# NASA TECHNICAL MEMORANDUM

NASA TM X-64926

(NASA-TM-X-64926) A STEERING LAW FOR THREE DOUBLE-GIMBAL CONTROL MOMENT GYRO SYSTEMS (NASA) 34 p HC $3.75 CSCL 17G

N75-23925

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G3/35 21798

# A STEERING LAW FOR THREE DOUBLE-GIMBAL CONTROL MOMENT GYRO SYSTEMS

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March 1975

NASA

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| 1. Report No. | 2. Government Accession No. | 3. Recipient's Catalog No. |
|---|---|---|
| NASA TM X-64926 | | |

4. Title and Subtitle

A Steering Law for Three Double-Gimbal Control Moment Gyro Systems

5. Report Date
March 1975

6. Performing Organization Code

7. Author(s)

Tsuneo Yoshikawa

8. Performing Organization Report No.

9. Performing Organization Name and Address

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

10. Work Unit No.

11. Contract or Grant No.

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D.C. 20546

13. Type of Report and Period Covered

Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes This article was written while the author held an NRC Resident Research Associateship at the Systems Dynamics Laboratory, Science and Engineering, Marshall Space Flight Center, NASA, Huntsville, Alabama, on leave from Faculty of Engineering, Kyoto University, Kyoto, Japan.

16. Abstract


A steering law for three double-gimbal control moment gyro (CMG) systems is proposed whose principle is very simple. This steering law is applicable to systems with almost any configuration of CMGs and the CMG-out operation needs no special modification.

Examples of three double gimbal CMG systems in an orthogonal configuration and in a parallel configuration are shown along with the results of digital simulations. Simulation results show that any command torque can always be met except when the system is in a singular state and that, whenever the system is in, or close to, a singularity, the steering law drives the system state out of the vicinity of the singularity.


17. Key Words (Suggested by Author(s))

18. Distribution Statement

Unclassified-unlimited

Tsuneo Yoshikawa

| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price |
|---|---|---|---|
| Unclassified | Unclassified | 34 | NTIS |

MSFC - Form 3292-1 (Rev December 1972)

For sale by National Technical Information Service, Springfield, Virginia 22151

# ACKNOWLEDGMENT

The author would like to express his sincere thanks to Dr. Michael T. Borelli for his discussions and advice in writing this report. The author is also grateful to Mr. H. F. Kennel and Dr. J. R. Glaese for their comments on this report.

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# DEFINITION OF SYMBOLS

| Symbol | Definition |
|---|---|
| $a_{\alpha i}$ | $= s\alpha_i \, c\alpha_i$ |
| $a_{\beta i}$ | $= s\beta_i \, c\beta_i$ |
| $b_{\alpha i}$ | $= c^2\alpha_i - s^2\alpha_i$ |
| $b_{\beta i}$ | $= c^2\beta_i - s^2\beta_i$ |
| $c\delta_\ell$ | $= \cos\delta_\ell$ |
| $C(\underline{\delta})$, C | coefficient matrix of the basic equation for CMG systems |
| $C^\#$ | pseudoinverse of C |
| $c_p$ | $= C(\underline{\delta} + \Delta\underline{\delta})$ where $\Delta\underline{\delta}$ is a small perturbation of $\underline{\delta}$ |
| $d_{ii}$ | $= g_{jj}g_{kk} - g_{jk}^2$, $i \neq j$, $j \neq k$, $k \neq i$ |
| $d_{ij}$ | $= g_{ik}g_{kj} - g_{ij}g_{kk}$, $i \neq j$, $j \neq k$, $k \neq i$ |
| $f(\underline{\delta})$ | criterion function |
| G | $= CC^T$ |
| $g_{ij}$ | entries of G |
| $_\ell g'_{ij}$ | $= \partial g_{ij}/\partial\delta_\ell$ |
| $\underline{h}$ | total angular momentum vector |
| $\underline{h}_i$ | angular momentum vector of the ith CMG |
| $h_i$ | magnitude of $\underline{h}_i$ |
| I | identity matrix |
| i, j, k | positive integers (= 1, 2, 3) |

# DEFINITION OF SYMBOLS (Continued)

| Symbol | Definition |
|---|---|
| $\underline{k}$ | arbitrary constant vector |
| $k_1, k_2, k_3$ | constants related to the selection of $\underline{k}$ |
| $n$ | dimension of the gimbal angle vector |
| $\underline{r}$ | gimbal rate command vector |
| $\underline{r}_0$ | gimbal rate command for zero command torque |
| $r_g$ | limit of the gimbal rate |
| $\underline{r}_t$ | $= C^{\#}\underline{t}_c$ |
| $s\delta_\ell$ | $= \sin \delta_\ell$ |
| $\underline{t}$ | output torque vector |
| $\underline{t}_c$ | command torque vector |
| $t_x, t_y, t_z$ | coordinate of $\underline{t}_c$ in XYZ-coordinate system |
| $w$ | the value of the criterion function $f(\underline{\delta})$ |
| $\dot{w}$ | time derivative of $w$ |
| $\dot{w}_0$ | $\dot{w}$ for zero command torque |
| $\dot{w}_{0max}$ | maximum of $\dot{w}_0$ under the constraint $\Vert\underline{r}_0\Vert \leq 1$ |
| $X_i, Y_i, Z_i$ | axes of a coordinate system regarding the ith CMG |
| X, Y, Z | axes of a coordinate system regarding the CMG system |
| $\alpha_i$ | outer gimbal angle of the ith CMG |
| $\alpha_{i0}$ | reference gimbal angle of the ith CMG |

# DEFINITION OF SYMBOLS (Concluded)

| Symbol | Definition |
|---|---|
| $\beta_i$ | inner gimbal angle of the ith CMG |
| $\beta_0$ | reference inner gimbal angle |
| $\underline{\delta}$ | gimbal angle vector |
| $\delta_\ell$, $\ell = 1, 2, \ldots, n,$ | $\ell$th component of $\underline{\delta}$ |
| $\Delta$ | sampling interval |
| $\Delta\delta$ , $\Delta\delta'$ | small positive numbers |
| $\tau$ | time variable |
| $\underline{\xi}$ | partial derivative vector of $f(\underline{\delta})$ with respect to $\underline{\delta}$ $(= [\xi_1 \vdots \xi_2 \vdots \ldots \vdots \xi_n]^T)$ |
| $\xi_\ell$, $\ell = 1, 2, \ldots, n,$ | partial derivative of $f(\underline{\delta})$ with respect to $\delta_\ell$ |

TECHNICAL MEMORANDUM X-64926

# A STEERING LAW FOR THREE DOUBLE-GIMBAL CONTROL MOMENT GYRO SYSTEMS

## I. INTRODUCTION

Control moment gyro (CMG) systems are effective actuators for attitude control of a large variety of spacecraft. In Skylab, a double-gimbal CMG system was used successfully. Also, in some future projects, like the Large Space Telescope (LST) and the High Energy Astronomy Observatory (HEAO), CMG systems are proposed as the primary system for attitude control.

In this report, a new steering law, whose principle is very simple, for three double-gimbal CMG systems is proposed. It is applicable to almost any configuration, and the CMG-out operation requires no special consideration. Although several steering laws for double-gimbal CMG systems have been developed in the past including one that was used in the Skylab [1-3], the basic principles of these steering laws are more complicated than that of the proposed steering law.

The main idea is utilization of a formula for the general solution to a set of linear algebraic equations whose number is less than the unknown variables. The general solution is given in terms of the pseudoinverse of the coefficient matrix for the set of linear algebraic equations and it contains an arbitrary vector. This arbitrary vector is used to obtain a desirable momentum distribution among the CMGs.

In Section II, a general description of the steering law is given. In Section III, two examples are worked out. The first example is a three double-gimbal CMG system with an orthogonal configuration and the second one is a three double-gimbal CMG system with a parallel configuration. In both examples, digital simulation results are given to illustrate the effectiveness of the steering law.

# II. DESCRIPTION OF THE STEERING LAW

The output torque of any CMG system can be described by

$$\underline{t} = \dot{\underline{h}} = C(\underline{\delta})\ \dot{\underline{\delta}}$$

where $\underline{\delta}$ is the n-dimensional gimbal angle vector (in the case of three double-gimbal CMGs, $n = 6$), $\underline{t}$ is the three-dimensional output torque vector, $\underline{h}$ is the three-dimensional total angular momentum vector, and $C(\underline{\delta})$ is a coefficient matrix, each entry of which is a function of $\underline{\delta}$. $\dot{\underline{h}}$ and $\dot{\underline{\delta}}$ denote the time derivatives of $\underline{h}$ and $\underline{\delta}$, respectively. It is assumed that there are no gimbal stops on the CMGs.

Given the present state $\underline{\delta}$ of the system, it is required that one obtain a gimbal rate command $\underline{r}$ which satisfies

$$\underline{t}_c = C(\underline{\delta})\,\underline{r} \tag{1}$$

where $\underline{t}_c$ is the command torque. The general solution to the above equation is given by [4] [$C(\underline{\delta})$ is also denoted by $C$ hereafter]

$$\underline{r} = C^{\#}\underline{t}_c + [I - C^{\#}C]\underline{k} \tag{2}$$

where $C^{\#}$ is the pseudoinverse of $C$, $I$ is the $n \times n$ identity matrix and $\underline{k}$ is an arbitrary constant vector. The second term can be used for momentum distribution.

It is assumed that a proper scalar criterion function which measures the desirability of the momentum distribution of the present state $\underline{\delta}$ is given,

$$w = f(\underline{\delta}) \quad , \tag{3}$$

and it is also assumed that the larger the value of $w$, the better the momentum distribution. The time derivative of $w$ is given by

$$\dot{w} = \underline{\xi}^T \underline{\dot{\delta}}$$

where the superscript $T$ denotes transpose and

$$\underline{\xi}^T = [\xi_1, \xi_2, \ldots, \xi_n]$$

$$\xi_\ell = \frac{\partial f(\underline{\delta})}{\partial \delta_\ell} \quad , \qquad \ell = 1, 2, \ldots, n \quad . \tag{4}$$

Under the assumption that the gimbal rate command (2) is perfectly realized, that is, $\underline{\dot{\delta}} = r$, the time derivative of $w$ is given as follows:

$$\dot{w} = \xi^T C^{\#} \underline{t}_c + \underline{\xi}^T [I - C^{\#} C] \underline{k} \quad . \tag{5}$$

In order to increase the value of $w$, it will be best to select $\underline{k}$ as

$$\underline{k} = \underline{\xi} k_1 \tag{6}$$

where $k_1$ is a positive constant. The reason for the selection of equation (6) follows: From equations (2), (5) and (6), one obtains

$$\underline{r} = C^{\#} \underline{t}_c + [I - C^{\#} C] \, \underline{\xi} \, k_1 \tag{7}$$

$$\dot{w} = \underline{\xi}^T C^{\#} \underline{t}_c + \underline{\xi}^T [I - C^{\#} C] \, \underline{\xi} \, k_1 \quad , \tag{8}$$

and the second term in the right-hand side of equation (8) becomes nonnegative (because $[I-C^{\#}C]$ is idempotent), contributing for an increase of the value $w$.

Equation (7) is the basic equation of the steering law. It involves the selection of only one scalar constant $k_1$.

In summary, the main procedure of the steering law is given as follows:

1. Calculate $C^{\#}$.

2. Calculate $\underline{\xi}$.

3. Calculate $\underline{r}$ by equation (7).

Remark 1: The meaning of the constant $k_1$ will be considered. If $\underline{t}_c = \underline{0}$ (zero vector), then

$$\underline{r}_0 = \underline{r}\Big|_{\underline{t}_c = \underline{0}} = [I - C^{\#}C]\,\underline{\xi}\,k_1 \tag{9}$$

$$\underline{\dot{w}}_0 = \underline{\dot{w}}\Big|_{\underline{t}_c = \underline{0}} = \underline{\xi}^T[I - C^{\#}C]\,\underline{\xi}\,k_1 \quad . \tag{10}$$

The maximum of $\dot{w}_0$ under the constraint $\|\underline{r}_0\| \le 1$ ($\|\ \|$ denotes norm) is given by

$$\dot{w}_{0max} = \max_{\|\underline{r}_0\| \le 1} \dot{w}_0 = \{\underline{\xi}^T[I - C^{\#}C]\,\underline{\xi}\}^{1/2} \quad . \tag{11}$$

Hence from equations (9) and (11)

$$\|\underline{r}_0\| = k_1\dot{w}_{0max} \quad . \tag{12}$$

Therefore, if $\underline{t}_c = \underline{0}$, the magnitude of $\underline{r}$ becomes proportional to $\dot{w}_{0max}$ and $k_1$ is the proportional coefficient. In practical applications, in order to prevent $\|\underline{r}_0\|$ from becoming too large, a limitation for $k_1$ should be given. One candidate for this is $\|\underline{r}_0\| \le k_2 r_g$, i.e.,

$$k_1 \le \{\underline{\xi}^T [I - C^{\#} C] \, \underline{\xi}\}^{-1/2} k_2 r_g \tag{13}$$

where $r_g$ (deg/sec) is the hardware limit for the gimbal rate and $k_2$ is a constant $(0 \le k_2 \le 1)$.

Remark 2: If $\det(CC^T) \neq 0$, then

$$C^{\#} = C^T (CC^T)^{-1} \quad . \tag{14}$$

On the other hand if $\det(CC^T) = 0$, the calculation of $C^{\#}$ is rather complicated. A simple way to overcome this difficulty will be to approximate $C^{\#}$ by $C_p^{\#}$ where $C_p = C(\underline{\delta} + \Delta\underline{\delta})$ and $\Delta\underline{\delta}$ is a small perturbation of $\underline{\delta}$ such that $\det(C_p C_p^T) \neq 0$. For double-gimbal CMG systems $\det(CC^T) = 0$ happens only when all the momentum vectors of the CMGs are on the same line (parallel or antiparallel). Such a situation can hardly happen except when $n = 4$ (i.e., two double-gimbal CMGs). Also one of the main purposes of the momentum distribution is to avoid such a situation. Hence the small error caused by the approximation of $C^{\#}$ by $C_p^{\#}$ will be negligible in practical applications.

Remark 3: In the following section, the root of the determinant of $CC^T$ will be taken as the criterion function $f(\underline{\delta})$:

$$f(\underline{\delta}) = \sqrt{\det G}$$

where

$$G = CC^T = [g_{ij}] \quad , \quad i, j = 1, 2, 3 \quad .$$

The reason for this selection of $f(\underline{\delta})$ will be given later. For this criterion function, $\xi_\ell$ is given by

$$\xi_\ell = \frac{\partial f(\underline{\delta})}{\partial \delta_\ell} = \frac{1}{2\sqrt{\det G}} \sum_{i,j=1}^{3} {}_\ell g'_{ij}\, d_{ij} \quad , \quad \ell = 1, 2, \ldots, \ell \tag{15}$$

where

$${}_\ell g'_{ij} = \partial g_{ij}/\partial \delta_\ell$$

$$d_{ii} = g_{jj} g_{kk} - g_{jk}^2 \tag{16a}$$

$$d_{ij} = g_{ik} g_{kj} - g_{ij} g_{kk} \quad , \quad (i,j,k) = (1,2,3),\ (2,3,1) \text{ or } (3,1,2) \quad . \tag{16b}$$

Using the fact that $g_{ij} = g_{ji}$ and $d_{ij} = d_{ji}$, equation (15) can be rewritten as follows:

$$\xi_\ell = [0.5({}_\ell g'_{11}\, d_{11} + {}_\ell g'_{22}\, d_{22} + {}_\ell g'_{33}\, d_{33}) + {}_\ell g'_{12}\, d_{12} + {}_\ell g'_{13}\, d_{13}$$

$$+ {}_\ell g'_{23}\, d_{23}]/\sqrt{\det G} \quad . \tag{17}$$

## III. EXAMPLES

In this section, two examples of three double-gimbal CMG systems will be given to show the effectiveness of the steering law.

Example 1: For a three double-gimbal CMG system in an orthogonal configuration, several notations will be introduced. For the ith CMG ($i = 1,2,3$), an orthogonal coordinate system ($X_i\, Y_i\, Z_i$) is used, where $Z_i$ axis is the outer gimbal pivot axis and $X_i$ and $Y_i$ axes are fixed with respect to the spacecraft. $\alpha_i$ and $\beta_i$ are the outer and inner gimbal angles, respectively, (in degrees) of the ith CMG, as shown in Figure 1. The angular momentum vector of the ith CMG is denoted by $\underline{h}_i$ and the total momentum by $\underline{h}(=\sum_{i=1}^{3} \underline{h}_i)$. $h_i$ denotes the magnitude of $\underline{h}_i$. Then, in $X_iY_iZ_i$ coordinate system,

$$\underline{h}_i = \begin{bmatrix} h_i\, c\alpha_i\, c\beta_i \\ h_i\, s\alpha_i\, c\beta_i \\ h_i\, s\beta_i \end{bmatrix}$$

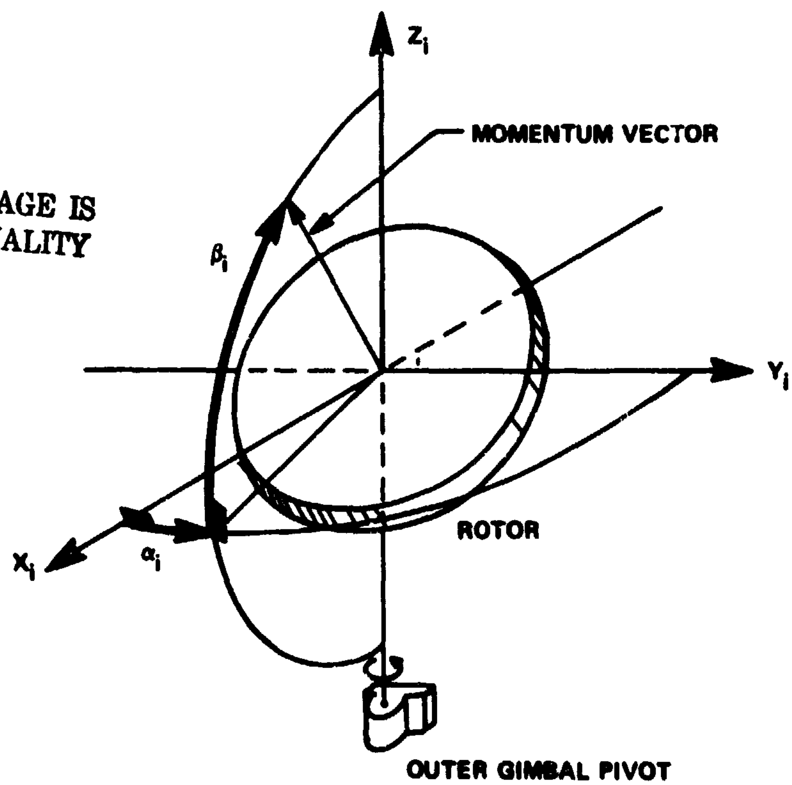




Figure 1. Outer and inner gimbal angles of a double-gimbal CMG.

and

$$\underline{\dot{h}}_i = \begin{bmatrix} -h_i \, s\alpha_i \, c\beta_i & -h_i \, c\alpha_i \, s\beta_i \\ h_i \, c\alpha_i \, c\beta_i & -h_i \, s\alpha_i \, s\beta_i \\ 0 & h_i \, c\beta_i \end{bmatrix} \begin{bmatrix} \dot{\alpha}_i \\ \dot{\beta}_i \end{bmatrix}$$

where $c\delta = \cos\delta$ , $s\delta = \sin\delta$ .

Now the three double-gimbal CMG system in an orthogonal configuration shown in Figure 2 is considered. Let

$$\delta_{2i-1} = \alpha_i \quad , \quad i = 1,2,3$$

$$\delta_{2i} = \beta_i \quad , \quad i = 1,2,3$$

then, in XYZ-coordinate system given in Figure 2,

$$\dot{C} = \left[ \begin{matrix} -h_1 \, s\alpha_1 \, c\beta_1 & -h_1 \, c\alpha_1 \, s\beta_1 & 0 & h_2 \, c\beta_2 \\ h_1 \, c\alpha_1 \, c\beta_1 & -h_1 \, s\alpha_1 \, s\beta_1 & -h_2 \, s\alpha_2 \, c\beta_2 & -h_2 \, c\alpha_2 \, s\beta_2 \\ 0 & h_1 \, c\beta_1 & h_2 \, c\alpha_2 \, c\beta_2 & -h_2 \, s\alpha_2 \, s\beta_2 \end{matrix} \right.$$

$$\left. \begin{matrix} h_3 \, c\alpha_3 \, c\beta_3 & -h_3 \, s\alpha_3 \, s\beta_3 \\ 0 & h_3 \, c\beta_3 \\ -h_3 \, s\alpha_3 \, c\beta_3 & -h_3 \, c\alpha_3 \, s\beta_3 \end{matrix} \right] .$$

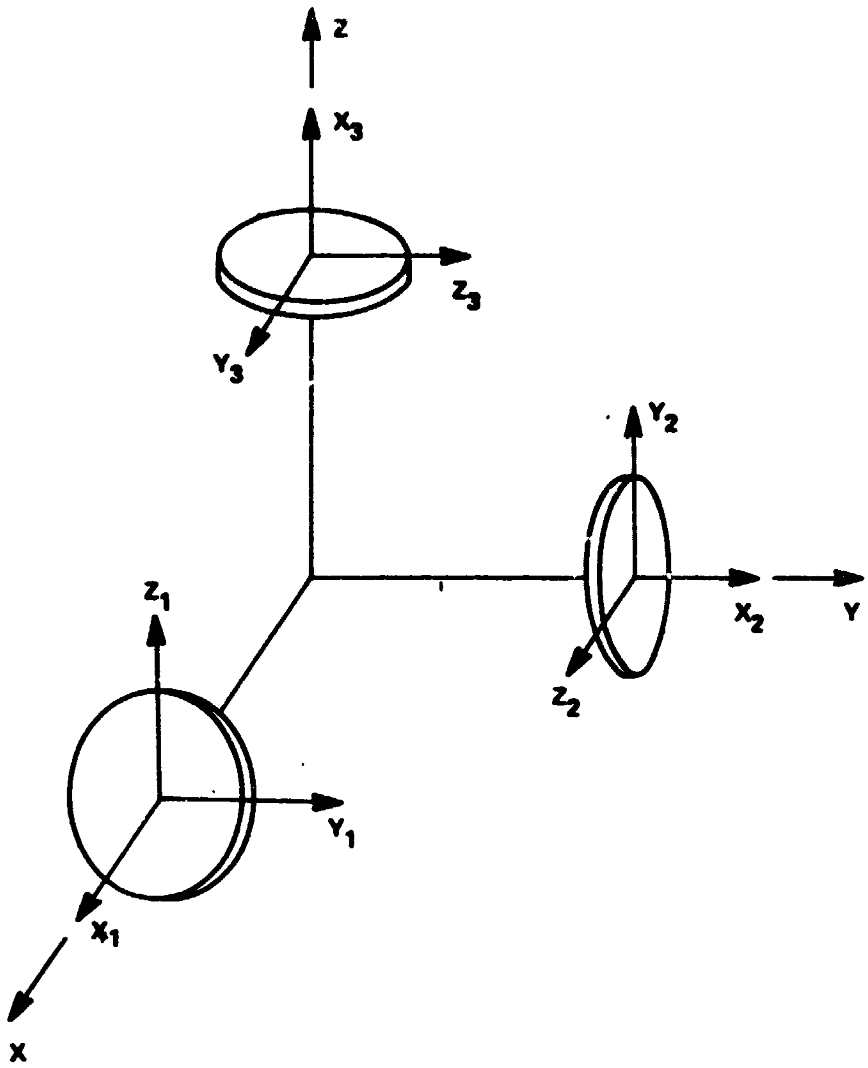


Figure 2. A three-CMG system in an orthogonal configuration.

Let $G = CC^T$ and $f(\underline{\delta}) = \sqrt{\det G}$, then ${}_{\ell}g_{ij}$'s are given by

$$ {}_{2i-1}g'_{ii} = -\,{}_{2i-1}g'_{jj} = 2h_i^2\, a_{\alpha i}\, b_{\beta i} $$

$$ {}_{2i-1}g'_{ij} = -h_i^2\, b_{\alpha i}\, b_{\beta i}\ , \quad {}_{2i-1}g'_{ik} = -h_i^2\, s\alpha_i\, a_{\beta i} $$

$$ {}_{2i-1}g'_{jk} = h_i^2\, c\alpha_i\, a_{\beta i}\ , \quad {}_{2i-1}g'_{kk} = 0 \tag{18a} $$

$$ {}_{2i}g'_{ii} = -{}_{2i}g'_{jj} = 2h_i^2\, b_{\alpha i}\, a_{\beta i} $$

$$ {}_{2i}g'_{ij} = 4h_i^2\, a_{\alpha i}\, a_{\beta i} \quad , \quad {}_{2i}g'_{ik} = -h_i^2\, c\alpha_i\, b_{\beta i} $$

$$ {}_{2i}g'_{jk} = -h_i^2\, s\alpha_i\, b_{\beta i} \quad , \quad {}_{2i}g'_{kk} = -2h_i^2\, a_{\beta i} \tag{18b} $$

where $(i,j,k) = (1,2,3)$, $(2,3,1)$ or $(3,1,2)$, and

$$ a_{\alpha i} = s\alpha_i\, c\alpha_i \quad , \quad a_{\beta i} = s\beta_i\, c\beta_i $$

$$ b_{\alpha i} = c^2\alpha_i - s^2\alpha_i \quad , \quad b_{\beta i} = c^2\beta_i - s^2\beta_i \quad . \tag{19} $$

Therefore, $\xi_\ell$'s are given by equations (16), (19), and

$$ \xi_{2i-1} = h_i^2\{[a_{\alpha i}(d_{ii} - d_{jj}) - b_{\alpha i}\, d_{ij}]\, b_{\beta i} - [s\alpha_i\, d_{ik} + c\alpha_i\, d_{jk}]\, a_{\beta i}\}/\sqrt{\det G} \tag{20a} $$

$$ \xi_{2i} = h_i^2\{[b_{\alpha i}(d_{ii} - d_{jj}) - d_{kk} + 4\, a_{\alpha i}\, d_{ij}]\, a_{\beta i} - [c\alpha_i\, d_{ik} - s\alpha_i\, d_{jk}]\, b_{\beta i}\}/\sqrt{\det G} \quad . \tag{20b} $$

Hence the steering law can be described as follows:

1. For a given $\underline{\delta}$, calculate $C(\underline{\delta})$ and $\det G$.
2. If $\det G = 0$, then let

$$\delta'_\ell = \delta_\ell + \Delta\delta \quad , \quad \ell = 1, 2, \ldots, 6$$

and go to step 1. If $\det G \neq 0$, go to step 3. $\Delta\delta$ is a small positive number (typically $\Delta\delta = 0.2$).

3. Calculate $C^{\#} = C^T G^{-1}$.

4. Calculate $\underline{\xi}$ from equations (16), (19), and (20).

5. Calculate $\underline{r}_t = C^{\#} \underline{t}_c$. If the absolute value of any component of $\underline{r}_t$ is larger than $r_g$, limit each component of $\underline{r}_t$ proportionally, such that the maximum absolute value of components of $\underline{r}_t$ is equal to $r_g$.

6. Calculate $r = \underline{r}_t + [I - C^{\#}C]\, \underline{\xi}\, k_1$

where

$$k_1 = \min\{k_3, k_2 r_g \{\underline{\xi}^T [I - C^{\#} C]\, \underline{\xi}\}^{-1/2}\}$$

and $k_2$ and $k_3$ are positive constants. If the absolute value of any component of $\underline{r}$ is larger than $r_g$, $k_1$ is decreased until the absolute value of every component of $\underline{r}$ becomes smaller than $r_g$.

This completes one cycle: After one unit time interval $\Delta$, the state of the system $\underline{\delta}$ is measured and the next cycle begins. In normal operation, the qualifying remarks mentioned in steps 5 and 6 will very seldom occur.

There are eight zero momentum stationary states. One is given by $\{\alpha_i = -45, \beta_i = 0, i = 1,2,3\}$ as shown in Figure 3, and the state which is symmetric to this state with respect to the origin is also a zero momentum stationary state. Others are symmetric to these two states with respect to the XY-plane, the YZ-plane, and the ZX-plane. In Figure 3, the outer circle shows the contour of the sphere with a unit radius and three inner circles denote the cross sections of the sphere by the XY-, YZ- and ZX-plane. The three arrows terminating on the sphere surface denote the three momentum vectors $\underline{h}_i$ $i = 1,2,3$.

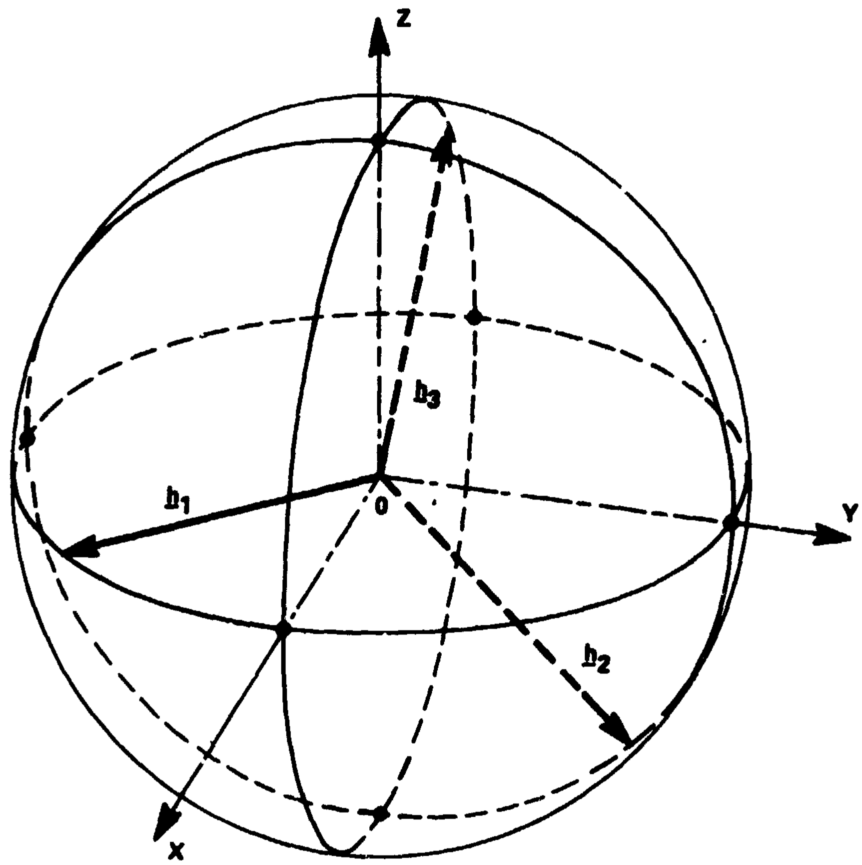


Figure 3. Zero momentum stationary state of the three-CMG system in an orthogonal configuration.

A digital simulation has been completed using a mini-computer. Values of constants were selected as follows:

$$h_i = 1 \ , \ i = 1,2,3, \ \text{(normalized)}$$

$$k_2 = 0.2 \ , \ k_3 = 0.1$$

$$r_g = 2 \ , \ \Delta = 8 \ , \ \Delta\delta = 0.2 \ .$$

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Figure 4 shows a response of the system to a torque command in the X axis direction with magnitude 0.01. The initial state is the zero momentum state shown in Figure 3. Dots on the unit sphere show the end points of the momentum vectors $\underline{h}_i$, i = 1, 2, 3, and arrows along the side of the dots show the time trajectory (broken lines indicate that the vectors are located on the back half of the sphere). The output torque followed the torque command satisfactorily.

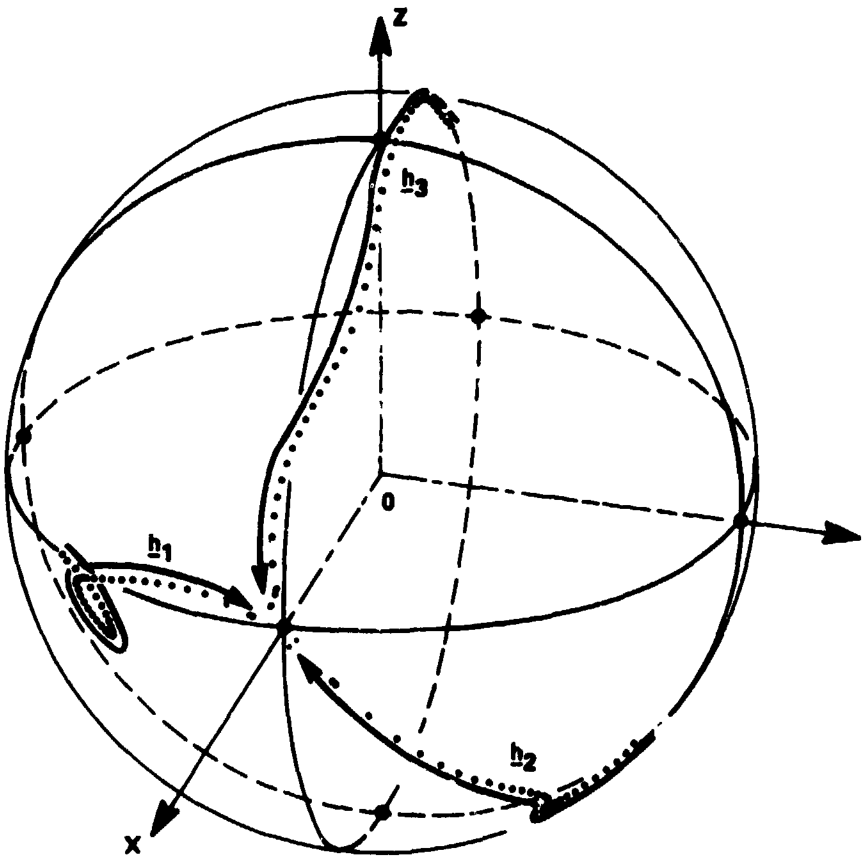



Figure 4. Response to an X directional torque command with magnitude 0.01 (orthogonal configuration).

If the direction of the command is exactly the opposite of a vector $\underline{h}_i$ all through the period of steering, there is a possibility for the system to get into a singular state. For example, if the initial state is $\{\alpha_i = -45, \beta_i = 0,$

$i = 1, 2, 3\}$ and the torque command is $\{t_x = 0.005,\ t_y = 0,\ t_z = -0.005\}$ (see Figure 5), then $\underline{h}_3$ does not move at all and the state falls into a singularity. In practice, however, such a situation can hardly happen because of the existence of disturbances on the spacecraft and noise in the sensors. Figure 6 shows a response to a torque command in the neighborhood of the most unfavorable direction. The initial state is $\{\alpha_1 = \alpha_2 = -45,\ \alpha_3 = -44,\ \beta_i = 0,\ i = 1, 2, 3\}$ and the torque command is $\{t_x = 0.005,\ t_y = 0,\ t_z = -0.005\}$. It is seen from the figure that by the compensation of $\underline{h}_1$ and $\underline{h}_2$, $\underline{h}_3$ is turned to a desirable direction.

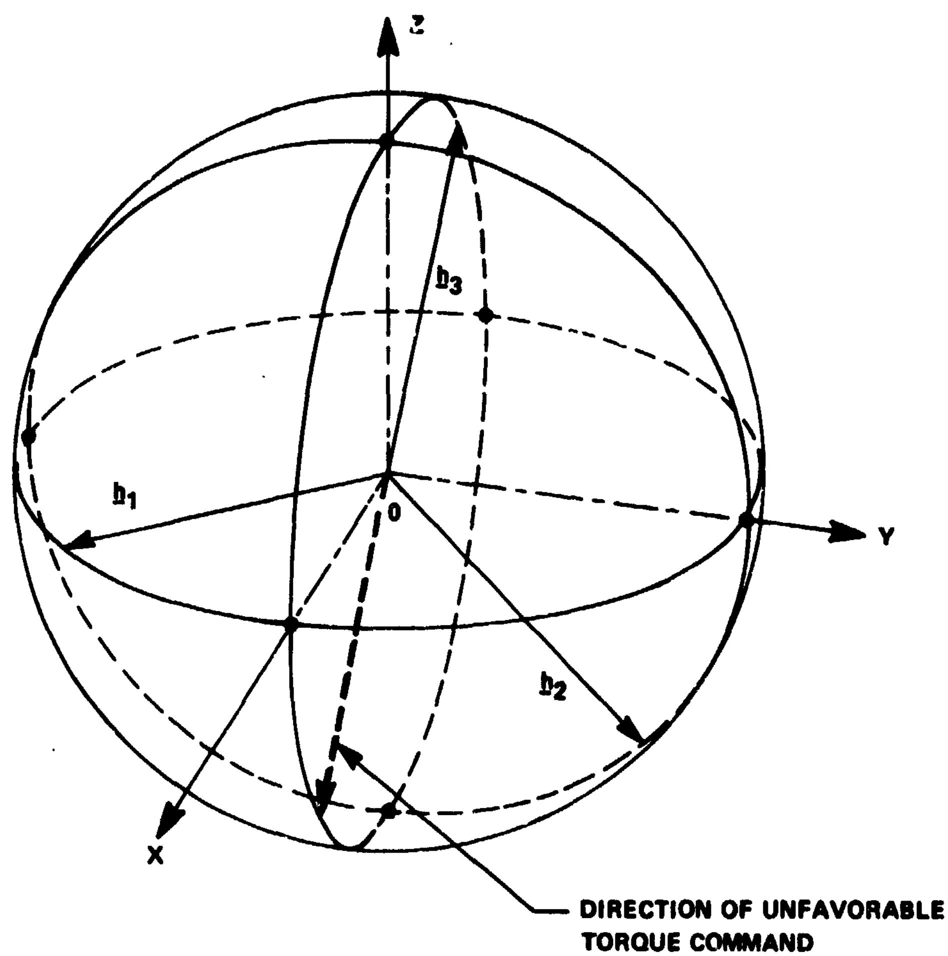



Figure 5. Possible direction of torque command for the system to get into a singular state (orthogonal configuration).

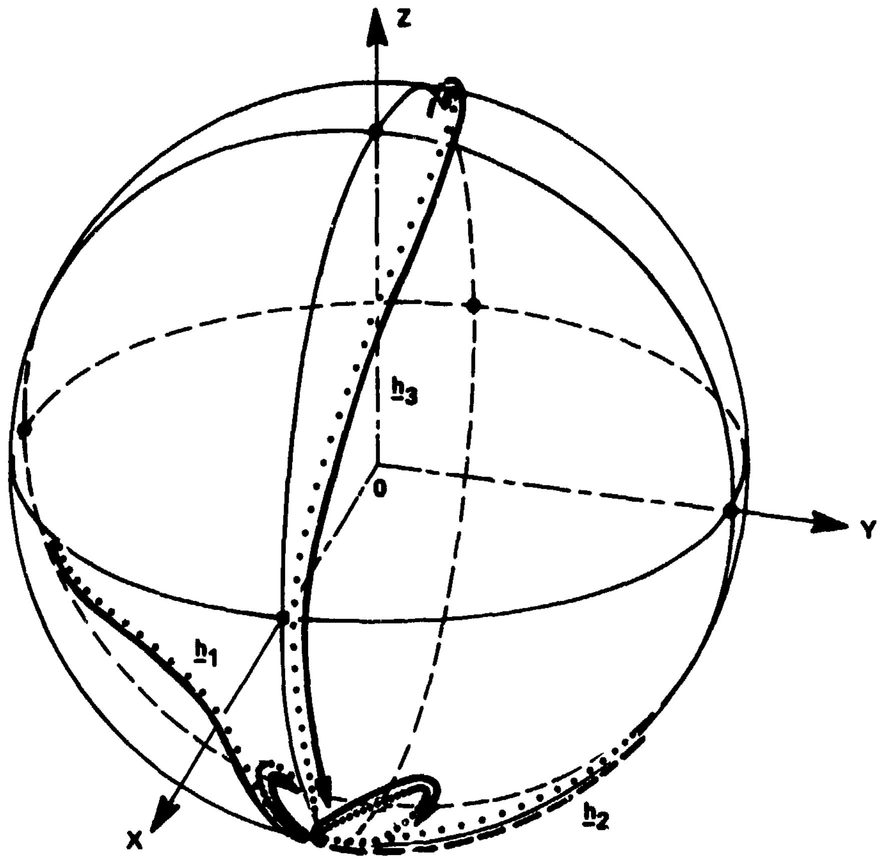
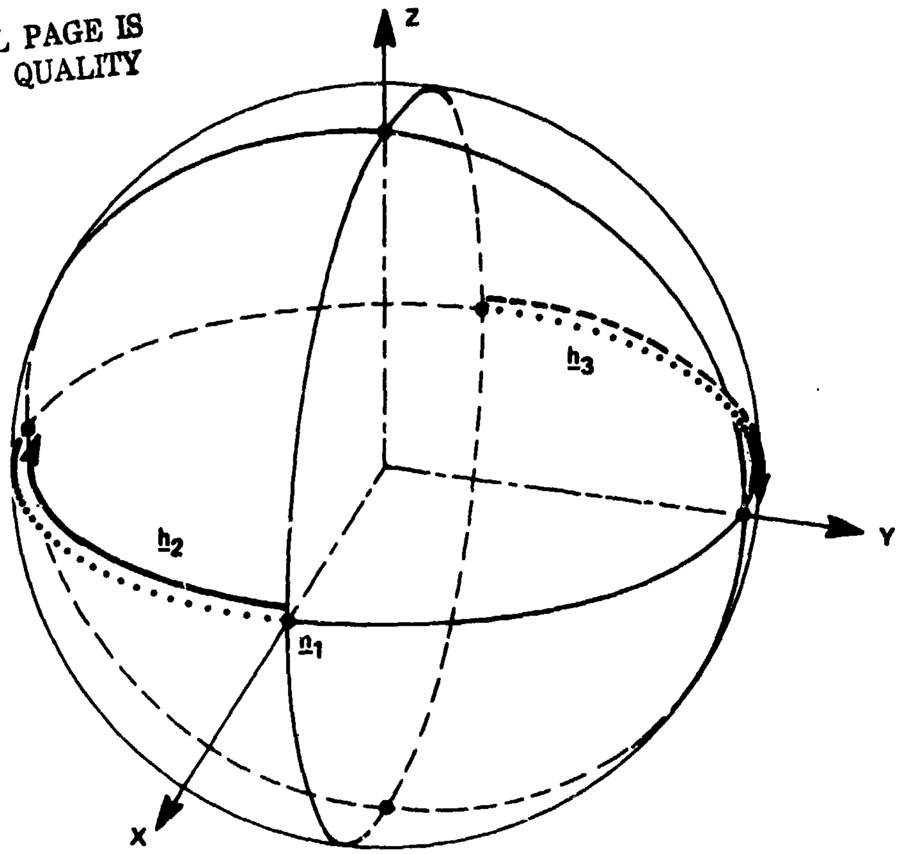



Figure 6. Momentum distribution process for an unfavorable torque command (orthogonal configuration).

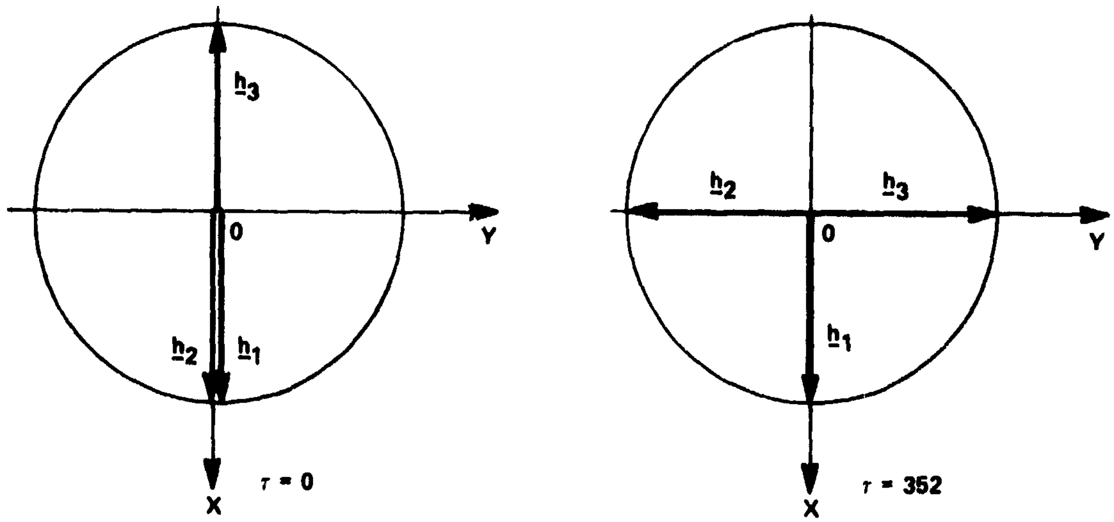
Moreover, even if the system falls into a singular state, the steering law has an ability to recover from it, as shown in Figure 7 where the initial state is $\{\alpha_1 = \beta_1 = \alpha_2 = \beta_3 = 0,\ \beta_2 = \alpha_3 = -90\}$ and the torque command is kept at zero. For a CMG-out operation, the only change that should be made is to set $h_i = 0$ when the ith CMG has failed. The control performance of this system was very satisfactory in every case simulated.

Example 2: A three double-gimbal CMG system in a parallel configuration. The same notation used in Example 1 is valid in this example. The three-CMG system in a parallel configuration[1] as shown in Figure 8 is considered. For this configuration, the matrix $C(\underline{\delta})$ is given by

1. Kennel, H. F.: Control Law for Parallel Mounted Double-Gimbal Control Moment Gyros. Second Annual Research and Technilogy Review, Marshall Space Flight Center, Oct. 1974.




a. Trajectory of the end points of momentum vectors.




b. Positions of momentum vectors in XY-plane at time $\tau$.

Figure 7. Recovery from a singular state (orthogonal configuration).

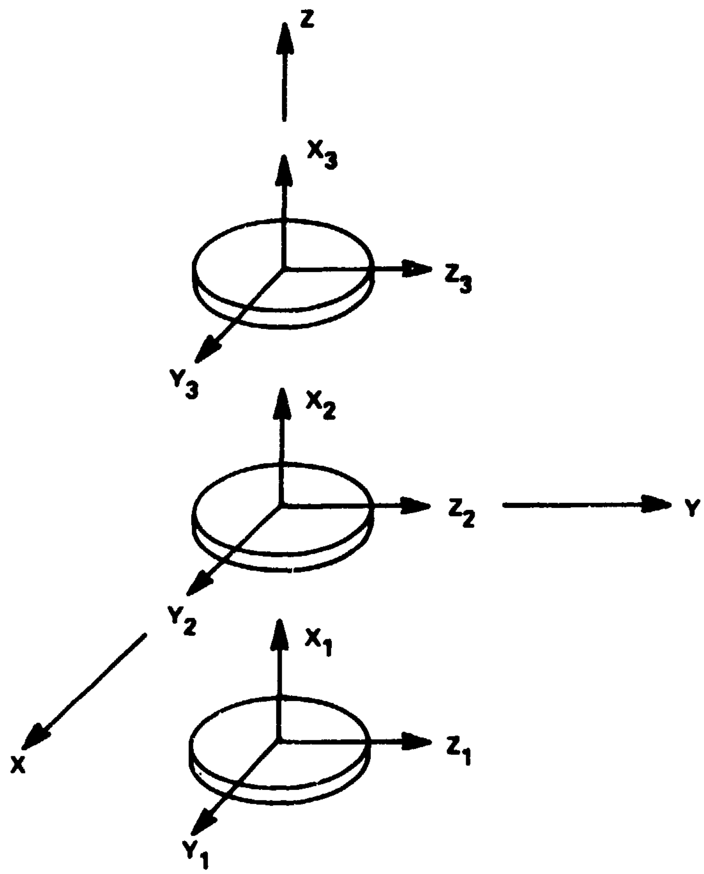



Figure 8. A three-CMG system in a parallel configuration.

$$C(\underline{\delta}) = \begin{bmatrix} -h_1 s\alpha_1 c\beta_1 & -h_1 c\alpha_1 s\beta_1 & -h_2 s\alpha_2 c\beta_2 & -h_2 c\alpha_2 s\beta_2 & -h_3 s\alpha_3 c\beta_3 & -h_3 c\alpha_3 s\beta_3 \\ h_1 c\alpha_1 c\beta_1 & -h_1 s\alpha_1 s\beta_1 & h_2 c\alpha_2 c\beta_2 & -h_2 s\alpha_2 s\beta_2 & h_3 c\alpha_3 c\beta_3 & -h_3 s\alpha_3 s\beta_3 \\ 0 & h_1 c\beta_1 & 0 & h_2 c\beta_2 & 0 & h_3 c\beta_3 \end{bmatrix} .$$

Let $f(\underline{\delta}) = \sqrt{\det G}$, then by a derivation similar to that in Example 1, $\xi_\ell$ is is given by equations (16), (19), and

$$\xi_{2i-1} = h_i^2\{[a_{\alpha i}(d_{11} - d_{22}) - b_{\alpha i} d_{12}] b_{\beta i}$$

$$- [s\alpha_i d_{13} + c\alpha_i d_{23}] a_{\beta i}\}/\sqrt{\det G} \qquad (21a)$$

$$\xi_{2i} = h_i^2\{[b_{\alpha i}(d_{11} - d_{22}) - d_{33} + 4 a_{\alpha i} d_{12}] a_{\beta i}$$

$$- [c\alpha_i d_{13} - s\alpha_i d_{23}] b_{\beta i}\}/\sqrt{\det G} \quad . \qquad (21b)$$

The procedure of the steering law is the same as that in Example 1 with the following exception: Use equation (21) instead of equation (20) in step 4, and instead of step 2 the following step should be used to ensure the avoidance of the case of $\det G = 0$:

If $\det G = 0$, then let

$$\delta'_\ell = \delta_\ell + \Delta\delta'(\ell - 3.5) \quad , \quad \ell = 1, 2, \ldots, 6$$

and go to step 1. If $\det G \neq 0$ go to step 3. $\Delta\delta'$ is a small positive number.

Zero momentum stationary state is given by $\{|\alpha_1 - \alpha_2| = |\alpha_2 - \alpha_3| = |\alpha_3 - \alpha_1| = 120, \beta_i = 0, i = 1,2,3\}$. One such state is shown in Figure 9; any other state can be obtained by a rotation of this state around the Z axis.

A digital simulation was performed with

$$k_2 = 0.2 \quad , \quad k_1 = 0.1$$

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$$r_g = 2 \quad , \quad \Delta = 8 \quad , \quad \Delta\delta' = 0.1 \quad .$$

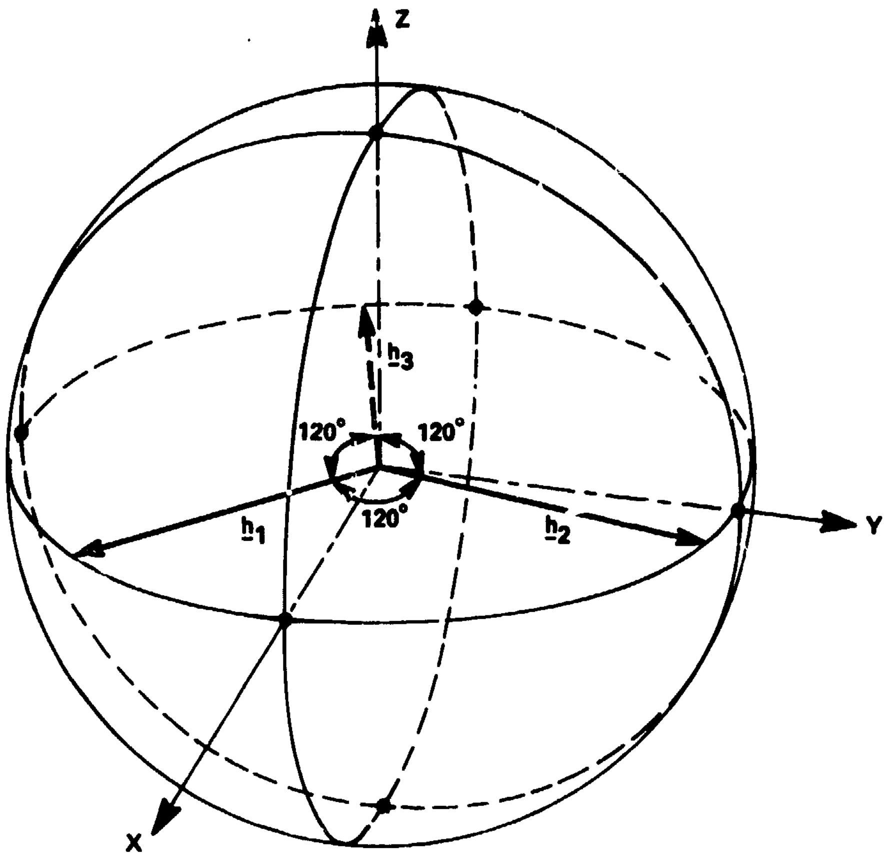


Figure 9. Zero momentum stationary state of the three-CMG system in a parallel configuration.

Figure 10 shows a typical behavior of the system; a torque command $\{t_x = t_y = t_z = 0.005\}$ is applied to the system with a zero momentum stationary state $\{\alpha_1 = 0,\ \alpha_2 = 120,\ \alpha_3 = -120,\ \beta_i = 0,\ i = 1,2,3\}$. Figure 11 shows a response to a torque command in the vicinity of the most unfavorable direction. The initial state is selected as $\{\alpha_1 = 60,\ \alpha_2 = 179,\ \alpha_3 = -60,\ \beta_i = 0,\ i = 1,2,3\}$, and the X-directional torque command with magnitude 0.005 is applied. The figure shows a very good momentum distribution procedure.

If the initial state is $\{\alpha_1 = 60,\ \alpha_2 = 180,\ \alpha_3 = -60,\ \beta_i = 0,\ i = 1,2,3\}$ and the torque command is kept exactly in X axis direction, the system will fall

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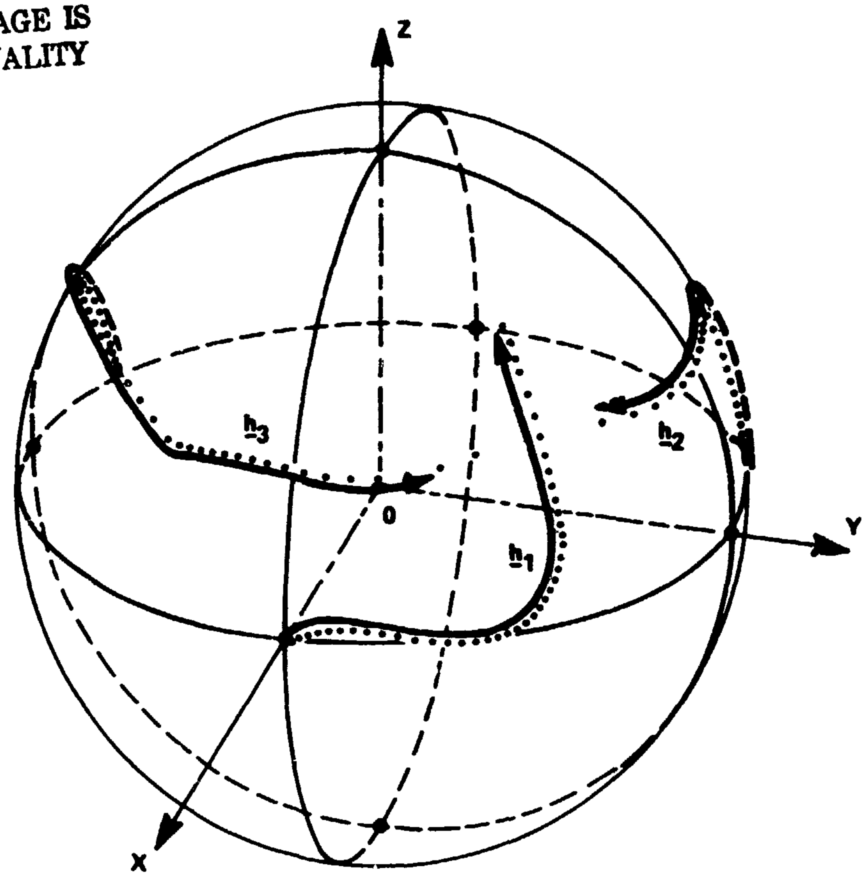
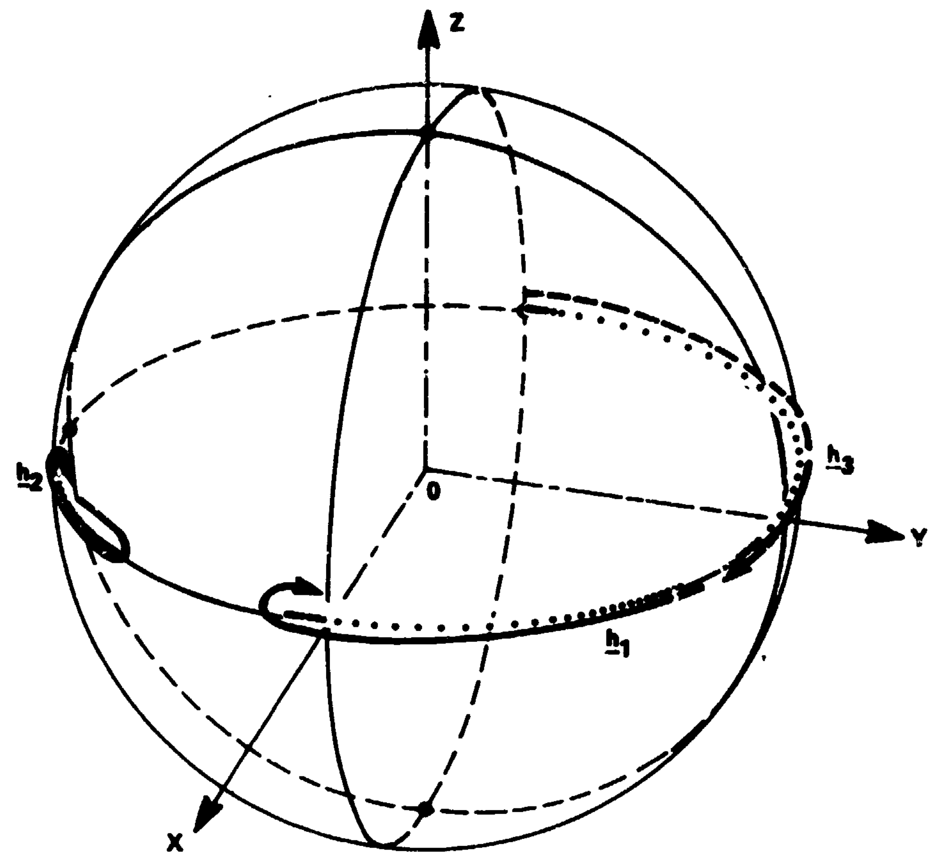



Figure 10. Response to torque command $t_x = t_y = t_z = 0.005$ (parallel configuration).

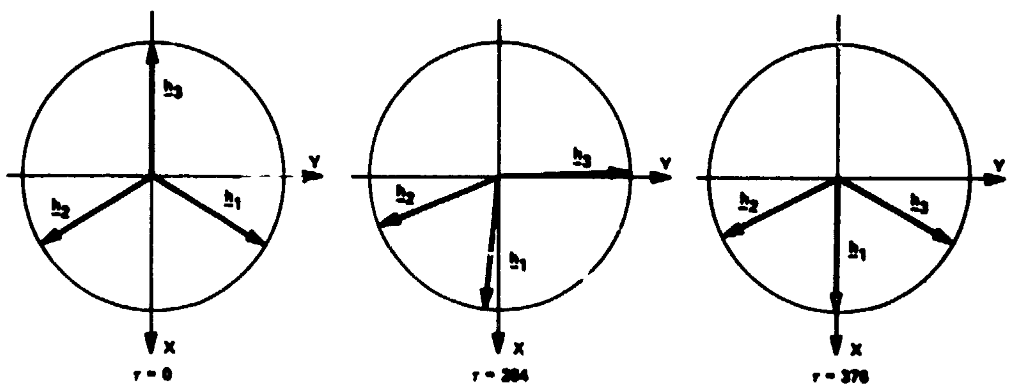
into the singular state shown in Figure 12(b), $\tau = 0$. As mentioned in Example 1, however, the possibility of occurrence of this situation is almost zero. Moreover, even if such a situation does occur, the steering law has the ability to recover from it, as shown in Figure 12 where the initial state is $\{\alpha_1 = \alpha_2 = 0, \alpha_3 = 180, \beta_i = 0, i = 1,2,3\}$ and the torque command is kept at zero.

## IV. DISCUSSION

Although the root of the determinant of $CC^T$ ($=\sqrt{\det G}$) has been used as the criterion function, any reasonable function of $\underline{\delta}$ can be a candidate for it. The reason for the selection of $\sqrt{\det G}$ in this report is as follows:




a. Trajectory of the ·ndpoints of momentum vectors.




b. Positions of momentum vectors in XY-plane at time $\tau$.

Figure 11. Response to an X directional torque command (parallel configuration).

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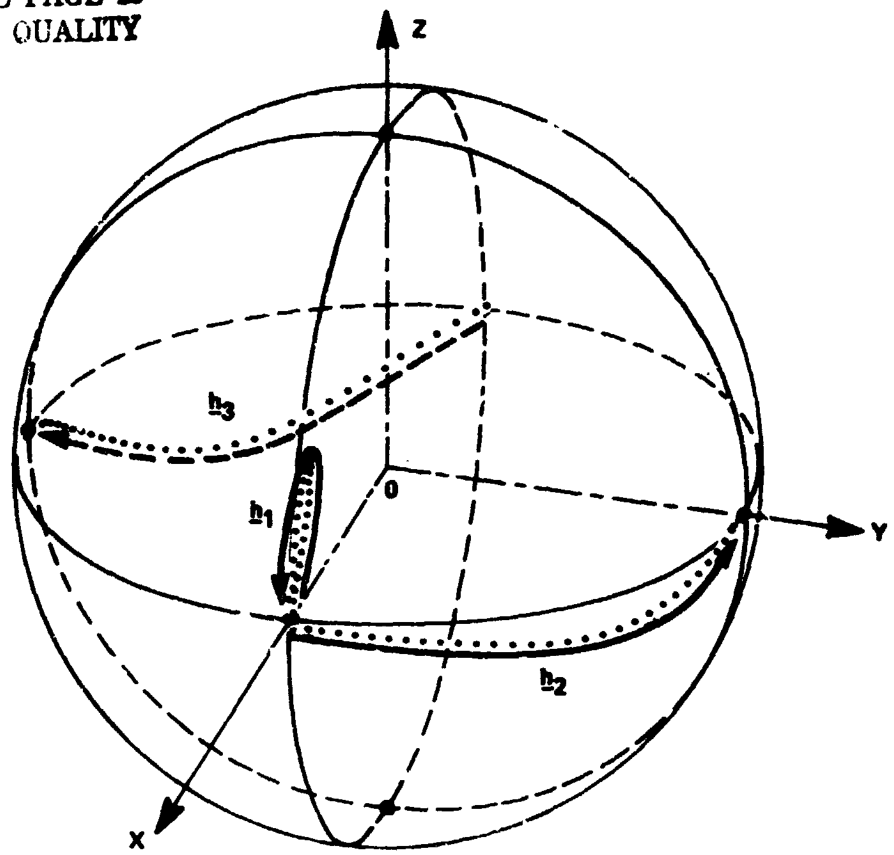
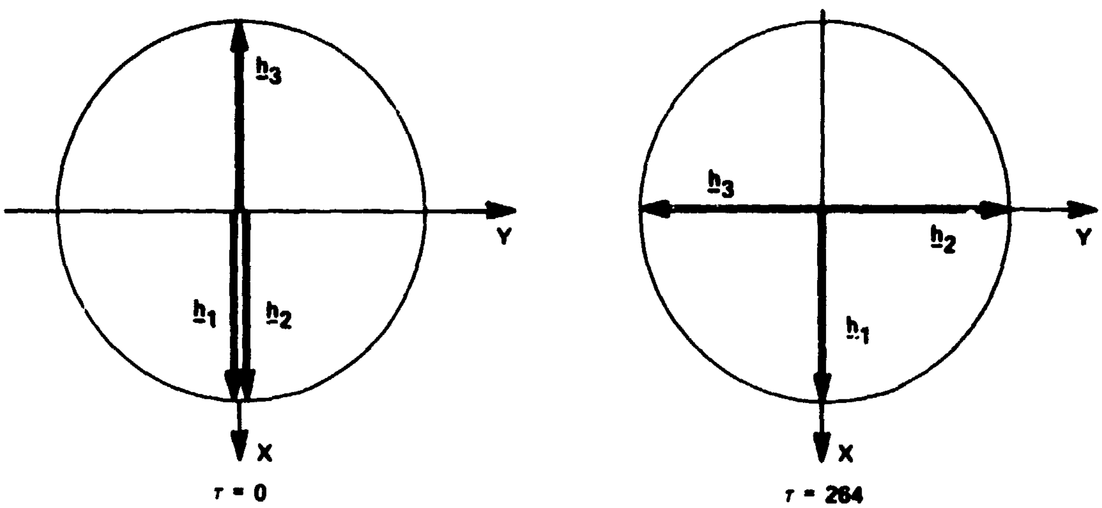



a. Trajectory of the end points of momentum vectors.




b. Positions of momentum vectors in XY-plane at time $\tau$.

Figure 12. Recovery from a singular state (parallel configuration).

1. detG is a scalar variable which is most directly related to the singularity of the system.

2. By digital simulation, it was found that $\sqrt{\det G}$ gives $\underline{\xi}$ which is large enough to drive the system out of the vicinity of the singular states, while detG gives very small $\underline{\xi}$ around the singular states.

3. So far, the author could not find any simpler criterion function which gives as good performance as $\sqrt{\det G}$.

For some specific CMG configurations, there may be simpler criterion functions. For example, for the parallel configur·.tion given in Example 2,

$$-\sum_{i=1}^{3} [(\alpha_i - \alpha_{i0})^2 + k(\beta_i - \beta_0)^2]$$

is a good candidate for $f(\underline{\delta})$, whe·· $\alpha_{i0}$, $i = 1,2,3$ and $\beta_0$ are reference angles that should be specified by the designer as functions of $\underline{\delta}$. Whcther or not this is really good for $f(\underline{\delta})$ could be determined only after investigating various methods of giving $\alpha_{i0}$ and $\beta_0$ and performing a digital simulation study. To find such a simpler criterion function is a topic for future investigation.

The author believes that the software of the proposed steering law is much simpler than any other steering laws developed so far. Rigorous comparison study should be performed, however, to give any objective statement about computer software. This is another direction of the future study.

Since any $\underline{r}$ which satisfies equation (1) can be expressed in the form of equation (2) by selecting an appropriate $\underline{k}$, any steering law which produces the same torque output as the torque command could be expressed in the form of equation (2). Also, if $\underline{k}$ is taken to be zero, equation (2) becomes a steering law which has usually been called the pseudoinverse steering law. Therefore, the steering law developed here may be called the "universal pseudoinverse steering law."

Systems utilizing only three double-gimbal CMGs have been developed in some detail. However, it is clear from the general discription given in

Section 2 that the steering law has the possibility of remaining effective for any number of double-gimbal and/or single-gimbal CMGs with any configuration (which result in $n \geq 3$), possibly with a proper selection of the criterion function. Reaction wheel systems are also expressed by (1) with a constant coefficient matrix C instead of a state-dependent matrix $C(\underline{\delta})$. Therefore, the approach taken in this report will be useful in obtaining steering laws for reaction wheel systems. Research in this direction should also be made in the future.

## V. CONCLUSION

A steering law, whose principle is very simple, for three double-gimbal CMG systems is proposed. This steering law is applicable to systems with almost any configuration of CMGs and the CMG-out operation needs no special modification.

To illustrate the effectiveness of the steering law, two examples of three double-gimbal CMG systems in an orthogonal configuration and in a parallel configuration were worked out in detail. Simulation results have shown that any command torque can always be met except when the system is in a singular state and that, whenever the system is in, or close to, a singularity, the steering law drives the system out of the vicinity of the singularity.

# REFERENCES


1. Chubb, W. B.; Schultz, D. N.; and Seltzer, S. M.: Attitude Control and Precision Pointing of the Apollo Telescope Mount. J. of Spacecraft and Rockets, Vol. 5, No. 8, Aug. 1968.

2. Kennel, H. F.: A Control Law for Double-Gimbaled Control Moment Gyros Used for Space Vehicle Attitude Control. NASA TM X-64536, Aug. 1970.

3. Chubb, W. B.; Kennel, H. F.; Rupp, C. C.; and Seltzer, S. M.: Flight Performance of Skylab Attitude and Pointing Control Systems. Proc. of AIAA Mechanics and Control of Flight Conference, Aug. 1974.

4. Boullion, T. L. and Odell, P. L.: Generalized Inverse Matrices. J. Wiley, New York, 1971.

# APPROVAL

## A STEERING LAW FOR THREE DOUBLE-GIMBAL CONTROL MOMENT GYRO SYSTEMS

By Tsuneo Yoshikawa

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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