ANALYSIS OF LARGE POWER SYSTEMS

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INTRODUCTION

This paper is a survey of computer-oriented power system analysis as practiced in the electric utility industry. Problems of the interconnected system in the western United States may be emphasized more than problems in other parts of the country because this is where the author gained most of his experience during eight years with the Bonneville Power Administration in Portland, Oregon. Of necessity, the survey can only highlight a few points. (An excellent source of background material is the July 1974 issue of the Proceedings of IEEE on computers in the power industry (ref. 1).

Power systems are interconnected facilities for generating, transmitting, and distributing electric energy. The backbone of a power system is a network of high-voltage overhead transmission lines that interconnect the powerplants with the load centers. Most lines are operated as balanced three-phase systems, but there are also high-voltage direct current links. Power systems usually extend over many states and comprise many utility companies. Figure 1 shows the major transmission lines in the western United States and Canada, which are owned by about 40 utility companies who have formed the Western Systems Coordinating Council (WSCC) for joint studies on a combined systems basis. Joint studies are made by a technical staff located in Salt Lake City, which utilizes the computer facilities of the University of Utah. Similar "power pools" or "regional electric reliability councils" exist in other parts of the United States (fig. 2).

In contrast to other forms of energy, there are few economic ways yet for storing electric energy on a large scale. One practical solution is the pumped-storage plant, where water is pumped into a reservoir at times of low load consumption and excess production at other plants, such as run-of-the-river hydroplants, and thermal plants running at minimum output. By and large, the energy demanded by the customers must therefore be generated at all times, following the load demand curve. Interconnections have been built primarily to take advantage of the greater diversity of load in a larger region to keep the total installed capacity as low as possible, and also to share reserve capacity in case of plant outages. Any imbalance between generation and utilization would require partial shedding of load through selective switching or voltage reduction. All generators on the system run in synchronism at 60 Hz in the United States and Canada, and any disturbance to the normal operation, such as an insulator flashover, leads to relative oscillations of the machines against one another. If these oscillations do not die out fast enough, then the system may become unstable and "collapse." Therefore, the system must not only be balanced between generation and load under normal operation, but must also be designed to be stable against small and large disturbances.

The growth of power systems is determined by the load demand, which has been doubling about every 10 years in the United States as well as in most other industrialized countries. Simple arithmetic will show that such a geometric progression cannot continue forever. The Club of Rome and others deserve credit for drawing attention to the limits of growth. There is reason to believe, however, that the portion of electric energy in the total energy consumption, which is about 25 percent today, may grow to about 50 percent by the year 2000, as more electric energy will be used for mass transportation, recycling, and other purposes. Therefore, electric power systems will continue to grow.

A reliable supply of electric energy without imposed restrictions on consumption requires (a) large amounts of capital expenditure and (b) careful planning of the total system expansion through analysis of the normal operation as well as of the effects of disturbances. The word "expansion" must be emphasized because planning is almost always concered with additions and modifications of an already existing system. Power system planning is partly science and partly art and must answer questions of the type "where and at what time should a powerplant (transmission line) be built, what should be its rating, and should few units of large size be used or more units of smaller size?" For such studies, the load growth must be forecast in great detail for the next 5 years and in less detail for the next 20 years, always, of course, with a degree of uncertainty. Advance planning is essential because lead times (time from the decision to build something to its going into operation) are almost 5 to 10 years now for powerplants and 2 to 4 years for transmission lines.

For those engaged in power system analysis and power system planning – the "software side" of power system research – it is well to remember that it is primarily the components of the system (such as powerplants, transmission lines, and circuit breakers) that require the most expensive research. Normally, it is not so much a matter of new technology but of continuity.

HISTORIC PERSPECTIVE

Systems analysis has always been important in the power industry simply because additions of powerplants and transmission lines require so much capital expenditure that their influence on the behavior of the overall system has to be analyzed before they are built. There is very little room for modifications after installation. There is reason to believe that power systems were the first systems studied in the modern sense of systems engineering.

The mathematical foundations of power system analysis are very old. The well-known node and mesh equations were already explained in Maxwell's books published in 1873 (ref. 2). He, in turn, relied on work done by Kirchhoff, Helmholtz, and others. It was recognized almost 50 years ago that meaningful system studies were impossible with hand calculations, which led to the development of specialized analog computers ("network analyzers") in the 1920's and 1930's. Some are still in use today, and some are still being built for special purposes (e.g., for studies of electromagnetic transients).

SOLUTION OF SPARSE NETWORK EQUATIONS

Many problems in power system analysis lead to the solution of a system of linear equations. The steady-state behavior of an electric network is normally described by node equations of the form

$$[Y]\ [V] = [I] \tag{1}$$

where $[Y]$ is the nodal admittance matrix; $[V]$, the vector of voltages from node to datum; and $[I]$, currents injected into the node from datum. All matrices are normally complex. The diagonal element Y_{ii} is the sum of all admittances of the branches connected to node i, and the off-diagonal element Y_{ik} is the negative admittance of the branch connecting nodes i and k. Since only few branches are connected to each node in power systems, $[Y]$ is "sparse" (it has only a few nonzero entries). Nodes represent powerplants, load centers, and major substations, while branches represent lines, cables, and transformers.

Gauss elimination[1] has clearly become the preferred method for solving linear equations of the form of equation (1). It requires less memory and time than the Gauss-Jordan diagonalization process. In power system analysis, the elimination process for matrix $[Y]$ is normally separated from the elimination process for the right-hand side $[I]$. This offers advantages if the system has to be solved repeatedly with the same matrix $[Y]$ but with different right-hand sides $[I]$, which is frequently the case. For such "repeat solutions," only the process for the right-hand sides must be repeated. Often, $[Y]$ is symmetric; in that case, only the upper triangular matrix must be stored.

All realistic power system problems are under the curse of dimensionality. Power flow studies must routinely by performed for networks of more than 1000 nodes with more than 2000 branches. The major breakthrough in the solution of equations for such large power systems came in the early 1960's with the exploitation of sparsity by ordered elimination (refs. 3,4). This has reduced solution times and memory requirements by almost a factor of 100 in smaller systems and much more in larger systems (see the ratio in fig. 3). It is believed that sparsity techniques were pioneered in the power industry, but mathematicians are well aware of it now (refs. 5–7). The following table illustrates the savings for a moderately sized system:

Matrix data:

Number of nodes	267
Number of branches	423
Number of nonzero elements above diagonal after elimination in 267 X 267 matrix	1015

Solution times (IBM 7040):

Triangular factorization of complex matrix	16.5sec
Repeat solution	1.6 sec

Without sparsity, the upper triangular matrix would have approximately 36,000 elements, which is approximately 36 times more than in the table. Figure 3 (from ref. 8) compares the numerical effort of straight-forward matrix inversion with that of ordered elimination with exploitation of sparsity for typical power network problems. Figure 4 (also from ref. 8) shows the network graph and the sparse matrix after triangularization for a 62-node water distribution network, which illustrates that sparsity can also be exploited in non-electric problems. Another example would be the analysis of pin-jointed mechanical structures such as transmission towers (ref. 9).

[1] Also called Gauss-Banachiewicz, triangulation, triangularization, triangular factorization, *LU* decomposition, Gauss-Doolittle, Crout, Cholesky, etc. (sometimes in modified forms).

Fault studies are made to find the fault currents if faults occur at various locations in the system. Most faults involve only one phase of the three phases, for example, a flashover across an insulator to the tower. Fault current values are needed to check whether they lie within the interrupting capacity of those circuit breakers that remove the faulted line selectively and temporarily from the system.[2] They are also needed to set the sensing devices for the circuit breaker tripping mechanism.

Classical fault studies solve the steady-state component of the fault current only, which is sufficient for the purposes mentioned before. The problem formulation leads to a system of linear equations of the form of equation (1). For balanced three-phase faults, a single-phase equivalent representation is used. For example, the voltage drop along the three phases of a line

$$-\begin{bmatrix} \frac{dV_a}{dx} \\ \frac{dV_b}{dx} \\ \frac{dV_c}{dx} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{2}$$

(where Z_s is self impedance and Z_m is mutual impedance) can be simplified for balanced conditions ($I_b = I_a e^{-j120°}$, $I_c = I_a e^{j120°}$, analogous for V) to the single-phase equivalent

$$-\frac{dV_a}{dx} = (Z_s - Z_m)I_a \; . \tag{3}$$

The solution for phases B and C is the same as that for phase A, except for phasor rotations with a factor $e^{\pm j120°}$. The expression $(Z_s - Z_m)$ is the equivalent impedance for balanced three-phase operation.

For unbalanced faults, such as flashover from one conductor to the tower, symmetrical components are used. This is a well-established technique in power system analysis (ref. 10) whereby the three coupled phase equations such as equation (2) are transformed into three decoupled equations:

$$-\begin{bmatrix} \frac{dV_0}{dx} \\ \frac{dV_1}{dx} \\ \frac{dV_2}{dx} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{4}$$

[2] The removal time ("dead time") is kept as short as possible (just long enough to permit arc extinction in case of flashovers); it is noticed by the consumer only as a brief flicker.

with the linear transformation,

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{5}$$

for V and one that is identical for I, with $a = e^{j120^\circ}$. The new variables 0,1,2 are called zero, positive, and negative sequence components. With symmetrical components, one has to solve N equations three times rather than $3N$ equations once, which saves memory and computer time. With sparsity techniques, however, the savings may no longer be as impressive because solution times increase about linearly with N rather than with N^3.

The techniques for classical fault studies are well developed, and it is unlikely that worthwhile improvements can be made.

POWER FLOW STUDIES

The flow of electric energy in interconnected ac systems is determined by the branch impedances, as expressed in Kirchhoff's laws, and cannot be controlled on an individual line. (There are exceptions, such as real power control with phase shifting transformers and high-voltage direct current links.) When lines are added to an existing system or when the effect of line outages is to be studied, it is therefore necessary to study the flows in the entire system. This is the power flow or load flow problem. Normally, only balanced conditions are studied with single-phase equivalents such as equation (3). Formulating the power flow equations in an N-node system leads to N–1 nodal equations of the form of equation (1), except that the current is a function of the voltages,

$$I_k = \frac{P_k - jQ_k}{V_k^*} \tag{6}$$

where P_k is the real power into node k, Q_k is the reactive power into node k, and V_k^* is the conjugate complex voltage from node k to ground. Normally, P_k and Q_k are specified, or P_k and $|V_k|$ are specified. Equation (6) not only makes the power flow equations nonlinear but also nonanalytic because of the conjugate complex term. Therefore, the complex derivative is not defined, and numerical techniques based on derivatives must use pairs of real equations rather than complex equations and rectangular or polar coordinates rather than complex variables.

The standard solution technique for power flow problems is now Newton's method (ref. 11). The system of power flow equations

$$[g([x])] = 0\,, \tag{7}$$

with the unknown vector $[x]$, is solved iteratively with the system of linearized equations:

$$\left[\frac{\partial g}{\partial x}\right][\Delta x] = -[g] \tag{8a}$$

and

$$\left[x^{(h)}\right] = \left[x^{(h-1)}\right] + [\Delta x] \,. \tag{8b}$$

The Jacobian matrix $[\partial g/\partial x]$ and the right-hand side $-[g]$ in equation (8a) is evaluated at the approximate solution point $[x^{(h-1)}]$. Newton's method only became practical for large power systems after sparsity techniques had been developed. The Jacobian matrix shows basically the same sparsity pattern as the admittance matrix in equation (1). Typically, a solution with good accuracy is reached in three to four iteration steps, independent of the size of the system.

The Jacobian matrix gives a linearized model of the power flow equations around the solution point; therefore, it is very easy to calculate first-order sensitivities with repeat solutions, provided the triangularized Jacobian matrix has been stored. The equations are simply

$$\left[\frac{\partial g}{\partial x}\right]\left[\Delta x\right] = -\left[\frac{\partial g}{\partial p}\right]\left[\Delta p\right] \tag{9}$$

with $[p]$ being those parameters for which the influence on the solution vector $[x]$ is sought.

The Jacobian matrix can also be used to calculate reduced gradients as the basis of optimization studies (refs. 12,13). (These techniques are beyond the scope of this paper.)

STABILITY STUDIES

Large disturbances, such as short circuits or powerplant outages, cause electromechanical transients in the form of relative oscillations between synchronous machines. These oscillations may be large enough to cause loss of synchronism in one or more machines. A generator that loses synchronism because of some disturbance is automatically disconnected from the system to avoid overheating and damage. Often, this increases the severity of the disturbance for other generators and, in turn, more generators may lose synchronism ("cascading outages"). This is the typical course of events in "blackouts."

Stability probems are more critical in geographically large systems (as in fig. 1) than in tightly meshed systems. Stability first became important in the 1930's when hydroelectric plants were built far away from the load centers. At that time, "swing curves" (= rotor oscillations as a function of time relative to synchronous speed) were calculated to the crest of the first swing because power systems had enough damping that synchronism was practically never lost on subsequent swings. Today, stability must be checked beyond the first swing because modern fast-acting excitation systems on generators have decreased the system damping. The interconnection of formerly disconnected power systems has also created new stability problems in the form of spontaneous oscillations, which are sometimes damped with supplementary control signals in exciters and turbine governors.

Lyapunov's second method has been proposed to find the region of stability directly without simulation. Many papers have been written on the subject because it is a challenging theoretical concept for scientific reasoning, but it is not yet a competitive alternative to the existing transient stability simulation programs. The region of stability obtained by Lyapunov's method is too conservative since the condition for stability is only sufficient but not necessary. In other words, the system might still be stable outside that region. The shortcomings of the method are overwhelming at this time and it seems questionable whether application to realistic power systems will ever be feasible (ref. 14), even though others disagree (ref. 15).

Since there are no practical methods yet for assessing stability directly, simulating the behavior as a function of time, for specific disturbances assumed by the planner, remains the only practical alternative. Its main drawback is that the question of system stability is only answered for the specific disturbance, starting from specific initial conditions. Production-type stability programs can solve systems with up to 2000 nodes and 600 generators step by step with about 10 to 20 different exciter models and 5 to 15 different turbine governor models.

In simulating electromechanical transients, two systems of equations must be solved simultaneously, namely, the system of power flow equations.[3]

$$[g([x],[y])] = 0 \tag{10}$$

and a system of differential equations,

$$\left[\frac{dy}{dt}\right] = [f([x],[y],t)] \tag{11}$$

which describe the dynamic behavior of the turbine-generator rotors and of the exciters and turbine governors.

Step-by-step solution methods for stability programs are classified in reference 16. Most programs solve the differential equations and the power flow equations alternatingly, using a prediction of power flow state variables to solve the differential equations over one time step and using these results to obtain the new power flow solution at the end of the time step. There is a large spread in the magnitude of the eigenvalues in equation (11), assuming that the equations are linear or linearized of the form $[dy/dt] = [A][y]$. Explicit methods such as fourth-order Runge-Kutta are very slow for such "stiff systems"; they require a small time step, dictated by the smallest time constants, which one would not anticipate from the smooth curves for the rotor oscillations. This "small time constant barrier" is being overcome with implicit integration schemes, such as the trapezoidal rule (ref. 16), which has been used quite successfully for electromagnetic transients in power systems since at least 1961 (see ref. 11 in ref. 17).

[3]The rotor oscillations are slow enough to permit the use of steady-state equations for the network part.

SUMMARY

Only some topics of power system analysis could be described. While they cover the "big three" problems of fault, power flow, and stability studies, there are many other analysis problems in power systems which are partly summarized in reference 1.

One topic omitted here, the computation of electromagnetic transients, would have provided a good example of transfer of knowledge from one discipline to another, which is at least partly the objective of this workshop. The method of characteristics described in reference 17, which has become the standard solution technique for traveling wave problems on transmission lines, was first used to study pressure waves in hydraulic systems in the late 1920's (see refs. 7 and 8 in ref. 17). It also illustrates that older techniques developed for hand calculations may still be valuable in our days of powerful computers, an observation which is also true for implicit integration with the trapezoidal rule.

The solution techniques for most power system problems are highly developed by now and relatively efficient because of the exploitation of sparsity. Improvements are still possible, of course.

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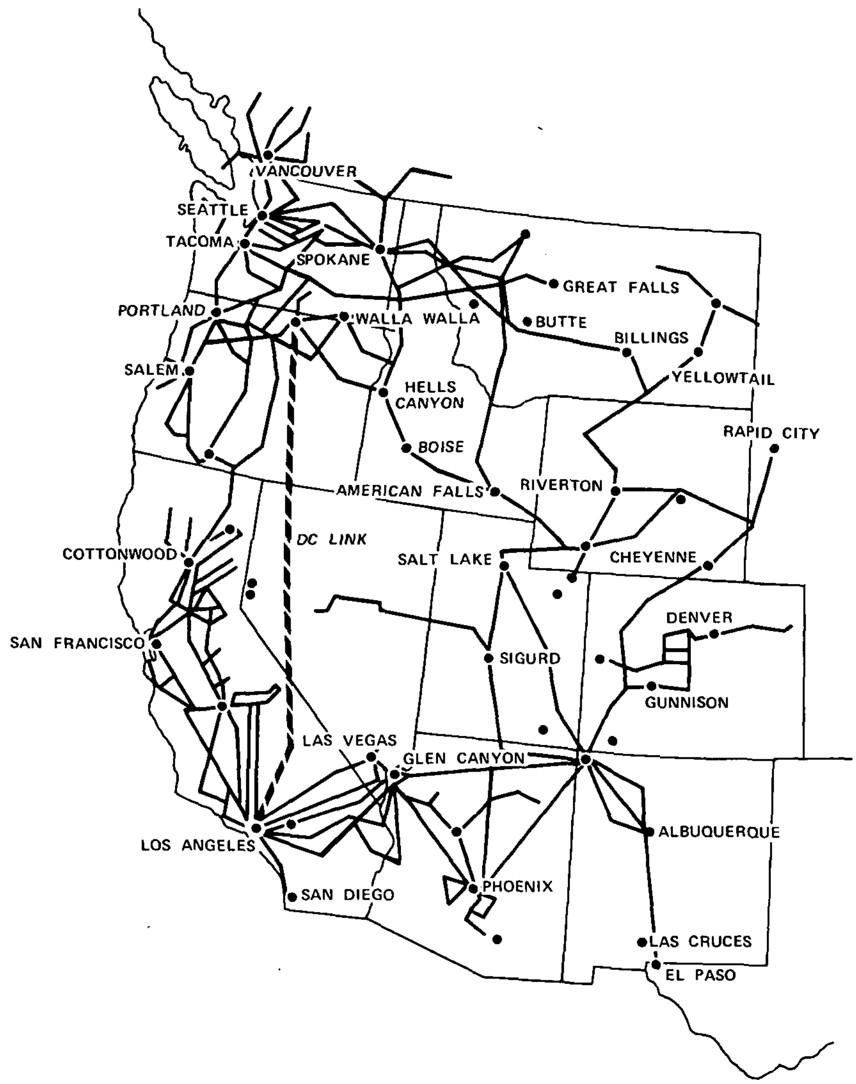

Figure 1.–Major transmission lines in the western United States. Source: Annual Report 1972 of the Western Systems Coordinating Council.

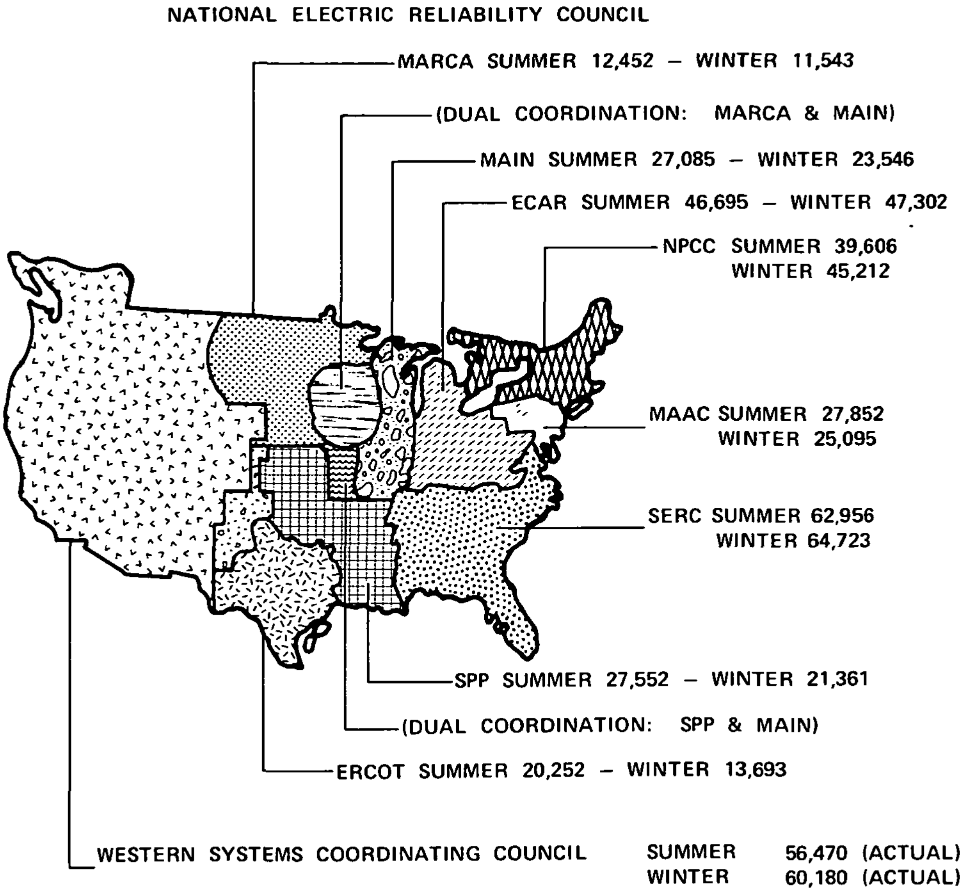

Figure 2.– Regional electric reliability councils. Peak loads in MW for Summer 1972 and Winter 1972/73 (forecast). Source: Sept. 1972 report of the National Electric Reliability Council.

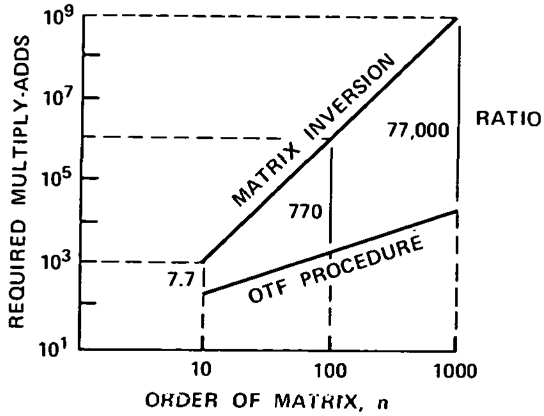

Figure 3.–Comparison of the numerical effort required by matrix inversion with that of ordered triangular factorization (OTF) for typical power network problems.

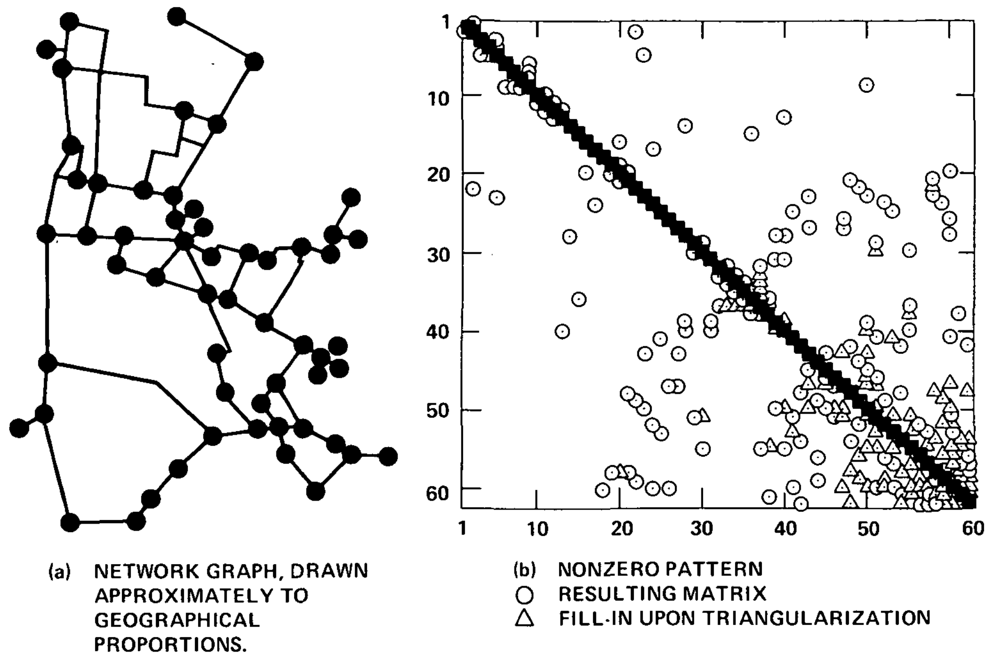

Figure 4.—Sample small 62-node water distribution network.