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CO₂ LASER RANGING SYSTEMS STUDY

FINAL REPORT

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system concept.

This report evaluates the conceptual design and error performance of a CO₂ laser ranging system on this basis. The study identifies ranging signal and subsystem processing alternatives, and their comprehensive evaluation yields preferred candidate solutions which are then analyzed to derive range and range rate error contributions. The performance results are presented in the form of extensive tables and figures that identify the ranging accuracy compromises as a function of the key system design parameters and subsystem performance indices. The ranging errors obtained are noted to be within the high accuracy requirements of existing NASA/GSFC missions with a proper system design.

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SUMMARY

This report presents the results of a CO₂ Laser Ranging Systems Study. The use of laser ranging techniques is motivated by the high-accuracy satellite tracking specifications (0.05 meters in range, 0.005 cm/sec in range rate) required in various NASA/GSFC missions. The use of CO₂ laser technology features high carrier (2.8×10^{13} Hz) and modulation frequencies for accurate ranging, plus CW analog modem compatibility for a unified independent range and range rate data extraction. The range data can be obtained from the propagation delays or phase offsets exhibited by a ranging event associated with the modulation signal, while the range rate data can be obtained from the doppler effect present in the received carrier signal. The independent extraction of range and range data has proven to be a desirable feature in tracking systems from orbital determination considerations.

The use of harmonic sidetone or pseudo-noise code ranging principles have been effectively employed in independent range and range rate tracking systems with the ranging event being identified as the tone zero-crossing or the code epoch. The tone signal involves a simpler subsystem processing for acquisition and tracking, though a set of harmonic tones is required to assure both fine accuracy and ambiguity resolution in the range data. The code signal provides both these features in a single signal, but at the expense of acquisition time and code detectability compromises. The use of a hybrid signal employing a high frequency tone for fine ranging purposes along with a reduced set of lower tones plus one code for ambiguity resolution optimizes the ranging signal design rationale.

The high accuracy tone and the ambiguity resolving signals (lower tones, code subcarrier) should modulate the carrier in a sequential mode, so as to minimize the power sharing compromise between the carrier component (range rate signal) and the modulation sidebands (range signal) in the ranging signal.

The choice of analog modem offers the conventional modulation options (amplitude and/or angle modulation, double or single sideband), plus coupling modulation as an available candidate for laser ranging systems. The receiver processing also offers the options of incoherent envelope detection or coherent product demodulation for the baseband signal extraction. The use of narrowband phase modulation along with coherent product demodulation are shown to provide an effective carrier vs. sideband power sharing compromise, as well as an efficient nonlinear processing that also exploits the available coherent carrier at the receiver. The use of double sideband signaling is also noted to provide error propagation protection when compared to a single sideband signal alternative. The PM carrier vs. sideband power sharing is quantitatively illustrated as a function of the modulation parameters. It is noted that a maximum phase modulation index of 1 radian assigns a minimum 58.55% of the available power to the carrier component and a maximum 38.73% to the major tone sidebands, but an extension to 1.4 radian would reverse the power sharing to a 32.14% minimum for the carrier and a 58.73% maximum for the tone sidebands. Such extended modulation index range is maintained in our CO₂ laser modulation evaluation, since it provides a significant improvement in the range vs. range rate accuracy compromise induced by the sideband vs. carrier power sharing.

The carrier and tone signals are tracked at the receiver via phase-locked loops. These loops involve a design compromise based on the joint interest in wideband tracking for effective doppler dynamic and oscillator jitter reproduction vs. narrowband tracking for additive thermal noise rejection. This tracking compromise is evaluated for 2nd and 3rd order loop realizations involving practical non-ideal integrators in their loop filters, and their loop design rationale and locking error performance are established. The range and range rate error contributions of these locking errors are then formulated after modeling the data extraction operations performed. The error analysis thus permits the tracking loop design to be guided by either a threshold locking error rationale or a tracking data error rationale. In either case, the loop design goals are to provide accurate range and range rate data, while maintaining the locking error below established threshold bounds to assure the signal reproduction.

The carrier loop design and range rate error performance is evaluated to illustrate that 3rd order loop realizations are required to maintain lock for the SNR conditions and orbital dynamic magnitudes expected in the missions. The additive thermal noise error contribution to range rate data is shown to be rather small, with oscillator noise effects yet to be evaluated. The tone loop design and range error performance is also evaluated assuming two possible design approaches. The first case chooses a dynamic locking error bound (μ_m degrees) to assure maximum doppler rate lock, and evaluates the corresponding dynamic ranging error bound (μ_r meters) as a function of the major tone frequency (F_m MHz). The second case reverses the design by choosing (μ_r) and evaluating (μ_m) for each (F_m). In

both cases, the rms locking error (σ_m degrees) and rms ranging error (σ_r meters) due to additive noise are evaluated as a function of the phase modulation index (β radians) and the major tone frequency (F_m MHz). The use of either 2nd or 3rd order loop realizations maintains lock for the tone loop design, and both design cases above are evaluated for both realizations in question. The results are tabulated and plotted as families of curves that show (σ_r) vs. (β) as a function of (F_m), with each family corresponding to one design approach (μ_m or μ_r bound) and one loop realization (2nd or 3rd order).

The tables that follow present some of the additive (thermal) noise range error magnitudes obtained under various logical design conditions, with oscillator noise effects yet to be evaluated. The tabulated results illustrate that small rms range errors within the accuracy specifications can be obtained by proper selection of the modulation frequency, modulation index, and loop order. The rms range error improvement capabilities of 3rd-order loop realizations are evident in each case. The functional dependence between the peak dynamic locking and ranging errors (μ_m , μ_r) is indicated at the end of each table to permit verification of the loop locking status and the dynamic range error contribution in each case.

RMS Additive-Noise Range Errors in Centimeters: $\mu_m = 1$ deg.

(a) Case of $\mu_m = 1$ deg and 2nd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	27.62	15.21	11.99
3	12.12	6.670	5.261
10	4.911	2.704	2.133
30	2.155	1.186	0.9355
100	0.8734	0.4808	0.3792
300	0.3831	0.2109	0.1664

(b) Case of $\mu_m = 1$ deg and 3rd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	9.618	5.295	4.176
3	4.219	2.323	1.832
10	1.710	0.9416	0.7426
30	0.7503	0.4131	0.3258
100	0.3041	0.1674	0.1321
300	0.1334	0.07345	0.05793

Note: $\mu_r = \frac{100 \mu_m}{2.4 F_m} =$ peak dynamic ranging error in cm.

RMS Additive-Noise Range Errors in Centimeters: $\mu_m = 5$ deg.

(a) Case of $\mu_m = 5$ deg and 2nd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	18.47	10.17	8.020
3	8.102	4.461	3.518
10	3.284	1.808	1.426
30	1.441	0.7932	0.6256
100	0.5840	0.3215	0.2536
300	0.2562	0.1411	0.1113

(b) Case of $\mu_m = 5$ deg and 3rd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	6.432	3.541	2.793
3	2.822	1.553	1.225
10	1.144	0.6297	0.4966
30	0.5017	0.2762	0.2179
100	0.2034	0.1120	0.08832
300	0.08922	0.04912	0.03874

Note: $\mu_r = \frac{100 \mu_m}{2.4 F_m} = \text{peak dynamic ranging error in cm.}$

RMS Additive-Noise Range Errors in Centimeters: $\mu_r = 1$ cm.

(a) Case of $\mu_r = 1$ cm and 2nd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	70.16	38.62	30.46
3	23.39	12.87	10.15
10	7.016	3.862	3.046
30	2.339	1.287	1.015
100	0.7016	0.3862	0.3046
300	0.2339	0.1287	0.1015

(b) Case of $\mu_r = 1$ cm and 3rd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	24.43	13.45	10.61
3	8.144	4.483	3.536
10	2.443	1.345	1.061
30	0.8144	0.4483	0.3536
100	0.2443	0.1345	0.1061
300	0.08144	0.04483	0.03536

Note: $\mu_m = \frac{2.4 \mu_r F_m}{100} = \text{peak dynamic locking error in deg.}$

RMS Additive-Noise Range Errors in Centimeters: $\mu_r = 5$ cm.

(a) Case of $\mu_r = 5$ cm and 2nd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	46.92	25.83	20.37
3	15.64	8.610	6.791
10	4.692	2.583	2.037
30	1.564	0.8610	0.6791
100	0.4692	0.2583	0.2037
300	0.1564	0.08610	0.06791

(b) Case of $\mu_r = 5$ cm and 3rd-order loop.

<u>F_m</u>	<u>$\beta=0.5$</u>	<u>$\beta=1.0$</u>	<u>$\beta=1.5$</u>
1	16.34	8.995	7.094
3	5.446	2.998	2.365
10	1.634	0.8995	0.7094
30	0.5446	0.2998	0.2365
100	0.1634	0.08995	0.07094
300	0.05446	0.02998	0.02365

Note: $\mu_m = \frac{2.4 \mu_r F_m}{100} = \text{peak dynamic locking error in deg.}$

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1. General Introduction

The use of CO₂ laser ranging techniques for high-accuracy satellite tracking applications is presently under consideration at NASA/GSFC. Various existing missions involving the GEOS and TIMATION spacecrafts are characterized by rather tight tracking accuracy requirements, such as 0.05 meters in range and 0.005 cm/sec in range rate. These magnitudes represent an improvement in the state-of-the-art ranging capabilities when compared to existing tracking systems. The use of CO₂ laser technology for these purposes represents a natural solution approach, since it is compatible with various analog modem methods, so that one can both exploit the ranging accuracy advantages of high modulating frequencies while maintaining a CW carrier component at 2.8×10^{13} Hz for accurate independent range rate data extraction. The availability of independent range and range rate data for orbital determination purposes has proven to be a desirable feature in tracking system design from error performance considerations.

A study program of the CO₂ laser ranging system was conducted to this effect. The objective was to perform a comprehensive evaluation of the available modulation methods and tracking system design for CO₂ laser ranging, and then to establish their fundamental ranging accuracy limitations when specific orbital dynamics to be tracked and representative SNR level conditions are assumed. The study thus includes both a qualitative analysis to establish the relative merits of various analog modems and subsystem processing alternatives, as well as a quantitative evaluation of the tracking and ranging error performance of the system.

The analytical evaluation and error performance results of this study program are presented in this report. The ranging system design principles that guide and govern the conceptual design and development of high-accuracy tracking systems are first presented to establish a natural foundation. The characteristics of independent range and range rate data extraction are identified, and the ranging signal design and subsystem processing considerations that follow are discussed for our laser ranging application. The range vs. range rate accuracy compromise involved is emphasized to motivate a careful attention to the sideband vs. carrier power sharing compromise in the analog modem candidates.

The ranging signal structure composition is analyzed to evaluate available options, and account for the fine ranging vs. ambiguity resolution operations involved. A hybrid signal composition that features the relative advantages of both harmonic sidetone and pseudo-noise code ranging principles is noted to optimize the ranging signal design compromises, along with the sequential application of the fine ranging and ambiguity resolution operations. The analog modem alternatives are carefully analyzed to establish their power sharing compromises and processing limitations that affect their ranging accuracy performance. The discussion includes all conventional AM, PM and AM/PM approaches, plus coupling modulation as an added option characteristic of CO₂ lasers. The power sharing and ranging error implications of envelope (incoherent) vs. product (coherent) demodulation methods are also evaluated for the modulation approaches in question. The specific limitations added by the laser modulation parameter values achievable are included in the discussion.

The tracking performance evaluation is specified for phase-locked extraction of the carrier (range rate) and tone (range) components of the ranging signal. The tone design parameters, loop design compromises, and tracking error analysis are identified for 2nd and 3rd-order loop realizations. The analysis permits a clear comparison of 2nd vs. 3rd-order loop performance, plus emphasizes the loop design and error performance distinctions between theoretical and practical limitations. The 3rd-order analysis features state-of-the-art technology, plus a unified formulation that readily guides natural extensions and potential improvements. The performance analysis is concentrated on orbital dynamics tracking in the presence of additive thermal noise, with the acknowledgement that oscillator noise effects should be ultimately included and may be significant with the high modulation and laser carrier frequencies involved.

The range and range rate data errors contributed by the loop tracking errors are next evaluated. The analysis requires the careful modeling of the range and range rate extractor processing to properly reflect the effect of additive phase disturbances on the raw data samples obtained. In particular, two range rate extractor realizations are detailed and analyzed to derive the corresponding data error expressions. The report concludes with a quantitative error analysis and discussion based on these results, using typical link parameter specifications to derive the numerical error values. Two different loop design approaches are exhaustively evaluated to account for both locking and ranging error bound options in the formulation. All the numerical results are tabulated for reference purposes, plus families of characteristic performance curves

are included to facilitate the error analysis and logical design of the CO₂ laser ranging system. A glossary of the basic symbols used in this report is included at the end of the report for reference purposes and easy accessibility.

2. Ranging System Design Principles

The conceptual design and development of a spacecraft tracking system is based on the measurement of certain parameters that reflect changes in the spacecraft trajectory, along with an orbital determination program that processes the measured data after some preliminary smoothing or editing operations. A conventional set of measured parameters employed at NASA/GSFC for high-accuracy tracking purposes consists of the slant range and/or range rate of the spacecraft relative to a ground station network used as reference points. The number of stations involved depends on the number of simultaneous measurements to be performed, the orbital dynamics to be tracked, and the accuracy performance to be obtained.

The ground-centralized tracking systems extract the tracking data at the ground stations from the ranging signals received from the spacecraft. These signals may have been originally transmitted by the stations and returned by the spacecraft (two-way tracking systems) or they may originate at the spacecraft itself (one-way tracking systems). Our application specifies a two-way ground-centralized tracking system, where the spacecraft returns a 2.8×10^{13} Hz laser ranging signal back to ground by means of a passive reflector. The simplified signal turnaround that results from a passive transponder bypasses the signal acquisition and processing issues that arise with an active transponder.

The range data (r) may be obtained from the round-trip propagation delay (t_p) of a suitable ranging event associated with the ranging signal. The time elapsed between the transmission and reception of the ranging event is measured at ground, and the range

is derived via the relation $r = \frac{c}{2} (t_r)$, where (c) is the speed of light. The repetition period of the ranging event must be obviously smaller than the round-trip propagation delay to provide unambiguous consecutive range measurements, in the absence of other available information. However, the ranging signal may include ambiguity-resolving features to relax this requirement. In any case, epoch variations in the form of signal processing delays or added timing disturbances will contribute a range measurement error that must be calibrated out if possible or acknowledged as an accuracy limitation otherwise. Any departure from free-space propagation in the space channel will also affect the conversion of the measured delay into true slant range data.

The range rate data ($\dot{r}$) may, in turn, be obtained from the round-trip doppler effect (f_d) exhibited by a ranging signal frequency (f), via the relation $\dot{r} = \frac{c}{2f} f_d$. The use of a large frequency value is motivated for the attenuation of various error contributions in the doppler measurement process, and the link frequency assignment in our application is indeed compatible with this spirit. The presence of frequency drifts will, in general, contribute an apparent doppler effect, and induce a range rate measurement error unless compensated for in the data extraction method. The doppler measurement itself may be affected by the presence of random phase jitter disturbances accompanying the ideal signal phase fluctuations, and unavoidable range rate errors will be introduced as a consequence. A departure from free-space propagation in the space channel will also serve as an accuracy limitation in the conversion of the measured doppler effect into true slant range rate data.

The derivation of range or range rate data from one another is obviously possible. However, the association of the aforesaid ranging event and doppler effect with different components in the ranging signal structure can provide range and range rate data independently, which often represents a desirable feature from error control and information reliability considerations. An unavoidable compromise in the power allocation and accuracy performance then results. The available signal power is distributed among the range and range rate signal components, so that their respective receiver SNR's can only be improved at the expense of one another. The net effect is that the additive noise contribution to range error decreases while its range rate error contribution increases, and vice versa, as various power sharing percentages are considered.

The problem of specifying quantitative preferences to optimize the independent range vs. range rate accuracy compromise becomes rather complex in general. One would have to account for the various options available in orbital dynamics, data rates, error magnitudes, orbital determination procedures, and state-of-the-art technology. The usual approach consists of considering each application in question, acknowledging the existing tracking accuracy capabilities as well as the projected improvements, and then investigating the variation of the range and range rate error magnitudes as a function of the subsystem design parameters to be specified. Such approach can indeed identify diminishing return conditions, where an incremental error improvement in one data is accompanied by a severe error degradation in the other. The method then yields a range of design parameter values that represent favorable choices, though

the order of magnitude of the range and range rate error requirements is assumed a-priori to guide the compromise evaluation.

The conceptual design of the ranging system thus begins with the identification of the ranging signal event and the carrier frequency selected to respectively provide the independent range and range rate tracking data. The ranging signal design then considers the integration of these features into a composite signal propagating through the space link. The ranging signal structure is specified following a careful evaluation of modulation methods available to the system designer, which concludes with a choice among the preferred candidate solutions and the assignment of numerical values to the modulation parameters involved. The parameter assignment involves an unavoidable compromise between the range and range rate accuracy performance of the system, since the available power budget must be shared by the ranging signal components providing the independent range and range rate data.

The use of sidetone zero-crossings or pseudo-noise code formats as the ranging events is compatible with the high-accuracy long-range requirements of our laser tracking application. The doppler effects present in the carrier or clock components associated with these signals are also capable of providing a high range rate resolution. These ranging principles have been reliably employed in the design and development of tracking systems for space applications, and represent state-of-the-art procedures for CW ranging purposes. These procedures are indeed compatible with CO₂ laser modulation principles, which can then benefit from the technological experience and facilities already available.

The selection of a ranging signal structure is also linked to the problem of subsystem specification, since the ground station must generate, transmit, receive and track the ranging signal, plus extract and process the range and range rate data. The ground oscillators used in the transmitter signal generation and receiver signal processing contribute long-term frequency drifts and short-term instabilities that ultimately limit the tracking performance and accuracy capabilities of the system. The analysis of oscillator noise effects is, of course, emphasized by the high frequencies (carrier and modulation) involved in our laser ranging system. The signal processing itself is equally critical in the wideband application envisioned, since the design of the receiver loops required to acquire and track the ranging signal components will involve an unavoidable subsystem design compromise. Wide tracking bandwidths are required to reproduce the large doppler dynamics caused by the spacecraft orbital motion on the high frequencies employed, yet narrow loop bandwidths are also desired to restrict the additive noise filtered along with the signal phase fluctuations. The dynamic range of the received signal level and the effects of corresponding SNR variations on the tracking performance must also be accounted for in the subsystem design to assure above-threshold operation and smooth departures from nominal operating conditions.

The specification of the range and range rate data extraction and processing following the received signal reproduction also represents a significant subsystem design consideration. The range extractor must process the high-accuracy and ambiguity-resolving components of the range signal, so as to efficiently provide the range data without compromising tracking performance insofar as

possible. The range rate data extraction can be conveniently based on doppler cycle-counting methods, which have proven their effectiveness in space tracking applications. The design parameters involved in the measurement and arithmetic units of these data extractors must be carefully chosen to minimize the bias and random error contributions to the range and range rate samples extracted. The output data rates and formats must, in turn, be compatible with the pre-processing smoothing and orbital determination programs employed to track the spacecrafts in question.

3. Ranging Signal Alternatives

The ranging signal design consists of first identifying the ranging signal event and the doppler signal carrier respectively used to derive the independent range and range rate data, and then specifying the modulation principles employed to incorporate their characteristics into a unified ranging signal structure. The quantitative choice of modulation parameters and frequencies can then be based on the accuracy performance requirements of the application in question. There are two basic ranging signal design approaches that are employed in range and range rate tracking systems: harmonic sidetone ranging and pseudo-noise code ranging. These ranging signal alternatives are next discussed, along with their ranging system design implications and joint application capabilities.

3.1 Harmonic Sidetone Ranging Signals

The use of continuous sinusoidal waveforms for ranging purposes is based on identifying the ranging event as the zero-crossing of the sinusoidal pulses. The period (T_s) of the sinusoid then establishes the maximum unambiguous distance that can be resolved as $\left(\frac{c}{2} T_s\right)$ in two-way ranging, where (c) is the speed of light. The use of large periods (low frequencies) is incompatible with high-accuracy requirements, since range errors contributed by random noise effects are inversely proportional to the tone frequency (f_s). For example, a random phase error ($\Delta\phi$) rad in the tone reproduced at the ground receiver corresponds to a range error $\Delta r = \left(\frac{c}{2}\right) \frac{\Delta\phi}{2\pi f_s}$ in two-way ranging. The range error will thus increase with lower frequencies (larger periods) if the phase error in question is

frequency-independent, which is the case for random noise error contributions.

The design compromise between maximum ranging distance and high-accuracy performance is resolved by including other harmonically related sinusoids of lower frequencies in the ranging signal. The highest frequency tone (major tone) is then used to provide small random error contributions to the range data extracted, while the remaining tones (minor tones) are used to resolve the range measurement ambiguities caused by the major tone period. The ambiguity resolving tones are usually harmonically related, i.e., each tone frequency is an integer factor smaller than the preceding one of higher frequency. Every lower frequency tone is then used to resolve the ranging ambiguities of the preceding one of higher frequency, with the lowest frequency tone being capable of performing an unambiguous range measurement by itself. This last tone would yield an excessive random error contribution if used to provide the range data, but its role is simply to resolve the ranging ambiguities of the preceding tone so that its error tolerance is considerably relaxed.

The use of only two tones is theoretically possible: a high-frequency tone (f_h) for accuracy resolution, and a low-frequency tone (f_l) for ambiguity resolution. The practical problem lies in the tight phase error tolerance of the latter if expected to resolve the ranging ambiguities of the former. Indeed, the maximum time error permitted for the low tone is a period ($1/f_h$) of the high tone, so that the low tone phase error magnitude is bounded by $(2\pi f_l) (1/f_h) = 2\pi(f_l/f_h)$ rad. The frequency ratio (f_h/f_l) has a large value for the high accuracy and long distance requirements

of spacecraft tracking, and the phase error bound would be exceeded in practice with just two ranging tones. The use of successive minor tones as explained above then relaxes the phase error bound, since the time error tolerance would then be the period of the next tone rather than that of the major tone. The frequency ratio and total number of minor tones employed is thus governed by the phase error performance in the reproduction of each tone at the ground receiver, so as to permit the successful resolution of the ranging ambiguities of the preceding tone.

The mean-square phase error contribution from random noise effects is inversely proportional to the SNR existing in the tracking loop reproducing the major tone. The presence of the minor tones thus limits the range accuracy performance by taking power away from the major tone loop SNR. This effect can be somewhat reduced by choosing modulation parameters that result in an optimum power sharing between the major and minor tones to achieve the best compromise in range accuracy vs. ambiguity resolution. However, the sequential (rather than simultaneous) transmission of the tones completely eliminates this power sharing compromise and shall be assumed. The ambiguity resolution and fine ranging operations then occur sequentially, and only the major tone modulates the carrier during the fine range measurement. Of course, the power sharing between the tone (range signal) and carrier (range rate signal) remains as an accuracy performance compromise.

The major tone frequency employed is selected from accuracy considerations as previously explained, with higher frequencies yielding smaller random measurement errors for given loop SNR conditions. However, the use of higher frequencies is accompanied by

larger doppler dynamics and oscillator noise effects, which require wider acquisition and tracking bandwidths. These larger bandwidths then reduce the loop SNR and increase the random noise error contribution. An optimum loop design should thus jointly minimize the dynamic lag, oscillator noise and random noise phase errors in the reproduced major tone. Yet the loop optimization process must account for the different statistical nature of the phase error processes involved, so as to distinguish between bias and random errors in the range data extracted. An analogous discussion applies to the minor tone loop design, except that the phase error tolerance is established from ambiguity resolution rather than range accuracy considerations.

The lowest minor tone frequency employed is selected to unambiguously resolve the maximum range expected, and can result in rather low modulating frequencies. The modulation of these low frequencies on the signal carrier can result in signal components rather close to the carrier frequency, which may hinder the desired carrier extraction and ranging signal demodulation at the ground receiver. This situation can be avoided by exhibiting the lowest tone as a complemented bias on another frequency suitable for the signal generation and extraction operations. The use of SSB modulation on another higher frequency tone acting as a subcarrier has been used effectively in ranging system design under the terminology of doubly-complemented sidetones. This technique is applicable to any minor tone whose frequency is too low for a reliable system processing without double complementation.

3.2 Pseudo-Noise Code Ranging Signals

The ranging event in pseudo-random signal consists of the waveform structure itself, and the range measurement is based on the time shift required to match the transmitted and received waveforms. The notion of matching requires a characteristic property or parameter associated with the waveform that provides a reliable recognition of zero delay, plus easily discriminates when time shifted replicas are present. The use of the waveform auto-correlation function for time delay discrimination represents a natural choice since its magnitude peaks at the origin. An arbitrary analog pseudo-random waveform with a peaked auto-correlation function is perfectly valid, but it may introduce excessive storage and processing requirements, since the waveform duration (repetition period) is selected to provide unambiguous ranging to the spacecraft. The use of a coded waveform relaxes this consideration, since the number of digits and not the period in seconds governs the storage and processing complexity.

The use of pseudo-noise codes is thus motivated as an alternative ranging signal. Their autocorrelation function at integer shifts of a symbol duration is given by

$$R(\tau) = \begin{cases} 1 & \text{for } \tau = k P \\ -\frac{1}{L} & \text{for } \tau \neq k P \end{cases}$$

where (k) is an integer, (p) is the code period, and $L = (2^n - 1)$ is the code length or number of code digits in a period. Such auto-correlation function can easily discriminate the matched and mismatched states between transmitted and received codes. The

code period (P) is selected to provide the unambiguous ranging, while the symbol duration (P/L) determines the code bandwidth occupancy. The use of short symbols is desired since the range errors contributed by random noise effects are inversely proportional to the symbol rate (L/P). The latter plays a role analogous to the tone frequency in error control, but without any ambiguity resolution limitations. Indeed, the use of high symbol rates (short symbols) and long code periods are simultaneously possible by having a large code length (L). This represents an extra degree of freedom relative to sidetone ranging, where the tone frequency and the ranging event period were essentially the same parameter for design purposes.

The code length (L) governs the duration of its acquisition or matching process. The maximum number of (L) trials required can be reduced by replacing the code by a boolean combination of subcodes of smaller relatively prime lengths (L_i). The resultant code length is given by the sum $L = \sum L_i$, while the resultant code period is given by the product $P = \prod P_i$. The original code period and unambiguous range capability can thus be maintained with a substantial reduction in the maximum number of test trials involved by acquiring the subcodes separately. However, each subcode matching process will not exhibit the same detection immunity present with the original code autocorrelation function, and therein lies a compromise of acquisition time vs. code detectability in the presence of noise.

The repetition of the code matching process for consecutive range measurements can be avoided by tracking the code after acquisition. This may be implemented with delay-locked loops which track

a demodulated code, or with code-clock double loops that also extract a coherent clock component. Both these processors represent tracking cross-correlator realizations analogous to the conventional phase-locked loops employed to track the sinusoidal tones. However, the simpler processing and reliable experience available with the conventional loops represents an advantage of tone ranging over code ranging from acquisition and tracking considerations.

The long-range ambiguity resolution advantages of code ranging can be maintained without sacrificing the desired tone ranging features by having the code replace the lower frequency tones. This hybrid ranging signal then derives the fine range measurement from the major tone, and resolves range ambiguities with some intermediate minor tones plus the code. One of these minor tones can also be used as a code subcarrier to derive a clock component. The reduction in the number of minor tones employed when compared to the pure sidetone case is particularly desirable in our application, where the large major tone frequency required from accuracy considerations would induce a large number of such minor tones to effectively resolve all ambiguities.

4. Signal Modulation and Processing

The use of harmonic sidetone and/or pseudo-noise code as ranging signals represents an effective approach towards an independent range and range rate system design. A hybrid ranging signal composition is most advantageous to exploit both the tone and code signal features. The sequential application of the fine ranging and ambiguity resolving operations is also desirable to minimize the power sharing compromise between the range and range rate signal components. Under these conditions, the ranging signal structure during the fine ranging operation consists of the laser carrier modulated by a single ranging tone whose frequency is large enough to meet the range accuracy requirements. The doppler effect in the received carrier then provides the range rate data, and the phase offsets in the demodulated tone provide the range data independently.

The modulation of the ranging tone on the carrier presents various choices to the system designer. The use of amplitude modulation or narrowband phase modulation results in a double sideband spectrum around the carrier frequency, without any baseband signal bandwidth expansion. The use of a single sideband spectrum would further reduce the channel bandwidth, but the demodulation process may introduce a severe degradation in the range accuracy performance as illustrated below. The use of wideband phase modulation is not effective since it expands channel bandwidth while setting a limitation on carrier power allocation. The use of coupling modulation represents another alternative in laser ranging systems, and its characteristics are also discussed below.

4.1 Amplitude Modulation Systems

The use of AM techniques can be considered with the provision that a carrier component is maintained to permit the independent range rate data extraction at the receiver. The AM signal without carrier suppression can be formulated as

$$v_{AM}(t) = A \left[1 + \alpha m(t) \right] \cos(\omega_c t + \theta_c) \quad (4.1a)$$

$$= A \cos(\omega_c t + \theta_c) \pm \frac{\alpha A}{2} \cos[(\omega_c \pm \omega_m)t + \theta_c \pm \theta_m]$$

$$\text{for } m(t) = \sin(\omega_m t + \theta_m) \quad (4.1b)$$

The modulation factor (M.F.) is defined as (α) times the peak value of the modulation $m(t)$, which reduces to $M.F. = \alpha$ in our sinusoidal case. In turn, the peak factor (P.F.) is defined as the ratio of the peak to rms values of the modulation $m(t)$, which reduces to $P.F. = \sqrt{2}$ in our case. The average power in the AM signal can then be evaluated in terms of these factors to yield

$$\overline{v_{AM}^2(t)} = \frac{A^2}{2} \left[1 + \alpha^2 \overline{m^2(t)} \right] = \frac{A^2}{2} \left[1 + \left(\frac{M.F.}{P.F.} \right)^2 \right] \quad (4.2a)$$

$$= \frac{A^2}{2} \left(1 + \frac{\alpha^2}{2} \right) \text{ for } m(t) = \sin(\omega_m t + \theta_m) \quad (4.2b)$$

where the derivation assumes modulation frequencies well below the carrier frequency, plus zero d-c value in the modulation. Under these conditions, the total average power decomposes into a carrier term $\frac{A^2}{2}$ plus a sideband term $\frac{A^2}{2} \left(\frac{M.F.}{P.F.} \right)^2$. The ratio (M.F./P.F.) then governs the power allocation between the sideband (range) and the carrier (range rate) components in the ranging signal. Such ratio reduces to $\alpha^2/2$ in our sinusoidal modulation case, so that the $M.F. = \alpha$ becomes the power sharing design parameter.

The use of envelope demodulation for the modulation extraction at the receiver requires $M.F. \leq 1$, so as to avoid carrier phase reversal and permit the modulation recovery. The conditions $M.F. \leq 1$ and $P.F. \geq 1$ then illustrate that no more than 50% of the total power can be allocated to the sidebands in an AM signal with envelope detection. In the sinusoidal modulation case, the conditions become $M.F. = \alpha \leq 1$ and $P.F. = \sqrt{2}$, so that no more than 33% of the total power is available to the sideband (range) components. These constraints on the power allocation between the sidebands (range) and carrier (range rate) components are a direct consequence of the envelope demodulation assumed. The interest in such receiver processing is further diminished by noting that the carrier extraction is anyway required for range rate purposes regardless of the tone demodulation process. The carrier extraction via phase-locked techniques ideally yields a coherent carrier replica at the receiver, which can then be used for product demodulation purposes. This approach does not impose any a-priori conditions on the modulation factor, hence relaxing the power sharing limitation inherent in envelope demodulation.

The product demodulation of the AM signal with a coherent carrier yields the modulation at the receiver. In the sinusoidal modulation case, the demodulated tone may be jointly tracked and filtered with a phase-locked loop to permit continuous range estimation. The modulation parameter (α) is then selected to provide enough sideband SNR for the tone phase-locked extraction and accurate range measurement, while maintaining enough carrier SNR for the carrier phase-locked extraction and accurate range rate measurement. The quantitative analysis should account for the 3db improvement

in the tone SNR due to the coherent sideband folding, as well as the degradation introduced by the lack of ideal coherence in the reproduced carrier replica. A carrier locking error ($\Delta\phi$) causes an amplitude attenuation by $\cos(\Delta\phi)$ in the demodulated tone, with a corresponding SNR degradation by a $\cos^2(\Delta\phi)$ factor.

4.2 Angle Modulation Systems

The constant envelope features of PM or FM techniques have motivated their use in existing range and range rate tracking systems. The AM signaling approach may require a low-level operation to avoid nonlinear distortion or saturation effects, whereas angle modulation permits efficient nonlinear amplification provided AM/PM conversions are controlled. The distinction between PM and FM systems is particularly absent in the narrowband sinusoidal modulation case, using coherent product demodulation at the receiver for the tone extraction.

The use of a low modulation index (β) in the PM signal establishes a correspondence with the AM case when product demodulation is employed. The PM signal may be formulated:

$$v_{PM}(t) = A \cos [\omega_c t + \theta_c + \beta m(t)] \quad (4.3)$$

$$\approx A \cos (\omega_c t + \theta_c) - \beta A m(t) \cos (\omega_c t + \theta_c) \text{ for } \beta \ll 1 \quad (4.4a)$$

$$= A \cos (\omega_c t + \theta_c) \mp \frac{\beta A}{2} \sin \left[(\omega_c \pm \omega_m) t + \theta_c \pm \theta_m \right] \\ \text{for } \beta \ll 1, m(t) = \sin (\omega_m t + \theta_m) \quad (4.4b)$$

so that a comparison of (4.4a) and (4.4b) with the corresponding AM signals expressions in (4.1a) and (4.1b) shows that the only difference is a quadrature carrier in the PM case vs a cophasal carrier in the AM case. Such distinction does not affect the power

content, so that the results of (4.2a) and (4.2b) are also applicable with the parameters (β) and (α) interchanged. The disadvantage of low-index PM signaling is then that the power allocated to the sideband (range) components is a small fraction of the total power available.

The use of arbitrary index values does not have a general compact formulation, but for sinusoidal modulation it can preserve the narrowband spectrum (carrier plus one sideband pair) without introducing extreme power sharing constraints. The PM signal can be decomposed as a carrier plus an infinity of sideband pairs as

$$v_{PM}(t) = A \cos \left[\omega_c t + \theta_c + \beta \sin(\omega_m t + \theta_m) \right] \quad (4.5a)$$

$$= A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos \left[(\omega_c \pm n\omega_m t) + \theta_c \pm n\theta_m \right] \quad (4.5b)$$

where image sidebands have the same magnitude since the Bessel coefficients satisfy the symmetric relation $J_{-n}(\beta) = (-1)^n J_n(\beta)$.

The total average signal power is always given by $A^2/2$ regardless of the index value, since $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$ for all β . Each individual sideband component power is down by a factor $J_n^2(\beta)$ from the total power, so that the modulation index (β) governs the power sharing distribution between the carrier and all the sidebands. The use of $0 < \beta < 1.5$ further assures that only the carrier ($n = 0$) and the first sideband pair have a significant power allocation as illustrated by Figure 1 and Table 1. This case results in a narrowband spectrum without the $\beta \ll 1$ constraint as a consequence of the Bessel functions involved. The use of larger modulation indices would waste power in higher order sidebands, so that wideband angle modulation is not considered.

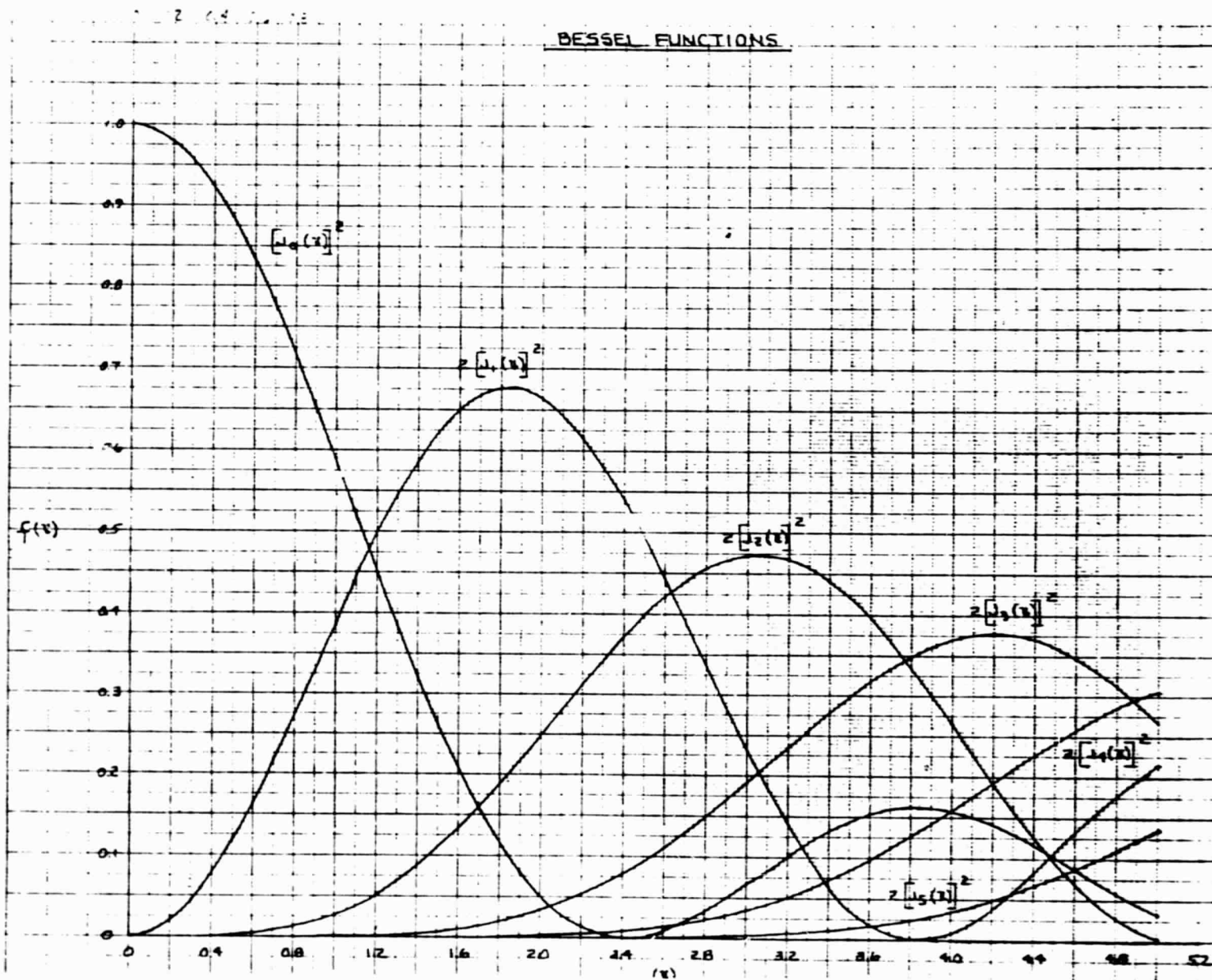


Figure 1(a)
PM Carrier vs. Sideband Power Allocation

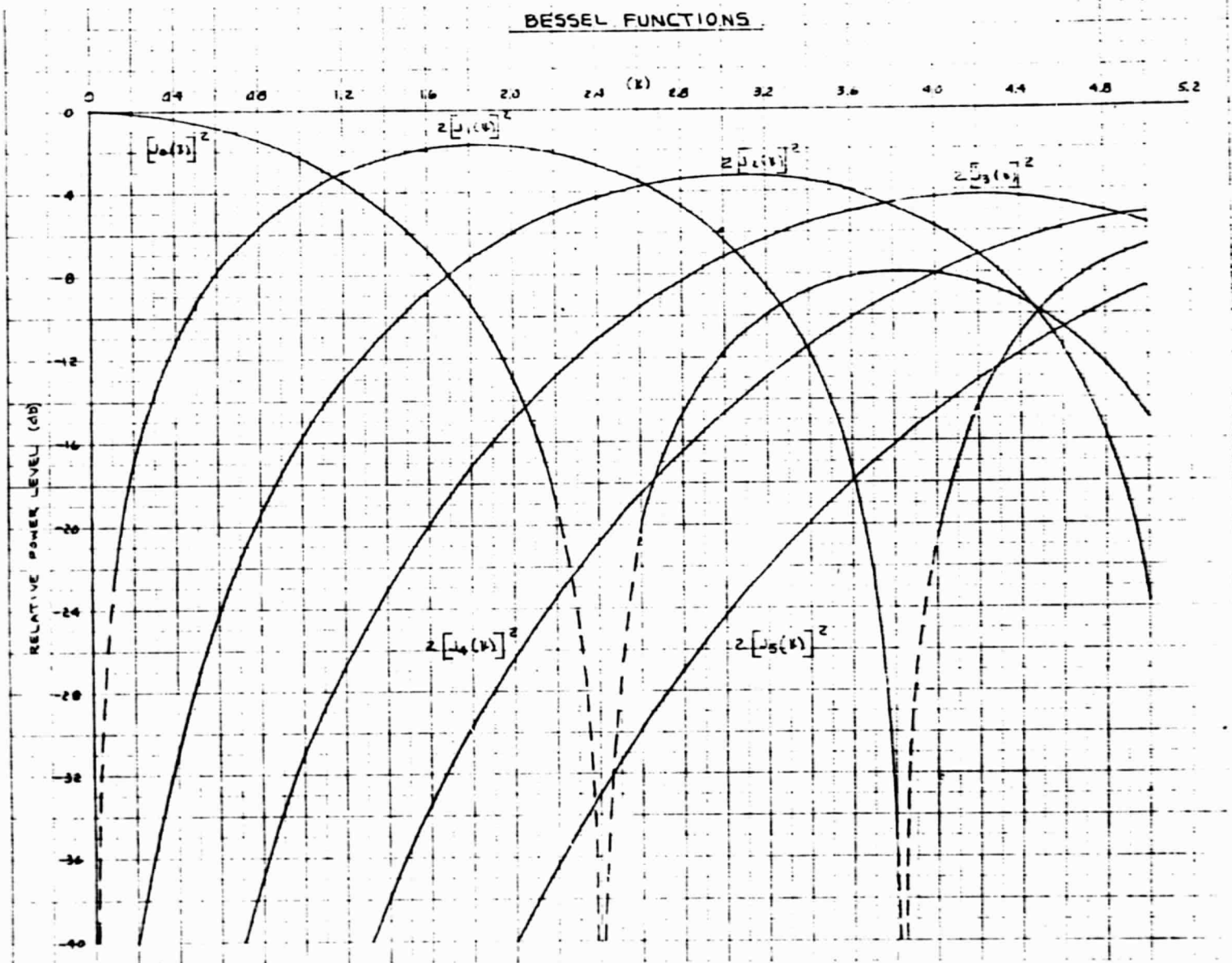


Figure 1(b)

PM Carrier vs. Sideband Power Allocation

TABLE 1
PM Carrier vs Sideband Power Sharing

β	$J_0^2(\beta)$	$2J_1^2(\beta)$	$\sum_{n=2}^{\infty} 2J_n^2(\beta)$
0.0	100.00%	0.00%	0.00%
0.1	99.50	0.50	0.00
0.2	98.01	1.98	0.01
0.3	95.57	4.39	0.04
0.4	92.24	7.68	0.08
0.5	88.08	11.74	0.18
0.6	83.17	16.44	0.39
0.7	77.65	21.65	0.70
0.8	71.62	27.20	1.18
0.9	65.21	32.95	1.84
1.0	58.55	38.73	2.72
1.1	51.78	44.35	3.87
1.2	45.04	49.66	5.30
1.3	38.46	54.49	7.05
1.4	32.14	58.73	9.13
1.5	26.19	62.25	11.56
1.6	20.73	64.96	14.31
1.7	15.84	66.77	17.39
1.8	11.56	67.63	20.81
1.9	7.94	67.56	24.50
2.0	5.01	66.52	28.47

The tone demodulation is again performed via phase-locked carrier extraction followed by coherent product demodulation and phase-locked tone filtering. The only distinction with the product demodulation operation of AM signals consists of a quadrature phase shift on the reproduced carrier to derive a coherent carrier replica. The modulation index (β) is selected to provide enough carrier and tone SNR for locking purposes, as well as accurate range and range rate data. Again, the 3db SNR processing gain, and the $\cos^2(\Delta\phi)$ tone SNR degradation in the presence of carrier locking errors ($\Delta\phi$) have to be accounted for in the analysis. The use of narrowband PM can thus match the signal bandwidth, power sharing and receiver detection characteristics of AM with coherent product demodulation, plus provide a constant envelope signal for effective processing.

4.3 Single Sideband Systems

The spectral occupancy of the ranging signal can be minimized by using SSB signals. Such signals can be obtained by either filtering the undesired sideband from a DSB spectrum, or by the phase shift method of SSB signal generation. The latter is based on the recognition that a SSB signal is a joint AM-PM signal, which can be decomposed as the sum of two AM-DSB signals on quadrature carriers. The two quadrature modulations are related via a Hilbert Transform, which simplifies to a simple 90° phase shift in the sinusoidal modulation case. The net effect of the dual modulation is the cancellation of the undesired sideband component.

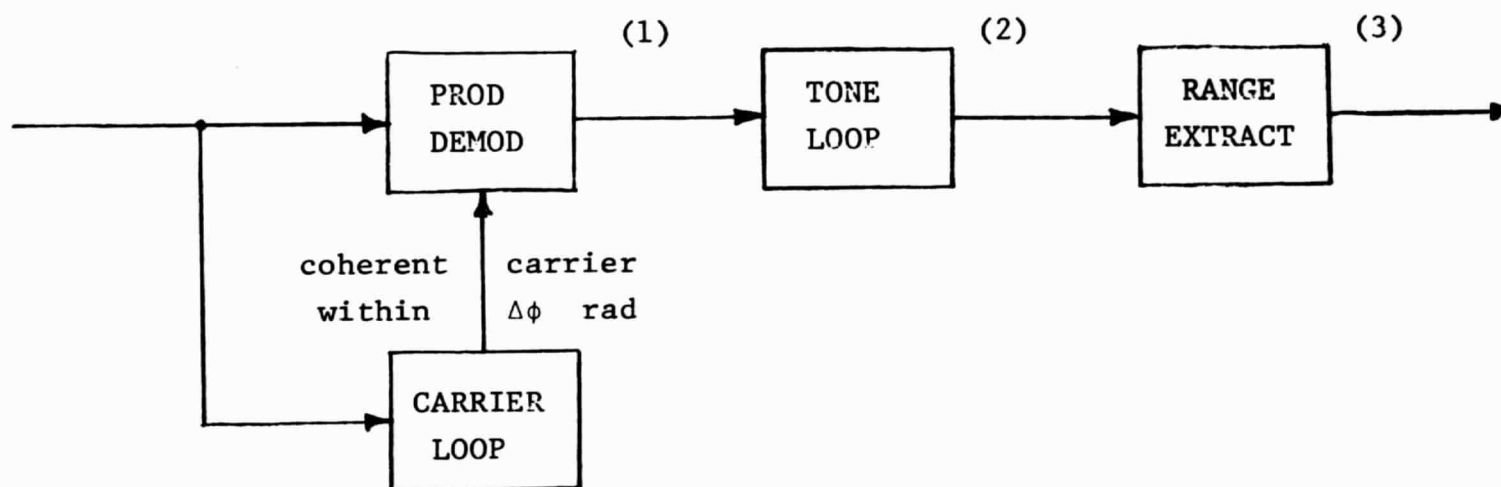
The tone demodulation at the receiver proceeds as before through coherent product demodulation. There is no 3 db SNR

processing gain since no sideband folding occurs, but this is exactly compensated by the extra amount of power that can be put in the single sideband component since the other sideband is not transmitted. However, the presence of a carrier locking error will produce rather different and harmful effects when compared to the DSB case. The product demodulation will now transfer the carrier phase error ($\Delta\phi$) to the demodulated tone as an additive phase shift, rather than as the amplitude attenuation effect obtained in the DSB case.

A comparison of both effects is illustrated in Figure 2. The SSB case introduces a time error of $\Delta t = \frac{\Delta\phi}{\omega_m}$ in the reproduced tone zero crossing, with a corresponding range error of $\Delta r = \frac{c}{2\omega_m} (\Delta\phi)$. Notice that these errors are essentially magnified by the carrier/tone frequency ratio since the phase error ($\Delta\phi$) pertains to the carrier reproduction, yet it is divided by the tone frequency (not the carrier frequency) to produce the time and range errors. The fact that both carrier and tone locking errors appear as additive range errors via the same conversion factor is a considerable degradation relative to the DSB case. The latter limited the carrier effect to a decrease in the tone SNR by $\cos^2(\Delta\phi)$, which only increases the rms error contribution of thermal noise alone by a $\sec(\Delta\phi)$ factor, but does not introduce any additive magnified error effects. The use of SSB signals is not employed in independent range and range rate tracking systems on this basis.

4.4 Coupling Modulation Systems

The previous approaches have always identified the range tone as the modulating function on the carrier. The modulation process



(1) DSB: $\cos(\Delta\phi) \sin(\omega_m t + \theta_m)$
 SSB: $\sin(\omega_m t + \theta_m + \Delta\phi)$

(2) DSB: $\sin(\omega_m t + \theta_m + \Delta\theta_m)$
 SSB: $\sin(\omega_m t + \theta_m + \Delta\theta_m + \Delta\phi)$

(3) DSB: rms range error due to additive noise increases by a $\sec(\Delta\phi)$ multiplicative factor
 SSB: total range error increases by a $\Delta\phi$ additive term.

Figure 2: Carrier Locking Error Effects in DSB vs. SSB Systems

produced a carrier plus first-order sidebands as the desired spectrum, with the tone frequency appearing as the differential frequency between carrier and sidebands. Such spectrum could also have been obtained by having the modulating function be another periodic function containing the tone frequency as the fundamental or a harmonic of it. For example, a rectangular pulse train with its fundamental as the tone produces a PM spectrum that can be decomposed in the form of (4.5b), except that the coefficients are now given by:

$$C_n(\beta) = \frac{\beta}{\beta+n} \operatorname{sinc} \frac{\pi}{2} (\beta - n) \text{ instead of the } J_n(\beta) \text{ Bessel coefficients.}$$

The use of coupling modulation is representative of such approach. It essentially consists of an AM system, where the modulating function consists of a sinusoidal functional of the tone frequency. The modulated waveform can be decomposed as a carrier plus sidebands as follows:

$$v_{cm}(t) = A \sin \left[\gamma_0 + \gamma_1 \sin (\omega_m t + \theta_m) \right] \cos (\omega_c t + \theta_c) \quad (4.6a)$$

$$\begin{aligned} &= A_0(\sin \gamma_0) J_0(\gamma_1) \cos (\omega_c t + \theta_c) \\ &+ A_0(\sin \gamma_0) \sum_{\substack{n=2 \\ \text{even}}}^{\infty} 2J_n(\gamma_1) \cos \left[(\omega_c \pm n\omega_m) t + \theta_c \pm n\theta_m \right] \\ &+ A_0(\cos \gamma_0) \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} 2J_n(\gamma_1) \cos \left[(\omega_c \pm n\omega_m) t + \theta_c \pm n\theta_m \right] \quad (4.6b) \end{aligned}$$

The power sharing distribution is governed by the two modulation parameters (γ_0) and (γ_1). The case of $\gamma_0 = 0$ yields a suppressed carrier, which is undersired in an independent range and rate system, plus prevents coherent product demodulation even if no range rate data is to be extracted. The case of $\gamma_0 = \frac{\pi}{2}$ suppresses all odd sidebands, so that the logical power sharing compromise involves

$J_0^2(\gamma_1)$ for the carrier vs. $2J_2^2(\gamma_1)$ for the $n = 2$ sidebands. This case yields an inferior power sharing option when compared to the $J_0^2(\beta)$ vs. $2J_1^2(\beta)$ compromise existing in PM systems. The minimum bandwidth spectral allocation of a carrier plus first order ($n = 1$) sidebands results in a power sharing compromise of $J_0^2(\gamma_1) \sin^2 \gamma_0$ vs. $2J_1^2(\gamma_1) \cos^2 \gamma_0$, which again represents an inferior option relative to PM systems. In any case, it should be noted that coupling modulation does not yield a constant total power waveform, since the relation $J_0^2(\gamma_1) + 2 \sum_{n=1}^{\infty} J_n^2(\gamma_1) = 1$, cannot be applied; i.e., the total average signal power is given by

$$\overline{v_{cm}^2(t)} = \frac{A^2}{2} \left\{ (\sin^2 \gamma_0) \left[J_0^2(\gamma_1) + 2 \sum_{\substack{n=2 \\ \text{even}}}^{\infty} J_n^2(\gamma_1) \right] + (\cos^2 \gamma_0) \left[2 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} J_n^2(\gamma_1) \right] \right\} \quad (4.7)$$

4.5 Laser Modulation Specifications

The previous discussions on the ranging signal structure and the power sharing compromises involved should also account for the allowable parameter values. The laser specifications limit the modulation frequency to 5 MHz for AM and 20 MHz for PM, and the channel bandwidth to 300 MHz. The modulation index is also restricted to 1 rad maximum in the PM case. The lower modulation frequency allowed for AM will restrict its random range error accuracy performance, since the latter has rms values inversely proportional to the tone frequency. However, a higher modulation frequency will imply larger doppler dynamics to be tracked by the tone loop, hence wider loop bandwidths and increased phase noise locking errors due to

additive thermal noise. The error analysis should thus carefully account for both locking and accuracy limitations when selecting the tone frequency. A quantitative evaluation is required to establish which of these two effects plays a more predominate role.

The channel bandwidth limitation is a secondary effect since narrowband conditions are present in AM and are desired in PM for effective power sharing. The modulation index restriction in PM is a rather important specification since it limits tone power allocation (and range accuracy performance) as previously illustrated in Table 1. The maximum index value of 1 radian would assign 58.55% of the available total power to the carrier, 38.73% to the tone sidebands, and 2.72% to the higher order sidebands. An increase of the allowable index value to 1.4 radian would essentially reverse the allocation to 32.14% to the carrier, 58.73% to the tone sidebands, and 7.05% to the higher order sidebands. Such an extension of the modulation index range is noted to result in a significant improvement in the power sharing options available in PM systems. The quantitative analysis shall include such extended range to project error performance capabilities of the laser ranging system under consideration.

5. Phase-Locked Loop Tracking Performance

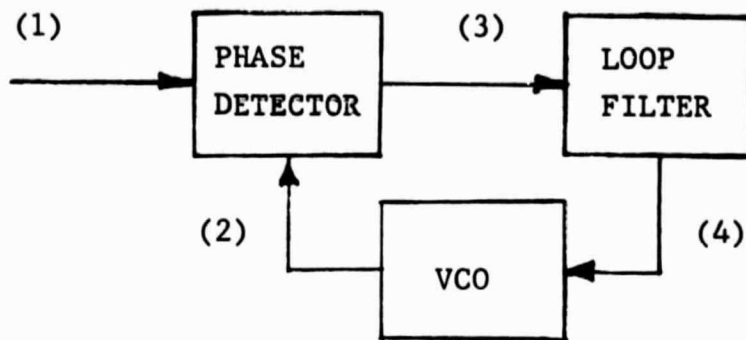
The ranging signal spectrum essentially consists of a carrier component plus a range tone sideband pair during the fine ranging operation. The receiver then phase locks to the carrier component and uses it to coherently product demodulate the range tone. The latter is also tracked with a phase locked loop and used to provide range data, while the extracted carrier provides independent range rate data. It should be emphasized that even if no independent range rate data is to be derived, the carrier component must have sufficient power to permit carrier phase lock in the presence of additive noise, so as to assure an effective coherent demodulation of the range tone. Hence, the carrier power allocation is established from carrier lock and range rate accuracy considerations (whichever predominates). while the tone power allocation is established from tone lock and range accuracy considerations (whichever predominates).

The phase locked loop designs must account for both the acquisition and tracking operations. The acquisition design must assure that the loop can acquire the maximum doppler plus drift conditions expected in the presence of additive noise with a low probability of failure in a moderate time. The design is based on existing data on the probability of acquisition as a function of the loop SNR, loop parameters and initial frequency offset to be captured. The use of manual or automatic acquisition aids such as VCO sweeps can be exploited to considerably ease the acquisition process and relax its loop design compromises. Once the signal acquisition has been successfully accomplished, the loops behave as equivalent linear time invariant filters on the input signal phase fluctuations plus additive or oscillator noise effects. The tracking

design is then based on maintaining the locked state with a small phase error. The loop design involves a basic compromise: wideband loops are effective in tracking doppler dynamic and oscillator jitter phase inputs, yet narrowband loops are desired for additive phase noise rejection. The phase error contributions of these superimposed effects should maintain the locked state and validate the linear filter model.

The baseband block diagram and linear phase transfer model applicable to phase-locked loops is shown in Figures 3(a) and 3(b). The input signal $s(t)$ exhibits a time-varying phase $\phi_i(t)$ assumed to include all oscillator jitter (including the VCO effects) besides the doppler phase dynamics to be reproduced. The VCO reference signal $r(t)$ produces an estimate of such input phase in its output phase $\phi_o(t)$, and the phase detector develops an error signal $u(t)$ which is a (2π) periodic function $f(\phi)$ of the phase estimation error $\phi(t) = \phi_i(t) - \phi_o(t)$. The additive noise $n(t)$ accompanying the input signal is decomposed as cophasal and quadrature components about the nominal signal phase, and it contributes an additive noise component $n'(t)$ at the phase detector output. The loop filter $F(s)$ produces a VCO control signal $v(t)$ which ideally causes a proportionate change in the VCO offset frequency $\dot{\phi}_o(t)$ from its free-running value.

The small phase error conditions prevailing in the locked state permit the linearization of the phase detector characteristic $f(\phi) \approx \phi$ to obtain the linear phase transfer model. The output phase estimate $\phi_o(s)$ and phase error $\phi(s)$ transforms are then related to the signal phase $\phi_i(s)$ and equivalent phase noise $\phi_n(s)$ input transforms through the transfer functions



$$(1) \quad s(t) + n(t) = A \sin \left[\omega_c t + \phi_i(t) \right] + n_c(t) \cdot \sin \omega_c t + n_q(t) \cdot \cos \omega_c t$$

$$(2) \quad r(t) = \cos \left[\omega_c t + \phi_o(t) \right] = \cos \left[\omega_c t + K_v \int_{-\infty}^t v(t) dt \right]$$

$$(3) \quad u(t) = AK_p f \left[\phi_i(t) - \phi_o(t) \right] + K_p n'(t)$$

$$(4) \quad v(t) = F(s) u(t)$$

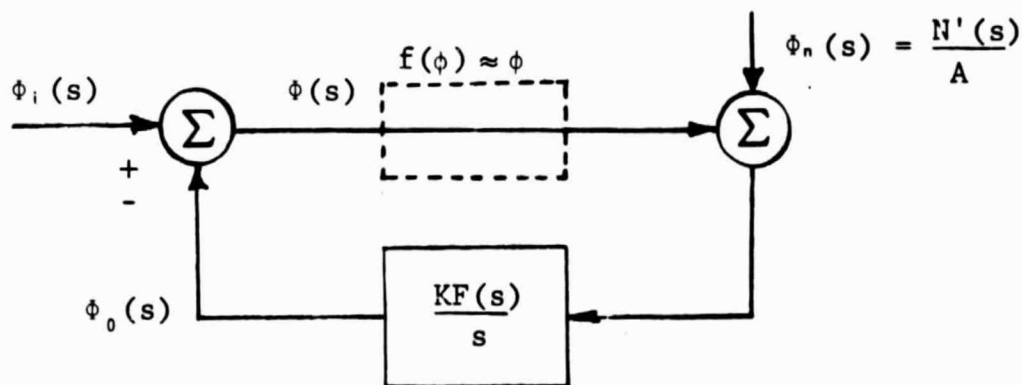


Figure 3: Phase-Locked Loop Block Diagram and Linear Models

$$\phi_o(s) = H(s) \left[\phi_i(s) + \phi_n(s) \right] = \frac{KF(s)}{s + KF(s)} \left[\phi_i(s) + \phi_n(s) \right] \quad (5.1)$$

$$\begin{aligned} \phi(s) &= \left[1 - H(s) \right] \phi_i(s) - H(s) \phi_n(s) \\ &= \frac{s}{s + KF(s)} \phi_i(s) - \frac{KF(s)}{s + KF(s)} \phi_n(s) \end{aligned} \quad (5.2)$$

The closed-loop transfer function $H(s)$ represents the equivalent filter estimating the signal phase fluctuations in the presence of additive noise. The loop design consists of specifying the loop gain K and loop filter $F(s)$ to yield an effective phase estimation. The first error term in (5.2) consists of the inability of the loop to ideally reproduce the signal phase fluctuations, and it includes the dynamic lag and oscillator noise errors. The second error term in (5.2) consists of the extra phase error contributed by the additive noise passed by the equivalent loop filter. The loop design compromise is illustrated in this formulation: a wideband $H(s)$ reproduces the high frequency content in $\phi_i(s)$, but a narrowband $H(s)$ rejects the high frequencies in $\phi_n(s)$.

The steady-state dynamic lag error when tracking polynomial step excitations of the form $\phi_i(t) = \frac{1}{n!} M_n t^n$ for $t \geq 0$ can be evaluated from the final value theorem as

$$\phi_{ss} = \lim_{s \rightarrow 0} s \left[1 - H(s) \right] \phi_i(s) = \lim_{s \rightarrow 0} \frac{M_n s^{1-n}}{s + KF(s)} \quad (5.3)$$

Such input phases may be obtained from a power series expansion of the orbital dynamics and corresponding doppler dynamics; so that M_0 represents a constant phase offset (constant range), M_1 is a constant doppler shift (constant velocity), M_2 is a constant doppler

rate (constant acceleration), etc. The loop filter $F(s)$ is assumed to have no ideal integrators, so that $F(0) = 1$ and the steady-state error is:

$$\phi_{ss} = \frac{M_n}{K} \lim_{s \rightarrow 0} s^{1-n} = \begin{cases} 0 & \text{if } n = 0 \\ \frac{M_1}{K} & \text{if } n = 1 \\ \infty & \text{if } n \geq 2 \end{cases} \quad (5.4)$$

The assumption of ideal integrators in the loop filter would yield different results. If (m) integrators are assumed so that $F(s)$ behaves as s^m/K when $s \rightarrow 0$, then the steady-state error becomes

$$\phi_{ss} = \frac{M_n}{K} \lim_{s \rightarrow 0} s^{1+m-n} = \begin{cases} 0 & \text{if } n < m+1 \\ \frac{M_n}{K} & \text{if } n = m+1 \\ \infty & \text{if } n > m+1 \end{cases} \quad (5.5)$$

so that $(m+1)$ - th order phase dynamics could be tracked with finite steady-state error (and m -th order with zero error) if (m) ideal integrators are present in the loop filter. The non-ideal integrator case can only track 2nd-order phase dynamics (constant doppler shift) with finite steady-state error (and constant phase offsets with zero error). The distinction is rather important since it invalidates the use of a steady-state error analysis based on ideal integrators, when the latter are recognized as absent in practical loop filter realizations.

The use of a steady-state error analysis is questionable by itself. The power series expansion is valid about a point in the orbital motion, so that letting $t \rightarrow \infty$ may be incongruous since the orbital dynamics change with time. For example, the non-ideal

integrator result indicates that a constant doppler rate cannot be tracked forever without a time-increasing phase error, so that the locked state and the loop linear model would be lost. Yet this does not exclude that such doppler rate cannot be tracked for a finite amount of time within the locked state, which may suffice if the tracking time in question is larger than the orbital time that validates the constant doppler rate assumption. As time elapses, the doppler rate does not remain constant, and the dynamic tracking error should be reformulated. Notice also that tracking a peak doppler rate magnitude for a short time is not necessarily more difficult than tracking a moderate magnitude for a longer time.

The use of an rms error analysis for the doppler dynamic phase inputs under consideration should be avoided. The rms evaluation consists of the (magnitude-square) time-averaging of the transient error response to these phase input excitations. Such error averaging includes transient overshoots, damped oscillations and steady-state error sections. An infinite steady-state error would always yield an infinite rms error, so finite error magnitudes would be limited to ideal integrator designs, or non-ideal integrators with constant phase offset or doppler shift inputs. Also, a finite rms error derived from the linear model analysis cannot assure that the error peaks present in the transient overshoot and ringing phenomena are within the locking bound (which would invalidate the linear model and the error analysis). Finally, the transient time-averaging would refer to orbital time, yet the rms error contribution of additive noise represents a stochastic ensemble averaging applicable to only one orbital point (since the

path loss fluctuation with orbital motion would change the SNR conditions) so that the joint accounting of rms dynamic plus noise effects would suffer interpretation.

The rms additive noise error follows from (5.2) in terms of the power density spectrum $S_{\phi_n}(\omega)$ of the equivalent phase noise input

$$\sigma_{\phi}^{(add)} = \sqrt{\int_{-\infty}^{\infty} S_{\phi_n}(\omega) \cdot |H(j\omega)|^2 \frac{d\omega}{2\pi}} = \sqrt{\int_{-\infty}^{\infty} \frac{S_n(\omega)}{A^2} \cdot |H(j\omega)|^2 \frac{d\omega}{2\pi}} \text{ rad}^2 \quad (5.6)$$

The power density spectrum $S_n'(\omega)$ of the phase detector baseband noise $n'(t)$ may be evaluated in terms of that of the input noise as a function of the VCO output phase fluctuations. If the closed-loop equivalent filter is narrowband relative to the loop input passband, the VCO phase effect will approximate an orthogonal transformation that preserves the baseband noise spectra. Under such conditions, the power density spectrum of $n'(t)$ is similar to those of $n_c(t)$ and $n_q(t)$; so that a two-sided N_0 watts/Hz white spectrum may be assumed when the noise input $n(t)$ has a one-sided N_0 watts/Hz white spectrum. The rms additive noise error can then be expressed in terms of the two-sided loop noise bandwidth (B_n in Hz) and the corresponding loop SNR via

$$B_n = \int_{-\infty}^{\infty} |H(j\omega)|^2 \frac{d\omega}{2\pi} \text{ Hz} \quad (5.7)$$

and

$$\sigma_{\phi}^{(add)} = \sqrt{\frac{N_0}{A^2} \int_{-\infty}^{\infty} |H(j\omega)|^2 \frac{d\omega}{2\pi}} = \sqrt{\frac{N_0 B_n}{A^2}} = \frac{1}{\sqrt{2(\text{SNR})_{B_n}}} \text{ rad} \quad (5.8)$$

The rms oscillator noise error also follows from (5.2) by using the power density spectrum $S_{\phi_i}(\omega)$ of the equivalent oscillator phase noise at the loop input to obtain the relation

$$\sigma_{\phi}^{(osc)} = \sqrt{\int_{-\infty}^{\infty} S_{\phi_i}(\omega) \cdot |1 - H(j\omega)|^2 \frac{d\omega}{2\pi}} \quad (5.9)$$

Such error contribution cannot be given a more compact formulation without further a-priori information, since the oscillator noise processes superimposed at the loop input depend on both the signal sources and subsystem processing employed in a given application.

The selection of loop gain K and loop filter $F(s)$ control these error magnitudes only through their effects on the equivalent transfer functions $H(s)$ and $1 - H(s)$. The latter specify loop order as the degree of their denominator characteristic polynomial in the (s) frequency domain. The next subsections shall consider loop filter realizations that result in second or third order loops. Their dynamic lag and additive noise error contributions shall be analyzed towards an optimum loop design, with the oscillator noise effects yet to be accounted for. Once these tracking errors are formulated, their corresponding range and range rate error contributions can be evaluated in the sections that follow.

The set of loop design parameters thus consists of the loop gain and the loop filter time constants or transfer functions. However, the following two results are applicable to all loop orders and filter realizations that satisfy $F(0) = 1$ (non-ideal integrators):

- (a) The steady-state dynamic lag error when tracking a constant frequency offset of Ω rad/sec is essentially given by $\frac{\Omega}{K}$ rad in the locked state (see (5.4)); i.e., $f^{-1}(\frac{\Omega}{K}) \approx \frac{\Omega}{K}$ for small errors.

(b) The rms additive noise error in the locked state is given by (5.7) in the presence of wideband additive white noise.

The importance of these results is that different loops will exhibit the same steady-state dynamic lag error (to a constant frequency offset) and the same rms additive noise locking error provided they have the same loop gain (K) and loop noise bandwidth (B_n), regardless of the other loop parameter values and the loop orders in question.

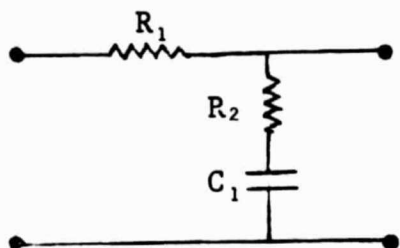
5.1 Second-Order Loops

The class of 2nd-order loops under consideration is characterized by the presence of a passive or active lag network as their loop filters. The loop filter realizations are illustrated in Figure 4, along with their corresponding transfer functions. The active case provides an increase in the realizable pole time constant with the operational amplifier gain as the magnification factor. Both the passive and active realizations have been employed in existing tracking systems. The existence of non-ideal integrators is emphasized in both cases, with the ideal integrator being closer approximated in the active case due to the inclusion of the amplifier gain in the time constant. The loop filter and closed-loop transfer functions can in both cases be formulated as

$$F(s) = \frac{1 + \tau_1 s}{1 + \tau_2 s}, \quad \tau_1 < \tau_2 \quad (5.10)$$

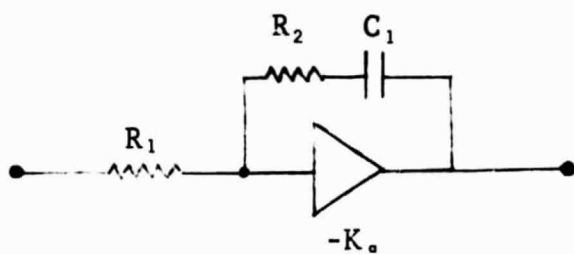
and

$$H(s) = \frac{1 + \tau_1 s}{1 + (\tau_1 + \frac{1}{K})s + \frac{\tau_2}{K}s^2} \quad (5.11)$$



$$F(s) = \frac{1 + R_2 C_1 s}{1 + (R_1 + R_2) C_1 s}$$

$$\approx \frac{1 + R_2 C_1 s}{1 + R_1 C_1 s}, \quad R_1 \gg R_2$$



$$F(s) = \frac{1 + R_2 C_1 s}{1 + [(1 - K_o) R_1 + R_2] C_1 s}$$

$$\approx \frac{1 + R_2 C_1 s}{1 + K_o R_1 C_1 s}, \quad K_o \gg 1, \quad R_1 \gg R_2$$

Figure 4: Second Order Loop Filter Realizations

The set of loop design parameters may then be taken as the trio (K, τ_1, τ_2) . The alternate set (K, ω_r, ζ) is more often employed based on servo-loop analogies, where (ω_r) is the resonance frequency in rad/sec and (ζ) is the damping factor specified by

$$\omega_r^2 = \frac{K}{\tau_2}, \quad \zeta = \frac{1}{2} \left(\tau_1 + \frac{1}{K} \right) \omega_r \quad (5.12)$$

The equivalent (two-sided) loop noise bandwidth $(B_n \text{ in Hz})$ can then be evaluated in terms of either set to yield

$$B_n = \frac{1}{2} \left(\frac{1}{\tau_1} + \frac{K\tau_1}{\tau_2} \right) \left(1 + \frac{1}{K\tau_1} \right)^{-1} = \frac{1 + 4\zeta^2 - \frac{4\zeta\omega_r}{K} + \left(\frac{\omega_r}{K} \right)^2}{4\zeta} \omega_r \quad (5.13a)$$

$$\approx \frac{1}{2} \left(\frac{1}{\tau_1} + \frac{K\tau_1}{\tau_2} \right) = \frac{1 + 4\zeta^2}{4\zeta} \omega_r \text{ for } K\tau_1 \gg 1, K \gg \omega_r \quad (5.13b)$$

The variation of the loop noise bandwidth (B_n) with the damping factor (ζ) is quite small for $0.5 \leq \zeta \leq 1$, which represents conventional 2nd-order loop designs; e.g., $B_n/\omega_r = (1, 3/2\sqrt{2}, 5/4)$ for $\zeta = (1/2, 1/\sqrt{2}, 1)$. It should be emphasized that the results above have the units of B_n in Hz vs. ω_r in rad/sec; e.g., $B_n \text{ (Hz)} = \omega_r \text{ (rad/sec)}$ for $\zeta = 1/2$. Notice that the set (K, B_n, ζ) represents still another option for the design parameters in a 2nd-order loop. This last set is preferable to the others because it features the loop gain and loop bandwidth as the design parameters which was previously motivated. An equivalent set that maintains this feature consists of (K, B_n, a) , where the last parameter satisfies $0 < a < 2$, and is suggested by the formulation of (5.13b) as follows:

$$\frac{1}{\tau_1} \approx a B_n, \quad \frac{K\tau_1}{\tau_2} \approx (2 - a) B_n \quad (5.14)$$

or

$$a \approx \frac{2}{1 + \frac{K\tau_1^2}{\tau_2}} \approx \frac{2}{1 + 4\zeta^2} \text{ for } K \gg a B_n \quad (5.15)$$

The set (K, B_n, a) could have been directly obtained from the closed-loop transfer function of (5.11). The interest in exhibiting the loop noise bandwidth as an independent design parameter suggests the transformation

$$\tau_1 = \frac{1}{a B_n}, \quad \frac{\tau_2}{K} = \frac{1}{b B_n^2} \quad (5.16)$$

which can be introduced in the expressions of (5.13b) to derive the following functional dependence for $K\tau_1 \gg 1$:

$$b \approx a(2 - a), \quad K \gg a B_n \quad (5.17)$$

Notice that the approach consists of first formulating the time constants $(\tau_1, \frac{\tau_2}{K})$ of $H(s)$ as inverse functions of the bandwidth but including an exponent on the latter to match that of the s -polynomial term. The set (K, τ_1, τ_2) is thus transformed into the set (K, B_n, a, b) , and then the dependence $b(a)$ is acknowledged to obtain the set (K, B_n, a) with three independent design parameters. Notice that in reality (b) is dependent on all three of these independent parameters as obtained from (5.13a), but the use of the approximation $K\tau_1 \gg 1$ and its result (5.13b) yields the simplified one-parameter dependence of (5.17). It is evident that this approach can be generalized to higher-order loop transfer

functions to free the loop noise bandwidth as an independent design parameter, and therein lies the motivation for this set.

It should be noted that the approximate results obtained under the condition $K\tau_1 \gg 1$ become exact when ideal integrators are assumed in the loop filter. The deletion of the (1) constant and the (s/K) term in the denominator of (5.10) and (5.11) respectively correspond to this assumption, from which the expression of (5.13b) then follows as an exact result. Hence, the loop noise bandwidth and rms additive noise error are essentially similar in the ideal vs. non-ideal integrator cases. However, their steady-state dynamic lag errors are considerably different in nature as illustrated below. The error transfer function governing such performance in the non-ideal integrator case is given by

$$1 - H(s) = \frac{\frac{1}{K} s + \frac{\tau_2}{K} s^2}{1 + (\tau_1 + \frac{1}{K})s + \frac{\tau_2}{K} s^2} \quad (5.18)$$

while both (s/K) terms in the numerator and denominator are absent for the ideal integrator case. The $(1/K)$ term may, indeed, be neglected in the denominator of (5.18) for $K\tau_1 \gg 1$, but not in its numerator where it appears alone and may play a significant role in the steady-state error behavior.

The steady-state dynamic lag error of a 2nd-order loop with non-ideal integrators is Ω/K rad for a constant doppler shift (Ω rad/sec) but infinite for a constant doppler rate ($\dot{\Omega}$ rad/sec²) or higher dynamic inputs. However, the steady-state error behavior

can still be studied for any input by performing a long division of (5.18) to derive the pertinent error coefficients. The expansion and truncation process yields the following results for the two inputs under consideration:

$$\phi_{ss}(t) = \frac{\dot{\Omega}}{K} \quad (5.19)$$

and

$$\begin{aligned} \phi_{ss}(t) &= \frac{\dot{\Omega}}{K} \left(t + \tau_2 - \tau_1 - \frac{1}{K} \right) \\ &\approx \frac{\dot{\Omega}}{K} (t + \tau_2) = \dot{\Omega} \left(\frac{t}{K} + \frac{1}{\omega_r^2} \right) \\ &= \frac{\dot{\Omega}}{K} t + \left(\frac{1 + 4\zeta^2}{4\zeta} \right)^2 \frac{\dot{\Omega}}{B_n^2} \approx \frac{\dot{\Omega}}{K} t + \frac{\dot{\Omega}}{a(2 - a)B_n^2} \text{ rad} \quad (5.20) \end{aligned}$$

The doppler rate error expression is noted to include a linearly time increasing term which causes the infinite steady-state error result. Such term is directly contributed by the (s/K) term in the numerator of (5.18), and would be absent if ideal integrators were assumed. Hence, the ideal integrator case could track a constant doppler rate forever with a finite constant steady-state error, but the non-ideal integrator case would result in the unlocked state as the tracking time increases. The conditions of high loop gains and short-lasting doppler rates relative to the tracking times are required if the non-ideal integrator error of (5.20) is to be approximated by its constant term only. In any case, the constant term is minimized to the value $\dot{\Omega}/B_n^2$ by the design $\zeta = 1/\sqrt{2}$, $a = 1$, $B_n \text{ (Hz)} = \omega_r \text{ (rad/sec)}$, which shall be assumed for the 2nd-order loops in question.

5.2 Third-Order Loops

The class of 3rd-order loops under consideration is essentially characterized by a cascade of two passive or active lag networks as their loop filters. These loop filter realizations are illustrated in Figure 5, along with their corresponding transfer functions. Again the active networks are shown to provide an extension of the realizable pole time constants. The all-active realization has not been employed in existing tracking systems to the best of our knowledge. It should be noted that all the cases under consideration yield two real-axis poles in the s-plane, yet the formulation that follows shall be generalized to include the possibility of complex conjugate roots resulting from a different loop filter realization. The loop filter and closed-loop transfer functions can then be formulated as

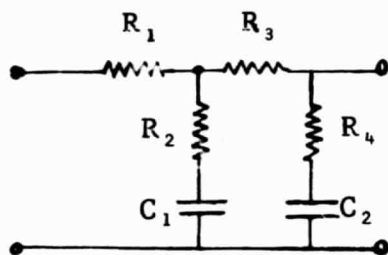
$$F(s) = \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + \tau_2 s + (\tau_4 s)^2} \quad \tau_3 < \tau_4 \quad (5.21)$$

and

$$H(s) = \frac{1 + \tau_1 s + (\tau_3 s)^2}{1 + (\tau_1 + \frac{1}{K})s - (\tau_3^2 + \frac{\tau_2}{K})s^2 + \frac{\tau_4^2}{K}s^3} \quad (5.22)$$

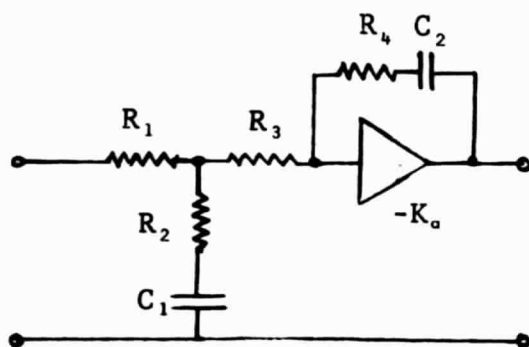
The set of loop design parameters may be taken as $(K, \tau_1, \tau_2, \tau_3, \tau_4)$, but such set would reflect the loop filter rather than the closed-loop performance. The interest is to choose a set that includes the pair (K, B_n) as previously indicated. The loop noise bandwidth may be evaluated using (5.7) to yield

$$B_n = \frac{K}{2} \left(\frac{\tau_3}{\tau_4} \right)^2 \frac{\left(\tau_1 + \frac{1}{K} \right) - \frac{1}{K} \left(\frac{\tau_4}{\tau_3} \right)^2 \left[1 - \left(\frac{\tau_1}{\tau_3} \right)^2 - \frac{\tau_2}{K \tau_3^2} \right]}{\left(\tau_1 + \frac{1}{K} \right) \left(1 + \frac{\tau_2}{K \tau_3^2} \right) - \frac{1}{K} \left(\frac{\tau_4}{\tau_3} \right)^2} \quad (5.23a)$$



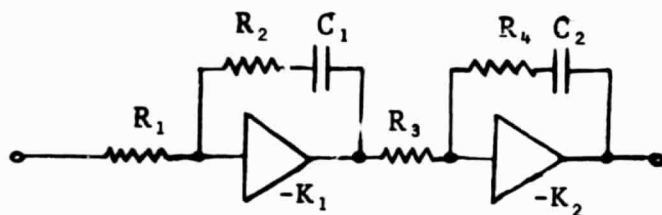
$$F(s) = \frac{(1 + R_2 C_1 s)(1 + R_4 C_2 s)}{[1 + (R_1 + R_2) C_1 s][1 + (R_3 + R_4) C_2 s]}$$

$$\approx \frac{1 + R_2 C_1 s}{1 + R_1 C_1 s} \cdot \frac{1 + R_4 C_2 s}{1 + R_3 C_2 s}$$



$$F(s) = \frac{-K_0 (1 + R_2 C_1 s)(1 + R_4 C_2 s)}{1 + \left\{ (R_1 + R_2) C_1 + [(K_0 + 1)(R_1 + R_3) + R_4] C_2 \right\} s + \left\{ (K_0 + 1) R_1 R_2 + [(K_0 + 1) R_3 + R_4] (R_1 + R_2) \right\} C_1 C_2 s^2}$$

$$\approx -K_0 \frac{1 + R_2 C_1 s}{1 + R_1 C_1 s} \cdot \frac{1 + R_4 C_2 s}{1 + K_0 R_3 C_2 s}$$



$$F(s) = \frac{K_1 K_2 (1 + R_2 C_1 s)(1 + R_4 C_2 s)}{\left\{ 1 + [(K_1 + 1) R_1 + R_2] C_1 s \right\} \cdot \left\{ 1 + [(K_2 + 1) R_3 + R_4] C_2 s \right\}}$$

$$\approx K_1 K_2 \frac{1 + R_2 C_1 s}{1 + K_1 R_1 C_1 s} \cdot \frac{1 + R_4 C_2 s}{1 + K_2 R_3 C_2 s}$$

Figure 5: Third Order Loop Realizations

$$\approx \frac{K}{2} \left(\frac{\tau_3}{\tau_4} \right)^2 \frac{\tau_1 - \frac{1}{K} \left(\frac{\tau_4}{\tau_3} \right)^2 \left[1 - \left(\frac{\tau_1}{\tau_3} \right)^2 \right]}{\tau_1 - \frac{1}{K} \left(\frac{\tau_4}{\tau_3} \right)^2} \quad \text{for } K\tau_1 \gg 1 \quad (5.23b)$$

$$K\tau_3^2 \gg \tau_2$$

where the approximate result of (5.23b) becomes exact if ideal integrators are assumed in the loop filter; i.e., the deletion of the (1) and $(\tau_2 s)$ terms in the (5.21) denominator, and of the $(1/K)$ and $(\tau_2 s^2/K)$ terms in the (5.22) denominator, yields (5.23b) as an exact result from (5.7).

The parameter transformation of interest is motivated by the closed-loop transfer function of (5.22) along with $K\tau_1 \gg 1$ and $K\tau_3^2 \gg \tau_2$ as follows:

$$\tau_1 = \frac{1}{aB_n}, \quad \tau_3^2 = \frac{1}{bB_n^2}, \quad \frac{\tau_4^2}{K} = \frac{1}{cB_n^3} \quad (5.24)$$

which replaces the original set by the set $(K, B_n, \tau_2, a, b, c)$. The non-ideal integrator parameter τ_2 has a negligible effect on the closed-loop transfer function and loop noise bandwidth expression, but becomes relevant in the error transfer function $1 - H(s)$ and the steady-state dynamic error analysis. The trio (a, b, c) contains only two independent parameters so as to maintain the same degree of freedom as in the original set. The dependent parameter is actually a complicated expression of all the parameters in the new set, but the approximating conditions in (5.32b) establish a two-parameter dependence $c(a, b)$ as the positive real solutions of the quadratic equation

$$c^2 + \left[\frac{b^2}{a} - (a + 2) \right] c - 2ab^2 \approx 0 \quad (5.25)$$

with an equal sign holding if ideal integrators are assumed. The situation is analogous to the pair (a,b) of 2nd-order loops, where the relation $b \approx 2 - a$ established their dependence. The equation (5.25) can be noted to be quadratic for either of the three parameters as a function of the other two, which can be resolved to eliminate one of them and reduce the new set to a five-parameter independent set equivalent to the original set. Hence, any trio of positive real numbers (a,b,c) satisfying (5.25) correspond to a 3rd-order loop design having the loop gain (K) and loop noise bandwidth (B_n) yet to be specified.

Another equivalent set of design parameters applicable to 3rd-order loops shall now be discussed. Its motivation arises from practical considerations: some 3rd-order loop realizations presently employed in existing tracking systems exhibit a design formulation which can be identified as a special case of this equivalent set. The loop design parameters are now transformed using

$$\tau_1 = \frac{x}{\omega_0}, \quad \tau_2 = \frac{y}{\omega_0}, \quad \frac{\tau_2^2}{K} = \frac{z}{\omega_0^3} \quad (5.26)$$

which may be compared to (5.24) for reference purposes. The trio (x,y,z) is a dependent set that must be reduced to an independent pair, which along with (K, ω_0 , τ_2) yields the five-parameter design set. For every trio (a,b,c) there exists a trio (x,y,z), which establishes the correspondence between B_n and ω_0 (and between the two sets) from the relation

$$B_n = \omega_0 \left(\frac{y}{2z} \right) \frac{\left(x + \frac{\omega_0}{K} \right) - \frac{z}{y} \left(1 - \frac{x^2}{y} - \frac{\tau_2 \omega_0^2}{yK} \right)}{\left(x + \frac{\omega_0}{K} \right) \left(1 + \frac{\tau_2 B_n^2}{yK} \right) - \frac{z}{y}} \quad (5.27a)$$

$$\approx \omega_0 \left(\frac{y}{2z} \right) \frac{xy - z \left(1 - \frac{x^2}{y} \right)}{xy - z} \quad \begin{array}{l} \text{for } Kx \gg \omega_0 \\ Ky \gg \tau_2 \omega_0^2 \end{array} \quad (5.27b)$$

The correspondence between the sets (K, B_n, τ_2, a, b) and $(K, \omega_0, \tau_2, x, y)$ for 3rd-order loops is analogous to that of the sets (K, B_n, a) and (K, ω_r, ζ) for 2nd-order loops. The sets containing the (ω_0) or (ω_r) parameters are based on critical (break) frequencies appearing in the closed-loop transfer function, while the sets containing the (B_n) parameter are based on additive noise locking error content through the loop noise bandwidth. All loops (perhaps of different order) with the same (B_n) value will exhibit the same rms additive noise error in the locked state, so that their relative merits can be established from dynamic lag error performance considerations. The remaining loop parameters besides (B_n) can be selected to minimize the dynamic error with the knowledge that their selection will not alter the additive noise error content (which conversely happens with the other set).

The steady-state dynamic lag error is again Ω/K rad for a constant doppler shift (Ω rad/sec) excitation, and infinite for a constant doppler rate ($\dot{\Omega}$ rad/sec²) or higher dynamic inputs. These results are a consequence of the non-ideal integrators involved, since both steady-state errors above are zero for the ideal integrator case. The error transfer function is given by

$$1 - H(s) = \frac{\frac{1}{K} s + \frac{\tau_2}{K} s^2 + \frac{\tau_4}{K} s^3}{1 + \left(\tau_1 + \frac{1}{K} \right) s + \left(\tau_3 + \frac{\tau_2}{K} \right) s^2 + \frac{\tau_4}{K} s^3} \quad (5.28)$$

and the (s/K) and $(\tau_2 s^2/K)$ numerator terms are absent in the ideal integrator case. The steady-state error pattern may again be analyzed by long-division to yield the following results for the two input excitations in question:

$$\phi_{ss}(t) = \frac{\dot{\Omega}}{K} \text{ rad} \quad (5.29)$$

and

$$\begin{aligned} \phi_{ss}(t) &= \frac{\dot{\Omega}}{K} \left(t + \tau_2 - \tau_1 - \frac{1}{K} \right) \approx \frac{\dot{\Omega}}{K} (t + \tau_2 - \tau_1) \\ &= \frac{\dot{\Omega}}{K} \left(t + \tau_2 - \frac{\pi}{\omega_0} \right) = \frac{\dot{\Omega}}{K} \left(t + \tau_2 - \frac{1}{aB_n} \right) \text{ rad} \end{aligned} \quad (5.30)$$

The doppler rate error again increases linearly with time, so that a limit on the doppler rate duration or the tracking time is required to maintain lock. For small tracking times, the error may be approximated by its constant term and the loop analysis can proceed. Notice that the constraint $\tau_1 \ll \tau_2$ valid for 2nd-order loops (see (5.20)) is not a-priori evident now. Moreover, even if such condition is assumed to hold, the constant term $(\dot{\Omega}\tau_2/K)$ cannot be claimed to have any specific functional dependence on the loop noise bandwidth (B_n) , since the time constant (τ_2) represents a non-ideal integrator parameter in the loop filter. This implies that more specific information concerning the non-ideal integrator approximation in the loop filter realization should be included in the formulation to proceed with the 3rd-order loop analytical design.

5.3 The Mallinckrodt Third-Order Loop

The generalized 3rd-order loop formulation presented in the previous section can now be used to account for the scarce 3rd-order loop realizations that have received significant attention in the literature. These particular designs have been motivated by certain specific features they exhibit, but the aforesaid formulation provides a unified analytical approach from which these loops emerge merely as special cases of a wide class of 3rd-order loop realizations. Indeed, these few existing 3rd-order loop designs represent specific assignments for the 3rd-order loop parameters (a,b,c) or (x,y,z) previously discussed. The fact that these loops have been identified as possible solutions of the quadratic equation (5.25) considerably extends the number of potential 3rd-order loop realizations and puts the few existing cases in the proper perspective.

The main effort in 3rd-order loop analysis and design has concentrated on the Wiener, Pool and Mallinckrodt loops. The Wiener loop was derived from the analytical minimization of the total mean-square locking error (dynamic transient plus additive noise) in the presence of the doppler rate excitation plus white noise and using the loop linear model. The optimum loop filter and closed-loop transfer functions thus obtained yield a 3rd-order loop with two ideal integrators in the loop filter. The resultant loop design parameters are given by $(a,b,c) = \left(\frac{3}{10}, \frac{9}{50}, \frac{27}{125} \right)$ or $(x,y,z) = (2,2,1)$. This loop has received considerable attention yet the mean-square transient plus noise error can be severely questioned as a performance index as previously discussed. The interest in transient as opposed to steady-state errors is dubious

after acquisition has occurred and the loop maintains lock. Also, a bounded mean-square (time-averaged) transient error does not assure peak error overshoots below the locking values that validate the linear model. Finally, such transient mean-square error is not bounded when the non-ideal integrator approximations are acknowledged in practice, so that another performance index and optimization criterion becomes necessary.

The Pool and Mallinckrodt loop designs were motivated by certain dynamic transient response features (to the best of our knowledge). They also exhibit two ideal integrators in their loop filters, thus sharing their dynamic tracking degradation when non-ideal integrators are employed in practice. Their loop designs correspond to the assignments $(a,b,c) = (1/5.27, 1/6.91, 1/6.42)$ or $(x,y,z) = (2\sqrt{2}, 2, 1)$ in the Pool case, and $(a,b,c) = (1/8.91, 1/19.85, 1/13.10)$ or $(x,y,z) = (6,9,4)$ in the Mallinckrodt case. A comparison of the three 3rd-order loop designs in question is summarized in Table 2. Their basic distinction is reflected in the pole-zero patterns: the Wiener loop contains complex conjugates in both transfer functions, the Pool loop only in $H(s)$ but not in $F(s)$, and the Mallinckrodt loop in neither.

In any case, there exists a significant degradation in the dynamic error performance when the practical non-ideal integrators are acknowledged. The relative merits of any loop will then be governed by its effectiveness in reducing or maintaining such degradation within tolerable error bounds. This issue shall now be evaluated in detail for the Mallinckrodt loop realization only. This loop has been selected based on the following arguments:

TABLE 2

Third-Order Loop Design with Ideal Integrators

	Wiener	Mallinckrodt	Pool
$H(s)$	$\frac{1 + \frac{2s}{\omega_0} + \frac{2s^2}{\omega_0^2}}{1 + \frac{2s}{\omega_0} + \frac{2s^2}{\omega_0^2} + \frac{s^3}{\omega_0^3}}$	$\frac{1 + \frac{6s}{\omega_0} + \frac{9s^2}{\omega_0^2}}{1 + \frac{6s}{\omega_0} + \frac{9s^2}{\omega_0^2} + \frac{4s^3}{\omega_0^3}}$	$\frac{1 + \frac{2\sqrt{2}s}{\omega_0} + \frac{2s^2}{\omega_0^2}}{1 + \frac{2\sqrt{2}s}{\omega_0} + \frac{2s^2}{\omega_0^2} + \frac{s^3}{\omega_0^3}}$
$KF(s)$	$\frac{1 + \frac{2s}{\omega_0} + \frac{2s^2}{\omega_0^2}}{s^2/\omega_0^3}$	$\frac{1 + \frac{6s}{\omega_0} + \frac{9s^2}{\omega_0^2}}{4s^2/\omega_0^3}$	$\frac{1 + \frac{2\sqrt{2}s}{\omega_0} + \frac{2s^2}{\omega_0^2}}{s^2/\omega_0^3}$
$\frac{B_n}{\omega_0}$	$\frac{5}{3}$	$\frac{297}{200}$	$\frac{4\sqrt{2} + 3}{4\sqrt{2} - 1}$
(x, y, z)	$(2, 2, 1)$	$(6, 9, 4)$	$(2\sqrt{2}, 2, 1)$
(a, b, c)	$\left(\frac{3}{10}, \frac{9}{50}, \frac{27}{125}\right)$	$\left(\frac{1}{8.91}, \frac{1}{19.85}, \frac{1}{13.10}\right)$	$\left(\frac{1}{5.27}, \frac{1}{6.91}, \frac{1}{6.42}\right)$
pole-zero of $H(s)$	$\frac{2\omega_0\left(s + \frac{\omega_0}{2} \pm j\frac{\omega_0}{2}\right)}{(s + \omega_0)\left(s + \frac{\omega_0}{2} \pm j\frac{\sqrt{3}\omega_0}{2}\right)}$	$\frac{9\omega_0\left(s + \frac{\omega_0}{3}\right)^2}{4\left(s + \frac{\omega_0}{4}\right)(s + \omega_0)^2}$	$\frac{2\omega_0\left(s + \frac{\omega_0}{\sqrt{2}}\right)^2}{(s + 0.4753\omega_0)(s + 0.7623\omega_0 \pm j 1.2376\omega_0)}$
pole-zero of $KF(s)$	$\frac{2\omega_0}{s^2} \left(s + \frac{\omega_0}{2} \pm j\frac{\omega_0}{2}\right)$	$\frac{9\omega_0}{4s^2} \left(s + \frac{\omega_0}{3}\right)^2$	$\frac{2\omega_0}{s^2} \left(s + \frac{\omega_0}{\sqrt{2}}\right)^2$

(a) It contains real poles and zeroes in $F(s)$, so that the networks of Figure 5 are fully applicable; (b) It has been successfully developed and employed in existing tracking systems so that it represents state-of-the-art technology; (c) Its analytical development can be easily extended to the all-active case of Figure 5 (c) to reflect potential improvements that motivate its future implementation.

The practical implementation of the Mallinckrodt loop shall be traced for its existing realization based on the passive plus active case of Figure 5 (b). The ideal vs. actual loop filter transfer functions are given by

$$F_{id}(s) = \frac{(1 + 3s/\omega_0)^2}{4K_p K_v s^2 / \omega_0^3} \quad (5.31)$$

and

$$F_{ac}(s) = K_o \cdot \frac{1 + R_2 C_1 s}{1 + (R_1 + R_2) C_1 s} \cdot \frac{1 + R_4 C_2 s}{1 + [R_3(1 + K_o) + R_4] C_2 s} \quad (5.32a)$$

$$\approx K_o \cdot \frac{1 + R_2 C_1 s}{1 + R_1 C_1 s} \cdot \frac{1 + R_4 C_2 s}{1 + K_o R_3 C_2 s} \quad (5.32b)$$

The loop filter zeroes are specified in terms of the critical frequency (ω_0) by the relation

$$R_2 C_1 = R_4 C_2 = \frac{3}{\omega_0} \quad (5.33)$$

while the high frequency gain establishes the condition

$$K_o \cdot \frac{R_2}{R_1} \cdot \frac{R_4}{K_o R_3} = \frac{9\omega_0}{4K_p K_v} \quad (5.34)$$

which specifies the loop filter poles as

$$R_1 C_1 = R_2 C_1 \left(\frac{R_1}{R_2} \right) = \frac{3}{\omega_0} \left(\frac{R_1}{R_2} \right) = \frac{3\lambda}{\omega_0} \quad (5.35a)$$

$$K_a R_3 C_2 = R_4 C_2 \left(\frac{K_a R_3}{R_4} \right) = \frac{3}{\omega_0} \left(\frac{K_a R_3}{R_4} \right) = \frac{4K}{3\omega_0^2} \left(\frac{R_2}{R_1} \right) = \frac{4K}{3\lambda\omega_0^2} \quad (5.35b)$$

The doppler rate steady-state error of (5.30) can next be evaluated by noting that the loop filter realization corresponds to

$$\tau_1 = R_2 C_1 + R_4 C_2 = \frac{6}{\omega_0} \quad (5.36a)$$

$$\tau_2 = R_1 C_1 + K_a R_3 C_2 = \frac{3\lambda}{\omega_0} + \frac{4K}{3\lambda\omega_0^2} \quad (5.36b)$$

which yields the following results for the dynamic lag error contribution

$$\phi_{ss} = \frac{\Omega}{K} \text{ rad (doppler shift)} \quad (5.37a)$$

$$\phi_{ss} = \frac{\dot{\Omega}}{K} \left(t + \frac{3\lambda - 6}{\omega_0} + \frac{4K}{3\lambda\omega_0^2} - \frac{1}{K} \right) \text{ rad (doppler rate)} \quad (5.37b)$$

The time dependent term in (5.37b) may be neglected under small tracking time conditions, and the constant term is noted to depend on the relative values of the (K, ω_0, λ) parameters. The conventional loop designs are based on large (K/ω_0) values such that the $(4K/3\lambda\omega_0^2)$ term predominates, though the $(3\lambda/\omega_0)$ term cannot be neglected in general. We shall assume that the doppler rate error can indeed be approximated by $\phi_{ss} \approx (\dot{\Omega}/3\lambda\omega_0^2)$ rad and use $\lambda = 200$ to reflect existing loop designs. Under these conditions, the doppler rate error simplifies to $\phi_{ss} \approx (\dot{\Omega}/150\omega_0^2)$ rad, which can be expressed in terms of the loop noise bandwidth using $B_n = (297/200)\omega_0$

for the Mallinckrodt loop. A comparison of this result with the 2nd-order loop case shows a reduction in the 3rd-order loop dynamic error by a factor of 68 for the same noise bandwidth, or a reduction of the 3rd-order loop noise bandwidth by a factor of $\sqrt{68}$ for the same dynamic error.

6. Range and Range Rate Data Errors

The previous section has illustrated the various phase errors introduced in the range (tone) and range rate (carrier) signals by the received tracking loops. These signals are then fed to their respective range and range rate extractors that provide the raw tracking data samples corresponding to successive independent measurements. A phase shift $\Delta\phi$ on the reproduced tone induces a zero crossing (ranging event) time error $\Delta t = \frac{\Delta\phi}{\omega_m}$ and range error $\Delta r = \frac{c}{2} (\Delta t)$. Hence, any tone loop locking errors are directly converted into a range error via a $(c/2\omega_m)$ multiplier. For example, the steady-state locking error to a tone doppler shift (Ω_m rad/sec) and doppler rate ($\dot{\Omega}_m$ rad/sec) were respectively derived as

$$\phi_{ss} = \frac{\Omega_m}{K} \text{ rad} \quad (6.1a)$$

and

$$\phi_{ss} \approx \frac{\dot{\Omega}_m}{\kappa B_n^2} \text{ rad (small tracking times)} \quad (6.1b)$$

where $\kappa = 1$ for 2nd-order loops with $\zeta = 1/2$, and $\kappa = 68$ for 3rd-order Mallinckrodt loops with $\lambda = 200$ and K/B_n large. The corresponding dynamic range errors are then given by

$$\mu_r = \frac{c}{2\omega_m} \cdot \frac{\Omega_m}{K} = \frac{\dot{r}}{K} \quad (6.2a)$$

and

$$\mu_r \approx \frac{c}{2\omega_m} \cdot \frac{\dot{\Omega}_m}{\kappa B_n^2} = \frac{\ddot{r}}{\kappa B_n^2} \text{ (small tracking times)} \quad (6.2b)$$

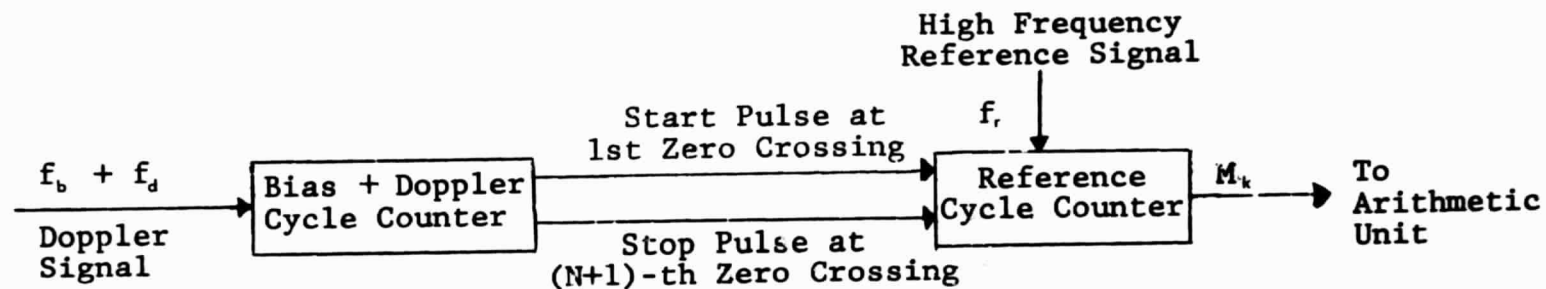
where $\dot{r}$ and $\ddot{r}$ represent the slant velocity (range rate) and acceleration (range rate derivative) of the spacecraft. In turn, the additive noise locking error of (5.8) yields an rms range error

that can be expressed in terms of the tone loop SNR (regardless of the loop order) as

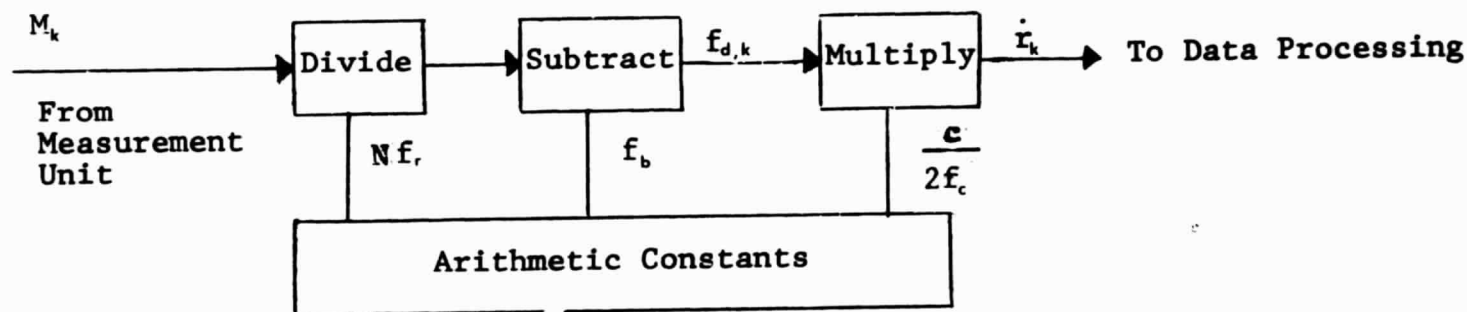
$$\sigma_r = \frac{c}{2\omega_m} \sigma_\phi = \frac{c}{2\sqrt{2\omega_m}} (\text{TNR})^{-\frac{1}{2}} \quad (6.3)$$

The range rate error is next evaluated by analyzing the two-way carrier doppler measurement operation. The doppler measurement can be reliably performed via cycle counting techniques: the number (N) of a bias (f_b) plus doppler (f_d) cycles in a time interval (T) satisfies the relation $N = (f_b + f_d) T$, so that measurement of one parameter from the pair (N, T) while the other remains a known constant resolves the doppler effect as $f_d = \frac{N}{T} - f_b$. The bias frequency assures a positive value for (N), if set larger than the peak doppler magnitude. The presence of doppler rates results in an average doppler measurement based on this procedure. The successive average doppler measurements ($f_{d,k}$) then yield average range rate data $\dot{r}_k = \frac{c}{2f_c} (f_{d,k})$ in discrete time samples (k) at a data rate of (1/T) samples per second. The choice of parameter to be measured and parameter to be fixed from the (N, T) pair yields the two range rate extractor realizations shown in Figures 6 and 7.

The Fixed-N realization of Figure 6 measures the counting times (T_k) in units (M_k) quantized to the period ($1/f_r$) of a high frequency reference (f_r). The first counter has a capacity of (N) cycle counts, and the 1st and (N + 1)-th zero crossing of its bias plus doppler input trigger the start and stop pulses for the second counter. The number (M_k) of reference cycles that occur in-between these two pulses then measures their time separation in ($1/f_r$) second units. The corresponding average doppler and range rate measurement are then obtained via the arithmetic computation

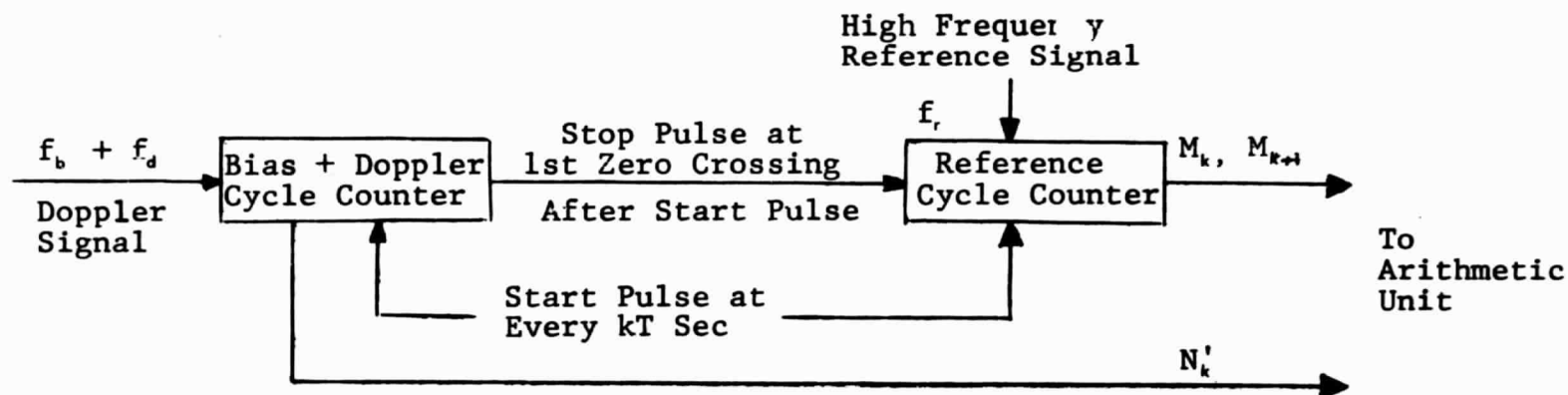


(a) Measurement Unit

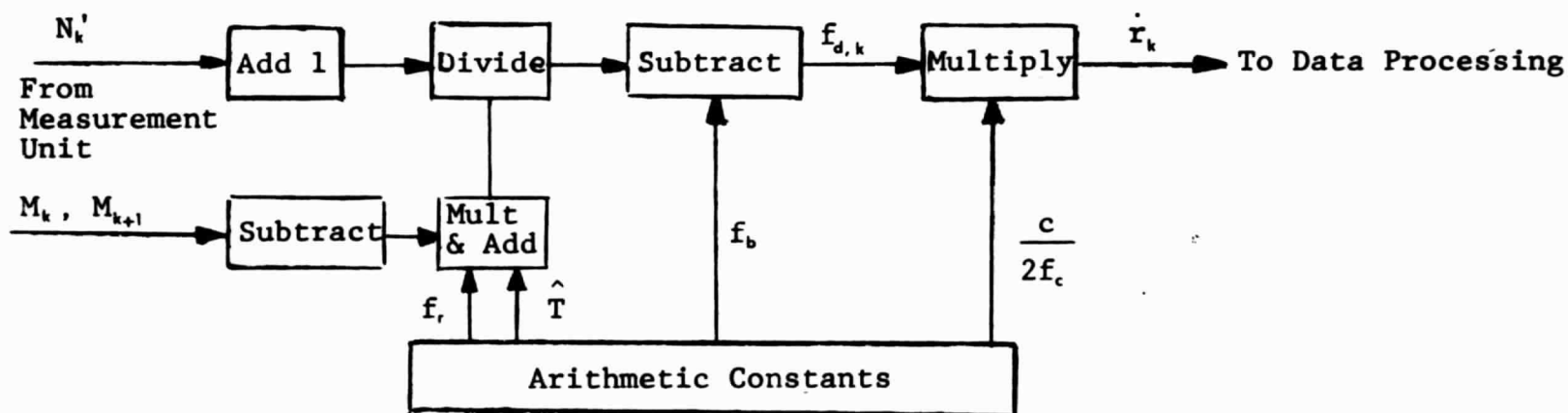


(b) Arithmetic Unit

Figure 6: Fixed-N Range Rate Extractor Realization



(a) Measurement Unit



(b) Arithmetic Unit

Figure 7: Fixed-T Range Rate Extractor Realization

$$\dot{r}_k = \frac{c}{2f_c} (f_{d,k}) = \frac{c}{2f_c} \left(\frac{N}{M_k} f_r - f_b \right) \quad (6.4)$$

The distinction between the measurement and arithmetic operations is quite relevant for error analysis. The actual parameter being measured is M_k , so that measurement errors can be accounted through their effects in altering this parameter. The presence of undesired phase jitter accompanying the bias plus doppler input shall advance or retard its zero crossings. The range rate measurement error can then be formulated from the reference error count ΔM_k caused by a time shift in the 1st and $(N + 1)$ -th zero crossings, assuming the jitter is bounded so that cycle slippage effects can be neglected. Any trigger pulse jitter or reference cycle quantization effects can be similarly analyzed to evaluate the measurement error induced. It should be emphasized that an equivalent phase jitter $\phi(t)$ on the input signal alters the count M_k only if there is a differential phase shift $(\Delta\phi)_T = \phi(T) - \phi(0)$ between the 1st and $(N + 1)$ -th zero crossing, since otherwise the latter are advanced or retarded by the same amount.

The reference count and range rate measurement errors introduced by a differential phase shift $(\Delta\phi)_T$ are given by

$$\Delta M_k = f_r (\Delta T_k) = \frac{f_r}{2\pi(f_b + f_d)} (\Delta\phi)_T \quad (6.5)$$

and

$$\begin{aligned} \Delta \dot{r} &= -\frac{c}{2f_c} (\Delta f_d) = \frac{c}{2f_c} \cdot \frac{Nf_r}{M^2} \cdot (\Delta M) \\ &= \frac{c}{2f_c} \cdot \frac{f_b + f_d}{2\pi N} (\Delta\phi)_T = \frac{c}{4\pi f_c T} (\Delta\phi)_T \end{aligned} \quad (6.6)$$

so that if the phase jitter $\phi(t)$ is assumed to be a WSS stochastic process with autocorrelation function $R_\phi(t)$, then the rms range rate error in a single measurement is given by

$$\sigma_r = \frac{c}{\sqrt{2}\omega_c T} \left[R_\phi(0) - R_\phi(T) \right]^{\frac{1}{2}} \quad (6.7a)$$

$$\approx \frac{c}{\sqrt{2}\omega_c T} \sigma_\phi \text{ if } R_\phi(T) \ll R_\phi(0) \quad (6.7b)$$

The compact result of (6.7b) is obtained when the effective correlation time of the phase jitter process is small relative to the counting time (T), so that the phase errors in the 1st and (N + 1)-th zero crossings are independent and add mean-square wise. Such correlation time is governed by the shape of the autocorrelation function $R_\phi(t)$, which is affected by the equivalent phase filter processing the phase jitter. These filters correspond to the closed-loop transfer function of the previous section, so that the assumption of large $B_n T$ products validates (6.7b) and permits the following expression for additive white noise effects in terms of the carrier loop SNR when lock is maintained:

$$\sigma_r \approx \frac{c}{2\omega_c T} (\text{CNR})_{B_n}^{-\frac{1}{2}} \quad (B_n T \text{ large}) \quad (6.8)$$

The Fixed-T realization of Figure 7 measures the number (N'_k) of bias plus doppler cycles that occur in a T-second interval. The first counter has start pulses every T-seconds, and registers the integer number (N_k) of bias plus doppler cycles in this interval, but loses the fractional counts corresponding to the beginning (δ_k) and end (μ_k) of the interval as illustrated in Figure 8. The

beginning time can be measured in real time using a second reference counter that starts at the beginning of the T-second interval and stops at the 1st zero crossing of the input signal. The number (M_k) of reference counts then measures this beginning time (δ_k) in $(1/f_r)$ second units as $\delta_k = M_k/f_r$. Conversely, the end time (μ_k) cannot be measured in real time since the first counter cannot tell a-priori which shall be the last ($N_k + 1$) zero crossing to be registered. However, the assumption of a constant doppler effect permits to resolve the end time from the next beginning time as $\mu_k = \frac{1}{f_b + f_{d,k}} - \delta_k +$

The net number of bias plus doppler cycles is thus given by $N'_k = N_k + (f_b + f_{d,k}) (\delta_k + \mu_k)$, which can be expressed in terms of the measured parameters (N'_k, M_k, M_{k+1}) and equated to $N_k = (f_b + f_{d,k}) T$ to resolve the doppler and range rate measurement as

$$\dot{r}_k = \frac{c}{2f_c} (f_{d,k}) = \frac{c}{2f_c} \left[\frac{N_k + 1}{T + \frac{1}{f_r} (M_{k+1} - M_k)} - f_b \right] \quad (6.9)$$

It can again be noted that the actual measured parameters are counts (N_k, M_k, M_{k+1}) while the conversion to range rate data involves an arithmetic computation. The measurement errors can thus be accounted through their effects on these counts. The analysis is more complicated than in the previous case since now the zero crossings advanced or retarded by an input phase jitter can alter these counts in various cumulative or compensating ways. The possible cases and their error formulation are summarized in Figure 9 and Table 3, which conclude with the same range rate error expression (6.6) and corresponding rms value (6.7) as before.

A basic distinction between the two cases lies in the parameter T appearing in these error expressions. The counting times (T_k) in

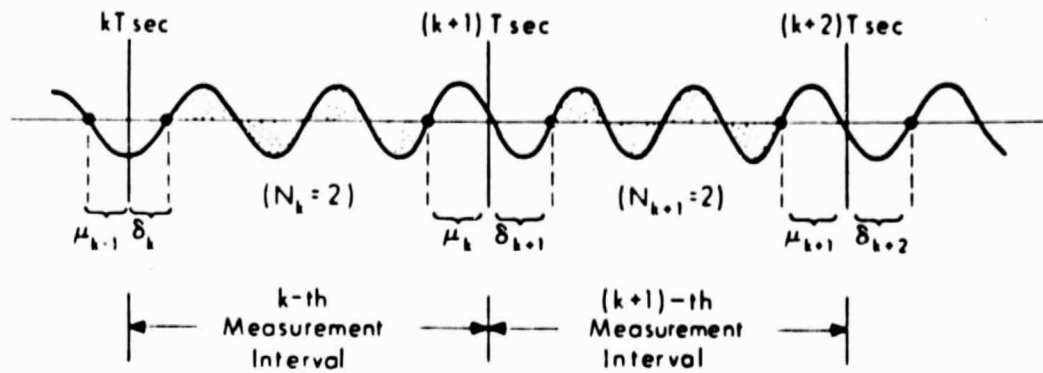


Figure 8: Description of Fixed-T Measurement

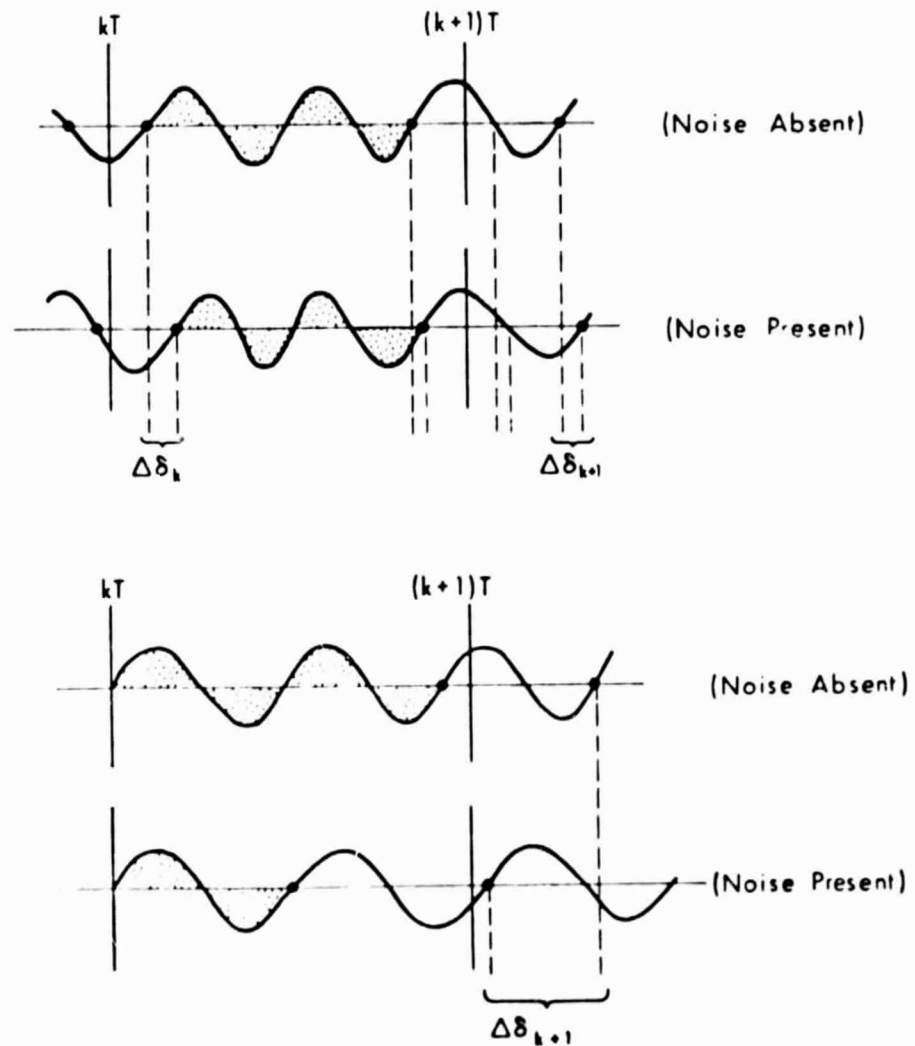


Figure 9: Details of Fixed-T Measurement
 $(\Delta N_k = 0 \text{ top and } \Delta N_k \neq \text{bottom})$

TABLE 3
Fixed-T Measurement Error Cases

<u>Transition at</u>		<u>$\Delta N'_k$</u>	<u>ΔM_k</u>	<u>ΔM_{k+1}</u>
<u>kT_o</u>	<u>$(k+1)T_o$</u>			
fwd	fwd	0	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$
fwd	bwd	2	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$
fwd	none	1	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$
bwd	fwd	-2	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$
bwd	bwd	0	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$
bwd	none	-1	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$
none	fwd	-1	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} - 1) \frac{f_r}{f_b + f_d}$
none	bwd	1	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi} + 1) \frac{f_r}{f_b + f_d}$
none	none	1	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$	$(\frac{\Delta\phi}{2\pi}) \frac{f_r}{f_b + f_d}$

the Fixed-N case varies from measurement to measurement with the average doppler, so that a constant output data rate ($1/T_0$) can only be achieved by having $T_k < T_0$ and introducing dead zones of variable duration $U_k = T_0 - T_k$ where the cycle counting is not performed. This effect is absent in the Fixed-T case, where $T_k = T_0$ is applicable. The loss of available doppler information has motivated the terminology "destructive counting" for the Fixed-N extractor, vs. "nondestructive counting" for the Fixed-T extractor. This dead zones usually eliminates error correlation over successive measurements in the Fixed-N case, while such correlation is present in the Fixed-T case and can be exploited in certain data smoothing procedures on the raw data samples obtained.

7. Error Analysis and Performance Evaluation

The formulation of the previous sections shall now be applied to derive quantitative range and range rate error results. However, the error expressions in question were contingent on the carrier and tone tracking loops maintaining lock, so that the loop tracking performance must be evaluated along with the error magnitudes to validate the results. Also, the joint reduction of dynamic and noise locking error contributions involve a loop design compromise based on the loop noise bandwidth parameter. With reference to the carrier loop, the dynamic lag error of (6.1b) should be bounded to maintain lock in the presence of doppler rate tracking by allowing a sufficiently large loop bandwidth. The minimum bandwidth value that maintains lock should then be used to limit the rms noise locking error of (5.8) and the range rate error of (6.7b). The carrier loop design and range rate error formulation based on this approach is summarized in Table 4 for PM ranging.

With reference to the tone loop design, the situation is more complicated because the dynamic lag locking error also contributes to range error. Two alternative design approaches shall be considered:

- (a) The minimum loop bandwidth is obtained by bounding the peak dynamic locking error of (6.1b), as in the carrier loop case. The loop bandwidth and dynamic locking error then become dependent on the tone frequency assignment through the doppler rate parameter ($\dot{\Omega}_m$). The dynamic range error of (6.2b) and the rms noise locking error of (5.8) will then also vary with the tone frequency since the loop bandwidth does.

The rms noise range error of (6.3) will exhibit a $(\omega_m)^{-3/4}$ functional dependence on the tone frequency rather than a simple $(\omega_m)^{-1}$ dependence as a consequence of the loop bandwidth variation. The tone loop design and range error formulation based on this approach is summarized in Table 5 for PM ranging.

(b) The minimum loop bandwidth is obtained by bounding the peak dynamic range error of (6.2b). The loop bandwidth and dynamic range error then become independent of the tone frequency assignment. However, the peak dynamic locking error of (6.1b) will then vary with the tone frequency through the doppler rate parameter $(\dot{\omega}_m)$, unlike the previous case where the loop bandwidth dependence maintained such error invariant. The rms noise locking of (5.8) will also be independent of the tone frequency, while the rms noise range error of (6.3) will exhibit the simple $(\omega_m)^{-1}$ functional dependence. The tone loop design and range error formulation based on this approach is summarized in Table 6 for PM ranging.

The basic distinction between the two design approaches can be appreciated from the table notes. The dynamic locking error (μ_m) is tone frequency independent in Table 5, but not in Table 6. Conversely, the noise locking error (σ_m) is tone frequency dependent in Table 5, but not in Table 6. The design approach of Table 5 may be supported from tracking considerations, since the loop

bandwidth is varied to account for different doppler rates to be tracked as different tone frequencies are used. The induced functional dependence $(\omega_m)^{1/4}$ of the noise locking error (σ_m) then corresponds to the loop design compromise of wider bandwidths (more noise locking errors) if the same dynamic locking status is desired regardless of the tone frequency assignment. The design approach of Table 6 may be supported from range accuracy considerations, since it features the same dynamic range error (μ_r) regardless of the tone frequency assignment, though at the expense of a variable dynamic locking status with the tone frequency. In both cases, the noise range error (σ_r) decreases with higher tone frequencies but through a different functional dependence. The carrier locking error effects would contribute a $(\cos^2 \Delta\phi)$ attenuation in the tone power level, and a $(\sec \Delta\phi)$ increase in the tone noise locking (σ_m) and ranging (σ_r) errors. This effect is not included in the formulation for simplicity, so that the numerical results will be conditioned on maintaining good carrier lock.

The tabulated formulation shall now be applied to perform a comprehensive range and range rate error analysis of the CO₂ laser ranging system. The link parameters under consideration have been specified in Table 7 for reference purposes. The carrier and tone sideband levels will be lower than the signal to noise density specified in the table by an amount governed by the modulation index (β) as previously discussed. The peak orbital acceleration shall be assumed to be $\ddot{r} = 60.15 \text{ m/sec}^2$ based on existing satellite (GEOS) orbital predictions. The peak carrier doppler rate is then $\dot{\Omega}_c = 224.56\pi \times 10^5 \text{ rad/sec}^2$ for the laser frequency specified. The carrier loop and range rate error performance then follows from

the formulation of Table 4 upon assumption of peak dynamic locking error bounds. The three bounds given by $\mu_c = 1, 5, 10$ degrees shall be assumed for both the 2nd ($\kappa = 1$) and 3rd ($\kappa = 68$) carrier loop realizations under consideration. An averaging time $T = 1$ second is assumed in the range rate extraction and rms additive noise error tabulation.

The 2nd-order loop results are first presented in Table 8 as a function of the modulation index. The tabulation of results was stopped when the loop CNR fell below 7 db, since the loop threshold is usually assumed in the 7-10 db range. It should be noted that the wider bandwidth (smaller dynamic locking error) case of Table 8(a) results in essentially below threshold operation even with all the signal power concentrated on the carrier component (no range data). The situation is not significantly improved with the lower bandwidth (larger dynamic locking error) values of Tables 8(b) and 8(c), where only marginal above threshold levels and noise locking errors are obtained, and mainly with the signal power concentrated on the carrier component. The use of a narrower bandwidth to provide a reliable above-threshold performance is not acceptable, since the peak dynamic locking error would exceed reliable locking bounds. These results thus force the conclusion that 2nd-order loops should not be employed for the carrier loop design. The 3rd-order loop results are next presented in Table 9 following the same format. The CNR levels and corresponding noise locking errors are substantially improved to permit reliable above-threshold operation, except at the larger indices where the signal power is concentrated on the tone at the carrier expense.

The tone loop performance and range error results are next presented. The dynamic locking error bound approach of Table 5 yields the 2nd and 3rd order loop results of Tables 10 and 11, respectively. The bounds of $\mu_m = 1, 5, 10$ degrees were again used, and the results were tabulated only for $F_m = 1$ MHz for simplicity. In turn, the dynamic range error bound approach of Table 6 yields the 2nd and 3rd order loop results of Tables 12 and 13, respectively. The bounds of $\mu_r = 0.01$ and 0.05 meters were used, and again the results were tabulated only for $F_m = 1$ MHz for simplicity. The effect of varying the tone frequency is acknowledged in the notes as before, and the actual numerical error values obtained as the tone frequency varies shall be included in the performance curves that follow this tabulation. In all the cases it can be verified that the TNR is sufficiently high and the noise locking errors are sufficiently low to provide a reliable tone loop lock.

The range rate error performance obtained with the 3rd-order carrier loop realization (Table 9) is illustrated in Figure 10 as a function of the modulation index (β) and the dynamic locking error bound ($\mu_c = 1, 5, 10$ deg). The use of a weaker bound than 10 deg is not recommendable, since there must be an error margin to account for additive and oscillator noise locking errors within the locked state. The use of a tighter bound than 1 deg is also undesirable, since the latter provides a sufficient error margin and a tighter bound would only increase the loop noise bandwidth, the additive noise locking error and the range rate additive noise error. The results illustrate that a low range rate error contribution from additive noise effects can be expected while maintaining a reliable dynamic error lock. The interest to account for

additional loop bandwidth constraints and potential carrier locking and range rate error degradations from these results is further emphasized by the low error magnitude obtained.

The range error performance obtained is next illustrated for both 2nd and 3rd order tone loop realizations, and also using both of the design approaches in Tables 5 and 6. The curves shall plot the rms additive noise error (σ_r) vs. the modulation index (β) as a function of the tone frequency (F_m) employed. The relation $\mu_m(\text{deg}) = 2.4 F_m(\text{MHz}) \cdot \mu_r(\text{met})$ is always satisfied as previously noted, so that there is a one-to-one correspondence between the dynamic range error (μ_r) and the tone frequency (F_m) in Table 5, and between the dynamic locking error (μ_m) and the tone frequency (F_m) in Table 6. The tone frequency dependence of the plots then also becomes either a dynamic range error or a dynamic locking error dependence for the formulations in Tables 5 or 6 respectively. Hence, each curve in the (σ_r vs. β) plots presented as a family corresponds to a pair (F_m , μ_r) or (F_m , μ_m) case. In turn, each family of curves presented in a different page corresponds to the option of 2nd vs. 3rd order loops ($\kappa = 1$ or 68) and the option of a dynamic locking vs. ranging bound design approach (Table 5 or 6).

The curves of Figures 11(a) - (f) are based on the formulation of Table 5, with the corresponding 2nd and 3rd order loop cases presented back to back to facilitate their comparison. The curves of Figures 12(a) - (d) are based on the formulation of Table 6, and follow the same format. It can be concluded that the peak dynamic and rms noise range errors (μ_r, σ_r) can both be maintained within the accuracy requirements using various moderate tone frequency and modulation index assignments, with either 2nd

or 3rd order tone loop realizations that maintain the peak dynamic and rms noise errors (μ_m, σ_m) within locking bounds. The choice of modulation index should be also guided by the range rate error results of Figure 10. The range error curves are asymptotically flat at high indeces but rise steeply at low indeces, with the opposite pattern existing for the range rate error curves. Hence, a variation in the modulation index can be directly evaluated as an rms range vs. range rate error compromise, with the quantitative trade-offs given by the curves. Such trade-offs are a direct consequence of the power sharing compromise between the sidebands (tone) and carrier components of the PM ranging signal.

TABLE 4

Carrier Loop Design and Range Rate Error Formulation

- (1) Maximum acceleration: $\ddot{r} = \text{given (met/sec}^2\text{)}$
- (2) Maximum doppler rate: $\dot{\Omega}_c = \frac{2\ddot{r}}{c} \omega_c \text{ (rad/sec}^2\text{)}$
- (3) Dynamic locking error bound: $\mu_c = \frac{\dot{\Omega}_m}{\kappa B_n^2} = \text{chosen (rad)}$
- (4) Loop noise bandwidth: $B_n = \left(\frac{\dot{\Omega}_c}{\kappa \mu_\phi} \right)^{\frac{1}{2}} = \left(\frac{2\ddot{r}\omega_c}{c\kappa\mu_\phi} \right)^{\frac{1}{2}} \text{ (Hz)}$
- (5) Noise locking error: $\sigma_c = \frac{1}{\sqrt{2}} \left(\frac{N_o B_n}{P_c} \right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} \left(\frac{N_o}{P_c} \right)^{\frac{1}{2}} \left(\frac{2\ddot{r}\omega_c}{c\kappa\mu_\phi} \right)^{\frac{1}{2}} \text{ (rad rms)}$
- (6) Noise range rate error: $\sigma_{\dot{r}} \approx \frac{c}{\sqrt{2}\omega_c T} \sigma_c = \frac{c}{\omega_c T} \left(\frac{N_o}{P_c} \right)^{\frac{1}{2}} \left(\frac{2\ddot{r}\omega_c}{c\kappa\mu_\phi} \right)^{\frac{1}{2}} \text{ (met/sec)}$

- Notes: (a) B_n directly proportional to $\omega_c^{\frac{1}{2}}$
- (b) σ_c directly proportional to $\omega_c^{\frac{1}{2}}$
- (c) $\sigma_{\dot{r}}$ directly proportional to $\omega_c^{\frac{1}{2}}$ for $P_c T$ large
- (d) $\text{CNR} = \frac{P_c}{N_o B_n} = \frac{J_o^2(\beta) P_c}{N_o B_n}$ for PM ranging

TABLE 5

Tone Loop Design and Range Error Formulation: Dynamic Locking Error Bound

- (1) Maximum acceleration: $\ddot{r} = \text{constant (met/sec}^2\text{)}$
- (2) Maximum doppler rate: $\dot{\Omega}_m = \frac{2\ddot{r}}{c} \omega_m \text{ (rad/sec}^2\text{)}$
- (3) Dynamic locking error bound: $\mu_m = \frac{\dot{\Omega}_m}{K B_n^2} = \text{chosen (rad)}$
- (4) Loop noise bandwidth: $B_n = \left(\frac{\dot{\Omega}_m}{K \mu_m} \right)^{\frac{1}{2}} = \left(\frac{2\ddot{r}\omega_m}{cK\mu_m} \right)^{\frac{1}{2}} \text{ (Hz)}$
- (5) Dynamic range error: $\mu_r = \frac{\ddot{r}}{K B_n^2} = \frac{c\mu_m}{2\omega_m} \text{ (met)}$
- (6) Noise locking error: $\sigma_m = \frac{1}{\sqrt{2}} \left(\frac{2N_o B_n}{4P_m} \right)^{\frac{1}{2}} = \frac{1}{2} \left(\frac{N_o}{P_m} \right)^{\frac{1}{2}} \left(\frac{2\ddot{r}\omega_m}{cK\mu_m} \right)^{\frac{1}{4}} \text{ (rad rms)}$
- (7) Noise range error: $\sigma_r = \frac{c}{2\omega_m} \sigma_m = \frac{c}{4\omega_m} \left(\frac{N_o}{P_m} \right)^{\frac{1}{2}} \left(\frac{2\ddot{r}\omega_m}{cK\mu_m} \right)^{\frac{1}{4}} \text{ (met rms)}$

Notes: (a) $\mu_r = \mu_m / (2.4 F_m) \text{ (met)}$ if $\mu_m \text{ (deg)}$ and $F_m \text{ (MHz)}$

(b) μ_m independent of F_m

(c) μ_r inversely proportional to F_m

(d) B_n directly proportional to $F_m^{\frac{1}{2}}$

(e) TNR inversely proportional to $F_m^{\frac{1}{2}}$

(f) σ_m directly proportional to $F_m^{\frac{1}{4}}$

(g) σ_r inversely proportional to $F_m^{\frac{3}{4}}$

(h) $\text{TNR} = \frac{2P_m}{N_o B_n} = \frac{2J_1^2(\beta)P_i}{N_o B_n}$ for PM ranging (3 db processing gain)

TABLE 6

Tone Loop Design and Range Error Formulation: Dynamic Range Error Bound

- (1) Maximum acceleration: $\ddot{r} = \text{given}$ (met/sec²)
- (2) Maximum doppler rate: $\dot{\Omega}_m = \frac{2\ddot{r}}{c} \omega_m$ (rad/sec²)
- (3) Dynamic range error bound: $\mu_r = \frac{\ddot{r}}{\kappa B_n^2} = \text{chosen}$ (met)
- (4) Loop noise bandwidth: $B_n = \left(\frac{\ddot{r}}{\kappa \mu_r} \right)^{\frac{1}{2}}$ (Hz)
- (5) Dynamic locking error: $\mu_m = \frac{\dot{\Omega}_m}{\kappa B_n^2} = \frac{2\mu_r \omega_m}{c}$ (rad)
- (6) Noise locking error: $\sigma_m = \frac{1}{\sqrt{2}} \left(\frac{2N_o B_n}{4P_m} \right)^{\frac{1}{2}} = \frac{1}{2} \left(\frac{N_o}{P_m} \right)^{\frac{1}{2}} \left(\frac{\ddot{r}}{\kappa \mu_r} \right)^{\frac{1}{4}}$ (rad rms)
- (7) Noise range error: $\sigma_r = \frac{c}{2\omega_m} \sigma_m = \frac{c}{4\omega_m} \left(\frac{N_o}{P_m} \right)^{\frac{1}{2}} \left(\frac{\ddot{r}}{\kappa \mu_r} \right)^{\frac{1}{4}}$ (met rms)

Notes: (a) $\mu_m = 2.4 \mu_r F_m$ (deg) if μ_r (met) and F_m (MHz)

(b) μ_r , B_n , TNR, σ_m independent of F_m

(c) μ_m directly proportional to F_m

(d) σ_r inversely proportional to F_m

(e) $\text{TNR} = \frac{4P_m}{2N_o B_n} = \frac{4J_1^2(\beta)P_i}{2N_o B_n}$ for PM ranging (3 db processing gain)

TABLE 7
Link Parameter Specifications

Carrier Frequency	2.8×10^{13} GHz
Transmitter Power	23 dbw (200 w)
Transmitter Antenna Gain	100 db
Receiver Antenna Gain	101 db
Transponder Reflector Gain	162 db
Path Loss (two-way)	-493.78 db
Miscellaneous Losses	-26.40 db
Receiver Signal Level	-134.18 dbw
Noise density	-190 dbw - Hz
Signal to Noise Density	55.82 db/Hz

TABLE 8
Range Rate Error Analysis (Second-Order Carrier Loop)

8(a) Carrier Loop: PLL₂, $\mu_c = 1$ deg, $B_c = 63.92$ KHz

β (rad)	CNR (db)	σ_c (deg)	σ_r (met/sec)
0.1	7.74	16.61	3.459×10^{-7}
0.2	7.67	16.74	3.485×10^{-7}
0.3	7.56	16.95	3.529×10^{-7}
0.4	7.41	17.26	3.593×10^{-7}
0.5	7.21	17.66	3.677×10^{-7}

8(b) Carrier Loop: PLL₂, $\mu_c = 5$ deg, $B_c = 28.59$ KHz

β (rad)	CNR (db)	σ_c (deg)	σ_r (met/sec)
0.1	11.23	11.11	2.313×10^{-7}
0.2	11.17	11.19	2.331×10^{-7}
0.3	11.06	11.34	2.360×10^{-7}
0.4	10.91	11.54	2.403×10^{-7}
0.5	10.70	11.81	2.459×10^{-7}
0.6	10.46	12.15	2.530×10^{-7}
0.7	10.16	12.58	2.619×10^{-7}
0.8	9.81	13.10	2.727×10^{-7}
0.9	9.40	13.72	2.857×10^{-7}
1.0	8.93	14.48	3.016×10^{-7}
1.1	8.40	15.40	3.207×10^{-7}
1.2	7.79	16.51	3.438×10^{-7}
1.3	7.11	17.87	3.721×10^{-7}

8(c) Carrier Loop: PLL₂, $\mu_c = 10$ deg, $B_c = 20.21$ KHz

<u>β (rad)</u>	<u>CNR (db)</u>	<u>σ_c (deg)</u>	<u>σ_f (met/sec)</u>
0.1	12.74	9.343	1.945×10^{-7}
0.2	12.67	9.413	1.960×10^{-7}
0.3	12.56	9.533	1.985×10^{-7}
0.4	12.41	9.704	2.020×10^{-7}
0.5	12.21	9.931	2.068×10^{-7}
0.6	11.96	10.22	2.128×10^{-7}
0.7	11.66	10.58	2.202×10^{-7}
0.8	11.31	11.01	2.293×10^{-7}
0.9	10.90	11.54	2.403×10^{-7}
1.0	10.44	12.18	2.536×10^{-7}
1.1	9.90	12.95	2.696×10^{-7}
1.2	9.30	13.89	2.891×10^{-7}
1.3	8.61	15.03	3.129×10^{-7}
1.4	7.83	16.44	3.423×10^{-7}

TABLE 9

Range Rate Error Analysis (Third-Order Carrier Loop)

9(a) Carrier Loop: PLL₃, $\mu_c = 1$ deg, $B_c = 7.751$ KHz

<u>β (rad)</u>	<u>CNR (db)</u>	<u>σ_c (deg)</u>	<u>σ_r (met/sec)</u>
0.1	16.90	5.786	1.205×10^{-7}
0.2	16.84	5.829	1.214×10^{-7}
0.3	16.73	5.903	1.229×10^{-7}
0.4	16.57	6.009	1.251×10^{-7}
0.5	16.37	6.150	1.280×10^{-7}
0.6	16.12	6.328	1.318×10^{-7}
0.7	15.83	6.549	1.364×10^{-7}
0.8	15.47	6.819	1.420×10^{-7}
0.9	15.07	7.147	1.488×10^{-7}
1.0	14.60	7.542	1.570×10^{-7}
1.1	14.07	8.020	1.670×10^{-7}
1.2	13.46	8.599	1.790×10^{-7}
1.3	12.77	9.307	1.938×10^{-7}
1.4	11.99	10.18	2.120×10^{-7}
1.5	11.11	11.28	2.348×10^{-7}
1.6	10.09	12.67	2.639×10^{-7}
1.7	8.92	14.50	3.019×10^{-7}
1.8	7.55	16.97	3.534×10^{-7}

9(b) Carrier Loop: PLL₃; $\mu_c = 5$ deg, $B_c = 3.467$ KHz

<u>β (rad)</u>	<u>CNR (db)</u>	<u>σ_c (deg)</u>	<u>σ_f (met/sec)</u>
0.1	20.40	3.869	0.8056×10^{-7}
0.2	20.33	3.898	0.8116×10^{-7}
0.3	20.22	3.948	0.8219×10^{-7}
0.4	20.07	4.019	0.8367×10^{-7}
0.5	19.87	4.112	0.8562×10^{-7}
0.6	19.62	4.232	0.8811×10^{-7}
0.7	19.32	4.380	0.9119×10^{-7}
0.8	18.97	4.560	0.9495×10^{-7}
0.9	18.56	4.779	0.9951×10^{-7}
1.0	18.09	5.044	1.050×10^{-7}
1.1	17.56	5.363	1.117×10^{-7}
1.2	16.95	5.751	1.197×10^{-7}
1.3	16.27	6.224	1.296×10^{-7}
1.4	15.49	6.809	1.418×10^{-7}
1.5	14.60	7.541	1.570×10^{-7}
1.6	13.59	8.475	1.764×10^{-7}
1.7	12.42	9.697	2.019×10^{-7}
1.8	11.05	11.35	2.363×10^{-7}

9(c) Carrier Loop: PLL₃; $\mu_c = 10$ deg, $B_c = 2.451$ KHz

β (rad)	CNR (db)	σ_c (deg)	σ_f (met/sec)
0.1	21.90	3.254	0.6774×10^{-7}
0.2	21.84	3.278	0.6825×10^{-7}
0.3	21.73	3.320	0.6921×10^{-7}
0.4	21.57	3.379	0.7036×10^{-7}
0.5	21.37	3.458	0.7200×10^{-7}
0.6	21.12	3.559	0.7409×10^{-7}
0.7	20.83	3.683	0.7668×10^{-7}
0.8	20.47	3.835	0.7984×10^{-7}
0.9	20.07	4.019	0.8368×10^{-7}
1.0	19.60	4.241	0.8830×10^{-7}
1.1	19.07	4.510	0.9390×10^{-7}
1.2	18.46	4.836	1.007×10^{-7}
1.3	17.77	5.234	1.090×10^{-7}
1.4	16.99	5.725	1.192×10^{-7}
1.5	16.11	6.341	1.320×10^{-7}
1.6	15.09	7.126	1.484×10^{-7}
1.7	13.92	8.155	1.698×10^{-7}
1.8	12.55	9.546	1.987×10^{-7}

TABLE 10

Range Error Analysis (Second-Order Tone Loop, Dynamic Locking Error Bound)

10 (a) Tone Loop: PLL_2 , $\mu_m = 1$ deg, $F_m = 1$ MHz, $B_m = 12.02$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	22.00	3.218	1.340
0.2	27.99	1.615	0.6725
0.3	31.45	1.083	0.4511
0.4	33.88	0.8198	0.3413
0.5	35.72	0.6633	0.2762
0.6	37.18	0.5605	0.2334
0.7	38.37	0.4884	0.2034
0.8	39.37	0.4357	0.1814
0.9	40.20	0.3958	0.1648
1.0	40.90	0.3652	0.1521
1.1	41.49	0.3412	0.1421
1.2	41.98	0.3225	0.1343
1.3	42.38	0.3078	0.1282
1.4	42.71	0.2965	0.1235
1.5	42.96	0.2880	0.1199
1.6	43.14	0.2820	0.1174
1.7	43.26	0.2781	0.1158
1.8	43.32	0.2763	0.1151

Note: $\mu_r = \mu_m / (2.4 F_m) = 0.416$ met for $\mu_m = 1$ deg and $F_m = 1$ MHz. F_m scaling decreases (μ_r , TNR^2 , $\sigma_r^{4/3}$) and increases (B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

10(b) Tone Loop: PLL₂, $\mu_m = 5$ deg, $F_m = 1$ MHz, $B_m = 5.375$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	25.49	2.152	0.8960
0.2	31.48	1.080	0.4497
0.3	34.95	0.7245	0.3017
0.4	37.37	0.5482	0.2283
0.5	39.21	0.4436	0.1847
0.6	40.67	0.3748	0.1561
0.7	41.87	0.3266	0.1360
0.8	42.86	0.2914	0.1213
0.9	43.69	0.2647	0.1102
1.0	44.39	0.2442	0.1017
1.1	44.98	0.2282	0.09502
1.2	45.47	0.2157	0.08980
1.3	45.88	0.2059	0.08571
1.4	46.20	0.1983	0.08256
1.5	46.46	0.1926	0.08020
1.6	46.64	0.1886	0.07851
1.7	46.76	0.1860	0.07745
1.8	46.82	0.1848	0.07695

Note: $\mu_r = \mu_m / (2.4F_m) = 2.08$ met for $\mu_m = 5$ deg and $F_m = 1$ MHz.

F_m scaling decreases (μ_r , TNR^2 , $\sigma_r^{4/3}$) and increases (B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

10(c) Tone Loop: PLL₂, $\mu_m = 10$ deg, $F_m = 1$ MHz, $B_m = 3.801$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	27.00	1.810	0.7535
0.2	32.99	0.9082	0.3781
0.3	36.45	0.6093	0.2537
0.4	38.88	0.4610	0.1919
0.5	40.72	0.3730	0.1553
0.6	42.18	0.3152	0.1312
0.7	43.37	0.2747	0.1144
0.8	44.37	0.2450	0.1020
0.9	45.20	0.2226	0.09269
1.0	45.90	0.2054	0.08550
1.1	46.49	0.1919	0.07990
1.2	46.98	0.1814	0.07551
1.3	47.38	0.1731	0.07208
1.4	47.71	0.1667	0.06943
1.5	47.96	0.1620	0.06744
1.6	48.14	0.1586	0.06602
1.7	48.26	0.1564	0.06512
1.8	48.36	0.1554	0.06470

Note: $\mu_r = \mu_m / (2.4 F_m) = 4.16$ met for $\mu_m = 10$ deg and $F_m = 1$ MHz.
 F_m scaling decreases (μ_r , TNR², $\sigma_r^{4/3}$) and increases
(B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

TABLE 11

Range Error Analysis (Third-Order Tone Loop, Dynamic Locking Error Bound)

11(a) Tone Loop: PLL_3 , $\mu_m = 3$ deg, $F_m = 1$ MHz, $B_m = 1.458$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	31.16	1.121	0.4666
0.2	37.15	0.5624	0.2342
0.3	40.62	0.3773	0.1571
0.4	43.04	0.2855	0.1189
0.5	44.88	0.2310	0.09618
0.6	46.34	0.1952	0.08127
0.7	47.54	0.1701	0.07082
0.8	48.53	0.1517	0.06317
0.9	49.36	0.1378	0.05740
1.0	50.06	0.1272	0.05295
1.1	50.65	0.1188	0.04948
1.2	51.14	0.1123	0.04676
1.3	51.55	0.1072	0.04463
1.4	51.87	0.1033	0.04299
1.5	52.12	0.1003	0.04176
1.6	52.31	0.09819	0.04089
1.7	52.43	0.09686	0.04033
1.8	52.43	0.09623	0.04007

Note: $\mu_r = \mu_m / (2.4 F_m) = 0.416$ met for $\mu_m = 1$ deg and $F_m = 1$ MHz. F_m scaling decreases (μ_r , TNR², σ_r) and increases(B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

11(b) Tone Loop: PLL₃, $\mu_m = 5$ deg, $F_m = 1$ MHz, $B_m = 0.6518$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ (met)
0.1	34.66	0.7494	0.3120
0.2	40.64	0.3761	0.1566
0.3	44.11	0.2523	0.1051
0.4	46.53	0.1909	0.07949
0.5	48.37	0.1545	0.06432
0.6	49.84	0.1305	0.05435
0.7	51.03	0.1137	0.04736
0.8	52.02	0.1015	0.04255
0.9	52.86	0.09218	0.03838
1.0	53.56	0.08504	0.03451
1.1	54.15	0.07947	0.03309
1.2	54.64	0.07510	0.03127
1.3	55.04	0.07169	0.02985
1.4	55.37	0.06905	0.02875
1.5	55.62	0.06707	0.02793
1.6	55.80	0.06567	0.02734
1.7	55.92	0.06477	0.02697
1.8	55.98	0.06435	0.02680

Note: $\mu_r = \mu_m / (2.4 F_m) = 2.08$ met for $\mu_m = 5$ deg and $F_m = 1$ MHz.

F_m scaling decreases (μ_r , TNR², σ , ^{1/3}) and increases (B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

11(c) Tone Loop: PLL₃, $\mu_m = 10$ deg, $F_m = 1$ MHz, $B_m = 0.4609$ Hz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	36.16	0.6302	0.2624
0.2	42.15	0.3163	0.1317
0.3	45.62	0.2122	0.08834
0.4	48.04	0.1605	0.06684
0.5	49.88	0.1299	0.05408
0.6	51.34	0.1098	0.04570
0.7	52.54	0.09565	0.03983
0.8	53.53	0.08532	0.03552
0.9	54.36	0.07752	0.03228
1.0	55.06	0.07151	0.02978
1.1	55.65	0.06683	0.02782
1.2	56.14	0.06315	0.02630
1.3	56.55	0.06028	0.02510
1.4	56.87	0.05807	0.02418
1.5	57.12	0.05640	0.02348
1.6	57.31	0.05522	0.02299
1.7	57.43	0.05447	0.02268
1.8	57.48	0.05411	0.02253

Note: $\mu_r = \mu_m / (2.4 F_m) = 4.16$ met for $\mu_m = 10$ deg and $F_m = 1$ MHz.

F_m scaling decreases (μ_r , TNR^2 , $\sigma_r^{1/3}$) and increases (B_m^2 , σ_m^4) proportionately, (μ_m remains invariant).

TABLE 12

Range Error Analysis (Second-Order Tone Loop, Dynamic Range Error Bound)

12(a) Tone Loop: PLL_2 , $\mu_r = 0.01$ met, $B_m = 77.56$ Hz, $F_m = 1$ MHz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	13.90	8.174	3.404
0.2	19.89	4.102	1.708
0.3	23.36	2.752	1.146
0.4	25.78	2.082	0.8671
0.5	27.62	1.685	0.7016
0.6	29.08	1.424	0.5928
0.7	30.28	1.241	0.5166
0.8	31.27	1.107	0.4608
0.9	32.10	1.006	0.4187
1.0	32.80	0.9276	0.3862
1.1	33.39	0.8668	0.3609
1.2	33.88	0.8192	0.3411
1.3	34.29	0.7820	0.3256
1.4	34.61	0.7532	0.3136
1.5	34.86	0.7316	0.3046
1.6	35.05	0.7163	0.2982
1.7	35.17	0.7065	0.2942
1.8	35.22	0.7020	0.2923

Note: $\mu_m = 2.4 \mu$, $F_m = 0.024$ deg for $\mu_r = 0.01$ met and $F_m = 1$ MHz.

F_m scaling increases μ_m and decreases σ_r proportionately
 (μ_r , B_m , TNR, σ_m remains invariant).

12(b) Tone Loop: PLL₂, $\mu_r = 0.05$ met, $B_m = 34.68$ Hz, $F_m = 1$ MHz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	17.40	5.466	2.276
0.2	23.38	2.743	1.142
0.3	26.85	1.840	0.7663
0.4	29.27	1.393	0.5798
0.5	31.11	1.127	0.4692
0.6	32.58	0.9521	0.3965
0.7	33.77	0.8297	0.3455
0.8	34.76	0.7401	0.3082
0.9	35.60	0.6724	0.2800
1.0	36.30	0.6203	0.2583
1.1	36.89	0.5797	0.2414
1.2	37.38	0.5478	0.2281
1.3	37.78	0.5229	0.2177
1.4	38.11	0.5037	0.2097
1.5	38.36	0.4893	0.2037
1.6	38.54	0.4790	0.1994
1.7	38.66	0.4725	0.1967
1.8	38.72	0.4694	0.1955

Note: $\mu_m = 2.4 \mu_r$, $F_m = 0.12$ deg for $\mu_r = 0.05$ met and $F_m = 1$ MHz.
 F_m scaling increases μ_m and decreases σ_r proportionately
(μ_r , B_m , TNR, σ_m remains invariant).

TABLE 13

Range Error Analysis (Third-Order Tone Loop, Dynamic Range Error Bound)

13 (a) Tone Loop: PLL₃, $\mu_r = 0.01$ met, $B_m = 9.405$ Hz, $F_m = 1$ MHz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	23.06	2.847	1.185
0.2	29.05	1.429	0.5948
0.3	35.52	0.9584	0.3991
0.4	34.94	0.7252	0.3019
0.5	36.78	0.5867	0.2443
0.6	38.24	0.4958	0.2064
0.7	39.44	0.4321	0.1799
0.8	40.43	0.3854	0.1605
0.9	41.26	0.3502	0.1458
1.0	41.96	0.3230	0.1345
1.1	42.55	0.3019	0.1257
1.2	43.04	0.2853	0.1188
1.3	43.45	0.2723	0.1134
1.4	43.77	0.2623	0.1092
1.5	44.03	0.2548	0.1061
1.6	44.21	0.2494	0.1039
1.7	44.33	0.2466	0.1024
1.8	44.39	0.2444	0.1018

Note: $\mu_m = 2.4 \mu_r$, $F_m = 0.024$ deg for $\mu_r = 0.01$ met and $F_m = 1$ MHz. F_m scaling increases μ_m and decreases σ_r proportionately,(μ_r , B_m , TNR, σ_m remains invariant).

13(b) Tone Loop: PLL₃, $\mu_r = 0.05$ met, $B_m = 4.206$ Hz, $F_m = 1$ MHz

β (rad)	TNR (db)	σ_m (deg)	σ_r (met)
0.1	26.56	1.904	0.7926
0.2	32.55	0.9554	0.3978
0.3	36.01	0.6409	0.2669
0.4	38.44	0.4849	0.2019
0.5	40.27	0.3924	0.1634
0.6	41.74	0.3316	0.1381
0.7	42.93	0.2889	0.1203
0.8	43.93	0.2577	0.1073
0.9	44.76	0.2342	0.09750
1.0	45.46	0.2160	0.08995
1.1	46.05	0.2019	0.08405
1.2	46.54	0.1908	0.07943
1.3	46.94	0.1821	0.07582
1.4	47.27	0.1754	0.07304
1.5	47.52	0.1704	0.07094
1.6	47.70	0.1668	0.06945
1.7	47.82	0.1645	0.06851
1.8	47.88	0.1635	0.06807

Note: $\mu_m = 2.4 \mu_r$, $F_m = 0.12$ deg for $\mu_r = 0.05$ met and $F_m = 1$ MHz.

F_m scaling increases μ_m and decreases σ_r proportionately,

(μ_r , B_m , TNR, σ_m remains invariant).

Figure 10: Range Rate Error Results

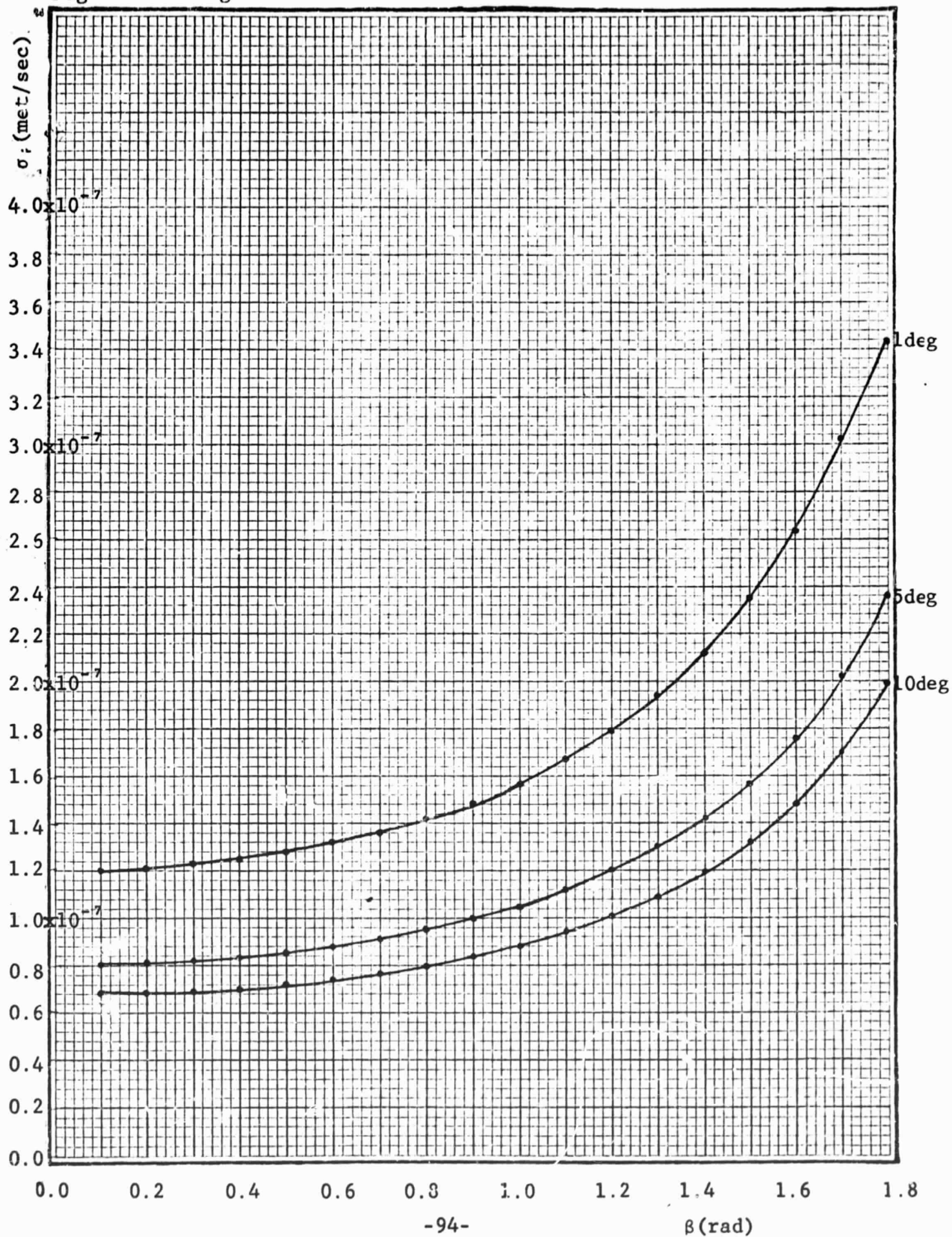


Figure 11(a) Range Error Results (PLL = 2nd, $\mu_m = 1$ deg)

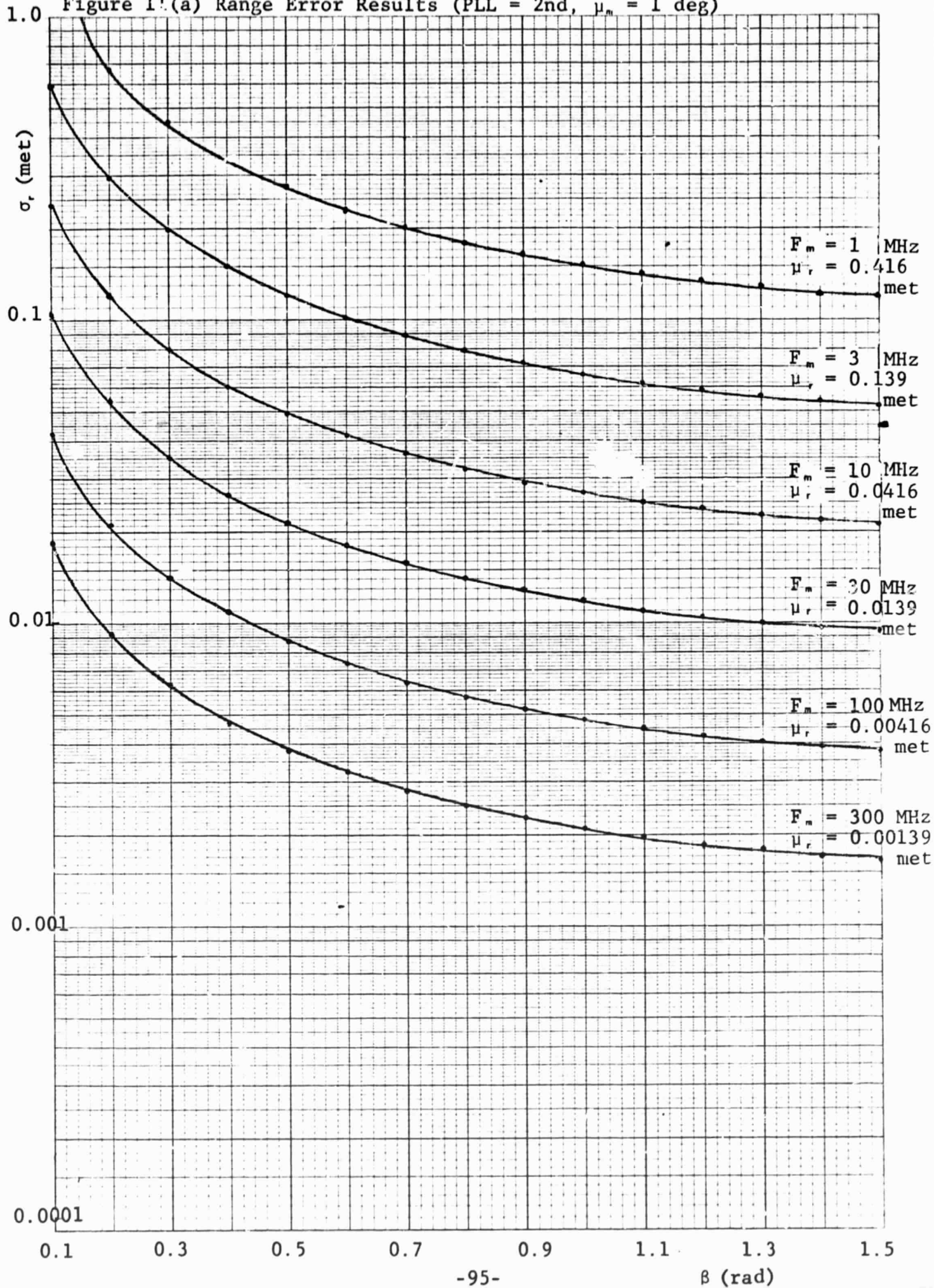


Figure 11(b) Range Error Results (PLL = 3rd, $\mu_m = 1$ deg)

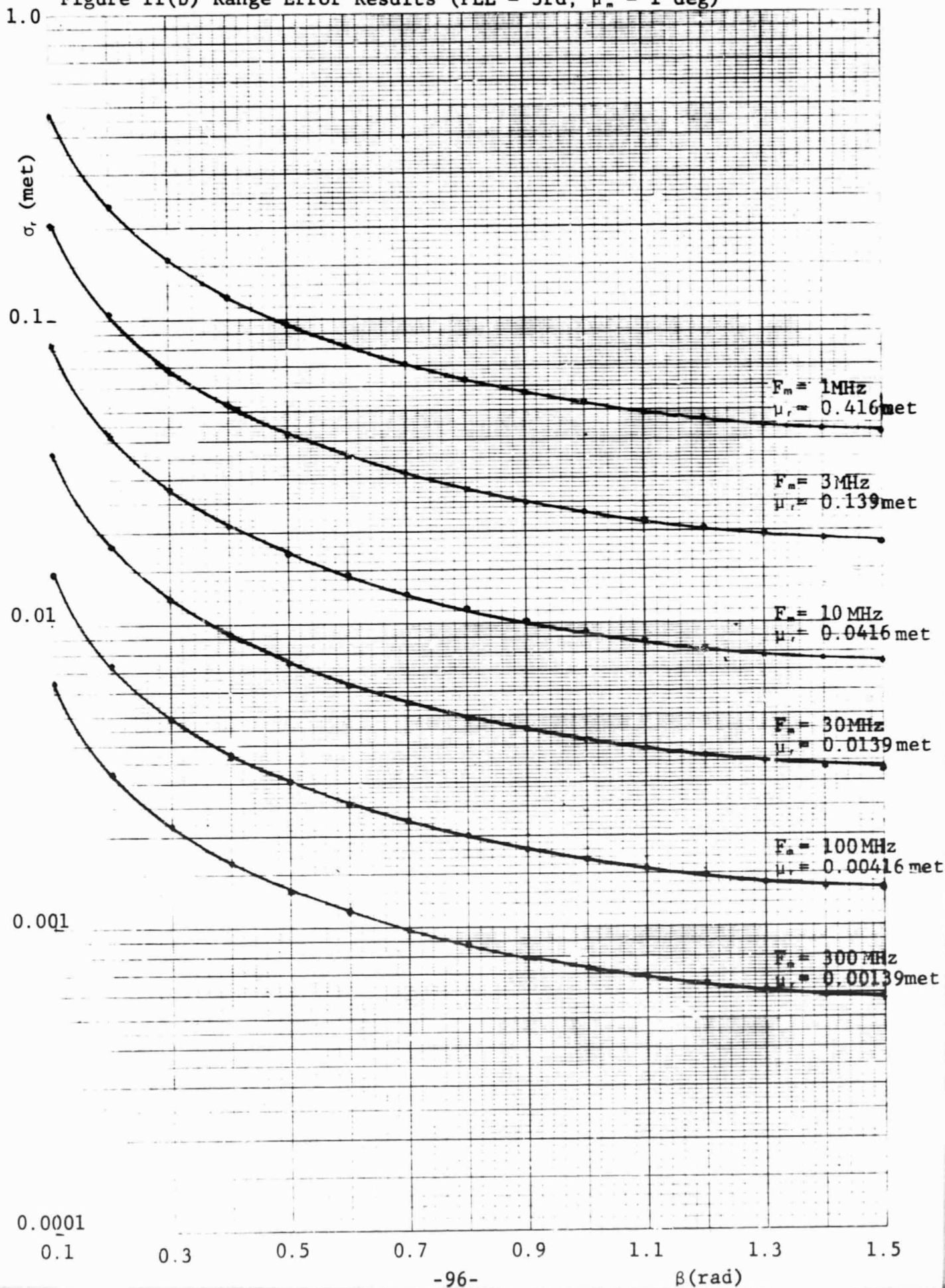
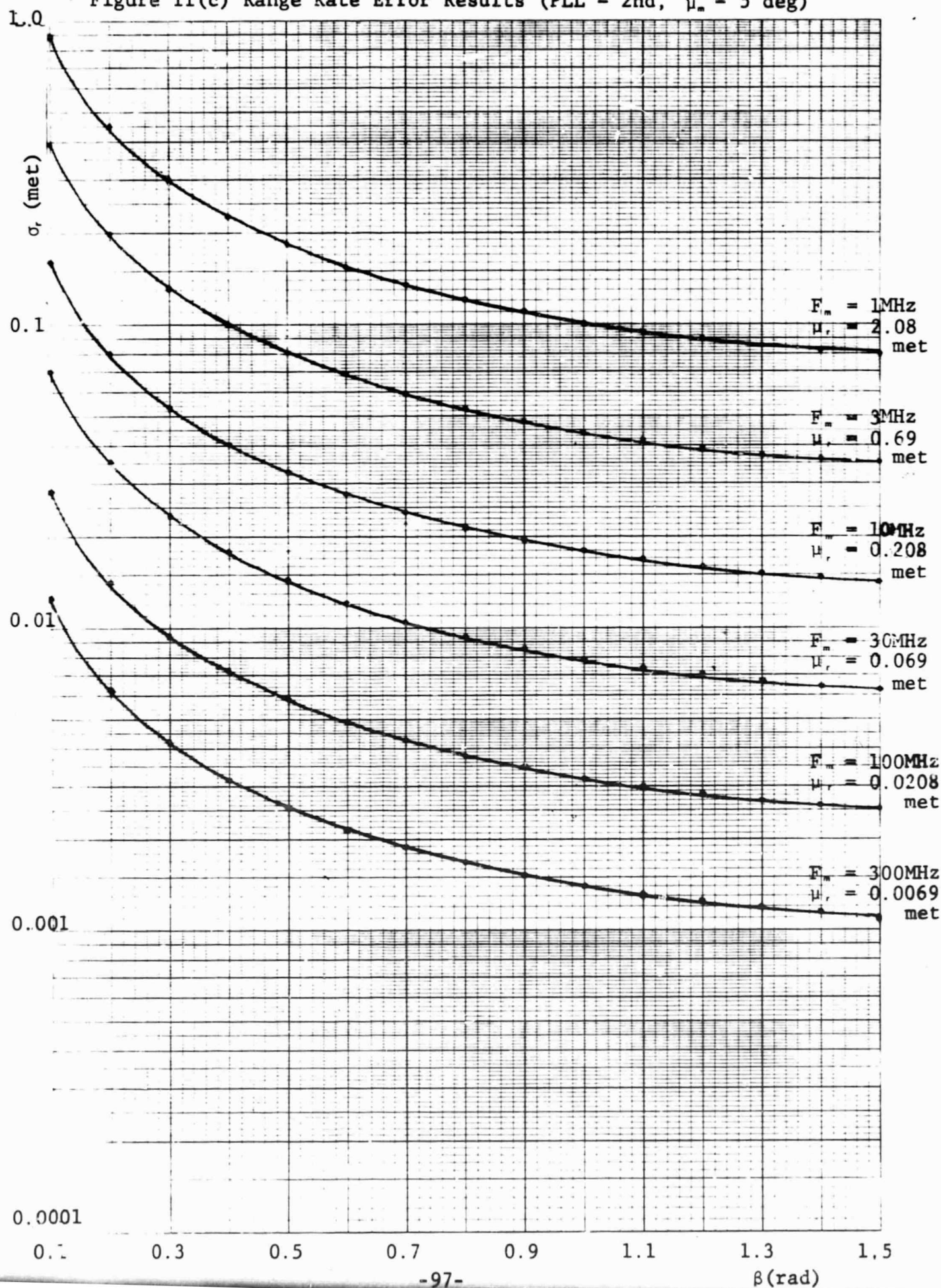


Figure 11(c) Range Rate Error Results (PLL = 2nd, $\mu_n = 5$ deg)



1.0 Figure 11(d) Range Error Results (PLL = 3rd, $\mu_n = 5$ deg)

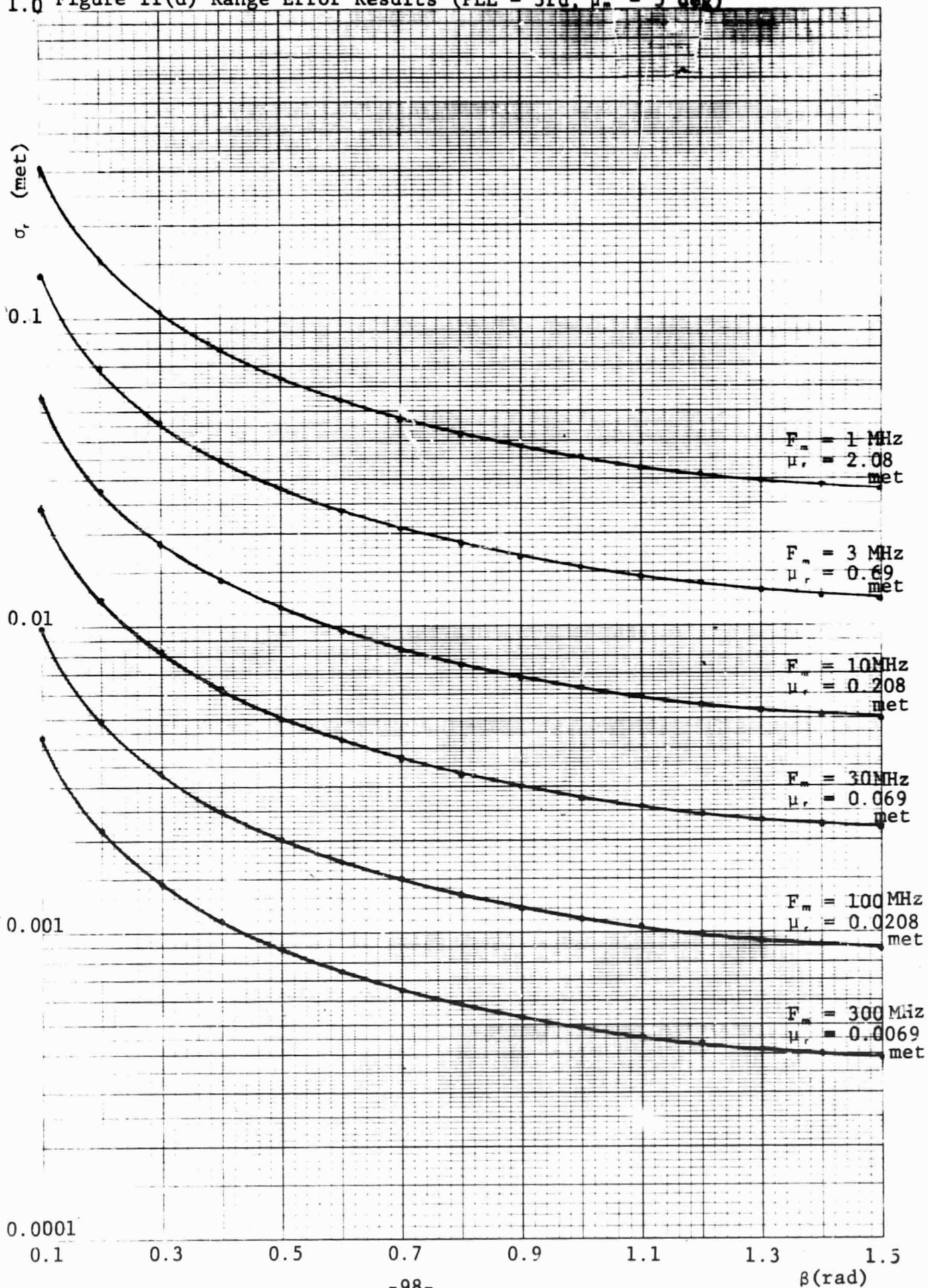


Figure 11(e) Range Error Results (PLL = 2nd, $\mu_m = 10$ deg)

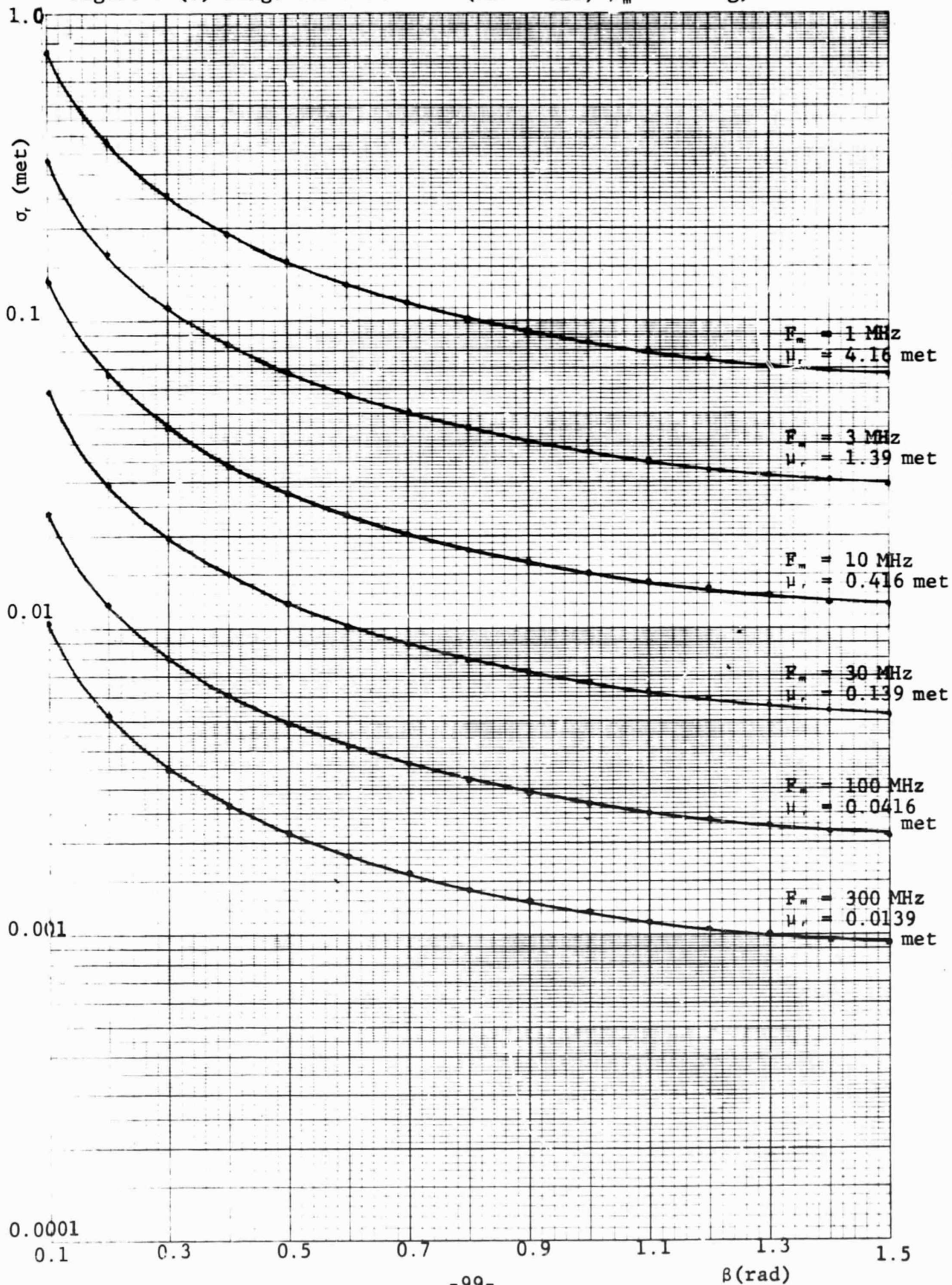


Figure 11(f) Range Error Results (PLL = 3rd, $\mu_m = 10$ deg)

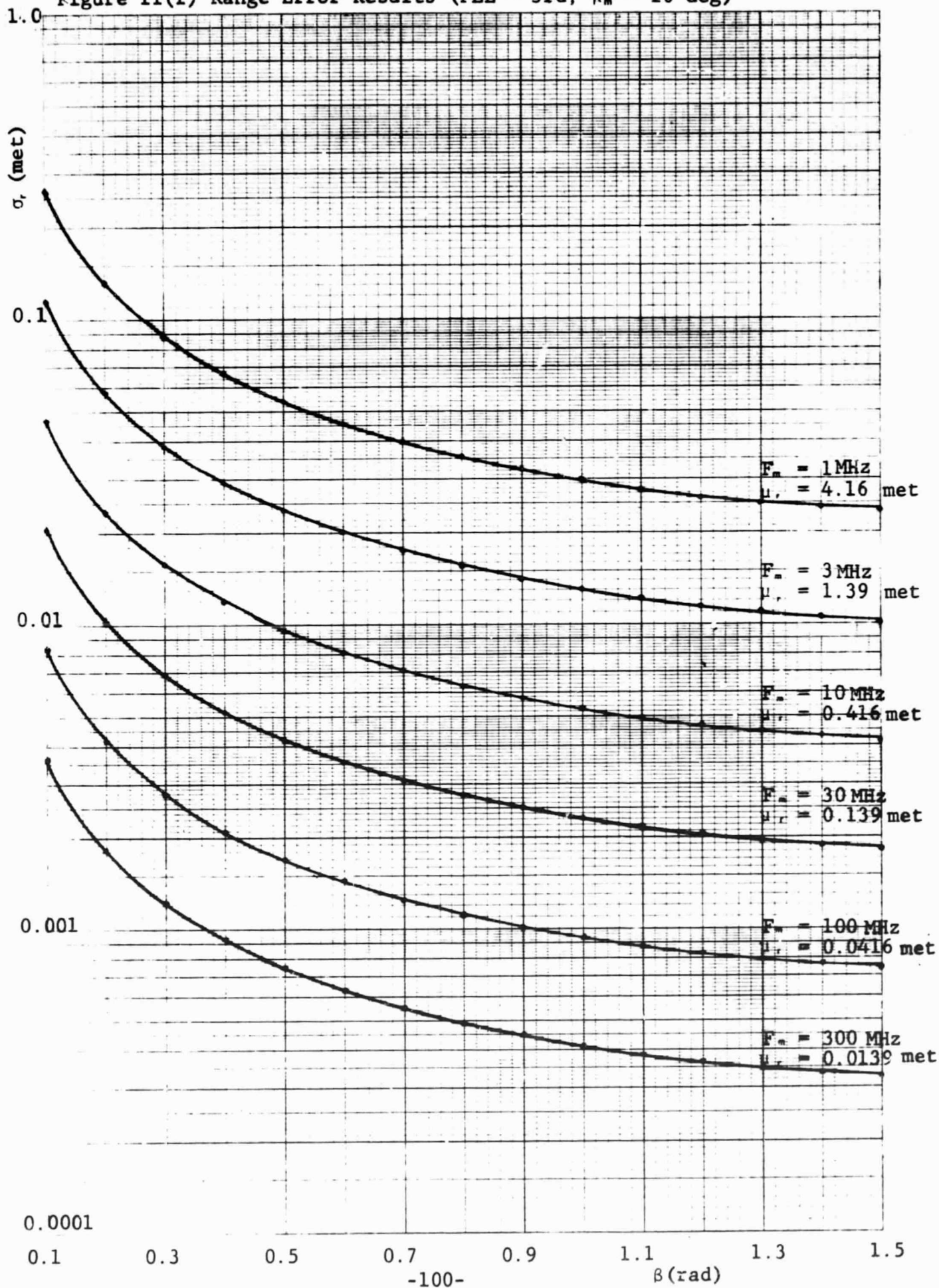


Figure 12(a) Range Error Results (PLL = 2nd, $\mu_r = 0.01$ met)

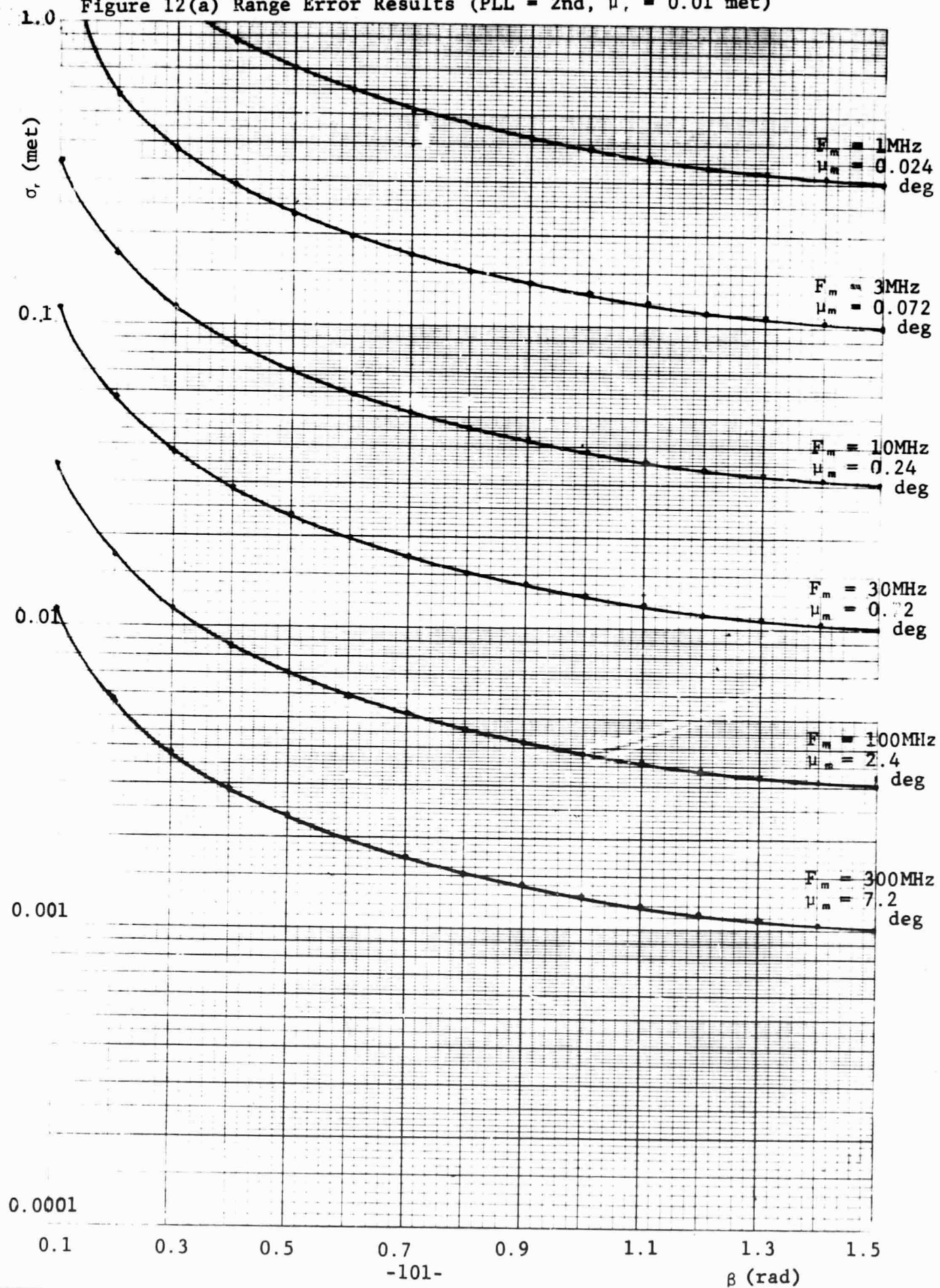


Figure 12(b) Range Error Results (PLL = 3rd, $\mu_r = 0.01$ met)

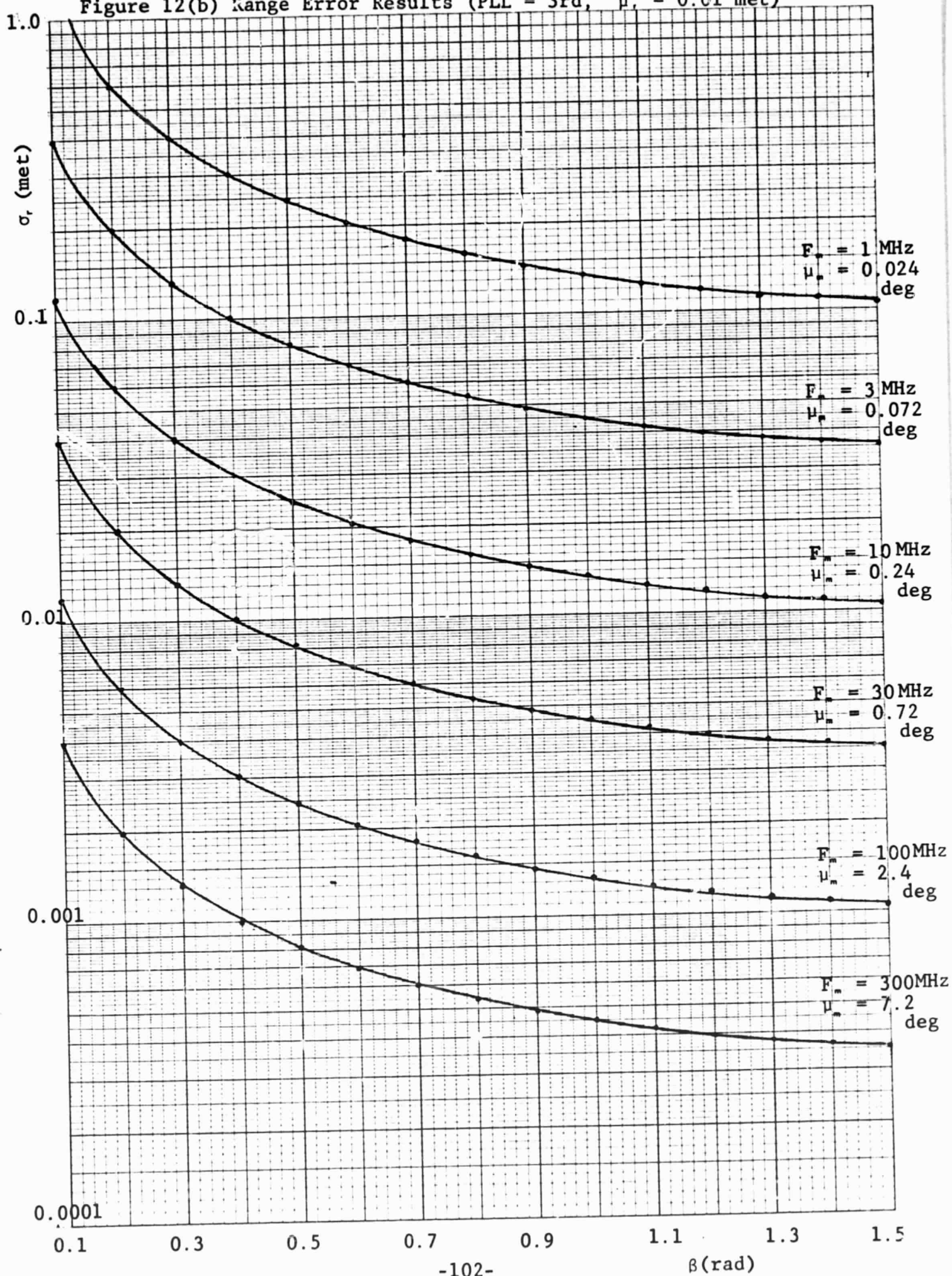


Figure 12(c) Range Error Results (PLL = 2nd, $\mu_r = 0.05$ met)

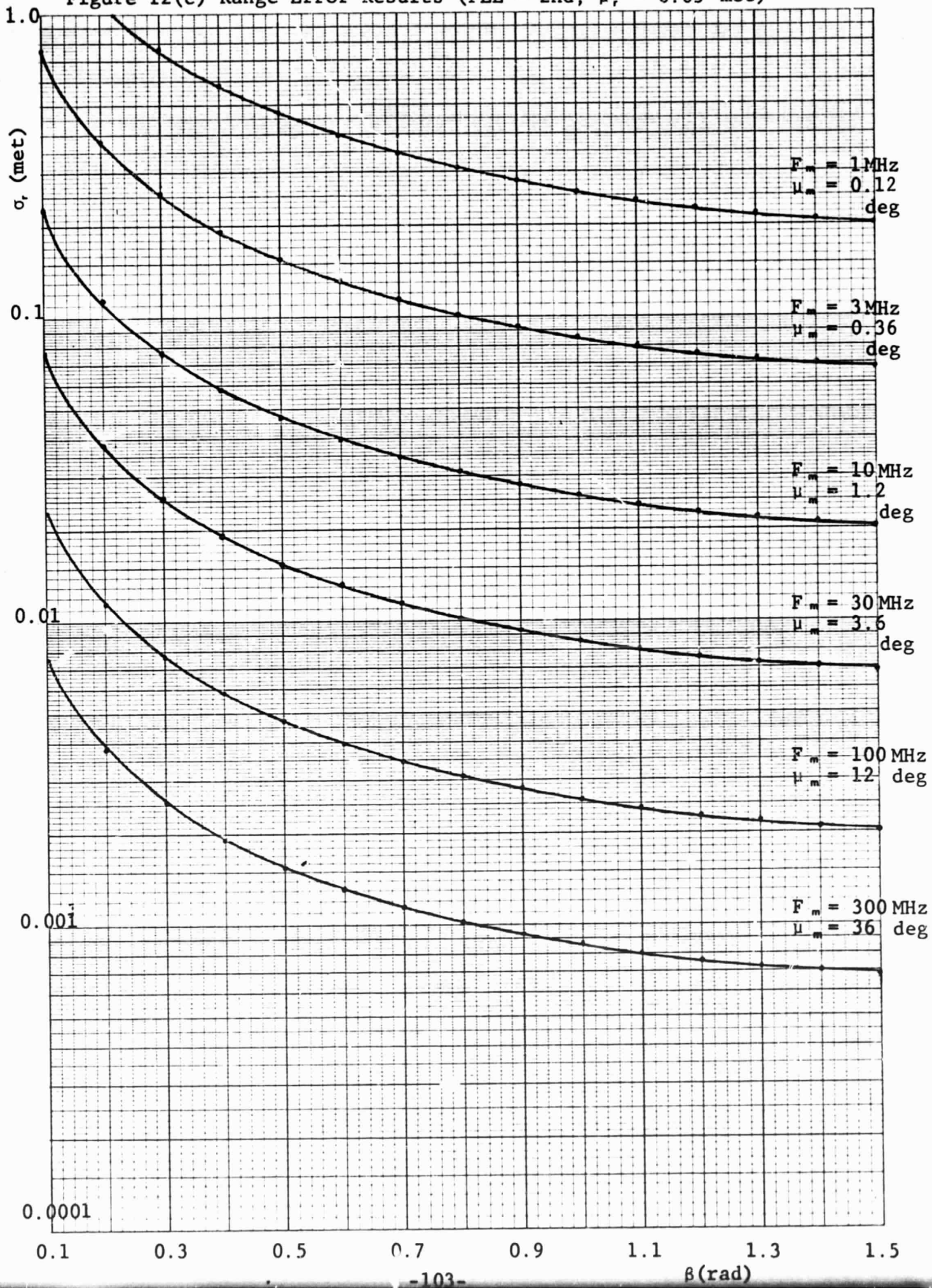
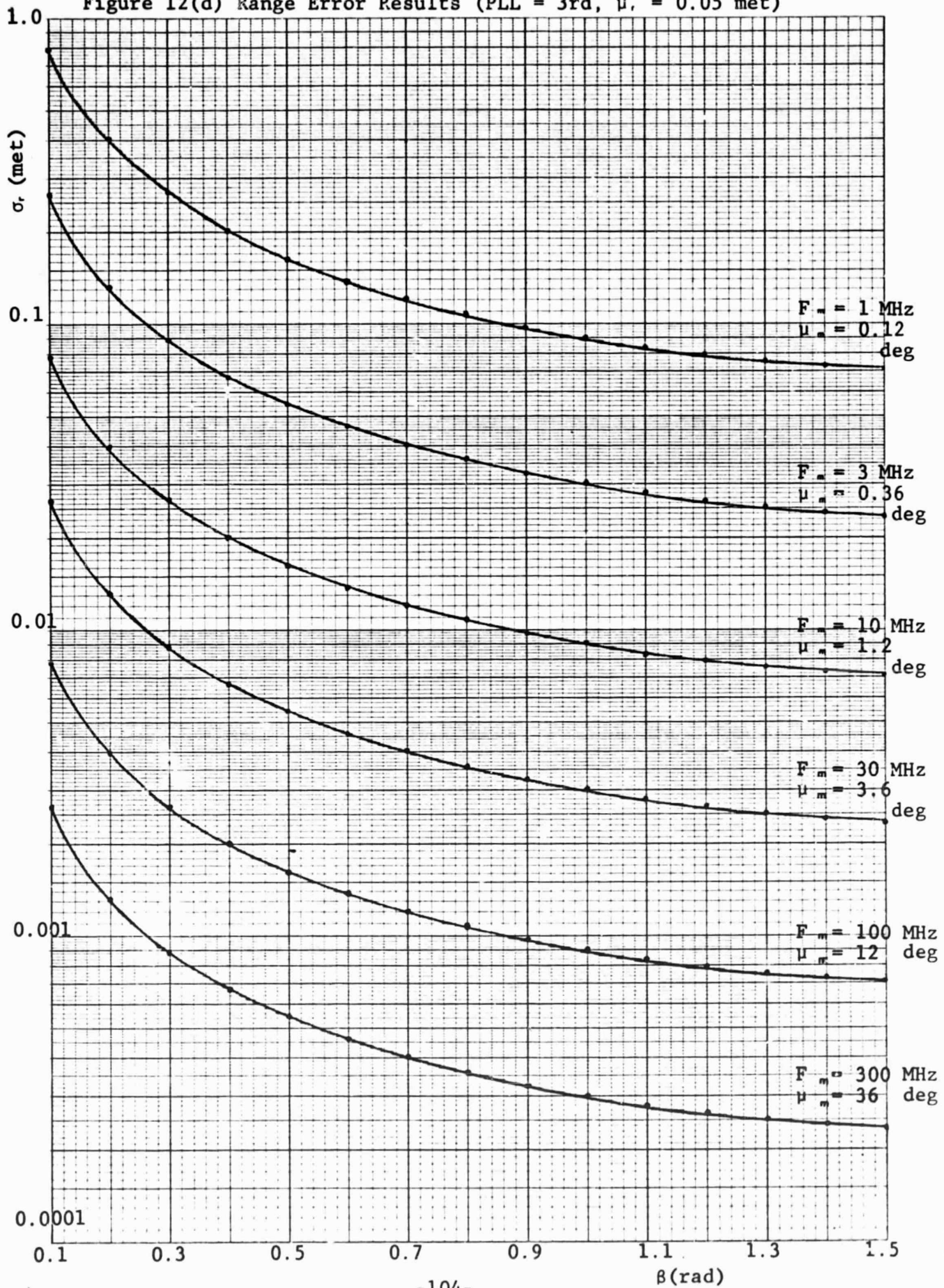


Figure 12(d) Range Error Results (PLL = 3rd, $\mu_r = 0.05$ met)



8. Conclusions and Recommendations

This report has covered in detail the work performed on the CO₂ laser ranging system. A system design based on independent range and range rate data was originally motivated, and the signal structure and subsystem processing were evaluated to this effect. The use of a high frequency range tone was recommended for the fine ranging operation, with lower subharmonic tones and a pseudo-noise code accounting for the ambiguity resolution operation. The fine ranging and ambiguity resolution operations should be performed sequentially to minimize the power sharing compromise between the sidebands (range signal) and the carrier (range rate signal) components.

The fine range tone modulation on the carrier can be based on AM/DSB, PM or coupling modulation methods, along with efficient coherent product demodulation techniques at the receiver. The carrier vs. sideband power sharing compromise was evaluated for these cases. The PM system features the advantages of a DSB narrow-band spectrum constant-envelope processing, plus an effective power sharing compromise. The use of SSB systems is undesired due to the harmful carrier locking error propagation effects. The coherent product demodulation exhibits phase-locked extraction of the carrier and demodulated tone components. The phase-locked loops act as lowpass filters on the input signal plus noise phase fluctuations, thus inducing a loop design compromise of wideband dynamic tracking potential vs. narrowband noise rejection capabilities.

The loop design and tracking performance was evaluated for 2nd and 3rd order loop realizations. The dynamic tracking limitations

of non-ideal vs. ideal integrators in the loop filters were carefully identified, and the locking error analysis accounted for the different nature of dynamic vs. noise error contributions. A conventional 2nd order loop and a state-of-the-art 3rd order loop were finally assumed as candidate realizations for the carrier and tone loop reproduction. The range and range rate accuracy limitations contributed by their dynamic tracking and additive noise errors were formulated in terms of the locking status and the SNR conditions prevailing in these loops. The loop design approach and resultant ranging error performance was then formulated to comply with both 2nd and 3rd order loop realizations, and to include the alternative control of dynamic locking vs. dynamic ranging error bounds.

The rms range and range rate error contributions of additive noise were evaluated as a function of the modulation index, the tone frequency, the loop realization, and the dynamic locking or ranging error bound. The existing loop locking errors and SNR conditions were tabulated for lock verification purposes, and comprehensive range and range rate error performance curves were provided to guide the system design and performance compromises. A 2nd-order carrier loop design could only provide below threshold or marginal above threshold locking conditions, and cannot be recommended. Conversely, a 3rd order carrier loop design exhibited a reliable locking performance, plus yielded a negligible range rate error contribution from additive noise effects. Both 2nd and 3rd order tone loops were also capable of maintaining lock while yielding acceptable additive noise range errors for various moderate modulation index and tone frequency values. The logical

assignment of modulation parameters was then governed by the range vs. range rate accuracy compromise set by the system designer.

The encouraging error performance thus obtained suggests the natural extension of the analysis to investigate the effects of reference and voltage-controlled oscillator noise. Both the carrier and tone frequencies involved in the laser ranging system are sufficiently high to motivate that such effects be accounted for in the locking and ranging error analysis. The oscillator phase noise inputs to the tracking loops will introduce additional locking constraints that may affect the resultant range and range rate error magnitudes. The oscillator noise errors will appear superimposed on the additive noise errors as an additive random disturbance, plus the additive noise errors may themselves vary due to the loop redesign that may be required to maintain a locked status.

The carrier or tone loop tracking performance may also be improved by considering natural extensions of the 3rd-order loop realization assumed. The use of different parameter values in the Mallinckrodt loop filter approximation based on a passive plus active network should be investigated, along with the additional realistic improvements that can be expected with an all-active network realization. Even though such modifications have not been implemented (to the best of our knowledge), they represent simple logical extensions of the state-of-the-art technology and merit investigation based on their theoretical locking and ranging error improvement potential.

9. Glossary of Basic Symbols

A	= signal amplitude (volts)
B_c	= two-sided carrier loop noise bandwidth (Hz)
B_m	= two-sided tone loop noise bandwidth (Hz)
B_n	= two-sided loop noise bandwidth (Hz)
$F(s)$	= loop filter transfer function
F_m	= tone frequency (MHz)
$H(s)$	= closed-loop transfer function
$J_n(\beta)$	= Bessel function coefficient
K	= loop gain (sec^{-1})
K_f	= loop filter gain (volt/volt)
K_p	= phase detector gain (volt/rad)
K_v	= vco gain (rad/sec-volt)
L	= pseudo-noise code length (symbols)
M	= reference cycle count (cycles)
M_n	= doppler dynamic magnitude (rad, rad/sec, rad/sec^2 , etc.)
N	= doppler plus bias cycle count (cycles)
N_0	= one-sided additive thermal noise density (watts/Hz)
P	= pseudo-noise code period (sec)
P_c	= carrier component power (watts)
P_m	= tone component power (watts)
P_t	= total signal power (watts)
T	= doppler measurement interval (sec)
T_o	= output range rate data period (sec)

c = speed of light (met/sec, cm/sec)

f = frequency unit (Hz)

f_b = bias frequency (Hz)

f_c = carrier frequency (Hz)

f_d = doppler effect (Hz)

f_m = tone frequency (Hz)

f_r = reference frequency (Hz)

r = slant range (met)

$\dot{r}$ = slant velocity or range rate (met/sec)

$\ddot{r}$ = slant acceleration (met/sec²)

s = laplace operator

t = time unit (sec)

t_p = propagation delay (sec)

Ω = doppler shift (rad/sec)

$\dot{\Omega}$ = doppler rate (rad/sec²)

Ω_c = carrier doppler shift (rad/sec)

$\dot{\Omega}_c$ = carrier doppler rate (rad/sec²)

Ω_m = tone doppler shift (rad/sec)

$\dot{\Omega}_m$ = tone doppler rate (rad/sec²)

α = AM sinusoidal modulation factor

β = PM sinusoidal modulation index (rad)

γ = coupling modulation factor

κ = 1 (2nd-order loop) or 68 (3rd-order loop)

λ = 3rd-order loop parameter