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STEADY-STATE AND TRANSIENT ANALYSIS OF A SQUEEZE FILM DAMPER BEARING FOR ROTOR STABILITY

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16. Abstract

This report presents a study of the steady-state and transient response of the squeeze film damper bearing. Both the steady-state and transient equations for the hydrodynamic bearing forces are derived. The steady-state equations are used to determine the bearing equivalent stiffness and damping coefficients. These coefficients are used to find the bearing configuration which will provide the optimum support characteristics based on a stability analysis of the rotor-bearing system. The effects of end seals and cavitated fluid film are included. The transient analysis of rotor-bearing systems is performed by coupling the bearing and journal equations and integrating forward in time. The effects of unbalance, cavitation, and retainer springs are included in the analysis. Methods of determining the stability of a rotor-bearing system under the influence of aerodynamic forces and internal shaft friction are discussed. Particular emphasis is placed on solving the system characteristic frequency equation, and stability maps produced by using this method are presented. The study shows that for optimum stability and low force transmissability the squeeze bearing should operate at an eccentricity ratio $\epsilon < 0.4$.

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NOMENCLATURE

SYMBOL	DESCRIPTION	UNITS
C	Bearing Clearance	in.
C_o	Equivalent bearing damping	lb-sec/in
C_{xx}, C_{xy}, C_{yy}, C_{yx}	Bearing damping	lb-sec/in
e	Journal Eccentricity	in.
EMU	Ratio of unbalance eccentricity to bearing clearance	---
Fx	Force component in x-direction	lbf
Fy	Force component in y-direction	lbf
F_r	Force component in radial direction	lbf
F_θ	Force component in tangential direction	lbf
FMAX	Maximum hydrodynamic force	lbf
FU	Force due to rotating unbalance	lbf
FURATIO	Ratio of FMAX to FU	---
h	Fluid film thickness	in.
$\vec{i}$, $\vec{j}$, $\vec{k}$	Unit vectors in fixed coordinate system	---
k_o	Equivalent bearing stiffness	lb/in
KR	Retainer Spring Stiffness	lb/in
k_{xx}, k_{xy}, k_{yy}, k_{yx}	Bearing Stiffness	lb/in
L	Bearing length	in.
N	Rotor speed	RPM
$\vec{n}_r$, $\vec{n}_\theta$	Unit vecotors in rotating coordinate system	---

SYMBOL	DESCRIPTION	UNITS
P	Pressure	lb/in^2
PMAX	Maximum hydrodynamic pressure	lb/in^2
Q	Aerodynamic cross coupling	lb/in
R	Bearing radius	in
t	Time	sec
W	Weight	lbf
x,y,z	Displacements	in.
μ	Viscosity	$lb\text{-}sec/in^2$
θ,θ'	Angular measure	---
$\dot{\phi}$	Journal precession rate	sec^{-1}
ω	Angular velocity	sec^{-1}

CHAPTER 1

INTRODUCTION

Modern turbomachines are highly complex systems. Current design trends are producing machines that consist of several process stages joined together. The rotors in these machines are highly flexible shafts, often mounted in more than two bearings, that rotate at very high speeds. It is not uncommon to see machines that operate above the second critical speed. As a result the system dynamics are very complicated.

One of the major problems encountered in these machines is instability produced by aerodynamic forces on impeller wheels, friction in the stressed rotor and hydrodynamic forces in the bearings. The instability is characterized by large amplitude whirl orbits and often results in bearing or total machine failure. It is often aggravated by unbalance and other external forces transmitted to the machine. Production losses from failed machines are very high and it may take many months to repair or replace the failed unit. In addition operator safety is jeopardized when machines fail and occasional loss of life occurs.

From the earliest investigations of rotor instability, it has been known that the use of flexible, damped supports has an effect on instability and can eliminate it or alter the speed at which it occurs. Recent research has produced a large body of knowledge on the use of these supports and their effect on instability.

The squeeze film damper bearing is one type of flexible support that is currently being investigated. This study examines the squeeze bearing and through computer simulation shows its effects on several rotor-bearing systems. The equations for the hydrodynamic bearing forces are developed in both fixed and rotating coordinate systems. The use of two coordinate systems allows for both steady-state and transient analysis of bearing performance. This results in more efficient bearing analysis and a savings in time and money when experimental testing of the bearings is conducted.

The steady-state behavior of the bearing results in the formulation of bearing stiffness and damping coefficients which can be used to set the bearing configuration. This is accomplished by comparing the coefficients with required values obtained from a stability analysis of the rotor-bearing system. Several methods of determining the system stability are discussed. The effects of end seals and cavitation of the fluid film are also included in the steady-state coefficients.

The transient analysis is very useful in determining the bearing response to particular forms of external and internal forces as noted previously. Also the effect of bearing retainer springs and fluid film cavitation can be found. The transient response is found by tracking the journal motion forward in time by integrating the equations of motion under the influence of the system forces.

The limitations of and assumptions used in deriving the steady-state and transient equations are discussed in order to obtain meaningful interpretation of the results and to establish useful design criteria.

Dr. R. Gordon Kirk developed the computer programs used to perform the transient analysis in this report.

CHAPTER 2

THEORETICAL ANALYSIS

2.1 REYNOLDS EQUATION

The configuration of the squeeze film damper bearing is shown in Figure (2-1) where the clearance has been exaggerated. Both fixed and rotating coordinate systems are shown, and the bearing equations are derived for both systems. The definitions of the various parameters are listed in the nomenclature section of this report.

The basic bearing equation is the Reynolds equation which is derived from the Navier-Stokes equations for incompressible flow. With the proper bearing parameters the equation for the fluid film forces are derived. [1]

The Reynolds equation for the short, plain journal bearing is given in both fixed and rotating coordinates by:

Fixed coordinates:

$$\frac{\partial}{\partial Z}\left[\frac{h^3}{6\mu}\frac{\partial P}{\partial Z}\right] = (\omega_b + \omega_j)\frac{\partial h}{\partial \theta} + 2\frac{\partial h}{\partial t} \tag{2-1}$$

Rotating coordinates:

$$\frac{\partial}{\partial Z}\left[\frac{h^3}{6\mu}\frac{\partial P}{\partial Z}\right] = (\omega_b + \omega_j - 2\dot{\phi})\frac{\partial h}{\partial \theta'} + 2\frac{\partial h}{\partial t} \tag{2-2}$$

As shown in Figure (2-1), the angle θ in the fixed coordinate expression is measured from the positive x-axis in the direction of rotation whereas the angle θ' in the rotating coordinate expression is measured from the line of centers in the

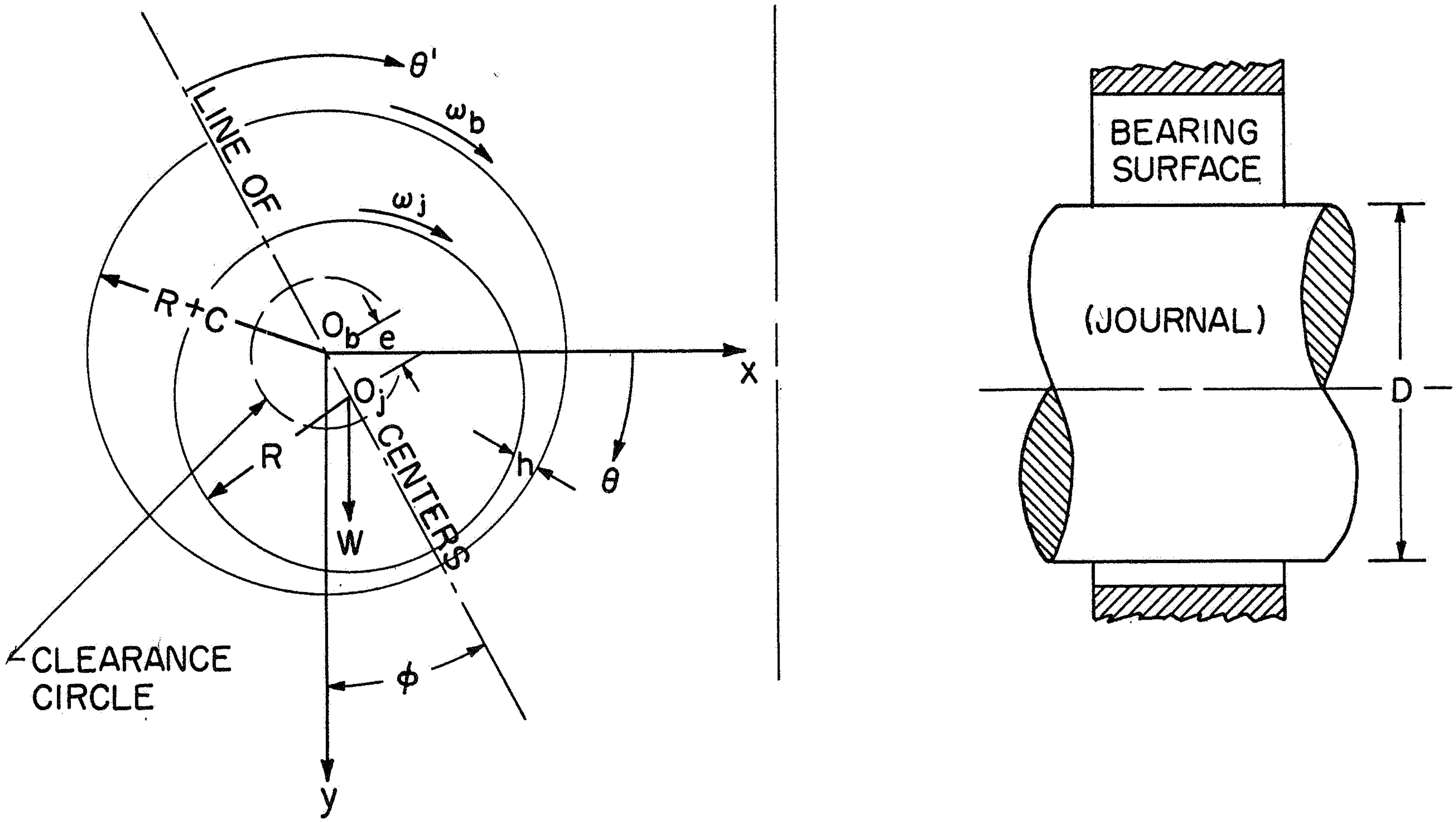

Figure 2-1 Squeeze Film Damper Bearing Configuration in Fixed and Rotating Coordinate Systems

direction of rotation. The assumptions used in the derivation of equations (1) and (2) include:

1. The fluid inertia terms in the Navier-Stokes equations have been neglected due to their small magnitude.
2. Body forces in the fluid film have been neglected.
3. The fluid viscosity is constant.
4. The flow in the radial direction has been neglected, that is, the short bearing approximation has been used.

Figure (2.2) shows a comparison of the short bearing solution and the general solution of the Reynolds equation solved by a finite difference technique for the plain journal bearing under steady state conditions. It can be seen that the short bearing solution is highly accurate for a wide range of eccentricities for $L/D < 1/4$ and is acceptable for L/D values up to 1 if the eccentricy ratio is low. The normal design range of the squeeze film bearings will be $L/D < 1/2$ and eccentricity ratios < 0.4.

By assuming the bearing is perfectly aligned (h not a function of Z) equations (1) and (2) are integrated directly to yield expressions for the fluid film forces.

2.2 BEARING FORCES IN FIXED COORDINATES

For the plain bearing with full end leakage the appropriate boundary conditions are:

$$P(\theta,0) = P(\theta,L) = 0 \tag{2-3}$$

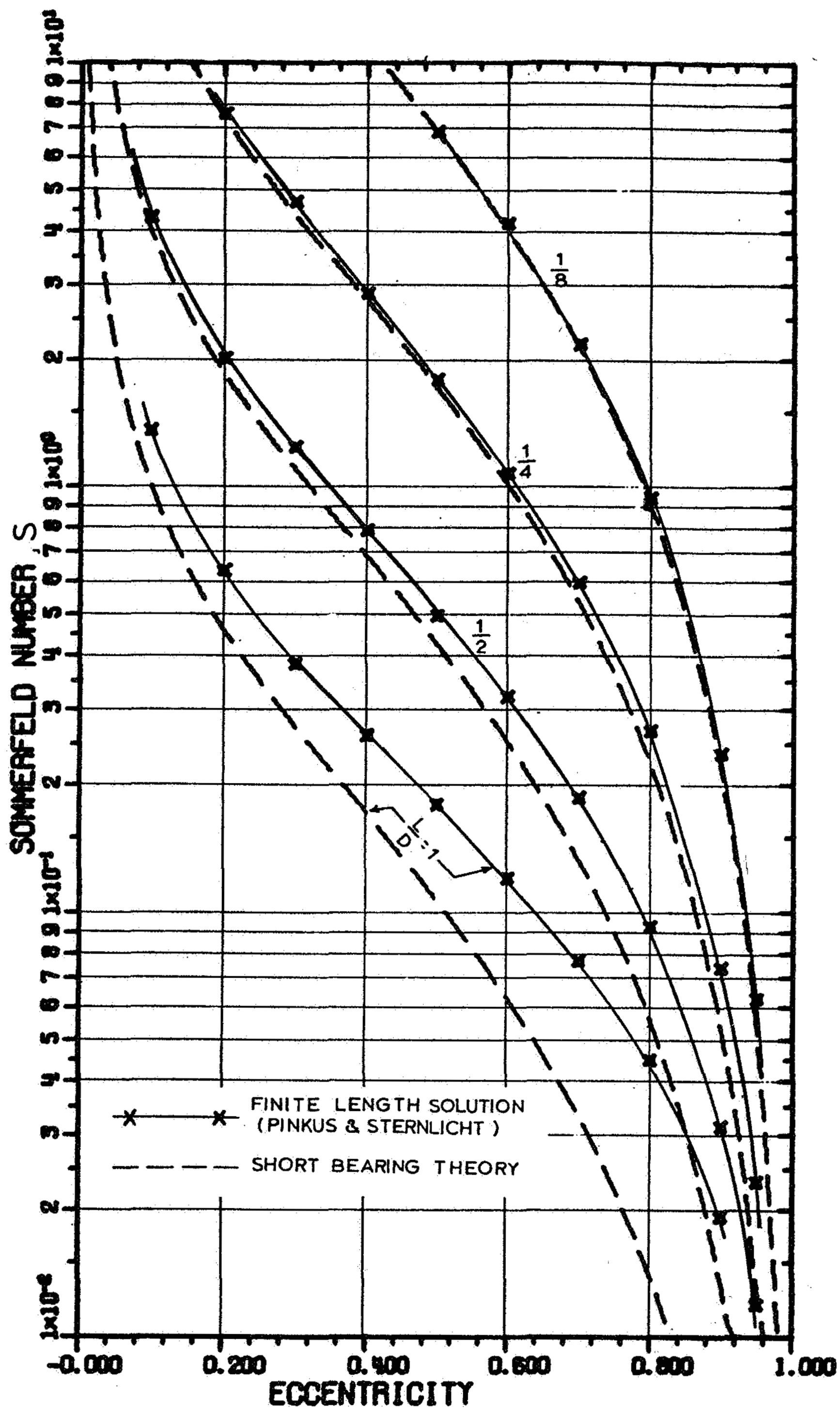

Figure 2-2 Comparison of Finite Length and Short Bearing Solutions

In the fixed coordinate system the film thickness, h, is given by:

$$h = c - x \cos\theta - y \sin\theta \tag{2-4}$$

Substituting into equation (2-1) and integrating yields:

$$P(\theta,Z) = \frac{3\mu}{h^3}\left[Z^2 - LZ\right]\left[(\omega_b + \omega_j)\frac{\partial h}{\partial\theta} + 2\frac{\partial h}{\partial t}\right] \tag{2-5}$$

Differentiation of equation (2-4) yields:

$$\frac{\partial h}{\partial\theta} = x \sin\theta - y \cos\theta \tag{2-6}$$

$$\frac{\partial h}{\partial t} = -\dot{x}\cos\theta - \dot{y}\sin\theta \tag{2-7}$$

The incremental force acting on the journal is:

$$\vec{\Delta F} = -P(\theta,Z)Rd\theta dZ \ |\vec{n}_r \tag{2-8}$$

The relationship between the unit vectors in the fixed and rotating reference frames is:

$$\vec{i} = \cos\theta \ |\vec{n}_r - \sin\theta \ |\vec{n}_\theta \tag{2-9}$$

$$\vec{j} = \sin\theta \ |\vec{n}_r + \cos\theta \ |\vec{n}_\theta \tag{2-10}$$

$$|\vec{n}_r = \cos\theta \ \vec{i} + \sin\theta \ \vec{j} \tag{2-11}$$

$$|\vec{n}_\partial = -\sin\theta \ \vec{i} + \cos\theta \ \vec{j} \tag{2-12}$$

Substituting equation (2-11) into equation (2-8) yields:

$$\Delta\vec{F} = -P(\theta,Z)Rd\theta dZ(\cos\theta\,\vec{i} - \sin\theta\,\vec{j}) \tag{2-13}$$

The elemental x and y force components may be found by taking the dot product of equation (2-13) with unit vectors in the x and y directions.

$$\Delta\vec{F}_x = (\Delta\vec{F}\cdot\vec{i})\vec{i} = -(P(\theta,Z)Rd\theta dZ\cos\theta)\,\vec{i} \tag{2-14}$$

$$\Delta\vec{F}_y = (\Delta\vec{F}\cdot\vec{j})\vec{j} = -(P(\theta,Z)Rd\theta dZ\sin\theta)\,\vec{j} \tag{2-15}$$

The total force components in the x and y directions are found by integrating equations (2-14) and (2-15) over the entire journal surface.

$$F_x = -\int_0^{2\pi}\int_0^{L} P(\theta,Z)R\cos\theta\,dZ\,d\theta \tag{2-16}$$

$$F_y = -\int_0^{2\pi}\int_0^{L} P(\theta,Z)R\sin\theta\,dZ\,d\theta \tag{2-17}$$

Substituting the expressions for $\frac{\partial h}{\partial \theta}$ and $\frac{\partial h}{\partial t}$ into the pressure equation and integrating around the bearing circumference gives:

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{-\mu RL^3}{2}\int_0^{2\pi} \frac{(\omega_b+\omega_j)(x\sin\theta - y\cos\theta) - 2(\dot{x}\cos\theta + \dot{y}\sin\theta)}{(c - x\cos\theta - y\sin\theta)^3}\begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} d\theta \tag{2-18}$$

The above equation is applicable to the evaluation of the forces developed in the plain journal bearing as well as the squeeze film damper bearing for arbitrary values of journal dispacement, velocity, and shaft and bearing housing angular velocities. Hence the analysis can also be used for the general floating bush bearing with rotation.

For the case of the squeeze film damper where the journal and housing are constrained from rotating, ($\omega_b = \omega_j = 0$), the force expressions become

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{-\mu R L^3}{2} \int_0^{2\pi} \frac{-2(\dot{x}\cos\theta + \dot{y}\sin\theta)}{(c - x\cos\theta - y\sin\theta)^3} \begin{Bmatrix} \cos\theta \\ \sin\theta \end{Bmatrix} d\theta \qquad (2\text{-}19)$$

These non-linear fluid film forces are easily combined with the rotor-bearing system dynamical equations providing a complete non-linear dynamical analysis of the system. Because the bearing force equations are written in fixed Cartesian coordinates a transformation from one coordinate system to another is not required. This is very important for conservation of computation time since the bearing pressure profile must be integrated at each time step of the system motion.

2.3 BEARING CAVITATION

If the complete pressure profile is calculated without regard to cavitation or rupture of the film, then the bearing pressure will be similar to Figure (2.3). This figure represents the three dimensional pressure generated in the bearing.

The exact mechanism causing cavitation in fluids is not

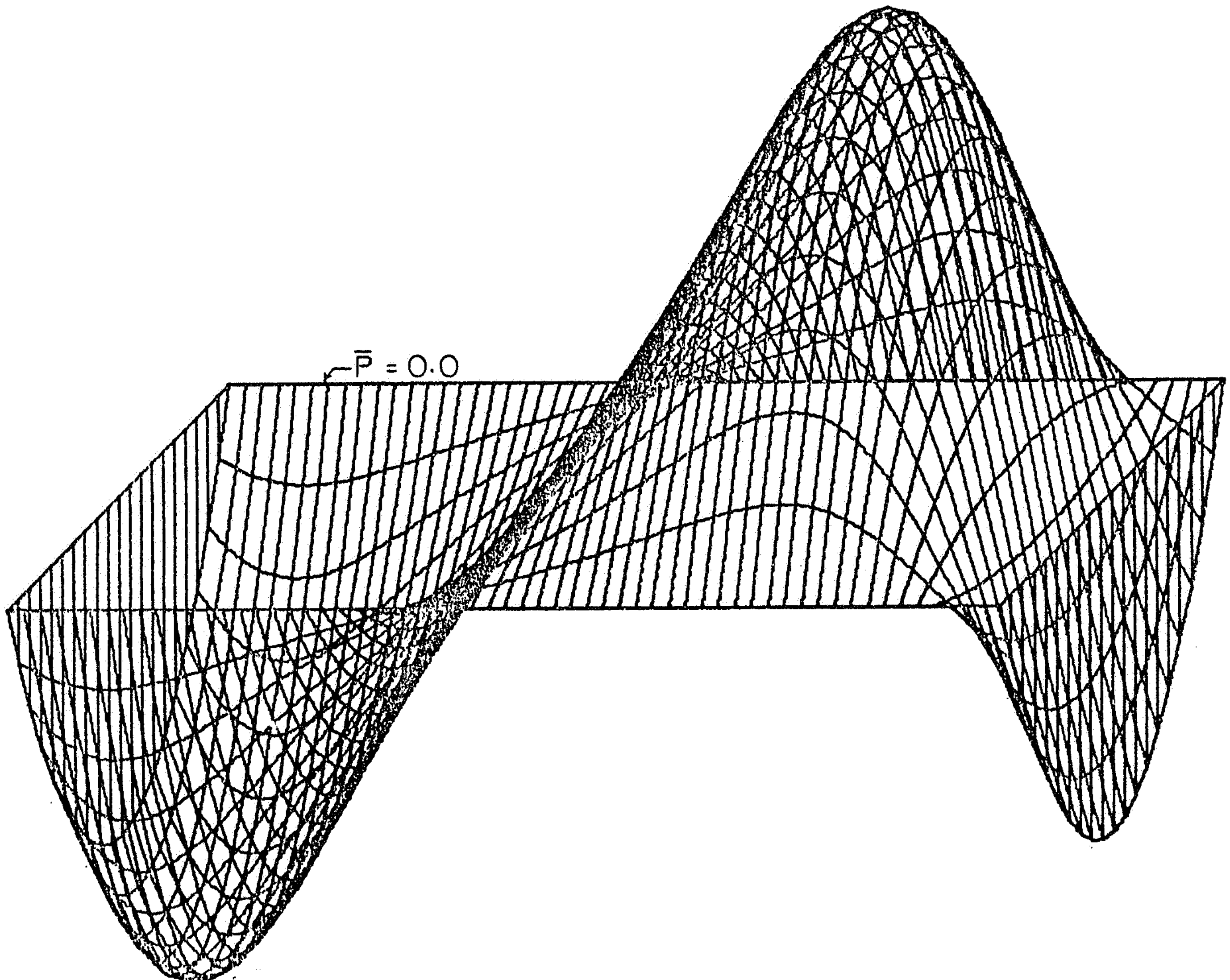

Figure 2-3 Uncavitated Pressure Profile Showing Region of Negative Hydrodynamic Pressure

fully known. It is known that film rupture is influenced by gas and solid content of the fluid. Recent investigations have shown that a fluid may stand large tensile stresses [2], and its ability to withstand rupture is dependent on its past history. In this investigation it is assumed that cavitation occurs when the pressure in the film drops below ambient pressure. The cavitated film then extends over only a section of the bearing circumference as shown in Figure (2-4). Recent experimental research has shown that cavitation in the squeeze bearing occurs in streamers of bubbles which extend around the entire bearing [3]. These streamers initially appear at the center of the bearing and extend outward as the rotor speed increases. It is beyond the scope of this present research to analyze this type of cavitation effect. Therefore the conventional cavitated film is assumed to occur when $P<P_c$ where P_c is the assumed cavitation pressure.

When evaluating the integral of equation (2-19), negative pressures are equated to zero if the film is assumed to cavitate. If the oil supply pressure is sufficiently high and suitable operating conditions exist the film does not cavitate.

2.4 BEARING FORCES IN ROTATING COORDINATES

The Reynolds equation in rotating coordinates was given by equation (2-2). Assuming steady-state circular synchronous precession of the journal about the bearing center and no axial misalignment, equation (2-2) can be integrated in closed form. The resulting equations for the bearing forces give the equivalent stiffness and damping of the bearing.

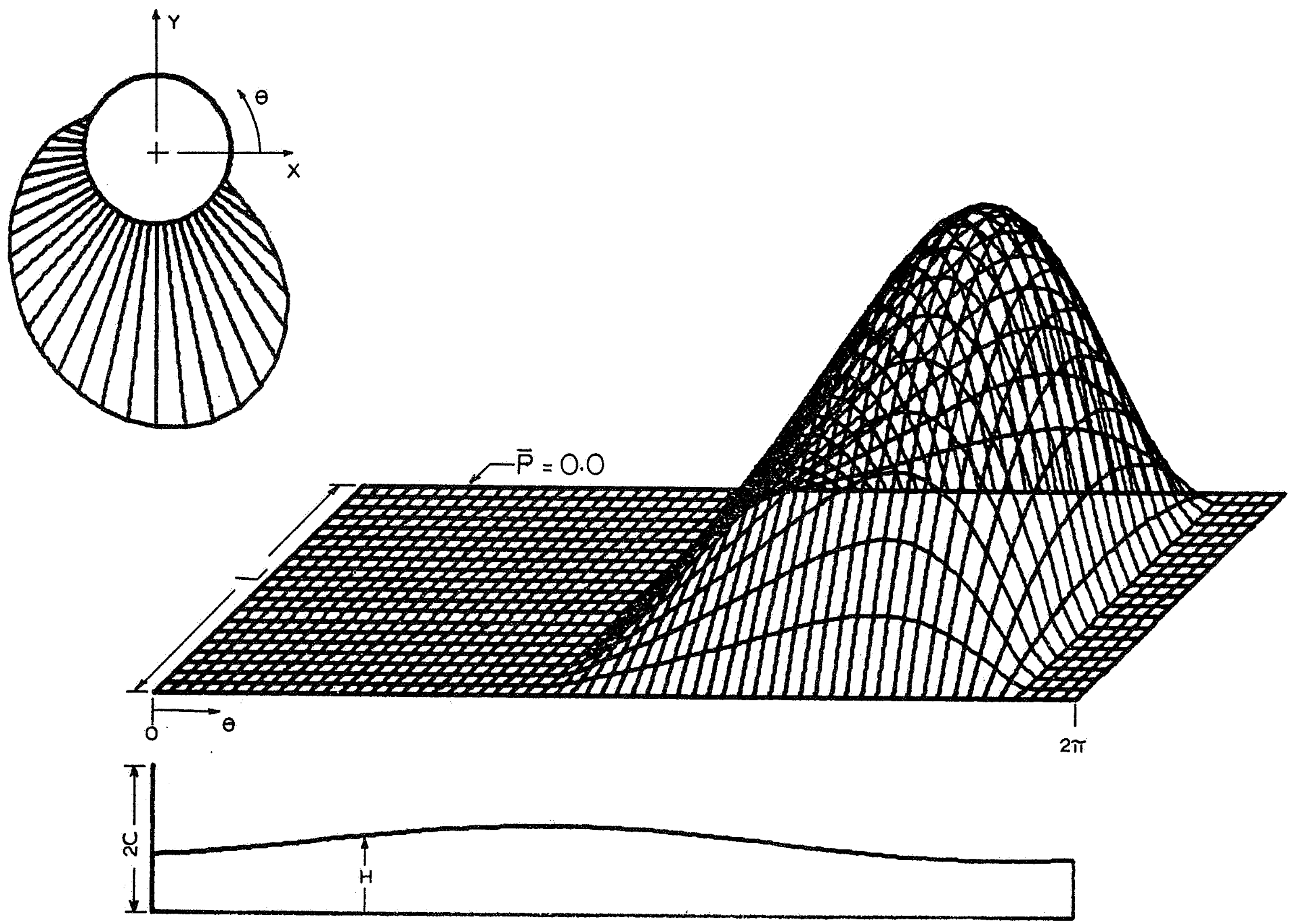

Figure 2-4 Cavitated Pressure Profile With Negative Hydrodynamic Pressure Equated to Zero

Applying the boundary conditions:

$$P(\theta,0) = P(\theta,L) = 0 \tag{2-20}$$

and integrating equation (2-2) yields:

$$P(\theta,Z) = \frac{3\mu}{h^3}\left[-2\dot{\phi}\frac{\partial h}{\partial \theta'} + 2\frac{\partial h}{\partial t}\right]\left[Z^2 - LZ\right] \tag{2-21}$$

where θ' is measured from the line of centers in the direction of rotation as shown in Figure (2-1).

In rotating coordinates the film thickness, h, is given by:

$$h = c(1 + \varepsilon \cos \theta') \tag{2-22}$$

where the eccentricty ratio, ε, is defined as:

$$\varepsilon = \frac{e}{c} \tag{2-23}$$

Differentiation of equation (2-22) gives:

$$\frac{\partial h}{\partial \theta'} = -c\ \varepsilon \sin \theta' \tag{2-24}$$

and

$$\frac{\partial h}{\partial t} = c\dot{\varepsilon} \cos \theta' \tag{2-25}$$

Substituting the expressions for h and its derivitives into equation (2-21) and integrating gives the fluid film force:

$$\vec{F} = \frac{\mu R L^3}{c^2} \int_{\theta_1'}^{\theta_2'} \frac{(\dot{\phi}\ \varepsilon \sin \theta' + \dot{\varepsilon} \cos \theta')}{(1 + \varepsilon \cos \theta')^3}\, d\theta' \ |\vec{n}_r \tag{2-26}$$

The transformation into the $\vec{n}_r$, $\vec{n}_\theta$ coordinates is

$$|\vec{n}_r = -\cos\theta' \ \vec{n}_r - \sin\theta' \ \vec{n}_\theta \qquad (2\text{-}27)$$

The components of the fluid film force in the radial and tangential directions, $\vec{n}_r$ and $\vec{n}_\theta$ are found by taking the dot product of the force with unit vectors in the radial and tangential directions. Thus:

$$\vec{F}_r = (\vec{F} \cdot \vec{n}_r)\vec{n}_r \qquad (2\text{-}28)$$

and

$$\vec{F}_\theta = (\vec{F} \cdot \vec{n}_\theta)\vec{n}_\theta \qquad (2\text{-}29)$$

and the force components are:

$$\begin{Bmatrix} F_r \\ F_\theta \end{Bmatrix} = \frac{-\mu R L^3}{c^2} \int_{\theta_1'}^{\theta_2'} \frac{(\dot{\phi}\ \varepsilon \sin\theta' + \dot{\varepsilon} \cos\theta')}{(1 + \varepsilon\cos\theta')^3} \begin{Bmatrix} \cos\theta' \\ \sin\theta' \end{Bmatrix} d\theta' \qquad (2\text{-}30)$$

The limits of integration, θ_1' and θ_2', define the area over which a positive pressure profile exists and are dependent on the type of journal motion and whether or not cavitation occurs.

It is assumed that the journal is precessing in steady-state circular motion about the origin. Therefore $\mathring{\varepsilon} = 0$ and the pressure expression, equation (2-21) becomes:

$$P(\theta', Z) = \frac{6\mu\omega\varepsilon \sin\theta'}{c^2(1 + \varepsilon\cos\theta')^3}\left[Z^2 - ZL\right] \qquad (2\text{-}31)$$

The maximum pressure in the axial direction occurs at $Z = L/2$ and equation (2-31) is rewritten as:

$$P(\theta', L/2) = \frac{-3\mu L^2 \omega\varepsilon \sin\theta'}{2c^2(1 + \varepsilon\cos\theta')^3} \tag{2-32}$$

By differentiating equation (2-32) with respect to θ' and equating to zero, the tangential location of the maximum pressure may be found. The angle, θ' max., where the pressure is maximum is given by:

$$(1 + \varepsilon\cos\theta'_{max})\cos\theta'_{max} + 3\,\varepsilon\sin^2\theta'_{max} = 0 \tag{2-33}$$

The angle θ'_{max} varies with ε and shifts from

$$\theta'_{max} = \frac{3\pi}{2} \quad \text{when} \quad \varepsilon = 0$$

to

$$\theta'_{max} = \pi \quad \text{when} \quad \varepsilon = 1$$

and the maximum pressure is given by:

$$P_{max} = \frac{-3\mu L^2 \omega\varepsilon \sin\theta'_{max}}{2c^2(1 + \varepsilon\cos\theta'_{max})^3} \tag{2-34}$$

The pressure expressed by equation (2-32) is positive over the region $\theta' = \pi$ to $\theta' = 2\pi$ and for the cavitated fluid film the limits of integration in equation (2-30) are:

$$\theta'_1 = \pi, \quad \theta'_2 = 2\pi \tag{2-35}$$

The radial and tangential components of the fluid film force are given by:

$$\begin{Bmatrix} F_r \\ F_\theta \end{Bmatrix} = \frac{-\mu R L^3 \varepsilon \omega}{c^2} \int_{\pi}^{2\pi} \frac{\sin\theta'}{(1 + \varepsilon\cos\theta')^3} \begin{Bmatrix} \cos\theta' \\ \sin\theta' \end{Bmatrix} d\theta' \tag{2-36}$$

The integrals in equation (2-36) were integrated using Booker's method [4]. The resulting force components are:

$$F_r = \frac{-2\mu R L^3 \varepsilon \omega e}{c^3 (1 - \varepsilon^2)^2} \tag{2-37}$$

and

$$F_\theta = \frac{-\mu R L^3 \pi e \omega}{2c^3 (1 - \varepsilon^2)^{3/2}} \tag{2-38}$$

The force in equation (2-37) appears as a stiffness coefficient times a displacement acting in line of the displacement towards the bearing center. The equivalent bearing stiffness is:

$$K_0 = \frac{2\mu R L^3 \varepsilon \omega}{c^3 (1 - \varepsilon^2)^2} \tag{2-39}$$

Since the journal is precessing and not rotating, every point in the journal has a velocity equal to $e\omega$. The force in equation (2-38) therefore appears as a damping coefficient times a velocity acting in the direction opposite the journal motion.

The equivalent bearing damping is:

$$C_0 = \frac{\mu R L^3 \pi}{2c^3 (1 - \varepsilon^2)^{3/2}} \tag{2-40}$$

For the uncavitated film the limits of integration in equation (2-36) become:

$$\theta_1' = 0, \ \theta_2' = 2\pi \tag{2-41}$$

Integrating and evaluating at those limits yields force components given by:

$$F_r = 0 \tag{2-42}$$

$$F_\theta = \frac{-\mu R L^3 \pi e \omega}{c^3 (1 - \varepsilon^2)^{3/2}} \tag{2-43}$$

It is therefore evident that a complete fluid film does not produce an equivalent bearing stiffness but doubles the damping of the cavitated film.

Although the equations for the bearing characteristics were derived for a plain bearing with no circumferential oil groove they are applicable to other bearing configurations. For example Figure (2-5a) represents a plain bearing with circumferential oil groove and full end leakage.

The total length of the bearing, L, corresponds to the length of the plain bearing with no oil groove. The bearing in Figure (2.5a) consists of two plain bearings without an oil groove whose length is L/2. Thus the bearing parameter L in the equations may be replaced by L/2 and the equations multiplied by 2 to obtain the total effect of the two half bearings. The net effect is to decrease the maximum pressure by a factor of 2:

$$\frac{2\left(\frac{L}{2}\right)^2}{L^2} = \frac{1}{2} \tag{2-44}$$

Similarly the damping and stiffness values are decreased by a factor of 4:

$$\frac{2\left(\frac{L}{2}\right)^3}{L^3} = \frac{1}{4} \tag{2-45}$$

The bearing represented in Figure (2.5b) is a plain bearing with circumferential oil groove and end seals to prevent end leakage. If there is no end leakage the boundary conditions are:

$$\left.\frac{\partial P}{\partial Z}\right)_{Z\,=\,0,L} = 0 \tag{2-46}$$

and the net effect leaves the pressure and bearing characteristic equations unchanged.

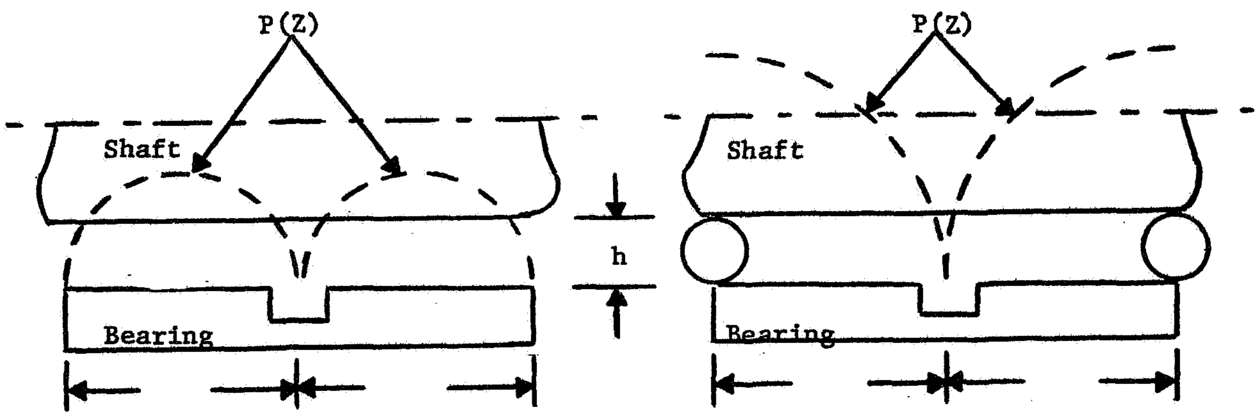

Figure 2-5 a. Axial Pressure Distribution of Bearing With Circumferential Oil Groove
b. Axial Pressure Distribution of Bearing With End Seals and Circumferential Oil Groove

The bearing equations derived in this section are summarized in Table (2-1). Also included in the table are the equations for pure radial squeeze motion. For this type of operation $\dot{\phi}$ = o and results from a purely unidirectional load on the journal. The radial and tangential force components are derived from equation (2-30) where only the term containing $\overset{\circ}{\varepsilon}$ in the integral is retained. The pressure equation is also modified to include only the $\overset{\circ}{\varepsilon}$ term. The maximum pressure occurs at $\theta = \pi$ for all values of journal eccentricity. Examination of the pressure equation reveals that the hydrodynamic pressure is positive only in the region $\theta' = \frac{\pi}{2}$ to $\frac{3\pi}{2}$. These values of θ' are the limits of integration in equation (2-30) for the cavitated film.

The table also shows that for purely radial motion no bearing stiffness is obtained in either the cavitated or uncavitated bearing. Thus if this type of motion exists retainer springs must be included to provide support flexibility.

For the case of circular journal precession, the table shows the stiffness and damping of the cavitated film and damping of the uncavitated film remain essentially constant for low eccentricity ratios. As the eccentricity ratio increases above 0.4 there is a rapid increase in these properties and they approach infinity as ε approaches 1. This variation of stiffness in the cavitated film is very important. As the eccentricity becomes large the support becomes more rigid with a corresponding increase in the rotor critical speed. If the rotor critical speed is increased above the operating speed, the phase angle between the

TYPE OF MOTION	MAXIMUM PRESSURE	EQUIVALENT DAMPING Ko (lb/in)	EQUIVALENT DAMPING Co (lb-sec/in)
CIRCULAR SYNCHRONOUS PRECESSION $\dot{\phi} = \omega,\ \dot{\varepsilon} = 0$ CAVITATED FILM	$\frac{-3\mu L^2 \omega\varepsilon \sin\theta_m}{2c^2(1 + \varepsilon\cos\theta_m)^3}$ where θ_m is given by: $(1+\varepsilon\cos\theta_m)\cos\theta_m + 3\varepsilon\sin^2\theta_m = 0$	$\frac{2\mu RL^3\varepsilon\omega}{c^3(1-\varepsilon^2)^2}$	$\frac{\mu RL^3 \pi}{2c^3(1-\varepsilon^2)^{3/2}}$
UNCAVITATED FILM		0	$\frac{\mu RL^3 \pi}{c^3(1-\varepsilon^2)^{3/2}}$
PURE RADIAL SQUEEZE MOTION $\dot{\phi} = 0,\ \dot{\varepsilon} \neq 0$ CAVITATED FILM	$\frac{-3\mu L^2\dot{\varepsilon}\cos\theta_m}{2c^2(1+\varepsilon\cos\theta_m)^3}$ $\theta_m = \pi$	0	$\frac{\mu RL^3[\pi - \cos^{-1}(\varepsilon)](2\varepsilon^2+1)}{c^3(1-\varepsilon^2)^{5/2}}$
UNCAVITATED FILM		0	$\frac{\mu RL^3\pi(2\varepsilon^2+1)}{c^3(1-\varepsilon^2)^{5/2}}$

Table 2-1. Summary of Equivalent Stiffness and Damping Coefficients for Squeeze Film Damper Bearings.

rotor unbalance vector and amplitude vector becomes less than 90°. When this condition occurs the force transmitted through the support structure will always be greater than the unbalance load. With an uncavitated film this problem does not occur because no bearing stiffness is generated. To obtain the stiffness required to stabilize a rotor (see Chapter 3) it is necessary to use retainer springs in the support bearings.

One of the most significant parameters affecting damper performance is the length to clearance ratio. The stiffness and damping coefficients vary as $(L/C)^3$ and therefore either doubling the bearing length or decreasing the clearance by 1/2 will increase the coefficients by a factor of 8.

CHAPTER 3

ROTOR-BEARING STABILITY AND STEADY-STATE ANALYSIS

3.1 ROTOR-BEARING STABILITY

After Jeffcott's [5] analysis in 1919 of the single mass flexible rotor on rigid bearings, manufacturers began producing light, flexible rotors operating above the first critical speed. However, some manufacturers encountered severe operating difficulties with some of their designs. These machines underwent violent whirling while running above the critical speed and often failed.

Experimental and analytical investigations by Newkirk and Kimball [6] [7] revealed that the whirl instability was not caused by unbalance in the rotor, but by internal shaft effects such as internal friction. Kimball theorized that forces normal to the plane of the deflected rotor could be produced by alternating stresses in the metal fibers of the shaft. In light of this theory, Newkirk concluded also that the same normal forces could be produced by shrink fits on the rotor shaft. By incorporating these forces in Jeffcott's model Newkirk showed that the rotor was unstable above twice the rotor critical speed.

Further investigation by Newkirk showed cases of rotor instability which were not produced by shaft effects but by effects in the journal bearings. [8] One cause of journal bearing instability was later shown to be due to lack of radial stiffness in the bearing and the instability occured at twice the rotor critical speed. These instabilities were especially common in lightly loaded rotors and larger bearing loads tended to promote stability. The effect of the larger loads is to cause cavitation of the fluid film which results in a radial stiffness component of the bearing forces being produced. [9], [10], [11]

In 1965 Alford reported on the effects of aerodynamic forces on rotors [12]. He showed that these forces couple the rotor equations of motion and can produce instability. He also noted that labyrinth seals and balance pistons also produce forces that can promote instability.

Recent investigators including Gunter, Kirk and Choudhury [13] [14][15] have analyzed the effects of support flexibility and damping on reducing rotor instability produced by the forces just described. As a result they have derived stability criteria for determining the necessary support characteristics.

One of the most general methods for determining rotor stability is to derive the characteristic frequency equation of the system. The stability is given by the roots of this equation. The real part of the root corresponds to an exponentially increasing or decreasing function of time. Thus a positive real part indicates instability whereas a negative real part indicates a stable system. This type of stability analysis of a rotor-bearing system therefore requires that the characteristic equation be known. This equation is not always easy to obtain.

The characteristic equation is derived from the homogeneous second order differential equations of motion of the system [15].

By assuming solutions of the form

$$x_i = A_i e^{\lambda t} \qquad i = 1, 2, \ldots, n$$

and differentiating, the equations are substituted back into the equations of motion. This produces a matrix known as the characteristic matrix. The determinant of this matrix gives the characteristic equation, a polynomial of degreee 2n in λ, where n is the number of degrees of freedom of the system.

The computer program SDSTB [16] was used to produce the stability maps shown in this chapter. The program calculates the characteristic equation for a three-mass symmetric flexible rotor mounted in journal bearings and supported in squeeze film damper bearings. The rotor-bearing model is shown in Figure (3-1). The rotor is assumed to remain stationary in the axial direction so the rotor has six degrees of freedom and the characteristic equation is therefore of degree twelve. The characteristic matrix is shown in Figure (3-2). The determinant of this matrix gives the characteristic equation. The unknown variable in this equation is λ, the natural frequency of the system. An examination of the characteristic matrix shows that the coefficients of λ are functions of the rotor and bearing properties as well as internal shaft friction, absolute rotor damping and aerodynamic cross coupling. The natural frequencies and stability of the system are found by finding the roots of this equation.

The journal and support bearing characteristics can either be inserted directly as linear coefficients or they may be calculated

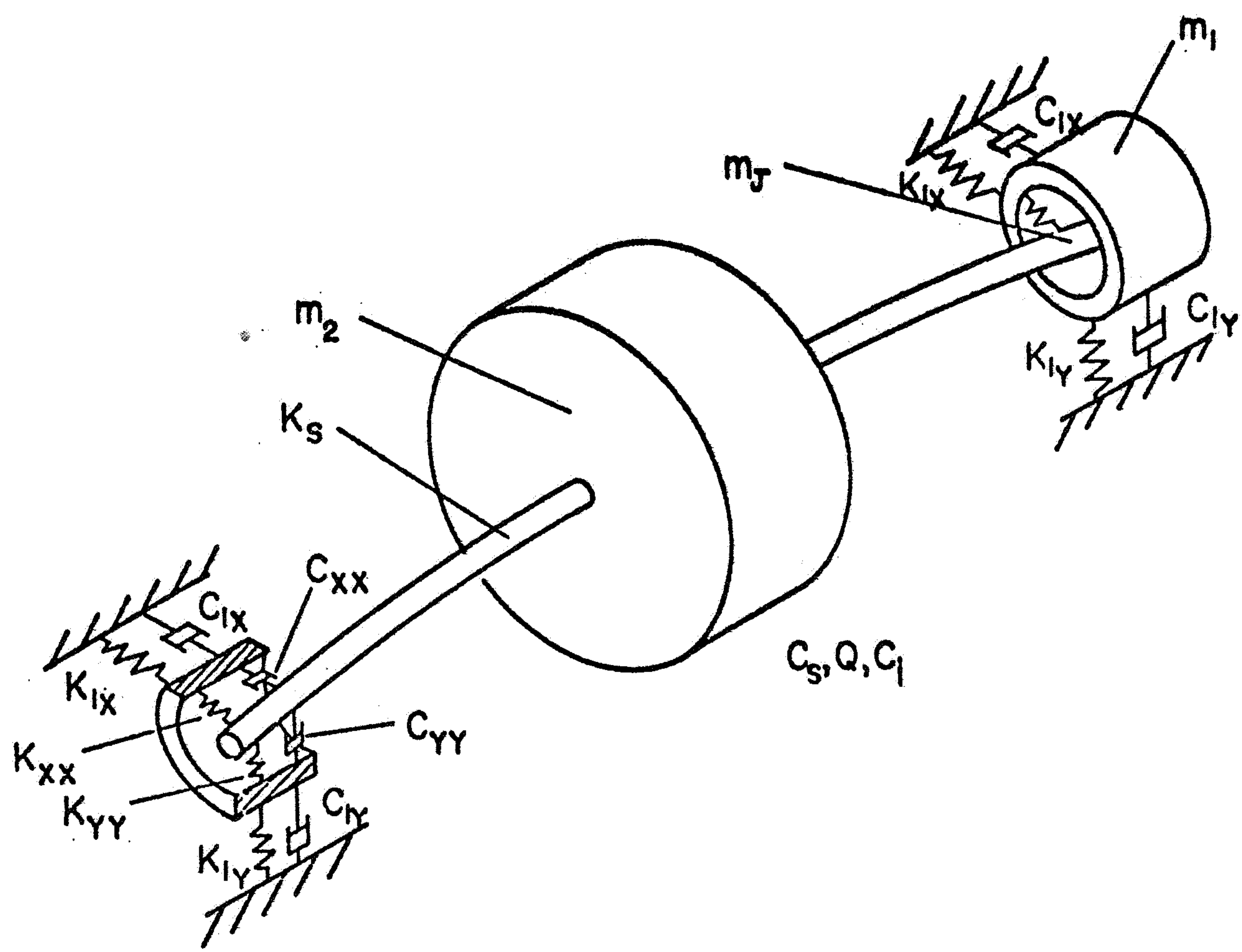

Figure 3-1 Three-Mass Flexible Rotor Mounted in Flexible, Damped Supports

$$
\begin{array}{c}
x_2: \\ \\ \\ y_2: \\ \\ \\ x_J: \\ \\ \\ y_J: \\ \\ \\ x_b: \\ \\ \\ y_b:
\end{array}
\begin{bmatrix}
\begin{matrix} m_2\lambda^2 \\ +(IC+cs)\lambda \\ +k_s \end{matrix} & Q+\omega IC & \begin{matrix} -\lambda IC \\ -k_s \end{matrix} & -\omega IC & 0 & 0 \\
-Q-\omega IC & \begin{matrix} m_2\lambda^2 \\ +(IC+cs)\lambda \\ +k_s \end{matrix} & \omega IC & \begin{matrix} -\lambda IC \\ -k_s \end{matrix} & 0 & 0 \\
\begin{matrix} -\lambda IC/2 \\ -(k_s+\omega IC)/2 \end{matrix} & 0 & \begin{matrix} m_J\lambda^2 \\ +(c_{xx}+IC/2)\lambda \\ +(k_s+\omega IC)/2 \\ +kxx \end{matrix} & \begin{matrix} c_{xy}\lambda \\ +k_{xy} \end{matrix} & \begin{matrix} -c_{xx}\lambda \\ -k_{xx} \end{matrix} & \begin{matrix} -c_{xy}\lambda \\ -k_{xy} \end{matrix} \\
0 & \begin{matrix} -\lambda IC/2 \\ -(k_s-\omega IC)/2 \end{matrix} & \begin{matrix} c_{yx}\lambda \\ +k_{yx} \end{matrix} & \begin{matrix} m_J\lambda^2 \\ +(c_{yy}+IC/2)\lambda \\ +(k_s-\omega IC)/2 \\ +k_{yy} \end{matrix} & \begin{matrix} -c_{yx}\lambda \\ -k_{yx} \end{matrix} & \begin{matrix} -c_{yy}\lambda \\ -k_{yy} \end{matrix} \\
0 & 0 & \begin{matrix} -c_{xx}\lambda \\ -k_{xx} \end{matrix} & \begin{matrix} -c_{xy}\lambda \\ -k_{xy} \end{matrix} & \begin{matrix} m_b\lambda^2 \\ +(c_{xx}+c_{1xx})\lambda \\ +(k_{xx}+k_{1xx}) \end{matrix} & \begin{matrix} (c_{xy}+c_{1xy})\lambda \\ +(k_{xy}+k_{1xy}) \end{matrix} \\
0 & 0 & \begin{matrix} -c_{yx}\lambda \\ -k_{yx} \end{matrix} & \begin{matrix} -c_{yy}\lambda \\ -k_{yy} \end{matrix} & \begin{matrix} (c_{yx}+c_{1yx})\lambda \\ +(k_{yx}+k_{1yx}) \end{matrix} & \begin{matrix} m_b\lambda^2 \\ +(c_{yy}+c_{1yy})\lambda \\ +(k_{yy}+k_{1yy}) \end{matrix}
\end{bmatrix}
\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Figure 3-2 Characteristic Matrix For Three-Mass Model Including Aerodynamic Cross-Coupling Internal Shaft Friction and Absolute Shaft Damping in the program from the bearing parameters by solving for the equilibrium positions of the journal and support. These characteristics are non-linear functions of the journal eccentricity. The stability maps in this chapter were produced with the linearized journal and support bearing characteristics given as input data to the program. The assumption of linear bearing characteristics is useful because for low eccentricity the characteristics do not vary greatly with changes in eccentricity. This assumption allows a large savings in computer time. If the non-linear characteristics are calculated, the amount of computer time increases because an iterative procedure is used to find the equilibrium position.

As an example of how a stability map is produced, consider the following system:

ROTOR CHARACTERISTICS

ROTOR WEIGHT	675 lbs
JOURNAL WEIGHT	312 lbs (each)
SUPPORT WEIGHT	15 lbs (each)
SHAFT STIFFNESS	280000 lb/in
SHAFT DAMPING	.10 lb-sec/in
INTERNAL DAMPING	0.0 lb-sec/in
ROTOR SPEED	10000 RPM

BEARING CHARACTERISTICS

k_{xx}	1.287×10^6 lb/in
k_{yy}	1.428×10^6 lb/in
C_{xx}	1200 lb-sec/in

C_{yy}	1290 lb-sec/in
$k_{xy} = k_{yx}$	0.0 lb/in
$C_{xy} = C_{yx}$	0.0 lb-sec/in

Two values of aerodynamic cross coupling were selected, Q = 20000 lb/in and Q = 100,000 lb/in. For each value of Q, several values of support stiffness were selected ranging from 50,000 lb/in to 500,000 lb/in. For each value of support stiffness a range of support damping values from 0 to 10000 lb-sec/in was used. Using this method a stability contour was found for a given value of aerodynamic cross coupling and support stiffness. The rotor and bearing characteristics remained unchanged.

Figures (3-3) and (3-4) show the stability maps for the above system for the two values of aerodynamic cross coupling. There is an intermediate range of support damping values for which the sytem is stable for a given value of the support stiffness. As the stiffness is increased the system becomes less stable. With Q = 20000 lb/in the optimum amount of damping ranges from 500 to 2500 lb-sec/in as the stiffness increases from 50000 to 500000 lb/in. For damping less than 100 lb-sec/in the system is unstable for all values of stiffness. The same is true if the damping exceeds 10000 lb-sec/in.

For Q = 100000 lb/in the optimum damping is 1000 lb-sec/in and does not shift over the stiffness range selected. When the stiffness reaches 250000 lb/in the system is unstable for all values of damping.

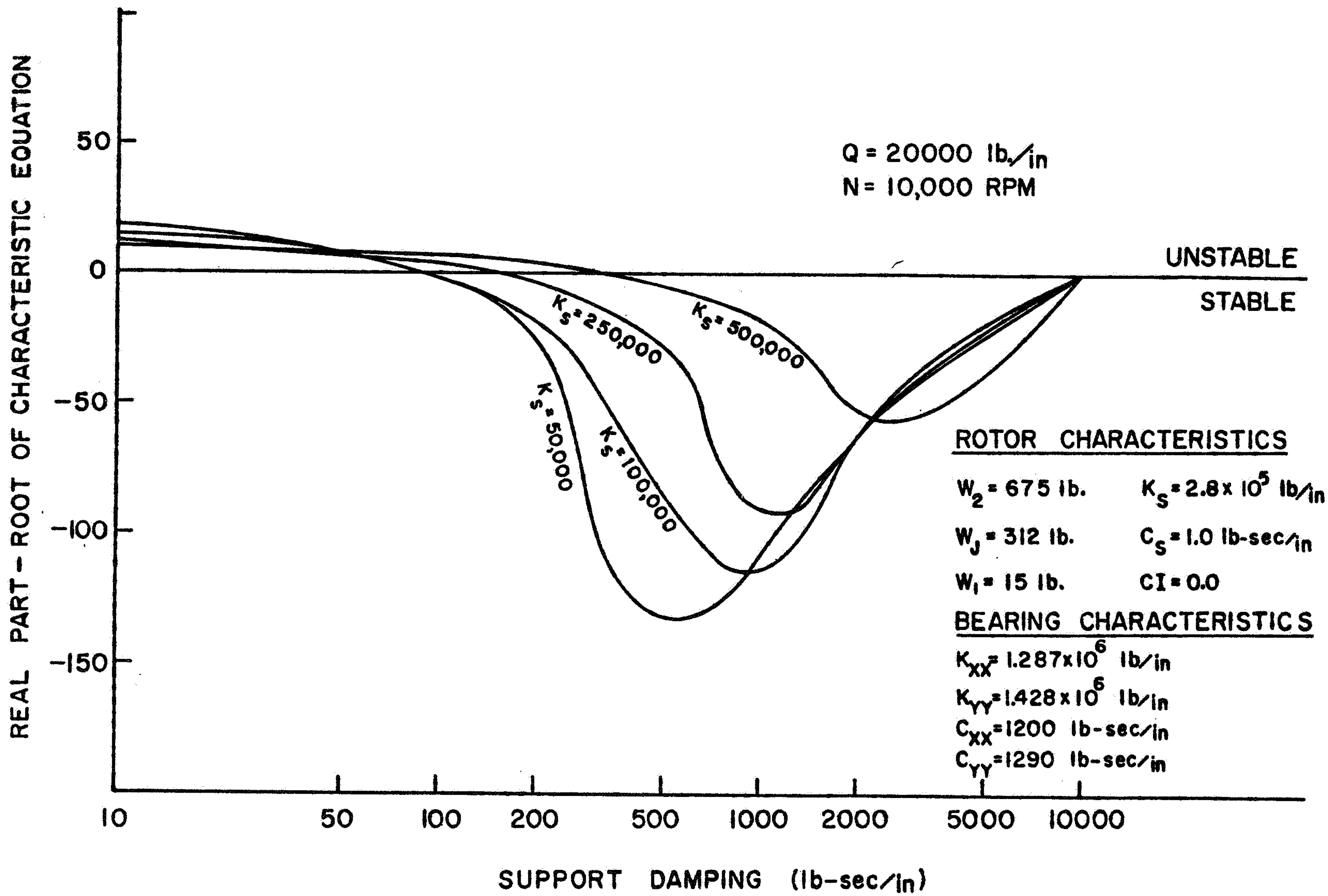

Figure 3-3 Stability of a Flexible Rotor With Aerodynamic Cross Coupling (Q = 20,000 lb/in., N = 10,000 RPM)

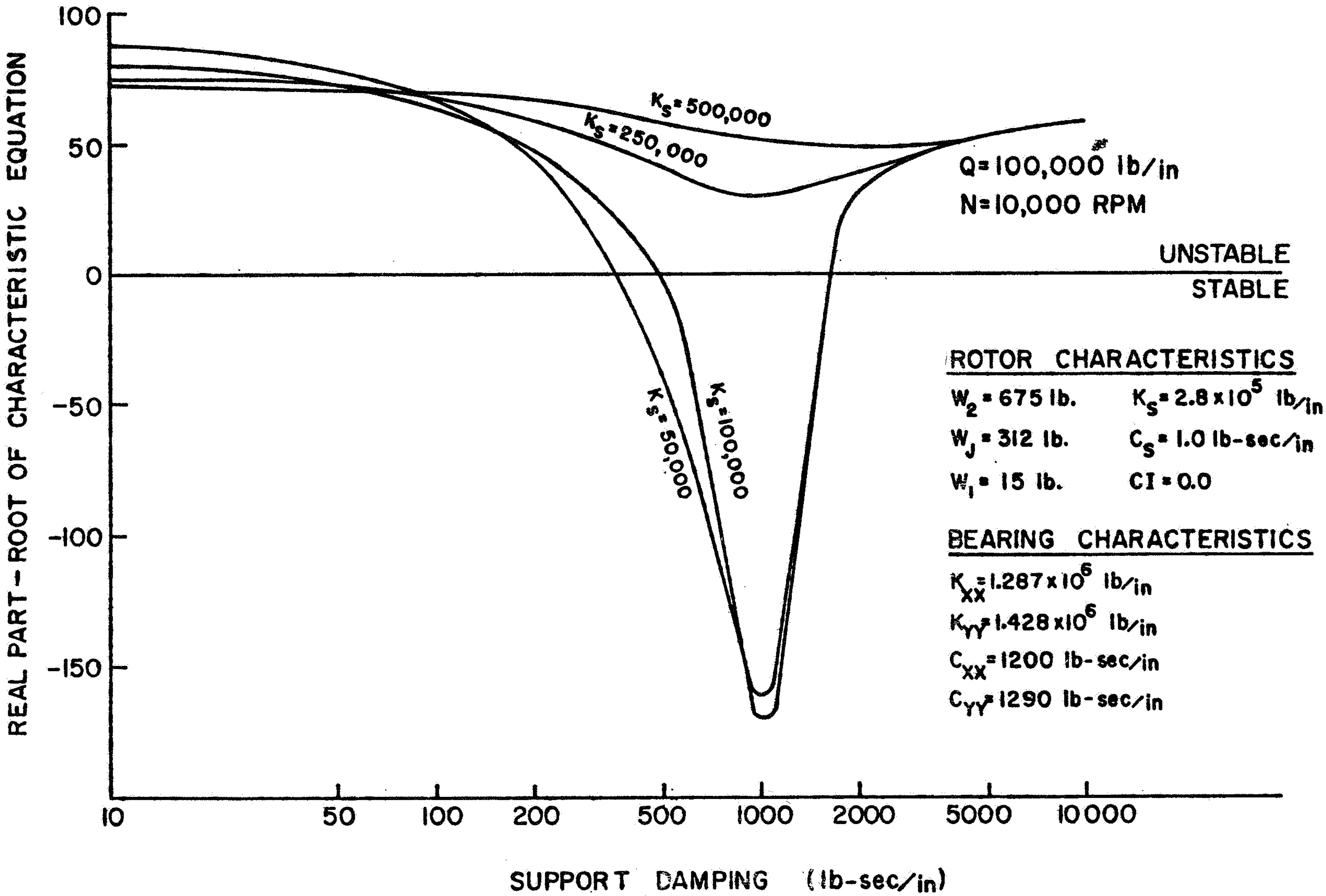

Figure 3-4 Stability of a Flexible Rotor With Aerodynamic Cross Coupling (Q = 100,000 lb/in., N = 10,000 RPM)

The stability calculation depends upon the accuracy of the root solving technique applied to the characteristic equation. For symmetric bearing and support characteristics repeated roots of the equation occur and many root solving routines break down under this condition. Although no cases have been encountered where the root solving routine has failed, it is possible that some combinations of rotor-bearing properties might cause this to happen. However because root solving routines are generally easy to obtain it would be easy to replace the one currently in SDSTB if such a situation arose.

The amount of computer time required to solve the characteristic equation depends upon the root solving technique and the order of the characteristic equation. Many studies do not require extensive stability maps and it is only necessary to determine whether the system is stable and not how stable. In these cases application of the Routh stability criteria [15] gives the required information without solving for the roots of the characteristic equation. This results in a savings of computer time. The option of using only the Routh stability criteria is available in SDSTB.

The method of determining the stability of the system from the characteristic equation becomes less practical when the order of the system is large. The elements of the characteristic matrix must be found from the equations of motion and unless the determinant is found by a computer routine, the characteristic equation must be expanded by hand. Therefore complicated systems may require a prohibitive amount of formulation time.

Recent research in transfer matrix and finite element techniques for determining the stability and natural frequencies of rotor bearing systems has been directed toward overcoming these difficulties [17] [18]. However, because iterative procedures are required in the solution, higher order modes may be expensive to obtain from the standpoint of computer time. The type of method used will depend on the amount of computer funds available and the availability of programs using the various techniques.

3.2 STEADY-STATE ANALYSIS

The steady-state stability maps just discussed provide information on the support characteristics needed to promote stability in a given rotor-bearing system. There remains the problem of relating these characteristics to the actual support bearing. The squeeze bearing equations derived in Chapter 2 in rotating coordinates are used to determine the preliminary bearing design. As noted in Chapter 2, these equations were derived assuming steady-state circular, synchronous precession of the journal.

The bearing characteristics, stiffness, damping and pressure are functions of the amplitude of the journal orbit, fluid viscosity and bearing geometry. The addition of oil supply grooves, end seals and cavitation affect the bearing characteristics.

The steady-state equations have been programmed on a digital computer. This program, SQFDAMP, analyzes three basic bearing configurations:

1. Plain bearing, no oil supply groove or end seals.
2. Bearing with oil supply groove but without end seals.
3. Bearing with both oil supply groove and end seals.

Both cavitated and uncavitated fluid films can be analyzed. A listing of the program with a description of the input data requirements and sample output are contained in Appendix A.

The program calculates the bearing characteristics and plots them as functions of the journal eccentricity ratio, ε. By varying the bearing parameters the designer is able to determine the bearing characteristics and select a bearing configuration that will provide the stability requirements of the system under consideration.

Figures (3-5) - (3-7) show the characteristics for a bearing being considered for the 675 lb rotor system described earlier. The bearing has an oil supply groove and end seals, and the fluid film is assumed to be cavitated. The bearing parameters are, length, 1.0 inches, radius, 1.2 inches and fluid viscosity 10 microreyns. For the case where Q = 20000 lb/in it was determined that the optimum support damping is about 500 lb-sec/in and the support stiffness should be less than 100000 lb/in. Because it is desirable to keep the eccentricity ratio of the journal low, Figures (3-5) and (3-6) reveal that this bearing will provide the necessary stiffness and damping characteristics with a clearance of about 4 mils at an eccentricity ratio of ε = .10 to .20.

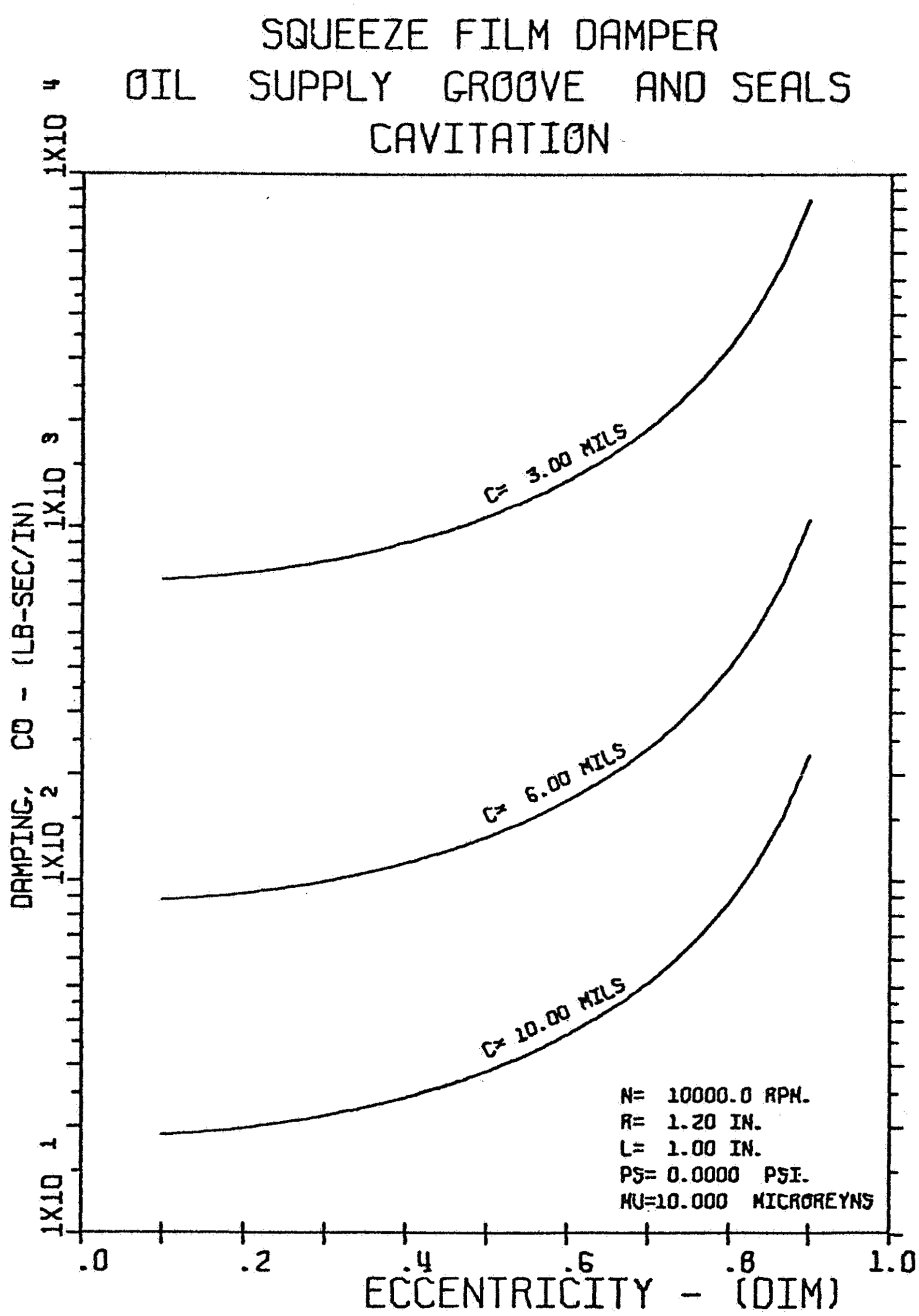

Figure 3-5. Damping Coefficient for Squeeze Film Bearing With Cavitated Film - End Seals and Oil Supply Groove Included.

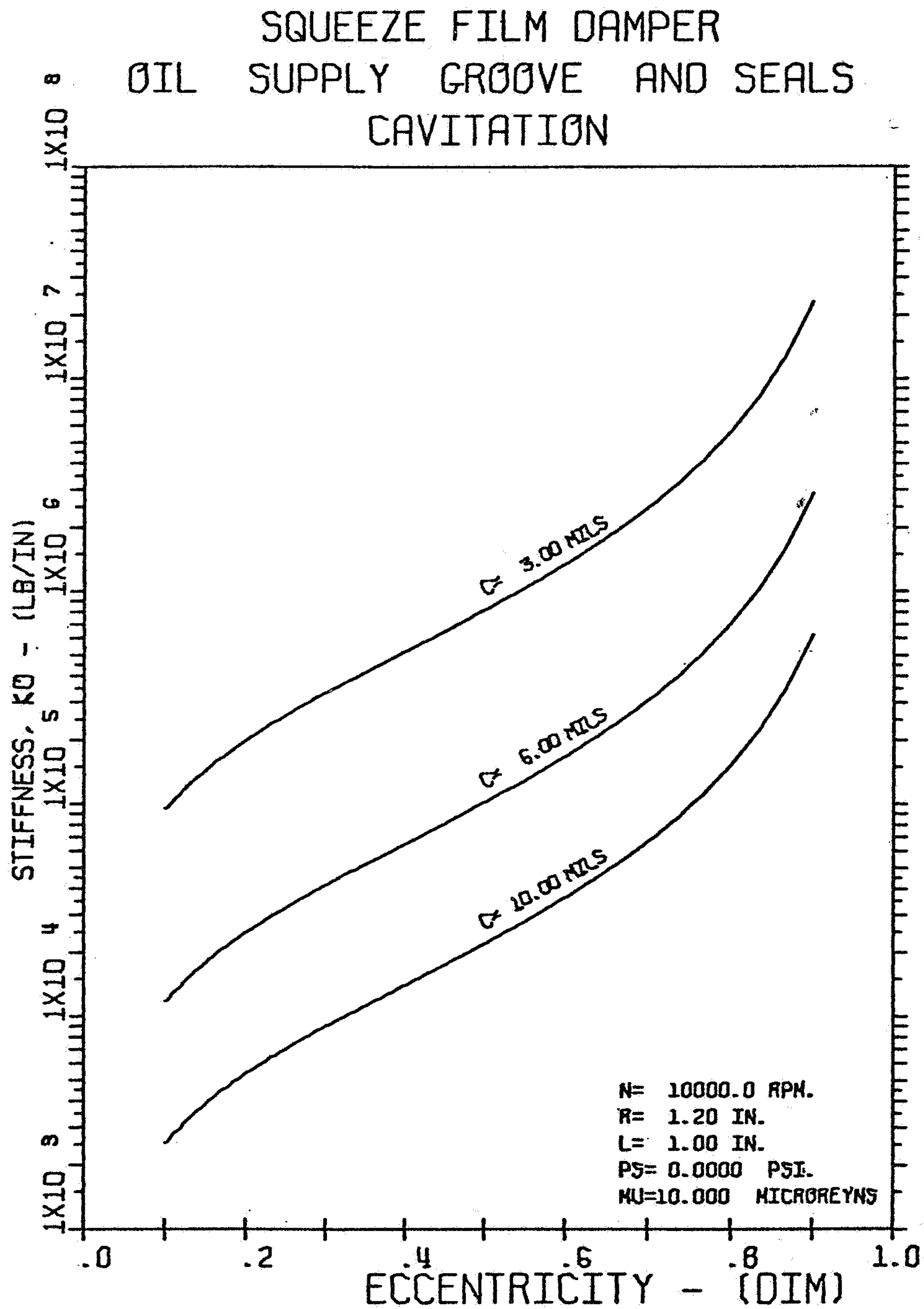

Figure 3–6. Stiffness Coefficient for Squeeze Film Bearing With Cavitated Film - End Seals and Oil Supply Groove Included.

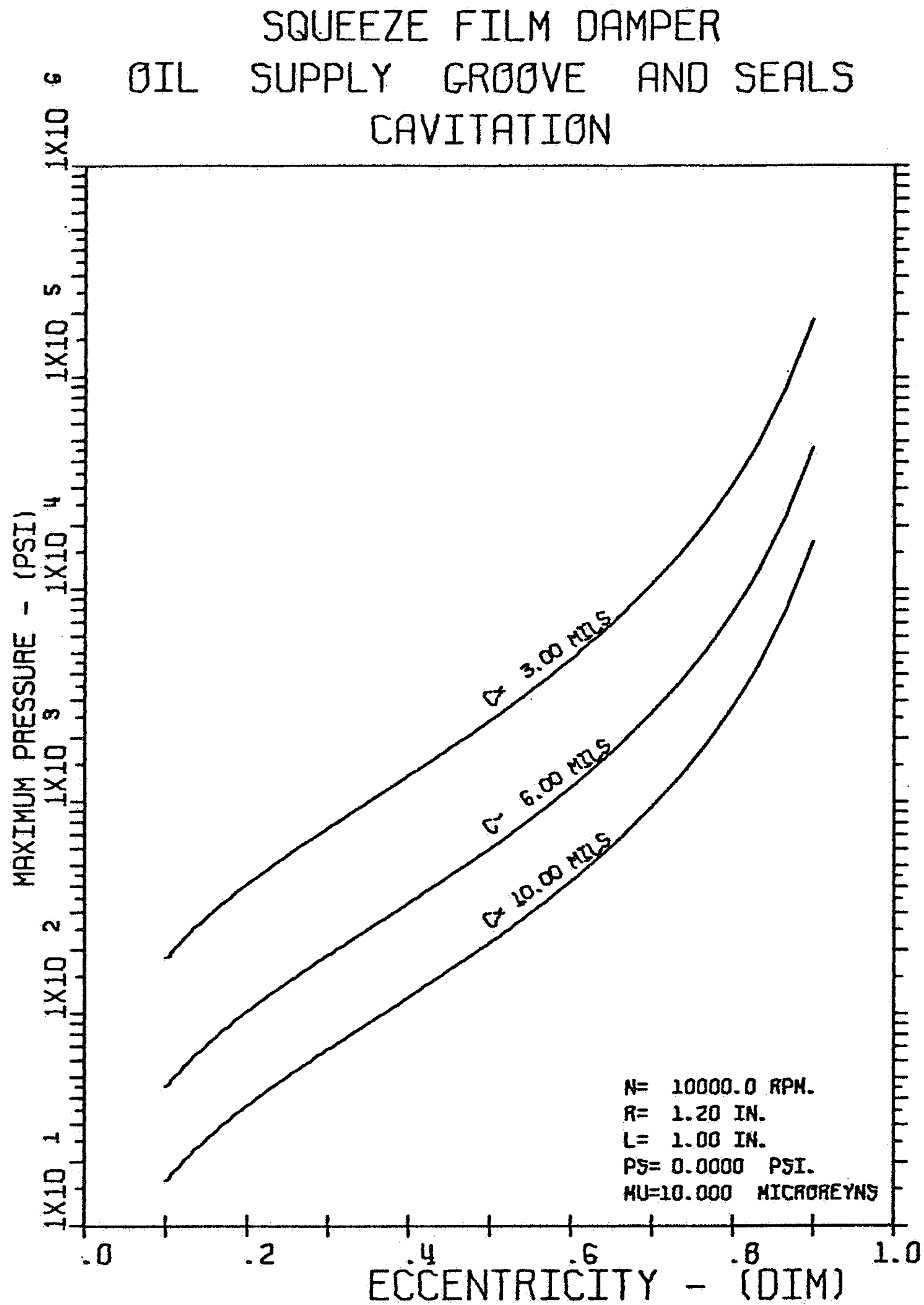

Figure 3-7. Maximum Pressure for Squeeze Film Bearing with Cavitated Film - End Seals and Oil Supply Groove Included.

This corresponds to a journal orbit of 0.4 to 0.8 mils amplitude. The maximum hydrodynamic pressure in the bearing is about 100 psi for this clearance. If the fluid film cavitates, the resulting characteristics are shown in Figures (3-8) and (3-9). A slightly larger clearance, 5.0 mils, will produce the optimum damping. However, because the uncavitated film does not produce an equivalent stiffness, retainer springs must be incorporated in the bearing. If the end seals are flexible the required spring rate may be obtained from them.

One advantage of the uncavitated film is that if the journal eccentricity ratio should become very large, there is no rise in stiffness that could cause the system to become unstable or raise the critical speed above the operating speed. Figures (3-5) and (3-8) indicate that even at eccentricity ratios of 0.9 the damping value still remains acceptable. For the cavitated film at $\varepsilon = .9$, the stiffness exceeds 2,000,000 lb/in and the system would be bordering on instability. For both films the maximum pressure exceeds 70000 psi. at $\varepsilon = 0.9$ and this large a rotating pressure field could result in bearing failure.

If Q = 100000 lb/in, the bearing characteristic graphs reveal that for a cavitated film the clearance must be as small as possible because of the limitations on stiffness shown in Figure (3-4). With a 3.0 mil clearance, the stiffness is 250000 lb/in at $\varepsilon = 0.23$ and this stiffness will produce system instability. For Q = 100000 lb/in it is desirable to study other bearing lengths and radii to obtain a cavitated bearing which pro-

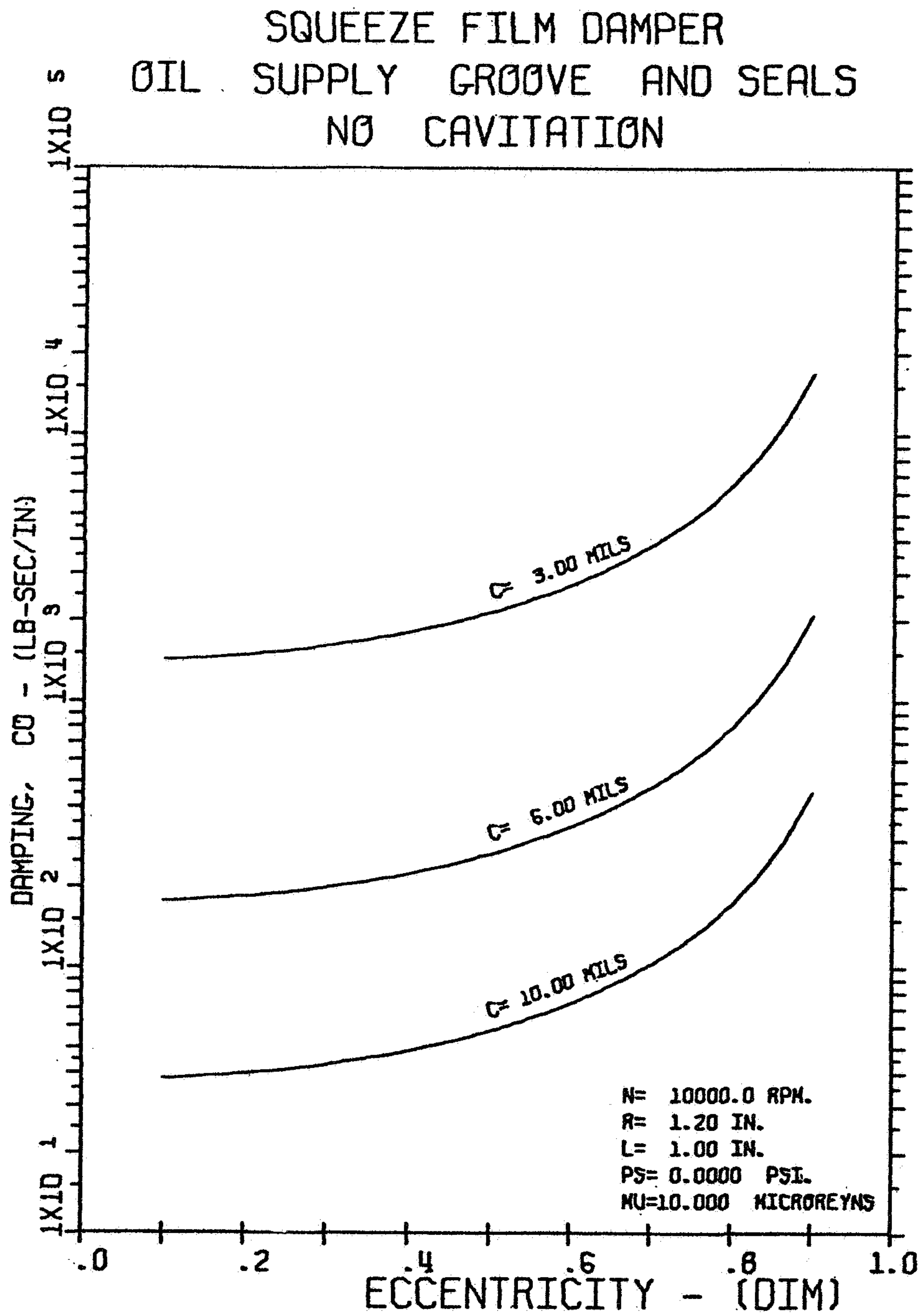

Figure 3-8. Damping Coefficient for Squeeze Film Bearing with Uncavitated Film -- End Seals and Oil Supply Groove Included

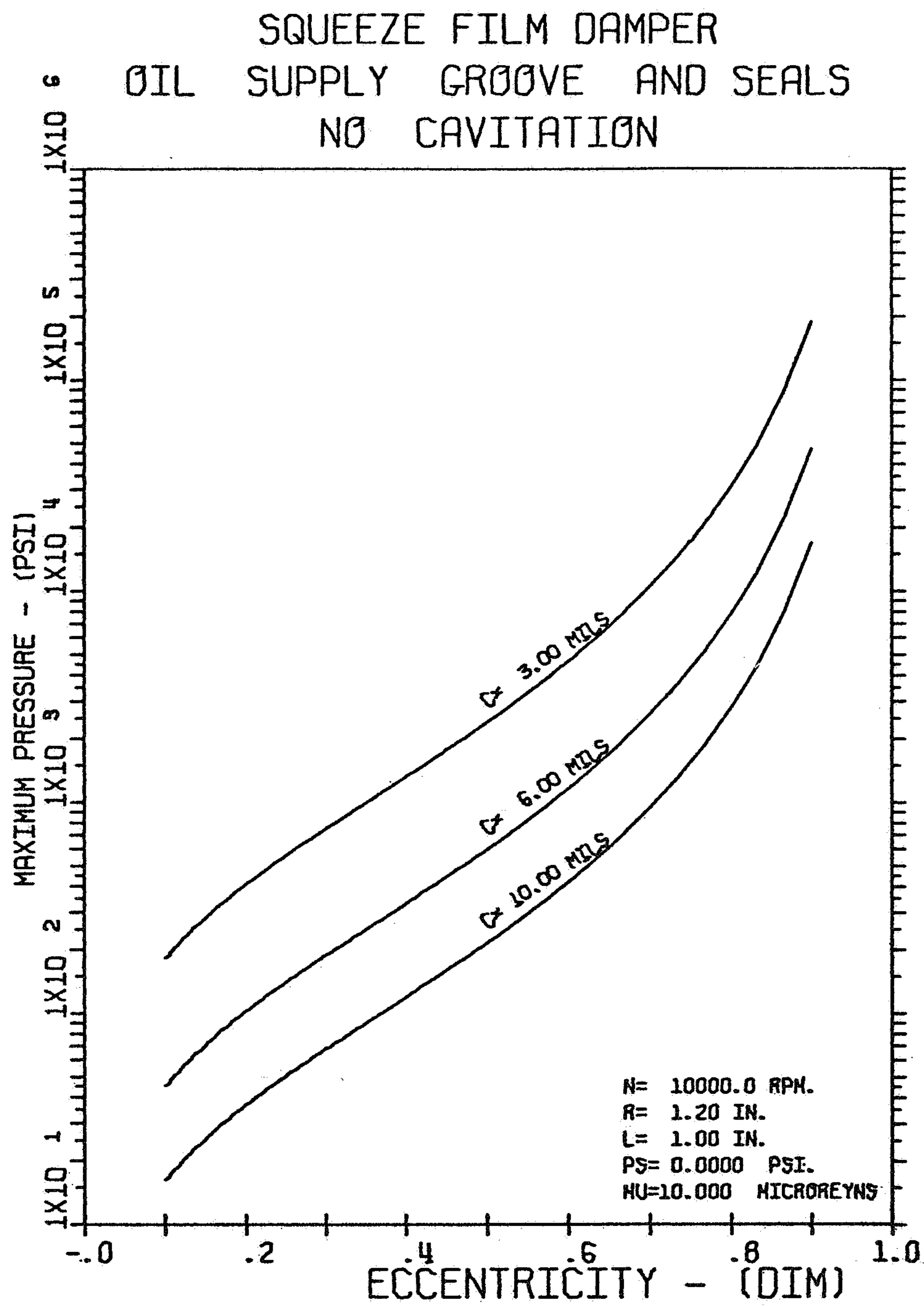

Figure 3-9. Maximum Pressure for Squeeze Film Bearing with Uncavitated Film - End Seals and Oil Supply Groove Included.

duces the required damping at low eccentricity ratios and also produces a stiffness of 100000 lb/in or less. For an uncavitated film a 3.0 mil clearance provides adequate damping.

The steady-state bearing characteristic equations used in conjunction with the stability analysis based on steady-state motion provide an excellent means of determining bearing configurations. Using the methods just described, good preliminary designs can be obtained which can be more thoroughly analyzed. The steady-state analysis described here and the transient analysis described in the next chapter provide bearing design criteria which will eliminate the experimental testing of unsuitable designs. The costs of the analysis more than offset the experimental losses when designs are tested that result in bearing or machine failure.

CHAPTER 4

TRANSIENT ANALYSIS

4.1 INTRODUCTION

There are a number of operating conditions in which the squeeze bearing journal does not orbit the bearing center in circular synchronous precession. These conditions can occur when there is a unidirectional load on the rotor or when there is a suddenly applied load such as the application of unbalance when blade loss occurs. Intermittent or cyclic forces transmitted to the machine from nearby equipment can also result in non-linear orbiting. Under these conditions the bearing stiffness and damping coefficients developed in Chapter 2 are no longer applicable and a time-transient analysis of the bearing is necessary to determine the squeeze bearing support's ability to restablize the system.

The equation for the squeeze film damper fluid film forces, in fixed coordinates, equation (2-19) provides a useful means of determining the time dependent transient behavior of a rotor bearing system. The force equations have been programmed on a digital computer and combined with the journal equations of motion. The resulting program, BRGTRAN [19] tracks the journal motion forward in time under the influence of the bearing forces, journal weight and unbalance. The program is capable of including the effects of retainer springs and cavitation.

Because both the bearing pressure equation and the journal equations of motion must be integrated, the accuracy of the simulation depends upon the numerical integration method used. Two integration methods, Adam-Bashforth-Moulton Predictor-Corrector, and 4th Order Runge- Kutta, are provided as options in BRGTRAN Although the 4th Order Runge-Kutta method is highly accurate, four functional evaluations of the pressure equation are required for each step in time. Even though the required time step size may be larger using the Runge-Kutta method, the total computer time required is still greater than other methods because of the many functional evaluations.

The Adams-Bashforth-Moulton method has been found to be sufficiently accurate for most cases run if the time step has been made sufficiently small, or about 0.01 cycles. At low eccentricities no problems are encountered in the integration process and the solutions are accurate for both methods. The changes in the bearing forces are relatively small from one time step to the next and even a very simple integration method such as the Modified Euler Method provides reasonable accuracy. However at high eccentricities the bearing forces change drastically with even a very small change in eccentricity. Even the more sophisticated integration schemes lose accuracy when the eccentricity is high unless the time step is made very small. The amount of computer time and core storage required for a small time step becomes prohibitive. This is especially true in light of the fact that at high eccentricities the validity of the short bearing approximation used in reducing the Reynolds equation is doubtful. In using the short bearing approximation it was assumed that the pressure gradient in the tangential direction is small, and this assumption may be violated at high eccentricities.

It was also shown in Chapters 2 and 3 that at high eccentricities the bearing stiffness becomes very large. This can result in system instability as indicated in the stability maps, Figures (3-3) and (3-4), or in raising the system critical speed above the operating speed causing large forces to be transmitted to the machine structure. For these reasons the design criteria of $\varepsilon < 0.4$ was established and it is unnecessary to use excessive computer time to obtain greater accuracy at higher eccentricites. The information provided at these eccentricites is very useful in showing trends in the ability of a bearing to perform adequately and should be used with this restriction kept in mind. Also recent transient analysis has been performed using hybrid computer simulation thereby avoiding the difficulties inherent in numerical integration. [20]

4.2 ANALYSIS

Using a time transient bearing program, a design engineer can make an analysis of the bearing effects without resorting to either a complete time-transient analysis of the entire rotor-bearing system or a costly experimental program during the preliminary design stage. During later design stages when the bearing configuration has been tentatively fixed, a more complete theoretical and experimental analysis of the entire system may be performed. The bearing force calculations have also been incorporated as a subroutine in a computer program which analyzes the transient behavior of certain rotor-bearing models [21]. By using these programs to determine the bearing parameters experimental verification of the bearing effects can be performed with more assurance that the design is feasible, and costly and time consuming machine prototype failures can be reduced.

The computer program BRGTRAN was recently used as part of an analysis of an existing turbomachine which had suffered frequent bearing failures. The manufacturer had decided to use a squeeze film damper bearing to reduce the vibration amplitudes at the failing bearings. Without performing a complete analysis of the bearing effects, an experimental program was initiated where various bearing configurations were installed on a test machine. The damper bearings used did not dampen out the vibrations and much time and money was lost during the project.

The following discussion of the computer simulation of this system shows the effect of varying the rotor-bearing parameters including bearing length, unbalance, cavitation and retainer springs.

In this study of the problem, the analysis made using BRGTRAN showed why the bearings used were unable to reduce vibration amplitudes. Figure (4-1) shows the journal orbit in a cavitated

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. S25741

W =	73.7	LBS	N =	16800	RPM
L =	.450	IN	R =	2.550	IN
C =	4.00	MILS	MU =	.382	MICROREYNS
PS =	0.00	PSI	FMAX=	7405.4	LBS
WX =	0.00	LBS	WY =	0.00	LBS
FU =	1181.91	LBS	EMU =	.50	
KRX =	0	LB/IN	KRY =	0	LB/IN
TRD =	6.27		PMAX=	13688.97	PSI

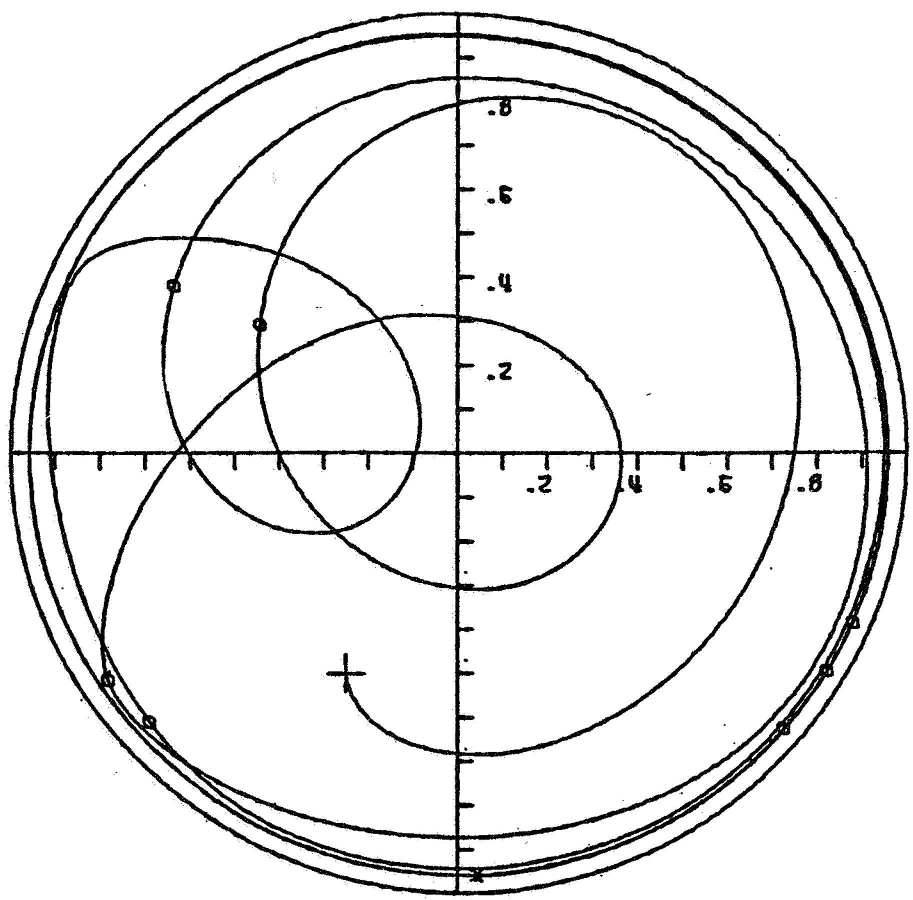

Figure 4-1. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.45 IN. - Unbalance Eccentricity = 0.002 IN. - No Retainer Spring.

film during the first 10 cycles of transient motion. The bearing parameters are:

LENGTH	- 0.45 inches
RADIUS	- 2.55 inches
CLEARANCE	- 0.004 inches
FLUID VISCOSITY	- 0.38×10^{-6}
JOURNAL WEIGHT	- 74 lbs.
UNBALANCE ECCENTRICITY	- 0.002 inches
RETAINER SPRING RATE	
Kxx	- 0 lb/in
Kyy	- 0 lb/in
N	- 16800 RPM

In the transient orbit figures a standard right-hand coordinate system has been adopted with positive journal rotation in the counterclockwise direction. The asterisk on the orbit represents the point where the maximum force is generated. The small dots represent timing marks denoting one revolution of shaft motion. These marks can be used to determine the relative location of the unbalance with respect to the amplitude in the x-direction. The timing mark sensor is assumed to be located on the positive x-axis, and the phase angle is measured from the x-axis to the timing mark in the clockwise direction.

Returning to Figure (4-1), it is seen that the journal very quickly spirals out to an eccentricity ratio $\varepsilon = 0.95$, where a limit cycle is formed due to the non-linearity of the bearing forces. The maximum force transmitted through the support structure is 7405 lbs, which is over 6 times the rotating unbalance load of 1181 lbs. as shown by the parameter TRD Figure (4-1). This large rotating force will eventually lead to bearing failure and is therefore undesirable.

The phase angle between the maximum amplitude in the x direction and the timing mark is approximately 30°, and this indicates that the precession rate is less than the natural frequency of the bearing. This has been caused by the large stiffness developed in the bearing. Figure (4-2) shows that the stiffness is approximately 487,000 lb/in. The damping is given in Figure (4-3) as 71.5 lb-sec/in.

One possible design change considered was to increase the bearing length. Figure (4-4) shows the effect of increasing the bearing length of 0.90 inches, all other parameters remaining the same. The journal still spirals outward, however the limit cycle is produced at an eccentricity ratio, ε = 0.88. This results in a reduction in the force transmitted to the support from 7405 to 3371 lbs., but it is still greater than the rotating unbalance load, and is undesirable since bearing failure will result. The phase angle has shifted from 30° to 60° and this would have been accompanied by an increase in the force transmitted except that the stiffness and damping values have also changed with the net effect being a reduction in the transmissability. The stiffness and damping coefficients are shown in Figures (4-5) and (4-6) to be 880000 lb/in and 22 lb-

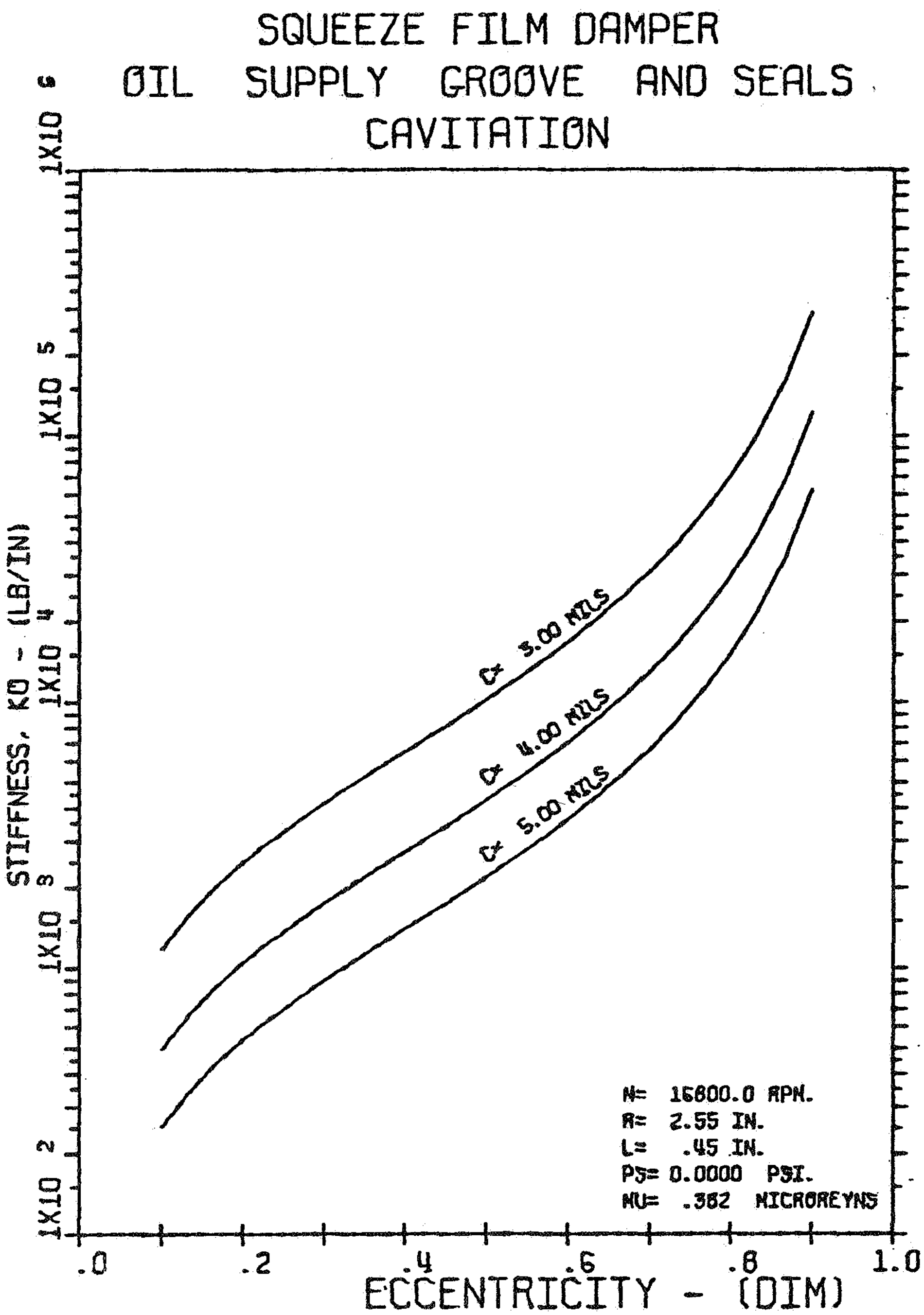

Figure 4-2. Stiffness Coefficient for Squeeze Film Bearing of Figure (4-1).

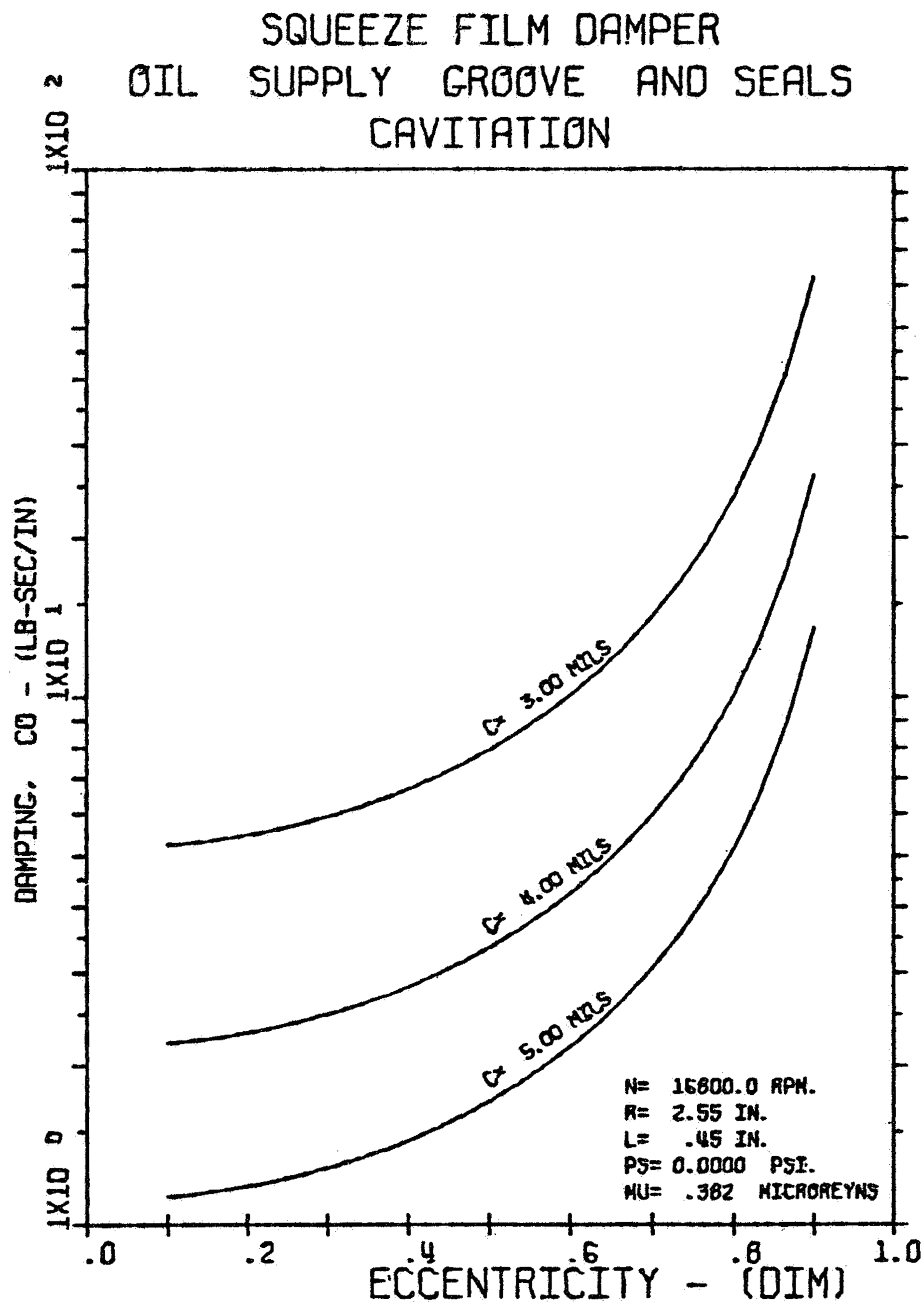

Figure 4-3. Damping Coefficient for Squeeze Film Bearing of Figure (4-1).

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 32972

W =	73.4 LBS	N =	16800 RPM
L =	.900 IN	R =	2.550 IN
C =	4.00 MILS	MU =	.382 MICROREYNS
PS =	0.00 PSI	FMAX=	3371.8 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	1177.10 LBS	EMU =	.50
KRX =	0 LB/IN	KRY =	0 LB/IN
TRD =	2.86	PMAX=	3828.76 PSI

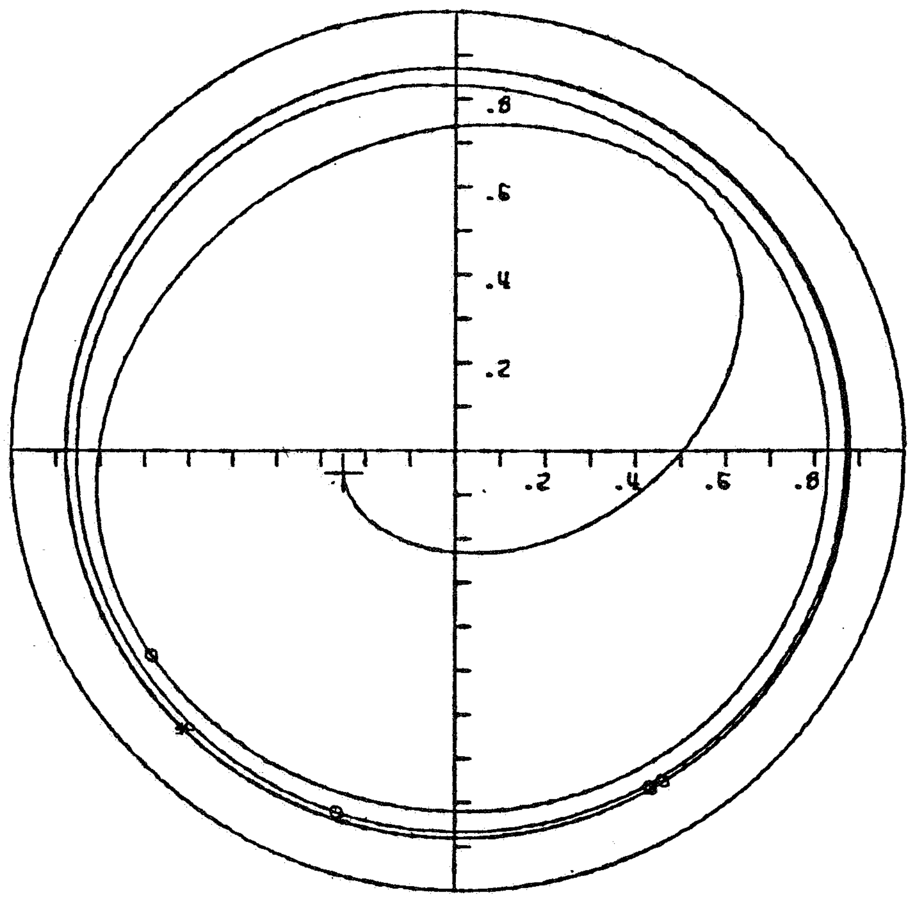

Figure 4-4. Unbalanced Rotor in Cavitated Squeeze Film Bearing L = 0.90 IN. - Unbalance Eccentricity = 0.002 IN. - No Retainer Spring.

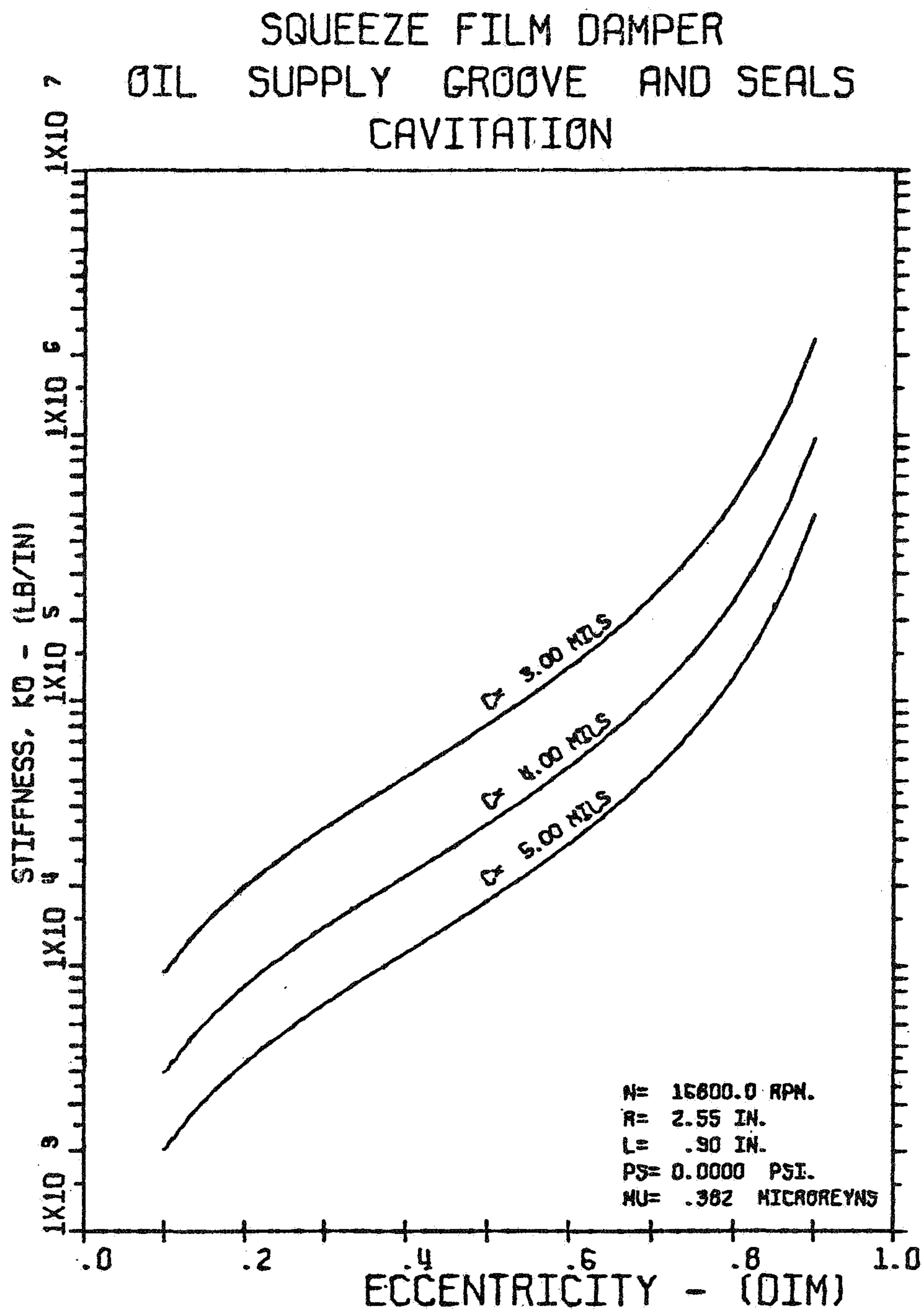

Figure 4-5. Stiffness Coefficient for Squeeze Film Bearing of Figure (4-4).

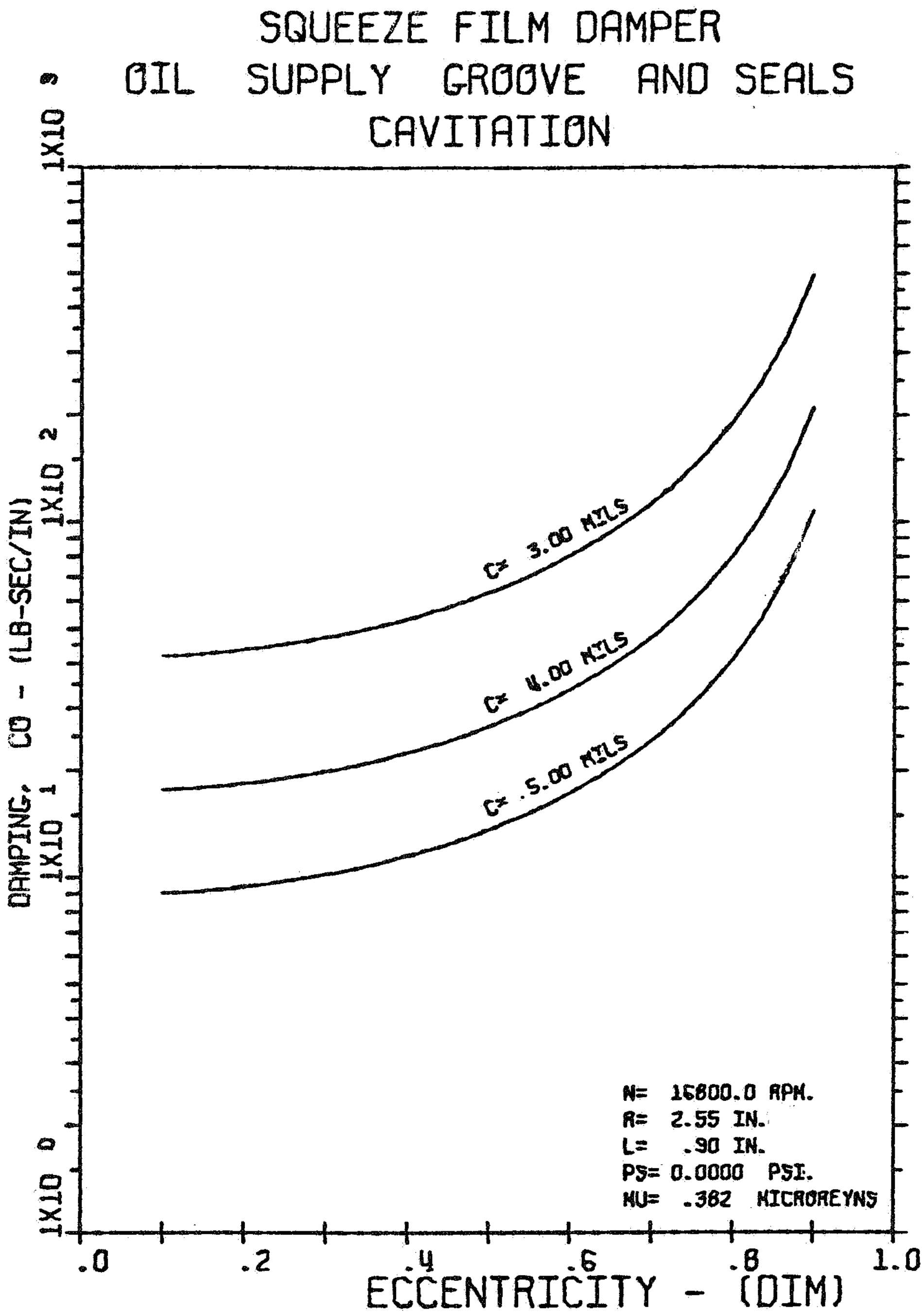

Figure 4-6. Damping Coefficient for Squeeze Film Bearing of Figure (4-4).

sec/in respectively.

Figure (4-7) shows the first 10 cycles of motion for the bearing configuration of Figure (4-1) with the addition of retainer springs with a stiffness of 123000 lb/in. Only a very slight improvement in force transmission results, and the bearing is still unacceptable.

The effect of changing the rotating unbalance load is shown in Figure (4-8). The bearing configuration is the same as Figure (4-4) except the unbalance has been reduced by one half. The journal orbit has been greatly reduced and the force transmitted to the support structure is reduced to only 80% of the unbalance load. The journal is now precessing about a point offset from the bearing center.

By adding retainer springs to the bearing of Figure (4-8) the motion in Figure (4-9) results. The retainer spring rate is 123,000 lb/in. The journal is orbiting about the center of the bearing at an eccentricity ratio of $\varepsilon = 0.35$. The journal motion is stabilizing more quickly than without the retainer springs and the journal is precessing synchronously as indicated by the small dots on the or bit which represents one cycle of motion. The transmitted force has been further reduced to only 65% of the unbalance load. From the standpoint of producing a small amplitude orbit and attenuating the unbalance load such a bearing configuration is desirable.

If the unbalance eccentricity is again increased to 0.002 inches, the bearing configuration of Figure (4-9) is no longer

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 925763

W =	73.7 LBS	N =	16800 RPM
L =	.450 IN	R =	2.550 IN
C =	4.00 MILS	MU =	.382 MICROREYNS
PS =	0.00 PSI	FMAX=	7331.9 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	1181.91 LBS	EMU =	.50
KRX =	123000 LB/IN	KRY =	123000 LB/IN
TRD =	6.20	PMAX=	12510.22 PSI

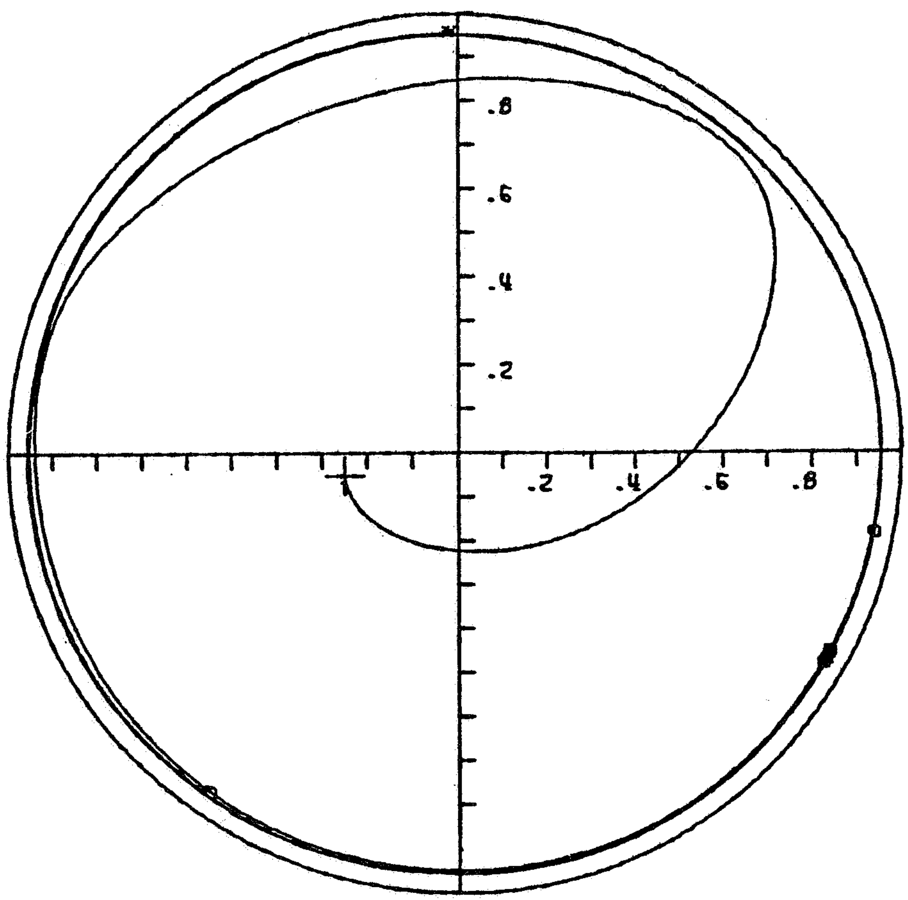

Figure 4-7. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.45 IN. - Unbalance Eccentricity = 0.002 IN. - Retainer Spring Stiffness, KR = 123,000 LB/IN.

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO.

W =	73.7 LBS	N =	16800 RPM
L =	.900 IN	R =	2.550 IN
C =	4.00 MILS	MU =	.382 MICROREYNS
PS =	0.00 PSI	FMAX=	470.0 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	590.96 LBS	EMU =	.25
KRX =	0 LB/IN	KRY =	0 LB/IN
TRD =	.80	PMAX=	52.28 PSI

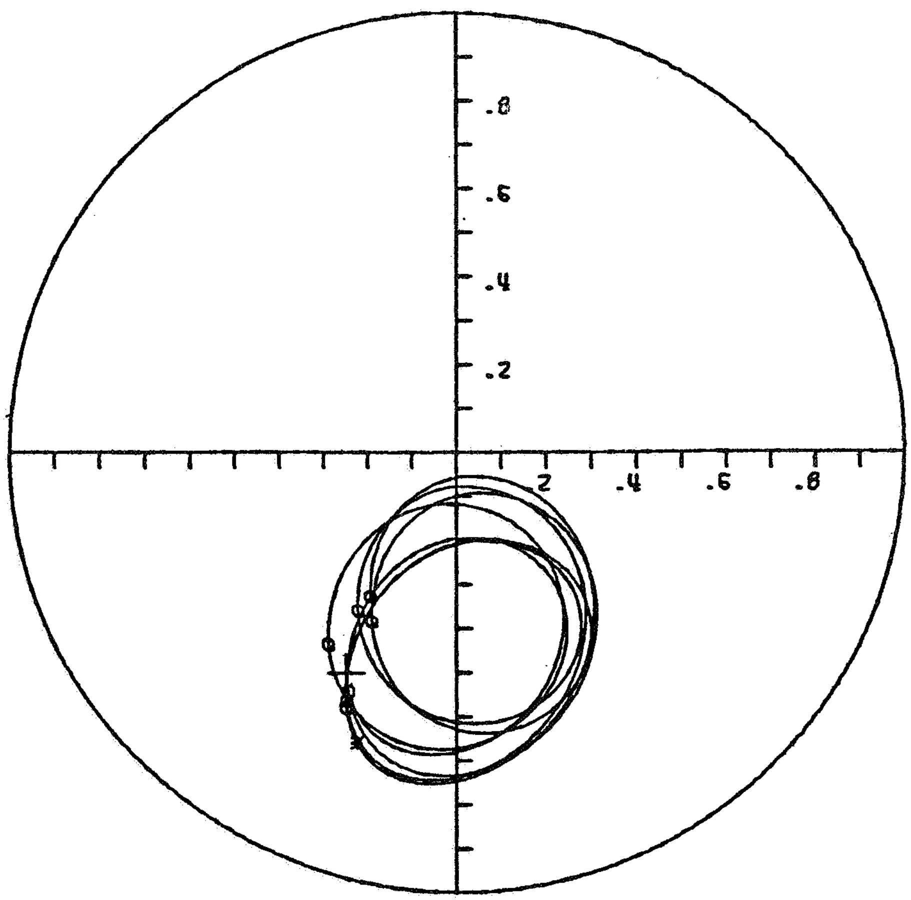

Figure 4-8. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.90 IN. - Unbalance Eccentricity = 0.001 IN. - No Retainer Spring

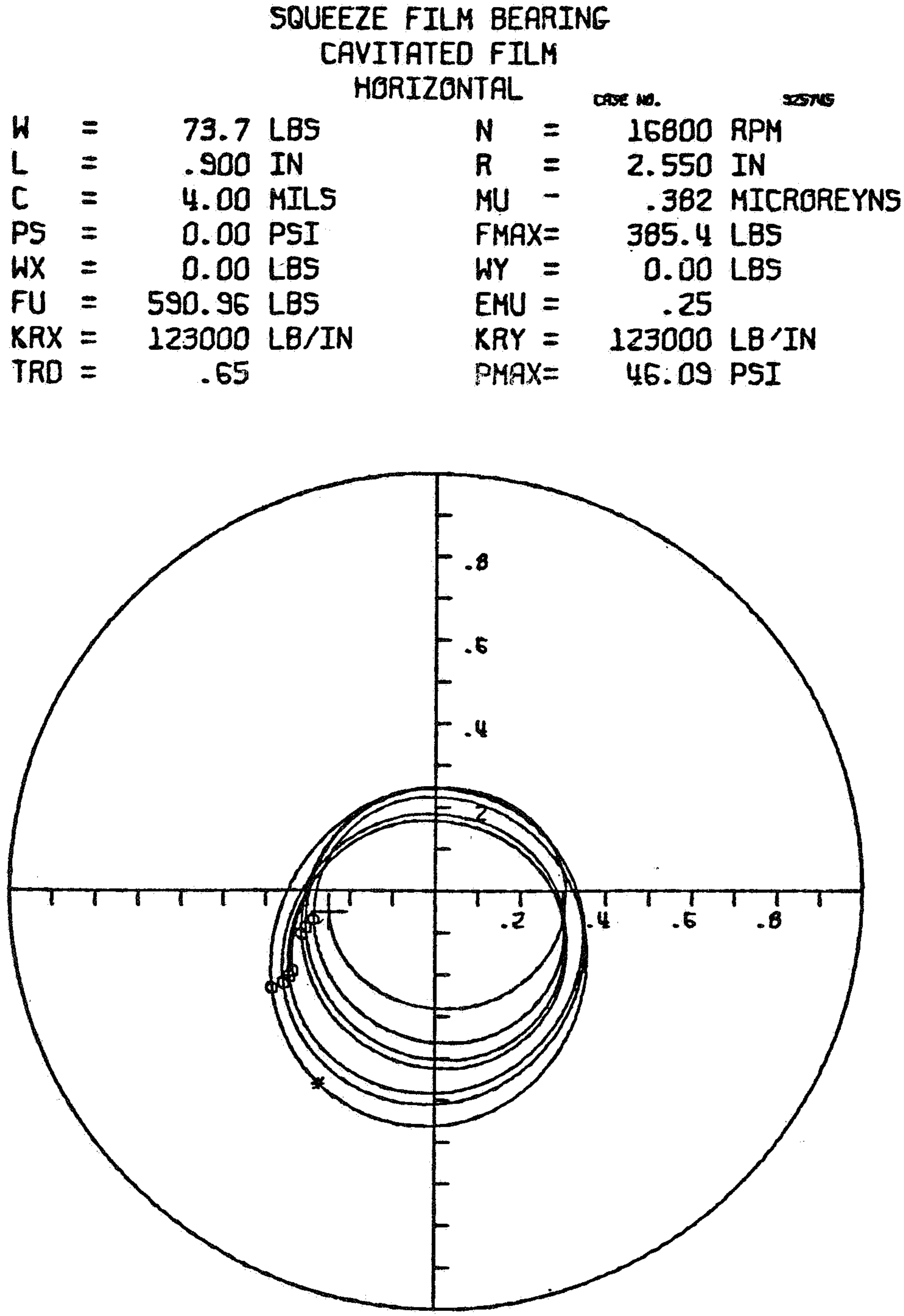

Figure 4-9. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.90 IN - Unbalance Eccentricity = 0.001 IN. - Retainer Spring Stiffness, KR = 123,000 LB/IN.

acceptable. The resulting motion is shown in Figure (4-10) and a large amplitude limit cycle with large force transmission again occurs. However if the fluid film does not cavitate the bearing performance improves and is marginally acceptable as shown in Figure (4-11). The journal is orbiting at $\varepsilon = 0.65$ and the transmitted force is only 5% less than the unbalance load.

Although this particular analysis includes only the journal weight and the unbalance loading, it shows the usefulness of this program in determining the bearing effects with different bearing parameters. The ability to perform the analysis without extensive preliminary experimental work provides a great savings in time and money. By systematically varying the bearing parameters design guidelines are established.

For instance Figures (4-12 - (4-15) shows the effect of varying the unbalance on a 675 lb. journal operating in a squeeze bearing with a 7 mil clearance. The unbalance eccentricity is increased from 1.75 to 3.5 mils with an accompanying increase in the journal amplitude and the force transmitted to the support structure. Although the exact unbalance may not be known precisely for a given rotor, a design estimate can be made based on the effect of a suddenly applied known unbalance due to blade loss or loss of chemical deposits from the blade surfaces. Prior to the sudden unbalance the rotor is assumed to be perfectly balanced. The ability of the bearing to reduce the amplitude to tolerable

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO.

W =	73.7	LBS	N =	16800	RPM
L =	.900	IN	R =	2.550	IN
C =	4.00	MILS	MU =	.382	MICROREYNS
PS =	0.00	PSI	FMAX=	3641.9	LBS
WX =	0.00	LBS	WY =	0.00	LBS
FU =	1181.91	LBS	EMU =	.50	
KRX =	123000	LB/IN	KRY =	123000	LB/IN
TRD =	3.08		PMAX=	3334.52	PSI

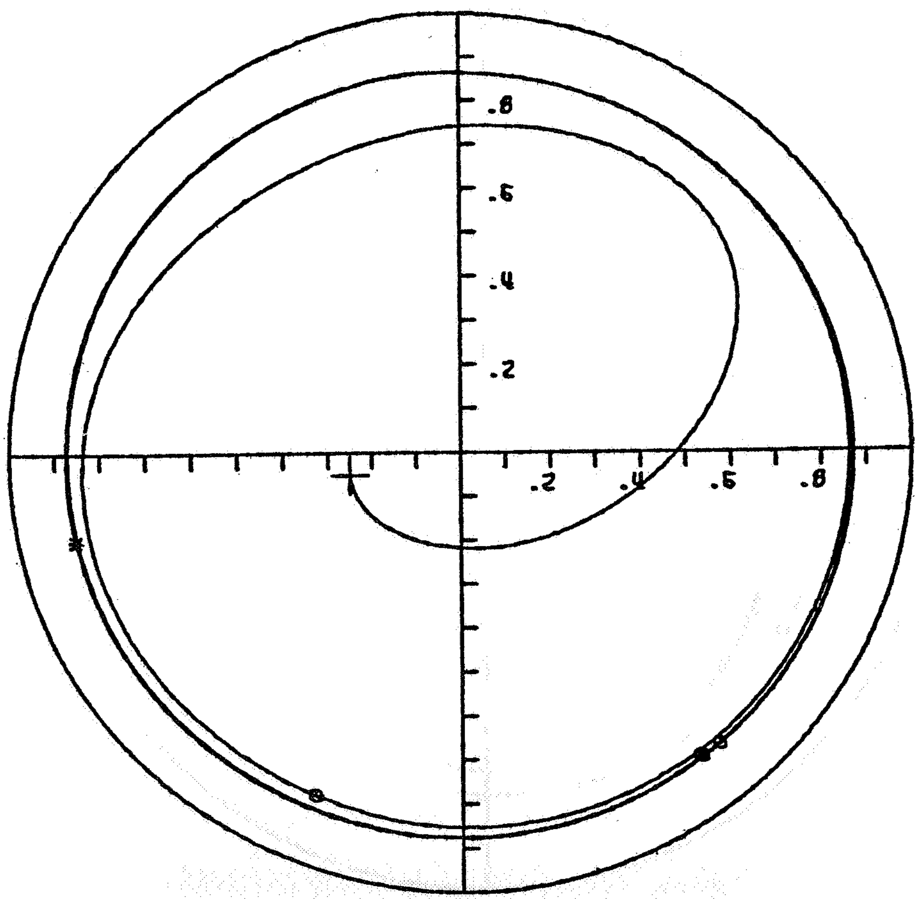

Figure 4-10. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.90 IN. - Unbalance Eccentricity = 0.002 IN. - Retainer Spring Stiffness, KR = 123,000 LB/IN.

SQUEEZE FILM BEARING
UNCAVITATED FILM
HORIZONTAL

CASE NO. 330745

W	=	73.7 LBS	N	=	16800 RPM
L	=	.900 IN	R	=	2.550 IN
C	=	4.00 MILS	MU	=	.382 MICROREYNS
PS	=	4000.00 PSI	FMAX	=	1123.5 LBS
WX	=	0.00 LBS	WY	=	0.00 LBS
FU	=	1181.91 LBS	EMU	=	.50
KRX	=	123000 LB/IN	KRY	=	123000 LB/IN
TRD	=	.95	PMAX	=	390.37 PSI

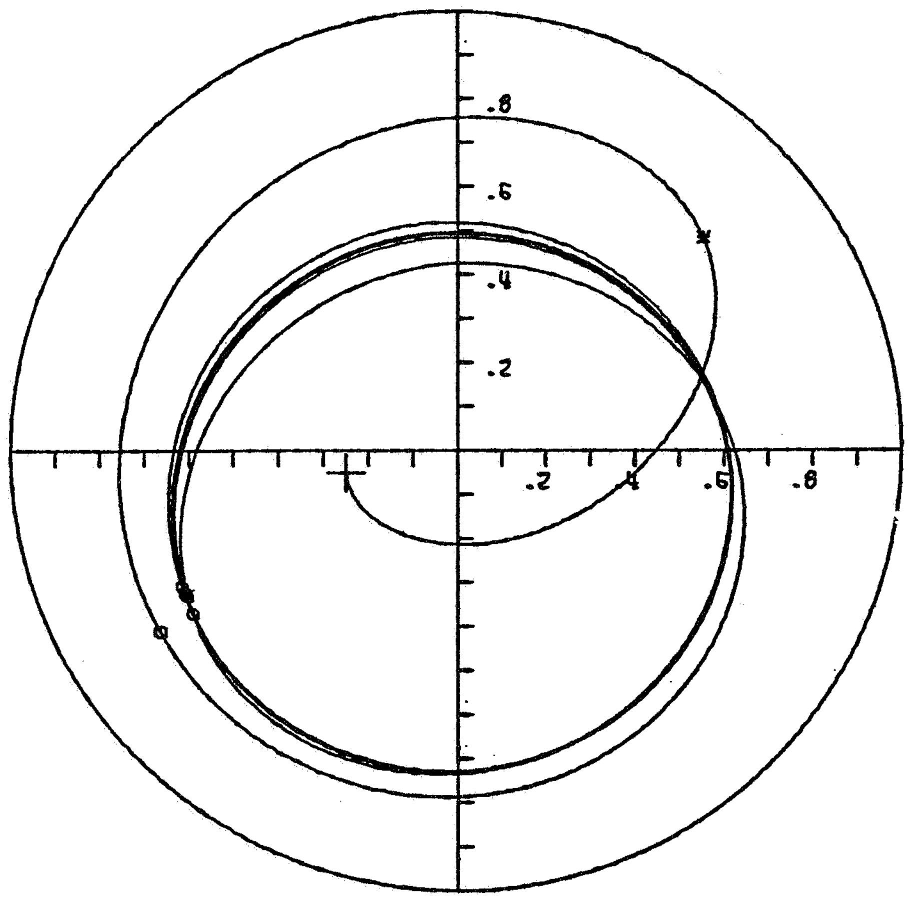

Figure 4-11. Unbalanced Rotor in Uncavitated Squeeze Film Bearing - L = 0.90 IN. - Unbalance Eccentricity = 0.002 IN. - Retainer Spring Stiffness, KR = 123,000 LB/IN.

SQUEEZE FILM BEARING
CAVITATED FILM
VERTICAL

CASE NO. 330744

W =	675.0 LBS	N =	10500 RPM
L =	2.000 IN	R =	3.500 IN
C =	7.00 MILS	MU =	2.490 MICROREYNS
PS =	0.00 PSI	FMAX=	2725.1 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	3699.90 LBS	EMU =	.25
KRX =	50000 LB/IN	KRY =	50000 LB/IN
TRD =	.74	PMAX=	131.98 PSI

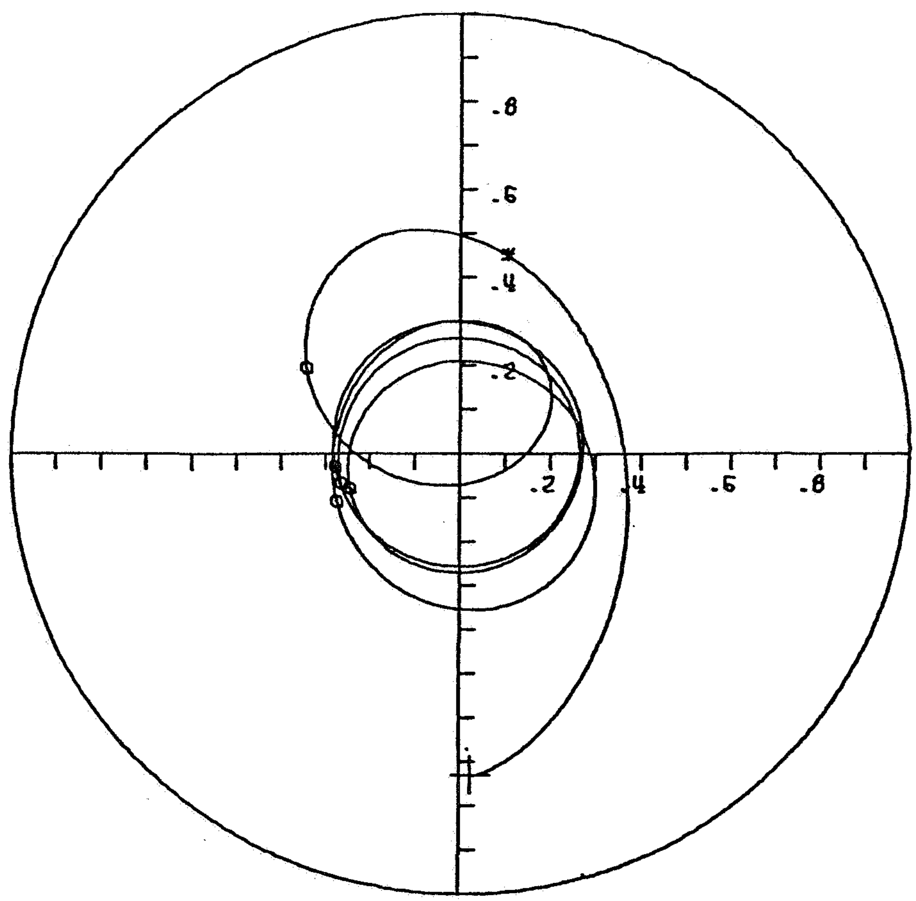

Figure 4-12. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Ecdentricity = 1.75 Mils

SQUEEZE FILM BEARING
CAVITATED FILM
VERTICAL

CASE NO.

W =	675.0	LBS	N =	10500	RPM
L =	2.000	IN	R =	3.500	IN
C =	7.00	MILS	MU =	2.490	MICROREYNS
PS =	0.00	PSI	FMAX=	4368.2	LBS
WX =	0.00	LBS	WY =	0.00	LBS
FU =	4439.88	LBS	EMU =	.30	
KRX =	50000	LB/IN	KRY =	50000	LB/IN
TRD =	98		PMAX=	181.61	PSI

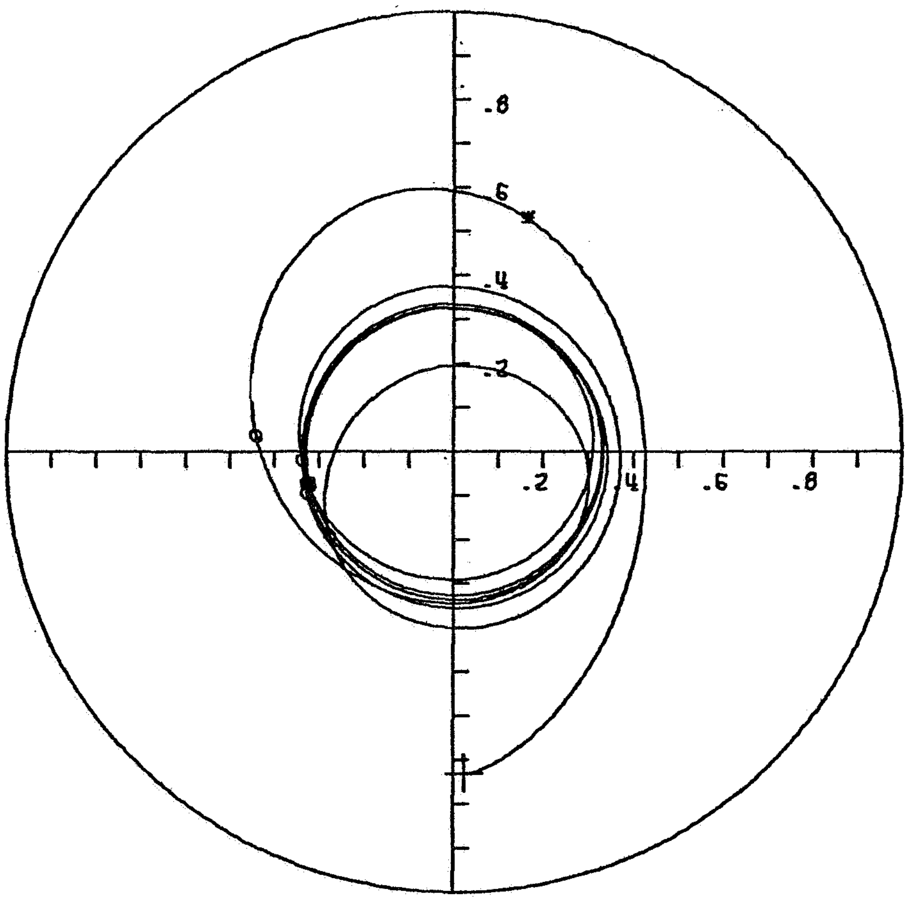

Figure 4-13. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 2.10 Mils.

SQUEEZE FILM BEARING
CAVITATED FILM
VERTICAL

CASE NO. 330742

W =	675.0 LBS	N =	10500 RPM
L =	2.000 IN	R =	3.500 IN
C =	7.00 MILS	MU =	2.490 MICROREYNS
PS =	0.00 PSI	FMAX=	6287.7 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	5179.86 LBS	EMU =	.35
KRX =	50000 LB/IN	KRY =	50000 LB/IN
TRD =	1.21	PMAX=	270.45 PSI

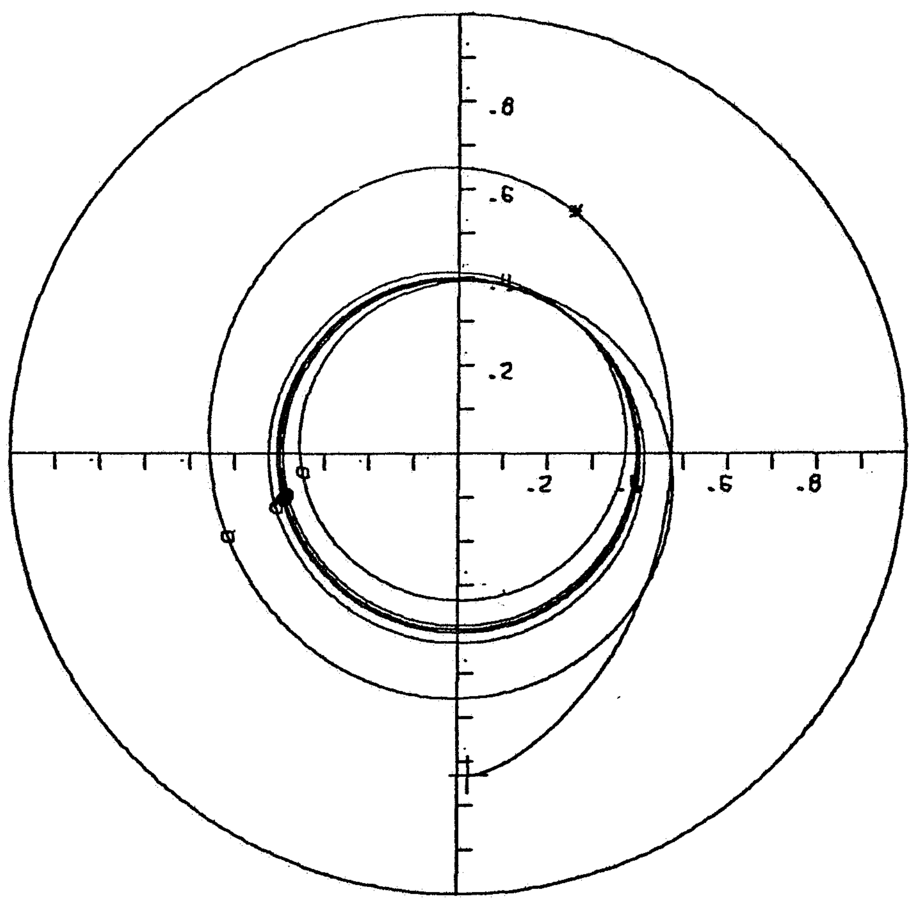

Figure 4-14. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 2.45 Mils.

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 3257411

W =	675.0 LBS	N =	10500 RPM
L =	2.000 IN	R =	3.500 IN
C =	7.00 MILS	MU =	2.490 MICROREYNS
PS =	0.00 PSI	FMAX=	15891.4 LBS
WX -	0.00 LBS	WY =	0.00 LBS
FU =	7399.81 LBS	EMU =	.50
KRX =	50000 LB/IN	KRY =	50000 LB/IN
TRD =	2.15	PMAX=	4699.92 PSI

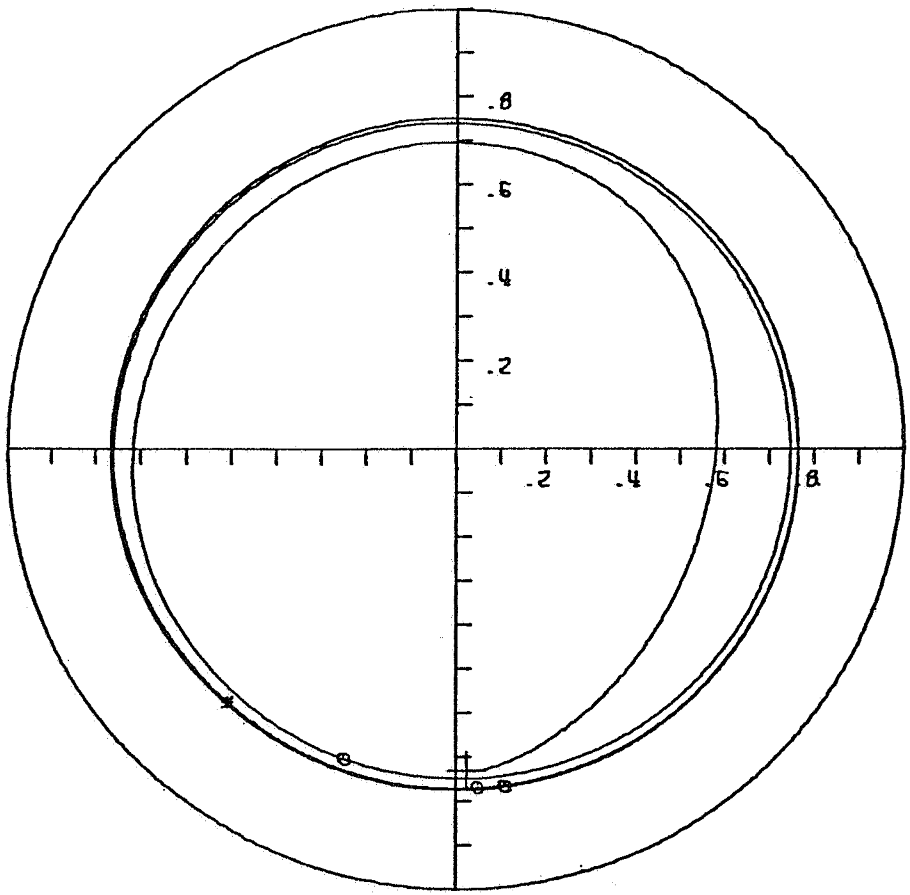

Figure 4-15. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 3.50 Mils.

levels until the machine is shut down or its operating conditions changed can be determined. Note the shift in phase angle from 180° to 90° as the eccentricity increases.

It has been shown that retainer springs help center the journal and reduce the vibration amplitude. They may also be used to prevent oil leakage from the end of the bearing if they are of the O ring type. As noted in Chapter 2, the uncavitated film provides no equivalent stiffness when the journal is operating in synchronous precession about the bearing center. In this case the use of a retainer spring to provide a restoring force in the bearing is necessary. Although an increase in stiffness results in centering the journal in the bearing, (see Figures (4-16) (4-19) the magnitude of the transmitted force is minimized for some intermediate value of stiffness which depends on the bearing parameters and loading. Also the ability of the journal to quickly return to a stable, steady state operating condition is impaired with increasing stiffness. Both of these operating conditions were indicated by the stability maps in Chapter 3.

After the preliminary bearing design, a more thorough analytic study may be made by incorporating the damper bearing effects into a program which includes the dynamic effects of the entire system. Although such programs are not readily available for many complex systems, one has been developed for a three mass rotor in journal bearings on a squeeze film bearing support as noted earlier. The rotor bearing model is shown in Figure (3-1). This program may be used to calculate the ability of squeeze

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 3257412

W =	675.0 LBS	N =	10500 RPM
L =	2.000 IN	R =	3.500 IN
C =	15.00 MILS	MU =	2.490 MICROREYNS
PS =	0.00 PSI	FMAX=	1867.2 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	951.40 LBS	EMU =	.03
KRX =	0 LB/IN	KRY =	0 LB/IN
TRD =	1.96	PMAX=	381.39 PSI

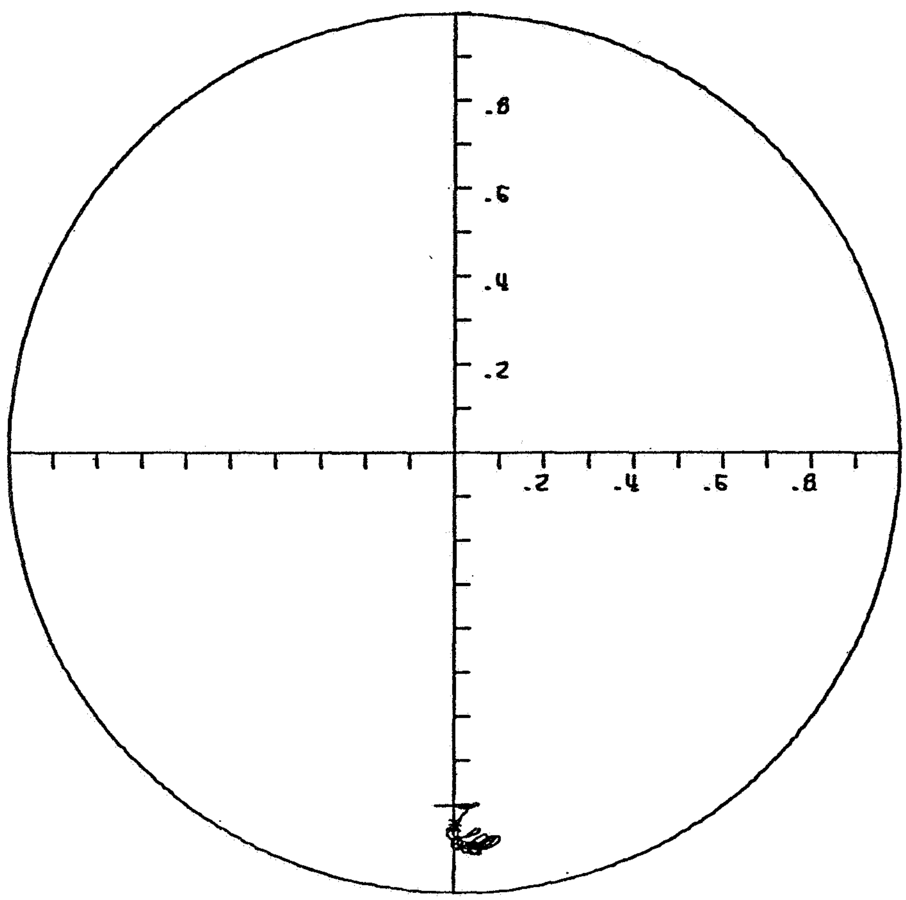

Figure 4-16. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, KR = 0 LB/IN.

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 3257113

W =	675.0 LBS	N =	10500 RPM
L =	2.000 IN	R =	3.500 IN
C =	15.00 MILS	MU =	2.490 MICROREYNS
PS =	0.00 PSI	FMAX=	1054.1 LBS
WX =	0.00 LBS	WY =	0.00 LBS
FU =	951.40 LBS	EMU =	.03
KRX =	50000 LB/IN	KRY =	50000 LB/IN
TRD =	1.11	PMAX=	99.41 PSI

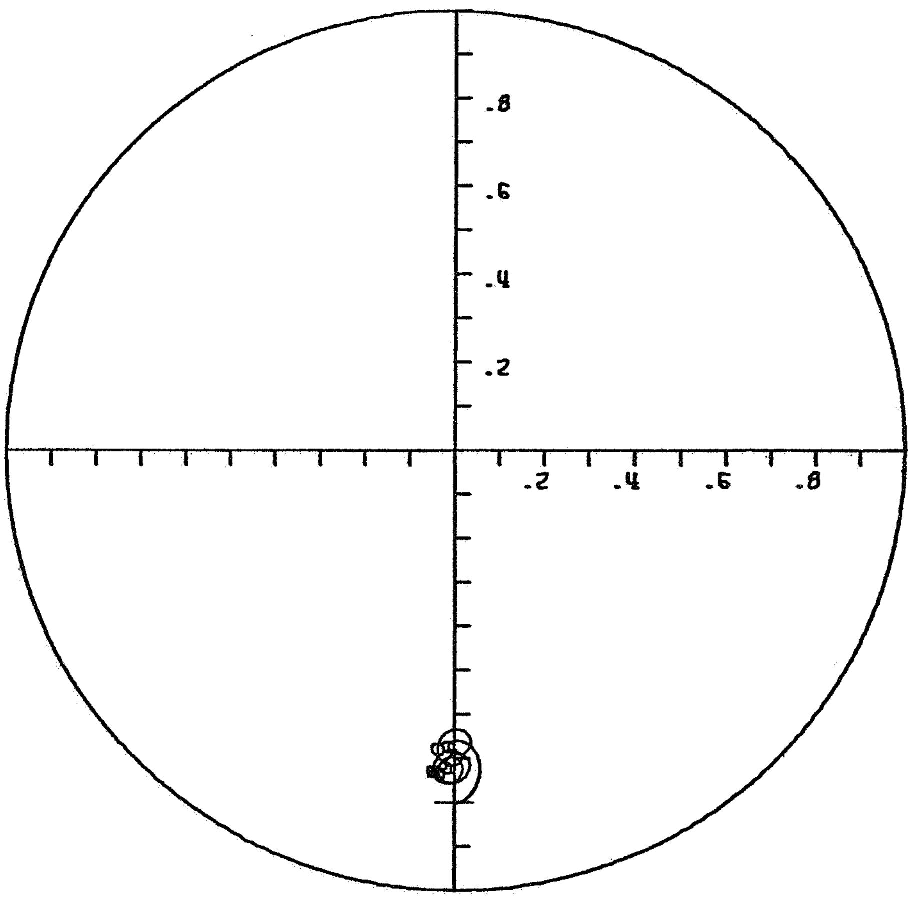

Figure 4-17. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, KR = 50,000 LB/IN.

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO.

W =	675.0	LBS	N =	10500	RPM
L =	2.000	IN	R =	3.500	IN
C =	15.00	MILS	MU =	2.490	MICROREYNS
PS =	0.00	PSI	FMAX=	936.7	LBS
WX =	0.00	LBS	WY =	0.00	LBS
FU =	951.40	LBS	EMU =	.03	
KRX =	100000	LB/IN	KRY =	100000	LB/IN
TRD =	.98		PMAX=	6.10	PSI

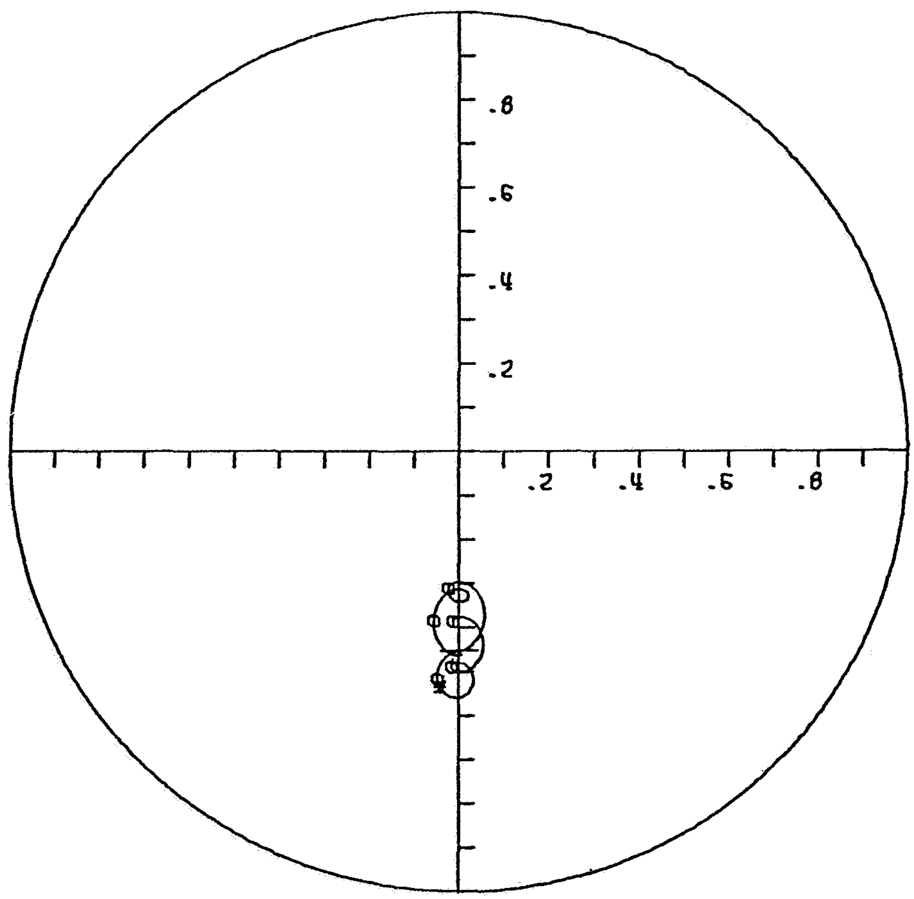

Figure 4-18. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, KR = 100,000 LB/IN.

SQUEEZE FILM BEARING
CAVITATED FILM
HORIZONTAL

CASE NO. 3257U15

W	=	675.0 LBS	N	=	10500 RPM
L	=	2.000 IN	R	=	3.500 IN
C	=	15.00 MILS	MU	=	2.490 MICROREYNS
PS	=	0.00 PSI	FMAX	=	1029.3 LBS
WX	=	0.00 LBS	WY	=	0.00 LBS
FU	=	951.40 LBS	EMU	=	.03
KRX	=	200000 LB/IN	KRY	=	200000 LB/IN
TRD	=	1.08	PMAX	=	9.18 PSI

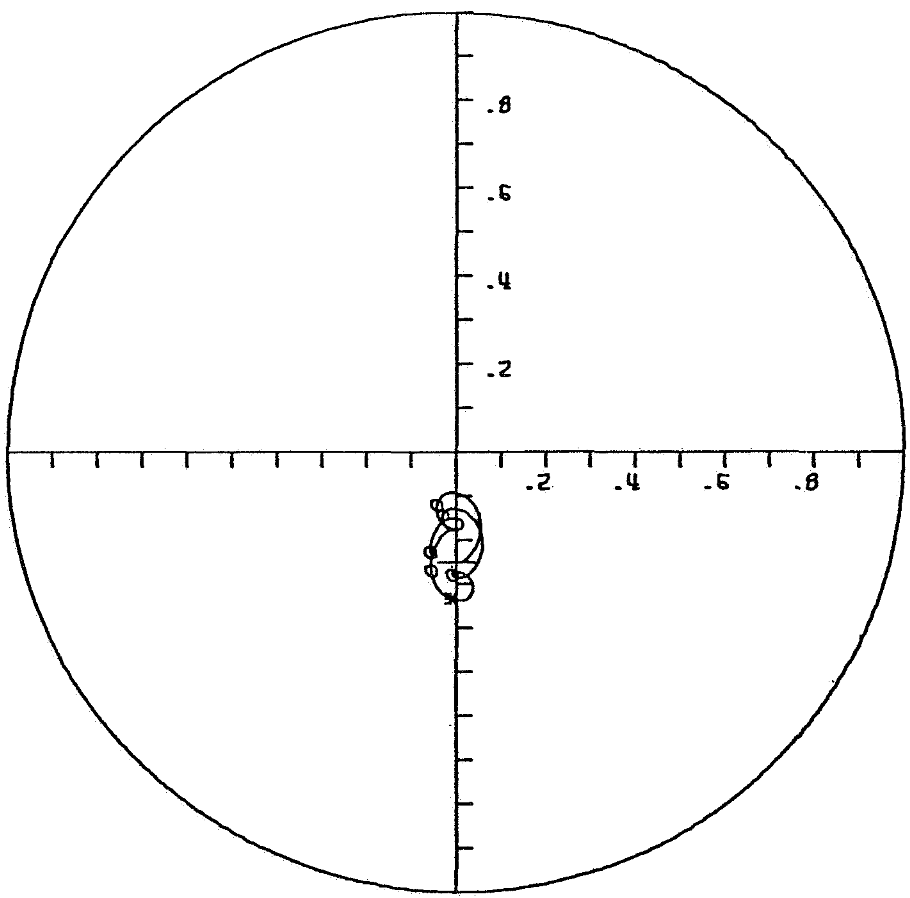

Figure 4-19. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, KR = 200,000 LB/IN.

film bearings to stabilize a multi-mass rotor. In addtion, information may be obtained to verify the stability maps obtained by other means (see Chapter 3). Because the bearing is initially designed using the criteria derived from a stability analysis such verification is useful in judging the overall worth and limitations of the analytical design process. Using these analytical methods leads to a more efficient testing program because unacceptable bearing designs are eliminated before testing begins.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

The problem of rotor stability is of current importance because of the high speeds and complex dynamics of modern rotor bearing systems. Damped flexible supports have a great effect on the ability of a system to suppress unstable whirl. Therefore there is a need for methods of predicting rotor instability and obtaining bearing designs that provide the necessary support characteristics.

5.1 PREDICTING ROTOR INSTABILITY

Methods for determining the stability of rotor bearing systems include:

1. Using the rotor-bearing system equations of motion including the effects of aerodynamic forces, shaft damping and internal friction to obtain the system characteristic equation. The roots of this equation show the stability and natural frequencies of the system.
2. Using finite element and transfer matrix methods to obtain the system stability and natural frequencies.
3. Applying Routh stability criterion to the system characteristic equation.

The third method does not yield information on relative stability but only determines whether or not the system is absolutely stable. The first two methods provide relative stability information that can be used to plot stability contours as functions of the support characteristics. The stability maps presented show that for a given value of support stiffness there is a range of damping values which will stabilize the system. If the support stiffness becomes too large, the system will be unstable for all values of support damping. The optimum stiffness and damping values for a particular system depend upon the rotor-bearing properties and the nature and magnitude of the forces acting on the system that produce instability.

5.2 DETERMINING THE STIFFNESS AND DAMPING COEFFICIENTS OF THE SQUEEZE FILM DAMPER BEARING

The assumption of steady state circular synchronous precession of the journal and the use of a rotating coordinate system allow the bearing forces to be equated to equivalent stiffness and damping forces. This establishes stiffness and damping coefficients for the bearing. These coefficients are functions of the bearing geometry, the use of end seals and cavitation of the fluid film. The coefficients obtained from the steady state bearing analysis are compared with the values from the stability maps to determine a bearing configuration that promotes system stability.

The stiffness and damping coefficients are non-linear functions of the journal eccentricity. However for $\varepsilon<0.4$ the coefficients do not change appreciably with changes in eccentricity.

For values of $\varepsilon>0.4$ a rapid increase in the coefficients occurs. The high stiffness developed can cause system instability or raise the system critical speed above the operating speed resulting in force transmissabilities greater than 1. For these reasons a design criteria of $\varepsilon<0.4$ has been established. If the fluid film does not cavitate a radial stiffness is not developed and must be supplied by retainer springs.

5.3 TRANSIENT ANALYSIS

The bearings designed using steady state analysis are further analyzed using transient response programs. The motion of a system under the influence of unbalance and other external forces is monitored. Effect of retainer springs to preload the bearing can be determined and the bearing design further refined.

The accuracy of transient response programs are dependent on the accuracy of the numerical integration methods employed. At high eccentricities very small integration step sizes are required to retain high accuracy in the solution because of the rapid variation in the bearing forces. The cost of obtaining high accuracy at high eccentricity does not justify using small time steps or complicated integration methods requiring many functional evaluations because optimum bearing design requires operation at low eccentricities. The information provided by simpler integration methods is sufficient to indicate trends in bearing operation at high eccentricities.

5.4 ADVANTAGES OF BEARING SIMULATION

The analytic simulation of the squeeze film damper bearing eliminates many bearing designs that would result in bearing or machine failure in a test installation. Because the cost of constructing and instrumenting a test rig is very high, preventing the failure of these machines is important. The time involved in manufacturing and testing bearings is also great and the elimination of unsuitable designs by analytic procedures results in a substantial savings in time and money.

A good test program is essential, however, to determine the actual rotor-bearing response under various conditions. The analytic simulation provides a means of interpreting the test data. Often the nature of the actual system excitation is unknown and the actual system response must be used to infer the nature of these excitations. Where it is possible to systematically vary the excitation the experimental results provide a check on the accuracy and limitations of the analytic simulation.

5.5 LIMITATIONS OF ANALYTICAL INVESTIGATIONS

In any analytical investigation it is very important to know the assumptions made in analyzing the problem. In deriving the bearing equations for this study several assumptions were made. To obtain the Reynolds equation from the Navier-Stokes equations it was assumed that:

1. The viscosity is constant
2. The flow is steady state
3. The fluid inertia terms are negligibly small

4. The density is constant

5. There is no flow in the radial direction

6. There is no pressure gradient in the radial direction

The Reynolds equation obtained was modified by using the short bearing approximation. The assumption made was that the pressure gradient in the tangential direction is small and when multiplied by h^3 it is very small in comparison to other terms containing only h. This assumption is valid only if the tangential pressure gradient is small, and at high eccentricity ratios this assumption may not be valid. For this reason the design criterion is that the eccentricity ratio be less than 0.4. In addition the short bearing approximation is valid for $\frac{L}{D} < 0.5$. For ratios greater than this the axial pressure gradient is not large compared to the tangential gradient. For $\frac{L}{D} > 0.5$ either the long bearing approximation or finite bearing techniques should be used.

The conditions under which cavitation occurs must be modified in light of the current experimental results. In this study cavitation conditions were assumed to be the same in squeeze bearings as in journal bearings.

5.6 RECOMMENDATIONS FOR FUTURE RESEARCH

1. Construct an experimental rotor with squeeze damper supports to provide data to verify the analytic squeeze bearing model.

2. Conduct analytic and experimental research into the conditions under which the squeeze bearing fluid film cavitates

and to determine how cavitation propagates through the film.

3. Investigate the heat transfer characteristics of the bearings and provide modifications to the bearing programs to account for variable viscosity of the lubricant.

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APPENDIX A

DESCRIPTION OF PROGRAM SQFDAMP

This program analyzes the stiffness, damping and pressure characteristics of the squeeze film damper bearing. Three bearing configurations may be analyzed:

1. Plain bearing without end seals or circumferential oil supply groove.
2. Bearing without end seals but with circumferential oil supply groove.
3. Bearing with both end seals and circumferential oil supply groove.

In addition the fluid film may be assumed to be cavitated or uncavitated. If cavitated the film extends from $\theta = \frac{\pi}{2}$ to $\theta = \frac{3\pi}{2}$. If uncavitated it extends from $\theta = o$ to $\theta = 2\pi$. The θ is measured from the line of centers of the bearing in the direction of journal precession. The evaluation of the bearing characteristics assumes that the journal precesses synchronously about the bearing center in a circular orbit.

The following is a description of the program input data:

CARD 1 -80 column free-field comment card.

CARD 2 -80 column free-field comment card.

CARD 3 -Namelist/BRGTYPE/ TYP, CAV, PS

TYP - 0 for bearing type 1 (see above)
- 1 for bearing type 2 (see above)
- 2 for bearing type 3 (see above)

CAV - 0 for uncavitated film
- 1 for cavitated film

PS - oil supply pressure, psi.

CARD 4 Namelist/BEARING/ L,R, MU, N

L - Bearing length, in.

R - Bearing radius, in.

MU - Lubricant viscosity, microreyns

N - Rotor speed (journal precession rate), RPM

CARD 5 Namelist/ECRATIO/ES,EF

ES - initial journal eccentricity ratio ES>0

EF - final journal eccentricity ratio EF<1

CARD 6 Namelist/CLEARNC/ C(I), NC

C(I) - clearance

NC - Number of clearance values

CARD 7 Namelist/PLOTSEM/ CS, PC, PK, PP

CS - Plot control, .T. if plot desired, otherwise .F.

PC - .T. if damping plot desired, otherwise .F.

PK - .T. if stiffness plot desired, otherwise .F.

(if CAV=0, PK = .F.)

PP - .T. if pressure plot desired, otherwise .F.

Sample input data.

CARD	DATA
1	SAMPLE DATA FOR SQFDAMP
2	1 MARCH 1973
3	$BRGTYP TYP = 0 , CAV = 1, PS = 0.0$
4	$BEARING L = 0.90, R = 2.55, MU = 0.382, N = 16800.0$
5	$ECRATIO ES = 0.1, EF = 0.9$

6 $CLEARANC C(1) = .003, C(2) = .004, C(3) = .005
C(4) = .006, NC = 4$

7 $PLOTSEM CS = .T., PC = .T., PK = .T., PP = .T.$

The following is a listing of the program and a sample output.

```
      PROGRAM SQFDAMP(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
C
C                        VERSION - 13 MARCH 1973
C
      REAL L,MU,KO,N
      LOGICAL CRING,CS,PC,PK,PP
      INTEGER COMENT1,COMENT2,TYP
      INTEGER CAV
      DIMENSION EO(135),CO(135),KO(135),PMAX(135),THETAM(135),C(5),
     1COMENT1(8),COMENT2(8)
      NAMELIST/BRGTYPE/ TYP,CAV,FS
      NAMELIST/BEARING/ L,R,MU,N
      NAMELIST/ECRATIO/ ES,EF
      NAMELIST/CLEARNC/ C,NC
      NAMELIST/PLOTSEM/ CS,PC,PK,PP
      COMMON NEO
C
C     READ DATA
C
      K=1
  900 READ(5,1) COMENT1
    1 FORMAT(8A10)
      IF(EOF,5) 9999,2
    2 READ(5,1) COMENT2
      READ(5,BRGTYPE)
      READ(5,BEARING)
      READ(5,ECRATIO)
      READ(5,CLEARNC)
      READ(5,PLOTSEM)
      IF(.NOT. CS) GOTO 3
      IF(K.EQ.0) GOTO 3
      CALL CALCOMP(1)
C
C     SET UP NEW BLOCK AND MOVE ORIGIN
      CALL PLOT(2.0,0.5,-3)
C
C     WRITE PROGRAM DESCRIPTION
C
    3 WRITE(6,4)
    4 FORMAT(1H1,35X,27H****SQUEEZE FILM DAMPER****/)
      WRITE(6,5)
    5 FORMAT(1X,
     158HTHIS PROGRAM ANALYZES THE STIFFNESS, DAMPING AND PRESSURE /1X,
     258HCHARACTERISTICS OF THE SQUEEZE FILM DAMPER BEARING. THREE /1X,
     358HBEARING CONFIGURATIONS MAY BE ANALYZED-                   /1X,
     458H  0 - PLAIN BEARING WITHOUT END LEAKAGE SEALS OR CIRCUM-  /1X,
     558H      FERENTIAL OIL SUPPLY GROOVE                         /1X,
     658H  1 - BEARING WITHOUT END LEAKAGE SEALS BUT WITH CIRCUM-  /1X,
     758H      FERENTIAL OIL SUPPLY GROOVE                         /1X,
     858H  2 - BEARING WITH BOTH END LEAKAGE SEALS AND SIRCUMFER-  /1X,
     958H      ENTIAL OIL SUPPLY GROOVE                            /1X,
     158HIN ADDITION, THE FILM MAY BE ASSUMED TO BE EITHER CAVI-   /1X,
     258HTATED OR UNCAVITATED. IF CAVITATED THE FILM IS ASSUMED    /1X,
     358HTO EXTEND FROM THETA=PI/2 TO THETA=3PI/2, WHERE THETA     /1X,
     458HIS MEASURED FROM THE LINE OF CENTERS IN THE DIRECTION OF  /1X,
     558HJOURNAL PRECESSION. THE EVALUATION OF THE BEARING CHARAC- /1X,
     658HTERISTICS ASSUMES THAT THE JOURNAL PRECESSES SYNCHRONOUS- /1X,
```

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      758HLY ABCUT THE BEARING CENTER.                                   /1X, SQFD0670
      858HTHE FCLLOWING IS A DESCRIPTICN CF THE INPUT PARAMETERS-       //) SQFD0680
      WRITE(6,6)                                                              SQFD0690
6     FORMAT(1X,                                                              SQFD0700
      158HCARD 1. -80 COLUMN FREE-FIELD CCMMENT CARD                    /1X, SQFD0710
      258HCARD 2. -80 COLUMN FREE-FIELD CCMMENT CARD                    /1X, SQFD0720
      358HCARD 3. -NAMELIST/BRGTYPE/ TYP,CAV,PS                         /1X, SQFD0730
      458H              TYP - 0 FCR BEARING TYPE 0 (SEE ABOVE)          /1X, SQFD0740
      558H                    1 FCR BEARING TYPE 1 (SEE ABOVE)          /1X, SQFD0750
      658H                    2 FOR BEARING TYPE 2 (SEE ABOVE)          )     SQFD0760
      WRITE(6,8)                                                              SQFD0770
8     FORMAT(1X,                                                              SQFD0780
      158H              CAV - 0 FCR UNCAVITATED FILM                    /1X, SQFD0790
      258H                    1 FCR CAVITATED FILM                      /1X, SQFD0800
      358H                    IF CAV=0 PK=.F. (SEE CARD 7)              /1X, SQFD0810
      458H              PS  - OIL SUPPLY PRESSURE, PSI                  /1X, SQFD0820
      758HCARD 4. -NAMELIST/BEARING/ L,R,MU,N                           /1X, SQFD0830
      858H              L   - BEARING LENGTH, IN.                       /1X, SQFD0840
      958H              R   - BEARING RADIUS, IN.                       /1X, SQFD0850
      158H              MU  - LUBRICANT VISCOSITY, MICROREYNS           /1X, SQFD0860
      258H              N   - RCTCR SPEED (JOURNAL PRECESSICN RATE,RPM)/1X, SQFD0870
      358HCARD 5. -NAMELIST/ECRATIO/ ES,EF                              /1X, SQFD0880
      458H              ES  - INITIAL JCURNAL ECCENTRICITY RATIO, ES>0./1X, SQFD0890
      558H              EF  - FINAL JOURNAL ECCENTRICITY RATIO, EF<1.0 /1X, SQFD0900
      658HCARD 6. -NAMELIST/CLEARNC/ C(I),NC                            /1X, SQFD0910
      758H              C(I)- CLEARANCE VALUES, IN.   0<I≤5             /1X, SQFD0920
      858H              NC  - NUMBER OF CLEARANCE VALUES                )     SQFD0930
      WRITE(6,7)                                                              SQFD0940
7     FORMAT(1X,                                                              SQFD0950
      158HCARD 7. -NAMELIST/PLOTSEM/ CS,PC,PK,PP                        /1X, SQFD0960
      258H              CS  - PLOT CONTROL, .T. IF PLOT DESIRED,        /1X, SQFD0970
      358H                    OTHERWISE .F.                             /1X, SQFD0980
      458H              PC  - .T. IF DAMP PLOT DESIRED, OTHERWISE .F.  /1X, SQFD0990
      558H              PK  - .T. IF STIFF. PLOT DESIRED, OTHERWISE .F./1X, SQFD1000
      658H              PP  - .T. IF PRESS PLOT DESIRED, OTHERWISE .F. /1X, SQFD1010
      758HSAMPLE DATA                                                   /1X, SQFD1020
      858HCOMMENT CARD 1                                                /1X, SQFD1030
      958HCOMMENT CARD 2                                                /1X, SQFD1040
      158H $BRGTYPE TYP=0,CAV=1,PS=0.0$                                 /1X, SQFD1050
      258H $BEARING L=0.90,R=2.55,MU=0.382,N=16800$                     /1X, SQFD1060
      358H $ECRATIO ES=0.1,EF=0.9$                                      /1X, SQFD1070
      458H $CLEARNC C(1)=.003,C(2)=.004,C(3)=.005,C(4)=.006,NC=4$       /1X, SQFD1080
      558H $PLOTSEM CS=.T.,PC=.T.,PK=.T.,PP=.T.$                        )     SQFD1090
C                                                                             SQFD1100
C     WRITE OUT INPUT DATA                                                    SQFD1110
C                                                                             SQFD1120
      WRITE(6,25) COMENT1,CCMENT2                                             SQFD1130
25    FORMAT(1H1,1X,2(8A10/1X))                                               SQFD1140
      IF(TYP.EQ.0) WRITE(6,27)                                                SQFD1150
      IF(TYP.EQ.1) WRITE(6,28)                                                SQFD1160
      IF(TYP.EQ.2) WRITE(6,29)                                                SQFD1170
      IF(CAV.EQ.0) WRITE(6,31)                                                SQFD1180
      IF(CAV.EQ.1) WRITE(6,32)                                                SQFD1190
27    FORMAT(/1X,                                                             SQFD1200
      158HPLAIN BEARING, NO END SEALS OR OIL SUPPLY GROOVE              /)    SQFD1210
28    FORMAT(/1X,                                                             SQFD1220
      158HBEARING WITH NO END SEALS BUT WITH OIL SUPPLY GROOVE          /)    SQFD1230
29    FORMAT(/1X,                                                             SQFD1240
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     158HBEARING WITH END SEALS AND OIL SUPPLY GROOVE                    SQFD1250
  31  FORMAT(1X,16HUNCAVITATED FILM/)                                    SQFD1260
  32  FORMAT(1X,14HCAVITATED FILM/)                                      SQFD1270
      WRITE(6,10) L,R,N,MU,PS,ES,EF,NC                                   SQFD1280
  10  FORMAT(////1X,15HBEARING LENGTH=,F8.2,                             SQFD1290
     128H  INCHES      BEARING RADIUS=,F8.2,12H  INCHES                  SQFD1300
     21X,15HN=             ,F8.1,                                        SQFD1310
     328H  RPM         MU             ,F8.3,12H  MICROREYNS/             SQFD1320
     41X,15HPS=            ,F8.1,                                        SQFD1330
     528H  PSI         ES=            ,F8.3/                             SQFD1340
     61X,15HEF=            ,F8.3,                                        SQFD1350
     728H              NC=            ,I8//)                             SQFD1360
      WRITE(6,11) (C(J),J=1,NC)                                          SQFD1370
  11  FORMAT(58X,19HCLEARANCES - INCHES//5(64X,F7.5/)////)               SQFD1380
C                                                                        SQFD1390
C     INITIALIZE CONSTANTS                                               SQFD1400
C                                                                        SQFD1410
      NEO=25                                                             SQFD1420
      DELE=(EF-ES)/(NEO-1)                                               SQFD1430
      PI=3.55/1.13                                                       SQFD1440
      W=N*0.10472                                                        SQFD1450
      MU=MU*1.0E-6                                                       SQFD1460
C                                                                        SQFD1470
C     OUTER LOOP FOR CLEARANCE VALUES                                    SQFD1480
C                                                                        SQFD1490
      DO 100 II=1,NC                                                     SQFD1500
      WRITE(6,12) C(II)                                                  SQFD1510
  12  FORMAT(1H1,13X,6H C=    ,F10.4,4H IN.//                            SQFD1520
     11X,94H          EO                       CO                   KO   SQFD1530
     2           PMAX                  THETA/                            SQFD1540
     31X,97H        (DIM)                    LB-SEC/IN            LB/IN  SQFD1550
     4         LB/IN**2                DEGREES)                          SQFD1560
C                                                                        SQFD1570
C     INNER LOOP FOR ECCENTRICITY RANGE                                  SQFD1580
C                                                                        SQFD1590
C                                                                        SQFD1600
      DO 20 I=1,NEO                                                      SQFD1610
      KK=I+(I-1)*(NC-1)+II-1                                             SQFD1620
      EO(KK)=(I-1)*DELE      + ES                                        SQFD1630
      IF(CAV.EQ.0) GOTO 26                                               SQFD1640
      CO(KK)=  (MU*(L**3)*R/(2.*(C(II)**3)))                             SQFD1650
      CO(KK)=CO(KK)*(PI/((1.-EO(KK)**2))**(3./2.))                       SQFD1660
      KO(KK)=(2.*MU*W*R*((L/C(II))**3)*EO(KK))                           SQFD1670
      KO(KK)=KO(KK)/((1.-(EO(KK))**2)**(2))                              SQFD1680
  26  THETAM(KK)=270.443 - 191.831*EO(KK) + 218.223*(EO(KK)**2)          SQFD1690
      THETAM(KK)=THETAM(KK) - 114.803*(EO(KK)**3)                        SQFD1700
      THETA=THETAM(KK)/57.29577751                                       SQFD1710
      PMAX(KK)=-1.5*((L/C(II))**2)*MU*W*EO(KK)*SIN(THETA)                SQFD1720
      PMAX(KK)=PMAX(KK)/((1.+EO(KK)*COS(THETA))**3)                      SQFD1730
      IF(CAV.EQ.1) GOTO 40                                               SQFD1740
      KO(KK)=0.0                                                         SQFD1750
      CO(KK)=MU*((L/C(II))**3)*R*PI                                      SQFD1760
      CO(KK)=CO(KK)/((1.-EO(KK)**2)**(3./2.))                            SQFD1770
  40  IF(TYP.NE.1) GOTO 20                                               SQFD1780
      PMAX(KK)=PMAX(KK)/2.                                               SQFD1790
      CO(KK)=CO(KK)/4.                                                   SQFD1800
      KO(KK)=KO(KK)/4.                                                   SQFD1810
  20  CONTINUE                                                           SQFD1820
```

```
300   NN=NEO*NC                                                      SQFD1830
      DO 30 III=II,NN,NC                                             SQFD1840
      I=III                                                          SQFD1850
      WRITE(6,13) EO(I),CO(I),KO(I),PMAX(I),THETAM(I)                SQFD1860
13    FORMAT(5(5X,F12.3,5X))                                         SQFD1870
30    CONTINUE                                                       SQFD1880
100   CONTINUE                                                       SQFD1890
      IF(.NOT. CS) GOTO 1000                                         SQFD1900
      NP1=NC*NEO+1                                                   SQFD1910
      NP2=NC*NEO+NC                                                  SQFD1920
      DO 400 I=NP1,NP2                                               SQFD1930
400   EO(I)=0.0                                                      SQFD1940
      NP1=NP1+NC                                                     SQFD1950
      NP2=NP2+NC                                                     SQFD1960
      DO 500 I=NP1,NP2                                               SQFD1970
500   EO(I)=0.16667                                                  SQFD1980
      NP=NC*NEO                                                      SQFD1990
      CALL PLOTTER(NP,L,R,MU,N,PS,ORING,COMENT1,COMENT2,NC,C,        SQFD2000
     1EO,CO,KO,PMAX,PC,PK,PP,TYP,CAV)                                SQFD2010
      K=0                                                            SQFD2020
1000  GOTO 900                                                       SQFD2030
9999  STOP                                                           SQFD2040
      END                                                            SQFD2050
```

```
      SUBRCUTINE PLOTTER(NP,L,R,MU,N,PS,ORING,COMENT1,COMENT2,NC,C,
     1EO,CO,KO,PMAX,PC,PK,PP,TYP,CAV)
C
C     THIS SUBROUTINE PLCTS CO,KO,AND PMAX AS FUNCTIONS OF EO
C     EACH SET OF CLEARANCE VALUES REQUIRES CNE BLOCK
C
      DIMENSICN C(5),COMENT1(8),COMENT2(8),CLABEL(2)
      INTEGER COMENT1,COMENT2,CLABEL,TYP
      INTEGER CAV
      REAL L,MU,N,KO,MU1
      LOGICAL CRING,PC,PK,PP
      DIMENSICN EO(135),CO(135),KO(135),PMAX(135)
      COMMON NEO
      DATA CLABEL(1),CLAEEL(2)/
     110HC=        M,10HILS        /
C
      NNP=NP/NC
      DELE=(EC(NP)-EO(1))/NNP
      NEO=NNP
      MU1=MU*1.0E6
      IF(.NOT. PC) GOTO 101
C
C     PLOT CO
C
      JAXIS=1
      CALL GRID(L,R,N,MU1,PS,CCMENT1,CCMENT2,JAXIS,EO,CC,KO,PMAX,
     1NP,TYP,CAV)
      NP1=NC*NEO+1
      NP2=NC*NEO+NC
      DO 400 I=NP1,NP2
  400 CO(I)=0.0
      NP1=NP1+NC
      NP2=NP2+NC
      DO 500 I=NP1,NP2
  500 CO(I)=1.0
      DO 30 I=1,NC
      CALL LINE(EO(I),CO(I),NNP,NC,0,0)
C
C     DECIDE WHETHER LABELING SHOULD BE OVER CURVE OR
C     AT END OF CURVE.  IF MAXIMUM ECCENTRICITY [ 0.7
C     PLACE LABELING OVER CURVE, OTHERWISE AT END
C
      IF(EC(NE) .GE. 0.7) GCTO 60
C
C     DETERMINE THE X AND Y CCCRDINATES OF THE LOWER
C     LEFT HAND CORNER CF THE LABELING AND THE ANGLE
C     IT MAKES WITH THE X-AXIS
C
      XX=(EO(NE)/0.16667) + 0.1
      III=NP-NC+I
      YY=CO(III) + 0.1
      THETA=ATAN((CO(III)-CC(III-10*NC))/((EO(III)-EO(III-10*NC))*6.0))
      THETA1=THETA*57.2958
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
C
C     DETERMINE THE X ANC Y COORDINATES OF THE LOWER
C     LEFT HAND CORNER CF THE CLEARANCE VALUE TO BE
```

```
C     PLOTTED
C
      XX=XX+0.26*COS(THETA)
      YY=YY+0.26*SIN(THETA)
      CV=1000.0*C(I)
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
      GOTO 30
C
C     DETERMINE THE X AND Y COORDINATES OF THE LOWER
C     LEFT HAND CORNER OF THE LABELING AND THE ANGLE
C     IT MAKES WITH THE X-AXIS
C
60    XX=EO(NC*((NEO-1)/2)+1)*6.0
      III=((NEO-1)/2)*NC+I
      YY=CO(III) + 0.1
      IP=(IFIX(0.13129/DELE))*NC +III
      THETA=ATAN((CO(IP)-CO(III))/((EO(IP)-EO(III))*6.0))
      THETA1=THETA*57.2958
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
C
C     DETERMINE THE X AND Y COORDINATES OF THE LOWER
C     LEFT HAND CORNER OF THE CLEARANCE VALUE TO BE
C     PLOOTED
C
      XX=XX+ 0.26*COS(THETA)
      YY=YY + 0.26*SIN(THETA)
      CV=1000.0*C(I)
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
30    CONTINUE
      CALL PLOT(15.0,0.0,-6)
101   IF(.NOT. PK) GOTO 201
      JAXIS=2
      CALL GRID(L,R,N,MU1,PS,COMENT1,COMENT2,JAXIS,EO,CO,KO,PMAX,
     1NP,TYP,CAV)
      NP1=NC*NEO+1
      NP2=NC*NEO+NC
      DO 401 I=NP1,NP2
401   KO(I)=0.0
      NP1=NP1+NC
      NP2=NP2+NC
      DO 501 I=NP1,NP2
501   KO(I)=1.0
      DO 40 I=1,NC
      CALL LINE(EO(I),KO(I),NNE,NC,0,0)
      IF(EO(NP) .GE. 0.7) GOTO 70
      XX=EO(NP)*6.0 +0.1
      III=NP-NC+I
      YY=KO(III) + 0.1
      THETA=ATAN((KO(III)-KO(III-10*NC))/((EO(III)-EO(III-10*NC))*6.0))
      THETA1=THETA*57.2958
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
      XX=XX+ 0.26*COS(THETA)
      YY=YY + 0.26*SIN(THETA)
      CV=1000.0*C(I)
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
      GOTO 40
70    XX=EO(NC*((NEO-1)/2)+1)*6.0
      III=((NEO-1)/2)*NC+I
```

```
      IP=(IFIX(0.13129/DELE))*NC +III                                   SQFD3210
      YY=KO(III)+0.1                                                    SQFD3220
      THETA=ATAN((KO(IP)-KO(III))/((EO(IP)-EO(III))*6.0))               SQFD3230
      THETA1=THETA*57.2958                                              SQFD3240
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)                           SQFD3250
      XX=XX+0.26*COS(THETA)                                             SQFD3260
      YY=YY+0.26*SIN(THETA)                                             SQFD3270
      CV=1000.0*C(I)                                                    SQFD3280
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)                           SQFD3290
   40 CONTINUE                                                          SQFD3300
      CALL PLOT(15.0,0.0,-6)                                            SQFD3310
  201 IF(.NOT. PP) RETURN                                               SQFD3320
      JAXIS=3                                                           SQFD3330
      CALL GRID(L,R,N,MU1,PS,CCMENT1,CCMENT2,JAXIS,EO,CO,KO,PMAX,       SQFD3340
     1NP,TYP,CAV)                                                       SQFD3350
      NP1=NC*NEO+1                                                      SQFD3360
      NP2=NC*NEO+NC                                                     SQFD3370
      DO 402 I=NP1,NP2                                                  SQFD3380
  402 PMAX(I)=0.0                                                       SQFD3390
      NP1=NP1+NC                                                        SQFD3400
      NP2=NP2+NC                                                        SQFD3410
      DO 502 I=NP1,NP2                                                  SQFD3420
  502 PMAX(I)=1.0                                                       SQFD3430
      DO 50 I=1,NC                                                      SQFD3440
      CALL LINE(EO(I),PMAX(I),NNF,NC,0,0)                               SQFD3450
      IF(EO(NP) .GE. 0.7) GOTO 80                                       SQFD3460
      XX=EO(NP)*6.0 +0.1                                                SQFD3470
      III=NP-NC + I                                                     SQFD3480
      YY=PMAX(III) + 0.1                                                SQFD3490
      THETA=ATAN((PMAX(III)-PMAX(III-10*NC))/                           SQFD3500
     1((EO(III)-EO(III-10*NC))*6.0))                                    SQFD3510
      THETA1=THETA*57.2958                                              SQFD3520
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)                           SQFD3530
      XX=XX+0.26*COS(THETA)                                             SQFD3540
      YY=YY+0.26*SIN(THETA)                                             SQFD3550
      CV=1000.0*C(I)                                                    SQFD3560
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)                           SQFD3570
      GOTO 50                                                           SQFD3580
   80 XX=EO(NC*((NEO-1)/2)+1)*6.0                                       SQFD3590
      III=((NEO-1)/2)*NC+I                                              SQFD3600
      YY=PMAX(III)+0.1                                                  SQFD3610
      IP=(IFIX(0.13129/DELE))*NC +III                                   SQFD3620
      THETA=ATAN((PMAX(IP)-PMAX(III))/((EO(IP)-EO(III))*6.0))           SQFD3630
      THETA1=THETA*57.2958                                              SQFD3640
      CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)                           SQFD3650
      XX=XX+0.26*COS(THETA)                                             SQFD3660
      YY=YY+0.26*SIN(THETA)                                             SQFD3670
      CV=1000.0*C(I)                                                    SQFD3680
      CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)                           SQFD3690
   50 CONTINUE                                                          SQFD3700
      CALL PLOT(15.0,0.0,-6)                                            SQFD3710
      RETURN                                                            SQFD3720
      END                                                               SQFD3730
```

```
      SUBROUTINE GRID(L,R,N,MU1,PS,COMENT1,CCMENT2,JAXIS,EO,CO,KO,PMAX, SQFD3740
     1NP,TYP,CAV)                                                      SQFD3750
C                                                                      SQFD3760
C     THIS SUBROUTINE PLOTS THE GRID AND LABELS THE PLOTS              SQFD3770
C                                                                      SQFD3780
      DIMENSION COMENT1(8),COMENT2(8),EO(135),CO(135),KC(135),         SQFD3790
     1PMAX(135),LEGEND1(2),LEGEND2(2),LEGEND3(2),LEGEND4(2),LEGEND5(3), SQFD3800
     2HEADER1(2),HEADER2(3),XLABEL(2),YLABEL1(3),YLABEL2(3),           SQFD3810
     3YLABEL3(3)                                                       SQFD3820
      REAL L,N,MU1,KO                                                  SQFD3830
      INTEGER CAV                                                      SQFD3840
      INTEGER TYP,HEADER3(3),HEADER4(3),HEADER5(2),HEADER6(2)          SQFD3850
      INTEGER COMENT1,CCMENT2,LEGEND1,LEGEND2,LEGEND3,LEGEND4,         SQFD3860
     1LEGEND5,HEADER1,HEADER2,XLABEL,YLABEL1,YLABEL2,YLABEL3           SQFD3870
      COMMON NEO                                                       SQFD3880
      DATA LEGEND1(1),LEGEND1(2),LEGENC2(1),LEGEND2(2),                SQFD3890
     1LEGEND3(1),LEGEND3(2),LEGEND4(1),LEGEND4(2),LEGEND5(1),          SQFD3900
     2LEGEND5(2),LEGEND5(3),HEADER1(1),HEADER1(2),                     SQFD3910
     3HEADER2(1),HEADER2(2),HEADER2(3),XLABEL(1),XLABEL(2),            SQFD3920
     4          YLABEL1(1),YLABEL1(2),YLABEL1(3),YLABEL2(1),           SQFD3930
     5YLABEL2(2),YLABEL2(3),YLABEL3(1),YLABEL3(2),YLABEL3(3)/          SQFD3940
     610HN=          ,10H  RPM.    ,10HR=        I,10HN.         ,     SQFD3950
     710HL=         I,10HN.          ,10HPS=        ,10H  PSI.    ,    SQFD3960
     810HMU=         ,10H MICROREYN,10HS           ,10HSQUEEZE FI,     SQFD3970
     910HLM DAMPER ,10HNC SEALS C,10HR OIL SUPP,10HLY  GROOVE,         SQFD3980
     110HECCENTRICI,10HTY - (DIM),10HDAMPING,  ,10HCO - (LB-S,         SQFD3990
     210HEC/IN)    ,10HSTIFFNESS,,10H KO - (LB/,10HIN)       ,         SQFD4000
     310HMAXIMUM PR,10HESSURE - (,10HPSI)      /                       SQFD4010
      DATA HEADER3(1),HEADER3(2),HEADER3(3),HEADER4(1),                SQFD4020
     1HEADER4(2),HEADER4(3),HEADER5(1),HEADER5(2),                     SQFD4030
     2HEADER6(1),HEADER6(2)/                                           SQFD4040
     310HOIL SUPPLY,10H GROOVE  -,10H  NO SEALS,10HOIL - SUPPL,        SQFD4050
     410HY  GROOVE ,10H AND SEALS,10H    NO  CAV,10HITATION    ,       SQFD4060
     510H     CAVIT,10HATION     /                                     SQFD4070
      ICYCLES=0                                                        SQFD4080
      LOEXP=0                                                          SQFD4090
      GO TO (100,200,300),JAXIS                                        SQFD4100
  100 CALL LOGSCAL(CO,NP,1,8.0,LCEXP,ICYCLES,NP+2)                     SQFD4110
      GOTO 150                                                         SQFD4120
  200 CALL LOGSCAL(KO,NP,1,8.0,LCEXP,ICYCLES,NP+2)                     SQFD4130
      GOTO 150                                                         SQFD4140
  300 CALL LOGSCAL(PMAX,NF,1,8.0,LOEXP,ICYCLES,NP+2)                   SQFD4150
  150 CONTINUE                                                         SQFD4160
      NNC=(NP/NEO)+1                                                   SQFD4170
      CALL AXIS1(0.0,0.0,XLABEL,-20,6.0,0.0,EO(NP+1),EO(NP+NNC),16.667) SQFD4180
      GO TO(101,201,301),JAXIS                                         SQFD4190
  101 CALL LOGAXIS(0.0,0.0,YLABEL1,26,+8.0,90.0,LOEXP,ICYCLES)         SQFD4200
      GOTO 160                                                         SQFD4210
  201 CALL LOGAXIS(0.0,0.0,YLABEL2,23,+8.0,90.0,LOEXP,ICYCLES)         SQFD4220
      GOTO 160                                                         SQFD4230
  301 CALL LOGAXIS(0.0,0.0,YLABEL3,24,+8.0,90.0,LOEXP,ICYCLES)         SQFD4240
  160 CALL PLOT(0.0,8.0,3)                                             SQFD4250
      CALL PLOT(6.0,8.0,2)                                             SQFD4260
      CALL LOGAXIS(6.0,0.0,2H  ,0,+8.0,-90.0,LOEXP,ICYCLES)            SQFD4270
C                                                                      SQFD4280
C     PLOT LEGENDS AND HEADINGS                                        SQFD4290
C                                                                      SQFD4300
```

```
      CALL SYMBOL(1.29,8.96,0.21,HEADER1,0.0,19)                      SQFD4310
      IF(TYP.EQ.0) CALL SYMBOL(0.30,8.57,0.21,HEADER2,0.0,30)         SQFD4320
      IF(TYP.EQ.1) CALL SYMBOL(0.30,8.57,0.21,HEADER3,0.0,30)         SQFD4330
      IF(TYP.EQ.2) CALL SYMBOL(0.30,8.57,0.21,HEADER4,0.0,30)         SQFD4340
      IF(CAV.EQ.0) CALL SYMBOL(1.20,8.18,0.21,HEADER5,0.0,20)         SQFD4350
      IF(CAV.EQ.1) CALL SYMBOL(1.20,8.18,0.21,HEADER6,0.0,20)         SQFD4360
      CALL SYMBOL(4.0,1.0,0.10,LEGEND1,0.0,16)                        SQFD4370
      CALL SYMBOL(4.0,0.8,0.10,LEGEND2,0.0,12)                        SQFD4380
      CALL SYMBOL(4.0,0.6,0.10,LEGEND3,0.0,12)                        SQFD4390
      CALL SYMBOL(4.0,0.4,0.10,LEGEND4,0.0,16)                        SQFD4400
      CALL SYMBOL(4.0,0.2,0.10,LEGEND5,0.0,21)                        SQFD4410
      CALL NUMBER(4.26,1.0,0.10,N,0.0,4HF8.1)                         SQFD4420
      CALL NUMBER(4.26,0.8,0.10,R,0.0,4HF5.2)                         SQFD4430
      CALL NUMBER(4.26,0.6,0.10,L,0.0,4HF5.2)                         SQFD4440
      CALL NUMBER(4.26,0.4,0.10,PS,0.0,4HF7.4)                        SQFD4450
      CALL NUMBER(4.26,0.2,0.10,MU1,0.0,4HF6.3)                       SQFD4460
      CALL SYMBOL(-0.43,-0.75,0.10,COMENT1,0.0,80)                    SQFD4470
      CALL SYMBOL(-0.43,-0.90,0.10,COMENT2,0.0,80)                    SQFD4480
      RETURN                                                          SQFD4490
      END                                                             SQFD4500
```

```
      SUBROUTINE LOGSCAL(Y,N,K,S,LO,CYCLES,NN)
C
C    THIS SUBROUTINE TAKES AN ARRAY OF DATA *Y* AND SCALES ITS VALUES FO
C    A LOGARITHMIC PLOT. THERE ARE TWO ENTRIS---
C          LOGSCAL IS USED IF THE RANGE OF DATA VALUES ARE UNKNOWN
C
C          LOGCYC IS USED IF THE RANGE OF DATA ARE KNOWN AND CAN BE PROVI
C
C    INPUT PARAMETERS FOR BOTH ROUTINES********************************
C    Y= THE ARRAY IN WHICH THE DATA TO BE SCALED IS STORED
C    N=THE NUMBER OF THE POINTS IN THE ARRAY
C    S= THE LENGTH OF THE AXIS IN INCHES
C    K=A REPETITION FACTOR FOR ACCESSING DATA IN Y
C    INPUT PARAMETERS FOR LOGCYC BUT OUTPUT PARAMETERS OF LOGSCAL
C    CYCLES =THE NUMBER OF CYCLES OF REPETITIONS OF THE GRAPH
C    LO=THE LOWEST EXPONENT OF OF THE DATA
C                IT IS AN OUTPUT PARAMETER OFLOGSCAL ***INPUT FOR LOGCYC
C          IE THE DIFFERENCE BETWEEN HIGHEST AND LOWEST EXPONENTS + ONE
C          IE.  IF Y(I) IS GREATER THAN 1.0*10**4 THEN LO =4
C    INTERNAL VARIABLES
C    HI =THE MAXIMUN VALUE OF THE DATA
C    AHI AND ALO ARE REAL VALUES FOR INTEGER HI AND LO
C    CL =A SCALE FACTOR
C    ******************************************************************
      INTEGER HI, CYCLES
      DIMENSION Y(NN)
      AHI=-10.**321
      ALO=10.**321
C
C    FIND THE MAXIMUM OF THE DATA
C
5     DO 10 I=1,N,K
          AHI=AMAX1(AHI, Y(I))
          ALO=AMIN1(ALO, Y(I))
10    CONTINUE
C
C    COMPUTE LO HI AND CYCLES
C
      ALO=ALOG10(ALO)
      AHI=ALOG10(AHI)
      LO=IFIX(ALO)
      IF(ALO.LT.0.0) LO=LO-1
      HI=IFIX(AHI)+1
      CYCLES=HI-LO
      GOTO 11
C
C    THIS ENTRY POINT IS USED WHEN LO AND CYCLES ARE KNOWN
C
      ENTRY LOGCYC
11    CL=S/CYCLES
      DO 20 I=1, N, K
C
C    GET LOG 10 OF THE DATA AND ADJUST TO SCALE
C
          IF( Y(I).LE.0.) PRINT 30
30    FORMAT(1X,*NEGATIVE OR 0 VALUES HAVE BEEN ACCESSED BY THE SCALING
     1SUBROUTINE************ THE ABSOLUTE VALUE OF THE VALUE HAS BEEN USS
```

```
 1ED FOR PLOTTING PURPOSES)                                        SQFD5080
      Y(I)=ALOG10(ABS(Y(I)))                                       SQFD5090
20    Y(I)=(Y(I)-LC)*CL                                            SQFD5100
      Y(N+1)=C.0                                                   SQFD5110
      Y(N+2)=1.0                                                   SQFD5120
      RETURN                                                       SQFD5130
      END                                                          SQFD5140
```

```
      SUBROUTINE LOGAXIS(X, Y, IBCD, NCH, SZ, THTA, LOEXP, ICYCLES)
C    THIS ROUTINE CREATES A LOGARITHMIC AXIS WHICH MAY BE HORIZONTAL
C         (ALONG THE LENGHTH OF THE PAPER , OR THTA=0.0) OR VERTICAL
C         (ACROSS THE WIDTH OF THE PAPER  OR THTA =90.0), OR AT ANY
C         OTHER ANGLE. FURTHER THE AXIS CAN BE DRAWN WITH DESIRED LABELI
C         PRINTED
C     VARIABLES ****************
C    INPUT PARAMETERS
C     X,Y-THE COORDINATES OF THE BEGINNING POINT OF THE AXIS
C    IBCD=AN ARRAY OF HOLLRITH CHARACTERS PRINTED AS LABELING
C     NCHAR-NUMBER OF CHARACTERS IN IBCD TO PLOT
C    SIZE = THE LENGTH OF THE AXIS IN INCHES
C     THETA - THE ANGLE AT WHICH THE AXIS IS DRAWN
C     LOEXP-MINIMUM EXPONENT(POWER OF TEN) OF ANY VALUE TO BE PLOTTED
C     ICYCLES-NUMBER OF REPITITIONS OF THE AXIS TO BE PLOTTED
C    INTERNAL VARIABLES
C     FINE-DETERMINES FINE OR COARSE TICK MARKS
C     HGT-CONTAINS THE HEIGHTS OF THE TICK MARKS
C     COORD -CONTAINS LOGS USED TO MOVE THE PEN THE CORRECT LENGHT
C           WHILE THE GRAPH IS BEING DRAWN
C     ***************************************************
      DIMENSION COORD(65), HGT(65), IBCD(8)
      LOGICAL FINE
C
C    NOTE WE NEED THE ABSOLUTE VALUES OF SIZE, THETA, NCHAR
C
      SIZE=SZ
      NCHAR=NCH
      THETA=THTA
      Z=COORD(1)
      ZZ=ALOG(FLOAT(11)/10.)
      IF(Z.EQ.ZZ) GO TO 1
C
C     THIS PROCEDURE FILLS THE ARRAYS WITH NEEDED INFORMATION
C
      ITEN=6H  1X10
      K=10
      INCR=1
      DO 5  I=1,65
           K=K+INCR
           A=FLOAT(K)/10.
           COORD(I)=ALOG10(A)
           HGT(I)=.04
           J=K-(K/5)*5
           IF(J.EQ.0) HGT(I) =.06
           J=K-(K/10)*10
           IF(J.EQ.0) HGT(I)=.100
           IF(K.EQ.50) INCR=2
    5 CONTINUE
C
C    THIS SECTION SETS THE VALUES FOR THE DIFFERENT OPTIONS IN TICK
C     MARKS LABELING ETC.
C
    1 DIRECT=1.
      IF(NCHAR.LT.0)  DIRECT=(-1.)
      TICK =DIRECT
      IF(THETA.LT.0.) TICK =(-TICK)
```

```
      FINE=.FALSE.
      IF(SIZE.LT.0.) FINE =.TRUE.
      THETA=ABS(THETA)
      NCHAR =IABS(NCHAR)
      SIZE =ABS(SIZE)
      CSTHETA=COS(THETA* .017455)
      SNTHETA=SIN(THETA*.017455)
C
C    THIS DRAWS THE AXIS AT THE PROPER ANGLE AND LENGTH
C    IT USES COORD ARRAY FOR GIVING THE RIGHT LENGTH FOR TH AXIS AND TIC
C    MARKS     TICK, DIRECT AND FINE SET THE DIRECTION OF TICK MARKS,
C    DIRECTION OF THE LABELING ANDOF THENUMBER OF TICK MARKS
C
      CYCLESZ=SIZE/FLOAT(ICYCLES)
      V=.10*(-TICK)*SNTHETA+X
      W=TICK*.10*CSTHETA+Y
      CALL PLOT(V, W, 3)
      CALL PLOT(X, Y, 2)
      K=5
      IF(FINE)K=1
      DO 20 I=1,ICYCLES
          J=0
          CALL WHERE(C, D, IDUMMY)
   10          J=J+K
               A=COORD(J)*CSTHETA*CYCLESZ+C
               B=COORD(J)*SNTHETA*CYCLESZ+D
               CALL PLOT(A, B, 2)
               V=HGT(J)*(-TICK)*SNTHETA+A
               W=TICK*HGT(J)*CSTHETA+B
               CALL PLOT(V,W,1)
               CALL PLOT(A, B, 1)
               IF(J.LT.65) GO TO 10
   20     CONTINUE
      IF(NCHAR.EQ.0) GO TO 820
C
C    THIS ROUTINE PROVIDES LABELING IN THE PROPER DIRECTION,
C    FOR THE IDENTIFICATION OF THE TICK MARKS
C
      IQ=5
      IUL=40
  310 BB=.07*TICK +.21*DIRECT -.07 -(TICK+DIRECT)*.03
      DX=CSTHETA*(-.28)-BB*SNTHETA
      DY=SNTHETA*(-.28) +BB*CSTHETA
      XA=SIZE*CSTHETA+X+DX
      YA=SIZE*SNTHETA+Y+DY
      CALL SYMBOL(XA, YA, .14, ITEN, THETA, 6)
      I1=ICYCLES+LOEXP
      XA=CSTHETA*.72-SNTHETA*.05+XA
      YA=SNTHETA* .72+CSTHETA*.05+YA
      POWER=2HI3
      CALL NUMBER(XA, YA, .10, I1, THETA, POWER)
      II=ICYCLES
C
C     THIS MOVES THE PEN DOWN THE AXIS PUTTING THE TICK MARK LABELING
C     AT THE CORRECT POSTION
C
  320 K=IQ
          II=II-1
```

```
          INCR=-5                                                    SQFD6300
          I=IUL                                                      SQFD6310
 330      QA= (COORD(I)+ FLOAT(II))*CYCLESZ +.25                     SQFD6320
                XA=QA*CSTHETA+X+DX                                   SQFD6330
                YA=QA*SNTHETA+Y+DY                                   SQFD6340
                K=K-1                                                SQFD6350
                IF(I.EQ.40) INCR=-10                                 SQFD6360
                I=I+INCR                                             SQFD6370
                IF(I.GT.0) GO TO330                                  SQFD6380
          QA=FLOAT(II)*CYCLESZ                                       SQFD6390
          XA=QA*CSTHETA+X+DX                                         SQFD6400
          YA =QA*SNTHETA+Y+DY                                        SQFD6410
          CALL SYMBOL(XA, YA, .14, ITEM, THETA, 6)                   SQFD6420
          I1=II+LOEXP                                                SQFD6430
          XA=CSTHETA*.72-SNTHETA*.05+XA                              SQFD6440
          YA=SNTHETA*.72+CSTHETA*.05+YA                              SQFD6450
          POWER=2HI3                                                 SQFD6460
          CALL NUMBER(XA, YA, .10, I1, THETA, POWER)                 SQFD6470
      IF(II.GT.0) GO TO320                                           SQFD6480
 C                                                                   SQFD6490
 C    THIS PRINTS THE DESIRED ALPHA NUMERIC INFORMATION ABOVE OR BELOW SQFD6500
 C    THE AXIS AS IS NEEDED TO IDENTIFY THE AXIS                     SQFD6510
 C                                                                   SQFD6520
      BA=.5*SIZE-FLOAT(NCHAR) *.06                                   SQFD6530
      BB =TICK*.05 +DIRECT*.40-.07                                   SQFD6540
      XA=BA*CSTHETA-BB*SNTHETA+X                                     SQFD6550
      YA =BA*SNTHETA+BB*CSTHETA+Y                                    SQFD6560
      CALL SYMBOL(XA, YA, .14, IBCD, THETA, NCHAR)                   SQFD6570
 820  RETURN                                                         SQFD6580
      END                                                            SQFD6590
```

```
                                   ****SQUEEZE FILM DAMPER****

THIS PROGRAM ANALYZES THE STIFFNESS, DAMPING AND PRESSURE
CHARACTERISTICS OF THE SQUEEZE FILM DAMPER BEARING. THREE
BEARING CONFIGURATIONS MAY BE ANALYZED-
  0 - PLAIN BEARING WITHOUT END LEAKAGE SEALS OR CIRCUM-
      FERENTIAL OIL SUPPLY GROOVE
  1 - BEARING WITHOUT END LEAKAGE SEALS BUT WITH CIRCUM-
      FERENTIAL OIL SUPPLY GROOVE
  2 - BEARING WITH BOTH END LEAKAGE SEALS AND SIRCUMFER-
      ENTIAL OIL SUPPLY GROOVE
IN ADDITION, THE FILM MAY BE ASSUMED TO BE EITHER CAVI-
TATED OR UNCAVITATED. IF CAVITATED THE FILM IS ASSUMED
TO EXTEND FROM THETA=PI/2 TO THETA=3PI/2, WHERE THETA
IS MEASURED FROM THE LINE OF CENTERS IN THE DIRECTION OF
JOURNAL PRECESSION. THE EVALUATION OF THE BEARING CHARAC-
TERISTICS ASSUMES THAT THE JOURNAL PRECESSES SYNCHRONOUS-
LY ABOUT THE BEARING CENTER.
THE FOLLOWING IS A DESCRIPTION OF THE INPUT PARAMETERS-

CARD 1. -80 COLUMN FREE-FIELD COMMENT CARD
CARD 2. -80 COLUMN FREE-FIELD COMMENT CARD
CARD 3. -NAMELIST/BRGTYPE/ TYP,CAV,PS
            TYP - 0 FOR BEARING TYPE 0 (SEE ABOVE)
                  1 FOR BEARING TYPE 1 (SEE ABOVE)
                  2 FOR BEARING TYPE 2 (SEE ABOVE)
            CAV - 0 FOR UNCAVITATED FILM
                  1 FOR CAVITATED FILM
                  IF CAV=0 PK=.F. (SEE CARD 7)
            PS  - OIL SUPPLY PRESSURE, PSI
CARD 4. -NAMELIST/BEARING/ L,R,MU,N
            L   - BEARING LENGTH, IN.
            R   - BEARING RADIUS, IN.
            MU  - LUBRICANT VISCOSITY, MICROREYNS
            N   - ROTOR SPEED (J649E13 7IEC5S89C5 I1CE,RPM)
CARD 5. -NAMELIST/ECRATIO/ ES,EF
            ES  - INITIAL JOURNAL ECCENTRICITY RATIO, ES>0.
            EF  - FINAL JOURNAL ECCENTRICITY RATIO, EF<1.0
CARD 6. -NAMELIST/CLEARNC/ C(I),NC
            C(I)- CLEARANCE VALUES, IN.   0<I≤5
            NC  - NUMBER OF CLEARANCE VALUES
CARD 7. -NAMELIST/PLOTSEM/ CS,PC,PK,PP
            CS  - PLOT CONTROL, .T. IF PLOT DESIRED,
                  OTHERWISE .F.
            PC  - .T. IF DAMP PLOT DESIRED, OTHERWISE .F.
            PK  - .T. IF STIFF. PLOT DESIRED, OTHERWISE .F.
            PP  - .T. IF PRESS PLOT DESIRED, OTHERWISE .F.
SAMPLE DATA
COMMENT CARD 1
COMMENT CARD 2
 $BRGTYPE TYP=0,CAV=1,PS=0.0$
 $BEARING L=0.90,R=2.55,MU=0.382,N=16800$
 $ECRATIO ES=0.1,EF=0.9$
 $CLEARNC C(1)=.003,C(2)=.004,C(3)=.005,C(4)=.006,NC=4$
 $PLOTSEM CS=.T.,PC=.T.,PK=.T.,PP=.T.$
```

```
BEARING WITH END SEALS AND OIL SUPPLY GROOVE

UNCAVITATED FILM


BEARING LENGTH=       .31  INCHES      BEARING RADIUS=       .31  INCHES
N=                 7000.0  RPM         MU                  1.600  MICROREYNS
PS=                   0.0  PSI         ES=                  .100
EF=                  .900              NC=                     3

                                                CLEARANCES = INCHES

                                                       .00300
                                                       .00500
                                                       .01000
```

C= .0030 IN.

EC (DIM)	CO LB-SEC/IN	KO LB/IN	PMAX LB/IN**2	THETA DEGREES
.100	1.802	0.000	1.996	253.327
.133	1.824	0.000	2.752	248.473
.167	1.852	0.000	3.590	244.001
.200	1.888	0.000	4.536	239.887
.233	1.931	0.000	5.617	236.105
.267	1.983	0.000	6.871	232.629
.300	2.045	0.000	8.343	229.434
.333	2.119	0.000	10.089	226.494
.367	2.205	0.000	12.184	223.784
.400	2.306	0.000	14.725	221.279
.433	2.425	0.000	17.844	218.952
.467	2.566	0.000	21.719	216.779
.500	2.733	0.000	26.601	214.733
.533	2.933	0.000	32.844	212.789
.567	3.174	0.000	40.964	210.923
.600	3.468	0.000	51.735	209.107
.633	3.831	0.000	66.349	207.317
.667	4.288	0.000	86.720	205.528
.700	4.875	0.000	116.049	203.713
.733	5.650	0.000	159.976	201.848
.767	6.708	0.000	229.101	199.906
.800	8.220	0.000	345.012	197.862
.833	10.512	0.000	556.595	195.691
.867	14.299	0.000	991.914	193.367
.900	21.438	0.000	2067.593	190.864

C= .0060 IN.

EC (DIM)	CO LB-SEC/IN	KO LB/IN	FMAX LB/IN**2	THETA DEGREES
.100	.225	0.000	.499	253.327
.133	.228	0.000	.688	248.473
.167	.232	0.000	.898	244.001
.200	.236	0.000	1.134	239.887
.233	.241	0.000	1.404	236.105
.267	.248	0.000	1.718	232.629
.300	.256	0.000	2.086	229.434
.333	.265	0.000	2.522	226.494
.367	.276	0.000	3.046	223.784
.400	.288	0.000	3.681	221.279
.433	.303	0.000	4.461	218.952
.467	.321	0.000	5.430	216.779
.500	.342	0.000	6.650	214.733
.533	.367	0.000	8.211	212.789
.567	.397	0.000	10.241	210.923
.600	.433	0.000	12.934	209.107
.633	.479	0.000	16.587	207.317
.667	.536	0.000	21.680	205.528
.700	.609	0.000	29.012	203.713
.733	.706	0.000	39.994	201.848
.767	.839	0.000	57.275	199.906
.800	1.027	0.000	86.253	197.862
.833	1.314	0.000	139.149	195.651
.867	1.787	0.000	247.979	193.367
.900	2.680	0.000	516.898	190.864

C= .0100 IN.

EO (DIM)	CO LB-SEC/IN	KO LB/IN	PMAX LB/IN**2	THETA DEGREES
.100	.049	0.000	.180	253.327
.133	.049	0.000	.248	248.473
.167	.050	0.000	.323	244.001
.200	.051	0.000	.408	239.887
.233	.052	0.000	.506	236.105
.267	.054	0.000	.618	232.629
.300	.055	0.000	.751	229.434
.333	.057	0.000	.908	226.494
.367	.060	0.000	1.097	223.784
.400	.062	0.000	1.325	221.279
.433	.065	0.000	1.606	218.952
.467	.069	0.000	1.955	216.779
.500	.074	0.000	2.394	214.733
.533	.079	0.000	2.956	212.789
.567	.086	0.000	3.687	210.923
.600	.094	0.000	4.659	209.107
.633	.103	0.000	5.971	207.317
.667	.116	0.000	7.805	205.528
.700	.132	0.000	10.444	203.713
.733	.153	0.000	14.398	201.848
.767	.181	0.000	20.619	199.906
.800	.222	0.000	31.051	197.862
.833	.284	0.000	50.094	195.691
.867	.386	0.000	89.272	193.367
.900	.579	0.000	186.083	190.864

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