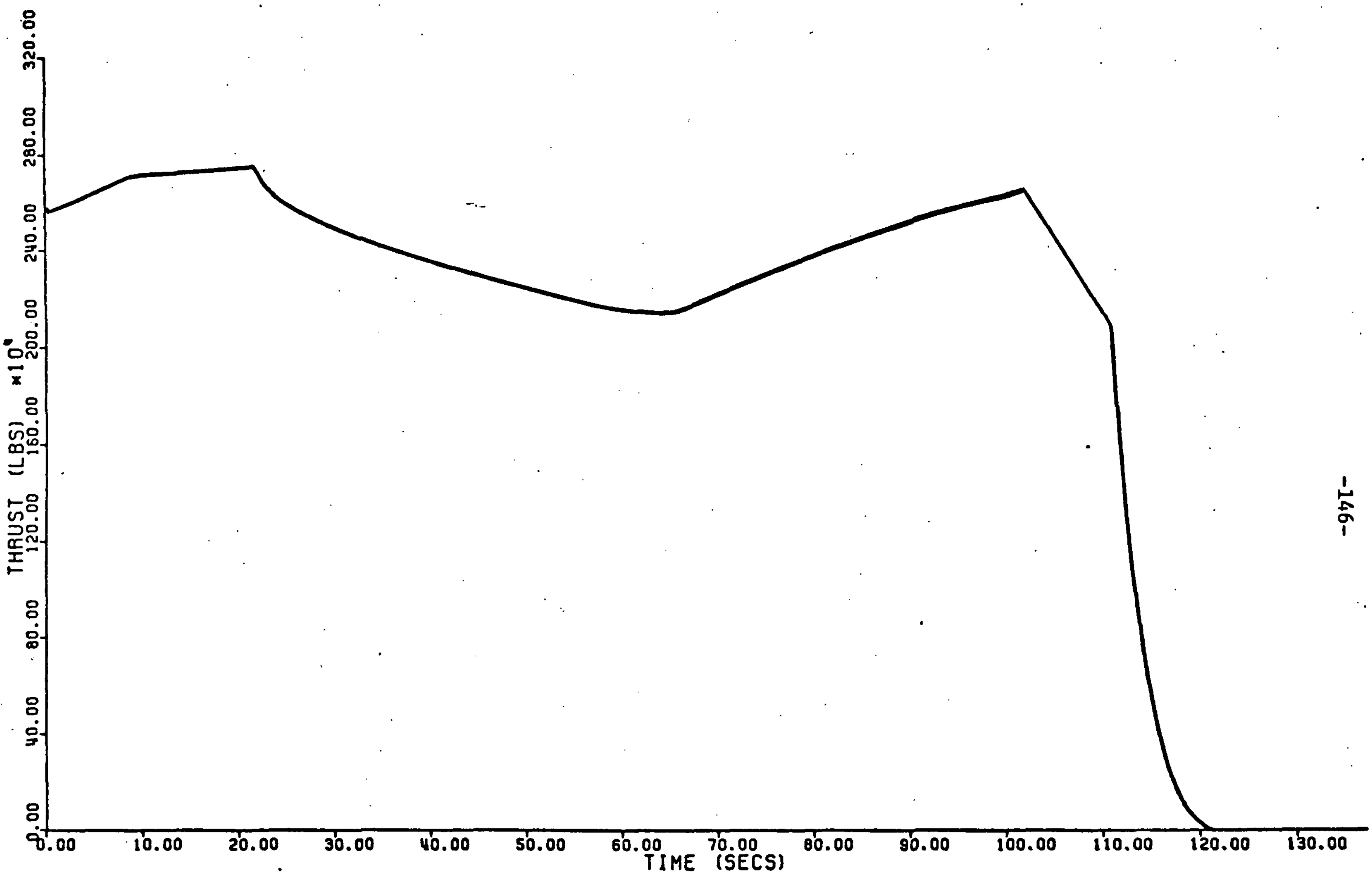

Figure A-26. Thrust versus time for two SRMs with length of circular perforated grain L_{Gci} difference of 3.0 inches (0.3%).

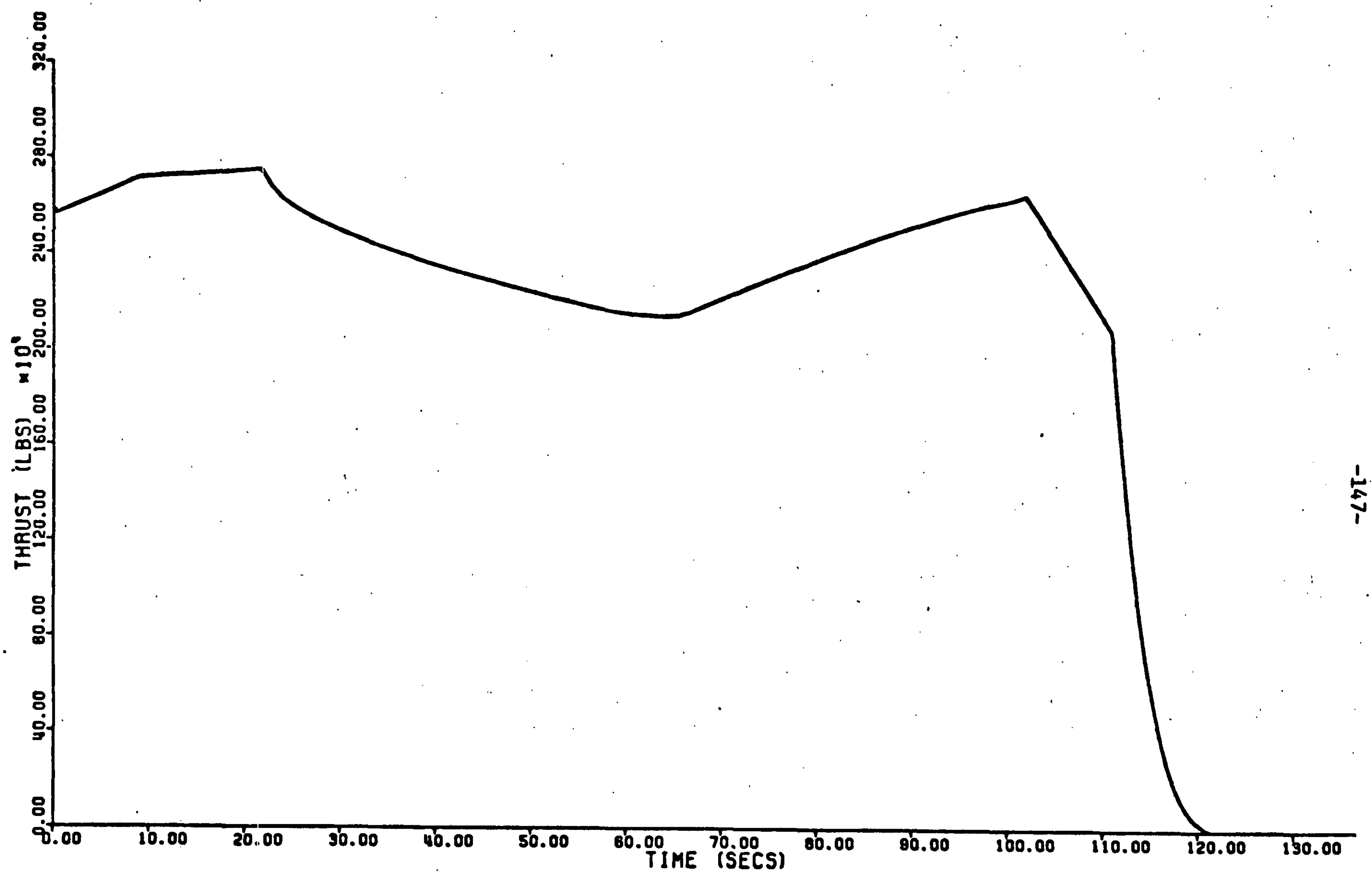

Figure A-27. Thrust versus time for two SRMs with length of grain L_{Gni} associated with θ_G difference of 3%.

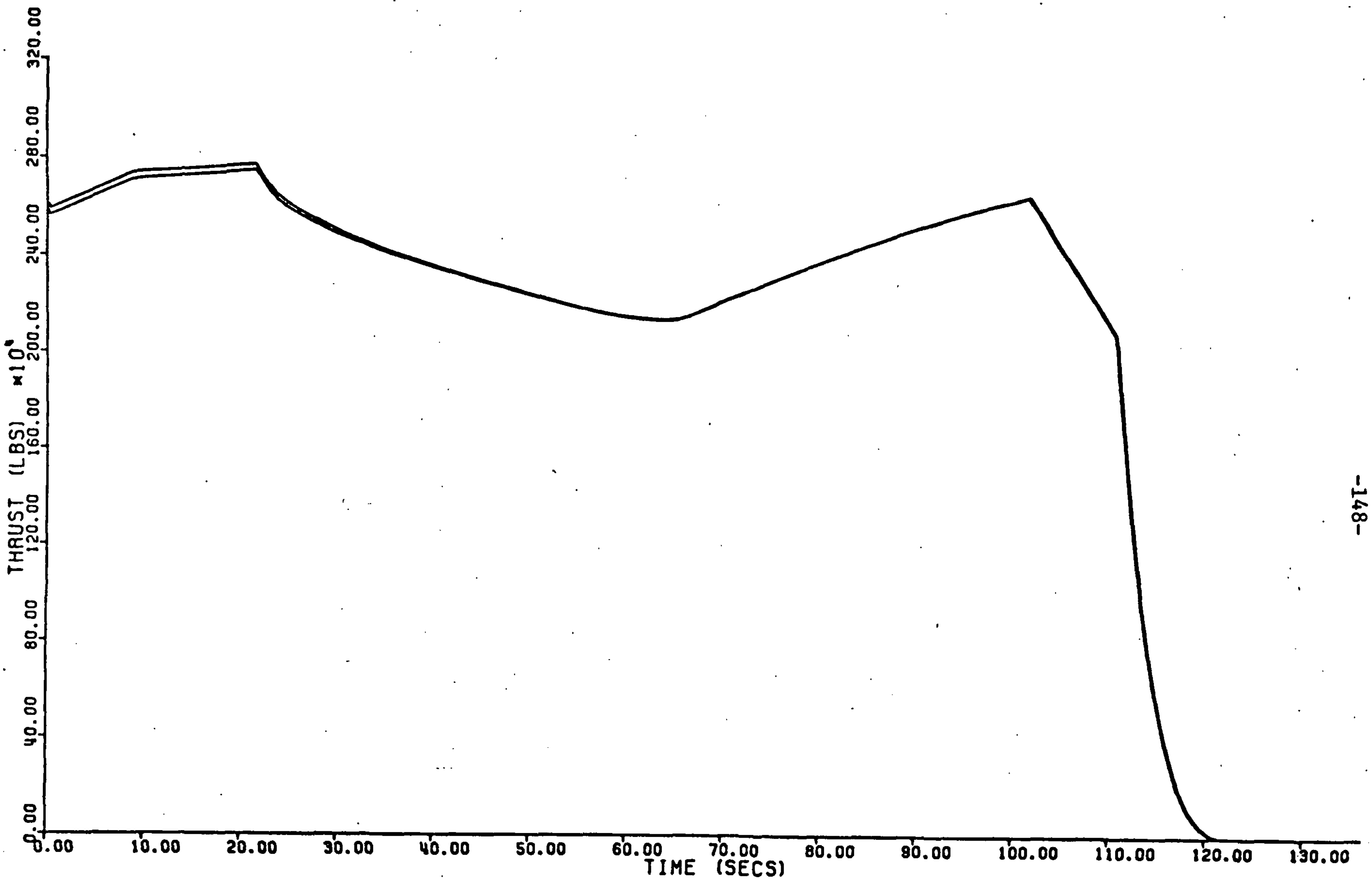

Figure A-28. Thrust versus time for two SRMs with length of star grain L_{Gsi} difference of 3.0 inches (1.6%).

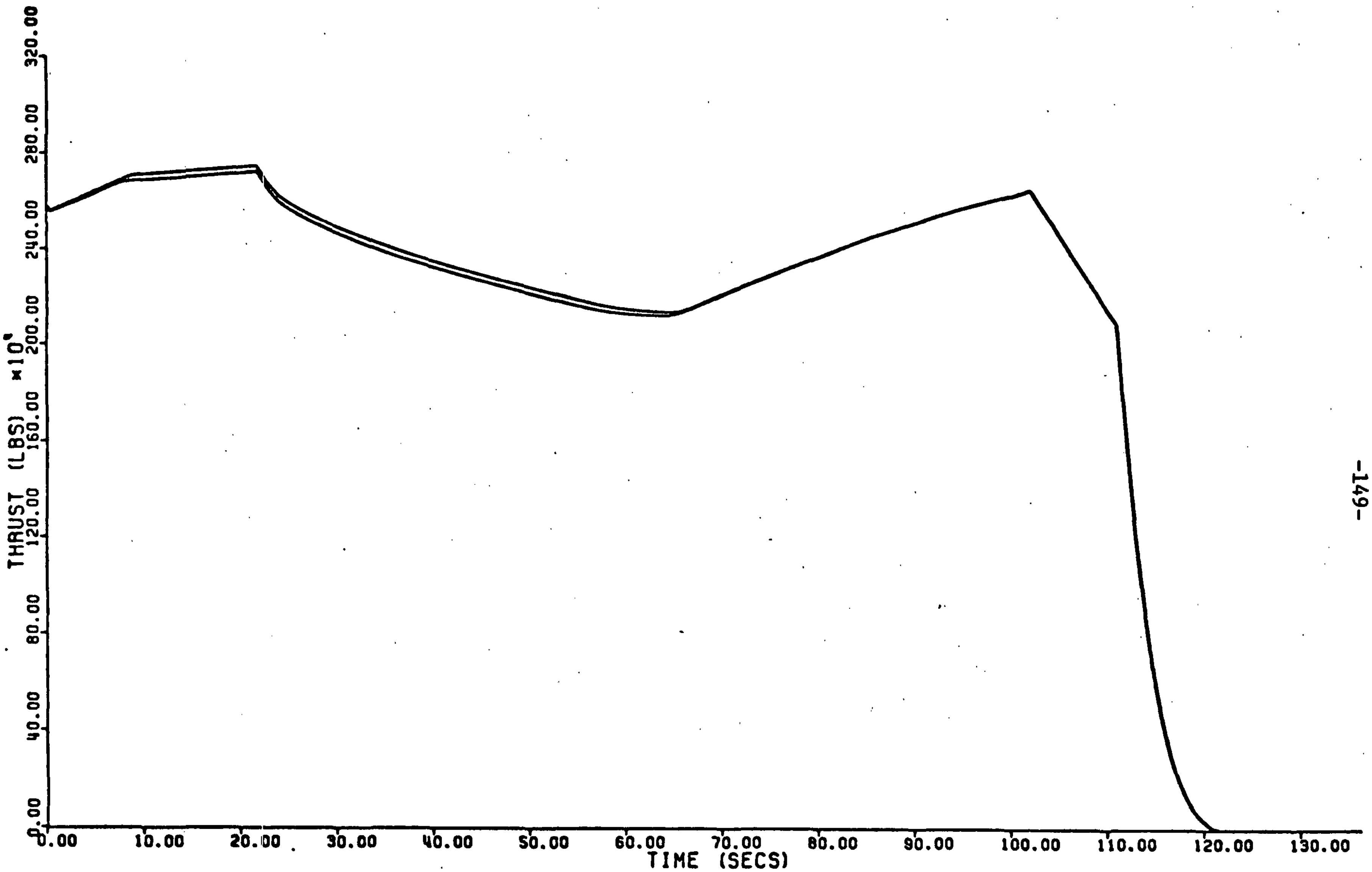

Figure A-29. Thrust versus time for two SRMs with radius of star grain fillet difference f of 0.3 inches (2.4%).

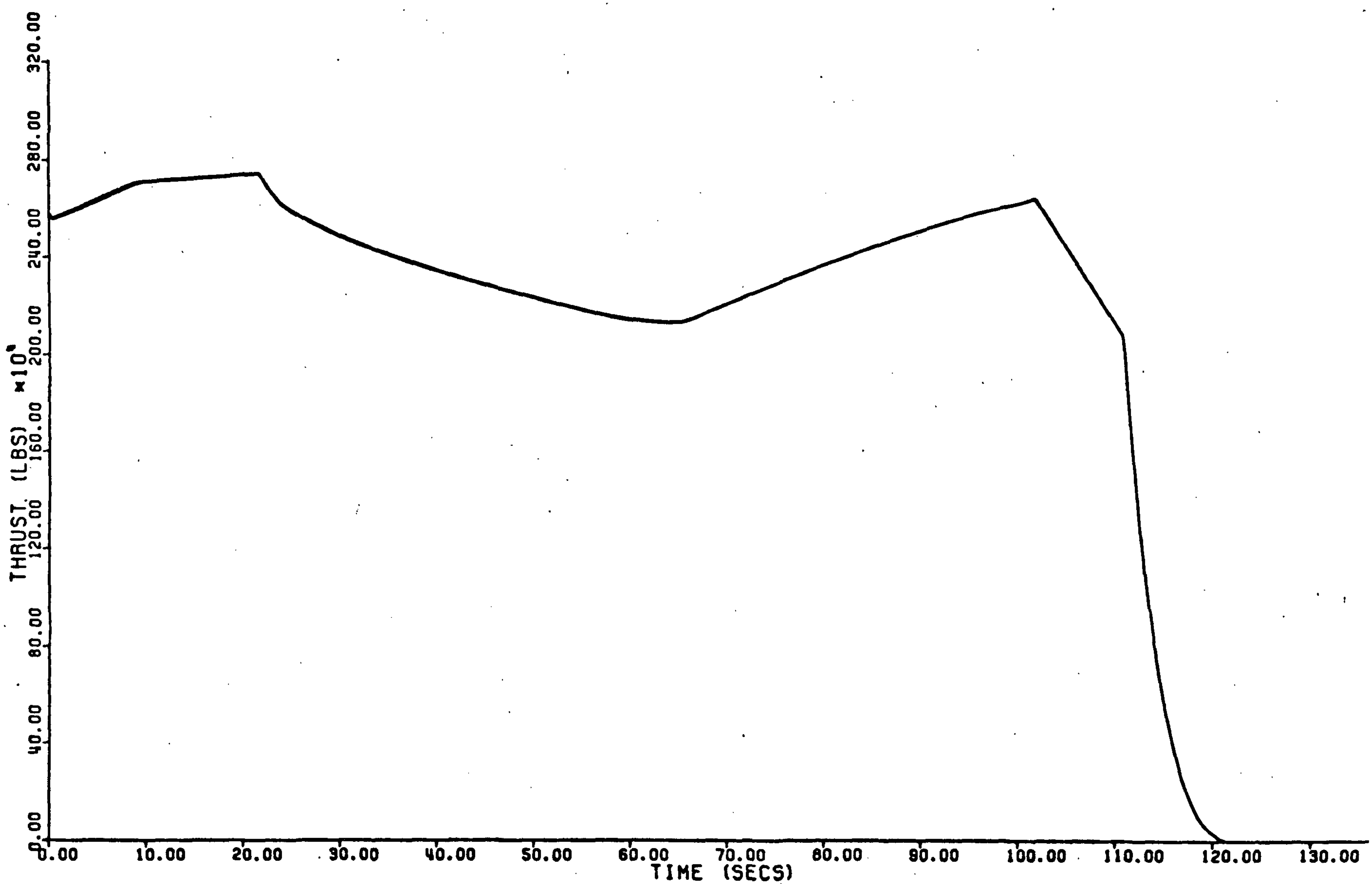

Figure A-30. Thrust versus time for two SRMs with average initial radius of star grain truncation R_p difference of 0.3 inches (2.4%).

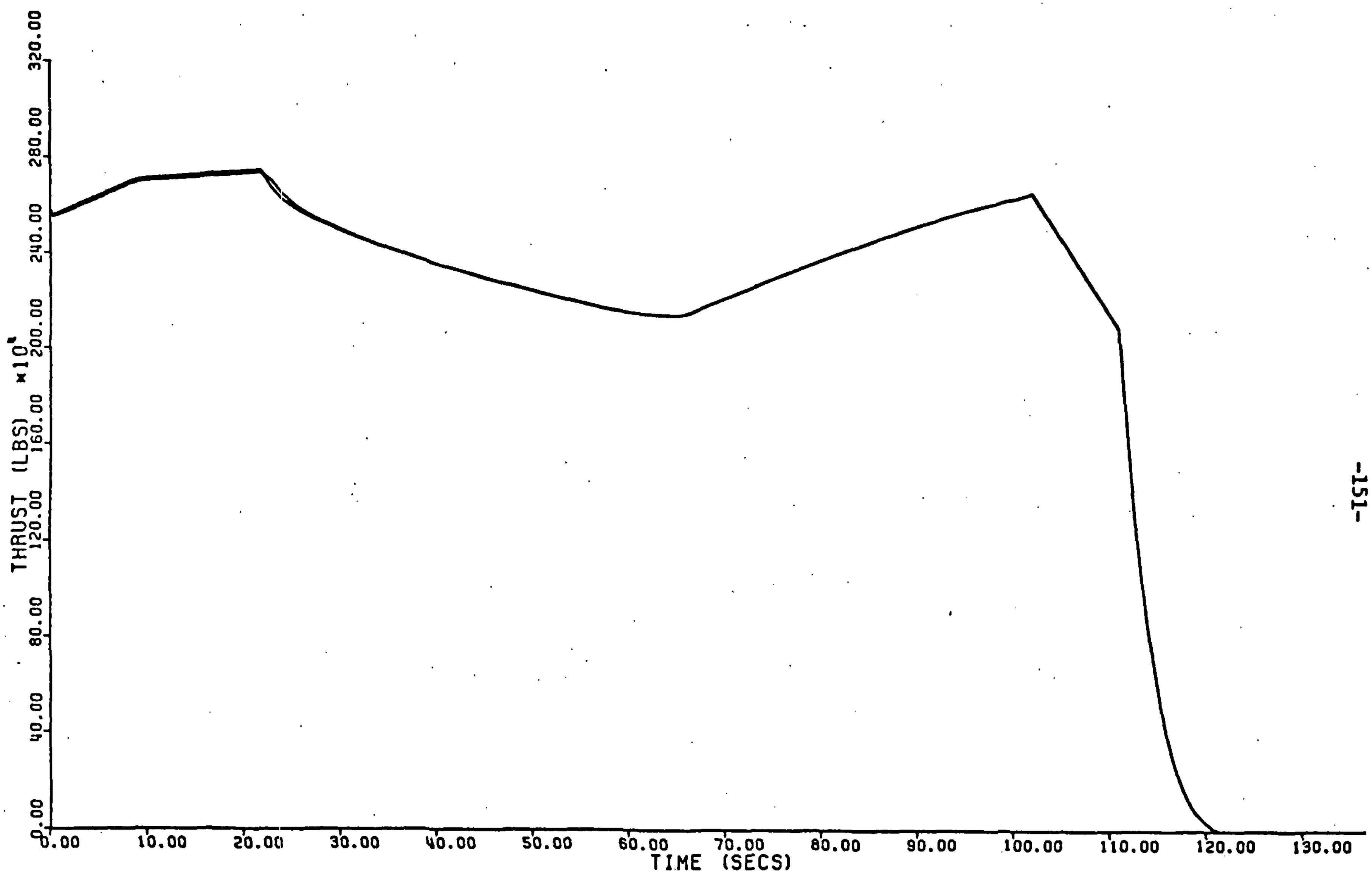

Figure A-31. Thrust versus time for two SRMs with average web thickness of truncated star grain τ_s of 0.3 inches (3.7%).

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