

NASA SP-371

NASA
SP-371
c.1

LOAN COPY: RETURN TO
AFWL TECHNICAL LIBRARY
KIRTLAND AFB, N.M.

TECH LIBRARY KAFB, NM

0063399

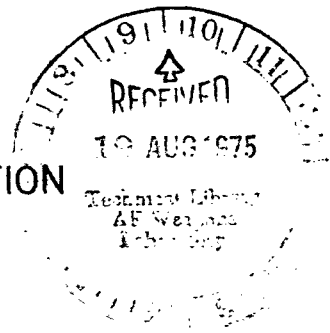


LARGE-SCALE DYNAMIC SYSTEMS


A seminar workshop held at
UTAH STATE UNIVERSITY
Logan, Utah
August 12-16, 1974



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION



2. u/u

1.
NASA/SP-371
TECH LIBRARY KAFB, NM

0063399

3.
**LARGE-SCALE
DYNAMIC SYSTEMS,**

The proceedings of the Utah State University-Ames Research Center Seminar Workshop on ~~Large Scale Dynamic Systems~~, held at Utah State University, Logan, Utah, August 12-16, 1974/

Prepared by Ames Research Center



Scientific and Technical Information Office
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C.
5. 1975/

For sale by the National Technical Information Service
Springfield, Virginia 22151 \$7.25

FOREWORD

The Utah State University — Ames Research Center Seminar Workshop on Large-Scale Dynamic Systems was held August 12-16, 1974, at Logan, Utah. The workshop was supported by Ames Research Center. These proceedings contain the papers presented at this workshop.

I wish to thank the participants for their cooperation in arranging the conference and for their promptness in supplying manuscripts. The support and encouragement given by Professor Ralph Johnson, Dean of the College of Science, and Professor Lawrence O. Cannon, Head of the Department of Mathematics, are gratefully acknowledged.

Special thanks are due Larry Sharp, who handled the many small details during the workshop, and also Wanda Sayer and Lynette Robinson, who cheerfully completed the almost impossible task of typing the manuscripts and rough drafts of the proceedings.

CLYDE F. MARTIN

PARTICIPANTS

Masanao Aoki
University of Illinois

Michael Athans
Massachusetts Institute of Technology

C. Y. Chong
Georgia Institute of Technology

Kai-Ching Chu
IBM, Thomas J. Watson Center

Luigi Cicolani
Ames Research Center

Hermann W. Dommell
University of British Columbia

Brian Doolin
Ames Research Center

Y. C. Ho
Harvard University

Trevor Hughes
Utah State University

Raymond Jacquot
University of Wyoming

Clyde Martin
Utah State University

John Richardson
Case Western Reserve University

Leonard Roberts
Ames Research Center

Larry Sharp
Utah State University

Dragoslav Siljak
Santa Clara University

Glen Smerage
Utah State University

Bruce Watkins
Utah State University

TABLE OF CONTENTS

	Page
Foreword	iii
List of Participants	iv
Summary	1
Brian Doolin and Clyde Martin Ames Research Center and Utah State University	
Simulation of Large-Scale Dynamic Systems	7
Leonard Roberts, Ames Research Center	
Hierarchical Models of Very Large Problems, Dilemmas, Prospects, and an Agenda for the Future	23
John M. Richardson, Jr., Case Western Reserve University	
An Informal Paper on Large-Scale Dynamic Systems	49
Y. C. Ho, Harvard University	
Analysis of Large-Scale Water Resources Systems	63
A. Bruce Bishop and Trevor C. Hughes, Utah State University	
Survey of Decentralized Control Methods	101
Michael Athans, Massachusetts Institute of Technology	
Feedback Control of a Single Irrigation Canal Reach	113
Raymond G. Jacquot, University of Wyoming	
Analysis of Large Power Systems	119
Herman W. Dommel, University of British Columbia	
Decentralized Regulation of Dynamic Systems	131
Kai-ching Chu, IBM Watson Research Center	
Large-Scale Systems: Complexity, Stability, Reliability	147
D. D. Šiljak, University of Santa Clara	
Digital Simulation of V/STOL Aircraft for Autopilot Research	163
L. S. Cicolani and George Meyer, Ames Research Center	
Some Notions of Decentralization and Coordination in Large-Scale Dynamic Systems	185
C. Y. Chong, Georgia Institute of Technology	
Some Aspects of Control of a Large-Scale Dynamic System	195
Masanao Aoki, University of Illinois	

**A SUMMARY OF THE PROCEEDINGS OF THE UTAH STATE UNIVERSITY –
AMES RESEARCH CENTER SEMINAR WORKSHOP ON
LARGE SCALE DYNAMIC SYSTEMS***

Brian Doolin

Ames Research Center

and

Clyde Martin

Utah State University

PURPOSE AND APPROACH

This seminar workshop was held on the campus of Utah State University during the week of August 12–16, 1974. The general purpose of the workshop was to discern classes of large-scale dynamic systems, with no attempt to solve their problems. It was felt that such a workshop was necessary to help bridge the gap between the theory that power has been substantiated in small problems where analytic models are readily verified and that required for the almost incomprehensibly large problems in aeronautics, economics, natural resource management, and many other fields which modern computational capability has emboldened researchers to face.

The approach adopted for the workshop contained three elements. One element was to examine three major examples of large-scale systems. It was hoped that an examination of these examples would result in a transfer of knowledge so that control theorists would gain a broader view of large systems and a knowledge of how the problems are being successfully handled. Principles or properties common to large systems could be extracted. Finally, it was hoped that studying the examples in the context of modern control theory would present directions of research that would lead to useful methods of analysis and synthesis. The remarks that follow show the workshop was somewhat successful in realizing these hopes.

The three technical areas covered in the examples were aeronautics, water resources, and electric power. Aeronautics technology was described in two papers. The opening paper reviewed the growing use, in the development of aircraft, of computer-based methods to coordinate procedures and concepts from several technical disciplines. The second illustrated the complexity in one of the simulations. Water resources management was also discussed in two papers: the first described the problem in general and the second illustrated the dynamic analysis of a canal reach. The development of analysis and its present status in the electric power distribution industry were covered in a single paper.

*Supported by NASA Grant No. NSG-2022.

A second element of the approach of the workshop was to review control theory relevant to large-scale systems. This was accomplished by the control theorists who explained the development of their ideas. The provocative paper by John Richardson, which discussed methodology in terms of examples from water eutrophication and world modeling, could be categorized as either example or theory.

The third element of the approach was a discussion period. Each afternoon, the participants freely aired their thoughts evoked by the papers presented and by restatements of the aims of the workshop.

WORKSHOP RESULTS

The most significant result of the workshop was the active transference of ideas and concepts during the afternoon discussion periods. Naturally, there was some fundamental disagreement on the significance of certain technical points, but this reflects the fact that the techniques to be used in the study of large-scale dynamic systems are not yet known.

Though the existence of large-scale systems as objects for understanding and management was unanimously affirmed, there was no consensus on a definition. Nor were acceptable quantitative measures of scale proposed. Various general properties from different points of view could be ascribed to them. From the viewpoint of developing analytic models, a system is large when its input-output behavior cannot be understood without curtailing it, partitioning it into modules, and aggregating its modularized subsystems. From a systems viewpoint, a system is large if it exceeds the capacity of a single control structure. This circumstance arises when there is too much data for a single-decision element to process in a timely fashion. A common solution is to decentralize the control structure at some cost in performance but with gains in reliability. Finally, there was concern that analytic models of some large systems could not be used to predict behavior over intervals of time much longer than required for observation. One reason for the concern was the experience of numerical difficulties due to the size of the model. Its predicted behavior would be accepted only if accompanied by some measure of error propagation. Another reason for the concern was that, when human response is involved, the model could not be made sufficiently complete.

The discussions led to a thorough review of the state of the theoretical knowledge of decentralized and connected systems. Centralization versus decentralization appears to be a major question in the control of large systems. It is now clear that quadratic optimality of linear systems implies centralized control and information at least under conditions of instantaneous and perfect transmission of information. Under real conditions, transformation and information is usually neither instantaneous nor perfect. Then decentralization offers the potential for increased reliability (in the sense of immunity from catastrophe) and subsystem stability. Sometimes, however, decentralization decreases the overall reliability by increasing the time required to detect failure. Then how to trade performance for reliability becomes obscure. Reliability has become an active area of research and one for which major results are almost certain to be obtained soon.

Although theory is developing on centralization versus decentralization and on optimization, reliability, and connective stability, there is a clear need for real numerical examples. If the control

theorists reached a consensus, it was that there is a strong continuing need to study particular examples in detail, both simple "textbook" examples and real detailed examples of systems currently being successfully analyzed and controlled.

The discussions of definitions, examples, and current theory led to an attempt to list dominant characteristics of large-scale dynamic systems (table 1). This table is in no way complete, and possibly cannot be completed. Also, the descriptors in the first column are not necessarily disjoint with respect to a given system. For example, a large system quite often exhibits both dynamic and static behavior. The last pair of descriptors in the column refers to the phenomenon that occurs in models of some large systems that, when a major structural perturbation occurs, the models before and after the perturbation bear almost no relation to each other. This phenomenon evidently occurs in power systems and is somehow related to the way subsystems are interconnected.

The columns "principles" and "questions" are closely related and there was sometimes discussion as to which column a particular item belonged. For example, there were questions about whether computability in the sense of predicting behavior is inherent to some models or a function of the model and designer. The column "methods" reflects the tools available at least at the theoretical level. The tools that will eventually be used in the study of large systems will almost certainly be at least based on this list.

RESEARCH NEEDS

The research needs are discussed under two categories: well defined or poorly defined. The well-defined problems are likely to be solved in the next few years if the past is any guide. Experience has shown that any problem that can be properly formulated mathematically and that makes sense as an engineering problem can be either solved or circumvented. Conversely, the poorly defined problems will require considerable time and no little luck to solve.

The first problem area expected to be well defined is optimization. The question of optimizing a large system is beset by many difficulties, not the least of which is that optimality may not be desirable. If total system optimization is sought, a major concern will be to show that the end result is reliable and stable under perturbations. Thus the critical question is the appropriate expression for a performance criterion. Without imposing special strictures, performance optimization itself implies highly centralized control. Yet large systems somehow argue toward decentralized structure. Preliminary results seem to indicate that imposing constraints on the rate at which information can pass through a unified structure or using parallel structures to desaturate control may allow optimization to yield decentralized structures. It seems therefore that any major advance in optimization theory for large systems will factor information structure, control saturation, or a structural measure of reliability into the performance criterion.

Decentralized structure can be assumed from the beginning in stability analysis. From that point of view, stability is in better shape than optimization, though it, too, faces major difficulties. Vector Lyapunov functions seem to yield the most promising approach at the moment, and seem to be a natural approach for some economic and biological systems. But Lyapunov functions that give sharp results are notoriously hard to achieve. Hence research is required to find stability tests that are well suited to large systems.

System reliability is an area that has begun to receive attention and an area in which major results and solutions are almost certain to be announced in the next few years. Reliability is being considered from several different viewpoints ranging from duplication of components at the simplest level to rather deep studies of the effect of decentralization. There are two different approaches on the effect of decentralization, one is roughly that it tends to decrease reliability by making detection more difficult and the other is that decentralization tends to increase reliability by making communication routes shorter.

There is not, at present, a general theory of aggregation. Aggregation techniques are used in practice, but generally in strictly an *ad hoc* fashion. Results that would indicate what information is lost by aggregation would be quite useful.

Many aspects of these problem areas undoubtedly will be solved or at least studied fruitfully in the near future. There is some suspicion, however, that a greater, less definable problem is acceptance of a developed theory of large-scale systems in economics, sociology, and ecology. The pressure toward acceptance is not so great in these areas as it is in aeronautics, for example, where improvements use technologies that obey demonstrable laws and the improvements must be as safe as analysis and testing can make them. In the softer areas, models are difficult to verify. It may not even be possible to demonstrate the validity for a predictive model whose adequacy is a central requirement in any theory of large systems. Hence it is not surprising that present models are not considered reliable in planning for more than short periods ahead. Extreme caution in accepting and using a theory is well justified considering the number of people that can be affected and the potential for catastrophe of decisions derived from improper models.

CONCLUSIONS

The workshop addressed all its objectives directly and obtained both positive and negative results. All participants unanimously affirmed the worth of the workshop. Such workshops serve a real need for transferring ideas and technology between groups that would not ordinarily meet. In fact, this workshop could have profited from the inclusion of specialists in more fields. Specialists in human performance, information transfer, and automata certainly would have made useful contributions. Above all, closer ties between mathematics and engineering would be desirable and would conserve effort in both disciplines.

TABLE 1.— ASPECTS OF LARGE-SCALE SYSTEMS STUDIES

Descriptors	Principles	Questions	Methods
Prescriptive Descriptive	Observability	Optimization	Math programming
Centralized Decentralized	Controllability	Stabilization	Variational methods
Off line On line	Invertibility	Reliability	Lyapunov methods
Open loop Closed loop	Decomposibility	Allocation	Graph theory
Static Dynamic	Information structure	Scheduling	Game theory
Deterministic Stochastic	Coordination	Model validation	Decision theory
Aggregated Detailed	Computability		Truncations
Fixed structure Variable structure	Reducibility		Singular perturbation
	Stability		

SIMULATION OF LARGE-SCALE DYNAMIC SYSTEMS

Leonard Roberts*

Ames Research Center, NASA

I am honored to be the first speaker at this workshop on large-scale dynamic systems. Also, I am impressed by the scope of the workshop: "to define the classes of large-scale dynamic systems, to extract principles common to known systems, and to develop theories for the rational analysis of large-scale systems."

Certainly, the need for a comprehensive study of large-scale systems is very evident. We need only look at the state of the existing dynamic systems on which we have come to depend critically in our daily lives to conclude that this attention is needed. Our society has become increasingly dependent, almost to the point of total reliance, on a series of networks, each of which is a complex dynamic system: ground and air transportation; energy distribution; a communication system (telephone, telegraph, radio, and television); and a system of water supply, distribution, and use. As each system is stretched to the breaking point by increasing demand, they interact with each other and with yet another complex dynamic system, the natural environment.

We have seen and felt the impact of these interactions in the past year. The effect of the energy crisis, which reduced the supply of crude oil in the United States by a rather small percentage (less than 10 percent for a period of only a few months) was disproportionate relative to the size of the reduction. Road and air traffic was disrupted; communication systems, used as a substitute for travel, were overloaded; power utilities could not operate at the same efficiencies because of an unwillingness to use the more costly fuel oil, and so on. The whole system was in fact so dynamic that it went into modes of vibration that were hardly thought possible.

Nor are the dynamics of some of the proposed solutions any better understood. Substituting hydrogen for jet propulsion fuel in aircraft, for example, would entail a cryogenic ground distribution system and a completely new generation of aircraft, the designs for which are only very vaguely defined and whose impact on the air transportation system has not yet been considered. The more extensive use of coal and nuclear energy may have environmental implications that we do not fully comprehend.

Despite the awesomeness of the task, the initial approach of considering the variety of large systems we are familiar with is the correct one. In each major dynamic system, there is a fund of practical experience that represents our best source of information from which to formulate governing principles. These systems should be classified and characterized expeditiously so that individual classes of dynamic systems can be studied, thereby avoiding the pursuit of global principles that may not in fact exist.

At this point, a word of caution is due with regard to the so-called "total systems approach." This term is much used and, while it has great merit as a statement of our intention to consider all the important interactions of the system, it is also well to remember that its success depends finally

*Director, Aeronautics and Flight Systems.

on our understanding of the component parts of the system. I suggest that, as we seek to describe the dynamic character of large-scale systems, we continue to check our representation of the component parts so that the answers do not become academic as a result of unrealistic assumptions.

The tools to carry out the investigation of large-scale dynamic systems are generally available and in good shape. The theoretical tools, even for the analysis of nonlinear dynamic systems, have been developed and successfully applied in the past. With the advent of large-capacity, high-speed computers over the past decade, we can now model nonlinear systems subjected to random effects with some degree of confidence.

My remarks to this point have been rather general and I think it might be a good idea if I were to be more specific. I have spent most of my professional career in and around aeronautical activities and I hope you will forgive me for concentrating on one example of a dynamic system – air transportation. Air transportation is an important national system and it also illustrates some of the ingredients normally found in large-scale systems modeling.

I would like to consider the modeling of aircraft and air transportation at various levels of completeness and discuss the effect on computer time and speed required and the amount of “people time” required. Figure 1 shows the scope of the considerations which must be given to the total task of describing the behavior of aircraft, the traffic environment in which it operates, and the air transportation system it serves. I will describe some of the work going on in NASA in each category and then make some observations that may apply to system modeling more generally.

AIRCRAFT DYNAMICS

Aircraft synthesis, even in its simplest form, requires that the aerodynamics, structure, and propulsion of the vehicle be represented sufficiently well so that the major weight and performance tradeoff studies can be made. The primary inputs to aircraft design are represented in figure 2. The interactions between these disciplinary modules are computed and aircraft concepts are configured through a control and optimization module. The design inputs and outputs are made by use of interactive graphics so that the operator can follow the influence of the changing design inputs on the configuration.

The kind of design synthesis or vehicle description shown in table 1 can be conducted at several levels, of course, and I have summarized the computer time and “people time” involved for several levels. At the conceptual design level, this effort can be as little as 1 man year and 5 hours of computer time (on a CDC 7600) and may involve only a small number of weight elements. At the detailed design level, 100 man years of effort may be required and 1000 hours of computer design with perhaps 5000 structural elements. A fourth level of design (not shown here), final design, is usually carried out before aircraft construction and may involve an increase in effort one or two orders of magnitude over that shown for detailed design.

Thus far, I have said little about the dynamic representation of an aircraft. The dynamic modeling of aircraft has received much attention in recent years and a great deal of new understanding has resulted from the combined theoretical and experimental approach used by NASA. Figure 3 illustrates the components of a dynamic model. The primary aerodynamic and structural elements

are represented in sufficient detail that the aircraft loads, the shape deformation that results, and the consequent changes in aerodynamic characteristics can be calculated. Similarly, the disturbance function and control functions are also represented so that the total aircraft behavior can be analyzed.

Here, again, the degree of complexity of the model can vary, depending on the accuracy of the representation required. In figure 4, an aircraft is subjected to a gust. On gust penetration, the aerodynamic forces on the aircraft are changed, causing the aircraft to follow a perturbed flight path. If the aircraft is properly designed, these motions will be damped out and the aircraft will return to steady level flight – it will be dynamically stable. Three levels of description of the aircraft motion are shown in the figure.

First, if the structural flexibility is not known, the first-order aircraft motion can be found from a *rigid body* analysis. As the aircraft description is refined, a *static flexibility* model can be used which permits the interaction between structural deformation and the aerodynamics to be investigated. Finally, the *dynamic flexibility* model is required to account for the effects of structural vibration and the unsteady airloads that result.

In figure 5, I have attempted to show the effect on computer time of modeling the more complex dynamics cases. Even the rigid model requires a substantial amount of computer time (about 20 min) because of the interaction between the rigid airplane dynamics and the aerodynamics. When structural flexibility is permitted, the computation time is increased severalfold (approximately 1 hour in the case shown here) because of the airplane change of shape. Dynamic flexibility, which introduces higher frequency modes, further increases the computation time required.

The dynamic behavior of an aircraft structure, although complex, is now well understood for conventional aircraft, both theoretically and experimentally. The means for providing dynamic stability through configuration design are known and the next major step may be to incorporate active control systems that can reduce the sizes of control surface and provide load alleviation, particularly for large flexible aircraft.

TERMINAL AREA SIMULATION

We return now to the question of modeling the motion of an aircraft. The accurate depiction of aircraft motion is particularly important during approach and landing and generally for operations in the terminal area. The piloting tasks in this busy phase of flight must be properly assessed to devise safe operating procedures in the terminal airspace.

Figure 6 shows the components used in a terminal area simulation. The simulation includes (a) a piloted simulation that represents the aircraft dynamics through cockpit motion and a changing visual scene and (b) an air traffic controller who provides traffic control instructions based on information derived from the air traffic situation generated within the computer.

The *piloted simulation* includes detailed aircraft dynamics and its guidance and navigation system; also, the cockpit included a general purpose graphics display to permit variations in

information format displayed to the pilot. The simulated aircraft can be given 3-dimensional or 4-dimensional guidance or can simply respond to vectoring commands from *ground control*. Superimposed on this interaction between pilot and controller are the effects of the environment: other aircraft, wind conditions, turbulent gusts, airspace constraints due to noise, etc. Wind models, for example, have variations in direction and magnitude with altitude. The aircraft in the terminal area may have position errors as a result of inaccuracies in navigation and guidance information, so that the sensitivity of the system to error magnitudes can be determined.

The resulting output from this simulation permits an evaluation of air traffic procedures, pilot and controller workloads, the identification of worst case weather conditions, and the influence of improved aircraft capabilities.

Figures 7 and 8 show some typical results using the simulation. The problem was to investigate the impact of introducing short takeoff and landing (STOL) aircraft on the air traffic control system. Such aircraft could maneuver in restricted airspace using steep curved approach paths rather than the conventional 3° glide slope used by conventional aircraft. The purpose of the simulation was to determine whether the controller could integrate the STOL traffic with the conventional traffic on an adjacent runway.

Figure 7 shows the kind of flight paths that resulted from the controller's first attempts when he used aircraft speed control as the primary means of achieving correct time of arrival at the runway. The complex maneuvers of the aircraft arriving from the south were necessary to ensure a proper separation distance when the first aircraft failed to meet the original arrival schedule as estimated by the controller. The large speed variation available to STOL aircraft makes such an estimation more difficult.

Figure 8 shows the same arrival situation flown with the help of a 4-dimensional navigation system aboard the aircraft. The controller's task is to assign and track the runway arrival times with the assistance of a ground-based computer. The communication workload is substantially reduced and the ability of the aircraft to meet tighter spacing requirements permits a virtual decoupling of the STOL traffic from the adjacent CTOL traffic.

To study the diverse problems in terminal area research, two other types of simulations are used, in addition to the one previously described. Table 2 summarizes each type in terms of elements simulated, computer requirements, and average costs for an experiment. The first type is used to establish the feasibility of a guidance, control, or air traffic concept and is run in fast time on a general purpose computer. By virtue of its modest computer requirements and low operating expense, it is the mainstay of the systems engineer in obtaining a preliminary evaluation of a concept. Although the dynamic models simulated in it may range from simple (as for an airport capacity simulation) to very complex (as for an aircraft guidance and control system simulation), this type of simulation has one basic limitation in air transportation research: the absence of human operators as active decision-makers. In a world where man-computer, man-machine, and man-to-man interactions are increasingly more complex, this limitation is unacceptable.

The next type, which is run in real time, permits participation of human operators, namely, one controller and one pilot. It is used to evaluate a concept that has shown above-average promise in the fast time studies. The results of the study described in table 3 were obtained with this type of interactive simulation.

If a moving-base rather than a fixed-based simulator is required for the piloted simulation, the cost of running an experiment increases by almost an order of magnitude (table 2). Computer requirements also increase by a smaller factor. However, considering the high costs of building and flight testing an aircraft, the moving-base simulator is an indispensable and cost effective tool for advanced aircraft research.

AIR TRANSPORTATION SYSTEM MODELING

With an understanding of the operational modes and constraints in the terminal airspace, one can take a broader look at the overall air transportation system model. Here the interest is in finding what influences the growth of air transportation. Clearly, such factors as demand, operating economics, and environmental constraints come into play. Figure 9 displays the major elements in the model. At the left is the model of the transportation system with its major components, the arena, the traveler, and the various travel modes including the automobile, airplane, rail, and bus. Each component can be modeled in various ways, ranging from the very simple to the very sophisticated. The next block represents the analysis phase of the transportation system. Although the analysis can yield a variety of results, interest here has been centered on the air mode performance and, in particular, on the criteria of merit shown on the right, namely, demand, economics, and environmental impact.

The demand for air transportation is best measured in terms of the number of passengers the system will attract and is of obvious importance if the system is to serve a useful purpose. The economics is readily measured in terms of return on investment. The environmental criteria would generally consist of several components, but here they are limited to one – the noise impact on the community surrounding the airport.

The remainder of the figure is concerned with optimization. The three figures of merit shown are used as feedbacks to implement an optimization procedure (as indicated at the bottom) in which the variable part of the air transportation system is to be chosen to achieve the desired objective – to maximize the number of passengers carried with a constraint on the investment return and noise impact.

The part of the analysis concerned with demand is the most complex and places the most severe requirements on the computer. Figure 10 illustrates the components in the demand analysis, generally known as a modal split analysis, which determines the division of travel demand among the competing modes.

The heart of the analysis is the modeling of the transportation system components: the arena, the traveler, and the travel modes. The first element, consisting of the origin and destination regions in the arena, is modeled by subdividing the regions into zones as indicated to reflect certain similarities of the population and their spatial distribution. For this purpose, the modeling includes data on the zonal boundaries, population, income distribution, number of hotels, travel demand as a function of business or nonbusiness trips and resident or nonresident traveler, time value distribution, and local travel functions. The second element, the traveler, is modeled to represent the differences between travelers within each zone. Some of the characteristics that are modeled include

the exact origin and destination within a zone, the trip purpose, desired departure time, sensitivity to frequency of service, car ownership, trip duration, party size, time value, and modal preference factors. Modeling of the third element, the travel modes, is more straightforward and includes such factors as trip cost, trip time, service frequency capacity, noise, investment required, and operating costs.

With this data base, the modal split analysis utilizes a Monte Carlo technique that selects from particular locations within zones an individual traveler whose characteristics are determined from distribution functions. The analysis examines the competitive situation between the various combinations of travel modes which could be used between the origin and destination points as indicated in the figure. The least cost alternative is determined and it is assumed that the traveler would utilize this alternative. This process is repeated for a large number of travelers and reliable statistics on the modal split are determined. In this way, the number of travelers attracted to each of the modes shown is determined.

The methodology for determining community noise impact has its greatest impact on the computer software requirements. This methodology is illustrated in figure 11 by a series of overlays. First is shown a photograph of a typical airport and the surrounding area. To determine the noise impact of aircraft operations at this airport, it was necessary to develop a land-use model for graphically describing land uses around the airport. Such a model is shown by the first overlay on which the surrounding area is categorized into residential, planned residential, and commercial or manufacturing (as indicated by the coded areas). Each of the irregularly shaped areas corresponds to a different land value. A computer model stores the geometry of these areas and the corresponding land values in dollars per unit area. The second overlay shows the noise contours generated by a mix of proposed CTOL aircraft.

Figure 12 shows two contours for NEF 30 and 35 which would result for the assumed mix of aircraft operating on takeoff and landing patterns shown. These NEF (noise exposure forecast) contours are generated by a computer program in which are combined noise source data, noise propagation, laws, and three-dimensional aircraft position data.

The noise impact is determined by combining the NEF contours with the stored land-use model; by means of a matrix comparison technique, intersections are located between contours and land parcels. By use of land value data stored in the computer, the dollar value of the impacted areas can be determined and used as a basis for determining buffer zone costs in terms of land acquisition or possibly for use in considering land-use changes.

With this kind of computer simulation, the economic viability and environmental impact of introducing a new transportation mode into a given arena can be determined. Clearly, this approach has a much broader application than to air transportation; conceivably, it could be adapted to ascertain the merits of introducing competing forms of energy (electric power vs. natural gas, for example) or to analyze the value of introducing a new water allocation program or a new communication system.

CONCLUDING REMARKS

My remarks have been concerned primarily with aircraft and air transportation whereas the interests of this workshop are in a much broader range of dynamic systems problems.

Let me conclude by suggesting some rules we have learned the hard way in aircraft and air traffic system analysis, which may be applicable more generally to the treatment of large dynamics systems.

- Define a hierarchy of models so that the problems to be studied can be compartmentalized and uncoupled to the extent that is realistically possible.
- Distill and simplify results at each level before proceeding to limit the complexity and reduce the cycle time of computation.
- Wherever possible, and particularly where complex physical phenomena are involved, check experimentally the validity of the results.
- If the judgment of a human controller of the system is critical, provide for his participation through simulation.
- Anticipate and incorporate major system tradeoff parameters early in the formation of the system definition (e.g., demand vs. capacity vs. environmental factors).

From the agenda of presentations to follow, I know there will be a great deal of interdisciplinary interaction of ideas. Much benefit will accrue from this and I can only encourage your attempt to formalize the principles that may govern the behavior of large-scale systems.

TABLE 1.— DEFINITION OF DESIGN LEVELS

	Conceptual	Preliminary	Detailed
Resources People time Computer time*	1-2 man years 2-5 hr	7-33 man years 75-100 hr	100-200 man years 1000+ hr
Complexity Aerodynamic elements Structural elements Weight elements	4 4 25	200 200 25-50	600 5000 50+

*CDC 7600

TABLE 2.— TYPES OF TERMINAL AREA SIMULATIONS

Type	Simulated elements	Application	Computer size, megabit	Cost thousands of dollars
Fast time (concept feasibility)	Aircraft dynamics, synthetic traffic	Guidance and control, capacity, delays	1	1
Real time (concept selection and evaluation)	Fixed-base simulator, pilot, single controller, synthetic traffic	Man-computer and pilot-computer interface	2	6
	Moving base simulator, pilot, single controller, synthetic traffic	As above plus pilot workload and handling qualities	4	40
Real time (detailed design)	Several controllers, real and pseudopilots,	Procedures for medium density hub	8	200
	Several controllers, real and pseudopilots, models of automated systems	Procedures for major hub	10-20 (ILLIAC?)	300-600

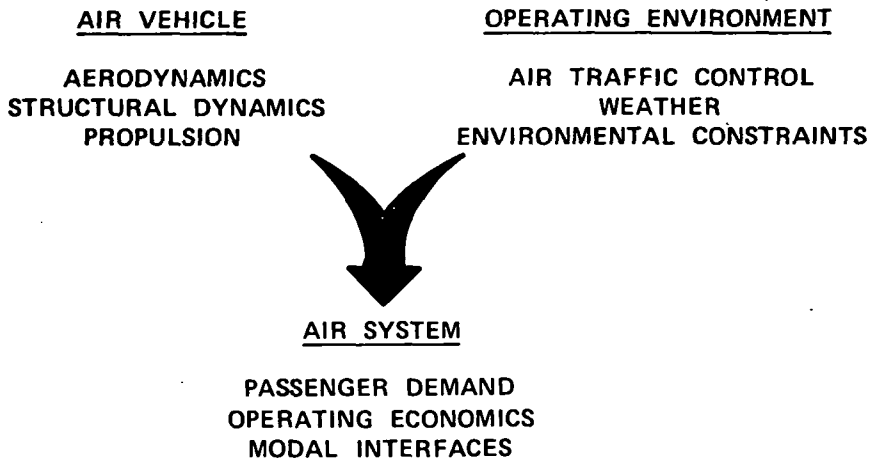


Figure 1.—Air transportation as a system.

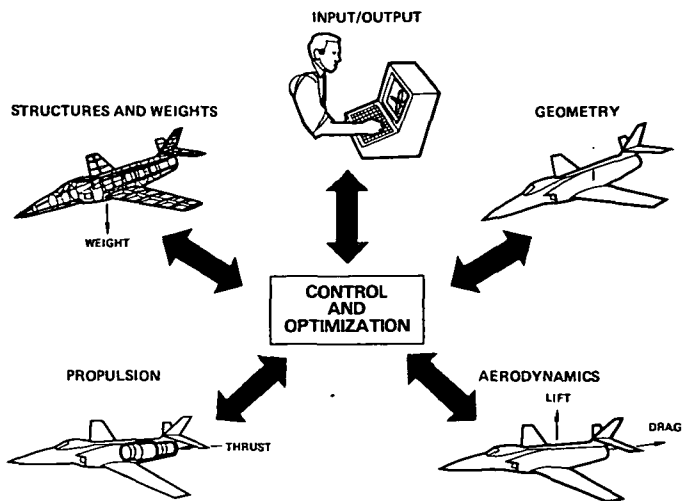


Figure 2.—Aircraft synthesis program.

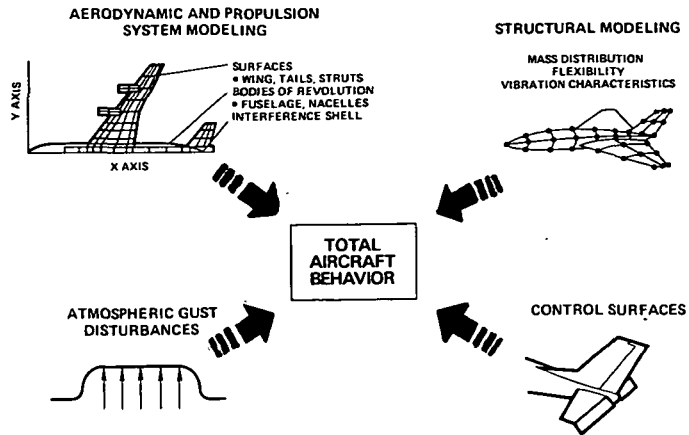


Figure 3.—Flight dynamics modeling of an aircraft system.

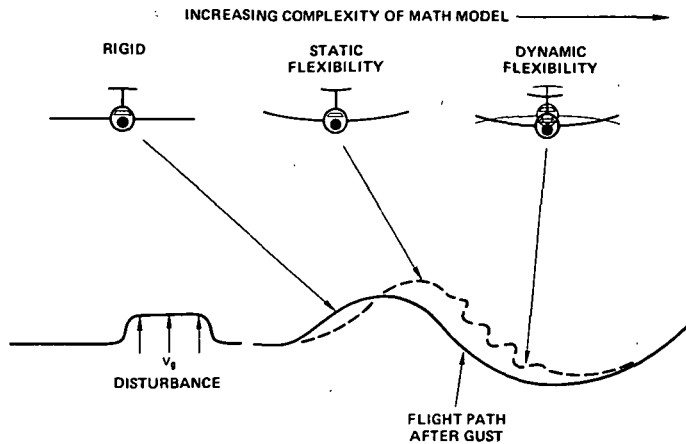


Figure 4.—Aircraft behavior models.

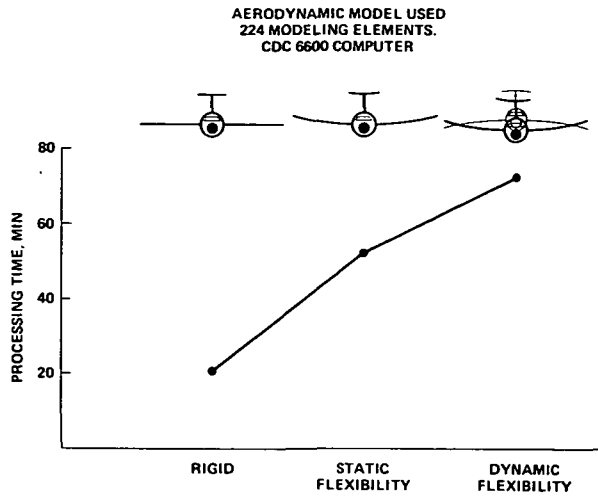


Figure 5.—Computer execution times.

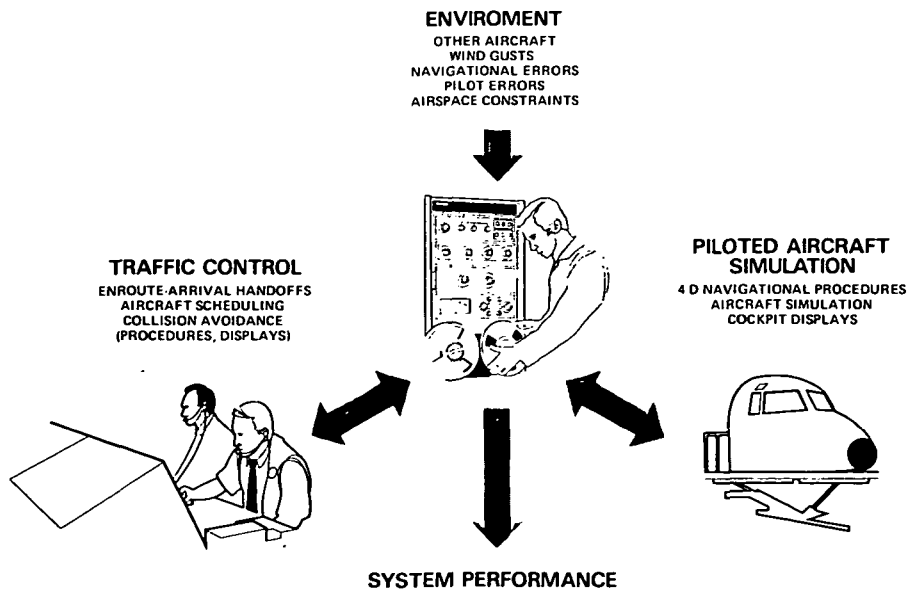
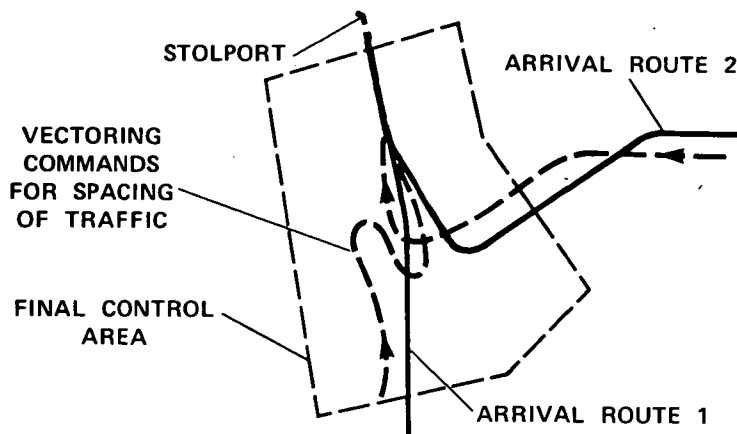
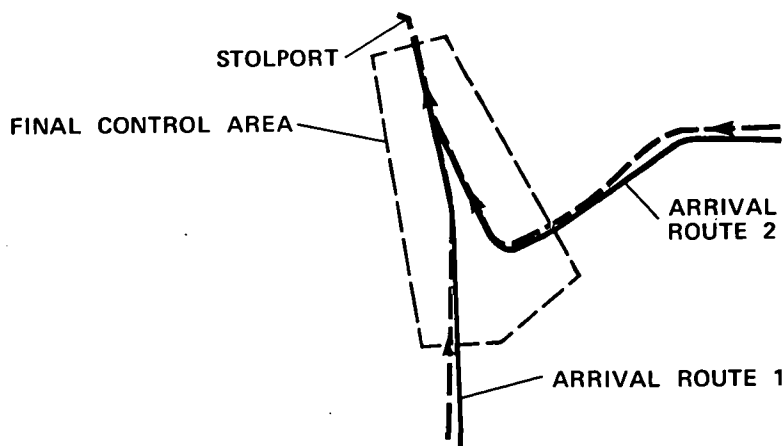


Figure 6.—Interactive terminal area simulation.



- HIGH PILOT AND CONTROLLER WORKLOAD
- LARGE TRACK ERRORS
- DELAYS

Figure 7.—Simulation results (present operating procedures for sequencing and spacing).



- REDUCED COMMUNICATIONS WORKLOAD
- INCREASED CAPACITY
- REDUCED DELAY AND FUEL CONSUMPTION

Figure 8.—Simulation results (on-board 4D navigation with computer-assisted sequencing).

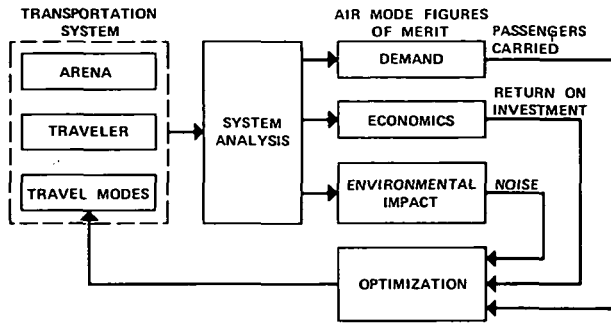


Figure 9.—Transportation system analysis.

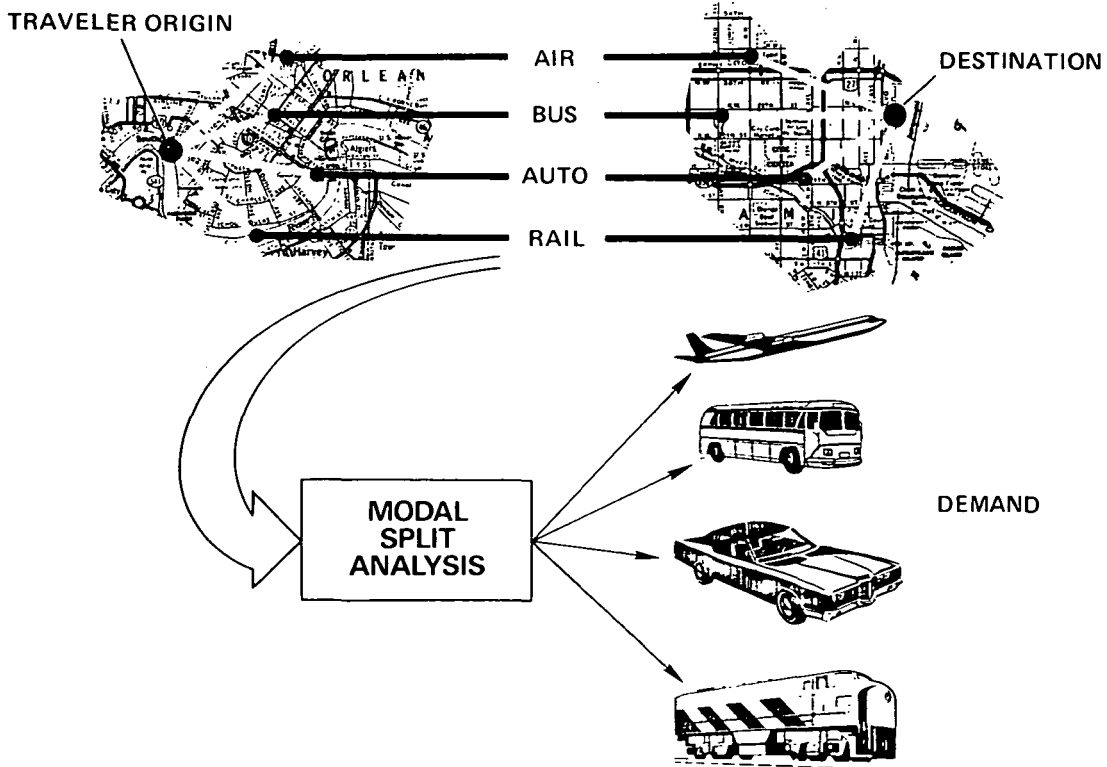


Figure 10.—Demand methodology.

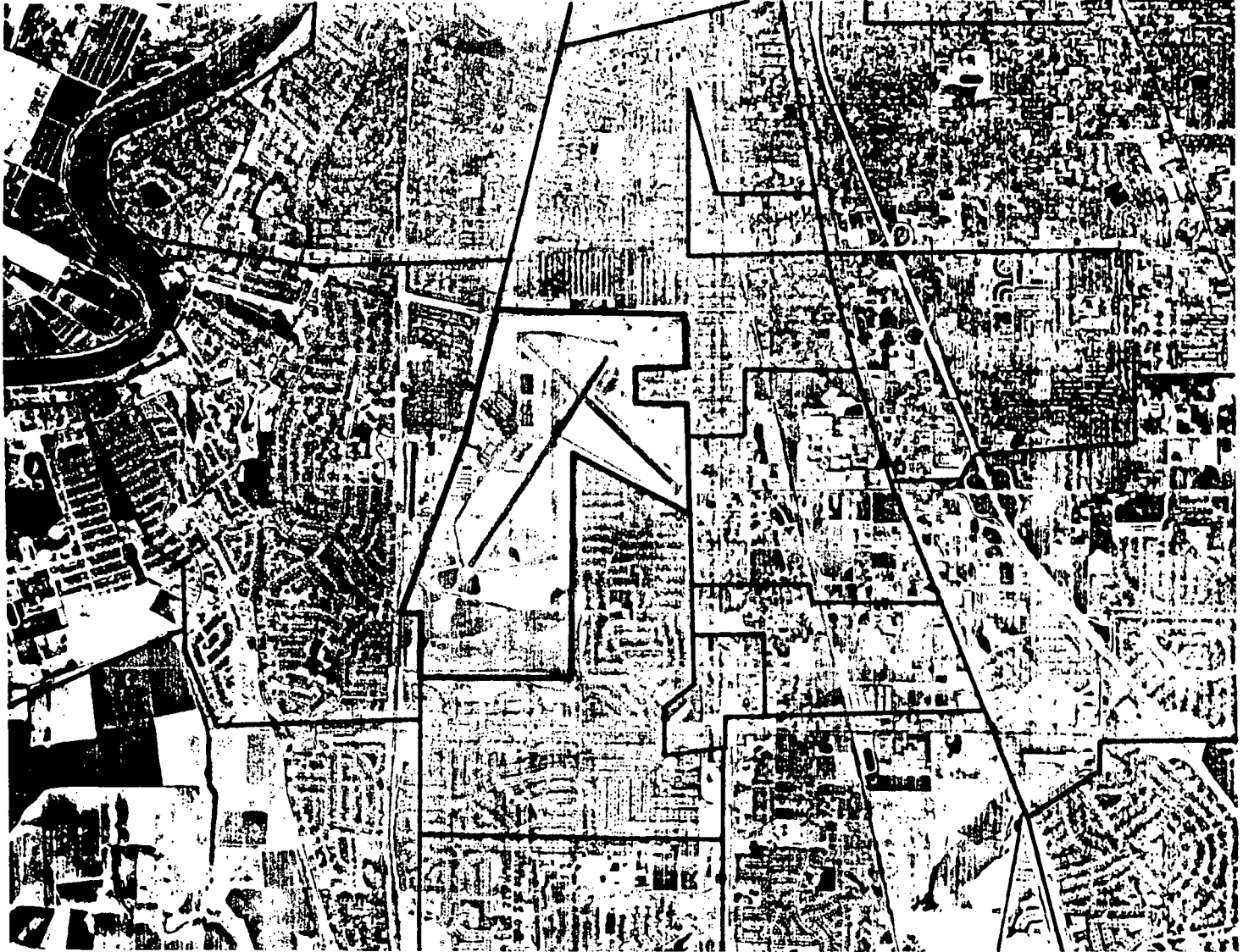


Figure 11.—Typical executive airport vicinity.

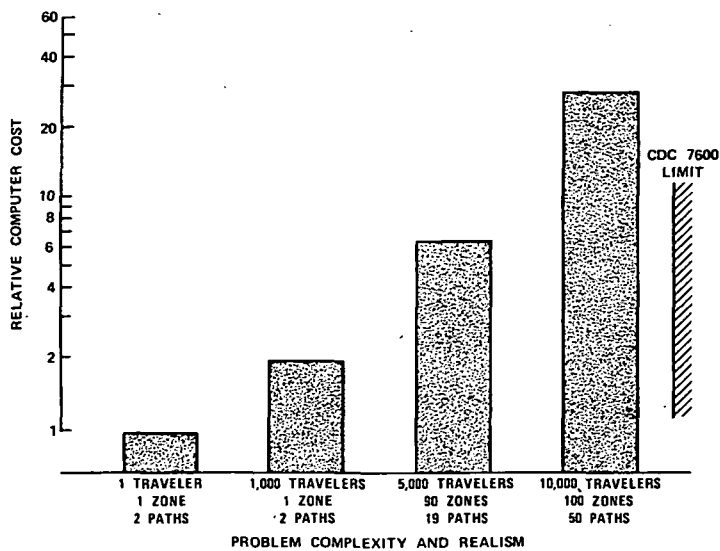


Figure 12.—Computer requirements (Monte Carlo techniques).

Page Intentionally Left Blank

HIERARCHIAL MODELS OF VERY LARGE PROBLEMS, DILEMMAS, PROSPECTS, AND AN AGENDA FOR THE FUTURE*

John M. Richardson, Jr.**

Case Western Reserve University, Cleveland Ohio

INTRODUCTION

During the past three years, an international group of scholars from many disciplines and from several universities in the United States and Europe has been engaged in developing a set of computer simulation models, focusing on the most critical problems of a global system.¹ The causes of these problems – population growth, environmental stress, and diminishing stocks of nonrenewable resources – are familiar to all of us. Equally familiar are the sorts of oversimplified discussions, ranging from predictions of inevitable doom to simple-minded assertions that no problems exist, which have attracted the widest public attention.

On this subject, where emotion has tended to dominate reason, the Multilevel World Modeling Project represents an application of systems methodology which, in our judgment, is particularly germane to the concerns of this workshop.² In this presentation, I should like to discuss the project from an historical and methodological perspective, with particular emphasis of those aspects of our experience which may be relevant to efforts of comparable scope.

THEORETICAL AND METHODOLOGICAL BACKGROUND

Under the direction of Donald P. Echman and, after his death, Mihalo D. Mesarovic, the Systems Research Center of Case Western University has a relatively long history of involvement in this newly developing field, having been concerned with the development of interdisciplinary

*The research described in this paper was conducted in connection with two projects – the Multilevel Regionalized World Modeling Project (conducted at Case Western Reserve University and the Technical University of Hannover, directed by M. D. Mesarovic and Eduard Pestel) and the Phosphorous Pollution Control Project supported by the Rockefeller Foundation.

**Associate Professor of Systems Science and Director, Systems Research Center Computing Laboratory.

¹The project is under the joint direction of Professor M. D. Mesarovic, Case Western Reserve University, Cleveland, Ohio, and Professor Eduard Pestel, Technical University, Hannover, Germany.

²Detailed results were first presented at a four-day meeting held recently (April 29–May 2) in Baden, Austria, under the auspices of the International Institute for Applied Systems Analysis (IIASA). A complete list of the reports presented at that seminar is appended. Copies are available through IIASA.

approaches to the modeling and control of large-scale systems for more than a decade. Of particular importance in this development has been the belief that meaningful application of the systems approach to large-scale problems would be greatly facilitated by the resolution of certain crucial theoretical issues regarding the representation of system structure (refs. 1 and 2).

The results of a number of theoretical papers written during the period from 1961 through 1969 were synthesized and integrated in a major work, "The Theory of Hierarchical Multilevel Systems" (ref. 3). In its first four chapters, this volume lays out a conceptual basis for the study of complex systems, based on hierarchical concepts. Three types of hierarchical structures are distinguished and described formally: levels of abstraction or *strata*, based on different levels of aggregation or complexity; levels of decision complexity or *layers* and levels of priority of action or *echelons*, which are characteristic of the structure of many large organizations. These structural notions are linked to a theory of coordination which is first presented algebraically and, in part II of the volume, using a more classical approach. Here the focus is on real time coordination; however, principles for realizing *satisfactory* as well as *optimal* system performance are discussed.

During the evolution of this body of theory, its generality has been explored through applications in a variety of areas, including organizational behavior (ref. 2), biological systems (ref. 4), artificial intelligence (ref. 5), urban systems (ref. 6), and water resource systems (ref. 7). Thus, the present application of the multilevel approach (refs. 8 and 9) should be regarded as further incremental steps in a lengthy (and continuing) evolutionary process. This process has involved, it should be emphasized, the development of organizational skills that are so essential for the success of large projects as well as technical skills in the application of a particular approach to systems theory. Indeed, the theoretical and organizational elements of the process (and of the present research) have been inextricably linked.

The multilevel approach may be viewed as comprising three principal components:

1. A set of heuristics of decomposing very large complex systems to make them more amenable to formal representation and to provide the basis for developing simulation models.
2. A body of mathematical theory that characterizes large complex systems in detail (more fully developed in ref. 10).
3. A set of methodologies for improving the behavior of complex systems, which includes various coordination strategies and algorithms for achieving, where appropriate, "satisfactory" system behavior as well as optimal system behavior.

During the past four years, the Center has devoted increasing attention to questions of public policy and policy analysis related to two very large systems, the Lake Erie Basin and the world. The goal has been to develop planning and decision-making tools that could be of real value for decision makers. As a consequence of this emphasis, there has been a need to carefully evaluate the tradeoffs between further development of the theory, particularly in the area of multilevel coordination, and the development of models that would be *problem relevant* but not, at least at the outset, analytically tractable.

For a model to be useful to decision makers, it must meet two sets of potentially conflicting criteria that might be labeled (a) comprehensiveness and reliability and (b) usefulness. Included in the first set of criteria are requirements such as

1. Comprehensiveness: The model should incorporate social, economic, and political as well as biophysical and technological variables.

2. State of the art: The model should reflect the state of the art in the respective disciplines primarily concerned with relevant subsystems as well as in system modeling techniques. Although the model is problem oriented, the theoretical paradigms of the respective disciplines should be appropriately taken into account.

3. Problem orientation: The model should focus on the specific problem under consideration and define system boundaries and policy alternatives accordingly.

4. Validity: The model should be able to "predict" the past behavior of the system with a high degree of accuracy. Assumptions and functional relationships in the model should be based on data whose quality meets generally accepted scientific standards of validity and reliability.

To be useful, the following criteria should be met:

1. Simplicity and comprehensibility: The model should be easy to understand, at least conceptually, so that policy makers (or their staffs) who have broad practical experience but are not necessarily familiar with specific scientific disciplines or modeling techniques will be persuaded of its utility and have confidence in its predictions.

2. Client orientation: The model should take into account the goals, values, and points of view of potential users. Moreover, particular care should be directed to ensuring that the goals and values of the modeler are not incorporated.

3. Timeliness: The policy recommendations derived from the model should be available at the time policy decisions are being made.

To date, there have been few systems models developed which focus on broad issues of public policy. Thus there is no consensus regarding the appropriate tradeoffs between these criteria. It is clear that procedures and standards developed in the context of real time systems, or systems where there are excellent data, or systems where there are no time constraints on the development of recommendations will not always be applicable. In the course of the two projects mentioned above, we developed a strategy for constructing multilevel regionalized models of large systems and subjecting them to scenario analysis. It is a strategy, we believe, worthy of consideration.

A MULTILEVEL REGIONALIZED MODEL OF THE LAKE ERIE BASIN

The multilevel regionalized model of the Lake Erie basin was developed in the context of a project with the following broad objectives:

(i) To develop efficient strategies for controlling phosphorus pollution on a regional basis, taking into account economic, societal, and political, as well as public health, scientific, and technological factors, with an awareness of the problems of implementation.

(ii) To develop models needed for these strategies, based on regional inventories and budgets of the distribution of phosphorus using data from, minimally, two regions that markedly contrast with respect to critical ecological, economic, population, and other factors.

(iii) To evaluate alternative strategies for regional control using methods of systems analysis.

The model that evolved is an excellent example of the way in which the multilevel approach provided a conceptual basis that guided the overall research strategy. Initially, five strata were identified and further decomposed into sectors (figs. 1 and 2). While similarities between this structure and the world model may be noted, the inclusion of an institutional regulatory stratum reflected the need to consider a set of normal governmental structures not present in the world system. Several months later, a nine-strata decomposition had leveled, corresponding even more closely to the world model, with the institutional regulatory stratum replaced by a more general institutional stratum (equivalent to the formal organizational stratum in the world model).

Before tracing the further evolution of the model, an additional characteristic of the multilevel approach, of particular importance in this project (but of some importance in the world project as well), should be mentioned.

Quite apart from the philosophic issue of whether multilevel hierarchial structures represent the most valid way to model large-scale systems, the approach is an extremely useful one from the practical standpoint of organizing a model development effort. First, one can draw upon the theories and skills of established disciplines, rather than evoking the hostility of their practitioner as more radically integrated systems modeling approaches often do.³ Each discipline can be given responsibility for a particular area, while the systems specialist focuses on those problems of synthesis, integration, and coordination he claims are the distinguishing concerns of his own profession. Second, it has been our experience that in large biophysical-social-technological systems, the "state of the art" relevant to the modeling of different strata may be quite different. During the initial phases of the model development process, submodels of the different strata normally develop quite differently. But integrative problems involving boundary definition, level of resolution, and interfacing can often be more easily resolved in terms of the concrete issues posed by well-developed submodels. Third, in the absence of crisis situations, strata have the property of partial decomposability (refs. 15–18). Thus, partial validation of submodels may be possible before the overall model is completely integrated. In crisis situations (the major focus of the Club of Rome Project), the couplings between strata tend to be much stronger (ref. 19).

In the Rockefeller Project, a model development strategy dictated by the above considerations was adopted. Specific subgroups, with commitments to traditional disciplines (ecology, chemical engineering, public health, economics, and political science) were given responsibility for submodel

³The systems dynamics methodology developed by Forrester (refs. 11 and 12) is a prime example of such an approach. Two applications of the methodology by Forrester (ref. 13) and Meadows *et al.* (ref. 14) have evoked considerable hostility.

development. The work of these subgroups provided the basis for the overall integrated model finally developed (refs. 20–22).

Figure 3 shows the structure of the model in its present form. Five strata are modeled explicitly. However, the “softer,” higher level strata are only considered by structuring scenarios for analysis. The model focuses on three variables judged to be of critical policy relevance: (a) the number of days during which anoxic conditions in the hypolimnons of Lake Erie’s Central or Eastern Basins, (b) the monthly oxygen depletion rate in the basin hypolimnons, and (c) the concentration of algal matter in the lake relative to the baseline period of 1950–1973.

Some of the scenarios examined include:

1. Baseline: No pollution control policies implemented after 1973.
2. Advanced waste treatment: Standards for advanced waste treatment specified in the 1973 Water Quality Act, Amendments achieved by 1980.
3. Detergent controls: Controls on the phosphorus content of domestic and industrial detergents implemented by 1980.
4. Regional control: Advanced treatment and detergent controls implemented for the Detroit-St. Clair region only.
5. Advanced waste treatment and detergent controls: Both advanced waste treatment and detergent controls implemented.

Some typical results obtained with the model are depicted in figures 17 and 18.

MULTILEVEL REGIONALIZED WORLD MODELING PROJECT: “PROBLEMATIQUE”⁴

The analysis of problems of a global system poses even more difficult challenges for systems methodology. The cluster of crises with which this project is concerned has been characterized by the Club of Rome as the “world problematique” to draw attention to the uniqueness and magnitude of the problems involved and to the extreme difficulty encountered in understanding the evolving situation, not to mention finding a remedy and the means to avoid disaster. In our judgment, they differ in several significant respects from other events in world history.

First, *the problems are global*: for some of the problems, for example, the energy crisis, this is quite obvious. For others, such as the threat of starvation in particular regions, the global character is felt either through sociopolitical or economic interdependence. The global character of the problems makes them very difficult to solve from the perspective of national or even regional institutions which have, more often than not, conflicting concerns.

⁴This discussion of the “problematique” is based on the remarks of Professor Pestel to the IIASA Symposium.

Second, *the changes are felt through the entire society*. Economic, technological, environmental, sociopolitical, and many other aspects appear to interact in such a way that what might appear to be a desirable strategy in one domain makes the situation only worse in others. This hinders the solutions of problems by traditional means which reflect only the concerns of a single discipline or domain (e.g., technology, economics, ecology, or the specialized fields of engineering).

Third, *there is a conflict between short- and long-range actions and goals*. A short-range solution often only compounds the long-range problem, making it worse when it reappears.

Fourth, *there are considerable delays between the time when a corrective action is applied and when its remedial effects are felt*. For example, a successful population-control policy aimed at achieving an equilibrium level of population will take 30 to 50 years and possibly more before the goal is reached.

Finally, in contrast with past crises, *the crises of the world problematique appear to result from actions that have been traditionally considered desirable*: to have a large family, to use as much energy as possible to save human labor, or to exploit nature to the utmost for the benefit of man. Thus, solutions must involve changes in values that have been traditionally considered sacrosanct.

For many, especially in Europe, the problematique first became a matter of attention and concern through Dennis and Dana Meadows' compelling and controversial book, *The Limits to Growth* (ref. 14), which reported on the results of the first "world modeling" project (initiated under the auspices of the Club of Rome). This project was based on the *systems dynamics* methodology, first developed by Professor Forrester at M.I.T. during the early 1950's.

Because of the wide familiarity with the "World I" and "World II" models of Forrester and Meadows, it will be useful to illustrate some significant characteristics of the multilevel approach by contrasting the major theses of the M.I.T. project with those of this project.

The theses of the M.I.T. project are roughly summarized as follows:

1. The world can be viewed as one system.
2. The system will "collapse" sometime in the middle of the next century.
3. To prevent collapse, an immediate slowdown of economic growth must be initiated, leading to no growth in a relatively short period of time.

By contrast, the most significant theses of the Regionalized Multilevel World Modeling Project are:

1. The world can be viewed only in reference to the prevailing differences in culture, tradition, and economic development. The world can be viewed as a system only in terms of interacting regions: a monolithic view of such a system is misleading.

2. Rather than a collapse of the world system as such, catastrophies or collapses on a regional level may occur (and, in the absence of positive remedial policies, *will* occur), possibly even long

before the middle of the next century, but in different regions, for different reasons, and at different times. Since the world is a system, such catastrophies will be felt profoundly throughout the entire world. Causes for such crises and potential catastrophies are the population, food, and economic relationships in Africa and South Asia; energy and raw material scarcity and production growth in the developed world; employment and population relationships in Latin America, etc.

3. The solution to such catastrophies of the world system is possible only in a global context and by appropriate global actions. If the framework for joint action is not developed, none of the regions will be able to avoid the consequences. For each region, its turn will come in due time.

4. Such a global solution can be implemented only through selective and balanced growth, not uniform, but greatly differentiated and diversified throughout the world. From the viewpoint of the total world system, this means growth analogous to organic growth rather than undifferentiated growth. It is irrefutable that the second type of growth is cancerous and would ultimately be fatal.

5. The delays in devising such global strategies are not only detrimental or costly, but deadly. What we are truly talking about is a "strategy for survival."

A STRATIFIED MODEL OF INTERACTING REGIONS IN A WORLD SYSTEM⁵

The original conception for the world model was published by Professors Mesarovic and Pestel in early 1972. The model has two principal and unique features, a *multilevel, multigoal structure and regionalization*. For each regional submodel, three general strata – the *norms stratum, organizational stratum, and causal stratum* – are identified. In addition, there are eight more specific strata. This structure (fig. 6) provides general guidelines for developing more specific problem-oriented models such as those focusing on food and energy problems.

The basic level of the models is the "causal stratum" containing elements such as the economy, resource levels, population dynamics, and technology developments. The causal stratum is designed to reflect the basic operation of model variables in areas where there is relatively little governmental intervention. The approach to modeling phenomena in this strata conforms closely to work of established discipline in the scientific community.

Because governments do, of course, often intervene to resolve (or at least attempt to resolve) pollution problems, overpopulation, energy shortages, and the like, these phenomena are incorporated in an organizational or institutional stratum. Thus the model is conceived as a true cybernetic control system with the organizational stratum attempting to maintain relative equilibrium within the causal stratum. In some of the models developed for the project, these phenomena were simulated. For example, Hughes (refs. 24 and 25) developed a model of crisis decision making in the energy sector (fig. 7). In other models, human interactors were used to provide the inputs from the organizational stratum to the causal stratum (fig. 8). In analyzing decision processes to incorporate them into the models, the concept of *multilayer*, hierarchical decomposition is often used.

The final layer of the model (in each region) is the normative stratum. In addition to the factors that influence decision making represented in the organizational stratum, decisions are also

⁵The discussion in this section owes a great deal to an excellent report by our colleague Barry B. Hughes (ref. 23).

shaped by the social values and other beliefs of decision makers. These determine, in large part, the final selection of a policy from a set of generally acceptable or satisfactory policies. The elements in this third stratum quite obviously pose the greatest difficulties for any attempt to completely simulate policy making. An alternative is to allow actual decision makers or other model users to introduce their own norms through interaction with the computer model.

Regionalization

The model has been subdivided into "regions" or groupings of countries similar with respect to the major political and economic variables of the model. That is, nations within a region are at approximately the same stage of economic development and share similar political structures. Regions need not be geographically contiguous. As noted above, regionalization is important because of major differences between the initial levels of major model variables (especially the gap between the rich and the poor) and in probable development patterns. In addition, the available and desirable policies in different regions may be quite different. Regions are interconnected via trade flows, population migrations, and other movements across regional boundaries.

Ten regions have been established since the research model building began: North America, Western Europe, Japan, Rest of the Developed World, Eastern Europe, Latin America, Middle East, Main Africa, South Asia and China (fig. 9).

APPLICATION OF THE MODEL TO THE WORLD FOOD CRISIS IN SOUTH ASIA

To gain a clearer understanding of the way in which the general principles and structures discussed above are actually implemented, it will be useful to discuss a more detailed analysis of a specific problem. From the overall set of submodels, a model that focuses on world food problems (called the Integrated Food Policy Analysis Model) has been developed. Unfortunately, the description of even this segment of the project cannot be very detailed. The complete model includes six major submodels and its detailed documentation runs to two volumes, each roughly the size of a telephone directory for an urban center of moderate size (ref. 26, 27).

Four of the major components of the food model – population submodel, economic submodel, land use submodel, and food production submodel – were developed, programmed, and validated individually before integration. The interrelationship between these components is shown in figure 10. Figure 11 is a more detailed representation. Two submodels were developed in Cleveland, one in Hannover and one jointly in Hannover and Cleveland.

Major Submodels

The *population submodel* was developed by K. H. Oehman and W. Paul of the Technical University, Hannover, under Dr. Pestel's direction (ref. 28). To present a highly resolved picture of demographic phenomena, the total population of each region is divided into 86 age groups. For each group, age specific fertility/mortality rates are defined as probability distributions. Regional

immigration and emigration are also defined on an age specific basis. Population distributions, fertility, mortality, and immigration patterns are sensitive to regional differences.

The *economic submodel* is a two-sector microeconomic model aggregated from a nine-sector model developed by M. D. Mesarovic, L. Klein (University of Pennsylvania), B. Hickman (Stanford University), T. Shook (Case Western Reserve), and P. Gille (Technical University, Hannover). The nine-sector model was based on a regionalized macroeconomic model developed by M. D. Mesarovic and K. Kominek (Case Western Reserve). The model provides an excellent example of the way in which detailed models of particular strata are modified to focus on specific problem areas in integrated models. For the nonfood sector, the production function is derived from the sectoral Cobb-Douglas production functions of the microeconomic model. There is no direct coupling to lower strata. However, the production function for the agricultural sector is based on linkages with the land use and food production submodels. Since variables in these submodels are in physical units rather than dollars, the strata must be coupled through a pricing mechanism that specifies dollar values for commodities and other factors of production. Prices are sensitive to scarcity of land, computed within the model, and to factors such as energy shortages through manipulation, of scenario variables.

In the *land use submodel*, six categories of land – cultivable but uncultivated, grazing, developed, cultivated grain, cultivated nongrain, and fish pond are defined. The rate of increase in cultivated land is determined by the amount of investment in land development and land development costs (which are affected by the market value of land). As population increases, the land is withdrawn from agricultural uses for urban and economic development.

Because the food production submodel computes information on 26 food types, it appears to be quite complex. The fairly high level of disaggregation permits an examination of the different uses of foodstuffs in different regions and allows for cost estimates of future dietary patterns in concrete terms. But the underlying rationale of the model is straightforward. Gross production levels in the three sectors – plants, livestock, and fish – are determined by the input level of land, capital, and other factors of production. Production levels for the various food types are determined by gross production levels with adjustments made for the utilization of some portion of the output for seed and livestock. In calculating the net food production level from which regional production of calories and protein is computed, household, marketing, and food processing losses are also considered. Protein and calories produced in the food production sector, along with imports (if any) are inputs to the population model. Both the land use model and food model were developed by P. Clapham and M. Warshaw (Case Western Reserve).

Scenario Analysis of the Food Crisis in South Asia

A detailed discussion of the scenario analysis completed to date is beyond the scope of this paper (see refs. 20 and 24). However, for illustrative purposes, some of the results from four scenarios, focusing on the capability of the South Asia region to implement “self-help” policies, are presented in figure 12. Descriptive titles of the scenarios are (1) *baseline*, a projection of historical trends; (2) *agrarian development*, an attempt to produce more food by shifting investment to the agricultural sector; (3) *population control*, a policy designed to limit births and gradually achieve a state of equilibrium over a 70-year period; and (4) *population control and agrarian development*, implementation of both (2) and (3).

AN AGENDA FOR THE FUTURE

As we approach the last quarter of this century, specialists in systems theory and methodology are proposing applications of their expertise to a growing number of economic, social, and political problems. Since the root of many problems facing contemporary society appears to be man's inability to manage the large, complex systems he has, at least in part, created, it is hoped that this new contribution will be significant. A framework for such applications and two specific examples were discussed in this paper.

Determining future research need has been defined as one of the principal objectives of this seminar. In this concluding section, I should like to reflect rather broadly on this issue, drawing from but not limiting myself to the areas that have already been discussed. This reflection takes the form of rather specific recommendations in three broad areas:

1. The development of more broadly focused, integrated systems models.
2. Resolution of certain technical problems that presently limit the application of systems models to broad issues involving public policy.
3. Creation of a receptive attitude on the part of decision makers to the kinds of issues raised by such models and to the proposed solutions.

Development of More Broadly Focused Integrated Models

In cooperation with social scientists, philosophers, and humanists, systems specialists should devote major attention to sharpening the way in which important issues involving public policy and human values are defined. The task of exploring such issues should be recognized as an integral and crucial part of the modeling process.

To date, systems specialists have shied away from attempts to incorporate the "soft" variables that are the concern of social scientists humanists as an integral component of their models. Those undertaking such attempts have been severely criticized and, in many instances, the criticism has been justified. Given the "state of the art" in the social sciences and humanities, it is not surprising that this should be the case. But "softness" of a particular variable or phenomena does not justify its exclusion from consideration (ref. 22). If a truly cooperative relationship can be developed between systems specialists, social scientists, and value-oriented scholars, the latter may become more sensitive to the precision that systems models require. At the same time, systems specialists will be compelled, through their deeper understanding of "soft" phenomena and value issues, to develop new structures to accommodate them.⁶

In training systems specialists, skill in judgment and in design, rather than purely mathematical computational skills, must be emphasized.

⁶ A particularly intriguing approach to this problem has recently been proposed by Bossel and Hughes (ref. 29).

Throughout this paper, the role of heuristic skills and the need for judgment in developing broadly focused systems models has been emphasized. The skills in applying systems methodology to new areas will not be sharpened by a repetitious examination and marginal modification of existing techniques and models. Instead, students must direct greater attention to a broad spectrum of cases involving successful application to new areas, focusing, in particular, on the design process in model development.⁷

The development of an organizing framework for integrated models must be a high priority objective.

During the past 10 years (as noted above), Mihajlo Mesarovic and his associates, including the author, have demonstrated the applicability of the multilevel approach to a variety of subject matter areas and have argued vigorously for its adoption as an organizing framework for systems modeling. Our proposal can be divided into three parts: (1) demonstration of the need for *some* organizing framework, (2) specification of criteria that *any* framework must meet, and (3) presentation of a specific framework, the multilevel approach, which meets the demonstrated need and conforms to the specified criteria. Until recently, however, even the need for such a framework has not been a major concern. Thus, it has been difficult to develop a frame of discourse in which the claims of the multilevel approach could be evaluated. We believe that the kinds of concerns to which the multilevel approach has responded should be regarded as more critical and central in systems theory and methodology.

Within the limits of their capabilities, systems specialists should attempt to promote a more favorable institutional environment for the cooperative enterprises necessary to develop integrated models.

In the United States, at least, much of the intellectual talent that must be mobilized to develop large integrated models is found in private and public universities. But it is difficult to conceive of an institution less suited to the kind of broadly focused cooperative effort this developmental process entails. A study in which the author participated identified similar problems in many Asian, European, and Latin American universities (refs. 31 and 32). Systems specialists cannot and should not be expected to resolve the problems of defensiveness, parochialism, and fragmentation which have plagued academic and research institutions since the time of Plato's Academy. But they can, at least, be especially sensitive to these problems when defining the boundaries of their own activities. Where appropriate and feasible, they should also work to strike down the impediments to cooperative research in the institutions of which they are a part.

Development of More Technically Sound Integrated Models

Major attention should be devoted to defining data requirements and encouraging necessary data-collection efforts to support the development of broad based integrated models.

Lack of adequate data poses a serious impediment to the development of broad-based models, especially in the areas of social, institutional, and political phenomena. Even where data seem

⁷ A program that reflects many of the same objectives has been presented in greater detail by Simon (ref. 30).

plentiful, they are often the wrong data. Existing data base structures necessarily have embedded within them the criteria of relevance germane to the eras and institutions that fostered their development. These criteria are often partly, or wholly, inconsistent with the requirements of contemporary systems modeling methodologies. One is often faced with the problem of developing what seems to be an accurate model structure for which little or no data are available, or developing a somewhat inaccurate model to fit the data available. The development of new models and supporting data bases are necessarily two sides of the same coin and must be recognized as such.

In cooperation with mathematicians and statisticians, systems specialists must direct attention to the development of new parameter estimation procedures appropriate for broad-based integrated models.

Related to the data problem is the problem of developing appropriate parameter estimation procedures where data are incomplete, but the model structure and problem require the inclusion of a theoretically significant variable. While existing estimation procedures in econometrics, operations research, statistics, and engineering are a useful starting point, it should be recognized that they have been devised to meet a quite different set of problems from those that may be encountered in the future. Again, this is especially true where social and institutional phenomena are the object of concern or where the time horizon of the model is measured in decades.

In cooperation with philosophers of science, statisticians, and mathematicians, systems specialists should develop more appropriate validation procedures for broad-based integrated models, especially those involving long-term forecasting.

Many of the models being developed purport to make policy-relevant predictions about events that will occur 10, 20, or even 50 years in the future. What kinds of legitimate statements can be made to decision makers regarding the probability that a particular prediction or scenario is, in some sense, valid? Presently, a degree of validity is claimed for models if they "fit" historical data. But it is by no means certain that this approach to validation is sufficient given the fact that both parameter and structural changes may be imposed on a model to explore a particular set of alternatives. It may well be that a fundamentally different approach⁸ from that presently available will be required to deal with the validation of these models.

Development of a Receptive Attitude on the Part of Decision Makers toward Recommendations Based on Systems Models

The claims made for the usefulness of systems models should not exceed a conservative assessment of their actual utility.

For broad-based integrated models to achieve the potential envisioned by their advocates, considerable progress must be made in resolving the technical problems discussed previously. Model builders must be able to offer confidently the recommendations based on their models and decision makers must be able to accept them with equal confidence. Nothing could be more harmful than

⁸"Fundamentally different approach" is used in the sense suggested by Kuhn (ref. 33).

for systems specialists to persuade decision makers to accept their recommendations in a new area before they have meaningful recommendations to give.

A concern with long-term, broadly defined goals must become institutionalized, preferably within existing local, state, and national governments.

In the United States, at least, there are few individuals at any level of government who think in concrete terms beyond the next election. Given the existing structure of many political institutions, this is often rational behavior. In view of this, perhaps new structures will be needed to accommodate these kinds of goals and objectives. Should this be true, politically oriented systems engineers should be able to play an active role in designing such structures.

A CONCLUDING THOUGHT

One of the major problems facing the systems field profession may be broadly defined as the problem of technology transfer – both to potential users and to other professions whose assistance is crucial to the development of broad-based integrated models. For several years, the author devoted considerable attention to the problem of transferring American agricultural technology to Asia, Africa, and Latin America. There are at least two lessons learned from that experience which may be of some relevance.

First is the importance of a “demonstration effect.” A demonstration effect is an event – often fortuitous – which provides concrete, highly visible evidence that the new technology could solve a particular problem that has been of major concern to the community. Substantial evidence exists that such an event is a necessary, though not a sufficient, condition for technology transfer.

The second lesson is the importance of communication, and the complexity of the interface between specialists developing a new technology and the client groups they are attempting to serve. In the United States, the farmer is two steps away – via extension agents and experiment stations – from academic agriculturalists in universities. We found that similar “interface” mechanisms adapted to potential recipient cultures were essential to the process of technology transfer, innovation, and change. While the analogy to problems faced by contemporary systems specialists may not be straightforward, they should recognize that a serious communication problem presently exists. Moreover, it is the systems specialist who must adapt to the attitudes and perceptions of potential clients rather than the reverse. In the long run, solving the communication problem may be more important and more difficult than any discussed previously.

REFERENCES

1. Eckman, D. P.; and Mesarovic, M. D.: On Some Basic Concepts of the General Systems Theory (Paper Presented at the Third International Congress on Cybernetics, 1961).
2. Mesarovic, M. D.: A General Systems Approach to Organization Theory. SRC 2-A-61-2 (CWRU, 1961).
3. Mesarovic, M. D.; Macko, D.; and Takahara, Y.: Theory of Hierarchical Multilevel Systems (New York: Academic Press, 1970).
4. Mesarovic, M. D.: Systems Theory and Biology – A View of a Theoretician, in M. D. Mesarovic, ed., Systems Theory and Biology (New York: Springer Verlag, 1968).
5. Mesarovic, M. D.: Systems Theoretic Approach to a Formal Theory of Problem Solving, in R. Banerji and M. D. Mesarovic eds., Theoretical Approaches to Non Numerical Problem Solving (New York: Springer Verlag, 1970).
6. Richardson, J. M., Jr.; and Pelsoci, T.: The Multilevel Approach and the City: A Proposed Strategy for Research, in M. D. Mesarovic and A. Feisman, ed., Systems Approach and the City (Amsterdam: North Holland, 1972).
7. Haimes, Y. Y.: Decomposition and Multilevel Approach in the Modelings and Management of Water Resources Systems, in D. M. Himmelblau ed., Decomposition of Large Scale Systems (Amsterdam: North Holland, 1973).
8. Mesarovic, M. D., ed.: Construction of Multilevel Systems Model for Regional Approach to Phosphorus Pollution Control, Complex Systems Institute and Systems Research Center, Case Institute of Technology, Case Western Reserve University, Cleveland, Ohio, January 30, 1973.
9. Cardenas, M.; and Mesarovic, M. D.: Overview of the Multilevel, Hierarchical Model, in M. D. Mesarovic, ed., Multilevel Systems Model for Interactive Mode Policy Analysis in Pollution Control – I. (CWRU, 1974).
10. Mesarovic, M. D.; and Takahara, Y.: Mathematical Theory of General Systems (Amsterdam, North Holland, 1974).
11. Forrester, J. W.: Industrial Dynamics (Cambridge: M.I.T. Press, 1961).
12. Forrester, J. W.: Principles of Systems (Cambridge: Wright-Allen, 1969).
13. Forrester, J. W.: Urban Dynamics (Cambridge: M.I.T. Press, 1969).
14. Meadows, D. *et al.*: The Limits to Growth (New York, Universe, 1972).
15. Simon, H. A.; and Ando, A.: Aggregation of Variables in Dynamic Systems, in A. Ando, F. M. Fisher and H. A. Simon, Essays of the Structure of Social Science Models (Cambridge: M.I.T. Press, 1963).
16. Ando, A.; and Fisher, F. M.: Near Decomposability, Partition and Aggregation, and the Relevance of Stability Discussions, in A. Ando, F. M. Fisher and H. A. Simon, Essays of the Structure of Social Science Models (Cambridge: M.I.T. Press, 1963).

17. Ando, A.; and Fisher, F. M.: Two Theorems on Ceteris Paribus in the Analysis of Dynamic Systems, in A. Ando, F. M. Fisher and H. A. Simon, *Essays of the Structure of Social Science Models* (Cambridge: M.I.T. Press, 1963).
18. Crecine, J.: *Governmental Problem Solving: A Computer Simulation of Municipal Budgeting* (Chicago: Rand McNally and Company, 1969).
19. Mesarovic, M. D.; and Pestel, E.: *Strategy for Survival – A Project to Develop an Alternative Model of the World System*, *Bild und Wissenschaft*, forthcoming in 1973.
20. Mesarovic, M. D. *et al.*: *Scenario Analysis with an Integrated Model of the Lake Erie Basin*, Systems Research Center, Case Western Reserve University, Cleveland, Ohio, January, 1974.
21. Richardson, J. M., Jr.; and Klabbers, J.: *A Policy Oriented Model of the Eutrophication Problem in the Lake Erie Ecosystem*, SRC Report 74-1, Systems Research Center, Case Western Reserve University, Cleveland, Ohio, March, 1974.
22. Richardson, J. M., Jr. *et al.*: *Scenario Analysis with an Integrated Model of the Lake Erie Basin*, Systems Research Center, Case Western Reserve University, Cleveland, Ohio, January 1974.
23. Hughes, B. B.: *Current Status of the Mesarovic Pestel World Model Project*, Systems Research Center, Case Western Reserve University, Cleveland, Ohio, March 1974.
24. Hughes, B. B.: *Regionalization and Decision Making in a World Model* (CWRU, 1972).
25. Mesarovic, M. D.; and Pestel, E.: *A Goal Seeking and Regionalized Model for Analysis of Critical World Relationships – The Conceptual Foundation*, *Kybernetics*, Vol. 1, 1972.
26. Richardson, J. M., Jr. *et al.*: *Scenario Analysis of the World Food Problem, 1975–2020, Using the Integrated Food Policy Analysis Model* (CWRU, 1974).
27. Richardson, J. M., Jr.; and Mesarovic, M. D.: *A Proposed Strategy and Recommendations for Dealing with the World Food Crisis*, *Testimony to the Subcommittee on Growth and Its Implications for the Future of the Committee on Merchant Marine and Fisheries of the United States House of Representatives*, Washington, D.C., June 13, 1974.
28. Oehman, K. H.; and Paul, W.: *World Population Model* (T. U. Hannover, 1974).
29. Bossel, H. H.; and Hughes, B. B.: *Simulation of Value-Controlled Decision Making: Approach and Prototype*, SRC Report 73-11, Systems Research Center, Case Western Reserve University, Cleveland, Ohio, November 1973.
30. Simon, H. A.: *The Science of Design: Creating the Artificial*, in *The Sciences of the Artificial*, (Cambridge: M.I.T. Press, 1969).
31. Richardson, J. M., Jr.: *Partners in Development: AID University Relations, 1950–1966*, East Lansing: Michigan State University Press, 1969).
32. Baldwin, I. L.; Rigney, J.; Roskelly, R. W.; and Thomas, B.: *Building Institutions to Serve Agriculture*, CIC-AID Project, 1969.
33. Kuhn, T. S.: *The Structure of Scientific Revolutions* (Chicago: University of Chicago Press, 1962).

FURTHER READING

Reports in the Proceedings of the Seminar on THE REGIONALIZED MULTILEVEL WORLD MODEL at the International Institute for Applied Systems Analysis, Laxenburg, Austria, April 29–May 3, 1974.*

Available upon request from the IIASA.

TITLE	AUTHOR
Methodology	
1. Objectives, Motivation and Conceptual Foundations	M. Mesarovic, E. Pestel
2. A Goal Seeking and Regionalized Model for Analysis of Critical World Relationships – The Conceptual Foundation, <i>Kybernetes Journal</i> , 1972	M. Mesarovic, E. Pestel
3. Interactive Mode Analysis of Energy Crisis Using Multilevel World Model; <i>Futures</i> , August 1973	B. Hughes, P. Gille R. Pestel, T. Shook M. Mesarovic
4. Simulation of Value Controlled Decision Making: Approach and Prototype	H. Bossel, B. Hughes
5. Human Computer Decision Making: Notes Concerning the Interactive Mode	J. H. G. Klabbers
6. Coordination Principles for System Interactions	Y. Takahara
7. Conversational Use of Multi-Layer Decision Models	F. Rechenmann
8. PROMETHEE (Programmed support for models of Earth Trends, Hierarchical, Economic, Ecologic)	J. Mermet
Population	
9. Population Model, Vol. I and II	K. H. Oehmen, W. Paul
Economics	
10. Methodology for Construction of World Economic Model	M. Mesarovic, E. Pestel
11. Specification of Structure for a Macro-Economic World Model	B. Hickman, L. Klein M. Mesarovic
12. Computer Implementation of Macro-Economic World Model	P. Gille, K. Kominek R. Pestel, T. Shook W. Stroebele

*No further bibliography is given since all references to other relevant literature are in our reports.

13. Implementation of Micro-Economic Model T. Shook
14. Cobb-Douglas Production Function for the World Model
Project and a One Sector Growth Model Interpretation M. McCarthy, G. Shuttic
15. Statistical Analysis of Error Propagation in
World Economic Model G. Blankenship

Energy

16. Energy Model: Resources R. Bauerschmidt, R. Denton
H. H. Maier
17. Energy Model: Demand B. Hughes, N. Chu
18. Energy Model: Supply H. Bossel
19. Regionalized World Liquid Fuels Production and
Consumption from 1925–1965 N. Chu
20. A Description of the World Oil Model B. Hughes
21. Assessment of the World Oil Crisis
Using the Multi-Level World Model B. Hughes, M. Mesarovic
E. Pestel
22. Global Energy Model R. P. Heyes, R. A. Jerdonek
A. B. Kuper
23. Environmental Impact Assessment M. Gottwald, R. Pestel

Food

24. A Regionalized Food Model for the
Global System W. B. Clapham, Jr.
M. Warshaw
25. The Integrated Food Policy Analysis
Model: Structural Description and
Sensitivity Analysis M. Mesarovic
J. M. Richardson, Jr.
M. Warshaw
26. Scenario Analysis of the World Food
Problem, 1974–2025, Using the Integrated
Food Policy Analysis Model W. B. Clapham, Jr.
M. Mesarovic
J. M. Richardson, Jr.
M. Warshaw
27. A Model for the Relationship between
Selected Nutritional Variables and
Excess Mortality in Populations T. Weisman

Water Resources

28. Water Resources Model M. Cardenas

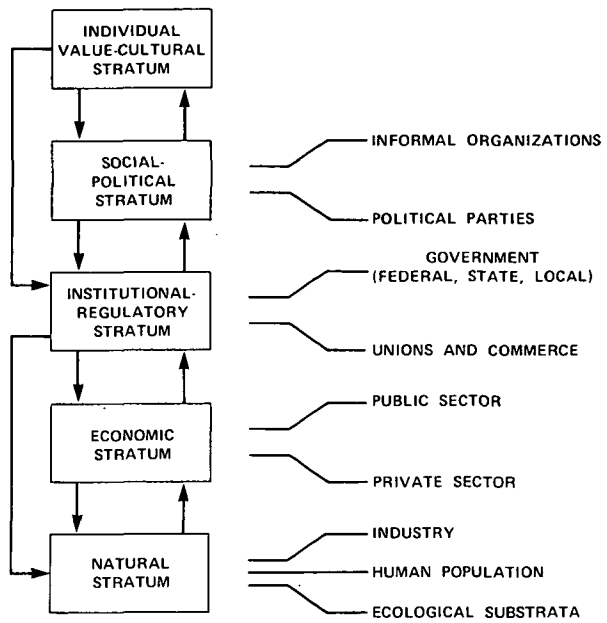


Figure 1.—Initial stratified decomposition of phosphorus management model.

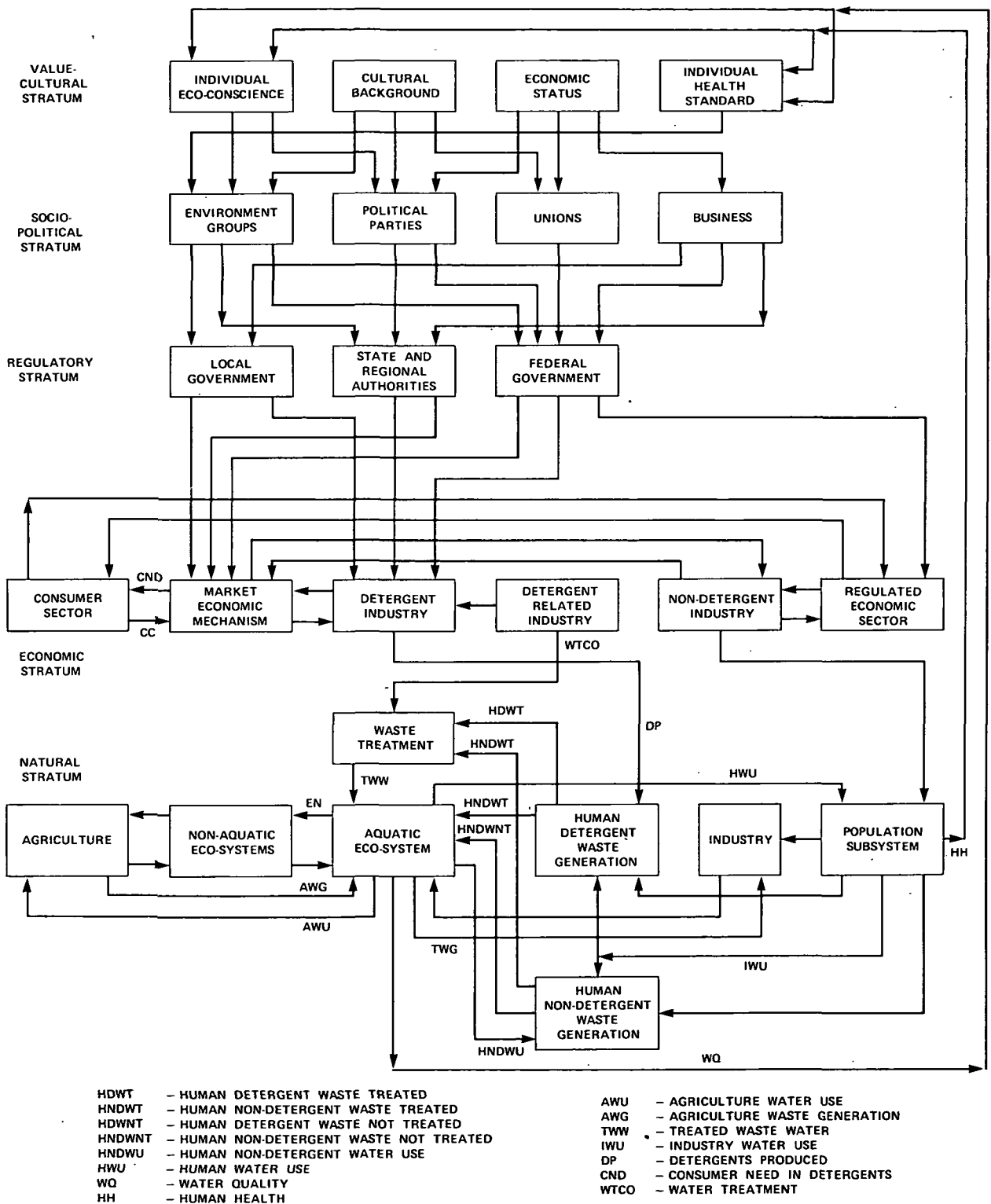


Figure 2.—Sector decomposition of phosphorus management model.

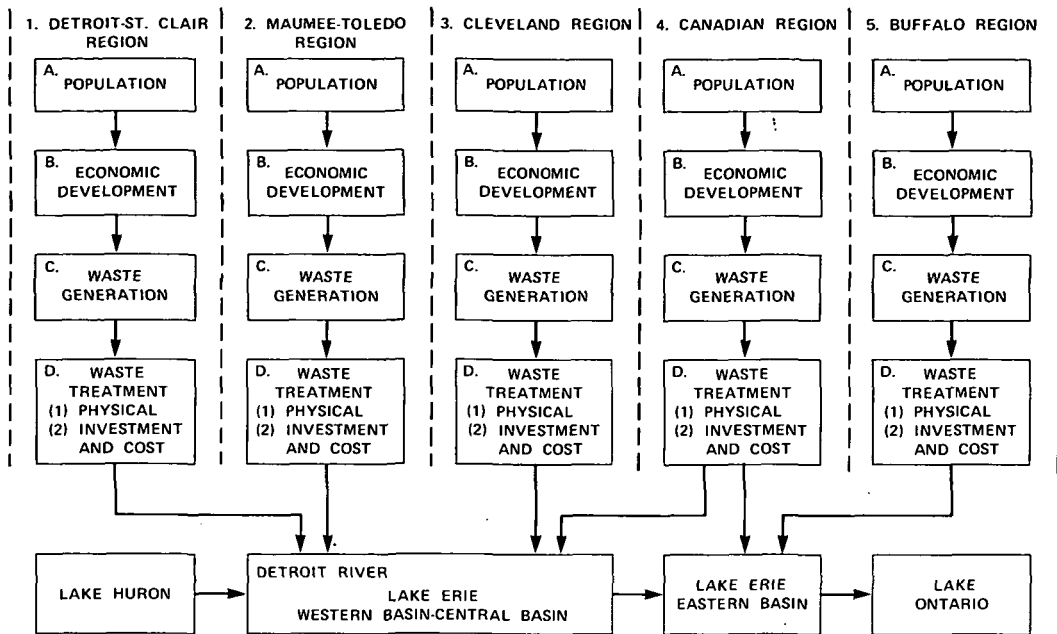


Figure 3.—Integrated model for interactive scenario analysis.

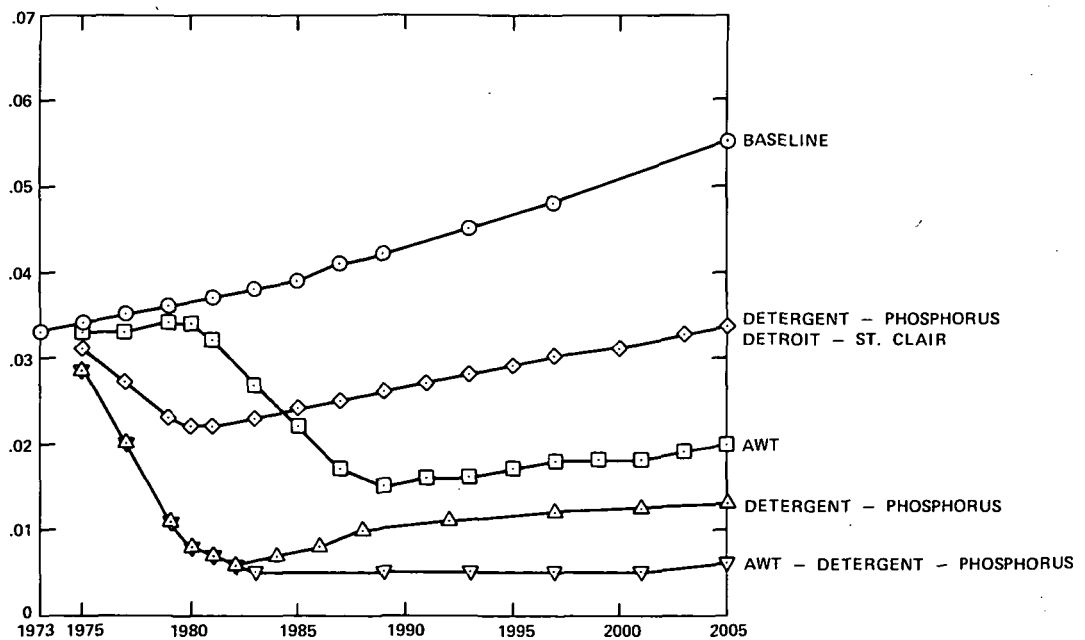


Figure 4.—Dissolved phosphorus concentration (DPC) central basin.

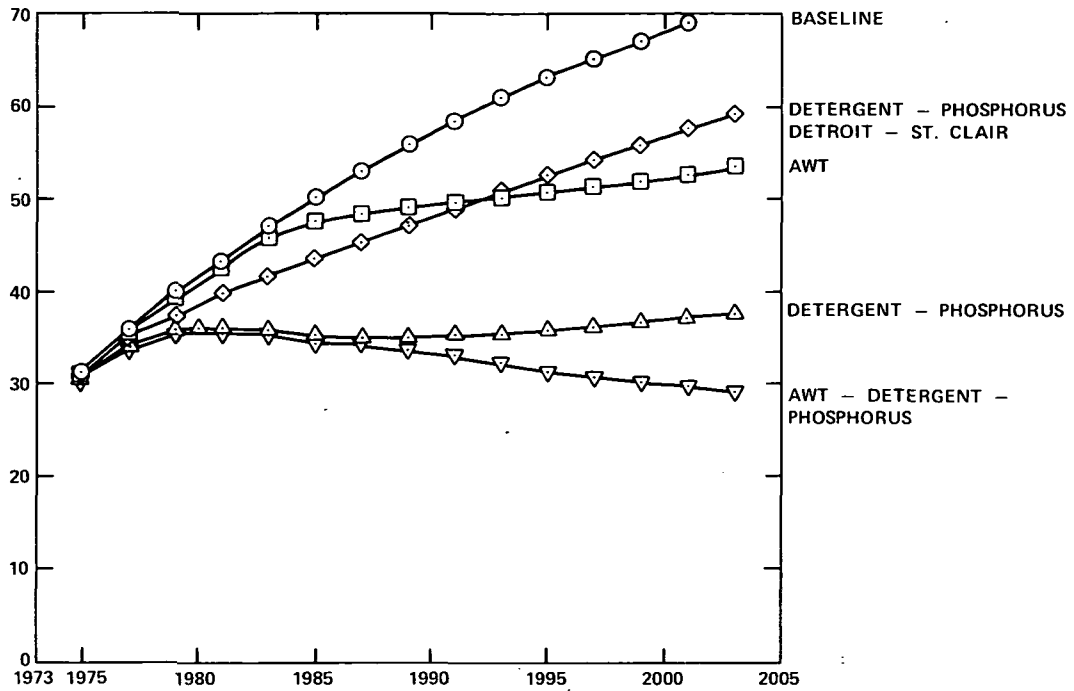


Figure 5.—Days of anoxia (DOA) central basin.

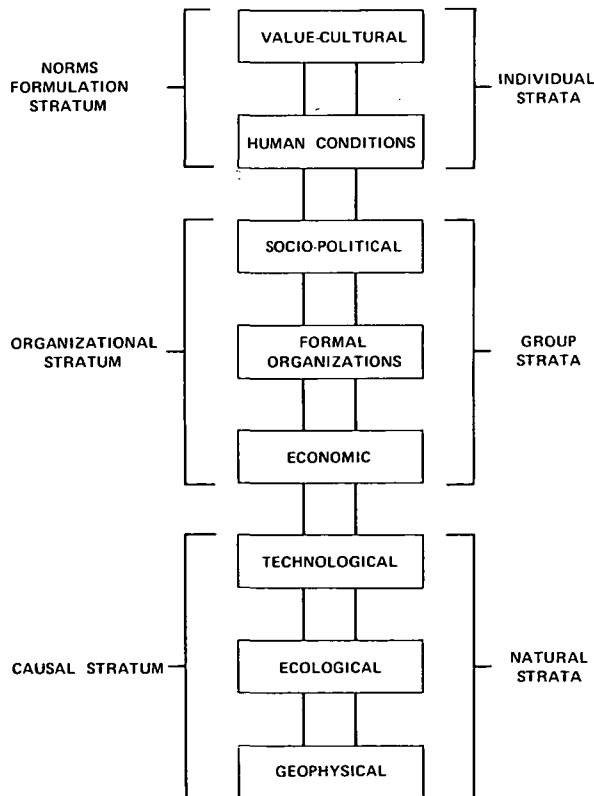


Figure 6.—General stratum for regional submodel.

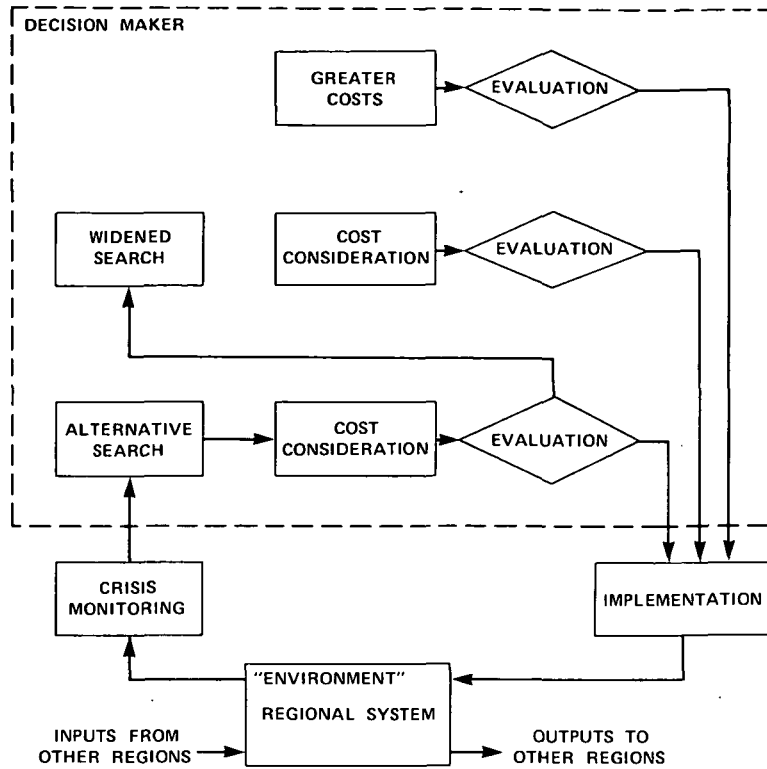


Figure 7.—Basic components of the Hughes crisis decision-making model*
 *Adapted from Hughes (ref. 23)

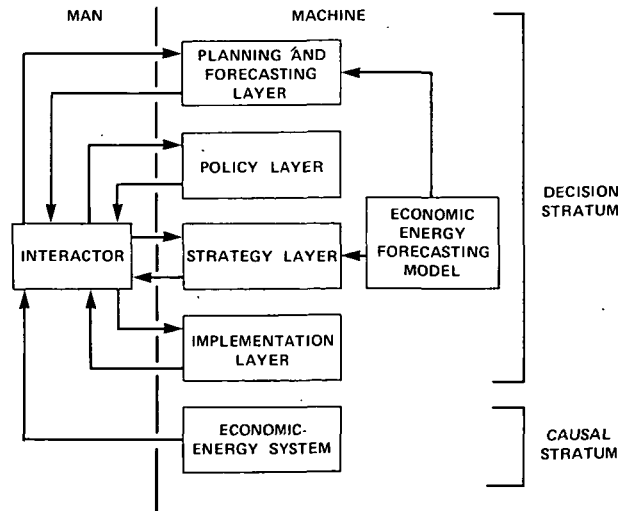


Figure 8.—Model with human interaction.

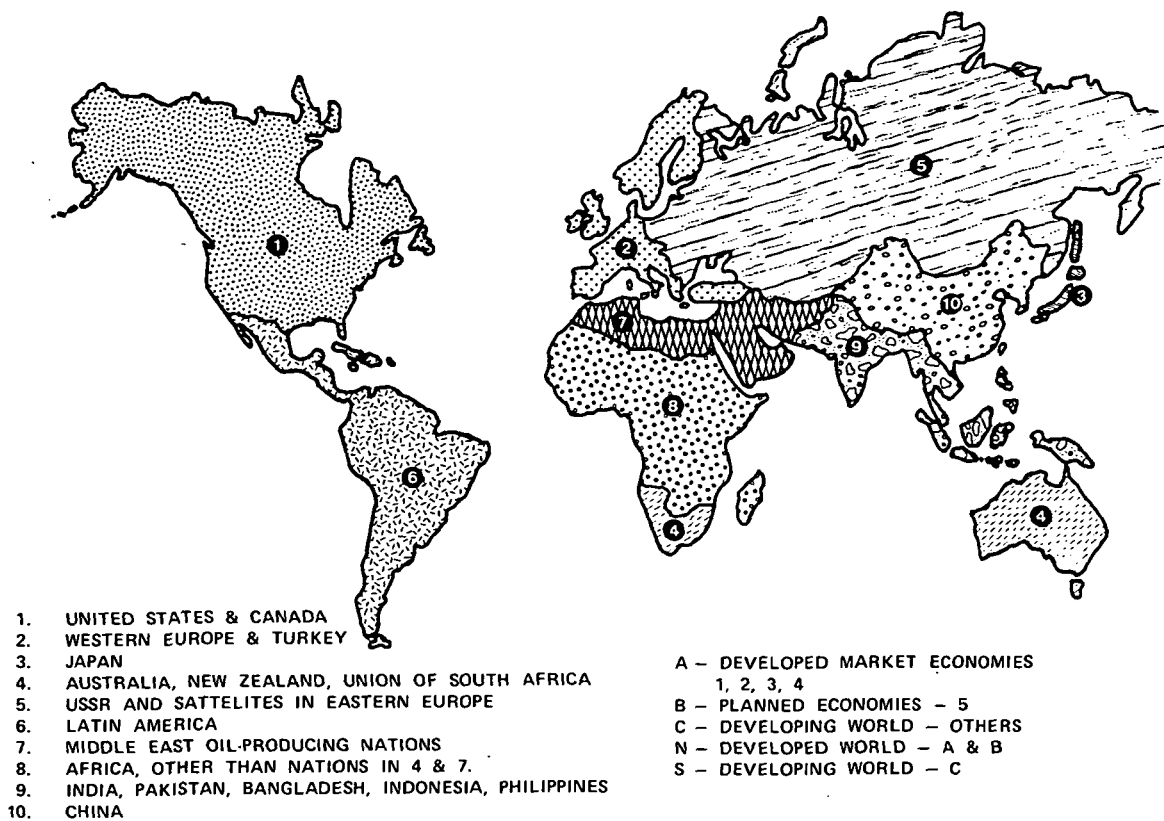


Figure 9.—Regionalization

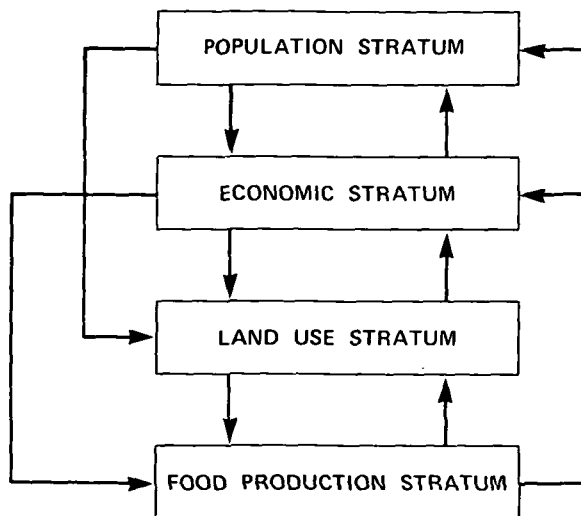


Figure 10.—Integrated food: policy analysis model basic strata.

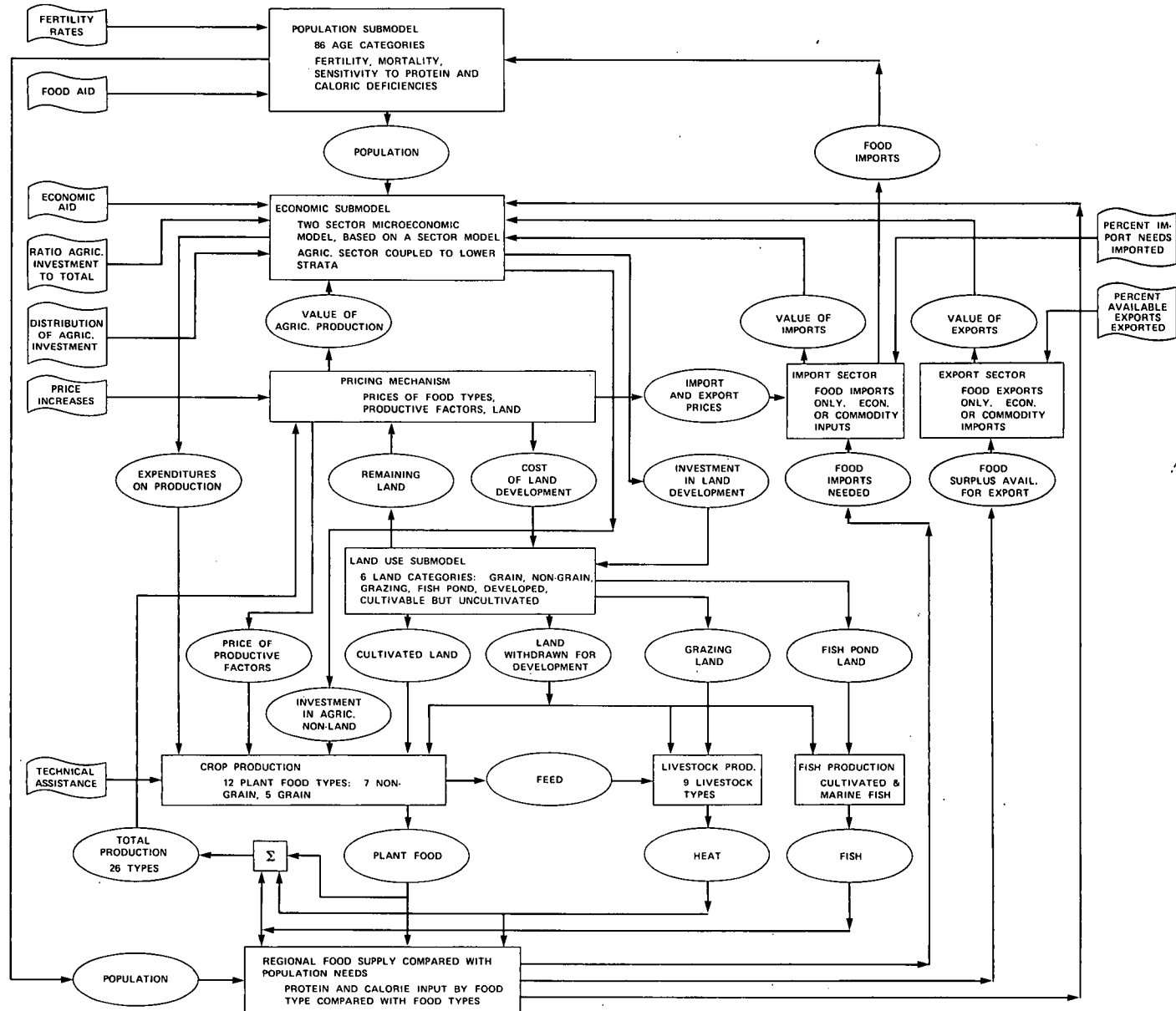


Figure 11.—Integrated food policy analysis model: structure and principal interrelationships between strata and sectors.

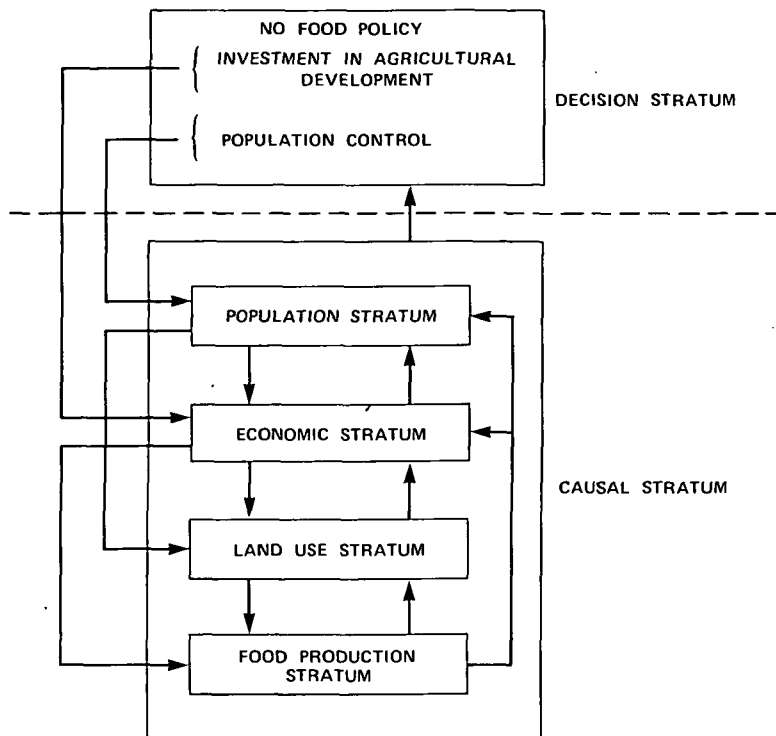


Figure 12.—Integrated food policy analysis model: regional policies.

Page intentionally left blank

AN INFORMAL PAPER ON LARGE-SCALE DYNAMIC SYSTEMS

Professor Y. C. Ho

Harvard University, Cambridge, Massachusetts

First, let us examine the title of this symposium. What is meant by "large scale" and "dynamic"? Figure 1 shows a system is large when it requires more than one decision maker to control it. Almost all interesting and difficult problems of a large-scale system are introduced by the fact that there is more than one controller involved; the reasons for having more than one decision maker or controller involved are several: the institutional bodies (e.g., local or regional government) often wish to retain their decision-making power against, say, the federal government; we have a natural aversion to dictatorship, which is another way of being centrally controlled; bureaucratic inertia often prevents us from controlling problems effectively in a centralized manner. These institutional constraints are quite familiar to anyone involved in politics, but they also occur naturally in any large organization. Communication difficulties arise; for example, because of the time required to transmit data from one place to another, by the time the central source receives the data it may be too late for effective control. An example would be a vehicle on Mars remotely controlled from Earth. A few seconds are required to transmit data from one place to another. The cost of transmission may make it no longer worthwhile to transmit all data to a centralized source. Finally, the computation time required to process the data at a centralized source may be too great. We have in mind particularly on-line control where the computation must be done quickly.

Figure 2 illustrates a decentralized control versus decomposition in computation. When we talk about decentralized control of large-scale dynamic systems we often have in mind on-line, real-time control (such as control of a power distribution network). There, you are talking about the time required to process the data is on the order of seconds and the response time of the systems is, at best, in minutes. On the other hand, many large-scale planning problems or decision-making problems can be done effectively off-line (such as planning of economic allocations or preliminary planning of a water resources system). For such problems, the computation time available is in hours or days and the response time of such a system may be in terms of days or years; therefore the control problems and planning problem are vastly different. Control problems are probably repetitive and day to day while planning problems are most often only one shot affairs. The system is planned and built and it lasts 20 to 50 years or maybe 20 or 50 months.

In planning, there are decomposition techniques for the computation, the best-known technique is the so-called "decomposition technique in large-scale mathematical programming." However, this is not really the thing of interest to us here. I am concerned only with decentralized control. First, there are some questions about the real usefulness of these decomposition techniques in mathematical programming. Consider a very large-scale planning problem that requires 10 hours of computer time, using the standard LP programming. With the decomposition technique, let's say you may be able to solve the problem in 1 hour. The difference between 10 hours and 1 hour of computation is really not that significant. Experience with these decomposition techniques often shows that, in terms of through-put, they offer little improvement because a special program must be written for each decomposition. A different planning problem requires its own special program. The process of collecting the data and processing it may take more than 10 hours. As a result, my experience with these decomposition techniques has shown that they are not used very often simply

because they are not economical. On the other hand, on-line, real time control is entirely different. It is not a question of whether you should use centralized or decentralized control; the institutional and communication constraints mentioned previously simply force you to use decentralized control.

Figure 3 defines what is meant by “dynamics” in a problem. Of course, most of you know intuitively what is meant by dynamics, but here I want to consider decomposition, particularly in terms of the problem it generates with decentralized control. A dynamic decision problem requires one to choose different decisions at different instances in time, based on different information available at the time. Often this is how a problem statement would appear in the language of game theory and decision theory. This is called extensive form formulation. It is the form we normally see when we first try to formulate the problem. On the other hand, for theoretical purposes, a control problem may be stated another way, namely, choose a strategy among all admissible strategies. What is a strategy? A strategy is a formula that tells you what decision you should make under all possible circumstances at all possible times. Mathematically, this means a map from the product space of the information available and time to the space of choices available. Once you have chosen a strategy, you have really indicated how you will behave under all possible circumstances. In principle, once a strategy is fixed you can always evaluate the cost of performance of a control system. And if you define the class of all possible strategies you are willing to consider, then you have essentially defined all possible performances with respect to each individual strategy. This then becomes an extremely simple-minded optimization problem, namely, pick the strategy that gives you the best performance – the normal form of formulation because, theoretically, it is a very clean statement of the problem and because the problems of dynamics and information have been suppressed in terms of a properly defined class of admissible strategies. For example, in the familiar language of control theory, suppose you want to use open-loop control only, that is, without any feedback information. Then, in normal form, we simply say the class of admissible strategy is the class of maps that are constant, that is, independent of information received in any given time. When we must choose a strategy among the class of constant strategies, it is the equivalent statement to open-loop control in extensive formulation.

Normal form formulation has many theoretical advantages, but it does not tell you how to solve the problem. In many instances, when there is more than one decision maker involved, particularly with game theory, you want to focus your attention on certain aspects of the problem peculiar to the fact that you are playing a game, without having to worry about a detailed solution, the dynamics, and the information. Often certain aspects of a control problem can be discussed in terms of normal form without detailed information (aspects of this problem are discussed later). Such concepts are not only applicable to purely static algebraic problems, but they apply equally in dynamic systems provided you realize we are working in the normal form. On the other hand, certain aspects of the dynamic information must be treated in the extensive form manner to show the problem areas.

Having thus defined the scope of the problem in terms of large-scale dynamic systems, let me first hasten to say that we know very little about decentralized control dynamics systems. Figure 4 is a very rough attempt to classify the different types of studies on decentralized control of dynamic systems in terms of whether the technique is deterministic or stochastic (they can also be classified according to what aim they have). For control problems, four types of questions can be asked. First a structural question – what is the right kind of model for the system? Once this is understood, how do you optimize the control? As often happens, you may not be able to solve this;

then you ask for something less, namely, can I do something to the system to make it stable? Finally, you may simply ask the basic question: "Is it feasible to do something about the system?"

In terms of decentralized control in the deterministic phase of the diagram, the first block covers questions such as foundation and philosophy of hierarchical control, organization, how to distribute the payoff among different decision makers, etc. Under optimization, we have mentioned large-scale mathematical programming already, economic problems such as optimal resource allocation by one supervisor among different departments, the vector payoff question when decision makers have different payoffs, and the Pareto optimality (sometime fashionably called Paretian analysis). It is just a different way of saying "How do you reconcile or trade off different objectives such as more guns or more butter?" Under stability, a whole class of problems come under the name of adjustment process. These are really interconnected dynamic systems, with one controller for each dynamic system. They all make adjustments to improve their performance and each adjustment would affect other systems. When a controller adjusts to improve his position, will this lead to a stable process that is good for everyone or will there be cut-throat competition? Finally, under feasibility, you have questions such as decentralized controllability – is it possible to control systems from one state to another in a decentralized manner?

On the stochastic side, the basic emphasis is essentially for questions dealing with the structure of information. We want to understand basically what is meant by information in a many-person decision problem. What do we mean by information structural properties and so forth? In stochastic optimization, specifically, terms such as team theory (another way of saying decentralized decision theory) and the question of value of information, and such appear.

So far as I know, no work has been done on the stochastic stability of decentralized systems or the feasibility question in stochastic models. Most of the discussion that follows concerns the structure of optimization for the stochastic phenomenon. My colleague and former co-worker, Dr. K. C. Chu, will discuss in another paper the role of team theory in decentralized control. I would like to discuss briefly the adjustment processes and decentralized stability or feasibility. This work, which appeared recently in the Russian literature, is, I think, quite interesting. Particularly, the adjustment process relates also to the work Professor Siljak discusses later.

The main part of this paper concerns the problem of information in general, and many-person decision problems, which, of course, includes that of decentralized control. There are really only two kinds of decision variables. First are the decisions made by human beings, u_1, \dots, u_n during different times at different places by different decision makers. Another set of variables is nature's decisions, ξ_1, \dots, ξ_m , often taken to be uncertain. These are noise in the measurement systems, disturbances in the control systems, or the flip of the coin, anything considered uncertain but given the probability of distribution. Every event under the sun is a function of human decisions as well as nature; these are the only fundamental variables in the problem. In dynamic systems, state variables are really secondary because the state of the system at a given time is a result of all past decisions made by the controller as well as the noise or disturbance that has occurred. Therefore, human and nature's decision variables are considered fundamental. Since every event is a function of these variables, the information available to a decision maker must also be a function of these two sets of variables. The particular function that relates u and ξ is called the *information structure* of the problem and these two sets of variables and definitions are shown in figure 5(a). Figure 5(b) defines what we mean by strategy. As mentioned before briefly, strategy is a mapping from information to decision, whether by adaptive control, stochastic control, or whatever. This is really the

most general definition of a control law — a prediction of behavior on the basis of information available at a particular time. Since information is defined as a function of u and ξ , this definition of strategy further relates information to u or defines the implicit equation $u_i = \gamma_i(\eta_i[u, \xi])$. If we specify the exact information structure (specify who knows what), furthermore specify what the strategies are (what each decision maker will do under all possible circumstances, i.e., all possible information patterns), and specify nature's decision for the given probability density ξ then the equation labeled (*) in figure 5(b) defines a set of equations that, when they have a solution, gives the actual decisions the human decision makers will make. The model represented by (*) can be extended to give even more general situations in game theory.

Some possible questions and problems associated with this set of equations are shown in figure 6. First, since the set of equations labeled (*) is implicit, the problem arises whether a solution exists. In fact, does (*) have a unique solution for a given information structure and for each possible and admissible strategy set Γ ? If there were not unique solutions, then we are in somewhat of a funny situation. If you tell what everybody knows and how everybody will behave based on what he knows, the outcome is undetermined because there is more than one possible solution. Given these two specifications γ and η (who knows what and how does every decision maker behave under all circumstances), there should be only one unique outcome. If the equation set has a unique solution, a problem will be well posed. Note that this type of question does not arise in the usual decision theory framework where dynamics does not occur (such a problem is called static or simple). In such cases, the information variable is always a function of what nature decides or uncertainty ξ , and does not depend on what other decision makers have done. For such a case, η is only a function of the nature's decision and uniqueness is trivial. If nature has acted according to some probability distribution, one's behavior is completely determined and unique. For a dynamic problem, what is known may depend on what other people have done in the past, so the information is not only a function of noise and disturbance but also of other people's controlled actions — therein the unique problem arises. An obvious question arises: what is a good information structure? This then involves the design of information systems. Whether one measurement system or one set of observations is better than another or whether you should observe one sample, three samples, and four samples, since each measurement is presumably with cost, this question of design of information system arises and the value of information or "who should know what."

A third question is the all familiar one, namely, what is a good control law? This is the usual optimization problem except here we are interested in the solution in a decentralized setting. Finally, since both the information structure and the control law are to be designed, which information structure will make the optimization problem easy to solve. (See Dr. Chu's paper for a discussion on this.)

Certain explicit results in team theory are related to this question. Figure 6 shows a particular fundamental difficulty in dynamic problems involving information structure just described. From earlier definitions, nature's decision is represented by the vector variable ξ , the basic variable in the problem, and a probability density function or distribution is introduced. But z_i and u_i are *not* the random variables for a given information structure unless the strategy is fixed. Consider a two-stage decision problem. The initial state is ξ_1 ; control u_1 is applied additively to yield an intermediate state x . Apply another control u_2 additively to intermediate state x to the terminal state y . A decision maker at time 1 knows the information z_1 , which is simply a perfect measurement of an initial state. The second decision maker knows z_2 at some time later, which is simply a noisy measurement of the intermediate state x or $\xi_1 + u_1 + \xi_2$. Nature's decisions are random choices of

the initial state and random noise ξ_2 . Note that z_2 is not a random variable at this point until the strategy of u_1 is defined. The strategy of u_1 is a function of z_1 , in this case ξ_1 . Once γ_1 is defined, u_1 becomes a random variable because it is a function of another well-defined basic random variable in the problem. If u_1 becomes a random variable, then z_2 also becomes a random variable because it is now a function of ξ_1, ξ_2 only. Furthermore, u_2 becomes a random variable with solution-dependent distributions. The distribution of u_2 depends on the particular γ_1 and γ_2 — the solutions we wish to obtain. Until the solution is known, γ_1 and γ_2 cannot be defined as random variables. But, until these γ terms can be defined as random variables, we cannot begin the solution process; this is the difficulty in such general information problems (fig. 8).

Assume that ξ_1, ξ_2 are random variables with very nice properties, for example, Gaussian. But this does not guarantee that z_2 is a nice random variable unless some additional restrictive assumptions are made about the control law γ_1 . Since γ_1 can be arbitrary, then z_2 will be a rather arbitrary function of ξ_1, ξ_2 (fig. 8), which certainly would not generally be Gaussian so the nice property was lost. Also, for a payoff function convex in u_1, u_2 in the problem, nothing can be said about the expected value of this convex function in u_1, u_2 . It is not known in fact, whether it is convex in γ_1 , a fact important in proving any kind of optimality property. This can be seen fairly easily. The expected value of a convex function in u_1, u_2 is basically a function of γ_1, γ_2 . In other words, once γ_1, γ_2 are fixed with the information structure (as defined earlier), the payoff is a function only of these control laws. But the control law γ_2 is a function of z_2 which, in turn, is a function of γ_1 . Therefore, the dependence of the payoff function J on γ_1 is rather intricate and depends on γ_1 explicitly (where u_1 is replaced by γ_1), but it also depends on γ_1 through γ_2 . Since γ_2 again is generally arbitrary, for functions originally convex in u_1 , there is no guarantee that it will be convex in γ_2 unless restrictions are placed on γ_2 (such as linear or otherwise monotonic properties). Again, since γ_1, γ_2 are, in fact, the answer we are looking for and they are assumed arbitrary in the beginning, there is no prior reason to assume they should be linear and so forth. As a result, the simplest problem of this type (as shown in fig. 7) cannot be solved. This point was first brought out in Witsenhausen's paper (ref. 7), which could be regarded as the starting point of all this research. So, in general, we must impose additional restrictions on the η information structure to avoid these difficulties and this is what I meant by the question raised earlier, that is, what kind of information structure would lead to easy solutions? (See references.)

The value of information should be discussed briefly. The value of information is simply the difference between the best performance with and without information. The difference, presumably (if it is possible to measure it in dollars), is the most one would be willing to pay for the information; this should be the basis for comparing different information system designs. This definition has several problem areas: in problems with multiple payoff, the definition of "best" requires more careful specification. Those familiar with game theory realize there are many different kinds of optimal solutions. One has to determine what is meant by best. Also, from the decision-theoretic viewpoint, one compares the expected value of the information, that is, the $E[VI]$ (fig. 9). Is this the only basis for comparison? and what is meant by more informative? This problem may appear to be fairly simple, but actually the more you look into it the more complicated it becomes. There is no uniform agreement on this definition of more informative. The expected value of information is often used as a basis for comparing whether one information system is more informative than another. Finally, perhaps a more disturbing question in the case of multiple decision making is that more information does not always lead to better payoffs, which is kind of counter intuitive when we are so used to thinking in terms of one player. The obvious answer is that if you get more information you can always ignore it and do what you did before without the information so you

could not possibly be worse off or get less payoff. The problem is that when more than one person is involved, sometimes you cannot ignore the information. This new information cannot be ignored because other people may not believe that you will ignore it and you cannot do what you did before because the other player, knowing that you have certain information, will alter his strategy. Once his strategy is altered, you can no longer dare to use your old strategy. Your strategy must be altered in the optimal manner, but his optimal payoff for the new information structure may, in fact, be worse than before. I have presented several examples (see reference list) to show that information may not always lead to a better payoff, and the question of the value of information is by no means settled.

I mentioned earlier some work in the Russian literature on decentralized control. In the last few years, they have been very interested in this line of research. Many controllers or decision makers or planners are involved in the problem, each controller having limited information. Imagine an indicator function that depends on what everybody else is doing. This function would indicate that if all values remain the same the control is moved either up or down or left or right, then you are moving away from the goal. A natural reaction at this point would be to change the parameters u_i according to the indicator function, that is, make the rate of change of u_i proportional to the indicator function or, more qualitatively, make the sign of the rate of change in u_i proportional to that of the indicator function. The value of the indicator function δ_i must be nonzero when u_i is not at the equilibrium goal. This situation is illustrated in figure 10 where all indicator functions equal zero at equilibrium, that is, where each goal is satisfied. Would such a single-minded adjustment process evolve in such a way that each goal would be satisfied. Perhaps the simplest example would be when the indicator function is a linear function of u : δ_i of u is simply in the product of u with q_i . These combined terms yield vector equation $u = Qu$. For the time rate of change of $\|u\|^2$ we have the quadratic form $\langle u'(Q + Q')u \rangle$. Clearly, if this matrix $Q + Q'$ is negative definite, then $\|u\|^2$ in the limit goes to zero because of the well-known Liapunov theorem. The system would then be stable, for then $u = 0$ and the indicator function Qu is zero also. Equivalently, $Q + Q'$ is negative definite if certain sufficient conditions are satisfied. The simplest condition is that the diagonal element of Q be larger than the sum of the absolute value of its row or its column (conditions 2 and 3), usually known as the Gersgorin circle theorem. When there is diagonal dominance, $Q < 0$.

The three conditions can be put into a different form. Instead of differential changes in u and δ consider finite changes Δu_i and $\Delta \delta_i$, for which conditions 1, 2, and 3 imply conditions I, II, and III in figure 11. Conditions I, II, and III are, in fact, conditions of stability for those adjustments when the indicator functions are not differentiable and nonlinear. Often conditions I, II, and III are much more applicable than condition (i), (ii), and (iii). The reference list offers a whole set of examples drawing from electric circuits, resource allocation, game theory, etc., which are formulated to show the generality of conditions I, II, and III.

Figure 12 poses the question of feasibility of adjustment control. Let us define x_i as the amount of the i th commodity in an economy. The production of each commodity requires input from other commodities, for example, producing machine parts requires input of steel, fuel, labor, etc. We shall define a matrix A with elements α_{ij} , the amounts of i th commodity needed to produce one unit of the j th commodity. Then the net amount of i th commodity produced is defined as y_i , which is simply the gross amount x_i minus the amount needed to produce other commodities $\sum_j \alpha_{ij}x_j$. In matrix form, $y = (I-A)x$ is the well-known Leontiff input/output economy. Now the question you may raise at this point is whether the economy is productive, that is, is $y < 0$ for some $x > 0$? This

question is of interest in terms of decentralized control because if this is possible (i.e., if every positive y_i has a corresponding x_i), then decentralized control is not impossible. Whenever a positive y_i is required that can be accomplished by a positive x_i . (So if production units only have to worry locally about its product and its quotas that you don't need some sort of coordinating structure.) This condition for productive economy has specific answers for certain conditions on the matrix A , which affects the possibility of decentralized control. This is a simple example of the feasibility of adjustment, which can be generalized in many ways. Furthermore, x and y need not be interpreted as productions. For example, the component vector y_1 of y is quality of education and y_2 is the cost of education for vector x ; x_1 is payment to the teacher and x_2 is the load the teachers take on. Does there exist a combination teacher pay and teacher load that will simultaneously increase the quality of education as well as lower the cost or at least maintain the cost? This question depends on the matrix A and can be generalized if y relates nonlinearly to x (see reference list).

Although this paper has been a rather rambling and somewhat disorganized survey of the subject of decentralized, large-scale dynamic systems, I think the field of large-scale systems control is very important. However, the results are very scattered at this point and certainly we are unable to claim a very unified picture of the whole field. I apologize for not being able to present a more coherent talk or discussion on this matter as a whole, but I hope the rest of the conference will try to make up for this. Thank you very much.

FOR FURTHER READING

1. Ho, Y. C.; and Chu, K. C.: Information Structure in Dynamic Multi-Person Control Problems. *Automatica*, July 1974.
2. Ho, Yu-Chi; and Sun, Fang-Kuo: Value of Information in Two-Person Zero Sum Problems. *Journal of Optimization Theory and Applications*, to appear.
3. Basa, Tamer; and Ho, Yu-Chi: Informational Properties of the Nash Solutions to Two Stochastic Nonzero-Sum Games. *Journal of Economic Theory*, vol. 7, no. 4, April 1974.
4. Ho, Y. C.; Blau, I.; and Basar, T.: A Tale of Four Information Structures. *Proceedings of IRIA Symposium on Control Theory*, June 1974; *Springer-Verlag Notes on Mathematical Systems and Economics*, October 1974 to appear.
5. Malishevskii, A. V.: Natural Systems. *Automation and Remote Control*, May 1974.
6. Malishevskii, A. V.: Models at Joint Operation of Many Goal Oriented Elements, I and II. *Automation and Remote Control*, May-June 1973.
7. Witsenhausen, H. S.: A Counter-example in Stochastic Optimum Control. *SIAM J. Control*, vol. 6, no. 1, 1968.

A SYSTEM IS "LARGE" WHEN IT REQUIRES MORE THAN ONE DECISION MAKER TO CONTROL IT.

I) INSTITUTIONAL CONSTRAINTS:

- LOCAL OR REGIONAL AUTONOMY VS. CENTRALIZED CONTROL
- AVERSION TO DICTATORSHIP
- BUREAUCRATIC INERTIA AGENT CENTRALIZED CONTROL

II) COMMUNICATION DIFFICULTIES:

- TIME REQUIRED FOR TRANSMISSION (EARTH - MARS)
- COST OF TRANSMISSION
- COST OF CENTRALIZED PROCESSING IN AVAILABLE TIME (ON-LINE CONTROL)

Figure 1.— Large scale dynamic systems.

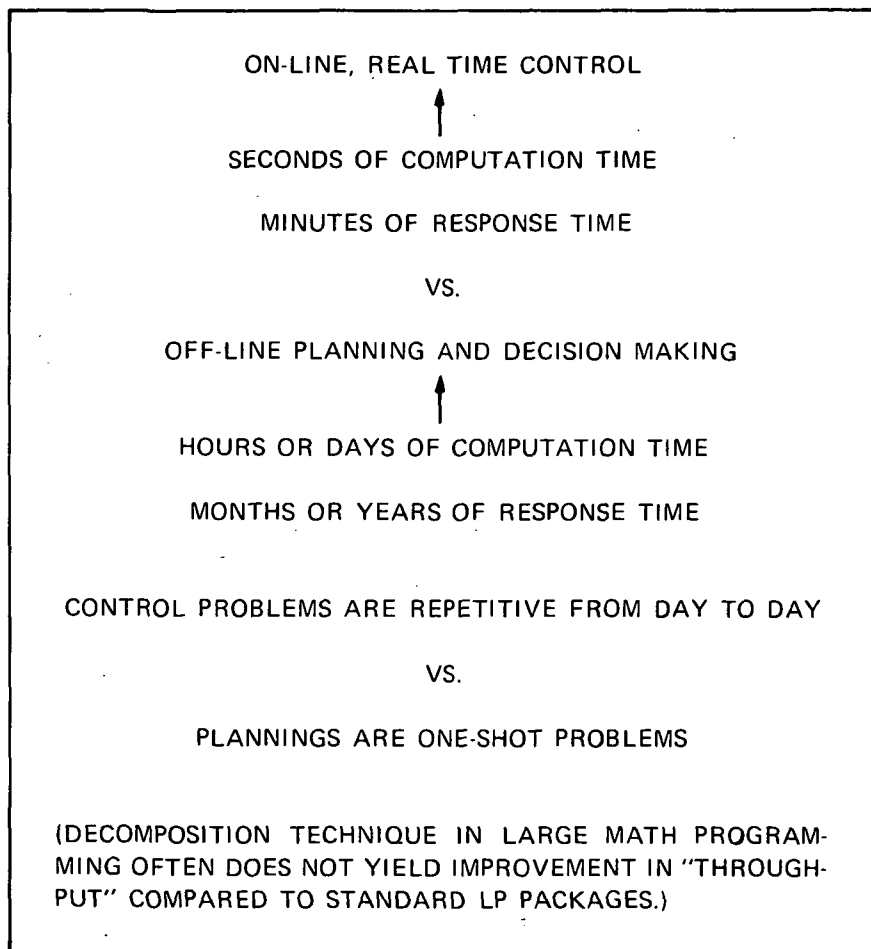


Figure 2.— Decentralized control vs. decomposition in computation.

Choosing many decisions at different instants of time based on different information available

---- EXTENSIVE form formulation (usual statement of a problem)

Choosing a strategy (which is a recipe: information x time choice) among all admissible strategies.

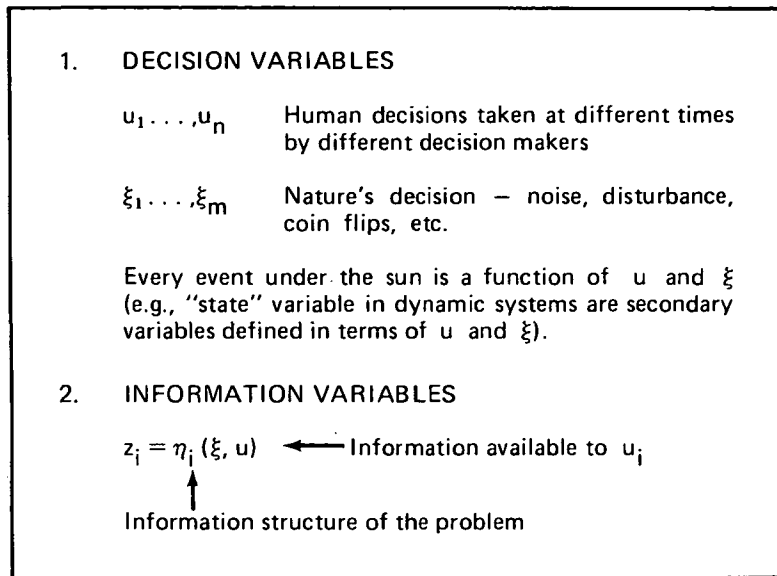
---- NORMAL form formulation has the advantage (theoretically) of suppressing the detail difficulties involving dynamics and information in the word "admissible" class of strategies. Relationship between performance vs. strategy choices are clear cut and can be analyzed without being able to solve the problem.

---- Main-stream game theory approach.

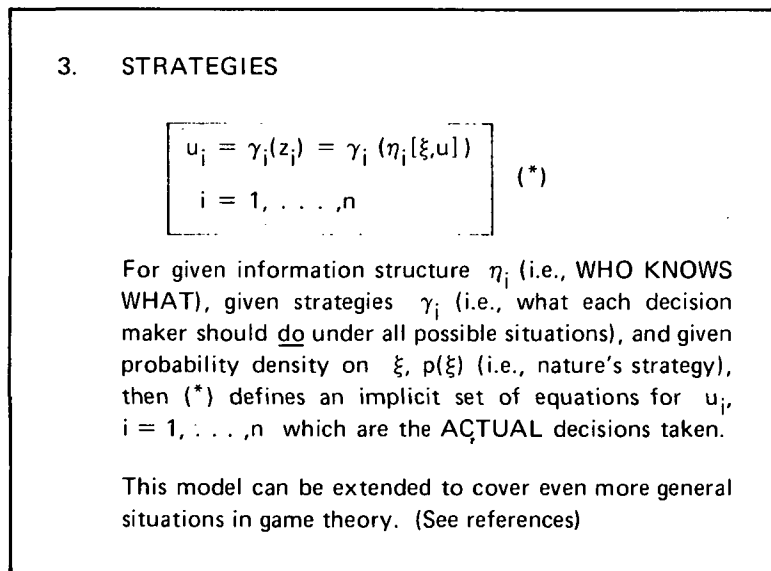
Figure 3.— DYNAMICS in a problem.

	Deterministic	Stochastic
Structural	Hierarchy, organizational, payoff distribution, etc.	Information: structure and properties
Optimization	Large-scale multipayoff, resource allocation, vector payoff, Paretian analysis	Team theory value of information
Stability	Adjustment processes	?
Feasibility	Decentralized controllability, etc.	?

Figure 4.— Decentralized control of dynamic systems.



(a) Design variables and information variables.



(b) Strategies.

Figure 5.— General problem of many person decision making.

1. Does (*) have a unique solution for given η and every admissible γ ? Is the problem well posed?
2. What is a "good" η ?
 - Design of information system. Who should know what?
3. What is a "good" γ ?
 - Usual optimization question in decentralized setting.
4. What choice of η will make good γ easy to solve?

Figure 6.— Questions and problems for $u_i = \gamma_i(\eta_i[\xi, u])$.

ξ (nature's decision) is the basic random variable with $p(\xi)$

$z_1 = \eta_1(\xi, u)$ and $u_1 = \gamma_1(\eta_1[\xi, u])$ are not random variables for a given η unless γ is fixed.

Example:

u_1 knows $z_1 = \xi_1$ ← measurements at $t = 1,2$

u_2 knows $z_2 = x + \xi_2$

$$= \xi_1 + u_1 + \xi_2$$

$$= \xi_1 + \gamma_1(\xi_1) + \xi_2$$

u_2 is a random variable only when γ_1 is fixed.

z_2 is a random variable only when γ_1 is fixed.

∴ information and decisions are random variables with solution-dependent distributions!

Figure 7.— Difficulties in dynamic information problems.

\therefore Even if ξ_1 and ξ_2 are random variables with nice properties (e.g., Gaussian), z_2 is not "nice" since γ_1 can be arbitrary. Also, a convex $L(u_1, u_2) \not\Rightarrow E\{L(u_1, u_2)\}$ is convex in γ_1 .

$\therefore E\{L(u_1, u_2)\} = J(\gamma_1, \gamma_2)$
 $= J(\gamma_1, \gamma_2 [\xi_1 + \gamma_1(\xi_1) + \xi_2])$

not convex in γ_1 unless γ_2 is linear or otherwise possesses nice properties.

"Must impose additional restrictions on η , the information structure, to avoid these difficulties!"

All information is centralized. See references for other examples.

Figure 8.— General information problems.

The "best" you can do with the information
 — the "best" you can do without the information
 = VI

Basis for comparing information system design.

- In problems with multipayoff, definition of the "best" requires more careful specification.
- Is $\epsilon[VI]$ the only basis for comparison? Definition of "more informative?"
- Does more information always lead to better payoffs?

Figure 9.— Value of Information.

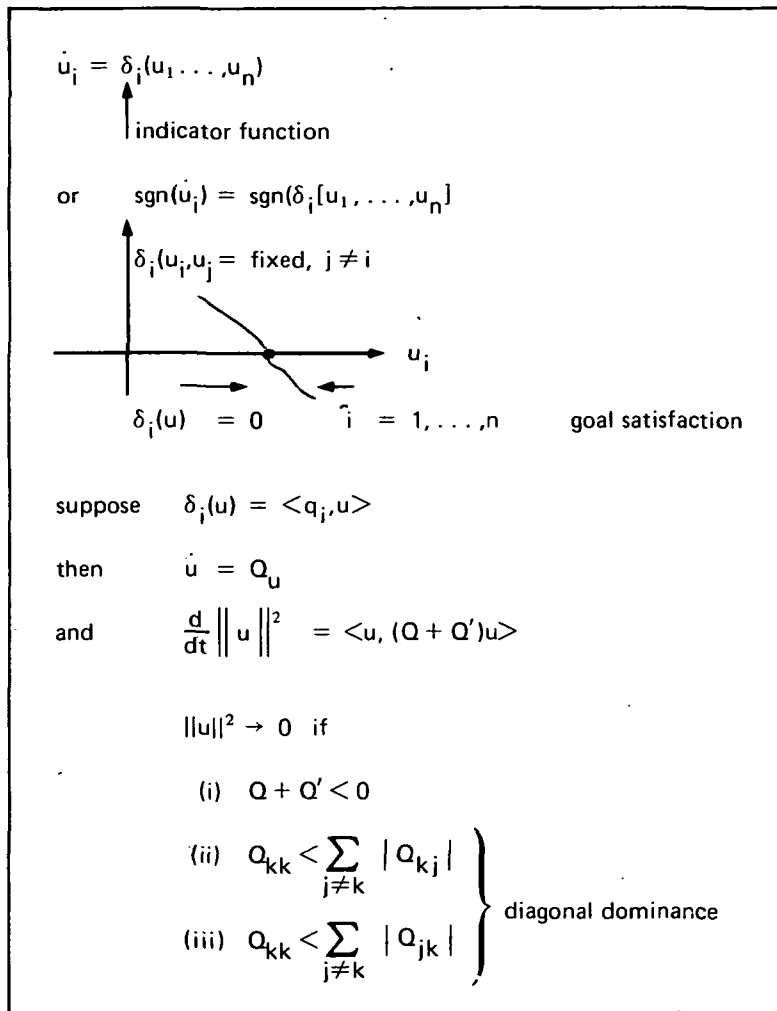


Figure 10.— Stability of goal-oriented adjustment processes.

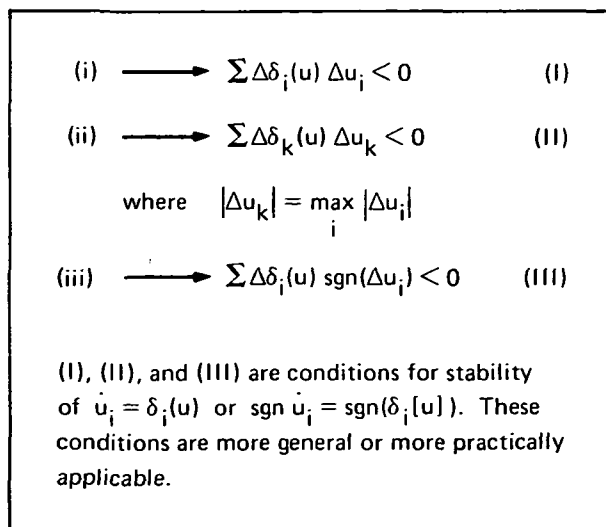


Figure 11.—Finite changes vs. differential changes.

Ex. x_i = levels of production of ith commodity

α_{ij} = amount of ith commodity needed to produce
one unit of jth commodity

$\sum_{j=1}^n \alpha_{ij} x_j$, total amount of ith commodity needed

$y_i \equiv x_i - \sum_{j=1}^n \alpha_{ij} x_j$, net production of ith commodity

or

$y = (I - A)x$, the Leontief in/out model

Question: Does an $x > 0$ exist for every $y > 0$? If so, decentralized control is possible – each unit has to increase its own production level.

Figure 12.— Feasibility of adjustment or control.

ANALYSIS OF LARGE-SCALE WATER RESOURCES SYSTEMS

A. Bruce Bishop* and Trevor C. Hughes*

Utah State University, Logan, Utah

WATER RESOURCES SYSTEMS

Water resources systems involve a host of physical, environmental, and societal aspects that must be considered in making decisions among alternative proposals for use of water and related resources. Thus, a water resources system can be viewed not only as a physical system, but also as agricultural, biological, economic, political, legal, and sociological systems. The numerous complex and interacting factors place most water resources problems in the category of "large-scale" systems. Systems analysis provides a way to undertake the formidable problems of choice involved in planning and managing such large-scale systems.

This paper examines the nature and characteristics of water resources as large-scale systems and indicates how systems analysis and modeling techniques have been used in dealing with various water problems. Accordingly, the first part of this paper discusses the systems characteristics of water and related resources, including objectives, components, types of inputs and outputs, and spatial and temporal dimensions. Against this background, the various mathematical modeling and systems techniques used in analyzing large-scale water resources systems are then summarized. Finally, the balance of the paper describes three example applications in greater detail.

Characterization of Water Resources Systems

A basin-wide network of water sources, users, and polluters is a highly interactive system. The overall system can be viewed in terms of hierarchical levels of interdependent basins, regions, and connected systems serving various purposes and objectives (fig. 1).

A broad classification of the characteristics of water resources systems is given in table 1 under the headings of objectives, scope, space and time:

1. "Objectives" specify the levels of performance or achievement to be obtained through planning, use, and management of the resources. The Water Resources Council (1973) has recognized national economic development and environmental enhancement as the two general planning objectives, with many detailed subobjectives.

2. "Scope" refers to the variety of the sectors of water use encompassed in the analysis of the system. This may range from any single sector from among those listed in table 1 to any set of multiple combinations of these sectors.

3. "Space" denotes the geographical extent of the water resources system, usually introducing other complexities in system formulation relating to physical, political, social, and economic

*Associate Professor, Civil and Environmental Engineering, Utah Water Research Laboratory.

boundaries. Three levels of geographical extent commonly used in water planning are (a) local or single basin (e.g., a single urban community, a watershed), (b) regional or multibasin (e.g., a multi-urban community in geographical proximity), and (c) statewide or river basin (e.g., state of Utah, Rocky Mountain states, the Colorado River Basin, the Columbia River Basin consisting of numbers of smaller basins).

4. "Time," as a characteristic of water resources systems, identifies the temporal distribution of the activities and impacts associated with construction, as well as the time flow of investment capital, and operation and maintenance funds necessary for realization of plans.

Translating water resources system characteristics into the kinds of alternatives to be considered in planning water projects involves structural (construction) and/or nonstructural (policy) aspects. Figure 2 is a brief summary of the features that describe an alternative's physical characteristics for structural solutions, or policy configurations for nonstructural solutions as related to the descriptive characteristics and scope of water resources systems.

Overview of Systems Modeling in Water Resources

There are two primary purposes of water resource systems modeling: (1) to describe the nature and attributes of the physical system and predict its response to changes in inputs and (2) to analyze various alternatives for planning and managing the system.

As figure 3 indicates, a further breakdown of these systems modeling purposes for planning and management includes: (1) allocation decisions, such as distributing water available to various users or purposes, operating rules for a reservoir, and budget allocations or investment policy and (2) pollution abatement or treatment strategies such as regulating pollutant discharges, allocating stream assimilative capacity to various users, and determining optimum capacity and location of various treatment plants within the system; and for systems description (1) hydrologic models to better understand various phenomena in the hydrologic cycle, including changes due to man-made effects on various components of the system, and (2) stream quality descriptions to better understand stream behavior under various pollutant loadings. Figure 3 also notes the general types of input and output information for these modeling purposes at local, regional, and basin levels.

For many large-scale systems models, management and descriptive purposes are integrated. The purposes of system analysis and modeling is to effectively determine the extent of system boundary, the necessary data inputs, appropriate system relationships and applicable mathematical techniques, and the desired system outputs.

The generalized system structure shown in figure 4 is, of course, applicable to water resources. The system receives inputs – controlled or uncontrolled – that affect the interaction of operation of system elements. Outputs are produced which, if the system is dynamic, may affect the inputs. The decision variables are manipulated to achieve some objective subject to any system constraints. These elements as related specifically to water resources systems are summarized as follows:

Inputs to Water Resources Systems:

A. Water sources

1. Surface sources; for example, surface water flow, sedimentation, or salt load, precipitation
2. Underground sources
3. Imported sources; for example, desalting water, imported water
4. Reuse and recycling; for example, treated water from treatment plant, recycling water in irrigation

B. Other natural resources

1. Land
2. Minerals, etc.

C. Economic resources

Outputs of Water Resources Systems:

A. Water allocation to user sectors

1. Municipal
2. Agriculture
3. Industry
4. Hydroelectric power
5. Flood control
6. Navigation
7. Recreation
8. Fishery and wildlife habitats

B. Quantity and quality of the water resources system

1. Flow of stream
2. Quality of stream

System Decision Variables:

A. Management and planning

1. Operating strategies
2. Land use zoning
3. Regional coordination and allocation policy
4. Number and location of treatment plants
5. Sequence of treatments and treatment level achieved

B. Investment policy

1. Budget allocation to various subsystems
2. Timing of investment; for example, stages of development, interest rate
3. Taxing and subsidy strategies

System Constraints on Systems Performance:

1. Economic constraints; for example, budget, B/C ratio
2. Political constraints; for example, tradeoff between regions
3. Law; for example, water right
4. Physical and technology constraints; for example, probability of water availability
5. Standards; system output may have to meet certain standards; for example, effluent standard from treatment plant
6. Time available to bring facilities into operation

System Physical and Engineering Components:

A. Planning and management system components

1. Dam and control structures
2. Levees and other protecting structures
3. Distribution or collection systems comprised of
 - (a) Canals
 - (b) Pipes
 - (c) Pumping stations and other control structures
4. Treatment plants

B. Descriptive system components

1. Physical properties of stream; for example, roughness, slope
2. Biochemical properties of stream; for example, rate of aeration, rate of self-regeneration
3. Chemical properties of stream; for example, hardness, pH

System complexities generally increase as the number of input and output combinations increase. An example of the simplest input-output combination would be a single water source being allocated to a single use sector. As shown schematically in figure 5, the complexity increases as more components, constraints, and decision variable are considered; until the most complex systems of multiple inputs and multiple outputs are achieved.

Mathematical Techniques Used in Water Resources Systems

Various mathematical techniques have been applied in analyzing the water resources systems discussed previously. Figure 6 gives an overview of the mathematical modeling techniques that have been applied for various input-output combinations and levels of water resources systems. Application of systems modeling techniques used to solve various water resources quantity/quality

problems are further summarized in tables 2 and 3. (References on the use of mathematical models in analyzing water resources systems are organized according to types of models in the bibliography.)

EXAMPLES OF PLANNING MODEL

Three examples of water resources systems models (developed at Utah State University) were selected to demonstrate the large variability in resolution of input and output as the scope of the problem varies, and to indicate the interface between models of various scopes within a common geographic region. The three example problems are a local, single-purpose (municipal water supply) planning problem; a regional (Salt Lake County), multipurpose planning problem; and a statewide (Utah) water resource allocation problem. The relationship of the three problems is shown in Figure 7.

A complete description of the three problems and resulting models is not possible here because of space limitations; however, a limited description is presented to give a general idea of the approach used and results obtained. Table 4 compares certain characteristics of the three models.

Local, Single-Purpose Development: Municipal Water Supply Planning Using Mixed Integer Programming (MIP)

This type of model has been applied to the water supply planning problem for Bountiful City, Utah, and also to a hypothetical municipal problem (Hughes, 1972). The hypothetical problem was summarized by Hughes and Clyde (1973) and that paper is quoted freely in the following brief description of the model. This single-purpose, multiple source problem is presented as an example of a relatively finely scaled model that includes consideration of individual facilities, their seasonal variation in load factors, and uncertainty in both supply and demand.

General problem structure – A planner for an urban water supply utility is faced with what is basically a transportation problem. The system has several potential water sources, related water supply facilities, and several service zones. The problem is to determine the least cost way of providing the desired quality of service. In mathematical programming terms, the question is: What level of flow from source i should be delivered to destination j for all possible i and j ?

The problem can be expressed as a mixed integer programming (MIP) problem with J integer decision variables, $K-J$ continuous decision variables, and I constraints as:

$$\text{Minimize: } \sum_{j=1}^J C_j X_j + \sum_{j=J+1}^K \hat{C}_j Y_j$$

$$\text{Subject to: } \sum_{j=1}^J \hat{A}_{ij} X_j + \sum_{j=J+1}^K A_{ij} Y_j \leq B_i; i = 1, 2, \dots, I$$

$$X_j \text{ integer } (j = 1, 2, \dots, J); X_j \geq 0; Y_j \geq 0$$

in which X_j is an interger variable associated with a particular type and capacity of physical facility; Y_j is a continuous variable indicating production level during a given time period; C_j is the annual fixed cost (capital investment cost) of facility X_j ; \hat{C}_j is the unit cost (operating cost) of producing water with facility X_j at operating level Y_j ; B_i is a constant indicating demand or capacity limitation; A_{ij} is generally a facility or source capacity; and \hat{A}_{ij} is a technical coefficient (0 or 1) that determines whether the related Y_j is active in constraint i .

The MIP formulation of the problem provides several important capabilities a strictly LP structure lacks. For example, MIP can handle the problem of selecting a particular facility capacity from among several fixed standard sizes; for example,

$$Y \leq A_1 \text{ or } A_2 \text{ or } \dots, A_j$$

where Y represents actual flow in a pipeline (or production of a well or treatment plant) and the A_j terms represent the capacity of a standard pipe size to be considered. The problem can be restated as

$$Y \leq A_1 X_1 + A_2 X_2 + \dots + A_J X_J; \sum X_j = 1; X_j = \text{integer.}$$

The X_j terms represent a particular diameter of pipe and can therefore be associated with the capital investment for that size pipeline and the Y variable can be associated with the operational cost of supplying water. Only one X_j can assume a positive value and that level is fixed at unity. The solution to this problem (after adding other constraints that require a minimum demand to be satisfied and a least cost objective function) will yield the optimum pipe diameter. The difficulty with LP models is to select a single unit cost for each variable before anything is known about capacity or use factors that will occur.

The computer program used in this research is called BBMIP (Branch-Bound mixed integer programming). It was written for IBM by Shreshian (1969) and is based on the Branch-Bound concept first developed by Land and Doig (1960). This algorithm was selected because it was available to the writers in an easily usable form. Many other MIP algorithms have been developed and the reader is referred to the recent state-of-the-art survey by Geoffrion and Marsten (1972) for a description and comparison of several other MIP algorithms.

Model application— The system has two service zones and three possible sources of water (fig. 8). The ground water is brackish and must be blended with equal parts of water from either of the other sources.

The expected values of demand are given in figure 9 along with the four discrete seasonal levels by which average demand is represented in the model. The model uses these average seasonal levels of demand to compute average seasonal operating costs. The level of capital investment, however, is determined by the probability of daily peaks during season 2. The stochastic supply hydrograph is not shown.

Table 5 summarizes the purpose of each group of constraints and table 6 shows the complete MIP model.

The variables and notation used are not defined here. They include integer variables representing three alternate sizes of conduit, three sizes of treatment plants, the number of wells and booster stations, and two sizes of storage reservoirs. A separate continuous variable is associated with the use level of each type of facility described above for each season of the year and with reservoir storage in both average and critical years.

The approach used to handle uncertainty is a type of chance constrained programming for normally distributed random variables developed by Charnes and Cooper (1963). It consists of specifying the acceptable frequency for a constraint not to be met (in this case, the frequency of a water shortage). For example, if $AX \geq B$ must hold with at least α probability, a single constraint would be

$$\text{Probability} \left(\sum_j A_{ij} X_j \geq B_i \right) \geq \alpha$$

where $1 - \alpha$ is the risk level and the uncertainty is present in B_i . If B_i (supply or demand) is normally distributed with expected value $E(B_i)$ and standard deviation σ_{B_i} , the procedure is to normalize the random variables. The constraint is then

$$\text{Probability} \frac{\sum_j A_{ij} X_j - E(B_i)}{\sigma_{B_i}} \geq \frac{B_i - E(B_i)}{\sigma_{B_i}} \geq \alpha.$$

But the right-hand side is distributed as the standard normal variate with a mean of 0 and a standard deviation of unity. The following constraint will then be satisfied at α probability:

$$\sum_j A_{ij} X_j \geq K_\alpha \sigma_{B_i} + E(B_i)$$

in which K_α is the value of the standard normal at α probability level. The right-hand-side terms are now all constants that allow use of the constraint in MIP format. For distributions other than normal, procedures such as linearizing the data are useful as described by Hughes (1972).

Results of model applications— The optimal solution gives selected capacity for each type of facility. The continuous variable activity levels also indicate optimum operating rules. For example, analysis of the sample problem solution revealed the following rules for minimizing costs (refer to figs. 8 and 9):

1. Use the imported water at the full conduit capacity except when this source alone exceeds demand.
2. Use local ground water to supplement imported water as demand requires to the maximum possible extent within the dilution constraints except in service zone 2 during seasons 2 and 3.

3. During seasons 2 and 3, treated water has a smaller variable cost than ground water delivered to zone 2. In all other situations, treated water production should be limited to that required to supplement the other two sources.

To regulate reservoir spills as an operator would in the face of hydrologic uncertainty (i.e., to maximize reservoir storage), a small cost penalty was applied to spills that decreased as seasons progressed.

An important side effect of this introduction of nonequal costs in place of the original zero cost coefficients was to dramatically reduce the number of iterations required to establish optimality. This is logical since, with an excess of water available to the reservoir, there is an infinite number of ways to schedule spills during the various seasons and therefore an infinite number of optimal solutions when no priority of costs is associated with these spills.

Regional Multipurpose Planning Using the Transportation Problem Form of Linear Programming (LP)

Several papers by Bishop and others develop a systems approach to regional water resource planning which links water supply and waste water treatment by means of the recycling concept. This approach departs from the traditional concept of planning to satisfy water demand and then, as a separate function, planning to dispose of waste water. Increasing water demands coupled with the projected large increases in quality standards for waste water return flows have combined to make recycling of water economically feasible. This situation has justified the type of model described here which treats return flows from any type of water use as a potential source of supply for the same use (recycling) or for a different use (sequential reuse). This model concept and the example problem are taken from Bishop and Hendricks (1971).

The components of the water resource system are shown in the matrix of figure 10, including both sources of supply and demand requirements. The water supply sources are indicated by row headings, and each origin of water is classified as (1) primary or base supply, (2) secondary or effluent supply, or (3) supplementary or imported supply. Each row represents a different possible origin of supply. The system of water users, indicated by the column headings in figure 10, are grouped into the broad sectors of municipal, industrial, agriculture demand, or other. Both the sectors of water use, the columns, the supply categories, and the rows can be specified to any degree of refinement desired.

In the context of broad system planning, the matrix of water supply sources and demand sector requirements depicts all possible combinations for satisfying the aggregate system demand with the aggregate available supply. Thus, each element in the matrix represents a possible means of satisfying all or part of the demand requirements of a sector with all or part of the water from a given source.

The problem of allocating water from various sources or origins to required points of use or destinations is closely related to the classical "transportation problem" (Gass, 1964). In the general transportation problem, a homogeneous product is available in the amounts a_1, a_2, \dots, a_m from each of m shipping origins and is required in amounts b_1, b_2, \dots, b_n by each of n shipping

destinations. The term x_{ij} represents the amount to be shipped from the i th origin to the j th destination. The cost of shipping a unit amount from the i th origin to the j th destination is c_{ij} , a constant, and must be known for all combinations. The structure of the problem is shown in figure 11.

The problem is to determine the amounts x_{ij} to be shipped over all routes so as to minimize costs. The mathematical statement of the transportation problem is to find values for the variables x_{ij} which minimize the total cost, TC :

$$TC = \sum_{i=1}^m \sum_{j=1}^n c_{ij}x_{ij}$$

subject to the constraints

$$\sum_{j=1}^n x_{ij} = a_i; \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j; \quad j = 1, 2, \dots, n$$

and each $x_{ij} \geq 0$.

Water reuse planning as a transportation problem— The format of the transportation problem is well adapted to the problem of water reuse planning. Water from several origins or categories of supply must be transported to various destinations or sectors of use at minimum cost. Since the effluent from a sector that is not consumptively used can be made available for reuse in the system, sectors of use (column vectors) also become origins of secondary supply (row vectors). Thus a waste treatment plant may be the destination of municipal effluent, while, at the same time, it becomes an origin for treated waste water available for reuse.

The costs incurred in allocating water from any origin to any destination depend on the water quality of the source, the quality requirement at the destination, and the facilities required to transport and to deliver the water from origin to destination. The sum of these components represents the cost, c_{ij} , between each origin and destination. The cost function guarantees that the quality constraints are fulfilled along with the quantity requirements.

Summary of model structure— The model for the general transportation problem for water reuse is summarized in figure 12. The structure of constraint equations and the types of availability and requirements are indicated for each of the water supply categories and use sectors. The special conditions for system balance and blending ratios are also specified.

Model application to Salt Lake County problem— The water reuse planning model has been applied to the Salt Lake County (the Lower Jordan River Basin) problem at several different levels of resolution. In a recent report (Bishop *et al.*, 1974), the basin was divided into seven subbasins.

This is the sort of detail necessary to produce solutions of value in the real world planning situation. Because of space limitations, however, the application described here (Bishop *et al.*, 1971) considers the basin a single geographic unit.

Figure 13 shows the source/use and alternatives. The numbers in the interior matrix are the unit costs of each source/use combination. The \$1000 costs are used to remove particular combinations from consideration. This compact matrix form derived from the transportation format is very convenient for displaying various model parameters. The tableau contrasts sharply, for example, with the expanded sparse matrixes of the regional, multipurpose planning problem and the statewide allocation problem. Figure 13 displays the unit costs, but the same tableau form can be used to display many other parameters such as water quality (required changes in BOD or in total dissolved solids for each source/use combination, for example) or activity variable levels of various solutions.

Application of the model requires supply and demand estimates for each time horizon of interest. The primary supply quantities do not change with time but return flows increase directly with demand requirements. Optimal solutions for each desired point in time are obtained by simply changing the constants in the last row and column and the unit costs.

Results of model application— The optimal activity levels for 1980, as one example, are shown in figure 14. Figure 15 is a flow diagram representing this solution.

The 1980 tableau shows a total use that is about twice the total primary supply. This ratio indicates that unless substantial amounts of water are imported into the basin (an alternative not included in this particular form of the model) substantial amounts of recycling will be required. In fact, significant amounts of agricultural water are being recycled at present.

Results for years 2000 and 2020 were also obtained but the figures are not shown here. Some useful aspects of the reuse model and the information it provides are as follows:

1. The optimum sequential and recycle reuse allocation from the primary and secondary sources of supply to satisfy user requirements.
2. Given a constant water supply and the projections of increased future demands, the solution points up the types of treatment process for water reuse to meet demands at the least cost.
3. The least cost allocation indicates the required capacities for treatment facilities and through parametric analysis the time at which they should be phased into the system.

In situations when a system presently includes treatment operations of given capacities, or when particular treatment facilities of certain capacities are proposed, an evaluation of optimal water reuse can be made by entering treatment operations with constraints less than or equal to specified plant capacities.

Statewide (Utah) Water Resource Allocation

Utah is generally considered to be an area of chronic water shortage. Production on approximately 2/3 of its irrigated land is limited by a partial water supply (usually early season high

runoff). There are also three million additional acres of land that could be added to agricultural production if water were available. Despite this situation, some two million acres of marshes, mud flats, and valley bottoms suffer from an excess of water and a large percent of Utah's share of Colorado River water continues to flow from the state unused. It is apparent that, despite the arid climate in many valleys, the real water resource problem is that of maldistribution of supply both seasonally and geographically rather than actual shortage on a statewide basis. A statewide water allocation model that addresses the question of economic implications of various water transfer policies is therefore an extremely valuable tool.

The approach to such a model in Utah has been a linear programming model. The LP approach is virtually dictated by the size of the problem. The initial model had a least cost for satisfying assumed fixed requirements. It included 340 decision variables and 205 constraints (King *et al.*, 1972). The model has since been expanded to incorporate a maximum net benefit objective function. The maximum parameter is agricultural production. Municipal and industrial (MI) and wetland requirements are still assumed as fixed. This change from a least cost to a maximum agricultural net benefit model required an increase in the model size by an order of magnitude. It now has over 4500 variables and 2100 constraints (Keith *et al.*, 1973).

Model description— The model shows the state as consisting of 10 origins and destinations for water (see fig. 17). The LP matrix is much too large to reproduce graphically. Figure 16 summarizes the types of constraints required.

Supply portion of model— The upper left portion of the figure consists essentially of the hydrographic, or supply model (the King 1972 model). This portion of the model defines the sustained yield available from surface and ground water in river basin. The cost of reservoir storage to ensure this yield at a given probability level and the cost of a reasonable amount of artificial ground water recharge is included in the model.

Interbasin transfer possibilities are incorporated to the extent necessary to analyze particular import proposals such as the Central Utah Project.

Parametric analysis of the supply model produces shadow price mappings for a particular region (fig. 17). The figure indicates variation in the marginal cost of supplying water as both MI and agricultural diversion vary. If MI use is assumed fixed at a particular level, an actual supply function for agricultural diversions such as the example in figure 18 can be developed. Since some 90 percent of water diversions in Utah are for agricultural purposes, such supply functions yield valuable planning information (particularly when linked to the demand functions described later).

Demand portion of model— The lower portion of figure 16 represents the demand portion of the model. The MI and wetlands constraints are very simple because requirements are fixed and water is allocated to these uses in a least cost manner, with the residual being allocated to agricultural production to the extent profitable. The agricultural demand portion of the model is extremely large. For each region, both existing and new agricultural land was classified according to type of soil and these classes were matched against productivity for six different crops. The constraints are further expanded by the fact that crop rotations are required, and some crops require more than one year to bring into production.

Results— The model has been run with several variations of constraints in regard to such things as inflow to Great Salt Lake and wetland requirements. These various system outflow constraints yield interesting information about the economically justifiable investment timing of importation plans such as the Central Utah Project. For example, figure 19 indicates that with present average inflow to Great Salt Lake and with some restriction of wetland inflow (salvage), use of Central Utah Project Water is limited to 30 percent of the Bonneville unit capacity until about the year 2010.

Parametric analysis of the model yields demand functions for each basin such as illustrated in figure 20.

SUMMARY

The field of systems engineering applications to water resource problems has been and still is experiencing an information explosion. Engineers at hundreds of different universities, government agencies, and private firms are applying numerous types of operations research tools to many different types and sizes of water resource problems.

The size and complexity of the models vary greatly with the purpose as well as the geographic scope of the problem. As shown in figure 2, for example, the expanded versions of the least cost Salt Lake County model have over 2000 variables, while the statewide least cost model has only 340 variables. Also, the single-purpose, municipal problem has a larger two-dimensional matrix (46×57) than the initial aggregated form of the Salt Lake County model (99×20). The relatively large size of the municipal model is caused by the resolution desired. The number of both variables and constraints in the municipal model was increased essentially by a factor of 4 when the year was divided into four seasons. Adding the uncertain capability to this model increased the number of variables by 24 percent and the number of constraints by 17 percent.

A generalization that can be made in regard to objective functions is that least cost objectives are inherently much simpler to model than are maximum benefit objectives because a least cost model must assume fixed (perfectly inelastic) demands. A net benefit model, however, must go through the rather complex exercise of matching marginal revenue and cost. For the statewide water resource allocation problem this required an order-of-magnitude increase in the number of both variables and constraints even though only one of the water use sectors (agriculture) was modeled in the net benefit manner.

Linear programming, interger programming, and even some dynamic programming algorithms are now routinely used to obtain optimal solutions to water resource problems. However, the water resource planning models that use these algorithms are often still developed on an ad hoc basis for each new application. The reasons for this apparently inefficient, fragmented approach are many. Certain types of problems such as municipal water supply planning have many common aspects and one would expect that a single generalized model could be developed which could then be applied to almost any planning problems within a particular category. Although this may be done eventually, most researchers have apparently found enough differences in the system structure, scope, or relative importance of particular parameters (or perhaps simply bias due to the researcher's background) to justify developing many related but not identical models.

FURTHER READING

Systems: Concepts, Methods, and Approach

- Affel, H. A., Jr. 1964. Systems Engineering. *International Science and Technology*, November.
- Buras, N. 1972. *Scientific Allocation of Water Resources*. American Elsevier Publishing Company, Inc., New York.
- Charnes, A., and W. W. Cooper. 1963. Deterministic Equivalents for Optimizing and Satisfying under Chance Constraints. *Operations Research*, II:18–39.
- Chestnut, H. 1967. *Systems Engineering Methods*. John Wiley & Sons, New York.
- Churchman, C. W. 1968. *The Systems Approach*. Delacorte Press, New York.
- de Lucia, R. J., H. A. Thomas, Jr., P. P. Rogers, M. B. Fiering, and R. P. Burden. 1971. *Systems Analysis in Water Resources Planning*. National Water Commission Report NWC-EES-71-018, Arlington, Virginia, July.
- de Neufville, R., J. Schaake, Jr., and J. H. Stafford. 1971. Systems Analysis of Water Distribution Networks. *Journal of the Sanitary Engineering Division, ASCE*, 97(SA6):825–842. Proc. Paper 8575, December.
- de Neufville, R., and J. A. Stafford. 1971. *Systems Analysis for Engineers and Managers*. McGraw-Hill, New York.
- de Neufville, R., and D. H. Marks, eds. 1973. *Systems Planning and Designs. A Reader of Case Studies*. Wiley & Sons, New York.
- Gass, Saul I. 1964. *Linear Programming – Methods and Applications*, 2nd ed. McGraw-Hill, New York.
- Hall, A. D. 1962. *A Methodology for Systems Engineering*. Van Nostrand, Princeton, New Jersey.
- Hall, W. A., and J. A. Dracup. 1970. *Water Resource Systems Engineering*. McGraw-Hill Book Co., Inc., New York.
- Hare, V. C., Jr. 1967. *Systems Analysis: A Diagnostic Approach*. Harcourt, Brace & World, New York.
- Kim, N. J. 1968. *Linear Programming with Random Requirements*. Utah Water Research Laboratory Report PRWG42-2T, Utah State University, Logan, Utah.
- Lasdon, L. S. 1970. *Optimization Theory for Large Systems*. Macmillan Company, New York.
- Land, A. H., and A. C. Doig. 1960. An Automatic Method for Solving Discrete Programming Problems. *Econometrica*, 28(3):497–520.
- McGraw, D., and Y. Y. Haines. 1974. A Parametric Approach to the Integrated System Identification and Optimization Problems. *Journal of Optimization Theory Appl.* 13(5):582–605.

Application of Mathematical Techniques

Linear Programming

- Anderson, Mark H., Jay C. Andersen, John E. Keith, and Calvin G. Clyde. 1973. *The Demand for Agricultural Water in Utah*. Utah Water Research Laboratory Report PRWG100-4. Utah State University, Logan, Utah.

- Bishop, A. B., and D. W. Hendricks. 1971. Water Reuse Systems Analysis. Journal of the Sanitary Engineering Division, ASCE, 97(SA1):41-57. Proc. Paper 7898, February.
- Bishop, A. B., D. W. Hendricks, and J. H. Milligan. 1971. Assessment Analysis for Water Supply Alternatives. Water Resources Bulletin, 7(3), June.
- Bishop, A. B., William J. Grenney, Rengasan Narayanan, and Stanley L. Klemetson. 1974. Evaluating Water Reuse Alternatives in Water Resources Planning. Utah Water Research Laboratory Report PRWG123-1. Utah State University, Logan, Utah.
- Clausen, George Samuel. 1970. Optimal Operation of Water-Supply Systems. Technical Report No. 1, Technical Reports on Hydrology and Water Resources. The University of Arizona, Tucson, Arizona.
- Clyde, C. G., B. C. Jensen, and J. H. Milligan. 1967. Optimizing Conjunctive Use of Surface Water and Groundwater. Proceedings, Symposium on Groundwater Development in Arid Basin, Utah State University, Logan, Utah.
- Clyde, C. G., Alton B. King, and J. C. Andersen. 1971. Application of Operations Research Techniques for Allocation of Water Resources in Utah. Utah Water Research Laboratory Report PRWG73-2. Utah State University, Logan, Utah.
- DeVries, R. M. 1968. An Application of Optimization in Planning the Use of Multiple Water Sources that Supply Municipal Water Demands. Unpublished Ph.D. dissertation. Utah State University, Logan, Utah.
- Dracup, J. A. 1966. The Optimum Use of a Groundwater and a Surface Water System: A Parametric Linear Programming Approach. Contribution No. 107, University of California, Los Angeles, California.
- deLucia, Russell J., *et al.* 1971. Systems Analysis in Water Resources Planning. Report No. PB 204 374. Meta Systems, Inc., Cambridge, Massachusetts.
- Drobny, N. L. 1971. Linear Programming Applications in Water Resources. Water Resources Bulletin, 7(6):1180-1193.
- Galler, W. S., and J. B. Gotaas. 1966. Optimization Analysis for Biological Filter Design. Journal of the Sanitary Engineering Division, ASCE, 92(SA1):163-182.
- Joeres, Erhard F., Jon C. Liebman, and Charles S. Revelle. 1971. Operating Rules for Joint Operation of Raw Water Sources. Reprint No. 1487, 7(2):225-235. University of Wisconsin, Madison, Wisconsin.
- Keith, John E., Jay C. Andersen, and Calvin G. Clyde. 1973. The Economic Efficiency of Inter-Basin Agricultural Water Transfers in Utah: A Mathematical Programming Approach. Utah Water Research Laboratory Report PRWG100-3. Utah State University, Logan, Utah.
- King, Alton B., Jay C. Andersen, Calvin G. Clyde, and Daniel H. Hoggan. 1972. Development of Regional Supply Functions and a Least-Cost Model for Allocating Water Resources in Utah: A Parametric Linear Programming Approach. Utah Water Research Laboratory Report PRWG100-2. Utah State University, Logan, Utah.
- Milligan, J. H., Jay C. Andersen, and C. G. Clyde. Colorado River Allocation Study. Utah Water Research Laboratory, Logan, Utah.
- Revelle, C. S., D. P. Loucks, and W. R. Lynn. 1968. Linear Programming Applied to Water Quality Management, Water Resources Research, 4(1):1-9.

Stephenson, D. 1970. Optimum Design of Complex Water Resources Projects. Journal of the Hydraulics Division, ASCE, 96(HY6):1229–1246. Proc. Paper 7320, June.

Thomas, H. A., Jr., and R. P. Burden. 1965. Indus River Basin Studies. Harvard University, Cambridge, Massachusetts. (Mimeographed)

Thomasin, R. V. 1965. Recent Results from a Mathematical Model of Water Pollution Control in the Delaware Estuary. Water Resources Research, 1(3):349–359.

Young, G. K. 1968. Reservoir Management: The Trade-off Between Low Flow Regulation and Flood Control. Water Resources Research, 4(3):507–511.

Dynamic Programming

Buras, N. 1963. Conjunctive Operation of Dams and Aquifers. Journal of Hydraulic Division, ASCE, 89(HY6):111–131.

Buras, N. 1965. A Three Dimensional Optimization Problem in Water Resources Engineering. Operation Research Quarterly, 16(4):419–427.

Butcher, W. S., Y. Y. Haimes, and W. A. Hall. 1969. Dynamic Programming for the Optimal Sequencing of Water Supply Projects. Water Resources Res., 5:1196.

Hall, W. A., W. S. Butcher, and A. Esogbue. 1968. Optimization of the Operation of a Multiple-Purpose Reservoir by Dynamic Programming. Water Resources Research, 4(3):471–477.

Hall, W. A., and D. T. Howell. 1963. The Optimization of Single-Purpose Reservoir Design with the Application of Dynamic Programming to Synthetic Hydrology Samples. Journal of Hydraulic Division, ASCE, 1:355–363.

Hall, W. A., and T. G. Roefs. 1966. Hydropower Project Output Optimization. Journal of Hydraulic Division, ASCE, 9(PO1):67–79.

Hall, W. A., and R. W. Shephard. 1967. Optimum Operations for Planning of a Complex Water Resources System. Water Resources Center Contribution No. 122. University of California, Los Angeles, California.

Liebman, Jon C., and Walter R. Lynn. 1966. The Optimal Allocation of Stream Dissolved Oxygen. Water Resources Research, 2(3):581–591.

Parikh, S. C. 1966. Linear Dynamic Decomposition Programming of Optimal Long-Range Operation of a Multiple Multi-Purpose Reservoir System. Proc. Fourth Inst. Conf. Operations Res., Boston, Massachusetts, September.

Nonlinear Programming

Bayer, M. B. 1974. Non-linear Programming in River Basin Modeling. Water Resources Bulletin, 10(2):311–317.

Higgins, John M., and Daniel A. Okun. 1972. Regional Development of Public Water Supply Systems. Water Resources Research Institute Report No. 72. University of North Carolina, Chapel Hill, N. C.

Hughes, T. C. 1972. A Mixed Integer Programming Approach to Planning Multiple Water Sources for Municipal Water Supply. Utah Water Research Laboratory Report PRWG73-3. Utah State University, Logan, Utah.

- Hughes, T. C., and Calvin G. Clyde. 1973. Municipal Water Planning – Mixed Integer Approach. *Journal of the Hydraulics Division*, HY11:2079–2095. November.
- Lauria, D. T. 1972. Water Supply Planning by Mixed Integer Programming. ESE Publication No. 298, Department of Environmental Sciences and Engineering, University of North Carolina, Chapel Hill, N. C.
- Liebman, J. C., and D. H. Marks. 1968. Balas' Algorithm for Zoned Uniform Treatment. *Journal of the Sanitary Engineering Division*, ASCE, Proc. Paper 6059, 96(SE4):585–593.
- Mulvihill, M. E., and J. A. Dracup. 1974. Optimal Timing and Sizing of a Conjunctive Urban Water Supply and Waste Water System with Non-linear Programming. *Water Resources Research*, 10(2):170–175.
- Shih, C. S., and W. L. Meir, Jr., 1972. Integrated Management of Quantity and Quality of Urban Water Resources. *Water Resources Bulletin*, 8(5):1006.
- Steiner, P. O. 1959. Choosing Among Alternative Public Investments in the Water Resources Field. *American Economic Review*, 69(5):916.
- Walker, W. R., and G. V. Skogerboe. 1973. The Impact of Water Quality Objectives of Urban Waste Water Supply Planning. *Water Resources Bulletin*, 9(5):861.
- Woolsey, R. E. D. 1969. An Application of Integer Programming to Optimal Water Resource Allocation for the Delaware River Basin. Unpublished Ph.D. dissertation. University of Texas, Austin, Texas.

Simulation

- Computer Application to System Analysis Lower Mekong River: Description of Streamflow Synthesis and Reservoir Regulation Program. August, 1967. U.S. Army, Engineering Division, North Pacific, Portland, Oregon. Computer Program No. 24-K3-G001.
- Goodman, A. S., and W. E. Dobbins. 1966. Mathematical Model for Water Pollution Control Studies. *Journal of the Sanitary Engineering Division*, ASCE, 92(SA6):1–19. Proc. Paper 5031, December.
- Grenney, W. J., and A. L. Huber. 1974. Mathematical Model Used for Water Quality Management in a Waste Load Allocation Study of Bear River. Utah Water Research Laboratory, Logan, Utah.
- Hill, R. W., E. K. Israelsen, A. L. Huber, and J. P. Riley. 1970. A Hydrologic Model of the Bear River Basin. Utah Water Research Laboratory, Logan, Utah.
- Hill, Robert W., Eugene K. Israelsen, and J. Paul Riley. 1973. Computer Simulation of the Hydrologic and Salinity Flow Systems Within the Bear River Basin. Utah Water Research Laboratory Report PRWG104-1. Utah State University, Logan, Utah.
- Hufschmidt, M. M., and M. B. Fiering. 1966. *Simulation Techniques for Design of Water-Resource Systems*. Harvard University Press, Cambridge, Massachusetts.
- Longenbaugh, R. A. 1967. Mathematical Simulation of a Stream-Aquifer System. Report CER 67-68 AL22, Department of Civil Engineering, Colorado State University, Ft. Collins, Colo.

Moreau, David H., Karl Elfers, Gilbert S. Nicholson, and Kuniyoshi Takeuchi. 1973. Regional Water Resource Planning for Urban Needs: Part I. Water Resources Research Institute Report No. 77. University of North Carolina, Chapel Hill, N. C.

Morn'ee, H. A. W., and W. N. Allan. 1959. Planning for the Ultimate Hydraulic Development of the Nile Valley. Proc. Inst. of Civil Engineering, 104:101-156.

Report on the Use of Electronic Computers for Integrating Reservoir Operations, Vol. 1. 1957. DATAmatic Corp. Prepared in cooperation with Raytheon Manufacturing Co. for U.S. Army Corps of Engineers. Missouri River Division.

Tedrow, A. C. Planning for the Oswego Basin in New York - A Systems Approach. In Natural Resources.

Multilevel Optimization

Haines, Y. Y. 1971. Modeling and Control of the Pollution of Water Resources Systems via the Multi-level Approach. Water Resources Bulletin, 7(1):104-112.

Haines, Y. Y., and W. S. Nainis. 1974. Coordination of Regional Water Resource Supply and Demand Planning Models. Water Resources Research. To appear.

Haines, Y. Y., R. L. Perrine, and D. A. Wismer. 1968. Identification of Aquifer Parameters by Decomposition and Multi-level Optimization. Contribution No. 123, Water Resources Center, University of California, Los Angeles, California.

Windsor, J. S., and V. T. Chow. 1972. Multi-reservoir Optimization Model. Journal of Hydraulics Division, ASCE, 98(HY10):1827-1845.

General Related Literature

Goeffrion, A. M., and R. E. Marsten. 1972. Integer Programming: A Framework and State-of-the-Art Survey. Management Science, 18(9):465-491.

Shreshian, R. 1969. Branch and Bound Mixed Integer Programming - BBMIP 360 D-15.2.005. Contributed Program Library, IBM, Program Information Department, Hawthorne, N. Y.

TABLE 1.—CHARACTERISTIC OF WATER RESOURCES SYSTEMS

<u>Objectives</u>	<u>Scope</u>	<u>Space</u>	<u>Time</u>
Single/multiple objectives	Single/multiple sectors or users	Single/multiple basin	Short/long term action
Economic	Municipal	River-basin	Investment
Environmental	Industrial	Subbasins	Construction
(Social)	Agricultural	Regions Urban areas	Operation
(Regional)	Aesthetic Recreational Fish/wildlife Hydropower, etc.		Maintenance

TABLE 2.— APPLICATIONS OF OPTIMIZATION MODELS IN WATER RESOURCES ALLOCATION

Geographical setting	Objective	Mathematical technique
Local	Least cost combination of available water supply sources for municipal uses and aquifer recharge	Linear
Local	Least cost of water supply system from available water sources	Mixed interger
Local	Least cost combination of unit process to remove a given amount of BOD	Linear
Regional	Least cost combination of available water resources to satisfy municipal, agriculture, and industrial requirements	Linear
Regional	Least cost alternatives to satisfy municipal and industrial water requirements	Nonlinear
Regional	Optional allocation policy of surface and ground water sources	Dynamic
Regional	Optional operating policy for system of reservoirs to maximize net benefits	Dynamic
Regional	Least cost combination of water treatment strategies to achieve desirable river quality standard	Multilevel
Statewide	Maximize net profit of agricultural rector at given municipal, industrial, and wetland requirement	Linear
River basin	Least cost sequencing of capacity expansion of water supply system	Dynamic
River basin	Maximization of hydroelectric power benefit given, flood control, water supply, navigation and recreation requirement	Simulation

TABLE 3.—APPLICATIONS OF OPTIMIZATION MODELS IN WATER QUALITY MANAGEMENT

Geographical setting	Objective	Mathematical technique
In-plant	Least cost combination of unit processes to remove a given amount of BOD	Linear
Local	Stage development over time of waste water treatment systems	Linear
Regional	Least cost of waste water collection and treatment and staging of construction for a region	Linear
In-plant	Least cost combination of inputs to production function to remove BOD	Nonlinear
Regional	Least cost regional waste water planning	Nonlinear
Local	Sequential capacity expansion of plants	Dynamic
Local	Multistage capacity expansion of water treatment systems	Dynamic
In-plant	Least cost combinations of unit processes to remove a given amount of BOD	Dynamic
In-plant	Serial multistage system of industrial waste treatment for BOD	Dynamic
Local	Minimum total annual cost to meet given treatment requirements	Dynamic
Regional	Sequencing of water supply projects to meet capacity requirements over time	Dynamic
Regional	Capacity expansion of large multilocation waste water treatment systems	Approximate & incomplete dynamic
River basin	Least cost selection of treatment levels to meet river quality standards using zones of uniform treatment level	Integer
Regional	Minimization of overall regional treatment costs to meet desired river quality standards. Determination of effluent charge pricing level	Nonlinear decomposition & multilevel approach

TABLE 4.— SUMMARY OF EXAMPLE PROBLEM COMPARISONS

	Problem No. 1 Local water supply source selection	Problem No. 2 Regional multipurpose model (Salt Lake County)	Problem No. 3 Statewide allocation model (Utah)
Water use sectors			
M&I	Yes	Yes	Yes
Agriculture	No	Yes	Yes
Recreation	No	Yes	Yes
Power	No	No	Yes
Required outflow	No	Yes	Yes (various assumed constraints)
Source of supply			
Primary:			
Local surface flow	Yes	Yes	Yes
Groundwater	Yes	Yes	Yes
Secondary:			
Municipal effluent	No	Yes	} Yes (combined)
Industrial waste	No	Yes	
Ag. return flow	No	Yes	
Supplementary:			
Imported water	Yes	Yes { added in	Yes
Desalinization	No	Yes { later model	No
Evp. suppression	No	No	No
Phreatophyte control	No	No	No
Art. recharge	Yes ¹	No	Yes ¹
Type and size of model			
Optimizing algorithm	Mixed interter programming	Transportation form of LP	Linear Programming
Objective function	Least cost	Least cost	Initially least cost, then maxi- mized net benefit to ag. production
Number of decision variables	46	99 to 2000	340 to 4500
Number of constraints	57	20 to 200	205 to 2100

TABLE 4.— SUMMARY OF EXAMPLE PROBLEM COMPARISONS — Continued

Type and size of model (cont.) Parametric analysis	No (except for interim solutions printed during search for optimal)	Yes	Yes
Resolution of model Spatial: (a) Supply facilities	Defines individual treatment, pumping & storage facilities.	Only large regional treatment plants and defined individually—others aggregated by service area.	No individual projects defined except central Utah importation. Entire basin supply treated as if at single point.
(b) Demand components	Service areas defined by elevation zones.	Region divided into subareas with average elevation assigned to each, but not individual cities.	State divided into 10 hydrologic regions (river basins). Entire basin demand treated as if at single point.
Temporal:	Four seasons of year plus peak day for both supply & demand.	Annual supply/demand only (model now being expanded to include seasonal peaks).	Annual supply/demand only.
Uncertainty Supply:	Probability of low flows defined per historic hydrologic data.	No—historic averages used	No—model is deterministic, however, yield probability for each basin was analyzed prior to selecting supply levels.
Demand:	Probability of high demands defined by historic use data	No—average annual use per person projected	No—estimated average annual requirements used.
Unit costs	Separation of fixed and variable costs allowing O&M costs to vary seasonally per associated use rates.	Single cost per unit volume of water represents average facility capacity and use factor; adjusted manually for after computer run for capacities greatly different than basis.	Single cost per unit volume of water represents average facility capacity and use factor; adjusted manually after computer run for capacities greatly different than basis.

TABLE 4.— SUMMARY OF EXAMPLE PROBLEM COMPARISONS — Concluded

Method of selecting range of capacities of new facilities	Several discrete alternatives included in model.	Varies—certain facilities fixed manually, others completely open to objective function.	Varies—certain facilities fixed manually, others completely open to objective function.
Reservoir storage simulation	Includes rough simulation by seasonal inflow, outflow, & evaporation continuity equation for both average and peak years.	Individual reservoirs not identified. Average annual yield assumed.	Reservoirs not identified. Storage requirement is estimated exogeneous to model.
Price elasticity of demand	Model assumes previous price levels will continue ∴ historic requirements are projected into future.	Model assumes previous price levels will continue ∴ historic requirements are projected into future.	Demand functions are developed for agricultural production only, M&I requirement is assumed fixed (perfectly inelastic).

¹ Fixed exogenous to model.

TABLE 5.— CONSTRAINT SUMMARY

Rows (1)	Purpose (2)
1-8	Supply \geq Demand (expected values)
9-26	Use \leq Developed facility capacity (expected values)
27-34	Ground-water dilution
35-39	Reservoir storage \leq Capacity developed
40-43	Reservoir spill-Carryover constraints (expected values)
44-57	Stochastic constraints

Levels:

Coordination of sub-basin systems and subbasin water transfers, operation of mainstem facilities

Operation of subbasin systems of dams, aqueducts, pumping stations, and treatment plants, system supplies

Operation of individual local systems, distribution networks, treatment plants, user demands

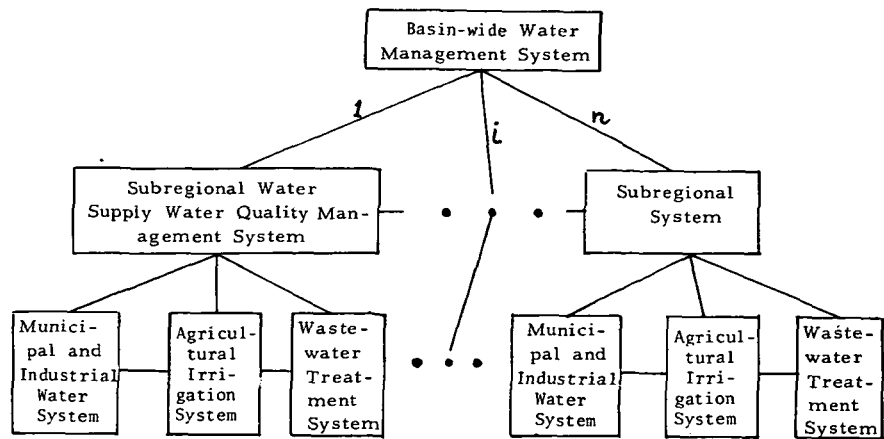


Figure 1.— Hierarchical structure of water resources system.

Planning Scope	Configuration of alternatives (single/multi-sectors)	
	Structural (physical)	Nonstructural (policy)
Descriptive characteristics		
Staging over time	construction/operation	phasing and implementation
Spatial features	location and scale	points of intervention in social, political or legal system
Operational arrangements	operating rules	administrative or organizational arrangements

Figure 2.— Considerations in the formulation of alternatives.

Geographical extent	Local	Regional	Statewide/Basin	
System Modeling Purposes Management and Planning Descriptive	Allocation (Quantity) Input: 1) Stream flows 2) Aquifers Output: 1) Allocation to specific users	Input: 1) Surface water 2) Ground-water Output: 1) Allocation to various sectors	Input: 1) Total availability of water Output: 1) Allocation and user types	
	Pollution Abatement (Quality) Input: 1) Pollution loads 2) Unit process Output: 1) Least cost combination	Input: 1) Loading 2) Treatment plant Output: 1) Location and number of treatment plants	Input: 1) Loading 2) Treatment plant Output: 1) Location and number of treatment plants	
	Hydrologic (Quantity) Input: 1) Point precipitation 2) Hourly, daily interval Output: 1) Hourly flow or daily flow	Input: 1) Average precipitation 2) Daily, weekly or monthly intervals Output: 1) Daily, weekly or monthly flow	Input: 1) Average precipitation 2) Daily, weekly or monthly intervals Output: 1) Daily, weekly or monthly flow	
	Stream Quality (Quality) Input: 1) Point Sources 2) Background levels Output: 1) Concent. along the stream read	Input: 1) Loading of pollutants Output: 1) Quality level of stream	Input: 1) Loading of pollutants Output: 1) Quality level of stream	

Figure 3.— Purposes of Systems Modeling.

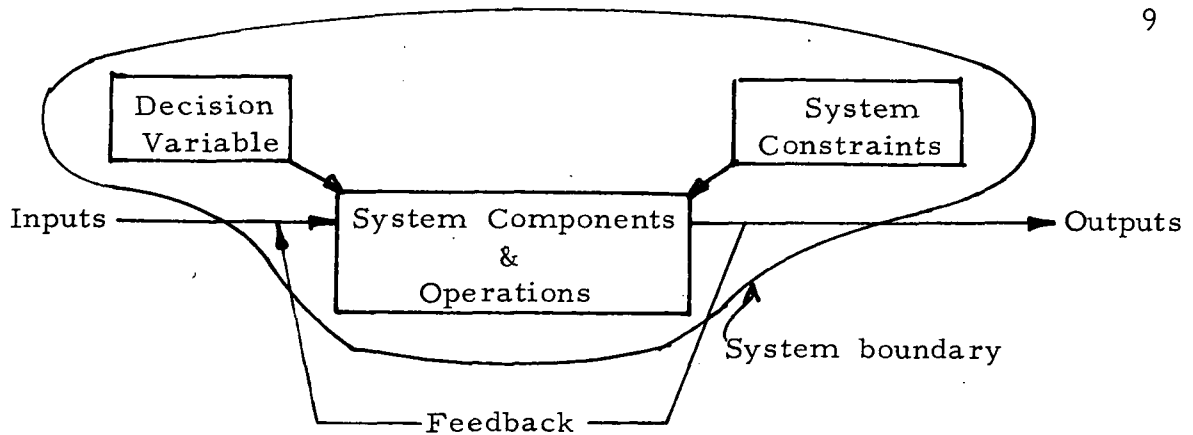


Figure 4.— Basic Water Resource System.

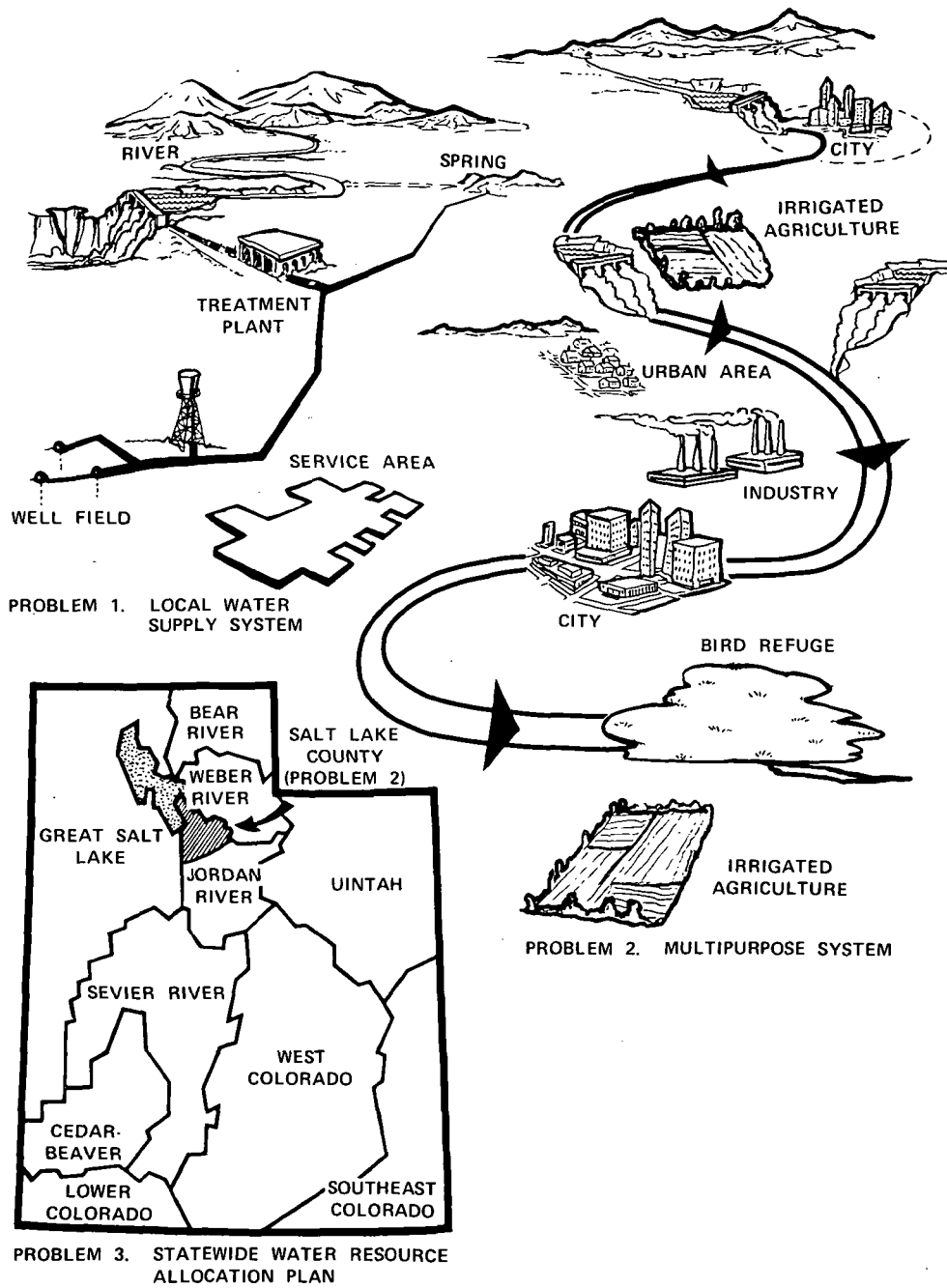


Figure 7.— Geographic relationship of example problems.

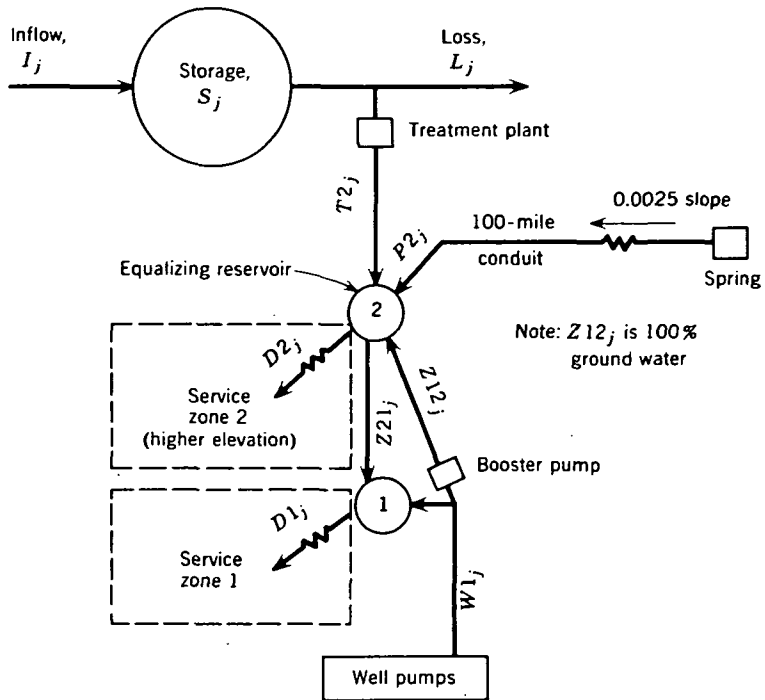


Figure 8.— Sample problem schematic.

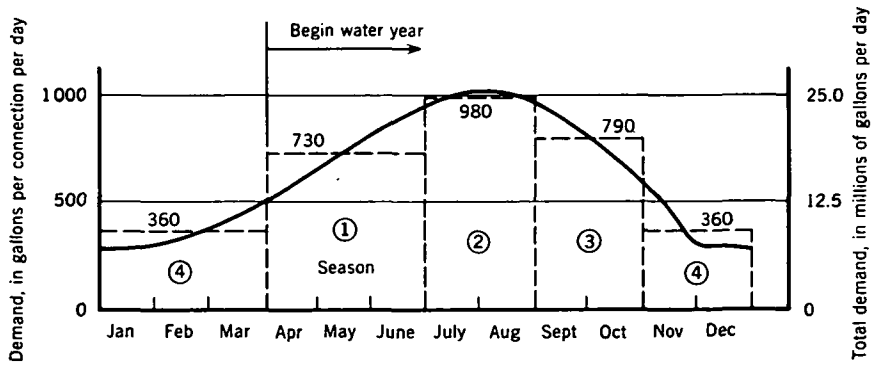


Figure 9.— Demand expected values.

Supply Origins		Demand Destinations		Municipal	Industrial	Agricultural	Recreation Wildlife Hydropower	System Outflow	Category Availabilities
		Municipal	Industrial	Agricultural	Recreation Wildlife Hydropower	System Outflow	Category Availabilities		
Primary Supply	Surface Water			{ initial allocation of primary supply }					annual outflow
	Groundwater								
Secondary Supply	Municipal Effluent	recycle reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	municipal waste system outflow
	Industrial Waste	sequential reuse	recycle reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	industrial wastewaters
	Agricultural Return Flow	sequential reuse	sequential reuse	recycle reuse	sequential reuse	sequential reuse	sequential reuse	sequential reuse	irrigation return flows
Supplementary Supply	Imported Water			{ allocation of supplementary supply }					annual importation
	Desalination of Sea Water								
	Use Sector Requirements	municipal diversion requirement	industrial diversion requirement	agricultural diversion requirement	miscellaneous diversion requirement	downstream outflow		Totals	

Figure 10.— Allocation alternatives for the water resource system.

i \ j	j		
	(1)	(2)	.	.	(j)	.	.	.	(n)	.	.	.
(1)	x_{11}	x_{12}			x_{1j}				x_{1n}			a_1
.												
.												
(i)	x_{i1}	x_{i2}			x_{ij}				x_{in}			a_i
.												
.												
(m)	x_{m1}	x_{m2}			x_{mj}				x_{mn}			a_m
	b_1	b_2			b_j				b_n			

Figure 11.— The transportation problem tableau.

	Diversion Requirements			Treatment Capacities				n System Outflow	Type of Control or RHS Value (Availabilities)
	1 Mun	2 Ind	... Ag	Primary Second.	j Tertiary	... Desalt	Blend		
1 Primary Supplies	Rows: $\sum_{j=1}^n x_{1j} = a_1$			Columns: $\sum_{i=1}^m x_{i2} = b_2$					$a_1 = \text{Availability}$
2 Supplementary Supplies									$\leq \text{Availability}$
Secondary Supplies Municipal Eff. Industrial Wastewater Agriculture Return F.	Rows: $\sum_{i=1}^n x_{3j} = a_3$			Columns: $\sum_{i=1}^m x_{i3} = b_3$					$\begin{cases} \text{Availability} \\ \text{(Diversion-CU)} \\ \text{Consumptive Use} \end{cases}$
i Treatment/or Blending Primary-Secondary Tert. Desalt m Blend	Rows: $\sum_{j=1}^n x_{ij} \begin{cases} \geq 0 \\ \leq a_i \end{cases}$ and Input = Output $\sum_{j=1}^n x_{ij} = \sum_{i=1}^m x_{ij}$			Columns: $\sum_{i=1}^m x_{ij} \begin{cases} \geq 0 \\ \leq b_j \end{cases}$					≥ 0 (No Plant Capacity Specified) $\leq \text{Plant Capacity}$
m+1 Special Conditions Blending Ratios Salt Loading m+r	$\sum \text{Salty Sources} = r_i \sum \text{(Fresh Sources)}$ $r_i = \text{ratio TDS of Salty Sources to Fresh Sources}$								
Type of Constraint or/RHS Value	= Requirements			≥ 0 (No Plant Capacity) $\leq \text{Plant Capacity}$				$\sum_{i=1}^n a_i = \sum_{j=1}^m b_j$	Total Supply (or) Demand

Figure 12.— Summary of model structure.

Demand Destinations Supply Origins	Municipal Requirement	Industrial Requirement	Agricultural Requirement	Secondary Treatment	Tertiary Treatment	Desalting ^d	Blending	Bird Refuge	System Outflow	Available Supply (1000 AF)
Water Allocation Costs \$/AF										
Surface water ^b supply	38 ^a	38	5	1000	1000	1000	38	5	0	83
Groundwater ^b supply	23	23	10	1000	1000	1000	23	10	0	48
Jordan River supply	108	10 ^b	5 ^b	1000	30	59	1000	0	0	270
Municipal ^c effluent	135 ^d	56	51	46	77	105	1000	46	46	74
Industrial effluent	115 ^d	39	10	29	1000	88	1000	29	29	87
Agricultural return flow	108 ^d	93	5	1000	30	59	1000	5	0	103
Secondary treatment	80 ^d	10	5	1000	33	59	1000	5	0	50
Tertiary treatment	1000	1000	10	1000	1000	1000	11	0	0	100
Desalting ^d	15	10	5	1000	1000	1000	11	5	0	56
Blending	0	0	5	1000	1000	1000	1000	5	0	100
Bird Refuge	1000	1000	1000	1000	1000	59	1000	1000	0	65
Demand requirement (1000 AF)	88	124	270	50	100	56	100	141	107	

^aCost in dollars per acre-ft.

Figure 13.— Transportation model tableau.

Supply Origins \ Demand Destinations	Municipal Requirement	Industrial Requirement	Agricultural Requirement	Secondary Treatment	Tertiary Treatment	Desalting	Blending	Bird Refuge	System Outflow	Available Supply (1000 AF)
Surface water supply	49						34			83
Groundwater supply	48									48
Jordan River supply		100			29			141		270
Municipal effluent		104							35	139
Industrial effluent			143							143
Agricultural return flow			81		5					86
Secondary treatment										
Tertiary treatment							34			34
Desalting										
Blending	68									68
Bird refuge									65	65
Demand requirement	165	204	224		34		68	141	100	

Total Cost = \$15,921,000
Total Diversions = 734,000 AF
Unit Cost = \$21.60/AF

Figure 14.— Year 1980 allocation pattern with no constraint on treatment plant capacities.

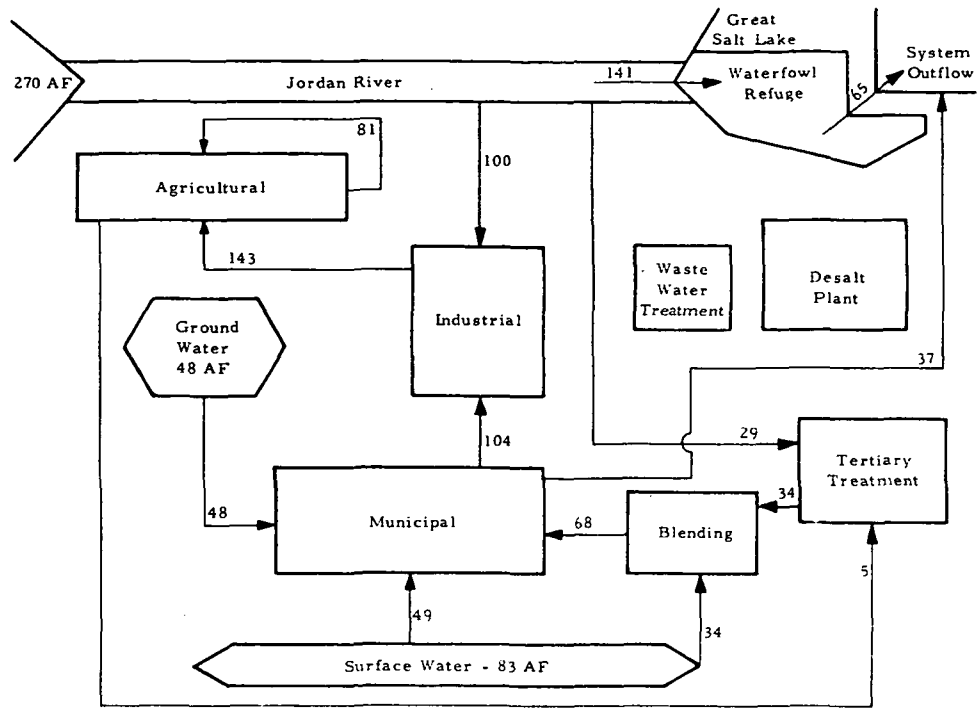


Figure 15.— Year 1980 allocation with constrained (or) unconstrained treatment capacities.

MODEL VARIABLES CONSTRAINTS	COLORADO RIVER TRANSFERS BU UI SA	LOCAL SURFACE WATER	GROUND WATER	STORED LSW	EVAPORATION LOSS	RETURN FLOW	GW RECHARGE	OUT-FLOW	M&I USE	WETLAND USE	PRODUCTION INPUTS	PRODUCTION OF CROPS BY LAND CLASS, CO., & HSU		
PROFIT (OBJ. FUNCTION)											COST M&I	COST WETLAND	COSTS OF PRODUCTION INPUTS	REVENUE FROM CROP SALES
LOCAL SURFACE WATER GROUNDWATER	WATER AVAILABILITIES WITHIN HSU'S													
DRAFT REQUIREMENT STORAGE REQUIREMENT EVAPORATION LOSS	RESERVOIR STORAGE AND EVAPORATION													
M&I WASTE WATER AGRICULTURAL RETURN	RETURN FLOWS													
WETLANDS SUPPLY	FREE GROUNDWATER FOR WETLANDS													
GROUNDWATER RECHARGE WATER TRANSFERS OUTFLOWS	WATER MOVEMENT RELATIONSHIPS													
M&I DEMANDS	LINKS M&I SUPPLY TO GIVEN M&I REQUIREMENTS													
WETLAND DEMANDS	LINKS WETLAND SUPPLY TO GIVEN WETLAND REQUIREMENTS													
AGRICULTURAL DEMANDS	LINKS AGRICULTURAL SUPPLY TO AGRICULTURAL DEMANDS												VARIABLE WATER REQUIREMENTS	WATER REQUIREMENTS BY CROP
PRESENTLY IRRIGATED LAND BY LAND CLASS, CO., HSU													PRESENT LAND REQUIREMENTS	PRESENT LAND IN PRODUCTION
TOTAL IRRIGABLE LAND BY LAND CLASS, CO., HSU													NEW LAND REQUIREMENTS	PRESENT AND NEW LAND IN PRODUCTION
PRODUCTIVITY LIMITS BY CROP													CROP PRODUCTION REQUIREMENTS	CROP YIELDS
ROTATION CONSTRAINTS														CROP ROTATION LIMITS
AMOUNT OF CROP SOLD														TOTAL CROP PRODUCTION

S
U
P
P
L
Y

D
E
M
A
N
D

Figure 16.— Diagrammatic representation of the programming model.

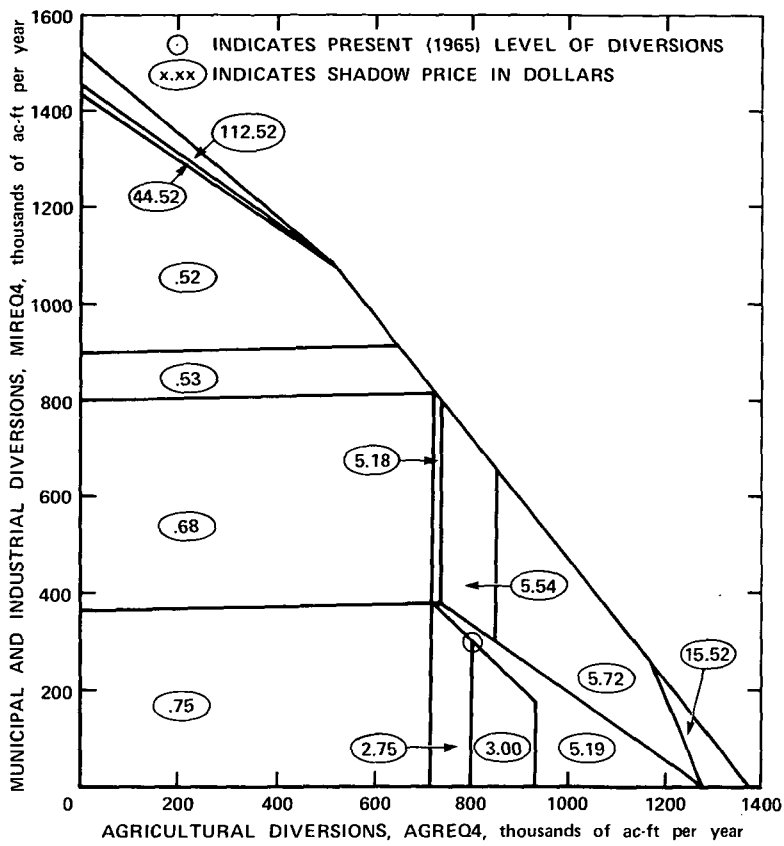


Figure 17.— Supply function mapping for HSU 4.

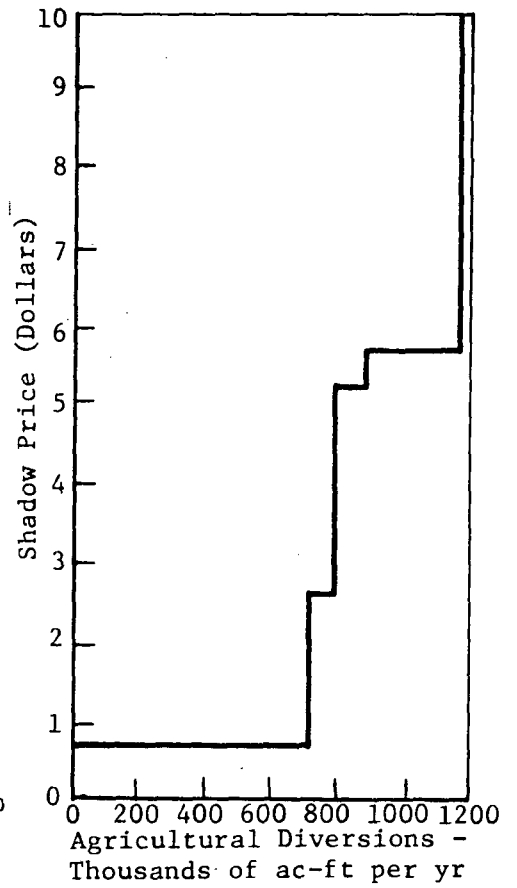


Figure 18.— Supply curve for HSU 4 for MI and wetland diversions for 1965.

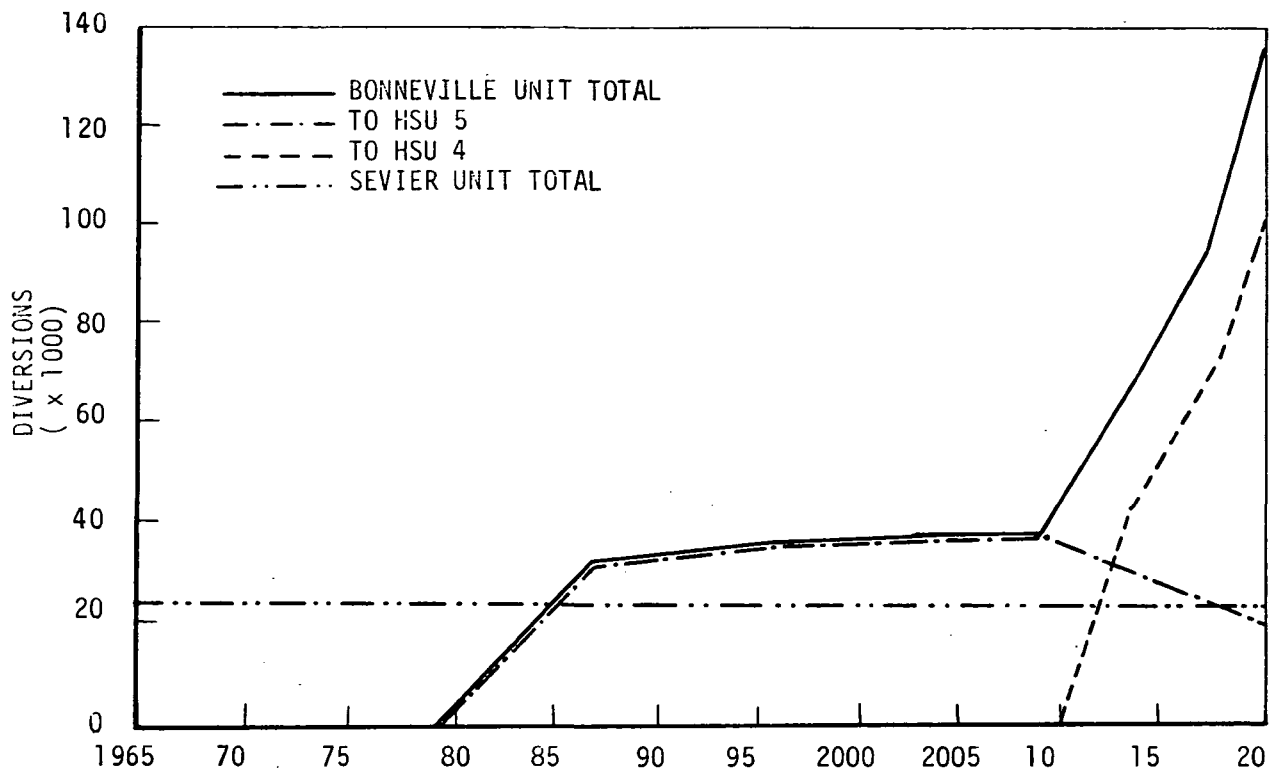


Figure 19.— Cup diversions, INFLO GSL < 1,014,000 with salvage.

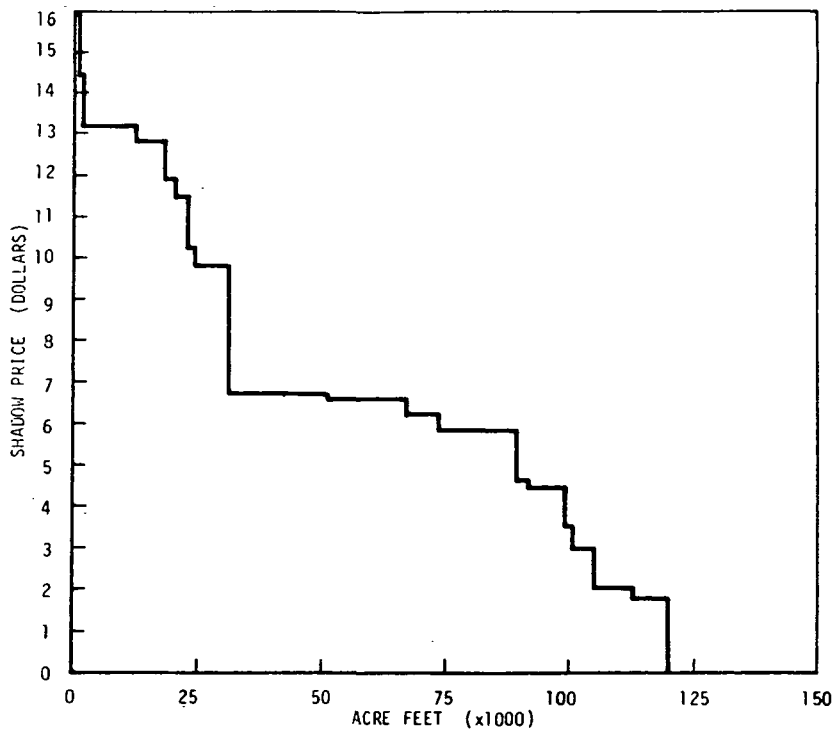


Figure 20.— Demand for agricultural water in HSU 4.

Page Intentionally Left Blank

SURVEY OF DECENTRALIZED CONTROL METHODS*

Michael Athans**

Massachusetts Institute of Technology
Cambridge, Massachusetts

INTRODUCTION

This paper presents an overview of the types of problems that are being considered by control theorists in the area of dynamic large-scale systems with emphasis on decentralized control strategies. A similar paper by Varaiya (ref. 1) indicated the interplay between static notions drawn from the mathematical economics, management, and programming areas and the attempts by control theorists to extend the static notions into the stochastic dynamic case. In this paper, we shall not elaborate upon the dynamic or team aspects of large-scale systems. Rather we shall concentrate on approaches that deal directly with decentralized decision making for large-scale systems.

Although a survey paper, the number of surveyed results is relatively small. This is due to the fact that there is still not a unified theory for decentralized control. What is available is a set of individual contributions that point out both "blind alleys" as well as potentially fruitful approaches.

What we shall attempt to point out is that future advances in decentralized system theory are intimately connected with advances in the so-called *stochastic control problem with nonclassical information pattern*. To appreciate how this problem differs from the *classical stochastic control problem*, it is useful to briefly summarize the basic assumptions and mathematical tools associated with the latter. This is done in section 2. Section 3 is concerned with certain pitfalls that arise when one attempts to impose a decentralized structure at the start, but the mathematics "wipes out" the original intent. Hence, one can draw certain conclusions about the proper mathematical formulation of decentralized control problems. Section 4 surveys some research (primarily carried out by the author and his students) that attempts to circumvent some of the pitfalls discussed in section 3. Section 5 presents some conclusions about future research.

CLASSICAL STOCHASTIC CONTROL PROBLEM

This section reviews in an informal way the classical stochastic control problem or the problem of stochastic control with classical information structure. Our main purpose is to indicate the types of assumptions one makes in this class of problems, the nature of the mathematical tools available, and the general structure of the solution. This overview is necessary so that one can see that the solution to the classical stochastic control problem leads to a completely centralized system.

*This research has been conducted in the Decision and Control Sciences Group of the MIT Electronic Systems Laboratory with support from AFOSR grant 72-2273, NASA/AMES grant NGL-22-009-124, and NSF grant GK-41647.

**Professor of Electrical Engineering and Director MIT Electronic Systems Laboratory.

From a technical point of view, the best survey of the issues associated with classical stochastic control is, in the opinion of the author, the paper by Witsenhausen (ref. 2). The classical stochastic control problem is the subject of many standard texts and monographs (see, e.g., refs. 3 to 7).

Figure 1 is an abstract block diagram of a centralized control system. One deals with a stochastic dynamical system (usually described by a set of stochastic difference or differential equations) and one makes possibly noisy measurements of certain of its variables. The time evolution of the system variables is influenced by stochastic disturbances and decisions (or control) variables generated in closed-loop feedback form by a *single* controller or decision maker.

The assumptions made in the classical stochastic control problem are as follows:

A2.1 There is a single controller or decision maker.

A2.2 The controller knows the mathematical form of the system dynamics (i.e., the stochastic differential or difference equations).

A2.3 The controller knows the relationship of the measurements to the system variables.

A2.4 The controller knows a priori the probability densities of all underlying stochastic variables (i.e., exogenous disturbances, uncertain system parameters, measurement errors, etc.).

A2.5 The controller wishes to minimize a well-defined scalar deterministic cost functional.

A2.6 The controller at any time t has *instant and perfect recall* of all past applied inputs or decisions and all past and present measurements.

Under these assumptions, classical stochastic control provides a well-defined rule that translates all information available to the controller at time t (i.e., the contents of assumptions A2.1 to A2.6) to an optimal control or decision at time t .

From a technical point of view the state variable (causal) description of the dynamical system, the use of a Bayesian rule to deal with the stochastic elements, and the use of stochastic dynamic programming blend well to yield the optimal stochastic control in a relatively straightforward *conceptual* manner. Actual calculations are generally very complex with the exception of the so-called Linear-Quadratic-Gaussian (LQG) problem (refs. 4-8).

DECENTRALIZED CONTROL: PITFALLS

This section presents the types of issues that arise when the basic assumptions of section 2 associated with classical centralized stochastic control are modified. There are several ways to depart from the basic framework of classical stochastic control. In this paper, we shall adopt the viewpoint of examining the issues when we wish to analyze some sort of hierarchical multilevel decentralized system (ref. 9).

One does not have to examine a complex hierarchical structure to understand the issues associated with decentralized control. Figure 2 presents the simplest possible case involving a two-level structure. We shall elaborate upon the structure implied in figure 2 to point out its general characteristics.

Case 3.1

Imagine for the time being that the “interaction” channel and the “coordinator” did not appear in figure 2. We are left with two “uncoupled” dynamical systems. If the framework of section 2 is adopted, we can postulate that each controller solves a classical stochastic control system.

Case 3.2

Next, let us still leave the “coordinator” out of figure 2, but restore the “interaction” channel. What we mean by “interaction” is that certain decision and/or state and/or output variables of each system influence the dynamic evolution of the other system. If this interaction is “weak,” then it is possible for both systems to operate non-optimally but still satisfactorily without altering the basic control strategy of Case 3.1, because the interactions are viewed as exogenous unknown disturbances and the inherent use of feedback tends to make the overall system response somewhat insensitive to weak, unmodeled disturbances.

This situation, namely, with weak interaction and the absence of coordination, has been analyzed by Chong, Kwong, and Athans (refs. 10-12). This research attempted to replace the weak interaction disturbances, which are actually correlated in time, with equivalent “fake white noise” inputs which are uncorrelated in time, and to evaluate techniques by which, in the LQG context, the covariance of the “fake white noise” can be selected.

Case 3.3

If the dynamic interaction between the systems is not negligible, then the performance of each system in figure 2 can be expected to deteriorate severely. To “cure” this performance degradation, one introduces the “coordinator” in figure 2.

Intuitively, in any physical large-scale system, the role of the coordinator is to receive some sort of information from the local subsystems and make some decisions to improve the performance compared to that under Case 3.2. The crucial question then is to make precise the role of the coordinator as a function of postulated strategies for the lower level subsystems.

It is possible, in any given physical situation, to specify the task of the coordinator in a reasonably good, but ad hoc way, so that the overall system performance is satisfactory for the specific application. However, the heart of decentralized control theory research is to formulate precise mathematical problems whose solution defines the *optimal task* of the coordinator, without destroying the intuitively appealing decentralized structure in figure 2.

Some Blind Alleys

The following assumptions sound reasonable from a physical point of view, but when they are incorporated in a mathematical framework, the mathematical solution destroys the decentralized structure. The pitfalls that the assumptions lead to are easily seen without resorting to complex mathematics, and the appreciation of the pitfalls provides valuable knowledge on how *not* to formulate a decentralized control problem.

We start with a list of assumptions, again keeping the structure of figure 2 in mind:

A3.1 Each local system neglects the interaction from the other system.

A3.2 Under A3.1, each local controller knows the dynamics and probabilistic information associated with his own system, has his own performance index, and has perfect recall of his own past measurements and controls. It follows (see section 2) that each local system can solve its own well-formulated classical stochastic control problem, and we assume that each local system applies in real time the optimal stochastic control obtained under these assumptions.

A3.3 The coordinator knows the dynamics of both subsystems, including the interaction, as well as all prior probabilistic information available to each local subsystem.

A3.4 The coordinator's cost functional (performance index, utility function) is a well-defined function of the cost functionals of each local subsystem (e.g., a weighted sum).

A3.5 At each instant of time, the coordinator can apply a dynamic control to each local system of the same nature of the local control.

A3.6 At each instant of time, each local subsystem transmits *instantly* and *without error* its measurements and controls to the coordinator; furthermore, the coordinator has perfect recall.

The key question is then: Under assumptions A3.1 to A3.6, what is the optimal decision rule for the coordinator? The answer is exceedingly simple. Under assumptions A3.3 to A3.6, the coordinator has a classical stochastic control problem for the entire system. Hence, so far as the coordinator is concerned, he must solve the overall optimal stochastic control problem (see section 2) and his optimal strategy is to (i) cancel the locally computed controls (see assumption A3.2) and (ii) substitute the global optimal controls.

Thus, the essential decentralized nature of the problem is destroyed. This points out that, even in the stochastic case, one cannot allow the coordinator full knowledge of everything because the mathematically optimal solution allows the coordinator to completely take over. This problem is even more serious if a complete deterministic framework for decentralized control is adopted (ref. 9).

Control-Sharing Strategies

So that the coordinator does not take over completely, some of assumptions A3.1 to A3.6 must be modified to deny the coordinator full knowledge of everything. Needless to say, there are

many ways to modify assumptions A3.1 to A3.6 and to attempt an analysis of the role of the coordinator.

In this section, we shall examine one variation because it has received some attention in the control literature, although not precisely in the context of this paper. Hence our remarks represent a reinterpretation of the research of Aoki (ref. 13) and Sandell and Athans (ref. 14).

One can argue that the coordinator can take over under assumptions A3.1 to A3.6 because assumption A3.6 allows the coordinator to have instantaneous access to all measurements of the local systems. This allows him (see assumption A3.3) to calculate the local controls to be used by the subsystems (see assumption A3.2) and cancel them (see assumption A3.5). Hence one may think that one way to prevent the coordinator from taking over is to deny to him the actual measurements of the local systems. Thus, we seek to modify assumption A3.6.

Assumption A3.6, however, deals not only with measurements but with controls. We can adopt the intuitive philosophy “do not flood your boss with day-to-day occurrences, but let him know your day-to-day decisions.” If we adopt this framework, we can replace assumption A3.6 with the following:

A3.6(M) At each instant of time, each local subsystem transmits *instantly* and *without error* ONLY its controls, but *not* its measurements to the coordinator; furthermore, the coordinator has perfect recall.

One can then pose a mathematical problem under assumptions A3.1 to A3.5 and A3.6(M) to find the optimal decision rule for the coordinator. The answer (refs. 13, 14) is both surprising and interesting: (i) the stochastic control problem for the coordinator is *not well defined*, in the sense that an optimal solution *does not exist*; and (ii) although an optimal solution does not exist, one can find ϵ optimal solutions in the sense that one can approach the unattainable optimal solution arbitrarily closely.

The way these ϵ optimal solutions are obtained is interesting and instructive because they indicate once more how *not* to formulate a decentralized control problem. We shall attempt to explain how this happens by a simple example.

From figure 2, let us suppose that the system operates in discrete time so that measurements and decisions are made at the values of the time index $t = 0, 1, 2, 3, \dots$. Let $z_1(t)$ denote the measurement and let $u_1(t)$ denote the control of system 1; for simplicity, assume that both $z_1(t)$ and $u_1(t)$ are scalars. Suppose that the following sequence of measurements has been made by system 1:

$$\left. \begin{aligned} t = 0, z(0) &= 6 \\ t = 1, z(1) &= 7 \\ t = 2, z(2) &= 8 \\ t = 3, z(3) &= 9 \end{aligned} \right\} \quad (1)$$

Under assumption A3.2, system 1 has a well-defined rule for generating its own optimal control. Suppose that, at the basis of the measurements of equation (1), the optimal local control for system 1 at the time $t = 3$ is

$$u_1^*(3) = 1.234 . \quad (2)$$

Under assumption A3.6(M), the control in equation (2) can be transmitted instantly and without error to the coordinator. However, the nature of the ϵ optimal solution indicates that system 1 *should not transmit* the control (eq. 2) to the coordinator. Rather, it should transmit and apply to the system a control of the following form:

$$\hat{u}_1(3) = 1.234000000 \dots 00006789. \quad (3)$$

The information conveyed to the coordinator when he receives the control (eq. 3) *without error* is very different from that contained in equation (2). Examination of equations (1) to (3) indicates that the past measurements (6, 7, 8, 9) have been *coded* in $u_1(3)$. From the front part of equation (3), the coordinator knows the control $u_1^*(3)$ of equation (2); from the tail end of equation (3), the coordinator knows exactly the past measurements of system 1. The string of zeros between the control (1.234) and the coded measurements (6789) is simply to guarantee that the application of $u_1(3)$, rather than $u_1^*(3)$, to the system results in an infinitesimal loss in system performance (i.e., the 000 ... 006789 part of the control is wiped out by the system uncertainty).

Hence, assumption A3.6(M) is not strong enough to prevent the coordinator from obtaining all the information he needs to take over for all practical cases.

Conclusions

The above discussion points out that, in stochastic decentralized problems, *instantaneous error-free* transmissions of either both controls and measurements or controls alone is *not* a realistic mathematical assumption because this allows the coordinator to take over and destroy the decentralized nature of the problem.

DECENTRALIZED CONTROL: PROMISING AVENUES

In this section, we discuss some recent results that appear to be useful toward building some elements of a theory for decentralized control. Once more the reader is referred to Variaya (ref. 1) for additional concepts and discussion.

Decentralization with Fixed Structure

Figure 3 depicts a specific decentralized structure somewhat different from that discussed in section 3. One is given an n th order, linear stochastic dynamic system, with two sets of measurements (z_1 and z_2) and two sets of controls (u_1 and u_2). It is decided *a priori* to select two dynamic controllers that generate stochastic controls in the manner illustrated in figure 3. It is assumed that

there is a single cost functional to be minimized, and one can view the job of the coordinator as defining the characteristics of the two controllers. This represents a variation on the dynamic team problem (see, e.g., Ho and Chu (refs. 15, 16)).

Because of the nonclassical information pattern and the general lack of knowledge for solving dynamic team problems, some additional assumptions have to be made so that the problem of designing the two controllers in figure 3 can be solved.

For the LQG continuous-time case, Chong and Athans (refs. 10, 17) fixed the structure of each controller to be linear and of the same dimension as the order of the dynamic system that was controlled. Furthermore, each controller was constrained so that its internal Kalman-Bucy filter would produce unconditional zero mean estimates of the state, ignoring the actions of the other controller. The parameter matrices of each dynamic controller could then be globally optimized by solving a deterministic matrix optimal control problem through the use of the matrix minimum principle (ref. 18). The discrete-time version of this problem was considered by Carpenter (ref. 19).

Two basic conclusions can be drawn from the above studies (see also Variaya (ref. 1)):

(i) The off-line computational effort for solving such decentralized problems is greater than that required for the centralized case.

(ii) Even in the LQG context, the *separation theorem* or *certainty equivalence principle* fails to hold.

Periodic Coordination

The discussion in section 3 indicates that for decentralized systems (fig. 3), one *cannot* provide the coordinator with instantaneous and error-free transmission of the local subsystem measurements and/or decisions to the coordinator; otherwise, the coordinator takes over and substitutes the globally optimal stochastic controls, thus overriding the decisions of the local controller.

One way to bypass this problem is to assume that the coordinator is allowed to “interfere” only occasionally. This notion of periodic coordination has been considered by Chong and Athans (refs. 20-22). To understand the intuitive notion of periodic coordination, suppose that assumptions A3.1 to A3.4 of section 3 are still valid, but assumptions A3.5 and A3.6 are replaced by the following (informal) one.

Periodic coordination structure – Suppose that the entire system operates in discrete time. For concreteness, we assume that the basic time unit is a day. Then the basic system operation is

(i) Assume that each lower-level system makes its measurements and generates its controls (decisions) *once a day*.

(ii) *Once a month*, all lower-level system measurements and controls are “mailed” without error to the coordinator.

The basic question is: what is the job of the coordinator at the beginning of each month? The mathematical approach adopted and the results obtained (refs. 20-22) have the following interpretation which we feel has certain intuitively appealing aspects.

Once a month, the coordinator has a threefold task with respect to each lower-level system:

(i) *Set it straight.* In a technical context, he corrects the estimates generated by the lower-level Kalman filters because these estimates are in error because each lower-level system neglects the interactions from the other lower-level systems.

(ii) *Change its directives,* in the sense that new time paths for the lower-level controls are given in an open-loop sense.

(iii) *Change its incentives,* in the sense that additional terms are added to each lower-level system cost functional to compensate for the fact that the global cost functional differs from each lower-level cost functional.

The main advantage of these results is that the mathematical theory itself suggests the tasks that must be performed periodically by the coordinator. The main disadvantage is that the coordinator must still solve a very large-scale stochastic optimization problem, although not as often as in the basic time frame of the lower level. Although for certain applications this approach may be feasible, it lacks the capability of somehow aggregating the information flow from the lower-level systems to the coordinator.

Nonetheless, because the theory itself suggests this mode of coordination (by changing directives and incentives), it provides strong motivation to postulate a specific framework for operating the lower-level systems. A preliminary formulation along these lines can be found in a recent paper by Athans (ref. 23).

The notion of delaying the information exchanged between different portions of a hierarchical system is intuitively appealing. Much more research is needed to understand its impact on decentralized control theory. However, the results of Sandell and Athans (ref. 14), in which it was shown that LQG problems with a unit-time step delay of information exchange admit a *linear optimal decision rule*, which can be calculated explicitly, appear to be promising so far as their applicability to decentralized control theory is concerned.

Remarks

Most of the results surveyed attempt in one way or another to present to both the coordinator and the local subsystems a classical stochastic control problem. Although research along these lines is useful, there is no definitive theory that deals directly with issues of aggregated information, decision making with partial information, or decision making with finite memory.

To adequately deal with these issues in the context of decentralized control, much additional research is needed in the area of *stochastic control with nonclassical information patterns*. The famous Witsenhausen counter-example (ref. 24), in which a simple LQG problem with nonclassical information pattern was shown to have a *nonlinear* optimal decision rule, points out the immense

difficulties associated with this class of problems. Witsenhausen (refs. 25, 26) has continued his fundamental investigations in this class of problems, but their implications in the context of decentralized control theory remain largely unexplored. The work on finite-state, finite-memory control of Sandell and Athans (refs. 27, 28) may be useful to aggregate the flow of information between the different levels of a hierarchy and to limit the computational complexity available to the coordinator. In addition, the recent results surveyed by Ho (ref. 29) pertaining to approaches in information structures when many decision makers are involved is of direct importance to decentralized control problems.

CONCLUSIONS

The main conclusions that one can draw are:

(1) Any purely deterministic approach to multilevel hierarchical dynamic systems is not apt to lead to realistic theories or designs.

(2) The flow of measurements and decisions in a decentralized system should not be instantaneous and error-free.

(3) Delays in information exchange in a decentralized system lead to reasonable approaches to decentralized control.

(4) A mathematically precise notion of aggregating information is not yet available.

(5) Research in nonclassical information structures is directly relevant to problems of decentralized control.

REFERENCES

1. Varaiya, P. P.: Trends in the Theory of Decision-Making in Large Systems, *Annals of Economic and Social Measurement*, vol. 1, no. 4, Oct. 1972, pp. 493-500.
2. Witsenhausen, H. S.: Separation of Estimation and Control for Discrete Time Systems. *Proceedings IEEE*, vol. 59, 1971.
3. Bellman, R.: *Dynamic Programming*, Princeton University Press, 1957.
4. Aoki, M.: *Optimization of Stochastic Systems*, Academic Press, 1967.
5. Hinderer, K.: *Foundations of Non-stationary Dynamic Programming with Discrete Time Parameter*. Springer-Verlag, Berlin, 1970.
6. Astrom, K.: *Introduction to Stochastic Control Theory*. Academic Press, 1970.
7. Kushner, H. J.: *Introduction to Stochastic Control*. Holt, Rinehart, and Winston, 1971.
8. Athans, M.: The Discrete Time Linear-Quadratic-Gaussian Stochastic Control Problem. *Annals of Economic and Social Measurement*, vol. 2, October 1972, pp. 449-491.
9. Mesarovic, M. D.; Macko, D.; and Takahara, Y.: *Theory of Hierarchical Multilevel Systems*. Academic Press, 1970 (see also the review of this book in *IEEE Transactions on Automatic Control*, vol. AC-17, 1972, pp. 280-281).
10. Chong, C. Y.: *On the Stochastic Control of Coupled Linear Systems*, S. M. Thesis, MIT, 1972.
11. Kwong, R. H.: *On a Stochastic Control Method for Weakly Coupled Systems*. S. M. Thesis, MIT, 1972.
12. Kwong, R. H.; Chong, C. Y.; and Athans, M.: On Stochastic Control System Design Methods for Weakly Coupled Large Scale Systems. *Proceedings 1972 Joint Automatic Control Conference*, pp. 220-229.
13. Aoki, M.: On Decentralized Linear Stochastic Control Problems with Quadratic Cost. *IEEE Transactions on Automatic Control*, vol. AC-18, June 1973, pp. 243-250.
14. Sandell, Jr., N. R.; and Athans, M.: Solution of Some Nonclassical LQG Stochastic Decision Problems. *IEEE Transactions on Automatic Control*, vol. AC-19, April, 1974, pp. 108-116.
15. Ho, Y. C.; and Chu, K. C.: Team Decision Theory and Information Structures in Optimal Control Problems; Part I. *IEEE Transactions on Automatic Control*, vol. AC-17, 1972.
16. Chu, K. C.: Team Decision Theory and Information Structures in Optimal Control Problems; Part II. *IEEE Transactions on Automatic Control*, vol. AC-17, 1972.
17. Chong, C. Y.; and Athans, M.: On the Stochastic Control of Linear Systems with Different Information Sets. *IEEE Transactions on Automatic Control*, vol. AC-16, October, 1971, pp. 423-430.

18. Athans, M.: The Matrix Minimum Principle. *Information and Control*, vol. 11, November 1967, pp. 592-606.
19. Carpenter, Jr., C. H.: Some Control and Communications Aspects of Stochastic Systems. S. M. Thesis, MIT, 1972.
20. Chong, C. Y.: On the Decentralized Control of Large-Scale Systems. Ph.D. Thesis, MIT, 1972.
21. Chong, C. Y.; and Athans, M.: On the Periodic Coordination of Linear Stochastic Systems. July 1974 (to appear).
22. Chong, C. Y.; and Athans, M.: Hierarchical Decomposition for a Stochastic Optimization Problem. Proceeding 1973 Princeton Conference on Information Sciences and Systems, Princeton, N.J., March 1973.
23. Athans, M.: Theoretical and Practical Problems in the Design of Stochastic Large Scale Systems. Proceedings Fourth Iranian Conference on Electrical Engineering, Shiraz, Iran, May 1974.
24. Witsenhausen, H. S.: A Counterexample in Stochastic Optimal Control. *SIAM Journal on Control*, vol. 6, 1968.
25. Witsenhausen, H. S.: On Information Structures, Feedback, and Causality. *SIAM Journal on Control*, vol. 9, 1971.
26. Witsenhausen, H. S.: A Standard Form for Sequential Stochastic Control. *J. Mathematical System Theory*, vol. 7, 1973.
27. Sandell, Jr., N. R.; and Athans, M.: Control of Finite State Finite Memory Stochastic Systems. Proceedings 1974 Princeton Conference on Information Sciences and Systems, Princeton, N.J., March 1974.
28. Sandell, Jr., N. R.: Control of Finite-State, Finite-Memory Stochastic Systems. Ph.D. Thesis, MIT, May 1974.
29. Ho, Y. C.: Information Structures in Many-Person Optimization Problems. *Annals of Economic and Social Measurement*, (submitted).

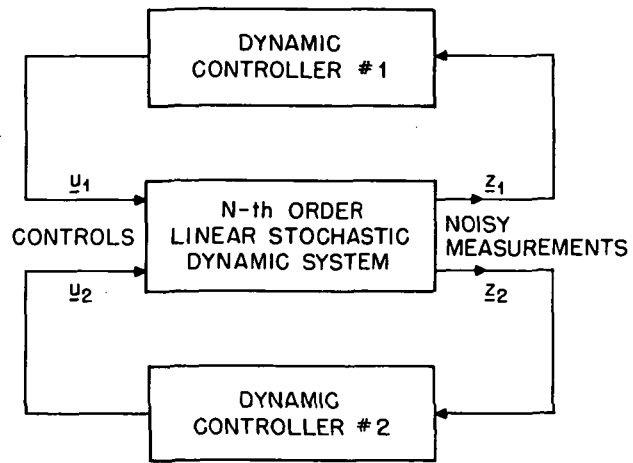


Figure 1.— Centralized control system.

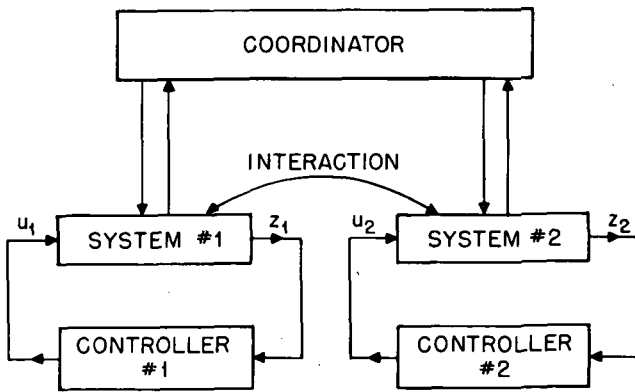


Figure 2.— Two level hierarchical structure.

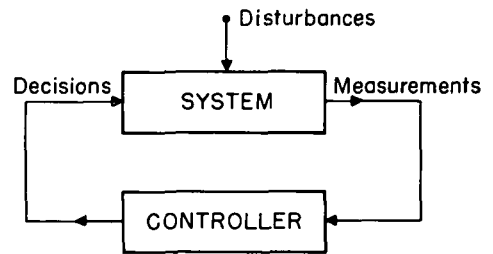


Figure 3.— Decentralized structure.

FEEDBACK CONTROL OF A SINGLE IRRIGATION CANAL REACH*

Raymond G. Jacquot**

University of Wyoming, Laramie, Wyoming 82071

INTRODUCTION

Increasing population and increasing nonagricultural land use have necessitated the irrigation of large arid regions throughout the world. Such irrigation can be achieved only with very complex water distribution systems. In this country, flow control is commonly accomplished with motor-driven gates that control the flow between two sections or reaches of a canal. In the past, control was usually achieved by scheduling the water use along the canal and raising or lowering the gates according to an *a priori* water use schedule. This is a good economical management policy if the demand is as assumed in the predetermined schedule. However, if the demand deviates from the schedule, a system of control which reflects dynamic demand must be sought. A pragmatic solution to the problem is the application of a feedback control scheme that will automatically adjust the reach gates in the face of dynamic demands to keep a constant head. As the first step in controlling a complete canal system, this paper discusses the application of feedback control to a single reach of such a canal.

A typical canal reach along with the associated parameters and variables is shown in figure 1(a) and (b).

Several techniques for automatic control of a canal head have been proposed in the past (ref. 1). The first, "upstream control," uses a feedforward technique in which the lower gate is controlled according to head measurement at the upper end. The second, "downstream control," controls the upper gate according to head measurement at the lower end of the canal reach. The latter scheme (to be discussed here) is illustrated in figure 2. The mathematical technique given here is applicable not only to downstream control but also to other methods of control that use multiple measurements and control efforts.

A stability analysis of a feedback-controlled canal in which the method of characteristics was used to approximate the nonlinear partial differential equations by a system of second-order ordinary differential equations was accomplished by Shand et al. (refs. 2,3). In such an analysis, the effect of higher-order system modes may be lost; hence higher mode instability may be missed by such approximations (ref. 4). Here an eigenfunction expansion technique is used for the spatial portion of the problem so that the distributed nature of the problem is maintained as long as necessary to check for higher mode instability. The instability in distributed parameter systems does not occur in a single open-loop mode since the feedback couples all the open-loop system modes together; however, one or more of these modes might be dominant.

*This discussion is based on the Ph.D. Dissertation of Tooraj Kia-Koojoori of the University of Wyoming directed by the author and C. T. Constantinides in 1973.

**Associate Professor, Department of Electrical Engineering.

This work considers the stability of the "downstream control" of a single reach of a canal. The eventual goal is to examine the control of a complete canal system or, for that matter, a complete irrigation system where the coupling between reaches is included in the model so that stability of the system as a whole must be considered.

Recently, Winn and Moore (ref. 5) considered the application of optimal regulator theory to the control of water pollution due to treatment system overflow. This type of technique should have some application to the irrigation canal problem; however, it is difficult to envision a meaningful performance index or cost function.

REACH MODEL AND CONTROL SYSTEM

The reach was modeled as a rectangular channel with a viscous, incompressible fluid in one-dimensional flow. A linear momentum balance and a mass balance for an elemental length of the reach yielded a pair of nonlinear partial differential equations. The associated boundary conditions are related to the flow under the gates at the ends of the reach and are also nonlinear. The work here was directed toward control of the water head about some predetermined, fixed level; hence the differential equations and boundary conditions were linearized about a fixed equilibrium flow condition. The perturbation system of linear equations yielded an eigenvalue-eigenfunction problem wherein the boundary conditions fix the eigenvalues (system open-loop poles) and the open-loop eigenfunctions. These open-loop poles are shown in figure 3 in the complex λ plane. Since there are two partial differential equations for the dynamics, the eigenfunctions form a function vector for each eigenvalue. These two partial differential equations represent the plant portion of this control system. The control effort for downstream control enters the problem as a time-dependent boundary condition at the upper end of the reach. This inhomogeneous boundary condition does not lend itself well to an eigenfunction representation, but with the technique of Brogan (ref. 6) the problem may be reduced to one having homogeneous boundary conditions, with the control effort appearing in one of the differential equations represented as a Dirac delta function at the system boundary.

The plant of the system can thus be represented in modal form and, with the arbitrary lumped parameter controller cascaded in the control loop, the stability of perturbations from equilibrium can be investigated by linear system techniques. The difficulty of applying standard linear system techniques is caused by the infinite number of poles in the open-loop plant. In this problem, the characteristic equation can be given in the same form as that for a lumped parameter control system:

$$1 + G(s)H(s) = 0 \quad (1)$$

where $G(s)$ is an infinite series representation of the distributed portion of the system and $H(s)$ is the transfer function of the lumped parameter controller. The values of s which satisfy equation (1) are the characteristic roots of the closed-loop system. The Nyquist criterion may be applied to study the stability of the system without evaluating the closed-loop characteristic roots.

This study was conducted only for a lowpass-type controller with the transfer function:

$$H(s) = \frac{K}{\tau s + 1} . \quad (2)$$

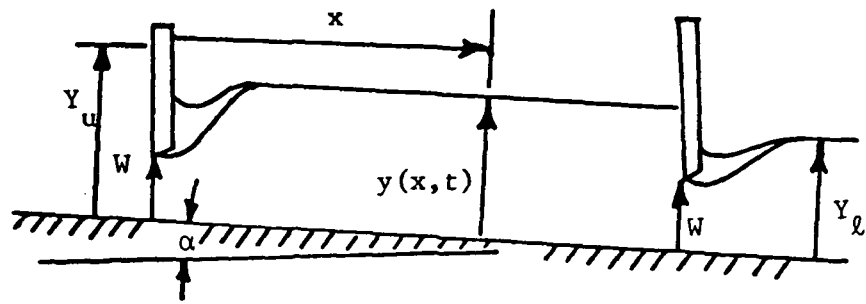
A typical Nyquist plot for this system is shown in figure 4 where Y_o is the depth of water above the upper gate and τ is the delay associated with a simple lowpass controller. Figure 5 illustrates a stability boundary for the system as a function of the differential head across the reach. This indicates that, for high flow rates, the gain of the plant is essentially higher, which allows a lower maximum gain in the controller for a stable system. A more detailed account of this work can be found in reference 7.

CONCLUSIONS

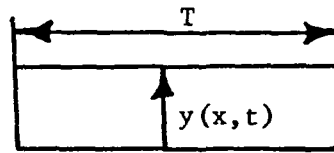
This paper has discussed the stability of the feedback control of a single reach of an irrigation canal. The stability study was accomplished by examining perturbations about a steady flow condition, which then allows the system to be treated as a linear system and thus frequency domain techniques are applicable.

REFERENCES

1. Braunagel, M. V.: Control System for Canal Water Level. Published by the Division of Design, Office of Chief Engineer, Bureau of Reclamation, Denver, Colorado, Aug. 1968.
2. Harder, J. A.; Shand, M. J.; and Buyalski, C. P.: Automatic Downstream Control of Canal Check Gates by the Hydraulic Filter Level Offset Method. Presented at the Fifth Technical Conference, U. S. Committee on Irrigation, Drainage and Flood Control, Denver, Colorado, Oct. 1970.
3. Shand, M. H.: Automatic Downstream Control Systems for Irrigation Canals. Ph.D. Thesis, University of California, Hydraulic Engineering Laboratory, Berkeley, California, 1971.
4. Athans, M.: Some Remarks on the Control of Distributed Parameter Systems. 1969 Joint Automatic Control Conference, Boulder, Colorado.
5. Winn, C. B.; and Moore, J. B.: The Application of Optimal Linear Regulator Theory to a Problem in Water Pollution. IEEE Trans. on Systems, Man and Cybernetics, vol. SMC-3, no. 5, Sept. 1973, pp. 450-455.
6. Brogan, W. L.: Optimal Control Theory Applied to Systems Described by Partial Differential Equations. Advances in Control Systems, ed. by C. T. Leondes, vol. 6, 1968, pp. 221-316.
7. Kia-Koojoori, T.; Jacquot, R. G.; and Constantinides, C. T.: Feedback Control of a Distributed Parameter System. Proc. of the 1974 Conference on Computers, Electronics and Control (CEC 1974), Calgary, Alberta, May 1974.



(a) Side view.



(b) End view.

Figure 1.— Typical irrigation canal reach.

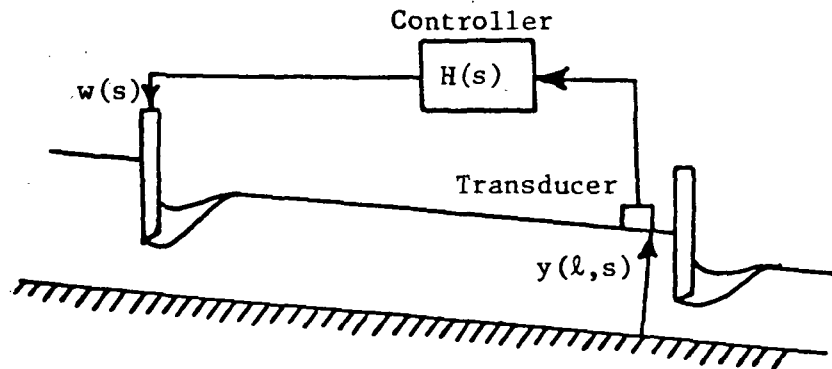


Figure 2.— “Downstream” control technique.

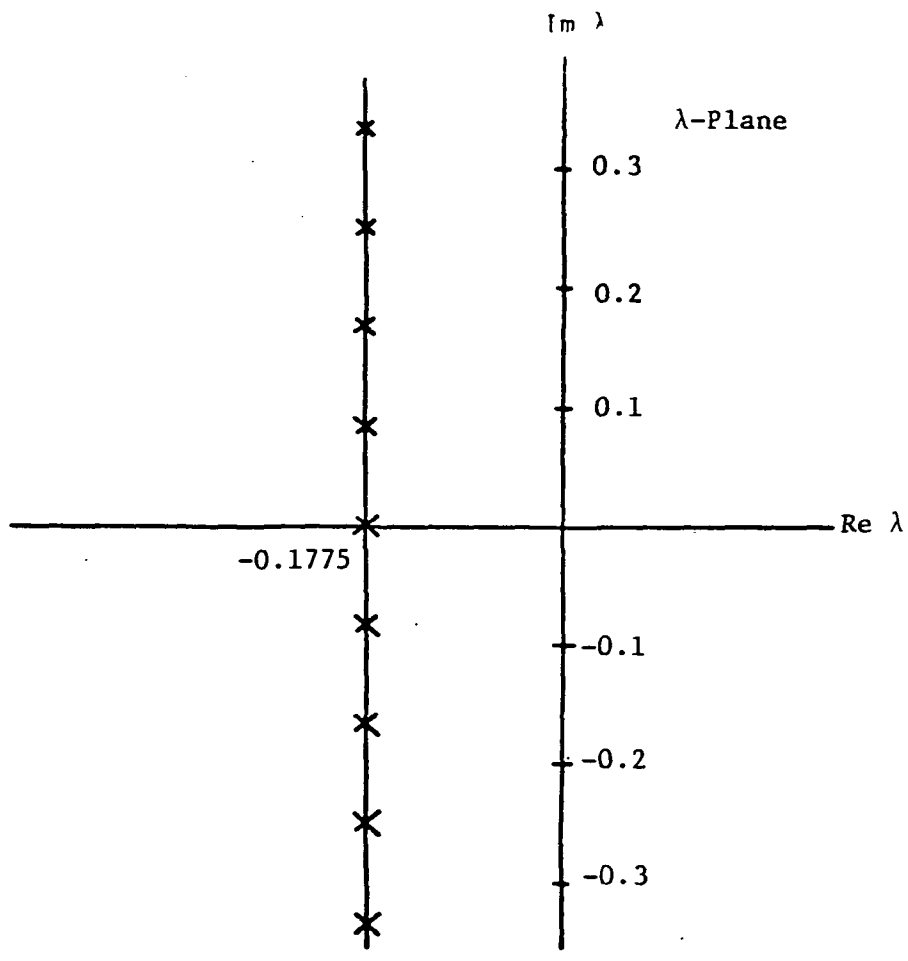


Figure 3.— Open-loop eigenvalue locations.

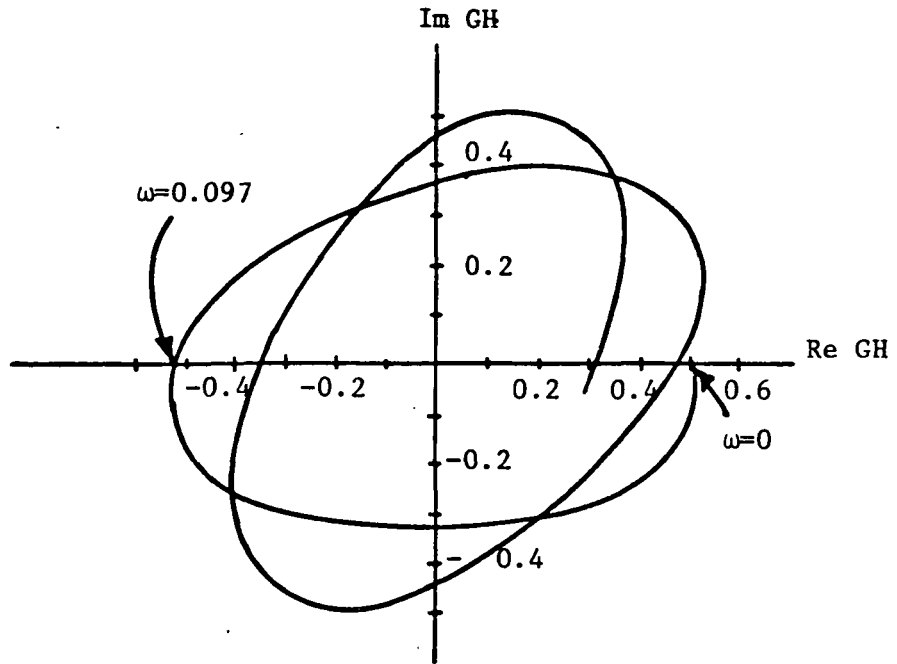


Figure 4.— Partial Nyquist plot for $Y_0 = 6.6$ and $\tau = 0$.

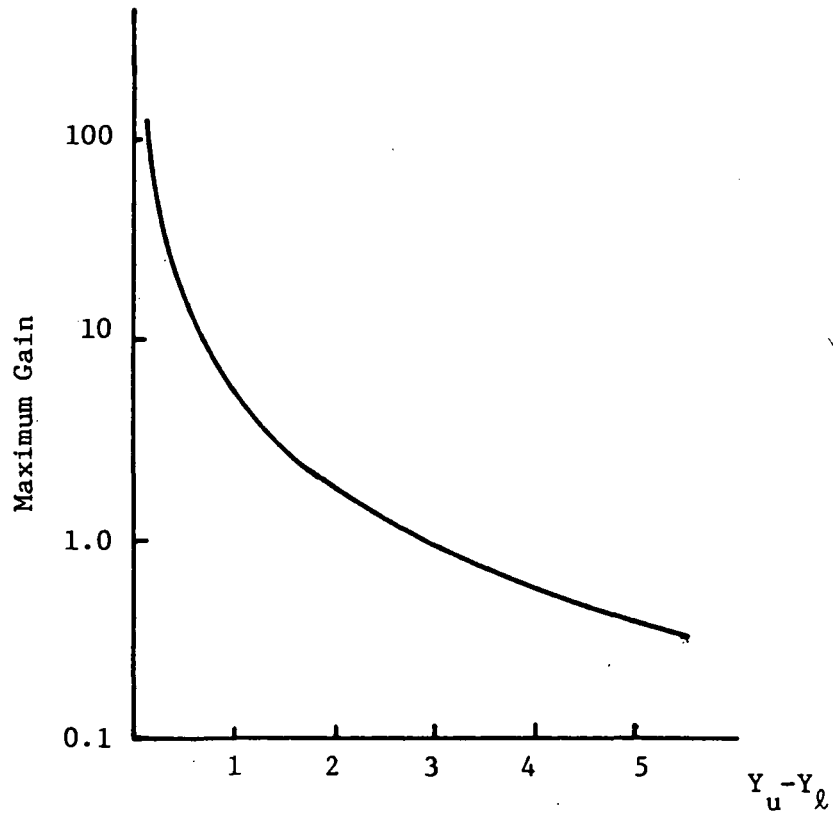


Figure 5.— Maximum stable gain $Y_0 = 6.6$ ft and $\tau = 0$.

ANALYSIS OF LARGE POWER SYSTEMS

Herman W. Dommel

University of British Columbia
Vancouver, B.C., Canada

INTRODUCTION

This paper is a survey of computer-oriented power system analysis as practiced in the electric utility industry. Problems of the interconnected system in the western United States may be emphasized more than problems in other parts of the country because this is where the author gained most of his experience during eight years with the Bonneville Power Administration in Portland, Oregon. Of necessity, the survey can only highlight a few points. (An excellent source of background material is the July 1974 issue of the Proceedings of IEEE on computers in the power industry (ref. 1).

Power systems are interconnected facilities for generating, transmitting, and distributing electric energy. The backbone of a power system is a network of high-voltage overhead transmission lines that interconnect the powerplants with the load centers. Most lines are operated as balanced three-phase systems, but there are also high-voltage direct current links. Power systems usually extend over many states and comprise many utility companies. Figure 1 shows the major transmission lines in the western United States and Canada, which are owned by about 40 utility companies who have formed the Western Systems Coordinating Council (WSCC) for joint studies on a combined systems basis. Joint studies are made by a technical staff located in Salt Lake City, which utilizes the computer facilities of the University of Utah. Similar "power pools" or "regional electric reliability councils" exist in other parts of the United States (fig. 2).

In contrast to other forms of energy, there are few economic ways yet for storing electric energy on a large scale. One practical solution is the pumped-storage plant, where water is pumped into a reservoir at times of low load consumption and excess production at other plants, such as run-of-the-river hydroplants, and thermal plants running at minimum output. By and large, the energy demanded by the customers must therefore be generated at all times, following the load demand curve. Interconnections have been built primarily to take advantage of the greater diversity of load in a larger region to keep the total installed capacity as low as possible, and also to share reserve capacity in case of plant outages. Any imbalance between generation and utilization would require partial shedding of load through selective switching or voltage reduction. All generators on the system run in synchronism at 60 Hz in the United States and Canada, and any disturbance to the normal operation, such as an insulator flashover, leads to relative oscillations of the machines against one another. If these oscillations do not die out fast enough, then the system may become unstable and "collapse." Therefore, the system must not only be balanced between generation and load under normal operation, but must also be designed to be stable against small and large disturbances.

The growth of power systems is determined by the load demand, which has been doubling about every 10 years in the United States as well as in most other industrialized countries. Simple

arithmetic will show that such a geometric progression cannot continue forever. The Club of Rome and others deserve credit for drawing attention to the limits of growth. There is reason to believe, however, that the portion of electric energy in the total energy consumption, which is about 25 percent today, may grow to about 50 percent by the year 2000, as more electric energy will be used for mass transportation, recycling, and other purposes. Therefore, electric power systems will continue to grow.

A reliable supply of electric energy without imposed restrictions on consumption requires (a) large amounts of capital expenditure and (b) careful planning of the total system expansion through analysis of the normal operation as well as of the effects of disturbances. The word "expansion" must be emphasized because planning is almost always concerned with additions and modifications of an already existing system. Power system planning is partly science and partly art and must answer questions of the type "where and at what time should a powerplant (transmission line) be built, what should be its rating, and should few units of large size be used or more units of smaller size?" For such studies, the load growth must be forecast in great detail for the next 5 years and in less detail for the next 20 years, always, of course, with a degree of uncertainty. Advance planning is essential because lead times (time from the decision to build something to its going into operation) are almost 5 to 10 years now for powerplants and 2 to 4 years for transmission lines.

For those engaged in power system analysis and power system planning – the "software side" of power system research – it is well to remember that it is primarily the components of the system (such as powerplants, transmission lines, and circuit breakers) that require the most expensive research. Normally, it is not so much a matter of new technology but of continuity.

HISTORIC PERSPECTIVE

Systems analysis has always been important in the power industry simply because additions of powerplants and transmission lines require so much capital expenditure that their influence on the behavior of the overall system has to be analyzed before they are built. There is very little room for modifications after installation. There is reason to believe that power systems were the first systems studied in the modern sense of systems engineering.

The mathematical foundations of power system analysis are very old. The well-known node and mesh equations were already explained in Maxwell's books published in 1873 (ref. 2). He, in turn, relied on work done by Kirchhoff, Helmholtz, and others. It was recognized almost 50 years ago that meaningful system studies were impossible with hand calculations, which led to the development of specialized analog computers ("network analyzers") in the 1920's and 1930's. Some are still in use today, and some are still being built for special purposes (e.g., for studies of electromagnetic transients).

SOLUTION OF SPARSE NETWORK EQUATIONS

Many problems in power system analysis lead to the solution of a system of linear equations. The steady-state behavior of an electric network is normally described by node equations of the form

$$[Y] [V] = [I] \quad (1)$$

where $[Y]$ is the nodal admittance matrix; $[V]$, the vector of voltages from node to datum; and $[I]$, currents injected into the node from datum. All matrices are normally complex. The diagonal element Y_{ii} is the sum of all admittances of the branches connected to node i , and the off-diagonal element Y_{ik} is the negative admittance of the branch connecting nodes i and k . Since only few branches are connected to each node in power systems, $[Y]$ is "sparse" (it has only a few nonzero entries). Nodes represent powerplants, load centers, and major substations, while branches represent lines, cables, and transformers.

Gauss elimination¹ has clearly become the preferred method for solving linear equations of the form of equation (1). It requires less memory and time than the Gauss-Jordan diagonalization process. In power system analysis, the elimination process for matrix $[Y]$ is normally separated from the elimination process for the right-hand side $[I]$. This offers advantages if the system has to be solved repeatedly with the same matrix $[Y]$ but with different right-hand sides $[I]$, which is frequently the case. For such "repeat solutions," only the process for the right-hand sides must be repeated. Often, $[Y]$ is symmetric; in that case, only the upper triangular matrix must be stored.

All realistic power system problems are under the curse of dimensionality. Power flow studies must routinely be performed for networks of more than 1000 nodes with more than 2000 branches. The major breakthrough in the solution of equations for such large power systems came in the early 1960's with the exploitation of sparsity by ordered elimination (refs. 3,4). This has reduced solution times and memory requirements by almost a factor of 100 in smaller systems and much more in larger systems (see the ratio in fig. 3). It is believed that sparsity techniques were pioneered in the power industry, but mathematicians are well aware of it now (refs. 5-7). The following table illustrates the savings for a moderately sized system:

Matrix data:

Number of nodes.....	267
Number of branches.....	423
Number of nonzero elements above diagonal after elimination in 267 X 267 matrix.....	1015

Solution times (IBM 7040):

Triangular factorization of complex matrix.....	16.5sec
Repeat solution.....	1.6 sec .

Without sparsity, the upper triangular matrix would have approximately 36,000 elements, which is approximately 36 times more than in the table. Figure 3 (from ref. 8) compares the numerical effort of straight-forward matrix inversion with that of ordered elimination with exploitation of sparsity for typical power network problems. Figure 4 (also from ref. 8) shows the network graph and the sparse matrix after triangularization for a 62-node water distribution network, which illustrates that sparsity can also be exploited in non-electric problems. Another example would be the analysis of pin-jointed mechanical structures such as transmission towers (ref. 9).

¹ Also called Gauss-Banachiewicz, triangulation, triangularization, triangular factorization, *LU* decomposition, Gauss-Doolittle, Crout, Cholesky, etc. (sometimes in modified forms).

FAULT STUDIES

Fault studies are made to find the fault currents if faults occur at various locations in the system. Most faults involve only one phase of the three phases, for example, a flashover across an insulator to the tower. Fault current values are needed to check whether they lie within the interrupting capacity of those circuit breakers that remove the faulted line selectively and temporarily from the system.² They are also needed to set the sensing devices for the circuit breaker tripping mechanism.

Classical fault studies solve the steady-state component of the fault current only, which is sufficient for the purposes mentioned before. The problem formulation leads to a system of linear equations of the form of equation (1). For balanced three-phase faults, a single-phase equivalent representation is used. For example, the voltage drop along the three phases of a line

$$-\begin{bmatrix} \frac{dV_a}{dx} \\ \frac{dV_b}{dx} \\ \frac{dV_c}{dx} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2)$$

(where Z_s is self impedance and Z_m is mutual impedance) can be simplified for balanced conditions ($I_b = I_a e^{-j120^\circ}$, $I_c = I_a e^{j120^\circ}$, analogous for V) to the single-phase equivalent

$$-\frac{dV_a}{dx} = (Z_s - Z_m)I_a \quad (3)$$

The solution for phases B and C is the same as that for phase A , except for phasor rotations with a factor $e^{\pm j120^\circ}$. The expression $(Z_s - Z_m)$ is the equivalent impedance for balanced three-phase operation.

For unbalanced faults, such as flashover from one conductor to the tower, symmetrical components are used. This is a well-established technique in power system analysis (ref. 10) whereby the three coupled phase equations such as equation (2) are transformed into three decoupled equations:

$$-\begin{bmatrix} \frac{dV_0}{dx} \\ \frac{dV_1}{dx} \\ \frac{dV_2}{dx} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (4)$$

²The removal time ("dead time") is kept as short as possible (just long enough to permit arc extinction in case of flashovers); it is noticed by the consumer only as a brief flicker.

with the linear transformation,

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (5)$$

for V and one that is identical for I , with $a = e^{j120^\circ}$. The new variables 0,1,2 are called zero, positive, and negative sequence components. With symmetrical components, one has to solve N equations three times rather than $3N$ equations once, which saves memory and computer time. With sparsity techniques, however, the savings may no longer be as impressive because solution times increase about linearly with N rather than with N^3 .

The techniques for classical fault studies are well developed, and it is unlikely that worthwhile improvements can be made.

POWER FLOW STUDIES

The flow of electric energy in interconnected ac systems is determined by the branch impedances, as expressed in Kirchhoff's laws, and cannot be controlled on an individual line. (There are exceptions, such as real power control with phase shifting transformers and high-voltage direct current links.) When lines are added to an existing system or when the effect of line outages is to be studied, it is therefore necessary to study the flows in the entire system. This is the power flow or load flow problem. Normally, only balanced conditions are studied with single-phase equivalents such as equation (3). Formulating the power flow equations in an N -node system leads to $N-1$ nodal equations of the form of equation (1), except that the current is a function of the voltages,

$$I_k = \frac{P_k - jQ_k}{V_k^*} \quad (6)$$

where P_k is the real power into node k , Q_k is the reactive power into node k , and V_k^* is the conjugate complex voltage from node k to ground. Normally, P_k and Q_k are specified, or P_k and $|V_k|$ are specified. Equation (6) not only makes the power flow equations nonlinear but also nonanalytic because of the conjugate complex term. Therefore, the complex derivative is not defined, and numerical techniques based on derivatives must use pairs of real equations rather than complex equations and rectangular or polar coordinates rather than complex variables.

The standard solution technique for power flow problems is now Newton's method (ref. 11). The system of power flow equations

$$[g([x])] = 0, \quad (7)$$

with the unknown vector $[x]$, is solved iteratively with the system of linearized equations:

$$\left[\frac{\partial g}{\partial x} \right] [\Delta x] = -[g] \quad (8a)$$

and

$$[x^{(h)}] = [x^{(h-1)}] + [\Delta x] . \quad (8b)$$

The Jacobian matrix $[\partial g/\partial x]$ and the right-hand side $-[g]$ in equation (8a) is evaluated at the approximate solution point $[x^{(h-1)}]$. Newton's method only became practical for large power systems after sparsity techniques had been developed. The Jacobian matrix shows basically the same sparsity pattern as the admittance matrix in equation (1). Typically, a solution with good accuracy is reached in three to four iteration steps, independent of the size of the system.

The Jacobian matrix gives a linearized model of the power flow equations around the solution point; therefore, it is very easy to calculate first-order sensitivities with repeat solutions, provided the triangularized Jacobian matrix has been stored. The equations are simply

$$\left[\frac{\partial g}{\partial x} \right] [\Delta x] = - \left[\frac{\partial g}{\partial p} \right] [\Delta p] \quad (9)$$

with $[p]$ being those parameters for which the influence on the solution vector $[x]$ is sought.

The Jacobian matrix can also be used to calculate reduced gradients as the basis of optimization studies (refs. 12,13). (These techniques are beyond the scope of this paper.)

STABILITY STUDIES

Large disturbances, such as short circuits or powerplant outages, cause electromechanical transients in the form of relative oscillations between synchronous machines. These oscillations may be large enough to cause loss of synchronism in one or more machines. A generator that loses synchronism because of some disturbance is automatically disconnected from the system to avoid overheating and damage. Often, this increases the severity of the disturbance for other generators and, in turn, more generators may lose synchronism ("cascading outages"). This is the typical course of events in "blackouts."

Stability problems are more critical in geographically large systems (as in fig. 1) than in tightly meshed systems. Stability first became important in the 1930's when hydroelectric plants were built far away from the load centers. At that time, "swing curves" (= rotor oscillations as a function of time relative to synchronous speed) were calculated to the crest of the first swing because power systems had enough damping that synchronism was practically never lost on subsequent swings. Today, stability must be checked beyond the first swing because modern fast-acting excitation systems on generators have decreased the system damping. The interconnection of formerly disconnected power systems has also created new stability problems in the form of spontaneous oscillations, which are sometimes damped with supplementary control signals in exciters and turbine governors.

Lyapunov's second method has been proposed to find the region of stability directly without simulation. Many papers have been written on the subject because it is a challenging theoretical concept for scientific reasoning, but it is not yet a competitive alternative to the existing transient stability simulation programs. The region of stability obtained by Lyapunov's method is too conservative since the condition for stability is only sufficient but not necessary. In other words, the system might still be stable outside that region. The shortcomings of the method are overwhelming at this time and it seems questionable whether application to realistic power systems will ever be feasible (ref. 14), even though others disagree (ref. 15).

Since there are no practical methods yet for assessing stability directly, simulating the behavior as a function of time, for specific disturbances assumed by the planner, remains the only practical alternative. Its main drawback is that the question of system stability is only answered for the specific disturbance, starting from specific initial conditions. Production-type stability programs can solve systems with up to 2000 nodes and 600 generators step by step with about 10 to 20 different exciter models and 5 to 15 different turbine governor models.

In simulating electromechanical transients, two systems of equations must be solved simultaneously, namely, the system of power flow equations.³

$$[g([x],[y])] = 0 \tag{10}$$

and a system of differential equations,

$$\left[\frac{dy}{dt} \right] = [f([x],[y],t)] \tag{11}$$

which describe the dynamic behavior of the turbine-generator rotors and of the exciters and turbine governors.

Step-by-step solution methods for stability programs are classified in reference 16. Most programs solve the differential equations and the power flow equations alternately, using a prediction of power flow state variables to solve the differential equations over one time step and using these results to obtain the new power flow solution at the end of the time step. There is a large spread in the magnitude of the eigenvalues in equation (11), assuming that the equations are linear or linearized of the form $[dy/dt] = [A][y]$. Explicit methods such as fourth-order Runge-Kutta are very slow for such "stiff systems"; they require a small time step, dictated by the smallest time constants, which one would not anticipate from the smooth curves for the rotor oscillations. This "small time constant barrier" is being overcome with implicit integration schemes, such as the trapezoidal rule (ref. 16), which has been used quite successfully for electromagnetic transients in power systems since at least 1961 (see ref. 11 in ref. 17).

³The rotor oscillations are slow enough to permit the use of steady-state equations for the network part.

SUMMARY

Only some topics of power system analysis could be described. While they cover the “big three” problems of fault, power flow, and stability studies, there are many other analysis problems in power systems which are partly summarized in reference 1.

One topic omitted here, the computation of electromagnetic transients, would have provided a good example of transfer of knowledge from one discipline to another, which is at least partly the objective of this workshop. The method of characteristics described in reference 17, which has become the standard solution technique for traveling wave problems on transmission lines, was first used to study pressure waves in hydraulic systems in the late 1920's (see refs. 7 and 8 in ref. 17). It also illustrates that older techniques developed for hand calculations may still be valuable in our days of powerful computers, an observation which is also true for implicit integration with the trapezoidal rule.

The solution techniques for most power system problems are highly developed by now and relatively efficient because of the exploitation of sparsity. Improvements are still possible, of course.

REFERENCES

1. Anon.: Proceedings of the IEEE, vol. 62, July 1974 (special issue on computers in the power industry).
2. Maxwell, J. C.: A Treatise on Electricity and Magnetism, vol. 1, Clarendon Press, Oxford, 1873 (reprinted by Dover Publications, New York).
3. Sato, N.; and Tinney, W. F.: Techniques for exploiting the sparsity of the network admittance matrix, IEEE Trans. Power Apparatus and Systems, vol. PAS-82, pp. 944–950, Dec. 1963.
4. Tinney, W. F.; and Walker, J. W.: Direct solutions of sparse network equations by optimally ordered triangular factorization, Proc. IEEE, vol. 55, pp. 1801–1809, Nov. 1967.
5. Willoughby, R. A. (Editor): Proceedings of the Symposium on Sparse Matrices and Their Applications. IBM Watson Research Center, Yorktown Heights, New York, 1969.
6. Reid, J. K. (Editor): Large Sparse Sets of Linear Equations. Academic Press, London, New York, 1971.
7. Rose, D.J.; Willoughby, R. A. (Editors): Sparse Matrices and Their Applications. Plenum Press, New York, 1972.
8. Tinney, W. F.; and Meyer, W. S.: Solutions of large sparse systems by ordered triangular factorization, IEEE Trans. Automatic Control., vol. AC-18, pp. 333–346, Aug. 1973.
9. Jensen, H. G.: Efficient matrix techniques applied to transmission tower design, Proc. IEEE, vol. 55, pp. 1997–2000, Nov. 1967.
10. Fortescue, C. L.: Method of symmetrical coordinates applied to the solution of polyphase networks, Trans. AIEE, vol. 37, p. 1027, 1918.
11. Stott, B.: Review of load-flow calculation methods, Proc. IEEE, vol. 62, pp. 916–929, July 1974.
12. Dommel, H. W.; and Tinney, W. F.: Optimal power flow solutions, IEEE Trans. Power Apparatus and Systems, vol. PAS-87, pp. 1886–1876, 1968.
13. Sasson, A.M.; and Merrill, H.M.: Some applications of optimization techniques to power system problems, Proc. IEEE, vol. 62, pp. 959–972, July 1974.
14. Foerst, K. R.: Feasibility study about the application of Lyapunov functions for transient stability monitoring, Bonneville Power Administration, Portland, Ore., memo to A. R. Benson, March 1971 (includes a critical review of 22 papers on the subject).
15. Williams, H. F.; Louie, S. A.; and Bills, G. W.: Feasibility of Liapunov functions for the stability analysis of electric power systems having up to 60 generators, IEEE Trans. Power Apparatus and Systems, vol. PAS-91, pp. 1145–1157, May/June 1972.
16. Dommel, H. W.; and Sato, N.: Fast transient stability solutions, IEEE Trans. Power Apparatus and Systems, vol. PAS-91, pp. 1643–1650, July/August 1972.
17. Dommel, H.W.; and Meyer, W.S.: Computation of electromagnetic transients, Proc. IEEE, vol. 62, pp. 983–993, July 1974.

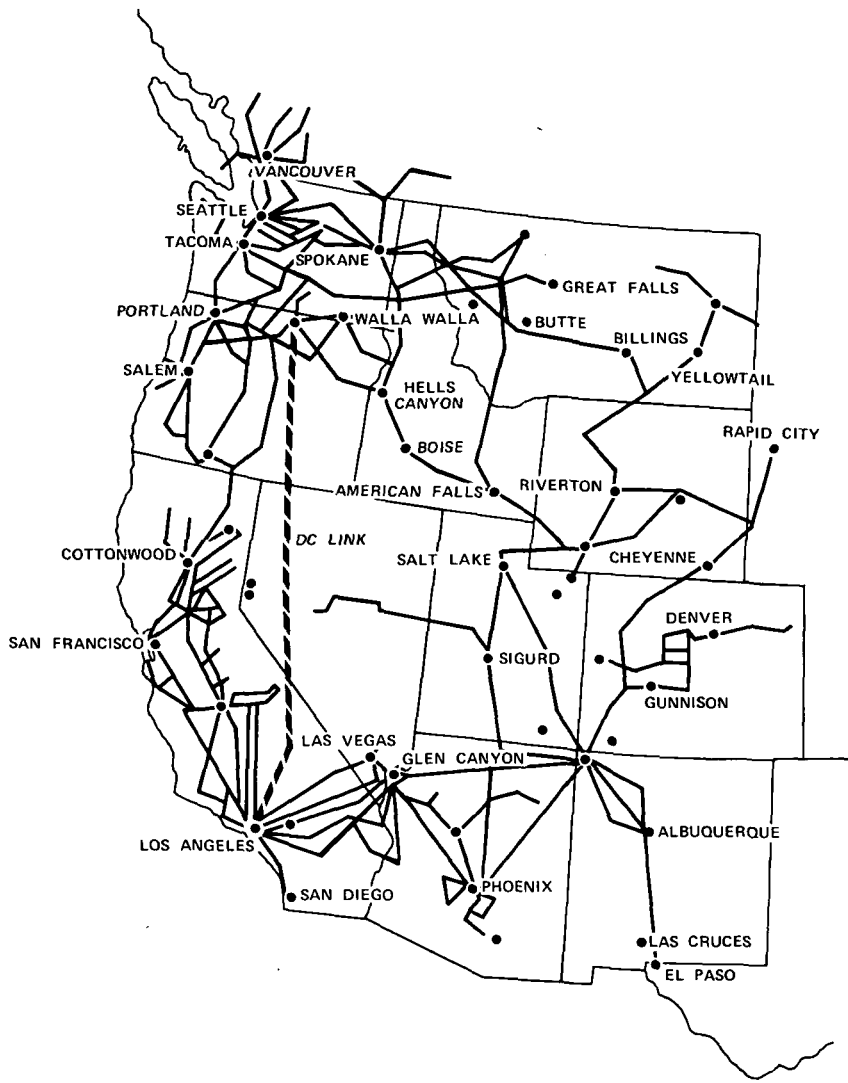


Figure 1.—Major transmission lines in the western United States. Source: Annual Report 1972 of the Western Systems Coordinating Council.

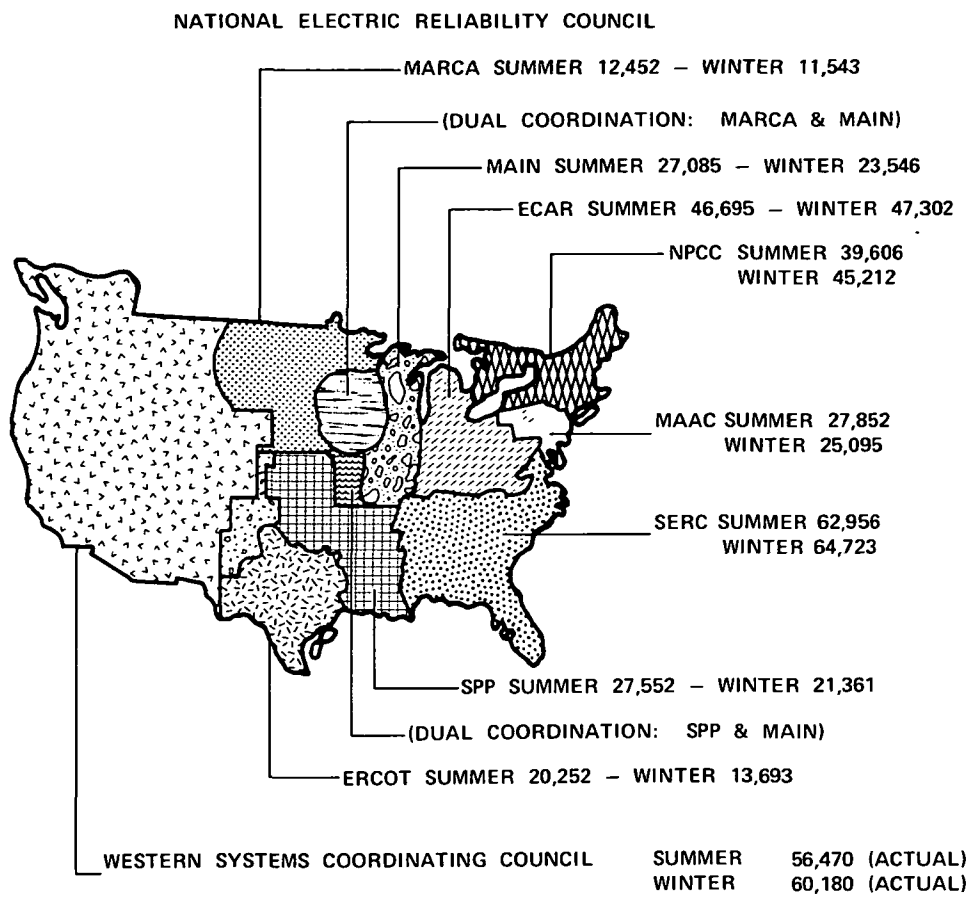


Figure 2.— Regional electric reliability councils. Peak loads in MW for Summer 1972 and Winter 1972/73 (forecast). Source: Sept. 1972 report of the National Electric Reliability Council.

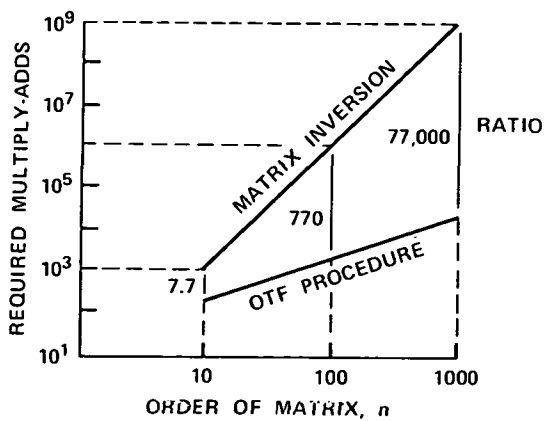


Figure 3.—Comparison of the numerical effort required by matrix inversion with that of ordered triangular factorization (OTF) for typical power network problems.

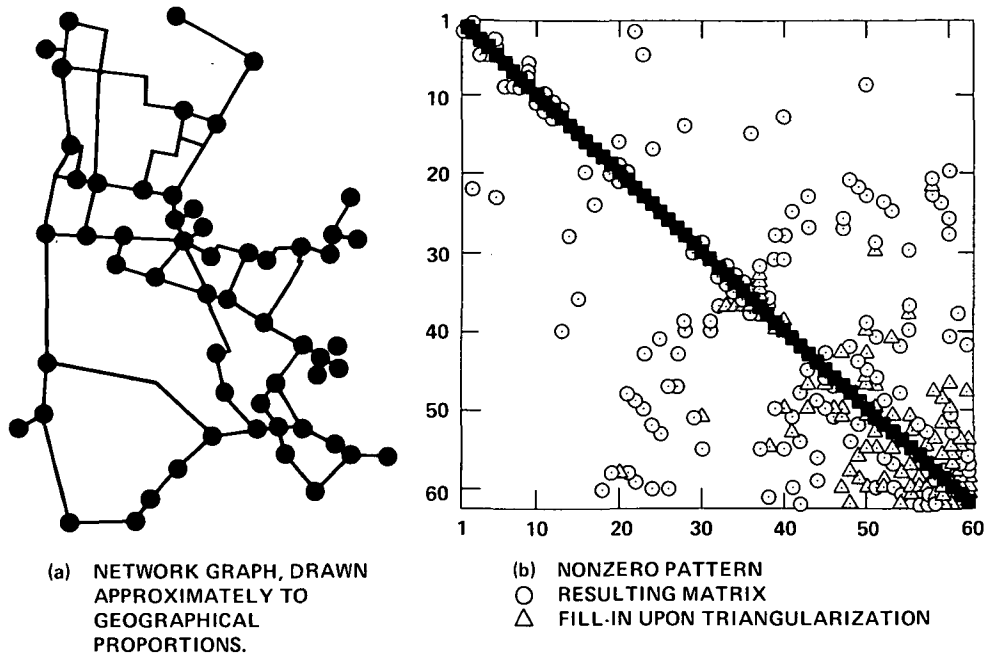


Figure 4.—Sample small 62-node water distribution network.

DECENTRALIZED REGULATION OF DYNAMIC SYSTEMS

Kai-ching Chu

IBM Watson Research Center, Yorktown Heights, New York 10598

INTRODUCTION

In classical control and decision-making problems, the system is handled in a centralized fashion, namely, there is a single supervisor who handles all the information processing and decision making for the entire system. The decisions of control policies and their implementation are all made according to the preference of this central supervisor.

By contrast, when a large-scale system is considered, such as those arising from the studies of socio-economic problems and electrical power systems, information processing and control policy decision are delegated to a set of agents. Generally, these agents have different information, different permissible control notices, and different preference orderings. These agents may act in complete independence; some may coordinate or supervise the actions of others and hence they form a certain hierarchical decision structure in the system. The behavior of the entire system will result from the interaction of all the decisions by these agents. Such an environment is called here decentralization decision making.

This paper deals with a special class of decentralized control problem in which the objectives of the agents are to steer the state of the system to certain desirable levels. Such a problem is called the decentralized regulation problem. Each agent is concerned about certain aspects of the state of the entire system. The following defines notation:

$$\begin{aligned} \text{State: } \quad x^T &= (x_1^T, x_2^T, \dots, x_n^T) & x_i &\in R^{n_i} \\ \text{Control: } \quad u^T &= (u_1^T, u_2^T, \dots, u_n^T) & u_i &\in R^{m_i}. \end{aligned}$$

The state variables are affected by the individual controls by the following differential equations:

$$\begin{aligned} \dot{x}_1 &= f_1(x, u) \\ \dot{x}_2 &= f_2(x, u) \\ &\vdots \\ \dot{x}_n &= f_n(x, u) . \end{aligned} \tag{1}$$

The objective of agent i is to use control u_i to affect the system so that state x_i can approach a certain desirable level as time $t \rightarrow \infty$. Without losing generality, we can assume that such desirable levels are zero for all agents.

The information available to agent i at any time is assumed to be a mapping of the present state x to his data space, that is, for the i th agent information:

$$y_i = h_i(x), \quad h_i : \prod_j R^{n_j} \rightarrow R^{p_i}. \quad (2)$$

Control strategy is assumed to be of a feedback form from information y_i to u_i , that is,

$$\text{Control: } u_i = \gamma_i[h_i(x)], \quad \gamma_i : R^{p_i} \rightarrow R^{m_i}. \quad (3)$$

The mapping γ : above is chosen from the permissible set of functions Γ_i . Two sets of questions arise from the above function:

(1) **Stabilization:** Given the structure (f_i, h_i, Γ_i) , is it feasible to find $\{\gamma_i\}$ such that $x_i \rightarrow 0$ as $t \rightarrow \infty$ for all i ? If it is not feasible, what kind of structure modification will enable us to make it feasible?

(2) **Optimization:** When it is feasible to regulate all the states, $x_i \rightarrow 0$ as $t \rightarrow \infty$, what will be the optimal $\{\gamma_i\}$ to achieve such goals with respect to certain performance criteria? What is the impact of various information structures to the performance of regulation?

To be more specific, we shall limit ourselves to linear time-invariant systems:

$$\dot{x} = Ax + \sum_{i=1}^n B_i u_i \quad (1')$$

where

$$\begin{aligned} x^T &= (x_1^T, \dots, x_n^T) \\ y_i &= H_i x \end{aligned} \quad (2')$$

and

$$u_i = F_i y_i \quad (3')$$

where F_i is a constant real matrix of appropriate dimension.

STABILITY AND COORDINATION

Agent i applies control $u_i = F_i z_i$ to regulate the state x_i so that its desirable level $x_i = 0$ will be achieved and maintained. Because the actions by all agents are coupled, the ability of agent i to regulate state x_i depends on the actions of the other agent. On the other hand, a control u_i of

agent i supposedly to decrease the deviation of x_i from zero may affect the other system state variables x_j . This paper discusses the interactions of these individual regulation actions.

Individual Stability

System state variable x_i is said to be *individually stable* with feedback control gain F_i if agent i applies the control $u_i = F_i x_i$ and all other agents take no actions ($F_j = 0 \forall j \neq i$) and $t \rightarrow \infty$ implies $x_i \rightarrow 0$. The collection of all such F_i is denoted by S_i .

Collective Stability

System state variable x is said to be *collectively stable* with feedback control gains (F_1, \dots, F_n) if, for all i , agent i applies control $u_i = F_i x_i$ and $t \rightarrow \infty$ implies $x_i \rightarrow 0$ for all i . The collection of all such $(F_1 x \dots, x F_n)$ is denoted by S_c .

The following two observations can be made:

- (1) $F_i \in S_i \forall i$ does not imply $(F_1 x \dots x F_n) \in S_c$
- (2) $F_i \notin S_i \forall i$ does not imply $(F_1 x \dots x F_n) \notin S_c$.

These two facts are easily demonstrated by the following scalar example:

$$\begin{aligned}\dot{x}_1 &= -x_1 + ax_2 + u_1 + u_2 \\ \dot{x}_2 &= -x_2 + ax_1 + u_1 + u_2.\end{aligned}$$

Both u_1 and u_2 influence x_1 and x_2 . Agent 1 has access only to information x_1 and agent 2 has access only to information x_2 , that is, $u_1 = f_1 x_1$ and $u_2 = f_2 x_2$. For collective stability, the example requires that $f_1 + f_2 < 1 - a$. For individual stability, it requires that $f_1 < 1 - a$ and $f_2 < 1 - a$. Set S_1, S_2 , and S_c are illustrated in figure 1(a) and (b). When $-1 < a < 1$, $S_1 \times S_2 \not\subset S_c$ and when $1 < a$, $S_1 \times S_2 \subset S_c$. For $-1 < a < 1$, when both agents use their $F_i \in S_i$ purposely to regulate their state, it is possible that none of them will achieve that goal. For $1 < a$, any individual action $F_i \in S$ will guarantee the results $x_i \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1$ and 2. For $1 < a$, it may also be interesting to note that it is possible to have $F_i \notin S_i$ for both $i = 1$ and 2 while $x_{1,2} \rightarrow 0$ as $t \rightarrow \infty$.

If $F_i \in S_i$ for all i implies $(F_1 x \dots x F_n) \in S_c$, the system is called coordinated. In a coordinated system, each agent need only to assess his stability regime S_i . If each of them pick an $F_i \in S_i$, everyone will achieve $x_i \rightarrow 0$.

A noncoordinated system can be made coordinated by imposing constraints on the permissible control policies each agent can use. Such constraints should be imposed by a certain *central coordinator* (planner). For instance, in the example given above, when $-1 < a < 1$, if both agents are first forbidden to pick a feedback gain $\geq (1 - a)/2$, the system becomes coordinated.

The concept of coordination in a system is important. In a coordinated system, agents have a greater degree of autonomy. Both the decision of his control policy and implementation of such control can be done by total decentralization. In a noncoordinated system, the solution of all control policies for each agent may require the help of a central coordinator, which may involve very difficult computation. Substituting $u_i = F_i y_i$ and $y_i = H_i x$ into $\dot{x} = Ax = \sum B_i u_i$, it becomes

$$\dot{x} = (A + \sum B_i F_i H_i) x \quad (4)$$

Obviously, the condition for collective stability is that all eigenvalues of $A + \sum B_i F_i H_i$ have negative real parts.

A more basic question to ask is that, for given structure matrices (A_i, B_i, H_i) , is it ever possible to find matrices (F_1, \dots, F_n) that stabilize the system? Fisher and Fuller (ref. 1), McFadden (refs. 2, 3), and Aoki (ref. 4) have all tried to answer this *stabilizability* problem in certain aspects. The general stabilizability problem for system equation (4) has been solved recently by Wang and Davison (ref. 5). The problem of finding conditions on (A_i, B_i, H_i, F_i) such that the system is coordinated, controlled by agents and can fully be decentralized is still open.

OPTIMIZATION OF DECENTRALIZED CONTROL

When the system can be stabilized collectively, we would like to consider how to optimize the choice of (F_1, \dots, F_n) . Costs are associated with the deviation of the states from their desirable values and the magnitudes of the control "forces." The objective of the central coordinator (planner) is to choose (F_1, \dots, F_n) to minimize these costs. Suppose that the costs are described by the following quadratic loss function:

$$J = \frac{1}{2} \int_0^x (x^T Q x + \sum u_i^T R_i u_i) dt \quad (5)$$

where $Q \geq 0$ and $R_i > 0 \forall i$. Since

$$\dot{x} = D x$$

where

$$D = A + \sum B_i F_i H_i \quad (6)$$

we have

$$x = e^{Dt} x_0 \quad (7)$$

where x_0 is the state deviation at $t = 0$. Then

$$\begin{aligned}
J &= \frac{1}{2} \int_0^{\infty} x_0^T e^{D^T t} (Q + \Sigma H_i^T F_i^T R_i F_i H_i) e^{Dt} x_0 dt \\
&= \frac{1}{2} \text{trace} \left[\int_0^{\infty} e^{D^T t} (Q + \Sigma H_i^T F_i^T R_i F_i H_i) e^{Dt} dt x_0 x_0^T \right].
\end{aligned} \tag{8}$$

It can be shown that, if D is stable, J can be expressed as (ref. 6)

$$J = \frac{1}{2} \text{trace} (K x_0 x_0^T) \tag{9}$$

where

$$KD + D^T K + Q + \Sigma H_i^T F_i^T R_i F_i H_i = 0. \tag{10}$$

Since D is linear in $\{F_i\}$, K solved by equation (10) is a rational function of $\{F_i\}$. The determination of optimal $\{F_i\}$ falls in the framework of classical nonlinear programming.

It can be shown (ref. 7) that the gradient at the optimal choice of $\{F_i\}$ must satisfy

$$0 = \frac{\partial J}{\partial F_i} = R_i F_i H_i L H_i^T + B_i K L H_i^T; \quad i = 1, \dots, n \tag{11}$$

where

$$LD^T + DL + x_0 x_0^T = 0. \tag{12}$$

The necessary condition of optimality (eqs. (10)–(12)) is given here as the generalization of the single-agent output optimization problems studied by Levine and Athans (ref. 8). Note that sometimes the initial value x_0 is not known exactly. The product $x_0 x_0^T$ in equations (9) and (12) should be replaced by its expected value. Furthermore, if we wish to find the optimal $\{F_i\}$ independent of the initial states, we could use an approach by assuming that the initial state is random and distributed uniformly in a sphere, namely, the expected value of $x_0 x_0^T$ is expressed as an identity matrix.

What is expressed in equations (10), (11), and (12) is the necessary condition of optimal $\{F_i\}$; it is entirely possible to have more than one solution to such conditions. In any case, the determination of optimal $\{F_i\}$ involves many structure parameters and we cannot possibly expect elegant solutions unless the system is of very low dimension or of high dimension with nice structures (e.g., in symmetry or repetition).

Since in practice only a high-dimension system justifies decentralized control, we should examine in the sequel some examples of high-dimension decentralized problems where the system is nicely structured. With such a system, we will be able to derive some results and to understand the relations between structure and control system behavior in more explicit terms.

SPECIAL LARGE-SCALE SYSTEMS

Sequential Systems (ref. 9)

The system can be decomposed into a sequence of subsystems, all identical in structure (see fig. 2). The interactions among subsystems depends only on the distance, that is, the difference in the subsystem indices.

Subsystem i : state x_i and control u_i

$$x^T = (\dots, x_{i-1}^T, x_i^T, x_{i+1}^T, \dots) \quad (13)$$

$$\dot{x}_i = \sum_j A_{i-j} x_j + \sum_j B_{i-j} u_j \quad (14)$$

$$y_i = \sum_j H_{i-j} x_j \quad (15)$$

For simplicity, the number of subsystems in the string is assumed to be infinite so that there are no ends in the string and the roles of all subsystems are identical. If a system comprises a large but finite number of subsystems, it can be treated as if the number were infinite by assuming fictitious subsystems at both ends of the string.

Since all subsystems are identical, the feedback gains used in each control can also be assumed identical, that is,

$$u_i = F y_i \quad \text{for all } i \quad (16)$$

Bilateral transformations can be used to convert the sequential subsystems into a lumped system in the z domain:

$$\{x_i\} \xrightarrow{z} X(z) = \sum_{i=-\infty}^{\infty} x_i z^{-i} \quad (17)$$

$$\{u_i\} \xrightarrow{z} U(z) = \sum_{i=-\infty}^{\infty} u_i z^{-i} \quad (18)$$

etc.

Optimal conditions of F can be obtained by converting equations (9), (10), (11), and (12) into appropriate counterparts involving a z transformation (ref. 9).

Large-Scale Systems in Arrays

This configuration is illustrated in figure 3. It is similar to the sequential system in the previous section, except that the subsystems are distributed in two-dimensional arrays. If the structures and interactions of all subsystems are all identical subject only to index translation, double bilateral z-transformation techniques can be used to solve the optimization problem. Such models could represent situations of street traffic control, huge electric network control, etc.

Regulation of Vehicular Strings (ref. 10)— A string of high-speed, densely packed vehicles are moving along a certain guideway. It is desirable to keep the spacings and velocities of all vehicles in the string as close as possible to certain predetermined values (see fig. 4). The position deviation of the k th vehicle from its predetermined reference is denoted by x_k . The dynamics of the k th vehicle can be described by the second-order, normalized differential equation:

$$\ddot{x}_k + \alpha \dot{x}_k = u_k ; \quad -\infty < k < \infty . \quad (19)$$

The total information data for the k th vehicle is denoted by a vector y_k given by

$$y_k = \sum_j H_{k-j} \begin{bmatrix} x_j \\ \dot{x}_j \end{bmatrix} ; \quad -\infty < k < \infty . \quad (20)$$

The structure of equation (20) includes the following special cases:

$$(i) \quad y_k^T = (x_{k-1}, x_k, x_{k+1}, \dot{x}_{k-1}, \dot{x}_k, \dot{x}_{k+1})$$

$$(ii) \quad y_k^T = (x_k, \dot{x}_k)$$

$$(iii) \quad y_k^T = (x_{k-1}, x_k, x_{k+1})$$

$$(iv) \quad y_k^T = (x_k)$$

$$(v) \quad y_k^T = (x_j, \dot{x}_j | -\infty < j < \infty) .$$

The linear feedback control used by each vehicle is of the same form:

$$u_k = F y_k ; \quad -\infty < k < \infty . \quad (21)$$

The regulation cost is assumed to be of the following form where the magnitude of the control forces and relative vehicle position errors are penalized.

$$J = \frac{1}{2} \int_0^{\infty} \sum_k \left[q(x_{k-1} - x_k)^2 + u_k^2 \right] dt . \quad (22)$$

Results— If $\alpha = 1$, $q = 10$, and the information is given as

$$z_k = (x_{k-1}, x_k, x_{k+1}, \dot{x}_{k-1}, \dot{x}_k, \dot{x}_{k+1})^T. \quad (23)$$

The optimal control for each vehicle is

$$u_k = f_0 x_k + f_1 (x_{k-1} + x_{k+1}) + g_0 \dot{x}_k + g_1 (\dot{x}_{k-1} + \dot{x}_{k+1})$$

where

$$f_0 = -4.06, \quad f_1 = 1.43, \quad g_0 = 1.94, \quad g_1 = 0.52. \quad (24)$$

The associated minimal cost is

$$J^* = 7.59. \quad (25)$$

Figure 5 represents the stability region as well as the sensitivity of J to various choices of feedback gains. Figure 6 shows the behavior of a string of 10 vehicles when the optimal control scheme is applied. The position of each vehicle is plotted relative to a coordinate system moving with the desired velocity. At $t = 0$, all vehicles are subject to random perturbations in position and velocity. Also, vehicle 5 is constantly driven with a sinusoidal disturbance force. For comparison, figure 7 shows the response of the 10 vehicles when the controllers chosen are too sensitive to the motion of the neighboring vehicles — the result is a chain collision.

INFORMATION STRUCTURE DESIGN

Structure Design versus Stabilizability

The problem of stabilizability for a given structure (A, B_i, H_i) was discussed under Stability and Coordination. A more basic problem is how the information structures H_i influence the stabilizability. In other words, if a system cannot be stabilized initially by the imposed decentralized scheme, what kind of change in information structure can induce stabilizability?

Example— In the vehicular string regulation problem presented earlier, it can be shown that, if the information of vehicle k 's state x_k is available to him, the system can be made stable by appropriate choice of the feedback gains. If the information available to the controllers is only velocity data with no position data, the entire system can never be stabilized by any choice of feedback gains.

Example — Assignment Problem (ref. 3)— In a special decentralized regulation problem,

$$\dot{x} = \Sigma B_i u_i. \quad (26)$$

If control agent i knows only about the i th component of x , the feedback will be of the following form:

$$u_i = F_i x_i \quad (27)$$

hence

$$x = [B_1, B_2, \dots, B_n] \begin{bmatrix} F_1 & & 0 \\ & F_2 & \\ 0 & & \dots & F_n \end{bmatrix} x. \quad (28)$$

A nonstabilizable system could possibly be stabilized by permitting the information available to each agent. It has been shown that if the matrix $[B_1, B_2, \dots, B_n]$ is nonsingular, it is always possible to stabilize the system of equation (28) by an appropriate permutation of the information structure, that is,

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = Px$$

where P is a permutation matrix (refs. 3, 11).

Structure versus Regulation Performance

As expressed in equation (5), the regulation performance of the entire system is given by a certain cost function J . When the regulation is feasible (stabilizable), it is interesting to ask how each piece of information contributes to the optimization of J . A useful concept of the *value of information* can be defined. Simply speaking, the value of information can be visualized as the difference between the optimal J^* with the given information and the optimal J^* obtained without that particular information.

Reconsider the vehicular string regulation problem. A list of certain information structures with the optimal J^* of each is given below (see also fig. 8).

Information available to the k th vehicle	Optimal J^*
Case A: $x_{k-1}, x_k, x_{k+1}, \dot{x}_{k-1}, \dot{x}_k, \dot{x}_{k+1}$	7.59
Case B: x_{k-1}, x_k, x_{k+1}	11.30
Case C: x_k, \dot{x}_k	8.13
Case D: x_k	11.81
Case E: $\{\phi\}$ (nothing)	∞
Case F: $\{x_j, \dot{x}_j \mid -\infty < j < \infty\}$	7.54

If each vehicle is provided with its own state information x_k , $J^*(D)$ is 11.81. The following are values of certain added pieces of information for the k th vehicle:

(i) \dot{x}_k	3.68	$J^*(D) - J^*(C)$
(ii) x_{k-1} and x_{k+1}	.50	$J^*(D) - J^*(B)$
(iii) $x_{k-1}, x_{k+1}, \dot{x}_{k-1}, \dot{x}_{k+1}$.54	$J^*(C) - J^*(A)$
(iv) \dot{x}_{k-1} and \dot{x}_{k+1}	.03	$J^*(iii) - J^*(ii)$
(v) $\{x_j, \dot{x}_j \mid \text{for all } j-k \geq 2\}$.05	$J^*(A) - J^*(F)$.

For regulating this vehicular system, the controller's own velocity data \dot{x}_k is much more important than the data x_{k-1} and x_{k+1} . Moreover, the remote input data $\{x_j, \dot{x}_j \mid |j-k| \geq 2\}$ do not significantly add to the optimization of the regulation performance. If the structure in case A is adopted, despite the fact that much less information is required than for case F, the performance index value is very close to the ultimate minimum.

In real implementation of various control schemes, it is important to consider the cost of installing various measurement and control mechanisms and the feasibility of establishing various links for data and controls. The concept of information value discussed here provides a quantitative method for measuring and comparing the relative merits of different information structures and hence the usefulness of the different information provided.

REFERENCES

1. Fisher, M. and Fuller, A. T.: On Stabilizability of Matrices and the Convergence of Linear Iterative Processes, *Proceedings of the Cambridge Philosophical Society*, vol. 54 (1958), 417-425.
2. McFadden, D.: On Hicksian Stability, Value, Capital and Growth, J. N. Wolf ed. 1968.
3. McFadden, D.: On the Controllability of Decentralized Macro-economic Systems; assignment problems, *Mathematical Systems Theory and Economics*, Volume I, pages 221-239, 1963 ed. by H. Kuhn and G. Szego.
4. Aoki, M.: On Feedback Stability of Decentralized Dynamics Systems, *Automatiza*, vol. 8, 163-173, 1972.
5. Wang, S. H.; Davison, E. J.: On the Stabilization of Decentralized Control Systems, *IEEE Trans. on Automatic Control*, vol. 18, no. 5, Oct. 1973, 473-478.
6. Bellman, R.: *Introduction to Matrix Analysis*, New York, McGraw Hill, 1970.
7. Chu, K. C.: Optimal Decentralize Regulation of Linear Dynamic Systems, IBM Research Division, RC Report 1974 (to appear).
8. Levine, W. and Athans, M.: On the Determination of the Optimal Constant Output Feedback Gains for Linear Multivariable Systems, *IEEE Trans. on Automatic Control*, vol. 15, Feb. 1970, 44-48.
9. Chu, K. C.: Optimal Decentralized Regulation for a String of Coupled Systems, *IEEE Trans. on Automatic Control*, vol. 19, 243-246, June 1974.
10. Chu, K. C.: Decentralized Control of High Speed Vehicular Strings, *Transportation Science*, Nov. 1974 (to appear), also IBM Research Report RC 4753, March 1974.
11. Corfinat, J. P. and Morse, A. S.: Stabilization with Decentralized Feedback Control, *IEEE Trans. on Automatic Control*, December 1973, 679-682.

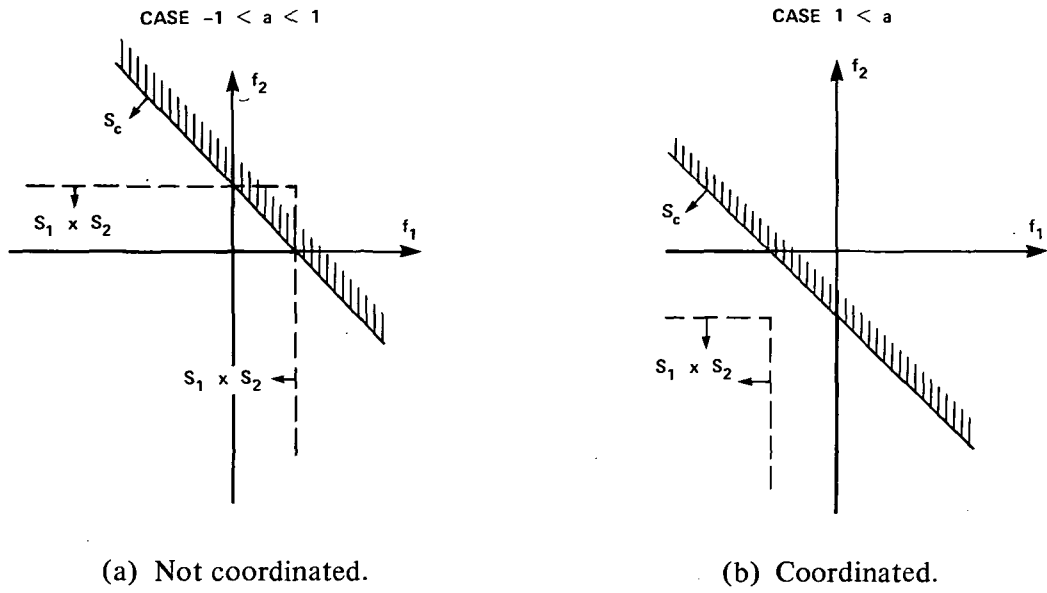


Figure 1.— Example relations between individually and collectively stable systems.

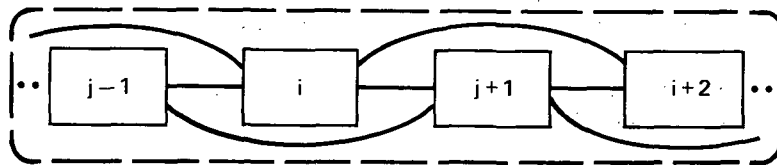


Figure 2.— A section of a sequential system and some of its interactions.

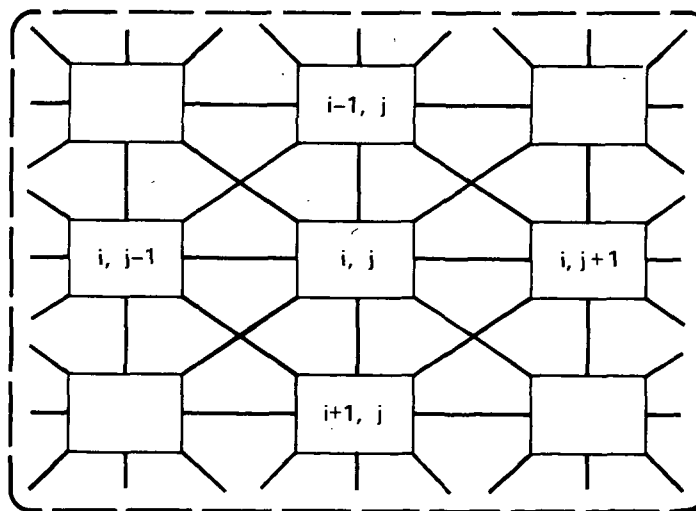


Figure 3.— A section of a planar array of identical subsystems and some of its interactions.

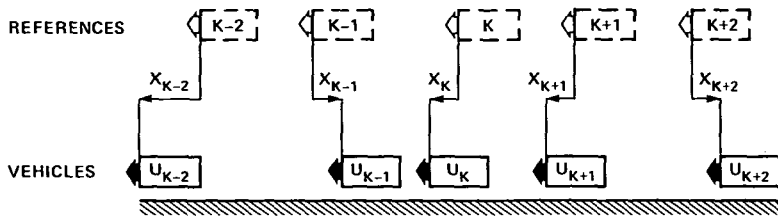
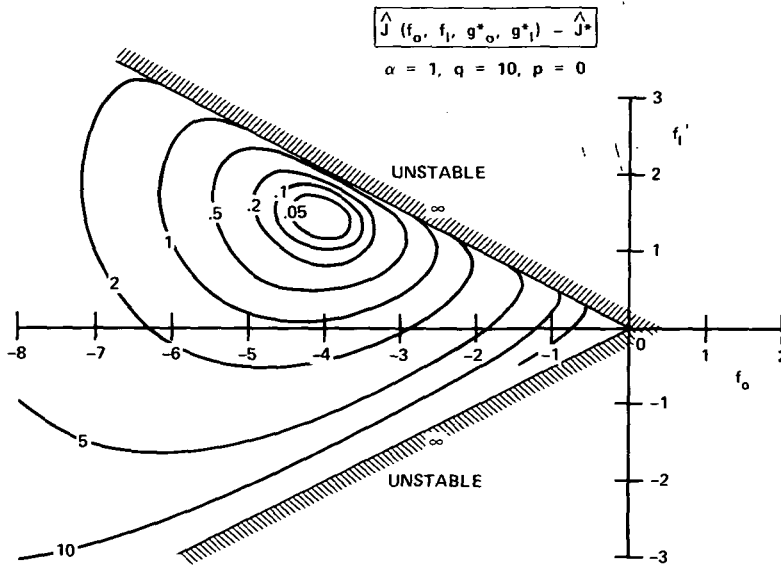
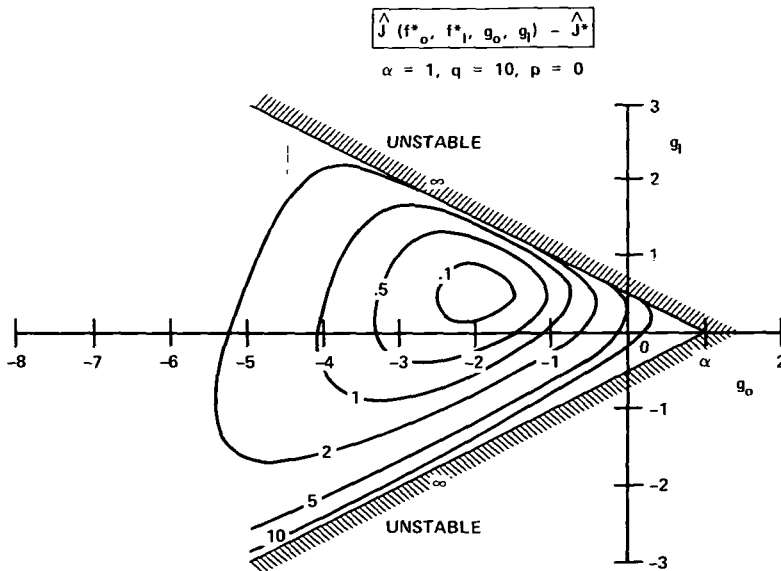


Figure 4.— Regulated vehicular strings: a sequential system.



(a) Not coordinated.



(b) Coordinated.

Figure 5.— Stability regions for a regulated vehicular string.

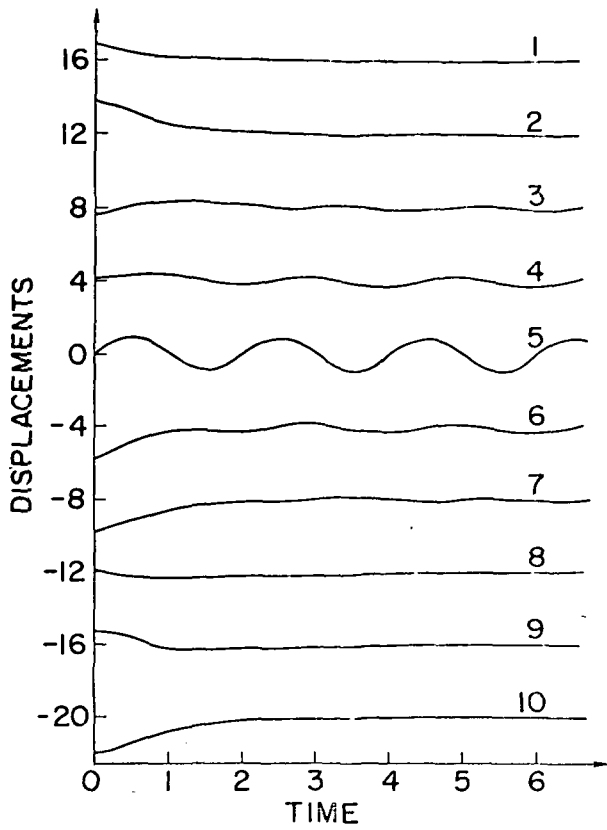


Figure 6.— Behavior of an optimally regulated string of 10 vehicles with the 5th vehicle oscillating.

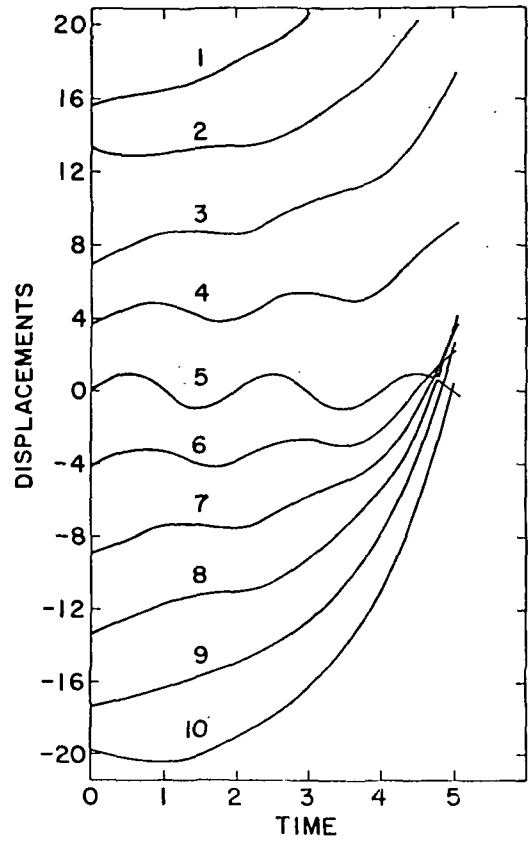


Figure 7.— Behavior of a vehicular string with excessive regulation.

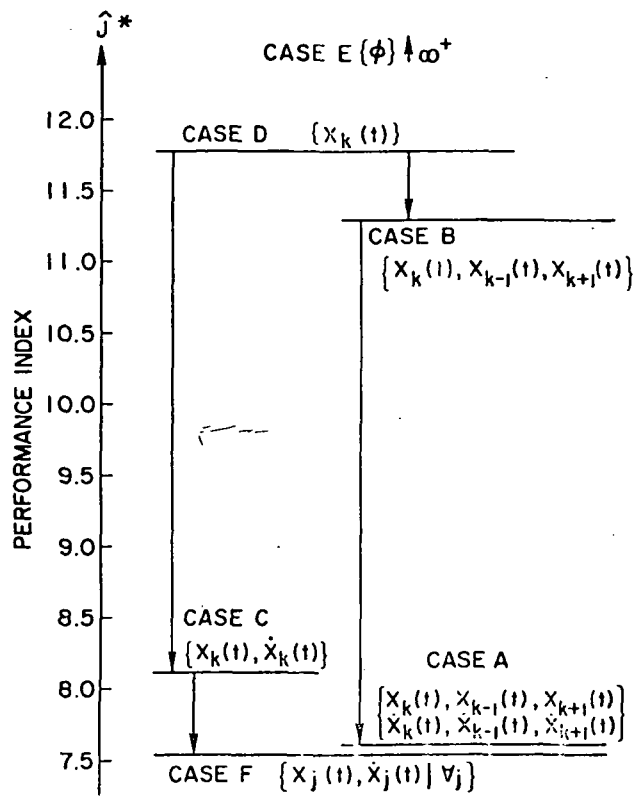


Figure 8.— Information value of various information structures in a regulated vehicular string.

Page Intentionally Left Blank

LARGE-SCALE SYSTEMS: COMPLEXITY, STABILITY, RELIABILITY*

D. D. Šiljak**

University of Santa Clara, Santa Clara, Calif. 95053

INTRODUCTION

“Roughly, by a complex system I mean one made up of a large number of parts that interact in a nonsimple way” is Simon’s description (ref. 1) of a complex system. He goes on further to say that “complexity frequently takes the form of hierarchy,” and by intuitive arguments he points out that the evolution of complex systems is highly reliable if it is carried out as a hierarchic process whereby complex systems are formed by interconnecting stable simple parts (subsystems).

The main objective of this work is to show rigorously that a complex system with (or designed to have) a competitive structure has highly reliable stability properties. The competitive models were studied in such diverse fields as economics (refs. 2, 3) and biology (ref. 4), arms race (ref. 5), and pharmacokinetics (ref. 6), and only recently it was shown (refs. 7–10) that in these various scientific disciplines, such models are “fail-safe” stable. In the framework of the connective stability concept (refs. 11–14), we will provide a definite support of Simon’s intuitive arguments. We will show that, under certain conditions, a stable complex system can tolerate a wide range of nonlinear, time-varying perturbations.

For large-scale systems, a competitive hierarchic model will be constructed by aggregating the stability properties of each subsystem so that stability of the model implies stability of the original system despite structural perturbations whereby subsystems are disconnected and again connected in various ways during the operation of the system. That is, from stability of each subsystem and stability of the aggregate model on the upper hierarchic level, we infer connective stability of the overall complex system. This result is remarkable in that it provides a natural setting for designing large-scale dynamic systems as competitive structures with highly reliable stability properties.

The stability investigations of competitive models are carried out by using the powerful mathematical concept of the comparison principle and vector Liapunov functions (ref. 15). The concept is extended here to include considerations of the connective stability aspects and the effects of the structural perturbations. Furthermore, the decomposition-aggregation methods developed in this context can take advantage of the special structural features of complex systems and reduce considerably the dimensionality of relevant stability problems.

*The research reported here was supported by NASA Ames Research Center, Moffett Field, Calif., under Grant NGR 05-017-010.

**Department of Electrical Engineering and Computer Science.

COMPETITIVE MODELS

Let us start with a linear constant dynamic system described by the differential equation*

$$\dot{x} = \bar{A}x \quad (1)$$

where $x(t) \in R^n$ is the state of the system and $\bar{A} = (\bar{a}_{ij})$ is an $n \times n$ constant matrix. By use of the connective stability concept (ref. 12), we derive the conditions under which stability is a highly reliable property of the system (1), and show that it remains stable despite a wide range of nonlinear, time-varying perturbations. In fact, by use of the modern mathematical machinery of the comparison principle (ref. 15), we show that, under relatively simple conditions, stability of the system (1) implies stability of a broad class of dynamic systems described by

$$\dot{x} = A(t, x)x. \quad (2)$$

In equation (2), $x(t) \in R^n$ is again the state of the system and the $n \times n$ matrix function $A: T \times R^n \rightarrow R^{n^2}$ is defined, bounded, and continuous on $T \times R^n$ so that the solutions $x(t; t_0, x_0)$ of equation (2) exist for all initial conditions $(t_0, x_0) \in T \times R^n$ and $t \in T_0$. The symbol T represents the time interval $(\tau, +\infty)$, where τ is a number or the symbol $-\infty$, and T_0 is the semiinfinite time interval $(t_0, +\infty)$. The matrix $A = (a_{ij})$ of equation (2) is an obvious modification of the matrix $\bar{A} = (\bar{a}_{ij})$ of equation (1), in which the constant elements \bar{a}_{ij} of \bar{A} are replaced by nonlinear, time-dependent functions $a_{ij} = a_{ij}(t, x)$.

To consider the connective aspect of stability, the elements a_{ij} of the matrix A in equation (1) are written:

$$a_{ij}(t, x) = -\delta_{ij}\psi_i(t, x) + e_{ij}(t)\psi_{ij}(t, \dot{x}) \quad (3)$$

where δ_{ij} is the Kronecker symbol and $\psi_i(t, x), \psi_{ij}(t, x) \in C^{(0,0)}(T \times R^n)$. In equation (3), $e_{ij} = e_{ij}(t)$ are elements of the $n \times n$ interconnection matrix $E = (e_{ij})$, which are $e_{ij}(t) \in C^0(T)$ and are restricted as $e_{ij}(t) \in [0, 1], \forall t \in T$ (ref. 12). For system (2), the element $e_{ij}(t)$ reflects the coupling between $\dot{x}_i(t)$ and $x_j(t)$ at each instant in time, that is, the time-dependent influence of the state $x_j(t)$ on the derivative of the state $x_i(t)$. Therefore, the interconnection matrix E represents the structural perturbations of the nonlinear matrix system (2).

In this section, we study asymptotic stability properties of system (2) under structural perturbations. More precisely, we investigate stability formulated as:

Definition 1. The equilibrium state $x = 0$ of system (2) is connectively asymptotically stable in the large if and only if it is asymptotically stable in the large for all interconnection matrices $E(t)$.

*With some obvious exceptions, lower case roman letters denote vectors, capital Roman letters denote matrices, and Greek letters denote scalars.

Before we turn to the derivation of the conditions for the kind of stability expressed by definition 1, we need the notion of the *fundamental interconnection matrix* \bar{E} (ref. 12). The matrix \bar{E} is a time-invariant, interconnection matrix in which the elements \bar{e}_{ij} take on binary values -1 if the j th state x_j influences the i th time derivative \dot{x}_i of the state x_i and 0 if x_j has no influence on \dot{x}_i . The matrix \bar{E} is a binary matrix (ref. 16) that reflects the basic structure of the system. Therefore, any interconnection matrix $E(t)$ is generated from \bar{E} by replacing the unit elements of \bar{E} by corresponding elements $e_{ij}(t)$ of $E(t)$.

The conditions for connective stability are expressed in terms of \bar{E} , but are valid for all E as required by definition 1. This is an important qualitative result since we show stability of a class of nonlinear, time-varying systems by proving stability of one member of that class which is a time-constant linear system.

To establish conditions for asymptotic connective stability, we assume that the elements $a_{ij}(t, x)$ of the matrix $A(t, x)$ are specified by equation (3) where $\psi_i(t, x)$ and $\psi_{ij}(t, x)$ are bounded functions on $T \times R^n$ and that there exist numbers $\alpha_i > 0$, $\alpha_{ij} \geq 0$ such that

$$\psi_i(t, x)|x_i| > \alpha_i \phi_i(|x_i|), \psi_{ij}(t, x)x_j \leq \alpha_{ij} \phi_j(|x_j|), \forall i, j = 1, 2, \dots, n; \forall (t, x) \in T \times R^n \quad (4)$$

and $\alpha_i > \alpha_{ii}$. In equation (4), $\phi_i : R_+^1 \rightarrow R_+^1$ are comparison functions that belong to the class $H : \phi_i(\rho) \in C^0(R_+^1)$, $\phi_i(0) = 0$ and $\phi_i(\rho) < \phi_i(\rho_2)$, $\forall \rho_1, \rho_2 : 0 \leq \rho_1 < \rho_2 < +\infty$ (ref. 17, 18).

By $\bar{A} = (\bar{a}_{ij})$, we denote the $n \times n$ constant matrix with the coefficients

$$\bar{a}_{ij} = -\delta_{ij}\alpha_{ij} + \bar{e}_{ij}\alpha_{ij} \quad (5)$$

where the elements \bar{e}_{ij} take the values 1 or 0 according to the matrix \bar{E} .

We prove the following:

Theorem 1. The equilibrium state $x = 0$ of the system (2) is connectively asymptotically stable in the large if the $n \times n$ constant matrix $\bar{A} = (\bar{a}_{ij})$ defined by equations (4) and (5) satisfies the conditions:

$$(-1)^k \begin{vmatrix} \bar{a}_{11} & \bar{a}_{12} & \dots & \bar{a}_{1k} \\ \bar{a}_{21} & \bar{a}_{22} & \dots & \bar{a}_{2k} \\ \dots & \dots & \dots & \dots \\ \bar{a}_{k1} & \bar{a}_{k2} & \dots & \bar{a}_{kk} \end{vmatrix} > 0, \quad \forall k = 1, 2, \dots, n. \quad (6)$$

Proof: Let us consider the function $v: R^n \rightarrow R_+^1$,

$$v(x) = \sum_{i=1}^n b_i |x_i| \quad (7)$$

as a candidate for Liapunov's function (ref. 19) for system (2) where $b_i > 0, i = 1, 2, \dots, n$ are yet unspecified numbers.

For $v(x)$, we have the inequalities

$$\phi_I(|x|) \leq v(x) \leq \phi_{II}(|x|), \quad \forall(t, x) \in T \times R^n \quad (8)$$

where ϕ_I and $\phi_{II} \in H$ are given as

$$\phi_I(|x|) = b_m |x|, \quad \phi_{II}(|x|) = n^{1/2} b_M |x| \quad (9)$$

and $b_m = \min_i b_i$ and $b_M = \max_i b_i$.

Since the derivative of $|x_i(t)|$ need not exist at a point where $x_i(t) = 0$, it is necessary to calculate the right-hand derivative $D^+|x_i(t)|$ with respect to equation (2) as proposed in reference 19. For this purpose, the functional σ_i is defined as

$$\sigma_i = \left\{ \begin{array}{l} 1, \text{ if } x_i > 0, \text{ or if } x_i = 0 \text{ and } \dot{x}_i > 0 \\ 0, \text{ if } x_i = 0 \text{ and } \dot{x}_i = 0 \\ -1, \text{ if } x_i < 0, \text{ or if } x_i = 0 \text{ and } \dot{x}_i < 0, \end{array} \right\} \quad (10)$$

where $x_i = x_i(t) \in C^1(T)$. Then

$$D^+|x_i(t)| = \sigma_i \dot{x}_i(t). \quad (11)$$

Using the constraints (4) and expression (11), we calculate the desired derivative as

$$\left. \begin{aligned} D^+v(x) &= \sum_{i=1}^n b_i \sigma_i \dot{x}_i(t) \\ &= \sum_{i=1}^n b_i \sigma_i \sum_{j=1}^n a_{ij}(t, x) x_j(t) \\ &\leq b^T \bar{A} w(x), \quad \forall(t, x) \in T \times R^n \end{aligned} \right\} \quad (12)$$

where $b = (b_1 \ b_2 \ \dots \ b_n)^T$ is a positive constant vector ($b > 0$) and the positive vector function $w: R^n \rightarrow R_+^n$ is defined as

$$w(x) = [\phi_1(|x_1|) \ \phi_2(|x_2|) \ \dots \ \phi_n(|x_n|)]^T. \quad (13)$$

From equation (5), we conclude that \bar{A} is a Metzler matrix (ref. 20), that is, it has negative diagonal elements ($\bar{a}_{ii} < 0$) and nonnegative off-diagonal elements ($\bar{a}_{ij} \geq 0, i \neq j$). We recall that a matrix \bar{A} is called a Hicks matrix (ref. 3) (equivalently, $-\bar{A}$ is an M matrix (ref. 21) if all even-order

principle minors of \bar{A} are positive and all odd-order principle minors of \bar{A} are negative. For a Metzler matrix \bar{A} , the Hicksian property is equivalent to the Sevastyanov-Kotelyanskii conditions (6) (ref. 22). Since \bar{A} is a Metzler matrix, the fact that \bar{A} satisfies inequalities (6) and is a Hicks matrix is equivalent (refs. 20, 21) to saying that, for any constant vector $c > 0$, there exists a constant vector $b > 0$ such that

$$c^T = -b^T \bar{A} . \quad (14)$$

Therefore, we can rewrite inequality (12) as

$$\begin{aligned} D^+v(x) &\leq -c^T w(x) \\ &\leq -c_m \sum_{i=1}^n \phi_i(|x_i|) \\ &\leq -\phi_{III}(\|x\|) , \quad \forall (t, x) \in T \times R^n \end{aligned} \quad (15)$$

where $c_m = \min_i c_i$ and $\phi_{III}(\|x\|) \in H$.

From (8) and (15) and reference 18, we conclude global asymptotic stability of $x = 0$ in equation (2). To show that stability is also connective, we need only notice that

$$A(t, x)x \leq \bar{A}w(x) , \quad \forall (t, x) \in T \times R^n \quad (16)$$

where inequality (16) is taken component-wise. Therefore, equation (15) holds for all $E(t)$. This proves theorem 1.

Note that the constraints (4) imply that $\psi_i(t, x) > 0, \forall (t, x) \in T \times R^n$. Positivity of $\psi_i(t, x)$ is absolutely essential for stability of equation (1) since it is easy to show that the Hicks conditions (6) imply $\bar{a}_{ij} < 0, \forall i = 1, 2, \dots, n$. With this in mind, we can rewrite the first condition in (4) as $|\psi_i(t, x)x_i| \geq \alpha_i \phi_{ii}(|x_i|)$, which looks similar to the second condition (4) except for the reversal of the inequality sign.

If the conditions (4) are simplified to

$$\psi_i(t, x) > \alpha_i , \quad |\psi_{ij}(t, x)x_j| \leq \alpha_{ij}|x_j| \quad \forall i, j = 1, 2, \dots, n; \forall (t, x) \in T \times R^n \quad (17)$$

where comparison functions $\phi_j(|x_j|)$ are chosen as $|x_j|$, then we can establish exponential property of connective stability as in reference 9. Furthermore, if we use the notion of absolute stability for the nonlinear matrix systems proposed by Persidskii (ref. 23), we can prove that Sevastyanov-Kotelyanskii inequalities (6) become both necessary and sufficient conditions for connective stability (ref. 9).

On the basis of the constraints (17), we define the following classes of continuous functions:

$$\left. \begin{aligned} \psi_i &= \{ \psi_i(t, x) : \psi_i(t, x) \geq \alpha_i \} \\ \psi_{ij} &= \{ \psi_{ij}(t, x) : |\psi_{ij}(t, x)| \leq \alpha_{ij} \} \end{aligned} \right\} \quad (18)$$

where α_i, α_{ij} are numbers as in (4). Then, we state:

Definition 2. The equilibrium state $x = 0$ of the system equation (2) is connectively, absolutely, and exponentially stable if and only if there exist two positive numbers Π and π independent of initial conditions (t_0, x_0) such that

$$\|x(t; t_0, x_0)\| \leq \Pi \|x_0\| \exp[-\pi(t - t_0)], \quad \forall t \in T_0 \quad (19)$$

for all $(t_0, x_0) \in T \times R^n$, all $\psi_i \in \Psi_i, \psi_{ij} \in \Psi_{ij}$, and all interconnection matrices $E(t)$.

To establish this kind of stability, we can use the following:

Theorem 2. The equilibrium state $x = 0$ of the system (2) is connectively, absolutely, and exponentially stable if and only if the $n \times n$ constant matrix $\bar{A} = (\bar{a}_{ij})$ defined by (5) and (17) satisfies conditions (6).

Proof: Let us consider again the function $v(x)$ in equation (7). When (4) is reduced to (17), the vector $w(x)$ in equation (13) becomes $[|x_1| |x_2| \dots |x_n|]^T$, and from (12) and (16) we get

$$D^+v(x) \leq \sum_{j=1}^n b_j |x_j| |\bar{a}_{jj}| + \sum_{j=1}^n |x_j| \sum_{\substack{i=1 \\ i \neq j}}^n b_i |\bar{a}_{ij}|, \quad \forall (t, x) \in T \times R^n. \quad (20)$$

Since \bar{A} is a Metzler matrix, the fact that it satisfies conditions (6) is equivalent (refs. 21, 24) to saying that there exists a positive vector $b = (b_1 \ b_2 \ \dots \ b_n)^T$, and a positive number π such that

$$|\bar{a}_{jj}| - b_j^{-1} \sum_{\substack{i=1 \\ i \neq j}}^n b_i |\bar{a}_{ij}| \geq \pi, \quad A_j = 1, 2, \dots, n \quad (21)$$

that is, \bar{A} is a quasidominant diagonal matrix (ref. 24).

From (20) and (21), we get the differential inequality

$$D^+v \leq -\pi v, \quad \forall t \in T_0, \quad \forall v \in R_+^1, \quad \forall E. \quad (22)$$

By integrating equation (22), we obtain

$$v[x(t)] \leq v(x_0) \exp[-\pi(t - t_0)], \quad \forall t \in T_0, \quad \forall (t_0, x_0) \in T \times R^n, \quad \forall E. \quad (23)$$

Using the well-known relationship between the Euclidean and absolute-value norms $\|x\| \leq |x| \leq n^{1/2} \|x\|$, we can rewrite (23) as

$$\left. \begin{aligned} \|x(t; t_0, x_0)\| &\leq \Pi \|x_0\| \exp[-\pi(t - t_0)] , \quad \forall t \in T_0 \\ \forall(t_0, x_0) &\in T \times R^n , \quad \forall \psi_i \in \Psi_i , \quad \forall \psi_{ij} \in \Psi_{ij} , \quad \forall E \end{aligned} \right\} \quad (24)$$

with

$$\Pi = n^{1/2} b_M b_m^{-1} \quad (25)$$

where $b_M = \max_i b_i$ and $b_m = \min_i b_i$.

Therefore, conditions (6) are sufficient for the absolute exponential property of connective stability of the equilibrium $x = 0$ in equation (2). This establishes the "if" part of theorem 2.

To prove the "only if" part of theorem 2, we select the particular system (2) specified by

$$\psi_i(t, x) = \alpha_i , \quad \psi_{ij}(t, x) = \alpha_{ij} , \quad \forall i, j = 1, 2, \dots, n \quad (26)$$

and the fundamental interconnection matrix \bar{E} . That is, the matrix $A(t, x)$ in equation (2) is taken as the constant Metzler matrix \bar{A} and system (2) is described by equation (1). If \bar{A} does not satisfy the Hicks conditions (6), the system (1) is unstable, and the equilibrium $x = 0$ of equation (2) is not stable $\forall \psi_i \in \Psi_i , \forall \psi_{ij} \in \Psi_{ij}$. This completes the proof of theorem 2.

HIERARCHIC MODELS

On the basis of the results obtained in the preceding section, we conclude that stability is a highly reliable property of competitive dynamic systems since it can tolerate a wide range of structural, nonlinear, and time-varying perturbations. Therefore, it would be desirable to define a class of noncompetitive dynamic systems for which a competitive model can be constructed so that their structural stability properties are implied by the same properties of the model. Such a construction is possible in the context of the hierarchic stability analysis, and we will show how the decomposition-aggregation scheme (refs. 11-14) can be used to form a competitive aggregate model for a large class of dynamic systems. The scheme is based on the modern mathematical machinery of the comparison principle and vector Liapunov functions (ref. 15). The decomposition-aggregation stability analysis not only provides a competitive aggregate model with structural stability properties, but it can also take advantage of the special structural features of complex dynamic systems and yield considerable simplification in the relevant stability investigations.

Let us immediately recall the fact stated in reference 7 that a natural generalization of the competitive models considered in reference 3 is represented by a differential equation

$$\dot{z} = u(t, z) \quad (27)$$

where the function $u : T \times R^s \rightarrow R^s$ is $u(t, x) \in C^{(0,0)}(T \times R^n)$ and $u(t, 0) = 0, \forall t \in T$. The function $u(t, x)$ belongs to the following class of functions:

$$K: u_i(t, a) \leq u_i(t, b), \quad \forall i = 1, 2, \dots, s$$

$$\forall \{(t, a), (t, b)\} \in T \times R^s \ni a_i = b_i, a_j \leq b_j (j = 1, 2, \dots, s; i \neq j).$$

The class of functions K , used by Kamke (ref. 25) in a formulation of the comparison principle, plays an important role in the stability analysis of dynamic systems by the vector Liapunov function. Therefore, the strong stability results obtained in the theory of differential inequalities and vector Liapunov functions can now be used to study stability properties of the competitive models.

In the following development, we will show how a competitive model described by equation (27) can be constructed for a dynamic system,

$$\dot{x} = f(t, x). \quad (28)$$

In (28), $x(t) \in R^n$ is the state vector of the system and the function $f: T \times R^n \rightarrow R^n$ is defined, bounded, and continuous on the domain $T \times R^n$ so that the solution $x(t; t_0, x_0)$ of equation (28) exists for all initial conditions $(t_0, x_0) \in T \times R^n$ and $t \in T_0$. Furthermore, we assume that $f(t, 0) = 0, \forall t \in T$, and $x = 0$ is the unique equilibrium state of the system (28). We derive conditions under which stability of the trivial solution $z = 0$ of equation (27) implies connective stability of the equilibrium $x = 0$ of the system (28).

To introduce the connective aspect of stability in the context of system (28), let us represent the state space R^n as

$$R^n = R_1^{n_1} \times R_2^{n_2} \times \dots \times R_s^{n_s} \quad (29)$$

so that the state vector has the form

$$x(t) = [x_1^T(t) x_2^T(t) \dots x_s^T(t)]^T \quad (30)$$

and $x_i(t) \in R^{n_i}$. The function $f(t, x)$ is further specialized by its components as

$$f_i(t, x) \equiv \hat{f}_i(t, x_i, e_{i_1}x_1, e_{i_2}x_2, \dots, e_{i_s}x_s), \quad i = 1, 2, \dots, s \quad (31)$$

where $e_{ij}(t)$ are elements of the $s \times s$ interconnection matrix $E(t)$. Now, we can broaden the scope of definition 1 to include the class of systems (28). We say that the equilibrium $x = 0$ of the system (28) is connectively stable if and only if it is stable (in the sense of Liapunov) for all interconnection matrices $E(t)$ (ref. 12).

To establish connective stability of the system, we will form the model (eq. (27)) where the "aggregate" function $u(t, x)$ has the form

$$u_i(t, z) \equiv \hat{u}_i(t, z_i, e_{i_1}z_1, e_{i_2}z_2, \dots, e_{i_s}z_s), \quad i = 1, 2, \dots, s \quad (32)$$

and $z_i \in R^1$. On the basis of (32), we define the model

$$\dot{z} = \bar{u}(t, z) \quad (33)$$

where the function $\bar{u}(t, z) \in K$ corresponds to the fundamental interconnection matrix \bar{E} . Then, stability of the trivial solution $z = 0$ of the aggregate competitive model (33) implies connective stability of the equilibrium $x = 0$ of the system equation (28).

More precisely, we prove the following:

Theorem 3. There exists a function $v(t, x)$ with the properties: $v(t, x) \in C^{(0,0)}(T \times R^n)$; $v(t, x)$ is locally Lipschitzian in x ; $v(t, 0) \equiv 0$, and $v(t, x) \geq 0$ on $T \times R^n$; the function $v: T \times R^n \rightarrow R_+^1$, defined as

$$v(t, x) = b^T v(t, x) = \sum_{i=1}^s b_i v_i(t, x) \quad (34)$$

for some constant vector $b > 0$ satisfies the inequalities

$$\phi_I(\|x\|) \leq v(t, x) \leq \phi_{II}(\|x\|), \quad \forall (t, x) \in T \times R^n \quad (35)$$

where $\phi_I, \phi_{II} \in H$ and $\phi_I(\rho) \rightarrow +\infty$ as $\rho \rightarrow +\infty$; the function

$$D^+ v(t, x) = \limsup_{h \rightarrow 0^+} \frac{1}{h} \{v[t+h, x+hf(t, x)] - v(t, x)\} \quad (36)$$

defined with respect to equation (28), satisfies a differential inequality

$$D^+ v(t, x) \leq \bar{u}[t, v(t, x)], \quad \forall (t, v) \in T \times R^s \quad (37)$$

where the function $\bar{u}: T \times R_+^s \rightarrow R^s$ is $\bar{u}(t, v) \in K$ and $\bar{u}(t, 0) = 0$.

Then, asymptotic stability in the large of $z = 0$ in the comparison equation (33) implies connective asymptotic stability in the large of $x = 0$ in equation (28), and

$$v(t, x) = [v_1(t, x) \ v_2(t, x) \ \dots \ v_s(t, x)]^T \quad (38)$$

is a vector Liapunov function for the system (28).

Proof: It is a well-known result (ref. 15) that under the conditions of the theorem, asymptotic stability of $z = 0$ of equation (33) implies the same stability property for $x = 0$ in equation (28). To show that stability of $x = 0$ is also connective, we notice that the assumption $v(t, x) \geq 0$ on $T \times R^n$ allows us to establish the inequality

$$u(t, z) \leq \bar{u}(t, z), \quad \forall (t, z) \in T \times R^s \quad (39)$$

which holds for all $E(t)$. Therefore, the differential inequality in equations (37) is also valid for all interconnection matrices $E(t)$, and stability of $x = 0$ is connective. This proves theorem 3.

An attractive property of theorem 3 is that it allows a reduction in dimensionality of stability problems in much the same way as does the original Liapunov theory. By theorem 3, we can prove stability of a system of order n by demonstrating stability of s lower-order systems and one system of order $s \leq n$ which involves the vector Liapunov function.

Another important feature of theorem 3 we will not take advantage of is that the components $v_i(t, x)$ of the vector Liapunov function $v(t, x)$ can satisfy the weaker requirements than the usual ones associated with scalar Liapunov functions. This is not to say, however, that vector Liapunov functions are automatically more flexible than scalar Liapunov functions as was demonstrated in reference 7.

Let us now show how to construct the competitive aggregate model (27) for a given system (28) by applying the decomposition-aggregation method proposed in reference 13. We start with decomposing the system (28) into s subsystems described by

$$\dot{x}_i = g_i(t, x_i) + h_i(t, x), \quad i = 1, 2, \dots, s \quad (40)$$

where $x_i(t) \in R^{n_i}$ is the state vector of the subsystem (40) and represents the i th "vector" component of the state vector $x(t)$ specified by equation (30).

In equation (40), the functions $g_i : T \times R^{n_i} \rightarrow R^{n_i}$ describe the "decoupled" subsystems:

$$\dot{x}_i = g_i(t, x_i), \quad i = 1, 2, \dots, s \quad (41)$$

obtained from equation (40) when all interaction functions $h_i : T \times R^n \rightarrow R^{n_i}$ among the subsystems are set to zero ($h_i(t, x) \equiv 0$). Each $g_i(t, 0) = 0, \forall t \in T$, so that $x_i = 0$ is the unique equilibrium state of every subsystem (41).

We assume that with each *decoupled subsystem* (41) we can associate a function $v_i : T \times R^{n_i} \rightarrow R_+^1$ such that $v_i(t, x_i) \in C^{(0,0)}(T \times R^{n_i})$, $v_i(t, x)$ satisfies a Lipschitz condition in x_i for a constant $\kappa_i > 0$, and

$$\left. \begin{aligned} \phi_{i1}(\|x_i\|) &\leq v_i(t, x_i) \leq \phi_{i2}(\|x_i\|) \\ D^+ v_i(t, x_i)_{(41)} &\leq -\phi_{i3}(\|x_i\|) \\ \forall i = 1, 2, \dots, s; \quad \forall t \in T, \quad \forall x_i \in R^{n_i} \end{aligned} \right\} \quad (42)$$

where $D^+ v_i(t, x_i)_{(41)} = \limsup_{h \rightarrow 0^+} \frac{1}{h} \{v_i[t+h, x_i + hg_i(t, x_i)] - v_i(t, x_i)\}$, the functions $\phi_{i1}, \phi_{i2}, \phi_{i3} \in H$ and $\phi_{i1}(\rho) \rightarrow +\infty$ as $\rho \rightarrow +\infty$.

We assume that the *interactions* $h_i(t, x)$ among the subsystems (41) have the form

$$h_i(t, x) \equiv h_i(t, e_{i1}x_1, e_{i2}x_2, \dots, e_{is}x_s) \quad (43)$$

and that there exist bounded functions $\xi : T \times R^n \rightarrow R_+^1$ such that

$$\left. \begin{aligned} \|h_i(t, x)\| &\leq \sum_{j=1}^s e_{ij}(t) \xi(t, x) \phi_{j3}(\|x_j\|), \\ \forall i = 1, 2, \dots, s; \quad \forall (t, x) &\in T \times R^n \end{aligned} \right\} \quad (44)$$

Let us define an $s \times s$ constant matrix $\bar{A} = (\bar{a}_{ij})$ by

$$\bar{a}_{ij} = -\delta_{ij} + \bar{e}_{ij} \kappa_i \alpha_{ij} \quad (45)$$

where \bar{e}_{ij} are the elements of the fundamental interconnection matrix \bar{E} , and the numbers $\alpha_{ij} \geq 0$ are computed as

$$\alpha_{ij} = \max \left\{ 0, \sup_{T \times R^n} \xi_{ij}(t, x) \right\} \quad (46)$$

Now we state:

Theorem 4. The equilibrium state $x = 0$ of the system (28) is connectively asymptotically stable in the large if the $s \times s$ constant matrix $\bar{A} = (\bar{a}_{ij})$ defined by equation (45) satisfies the conditions (6).

Proof: As in reference 13, consider the function $v : T \times R^n \rightarrow R_+^1$,

$$v(t, x) = \sum_{i=1}^s b_i v_i(t, x_i) \quad (47)$$

as a candidate for a Liapunov function for the system (28) where $b_i > 0, i = 1, 2, \dots, s$ are components of a positive (yet unspecified) vector b . Then define the function

$$D^+ v_i(t, x_i)_{(40)} = \limsup_{h \rightarrow 0^+} \frac{1}{h} \left\{ v_i(t+h, x_i + h[g_i(t, x_i) + h_i(t, x)]) - v_i(t, x_i) \right\} \quad (48)$$

which we compute with respect to subsystem equation (40) and obtain

$$D^+ v_i(t, x_i)_{(40)} \leq D^+ v_i(t, x_i)_{(41)} + \kappa_i \|h_i(t, x)\|, \quad i = 1, 2, \dots, s \quad (49)$$

since each $v_i(t, x_i)$ is Lipschitzian with a constant $\kappa_i > 0$ (ref. 15).

By using the interconnection conditions (44) and subsystem stability equations (42), from (49) we obtain the vector differential inequality:

$$D^+ v_{(40)} \leq \bar{A} w(v), \quad \forall (t, x) \in T \times R^n \quad (50)$$

which holds for all interconnection matrices $E(t)$.

As in equation (13), $w : R^S \rightarrow R^S$ is the comparison vector function:

$$w(v) = [\phi_{13}(v_1) \phi_{23}(v_2) \dots \phi_{s3}(v_s)]^T. \quad (51)$$

By using the same argument as in the proof of theorem 1, from equations (47) and (50), we obtain

$$\phi_I(\|x\|) \leq v(t, x) \leq \phi_{II}(\|x\|), \quad D^+v(t, x) \leq -\phi_{III}(\|x\|), \quad \forall (t, x) \in T \times R^n \quad (52)$$

where the functions

$$\phi_I(\|x\|) = b_m \sum_{i=1}^s \phi_{i1}(\|x_i\|),$$

$$\phi_{II}(\|x\|) = b_M \sum_{i=1}^s \phi_{i2}(\|x_i\|),$$

$$\phi_{III}(\|x\|) = c_m \sum_{i=1}^s \phi_{i3}(\|x_i\|)$$

all belong to the class H .

Since inequalities (52) hold for all interconnection matrices $E(t)$, the equilibrium state $x = 0$ of the system (28) is connectively asymptotically stable in the large. The proof of theorem 4 is complete.

By observing that $\bar{A}w(v) \in K$, theorem 4 follows directly from inequality (50) and theorem 3. In the proof of theorem 4, we used the scalar function $v = b^T v$ to show explicitly the multilevel nature of the analysis. The components v_i of the vector Liapunov function v are scalar Liapunov functions responsible for stability on the subsystem level, and the scalar function v is a scalar Liapunov on the aggregate overall system level.

If we strengthen the constraints imposed on the interactions $h_i(t, x)$ among the subsystems, we can infer exponential stability of the overall system from exponential stability of the subsystems.

Let us assume that the estimates (42) have the form

$$\left. \begin{aligned} \eta_{i1} \|x_i\| \leq v_i(t, x_i) \leq \eta_{i2} \|x_i\| \quad D^+v_i(t, x_i) \leq -\pi_i v_i(t, x_i), \\ \forall i = 1, 2, \dots, s; \quad \forall t \in T, \quad \forall x_i \in R^{n_i} \end{aligned} \right\} \quad (53)$$

where η_{i1} , η_{i2} , and π_i are all positive numbers. Inequalities (53) guarantee exponential stability of the decoupled subsystems (39).

We also assume that the interactions $h_i(t, x)$ among the subsystems satisfy the constraints

$$\|h_i(t, x)\| \leq \sum_{j=1}^s e_{ij}(t) \xi_{ij}(t, x) \|x_j\| \quad \forall i = 1, 2, \dots, s; \quad \forall (t, x) \in T \times R^n. \quad (54)$$

We again construct the aggregate $s \times s$ constant matrix $\bar{A} = (\bar{a}_{ij})$ defined by

$$\bar{a}_{ij} = -\delta_{ij}\pi_i + \bar{e}_{ij}\alpha_{ij}\kappa_i\eta_{j1}^{-1} \quad (55)$$

and prove the following:

Theorem 5. The equilibrium state $x = 0$ of the system (28) is connectively exponentially stable in the large if the $s \times s$ constant matrix $\bar{A} = (\bar{a}_{ij})$ defined by equation (55) satisfies the inequalities (6).

Proof: We again use the function $v(t, x)$ of equation (47) as a Liapunov function. By using the estimates (53) and (54), from (49) we obtain the following inequalities:

$$\left. \begin{aligned} D^+ v_i(t, x_i)_{(40)} &\leq -\pi_i v_i(t, x_i) + \kappa_i \sum_{j=1}^s \bar{e}_{ij} \alpha_{ij} \|x_j\| \\ &\leq -\pi_i v_i(t, x_i) + \kappa_i \sum_{j=1}^s \bar{e}_{ij} \alpha_{ij} \eta_{j1}^{-1} v_j(t, x_j) \\ i = 1, 2, \dots, s; \quad \forall t \in T, \quad \forall x \in R^n \end{aligned} \right\} \quad (56)$$

that are valid for all interconnection matrices $E(t)$.

From equation (56), we obtain the aggregate competitive model:

$$D^+ v_{(40)} \leq \bar{A} v, \quad \forall (t, x) \in T \times R^n \quad (57)$$

which is a linear differential inequality in v . Proceeding as in the proof of theorem 4, we derive the scalar inequality

$$D^+ v(t, x)_{(40)} \leq -\pi v(t, x), \quad \forall (t, x) \in T \times R^n, \quad \forall E(t) \quad (58)$$

where $\pi = \min_i \pi_i$.

Integrating inequality (58), we obtain

$$v[t, x(t)] \leq v(t_0, x_0) \exp[-\pi(t - t_0)], \quad \forall t \in T_0, \quad \forall (t_0, x_0) \in T \times R^n, \quad \forall E(t). \quad (59)$$

By use of the estimates (53), we can obtain from this inequality a further inequality that involves the solution $x(t; t_0, x_0)$ of the original overall system:

$$\|x(t; t_0, x_0)\| \leq \Pi \|x_0\| \exp[-\pi(t - t_0)], \quad \forall t \in T_0, \quad \forall (t_0, x_0) \in T \times R^n, \quad \forall E(t) \quad (60)$$

where $\Pi = s^{1/2} b_m b_M \eta_{m_1}^{-1} \eta_{M_2}$, $b_m = \min_i b_i$, $b_M = \max_i b_i$, $\eta_{m_1} = \min_i \eta_{i_1}$, $\eta_{M_2} = \max_i \eta_{i_2}$, and $\pi = \min_i \pi_i$. This proves theorem 5.

CONCLUSIONS

Two important conclusions result from this work. First, the competitive structures are an appropriate framework for constructing reliable complex systems. Secondly, via the diagonal dominance of the aggregate competitive model, the decomposition-aggregation method provides a good measure of complexity for stable, large-scale systems. Both results arose in the context of connective stability by applying the mathematical apparatus of the comparison principle and vector Liapunov function. One of the most attractive aspects of the results is that they are obtained in the Hicks-Metzler algebraic setting, which provides a rich environment for their application and further refinements.

A number of interesting problems were either not mentioned or were not explored in sufficient detail. For example, it is not clear how the decomposition and aggregation should be performed to balance the gains in simplification against the errors resulting from the approximation involved in the decomposition-aggregation process. Furthermore, it is of interest to investigate various implications (ref. 26) of the obtained results in the competitive analysis conducted in the field of economics (refs. 2, 3, 7) and ecosystems (refs. 4, 8, 10). In control systems (ref. 27), there are already some specific applications that indicate certain definite advantages of the competitive analysis and the decomposition-aggregation method in the multilevel stabilization and optimization of large-scale systems. A good deal of work remains to be done to explore the possibilities offered by the competitive structures in dynamic systems and to obtain important new results.

REFERENCES

1. Simon, H. H.: The Architecture of Complexity, Proceedings of the American Philosophical Society, 106(1962), 467–482.
2. Quirk, J.; and Saposnik, R.: Introduction to General Equilibrium Theory and Welfare Economics, McGraw-Hill, New York, 1968.
3. Arrow, K. J.; and Hahn, F. H.: General Competitive Analysis, Holden-Day, San Francisco, 1971.
4. May, R. M.: Stability and Complexity in Model Ecosystems, Monographs in Population Biology, No. 6, Princeton University Press, N. J., 1973.
5. Richardson, L. F.: Arms and Insecurity, Boxwood Press, Pittsburgh, 1960.
6. Bellman, R.: Topics in Pharmacokinetics, I: Concentration-Dependent Rates, Mathematical Biosciences, 6(1970), 13–17.
7. Šiljak, D. D.: Competitive Economic Systems: Stability, Decomposition, and Aggregation, Proc. of the 1973 Conference on Decision and Control, San Diego, 1973, 265–275.
8. Šiljak, D. D.: Connective Stability of Complex Ecosystems, Nature, 249(1974), 280.
9. Šiljak, D. D.: On Connective Stability and Instability of Competitive Equilibrium, Proc. 1974 JACC Conference, University of Texas, Austin, Texas, 1974, 223–228.
10. Šiljak, D. D.: When is a Complex Ecosystem Stable?, Mathematical Biosciences, (to be published).
11. Šiljak, D. D.: Stability of Large-Scale Systems Under Structural Perturbations, IEEE Transactions, SMC-2(1972), 657–663.
12. Šiljak, D. D.: On Stability of Large-Scale Systems Under Structural Perturbations, IEEE Transactions, SMC-3(1973), 415–417.
13. Grujić, Lj. T.; and Šiljak, D. D.: Asymptotic Stability and Instability of Large-Scale Systems, IEEE Transactions, AC-18(1973), 636–645.
14. Grujić, Lj. T.; and Šiljak, D. D.: Exponential Stability of Large-Scale Discrete Systems, International Journal of Control, 19(1974), 481–491.
15. Lakshmikantham, V.; and Leela, S.: Differential and Integral Inequalities, Vols. I and II, Academic Press, New York, 1969.
16. Harray, F.; Norman, R.; and Cartwright, D.: Structural Models: An Introduction to the Theory of Directed Graphs, Wiley, New York, 1965.
17. Hahn, W.: Stability of Motion, Springer-Verlag, New York, 1967.
18. Halanay, A.: Differential Equations: Stability, Oscillations, Time Lags, Academic Press, New York, 1966.

19. Rosenbrock, H. H.: A Lyapunov Function for Some Naturally Occurring Linear Homogeneous Time-Dependent Equations, *Automatica*, 1(1963), 97–109.
20. Arrow, K. J.: Price-Quality Adjustments in Multiple Markets with Rising Demands, *Proceedings of a Symposium on Mathematical Methods in the Social Sciences*, Stanford University Press, Stanford, California, 1966, pp. 3–15.
21. Fiedler, M.; and Pták, V.: On Matrices with Non-Positive Off-Diagonal Elements and Positive Principal Minors, *Czechoslovakian Mathematical Journal*, 12(1969), 382–400.
22. Gantmacher, F. R.: *The Theory of Matrices*, Vol. II, Chelsea, New York, 1960.
23. Persidskii, S. K.: Problem of Absolute Stability (in Russian), *Avtomatika i Telemekhanika*, 29(1969), 5–11.
24. Newman, P. K.: Some Notes on Stability Conditions, *Review of Economic Studies*, 72(1959), 1–9.
25. Kamke, E.: Zur Theorie der Systeme gewöhnlicher Differentialgleichungen, II, *Acta Mathematica*, 58(1932), 57–85.
26. Šiljak, D. D.: Connective Stability of Nonlinear Matrix Systems, *Proc. 1974 IEEE International Symposium on Circuits and Systems*, San Francisco, Calif., 1974, 37–41.
27. Čuk, S. M.; and Šiljak, D. D.: Decomposition-Aggregation Stability Analysis of the Spinning Skylab, *Proc. Seventh Asilomar Conference on Circuits, Systems, and Computers*, Pacific Grove, Calif., 1973, 204–211.

DIGITAL SIMULATION OF V/STOL AIRCRAFT FOR AUTOPILOT RESEARCH

L. S. Cicolani and George Meyer

Ames Research Center
Moffett Field, Calif. 94035

INTRODUCTION

Research is currently underway at Ames Research Center on developing trajectory control logic for a class of advanced aircraft known as vertical or short takeoff and landing aircraft (V/STOL) (ref. 1). From that work, simulations of such aircraft oriented to autopilot research are introduced as an example of a large-scale system.

The scope of this research is to proceed from basic theory to flight test on at least one research STOL aircraft now at Ames. Work is currently concerned with the augmentor wing jet STOL research aircraft (AWJSRA); this will be the source of some of the details of aircraft models discussed later. Since generalized digital flight-control hardware system (called STOLAND, ref. 2) has been designed for Ames and installed in the AWJSRA, the results of our research can be entered as control logic software in the STOLAND system and flight tested. The research is focused on controlling the vehicle during approach and landing in anticipation of an operational environment of high-density terminal area traffic where tight control over flight paths, both in space and time, is required.

Scale, in this paper, is sensed from the point of view of the autopilot research, that is, the size of the simulation is defined in relation to its use in the research. One measure of size significant to us was the ease with which the simulation could be used as a research tool by one or two persons. Another measure was its complexity as a system to be controlled. A generalized autopilot is a device for controlling the aircraft flight path throughout its flight regime and, as such, necessarily solves the aircraft equations of motion. If that solution is very difficult, then it reflects the complexity of the aircraft itself.

In our research, problems of both size and complexity were encountered and these derive mostly from the nature of the V/STOL aircraft itself. For the class of V/STOL aircraft, several new design elements contribute to problems of size and complexity; they are designed to use both engine power and major changes in structural configuration to augment the aerodynamic forces to achieve low landing speeds, and this results in nonlinear engine and configuration-dependence of aerodynamic modeling data. Often there is also a novel type of engine control not found on conventional aircraft; for example, the engines of the AWJSRA exhaust through a controllable nozzle that can be rotated more than 90° . This control is simple to simulate but adds a great deal of complexity to the flight-control problem.

The appropriate level of detail in simulating the flight dynamics is the rigid body motion of the aircraft which, by itself, is not a large number of degrees of freedom.

Problems of size as a working tool were attacked by use of a hierarchy of simulations that represent the aircraft with increasing completeness of detail in parallel with a corresponding hierarchical development of the autopilot, and also through various partitionings of the system equations. The control problem was also attacked through partitioning to divide it into problems half as large although still difficult to solve.

SYMBOLS

A	matrix mapping angular velocity into Euler attitude rates
A_g, B_g	matrices of wind gust state equations
A, B	matrices of approximate attitude state equation
AT	list atmospheric parameters: density, temperature, pressure, and speed of sound
ATT	(Φ, Θ, Ψ) , Euler attitude angles
b	wing span
c	wing cord
C_D, C_Y, C_L	dimensionless coefficients of drag, Y , and lift force
C_J	dimensionless coefficient of equivalent thrust
C_ℓ, C_m, C_n	dimensionless coefficients of roll, pitch, and yaw moments
d_i, d_{mi}	dimensionless gradients of aerodynamic force and moment
E_1, E_2, E_3	transformations for rotations about the i, j, k axes, respectively
f_0, f_i, f_{mi}	dimensionless force and moment vectors
F	flap deflection angle
F_B, FA_B, FE_B	forces – total, aerodynamic, engine, respectively, in body axes
I	aircraft inertia matrix in body axes
(I, J, K)	inertial reference frame (runway)
(i_b, j_b, k_b)	body axes reference frame
(i_w, j_w, k_w)	wind axes reference frame

l_E	location of engine in body axes
M	aircraft mass
\dot{M}_F	engine fuel flow
M_B, MA_B, ME_B	moments in body axes – total, aerodynamic, and engine moments, respectively
N	noise
Q	dynamic pressure
R_I	position, vector, inertial coordinates
S	wing area
T	engine thrust
TC	equivalent thrust of compressed engine air used to augment aerodynamics
u_i	control variable
UM	moment controls ($\delta_a, \delta_e, \delta_R$)
UF	force controls ($F, \delta th, \nu$)
VA	airspeed
VA_B	velocity vector with respect to air mass, body axes components
V_I	inertial velocity, inertial axes components
W_I, WS_I, WG_I	total, steady, gust winds, respectively; inertial axes components
XT	translational motion variables (R_I, V_I, \dot{V}_I)
XT_{REF}	translational variables for reference trajectory
XR	rotational motion variables ($\Phi, \Theta, \psi, \Omega_B$)
α	angle of attack, angle between body axis I_B and velocity vector
β	side-slip angle, angle between velocity vector and aircraft plane of symmetry
δa	aileron deflection angle
δe	elevator deflection angle

δi	angle variable, one of $\beta, \delta_a, \delta_e, \delta_R$
δR	rudder deflection angle
δth	throttle setting
(Φ, Θ, Ψ)	Euler attitude angles; roll, pitch, and heading angles, respectively
μ_j	angular rate variable, one of $\{\dot{\alpha}, \dot{\beta}, \Omega_B\}$
μ^*	nondimensionalizing factor for μ_j
τ	time constant
Ω_B	angular velocity, body axes components

Subscripts

B	quantity is a vector given in body axes
C	commanded value of the quantity
EX	extremes of the quantity (maximum or minimum)
I	quantity is a vector given in inertial axes components
W	quantity is a vector given in wind axes components

Superscript

(\cdot)	time derivative of the quantity
-----------	---------------------------------

SIMULATION USES IN THE RESEARCH

The digital simulation of the system serves as a tool for both computing and understanding during the process of solving the control problem. First, it contains a reference definition of the aircraft in the form of parameters and model data and functional relationships, as well as algorithmic specifications of the dynamic behavior of the aircraft algorithmically. All aspects of the aircraft influence the autopilot development so that it is advantageous for the researcher to become intimately familiar with the simulation. To promote these uses, the simulations should be easy to access, use, understand, and modify, and its structure should reflect the autopilot researcher's perception of aircraft flight at as many levels as possible. Secondly, aircraft motion is sufficiently complex that the autopilot design begins with ideas based on a simplified view of the aircraft followed by an iterative use of the simulation in which ideas are tested and unforeseen effects uncovered and then fed back into the design. Many effects understood only qualitatively are

quantitatively tested. During this process, the autopilot changes frequently while the remainder of the simulation program remains the same so that it will be useful to isolate the autopilot as a plug-in subsystem. Thirdly, the autopilot research from theory to flight test proceeds in stages of ever increasing integration with the complete flight-control problem and a hierarchy of simulations that correspondingly simulate the aircraft in increasing detail is necessary. In the first stage, since we are concerned with trajectory control, only a simplified simulation of the rotational dynamics is necessary. Later, attitude control and navigation is added, which requires simulation of both the translational and rotational degrees of freedom. A final stage of integration with flight hardware, displays, safety, and pilot opinion is required, and a large general purpose, man-in-the-loop laboratory simulation is used in this stage.

The main obstacle to these uses was the size of the available simulations. Although it was to be a tool for autopilot research, the simulation program for V/STOL aircraft with rigid-body dynamics threatened to be much larger than the autopilot program. However, the effort and labor required are proportional to size for a digital program, so it is essential to minimize size, especially in the critical early stages of the research. Much was gained by properly partitioning the simulation and several partitions that have proved advantageous to our work are described. Since these partitions are inevitably motivated by the nature of flight, a description of flight is first given and then the system is defined.

DESCRIPTION OF FLIGHT

The motion of a rigid aircraft can be thought of in terms of its six degrees of freedom, three of which describe the path of the center of gravity through space and three describe the rotational motion of the aircraft about its center of gravity.

The trajectory is defined in some inertial frame. Figure 1 shows a coordinate frame with origin at the touchdown point on the runway. This frame is suitable to describe terminal area approach paths, and it is an inertial frame if a flat earth is assumed. The trajectory is controlled through the aerodynamic and engine forces, which can be described by their body axis components. The body axes are a frame attached to the structure with (i_b, k_b) in the aircraft plane of symmetry (fig. 1). If the velocity vector is maintained in the plane of symmetry, then the aerodynamic forces as well as the engine forces all lie in the plane of symmetry. The aerodynamic force is normally given by its components along and perpendicular to the velocity vector (called drag and lift, respectively). This force vector is located with respect to body axes by the angle of attack, α , and is a strong function of α so that lift force is controlled by controlling α . The other major force is the engine thrust, whose magnitude is controlled by a throttle and, for the AWJSRA, its direction with respect to the body axes is controlled by a rotatable exhaust nozzle. In addition, compressed air from the engine is exhausted through a special flap to augment the aerodynamic lift during landing. Thus, control over the inertial path requires control over the inertial orientation of the plane of symmetry as well as forces in that plane.

The inertial orientation is usually defined by the Euler angles Φ , Θ , and ψ (fig. 2). Control over Φ and ψ provides control over the inertial orientation of the plane of symmetry and control over Θ provides control over the angle of attack and therefore the lift. Control over these attitudes is

obtained with the applied moments. For this purpose, small, movable aerodynamic surfaces are found at the extremes of the structure (fig. 2) – ailerons at the wing ends and tail and rudder at the end of the body – which provide control moments about three orthogonal directions. The terms δa , δe , δr refer to surface deflection angles. This location at the structural extremes also minimizes the disturbance forces associated with attitude control and hence minimizes the coupling of attitude controls with the center of gravity motion.

An experienced passenger may have noticed that there is little rotational motion in commercial flights or rather he may have noticed the discomfort when it is otherwise. The attitude is almost always steady or slowly varying, and this characterizes the attitude motion generated with the simulation. There are also small-amplitude variations about this steady motion, typically at high frequency compared to variations in the center-of-gravity motion because the aircraft has inherently faster dynamics and controls in the rotational degrees of freedom than in the linear degrees.

AIRCRAFT SIMULATION – MAIN SUBSYSTEMS

Figure 3 shows a subdivision of the simulation at its broadest level of subsystems and also the input-output relations among them. The autopilot outputs are commands to the six force and moment controls of the aircraft model. The aircraft model calculates the applied engine and aerodynamic forces and moments that are output to the equations of motion section which integrates these equations and calculates environmental parameters (such as winds, density, etc.). State information and environmental data are then fed back to the autopilot and aircraft model. In the process of simulation, the flight evolves by cycling through this loop, where each cycle corresponds to the integration step size used in the numerical integration of the equations of motion.

The principle intent of this modular structure is to minimize the programming labor needed for autopilot research by isolating the equations into blocks according to the frequency with which they will change. The equations of motion subsystems apply to all aircraft and all autopilots. The aircraft model changes rarely as new modeling data are obtained from flight identification, or it changes among groups working on different aircraft. Finally, the autopilot changes most frequently. The labor of plugging in a revised or alternate subsystem is minimized since there are very few interface variables.

Finally, the same structure can be used for all simulations in the anticipated hierarchy of simulations so that the autopilot can be transferred up the hierarchy with a minimum of plug-in labor, including, eventually, the aircraft flight-control system itself. While the interfaces have been minimized, complexity and large size remains internalized in the subsystems, particularly the aircraft model and the autopilot. To consider further partitioning of the system and also to define it, the equations for each of the three main subsystems are reviewed next.

Equations of Motion and Environment Subsystems

Figure 4 shows the equations of motion. Rigid body and flat earth are assumed as is appropriate in our case. These equations exhibit the nature of the dynamics without containing any detail about the aircraft or autopilot, so this subset of system equations will appear for any aircraft or

autopilot. Where vectors occur in the notation, a subscript indicates the coordinate frame in which its components are given, for example, R_I is the position vector in inertial coordinates.

The first two equations describe the trajectory in the inertial reference frame. The aircraft forces are assumed given in body axes and must be transformed to the inertial frame. During a simulation run, these are integrated to give the trajectory. The second pair of equations describe the rotational degrees of freedom. The angular acceleration equation is written in body axes because the aircraft matrix, I , is constant in this frame. The applied moments are assumed given in body axes also.

The system dynamics are partitioned later among the translational and rotational degrees of freedom. Attitude enters the trajectory motion through the transformation, T_{IB} , and to the extent that the force F_B depends on variables of the rotational motion, although this is not shown here. The translational degrees enter the rotational motion through M_B only, although this dependence is not yet shown.

Generally, the air mass will be in motion relative to the ground and this enters the problem because the aircraft forces are functions of the velocity relative to the air mass rather than the inertial velocity. The wind is modeled as the sum of a steady component (WB) and a gust component (WG) generated from a linear system driven by Gaussian distributed noise; this model is frequently used in simulations. Parameters of the atmosphere – density, temperature, pressure, and speed of sound – are also required for calculating aircraft forces and these are generated from an altitude-dependent, standard atmosphere model.

Finally, various functions of the motion must also be calculated in the simulation: the Euler angle transformation and the matrix, A , used in the equations of motion and also the velocity vector with respect to the air mass required by the aircraft model. This vector appears in the model in the form of its spherical coordinates (VA, α, β). Additional quantities used by the aircraft model in general are the dynamic pressure, Q and the angle rates α and β .

This group of equations is the subset of system equations that define the equations of motion subsystem of the simulation. This subsystem appears in any simulation independent of aircraft or autopilot. The inputs to this subsystem are the total aircraft forces and moments in body axes and the aircraft mass, and its outputs are the translational and rotational motion variables, wind, and atmospheric parameters, and various parameters of the velocity vector with respect to the air mass. Figure 5 is a block diagram of this subsystem.

Aircraft Model Subsystems – AWJSRA

The aircraft model subsystem is defined by its equations (fig. 6). This subsystem is the principal source of size and complexity in the simulation and in the control problem so the equations are given in sufficient detail to show these aspects. Where details of a specific aircraft occur, these are taken from the augmentor wing. The function of this subsystem is to calculate the aircraft forces and moments that arise from the engine and from the airflow over the aircraft structure.

Engine— The engine model requires first that sufficient data be stored to define fuel flow, thrust magnitude, and the equivalent augmentation thrust. This last is a fictitious thrust force that

measures the amount of compressed engine air exhausted through the trailing edge of the experimental flap of the augmentor wing. The stored functions are nonlinear functions of atmospheric parameters, airspeed, and throttle control. The fuel consumption accounts for slow variations in aircraft mass during flight. The engine dynamics that govern variations in thrust in response to throttle commands are modeled simply as a first-order system. However, engine dynamics are normally quite complex, reflected here in the form of a time constant that changes (depending on whether thrust is increasing or decreasing) and in the form of constraints on the extremes of magnitude, rate, and acceleration of thrust. The nozzle angle dynamics are taken simply as a constant rate. The parameter, C_J , is the dimensionless equivalent thrust and appears as a variable throughout the aerodynamic functions. The factor QS is commonly used throughout aerodynamics to make forces dimensionless. Finally, the engine force is given by the thrust and directed relative to the body axes frame by a unit vector given from the nozzle angle, and the engine moment is given straightforwardly (fig. 6). This description is specialized to the AWJSRA, particularly in the appearance of C_J and ν . The term C_J appears throughout the aerodynamics, and its contribution to modeling complexity is apparent. The nozzle appears only in the very simple manner shown, but it contributes major difficulties to the solution of the control problem, both conceptually and computationally. Engine models for other powered lift aircraft would also have special effects that would increase the complexity, although different in detail.

Actuators— The dynamic response of the aerodynamic control surface deflections to control commands is considered next (fig. 6). The actual hardware by which the pilot or autopilot communicates with these surfaces is a complex sequence of linkages, servomotors, hydraulic actuators, and the surface itself, as defined on an engineering schematic of some kind. However, we are not interested in this level of detail, so an input-output model of the actuator system is identified from its detailed description and the dominating dynamics separated from that and modeled in the simulation as a simple first- or second-order system. The remaining dynamics are much faster than those of the aircraft motion. The remaining features of concern are the limits on surface position and rate, resulting from aerodynamic or servomotor saturation, which are reflected in the control problem as limits on control power.

Aerodynamics— The principal source of size and complexity in our system is the aerodynamic model. Generally, the aerodynamic forces and moments are functions of time histories (up to the present) of motion and control variables. However, almost all flight can be conceived as being small departures from some steady flight condition and this motivates modeling the aerodynamic force as a sum of terms (fig. 6). The leading term, f_0 , is the force in steady flight and the remaining terms are superposed independent small effects that account for nonzero values of various motion and control variables. For completeness, the expression is premultiplied by a transformation $E_2(-\alpha)$ from wind axes components in which the aerodynamic force data are usually given, and the factor, QS , that separates the principal dependence of forces on air speed and also makes the vectors in the sum dimensionless.

The leading term, f_0 , is a function of α , F , C_J , and altitude, and it accounts for the principal dependence of aerodynamic forces on these variables. Although the $\{f_i\}$ terms are functions of several variables, the principal variations in flight and their difference for zero results from the variable δi , which is one of the angles $\{\beta, \delta a, \delta e, \delta R\}$. These terms are either linear or nearly so in the variable δi . For the controls, their usefulness is closely dependent on their near-linearity. The remaining terms $\{d_i\}$ account for various angular rates $\{\alpha, \beta, r_B\}$, whose effects are modeled as

linear and are made dimensionless by the appropriate factor from $\{\mu_j^*\}$. Similar remarks apply to the description of the moments.

The components of the force and moment vectors given in wind axes are individually identified in the working nomenclature as coefficients of drag, Y , and lift forces and as coefficients of roll, pitch, and yaw moments – one coefficient for each component of each vector in the sum. These coefficients are functions of varying degrees of complexity and sufficient data to define all the coefficients in the list are stored in the simulation. The gross properties of this collection of coefficients are examined next.

Aerodynamic coefficients – Figure 7 summarizes in a matrix the coefficients required for the aerodynamic mode. The columns itemize the terms that are summed to form the total force and moment and the rows itemize the wind axes components of each term. The resulting nomenclature of the coefficients thus identifies each contribution to aerodynamic forces and moments by source and by effect – this is the working nomenclature. A large body of classical literature exists which analyzes the origins and consequences to the flight dynamics of each coefficient.

The collection of coefficients divides about evenly into trivial and nontrivial ones. The nontrivial ones vary in complexity from constants to functions of four variables. Much of this complexity is characteristic of powered lift aircraft compared to conventional aircraft, for example, the engine parameter, C_J , appears in most of the coefficients. And parameters such as F , δa , which conventionally have linear additive effects, are now nonlinear and even nonadditive as for F .

In the simulation program, these coefficients are evaluated at each integration step and this constitutes a large part of the computational and storage load in a six-degree-of-freedom simulation. This situation is common to powered lift vehicles, and efficient computational methodology is a significant programming problem that is also reflected as a significant programming problem in the autopilot. The functions are handled by storing tabulated functions that are then interpolated during a simulation run. Figure 8 shows an example – a lift-drag polar that defines the leading force term by mapping values of the three independent variables (α , F , C_J) into values of C_L and C_D . For a conventional aircraft, we would have a single independent variable and therefore a single line on one plot. For the AWJSRA, two additional variables require first additional lines and then additional plots.

Half of the 60 possible coefficients are zero. This reflects the physical symmetry of the aircraft as well as the normal operation of the aircraft with the velocity vector in the plane of symmetry. If the rows and columns of the matrix of the coefficients are shuffled, the structure shown in figure 9 appears. As seen, the system of variables and of force and moment components almost divides into two independent subsystems (referred to as the longitudinal and lateral degrees of freedom), which separates motion in the plane of symmetry from motion lateral to it. If we were to linearize the equations of motion about an unaccelerated flight condition, this decomposition appears throughout the systems of equations, and this has been exploited heavily in the classical development of aircraft control since the small-disturbance behavior of the aircraft can thus be analyzed as two subsystems half the size.

This subdivision is now exploited in the nonlinear simulations, nor in our control problem, where we are interested in the inertial path of the vehicle as much as its local small-perturbation

dynamics. It will prove more rewarding in terms of solving the autopilot problem to exploit a division into linear and rotational degrees of freedom. Nevertheless, this shows that there are other partitions of the complete system into loosely coupled subsystems besides the ones to be focussed on here.

Figures 6 through 9 have defined the model of the aircraft forces and moments that result from the engine and aerodynamics. Their interrelationships are exhibited in a block diagram of the aircraft model (fig. 10). The internal blocks correspond to the main subroutines of the digital simulation of this subsystem; the engine is modeled in a single block while the aerodynamic description is divided into three blocks – actuators, forces, and moments.

The input-output relations among these blocks are arranged as two main parallel streams of cause and effect. This divides the calculations into quantities for which significant variations occur slowly or rapidly in time relative to each other. The lower stream contains slowly varying quantities – engine model, aerodynamic forces, and many coefficients of the moment equations which vary slowly with α , F , and C_j . Most calculations of the aircraft model can be separated into this stream. This partitioning is exploited when computation time is critical by carrying out the slow stream at, for example, half the frequency of the fast stream. This has been done in simulations operated in real time, primarily man-in-the-loop simulations. On the one hand, the computation time for a cycle must not exceed the corresponding real time increment being simulated, and that time increment should also be about 10 times the natural frequency of the dynamics being simulated. This partitioning of calculations was used when the computational load required for simulating powered lift vehicles exceeded the computer speed if carried out in a single stream. An alternate solution, also used, is a faster computer. A similar constraint on computational cycle time will occur for the autopilot in flight, and a similar partitioning of the control calculations into subsystems evaluated at different rates can be exploited there as well.

Autopilot– In the simulation of aircraft flight, the motion is always controlled, that is, the values of all controls must somehow be defined to operate the simulation. In the present case, where the simulation is used for autopilot development, experimental autopilot logic fills this role. Although the autopilot is not discussed in detail, it is useful to describe its functioning at a general level since this will motivate a useful partitioning of the aircraft simulation (fig. 11).

The purpose of the autopilot, in general, is to fly some reference path defined, for example, by air traffic control. To do this, the autopilot necessarily solves the equations of motion for the control settings that fly the reference path and corrects any errors from the path.

Figure 11 shows the input information to be the state variables divided into variables of the translational and rotational degrees of freedom, and also wind information and inputs from air traffic control that define the reference trajectory. The outputs are commands to the control actuators, subdivided into force and moment controls.

Internally, the autopilot logic is partitioned into trajectory control and attitude control. The attitude control logic calculates the moment control commands that execute attitude and angular velocity commands from the trajectory control logic. The trajectory control logic uses the force controls, UF , to control forces in the plane of symmetry; δ th and ν are engine controls and F , the flap deflection angle, controls lift through its dependence on wing configuration. It also uses the

attitude control subsystem to control the orientation of the plane of symmetry in inertial space and it uses pitch angle to control aerodynamic force through angle of attack.

The control problem is thus divided into two smaller problems based on a partitioning of the system into translational and rotational degrees of freedom. The trajectory is controlled by large engine and aerodynamic forces that inherently have slower dynamics than the control moments used by the attitude control subsystem. In addition, it is assumed that attitude commands from the trajectory control logic will have only low-frequency variations for the passenger-oriented operations considered here. Thus, for the controlled airplane, the rotational subsystem appears to the translational subsystem as a fast servo insofar as attitude response to attitude commands is concerned. This result can be used to simplify the rotational degrees of freedom and to obtain a relatively small simulation that has proved extremely useful in studying the trajectory control problem (fig. 12).

SIMPLIFIED SIX-DEGREE-OF-FREEDOM SIMULATION

Figure 12 shows the complete system – autopilot, aircraft model, and equations of motion – subdivided into translational and rotational subsystems that separate the trajectory variables and attitude variables. The translational subsystem is affected by the rotational subsystem through the attitude response to attitude commands and through a perturbation force that results from high frequency rotational dynamics and has negligible effect on the trajectory. Because of the faster rotational dynamics, the complete rotational subsystem can be removed and its input-output behavior obtained approximately from a linear second-order system (fig. 12).

Thus the simulation can be partitioned into a half-size simulation to study the trajectory control problem. This has proved an ideal tool, much smaller in size and computation time than the parent six-degree-of-freedom simulation. In fact, the size of the trajectory control logic program is twice that of the remainder of the simulation, which gives a comfortable ratio of the tool size to problem size for this portion of the research. If the complete six-degree-of-freedom simulation were to be used for the trajectory control research, then the reverse would be true, and the control logic program would be outweighed by the rest of the simulation by more than two to one.

HIERARCHY OF SIMULATIONS

Figure 13 shows some aspects of literal size revealed by the hierarchy of simulations. A six-degree-of-freedom simulation was available for our research needs, but the size of this object was a problem, as well as its organization, which was oriented toward general-purpose use with man-in-the-loop simulation. It was large in terms of the number of boxes of FORTRAN cards that could be carried about lightly, or the number of listings that could be spread out on a desk, or the number of sheets of paper required to write down a working definition of the system. In this regard, the report that defines the aircraft model has 96 pages of equations and tabulated function data (ref. 3). These sizes are significant in relation to the desire for easy accessibility, use, understanding, and modification during the critical early stages of the research when the central conceptual problems are solved.

In response to this problem of size, a hierarchy of simulations of increasing size and detail has been assembled or planned. First, the simplified six-degree-of-freedom simulation described in figure 12 is used to develop the trajectory control logic. The shaded and unshaded portions of the bar in figure 13 are the size of the trajectory control logic and the size of the remainder of the simulation, respectively, in feet of FORTRAN cards. Not only is the absolute size of this program quite small, but the size of the research portion of the simulation is twice that of the rest of the simulation program. This is a comfortable working ratio between the size of the tool used and the size of the problem being solved. Although the simulation is very much smaller than the six degrees of freedom, the control problem remains difficult.

The second member of the hierarchy is the six-degree-of-freedom simulation, used for attitude control and navigation studies, as well as for verification of the trajectory control logic in a more detailed simulation of the flight dynamics. This tool is much larger and the ratio of tool size to problem size is less favorable. (One response to tool size is to enlist a specialist to work the tool for you, but at the expense of some loss of familiarity with the system being controlled.)

The third member of the hierarchy of simulations is a large man-in-the-loop laboratory simulation on computers dedicated to such simulations. The added detail included a cockpit, displays, sensors, and also a copy of the STOLAND airborne computer and peripheral equipment that may be used for a thorough evaluation before any flight test. With a simulation of this size, a staff of specialists and assistants who work solely with the simulation is required.

The organizational size itself creates certain problems of inconvenience to the research. The demand for the simulator-STOLAND is sufficiently great that long lead time scheduling or inconvenient working hours become necessary and it is quite possible that an additional simulation, intermediate in size between the second and third members of the hierarchy, will be programmed to minimize the time needed on simulator-STOLAND.

SUMMARY

Simulations of V/STOL aircraft for autopilot research were introduced as an example of a large-scale system. Large, in this case, was perceived in relation to the uses of the simulation, that is, in terms of the research requirement for a simulation that was easy to work with and in terms of the complexity and difficulty of the control problem. Both problems were attacked and at least partially solved through partitioning of the system of equations and hierarchic organization of the system. The various organizations described are listed in figure 14.

First, a hierarchy of simulations was assembled — a sequence of increasingly detailed simulations of the aircraft paralleling a corresponding hierarchic development of the autopilot. The object was to have a digital simulation no larger than necessary at each stage of the research since such factors as ease of access, use, understanding, and modification are all proportional to size.

Second, a modular organization was given to the simulations, which were divided into three main subsystems — the autopilot, the aircraft model, and the equations of motion and environment section. These modules have a minimum of interface variables and provide for plug-in simplicity so

that frequent revisions of the autopilot and occasional changes of aircraft occurring in the research can be made.

Third, the dynamics of the system were subdivided into translational and rotational degrees of freedom, based on the different frequencies at which significant variations in motion variables and control forces and moments occur in the two subsystems. This is the basis of the autopilot partitioning into two smaller control problems. This frequency separation can also be used to satisfy the real-time constraint on the autopilot cycle time should the computational load for powered-lift aircraft prove too large in relation to computer speed. This was also the basis for devising a simplified six-degree-of-freedom simulation by replacing the rotational subsystem with its input-output relation. This has been a very convenient research tool for the trajectory control problem and has permitted a study of that problem independent of the attitude control problem.

Fourth, an alternate partitioning of the aircraft dynamics into longitudinal and lateral degrees of freedom was mentioned. This is used in the classical linearized analysis and separates the dynamics into motion in the plane of symmetry and lateral to it.

Finally, partitioning the aircraft model section into two subsystems whose variations were well separated in frequency permitted these subsystems to be evaluated at different rates during simulation. This has been used in real-time simulations to meet the real-time constraints on computation cycle time, especially for powered lift V/STOL aircraft.

REFERENCES

1. Meyer, G.; and Cicolani, L.: Methodology for Design of Active Controls for V/STOL Aircraft. NASA-FRC Symposium on Advanced Control Technology and Its Potential for Future Transport Aircraft. Los Angeles, July 9-11, 1974.
2. Young, L. S.; Hansen, Q. M.; Rouse, W. E.; and Osder, S. S. Development of STOLAND, A Versatile Navigation, Guidance and Control System. NASA TM X-62,183, Oct. 1972, Ames Research Center, Moffett Field, California.
3. Cleveland, W.; Vomaske, R.; and Sinclair, S. R. M.: Augmentor Wing Jet STOL Research Aircraft Digital Simulation Model. NASA TM X-62,149, April 1972.

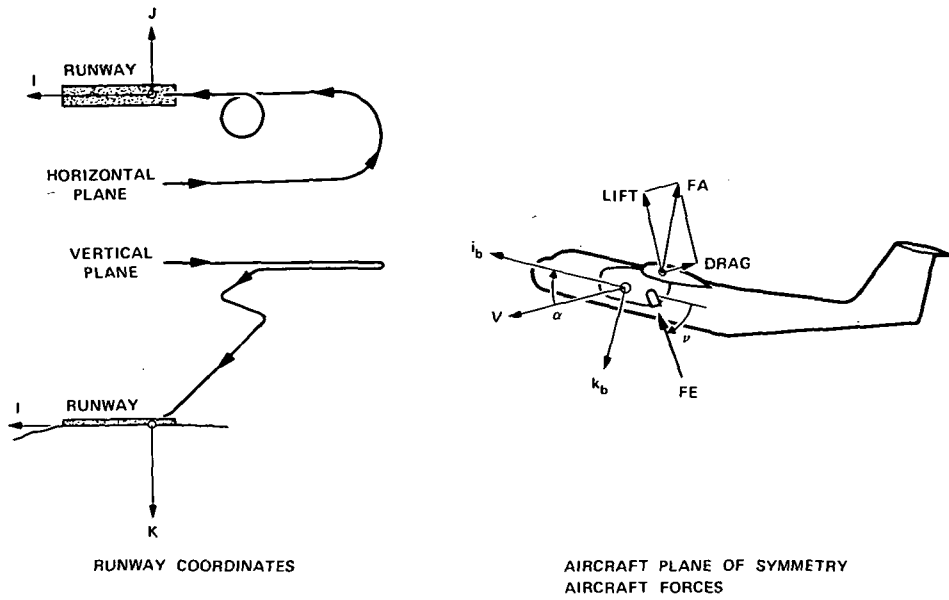


Figure 1.— Translational degrees of freedom.

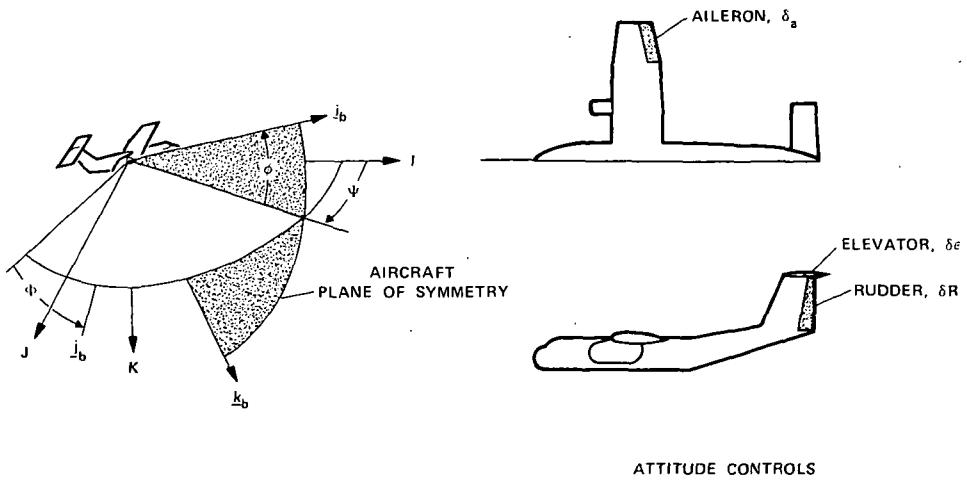
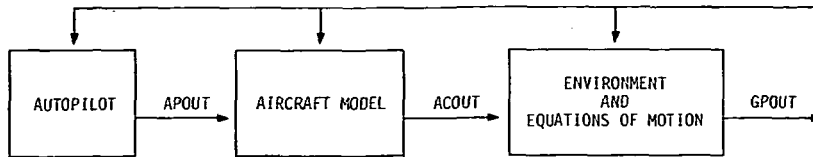


Figure 2.— Rotational degrees of freedom.



APOUT: $(F_c, \delta th_c, v_c, \delta a_c, \delta e_c, \delta R_c)$

ACOUT: (M, F_B, M_B)

GPOUT: $(R_I, v_I, \dot{v}_I, (\phi, \theta, \psi), \Omega_B,$
 $w_I, VA, \alpha, \beta, \dot{\alpha}, \dot{\beta}, AT)$

Figure 3.— Block diagram of AWJSRA aircraft simulation.

Translational degrees of freedom: position and velocity:

$$\dot{R}_I = v_I$$

$$\dot{v}_I = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + T_{IB}(\phi, \theta, \psi) \frac{1}{m} \Sigma F_B$$

Rotational degrees of freedom: attitude and angular velocity:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = A(\phi, \theta, \psi) \cdot \Omega_B$$

$$\dot{\Omega}_B = I^{-1} \Sigma M_B + I^{-1} \Omega_B \times I \Omega_B$$

Wind velocity:

$$w_I = WS_I(z) + WG_I$$

$$\begin{bmatrix} \dot{WG} \\ \dot{WG}_I \end{bmatrix} = A_g \begin{bmatrix} WG \\ WG_I \end{bmatrix} + B_g N_g ; \quad N_g = (N_x, N_y, N_z) \sim N(0,1)$$

Atmosphere:

$$AT = [\rho(z), T(z), p(z), a(z)]$$

Kinematic functions:

$$T_{IB} = E_1(\phi)E_2(\theta)E_3(\psi)$$

$$A = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix}$$

$$VA_B = T_{BI}(v_I - w_I)$$

(VA, α, β) = spherical coordinates of VA_B

$$Q = \frac{1}{2} \rho V^2$$

$$\dot{\alpha} = da/dt$$

$$\dot{\beta} = d\beta/dt$$

Figure 4.— Equations of motion — rigid body, flat earth.

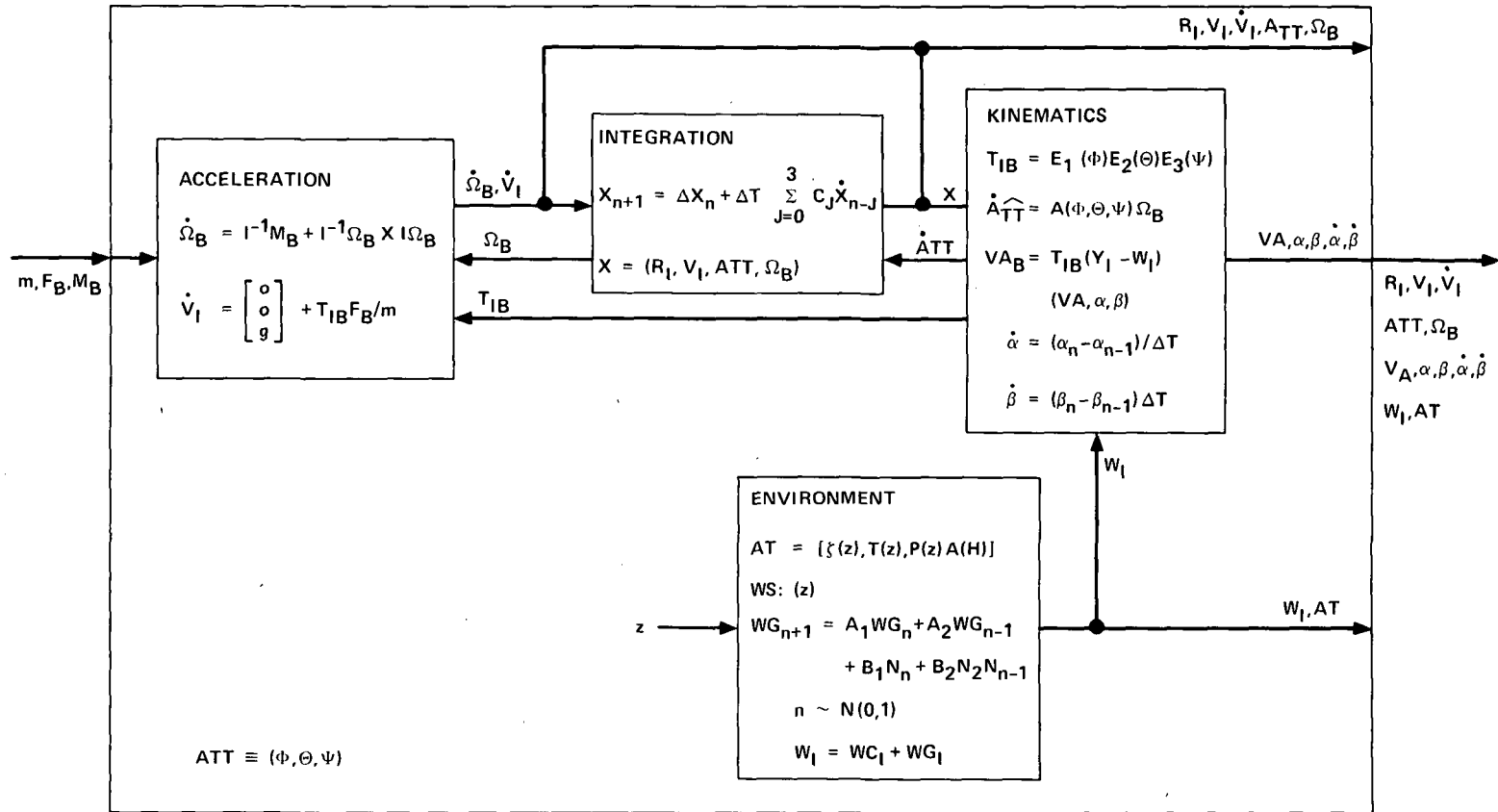


Figure 5.— Block diagram of equations of motion and environment model.

$$\Sigma F = FE+FA$$

$$\Sigma M = ME+MA$$

Engine

Stored data: $\dot{M}_F(AT, \delta th), T(AT, VA, \delta th), TC(AT, VA, \delta th)$

$$\dot{M} = -\dot{M}_F \quad \text{Constraints: } (T_{EX}, \dot{T}_{EX}(T), \ddot{T}_{EX}(T, \dot{T}))$$

$$\dot{T} = (\delta th - T)/\tau \quad \tau(\dot{T})$$

$$\dot{v} = \text{Const}, C_v \quad \text{Constraints: } (v_{EX})$$

$$C_J = TC/QS$$

$$FE_B = \dot{i}(v)T$$

$$ME = FE \times \xi_E$$

Actuators

$$\dot{u}_i = [u_i - u_i]/\tau_i \quad u_i \in (\delta a, \delta e, \delta R)$$

$$\text{Constraints: } (u_{iEX}, |u_i|_{\max})$$

$$|\dot{F}| = \text{const}, C_F \quad \text{Constraints: } (F_{EX})$$

Aerodynamics

$$FA_B = QSE_2(-\alpha) \left[f_0(z, \alpha, F, G_j) + \Sigma f_i(\delta i, \alpha, F, G_j) + \Sigma d_i(d, F, G_j) \frac{v_j}{v_j^*} \right]$$

$$MA_B = QS[\xi]E_2(-\alpha) \left[f_{m_0} + \Sigma f_{m_i} + \Sigma d_{m_i} \frac{v_j}{v_j^*} \right]$$

$$\delta i \in (\beta, \delta a, \delta e, \delta R)$$

$$f_i = 0 \text{ for } \delta i = 0; f_i \text{ monotonic in } \delta i$$

$$\left| \frac{\partial f_i}{\partial \delta i} \right| \delta i_{\max} \gg \left| \frac{\partial f_i}{\partial \alpha} \right| \alpha_{\max}, \text{ etc.}$$

$$u_j \in (\hat{\alpha}, \hat{\beta}, \hat{\rho}_B)$$

$$u_j^* \in \left\{ 2 \frac{VA}{b}, 2 \frac{VA}{c} \right\}$$

Stored data: $\left\{ C_{D_0}, C_{D_1}, \dots, C_{y_0}, C_{y_1}, \dots, C_{L_0}, C_{L_1}, \dots \right\}$

$$\left\{ C_{\xi_0}, C_{\xi_1}, \dots, C_{m_0}, C_{m_1}, \dots, C_{n_0}, C_{n_1}, \dots \right\}$$

Figure 6.— Aircraft model — AWJSRA.

Wind axes components		f_o, f_{m_o}	$f_{\delta e}$ $f_{m_{\delta e}}$	$f_{\delta a}$ $f_{m_{\delta a}}$	$f_{\delta R}$ $f_{m_{\delta R}}$	f_{β} $f_{m_{\beta}}$	d_{α} $d_{m_{\alpha}}$	d_{β} $d_{m_{\beta}}$	d_{ρ} $d_{m_{\rho}}$	d_{q} $d_{m_{q}}$	d_r d_{m_r}
FA	$-l_w \cdot FA$ (drag)	$C_{D_{WB}}$	0	$\Delta C_{D_{\delta a}}$	0	0	0	0	0	0	0
	$J_w \cdot FA$ (Y)	0	0	$\Delta C_{\delta a}$	$\Delta C_{y_{\delta R}}$	$C_{y_{\beta}} \cdot \beta$	0	$C_{y_{\beta}}$	$C_{y_{\rho}}$	0	C_{y_r}
	$-k_w \cdot FA$ (lift)	$C_{L_{WB}} + \Delta C_{L_T}$	$C_{L_{\delta e}} \cdot \delta e$	$\Delta C_{L_{\delta a}}$	0	0	$C_{L_{\alpha}}$	0	0	C_{L_q}	0
MA	$l_w \cdot MA$ (roll)	0	0	$\Delta C_{l_{\delta a}}$	0	$C_{l_{\beta}} \cdot \beta$	0	$C_{l_{\beta}}$	$C_{l_{\rho}}$	0	C_{l_r}
	$J_w \cdot MA$ (pitch)	$C_{m_{WB}} + \Delta C_{m_T}$	$C_{m_{\delta e}} \cdot \delta e$	$\Delta C_{m_{\delta a}}$	0	0	$C_{m_{\alpha}}$	0	0	C_{m_q}	0
	$K_w \cdot MA$ (yaw)	0	0	$\Delta C_{n_{\delta a}}$	$\Delta C_{n_{\delta R}}$	$C_{n_{\beta}} \cdot \beta$	0	$C_{n_{\beta}}$	$C_{n_{\rho}}$	0	C_{n_r}
Independent variables		$\alpha, \alpha(t-\tau)$ F, C_J, Z	α	δa α, F, C_J	δR	α, F, C_J			α, F, C_J		α, F, C_J

Figure 7.— Aircraft model – aerodynamic force coefficients.

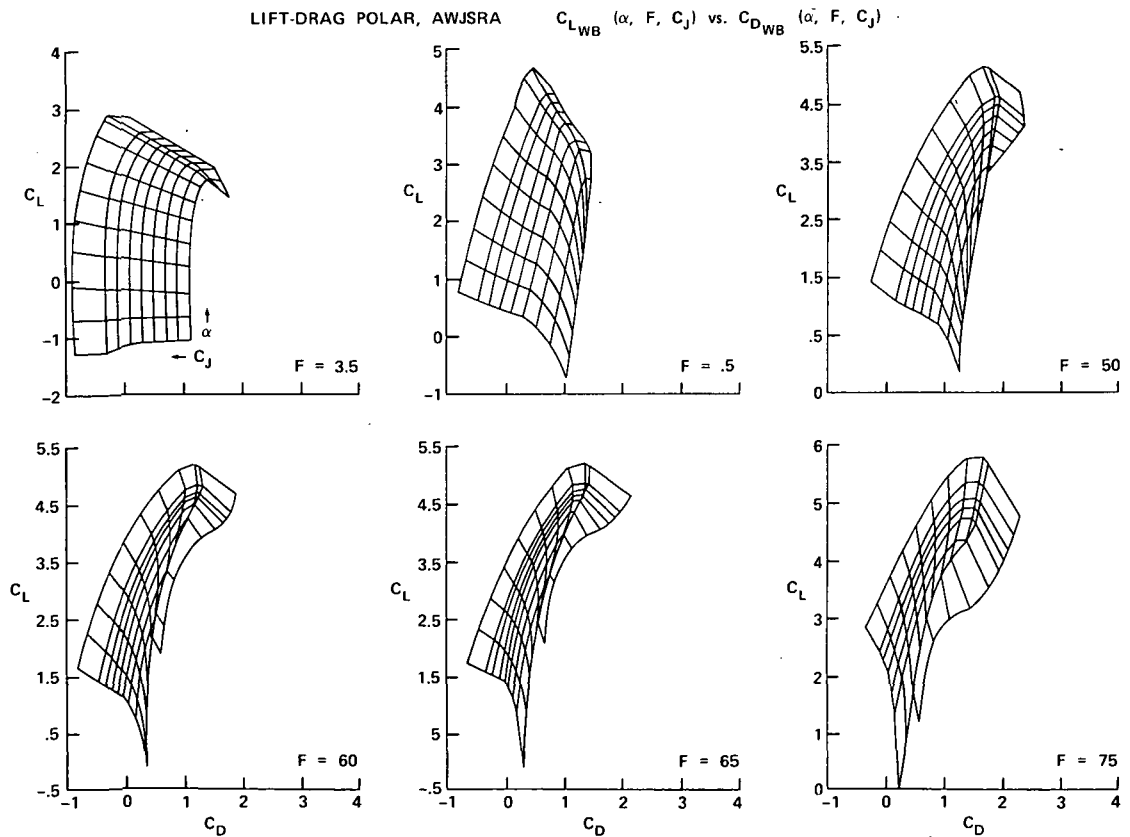


Figure 8.— Lift-drag polar, AWJSRA.

Coefficient	Parameters									
	α, F, C_J	δ_e	α	q	δa	δR	β	$\dot{\beta}$	ρ	r
(a) Longitudinal										
Drag, $-i_w \cdot FA$	$C_{D_{WB}}$	0	0	0	$\Delta C_{D_{\delta a}}$					
Pitch, $j_w \cdot MA$	$C_{m_{WB}} + C_{m_T}$	$C_{m_{\delta e}}$	$C_{m_{\alpha}}$	C_{m_q}	$\Delta C_{m_{\delta a}}$					
Lift, $-k_w \cdot FA$	$C_{L_{WB}} + \Delta C_{L_T}$	$C_{L_{\delta e}}$	$C_{L_{\alpha}}$	C_{L_q}	$\Delta C_{L_{\delta a}}$					
(b) Lateral										
Roll, $i_w \cdot MA$					$\Delta C_{l_{\delta a}}$	$\Delta C_{l_{\delta R}}$	$C_{l_{\beta}}$	$C_{l_{\dot{\beta}}}$	$C_{l_{\rho}}$	C_{l_r}
y, $j_w \cdot FA$					$\Delta C_{y_{\delta a}}$	$\Delta C_{y_{\delta R}}$	$C_{y_{\beta}}$	$C_{y_{\dot{\beta}}}$	$C_{y_{\rho}}$	C_{y_r}
Yaw, $k_w \cdot MA$					$\Delta C_{n_{\delta a}}$	$\Delta C_{n_{\delta R}}$	$C_{n_{\beta}}$	$C_{n_{\dot{\beta}}}$	$C_{n_{\rho}}$	C_{n_r}

Figure 9.— Aircraft model — coefficients for longitudinal and lateral degrees of freedom.

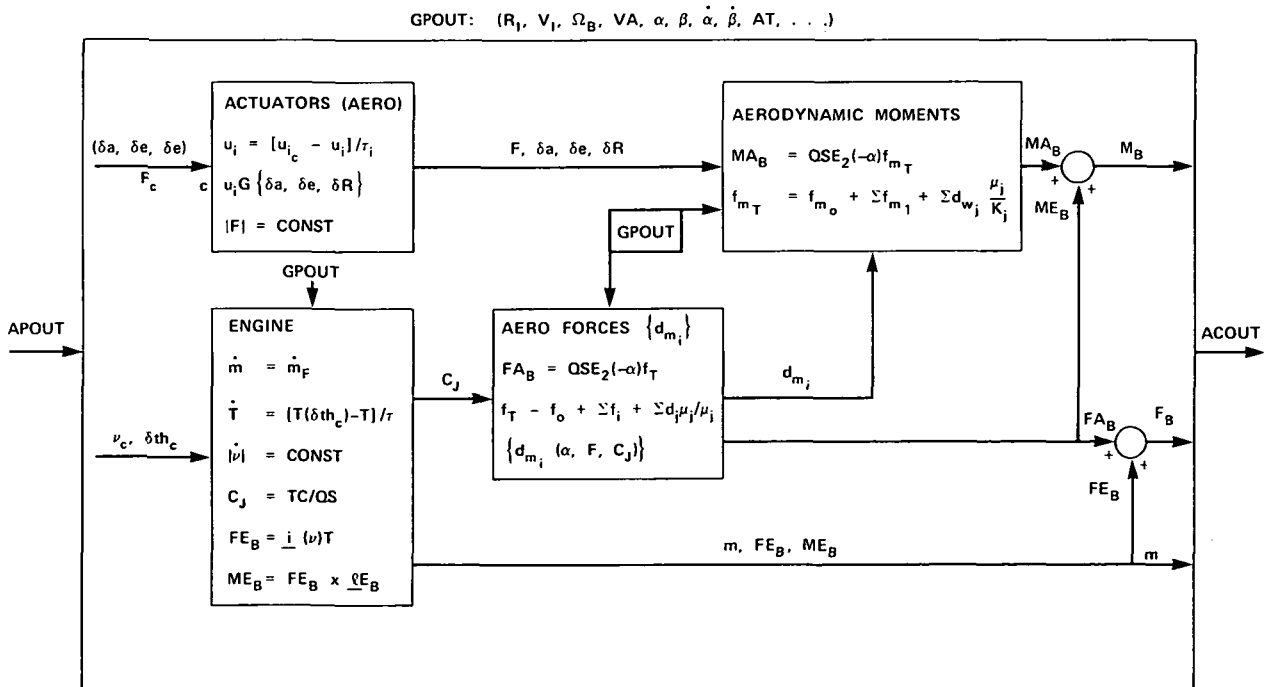


Figure 10.— Block diagram of aircraft model — (AWJSRA).

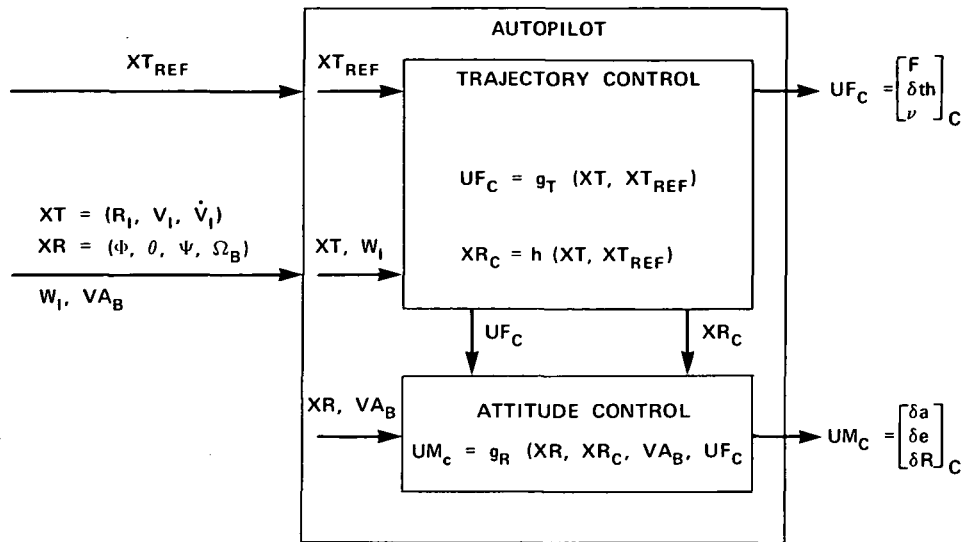


Figure 11.— Block diagram of autopilot.

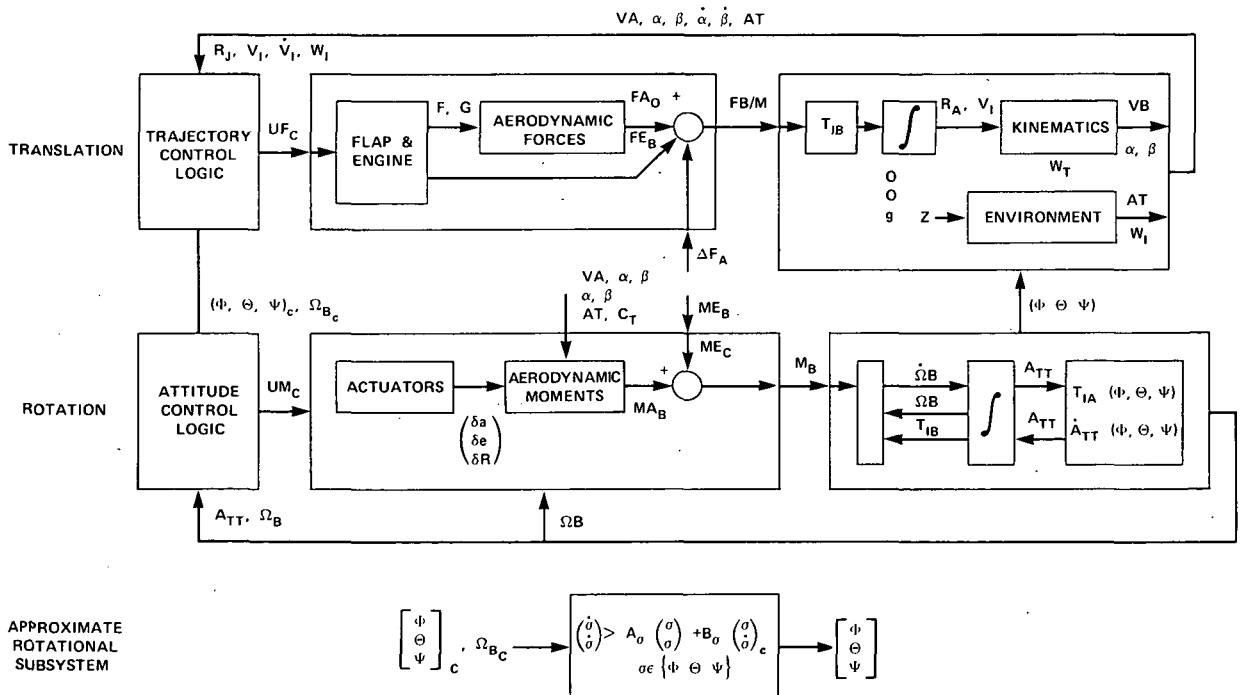


Figure 12.— Subsystems — translational and rotational degrees of freedom.

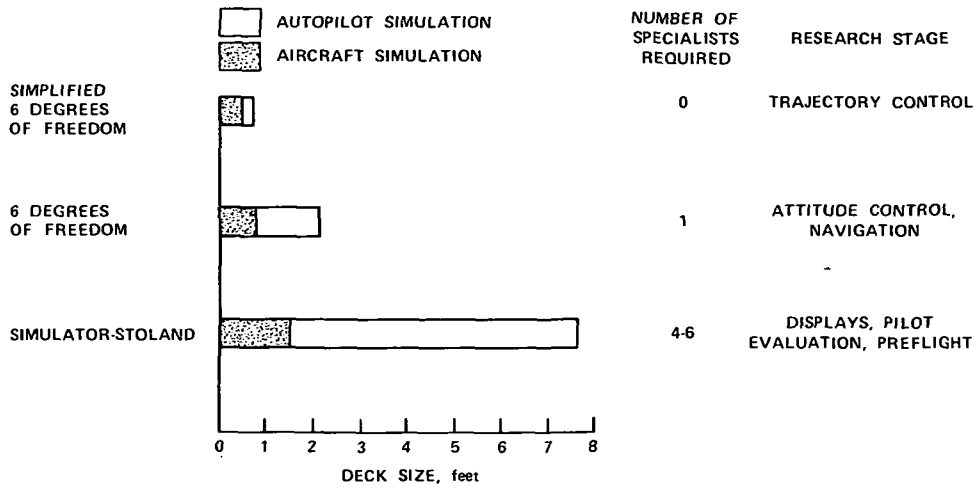


Figure 13.— Hierarchy of simulations.

1. HIERARCHY OF SIMULATIONS	LIMIT SIZE OF SIMULATIONS
2. MODULAR ORGANIZATION	PLUG-IN SIMPLICITY FOR AUTOPILOT OR AIRCRAFT CHANGES
3. TRANSLATIONAL/ROTATIONAL DECOMPOSITION	SUBDIVIDE CONTROL PROBLEM REDUCE COMPUTATION CYCLE TIME SIMPLIFIED SIX-DEGREE-OF-FREEDOM SIMULATION
4. LONGITUDINAL/LATERAL DECOMPOSITION	SUBDIVIDE LINEARIZED DYNAMICS
5. AIRCRAFT MODEL "SLOW"/"FAST" CALCULATIONS	REDUCE COMPUTATION CYCLE TIME

Figure 14.— Summary of organizations.

Page Intentionally Left Blank

SOME NOTIONS OF DECENTRALIZATION AND COORDINATION IN LARGE-SCALE DYNAMIC SYSTEMS

C. Y. Chong*

Georgia Institute of Technology, Atlanta, Georgia 30332

INTRODUCTION

In this paper, we shall discuss some notions of decentralization and coordination in the control of large-scale dynamic systems. Decentralization and coordination have always been important concepts in the study of large systems. Roughly speaking, decentralization is the process of dividing a large problem into subproblems so that it can be handled more easily. Coordination is the manipulation of the subproblem so that the original problem is solved. A great deal of literature is available dealing with these two topics, especially decentralization. In this paper, we shall discuss the various types of decentralization and coordination that have been used to control dynamic systems. Our emphasis will be to distinguish between on-line and off-line operations. This distinction is not, of course, unique. However, it helps to understand the results available by indicating the aspects of the problem which are decentralized. This discussion is informal and no attempt has been made to give precise definitions. Our main objective is to illustrate intuitively "what" is decentralized in the decision-making.

The hierarchical approach with a coordinator has been suggested as a possible way to control a large system. We propose a coordination scheme that is suitable for stochastic systems. This is discussed with respect to the various notions of decentralization.

INFORMATION AND COMPUTATION IN DECISION MAKING

In this section, we consider the control of a dynamic system. A dynamic system is given with a set of control inputs and a set of measurements. This is to be controlled by a decision maker (controller) in real time to achieve certain objectives. The objectives may be to optimize or stabilize the overall system.

The information available to the decision maker consists of two parts:

1. *Prior information* — this includes information on the system structure, values of the parameters, constraints on the controls, and so on. If the problem is stochastic, then it may also include the statistics of the uncertain quantities. The word "prior" is used in a relative sense. Prior information available now is usually deduced from measurement collected in the past.

*School of Electrical Engineering.

2. A posteriori information – This includes measurements obtained in real time as well as controls used in the past. When several controllers are present, a posteriori information includes messages communicated among the controllers.

The decision maker has to generate the controls in real time based on his prior information and the posteriori information. Figure 1 illustrates the structure of the control problem.

The job of the decision maker can be divided into two parts, depending on whether it is carried out on-line or off-line.

Off-line operation involves generating the control law using all available prior information. This, of course, also makes use of the knowledge about the availability of measurements. On-line operation involves generating the control value at any time using the control law and the posteriori information available at that time. Usually, the off-line operations involve human beings or computers doing the computation while the on-line operations involve hardware of onboard computers. For example, if the system is linear, the noises are Gaussian and the cost functional is quadratic, then the off-line operations involve the solution of Riccati's equations and the on-line operations involve only matrix-vector multiplications and additions for discrete time problems. Figure 1 is then modified to figure 2.

COMPLEXITY OF THE CONTROLLER

We shall discuss the complexity of the controller with respect to the information processing requirements. There are two operations which must be done in real time: transmitting the measurements to the control agents and computing the control values from these measurements. Then the complexity of the control system is reflected by the size of the communication system involved and by the amount of on-line computation required. If we want a quantitative measure, we can count the number of wires connecting the inputs to the measurements and evaluate the size of the real time computers. The off-line computation discussed in the previous section reflects the complexity involved in finding the control laws.

To build a control system, it is often necessary to put some constraints on the complexity of the controller. The complexity of the communication system can be constrained rather easily. This is the case studied in dynamic team theory on stochastic control with nonclassical information pattern (refs. 1 and 2). The on-line computation involved can also be constrained by using control laws of a particular form (e.g., linear control laws). It is difficult, however, to constrain the complexity of the off-line computation. As a matter of fact, the characterization of computational complexity is not easy (ref. 3). There is often a tradeoff between the complexities of the different operations. For example, if complete information about the system is allowed, satisfactory performance can often be obtained with very simple control laws that require simple off-line computation. On the other hand, if less information is available, the control laws are often more complicated.

VARIOUS TYPES OF DECENTRALIZATION

In the control of traditional small-scale systems, all measurements on the system are generally pooled together to generate the controls. Also, the control laws that govern the relationship between the measurements and the controls are usually determined in a centralized manner. In the control of large-scale systems, the centralized approach will give rise to serious problems of implementation. In addition, the decision problems themselves may be so complicated that they exceed the capacity of the fastest computers. Some kind of decentralization is therefore desirable. Based on the discussion in the previous section, two types of decentralization are distinguished. The first type deals with the real time operations and is used to reduce the complexity of implementation. The values of certain control variables will depend only on a subset of measurements. This reduces the complexity of the communication system required. We shall refer to this as *decentralized control*. The other type deals with the off-line operations and is used to reduce the complexity of finding the control laws. We shall refer to this as *decentralized off-line computation*. When decentralized off-line computation is used, the control laws can be visualized as being generated from several computers operating independently of each other.

We shall look at the various decentralized schemes considered in the literature and attempt to classify them according to the two types of decentralization discussed above.

Case 1: Centralized Control and Centralized Off-Line Computation

This is typical of traditional small-scale systems. The generation of the control values from the measurements as well as the determination of the control laws are handled by a single decision maker. The classical linear-quadratic-Gaussian problem and the pole allocation by centralized state feedback (ref. 4) all fall into this category.

Case 2: Decentralized Control and Centralized Off-Line Computation

This occurs when the information pattern is decentralized but the control laws are still computed in a centralized manner. Thus there is more than one control law, each of which transforms some set of measurements into a set of controls. On the other hand, the control laws are all determined by a single decision maker (fig. 3). Examples of decentralized control and centralized off-line computation include nonclassical, linear-quadratic-Gaussian problems (dynamic teams) (refs. 1, 2, 5) and stabilization of systems with decentralized feedback (refs. 6, 7). Usually, these are characterized by simple communication systems relatively complicated on-line and off-line computation. This is especially obvious in optimization problems. For the centralized control and centralized computation case of the linear-quadratic-Gaussian problem, the optimal control law consists of the optimal deterministic gain acting on the estimate of the Kaeman-Bucy filter. The on-line computation involves only a finite-dimensional filter. The off-line computation involves the solution of Riccati equations. For the nonclassical, linear-quadratic-Gaussian problem, however, the on-line computation is complicated because a filter of growing dimension is needed. The off-line computation is even more complicated (the solution is not known yet). We can explain the trade-off between complexity in communication and computation as follows. Since the off-line computation is

centralized, the decision maker will try to generate the missing measurements needed in the control by use of more complicated control laws.

Case 3: Centralized Control and Decentralized Off-Line Computation

The main interest here is to reduce the complexity of the computation of the control laws. A multilevel approach in finding the optimal control laws can be considered as part of this category (ref. 8).

Case 4: Decentralized Control and Decentralized Off-Line Computation

This case is the same as case 2 except that the control laws are also computed in a decentralized manner. We can regard both a posteriori information and prior information as decentralized. Figure 4 illustrates such a control configuration. Examples in this category include adjustment processes in resource allocation problems (ref. 9) where the off-line decentralized computation is usually conducted in an iterative manner and the decentralized stabilization of systems with unknown global structure (refs. 10, 11). Since decentralized (prior and a posteriori) information is the rule in large-scale systems, this approach deserves further investigation.

COORDINATION APPROACH IN LARGE-SCALE SYMAMIC SYSTEMS

The hierarchical approach has been proposed as a possible way to control large-scale systems (ref. 8). Much of the existing work deals with off-line computation of the optimal control strategies. The main function of the coordinator is to coordinate the decentralized off-line computation of the control laws. In references 12 and 13, a possible approach for the multilevel, hierarchical control of stochastic systems has been proposed. We shall relate this approach to the notions of decentralization discussed earlier. Some of the results are summarized. M. Athans also discusses this approach in another paper of this proceedings.

In a multilevel approach, there are (at least) two levels of decision makers. In the simplest case, a two-level hierarchy is considered. The local controller of each subsystem knows the local dynamics of the subsystem under his control. The local measurements of his subsystem are also available. At the higher level, the coordinator knows the global dynamics of all subsystems and their interactions. He also collects measurements periodically from the lower level. Thus we have the following information pattern.

	Prior information	Measurements
Coordinator	Centralized	Centralized periodic
Local controller	Decentralized	Decentralized

This reflects the intuitive notion that the local controllers have detailed but local information and the coordinator has coarse but global information. The local controllers send their measurements and controls to the coordinator periodically. The coordinator computes new coordinating

parameters based on this information and his prior information about the structure of the system. Using these coordinating parameters, the local controllers compute their control laws for the next period in a decentralized manner (fig. 5).

FORMULATION AND SUMMARY OF RESULTS

The linear-quadratic-Gaussian case is presented here. Consider a system consisting of N subsystems coupled together:

$$x_i(k+1) = A_{ii}x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k), \quad i = 1, \dots, N$$

where x_i is the state of the i th subsystem, u_i is the control, ξ_i is the driving noise, and v_i is the interaction from other subsystems given by

$$v_i(k) = \sum_{j \neq i} A_{ij} x_j(k)$$

The cost function is a sum of the cost functionals of the individual subsystems:

$$J = \sum_{i=1}^N J_i$$

$$J_i = E \left\{ x_i'(T) E_i x_i(T) + \sum_{k=0}^{T-1} x_i'(k) Q_i x_i(k) + u_i'(k) Q_i u_i(k) \right\}$$

The local measurement of the i th controller is

$$y_i(k) = C_i x_i(k) + \theta_i(k)$$

where $\theta_i(k)$ is the measurement noise. Let

$$Y_i(k) = \{y_i(0), \dots, y_i(k)\}$$

$$U_i(k) = \{u_i(0), \dots, u_i(k)\}$$

$$Y(k) = Y_1(k) \cup Y_2(k), \dots, \cup Y_N(k)$$

$$U(k) = U_1(k) \cup U_2(k), \dots, \cup U_N(k)$$

where n is an integer such that $k \geq n_Q$. This implies that the coordinator collects measurements from the lower level every Q units of time.

For each lower-level controller to control his system based on his information, some knowledge of v_i is required. The job of the coordinator is to provide this information so that v_i is the best estimate of the interaction based on his information, that is

$$\left\{ v_i(t) - \sum_{j \neq i} A_{ij} x_j(t) | Y_0(k) \right\} = 0 \quad t \geq k .$$

At the same time he would like to minimize the cost functional.

The coordinator can transmit any portion of his stored data to the lower level except the structure of the overall system. Thus, he can transmit the value of the matrices A_{ij} but not how they interact to form the overall system. Since $v_i(k)$ depends on $Y_0(k)$, we shall also let $u_i(k)$ depend on $Y_0(k)$. This implies that the a posteriori information of the coordinator can be used by the lower level. We then have the following problem. Given

$$x_i(k+1) = A_{ii} x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k)$$

$$J = \sum_{i=1}^N J_i .$$

Then each lower-level controller possesses the following information.

A priori information:

$$x_i(k+1) = A_{ii} x_i(k) + v_i(k) + B_i u_i(k) + \xi_i(k)$$

$$y_i(k) = C_i x_i(k) + \theta_i(k)$$

$$J_i = E \left\{ x_i'(T) F_i x_i(T) + \sum_{k=0}^{T-1} x_i'(k) Q_i x_i(k) + u_i'(k) R_i u_i(k) \right\} .$$

The statistics of $x_i(0)$, $\xi_i(k)$, $\theta_i(k)$, $k = 0, \dots, T-1$ are all known; $v_i(k)$ represents the interaction from the other subsystems, of which the i th controller is completely ignorant.

A posteriori information:

$$Y_i(k), U_i(k-1) .$$

The information of the coordinator consists of

A priori information:

$$x(k+1) = Ax(k) + Bu(k) + \xi(k)$$

$$y(k) = Cx(k) + \theta(k)$$

$$J = \sum_{i=1}^N J_i .$$

The statistics of all random quantities are known. Thus, the coordinator has complete a priori (structural) information of the system.

A posteriori information:

Let $Y_0(k)$ be the information available to the coordinator at time k :

$$Y_0(k) = Y(n_\varrho) \cup U(n_\varrho - 1),$$

$$E \left\{ v_i(t) - \sum_{j \neq i} A_{ij} x_j(t) \mid Y_0(k) \right\} = \theta \quad t \geq k,$$

$$u_i(k) = \gamma_i^k [Y_i(k), U_i(k-1), Y_0(k)],$$

$$v_i(k) = \eta_i^k [Y_0(k)].$$

Find the optimal strategies γ_i^k and η_i^k , $i = 1, \dots, N$, $k = 0, \dots, T-1$ so that J is minimized. Therefore,

- (1) The minimization is done with respect to both γ_i^k and η_i^k , for which γ_i^k gives the actual control and η_i^k generates estimate of the interaction needed in the control.
- (2) The term γ_i^k will not use the structure of the system since a decoupled model is given.
- (3) The coupled nature of the system is taken care of by constraint.

So far the coordinator shares almost all of his information, except the model of the entire system, with the lower-level controllers. However, by solving the preceding stochastic control problem (with nonclassical information pattern), it can be shown that only certain types of parameters need to be transmitted; specifically, (1) new global estimates $\bar{x}_i(r_\varrho/n_\varrho)$ that enable the local controllers to correct his local estimates and (2) parameters that change the objectives of the local controllers.

Although the results are intuitively attractive, more work remains to be done in the hierarchical control of stochastic systems. For example, in the approach taken, the coordinator knows the detailed structure of the entire system. It would be more realistic to assume that he is interested in an aggregated model. Also, other coordinating criteria for the coordination are possible.

SUMMARY

In this paper, we have presented various notions of decentralization and how these are related to the computation and information involved in the control of large-scale dynamic systems. Other attributes of decentralization such as reliability have been purposely omitted. Some attempt is also made to relate the various notions of decentralization to a proposed approach for the coordination of stochastic systems.

REFERENCES

1. Witsenhausen, H. S.: Separation of Estimation and Control for Discrete Time Systems, Proceedings of the IEEE, Vol. 59, pp. 1557–1566, Nov. 1971.
2. Ho, Y. C.; and Chu, K. C.: Tema Decision Theory and Information Structures in Optimal Control Problems; Part I, IEEE Transactions on Automatic Control, Vol. AC-17, 1972.
3. Chaitin, G. J.: Information-Theoretic Computational Complexity, IEEE Transactions on Information Theory, Vol. IT-20, 1974.
4. Wonham, W. M.: On Pole Assignment in Multi-Input Controllable Linear Systems, IEEE Transactions On Automatic Control, Vol. AC-12, 1967.
5. Sandell, N. R., Jr.; and Athans, M.: Solution of Some Nonclassical LQG Stochastic Decision Problems, IEEE Transactions on Automatic Control, Vol. AC-19, 1974.
6. Aoki, M.: On Feedback Stabilizability of Decentralized Dynamic Systems, Automatica 8, 1972.
7. Wang, S. H.; and Davison, E. J.: On the Stabilization of Multivariable Decentralized Control Systems, IEEE Transactions on Automatic Control, Vol. AC-18, 1973.
8. Mesarovic, M. D.; Macko, D.; and Takahara, Y.: *Theory of Hierarchical, Multilevel Systems*, Academic Press, 1970.
9. Camacho, A.: Externalities, Optimality and Informationally Decentralized Resource Allocation Processes, International Economic Review, Vol. 11, 1970.
10. Davison, E. J.: The Decentralized Stabilization and Control of a Class of Unknown Non-Linear Time-Varying Systems, Automatica, Vol. 10, 1974.
11. Siljak, D. D.: Stability of Large-Scale Systems Under Structural Perturbations, IEEE Transactions on Systems, Men, and Cybernetics, SMC-2, 1972.
12. Chong, C. Y.: On the Decentralized Control of Large-Scale Systems, Ph.D. Thesis, M.I.T. 1973.
13. Chong, C. Y.; and Athans, M.: On the Periodic Coordination of Linear Stochastic Systems, (to appear).

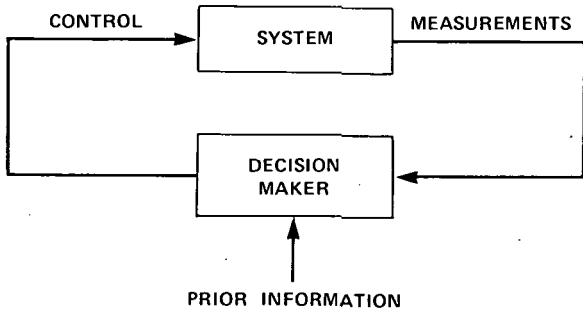


Figure 1.—Control problem I.

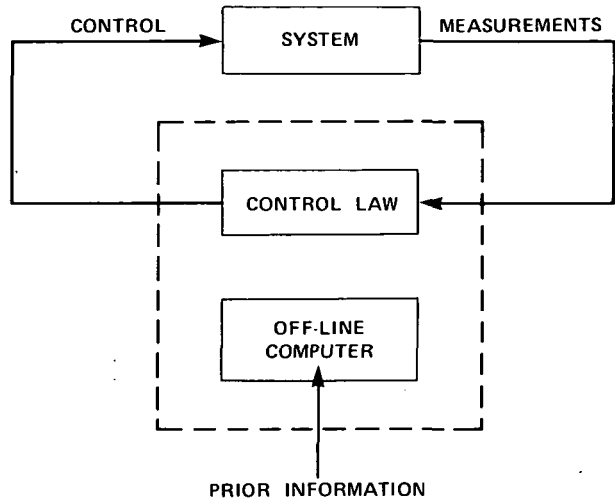


Figure 2.—Control problem II.

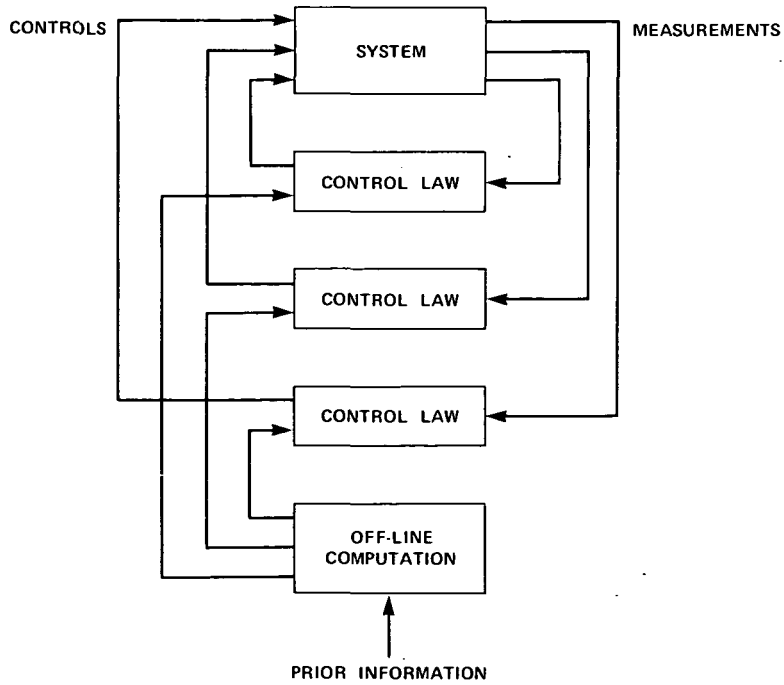


Figure 3.—Decentralized control and centralized off-line computation.

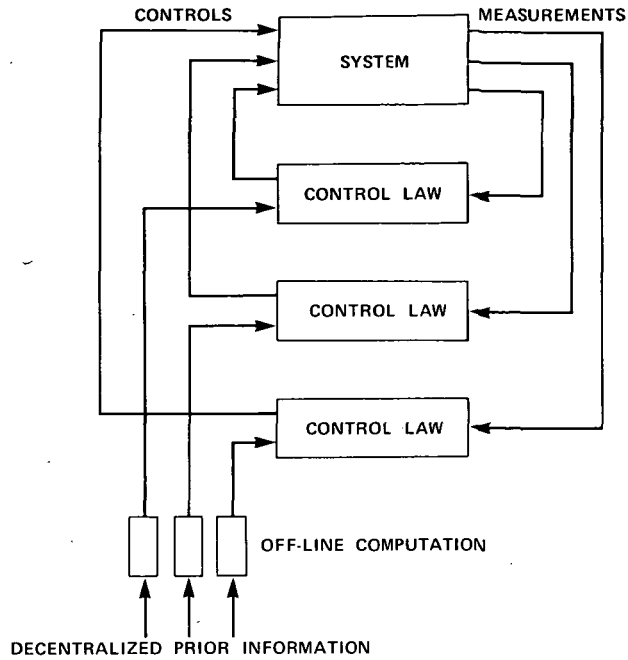


Figure 4.—Decentralized control and decentralized off-line computation.

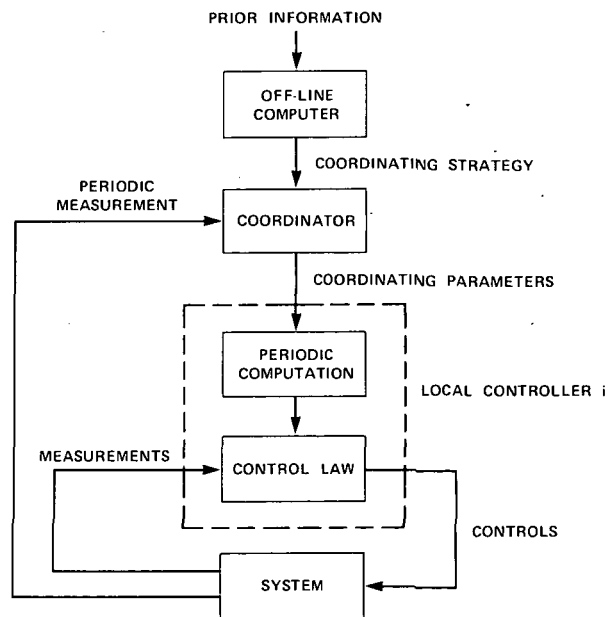


Figure 5.—Coordination.

SOME ASPECTS OF CONTROL OF A LARGE-SCALE DYNAMIC SYSTEM*

Masanao Aoki

University of Illinois
Urbana, Illinois

INTRODUCTION

Problems of understanding and dealing effectively with dynamic systems of high dimension (large-scale dynamic systems) have been objects of research for some time. The need for better techniques of predicting and/or controlling dynamic behavior of large-scale systems, involving both modeling and analysis, has been felt more intensely as demands have increased rapidly for designing and managing large-scale systems (such as interconnected network of power systems, water resources, computers, warehouses, or depots). Despite varied and intensive research, many questions remain about large-scale dynamic systems for which we do not have adequate answers.

If we assume perfect and centralized information, that is, if all "relevant" data on systems and environments¹ and complete problem descriptions are available for a single decision maker, then stabilization and control of large-scale systems are usually reducible to problems of constructing algorithms for efficient information processing (such as decomposition algorithms in large-scale mathematical programming) and associated problems of designing efficient information collection and transmission structures. Existing dynamic systems with centralized information structure are usually not "too" large and may have special structures because of the increasing cost of collecting information and processing it by a single decision maker as systems become larger.

For large-scale systems, it is therefore likely that several decision makers exist who influence the dynamic behavior of large-scale systems. These systems are called decentralized systems. If we drop the assumption of centralized information pattern, we must explicitly recognize and cope with the problem of interaction of decisions and information held by several decision makers.² In

*Research reported here was supported in part by AFOSR Grants 1328-67 and 72-2319.

¹We make the usual distinction of risk and uncertainty. The former is what is called purely stochastic and a special case of the latter is called a parameter adaptive (ref. 1).

²Another aspect of research on large-scale systems becomes important if we retain the centralized information pattern but drop the assumption of perfect information. This is the aspect of learning or adaptation in decision making. In reality, neither assumption is likely to hold exactly. Most large-scale systems we encounter are likely to be imperfectly modeled and are being acted on by more than one decision maker with nonidentical knowledge of the system.

decentralized systems, no single decision maker may have enough information needed for stable operation of large-scale systems, to say nothing of "optimal" operation of systems.³

Thus, we characterize and somewhat limit the scope of our investigation on large-scale dynamic systems as decentralized systems and as dynamic team problems. Sharing information (such as choice of message alphabets and estimation and partial reconstruction of information content held by other decision makers) and assigning control responsibility are some of the problems we have considered (see, for example (refs. 2-4)). Questions related to reliability, security, design of good measurement and control subsystems, or situations such as oligopolistic competition are not covered here.

In the next three sections, we summarize our findings in (1) control of large-scale systems by dynamic team with delayed information sharing. (2) dynamic resource allocation problems by a team (here we assume a hierarchical structure with a coordinator (central agent) who coordinates decision making by lower-level (local) decision makers), and (3) some problems related to construction of a model of reduced dimension.

OPTIMIZATION BY A DYNAMIC TEAM WITH DELAYED INFORMATION SHARING

Consider a team composed of N decision makers. Radner (ref. 5) proved that given a quadratic performance index for a team

$$F = -u'Qu + 2\mu'u, \quad u' = (u'_1, \dots, u'_N), \quad \mu' = (\mu'_1, \dots, \mu'_N)$$

where Q and μ may contain random variables, the (person-by-person satisfactory) optimal decisions are given as the unique solution of

$$E \left[Q_{ii} | \mathcal{H}_i \right] u_i + \sum_{j \neq i} E \left[Q_{ij} u_j | \mathcal{H}_i \right] = E \left[\mu_i | \mathcal{H}_i \right]; \quad i = 1, \dots, n \quad (1)$$

where \mathcal{H}_i is the information σ -field of decision maker i . This equation may be solved under a set of suitable assumptions by adapting an iterative solution technique used by Aoki (ref. 6)(see also ch. 2 of ref. 7).

To emphasize the aspect of learning or adaptation, suppose μ is imprecisely known. Suppose further decision maker i observes $\partial F / \partial u_i$ (i.e., his differential influence on F) through an additive noise. By sharing his observation with other decision makers with one period delay, the *pbps* decisions are given as the solution of a set of equations such as equation (1). The decisions consist of two parts:

$$u_t^d = u_{t-1}^c + \Delta u_t \quad (2)$$

³We exclude from our consideration game situations, where decision makers may have contradictory objectives or problems of incentive for cooperation.

where u_{t-1}^c is the decision vector with the centralized information structure and Δu_t is the correction term due to individual differences in the information set at time t .

Similar results were obtained independently by Sandell and Yoshikawa (refs. 8,9). The standard stochastic linear regulator problem with quadratic cost can be reformulated as a decentralized control problem by a team with delayed information-sharing where decision makers observe different linear combinations of the state vector through noise. The structure of the decentralized control is still of the form of equation (2).

It is also possible to obtain the decentralized optimal decision when some elements of Q are not known precisely. In this case, however, it is not possible to characterize the decentralized decision rules conveniently as in equation (2). (see ref. 7).

DYNAMIC RESOURCE ALLOCATION BY A TEAM

We now briefly describe our work which deals with a dynamic version of a stochastic resource allocation problem discussed by Groves and Radner (refs. 10, 11). The problem is to allocate a finite amount of resources to n subsystems to maximize some objective function cost for the whole team. The amount available for allocation depends on past decisions. Some parameters in the objective functions are imprecisely known.⁴ This brings in the interactions between control and information, the so-called "dual control effects."

Some approximations must be made to approximately evaluate contributions to the objective function (cost) from the future (cost-to-go). The conflict arises between control and information since the larger the control is, the smaller is the estimation error variance of the parameter value, while the total amount of resource available for control is finite. Open-loop feedback, certainty equivalent, and other schemes are compared (see ref. 2 for details).

AGGREGATION UNDER PERFECT INFORMATION

Another active area of research on large-scale systems is the following: construction of models of similar structure and/or with smaller dimension and use of these models to stabilize and control large-scale systems. Several reduced-order models may be used by a single decision maker or by several decision makers in decentralized large-scale problems (see, e.g., refs. 13 and 14).

In this section, we consider the feasibility of aggregating a collection of systems (called microsystems or subsystems) into larger systems (called macrosystems). One part of the problem may be called the aggregation problem of optimal systems (or optimal configurations). It relates to the gross characterization of optimal system performance characteristics. The other class relates to the possibility of constructing a model, called a macromodel, or aggregates of microsystems and optimizing such aggregates of systems via the macromodel.

⁴This problem has never been formulated in this manner so far as we can ascertain.

In many control problems of physical systems, systems are naturally divided into several distinct subsystems because of the physical makeup of the problem. Here the word "physical" should be interpreted broadly to include socioeconomic systems as well as the usual, truly physical control systems. Loosely speaking, the concept of aggregation implies that two or more such subsystems are combined to make a larger subsystem. One of the objectives of such aggregation is usually to reduce the number of variables in the state space description of the problem. More often the problem is already stated in an aggregated fashion because of the sheer necessity of limiting the number of variables. The latter is true, for example, in describing the economic behavior of a country by a dynamic macroeconomic model.

The concept of aggregation therefore complements that of decomposition. These two concepts, taken together, form two sides of a coin. In applying the concept of decomposition to a system to form a two-level structure, the subsystems in the lower level may be aggregated to a smaller number of subsystems with some advantages, for example. The concept of decomposition, with the resultant multilevel structure, is fairly well known and some computational algorithms have been proposed. The results available to date, however, are of a mathematical nature and somewhat of the nature of further developments of Lagrange multipliers and of generalized Kuhn-Tucker conditions.

Consider a collection of subsystems whose structures are alike. They differ only in their parameter values that specify the systems completely. The problem is to allocate a limited amount of resources so that the overall performance (return) from these subsystems is optimized (maximized, to be definite). The interactions among these subsystems come therefore from the common sources of resources. No interactions through dynamic interactions among the subsystems are considered.

Consider, for example, a network of power-generating stations. Optimal performance characteristics of each power station may have the same functional dependency on the key parameter, such as the amount of fuel available, load of the station, capacity, etc. (see refs. 15, 16). More precisely, we consider a macrosystem consisting of N subsystems so that the total return from this system F is related to the individual return of the i th subsystem g_i by

$$F(x^1, x^2, \dots, x^N) = \sum_1^N g_i(x^i)$$

where F and g_i are scalar valued and where x^i is an m vector. The term F is to be maximized over the domain D_N defined by

$$\sum_1^N x^i = x, \quad x^i \geq 0.$$

Among the class of problems with such separable criterion functions, we are interested particularly in the special case

$$g_i(x^i) = g(x^i, a^i), \quad 1 \leq i \leq N$$

namely, we are interested in problems where subsystems are all alike and the return from each subsystem differs from each other only in its parameter so that

$$\begin{aligned} \max \left\{ g(x^1, a^1) + g(x^2, a^2) \right\} &= g \left\{ x, h(a^1, a^2) \right\} \\ x^1 + x^2 &= x \\ x^1 \geq 0, \quad x^2 \geq 0 \end{aligned} \quad (3)$$

where $h(a^1, a^2)$ is a known function of a^1 and a^2 . This means that the functional form g is reproduced after aggregating two subsystems, a very desirable feature indeed. Equation (3) can be interpreted as follows: A controller is associated with each subsystem i , and its performance is optimized subject to the constraint that the given amount of resource be x^i . The optimal performance is denoted by $g(x^i, a^i)$, which are called the optimal performance characteristics by Kulikowski (ref. 15). Then the optimal performance of the aggregate of N such subsystems $g \left\{ x, h(a^1, \dots, a^N) \right\}$ is given by⁵

$$g \left\{ x, h(a^1, \dots, a^N) \right\} = \max \left[\sum_1^N g_i(x_i, a^i) \right]_{x \in D_N}.$$

Conditions for aggregating microvariables pertaining to dynamic behavior of subunits to obtain macrovariables for describing a set of subunits are usually rather restrictive. Since only a small class of problems permits perfect aggregation, investigating the effects of approximate aggregation is important. However, it is useful to know when microstate variables can be aggregated without error. In the dynamic resource allocation of section 2, this type of aggregation is possible for a class of problems. An example of perfect aggregation follows (ref. 14).

Consider a system composed of N subsystems with the optimal performance characteristic functions of a common type given as

$$A_i^\alpha B_i^\beta \dots Z_i^\omega = k_i \quad 1 \leq i \leq N \quad (4)$$

where $A_i, \dots, Z_i, \alpha, \beta, \dots, \omega$ and $k_i > 0$ are given real numbers. Assume that these N subsystems are coupled by the conditions

$$A = \sum_1^N A_i, \quad B = \sum_1^N B_i, \quad \dots, \quad Z = \sum_1^N z_i$$

where

$$0 \leq \underline{B} \leq B \leq \bar{B}, \quad \dots, \quad 0 \leq \underline{Z} \leq Z \leq \bar{Z}.$$

Interpret A as the total cost of control B, C, \dots, Z are the total amount of resources allocated to the N subsystems. They are constrained from below and above by $\underline{B}, \bar{B}, \dots, \underline{Z}$ and \bar{Z} . The problem is to minimize A .

⁵ In the multilevel approach (decomposition approach), the subsystems are on the first-level and the second-level controller assigns certain x^i to the i th subsystem.

The optimal performance characteristics of the total system can be shown to be of the same type as

$$A^\alpha B^\beta \dots Z^\gamma = k$$

where k is given by

$$k = \left(\sum_1^N k_i^q \right)^{1/q}, \quad \frac{1}{q} = \alpha + \beta + \dots + \omega$$

and the optimal parameter value settings are given by

$$B_i^*/B = \dots = Z_i^*/Z = (k_i/k)^q$$

where the starred variables indicate optimal values.

Actually, any number of subsystems can be combined or aggregated to form a larger subsystem without destroying the common functional form of the optimal performance characteristics.

For example, subsystems 1 and 2 can be combined to give

$$A_{12}^\alpha B_{12}^\beta \dots Z_{12}^\omega = k_{12}$$

where

$$A_{12} \triangleq A_1 + A_2$$

$$B_{12} \triangleq B_1 + B_2$$

$$Z_{12} = Z_1 + Z_2$$

$$k_{12} = (k_1^q + k_2^q)^{1/q}$$

and where

$$A_{12}^* = \min (A_1 + A_2)$$

$$B_1 + B_2 = B_{12}$$

$$Z_1 + Z_2 = Z_{12}$$

$$A_1^*/A_{12}^* = B_1^*/B_{12}^* = \dots = Z_1^*/Z_{12}^* = (k_1/k_{12})^q.$$

Thus, for this class of problems, instead of optimizing the system with respect to variables $B_i, C_i, \dots, Z_i, 1 \leq i \leq N$, some or all of the N subsystems can be aggregated and optimized separately to form a system with fewer subsystems without incurring any loss in the overall system performance. This fact will be referred to as a perfect aggregation. Unfortunately, the class of functions that permits this perfect aggregation is not large and must be essentially of the type given by equation (4). Therefore, some sort of approximate aggregation procedure must be developed. One such approximation may be to approximate optimal operating characteristics with exponential-type functions.

Note also that the subsystems of the class of systems considered by Kulikowski interact only indirectly through the allocation of common resources. No direct couplings of the dynamics of the subsystem through their inputs and outputs are considered. A network of power-generating stations has been mentioned as a possible system providing a motivation for such a model.

Another important example is a network of computing centers. A computer center may be taken to be the basic subsystem. A local network consisting of several such centers may be considered next. The allocation of total computing load has been considered for a computer center consisting of several central processing units with perhaps a common memory. What is being considered is not the detailed job assignment schedule but a gross characterization of optimal operating condition. Some job assignment schedule is assumed to have been adopted and the operating characteristics involving such variables as the description of the composition of the types of computing jobs, average times of executions, characterization of computing speed, etc., are assumed to be known.

What is being proposed is to characterize the operating characteristic of a network consisting of two or more computing centers with similar operating characteristics in terms of similar macro-variables when these computing centers obtain jobs for a common source. Optimal job allocation is considered in terms of the given operating characteristics of the individual computer centers. Of course, in applying the theory to be developed in the proposed work, a major problem is to identify the important variables and obtain adequate characterizations.

When the assumption of perfect information is relaxed, we can no longer perform perfect aggregation because of interactions between control decisions and information of various decision makers.

REFERENCES

1. Aoki, M.: Optimization of Stochastic Systems. Academic Press, 1967.
2. Aoki, M. and Li, M. T.: Partial Reconstruction of State Vectors in Decentralized Dynamic Systems. IEEE Trans. Aut. Control, AC-18, June 1973, pp. 289-292.
3. Aoki, M.: On Feedback Stabilizability of Decentralized Dynamic Systems. Automatica, 8, 1972, pp. 163-173.
4. Aoki, M., and Li, M. T.: Controllability and Stabilizability of Decentralized Dynamic Systems. Preprint, Joint Automatic Control Conference, Columbus, Ohio, 1973, pp. 278-286.
5. Radner, R.: Teams, in Decision and Organization, McGuire, C. B. and Radner, R. (eds.). North Holland, 1971.
6. Aoki, M.: Decentralized Linear Stochastic Control Problems with Quadratic Cost. IEEE Trans. Aut. Control, AC-18, 1973, pp. 243-250.
7. Toda, M.: Dynamic Team Decision Problems Under Uncertainty. Ph.D. Dissertation, Department of System Science, University of California, Los Angeles, 1974.
8. Sandell, N. R.; and Athans, M.: Solution of Some Nonclassical Stochastic Decision Problems. IEEE Trans. Aut. Control, AC-19, April 1974, pp. 415-441.
9. Yoshikawa, T.: Dynamic Programming Approach to Decentralized Stochastic Control Problems. submitted to 1974 Decision and Control, Phoenix, Arizona.
10. Groves, T.; and Radner, R.: Allocation of Resources in a Team. J. Econ. Theory, 4, 1972, pp. 415-441.
11. Groves, T.: Market Information and the Allocation of Resources in a Dynamic Team Model. Preprint 1972, Joint Aut. Control Conf.
12. Aoki, M.; and Toda, M.: Parameter Adaptive Resource Allocation Problem for a Decentralized Dynamic System. To be presented at 1974 Decision and Control Conference, Phoenix, November 1974.
13. Aoki, M.: Aggregation. Chapter 5 in Optimization Methods of Large-Scale Systems, D. A. Wismer (ed) McGraw-Hill, 1971, New York.
14. Aoki, M.: Aggregation and Informationally Decentralized Linear Dynamic Systems. Unpublished manuscript.
15. Kulikowski, R.: Optimum Control of Aggregated Multi-Level Systems. 1966 IFAC Congress Section 40, London, June 1966.
16. Kulikowski, R.: Decentralized Optimization of Large-Scale Dynamic Systems. 1969 IFAC Congress, Warsaw, June 1969.



745 001 C1 U G 750711 S00903DS
DEPT OF THE AIR FORCE
AF WEAPONS LABORATORY
ATTN: TECHNICAL LIBRARY (SUL)
KIRTLAND AFB NM 87117

POSTMASTER: If Undeliverable (Section 158
Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546