Department of Geodetic Science

BASIC RESEARCH AND DATA ANALYSIS FOR THE EARTH AND OCEAN PHYSICS APPLICATIONS PROGRAM AND FOR THE NATIONAL GEODETIC SATELLITE PROGRAM

Fourth Semiannual Status Report
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PREFACE

These projects are under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University, and are under the technical direction of Mr. James P. Murphy, Special Programs, Office of Applications, NASA Headquarters, Washington, D. C. The contracts are administered by the Office of University Affairs, NASA Headquarters, Washington, D. C. 20546.

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1. STATEMENT OF WORK

The statement of work includes data analysis and supporting research in connection with the following broad objectives:

(1) Provide a precise and accurate geometric description of the earth's surface.

(2) Provide a precise and accurate mathematical description of the earth's gravitational field.

(3) Determine time variations of the geometry of the ocean surface, the solid earth, the gravity field and other geophysical parameters.

2. ACTIVITIES RELATED TO THE NGSP
(Grant No. NGL 36-008-093)

2.1 Data Acquisition and Processing

The data of the WEST (West European Satellite Triangulation) and the ISAGEX (International Satellite Geodesy Experiment) programs are at our disposal. The purpose of this investigation is to utilize some or all of the above observations in order to improve the values of some station coordinates on the European continent which are presently included in the WN-14 solution and to assess the quality of the WN-14 solution with the help of the new data available. A detailed description of the data is given in the previous Semiannual Status Report. The current status of acquisition and processing is given below.

2.11 WEST Data

There are two sets of optical data available. One set contains the direction cosines of single ficticious images per plate including the standard deviations which were derived from polynomial fitting. The other set contains the direction cosines of seven fictitious images. All directions are given in the Greenwich Hour Angle Declination system.

2.11.1 Single Image Data Processing

Since no program was available at OSU to process single image data, the OSUGOP program (which was previously used to process BC-4 seven image data) was suitably modified by James P. Reilly. The subroutines READIN and ASD 360 had to be completely rewritten, while the subroutine FORMRN had to undergo only minor changes.

Transformation of variances: The variances of the observations were given in the form of standard deviations along and across the satellite trail. The modified subroutines require as input the standard deviation of the Greenwich hour angle multiplied with the cosine of the declination, the standard deviation of the declination and the covariance term. The variances were transformed as follows:

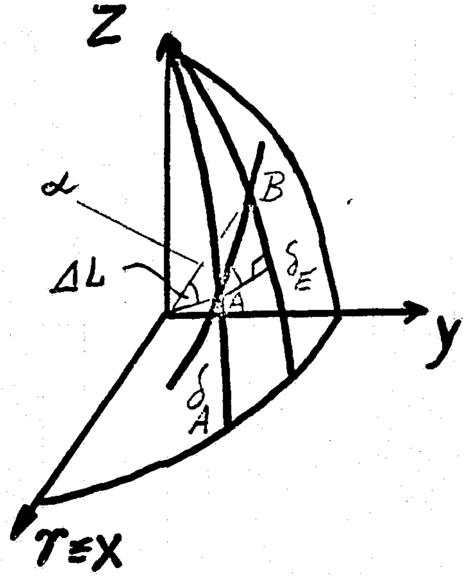

Fig. 1 Transformation of Variances

ΔL denotes length of the trail

δ_A, δ_E denote declination of satellite (beginning and end of trail)

α denotes rotation angle.

The actual rotation is approximated by a rotation around the point A where the satellite trail AB is taken to be a straight line.

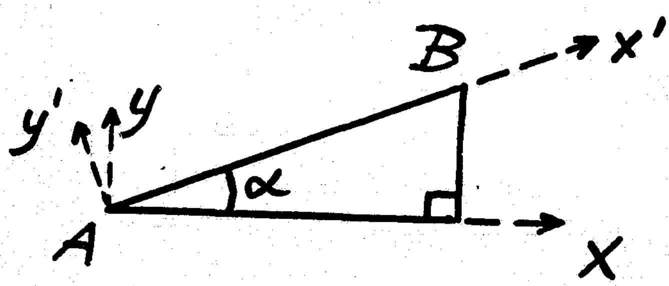

Fig. 2 Rotation to GHA System

We obtain the relation

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos(-\alpha) & \sin(-\alpha) \\ -\sin(-\alpha) & \cos(-\alpha) \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \tag{1}$$

where α is computed from the spherical relation

$$\sin\alpha = \frac{\sin(\delta_E - \delta_A)}{\sin \Delta L} \quad . \tag{2}$$

Using the given variance-covariance matrix

$$\Sigma_{x'y'} = \begin{pmatrix} \delta_{x'}^{2} & 0 \\ 0 & \delta_{y'}^{2} \end{pmatrix} \qquad (3).$$

where δ_x^2 is the variance along the trail and δ_y^2 is the variance across the trail, we obtain with relation (1) and the law for propagating the variances, the transformed variance-covariance matrix

$$\Sigma_{xy} = \begin{pmatrix} \delta_x^2 & \delta_{xy} \\ \delta_{xy} & \delta_y^2 \end{pmatrix} = \left(\begin{array}{c|c} \cos^2(-\alpha)\delta_1^2 + \sin^2(-\alpha)\delta_c^2 & \sin(-\alpha)\cos(-\alpha)(-\delta_1^2 + \delta_c^2) \\ \hline & \sin^2(-\alpha)\delta_1^2 + \cos^2(-\alpha)\delta_c^2 \end{array}\right)$$

It should be noted that δ_x denotes $\delta_{GHA} \cdot \cos\delta$ and that δ_y denotes δ_δ.

Some information about the observations stations: Station numbering: The whole set of observations contained 30 different tracking stations. They are listed in Table 1 and their relative location can be seen from Figure 3. Since the numbering system for the WEST stations and WN-14 stations are independent, the same station number was assigned to different stations. In order to avoid confusion the WEST stations were in part renumbered with a four digit number. The complete station number consists of four digits, where the first two digits were arbitrarily chosen as 87 and the last two digits were the same as in the corresponding WEST station number. In some cases even the second digit had to be changed. The following table presents both numbers. Further modification of the station numbers might become necessary.

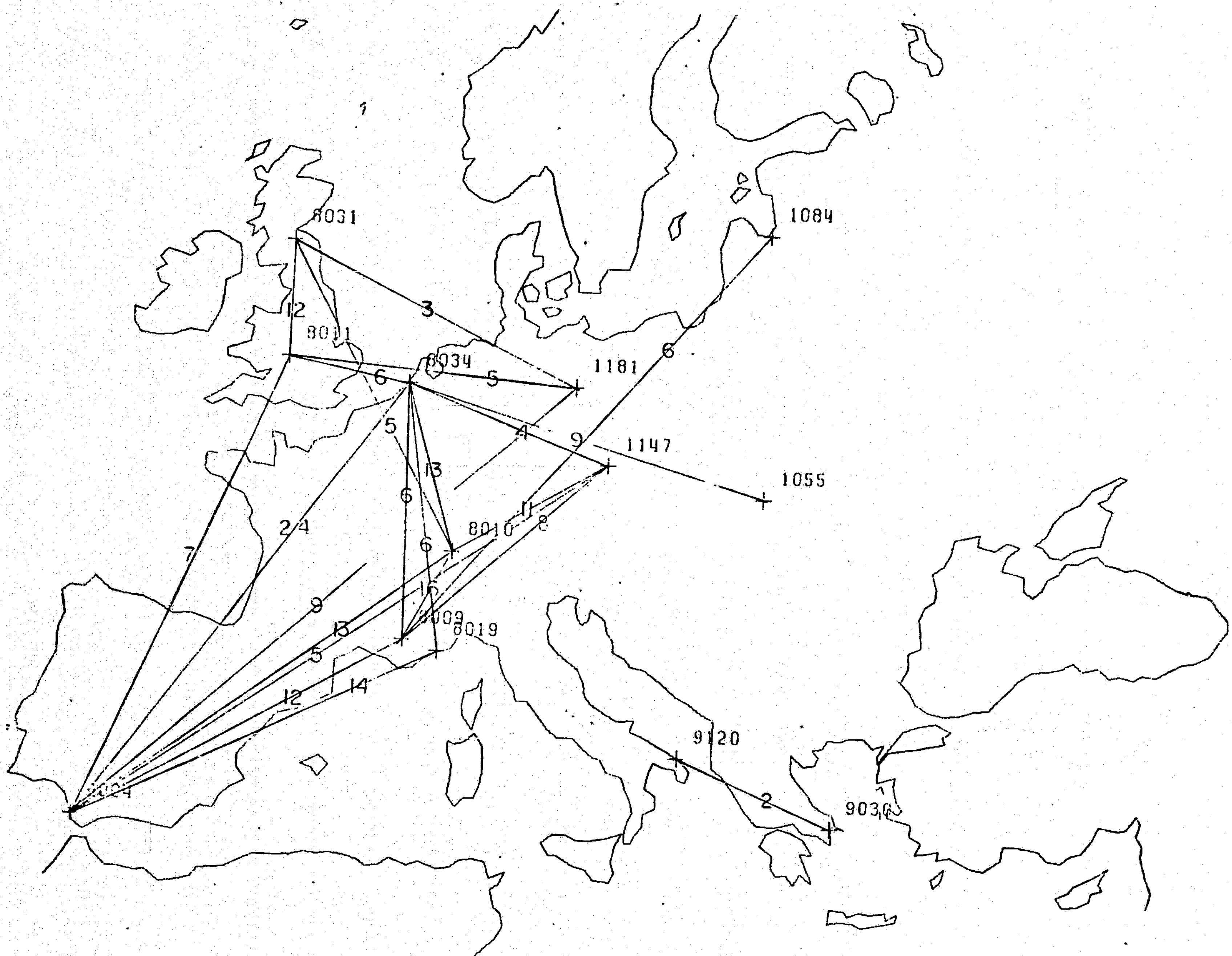

Fig. 3

West Station Numbers		Name
Modified	Original	
1001	1001	GRAZA
2001	2001	BRXOR
3001	3001	COPHN
5001	5001	MEUDN
5002	5002	STRBG
5003	5003	BRDUX
5004	5004	NICEM
5005	5005	GOULT
6004	6004	BRNSG
6005	6005	FRNFT
6010	6010	HOPBG
6012	6012	WSNDF
6110	6110	HOPBG
8004	8004	CATAN
8005	8005	OPICI
8006	8006	ORIAA
8007	8007	SRDIN
8008	9008	TANIA
8702	10002	MADRD
8703	10003	MADRI
8706	6006	KLSRH
8709	8009	CATNA
8712	11002	LOVCA
8721	12001	ZMWLD
8731	13001	EDNBG
8732	13002	MLVRN
8742	14002	TRMSO
8753	15001	REKVK
9001	9001	DELFT
9002	9002	DELFY

Table 1 Summary of Observation Stations, WEST

Relative constraints for nearby stations: For the adjustment computations it is important to establish the exact relationship between nearby stations which can be introduced as relative constraints. In most cases the following information was extracted from Circular Letters which were distributed by the International Association of Geodesy during the time of the WEST campaign. In some cases the relative location of observation stations could be established from Cartesian coordinates which are given in Ehrnsperger [4]. The coordinates are given up to millimeter and it seems reasonable to assume that they accurately render the relative position of observation stations. (The actual distance between nearby stations usually amounts to a few meters.) The details about some specific stations and our observations and conclusions are given below.

STATIONS 8702 (10002) MADRID - 8703 (10003) MADRI

Circular Letter No. 35: The new pillar 10003 MADRI (IGN camera) is 2.74m from 10002 MADRD. Ehrnsperger [4] gives the following coordinate differences: $\Delta x = -0.065$, $\Delta y = 2.696$, $\Delta z = 0.282$. Check: the linear distance as computed from the coordinate differences is 2.71 m. Conclusion: the above coordinate differences seem to be correct.

STATIONS 9001 DELFT - 9002 DELFY

Ehrnsperger [4] gives the following coordinate differences: $\Delta x = 3709.057$, $\Delta y = 1053.539$, $\Delta z = 2925.820$. Both stations are listed at the NASA Directory of Observation Stati.n Locations. Their differences agree with the above values.

STATIONS 6010 HOPBG - 6110 HOPBG

Circular Letter No. 31 confirms that both stations are identical. The change of the station numbers agrees with the convention during the WEST campaign that a change of the camera should be indicated by a change in the third digit of the station number. In this case station 6010 HOPBG was equipped with an IGN camera while 6110 HOPBG carried a BC-4 camera.

STATIONS 6004 BRNSG - 6012 WSNDF

Circular Letter No. 33: both stations are approximately 2m apart. Ehrnsperger [4] gives the following coordinate differences: $\Delta x = 1.785$, $\Delta y = -1.335$, $\Delta z = -0.161$. Check: the linear distance as computed from the coordinate differences is 2.23 m. The coordinates of station 6004 BRNSG are listed in the NASA Directory of Observation Station Locations and agree with those used by Ehrnsperger. Conclusion: the above coordinate differences seem to be correct.

STATIONS 8008 TANIA - 8709 CATNA

Ehrnsperger [4] gives the following coordinate differences: $\Delta x = 13333.95$, $\Delta y = 10080.89$, $\Delta z = -22966.80$.

STATIONS 8004 CATAN - 8008 TANIA

Circular Letter No. 37: the new station 8008 TANIA is approximately 12m south, southesat of station 8004 CATAN. Ehrnsperger gives the following coordinate differences: $\Delta x = -4.036$, $\Delta y = -8.238$, $\Delta z = 7.882$. Check: the distance as computed from the coordinate differences is 12.06m and points in the expected direction. Conclusion: the above coordinate differences seem to be correct.

OLD BC-4 SITE 6016 CATANIA

Circlular Letter No. 37: the old BC-4 site is 2.76m south of station 8004 CATAN. This information made the computation of relative constraints possible (assuming that both stations have the same heights): $\Delta x = -1.67$, $\Delta y = 0.00$, $\Delta z = 2.17$.

OLD BC-4 SITE 6065 HOHENPEISSENBERG

WEST observation station 6010 HOPBG is identical with station 8032 MUNICH according to [6]. The following relative constraint between the WN-14 station, 6065 HOHENPEISSENBERG, and the WEST station, 6010 HOPBG, were derived from geodetic coordinates given in [6]: $\Delta x = 21.26$, $\Delta y = 54.46$, $\Delta z = -24.52$.

In order to compare or to combine the WN-14 and WEST systems, common stations have to be identified. The following table of identical stations could be gathered:

WN-14 No.	Name	WEST No.	Reference
6006	TROMSO	8742	[7] No. 26
6016	CATANIA	8004*	[7] No. 37
6065	HOPBG	6110, 6010*	[6]
8009	DELFT	9001	[6]
8010	ZMDLD	8721	[6]
8011	MLVRN	8732	[6]
8019	NICE	5004	[6]
8030	MEUDN	5001	[6]

*See the specific information given for the old BC-4 sites.

Table 2 Identical Stations for WN-14 and West

Preliminary Adjustment Computations

Adjustment WEST No. 1

The purpose of the first adjustment was to find the adjusted variance of unit weight and to get a first insight into the quality of the data. The following input data and constraints were considered:

1) The transformed variance-covariance matrix was used as described previously.

2) For all identical stations as given in Table 2, the approximate coordinates of the WN-14 solution were used. All other station coordinates were transformed from the European Datum EU-50 to the OSU (WN-14) datum using the following parameters:

EU-50: A = 6378388.0m, 1/F = 297.0

OSU (WN-14): A = 6378155.0m, 1/F = 298.2494985

dx = -99.4, dy = -132.0, dz = -116.0, ds = 0.0675 ppm.

3) All relative constraints which were previously described were enforced by appropriate weighting. The weights are based on an assumed accuracy for the geodetic survey of approximately 1:50,000.

4) The origin of the coordinate system was defined by Inner Adjustment.

5) The scale was introduced through the base line 6016 CATANIA - 8742 TROMSO with an accuracy of 1 ppm.

Result of the adjustment:

a) A posteriori variance of unit weight: 35.5.

b) In all cases the adjusted coordinate differences of those stations which were connected by relative constraints do not deviate from the the constrained values.

Adjustment WEST No. 2

The purpose of this adjustment is to combine the WN-14 system and the WEST system. The adjustment is based on the following input:

1) All possible relative constraints.

2) Chord constraints between stations 6016 CATANIA and 8742 TROMSO.

3) The coordinates of all common stations between both systems (Table 2) are constrained to the adjusted WN-14 values using weights which were computed from corresponding variances.

4) The heights of all common stations are constrained to the values given in [3], Table 3.3-3.

Result of the adjustment: a) A posteriori variance of unit weight: 37.1.

b) Comparison of coordinates of identical stations:

Station No. WN-14	Station No. WEST	Δx (m)	σ_x WN-14 / σ_x WEST	Δy (m)	σ_y WN-14 / σ_y WEST	Δz (m)	σ_z WN-14 / σ_z WEST
6065		-0.56	2.02 1.83	0.25	2.24 2.24	0.81	2.35 2.04
6016		-0.06	1.81 1.67	1.47	2.19 2.18	0.07	2.24 2.08
6006	8742	-0.35	2.36 2.16	3.03	2.92 2.58	-0.94	2.89 2.31
8030	5001	-1.86	6.46 4.94	-14.51	9.66 7.77	0.79	5.80 2.40
8019	5004	0.69	4.12 3.50	-12.31	7.91 6.64	0.38	4.31 3.68
8010	8721	7.51	5.71 3.49	-9.90	8.28 5.02	-4.89	5.44 3.57
8011	8732	-6.70	8.86 3.61	-36.81	14.27 5.31	4.48	6.96 3.84
8009	9001	5.69	8.48 4.28	-6.92	10.07 5.38	-0.07	6.86 4.10

Table 3 Adjusted WN-14 Coordinates Minus the Coordinates of Adjusted West No. 2

c) In Tables 4 and 5 the results are given for a transformation of the coordinates obtained from adjustment WEST 2 to the WN-14 system.

2.11.2 Seven Image Data Processing

The seven image data received on cards was first transferred onto tape. This data gives direction cosines, event/stationwise. Some event numbers have been duplicated and there are some image numbers from 8 to 14. A second program read the data from the tape, transformed the direction cosines to right ascension and declination, generated additional parameters (such as numbers of stations in each event) required for input for the OSUGOP program and transferred the data in the new form onto disk. A third program has been made to read the single image data (for variances) from cards, match it with the modified seven image data (event and stationwise) and output the merged observational data giving α, δ and variances for the seven images. A variational parameter has been added to vary the variances of images 1, 2, 3, 5, 6, 7 with respect to the variance of the 4th image which is available in the single image data.

SOLUTION FOR 3 TRANSLATION, 1 SCALE AND 3 ROTATION PARAMETERS

(USING VARIANCES ONLY)

Dx METERS	Dy METERS	Dz METERS	DELTA (X1.D+6)	OMEGA SECONDS	PSI SECONDS	EPSILON SECONDS
-1.24	-1.22	-0.60	0.51	0.29	-0.01	-0.18
± 2.57	± 2.12	± 3.12	± 1.05	± 0.52	± 0.26	± 0.67

VARIANCE - COVARIANCE MATRIX

SO2= 0.79

0.659D+01	0.159D+01	0.493D+01	-0.241D-07	-0.558D-05	0.830D-06	0.578D-05
0.159D+01	0.451D+01	0.224D+01	-0.109D-05	-0.185D-05	0.660D-06	0.296D-05
0.493D+01	0.224D+01	0.975D+01	0.219D-06	-0.576D-05	0.142D-05	0.895D-05
-0.241D-07	-0.109D-05	0.219D-06	0.111D-11	0.179D-13	-0.185D-12	-0.880D-13
-0.558D-05	-0.185D-05	-0.576D-05	0.179D-13	0.643D-11	-0.140D-11	-0.675D-11
0.830D-06	0.660D-06	0.142D-05	-0.185D-12	-0.140D-11	0.159D-11	0.177D-11
0.578D-05	0.296D-05	0.895D-05	-0.880D-13	-0.675D-11	0.177D-11	0.105D-10

COEFFICIENTS OF CORRELATION

0.100D+01	0.292D+00	0.616D+00	-0.893D-02	-0.858D+00	0.256D+00	0.695D+00
0.292D+00	0.100D+01	0.338D+00	-0.488D+00	-0.343D+00	0.246D+00	0.431D+00
0.616D+00	0.338D+00	0.100D+01	0.668D-01	-0.727D+00	0.360D+00	0.884D+00
-0.893D-02	-0.488D+00	0.668D-01	0.100D+01	0.671D-02	-0.139D+00	-0.258D-01
-0.858D+00	-0.343D+00	-0.727D+00	0.671D-02	0.100D+01	-0.437D+00	-0.822D+00
0.256D+00	0.246D+00	0.360D+00	-0.139D+00	-0.437D+00	0.100D+01	0.432D+00
0.695D+00	0.431D+00	0.884D+00	-0.258D-01	-0.822D+00	0.432D+00	0.100D+01

Table 4

RESIDUALS

	DX	DY	DZ
6006	-1.7	-1.9	2.1
6016	1.8	-2.1	-0.1
6065	0.7	-1.3	-0.8
8009	-6.9	5.5	-0.4
8010	-7.8	8.5	4.5
8011	4.7	35.0	-5.6
8019	-0.7	10.8	-1.0
8030	0.7	12.7	-1.7

Table 5

After outputting the merged data onto a tape in a form compatible with the input requirement of OSUGOP, experiments similar to the ones done with the single image data will be performed.

2.12 ISAGEX Data

General Remarks

A detailed description of the ISAGEX data as obtained from the Centre National d'Etudes Spatiales is given in the previous Semiannual Status Report on pages 4 - 10. The data consist of laser ranges and optical observations.

It was already reported that no simultaneous laser range observations could be found. Therefore, efforts were made to further process the optical observations only. The preliminary results, which were already reported, indicated either a very poor quality of the ISAGEX data or a blunder in processing. It was therefore decided to completely re-examine the investigations done so far, starting with the preprocessed data as provided by Wolf Research and Development Corporation. The data did not include observations of MIDAS 4 and PAGEOS. These data were thus sent to Wolf Research and Development Corporation for preprocessing. However, it was learned during a telephone conversation with Ms. Donna Walls of the Wolf Corporation, that difficulties had arisen in obtaining the correct input data for preprocessing. It was consequently decided not to use the MIDAS 4 and PAGEOS data anymore for this investigation.

Current Status of Processing the ISAGEX Optical Data

The preprocessed data were tested for simultaneity. Allowing a time gap of 0.2ms, a total of 353 observations proved to be simultaneous, involving 13 different stations which are exclusively located in Europe.

In the next step the quality of the observations was tested by forming the normal equations using the OSUGOP program. As explained in the Reports of the Department of Geodetic Science No. 190, page 12, a so-called test distance can be used to specify rejection criteria for each observation; or, conversely, by looking at the computed test distance we can judge the quality of the observation. A large test distance indicates either bad quality of observations or poor approximate station coordinates. In our computations we used first, station coordinates in the European system ED 50 as extracted from the NASA Directory of Observation Station Locations and from various ISAGEX documents, and secondly, the approximate coordinates which were used for the WN-14 solution. If no WN-14 coordinates were available, approximately transformed coordinates were used. The coordinates of station 1147 ONDREJOV were extracted from [5]. Both computations showed only minor differences. In Figure 4 the distribution of the 13 remaining observation stations is given. Also the number of observations on each line with a test distance smaller than 15 arc sec are shown. It is important to note that only a few observations which qualify, exhibit such a large test distance, while the vast majority has a test distance of 1 arc sec or even less. The remaining observations have in most cases, test distances of several thousands arc sec, which probably indicates an error in data reduction. A closer inspection of those lines shows that the approximate coordinates do not cause such large test distances.

The following table lists the 13 qualifying stations:

ISAGEX No.	Name
1055	UZHGOROD
1184	RIGA
1147	ONDREJOV
1181	POTSDAM
8009	HAUTE PROVENCE
8010	ZIMMERWALD
8011	MALVERN
8019	NICE
8031	EARLY POINT
8034	YPBURG
9004	SAN FERNANDO
9030	DYONISOS
9120	SAN VITO

Table 6 ISAGEX Stations

Some Remarks on the Observation Stations:

The two data sets, WEST and ISAGEX, are independent sets, but the observations were made from common or nearby stations. It is, therefore, important to uniquely identify the observation stations. This investigation is still in progress and the following two tables are only of preliminary character:

ISAGEX No.	WN-14 No.	Name
9004	9004	SAN FERNANDO
8009	8015	HAUTE PROVENCE
8010	8010	ZIMMERWALD
8019	8019	NICE
8011	8011	MALVERN

Table 7 Apparently Common Stations between ISAGEX and WN-14

ISAGEX No.	Name	WEST No.	Name
8031	EARLY POINT	8731	EDNBG
8034	YPBURG	9002	YPBURG
8010	ZIMMERWALD	8721	ZIMMERWALD
8019	NICE	5004	NICEM
5005	GOULT	8009	HAUTE PROVENCE

Table 8 Apparently Common Stations between ISAGEX and WEST

Preliminary Adjustment Computations:

In order to further test the observations, various adjustment computations were carried out. From Figure 4 it can be seen that the following four stations do not form closed figures:

1055 UZHGOROD

1084 RIGA

9030 DYONISOS

9120 SAN VITO.

These stations have been neglected in subsequent computations.

Adjustment: ISAGEX 1

The adjustment is based on the following information:

1) Number of stations: 9

2) Standard deviation of the observation: 1 arc sec

3) The origin of the coordinate system is defined by Inner Adjustment

4) All five stations given in Table 7 have been constrained to the WN-14 coordinates where the weights were computed from the standard deviations given in the Reports of the Department of Geodetic Science No. 199, pages 118-145, as

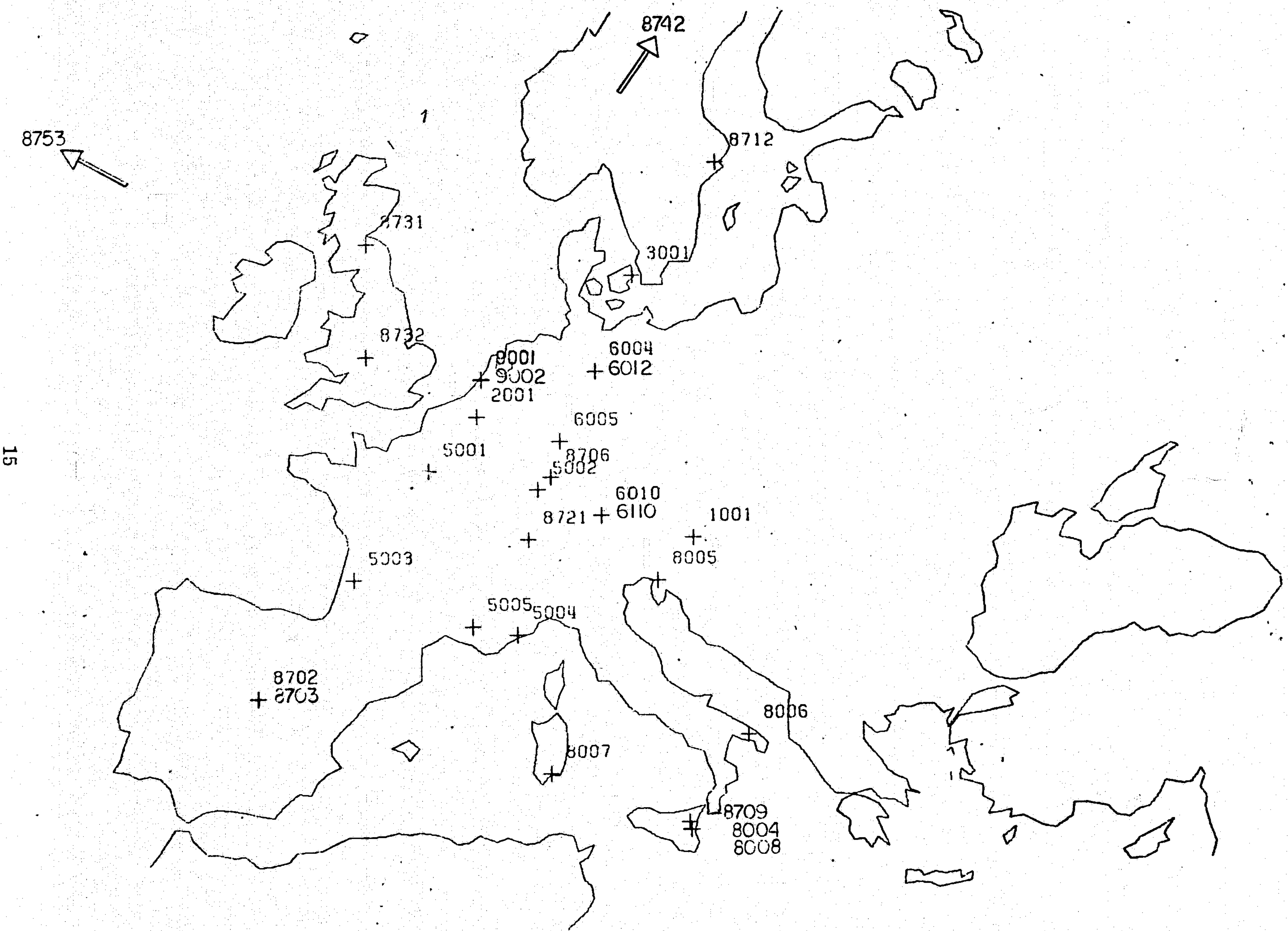
8742
8753
8712
8731
3001
8732
9001
9002
2001
6004
6012
6005
5001
8706
5002
6010
6110
8721
1001
8005
5003
5005
5004
8702
8703
8006
8007
8709
8004
8008

Fig. 4

$$\text{Weight} = 1/\sigma^2.$$

Result:

a) Variance of unit weight: 4.0

b) Comparison of coordinates: WN-14 - ISAGEX.

Station No.		Δx(m)	$\sigma_{x\,WN-14}$ / $\sigma_{x\,ISAGEX}$	Δy(m)	$\sigma_{y\,WN-14}$ / $\sigma_{y\,ISAGEX}$	Δz(m)	$\sigma_{z\,WN-14}$ / $\sigma_{z\,ISAGEX}$
WN-14	ISAGEX						
			5.71		8.28		5.44
8010	8010	26.21	7.51	-17.64	5.36	-15.1	5.40
			8.66		14.27		6.95
8011	8011	2.43	13.12	-60.88	14.65	-11.86	9.04
			4.19		8.00		4.38
8015	8009	-28.01	6.36	-21.47	5.91	25.31	6.87
			4.12		7.31		4.31
8019	8019	-20.02	6.31	44.15	10.74	-12.45	8.31

Table 9 WN-14 - Adjustment ISAGEX 1

2.13 Determination of Transformation Parameters and Network Distortions

The Fourteenth and Fifteenth Semiannual Status Reports contain various tables of transformation parameters and figures which give indications of network distortions. Investigations have been made so far for the North American Geodetic Datum NAD 27 and the Australian Geodetic Datum. Both reports also contain a detailed description of the procedure used in this investigation. During the present reporting period, computations have been carried out for the South American Datum SAD 69 using Doppler stations which were provided by the Defense Mapping Agency (Attachment 1). The results are given in Table 10 and Figures 5, 6, 7, 8, 9 and 10.

At this time the investigation regarding the transformation parameters and network distortions using the previously mentioned procedure can be considered complete. It is intended to publish all results and updated computer programs in a final report.

NWL9	SAD69
Δx (m)	-77.8 ± 2.7
Δy (m)	-12.4 ± 3.9
Δz (m)	-49.5 ± 2.6
$\Delta (10^{-6})$	-0.99 ± 0.55
$\omega ('')$	1.18 ± 0.33
$\psi ('')$	-0.90 ± 0.13
$\epsilon ('')$	0.16 ± 0.10
$\alpha ('')$	0.33 ± 0.10
$\xi ('')$	-0.48 ± 0.10
$\eta ('')$	-1.37 ± 0.35

Table 10 Datum Transformation Parameters

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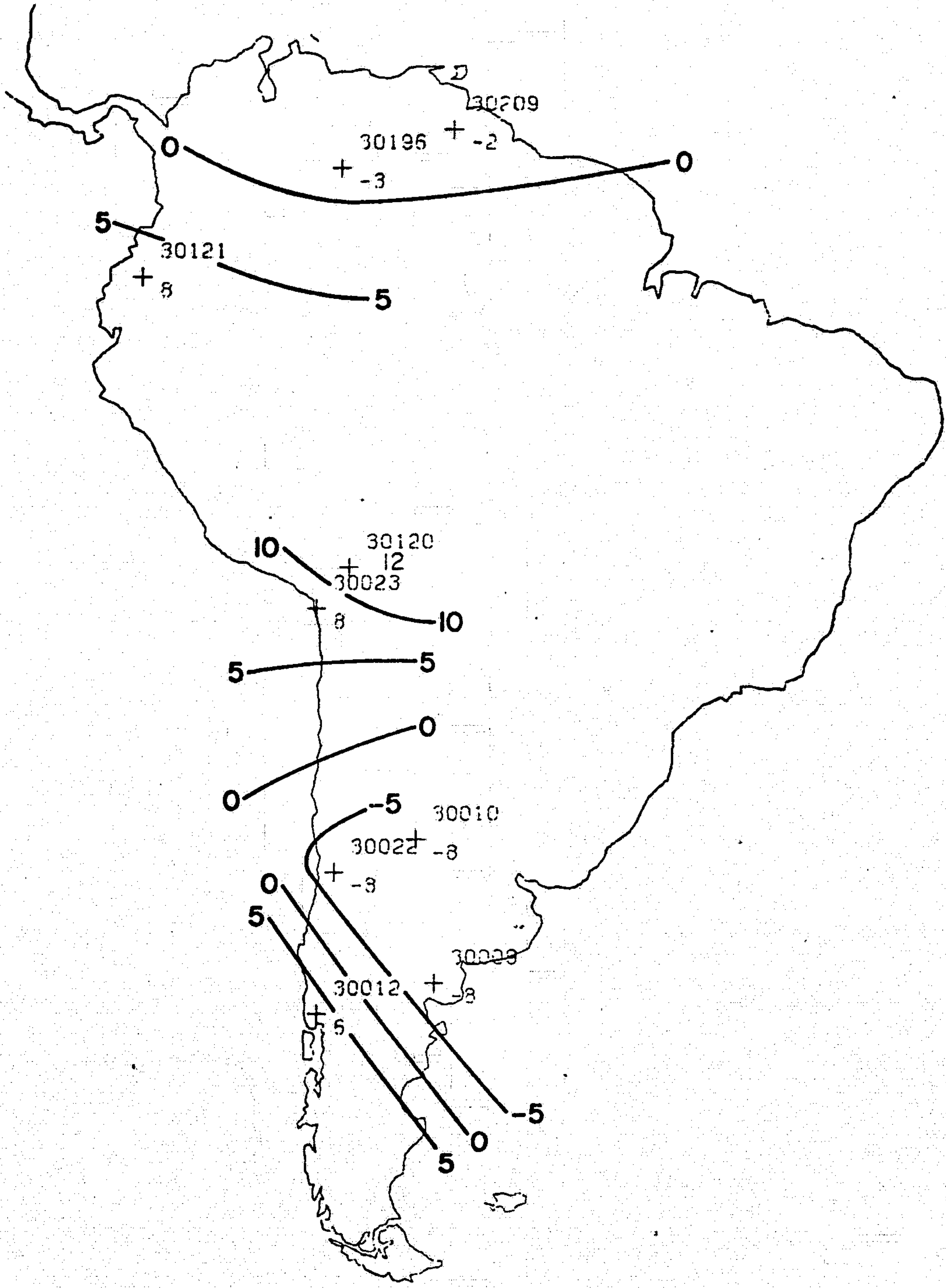
[6] NASA, 1973. "Directory of Observation Station Locations," Goddard Space Flight Center, Greenbelt, MD.

[7] Bureau International de l'Heure, "Circular Letters," distributed by the International Association of Geodesy during the WEST Program.

Latitude Differences After Transforming SAD69 to NWL9D (in Metres)
3 Transformation Parameters

Δu (m) = 80.4 ± 2.6
Δv (m) = −0.3 ± 2.6
Δw (m) = −40.3 ± 2.6



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April 1975

Fig. 5

Latitude Differences After Transforming SAD69 to NWL9D (in Metres)
7 Transformation Parameters (NWL9D−SAD69)

	MODEL Molodenskii	Veis
Δu (m) = −77.8 ± 2.7	$\omega('')$ = 1.18 ± 0.33	$\alpha('')$ = 0.33 ± 0.10
Δv (m) = −12.4 ± 3.9	$\psi('')$ = −0.90 ± 0.13	$\xi('')$ = −0.48 ± 0.10
Δw (m) = −49.5 ± 2.6	$\epsilon('')$ = 0.16 ± 0.10	$\eta('')$ = −1.37 ± 0.35
Δ (ppm) = −0.99 ± 0.55		

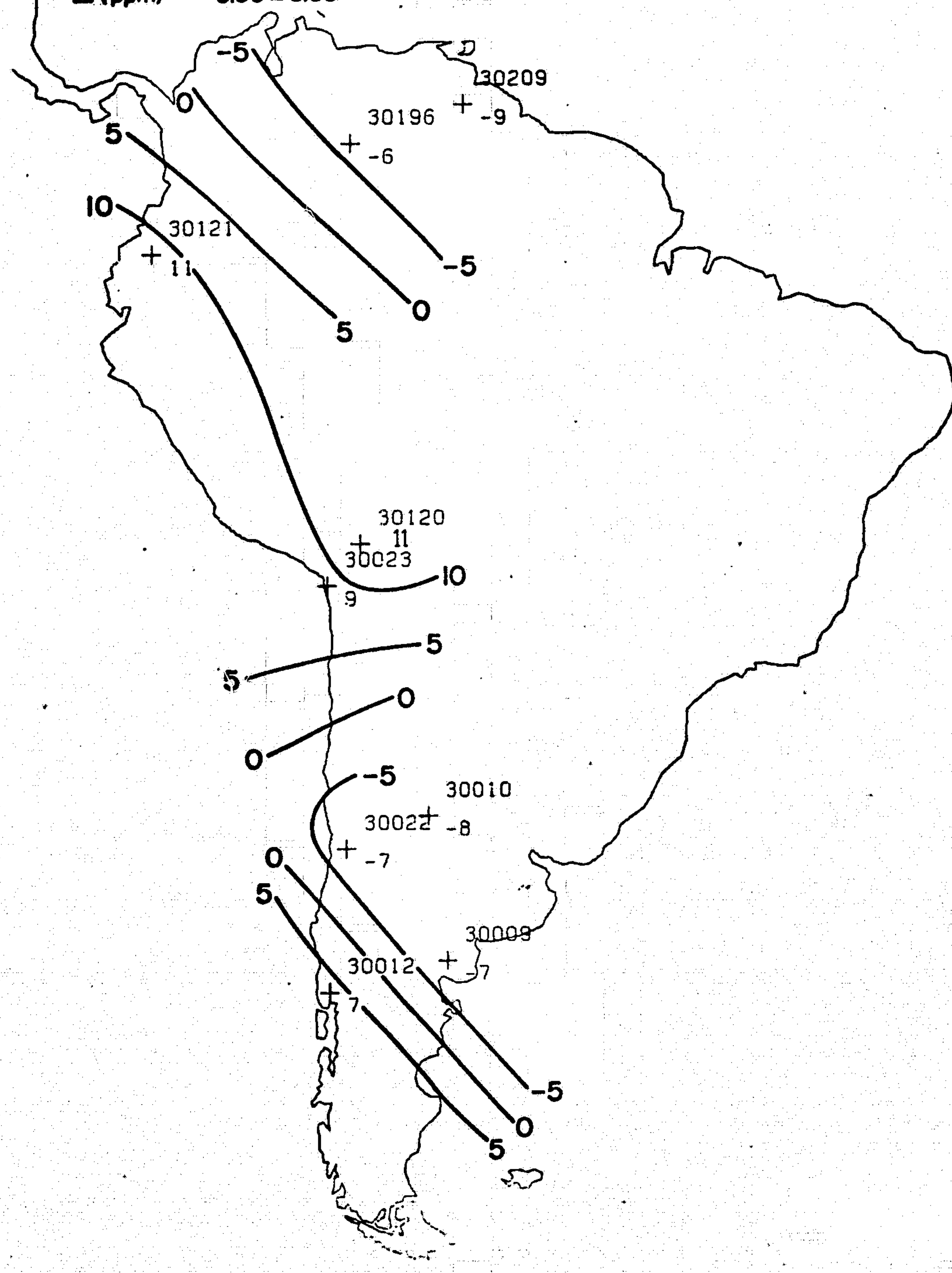

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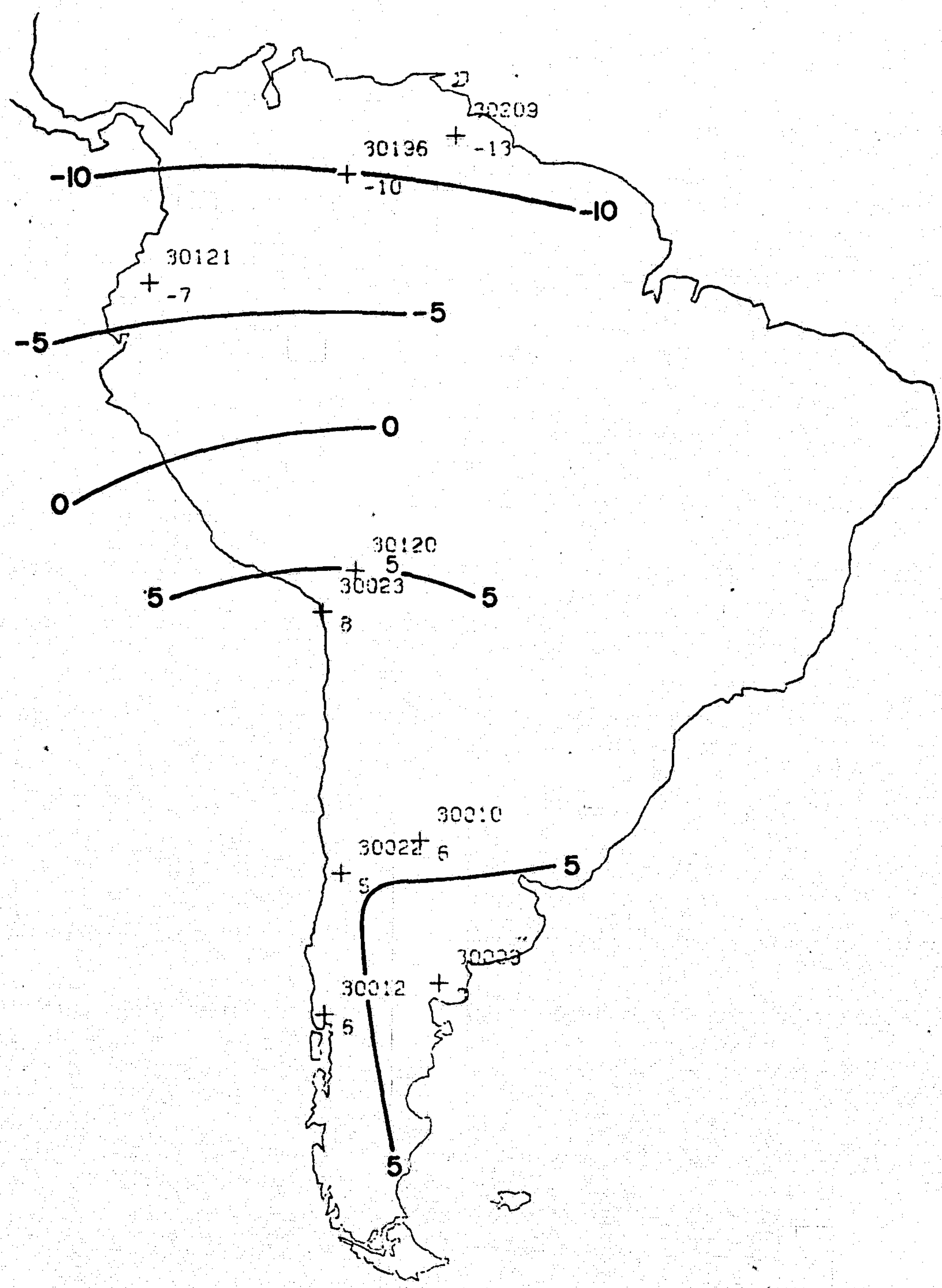
Fig. 6

Longitude Differences After Transforming SAD69 to NWL9D (in Metres)
3 Transformation Parameters

$\Delta u\,(m) = 80.4 \pm 2.6$
$\Delta v\,(m) = -0.3 \pm 2.6$
$\Delta w\,(m) = -40.3 \pm 2.6$



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April 1975

Fig. 7

Longitude Differences After Transforming SAD69 to NWL9D (in Metres)
7 Transformation Parameters (NWL9D−SAD69)

	MODEL Molodenskii	Veis
Δu (m) = −77.8 ±2.7	$\omega('')$ = 1.18±0.33	$\alpha('')$ = 0.33±0.10
Δv (m) = −12.4 ±3.9	$\psi('')$ = −0.90±0.13	$\xi('')$ = −0.48±0.10
Δw (m) = −49.5 ±2.6	$\epsilon('')$ = 0.16±0.10	$\eta('')$ = −1.37±0.35
Δ (ppm) = −0.99±0.55		

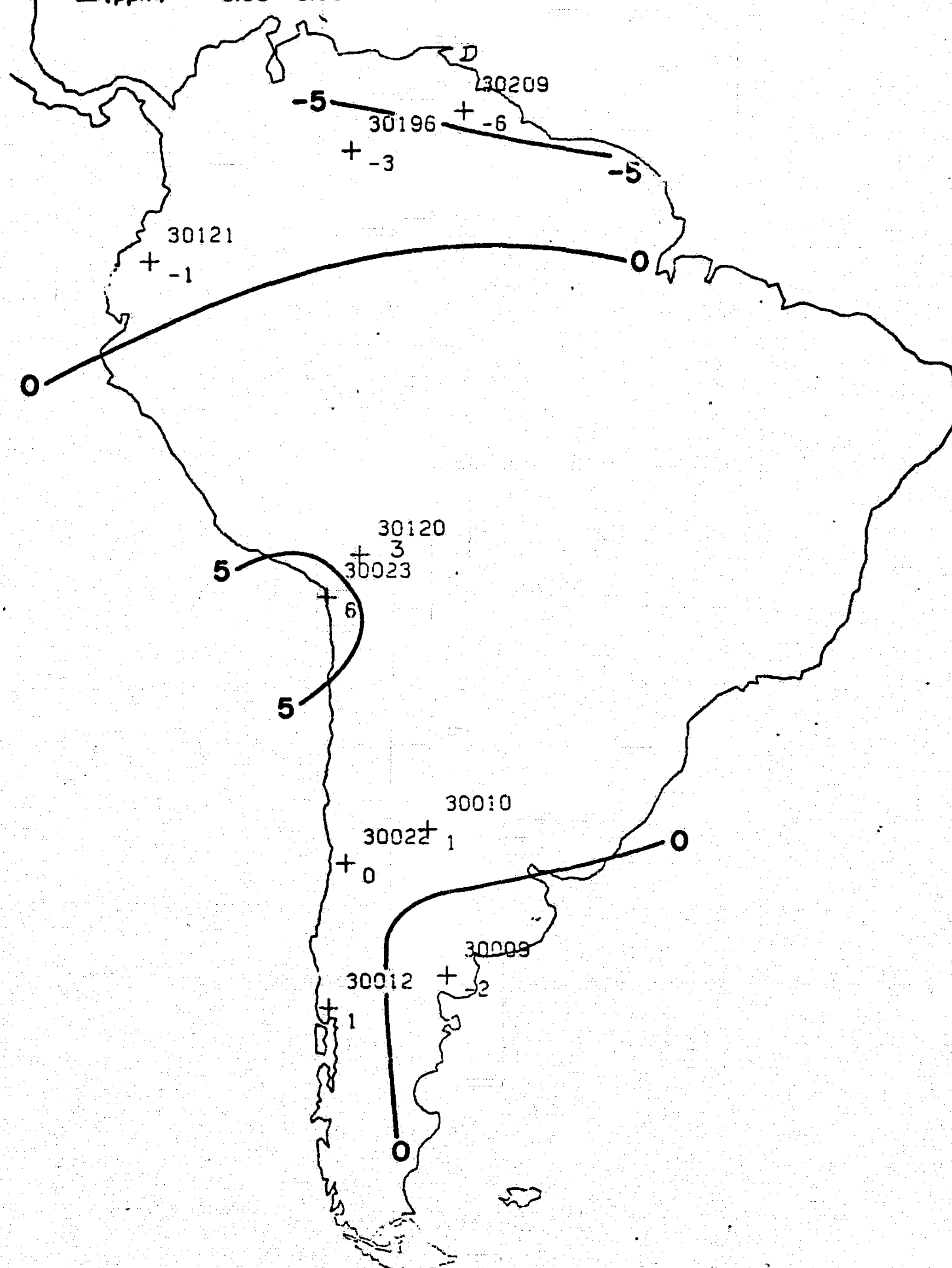

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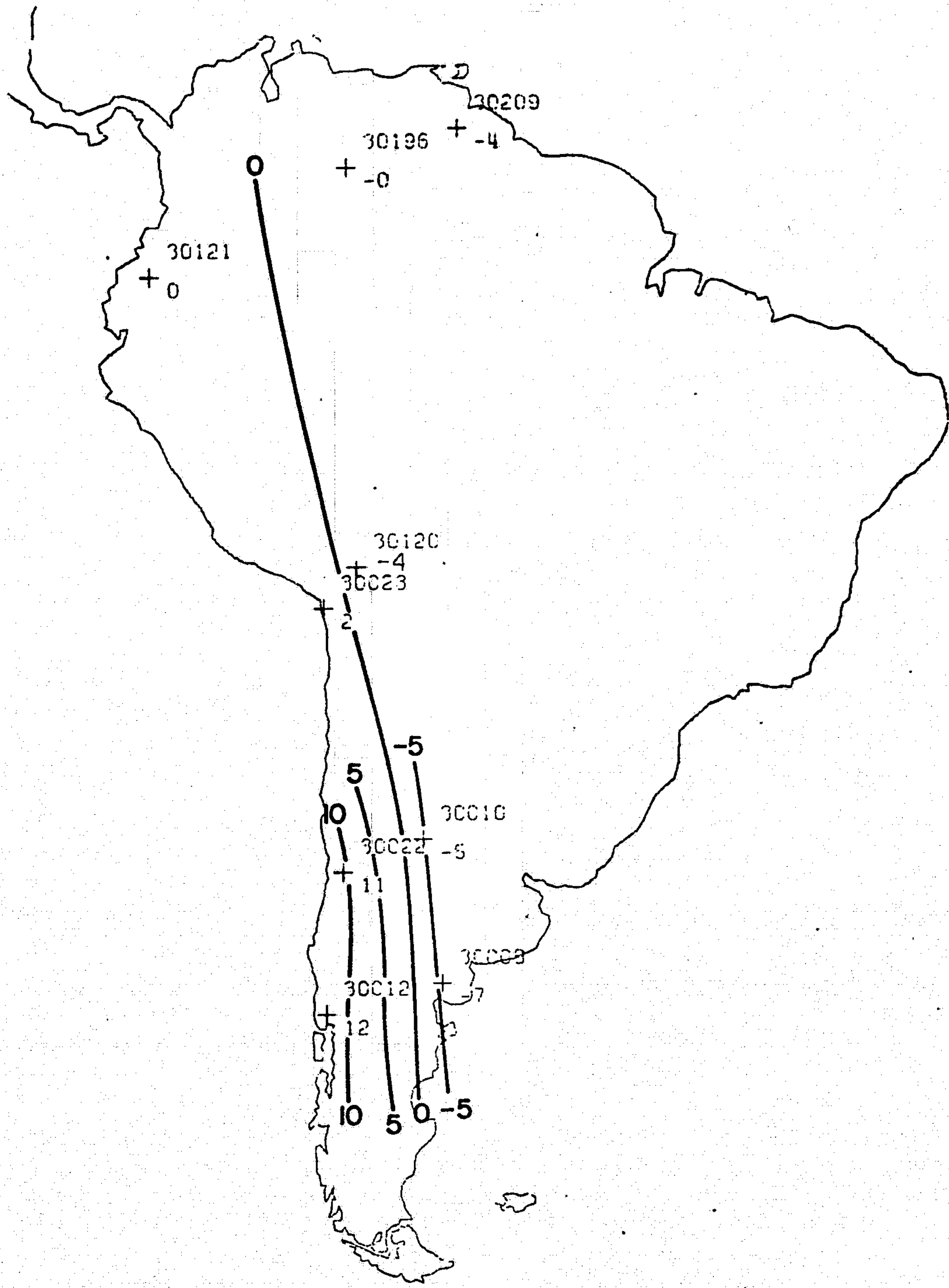
Fig. 8

Height Differences After Transforming SAD69 to NWL9D (in Metres)
3 Transformation Parameters

$\Delta u\,(m) = 80.4 \pm 2.6$
$\Delta v\,(m) = -0.3 \pm 2.6$
$\Delta w(m) = -40.3 \pm 2.6$



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April 1975

Fig. 9

Height Differences After Transforming SAD69 to NWL9D (in Metres)
7 Transformation Parameters (NWL9D−SAD69)

	MODEL Molodenskii	Veis
Δu (m) = −77.8 ± 2.7	$\omega('')$ = 1.18 ± 0.33	$\alpha('')$ = 0.33 ± 0.10
Δv (m) = −12.4 ± 3.9	$\psi('')$ = −0.90 ± 0.13	$\xi('')$ = −0.48 ± 0.10
Δw (m) = −49.5 ± 2.6	$\epsilon('')$ = 0.16 ± 0.10	$\eta('')$ = −1.37 ± 0.35
Δ (ppm) = −0.99 ± 0.55		

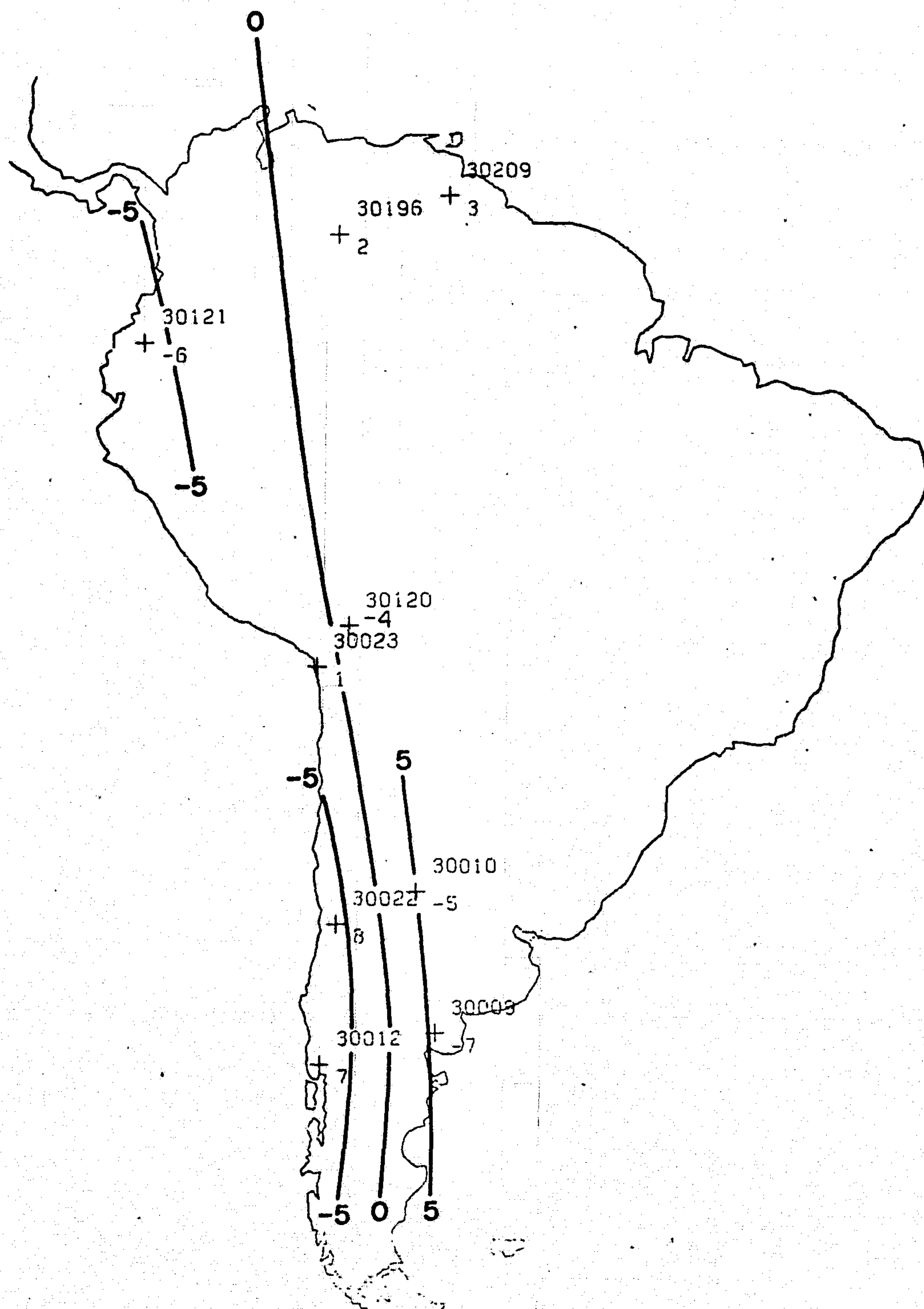

Fig. 10

DEPARTMENT of GEODETIC SCIENCE
Ohio State University
April 1975

3. ACTIVITIES RELATED TO EOPAP
(Grant No. NGR 36-008-204)

3.1 Sea Level Slopes Along the Continental Boundaries of the U.S.A.

The previous Semiannual Status Report sets out the statement of the problem and the conclusions reached in comparing the results of leveling as done by geodesists and oceanographers. These computations have been refined by using Prey reduction instead of free air reduction for gravity values. These modified computations, (see Attachment 2), do not change the conclusions reached earlier.

The subject was discussed in Washington, D. C. on June 16, 1975 at the meeting of the American Geophysical Union by the Subcommittee on the Discrepancy in the Geoids. The following points emerged from these discussions:

(i) The oceanographers indicated that further research effort in identifying the cause for discrepancy could be concentrated on the region very near the ocean surface.

(ii) The oceanographers agreed to supply further details about their method of work by giving detailed calculations at one of the stations.

(iii) The study could be extended to the continental coasts of both North and South America for which data is understood to be available.

The computations previously carried out were modified to depict the mean sea level with respect to the origin of geodetic leveling. This has been shown in Figure 11. This may be studied in reference to the previous Semiannual Status Report, Figures 3.1-3 and 3.1-5.

The graphics give no additional information. However, the agreement about the direction of slope between oceanic and geodetic leveling is now less obvious. Further investigations could be attempted after more data is received.

ORTHOMETRIC HEIGHT OF OCEAN SURFACE
WITH RESPECT TO ORIGIN OF GEODETIC LEVELING

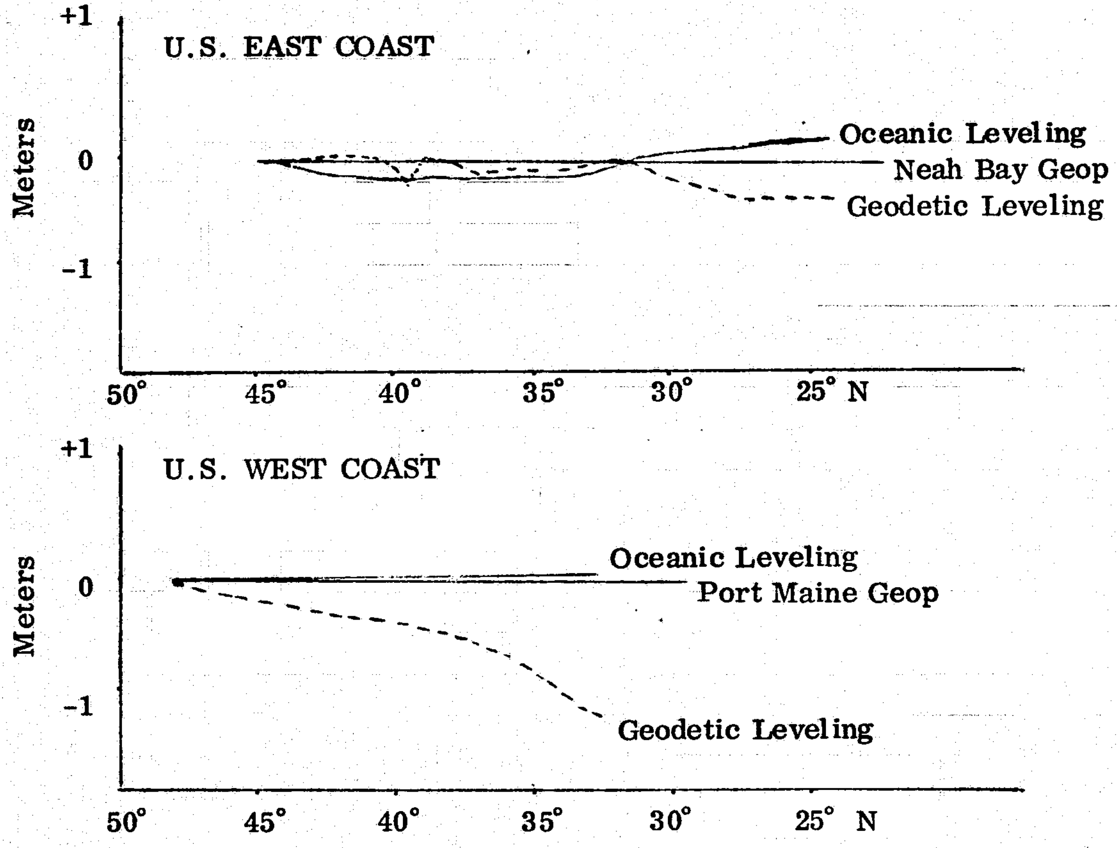

Fig. 11

3.2 Rotation of the Earth

The equations of motion of a rigid body about its center of mass are governed by the well known Euler's equation. Under the assumption of absolute rigidity, the relative positions of all mass particles constituting such bodies are constant, so the external form as well as the moments of inertia are fixed and independent of time.

It appears clear now that the assumption of rigidity for the earth is incorrect and a better modeling should be investigated. Because the mass distribution of the earth is subject to variations with time (i.e., tidal deformation, crustal motions, etc.), producing changes in its inertia tensor, the rotational dynamics of the earth are better studied by the Lagrange-Liouville equations.

The mathematical theory can be summarized in the following three equations, expressed in matrix notation by

$$\{L\} = [I]\{\dot{\omega}\} + [\dot{I}]\{\omega\} + \{\dot{h}\} + [\underline{\omega}][I]\{\omega\} + [\underline{\omega}]\{h\} \qquad (1)$$

where:

$\{L\}$ ≡ vector of external torques

$\{\omega\}$ ≡ rotation vector of the earth

$\{h\}$ ≡ relative angular momentum vector

$[I]$ ≡ earth's inertia tensor of the second order

$[\underline{\omega}]$ ≡ skew-symmetric matrix of the rotation vector.

All of the above vectors' components are referred to an arbitrary earth fixed system. Significant simplifications will be introduced when we choose the principal moments of inertia axis as the reference system. As usual, the derivatives respect to time are represented by a dot.

Equation (1) can be solved for the general case of a deformable earth by taking into consideration the variations of $[I]$ and $\{h\}$. For example, it is possible to write

$$[I] = [I_0] + [\Delta I]_P + [\Delta I]_R + [\Delta I]_T + [\Delta I]_E + \text{other effects} \qquad (2)$$

where:

$[I_0]$ ≡ initial value of the earth's tensor of inertia

$[\Delta I]_P$ ≡ contribution to $[I_0]$ due to crustal mass displacements (plate tectonics)

$[\Delta I]_R$ ≡ contribution to $[I_0]$ due to rotational deformation

$[\Delta I]_T \equiv$ contribution to $[I_0]$ due to tidal deformation

$[\Delta I]_E \equiv$ contribution to $[I_0]$ due to large earthquake faulting.

Similarly,

$$\{h\} = \{h_0\} + \{h\}_P + \{h\}_R + \{h\}_T + \{h\}_E + \text{other effects} . \tag{3}$$

Each of the above contributions to $[I_0]$ and $\{h_0\}$ is obtained after consideration of the particular adopted earth model.

For example, one may express:

$$[\Delta I]_P = \sum_{i=1}^{n} [\Delta I]_{P_i} \tag{4}$$

where n is the number of tectonic plates constituting the earth crust.

Likewise,

$$[\Delta I]_T = \sum_{j=1}^{m} [\Delta I]_{T_j} \tag{5}$$

where $j = 1, 2, \cdots m$ refers to the moon, sun and planets producing tidal deformations.

Thus, the solution $\{\omega\}$ of the differential equations (1) will provide the changes in the earth rotation vector after consideration of the latest geophysical theories.

The nature of these changes will depend on the hypothesis about the distribution of masses and its time variations. Clearly, secular changes in $\{\omega\}$, if any, will be produced by $[\Delta I]_P$. Periodic changes will be caused by $[\Delta I]_T$ and sudden variations in $\{\omega\}$ will be due to the effect of $[\Delta I]_E$.

The present intent is to answer the controversial question of secular drift of the "mean pole." The tectonic plate model given by [Solomon and Sleep] is used in the investigation for the computation of $[\Delta I]_P$ (see equation (4)).

Analytical expressions for the obtention of $[\Delta I]_{P_i}$ due to differential motion of the plates have been developed as well as formulas for the computation of $[\Delta I]_{P_i}$ and $\{h\}_{P_i}$.

A computer program is being written in order to obtain the contribution to $[I_0]$ of each independent plate $[\Delta I]_{P_i}$. This program integrates over an ellipsoid and assumes the Heiskanen theory of isostatic compensation for the crust. This hypothesis roughly agrees with the separation crust-mantle as postulated by seismologists (i.e., Mohorovičić or M discontinuity).

Also a computer program is under way in order to obtain good initial values of the earth inertia tensor $[I_0]$.

In the future, after the values of $[I_0]$, $[\Delta I]_P$, $[\dot{\Delta I}]_P$, $\{h\}_P$ and $\{\dot{h}\}_P$ are known, the differential equations (1) will be solved numerically, thus answering the question of a possible secular shift of the pole due to crustal unrest.

REFERENCES

Solomon, S. C. and N. Y. Sleep, 1974. "Some Simple Physical Models for Absolute Plate Motions," Journal Geophysical Research, 79, 2557-2567.

3.3 Close Grid Geodynamic Satellite (CLOGEOS) System

Some effort was made to help the investigations performed for Marshall Space Flight Center. The results are included in Attachments 3 and 4.

4. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Manohar G. Arur, Graduate Research Associate, part time

Athanasios Dermanis, Graduate Research Associate, part time

Michael Gildengorin, Graduate Research Associate, part time from 4/1/75

Alfred Leick, Graduate Research Associate, part time

Anne S. Mason, Research Assistant, part time through 1/5/75

Daniel McLuskey, Graduate Research Associate, part time through 1/17/75

Tomas Soler, Graduate Research Associate, part time

5. TRAVEL

Mueller, Ivan I.
Siena, Italy — March 29 - April 6, 1975
To attend Symposium on Mathematical Geodesy
(partial support)

Mueller, Ivan I.
Washington, D. C. — April 21, 1975
To attend meeting at NASA Headquarters

Manohar G. Arur
Washington, D. C. — June 16 - 20, 1975
To attend Annual Meeting of American Geophysical Union

6. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant No. NSR 36-008-003:

70 The Determination and Distribution of Precise Time
by Hans D. Preuss
April, 1966

71 Proposed Optical Network for the National Geodetic Satellite Program
by Ivan I. Mueller
May, 1966

82 Preprocessing Optical Satellite Observations
by Frank D. Hotter
April, 1967

86 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 1 of 3: Formulation of Equations
by Edward J. Krakiwsky and Allen J. Pope
September, 1967

87 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 2 of 3: Computer Programs
by Edward J. Krakiwsky, George Blaha, Jack M. Ferrier
August, 1968

88 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 3 of 3: Subroutines
by Edward J. Krakiwsky, Jack Ferrier, James P. Reilly
December, 1967

93 Data Analysis in Connection with the National Geodetic Satellite Program
by Ivan I. Mueller
November, 1967

OSU Department of Geodetic Science Reports published under Grant No. NGR 36-008-093:

100 Preprocessing Electronic Satellite Observations
by Joseph Gross
March, 1968

106 Comparison of Astrometric and Photogrammetric Plate Reduction Techniques for a Wild BC-4 Camera
by Daniel H. Hornbarger
March, 1968

110 Investigations into the Utilization of Passive Satellite Observational Data
by James P. Veach
June, 1968

114 Sequential Least Squares Adjustment of Satellite Triangulation and Trilateration in Combination with Terrestrial Data
by Edward J. Krakiwsky
October, 1968

118 The Use of Short Arc Orbital Constraints in the Adjustment of Geodetic Satellite Data
by Charles R. Schwarz
December, 1968

125 The North American Datum in View of GEOS I Observations
by Ivan I. Mueller, James P. Reilly, Charles R. Schwarz
June, 1969

139 Analysis of Latitude Observations for Crustal Movements
by M. G. Arur
June, 1970

140 SECOR Observations in the Pacific
by Ivan I. Mueller, James P. Reilly, Charles R. Schwarz, George Blaha
August, 1970

147 Gravity Field Refinement by Satellite to Satellite Doppler Tracking
by Charles R. Schwarz
December, 1970

148 Inner Adjustment Constraints with Emphasis on Range Observations
by Georges Blaha
January, 1971

150 Investigations of Critical Configurations for Fundamental Range Networks
by Georges Blaha
March, 1971

177 Improvements of a Geodetic Triangulation through Control-Points Established by Means of Satellite or Precision Traversing
by Narendra K. Saxena
June, 1972

184 Coordinate Transformation by Minimizing Correlations Between Parameters
by Muneendra Kumar
July, 1972

185 On the Geometric Analysis and Adjustment of Optical Satellite Observations
by Emmanuel Tsimis
August, 1972

187 Geodetic Satellite Observations in North America (Solution NA-9)
by Ivan I. Mueller, J. P. Reilly and Tomas Soler
September, 1972

188 Free Adjustment of a Geometric Global Satellite Network (Solution MPS-7)
by Ivan I. Mueller and M. C. Whiting
October, 1972

190 The Ohio State University Geometric and Orbital (Adjustment) Program (OSUGOP) for Satellite Observations
by James P. Reilly, Charles R. Schwarz and M. C. Whiting
December, 1972

191 Critical Configurations (Determinantal Loci) for Range and Range-Difference Satellite Networks
by E. Tsimis
January, 1973

193 Free Geometric Adjustment of the DOC/DOD Cooperative Worldwide Geodetic Satellite (BC-4) Network
by Ivan I. Mueller, M. Kumar, J. Reilly and N. Saxena
February, 1973

195 Free Geometric Adjustment of the Secor Equatorial Network (Solution SECOR-27)
by Ivan I. Mueller, M. Kumar and Tomas Soler
February, 1973

196 Geometric Adjustment of the South American Satellite Densification (PC-1000) Network
by Ivan I. Mueller and M. Kumar
February, 1973

199 Global Satellite Triangulation and Trilateration for the National Geodetic Satellite Program (Solutions WN 12, 14 and 16)
by Ivan I. Mueller and M. Kumar, J. P. Reilly, N. Saxena, T. Soler
May, 1973

216 Marine Geodesy, A Multipurpose Approach to Solve Oceanic Problems
by Narendra K. Saxena
October, 1974

The following papers were presented at various professional meetings:

"Report on OSU participation in the NGSP"
47th Annual meeting of the AGU, Washington, D. C., April 1966

"Preprocessing Optical Satellite Observational Data"
3rd Meeting of the Western European Satellite Subcommission, IAG, Venice, Italy, May 1967.

"Global Satellite Triangulation and Trilateration"
XIVth General Assembly of the IUGG, Lucerne, Switzerland, September 1967, (Bulletin Geodesique, March 1968).

"Investigations in Connection with the Geometric Analysis of Geodetic Satellite Data"
GEOS Program Review Meeting, Washington, D. C., Dec. 1967.

"Comparison of Photogrammetric and Astrometric Data Reduction Results for the Wild BC-4 Camera"
Conference on Photographic Astrometric Technique, Tampa, Fla., March 1968.

"Geodetic Utilization of Satellite Photography"
7th National Fall Meeting, AGU, San Francisco, Cal., Dec. 1968.

"Analyzing Passive-Satellite Photography for Geodetic Applications"
4th Meeting of the Western European Satellite Subcommission, IAG, Paris, Feb. 1969.

"Sequential Least Squares Adjustment of Satellite Trilateration"
50th Annual Meeting of the AGU, Washington, D. C., April 1969.

"The North American Datum in View of GEOS-I Observations"
8th National Fall Meeting of the AGU, San Francisco, Cal., Dec. 1969 and GEOS-2 Review Meeting, Greenbelt, Md., June 1970 (Bulletin Geodesique, June 1970).

"Experiments with SECOR Observations on GEOS-I"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"Experiments with the Use of Orbital Constraints in the Case of Satellite Trails on Wild BC-4 Photographic Plates"
GEOS-2 Review Meeting, Greenbelt, Md., June 1970.

"GEOS-I SECOR Observations in the Pacific (Solution SP-7)"
National Fall Meeting of the American Geophysical Union, San Francisco, California, December 7-10, 1970.

"Investigations of Critical Configurations for Fundamental Range Networks"
Symposium on the Use of Artificial Satellites for Geodesy, Washington, D.C., April 15-17, 1971.

"Gravity Field Refinement by Satellite to Satellite Doppler Tracking"
Symposium on the Use of Artificial Satellites for Geodesy, Washington, D.C., April 15-17, 1971.

"GEOS-I SECOR Observations in the Pacific (Solution SP-7)"
Symposium on the Use of Artificial Satellites for Geodesy, Washington, D.C., April 15-17, 1971.

"Separating the Secular Motion of the Pole from Continental Drift - Where and What to Observe?"
IAU Symposium No. 48, "Rotation of the Earth," Morioka, Japan, May 9-15, 1971.

"Geodetic Satellite Observations in North America (Solution NA-8)"
Annual Fall Meeting of the American Geophysical Union, San Francisco, California, December 6-9, 1971.

"Scaling the SAO-69 Geometric Solution with C-Band Radar Data (Solution SC 11)"
Annual Fall Meeting of the American Geophysical Union, San Francisco, California, December 6-9, 1971.

"The Impact of Computers on Surveying and Mapping"
Annual Meeting of the Permanent Committee, International Federation of Surveyors, Tel Aviv, Israel, May 1972.

"Investigations on a Possible Improvement of Terrestrial Triangulation by Means of Super-Control Points"
IAG International Symposium - Satellite and Terrestrial Triangulation, Graz, Austria, June, 1972.

"Free Adjustment of a Geometric Global Satellite Network (Solution MPS7)"
IAG International Symposium - Satellite and Terrestrial Triangulation, Graz, Austria, June, 1972.

"Conjugate Gradient Method (Cg-Method) for Geodetic Adjustments"
Annual Fall Meeting of the American Geophysical Union, San Francisco, California, December 3-6, 1972.

"Preliminary Results of the Global Satellite Triangulation Related to the NGSP"
Journees Luxembourgeoises de Geodynamique, Luxembourg, February 19-21, 1973.

"Present Status of Global Geometric Satellite Triangulation and Trilateration"
54th Annual Spring Meeting of the American Geophysical Union, Washington, D.C., April 16-20, 1973.

"Free Geometric Adjustment of the OSU/NGSP Global Network (Solution WN4)"
First International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Athens, Greece, May 14-21, 1973.

"Earth Parameters from Global Satellite Triangulation and Trilateration"
International Symposium on Earth's Gravitational Field and Secular Variations in Position, Sydney, Australia, November 26-30, 1973.

"Review of Problems Associated with Geodetic Datums"
International Symposium on Problems related to the Redefinition of North American Geodetic Networks, Fredericton, N.B., Canada, May 20-25, 1974.

"Marine Geodesy - Problem Areas and Solution Concepts"
International Symposium on Application of Marine Geodesy, Battelle Auditorium, Columbus, Ohio, June 3-5, 1974.

"Station Coordinates and Geodetic Datum Positions from the National Geodetic Satellite Program"
First Pan American Congress and the
Third National Congress of Photogrammetry, Photointerpretation and Geodesy, Mexico City, Mexico, July 7-12, 1974.

"Review of Classical Methods for the Determination of Geodetic Datums"
International Colloquium on Reference Coordinate Systems for Earth Dynamics (IAU Colloquium No. 26)
Torun, Poland, August 26-31, 1974.

"Global Satellite Triangulation and Trilateration Results"
Intercosmos Symposium on Results of Satellite Observations
Budapest, Hungary, October 21-24, 1974.

Attachment 1
2 Pages

DEFENSE MAPPING AGENCY
TOPOGRAPHIC CENTER
WASHINGTON, D.C. 20315

REPLY TO
ATTENTION OF:

DMATC-G(52321)

29 APR 1975

Mr. Alfred Leick
Department of Geodetic Science
Ohio State University
1958 Neil Avenue
Columbus, Ohio 43210

Dear Mr. Leick:

Reference is made to your letter to Dr. Kenneth I. Daugherty, dated 21 October 1974, requesting coordinates of Doppler stations in South America and your telephone conversation with Mr. John Love of this Center on 21 April 1975.

The attached data partially fulfill your request. Enclosed for your retention are copies (front and back) of Geodetic Summary cards containing South American datum (SAD) positional data and Doppler Receiver Geodetic Summary sheets containing satellite derived positions for the following stations:

STATION NO.	STATION LOCATION
30009 (T-009)	General Conesa, Rio Negro, Argentina
30010 (T-010)	Villa Dolores, Argentina
*30011	Tierra Del Fuego, Cerro Sombrero, Chile
30012	Frutillar Alto, Chile (Pre- and Post- Earthquake Values)
*30013	Punta Arenas, Chile
30022	Santiago, Chile
30023	Arica, Chile
30120	La Paz, Bolivia
30121	Quito, Ecuador
30196	Coromoto, Venezuela
30209	El Callao/Tumeremo, Venezuela

*1963 Provisional South Chile datum, not yet related to SAD.

Authorization from the governments concerned has been obtained to release the coordinates of the Doppler stations for your use at Ohio State University; publication of the coordinates would require further authorization.

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DMATC-G(52321)
Ohio State University

29 APR 1975

Positional data for Station No. 30122, Asuncion, Paraguay, are not furnished because authorization for its release has not been obtained. Upon receipt of authorization from the Government of Paraguay, data for Station No. 30122 will be forwarded.

Sincerely,

Richard P. Peat

RICHARD P. PEAT
Chief, Department of Geodesy

Enclosure
As stated

DOES MEAN SEA LEVEL SLOPE UP OR DOWN TOWARD NORTH?

Comments on the article of the same title by Irene Fischer [1975]

by

M. G. Arur and Ivan I. Mueller
Department of Geodetic Science
The Ohio State University, Columbus

1. Introduction

Geodesists and oceanographers have disagreed on the direction and magnitude of the North-South sea level slopes along the East and West Coasts of the United States.

There was some room to doubt the validity of the comparisons between the results of the geodesists and the oceanographers since they use different methods and different reference surfaces for the determination of these slopes [Fischer, 1975].

An attempt has now been made to compare the results of the oceanographers and geodesists by reducing them to the same terms.

2. Method of Calculation

The results of both geodetic and oceanic leveling have been reduced to the following compatible quantities for comparison at several stations along the two coasts (see Fig. 1):

(i) Geopotential differences between the mean sea level and the deep sea isobaric surface used as a reference surface in oceanic leveling.

(ii) Orthometric heights between the mean sea level and the same deep sea isobaric surface.

Values at the various stations for the anomalous geopotentials converted to dynamic heights of the mean sea level with respect to the standard ocean surface (0-decibar) of reference have been taken from the graphs of Sturges [1974]. Values at these stations for the orthometric height differences between the mean sea level and the reference geopotential surface used in geodetic leveling have been taken from the graphs of Balazs [1973].

Computations have been carried out for 21 stations along the U.S. East Coast and for 8 stations along the West Coast. The following assumptions have been made:

(i) The deep sea isobaric surface used as a reference in oceanic leveling is an equipotential surface.

(ii) Oceanic and geodetic leveling is in perfect agreement at Neah Bay on the West Coast and Port Maine on the East Coast, both having been used as references (origins) in geodetic leveling along the West and East Coasts respectively.

(iii) The gravity field of the earth is well described by the normal gravity field in the areas under investigation and gravity varies linearly with height/depth up to 2 km.

None of these assumptions will effect adversely the conclusions to be drawn.

2.1 Calculation of Geopotential Differences

2.11 Oceanic Leveling

In accordance with the notation in Fig. 1, the difference in the geopotential between the deep isobaric surface and the mean sea level as determined from oceanic leveling at an arbitrary station i , is

$$\Delta W_{0_i} = \Delta W_s + dW_{0_i} = g_{m_i} \times H_{s_i} + h_{D_i} \times 1000 \qquad (1)$$

where

ΔW_s is the standard geopotential difference between the deep sea isobaric surface and the standard ocean (0-db) surface. The value for this is $9704.032\,m^2 s^{-2}$ for 1000-db surface used as a reference on the West Coast [Montgomery, 1973]; and $19364.2\,m^2 s^{-2}$ for 2000-db surface used as a reference on the East Coast [Bjerknes and Sandstrom, 1910].

dW_{0_i} is the anomalous geopotential at station i, as determined by oceanographers.

h_{D_i} is the dynamic height at station i from the graphs of Sturges [1974] and is to be interpreted as per equation (1) above, 1000 being the constant value of gravity (in gals) used by Sturges to convert his original dW_{0_i} potential anomalies into metric units.

H_{s_i} is the orthometric height difference at station i corresponding to the standard geopotential difference ΔW_S.

g_{m_i} is the average normal gravity between the mean sea level and the deep sea isobaric surface at station i (in gals) based on the Geodetic Reference System 1967. It is computed from

$$g_{m_i} = 978.03185\,(1 + 0.005278895 \sin^2\varphi_i + 0.000023462 \sin^4\varphi_i - 0.0000002277 \frac{H_{0_i}}{2})\ \mathrm{cm\,s^{-2}} \tag{2}$$

where φ_i is the latitude and H_{0_i} is the orthometric height between the deep sea isobaric surface and the mean sea level (in meters). The constant 0.0000002277 ($\times$ 978) is the normal vertical gradient of gravity in water.

With the above notation ΔW_{0_i} can also be computed from

$$\Delta W_{0_i} = g_{m_i} \times H_{0_i}\,. \tag{3}$$

The quantities g_{m_i} and H_{0_i} are to be determined iteratively until equations (1), (2) and (3) are mutually satisfied.

2.12 Geodetic Leveling

The geopotential difference at the station i between the deep sea isobaric surface and the mean sea level may also be computed from the results of geodetic leveling as follows:

$$\Delta W_{0_i} = \Delta W - dW_{g_i} \tag{4}$$

where

ΔW is the geopotential difference between the deep sea isobaric surface and the reference geopotential surface along which the geodetic leveling is assumed to take place, and computed at the reference station (origin of leveling) in accordance with section 2.11.

dW_{g_i} is the difference of geopotential between the reference geopotential surface of geodetic leveling and the mean sea level at the computation station, or

$$dW_{g_i} \cong h_{g_i} \times g_{s_i} \quad , \tag{5}$$

where h_{g_i} is the orthometric height at the computation station between the reference geopotential surface and the mean sea level, obtained from the graphs of Balazs [1973], and g_{s_i} is the average gravity along h_{g_i} obtained approximately by inserting $H_{0_i} = 0$ in equation (2).

2.2 Results

Figures 2 and 3 show the orthometric heights H_{0_i} , i.e., the position of the mean sea level with respect to the deep isobaric surface as determined through oceanographic and geodetic leveling. The heights from oceanographic leveling were computed in accordance with section 2.11, i.e., through an iterative procedure to simultaneously fulfill equations (1), (2) and (3). The heights from geodetic leveling were computed from

$$H_{0_i} = H_i - h_{g_i} , \tag{6}$$

where H_i was also computed iteratively as H_{0_i} from oceanographic leveling, except using the potential difference ΔW instead of ΔW_{0_i}.

The differences between these two types of heights (oceanographic minus geodetic) near the mean sea level in terms of geopotential are shown in Figures 4 and 5..

3. Conclusions and Comments on the Paper by Fischer [1975]

(i) The results of oceanic and geodetic leveling are comparable. In terms of magnitude, the discrepancies as pointed out by oceanographers, between oceanic and geodetic leveling, unfortunately do exist.

(ii) If the deep sea isobaric surface is taken as the equipotential surface of reference, the results of both oceanic and geodetic leveling indicate that the ocean is sloping down from South to North along both the U.S. East and West Coasts.

(iii) The discrepancies are greater along the West Coast where the deep sea isobaric surface of reference is 1000-db, as compared to the East Coast where the reference surface is 2000-db. The geopotential discrepancy is predominantly negative and increases with the distance from the reference station.

(iv) Factors of some importance in the above comparisons are the lack of actual gravity data which made the use of normal gravity a necessity and the choice of deep sea isobaric surface. Both are likely to account for a very small part of the discrepancy in magnitude. The discrepancy about the direction of the slope seems to have been resolved.

(v) Some of the above conclusions are unfortunately at variance with the findings of Fischer [1975]. The following comments are offered.

In preparing her graphs (Figures 2, 3 and 4 of Fischer's article), she has computed normal orthometric corrections (Δh) in ten-degree meridional sections from Helmert's formula, multiplied by scale factors (k) so chosen that the sum of the corrections from the pole would yield an equatorial bulge of 2m, 1.6m and 1.8m, respectively, to match Sturges's profiles. According to Helmert's formula the equatorial bulge for 100 m separation at the pole is 53 cm, thus equipotential surfaces about 300 m apart at the pole will have a bulge of 1.6m at the equator, which is the equivalent of a k factor of 3.019.

Let us take the case of the West Coast and study the implication of considering 1.6m as the equatorial bulge as represented in Figure 3 in Fischer's article. This bulge and the use of the corresponding k factor (3.019) implies that she was in fact dealing with two equipotential surfaces which have an orthometric separation of about 301.9m at the poles and 303.5m at the equator. The upper surface is represented by the curve G in her graph and is thus referenced to the lower surface (let us call it $\overline{0}$), taken as a straight line about 300m below the ocean surface. Such a surface has no real meaning in the oceanographic or geodetic sense. It is too low to be considered as the theoretical 0-db surface (to which Sturge's profiles are referenced) and too high to be considered as the 1000-db surface (which is about 990m below the ocean surface). Since Fischer's G profile in fact refers to this $\overline{0}$ surface and Sturges's profile to the 0-db surface, the convergence of these two surfaces needs to be considered, i.e., when comparing the two profiles the convergence needs to be computed and added to Sturges's values to refer them also to the $\overline{0}$ surface.

Alternately, Sturges's values can be plotted with respect to a correctly depicted theoretical 0-db surface which would show a curvature with respect to $\overline{0}$, and in fact would be practically parallel to the G surface, being within a few meters from it.

If the plotting would have been done correctly as indicated above, it would have been obvious that the magnitude of the discrepancies between the oceanographic and geodetic leveling persists, and that the results of oceanographic and geodetic leveling have been correctly compared in the past.

The main point thus is that geodetic leveling is done with respect to an identifiable reference geopotential surface passing through the origin of leveling. Oceanic leveling provides dynamic heights with respect to the theoretical 0-db surface. These two equipotential surfaces, being within a few meters from each other, are practically parallel and they have a varying separation with respect to the deep ocean equipotential surface (such as the 1000-db surface) used as a reference in oceanic leveling. This varying separation is taken into account in the process of oceanic leveling. Thus for practical comparisons, geodetic and oceanic leveling provide values with respect to two parallel surfaces separated by the dynamic height of mean sea level at the reference station (origin) for geodetic leveling.

References

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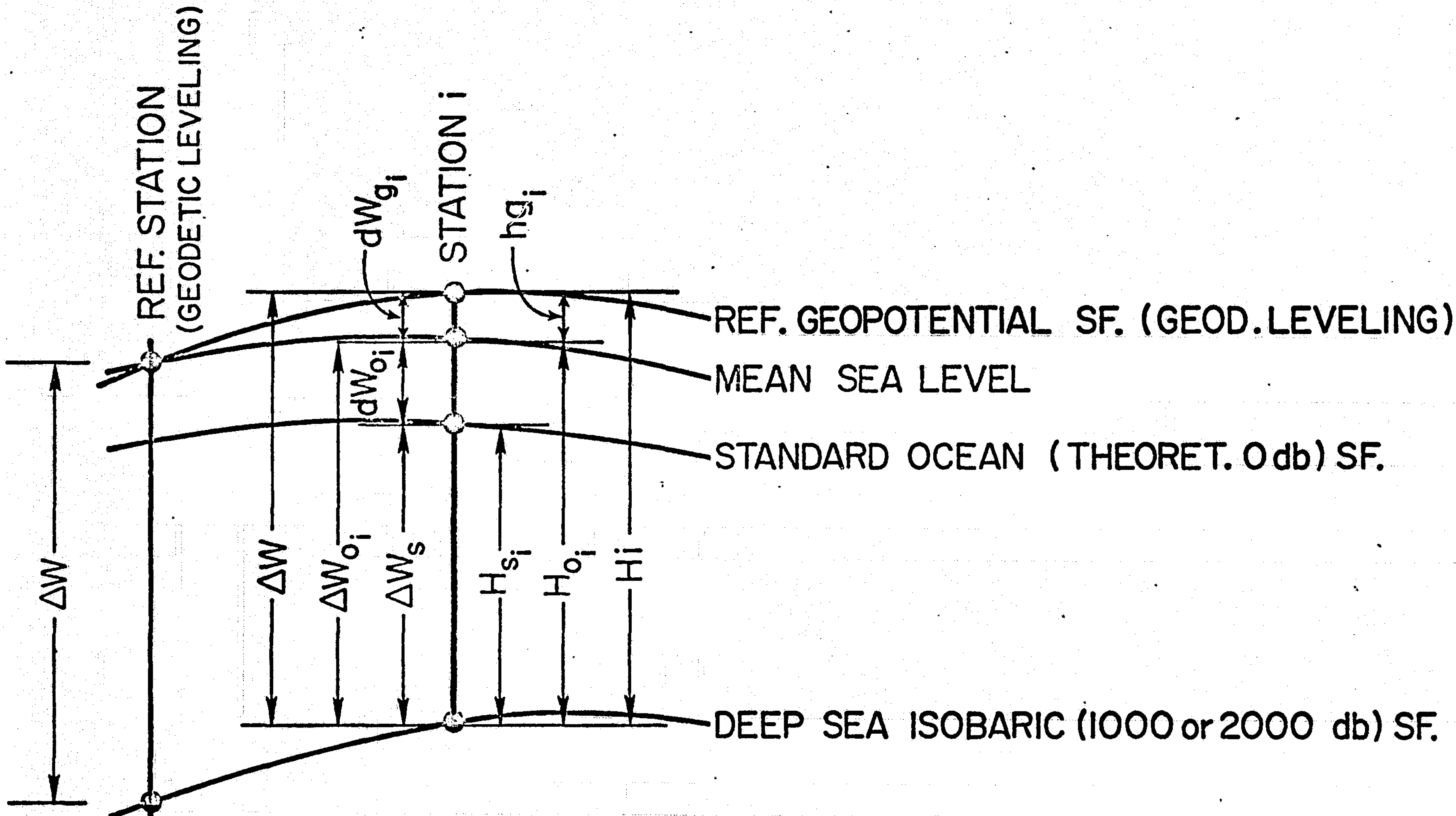
REF. STATION (GEODETIC LEVELING)
STATION i
dWgi
hgi
REF. GEOPOTENTIAL SF. (GEOD. LEVELING)
MEAN SEA LEVEL
dWoi
STANDARD OCEAN (THEORET. O db) SF.
ΔW
ΔW
ΔWoi
ΔWs
Hsi
Hoi
Hi
DEEP SEA ISOBARIC (1000 or 2000 db) SF.

Fig. 1

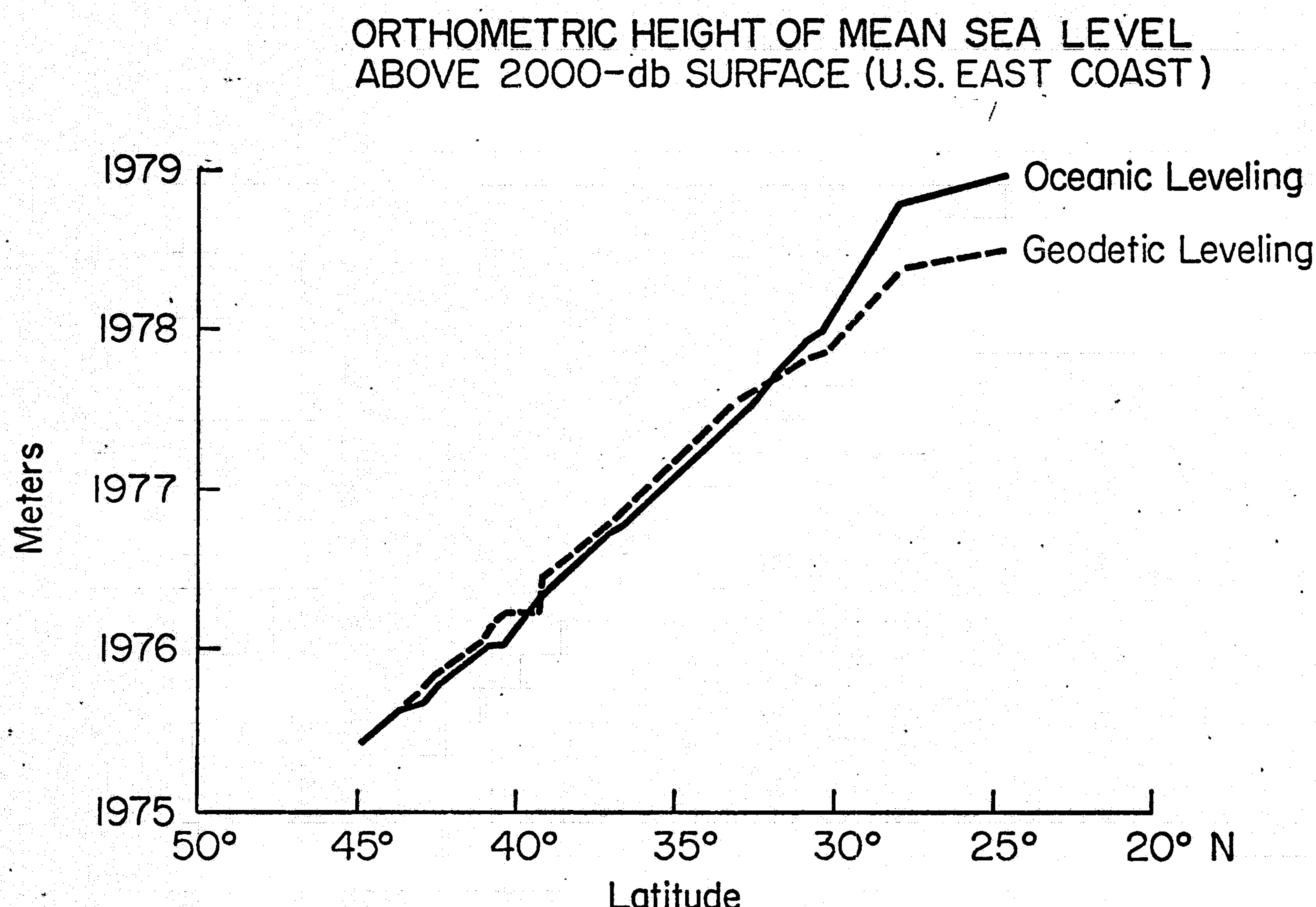
ORTHOMETRIC HEIGHT OF MEAN SEA LEVEL
ABOVE 2000-db SURFACE (U.S. EAST COAST)
Oceanic Leveling
Geodetic Leveling
Meters
1979
1978
1977
1976
1975
50°
45°
40°
35°
30°
25°
20° N
Latitude

Fig. 2

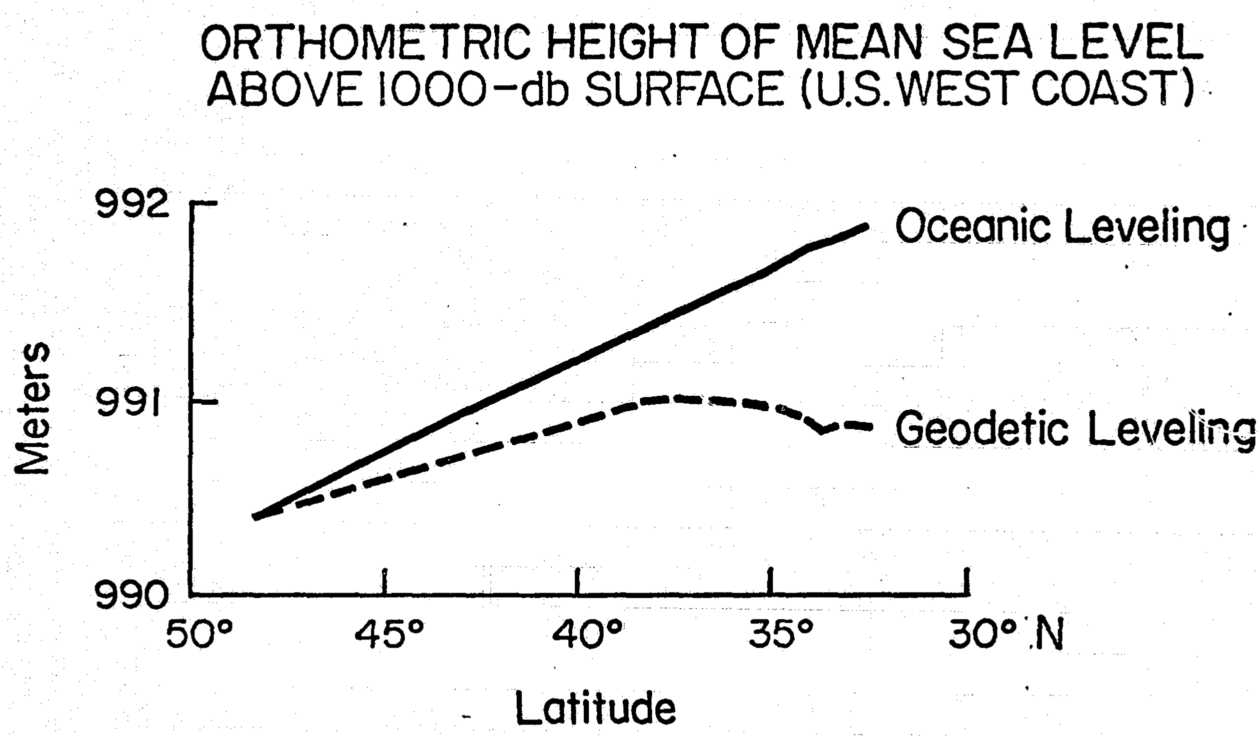
ORTHOMETRIC HEIGHT OF MEAN SEA LEVEL
ABOVE 1000-db SURFACE (U.S. WEST COAST)
992
991
990
Meters
Oceanic Leveling
Geodetic Leveling
50°
45°
40°
35°
30° N
Latitude

Fig. 3

GEOPOTENTIAL DIFFERENCE AS DETERMINED BY OCEANIC AND GEODETIC LEVELING (U.S. EAST COAST)

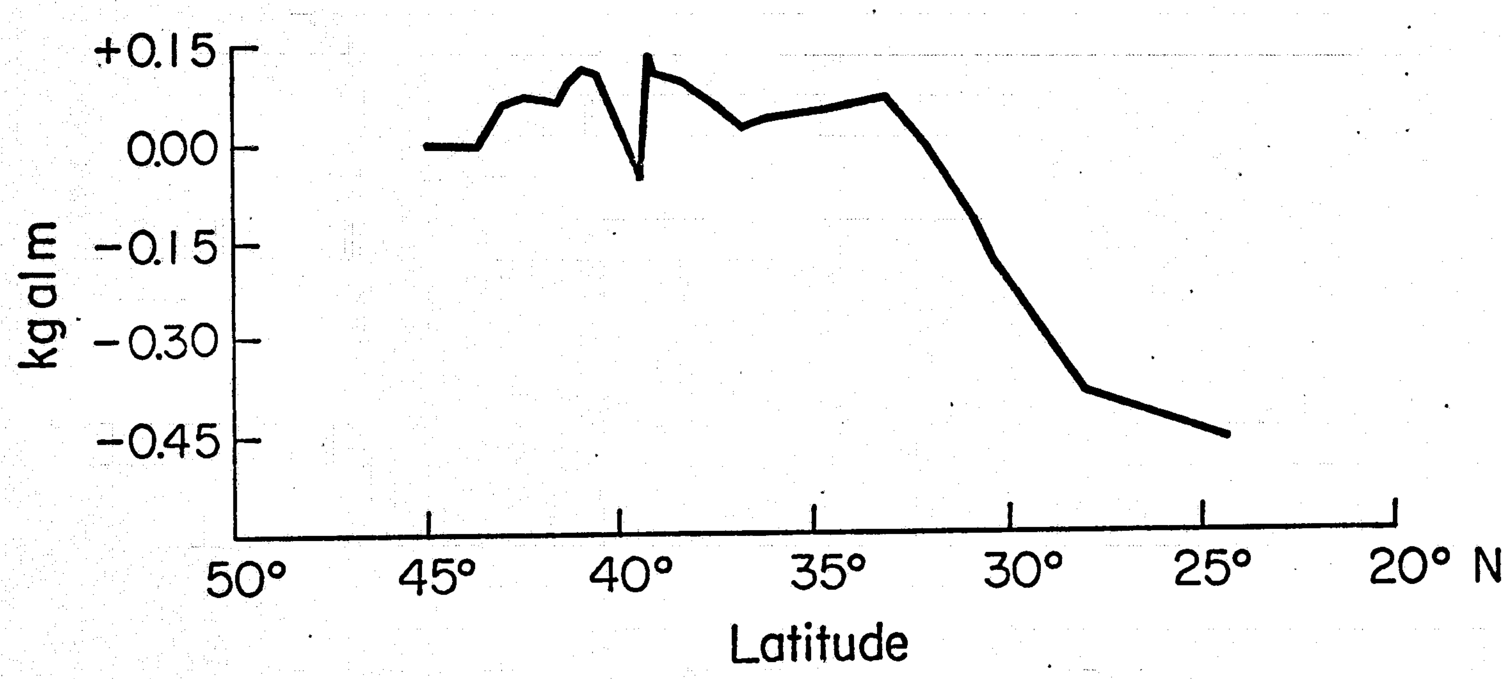

Fig. 4

GEOPOTENTIAL DIFFERENCE AS DETERMINED BY OCEANIC AND GEODETIC LEVELING (U.S. WEST COAST)

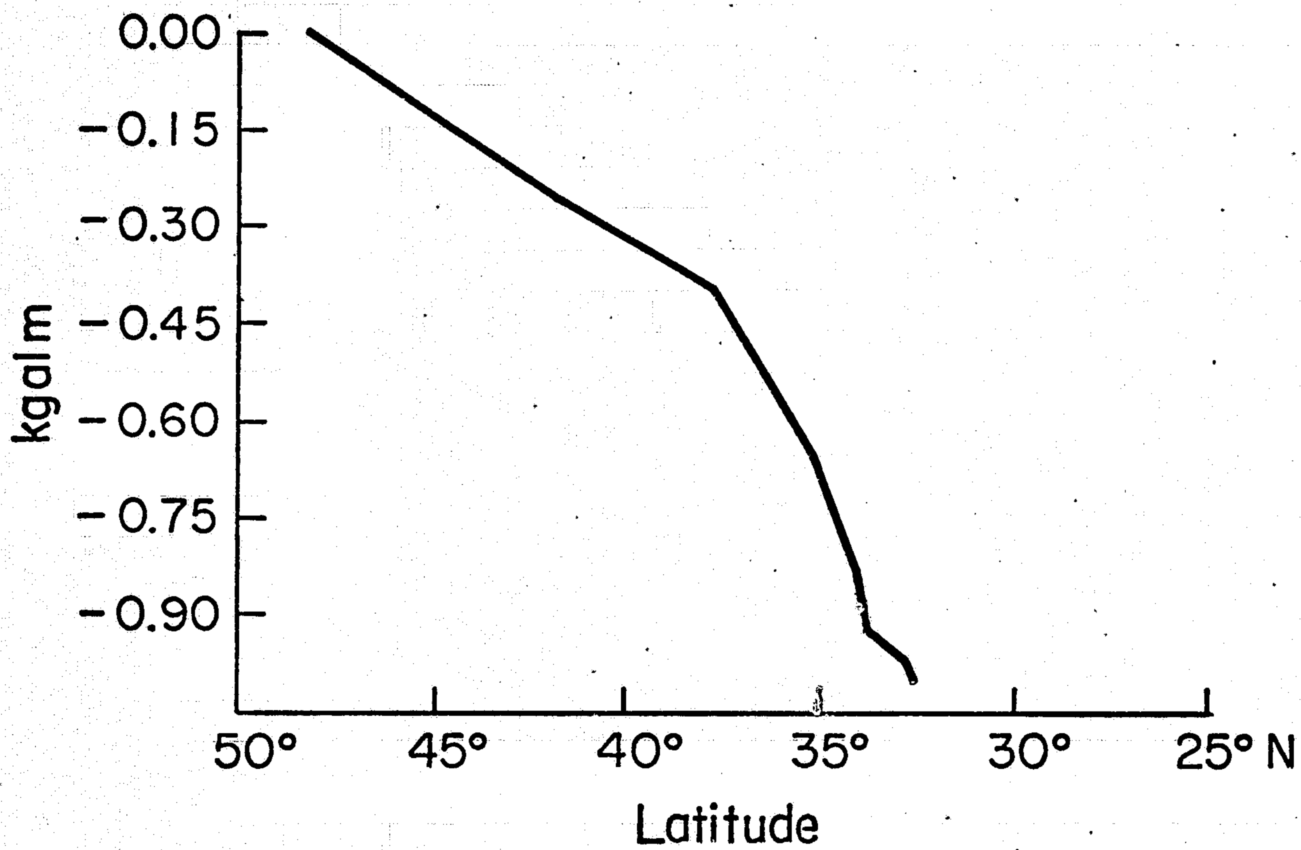

Fig. 5

Attachment 3

48 Pages

Department of Geodetic Science

CLOSE GRID GEODYNAMIC SATELLITE MEASUREMENT SYSTEM DEFINITION

Quarterly Status Report
Contract No. NAS 8-31195
OSURF Project No. 4105-A1

Period Covered: January 1 - April 15, 1975

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Alabama, 35812

The Ohio State University
Research Foundation
Columbus, Ohio, 43212

April, 1975

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PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University and is under the technical direction of Dr. Nicholas C. Costes, Code ES31, MSFC, Huntsville, Alabama. The contract is issued by Procurement Office, MSFC, and is administered through ONRRR, Columbus, Ohio.

1. Statement of Work

Perform an error analysis, based on assumed sets of satellite borne transmitting equipment and ground receivers, to determine the optimum use of such systems in connection with the science that can be obtained from CLOGEOS measurements.

2. Available Computer Programs

To achieve the goals vide para. 1, main computer programs available at The Ohio State University are briefly described below.

2.1 Goddard Trajectory Determining System (GTDS)

The Program GTDS, acquired recently at OSU from Goddard Space Flight Center, is extremely versatile and has numerous operating modes and capabilities [COSMIC, 74].

Space craft dynamics used in the program includes gravitational acceleration for Sun, Earth (up to 15x 15 non-spherical field), Moon (up to 4x 4 non-spherical field) and all the planets, drag acceleration and solar radiation model with shadow effect and variations depending on distance from sun [Wagner, et. al., 1972].

In this project, this program was mainly used for ephemeris generation.

2.2 The Ohio State University Geometric and Orbital (Adjustment) Program (OSUGOP)

The program OSUGOP performs basically as an adjustment program, using optical or range observations, in geometric/orbital mode [Reilly, et. al., 1972]. One important feature of the program is its capability to apply and obtain different constrained solutions, including 'inner' constraints [Blaha, 1971].

As the current error analysis was to be based on the 'estimable' quantities e.g., chords and angles between the locations of ground stations, a new subroutine 'CHECK' has been added to the program.

2.3 Short Arc Geodetic Adjustment Program (SAGA)

The latest version of this program, as developed by Duane Brown Associates, was obtained at OSU from Air Force Cambridge Research Laboratories [Brown and Trotter, 1973].

The program employs a power series solution using partitioned regression technique. Inner constraints capability has also been added in the present program.

2.4 Range Generation-Geometric - with 7 Records in Mode B Program (RGGR7-B)

The computer program RGGR7-B generates ranges from the short arcs on tape as generated by GTDS and given station locations. Time interval between the ranges, cut off angle or maximum zenith angle as well as the station locations and the model in which they observe the satellite per pass can be specified outside the program. The program has the capability to superimpose white noise of any standard deviation on the ranges. The ranges are written on tape in the GEOS range format as required for the input to the computer program OSUGOP.

3. Data Generation

3.1 Satellite Orbits

During the orientation meeting for CLOGEOS at Huntsville, Alabama on February 6, 1975, it was agreed upon that the possible approximate altitudes for investigation are 350 km, 600 km and 1000 km. However, during the period under report some alterations were made in the designated orbits.

The orbit generation was carried out in two steps - first a long arc for 126 hours was generated in inertial system and then taking the suitable orbit points several short arcs in the body fixed system were generated over the area under investigation.

The details about orbit delineation and the computer expenditure involved are given in Tables 3.1-1 to 3.1-5.

3.2 Range Generation

The computer program RGGR7-B and the short arcs (vide para. 3.1) were then used to generate ranges in geometric mode with Gaussian standard deviation of 10cm. During the range generation the density of orbit points was suitably altered between lower, middle and upper orbit to keep the number of ranges the same for each orbit.

During the range generation a new variable was introduced as under (Fig. 1):

Case A - When all the observing stations were located on the ellipsoid (h=0).

Case B - When some of the observation stations were raised from the ellipsoid to a maximum height of 100 meters.

Case C - Same as Case B except maximum height was made up to 1000 meters.

The station layout, orbital parameters, span of observations. general distribution, etc. are given in Fig. 2 and Fig. 3. For details of data generated and the computer expenses see Table 3.2-1.

The ranges were generated on three tapes RNGE01, RNGE02, and CSTP01 and the contents of files on these tapes are given in Tables 3.2-2, 3.2-3 and 3.2-4.

Tape Format

DCB=(RECFM=VBS, LRECL=40, BLKSIZE=12004)

Record Format

NN, IYMD, IH, IM, IS, RR, SIGR

where

NN = Station identification number

IYMD = Year, month and date of observation in packed format

$\left.\begin{array}{l}\text{IH = Hour}\\ \text{IM = Minute}\\ \text{IS = Seconds} * 10^4\end{array}\right\}$ of observation

RR = Range in meters

SIGR = Standard deviation of range in meters.

4. Simulated Solutions

The CLOGEOS error analysis was decided to be carried out in two main section viz., geometric and short arc modes. During the period under report the investigations were made only in geometric mode. Thirty-seven simulated solutions were computed as detailed in Tables 4-1 to 4-3.

5. Analysis and Conclusions (Preliminary)

In an error analysis of the type under consideration it is more realistic to analyze the estimable quantities. As the station coordinates fall under non-estimable quantities, error analysis for them would have been significantly dependent on the origin and the error propagating outwardly from it. Distances and angles are the only estimable quantities in the present error analysis and are defined as under:

$$\text{chord } R_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \tag{5.1}$$

$$\text{angle } \alpha_{ijk} = \cos^{-1} \frac{(x_i - x_j)(x_k - x_j) + (y_i - y_j)(y_k - y_j) + (z_i - z_j)(z_k - z_j)}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}\,\sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}} \tag{5.2}$$

where x_i, y_i, z_i are the rectangular coordinates of the i^{th} points. The current investigation deals with the distance analysis and the results have been broadly grouped as under.

5.1 Effect of Orbit And/Or Station Separation

Six solutions were run for 500 events each for cases A, B and C and the standard deviations for the best case $\sigma_{r_{12}}$ (where r_{ij} denotes the distance between station numbers i and j) and for the worst case $\sigma_{r_{19}}$ were plotted (Figures 4 and 5).

The $\sigma_{r_{ij}}$ -s show significant improvement either introducing station separation as in Case C or by introducing mixed satellite altitudes through eccentric or multiple arcs.

5.2 Effect of Orbital Height

Figures 6 (for $\sigma_{r_{12}}$) and 7 (for $\sigma_{r_{19}}$) show that the height of the orbit cannot improve solutions when observing stations have a 'near' critical configuration.

5.3 Effect of Number of Events

Figures 8 (for $\sigma_{r_{12}}$) and 9 (for $\sigma_{r_{19}}$) show that the number of events included in any solution had a significant influence. However, in any given station configuration, the less critical is the configuration the smaller the number of events needed to obtain a specified accuracy.

6. Personnel

Ivan I. Mueller, Project Supervisor, part time.

Muneendra Kumar, Graduate Research Associate, part time.

Boudewijn H.W. Van Gelder, Graduate Research Associate, part time.

Michelle Neff, Administrative Assistant, part time.

7. Travel

1. Project meetings at MSFC, Huntsville, Alabama.

   December 17, 1974 (Mueller and Van Gelder).

   February 6, 1975 (Mueller, Van Gelder and Kumar).

   See material distributed in Appendix A.

   April 17, 1975 (Mueller)

   Material presented at this meeting is identical to the one in this report.

2. January 30 - Feb. 2, 1975, New York City, (Mueller)

   To attend the AAAS meeting.

3. April 2-5, 1975, Siena, Italy, (Mueller)

   Mathematical Geodesy (Partial support). For a report on this meeting see Appendix B.

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Reilly, J.P., C.R. Schwarz and M.C. Whiting, (1972). "The Ohio State University Geometric and Orbital (Adjustment) Program (OSUGOP) for Satellite Observations", Reports of the Department of Geodetic Science No. 190, The Ohio State University, Columbus, OH.

Figure 1

HEIGHTS OF STATIONS

$h_{1,5,7} = 1.0$ m

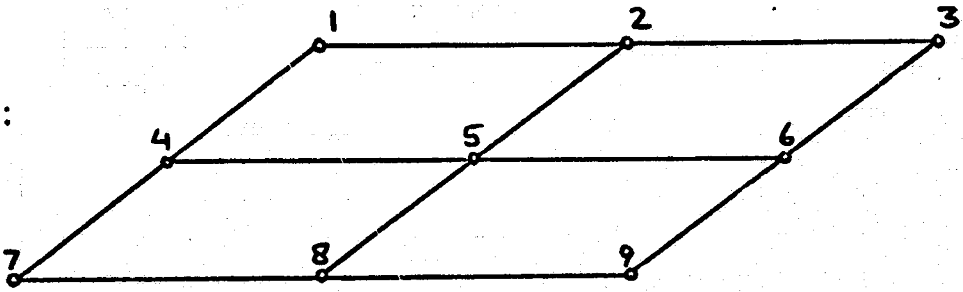
$h_{2,4,6,8} = 0.5$ m

$h_{3,9} = 0.0$ m

HEIGHT FACTOR : HF

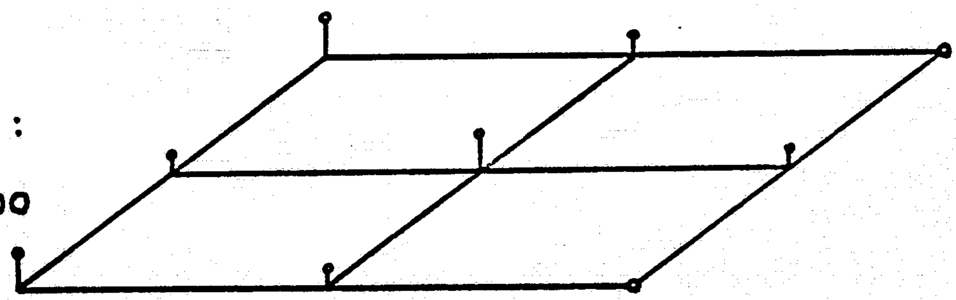
$$h'_i = HF * h_i$$

CASE A :
HF = 0



CASE B
HF = 100

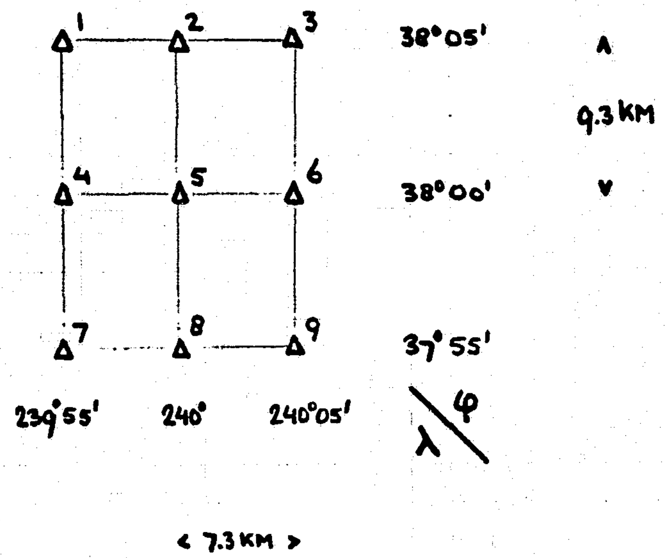
CASE C :
HF = 1000



GENERAL INFORMATION

STATION COORDINATES — GEODETIC REFERENCE SYSTEM 1967
$a = 6378.160$ KM
$1/f = 298.247\ 167\ 427$

STATION LAYOUT



ORBIT SATELLITE — $H = 392,\ 657$ AND 1007 KM
$i = 90°$, POLAR ORBIT
$e = 0.001$

AIRPLANE — $H = 9$ KM

OBSERVATIONS — FEB 1.0 – FEB 6.25, 1975
(126 HOURS)

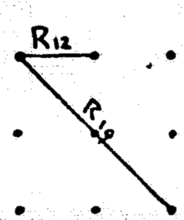
RANGING ACCURACY — $\sigma_r = 10$ CM

SIMULATED SOLUTIONS — 37

ANALYSIS — DISTANCES & ANGLES : ONLY ESTIMABLE QUANTITIES IF NO CONSTRAINTS ARE USED

BEST DISTANCE — R_{12}
WORST DISTANCE — R_{19}

$$\sigma_{R_{12}} \leq \sigma_{R_{ij}} \leq \sigma_{R_{19}}$$



DISTRIBUTION OF OBSERVATIONS

HEIGHT ORBIT	$\Delta\varphi$	$\Delta\lambda$	INTERVAL BETWEEN EVENTS TO OBTAIN 5000 EVENTS	AVERAGE LENGTH OF A PASS	NUMBER OF REQUIRED PASSES	NUMBER OF PASSES PER DAY	TOTAL OF DAYS REQUIRED
L=392 KM	19°.0	24°.0	1.0 SEC	4 MIN	21	2	$10\frac{1}{2}$
M=657 KM	27°.8	35°.2	1.0	6	14	3	5
U=1007 KM	37°.0	46°.8	1.0	9	10	4	$2\frac{1}{2}$

(IDEAL WEATHER CONDITIONS)

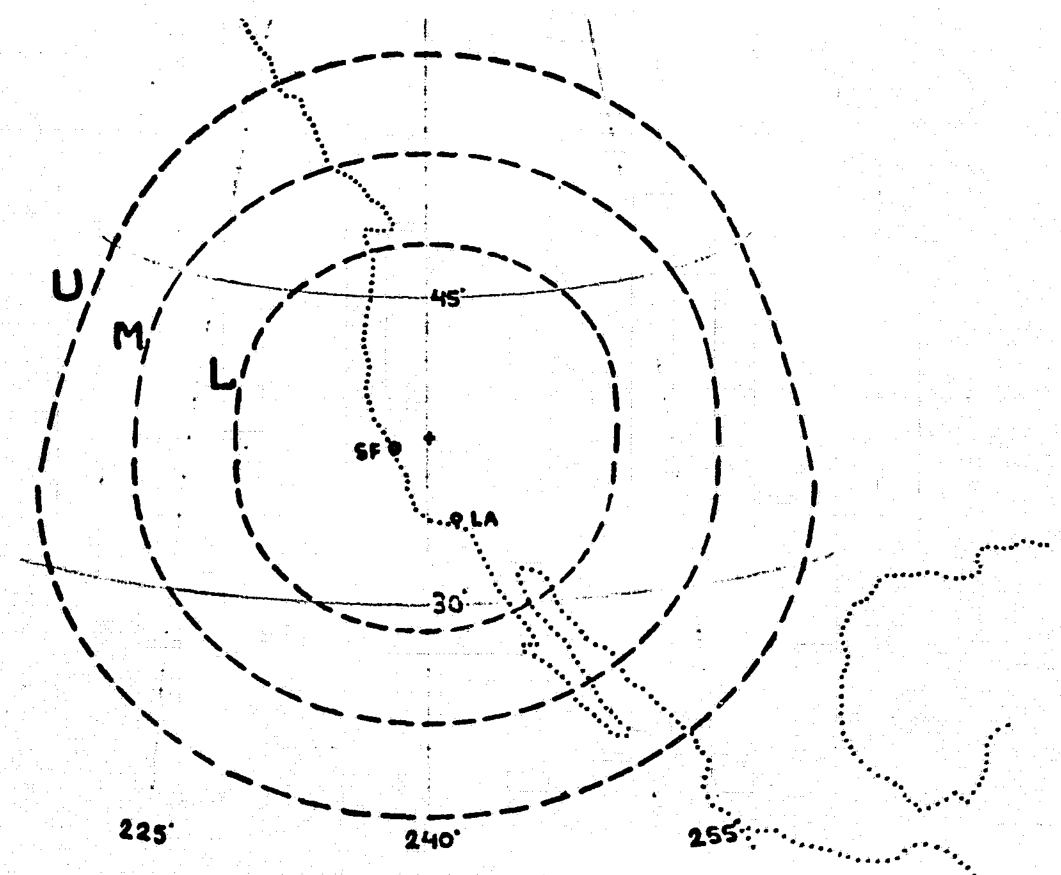
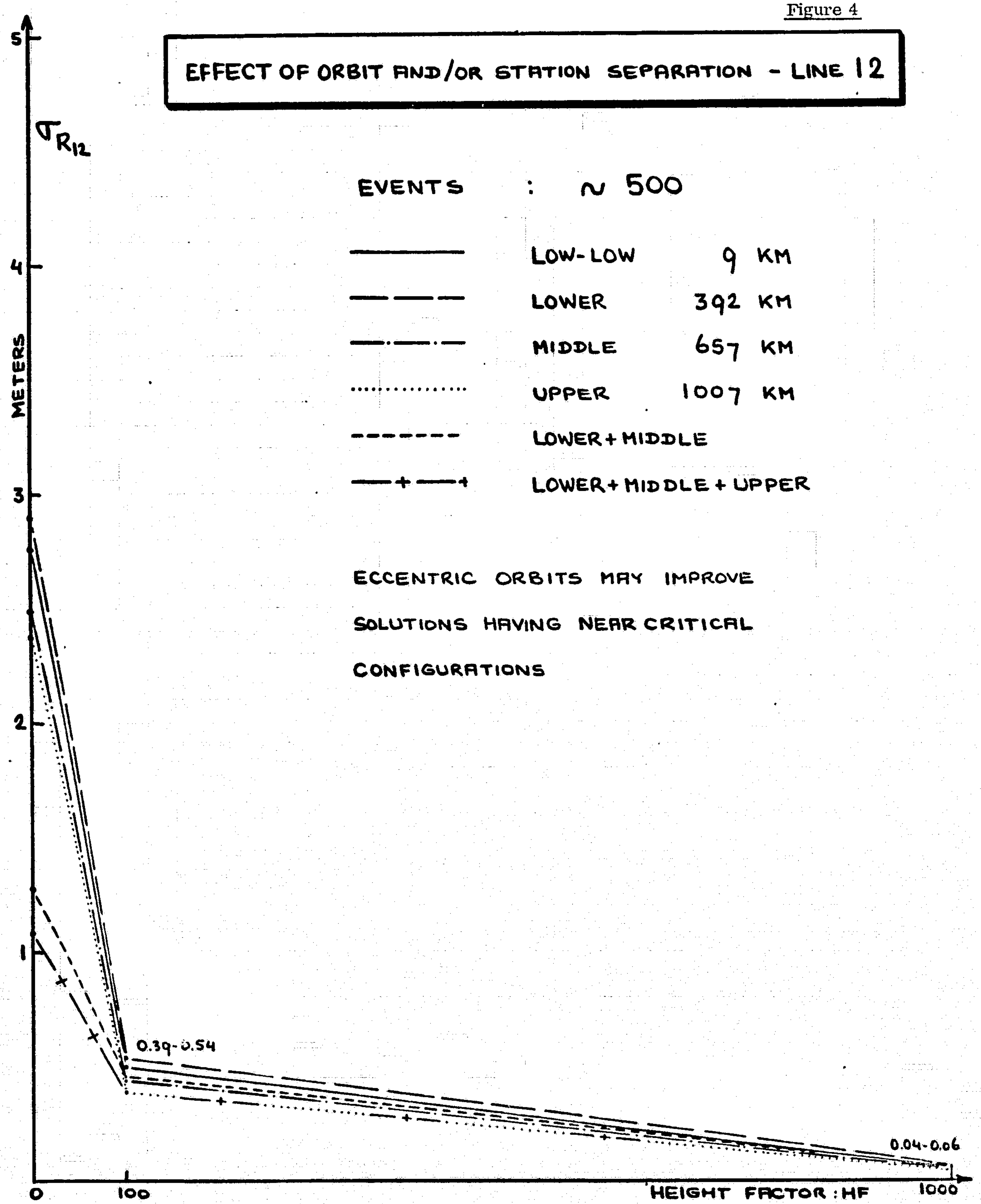
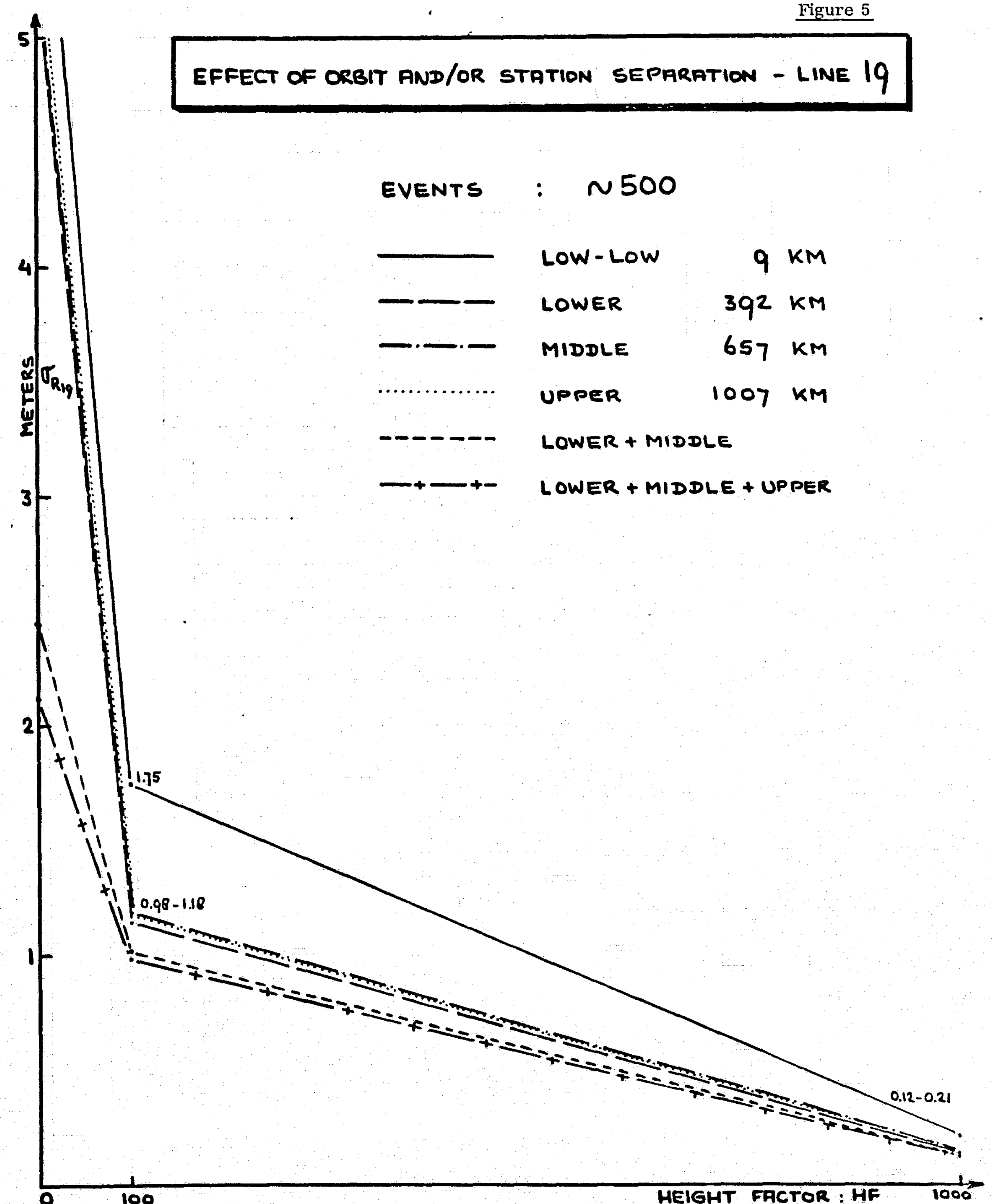


Figure 4



EFFECT OF ORBIT AND/OR STATION SEPARATION - LINE 12
$\sigma_{R_{12}}$
METERS
EVENTS : ~ 500
LOW-LOW 9 KM
LOWER 392 KM
MIDDLE 657 KM
UPPER 1007 KM
LOWER + MIDDLE
LOWER + MIDDLE + UPPER
ECCENTRIC ORBITS MAY IMPROVE SOLUTIONS HAVING NEAR CRITICAL CONFIGURATIONS
0.39-0.54
0.04-0.06
0
100
1000
1
2
3
4
5
HEIGHT FACTOR : HF

Figure 5



EFFECT OF ORBIT AND/OR STATION SEPARATION - LINE 19
EVENTS : ~500
LOW-LOW 9 KM
LOWER 392 KM
MIDDLE 657 KM
UPPER 1007 KM
LOWER + MIDDLE
LOWER + MIDDLE + UPPER
METERS
$\sigma_{R_{19}}$
5
4
3
2
1
1.75
0.98 - 1.18
0.12 - 0.21
0
100
1000
HEIGHT FACTOR : HF

Figure 6

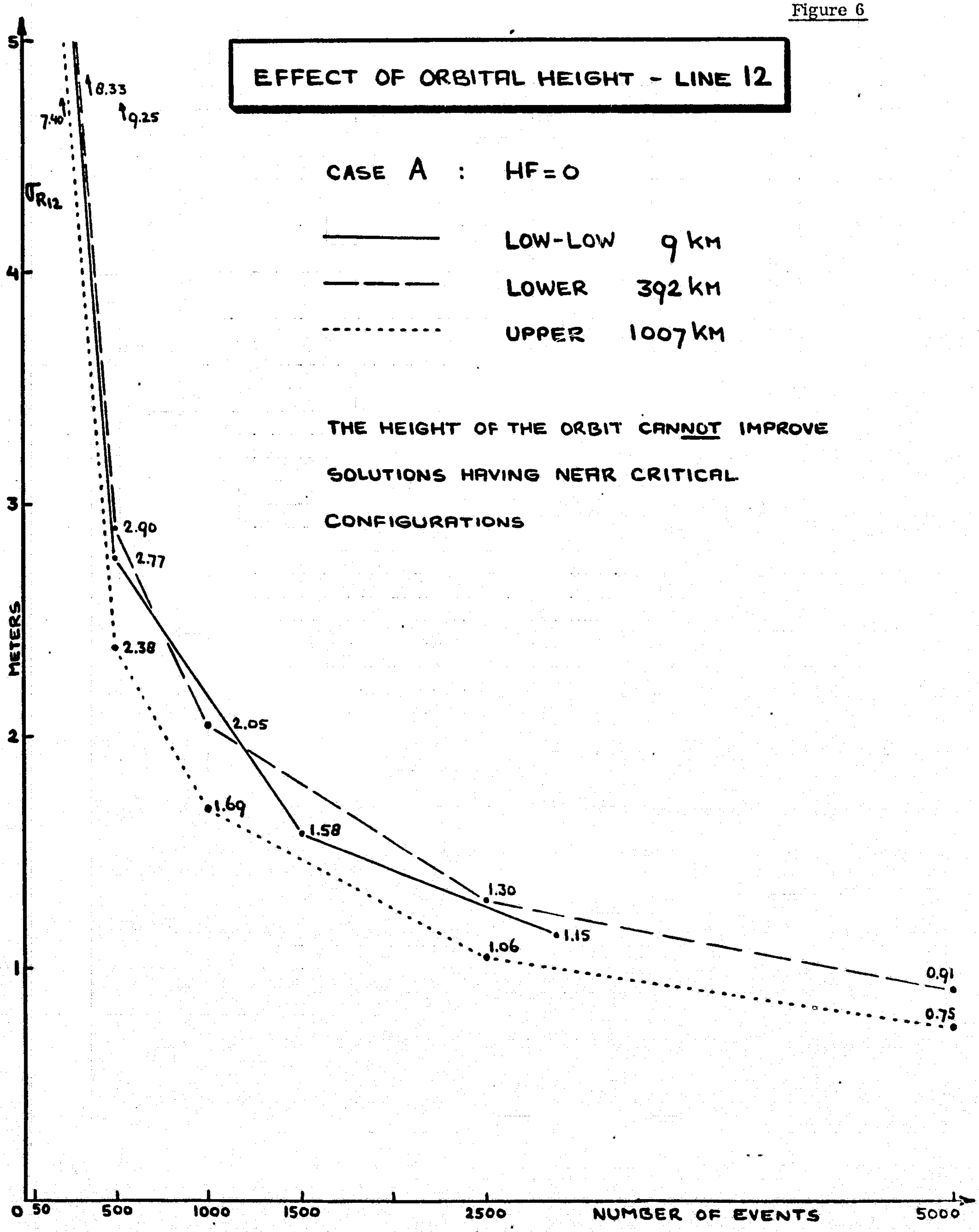

EFFECT OF ORBITAL HEIGHT - LINE 12
CASE A : HF = 0
LOW-LOW 9 KM
LOWER 392 KM
UPPER 1007 KM
THE HEIGHT OF THE ORBIT CANNOT IMPROVE SOLUTIONS HAVING NEAR CRITICAL CONFIGURATIONS
$\sigma_{R_{12}}$
METERS
NUMBER OF EVENTS
0
50
500
1000
1500
2500
5000
1
2
3
4
5
7.40
8.33
9.25
2.90
2.77
2.38
2.05
1.69
1.58
1.30
1.15
1.06
0.91
0.75

Figure 7

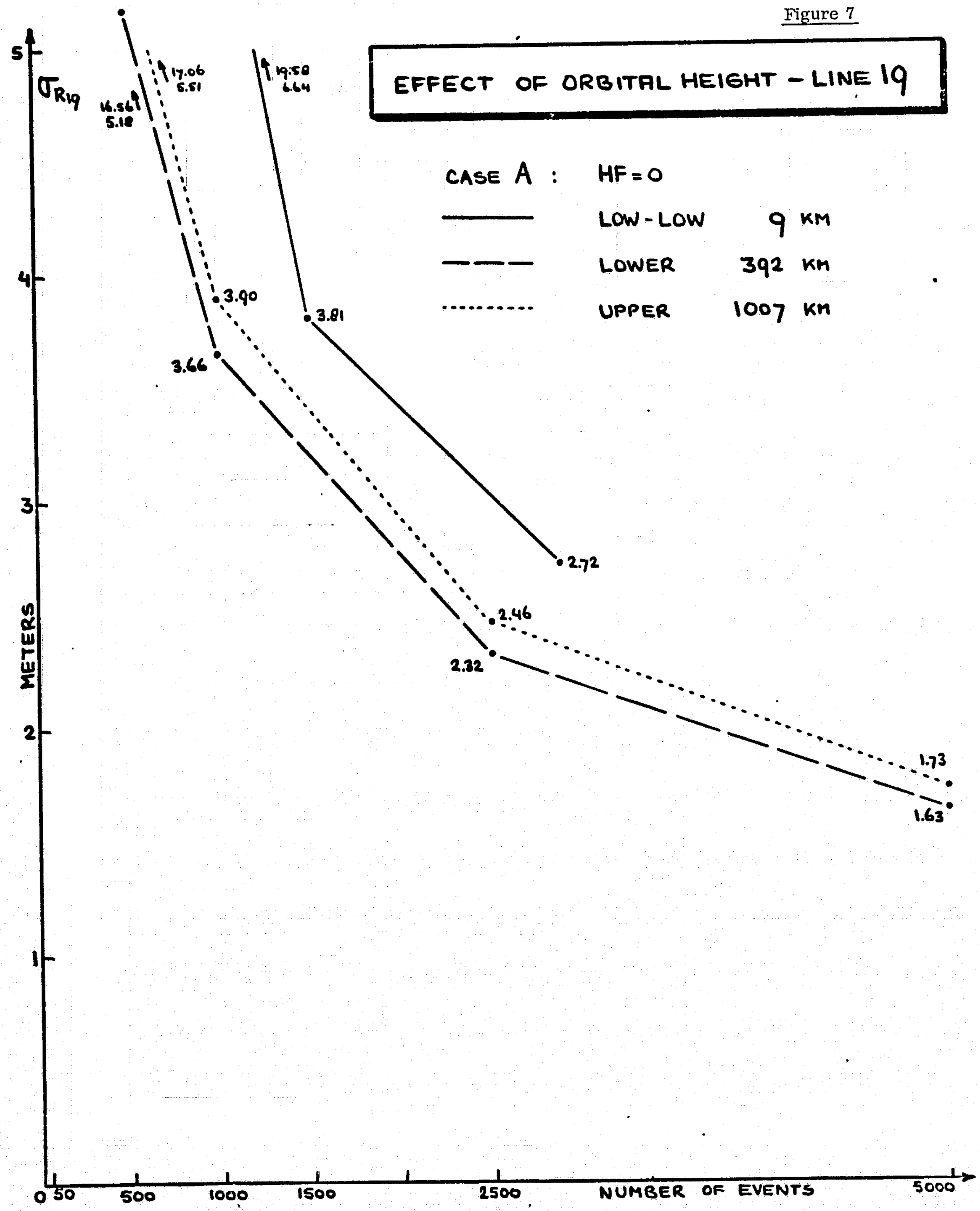
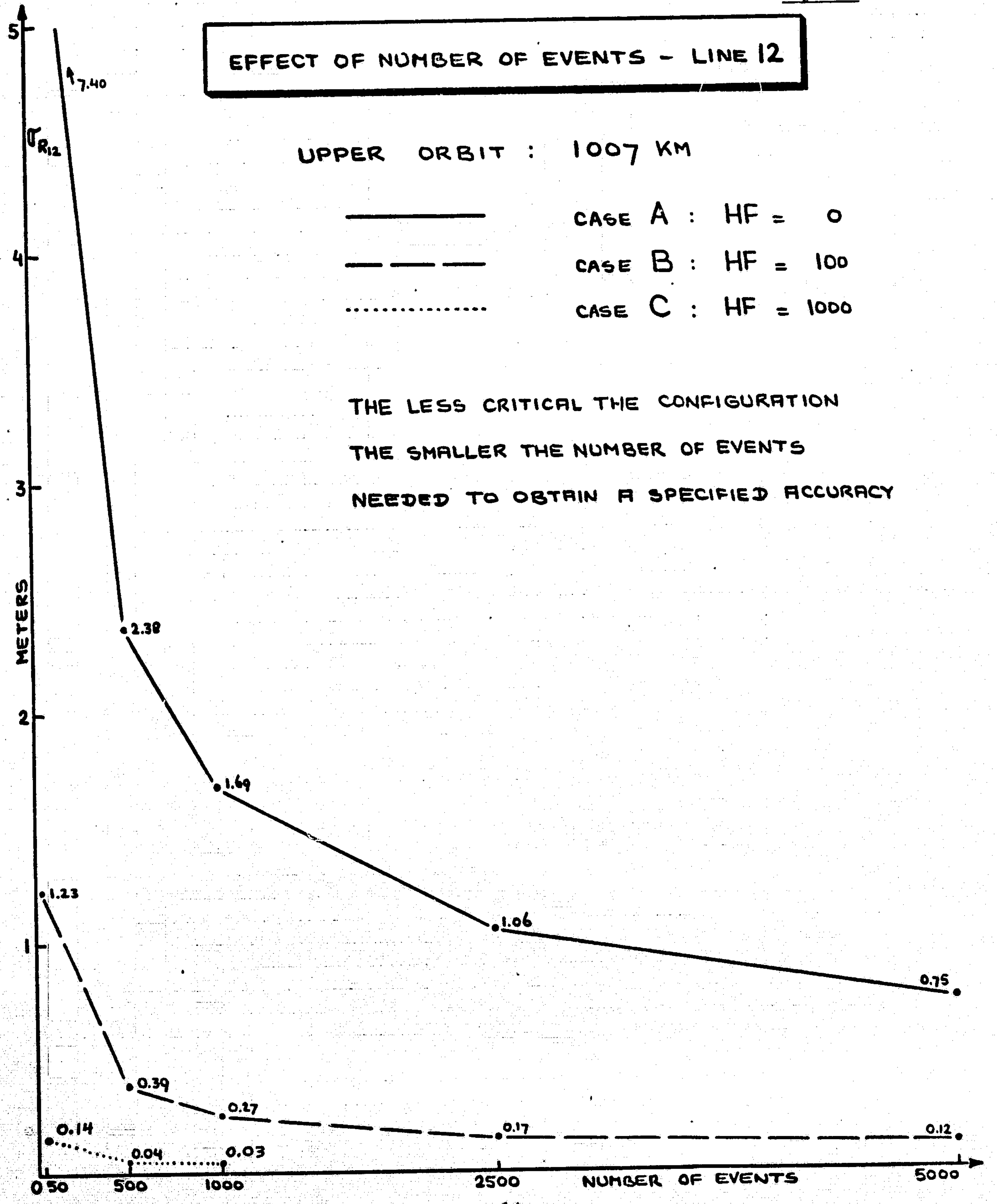
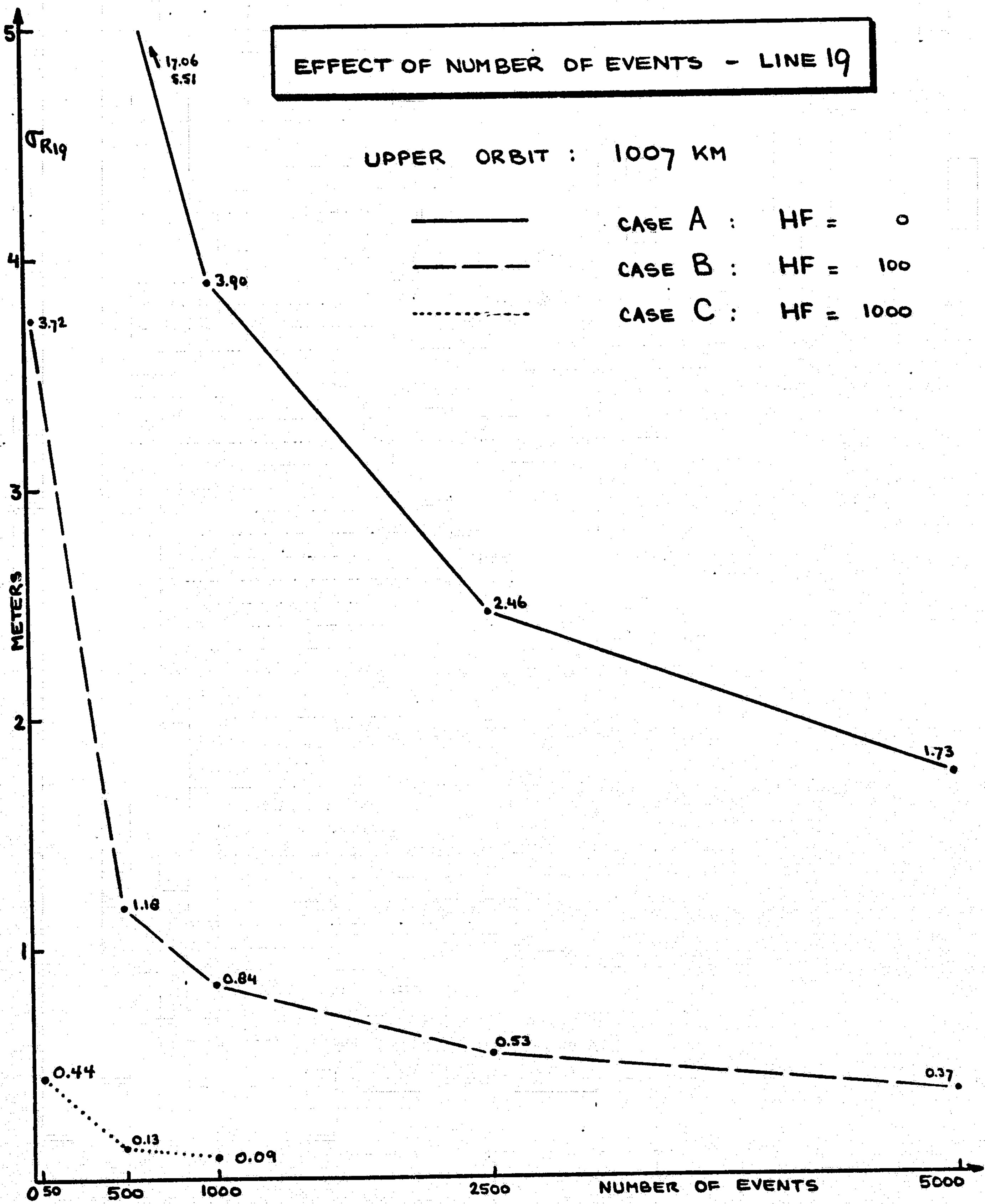

EFFECT OF ORBITAL HEIGHT - LINE 19
CASE A : HF = 0
LOW-LOW 9 KM
LOWER 392 KM
UPPER 1007 KM
σ_{R19}
METERS
NUMBER OF EVENTS
0
50
500
1000
1500
2500
5000
1
2
3
4
5
16.56
5.18
17.06
5.51
19.58
6.64
3.90
3.81
3.66
2.72
2.46
2.32
1.73
1.63

Figure 8



EFFECT OF NUMBER OF EVENTS - LINE 12
UPPER ORBIT : 1007 KM
CASE A : HF = 0
CASE B : HF = 100
CASE C : HF = 1000
THE LESS CRITICAL THE CONFIGURATION
THE SMALLER THE NUMBER OF EVENTS
NEEDED TO OBTAIN A SPECIFIED ACCURACY
$\sigma_{R_{12}}$
METERS
NUMBER OF EVENTS
5
4
3
2
1
0
50
500
1000
2500
5000
7.40
2.38
1.69
1.06
0.75
1.23
0.39
0.27
0.17
0.12
0.14
0.04
0.03

Figure 9



EFFECT OF NUMBER OF EVENTS - LINE 19
UPPER ORBIT : 1007 KM
CASE A : HF = 0
CASE B : HF = 100
CASE C : HF = 1000
σ_{R19}
METERS
NUMBER OF EVENTS
17.06
5.51
3.90
3.72
2.46
1.73
1.18
0.84
0.53
0.37
0.44
0.13
0.09
0
1
2
3
4
5
0
50
500
1000
2500
5000

Table 3.1-1

TAPE COGEOS IØ56

FILE	ORBIT	PASS	Δt sec	DATE	START PASS H	M	S	END PASS H	M	S	RECFM	LRECL	BLK SIZE
1	L	1	0.1	FEB1,1975	00	04	00.0	00	16	00.0	FB	1120	11200
	"	2	"	"	01	36	00.0	01	48	00.0	"	"	"
2	"	3	"	"	03	08	00.0	03	20	00.0	"	"	"
	"	4	"	"	11	16	00.0	11	29	00.0	"	"	"
3	"	5	"	"	12	48	00.0	13	01	30.0	VBS	2400	12004
	"	6	"	"	14	20	00.0	14	34	00.0	"	"	"
4	"	7	"	FEB2,1975	00	40	00.0	00	52	00.0	"	"	"
	"	8	"	"	02	12	00.0	02	24	00.0	"	"	"
5	"	9	"	FEB1,1975	15	52	00.0	16	06	00.0	"	"	"
	"	10	"	"	23	08	00.0	23	20	30.0	"	"	"
6	"	11	"	FEB2,1975	03	44	00.0	03	57	15.0	"	"	"
	"	12	"	"	11	52	00.0	12	06	30.0	"	"	"
7	"	13	"	"	13	24	00.0	13	38	45.0	"	"	"
	"	14	"	"	14	56	00.0	15	10	45.0	"	"	"
8	"	15	"	"	23	44	00.0	23	58	00.0	"	"	"
	"	16	"	FEB3,1975	01	16	00.0	01	30	00.0	"	"	"
9	"	17	"	"	02	48	00.0	03	02	30.0	"	"	"
	"	18	"	"	12	32	00.0	12	44	00.0	"	"	"
10	"	19	"	"	14	04	00.0	14	16	00.0	"	"	"
	"	20	"	"	15	36	00.0	15	49	00.0	"	"	"

TAPE CL6S29 J158

FILE	ORBIT	PASS	Δt sec	DATE	START PASS H	M	S	END PASS H	M	S	RECFM	LRECL	BLK SIZE
1	L	21	0.1	FEB4,1975	00	20	00.0	00	34	45.0	VBS	2400	12004
	"	22	"	"	01	52	00.0	02	07	00.0	"	"	"
2	"	23	"	"	03	28	00.0	03	40	00.0	"	"	"
	"	24	"	"	11	36	00.0	11	48	30.0	"	"	"
3	"	25	"	"	13	08	00.0	13	21	00.0	"	"	"
	"	26	"	"	14	40	00.0	14	53	00.0	"	"	"
4	"	27	"	"	23	28	00.0	23	40	00.0	"	"	"
	"	28	"	FEB5,1975	01	00	00.0	01	12	00.0	"	"	"
5	"	29	"	"	02	32	00.0	02	44	00.0	"	"	"
	"	30	"	"	12	12	00.0	12	25	00.0	"	"	"
6	"	31	"	"	13	44	00.0	13	57	00.0	"	"	"
	"	32	"	"	15	20	00.0	15	29	00.0	"	"	"
7	"	33	"	FEB6,1975	00	04	00.0	00	16	00.0	"	"	"
	"	34	"	"	01	36	00.0	01	48	00.0	"	"	"
	"	35	"	"	03	08	00.0	03	21	00.0	"	"	"

TAPE CSMAØ1 LØ52

FILE	ORBIT	PASS	Δt sec	DATE	START PASS H	START PASS M	START PASS S	END PASS H	END PASS M	END PASS S	RECFM	LRECL	BLK SIZE
1	M	1	0.1	FEB 1,1975	01	44	00.0	01	53	00.0	VBS	2400	12004
	"	2	"	"	03	20	00.0	03	30	45.0	"	"	"
2	"	3	"	"	13	36	00.0	13	45	30.0	"	"	"
	"	4	"	"	15	12	00.0	15	23	15.0	"	"	"
3	"	5	"	FEB 2,1975	00	32	00.0	00	42	00.0	"	"	"
	"	6	"	"	02	08	00.0	02	20	00.0	"	"	"
4	"	7	"	"	12	24	00.0	12	35	00.0	"	"	"
	"	8	"	"	14	00	00.0	14	12	45.0	"	"	"
5	"	9	"	FEB 3,1975	01	00	00.0	01	09	30.0	"	"	"
	"	10	"	"	02	36	00.0	02	47	15.0	"	"	"
6	"	11	"	"	12	52	00.0	13	02	00.0	"	"	"
	"	12	"	"	14	28	00.0	14	40	00.0	"	"	"
7	"	13	"	FEB 4,1975	01	24	00.0	01	36	30.0	"	"	"
	"	14	"	"	03	04	00.0	03	14	15.0	"	"	"
8	"	15	"	"	13	20	00.0	13	29	00.0	"	"	"
	"	16	"	"	14	56	00.0	15	07	30.0	"	"	"
9	"	17	"	FEB 5,1975	00	16	00.0	00	26	00.0	"	"	"
	"	18	"	"	01	52	00.0	02	04	00.0	"	"	"
10	"	19	"	"	12	08	00.0	12	18	30.0	"	"	"
	"	20	"	"	13	44	00.0	13	56	15.0	"	"	"
11	"	21	"	FEB 6,1975	00	44	00.0	00	53	00.0	"	"	"
	"	22	"	"	02	20	00.0	02	31	00.0	"	"	"

TAPE CSUAØ1 LØ67

FILE	ORBIT	PASS	Δt sec.	DATE	START PASS			END PASS			RECFM	LRECL	BLK SIZE
					H	M	S	H	M	S			
1	U	1	0.1	FEB1,1975	00	04	00.0	00	17	30.0	VBS	2400	12004
	"	2	"	"	01	48	00.0	02	03	30.0	"	"	"
2	"	3	"	"	03	32	00.0	03	48	30.0	"	"	"
	"	4	"	"	12	52	00.0	13	05	30.0	"	"	"
3	"	5	"	"	14	36	00.0	14	50	30.0	"	"	"
	"	6	"	FEB2,1975	00	36	00.0	00	50	45.0	"	"	"
4	"	7	"	"	02	20	00.0	02	36	00.0	"	"	"
	"	8	"	"	04	48	00.0	04	20	00.0	"	"	"
5	"	9	"	"	11	40	00.0	11	53	00.0	"	"	"
	"	10	"	"	13	24	00.0	13	38	00.0	"	"	"
6	"	11	"	"	15	08	00.0	15	23	30.0	"	"	"
	"	12	"	"	23	24	00.0	23	39	00.0	"	"	"
7	"	13	"	FEB3,1975	01	08	00.0	01	24	00.0	"	"	"
	"	14	"	"	02	52	00.0	03	09	00.0	"	"	"
8	"	15	"	"	12	12	00.0	12	26	00.0	"	"	"
	"	16	"	"	13	56	00.0	14	11	00.0	"	"	"
9	"	17	"	"	15	40	00.0	15	56	00.0	"	"	"
	"	18	"	"	23	56	00.0	00	11	30.0	"	"	"

TAPE CSUAØ2 L189

FILE	ORBIT	PASS	Δt sec.	DATE	START PASS			END PASS			RECFM	LRECL	BLK SIZE
					H	M	S	H	M	S			
1	U	19	0.1	FEB4,1975	01	40	00.0	01	56	30.0	VBS	2400	12004
	"	20	"	"	03	28	00.0	03	41	30.0	"	"	"
2	"	21	"	"	12	44	00.0	12	59	00.0	"	"	"
	"	22	"	"	14	28	00.0	14	44	00.0	"	"	"
3	"	23	"	FEB5,1975	00	28	00.0	00	44	00.0	"	"	"
	"	24	"	"	02	12	00.0	02	29	15.0	"	"	"
4	"	25	"	"	11	32	00.0	11	47	00.0	"	"	"
	"	26	"	"	13	16	00.0	13	31	45.0	"	"	"
5	"	27	"	"	15	00	00.0	15	17	00.0	"	"	"
	"	28	"	"	23	20	00.0	23	32	00.0	"	"	"
6	"	29	"	FEB6,1975	01	00	00.0	01	17	00.0	"	"	"
	"	30	"	"	02	48	00.0	03	02	45.0	"	"	"

Table 3.1-4

Long Arcs

Satellite Height (km)	Orbit Type	Time of Data Generation (Hrs)	Computer Expenses $
150	Polar	126	100.00
392	-do-	126	120.00
657	-do-	126	130.00
1007	-do-	126	150.00
			500.00

Table 3.1-5

Short Arcs

Satellite Height (km)	No. of Passes	Length of each Pass in Time	Density of Satellite points	Computer Expenses $
9	30	10 sec	10/sec	300.00
392	35	8 min	10/sec	1050.00
657	22	10 min	10/sec	770.00
1007	30	12 min	10/sec	1200.00
				3320.00

Table 3.2-1

Satellite Height (km)	No. of Passes	Case Type	Maximum Data Points Generated	Computer Expenditure $
9	30	A	3000	80.00
		B	1500	60.00
		C	1500	60.00
392	35	A	5000	180.00
		B	500	30.00
		C	500	30.00
657	22	A	500	30.00
		B	500	30.00
		C	500	30.00
1007	30	A	5000	180.00
		B	5000	180.00
		C	500	30.00
				$920.00

Table 3.2-2

TAPE RNGEØ1 MØ55

FILE	DATA	S.D. cm	ORBIT	PASSES	EVENTS	HF	Δt sec.	CASE	FROM: TAPE(S)	FILE	PASSES	EVENTS	Δt, sec.	CASE	FROM
1	RANGES	10	L	11	558	0	5	L005	CSTPØ1	1	6	302	5	L005^a	COGEOS, 10F, 20P
									CSTPØ1	2	5	256	5	L005^b	CLGS29, 7F, 15P
2	"	10	M	14	1090	0	5	M001	CSMAØ1	1-11	22	–	0.1	–	–
3	"	10	U	11	2029	0	5	U001^a	CSUAØ1	1-9	18	–	0.1	–	–
4	"	10	U	9		0	5	U001^b	CSUAØ2	1-6	12	–	0.1	–	–
5	"	10	M	14	546	0	10	M006	CSMAØ1	1-11	22	–	0.1	–	–
6	"	10	U	11	507	0	20	U006^a	CSUAØ1	1-9	18	–	0.1	–	–
7	"	10	U	9		0	20	U006^b	CSUAØ2	1-6	12	–	0.1	–	–
8	"	10	L	6	281	0	10	L011^a	COGEOS	1-10	20	–	0.1	–	–
9	"	10	L	5		0	10	L011^b	CLGS29	1-7	15	–	0.1	–	–
10	"	10	M	14	274	0	20	M011	CSMAØ1	1-11	22	–	0.1	–	–
11	"	10	L	6	186	0	15	L008^a	COGEOS	1-10	20	–	0.1	–	–
12	"	10	L	5		0	15	L008^b	CLGS29	1-7	15	–	0.1	–	–
13	"	10	M	14	181	0	30	M008	CSMAØ1	1-11	22	–	0.1	–	–
14	"	10	U	11	170	0	60	U008^a	CSUAØ1	1-9	18	–	0.1	–	–
15	"	10	U	9		0	60	U008^b	CSUAØ2	1-6	12	–	0.1	–	–
16	"	10	U	11	46	1000	240	U015^a	CSTPØ1	3	11	507	20	U004^a	CSUAØ1, 9F, 18P
17	"	10	U	9		1000	240	U015^b	CSTPØ1	4	9		20	U004^b	CSUAØ2, 6F, 12P
18	"	10	M	14	45	1000	120	M015	CSTPØ1	5	14	546	10	M004	CSMAØ1, 11F, 22P
19	"	10	L	6	42	1000	60	L015^a	CSTPØ1	6	6	558	5	L007^a	COGEOS, 10F, 20P
20	"	10	L	5		1000	60	L015^b	CSTPØ1	7	5		5	L007^b	CLGS29, 7F, 20P
21	"	10	U	11	5075	0	2	U025^a	CSUAØ1	1-9	18	–	0.1	–	–
22	"	10	U	9		0	2	U025^b	CSUAØ1	1-6	12	–	0.1	–	–
23	"	10	L	6	5572	0	.5	L016^a	COGEOS	1-10	20	–	0.1	–	–
24	"	10	L	5		0	.5	L016^b	CLGS29	1-7	15	–	0.1	–	–
25	"	10	LL	14	479	0	.15	LL004	CSTPØ1	26	14	1435	.05	LL001	CLLAØ1, 15F, 15P
26	"	10	LL	14	473	100	.15	LL005	CSTPØ1	27	14	1419	.05	LL002	CLLAØ1, 15F, 15P
27	"	10	LL	13	421	1000	.15	LL006	CSTPØ1	28	13	1261	.05	LL003	CLLAØ1, 15F, 15P
28	"	10	LL	14	54	0	1.35	LL007	CSTPØ1	26	14	1435	.05	LL001	CLLAØ1, 15F, 15P
29	"	10	LL	14	53	100	1.35	LL008	CSTPØ1	27	14	1419	.05	LL002	CLLAØ1, 15F, 15P
30	"	10	LL	13	47	1000	1.35	LL009	CSTPØ1	28	13	1261	.05	LL003	CLLAØ1, 15F, 15P

Table 3.2-3

TAPE RNGEØ2 N281

FILE	DATA	S.D. cm	ORBIT	PASSES	EVENTS	HF	Δt Sec.	CASE	FROM: TAPE(S)	FROM: FILE	FROM: PASSES	FROM: EVENTS	FROM: Δt, sec	FROM: CASE	FROM: FROM
1	RANGES	10	L	6	558	100	5	L006^a	COGEOS	1-10	20	–	0.1	–	–
2	"	10	L	5		100	5	L006^b	CLGS29	1-7	15	–	0.1	–	–
3	"	10	M	14	546	100	10	M005	CSMAØ1	1-11	22	–	0.1	–	–
4	"	10	U	11	507	100	20	U005^a	CSUAØ1	1-9	18	–	0.1	–	–
5	"	10	U	9		100	20	U005^b	CSUAØ2	1-6	12	–	0.1	–	–
6	"	10	L	6	281	100	10	L012^a	COGEOS	1-10	20	–	0.1	–	–
7	"	10	L	5		100	10	L012^b	CLGS29	1-7	15	–	0.1	–	–
8	"	10	M	14	274	100	20	M012	CSMAØ1	1-11	22	–	0.1	–	–
9	"	10	L	6	186	100	15	L009^a	COGEOS	1-10	20	–	0.1	–	–
10	"	10	L	5		100	15	L009^b	CLGS29	1-7	15	–	0.1	–	–
11	"	10	M	14	181	100	30	M009	CSMAØ1	1-11	22	–	0.1	–	–
12	"	10	U	11	170	100	60	U009^a	CSUAØ1	1-9	18	–	0.1	–	–
13	"	10	U	9		100	60	U009^b	CSUAØ2	1-6	12	–	0.1	–	–
14	"	10	U	11	5075	100	2	U002^a	CSUAØ1	1-9	18	–	0.1	–	–
15	"	10	U	9		100	2	U002^b	CSUAØ2	1-6	12	–	0.1	–	–
16	"	10	U	11	2538	0	4	U026^a	RNGEØ1	21	11	5075	2	U025^a	CSUAØ1, 9F, 18P
17	"	10	U	9		0	4	U026^b	RNGEØ1	22	9		2	U025^b	CSUAØ2, 6F, 12P
18	"	10	U	11	1016	0	10	U027^a	RNGEØ1	21	11	5075	2	U025^a	CSUAØ1, 9F, 18P
19	"	10	U	9		0	10	U027^b	RNGEØ1	22	9		2	U025^b	CSUAØ2, 6F, 12P
20	"	10	U	11	509	0	20	U028^a	RNGEØ1	21	11	5075	2	U025^a	CSUAØ1, 9F, 18P
21	"	10	U	9		0	20	U028^b	RNGEØ1	22	9		2	U025^b	CSUAØ2, 6F, 12P
22	"	10	U	11	52	0	200	U029^a	RNGEØ1	21	11	5075	2	U025^a	CSUAØ1, 9F, 18P
23	"	10	U	9		0	200	U029^b	RNGEØ1	22	9		2	U025^b	CSUAØ2, 6F, 12P
24	"	10	L	6	2786	0	1	L017^a	RNGEØ1	23	6	5572	0.5	L016^a	COGEOS, 10F, 20P
25	"	10	L	5		0	1	L017^b	RNGEØ1	24	5		0.5	L016^b	CLGS29, 7F, 15P
26	"	10	L	6	1115	0	2.5	L018^a	RNGEØ1	23	6	5572	0.5	L016^a	COGEOS, 10F, 20P
27	"	10	L	5		0	2.5	L018^b	RNGEØ1	24	5		0.5	L016^b	CLGS29, 7F, 15P
28	"	10	L	6	558	0	5	L019^a	RNGEØ1	23	6	5572	0.5	L016^a	COGEOS, 10F, 20P
29	"	10	L	5		0	5	L019^b	RNGEØ1	24	5		0.5	L016^b	CLGS29, 7F, 15P
30	"	10	L	6	57	0	50	L020^a	RNGEØ1	23	6	5572	0.5	L016^a	COGEOS, 10F, 20P
31	"	10	L	5		0	50	L020^b	RNGEØ1	24	5		0.5	L016^b	CLGS29, 7F, 15P

Table 3.2-4

TAPE CSTP∅1 C∅54

FILE	DATA	S.D. cm	ORBIT	PASSES	EVENTS	HF	Δt sec.	CASE	FROM TAPE(S)	FROM FILE	FROM PASSES	FROM EVENTS	FROM Δt, sec	FROM CASE	FROM FROM
1	RANGES	10	U	11	2029	1000	5	U003^{a}	CSUA∅1	1-9	18	–	0.1	–	–
2	"	10	U	9		1000	5	U003^{b}	CSUA∅2	1-6	12	–	0.1	–	–
3	"	10	U	11	507	1000	20	U004^{a}	CSUA∅1	1-9	18	–	0.1	–	–
4	"	10	U	9		1000	20	U004^{b}	CSUA∅2	1-6	12	–	0.1	–	–
5	"	10	M	14	546	1000	10	M004	CSMA∅1	1-11	22	–	0.1	–	–
6	"	10	L	6	558	1000	5	L007^{a}	COGEOS	1-10	20	–	0.1	–	–
7	"	10	L	5		1000	5	L007^{b}	CLGS29	1-7	15	–	0.1	–	–
8	"	10	L	6	281	1000	10	L013^{a}	COGEOS	1-10	20	–	0.1	–	–
9	"	10	L	5		1000	10	L013^{b}	CLGS29	1-7	15	–	0.1	–	–
10	"	10	M	14	274	1000	20	M013	CSMA∅1	1-11	22	–	0.1	–	–
11	"	0	L	6	558	1000	5	L014^{a}	COGEOS	1-10	20	–	0.1	–	–
12	"	0	L	5		1000	5	L014^{b}	CLGS29	1-7	15	–	0.1	–	–
13	"	10	L	6	186	1000	15	L010^{a}	COGEOS	1-10	20	–	0.1	–	–
14	"	10	L	5		1000	15	L010^{b}	CLGS29	1-7	15	–	0.1	–	–
15	"	10	M	14	181	1000	30	M010	CSMA∅1	1-11	22	–	0.1	–	–
16	"	10	U	11	170	1000	60	U010^{a}	CSUA∅1	1-9	18	–	0.1	–	–
17	"	10	U	9		1000	60	U010^{b}	CSUA∅2	1-6	12	–	0.1	–	–
18	"	10	U	11	509	100	20	U021^{a}	RNGE∅2	14	11	5075	2	U002^{a}	CSUA∅1, 9F, 18P
19	"	10	U	9		100	20	U021^{b}	RNGE∅2	15	9		2	U002^{b}	CSUA∅2, 6F, 12P
20	"	10	U	11	52	100	200	U022^{a}	RNGE∅2	14	11	5075	2	U002^{a}	CSUA∅1, 9F, 18P
21	"	10	U	9		100	200	U022^{b}	RNGE∅2	15	9		2	U002^{b}	CSUA∅2, 6F, 12P
22	"	10	U	11	2538	100	4	U023^{a}	RNGE∅2	14	11	5075	2	U002^{a}	CSUA∅1, 9F, 18P
23	"	10	U	9		100	4	U023^{b}	RNGE∅2	15	9		2	U002^{b}	CSUA∅2, 6F, 12P
24	"	10	U	11	1016	100	10	U024^{a}	RNGE∅2	14	11	5075	2	U002^{a}	CSUA∅1, 9F, 18P
25	"	10	U	9		100	10	U024^{b}	RNGE∅2	15	9		2	U002^{b}	CSUA∅2, 6F, 12P
26	"	10	LL	14	1435	0	.05	LL001	CLLA∅1	17-31	15	–	.05	–	–
27	"	10	LL	14	1419	100	.05	LL002	CLLA∅1	17-31	15	–	.05	–	–
28	"	10	LL	13	1261	1000	.05	LL003	CLLA∅1	17-31	15	–	.05	–	–
29	"	10	LL	27	2853	0	.05	LL010	CLLA∅1	17-45	29	–	.05	–	–

Table 4-1

Satellite Orbits Varied

Case Type	No. /Type of orbit used	No. of events per orbit	Total Events	No. of Simulated Solutions	Computer Expenses $
A, B and C	1(low-low)	500	500	3	18 x 50.00 = $900.00
	1(lower)	500	500	3	
	1(middle)	500	500	3	
	1(upper)	500	500	3	
	2(lower + middle)	250	500	3	
	3(lower+ middle+ upper)	133	500	3	

Table 4-2

No. of Events Varied

Case Type	Type of Orbit Used	No. of events included in the Simulated Solution							Total* number of Solutions	Computer Expenses $
	low-low	50	500	-	1500	-	3000	-	3	120.00
	lower	50	500	1000	-	2500	-	5000	4	400.00
A	middle	50	500	1000	-	2500	-	5000	4	400.00
	upper	50	500	1000	-	2500	-	5000	4	400.00
B & C	low-low	50	500	-	1500	-	-	-	4	80.00
										$1400.00

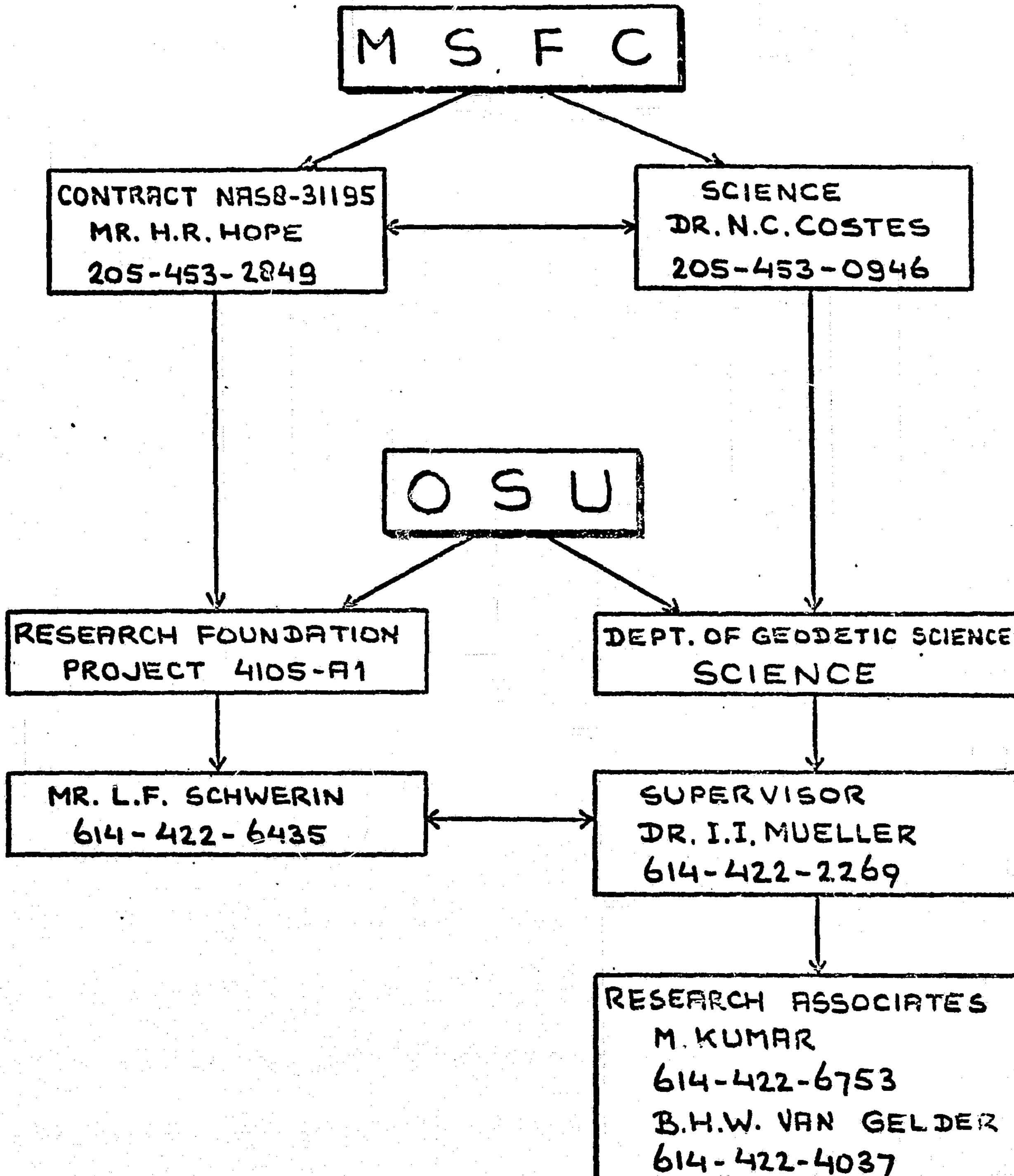
*Solution for 500 events excluded as already counted in previous Table.

STANDARD DEVIATIONS (M) OF R_{12} AND R_{19}

	CASE ↓ / EVENTS →	50		500		1500		3000	
		$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$
LOW-LOW 9 KM	A	8.33	19.58	2.77	6.64	1.58	3.81	1.15	2.72
	B	1.55	5.30	0.50	1.75	0.28	1.00		
	C	0.18	0.59	0.06	0.21	0.03	0.12		

	CASE	50		500		1000		2500		5000	
		$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$	$\sigma_{R_{12}}$	$\sigma_{R_{19}}$
LOWER 392 KM	A	9.25	16.56	2.90	5.18	2.05	3.66	1.30	2.32	0.91	1.63
	B	1.77	3.75	0.54	1.14	0.38	0.81	0.24	0.51	0.17	0.36
	C	0.20	0.41	0.06	0.13	0.04	0.09				
MIDDLE 657 KM	A			2.49	5.17						
	B	1.42	3.93	0.43	1.18	0.30	0.82	0.19	0.52	0.13	0.37
	C	0.16	0.42	0.05	0.13	0.03	0.09				
UPPER 1007 KM	A	7.40	17.06	2.38	5.51	1.69	3.90	1.06	2.46	0.75	1.73
	B	1.23	3.72	0.39	1.18	0.27	0.84	0.17	0.53	0.12	0.37
	C	0.14	0.44	0.04	0.13	0.03	0.09				
LOWER + MIDDLE	A			1.28	2.44						
	B			0.44	1.02						
	C			0.05	0.12						
LOWER + MIDDLE + UPPER	A			1.09	2.12						
	B			0.40	0.98						
	C			0.05	0.12						

APPENDIX A



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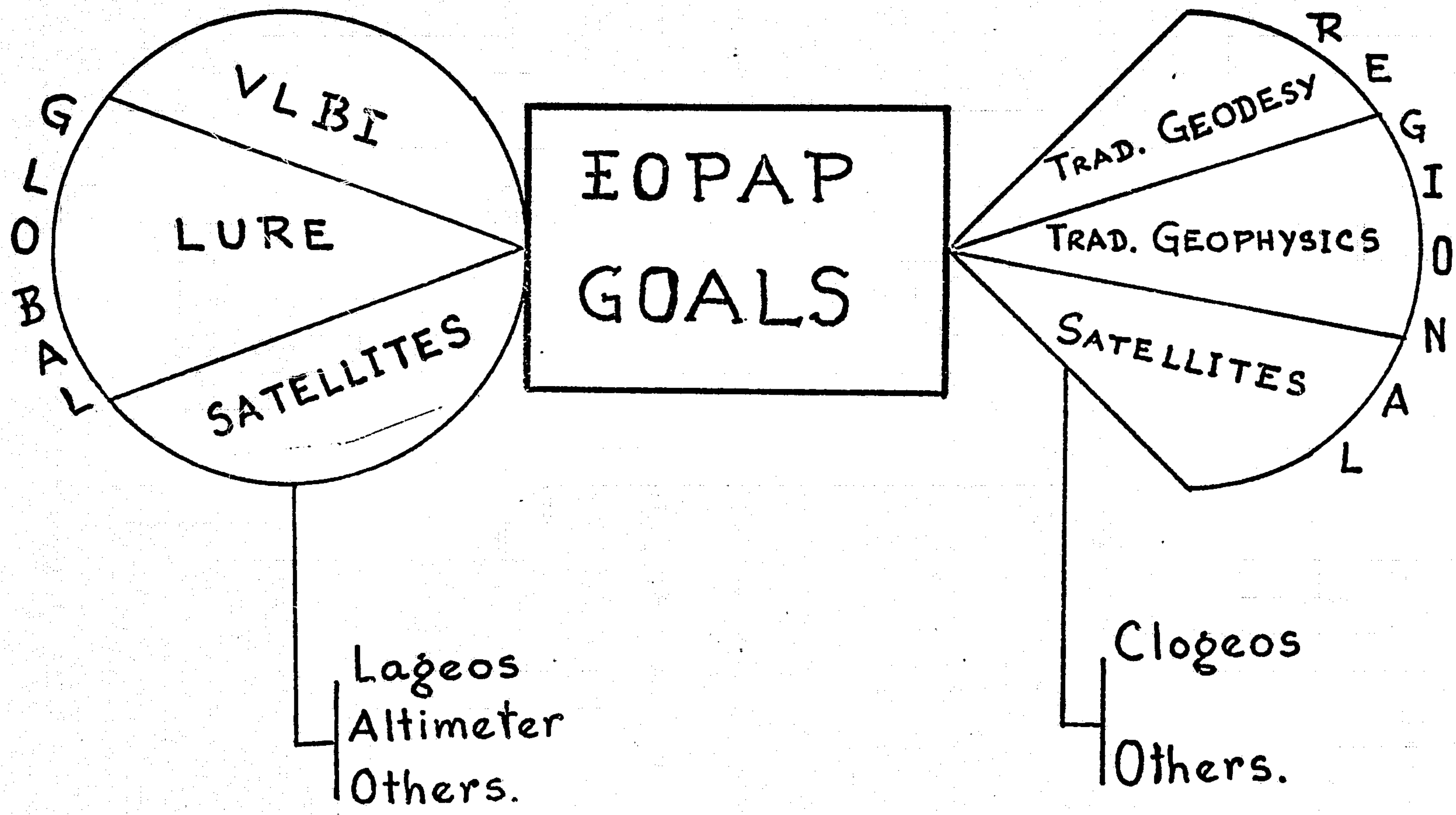
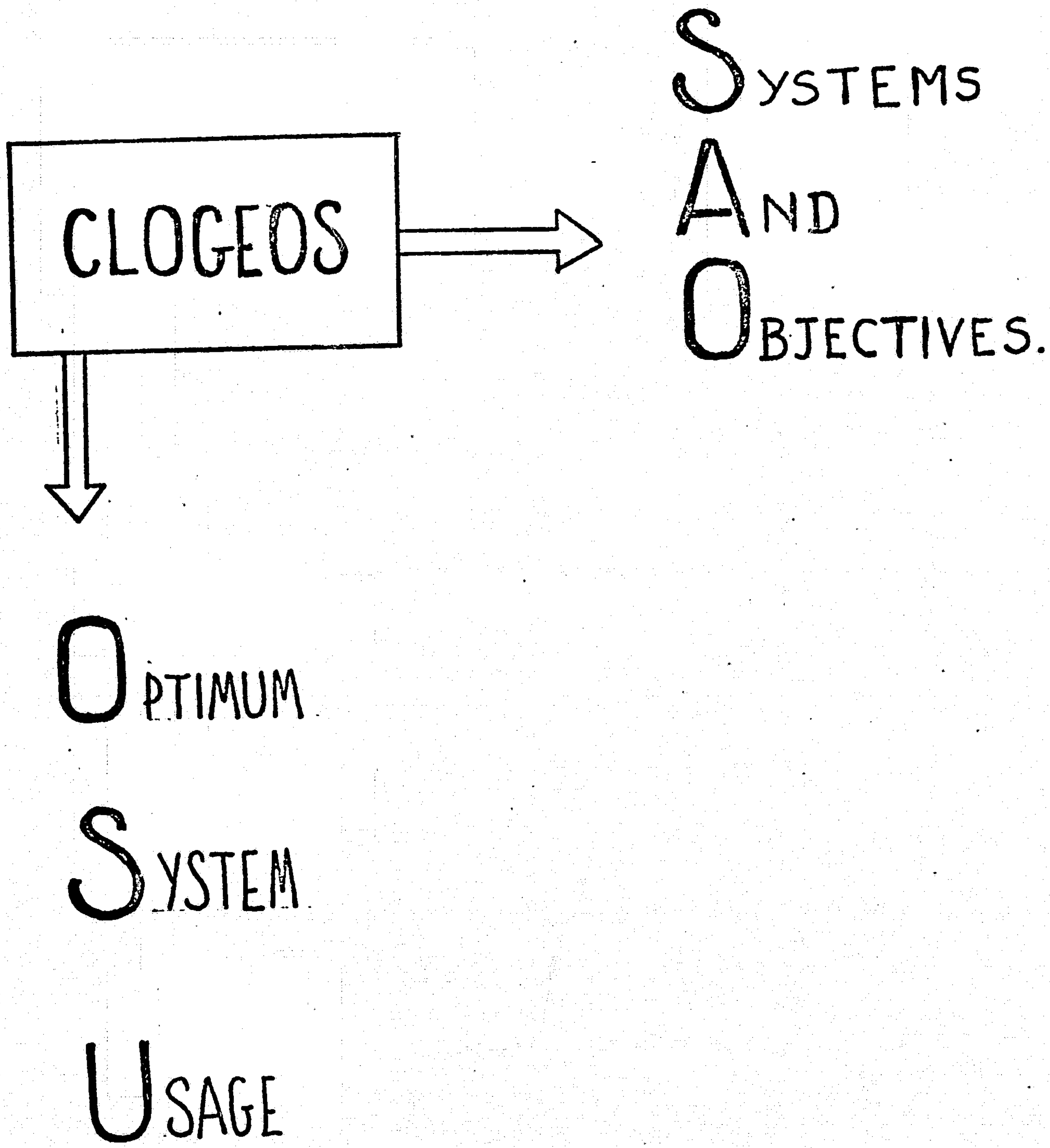
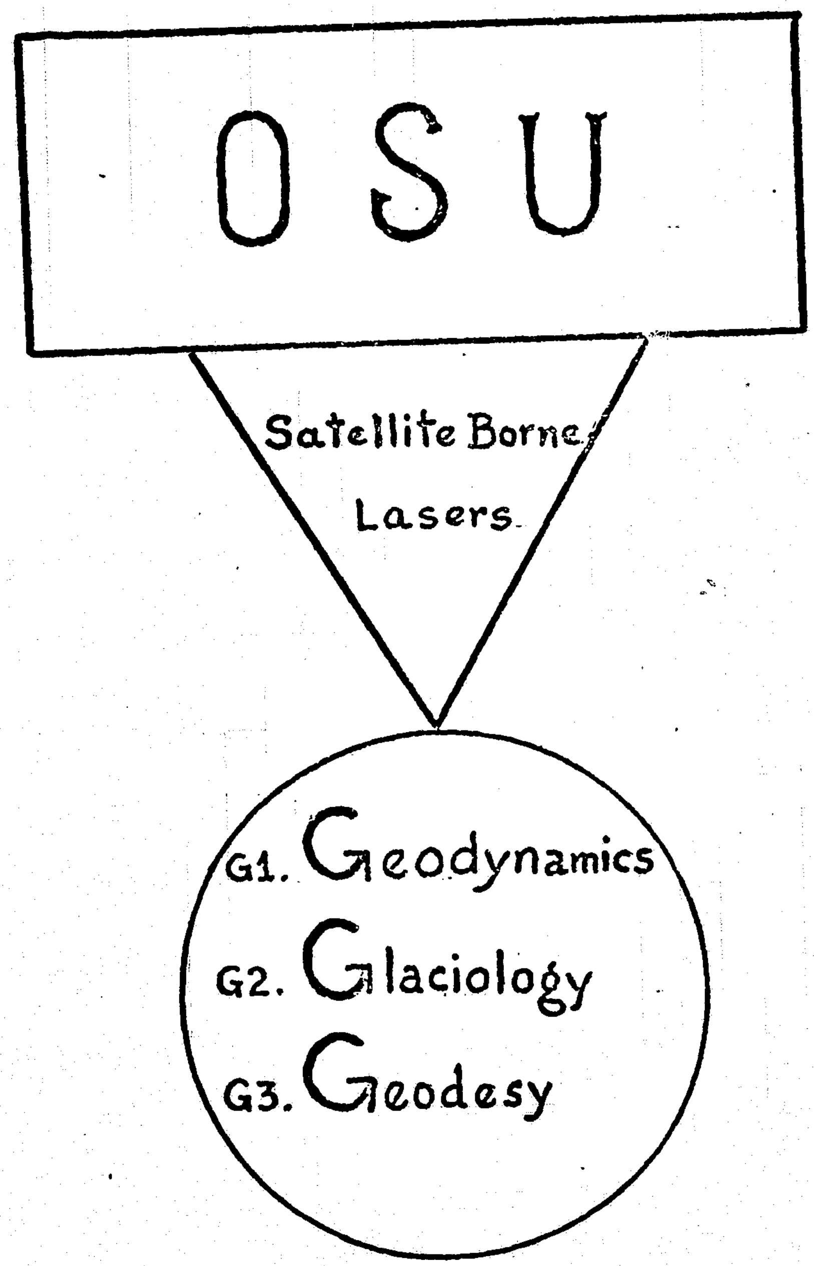
DEPARTMENT OF GEODETIC SCIENCE


GLOBAL
VLBI
LURE
SATELLITES
EOPAP
GOALS
REGIONAL
TRAD. GEODESY
TRAD. GEOPHYSICS
SATELLITES
Lageos
Altimeter
Others.
Clogeos
Others.

Appendix A – 2


CLOGEOS
Systems
And
Objectives.
Optimum
System
Usage


OSU
Satellite Borne Lasers
G1. Geodynamics
G2. Glaciology
G3. Geodesy

G1

Geodynamics

(i) Patterns of Strain Accumulation Through Monitoring Fault Motions

(ii) Accurate Delineation of Plates

(iii) Relative Movements Within Plate Interiors

G2

Glaciological & Related Usages

(i) Monitoring of Ice-cap Condition

(ii) Ice-floe Monitoring

(iii) Iceberg Monitoring

(iv) Glacier Monitoring

G3

Geodesy — Marine

(i) Ocean Bottom Control

Improved Navigation

Offshore Positioning

Mapping of Ocean Floor

Sea Slope Data

(ii) Buoy Monitoring

Positioning

Tsunami Warning

Current Monitoring

Open Sea Tidal Data

CLOGEOS

BASICALLY A SYSTEM OF

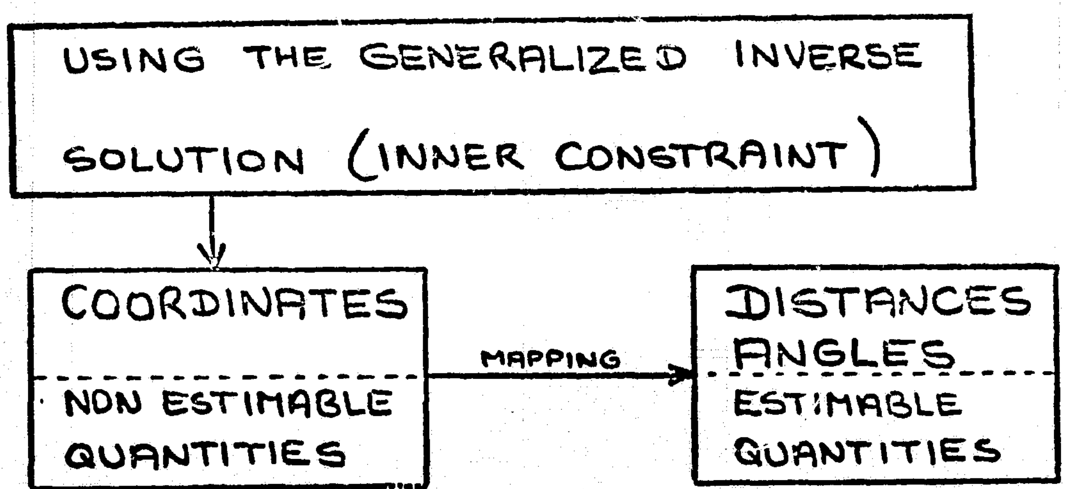
APPLICATIONS	
G_1 G_2 G_3	DETERMINATION OF (RELATIVE) POSITIONS (MOVEMENTS) OF POINTS AT THE EARTH'S SURFACE (IN LOCALIZED AREAS) AT A PRACTICAL COST

THE OHIO STATE UNIVERSITY
DEPARTMENT OF GEODETIC SCIENCE

A. RELATIVE POSITION OF POINTS

LACKS/DOES NOT NEED COORDINATE SYSTEM INFORMATION EXCEPT FOR A COORDINATE SYSTEM DETERMINED BY THOSE POINTS I.E. LOCAL COORDINATE SYSTEM

SYSTEM IS RANK DEFICIENT BY 7 (6)



B. POINTS IN A LOCALIZED AREA

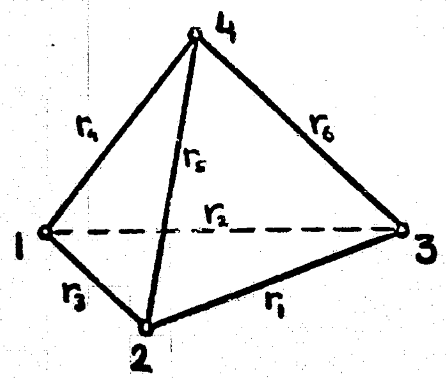
POINTS FORM A

2 DIM. SPACE IN 3 DIM. SPACE

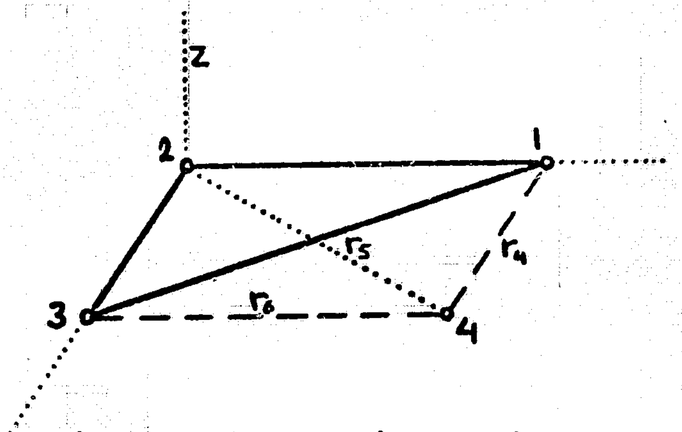
INSTABLE GEOMETRY

EXAMPLE

4 POINTS ARE DETERMINED IN RELATIVE POSITION BY 6 [12-(7-1)] QUANTITIES



NOW POINT 4 IN PLANE SPANNED BY 1,2 AND 3



$$dz_4 = \cdots dr_5 + \cdots dr_4 + \cdots dr_6 + \cdots$$

COEFFICIENT BECOMES ∞ WHEN 4 APPROACHES PLANE 1,2,3

HORIZONTAL POSITIONING PROBABLY MORE FEASIBLE THAN VERTICAL POSITIONING

DUE TO THIS CRITICAL GEOMETRY THE FOLLOWING MEASUREMENT SYSTEM WILL BE INVESTIGATED INITIALLY:

6 TERRESTRIAL POINTS OBSERVED SIMULTANEOUSLY FROM AT LEAST 4 DIFFERENT SATELLITE POSITIONS

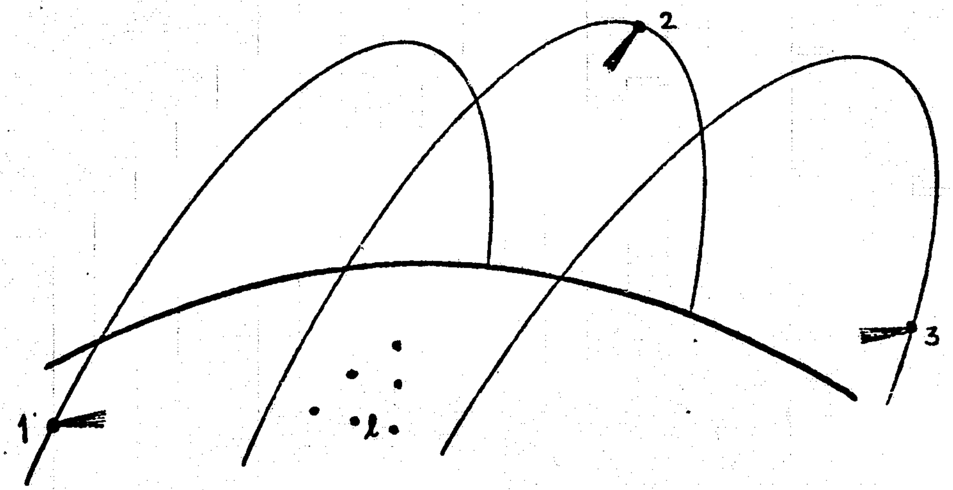
REF: OSU, JPL

C. FOR A LIMITED NETWORK

C1. GEOMETRIC MODE

MOST FAVORABLE CASE

QUASI SIMULTANEOUS MODE IN TRANSLOCATION



1,2,3,4,...,n

SYSTEM : RANGE
RANGE RATE
RANGE DIFF.

ORBIT	a, i
OBSERVATIONS	n
GROUND STATION CONFIGURATION	ℓ
ACCURACY OF OBSERVATIONS	$\sigma_r, \sigma_{\dot{r}}$

$\rightarrow \boxed{\sum_6}$ VAR/COVAR MATRIX WHICH DESCRIBES RELATIVE LOCATIONS OF 6 POINTS

IF CASE C1 LOOKS PROMISING LESS FAVORABLE CASES WILL BE INVESTIGATED (C2)

C2. SHORT ARC MODE

LESS FAVORABLE CASE

ORBITAL UNCERTAINTIES CAUSED BY

— GRAVITATIONAL FIELD OF THE EARTH

— ATMOSPHERIC DRAG

— SOLAR RADIATION PRESSURE

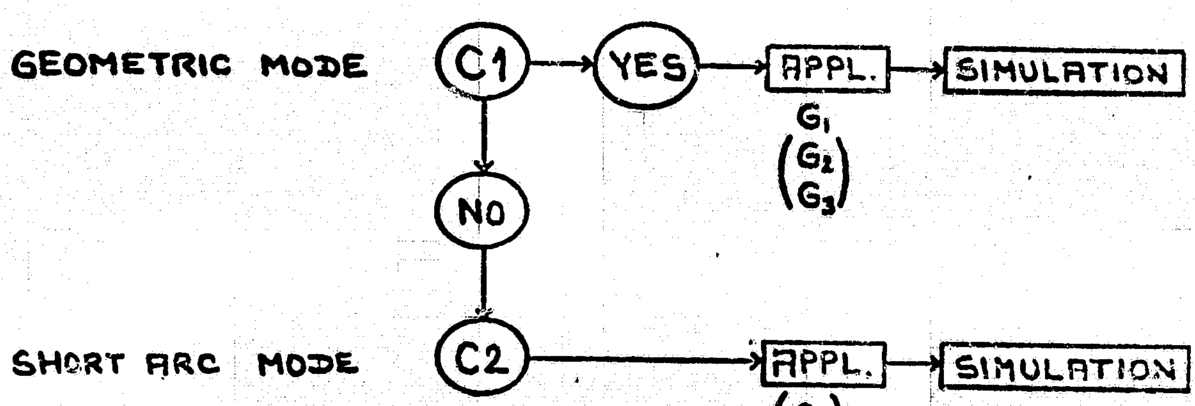
—

D. TRADE OFF BETWEEN

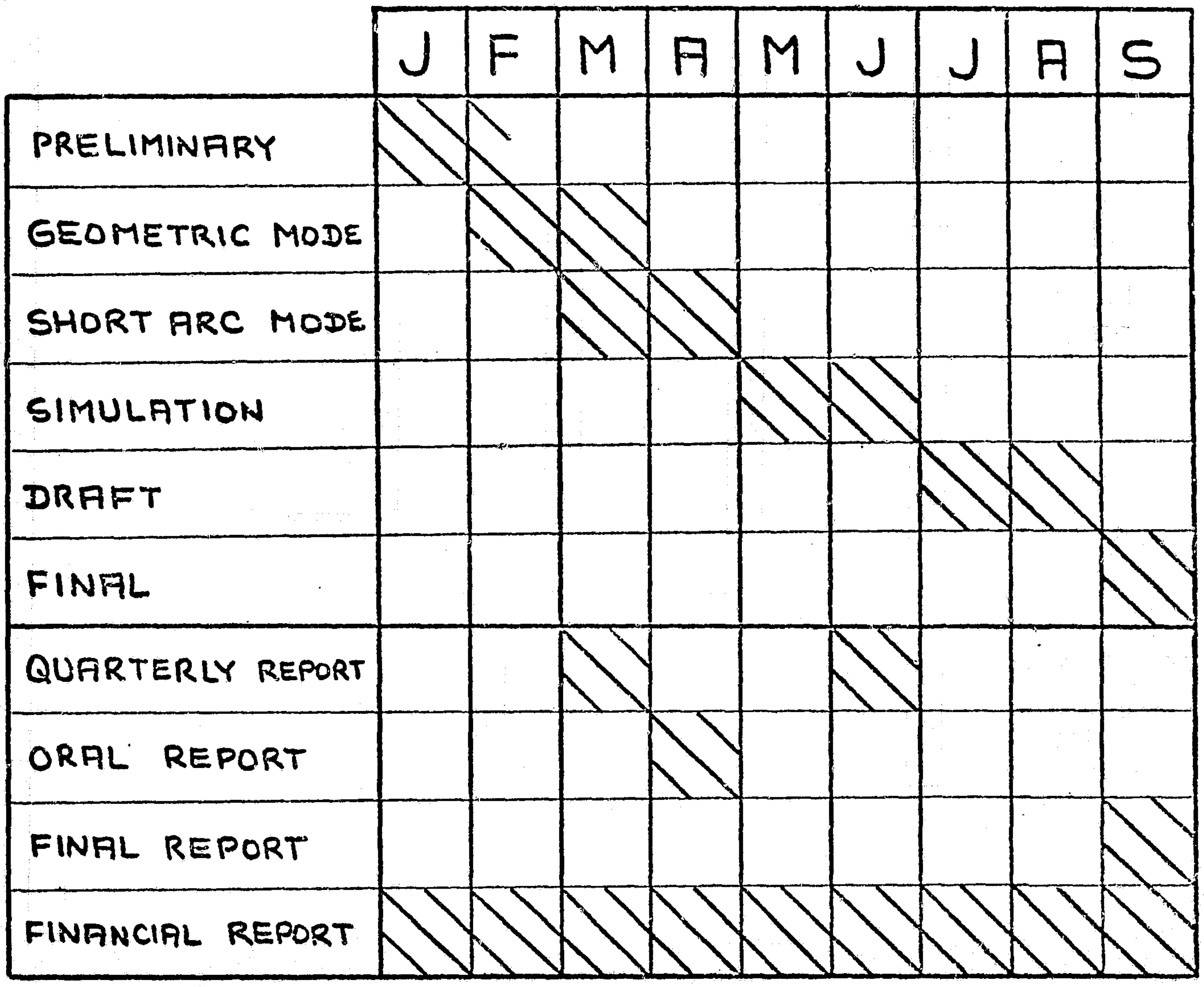
MAXIMUM ACCURACY

VS

APPLICATIONS



TIME TABLE


J
F
M
A
M
J
J
A
S
PRELIMINARY
GEOMETRIC MODE
SHORT ARC MODE
SIMULATION
DRAFT
FINAL
QUARTERLY REPORT
ORAL REPORT
FINAL REPORT
FINANCIAL REPORT

APPENDIX B

C-2

REPUBBLICA ITALIANA
COMMISSIONE GEODETICA ITALIANA
IL PRESIDENTE

Trieste, February 18, 1975

6th SYMPOSIUM ON MATHEMATICAL GEODESY
(3rd Hotine Symposium)

Siena, April 2-5, 1975

Dear Colleague,

Following to my letter of January 8, I am sending the Second (and last) Circular Letter concerning the 6th Symposium on Mathematical Geodesy (3rd Hotine Symposium) organized by the Italian Geodetic Commission under the aegis of S.S.G. No. 4.31 "Mathematical Techniques in Physical Geodesy".

The opening of the Symposium will take place at 9.30 A. M. of April 2.

The participation to the Symposium is not bound to any formality or payment of a registration fee. You are only required to fill in the enclosed reservation form, and send it directly to the Azienda Autonoma Turismo, Siena, as indicated. I recommend Hotels Minerva, Continental, Toscana, Chiusarelli and Pension Ravizza, that are close to the University in which the Symposium will take place (Via Banchi di Sotto 57, 1st floor).

The dead line for the hotel reservation is March 10; an earlier application will be highly appreciated.

In line with the former Hotine Symposia, no preliminary presentation of papers is required; however I will appreciate receiving the title of your contribution. The main accent will be on oral presentations and discussions on unusual aspects of Geodesy. The publication of the Proceedings of the Symposium is foreseen.

Looking forward to seeing you in Siena, and with my heartiest regards and wishes.

Yours very sincerely,

prof. Antonio Marussi

PRESSO L'ISTITUTO DI GEODESIA E GEOFISICA DELL'UNIVERSITÀ DEGLI STUDI DI TRIESTE
34100 TRIESTE · VIA DELL'UNIVERSITÀ 7 · TELEF. 69-563, 31-035 · TELEX 46014 PHYSICA TRIESTE

March 17, 1975

Professor Antonio Marussi
Direttore dell'Istituto di Geodesia e Geofisica
Dell'Università di Trieste
Via dell'Università 7
34100 Trieste
Italy

Dear Professor Marussi:

This is just a short note to let you know that I will be present at the Hotine Symposium in Siena. I do not plan to present a formal paper, but I would like to have some private discussions on the unusual aspects at the Close Grid Geodynamics Satellite Measurement System (CLOGEOS). This proposed system consists of a number of satellite-borne lasers and closely spaced reflectors on the ground for the primary purpose of monitoring fault motions, etc.

I am looking forward to seeing you again. With my best wishes to you and Mrs. Marussi

Sincerely yours,

Ivan I. Mueller
President/Geodesy

IIM/mn

6th SYMPOSIUM ON MATHEMATICAL GEODESY

(3rd Hotine Symposium)

Siena, April 2-5, 1975

PROGRAM

Wednesday - April 2, 1975, 15-17
Chairman: I. Mueller, Columbus/U.S.A.

1. Cartan and the holonomity problem I
   A. Marussi, University of Triests, Trieste/Italy

2. Cartan and the holonomity problem II
   N. Grossman, University of California at Los Angeles (UCLA), Los Angeles/U.S.A.

3. Cartan and the holonomity problem III
   F. Bocchio, University of Triests, Trieste/Italy

4. Cartan and the holonomity problem IV
   E. Grafarend, University of Bonn, Bonn/W-Germany

5. Cartan and the holonomity problem V
   J.G. Leclerc, University of Stockholm, Stockholm/Sweden and Quebec/Canada

Thursday, April 3, 1975, 9-12h

Chairman: T. Krarup, Copenhagen/Denmark

1. Utilization del documents cartographiques existants (anomalies de Bouguer, cartes d'altitude) pour une definition precise du potentiel dans l'espace exterieur au geoide vrai

   H.M. Dufour, IGN Paris/France

2. On the determination and application of gravity gradients in geodetic systems

   E. Groten, Technical University of Darmstadt, Darmstadt/Germany

3. Approximation of certain solutions of the exterior oblique derivative problem for the laplace equation

   K.J. Witsch, University of Bonn, Bonn/W-Germany

4. Boundary problems for the sphere

   E. Ecker, Technical University of Berlin, Berlin/W-Germany

5. Analytical continuation of a function from the length's surface upwards

   M. Pick, Academy of Science, Prag/CSSR

Thursday, April 3, 1975, 14-17h

Chairman: E. Grafarend, Bonn/Germany

1. Reflexive predictions

   A. Bjerhammar, University of Stockholm, Stockholm/Sweden

2. Determination of datum-shift parameters using least-squares collocation

   and

   A mass density covariance function consistent with the covariance functions of the anomalous potential

   C. Tscherning, Geodetic Institute Copenhagen/Denmark

3. A spherical harmonic expansion of the isostatic reduction potential

   G. Lachapelle, Geodetic Survey of Canada, Ottawa/Canada

4. Least squares collocation for large systems

   K.P. Schwarz, Technical University at Graz, Graz/Austria

Friday, April 4, 1975, 9-12h

Chairman: A. Bjerhammar, Stockholm/Sweden

1. Unusual aspects at the close grid geodynamics satellite measurement system (CLOGEOS)

   I. Mueller, The Ohio State University, Columbus/U.S.A.

2. Free adjustment of a torsion balance net

   G. Hein, Technical University of Darmstadt, Darmstadt/Germany

3. The nature of space near the earth

   N. Grossman, University of California at Los Angeles (UCLA), Los Angeles, U.S.A.

4. A general method for the computation of minimax-errors

   G. Heindl and F. Reinhart, Technical University of Munich, Munich/Germany

Friday, April 4, 1975, 14-17h

Chairman: H.M. Dufour, Paris/France

1. 3-d Mapping and mapping of the gravitational field

   A. Marussi, University of Trieste, Trieste/Italy

2. On a general potential-invariante representation of the geopotential field and applications,

   F. Bocchio, University of Trieste/Italy

3. Geodetic differential geometry

P. Defrise, University of Bruxelles, Bruxelles/Belgique
and E. Grafarend, University of Bonn, Bonn/Germany

Attachment 4
29 Pages

Department of Geodetic Science

CLOSE GRID GEODYNAMIC SATELLITE MEASUREMENT SYSTEM DEFINITION

Second Quarterly Status Report
Contract No. NAS 8-31195
OSURF Project No. 4105-A1

Period Covered: April 16 - June 30, 1975

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama 35812

The Ohio State University
Research Foundation
Columbus, Ohio 43212

July 1975

PREFACE

This project is under the supervision of Professor Ivan I. Mueller, Department of Geodetic Science, The Ohio State University and is under the technical direction of Dr. Nicholas C. Costes, Code ES31, MSFC, Huntsville, Alabama. The contract is issued by Procurement Office, MSFC and is administered through ONRRR, Columbus, Ohio.

1. Statement of Work

Perform an error analysis based on assumed set of satellite borne transmitting equipment and ground receivers to determine the optimum use of such systems in connection with the science that can be obtained from CLOGEOS measurements.

2. Data Generation

2.1 Range Generation

The computer program RGGR 7-B (as used for geometric mode) was suitably modified to generate ranges for analysis in short arc mode. Using the short arcs (see paragraph 3.1 of First Quarterly Status Report), ranges were generated with a Gaussian standard deviation of 10 cm. One set of errorless ranges was also generated.

For details of data generated and the computer expenses see Tables 2.1-1, 2.1-2 and 2.1-3.

3. Simulated Solutions

During the reporting period the investigations were mainly made in short arc mode. Thirty-one simulated solutions were computed as detailed in Table 3-2.

In addition, twelve simulated solutions (Table 3-3) were also computed in geometric mode. Computer expenses for simulated solutions are given in Table 3-1.

4. Analysis and Conclusions (Preliminary)

4.1 Geometric Mode

Reference to equations 5.1 and 5.2 of the First Quarterly Status Report, some further graphical analysis was performed to study the effect of orbit, station separation and number of events. Only the "typical" angles α_{ijk} and the distances r_{ij} were used in this analysis.

The above graphical analysis gave the same conclusion for the geometric mode as reported in the First Quarterly Status Report. However, the details will be included in the final report.

4.2 Short Arc Mode

4.2.1 Effect of Fundamental (LAGEOS) Stations Per Pass

Figure 4.2.1-1 shows the effect of inclusion of fundamental (LAGEOS) stations in short arcs. The reduction of LAGEOS stations to two per pass adversely effects the solution and for the directional stability, each pass must have at least two fundamental stations.

4.2.2 Effect of Grid (CLOGEOS) Stations Per Pass

Figure 4.2.2-1 and Tables 4.2.2-1 and 4.2.2-2 show the effect on recovery of relative position of grid (CLOGEOS) stations as the number of such stations varies in any satellite pass.

4.2.3 Effect of Orbital Height

Table 4.2.3-1 and Figures 4.2.3-1 and 4.2.3-2 show the effect of varying orbital height of the satellite. Even though the residuals in coordinates for the lower orbit (Table 4.2.3-1) are larger compared to upper orbit case, the overall recovery (Figures 4.2.3-1 and 4.2.3-2) is quite compatible in both cases.

4.2.4 Effect of Number of Events

Table 4.2.4-1 and Figures 4.2.4-1 and 4.2.4-2 show the effect of different number of events in a simulated solution. The results show that variations are not significant.

4.2.5 Effect of Observational Mode

Figure 4.2.5-1 shows the effect of simultaneous (grouped) versus the sequential observational mode in the short arc mode. The recovery is more or less the same in each case.

4.3 Geometric Mode vs. Short Arc Mode

Figures 4.3-1 and 4.3-2 show the comparative recovery in residuals in geometric (with 9 stations) and short arc mode (with 12 stations) for cases A and C.

In case A, the residuals for geometric mode are in meters, while all other residuals are in centimeters.

Figure 4.3-3 and Table 4.3-4 show the comparative recovery in residuals where one/three LAGEOS stations have been added in the geometric mode also.

5. Personnel

Ivan I. Mueller, Project Supervisor, part time
Muneendra Kumar, Graduate Research Associate, part time
Boudewijn H. W. VanGelder, Graduate Research Associate, part time
Michelle A. Neff, Administrative Assistant, part time

6. Travel

1. June 15 - 20, 1975, Washington, D. C. (Kumar)
   Annual Meeting of American Geophysical Union

REFERENCES

Blaha, Georges, (1971). "Inner Adjustment Constraints with Empahsis on Range Observations," Reports of the Department of Geodetic Science No. 148, The Ohio State University, Columbus, OH.

Brown, Duane C. and Jerry E. Trotter, (1973). "Extension to SAGA for Geodetic Reduction of Doppler Observations," Air Force Cambridge Research Laboratories No. AFCRL-TR-73-0177, Bedford, MA.

Goddard Trajectory Determining System (GTDS), (1974). User's Guide - GSC 11946, Vols. I & II, Computer Software Management and Information Center (COSMIC), University of Georgia, Athens, GA.

Goddard Trajectory Determining System (GTDS), (1972). Mathematical Specifications, Goddard Space Flight Center Report No. X-252-72-244, edited by W. E. Wagner and C. E. Velez, Greenbelt, MD.

Reilly, J.P., C.R. Schwarz and M.C. Whiting, (1972). "The Ohio State University Geometric and Orbital (Adjustment) Program (OSUGOP) for Satellite Observations", Reports of the Department of Geodetic Science No. 190, The Ohio State University, Columbus, OH.

Table 2.1-1

Long Arcs

Satellite Height (km)	Oribt Type	Time of Data Generation (hrs)	Computer Expenses $
392	Polar	126	120.00
657	-do-	126	130.00
1007	-do-	126	150.00
			500.00

Table 2.2-2

Short Arcs

Satellite Height (km)	No. of Passes	Length of each Pass in Time	Density of Satellite points	Computer Expenses $
392	26	8 min	1/sec	540.00
657	31	10 min	1/sec	775.00
1007	30	12 min	1/sec	900.00
				2,215.00

Table 2.1-3

Range Generation

Satellite Height (km)	No. of Passes	Case Type	Maximum Data Points Generated	Computer Expenditure $
392	26	A	5000	300.00
		C	500	50.00
1007	30	A	5000	380.00
		C	500	50.00
				700.00

Table 3-1

Simulated Solutions

Mode	No. of Solutions	Computer Expenses $
Short Arc	31	3,100.00
Geometric	12	720.00
		3,820.00

SHORT ARC MODE

TABLE 3-2 (1 of 4)

1

#	OR BIT	CASE	MODE	NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS: 3S ALL P.	3S SOME P.	2S	1S	0S	NUMBER OF GRID STATIONS PER PASS: ALL S.	SOME S.	Δt	DAYS	GRAV	σ_R	EVENTS: PASSES	NW EV/P	TOTAL EVENT	RNGE Ø3	PGRM Ø1	APPROX STAT	RESULTS: RESIDUALS IN M	SPREAD CM	REMARKS
1	U	C	S	✓					✓		6	3	SAGA	10	9	15	483	3	3	P	$0.02 < \Delta X < 0.07$ $0.05 < \Delta Y < 0.16$ $-0.45 < \Delta Z < -0.34$ $-0.06 < \Delta R < 0.06$	5 11 11 12	
2	U	A	S	✓					✓		6	3	SAGA	10	9	15	483	4	4	P	$0.04 < \Delta X < 0.07$ $0.01 < \Delta Y < 0.07$ $-0.37 < \Delta Z < -0.31$ $-0.06 < \Delta R < 0.00$	3 6 6 6	~1
3	U	A	S	✓					✓		6	5	SAGA	10	16	15	969	5	5	P	$0.03 < \Delta X < 0.07$ $-0.04 < \Delta Y < 0.03$ $-0.43 < \Delta Z < -0.37$ $-0.07 < \Delta R < 0.01$	4 7 6 8	
4	U	A	S	✓					✓		6	3	SIASA	0	9	15	483	6	6	P	$0.04 < \Delta X < 0.07$ $0.02 < \Delta Y < 0.08$ $-0.38 < \Delta Z < -0.32$ $-0.06 < \Delta R < 0.00$	3 6 6 6	~2
5	U	A	S	✓					✓		12	3	SAGA	10	9	8	239	7	7	P	$0.10 < \Delta X < 0.15$ $0.02 < \Delta Y < 0.08$ $-0.34 < \Delta Z < -0.29$ $-0.07 < \Delta R < 0.02$	5 6 5 9	
6	U	A	NS	✓					✓		lin12	3	SAGA	10	9	"8"	"243"	8	8	P	$0.07 < \Delta X < 0.11$ $0.00 < \Delta Y < 0.08$ $-0.34 < \Delta Z < 0.29$ $-0.06 < \Delta R < 0.02$	4 8 5 8	
7	U	A	NS	✓					✓		lin12	3	SAGA	10	9	"8"	"243"	8	8	NP(9)	$0.07 < \Delta X < 0.11$ $0.00 < \Delta Y < 0.08$ $-0.34 < \Delta Z < 0.29$ $< \Delta R <$	4 8 5	~6
8	L	A	S	✓					✓		3	5	SAGA	10	7	10	254	11	10	P	$0.39 < \Delta X < 0.51$ $0.81 < \Delta Y < 0.99$ $-1.60 < \Delta Z < -1.43$ $-0.09 < \Delta R < 0.01$	12 18 17 10	
9	L	A	NS		5			5	✓		lin12 lin9	5	SIASA	10	10	"8"		9,10 12	9	P	$-0.68 < \Delta X < 0.42$ $-5.04 < \Delta Y < 5.71$ $-6.25 < \Delta Z < 4.94$ $3.75 < \Delta R < 5.36$	O U T	

SHORT ARC MODE				NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS					NUMBER OF GRID STATIONS PER PASS						EVENTS						RESULTS		
#	OR BIT	CASE	MODE	3S ALL P.	3S SOME P.	2S	1S	0S	ALL S.	SOME S.	Δt	DAYS	GRAV	σ_R	PASSES	MIN EV/P	TOTAL EVENT	RNGE Ø3	PGRM Ø1	APPROX STAT	RESIDUALS IN M	SPREAD CM	REMARKS
10	U	A	S		4			5	✓		12	3	SAGA	10	9	8	239	7	11	P	$-0.56 < \Delta X < 1.53$ $-2.60 < \Delta Y < -2.28$ $-1.86 < \Delta Z < -1.58$ $-1.48 < \Delta R < 1.65$	OUT	
11	U	A	S			6		3	✓		12	3	SAGA	10	9	8	239	7	12	P	$-1.27 < \Delta X < 4.34$ $-3.59 < \Delta Y < 2.16$ $-2.06 < \Delta Z < 3.11$ $-2.06 < \Delta R < 0.55$	OUT	
12	L+U	A	S	✓					✓		12:U 3:L	3 + 5	SAGA	10	9 + 7	8	239 + 254 493	7 + 11	14	P	$-0.04 < \Delta X < 0.00$ $0.39 < \Delta Y < 0.51$ $-0.81 < \Delta Z < -0.70$ $-0.08 < \Delta R < -0.00$	4 12 11 8	~5+8
13	L	A	S	✓					✓		1	5	SAGA	10	7	29	758	13	15	P	$0.36 < \Delta X < 0.51$ $0.79 < \Delta Y < 0.98$ $-1.63 < \Delta Z < -1.47$ $-0.09 < \Delta R < 0.02$	15 19 16 11	
14	U	A	S		5	4			✓		12	3	SAGA	10	9	8	239	7	16	P	$0.27 < \Delta X < 0.33$ $0.10 < \Delta Y < 0.16$ $-0.18 < \Delta Z < -0.10$ $-0.09 < \Delta R < 0.03$	6 6 8 12	~5
15	L	A	S	✓					✓		3	5	GTDS 15x15	10	7	10	254	11	10	P	$0.39 < \Delta X < 0.51$ $0.81 < \Delta Y < 0.99$ $-1.60 < \Delta Z < -1.43$ $-0.08 < \Delta R < 0.01$	12 18 17 9	~8
16	U	A	S			9			✓		12	3	SAGA	10	9	8	239	7	17	P	$0.54 < \Delta X < 0.62$ $0.24 < \Delta Y < 0.28$ $0.03 < \Delta Z < 0.11$ $-0.12 < \Delta R < 0.03$	8 4 8 15	~5,14
17	U	A	S				9		✓		12	3	SAGA	10	9	8	239	7	18	P	$64.24 < \Delta X < 75.59$ $-17.25 < \Delta Y < -7.04$ $20.72 < \Delta Z < 23.53$ $-9.01 < \Delta R < 0.05$	OUT	~5,14,16
18	L	A	S	✓					✓		3	5	SPHE RI CAL	10	7	10	254	11	10	P	$1.31 < \Delta X < 1.59$ $1.27 < \Delta Y < 1.63$ $-0.59 < \Delta Z < -0.44$ $-0.25 < \Delta R < 0.01$	28 36 15 26	~8,15

Table 3-2 3

SHORT ARC MODE				NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS					NUMBER OF GRID STATIONS PER PASS						EVENTS						RESULTS		
#	ORBIT	CASE	MODE	3S ALL P.	3S SOME P.	2S	1S	0S	ALL S.	SOME S.	Δt	DAYS	GRAV	σ_R	PASSES	MIN EV/P	TOTAL EVENT	RNGE Ø3	PERM Ø1	APPROX STAT	RESIDUALS IN M	SPREAD CM	REMARKS
19	L	A	S	✓					✓		3	5	EL LIP SOI DAL	10	7	10	254	11	10	P	$0.35<\Delta X<0.47$ $0.74<\Delta Y<0.92$ $-1.63<\Delta Z<-1.45$ $-0.09<\Delta R<0.01$	12 18 18 10	~8,15,18
20	L	A	S	✓					✓		3	5	SAGA	10	7	10	254	11	10	P	$0.33<\Delta X<0.45$ $0.75<\Delta Y<0.94$ $-1.65<\Delta Z<-1.47$ $-0.08<\Delta R<0.00$	12 14 18 8	~8 INERTIAL STATE VECTORS
21	U	A	S	✓					✓		12	3	SAGA	10	9	8	239	7	7	P	$1.79<\Delta X<1.94$ $-0.53<\Delta Y<-0.46$ $0.50<\Delta Z<0.54$ $-0.14<\Delta R<0.00$	15 7 4 14	~5 $\sigma_{LAG}=5cm$
22	U	A	S	✓					✓		12	3	SAGA	10	9	8	239	7	7	P	$-0.06<\Delta X<0.07$ $-0.14<\Delta Y<0.01$ $-0.29<\Delta Z<-0.20$ $-0.09<\Delta R<0.04$	13 15 9 13	~5 $\sigma_{H_{100's}}=0$
23	L	A	S	✓					✓		3	5	GTDS 15x15	10	7	10	254	14	19	P	$0.39<\Delta X<0.51$ $0.01<\Delta Y<0.99$ $-1.60<\Delta Z<-1.43$ $-0.08<\Delta R<0.01$	12 18 17 9	~8 NO NON GRAV. FORCES
24	U	A	S			9				3	12	3	SAGA	10	9	8	239	7	20	P	$-28.79<\Delta X<31.14$ $-51.05<\Delta Y<-4.60$ $-5.64<\Delta Z<39.36$ $-29.53<\Delta R<7.79$	OUT	
25	U	A	S	✓						4	12	3	SAGA	10	9	8	239	7	21	P	$0.04<\Delta X<0.28$ $-0.06<\Delta Y<0.11$ $-0.41<\Delta Z<-0.26$ $-0.16<\Delta R<0.21$	24 17 15 37	
26	U	A	S			9				4	12	3	SAGA	10	9	8	239	7	22	P	$-6.78<\Delta X<-5.14$ $2.01<\Delta Y<3.05$ $-0.70<\Delta Z<0.21$ $-1.11<\Delta R<1.63$	1.64 1.04 0.91 2.74	
27	U	A	S	✓						6	12	3	SAGA	10	9	8	239	7	23	P	$0.07<\Delta X<0.21$ $-0.02<\Delta Y<0.09$ $-0.40<\Delta Z<-0.27$ $-0.15<\Delta R<0.10$	14 11 13 25	

#	SHORT ARC MODE			NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS					NUMBER OF GRID STATIONS PER PASS		Δt	DAYS	GRAV	σ_R	EVENTS			RNGE Ø3	FORM Ø1	APPROX STAT	RESULTS		REMARKS
	OR BIT	CASE	MODE	3S ALL P.	3S SOME P.	2S	1S	0S	ALL S.	SOME S.					PASSES	NW EV/P	TOTAL EVENT				RESIDUALS IN M	SPREAD CM	
28	U	A	S			9				7	12	3	SAGA	10	9	8	239	7	24	P	-3.67 < ΔX < -3.23 1.25 < ΔY < 1.85 -0.47 < ΔZ < -0.34 -0.18 < ΔR < 0.46	44 60 13 64	
29	U	A	S	✓						6	6	3	SAGA	10	9	15	483	4	25	P	0.03 < ΔX < 0.11 -0.02 < ΔY < 0.07 -0.40 < ΔZ < -0.30 -0.11 < ΔR < 0.05	8 9 10 16	
30	U	A	S			9				7	6	3	SAGA	10	9	15	483	4	26	P	-1.97 < ΔX < -1.66 0.64 < ΔY < 0.95 -0.70 < ΔZ < -0.57 -0.18 < ΔR < 0.24	31 31 13 42	
31	U	A	NS	✓						6	1in12	3	SAGA	10	9	"8"	"243"	8	27	P	0.04 < ΔX < 0.17 -0.02 < ΔY < 0.08 -0.35 < ΔZ < -0.26 -0.12 < ΔR < 0.08	13 10 9 20	
32																					< ΔX < < ΔY < < ΔZ < < ΔR <		
33																					< ΔX < < ΔY < < ΔZ < < ΔR <		
34																					< ΔX < < ΔY < < ΔZ < < ΔR <		
35																					< ΔX < < ΔY < < ΔZ < < ΔR <		
36																					< ΔX < < ΔY < < ΔZ < < ΔR <		

TABLE 3-3 (1 of 2)

GEOMETRIC MODE				NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS					NUMBER OF GRID STATIONS PER PASS							EVENTS			CSTP	PGRM	APPROX	RESULTS		REMARKS
#	ORBIT	CASE	MODE	3S ALL P.	3S SOME P.	2S	1S	0S	ALL S.	SOME S.	Δt	DAYS	GRAV	σ_R	PASSES	NW EV/P	TOTAL EVENT	Ø1	Ø1	STAT	RESIDUALS IN CM	SPREAD CM	σ IN CM
-8	U	C	S				9		✓		6	3		10	9	15	617	54		P	$-3 < \Delta X < 1$ $3 < \Delta Y < 5$ $3 < \Delta Z < 7$ $-5 < \Delta R < 3$	4 2 4 8	INNER C. $\sigma_X : 6$ $\sigma_Y : 15$ $\sigma_Z : 20$ $\sigma_R : 11$
-7	U	A	S	✓					✓		6	3		10	9	15	483	51		P	$-1 < \Delta X < 2$ $-3 < \Delta Y < 0$ $-1 < \Delta Z < 1$ $-2 < \Delta R < 2$	3 3 2 4	INNER C. $\sigma_X : 6$ $\sigma_Y : 3$ $\sigma_Z : 3$ $\sigma_R : 2$
-6	U	C	S	✓					✓		6	3		10	9	15	483	52		P	$-.0 < \Delta X < 2$ $-3 < \Delta Y < 0$ $-1 < \Delta Z < 1$ $-2 < \Delta R < 2$	2 3 2 4	INNER C. $\sigma_X : 5$ $\sigma_Y : 3$ $\sigma_Z : 3$ $\sigma_R : 2$
-5	U	A	S	✓					✓		6	3		10	9	15	483	51		P	$-1 < \Delta X < 1$ $-3 < \Delta Y < 0$ $-1 < \Delta Z < 1$ $-2 < \Delta R < 2$	2 3 2 4	LAGEOS C. $\sigma_X : 1$ $\sigma_Y : 1$ $\sigma_Z : 1$ $\sigma_R :$
-4	U	C	S	✓					✓		6	3		10	9	15	483	52		P	$-1 < \Delta X < 1$ $-3 < \Delta Y < 0$ $-1 < \Delta Z < 1$ $-2 < \Delta R < 2$	2 3 2 4	LAGEOS C. $\sigma_X : 1$ $\sigma_Y : 1$ $\sigma_Z : 1$ $\sigma_R :$
-3	L	A	S					11	✓		5	5		10	11		559			P	$-625 < \Delta X < 618$ $-414 < \Delta Y < 404$ $-218 < \Delta Z < 219$ $-1322 < \Delta R < 1326$	1243 818 437 2648	INNER C. $\sigma_X : 285$ $\sigma_Y : 212$ $\sigma_Z : 196$ $\sigma_R : 581$
-2	L	C	S					11	✓		5	5		10	11		559			P	$-10 < \Delta X < 10$ $-5 < \Delta Y < 6$ $-5 < \Delta Z < 7$ $-9 < \Delta R < 19$	20 11 12 28	INNER C. $\sigma_X : 6$ $\sigma_Y : 5$ $\sigma_Z : 5$ $\sigma_R : 13$
-1	U	A	S					20	✓		20	5		10	20		507			P	$-203 < \Delta X < 207$ $-234 < \Delta Y < 228$ $-302 < \Delta Z < 310$ $-378 < \Delta R < 669$	410 463 612 1047	INNER C. $\sigma_X : 255$ $\sigma_Y : 203$ $\sigma_Z : 227$ $\sigma_R : 569$
0	U	C	S					20	✓		20	5		10	20		507			P	$-6 < \Delta X < 7$ $-10 < \Delta Y < 9$ $-7 < \Delta Z < 9$ $-17 < \Delta R < 25$	13 19 16 42	INNER C. $\sigma_X : 6$ $\sigma_Y : 5$ $\sigma_Z : 5$ $\sigma_R : 14$

GEOMETRIC MODE

Table 3-3 (2 of 2)

(-1)

#	OR BIT	CASE	MODE	NUMBER OF PASSES WITH N FUNDAMENTAL STATIONS: 3S ALL P.	3S SOME P.	2S	1S	0S	NUMBER OF GRID STATIONS PER PASS: ALL S.	SOME S.	Δt	DAYS	GRAV	σ_R	EVENTS: PASSES	HW EV/P	TOTAL EVENT	CSTP Ø1	PGRM Ø1	APPROX STAT	RESULTS: RESIDUALS IN CM	SPREAD CM	REMARKS σ IN CM
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
																					$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		
-11	U	A	S			9			✓		6	3		10	9	15	506	55		P	$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		INNER C.
-10	U	C	S			9			✓		6	3		10	9	15	505	56		P	$<\Delta X<$ $<\Delta Y<$ $<\Delta Z<$ $<\Delta R<$		INNER C.
-9	U	A	S				9		✓		6	3		10	9	15	618	53		P	$-6 < \Delta X < 4$ $3 < \Delta Y < 6$ $3 < \Delta Z < 8$ $-10 < \Delta R < 2$	10 3 5 12	INNER C. σ_X : 6 σ_Y : 15 σ_Z : 19 σ_R : 11

EFFECT OF NUMBER OF GRID STATIONS PER PASS
(3 FUNDAMENTAL STATIONS PER PASS)

TABLE 4.2.2-1

#	ORBIT	HEIGHT FACTOR	OBSERVATIONAL MODE	FUNDAMENTAL STATIONS PER PASS	GRID STATIONS PER PASS	DAYS	EVENTS				Δt	$\sigma_{R\,cm}$	RESULTS		
							PASSES	MIN. EV. P. PASS	TOTAL EVENTS	TOTAL OBSERVAT.			RESIDUALS IN CM	SPREAD CM	σ
25	UPPER	0	S	3	4	3	9	8	239	1673	12	10	$4 < \Delta X < 28$ $-6 < \Delta Y < 11$ $-41 < \Delta Z < -26$ $-16 < \Delta R < 21$	24 17 15 37	
27	"	"	"	"	6	"	"	"	"	2151	12	"	$7 < \Delta X < 21$ $-2 < \Delta Y < 9$ $-40 < \Delta Z < -27$ $-15 < \Delta R < 10$	14 11 13 25	
29	"	"	"	"	"	"	"	15	483	4347	6	"	$3 < \Delta X < 11$ $-2 < \Delta Y < 7$ $-40 < \Delta Z < -30$ $-11 < \Delta R < 5$	8 9 10 16	
31	"	"	NS	"	"	"	"	"8"	243	2187	1 in 12	"	$4 < \Delta X < 17$ $-2 < \Delta Y < 8$ $-35 < \Delta Z < -26$ $-12 < \Delta R < 8$	13 10 9 20	
5	"	"	S	"	9	"	"	"	239	2868	12	"	$10 < \Delta X < 15$ $2 < \Delta Y < 8$ $-34 < \Delta Z < -29$ $-7 < \Delta R < 2$	5 6 5 9	

EFFECT OF NUMBER OF GRID STATIONS PER PASS
(2 FUNDAMENTAL STATIONS PER PASS)

TABLE 4.2.2-2

#	ORBIT	HEIGHT FACTOR	OBSERVATIONAL MODE	FUNDAMENTAL STATIONS PER PASS	GRID STATIONS PER PASS	DAYS	EVENTS				Δt	σ_{Rcm}	RESULTS		
							PASSES	MIN.EV. P.PASS	TOTAL EVENTS	TOTAL OBSERVAT.			RESIDUALS IN CM	SPREAD CM	σ
24	UPPER	0	S	2	3	3	9	8	239	1195	12	10	$-2879 < \Delta X < 3114$ $-5105 < \Delta Y < -460$ $-564 < \Delta Z < 3936$ $-2953 < \Delta R < 779$	5993 4645 4500 3732	
26	"	"	"	"	4	"	"	"	"	1434	"	"	$-678 < \Delta X < -514$ $201 < \Delta Y < 305$ $-70 < \Delta Z < 21$ $-111 < \Delta R < 163$	164 104 91 274	
28	"	"	"	"	7	"	"	"	"	2151	"	"	$-367 < \Delta X < -323$ $125 < \Delta Y < 185$ $-47 < \Delta Z < -34$ $-18 < \Delta R < 46$	44 60 13 64	
30	"	"	"	"	"	"	"	15	483	4347	6	"	$3 < \Delta X < 11$ $-2 < \Delta Y < 7$ $-40 < \Delta Z < -30$ $-18 < \Delta R < 24$	8 9 10 42	
16	"	"	"	"	9	"	"	8	239	2629	12	"	$54 < \Delta X < 62$ $24 < \Delta Y < 28$ $3 < \Delta Z < 11$ $-12 < \Delta R < 3$	8 4 8 15	

EFFECT OF ORBITAL HEIGHT

TABLE 4.2.3-1

#	ORBIT	HEIGHT FACTOR	OBSERVATIONAL MODE	FUNDAMENTAL STATIONS PER PASS	GRID STATIONS PER PASS	DAYS	EVENTS				Δt	$\sigma_{R_{cm}}$	RESULTS		
							PASSES	MIN.EV P. PASS	TOTAL EVENTS	TOTAL OBSERVAT.			RESIDUALS IN CM	SPREAD CM	σ
5	UPPER	O	S	3	9	3	9	8	239	2868	12	10	$10 < \Delta X < 15$ $2 < \Delta Y < 8$ $-34 < \Delta Z < -29$ $-7 < \Delta R < 2$	5 6 5 9	
8	LOWER	"	"	"	"	5	7	10	254	3048	3	"	$39 < \Delta X < 51$ $81 < \Delta Y < 99$ $-160 < \Delta Z < -143$ $-9 < \Delta R < 1$	12 18 17 10	
2	UPPER	"	"	"	"	3	9	15	483	5796	6	"	$4 < \Delta X < 7$ $1 < \Delta Y < 7$ $-37 < \Delta Z < -31$ $-6 < \Delta R < 0$	3 6 6 6	
13	LOWER	"	"	"	"	5	7	29	758	9096	1	"	$36 < \Delta X < 51$ $79 < \Delta Y < 98$ $-163 < \Delta Z < -147$ $-9 < \Delta R < 2$	15 19 16 11	

EFFECT OF NUMBER OF EVENTS

TABLE 4.2.4-1

#	ORBIT	HEIGHT FACTOR	OBSERVATIONAL MODE	FUNDAMENTAL STATIONS PER PASS	GRID STATIONS PER PASS	DAYS	EVENTS: PASSES	EVENTS: MIN. EV. P. PASS	EVENTS: TOTAL EVENTS	EVENTS: TOTAL OBSERVAT.	Δt	σR_{CM}	RESULTS: RESIDUALS IN CM	RESULTS: SPREAD CM	RESULTS: σ
5	UPPER	0	S	3	9	3	9	8	239	2868	12	10	$10 < \Delta X < 15$ $2 < \Delta Y < 8$ $-34 < \Delta Z < -29$ $-7 < \Delta R < 2$	5 6 5 9	
2	"	"	"	"	"	"	"	15	483	5796	6	"	$4 < \Delta X < 7$ $1 < \Delta Y < 7$ $-37 < \Delta Z < -31$ $-6 < \Delta R < 0$	3 6 6 6	
3	"	"	"	"	"	5	16	"	969	11628	"	"	$3 < \Delta X < 7$ $-4 < \Delta Y < 3$ $-43 < \Delta Z < -37$ $-7 < \Delta R < 1$	4 7 6 8	
8	LOWER	"	"	"	"	"	7	10	254	3048	3	"	$39 < \Delta X < 51$ $81 < \Delta Y < 99$ $-160 < \Delta Z < -143$ $-9 < \Delta R < 1$	12 18 17 10	
13	"	"	"	"	"	"	"	29	758	9096	1	"	$36 < \Delta X < 51$ $79 < \Delta Y < 98$ $-163 < \Delta Z < -147$ $-9 < \Delta R < 2$	15 19 16 11	

GEOMETRIC MODE VS. SHORT ARC MODE

TABLE 4.3-1

	#	ORBIT	HEIGHT FACTOR	OBSERVATION MODE	FUNDAMENTAL STATIONS PER PASS	GRID STATIONS PER PASS	DAYS	EVENTS: PASSES	EVENTS: MIN. EV. P. PASS	EVENTS: TOTAL EVENTS	EVENTS: TOTAL OBSERVAT.	Δt	$\sigma_{R\,cm}$	RESULTS: RESIDUALS IN CM	RESULTS: SPREAD cm	RESULTS: σ_{cm}
S.A.	2	UPPER	0	S	3	9	3	9	15	483	5796	6	10	$4 < \Delta X < 7$ $1 < \Delta Y < 7$ $-37 < \Delta Z < -31$ $-6 < \Delta R < 0$	3 6 6 6	
G.	-1	″	″	″	0	″	″	″	″	507	4563	″	″	$-203 < \Delta X < 207$ $-234 < \Delta Y < 228$ $-302 < \Delta Z < 310$ $-378 < \Delta R < 669$	410 462 612 1047	255 203 227 569
G.	-9	″	″	″	1	″	″	″	″	618	6180	″	″	$-6 < \Delta X < 4$ $3 < \Delta Y < 6$ $3 < \Delta Z < 8$ $-10 < \Delta R < 2$	10 3 5 12	6 15 19 11
G.	-7	″	″	″	3	″	″	″	″	483	5796	″	″	$-1 < \Delta X < 2$ $-3 < \Delta Y < 0$ $-1 < \Delta Z < 1$ $-2 < \Delta R < 2$	3 3 2 4	6 3 3 2
S.A.	1	″	1000	″	3	″	″	″	″	483	5796	″	″	$2 < \Delta X < 7$ $5 < \Delta Y < 16$ $-45 < \Delta Z < -34$ $-6 < \Delta R < 6$	5 11 11 12	
G.	0	″	″	″	0	″	″	″	″	507	4563	″	″	$-6 < \Delta X < 7$ $-10 < \Delta Y < 9$ $-7 < \Delta Z < 9$ $-17 < \Delta R < 25$	13 19 16 42	6 5 5 14
G.	-8	″	″	″	1	″	″	″	″	617	6170	″	″	$-3 < \Delta X < 1$ $3 < \Delta Y < 5$ $3 < \Delta Z < 7$ $-5 < \Delta R < 3$	4 2 4 8	6 15 20 11
G.	-6	″	″	″	3	″	″	″	″	483	5796	″	″	$-0 < \Delta X$ 2 $-3 < \Delta Y$ 0 $-1 < \Delta Z$ 1 $-2 < \Delta R$ 2	2 3 2 4	5 3 3 2

STATIONS

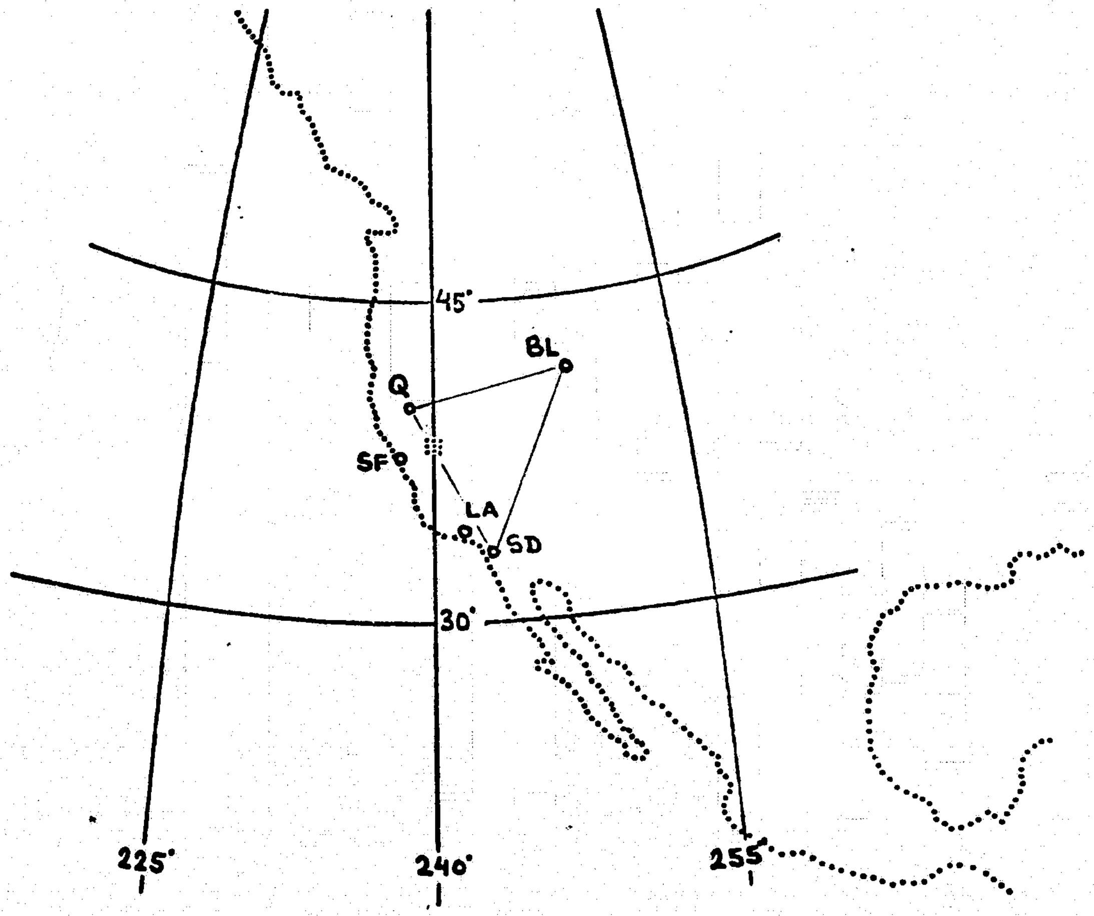
GRID STATIONS

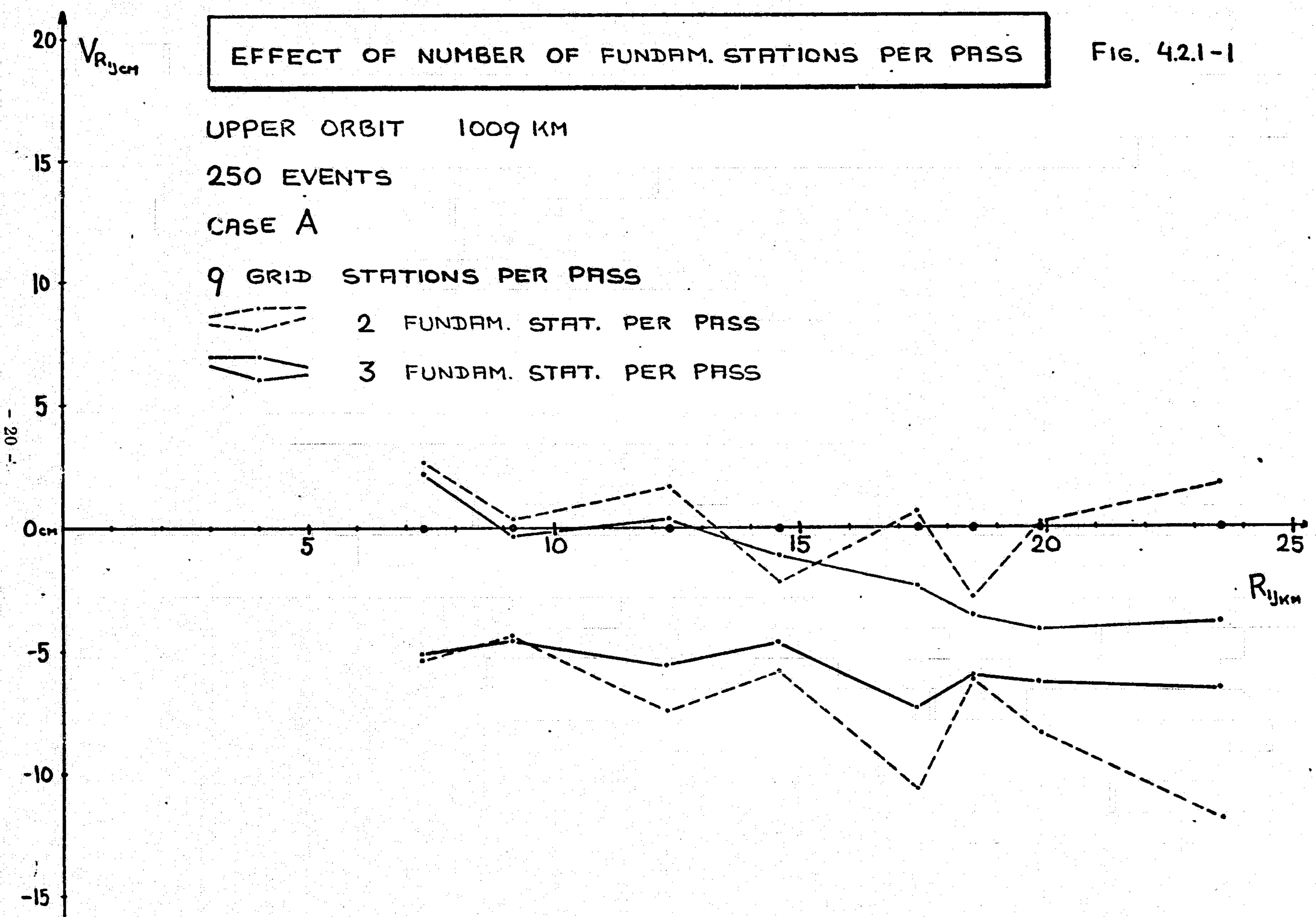
1005 $\varphi = 38°\ 0'$ $\lambda = 240°\ 0'$

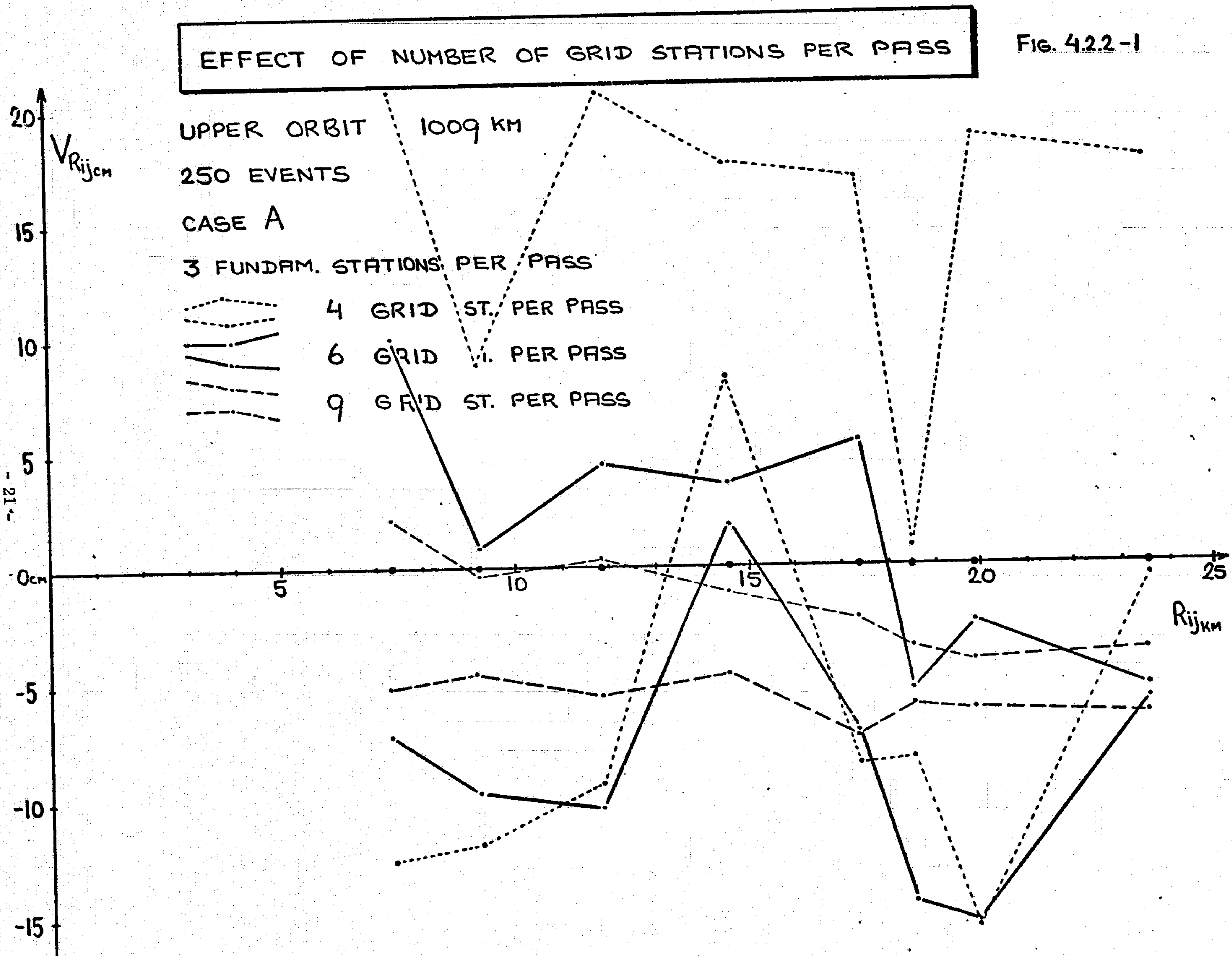
OTHER 8 GRID STATIONS ARE GROUPED AROUND 1005 AT 5' INTERVALS

FUNDAMENTAL STATIONS

1010	SAN DIEGO	$\varphi = 33°\ 0'$	$\lambda = 243°\ 0'$
1011	BEAR LAKE	$= 42°\ 0'$	$= 248°\ 30'$
1012	QUINCY	$= 40°\ 0'$	$= 239°\ 0'$



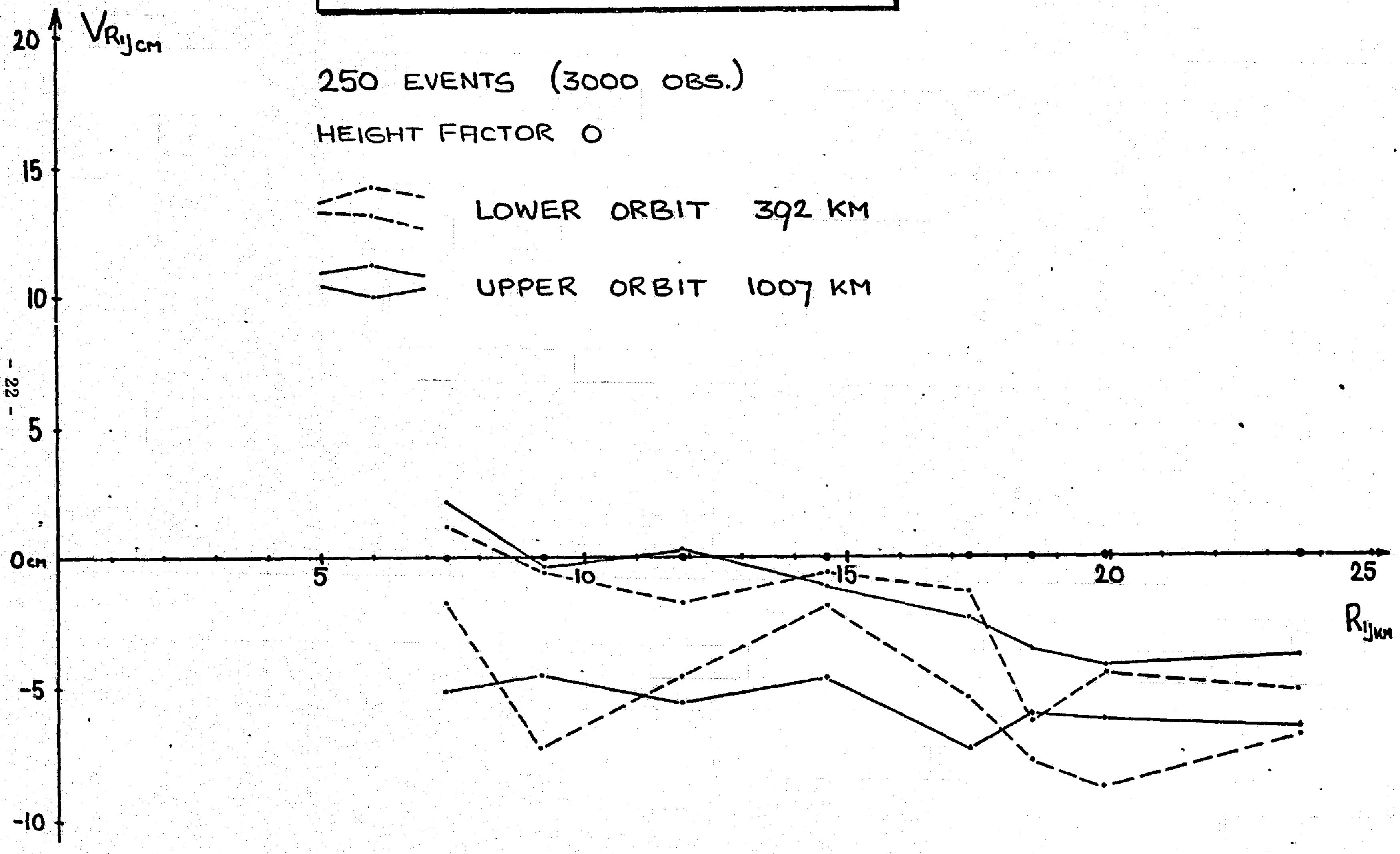

EFFECT OF NUMBER OF FUNDAM. STATIONS PER PASS
FIG. 4.2.1-1
UPPER ORBIT 1009 KM
250 EVENTS
CASE A
9 GRID STATIONS PER PASS
2 FUNDAM. STAT. PER PASS
3 FUNDAM. STAT. PER PASS
$V_{R_{ij}cm}$
$R_{ij\,km}$
20
15
10
5
0cm
-5
-10
-15
5
10
15
20
25



EFFECT OF NUMBER OF GRID STATIONS PER PASS
FIG. 4.2.2-1
UPPER ORBIT 1009 KM
250 EVENTS
CASE A
3 FUNDAM. STATIONS PER PASS
4 GRID ST. PER PASS
6 GRID " PER PASS
9 GRID ST. PER PASS
$V_{R_{ij}\,cm}$
$R_{ij\,KM}$
20
15
10
5
0cm
-5
-10
-15
5
10
15
20
25

EFFECT OF ORBITAL HEIGHT

FIG 4.2.3 - 1



EFFECT OF ORBITAL HEIGHT

FIG 4.2.3-2

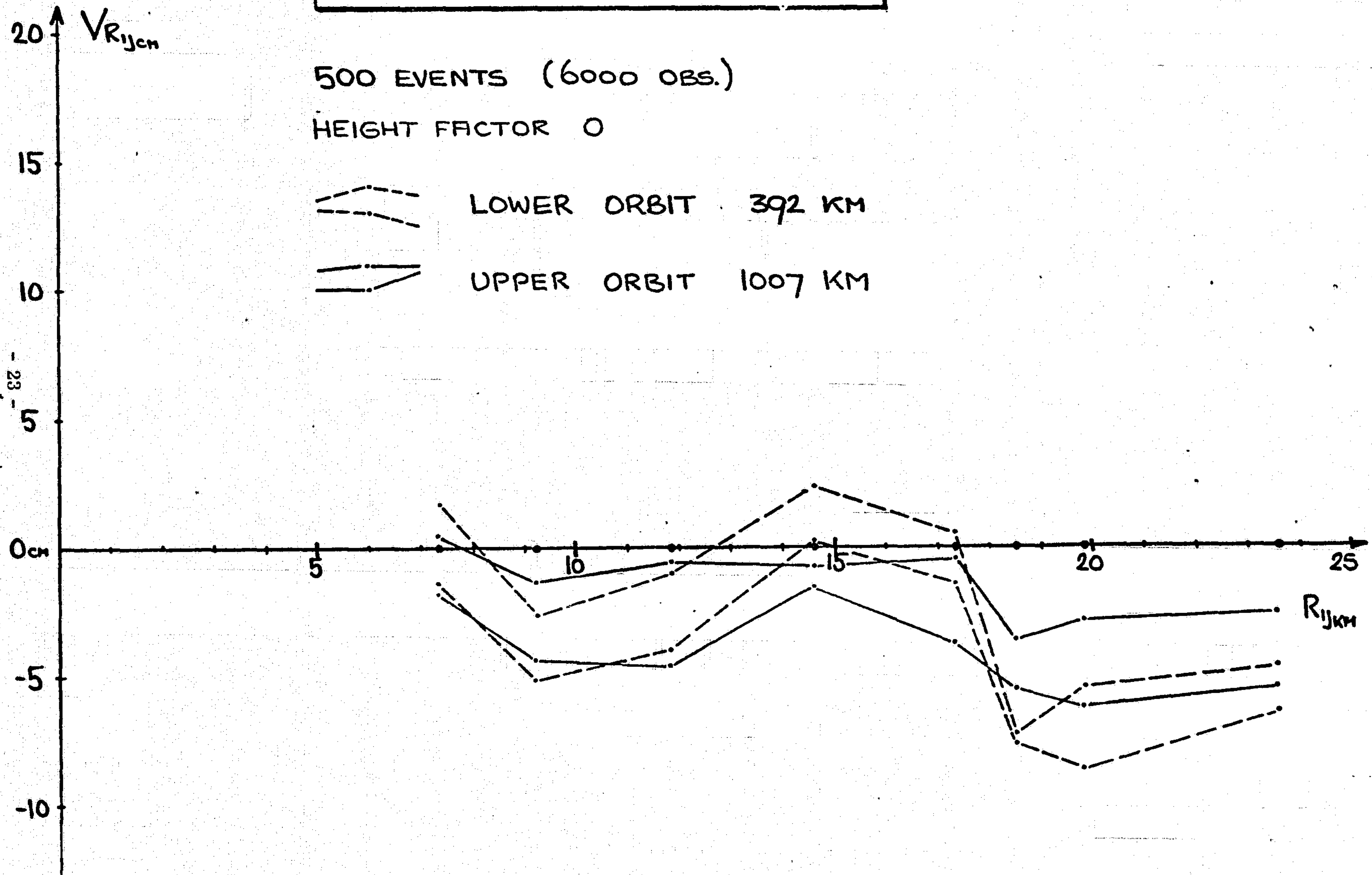
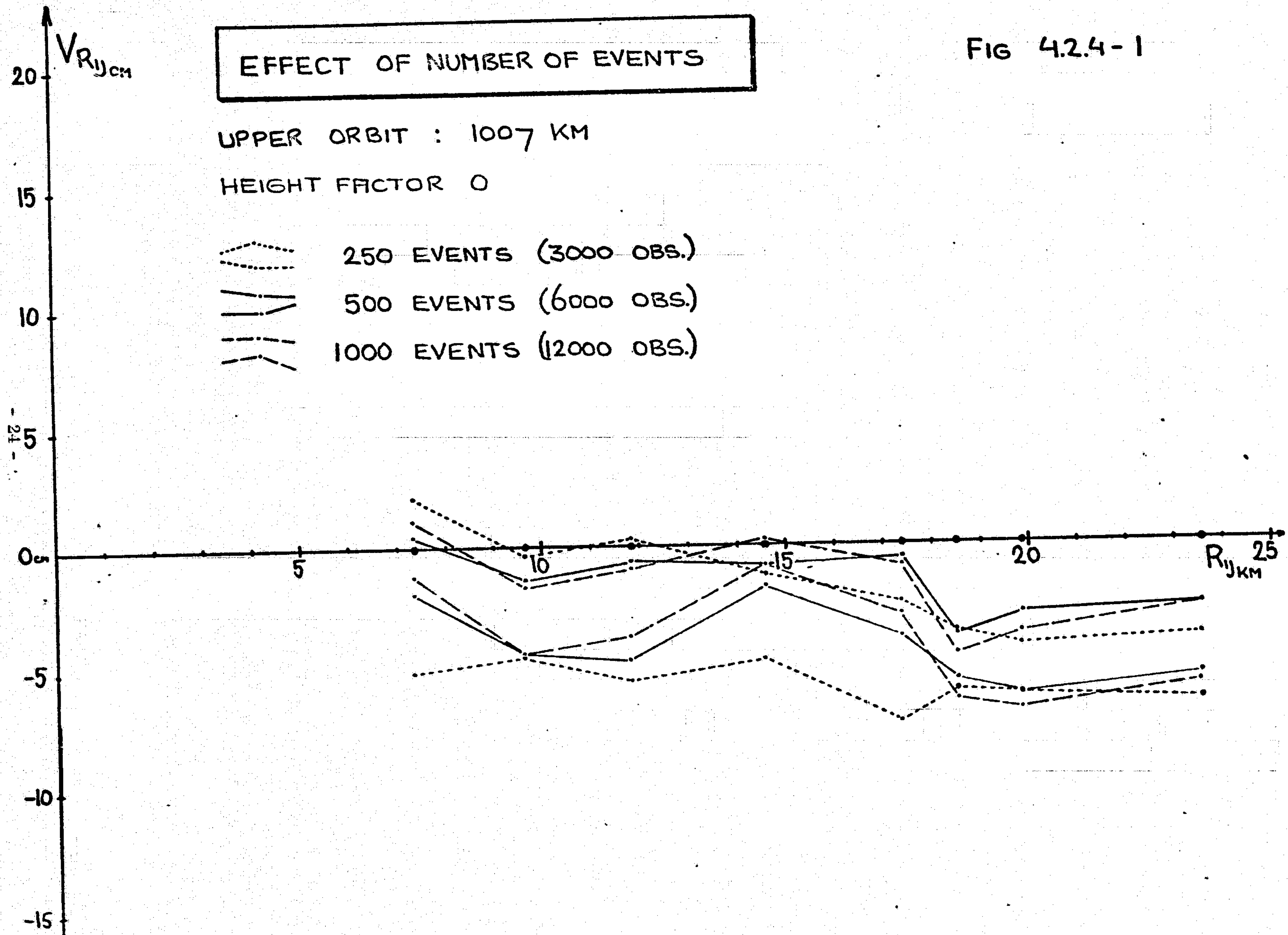
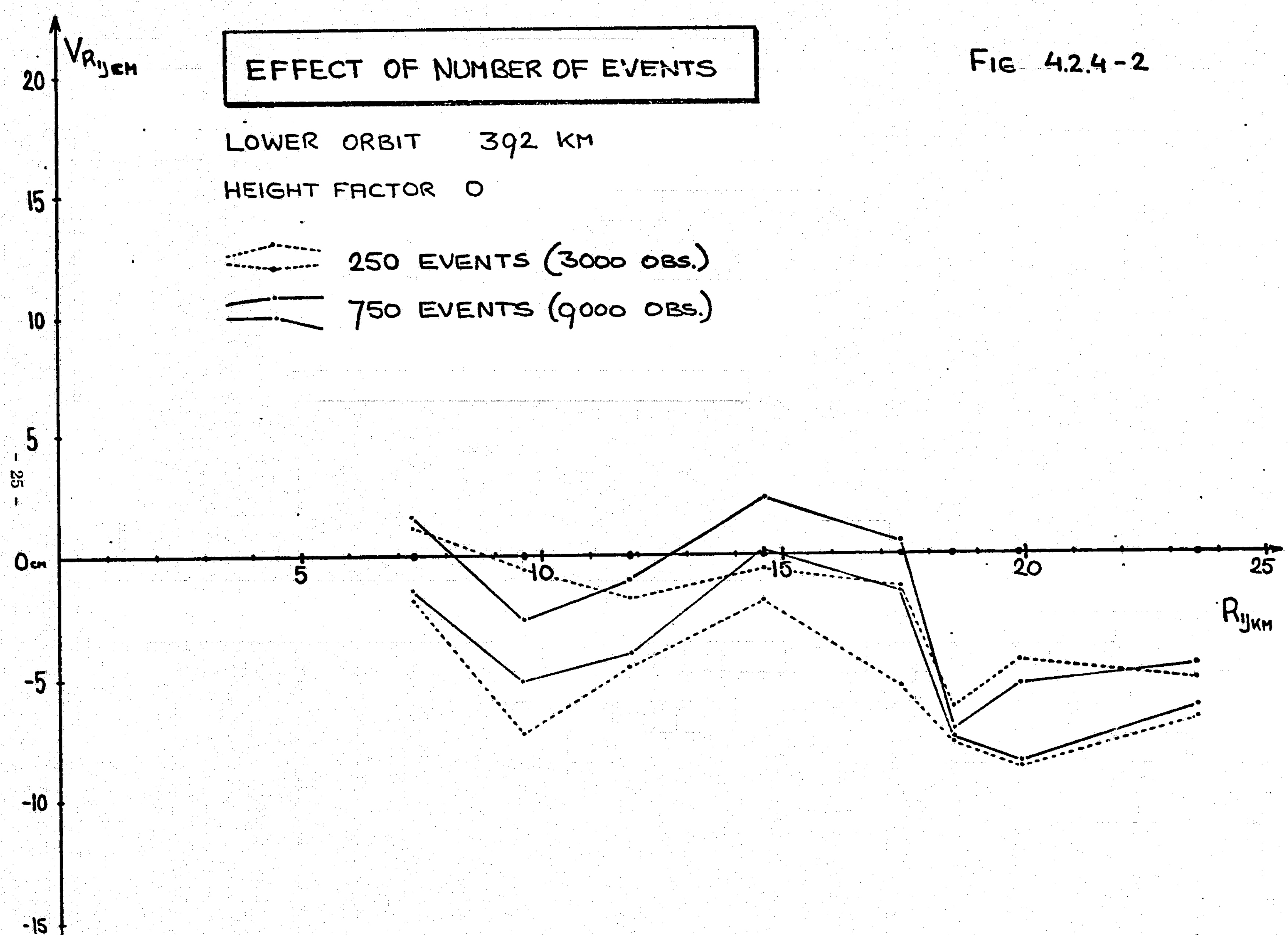


FIG 4.2.4-1


EFFECT OF NUMBER OF EVENTS
UPPER ORBIT : 1007 KM
HEIGHT FACTOR 0
250 EVENTS (3000 OBS.)
500 EVENTS (6000 OBS.)
1000 EVENTS (12000 OBS.)
$V_{R_{ij}cm}$
20
15
10
5
0cm
-5
-10
-15
5
10
15
20
25
$R_{ij}KM$



EFFECT OF NUMBER OF EVENTS
FIG 4.2.4-2
LOWER ORBIT 392 KM
HEIGHT FACTOR 0
250 EVENTS (3000 OBS.)
750 EVENTS (9000 OBS.)
$V_{R_{ij}CM}$
R_{ijKM}
20
15
10
5
0cm
-5
-10
-15
5
10
15
20
25

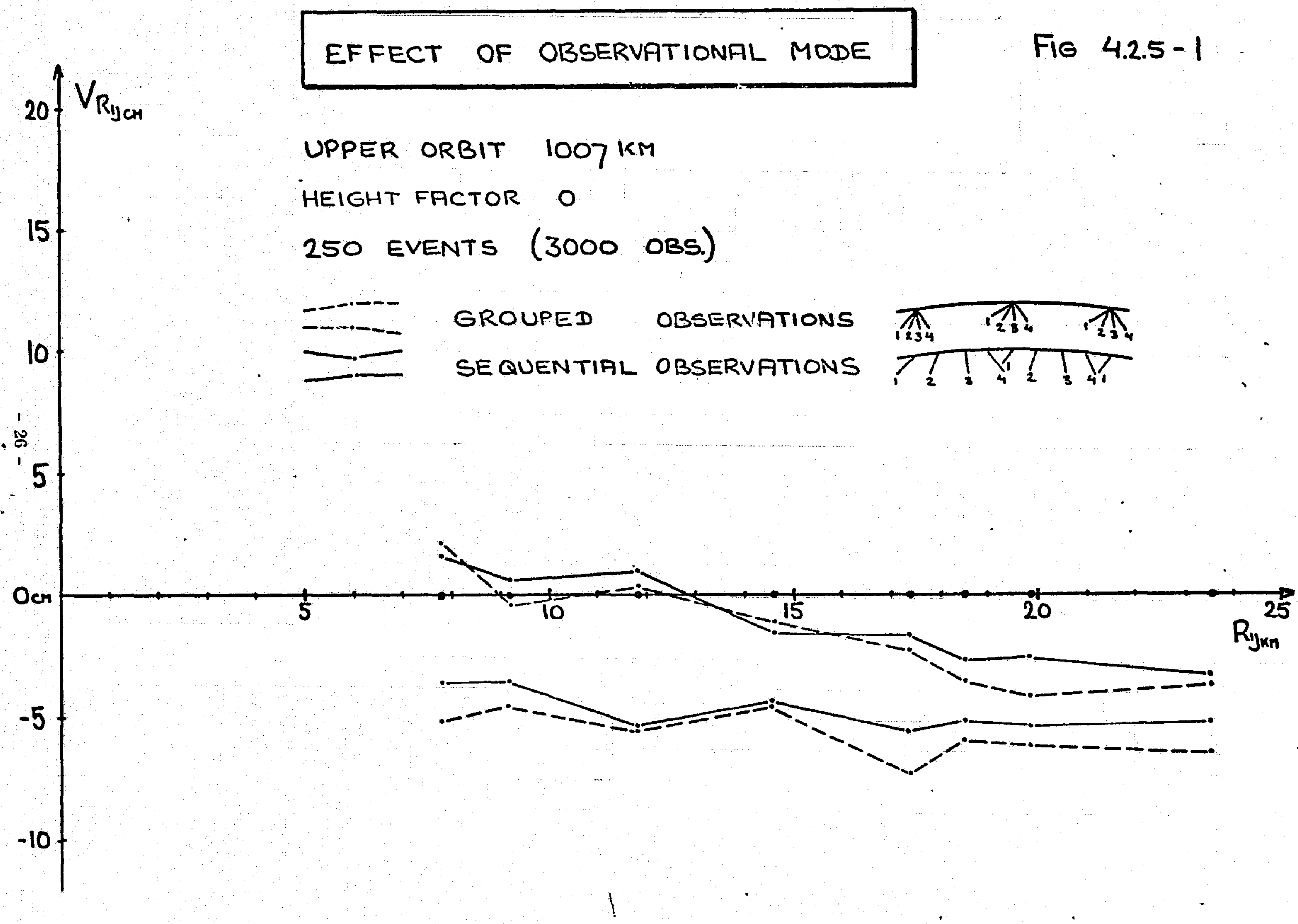
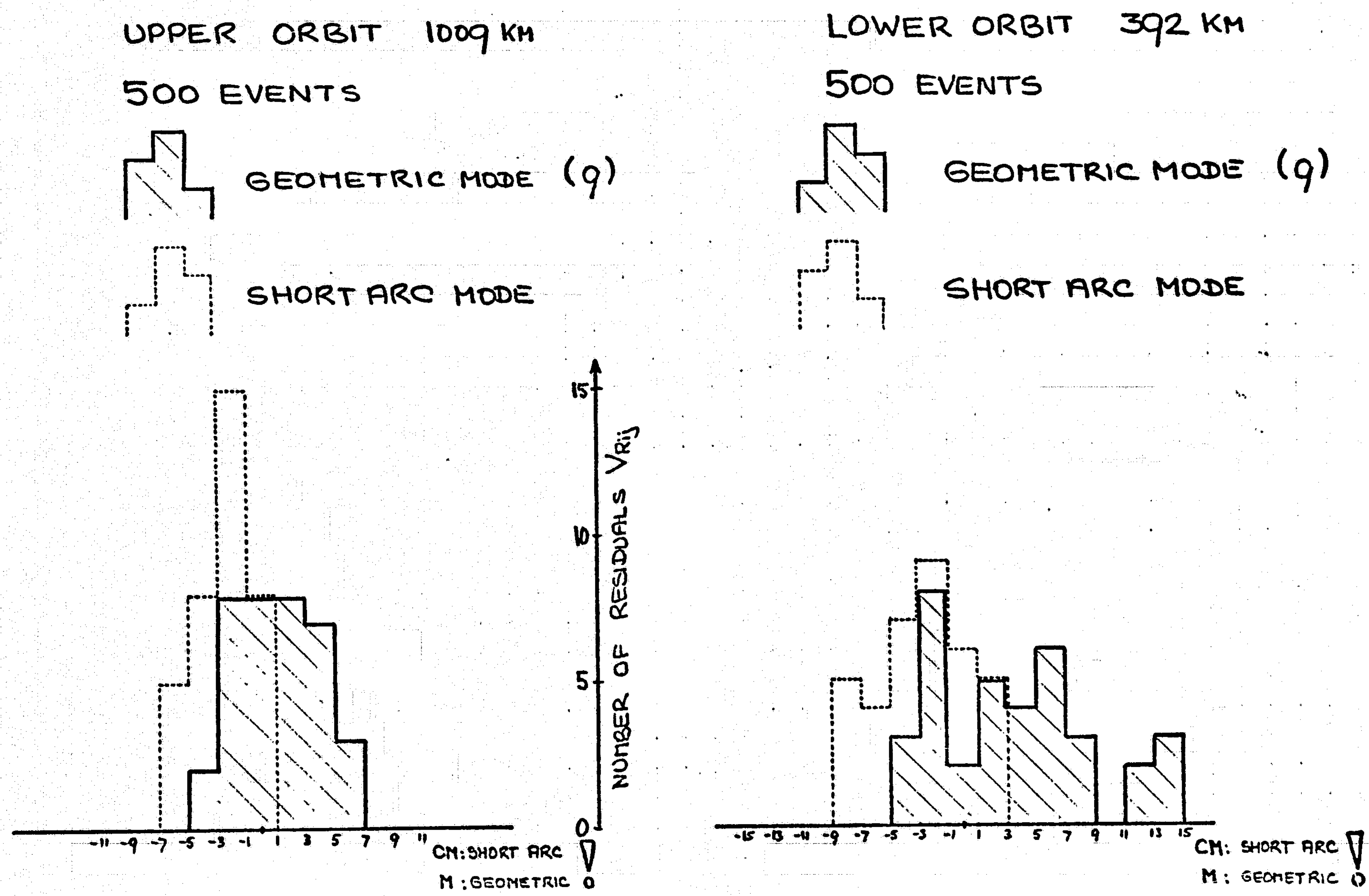

EFFECT OF OBSERVATIONAL MODE
UPPER ORBIT 1007 KM
HEIGHT FACTOR 0
250 EVENTS (3000 OBS.)
GROUPED OBSERVATIONS
SEQUENTIAL OBSERVATIONS
$V_{R_{ij}cm}$
20
15
10
5
0cm
-5
-10
5
10
15
20
25
$R_{ij}km$

FIG 4.2.5-1

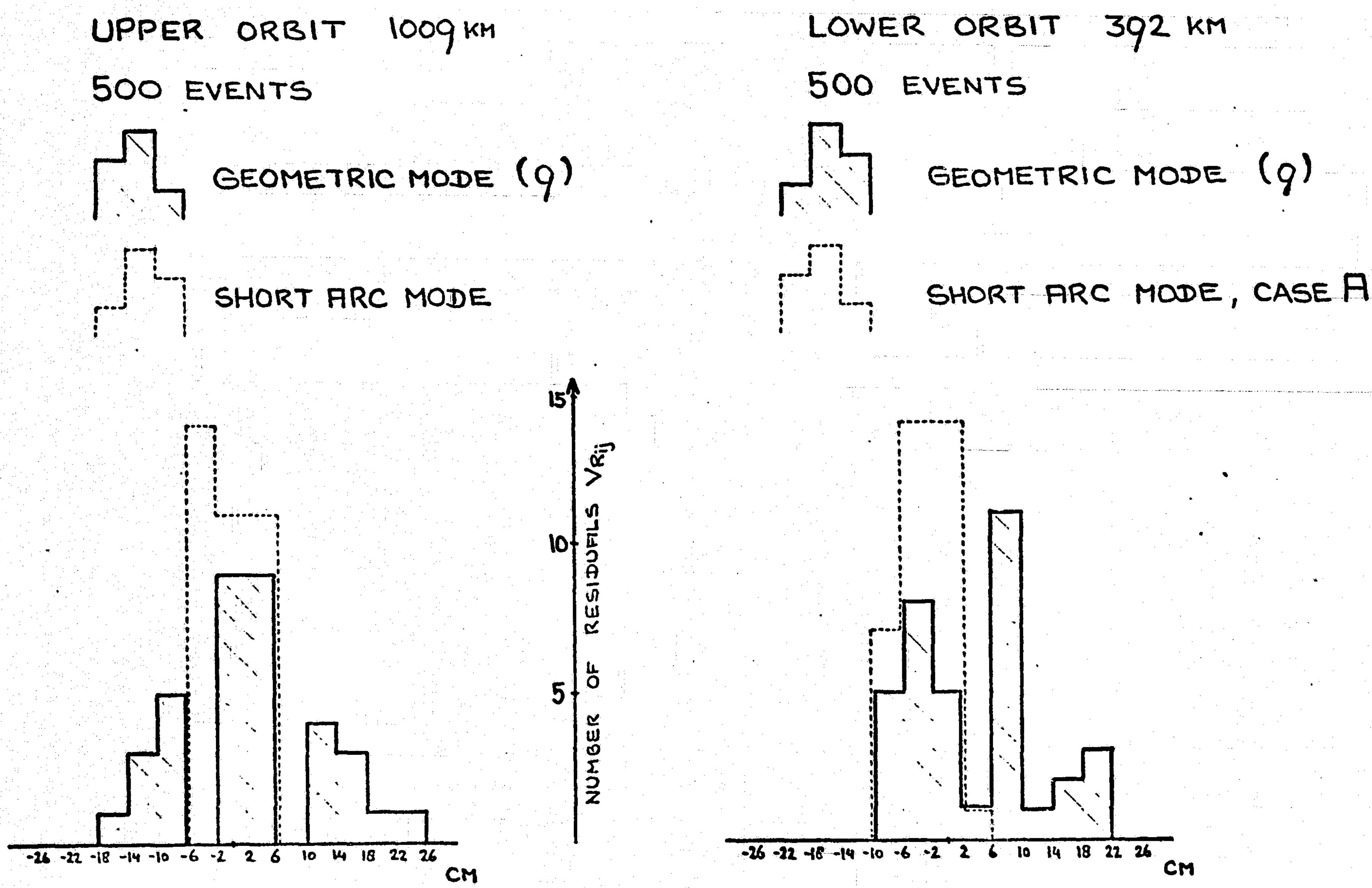
GEOMETRIC MODE VS. SHORT ARC MODE ; HEIGHT FACTOR 0

FIG 4.3-1



GEOMETRIC MODE VS. SHORT ARC MODE ; HEIGHT FACTOR 1000

FIG 4.3-2



GEOMETRIC MODE VS. SHORT ARC MODE

FIG 4.3-3