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LASER DOPPLER VELOCIMETER SYSTEM SIMULATION FOR SENSING AIRCRAFT WAKE VORTICES

PART III: THE PROBLEM OF REFLECTION FROM THE SECONDARY MIRROR

J.A.L. Thomson

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Principal Investigator: J. Alex Thomson
Phone: (415)848-3063

Procurement Officer: Ray Weems
Phone: (205)453-2857

Contracting Officer's Representative:
R. Milton Huffacker
Phone: (205)453-1595

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Various possibilities for alleviating the problem of reflection from the secondary mirror are discussed. Calculations are carried out to investigate the effects of obscuring the central portion of the secondary mirror. Two cases are treated: (1) a Gaussian distribution of reflectivity, and (2) a sharp-edged obscuration. Substantial reduction in reflection from the secondary mirror is found achievable, but the reduction was found highly sensitive to the form of the radial distribution of

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20. Abstract (cont'd)

reflectivity. In particular, the second case is found not effective. The total power reflected from the secondary mirror that is incident on the detector is estimated. Techniques for experimental testing of alleviation schemes are suggested.

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THE PROBLEM OF REFLECTION FROM THE SECONDARY MIRROR. I

1. The Problem

A portion of the outgoing laser beam from the central portion of the secondary mirror will return through the central hole in the primary and add to the return signal scattered back from the atmosphere (see Fig. 1).

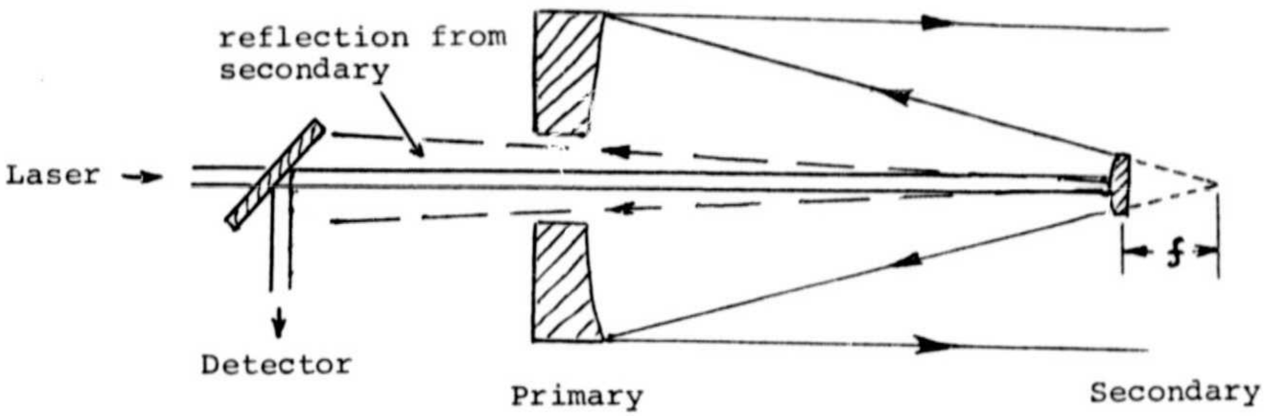

Figure 1. Cassegrainian Telescope Geometry

Discussions at a recent meeting[1] suggest that this strong reflected signal will either saturate the detector and/or exceed the dynamic range of the processing system.

Some possibilities for suppressing this problem have been suggested and include:

1) putting a hole or non-reflecting spot at the center of the secondary,

2) introducing quarter wave plates (somehow) to rotate the polarization of the unwanted scattered light so that it can be selectively rejected,

3) utilizing the fact that the secondary scatter forms a diverging beam whereas the atmospheric scatter beam is parallel,

4) making use of this scattered beam as the local oscillator signal,

5) decreasing the radius of curvature of the secondary (near its center only) to give greater divergence to the scattered beam.

In this note we will discuss item one.

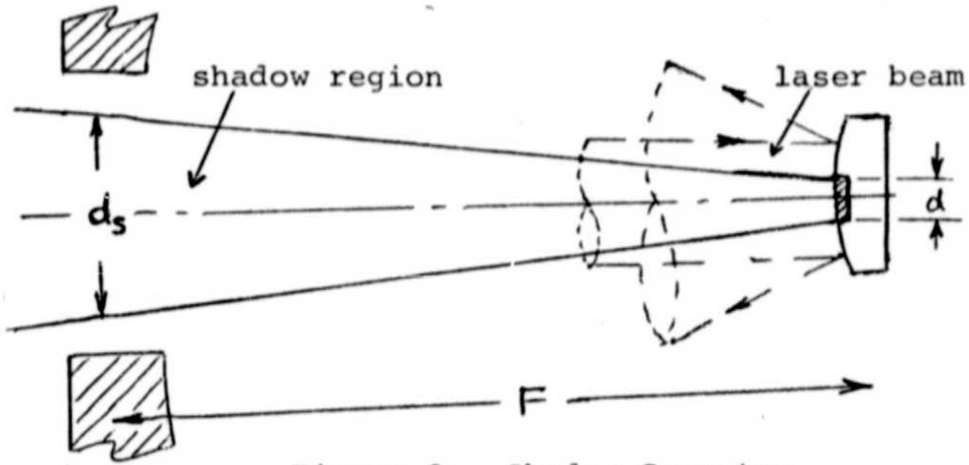

Figure 2. Shadow Geometry

A non-reflecting spot of diameter d will create a diverging shadow around the optic axis which will have a diameter $d_s = \frac{F}{f} d$ at the primary mirror. For the current system design ($F = 60$ cm, $f = 1.2$ cm) $d_s = 50d$.

Since the incident laser beam has a diameter of about 0.6 cm between e^{-2} points at the primary mirror, the geometric shadow of the spot should considerably exceed this dimension. If we take 1.2 cm as the minimum required shadow diameter at the primary mirror, a spot diameter of only 0.24 mm would be adequate if geometric optics were an accurate approximation. However, diffractive effects can be large for such small apertures, and this causes the shadow to "fill in". At the recent meeting at NASA-MSFC it was initially suggested that the Rayleigh distance (d^2/λ) was a proper measure of the maximum distance away from the secondary that was effectively shadowed (see Fig. 3). If this estimate were correct, shadowing would only be effective up to 10 cm distance for a 1 mm diameter spot (i.e., be of no value) and up to 90 cm for a 3 mm diameter spot (be marginally effective).

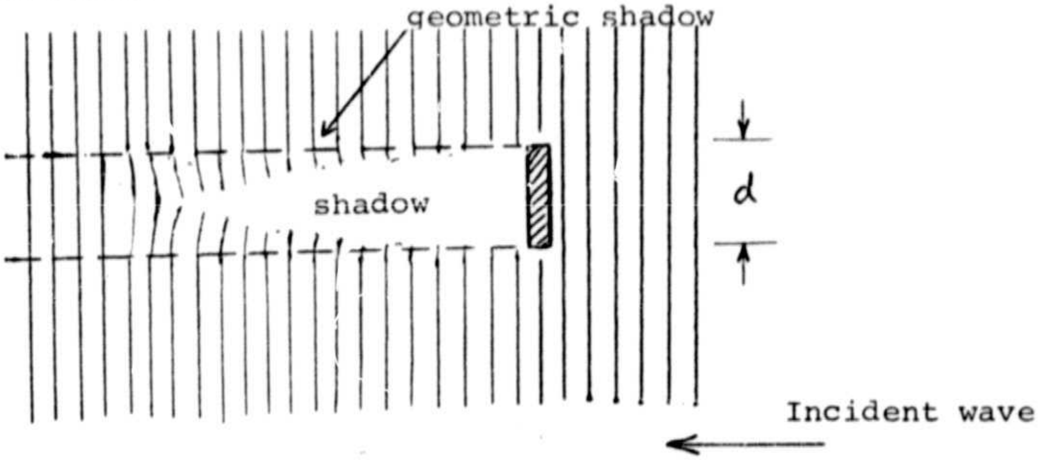

Figure 3. "Filling in" of the shadow of an object by diffraction

Joe Randall* indicated that this conclusion is appropriate only when the radius of curvature of the wave reflected from the secondary mirror is infinite (plane waves) and, for the case of interest, the fact that the secondary is convex with a relatively small radius of curvature (~2.4 cm) would cause substantial shadowing to occur even for small spot diameters.

This conclusion is verified in the following analysis. The reason for the difference is that the strong divergence

*NASA-MSFC Astrionics Laboratory

of the beam (resulting from the short focal length of the secondary) causes the geometric shadow radius to grow rapidly with distance away from the mirror, and this growth overwhelms the diffractive effects which are trying to reduce the effective shadow radius.

2. Calculation of the Effect of Obscuring the Central Portion of the Secondary Mirror

We use the usual Fresnel-Kirchoff approximation[2] to calculate the pattern of light reflected from the secondary. This approximation includes diffraction and is appropriate when the wave front normals all make small angles to the optic axis (paraxial approximation) and when the wavelength (0.00106 cm) is small compared to the various aperture dimensions (of order 0.1 cm or greater: the requirement for the physical optics approximation). These assumptions are well satisfied in the present case.

To simulate the effect of an opaque spot on the mirror, we will assume the mirror surface has an effective reflection coefficient $\sigma(r)$ which varies with distance from the optic axis. The wave incident on the mirror is assumed to be plane and to have a Gaussian intensity distribution

$$I = I_o \, e^{-2r^2/b^2} \tag{1}$$

where 2b is the diameter to the e^{-2} points and is about 6 mm in the present case.

The Fresnel-Kirchoff theory yields the following expression for the light wave amplitude at a distance z from the secondary mirror[2]:

$$\psi(x,y) = i\psi_o \int_o^{R_{sec}} e^{i\phi-\left(x'^2+y'^2\right)/b^2} \sqrt{\sigma(x',y')}\, dx'dy' \tag{2}$$

where

$$\phi = \frac{\pi}{\lambda z}\left((x-x')^2 + (y-y')^2\right) + \frac{\pi}{\lambda f}\left(x'^2 + y'^2\right)$$

ψ_o is the on-axis laser beam amplitude at the secondary. The point (x', y') is in the plane of the secondary mirror and (x,y) is in the plane of the primary mirror. R_{sec} is the radius of the secondary and can be set equal to infinity so long as it is significantly greater than b. $|\psi(r)|^2 = I(r)$ is the desired intensity distribution in the plane of the primary mirror.

We will treat two cases: 1) a Gaussian distribution of reflectivity that will allow explicit evaluation of Eq. (2) on-axis as well as off-axis, and 2) a sharp-edged obscuration but only on axis.

Gaussian Reflectivity Profile

We assume an effective reflectivity radial distribution given by (see Fig. 4)

$$\sigma(r') = \left[1 - \exp\left(-(r'/a)^2\right)\right]^2 \tag{3}$$

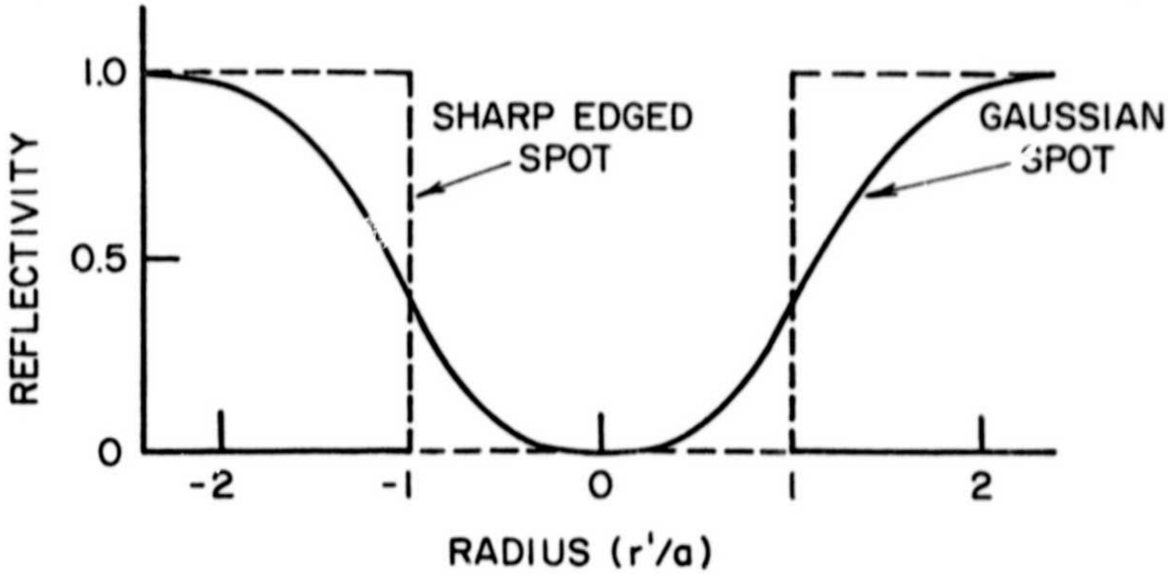

Fig. 4. Reflectivity Profiles for Obscuration of the Center of the Secondary Mirror

As a function of radius in the primary mirror plane the amplitude of the reflected wave may be expressed in the form

$$\frac{\psi}{\psi_o} = f\left(\frac{1}{b^2}\right) - f\left(\frac{1}{b^2} + \frac{1}{a^2}\right) \tag{4}$$

where

$$f(\xi) = \frac{(i\pi/\lambda z)\,\exp\left(-\left(\frac{\pi r^2}{\lambda z}\right)\Big/\left[(\lambda z/\pi)\,\xi - i(1 + z/f)\right]\right)}{\sqrt{\left(\xi - \frac{i\pi}{\lambda}\left(\frac{1}{z} + \frac{1}{f}\right)\right)\left(\xi - \frac{i\pi}{\lambda}\left(\frac{1}{z} + \frac{1}{f}\right)\right)}} \tag{5}$$

These two relations can be used to determine the intensity distribution. In Figure (5) we show the expected reduction of intensity for various values of the spot radius a.

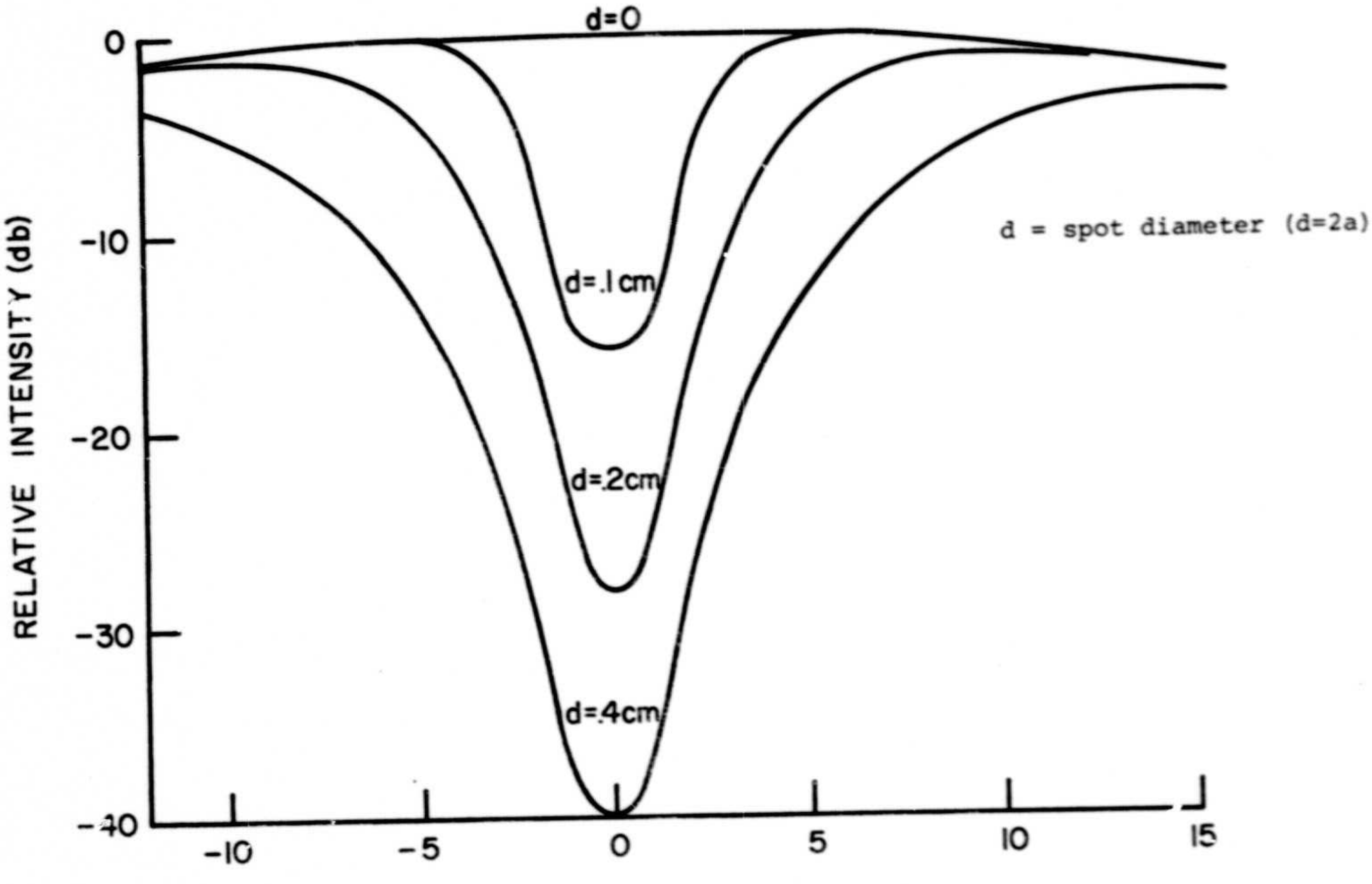

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Figure 5. Radial Distribution of Intensity for a Gaussian Spot

On-Axis Intensity

On the optic axis the reflected wave amplit de is given by

$$\psi = \frac{\pi i \psi_o}{\lambda z}\int_o^\infty e^{\left(\frac{i\pi}{\lambda}\left(\frac{1}{z}+\frac{1}{f}\right)-\frac{1}{b^2}\right)r^2}\sqrt{\sigma(r)}\,dr \tag{6}$$

For a sharp-edged aperture of radius a ($\sigma(r) = 0$ for $r<a$ and $\sigma(r) = 1$ for $r>a$) the amplitude integrates to be

$$\psi_{SE}(o) = \frac{i\pi\psi_o\left[\exp\left(-\frac{a^2}{b^2}+\frac{i\pi a^2}{\lambda}\left(\frac{1}{z}+\frac{1}{f}\right)\right)\right]}{\lambda z\left[\frac{1}{b^2}-\frac{i\pi}{\lambda}\left(\frac{1}{z}+\frac{1}{f}\right)\right]} \tag{7}$$

corresponding to an on axis intensity

$$\frac{I_{SE}(o)}{|\psi_o|^2} = \frac{e^{-2a^2/b^2}}{\left(1+\frac{z}{f}\right)^2+\left(\frac{\lambda z}{\pi b^2}\right)^2} \tag{8}$$

and a reduction factor (compared to the value for no modification of the secondary)

$$R_{SE}(o) = e^{-2a^2/b^2} \quad . \tag{9}$$

This expression agrees with that of Webb[4](Webb's Eq. 22) in the limit of an infinite radius (R_{sec}) of the secondary. For a finite radius secondary, additional terms enter into Eq. (9):

$$R_{SE}(o) = e^{-2a^2/b^2} + e^{-2R_{sec}^2/b^2} - 2e^{-(a^2+R_{sec}^2)/b^2} \cos\left(\frac{\pi}{\lambda}\left(\frac{1}{[illegible]} + \frac{1}{z}\right)\left(a^2 - R_{sec}^2\right)\right) \tag{10}$$

which is essentially the same* as Webb. The oscillating cosine term is due to the truncation of the outer edges of the laser beam and for our purposes is assumed negligibly small ($R_{sec} >> b$).

Since the spot radius should be less than the laser beam radius, the reduction is very weak and, in fact, is really no reduction at all: it is just the ratio of the laser beam intensity at the spot edge to that at the center. What has happened is that placing an opaque circular sharp-edged spot on the center of the secondary has not sensibly reduced the on-axis intensity <u>at all</u>. This is a well known "paradox" in optics where it has been well demonstrated(3) that a sharp-edged circular opaque aperture placed in front of a point source (the convex secondary mirror can be thought of as being exactly equivalent to locating a point source at the mirror focal point) produces a bright spot on the line through the source and the center of the disc. The brightness is restricted to axis and decays off axis. Of course very close to the disc where the paraxial approximation fails (a/z is not small) the brightness will diminish.

---

* Webb's expression is a factor of 2 larger, but this appears to be either incorrect or due to a different normalization.

The angular width is expected to be of the order of the diffraction angle based on the spot diameter ($\lambda/2a$). For the current design parameters (z = 60 cm, λ = 0.00106 cm) the diameter of this central bright portion is expected to be of the order

$$\frac{\lambda z}{2a} \sim 0.64 \left(\frac{0.1 \text{ cm}}{2a} \right) \text{ cm} . \tag{11}$$

A more precise calculation, similar to that by Webb, would be required to establish accurate values. For the present optical design and for reasonable spot diameters (1 to 3 mm) this central spot has a not insignificant width, and its presence mitigates against the use of a simple sharp-edged spot or hole. Methods for smoothing out (in radius) the sudden change in reflectivity are required.

For a smooth or "fuzzy" edged disc the bright spot of light will not develop. For the Gaussian reflectivity profile given in Eq. (3) the intensity reduction factor is

$$R_g(o) = \frac{1}{1 + \frac{a^2}{b^2} + \left[\frac{\pi a^2}{\lambda}\left(\frac{1}{f} + \frac{1}{z}\right)\right]^2} \tag{12}$$

which for the current design values (f = 1.2 cm, λ = 0.00106 cm, z = 60 cm, b = 0.3 cm) has the dependence

$$R_g(o) = \frac{1}{1 + (a/0.3)^2 + (a/0.02)^4} \quad \text{with } a \text{ in cm.} \tag{13}$$

In contrast to the sharp-edged spot this fuzzy edged spot gives very substantial reductions for spot radii greater than 0.2 mm. Reduction factors for the two profiles are shown in Figure (6).

3. Obscuration Losses

The central spot on the secondary will result in some losses by obscuration. These losses are estimated as the ratio of the effective (two way) transmission with the spot to that without the spot:

$$R_{blockage} = \left[\int \sigma(r) e^{-2r^2/b^2} rdr \Big/ \int e^{-2r^2/b^2} rdr \right]^2 \tag{14}$$

For the Gaussian spot this loss factor is

$$R_{g_{blockage}} = \left[\frac{1}{\left(1 + \frac{a^2}{b^2}\right)\left(1 + \frac{2a^2}{b^2}\right)} \right]^2 \tag{15}$$

and, for the sharp-edged spot,

$$R_{SE_{blockage}} = e^{-4a^2/b^2} \tag{16}$$

4. Experimental Testing of Alleviation Techniques

Because of the sensitivity of the reduction factor to the reflectivity profile and because of the importance of obtaining a substantial reduction, it is desirable to test given procedures before incorporating these into a design.

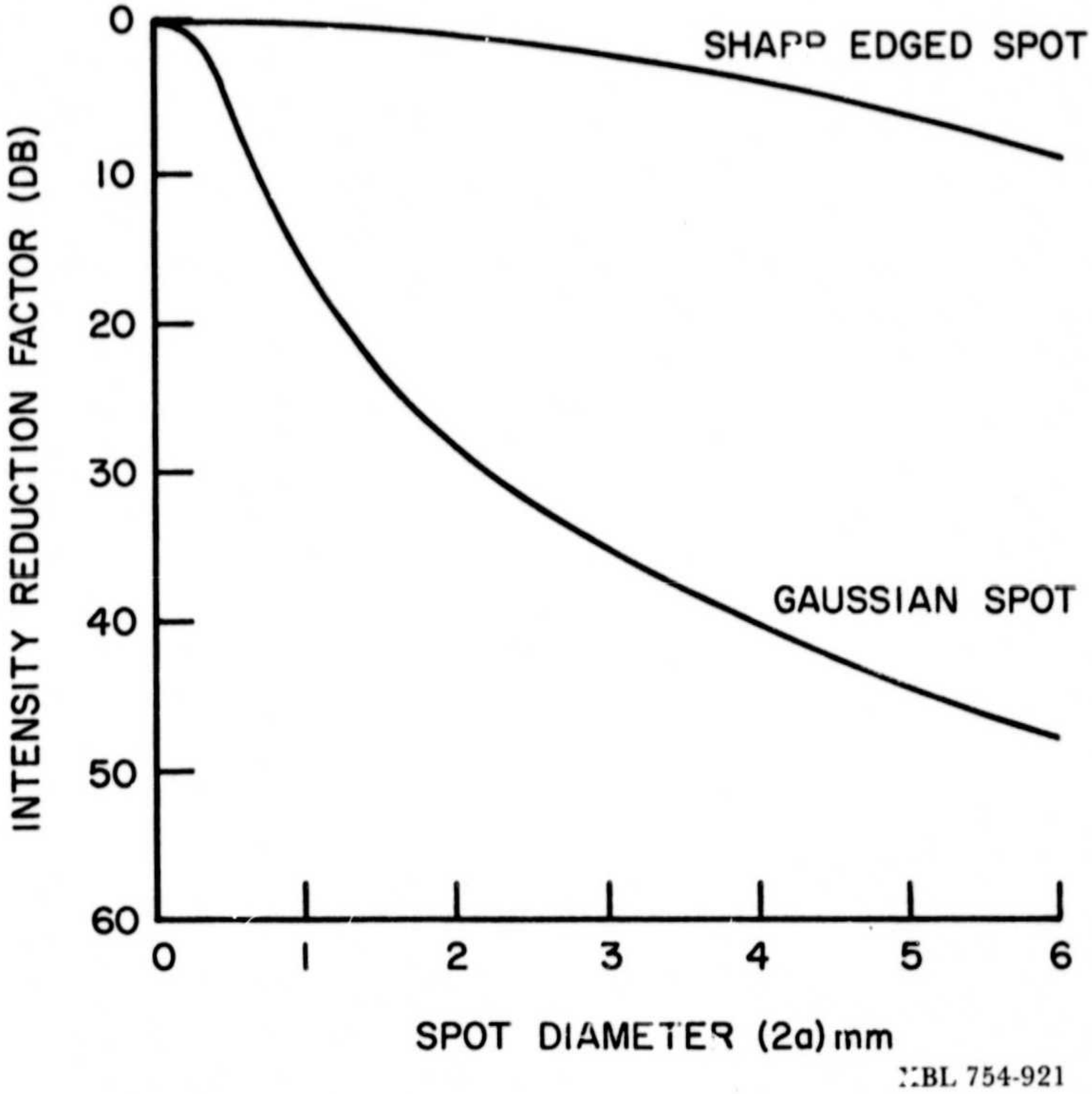

Figure 6. On-axis Reduction of Reflection from Secondary

It would be particularly useful to use a visible laser for such test procedures because of the easy diagnostics. Because diffraction is the effect of interest and because the physical optics approximations are well satisfied, this is readily possible. Reference to Eq. (2) shows that the wavelength always appears in the following dimensionless combinations:

$$\frac{(\text{lateral dimension})^2}{(\text{wavelength})(\text{longitudinal dimension})}$$

Thus testing of diffraction effects of given optical configurations can be done at different wavelengths <u>if</u> this quantity is held constant. For example, to scale from 10.6μ to 0.63μ (factor 16.8) all lateral dimensions could be reduced by the factor $\sqrt{16.8} = 4.1$ (i.e., the CO_2 laser beam from 6 mm to a He-Ne beam of 1.46 mm diameter). Here a CO_2 laser scale spot size of 2 mm diameter would be equivalent to a 0.5 mm spot diameter at the He-Ne scale. Alternately the lateral dimensions could be kept the same and all longitudinal distances (including focal lengths) scaled up by a factor 16.8 for test purposes (i.e., the secondary to primary distance from 60 cm to 10.1 meters; the secondary focal length from 1.2 cm to 20.2 cm). Note that secondary focal length might effectively be altered for the purposes of testing at 0.63 microns simply by introducing a converging lens (glass) in front of it. Of course most methods of changing the reflectivity of the secondary will yield different results at different wavelengths (except hole drilling) and this must be accounted for.

5. Conclusion/Recommendations

Substantial reductions in the level of the signal reflected directly back through the system from the secondary mirror appear readily achievable at modest cost to the signal to noise ratio. As suggested by Joe Randall of NASA-MSFC, the divergence of the reflected beam from the secondary can prevent the diffractive filling of the shadow of an opaque spot at the center of the secondary, and such a technique appears capable of yielding adequate reductions. However, the reduction achieved (on axis at least) is highly sensitive to the form of the radial distribution of reflectivity with the most easily achieved profile (a sharp-edged hole) being a very poor choice.

In Figure 7 we have replotted the data of Figure 6 to give the intensity reduction achievable as a function of the obscuration loss for the two profiles considered. It is thought that these profiles represent the extremes and that other reflectivity distributions will yield intermediate characteristics. These results infer that it is the smoothness and lack of high spatial frequency structure in the reflectivity profile that results in the low on-axis signal. Careful attention to this feature during the actual modification of the secondary mirror will be required to obtain effective results.

Because of the importance of the effect and because of the sensitivity of the reduction achieved to the reflectivity profile, it is suggested that experimental testing of various techniques be initiated as soon as possible. It appears that the phenomena are readily scaled to visible wavelengths and it is recommended that various techniques be tested first with a visible laser and scaled components. Further analysis should also be carried out to evaluate other alleviation techniques (items 2 to 5 in Section 1) as well as to investigate other aperturing (multiple ?) techniques. In addition, a value for the reduction required should be obtained from the detector/data processor characteristics.

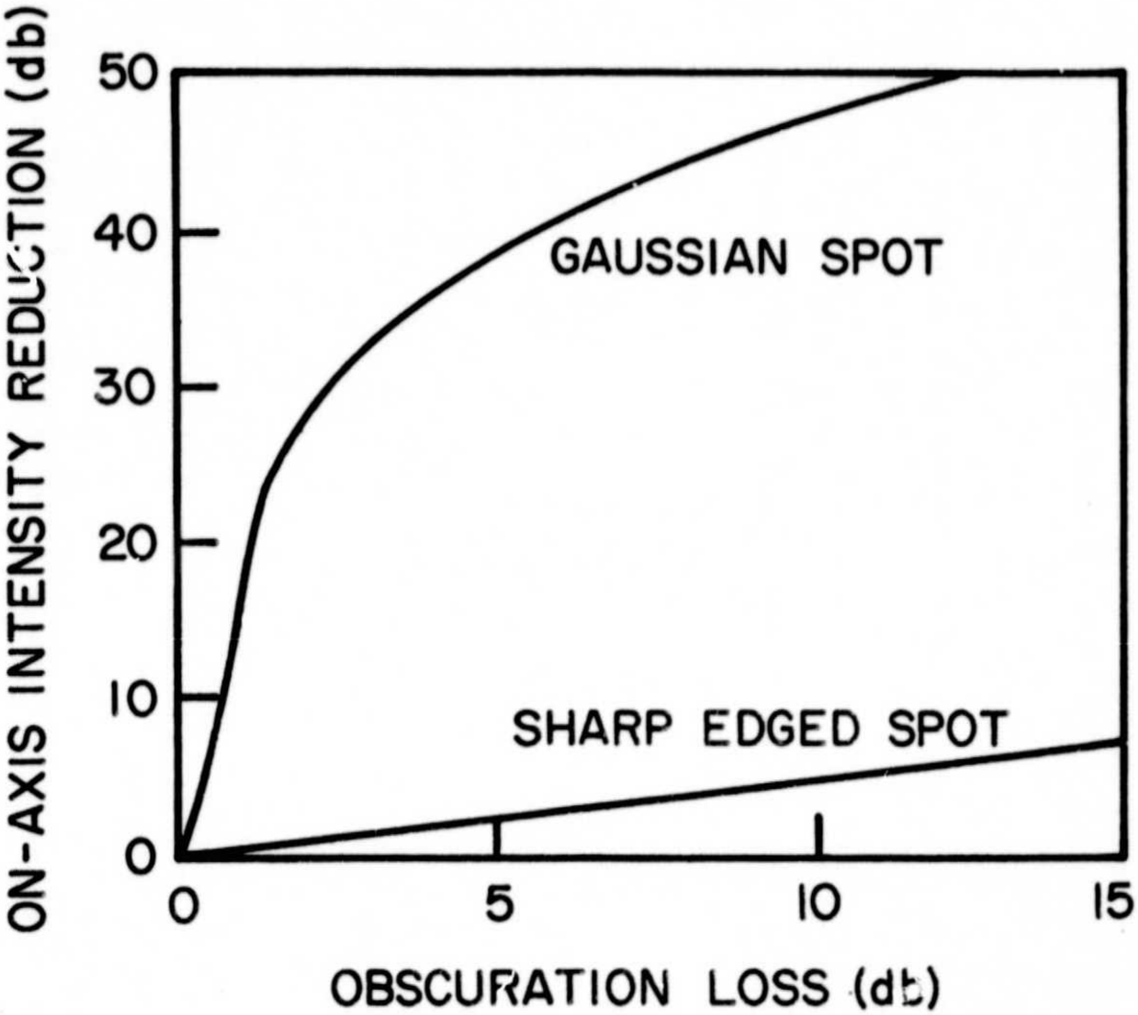

Figure 7. Achievable On-axis Intensity Reduction as a Function of Obscuration Loss (two-way)

6. References

1. Meeting on the Airport Warning System Design at NASA-MSFC Huntsville, June 25, 26, 1973.
2. See, for example, Born & Wolf "Optics".
3. See, for example, Jenkins & White "Principles of Optics".
4. Webb, W.E., "Near Field Antenna Patterns of Obstructed Cassegrainian Telescopes", 2nd Interim Progress Report on NASA Contract NAS8-25562, University of Alabama BER Report No. 143-70, January 1972.

THE PROBLEM OF THE REFLECTION FROM THE SECONDARY MIRROR. II

In a previous memorandum[1] we discussed methods for reducing the magnitude of the energy back scattered (reflected) from the secondary mirror and found that reductions of 30db or so should be achievable. In this note we estimate power levels.

The geometry is shown in Figure 1. The light reflected

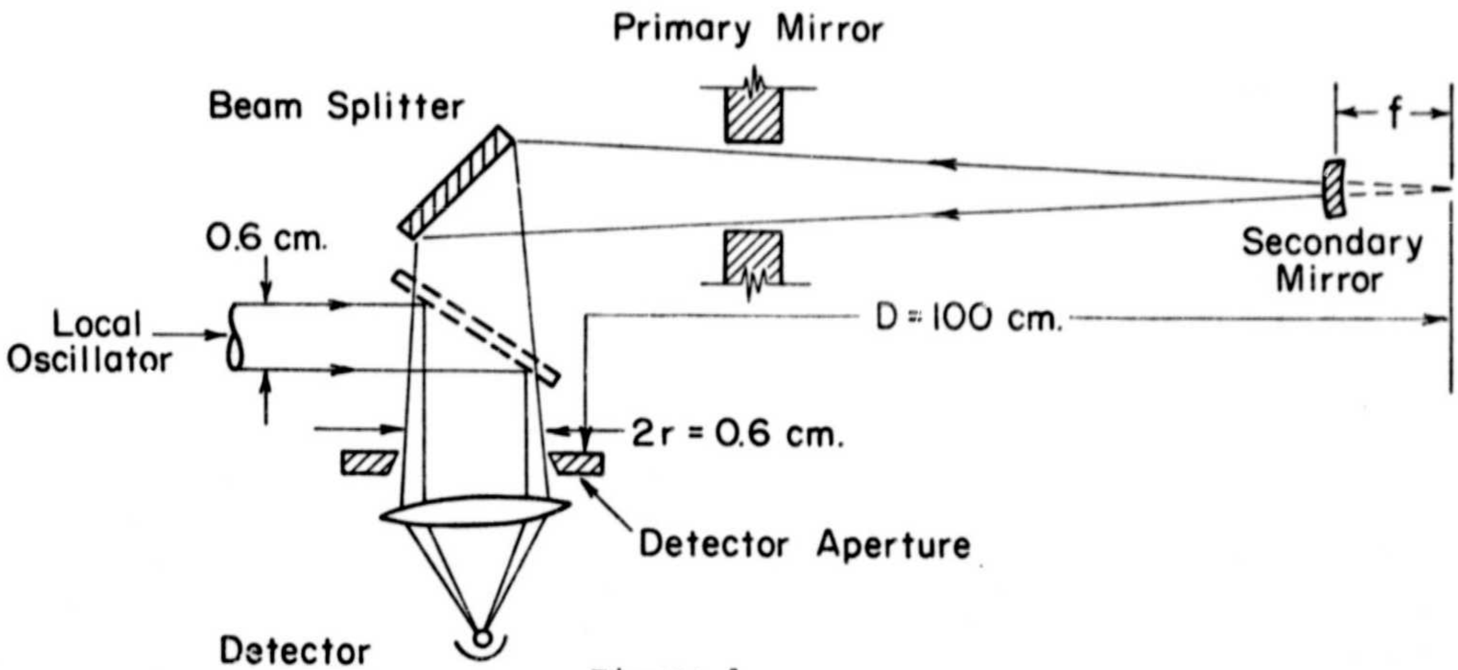

Figure 1.

from the secondary forms a diverging spherical wavefront with a virtual origin located a distance f behind the secondary mirror (f is the secondary focal length). The amplitude of this reflected wave at a distance D from the secondary and at a distance y off axis is

$$\psi = \sqrt{\bar{\sigma}}\psi_o \frac{f}{D} \exp(i\pi y^2/\lambda D)$$

where $\bar{\sigma}$ is the reflectivity of the central portion of the secondary. For a detector aperture of diameter 6mm located at a distance D (1.0 meters) from the secondary, the phase factor exhibits only a moderate variation across the aperture (Fresnel zone width: $(\sqrt{\lambda D} = 0.32$ cm)).

The heterodyne power for a gaussian-shaped local oscillator varies as

$$\frac{dP}{dA} = \frac{\bar{\sigma}}{4} P(0)_{laser} \quad (f^2/D^2) \Big/ \left[1 + \left(\frac{\pi r^2}{\lambda D}\right)^2\right] .$$

The factor of ¼ assumes a 50% loss each way at the beam splitter. Thus, for f=1.2 cm, D=100 cm and r=0.3 cm, the reduction due to beam divergence (f^2/D^2) is approximately -38.4 db and that due to depth of field $\left(\left[1 + (\pi r^2/\lambda D)^2\right]^{-1}\right)$ is -9.1 db. This latter effect may also be referred to as a heterodyne inefficiency, coherency, or out-of-focus effect. Thus, for a 20 watt incident laser power having a 0.3 cm beam radius $\left(P(0) = 141 \text{ watts/cm}^2\right)$, the total (incoherent) power incident on the detector aperture (area A_{det}) is

$$P_{incoherent} = P(0)_{laser} \quad (f/D)^2 \frac{\bar{\sigma}}{4} A_{det} \quad .$$

For a detector aperture diameter of 0.6 cm this becomes

$$P_{incoherent} = 1.44 \; \bar{\sigma} \text{ milliwatts} \quad .$$

The coherent power is lower by the factor $\left[1 + (\pi r^2/\lambda D)^2\right]^{-1}$ and is

$$P_{coherent} = 0.18\,\overline{\sigma}\text{ milliwatts.}$$

The doppler shift due to the secondary motion displaces this power from zero frequency by an amount equal to

$$\Delta f = 4\,\Delta x\,f_{scan}/\lambda$$

where Δx is the mirror displacement when scanned at a frequency f_{scan}. To scan from a range Z_{min} to infinite range the mirror must be moved through the distance $\Delta x \simeq F^2/Z_{min}$ where F is the primary focal length (0.6 meters). Thus to scan from a 50-meter range to infinity at a 5Hz rate (using a linear motion) the doppler offset is

$$\Delta f = 4F^2 f_{scan}/\lambda Z_{min}$$

$$= 13.6\text{ kHz}$$

The total doppler offset is twice this value (an offset occurs both on transmission and on reception).

In summary:

1) The total (incoherent) power reflected from the secondary that is incident on the detector is

about 1.4 milliwatts for a perfectly reflecting secondary mirror.

2) The heterodyne power is somewhat less (about 0.2 milliwatt).

3) The frequency broadening by the secondary mirror motion is about ±27 kHz for a 5 Hz scan rate with a minimum range of 50 m (non-linear scan motions may increase this bandwidth somewhat).

Conclusions

In the current design it is desired to keep the total power on the detector less than 1 milliwatt to avoid saturation. Thus, a reduction by a factor of 2 and preferably more is required. Although such reductions should be readily achievable[(1)] by modification of the reflectivity at the center of the secondary, simpler procedures may be adequate. In Reference (1) it was suggested that use be made of the fact that the scattered beam is diverging and also that the scattered beam could be used as the local oscillator. It appears possible to implement both of these suggestions by simply increasing the optical path between the secondary and the detector. In Figure 2 we show the total power on the detector and the heterodyne or coherence loss as a function of the secondary-detector separation D.

In the present design there is expected to be a 9 db reduction of the heterodyne signal below the total scattered power signal due to mismatch of the wavefronts. Increasing the distance to the detector from 1 meter to 3 or 4 meters should reduce the power on the detector by about an order of magnitude and reduce the heterodyne loss to be less than 2.5 db. If these numbers can be achieved in practice, it appears feasible to utilize the reflection from the secondary as the local oscillator. The doppler shift

Assumed Parameters for Present System

Primary Mirror: F=60 cm, D=30 cm
Secondary Mirror: f=1.2 cm, d>0.6 cm
Distance to detector: 100 cm from secondary
Detector aperture: 0.6 cm diameter
Laser: 20 watts, 0.6 cm diameter (e^{-2} points)
Beamsplitter loss: 50% each way
Range scan rate: 5Hz

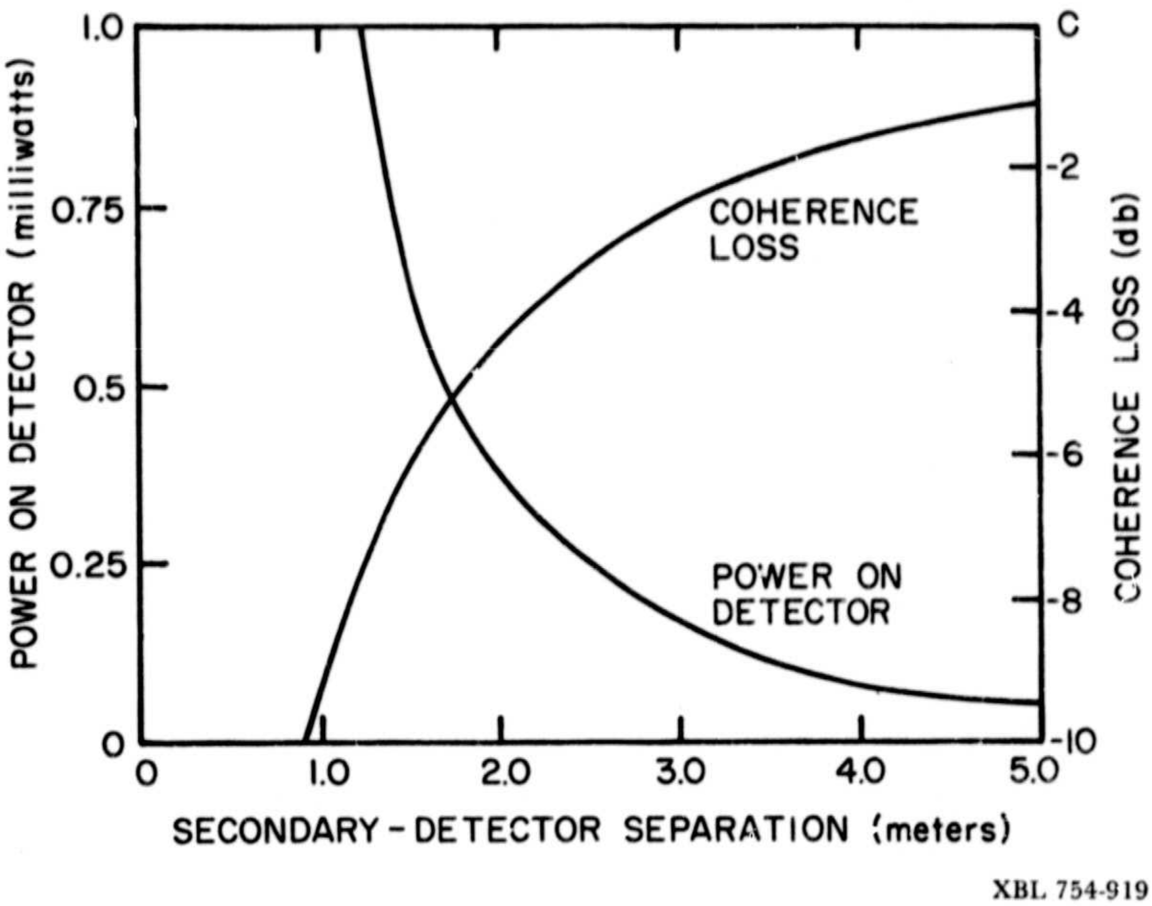

Figure 2. Effect of separation distance between secondary and detector.

introduced by the secondary motion will introduce a velocity error that varies progressively during the scan. However, the magnitude appears to be less than the planned filter bandwidths and should not be very significant.

References

1. Memorandum, "The Problem of Reflection from the Secondary Mirror", Physical Dynamics, July 1973. Subject matter incorporated into Part I of this report.