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Recently, considerable attention has been focused on the problem of interpreting the observations of a very small lunar dipole magnetic moment.[1,2,3]

Runcorn[4,5] in a series of papers has maintained that the observation of such a small surface dipole field ($\lesssim .05\gamma$ deduced from the Apollo 15 subsatellite magnetometer[3]) argues for the existence of a fairly strong interior lunar dipole moment in the past ($\gtrsim 3.2\times10^9$ years ago). His contention is that if an interior lunar magnetic field disappeared during the last 3.2×10^9 years, the exterior field of the moon would now be zero. This, he argues, is a direct result of a theorem of potential theory.

In the discussion below, I show for a very simple model of the moon, that if a primordial core magnetic field existed, it would give rise to a present day nonzero dipole external field. This conclusion contrasts with that of Runcorn.[4,5]

The general outline of the computation is as follows: I explicitly solve a potential problem for a differentiated planet with an intrinsic core magnetic field plus an induced mantle magnetic field. The mantle is assumed to be at least slightly ferromagnetic so that, after the core field's disappearance, a remanent permanent magnetization remains in the mantle. This magnetization is evaluated, and it is found to consist of two terms. One of these is identical to that considered by Runcorn, and produces zero external field. The second term is shown to produce a dipole external field. Some consequences of this result are discussed.

Consider a uniformly magnetized core of radius, a, embedded in a permeable mantle. (The uniform magnetization of the core is a simple idealization that results in an external dipole field). The core magnetization has the form $\underline{M}_o = M_o\hat{e}_3$. Let Φ $(\underline{x})$ be the scalar potential of the magnetic field, $\underline{H}$, such that $\underline{H} = -\nabla\Phi(\underline{x})$. In the mantle assume that $\underline{B} \simeq \mu\underline{H}$. Thus, Laplaces equation is satisfied everywhere except at the core-mantle and mantle-vacuum boundaries. At these boundaries the radial component of $\underline{B}$ and the tangential component of $\underline{H}$ are continuous. One immediately has, in spherical coordinates,

$$\Phi_C(\underline{x}) = \alpha r \cos\theta \tag{1}$$

$$\Phi_M(\underline{x}) = (\beta r + \gamma/r^2)\cos\theta \tag{2}$$

$$\Phi_V(\underline{x}) = \frac{\delta}{r^2}\cos\theta \tag{3}$$

where C, M, and V refer to core, mantle, and vacuum, respectively; and

$$\begin{aligned}
\alpha &= \beta + \gamma/a^3 \\
\beta &= -2(1-\mu)A \\
\gamma &= b^3(\mu+2)A \\
\delta &= 3\mu b^3 A \\
A &= 4\pi M_o a^3/D \\
D &= (2\mu+1)(\mu+2)b^3 - 2a^3(1-\mu)^2
\end{aligned} \tag{4}$$

and, where b is the radius of the planet. [Effects due to the diamagnetic plasma environment of the moon are ignored.]

One can now imagine that the core magnetic field dies out. The magnetization of the mantle in the absence of a core field is then

$$\underline{M}(\underline{x}) = \frac{(\mu-1)}{4\pi} \left[-\beta + \frac{2\gamma}{r^3}\right]\cos\theta\ \hat{e}_r + \left[\beta + \frac{\gamma}{r^3}\right]\sin\theta\ \hat{e}_\theta \tag{5}$$

The scalar potential, $\psi(\underline{x})$, of the resulting field, in the absence of any core field, is computed from[6]

$$\psi(\underline{x}) = -\nabla\cdot\int d^3x'\underline{M}(\underline{x})/|\underline{x} - \underline{x}'| \tag{6}$$

with the result

$$\psi_C(\underline{x}) = -\frac{2}{3}\frac{(\mu-1)}{a^3}(1 - a^3/b^3)\ \gamma r\ \cos\theta \tag{7}$$

$$\psi_M(\underline{x}) = \frac{(\mu-1)}{3}\ [\beta(a^3-r^3)-2\gamma(1-r^3/b^3)]\frac{\cos\theta}{r^2} \tag{8}$$

$$\psi_V(\underline{x}) = \frac{(\mu-1)}{3}(b^3-a^3)\ \beta\ \frac{\cos\theta}{r^2} \tag{9}$$

Equation (9) leads to a nonzero external dipole field. (This result can be easily generalized to include higher order moments than the dipole.) Runcorn's conclusion[4,5] that the external field is zero is based on his assertion that the potential of the magnetizing field has the form of (2), but with $\beta=0$. Clearly, if $\beta=0$ in (2), then $\psi_V(\underline{x}) = 0$. The purpose of this letter is to emphasize that, using an <u>internal</u> magnetizing field, it is quite easy to imagine situations in which the external field is <u>nonzero</u> after the core field has decayed to zero. The solution (7) - (9) is, in fact, a linear combination of the "interior" and "exterior" solutions discussed by Runcorn[5].

It is worth emphasizing that the conclusion that the external field is <u>not</u> zero can be derived without resort to the mathematical formalism outlined above in deriving (7) - (9). The nonzero result is a straightforward consequence of the linearity of the field equations of magnetostatics. Using the principle of superposition, the solution of the problem with zero core-field can be obtained from the solution with nonzero field, (1) - (4), by adding to the fields derivable from (1) - (4) the field of a uniformly magnetized sphere of radius, a, with magnetization $\underline{M}' = -M_o \hat{e}_3$. This is indicated schematically in Figure 1. It is obvious from this construction that the resulting external field is a dipole of reduced strength. It is also not difficult to show that the fields resulting from such a superposition are identical to those resulting from (7) - (8).

In a recent preprint, Stephenson, <u>et al.</u>[7] note that Runcorn's result is strictly true only if the magnetic susceptibility of the mantle is very small. They use a value of the susceptibility of 10^{-4}, and conclude that such effects can be ignored. From equation (9), and the definition of β, (eq.4), it is clear that the exterior field is of higher order in $(1-\mu)$ than the fields in the other two regions. However, one must be cautious about arguing that it is therefore negligible. Although μ has been treated as though it were a paramagnetic permeability in this simple derivation, it must, of necessity be ferromagnetic. To my knowledge the ferromagnetic permeability of the moon is not known. However, Dyal, <u>et al.</u>[8] have found a paramagnetic permeability $\mu \simeq 1.01$, that already is larger than the value of $1 + 10^{-4}$ used by Stephenson, et al.[7] This larger value for μ implies the existence of ferromagnetic material[8], the properties of which are undetermined.

The basic conclusion of this letter is that if the moon had a core magnetic dipole moment in the past that has died away, then, in general, a nonzero external dipole field would exist today. The strength of this dipole field would depend on details of the moon's evolution, which have not been considered here, and on details of the ferromagnetic properties of the lunar mantle that are as yet unknown.

I would like to acknowledge stimulating discussions with Drs. N. F. Ness and J. D. Scudder.

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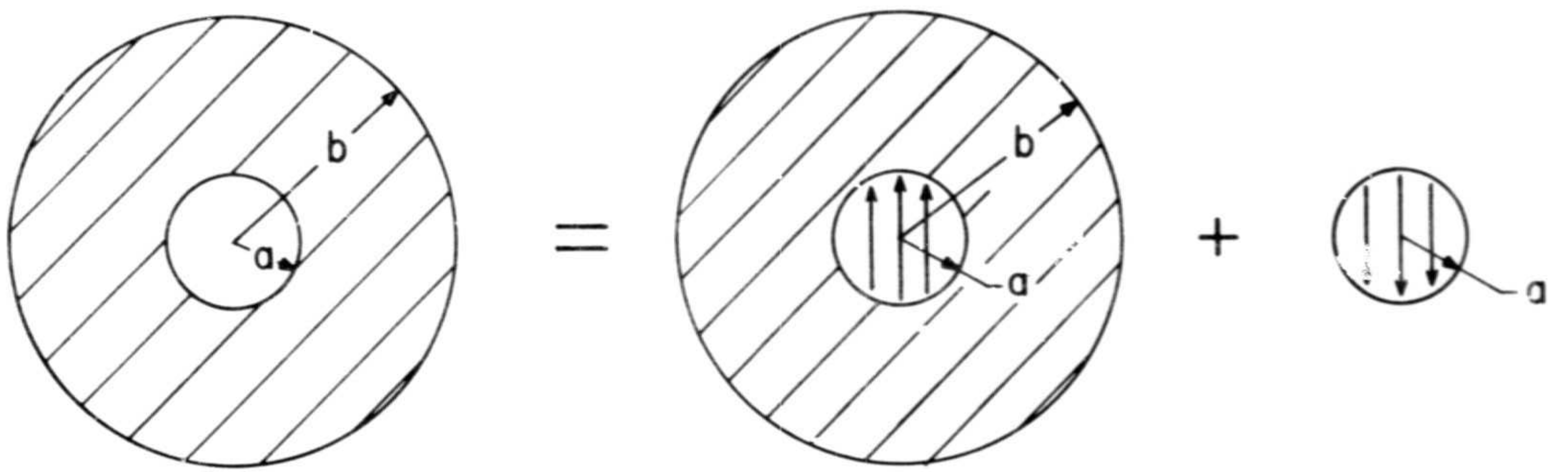
Figure 1 The solution indicated by eqs. (7)-(9) for a magnetized shell with no core field, (A), can be derived directly from the solution indicated by eqs. (1)-(3) for a body with a core magnetic field and a surrounding mantle, (B), by superimposing the fields due to a uniformly magnetized sphere, (C). It is clear from this construction that the external field of case (A) will, in general, be a dipole.



$\underline{M}_0 = 0\ ,\ r \leq a$

$\underline{M}$ is given by eq. 5 for $a \leq r \leq b$

(A)

$\underline{M}_0 = M_0\hat{e}_3\ ,\ r \leq a$

$\underline{B} \simeq \mu\underline{H},\ a < r \leq b$

(B)

$\underline{M}' = -M_0\hat{e}_3,\ r \leq a$

(C)