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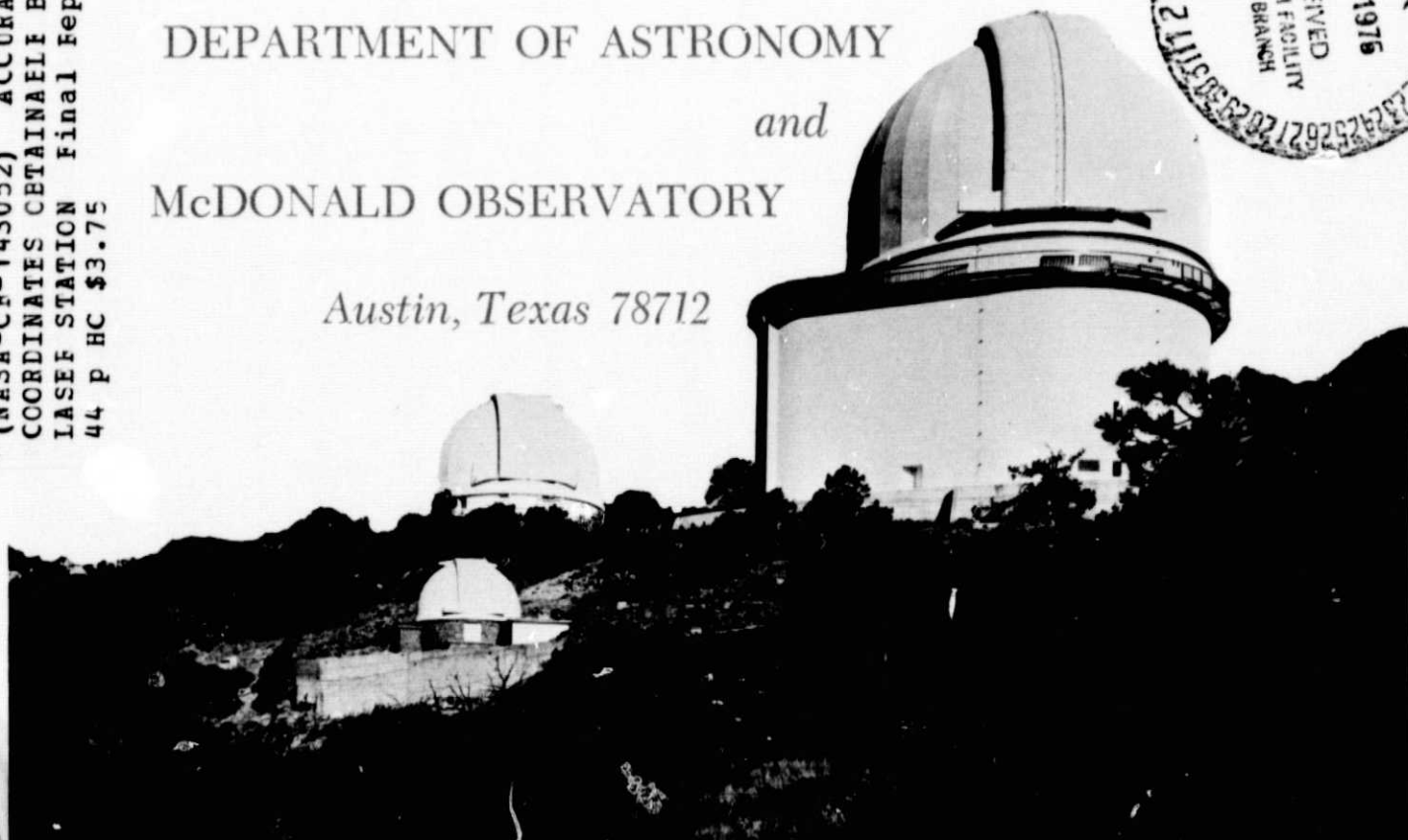


DEPARTMENT OF ASTRONOMY *and* McDONALD OBSERVATORY

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ACCURACY OF SITE COORDINATES OBTAINABLE BY A MOBILE
LUNAR LASER STATION: FINAL REPORT

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ABSTRACT

The accuracy with which a mobile lunar laser station can be located is the subject of a modelling study presented here. The influence of the number and accuracy of fixed lunar ranging stations, the uncertainty in polar motion, and data loss due to weather and similar factors has been considered, and the results are given in cartographic form. In general, all three coordinates (for coordinates to latitude $\pm 60^\circ$) are determined to better than the pole uncertainty, given three or more fixed sites and reasonable weather. This result indicates that one or more mobile stations will be extremely suitable for the study of geotectonics.

1. INTRODUCTION

Until recently, the lunar laser ranging project has concentrated on the improvement of the astronomical model: the lunar orbit, lunar rotation, and coordinates of the reflectors and of McDonald Observatory. Refinements in the physical model, particularly the librations, now permit the fitting of 5 years' observations to a continuous orbit with a mean residual of about 3 nsec (45 cm one-way range), the observations having formal uncertainties between 0.6 and 2.8 nsec [1]. These refinements have permitted also the first attempts to determine the variation in UTO at McDonald [2].

With the first lunar-laser determination of an intercontinental baseline [3], the commencement of regular operation of a fixed station in Hawaii, the advancement of fixed stations in Australia and France, and the soon-expected authorization of a mobile station, the lunar laser project has entered a new phase. This technique is on the verge of becoming a tool for global geophysics. In support of this aspect of the program, we have conducted a limited sensitivity study to estimate the formal precision with which a mobile station can find its own geocentric coordinates under a reasonable observing schedule.

The problem of modelling has been conceived in two parts, which we designate as geometric and statistical. The geometric is, quite simply, the uncertainty due to incomplete knowledge of the orientation of the terrestrial coordinate system and is essentially unrelated to the conditions at the mobile site; for the present study, we have included only the polar motion uncertainties in the geometric problem. The statistical uncertainty is provided by a covariance analysis of the

system of normal equations for the solution of the mobile station coordinates from an hypothetical observation set. Of course, the validity of such a process is at least in part dominated by the assumptions that go into it. We have tried to adopt a reasonable observing schedule and to account for the vagaries of weather in a way consistent with experience at McDonald, but more can be done in this direction.

2. THE GEOMETRIC PROBLEM

In the present study, we have considered two levels of knowledge for the polar motion, one in which the uncertainty is ± 50 cm, which is somewhat larger than the currently probable BIH errors, and the second with ± 5 cm uncertainty, a pessimistic estimate of the level to be expected from pole determinations based on a network of lunar ranging stations. In addition, it has been supposed that each fixed station is capable of determining its astronomical variation in latitude to the level of internal systematic errors (± 1 cm/sin ϕ for a 3-cm station). The justification for this supposition is in the averaging to be experienced over each pass of 6-10 hours, combined with the likelihood of correlation times in excess of one day for this phenomenon. The effect of this assumption is that the latitude of any point is better determinable along the meridian of a fixed station than would be permissible elsewhere. To see this, we will adopt temporarily an Earth-fixed coordinate system based on the true equator and the meridian of the fixed station. The BIH equations for the effects of pole displacements x and y are

$$\delta\lambda_e = - \{x \sin \lambda_e + y \cos \lambda_e\} \tan \varphi \quad (1)$$

$$\delta\varphi = + \{x \cos \lambda_e - y \sin \lambda_e\}$$

where λ_e and φ are east longitude and latitude, respectively. Although we suppose that the uncertainties $\delta x = \delta y = Y$, the additional information in the meridian of the fixed station makes it desirable to write the BIH-like system for the mobile site:

$$\delta\lambda_m = - \{x' \sin \Lambda_m + y' \cos \Lambda_m\} \tan \varphi_m \quad (2)$$

$$\delta\varphi_m = + \{x' \cos \Lambda_m - y' \sin \Lambda_m\}$$

where $\Lambda_m = \lambda_m - \lambda_f$, subscripts m and f denote mobile and fixed sites, x' and y' are the components of pole position in the new frame. Thus, if $X = |1/\sin \varphi|$, the resulting uncertainties are

$$U_f(\lambda_m) = \{|X \sin \Lambda| + |Y \cos \Lambda|\} \tan \varphi_m \quad (3)$$

$$U_f(\varphi_m) = \{|X \cos \Lambda| + |Y \sin \Lambda|\}$$

The total uncertainty assigned to the geometric aspect of the problem is then computed as the harmonic mean of the individual fixed station contributions

$$\frac{1}{U(\lambda_m)} = \sum_f \frac{1}{U_f(\lambda_m)}, \quad \frac{1}{U(\varphi_m)} = \sum_f \frac{1}{U_f(\varphi_m)} \quad (4)$$

If the effects of all fixed stations were identical, this would correspond to reducing the uncertainties by the square root of the number of fixed stations.

3. THE STATISTICAL PROBLEM

To sufficient accuracy for this study, one can represent the topocentric distance ρ of a reflector on the lunar surface by

$$\rho = R - \sigma \cos \delta \cos H - Z \sin \delta \quad (5)$$

where R is the projection on the line of sight of the geocentric reflector distance, σ the telescope spin axis distance, z its equatorial distance, H the local hour angle of the reflector and δ its topocentric declination. This relation readily gives the partial derivatives of topocentric distance with respect to telescope coordinates:

$$\frac{\partial \rho}{\partial \lambda} = \frac{\partial \rho}{\partial H} = \sigma \cos \delta \sin H$$

$$\frac{\partial \rho}{\partial \sigma} = - \cos \delta \cos H \quad (6)$$

$$\frac{\partial \rho}{\partial z} = - \sin \delta$$

Each observation gives an equation of condition relating the (observed minus computed) range residuals $\Delta \rho$ to the required improvements $\Delta \lambda$, $\Delta \sigma$, Δz to the estimated telescope coordinates:

$$\Delta \rho = \frac{\partial \rho}{\partial \lambda} \Delta \lambda + \frac{\partial \rho}{\partial \sigma} \Delta \sigma + \frac{\partial \rho}{\partial z} \Delta z \quad (7)$$

Each observation contributes to the system of normal equations in a classical Gaussian least squares analysis.

a) Scheduling

One of the features of the lunar motion that is at once an advantage and a nuisance is the fact of its slowness in its orbital motion. The advantages are related to the length of an observing passage, permitting the buildup of good statistics on the diurnal rotation of the Earth, which reflects into good stability for solutions of station longitude and spin axis distance. At the same time, however, it requires a long time to cover the full geometric range normal to the equator. For this reason, it is supposed that a mobile site will be occupied for a minimum of 24 days. Despite the fact that the mobile station will be a dedicated facility, we have conservatively adopted here the same daily schedule followed at McDonald: three ranging periods of 1 hour duration centered about hour angles -3h, 0h, +3h. This gives a total of 72 possible observations, but there are other boundary conditions that serve to reduce the number actually considered.

As a small step towards reality, the time of the "observation" is not fixed rigidly, but is situated within the available hour-wide interval by a random number generator function.

b) Zenith distance restriction

As a first restriction on the availability of an hypothetical observation, we adopt the geometric constraint imposed at McDonald by the Federal Aviation Administration: the laser may not be fired at

zenith distances greater than 70° . The official restriction may be less in many sites, but below this limit the air mass becomes so great that refraction modelling is a problem. The effect of this restriction is to reduce the available hour angle spread when the Moon is in the opposite hemisphere from the station. Again trying to be conservative, we simply exclude the corresponding "observation," rather than rescheduling closer to the meridian. An example of the results of this restriction is given in Appendix A.

c) Effects of weather

Optical techniques such as laser ranging are stopped by clouds and by atmospheric turbulence (the McDonald laser is not fired if the seeing is worse than 6"). Our program accepts as input a weather factor WFAC, which characterizes the incidence of usable sky. In using this factor, we try to include effects to account for both the medium-term randomness of weather with the fact of day-to-day correlation due to the size of weather patterns. This is accomplished by generating, for each observation window, a random number which is combined with a correlation function and compared with the weather factor. If the computed number is larger, the "observation" is nullified, contributing nothing to the system of normal equations, and weakening the statistical determination. The correlation function has two forms, chosen by experimentation and depending on the value of the weather factor. The derivation of this function is detailed in Appendix B, along with a series of large-scale sky-clearness maps that are a byproduct of this study.

d) Determination of statistical uncertainty

Finally, the effects of individual random numbers are reduced by averaging the results of ten computations. Admittedly, this is not an ideal means, but it seems to be a usable compromise between the desirable and the possible.

Each of the available observation times that is not eliminated by the zenith distance test or by the weather model contributes its share to the normal system. The result is a set of covariances that represent the determinability (in the language of statistics, the inverse weight) of each of the coordinates, or their expected uncertainty, due to the effects modelled.

4. RESULTS

The final uncertainty, in this study, is determined as the root mean square of the statistical and the geometric contributions. We present here a selection of cases that have been computed to illustrate the principal results. In all cases, we assume 3-cm (0.2 nsec) capability for fixed and mobile stations. This is a factor of 3 better than current McDonald operation, but equals the Hawaii design specification; eventual operation is expected at this level at McDonald also. The variation between cases depends uniquely on the polar motion model and the number and location of fixed stations. The results are given in graphical form, grouped by coordinate (radial distance, longitude, latitude, spin axis distance, equatorial distance). Note that radial distance is a function only of weather factors, thus only two charts are given. For longitude and latitude, we present the following cases:

a) Worst case -

This case assumes 50-cm polar motion information. All other effects are trivial in comparison.

b) Statistical case, WFAC = 0.5 -

Assumes no polar motion uncertainty, which is unrealistic, but permits one to see the intrinsic level of the statistical contribution when 50% of the observation periods are weathered out.

c) Statistical case, WFAC = 1.0 -

An unattainable best case, demonstrating the effect of a good weather site, relative to the preceding.

d) Three fixed stations (Texas, Hawaii, Australia), WFAC = 0.5 -

The most probable configuration in the immediate future, with a believable weather factor.

e) Three fixed stations, WFAC = 1.0 -

f) Four fixed stations, WFAC = 1.0 -

Showing the gain to be realized from an additional station, in southern France.

The last three cases are also illustrated in axial distance and equatorial distance.

5. DISCUSSION

The present study is neither definitive nor without its flaws. In general, we have neglected all systematic errors related to the orbit and rotation of the Moon. A justification for this is the existence of the network of fixed stations, which may be regarded as continuously monitoring these effects. The purpose of the mobile

station is purely terrestrial, and one can consider that the present discussions concern only the coordinates of the mobile station relative to the system defined by the fixed network. We do not consider this to be a serious problem. Systematic errors related to mobile station hardware is more serious: when the calculated coordinate uncertainties are smaller than those expected from the systematic error budget for the mobile station, then of course they should not be taken too seriously. Preliminary studies indicate that the estimate of 5 cm polar motion from a network of 3 cm fixed stations is reasonable, but more study is required in this area, and is indeed shortly to be undertaken. The use of a 50% weather factor is arbitrary, although it corresponds closely to the observation loss due to comparably random factors during several years' operations at McDonald.

Among the demonstrated advantages of lunar laser ranging for the study of secular variations in the orientation and distortion of the Earth is the combination of feasibility of daily operation over long periods of time and the extreme stability of the orbit of the Moon. It is not convincing to study long term secular changes by means of discontinuous models for the physical behavior of the observed object. It is precisely this point that makes our natural satellite a more suitable target for some purposes than any other Earth satellite. Present data analyses using a single uncorrected, unrectified, continuous orbit for the Moon over an interval of more than 5 years give mean residuals less than 40 cm without the determination of UT1 and polar motion from these data. The present study was undertaken within the context of that situation as a first step in evaluation of the potential utility of a mobile station for geodynamic studies.

The interpretation of the present results may be regarded, as with all covariance studies, as primarily qualitative rather than quantitative. Nonetheless it seems clear from the results that a mobile lunar laser ranging station will be capable of locating itself in all 3 coordinates to an accuracy comparable with its own observational uncertainty. The degree to which this can be achieved will depend, of course, on several factors not all of which concern the mobile site itself. In particular, it is absolutely essential that a high precision source of polar motion information be operational; we envision that this of course will be a network of fixed lunar laser ranging stations, some of which should be at relatively high latitude. The availability of good weather at the mobile site is important, quite naturally, but the requirements are not extreme.

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- [2] P. L. Bender, D. G. Currie, J. D. Mulholland, J. G. Williams, "Preliminary Determination of the Earth's Rotation From Lunar Laser Range Measurements" EOS 55, 1105, 1974 (abstract)
- [3] O. Calame, "Location of Lunakhod I and Chord Length McDonald-Crimea from Lunar Laser Range Measurements" COSPAR 1975, paper I.2.2

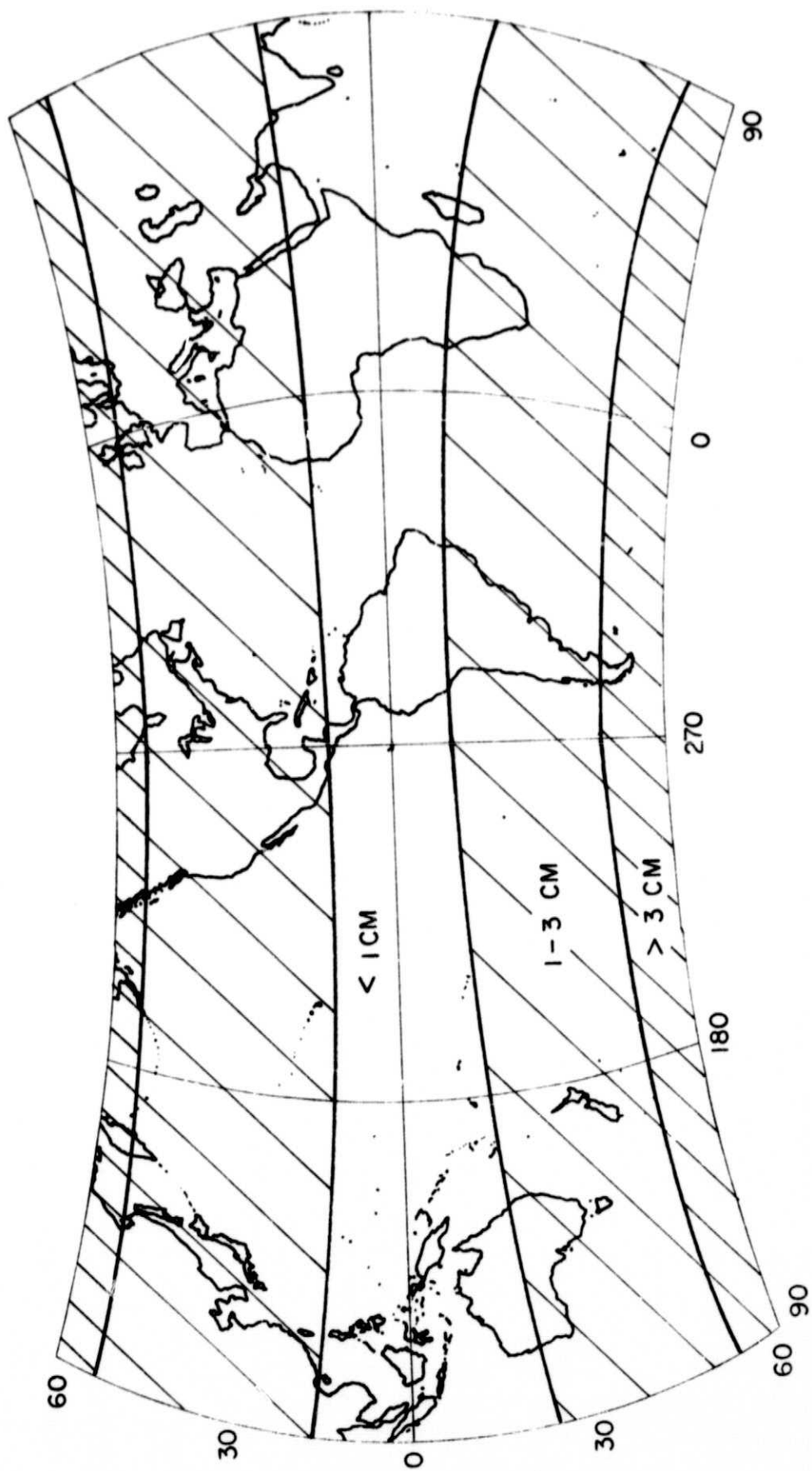
ACKNOWLEDGEMENTS

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and NGR 44-012-219 to the University of Texas. We thank P. L. Bender and G. V. Latham for helpful suggestions.

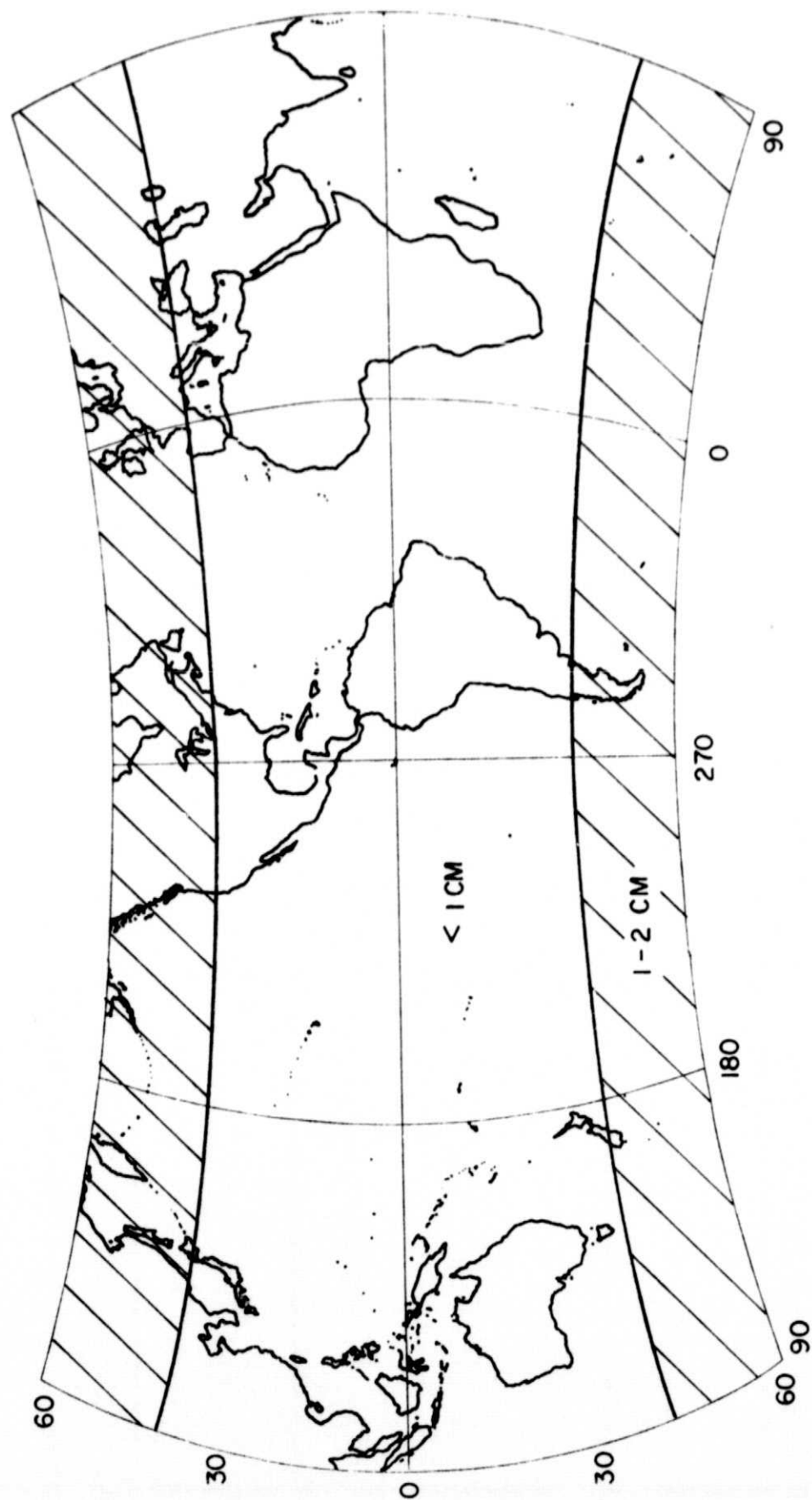
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RADIAL DISTANCE
3-CM MOBILE STATION



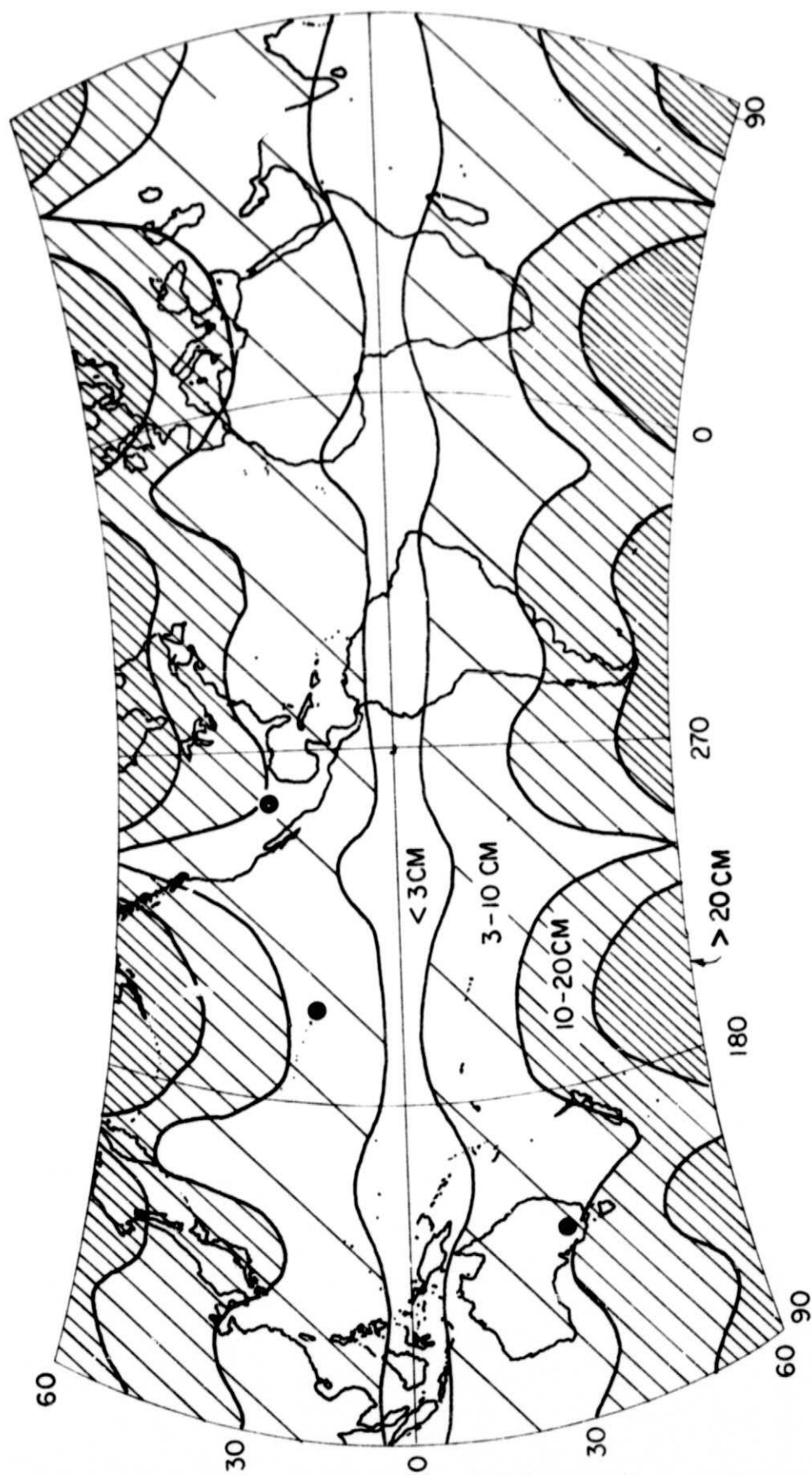
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RADIAL DISTANCE
3-CM MOBILE STATION



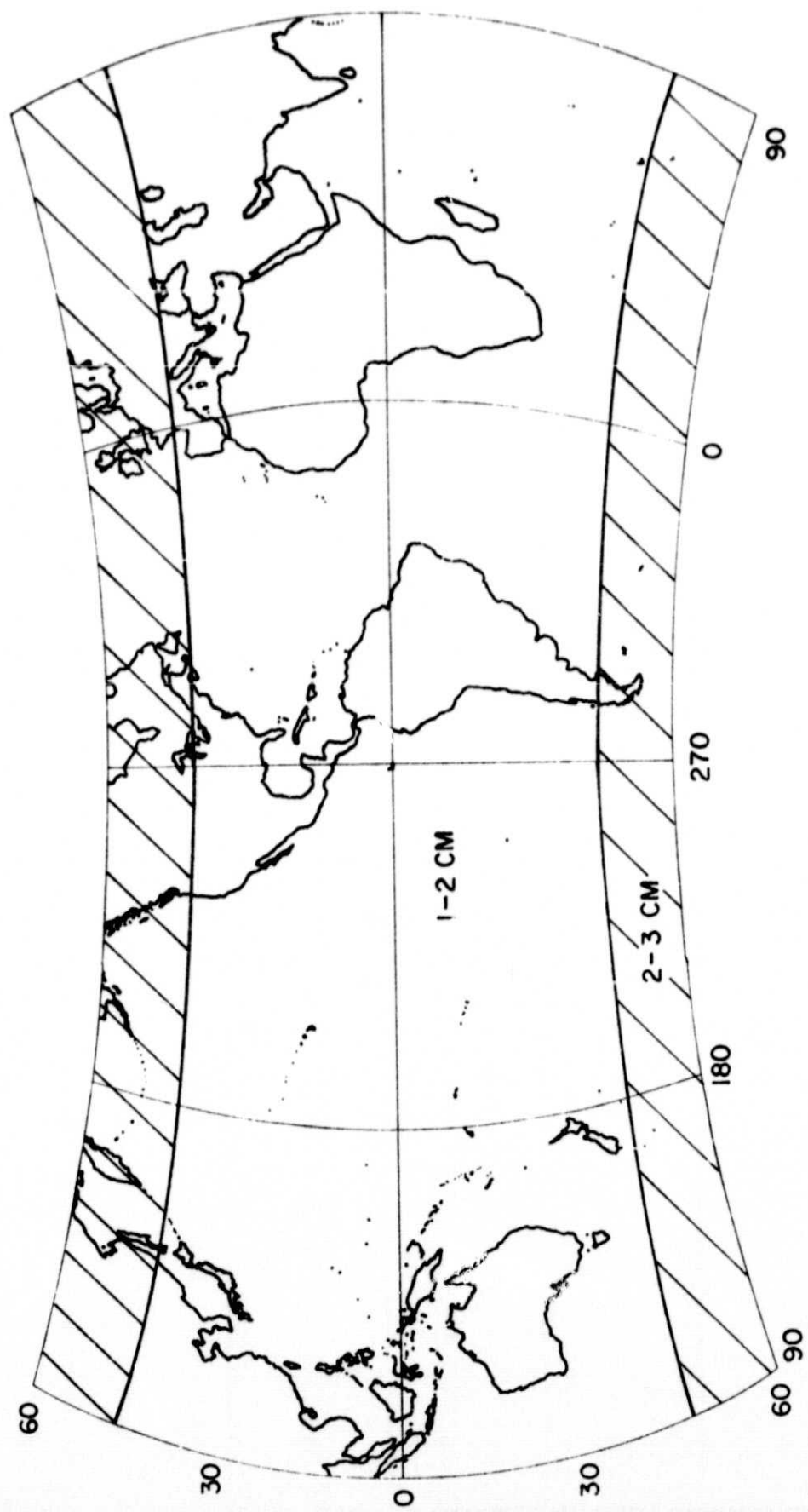
WORST CASE
B I H POLAR MOTION
THREE 3-CM STATION

LONGITUDE
3-CM MOBILE STATION



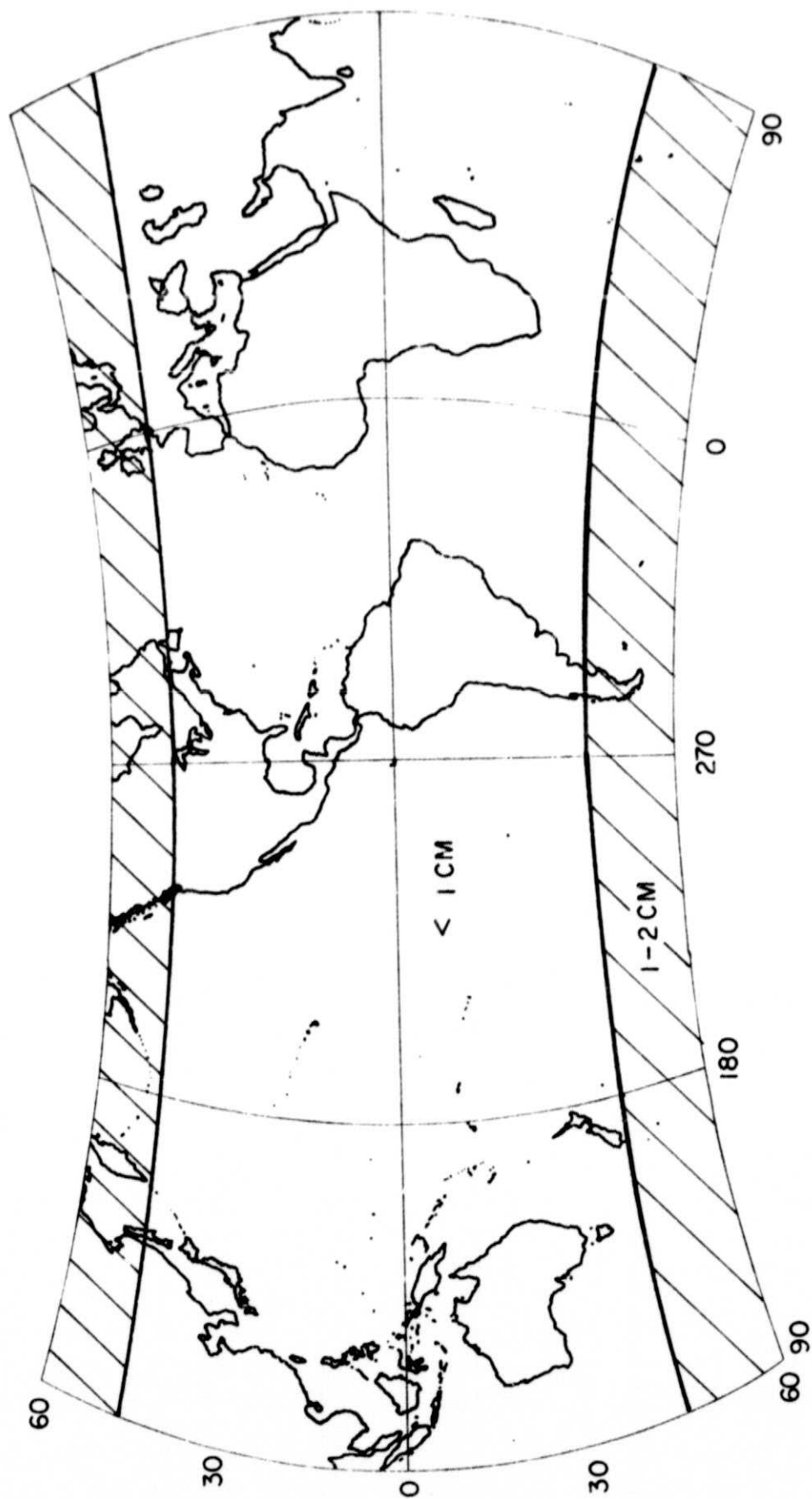
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WFAC = 0.5

LONGITUDE
3 - CM MOBILE STATION



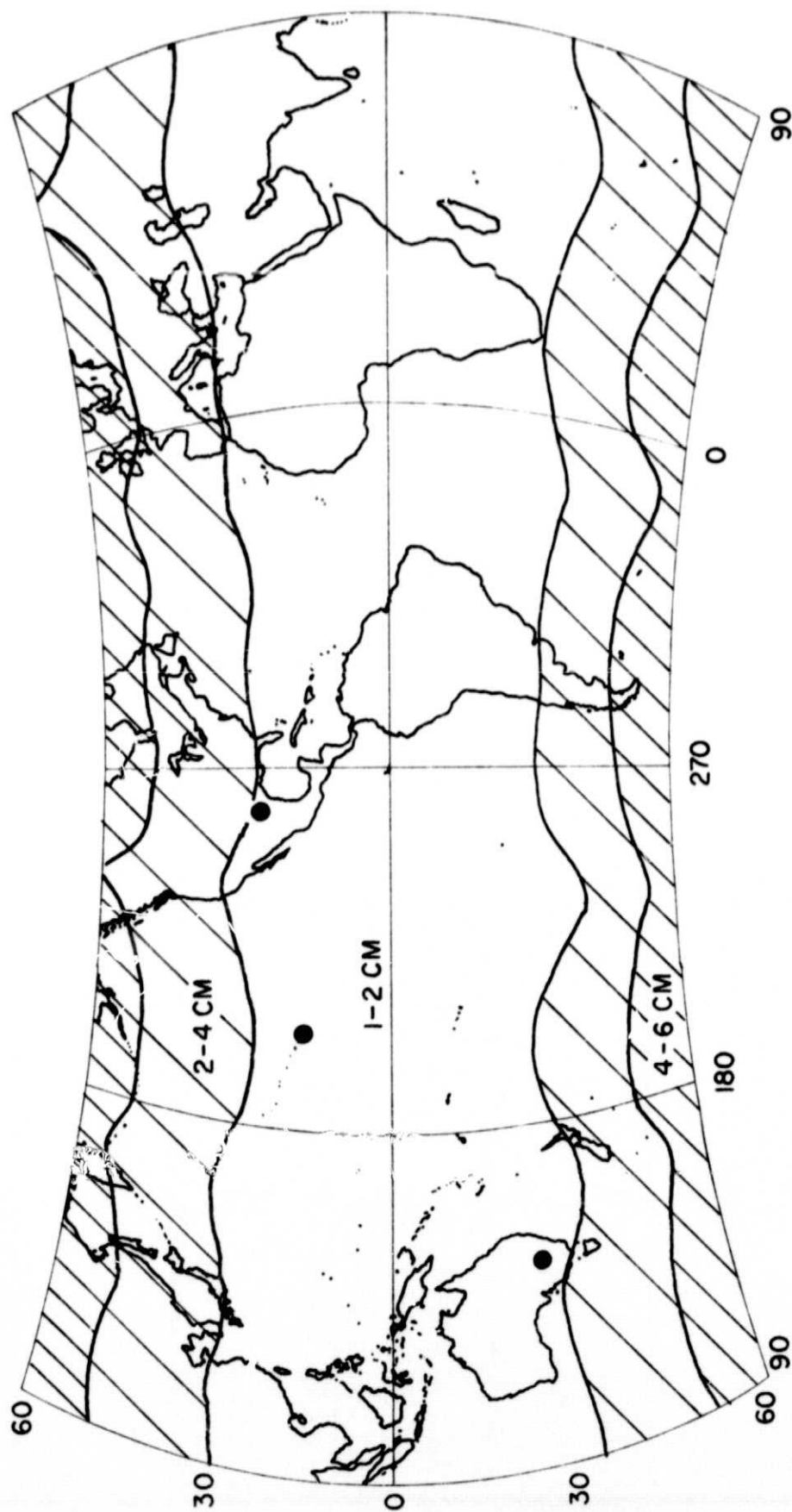
STATISTICAL UNCERTAINTY
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LONGITUDE
3-CM MOBILE STATION



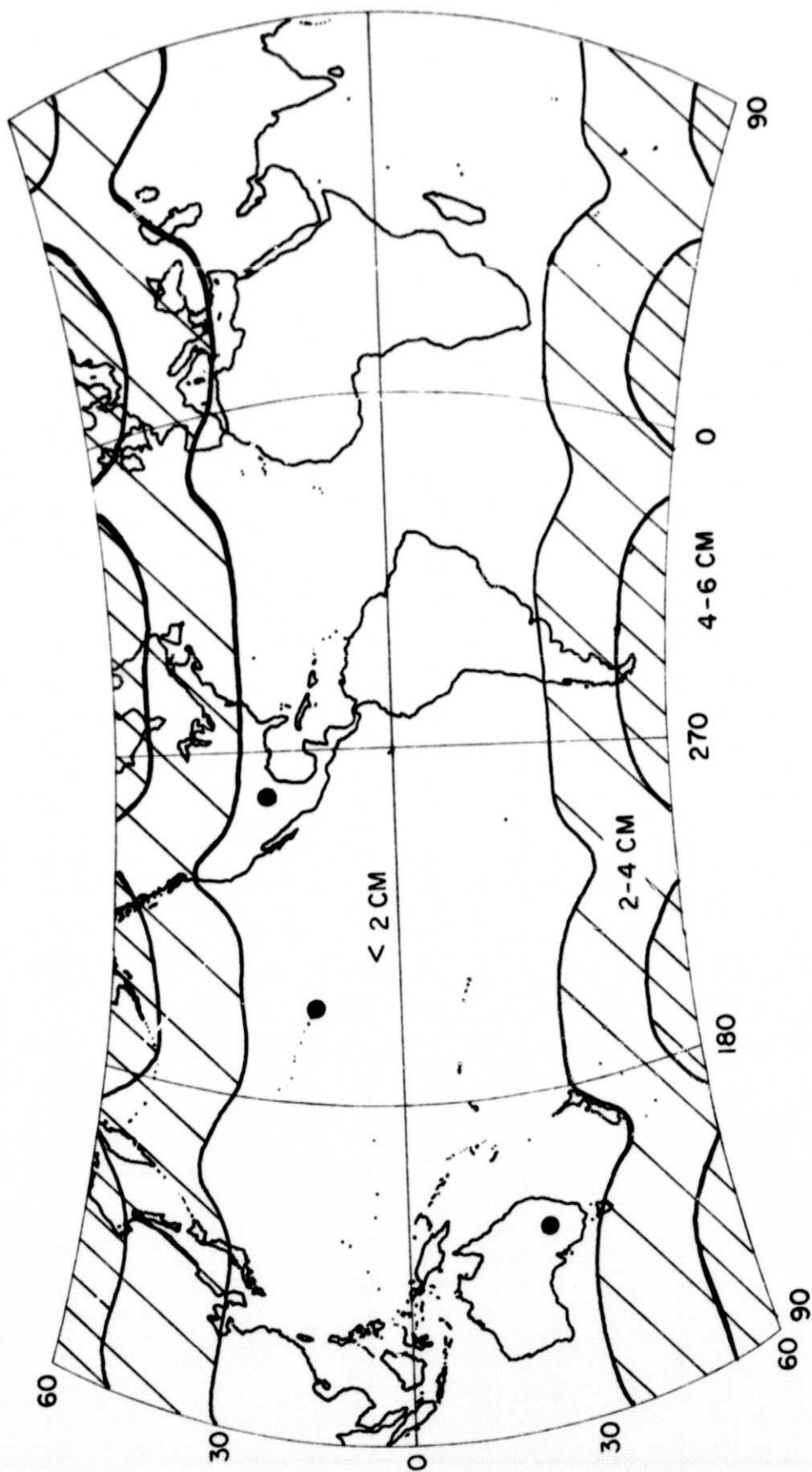
LONGITUDE
3-CM MOBILE STATIONS

THREE 3-CM STATIONS
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5-CM POLAR MOTION
WFAC = 0.5



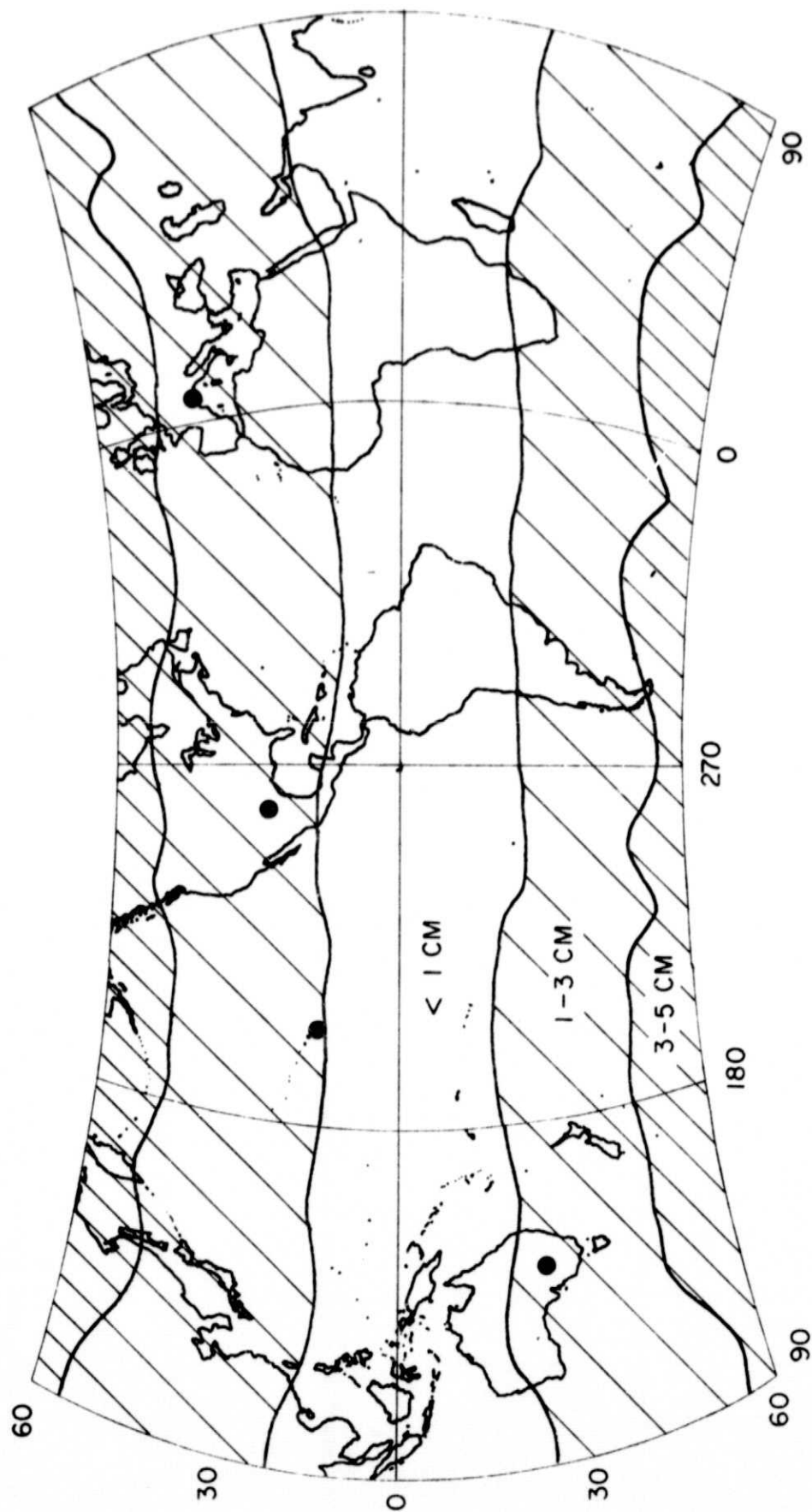
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5-CM POLAR MOTION
WFAC = 1.0

LONGITUDE
3-CM MOBILE STATION



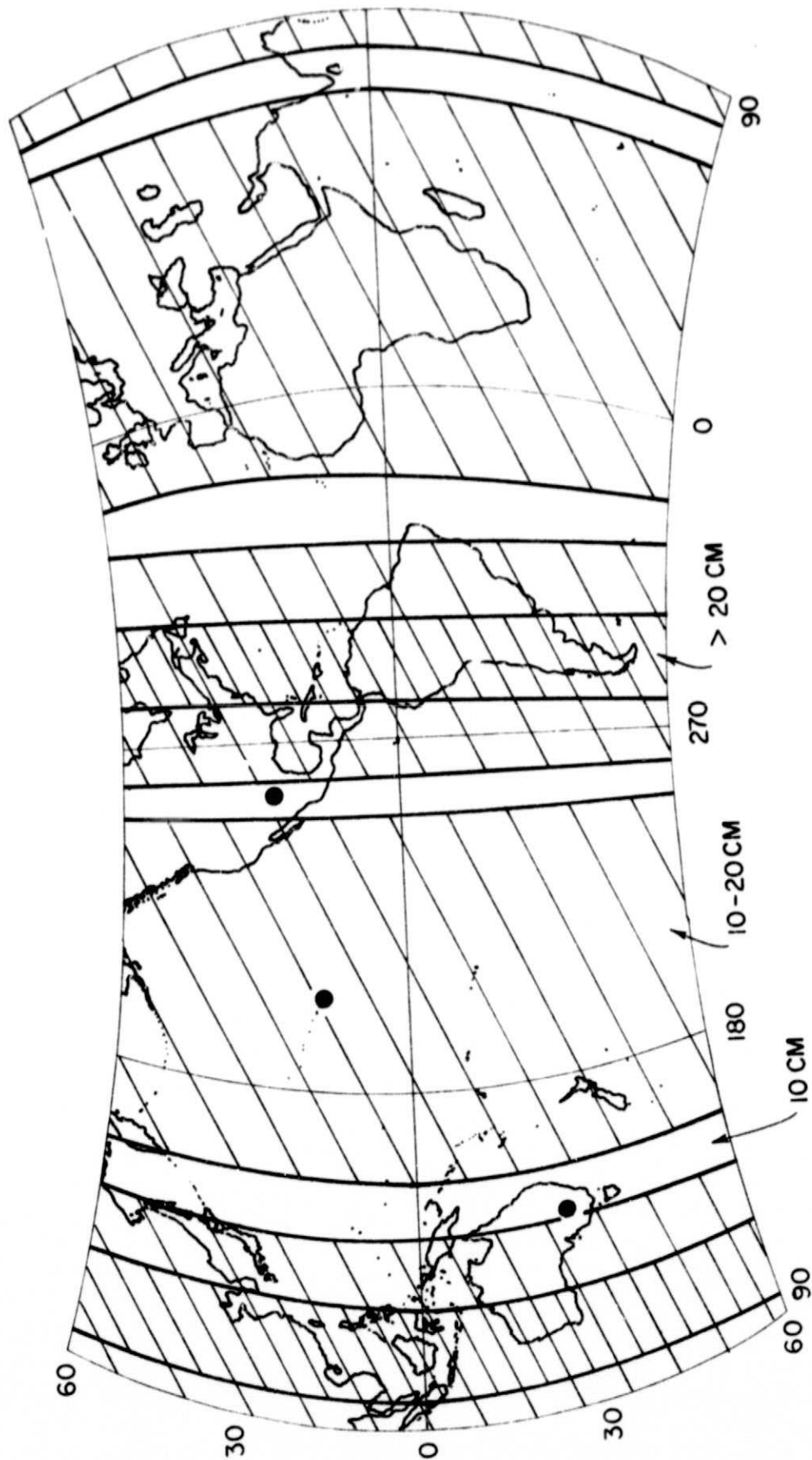
LONGITUDE
3-CM MOBILE STATION

FOUR 3-CM STATIONS
(TEX, HAW, AUS, FRA)
5-CM POLAR MOTION
WFAC = 1.0



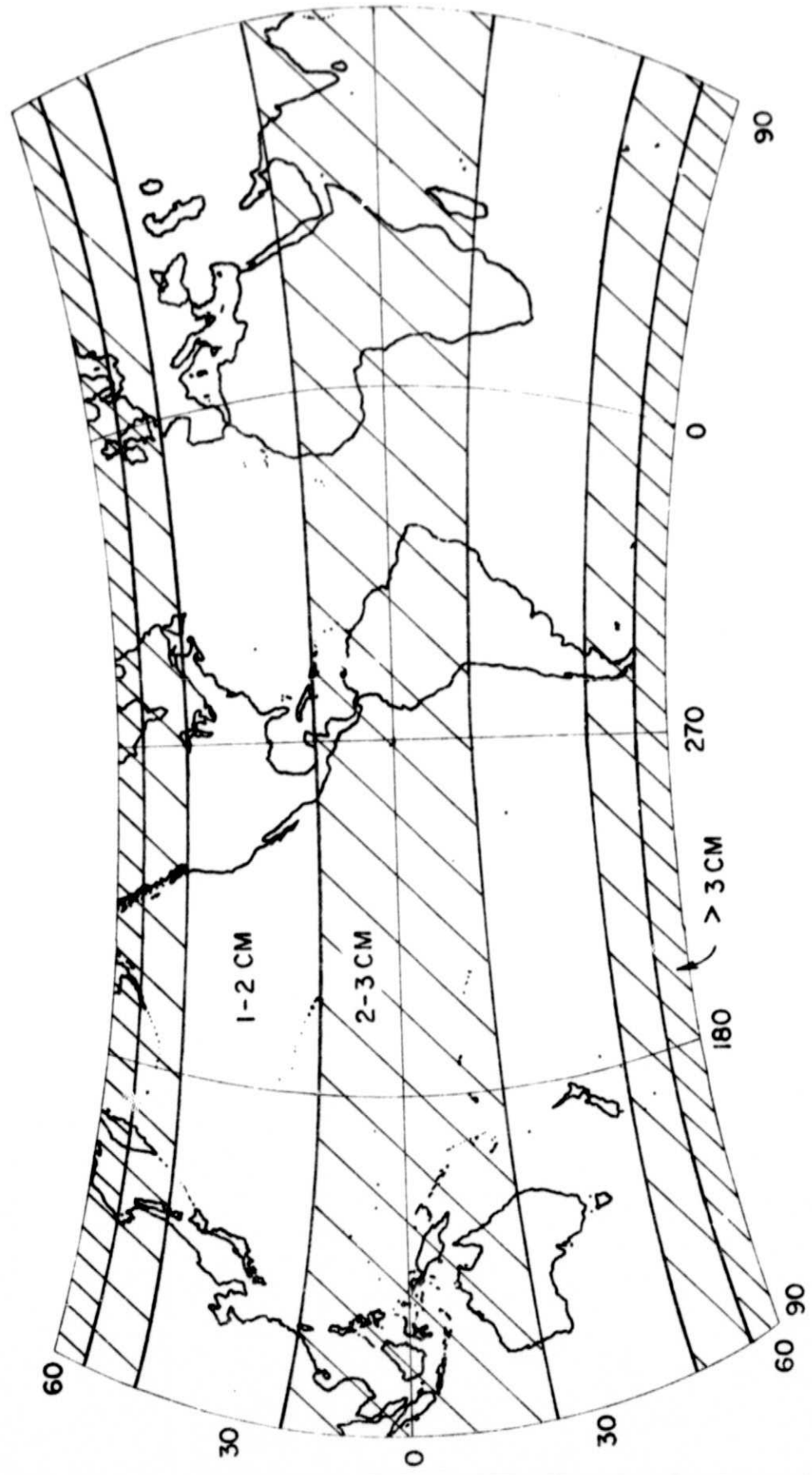
WORST CASE
BIH POLAR MOTION
THREE 3-CM STATIONS

LATITUDE
3-CM MOBILE STATION



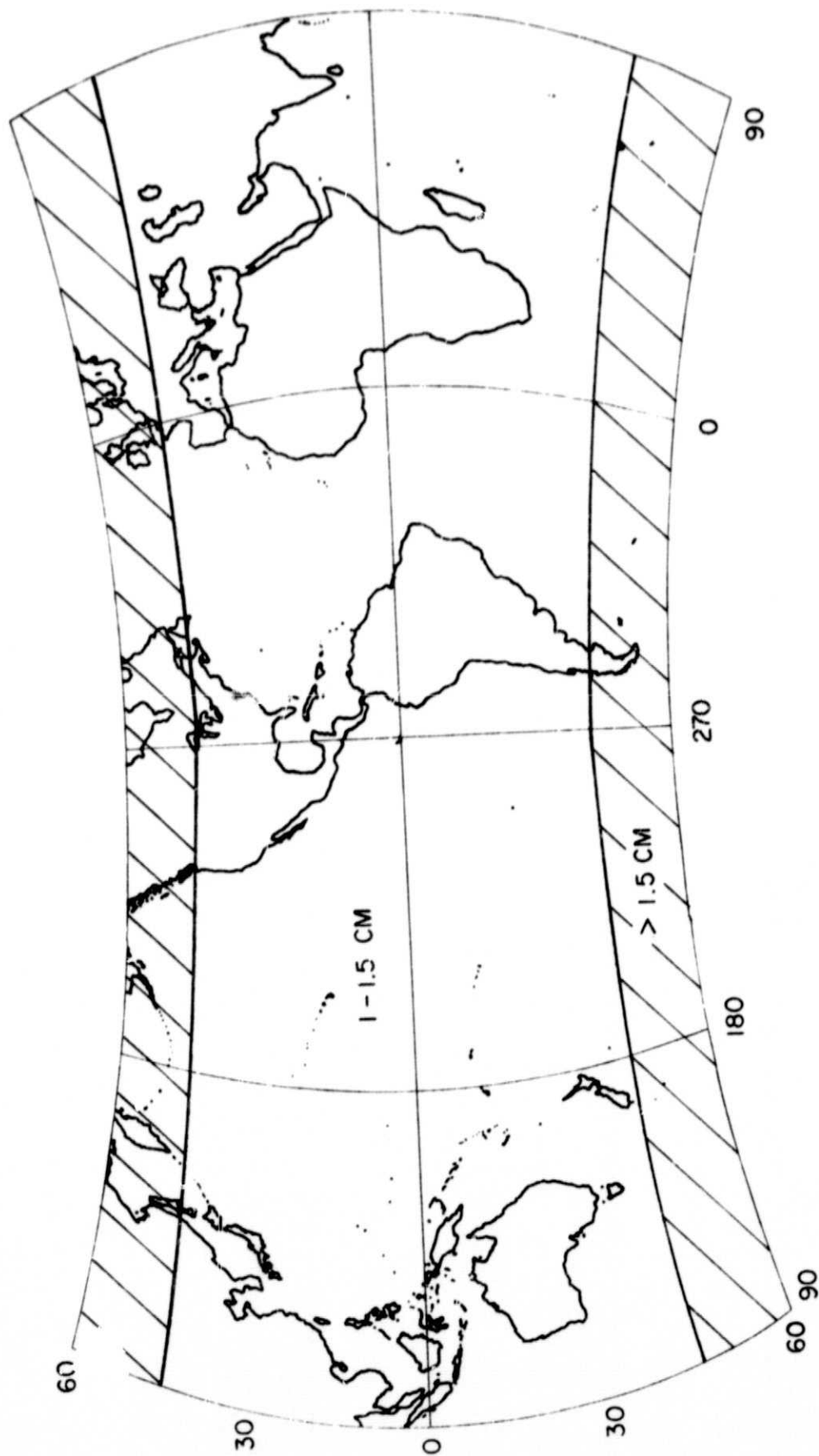
STATISTICAL UNCERTAINTY
WFAC = 0.5

LATITUDE
3-CM MOBILE STATION



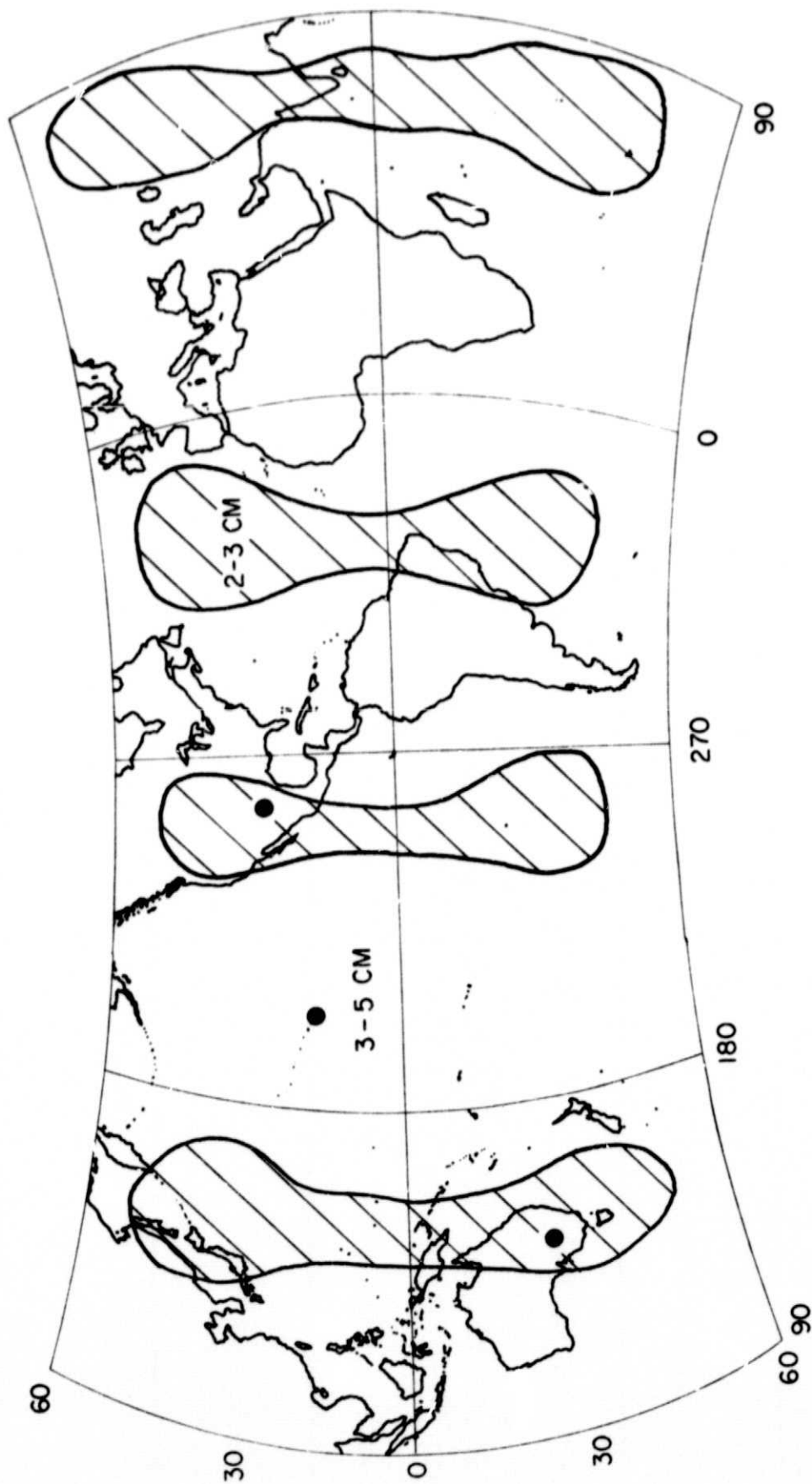
STATISTICAL UNCERTAINTY
WFAC = 1.0

LATITUDE
3-CM MOBILE STATION



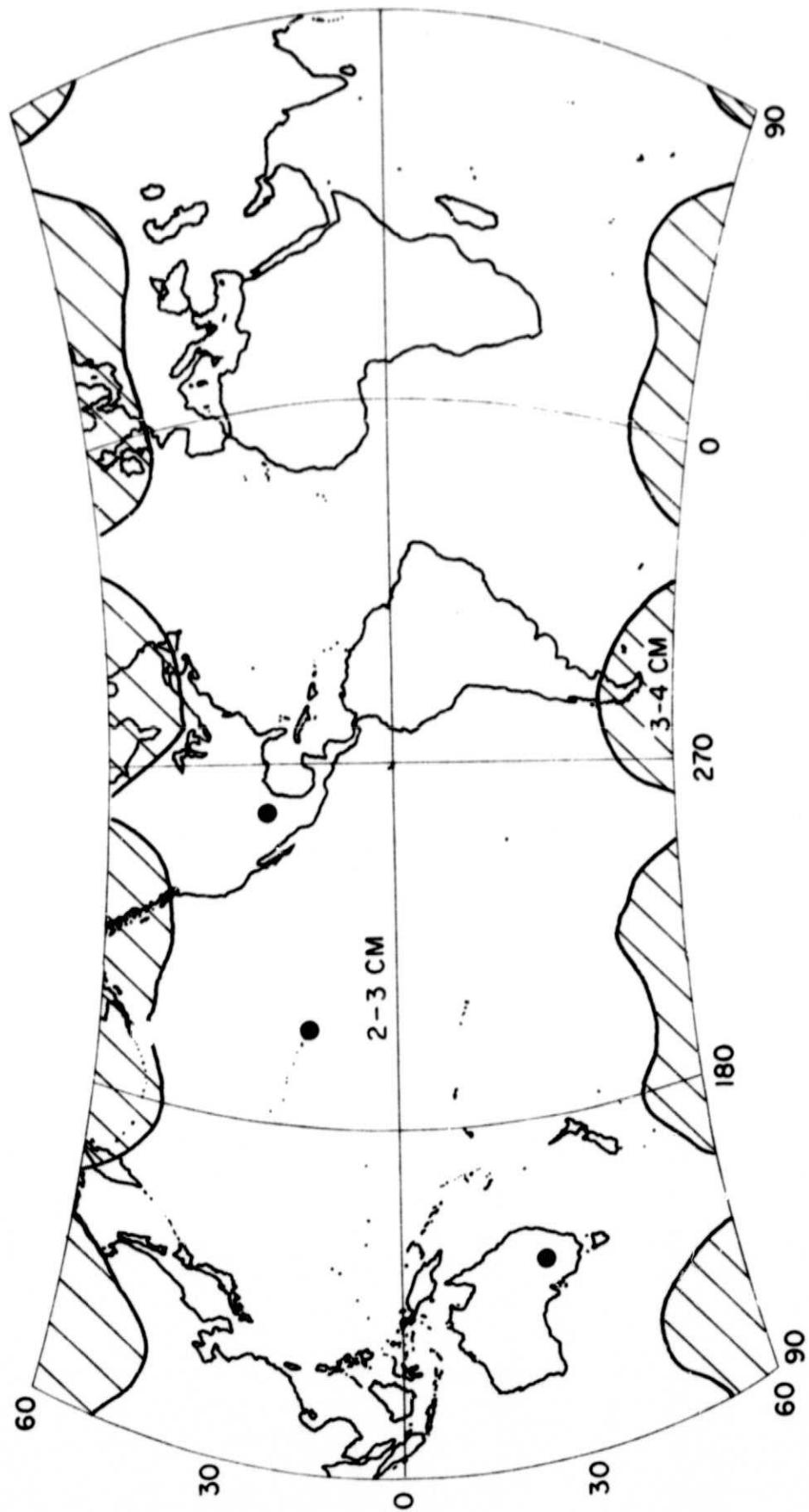
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5-CM POLAR MOTIC V
WFAC = 0.5

LATITUDE
3-CM MOBILE STATION



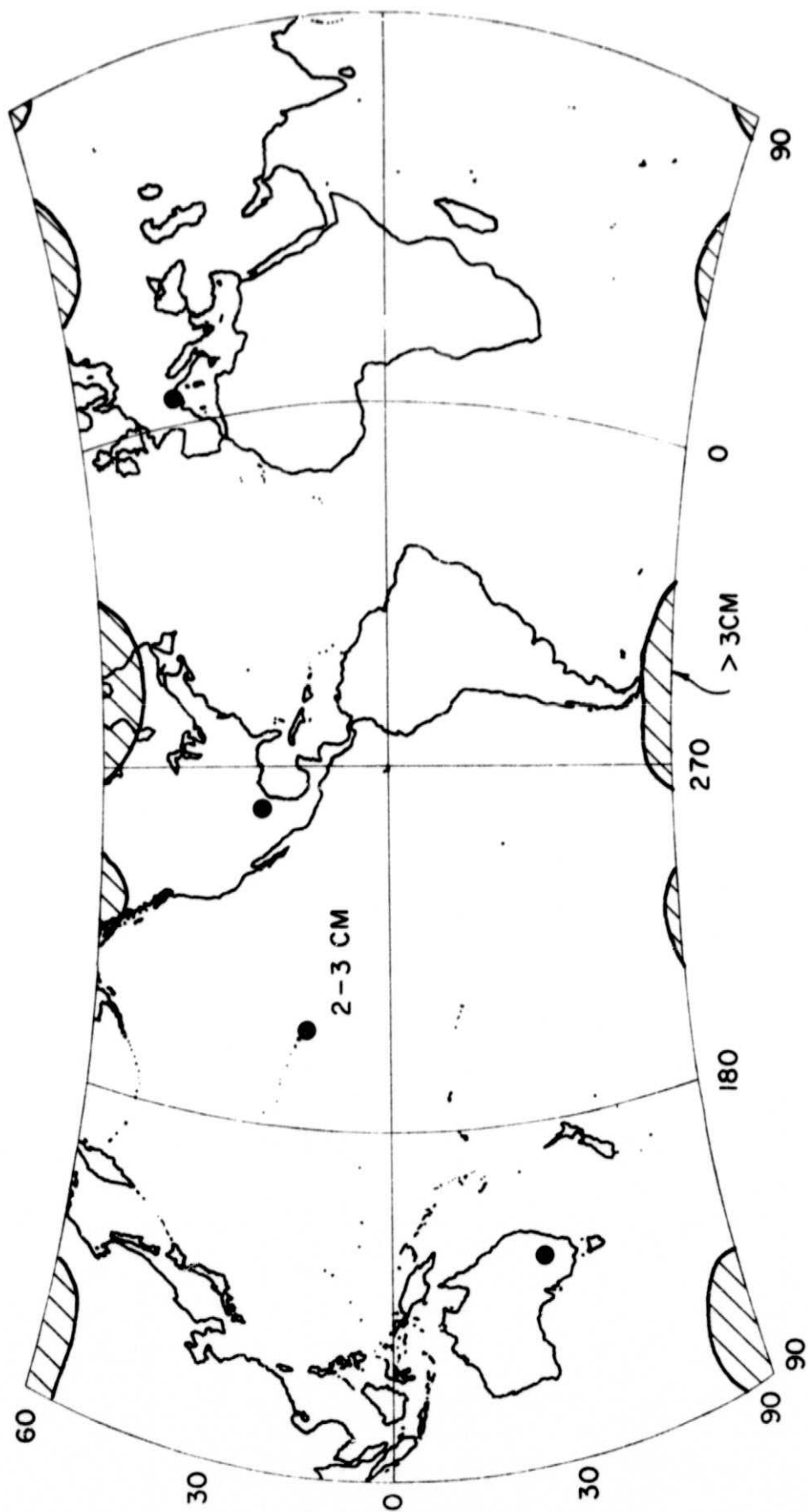
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 5-CM POLAR MOTION
 WFAC = 1.0

LATITUDE
 3-CM MOBILE STATION



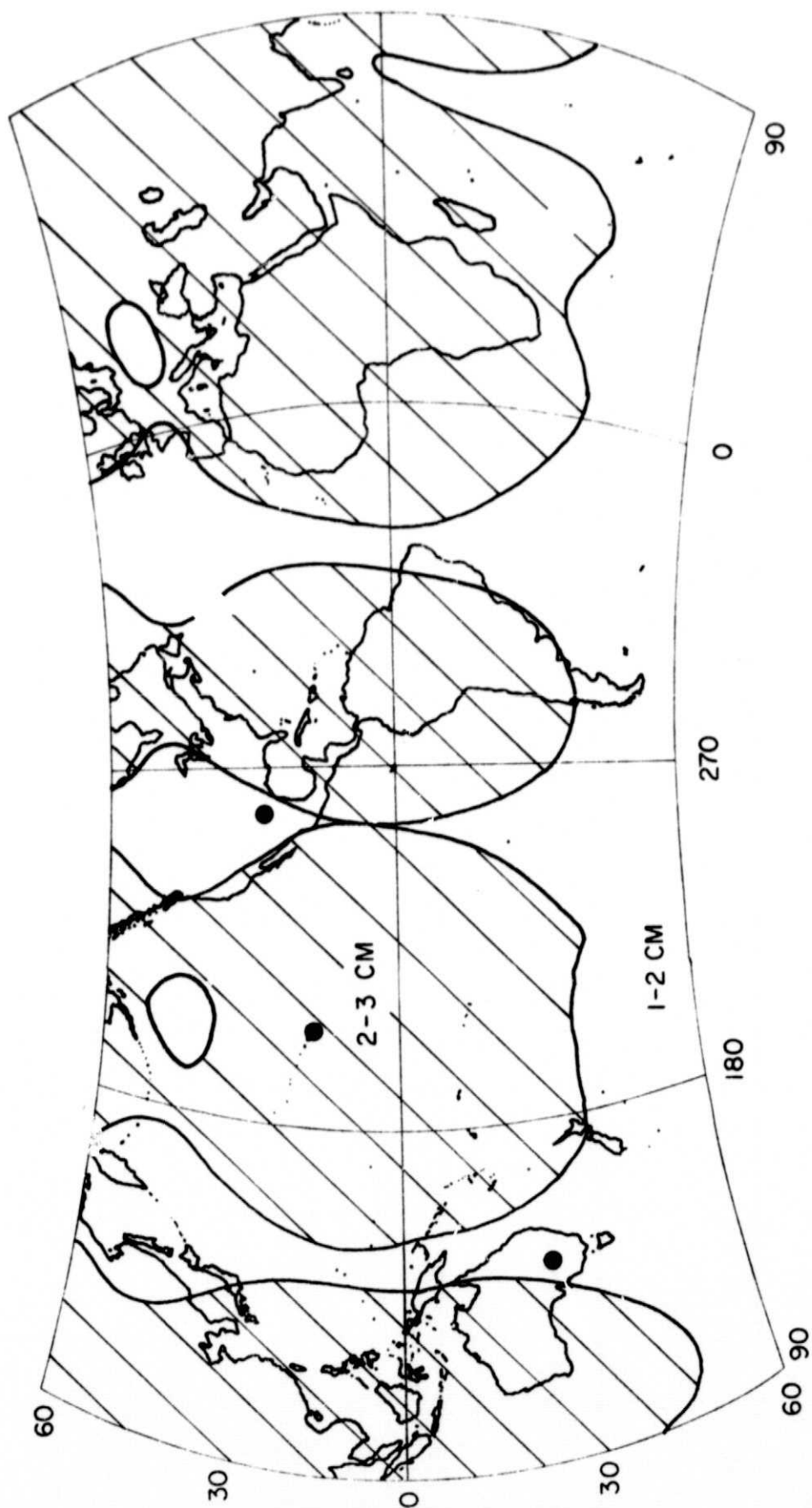
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 (TEX, HAW, AUS, FRA)
 5-CM POLAR MOTION
 WFAC = 1.0

LATITUDE
 3-CM MOBILE STATIONS



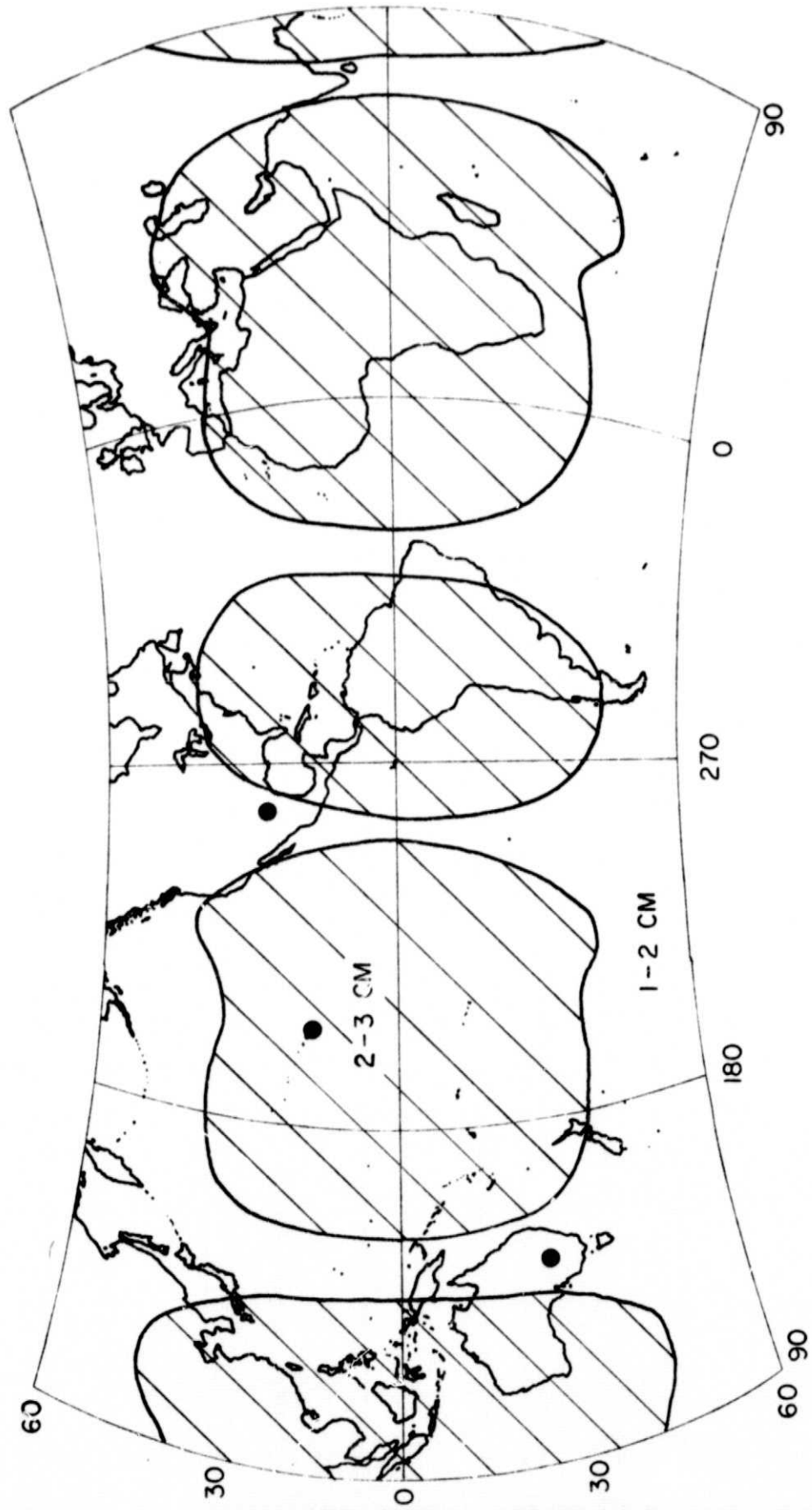
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 5-CM POLAR MOTION
 WFAC = 0.5

AXIAL DISTANCE
 3-CM MOBILE STATION



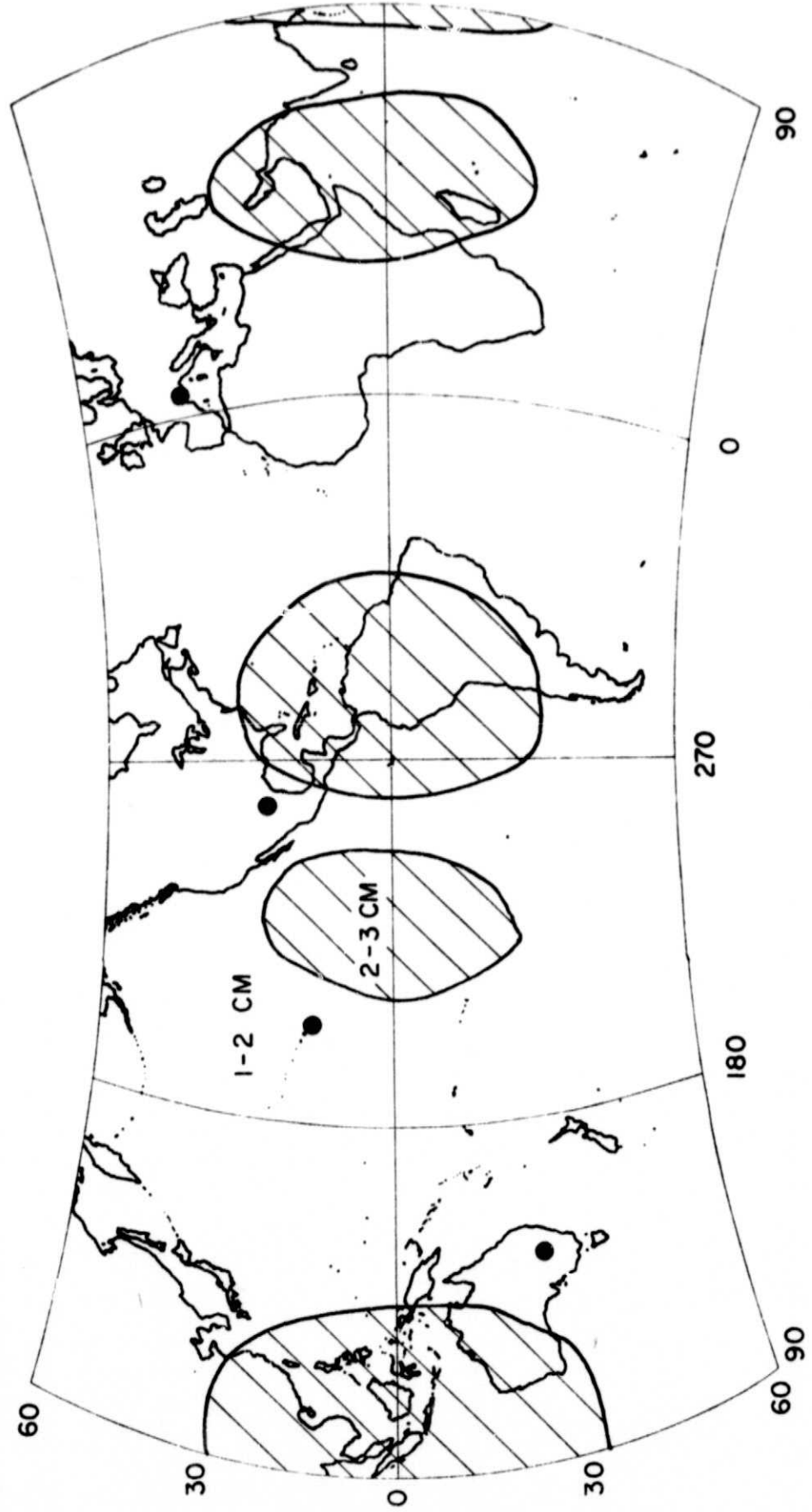
AXIAL DISTANCE
3-CM MOBILE STATION

THREE 3-CM STATIONS
(TEXAS, HAWAII, AUSTRALIA)
5-CM POLAR MOTION
WFAC = 1.0



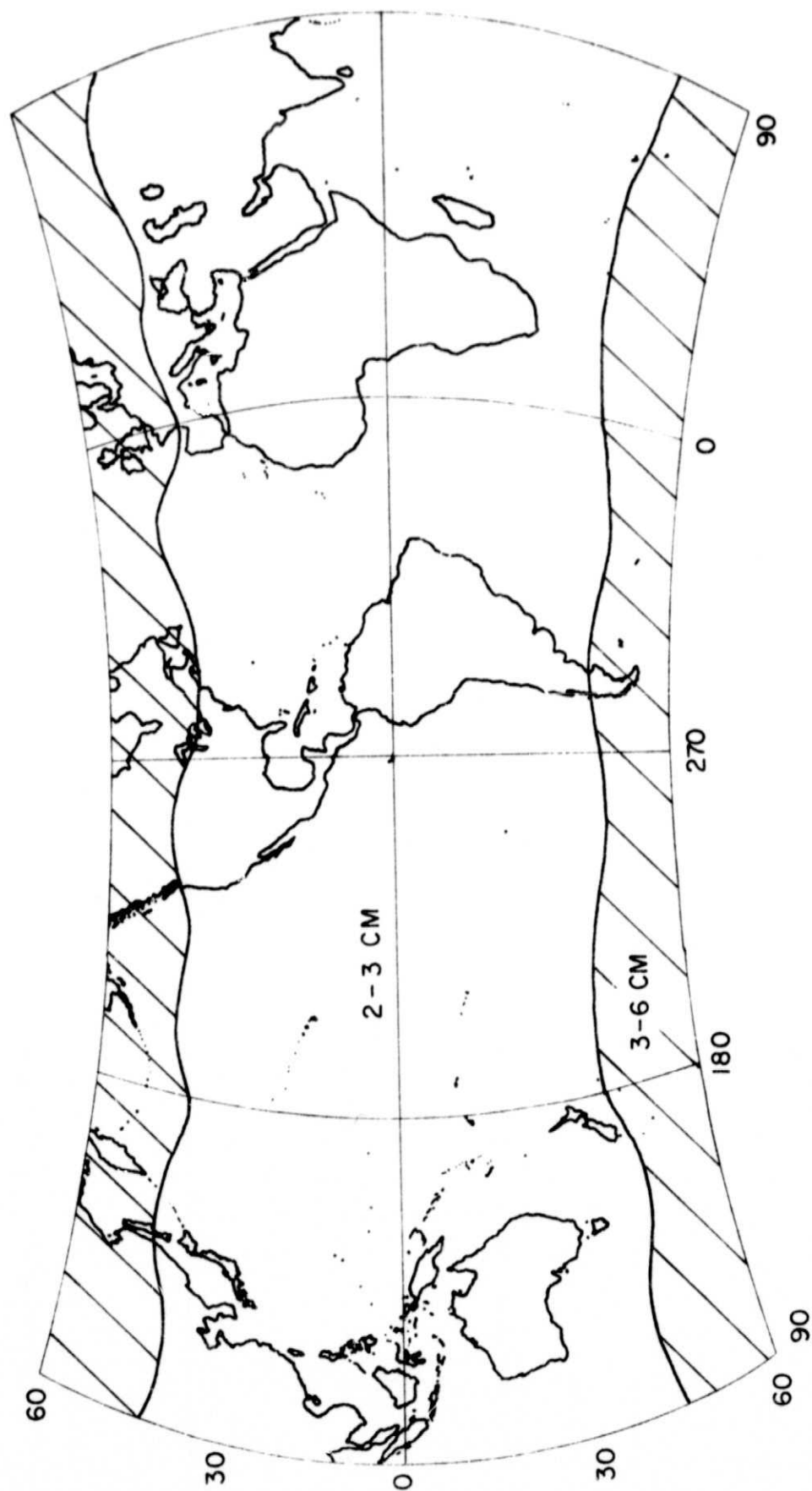
AXIAL DISTANCE

FOUR 3-CM STATION
(TEX, HAW, AUS, FRA)
5-CM POLAR MOTION
WFAC = 1.0



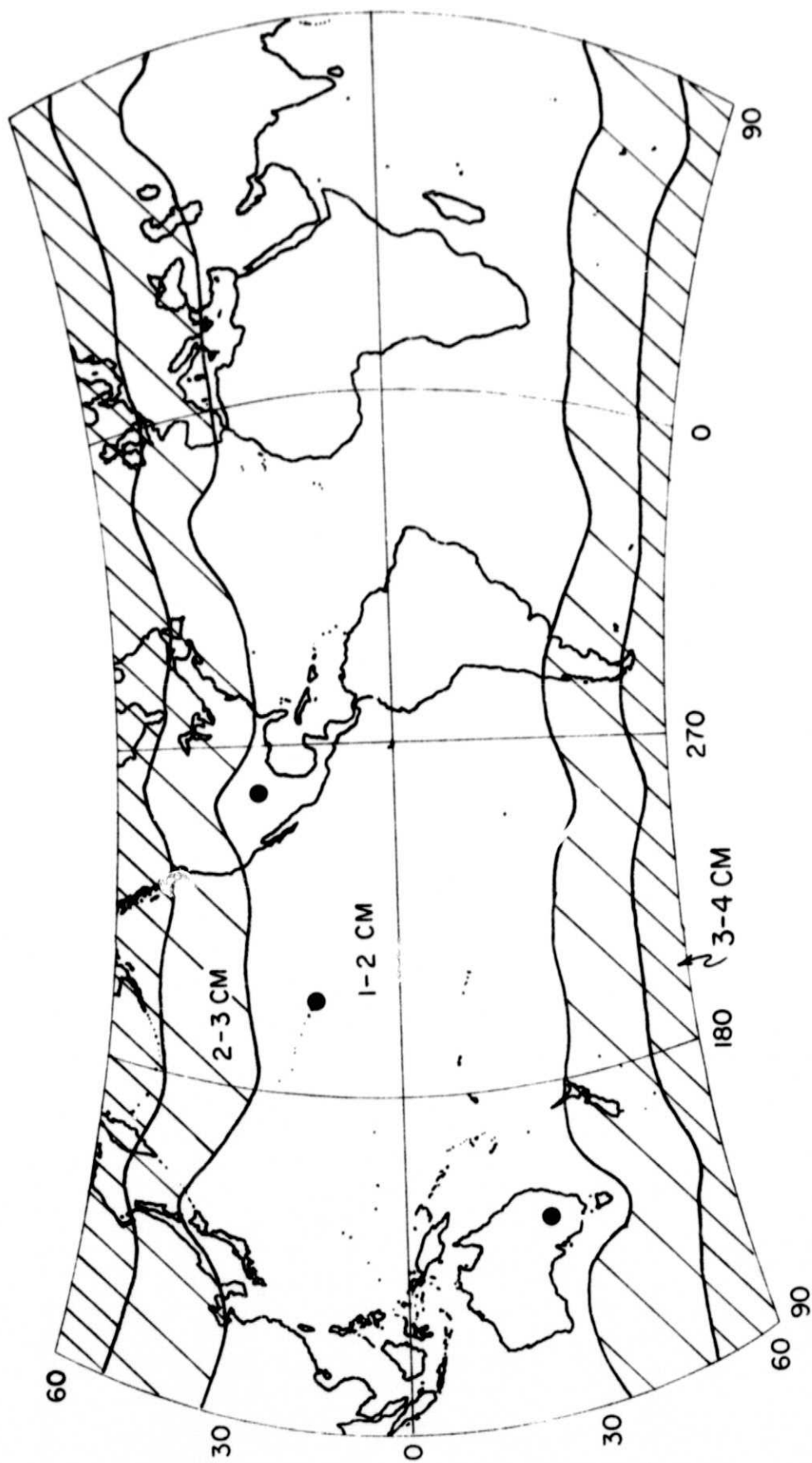
THREE 3-CM STATIONS
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5-CM POLAR MOTION
WFAC = 0.5

EQUATORIAL DISTANCE
3-CM MOBILE STATION



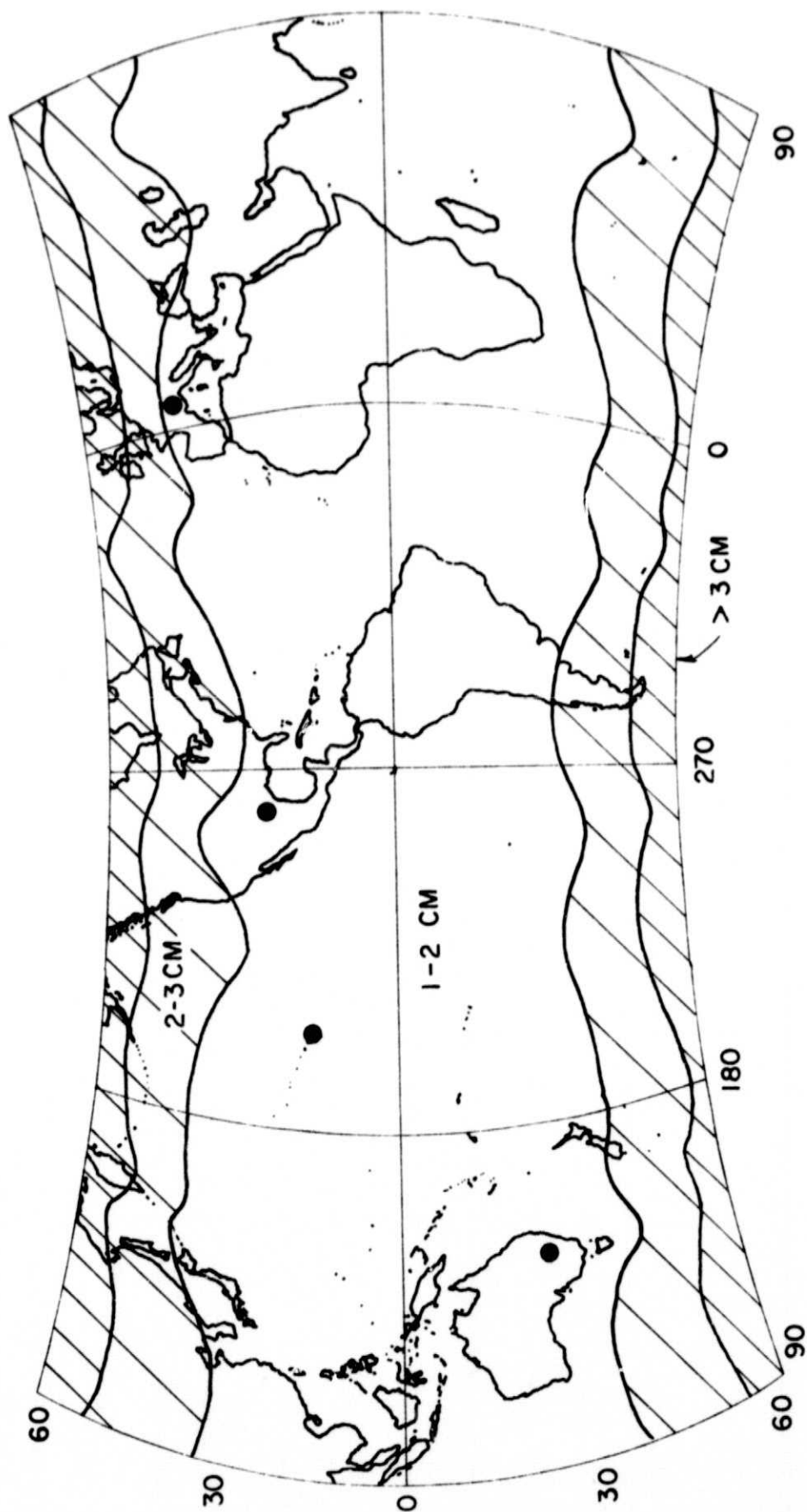
EQUATORIAL DISTANCE
3-CM MOBILE STATION

THREE 3-CM STATIONS
(TEXAS, HAWAII, AUSTRALIA)
5-CM POLAR MOTION
WFAC = 1.0



FOUR 3-CM STATIONS
 (TEX, HAW, AUS, FRA)
 5-CM POLAR MOTION
 WFAC = 1.0

EQUATORIAL DISTANCE
 3-CM MOBILE STATION



Appendix A: Effects of Zenith Distance Restriction

For the purposes of this study, the motion of the Moon in declination was modelled by a simple sine curve. For each of the 72 observing windows, the computed declination was combined with the lunar hour angle and the latitude of the mobile site to determine the zenith distance of the Moon at that instant. It is obviously necessary that there be some geometric limit that will eliminate the possibility of making some observations. If there were no stricter limitations, this would be provided by the horizon; i.e. the Moon cannot be observed when its zenith distance is greater than 90° . In fact, there are stricter limitations. They may be physical, such as the difficulty of making observations through the extended air mass near the horizon. They may be environmental, such as the 70° zenith distance imposed on laser operations at McDonald Observatory due to considerations of aircraft safety. Taking this latter as our criterion, we have automatically rejected any potential observation less than 20° above the horizon. This effect is obviously a function of the latitude of the mobile station, and it takes two general forms. Quite obviously, when the algebraic difference of the station latitude and lunar declination exceeds 70° , there is no possibility of an observation at any hour angle; that portion of the Moon's orbit is simply unavailable from that site. This will only occur for stations at latitudes higher than about $\pm 40^\circ$. Even under more favorable conditions, however, and over a much wider range of latitude, observations may be geometrically possible at meridian passage but not at the ± 3 hour windows. Thus, while virtually no observations are lost

at the equator, roughly 30% are eliminated by this criterion at latitude 45° , and fully half at latitude 60° . A station at one of the poles operating under this geometric constraint, would obtain observations only when the Moon was at its extreme northern declination and on the meridian. Table I illustrates the situation for the two intermediate latitudes. Note that some effect of the random variation within the one-hour windows is noticeable in the entries for $\delta = 45^\circ$ and days 13, 23 and 24.

Table I. Effect of 70° zenith distance limit at two latitudes

Day	$H = -3^h$		$H = 0$		$H = +3^h$	
	$\delta = 45$	$\delta = 60$	$\delta = 45$	$\delta = 60$	$\delta = 45$	$\delta = 60$
1	X	X	X	X	X	X
2	X	X	X	X	X	X
3	X	X	X	X	X	X
4	X	X	X	X	X	X
5	X	X	X	X	X	X
6	X	X	X	X	X	X
7	X	X	X	X	X	X
8	X	X	X	X	X	X
9	X	X	X	X	X	X
10	X	X	X	X	X	X
11	X	X	X	X	X	
12	X		X	X	X	
13	X		X	X		
14	X		X		X	
15			X			
16			X			
17			X			
18			X			
19			X			
20			X			
21			X			
22			X			
23	X		X	X		
24			X	X	X	

Appendix B: Weather Modelling

As indicated in the main text, the actual number of observations entered into the formation of the system of normal equations is reduced as a function of a weather factor WFAC assigned to that particular site. The mechanism by which this is accomplished is to use the assigned weather factor together with a correlation function intended to simulate the day-to-day nonrandomness of major weather patterns to obtain a specific weather index WF for that particular observation. WF is computed as the difference between a function of WFAC and the normalized product of a random number generator. If its value is negative then the observation is rejected as having been clouded out. Since WFAC has, by definition, maximum value unity, one would expect that the desired result of selecting $(72 \times \text{WFAC})$ observations could be achieved simply by defining WF as the difference between WFAC and the random number at each step. This, however, would achieve no correlation from one observation window to the next, which is clearly an unrealistic situation. We have attempted to introduce reasonably believable correlations on two levels, internal to a given day and from one day to the next. This required that the first observation window each day be treated differently from the succeeding ones. For these latter, it was found to be a very effective representation of the daily correlation to set

$$\text{WF} = \text{WFAC} - X + W$$

where X represents the random number and W the value of WF from the previous observation window. The situation for the initial window of each day was considerably more difficult. The simple scheme used

within a day, if extended to all observation windows, resulted in an unacceptably high level of snowballing and thus very poor overall representation. After an extensive heuristic study, it was concluded that no one single technique would suffice for the entire range of weather factors. To produce reasonable results, it was necessary to adopt two different functional relationships for WF to cover the range of interest for WFAC. For weather factors in the range 0.2 - 0.5, the following relation was used

$$WF = WFAC - X + \frac{W}{2} + (0.25 - WFAC^2)$$

while for higher weather factors, this relation was adopted

$$WF = WFAC - X + \frac{W}{3} + (0.23 - \frac{WFAC}{2})$$

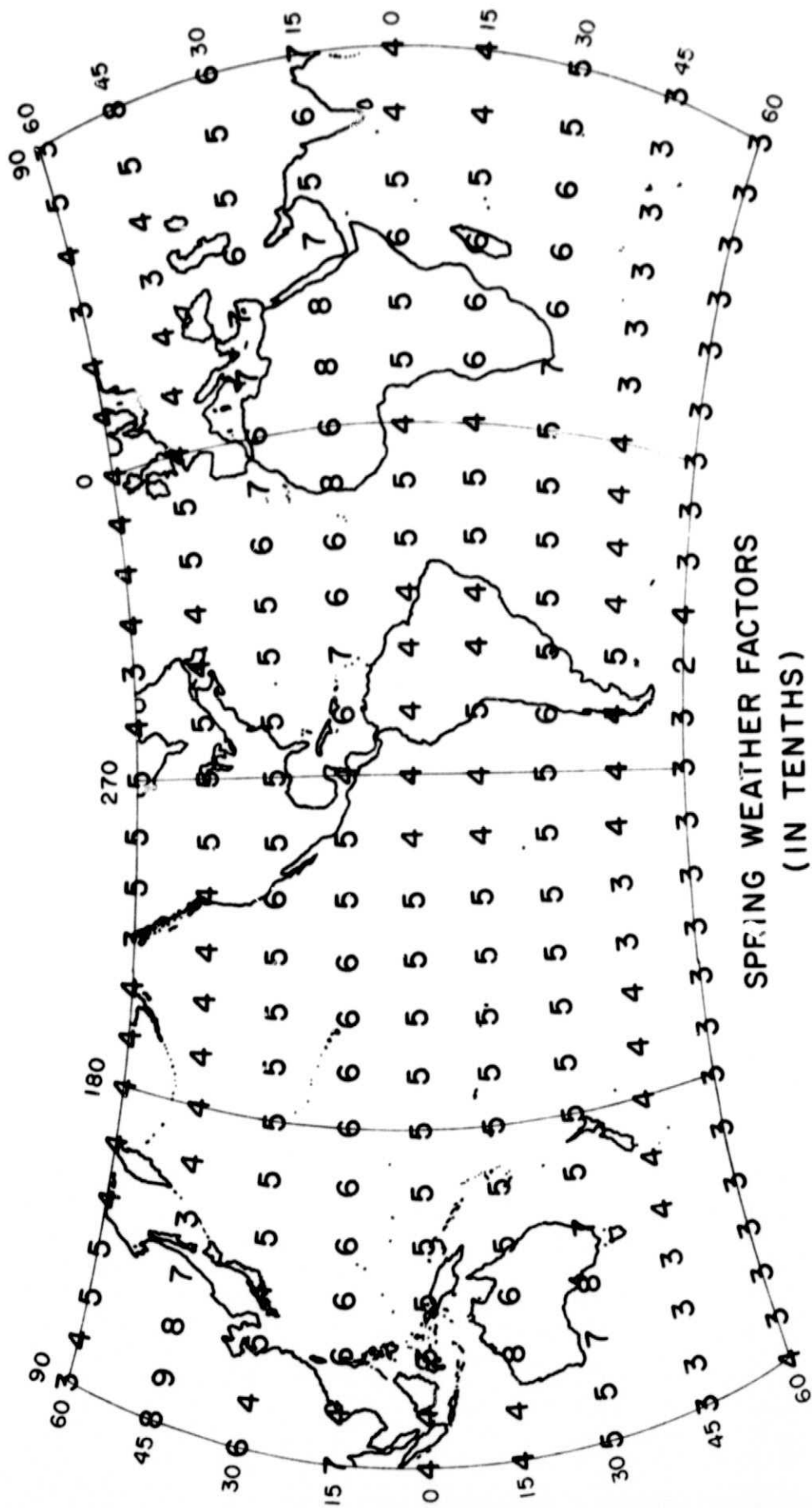
As before, X represents the random number and W represents the value of WF from the previous observation window, which in this case was the previous day. None of the schemes tried were capable of representing the case WFAC = 0.1, but this is not regarded as a serious drawback. Areas where the weather is so bad as that will be studiously avoided by a mobile station.

Sensitivity computations were performed over a wide range of values of the weather factor, and it was originally intended to produce maps based on large scale climatic records. It was pointed out, however, that it was virtually always possible to find specific sites in any region where the local weather was considerably better than the regional average. As a result of discussions with the requestors of this study, it was decided that only the two uniform cases of WFAC = 1.0 and 0.5 be included in the main body of this report. Nonetheless,

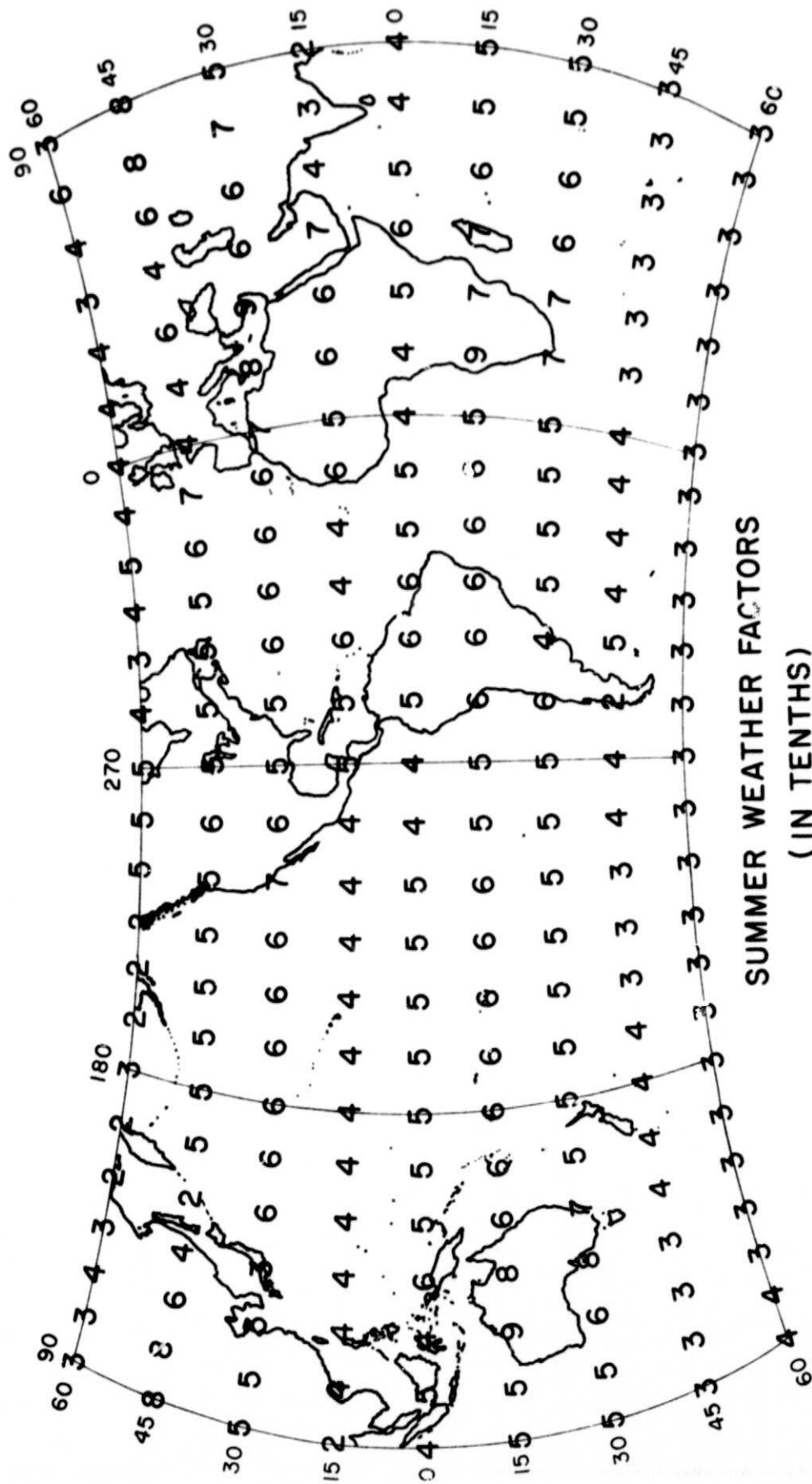
during the early stages of this study a certain amount of effort was expended on the compilation of global weather statistics. This information has been reduced to maps of clear sky fraction (given in tenths) over a 15° grid. These data have been compiled on a seasonal basis, and there is interesting information to be found here. Consequently, even though they were not used in the preparation of the results given here, we close this appendix by displaying the four seasonal clear sky maps plus a fifth showing the annual means.

References for Appendix B:

- Blair, T. A., Climatology, Prentice-Hall, Inc., New York, 1942.
- Borisov, A. A., Climates of the U.S.S.R., Aldine Publishing Company, Chicago, 1965.
- Clayton, H. H., World Weather, Macmillan Company, New York, 1923.
- Kendrew, W. G., Climate, Clarendon Press, Oxford, 1938.
- Landsberg, H. E., ed. in chief, World Survey of Climatology, Vol. 5, 8, 10, 11, 13, 14, Elsevier Publishing Company, New York, 1970 (main source).

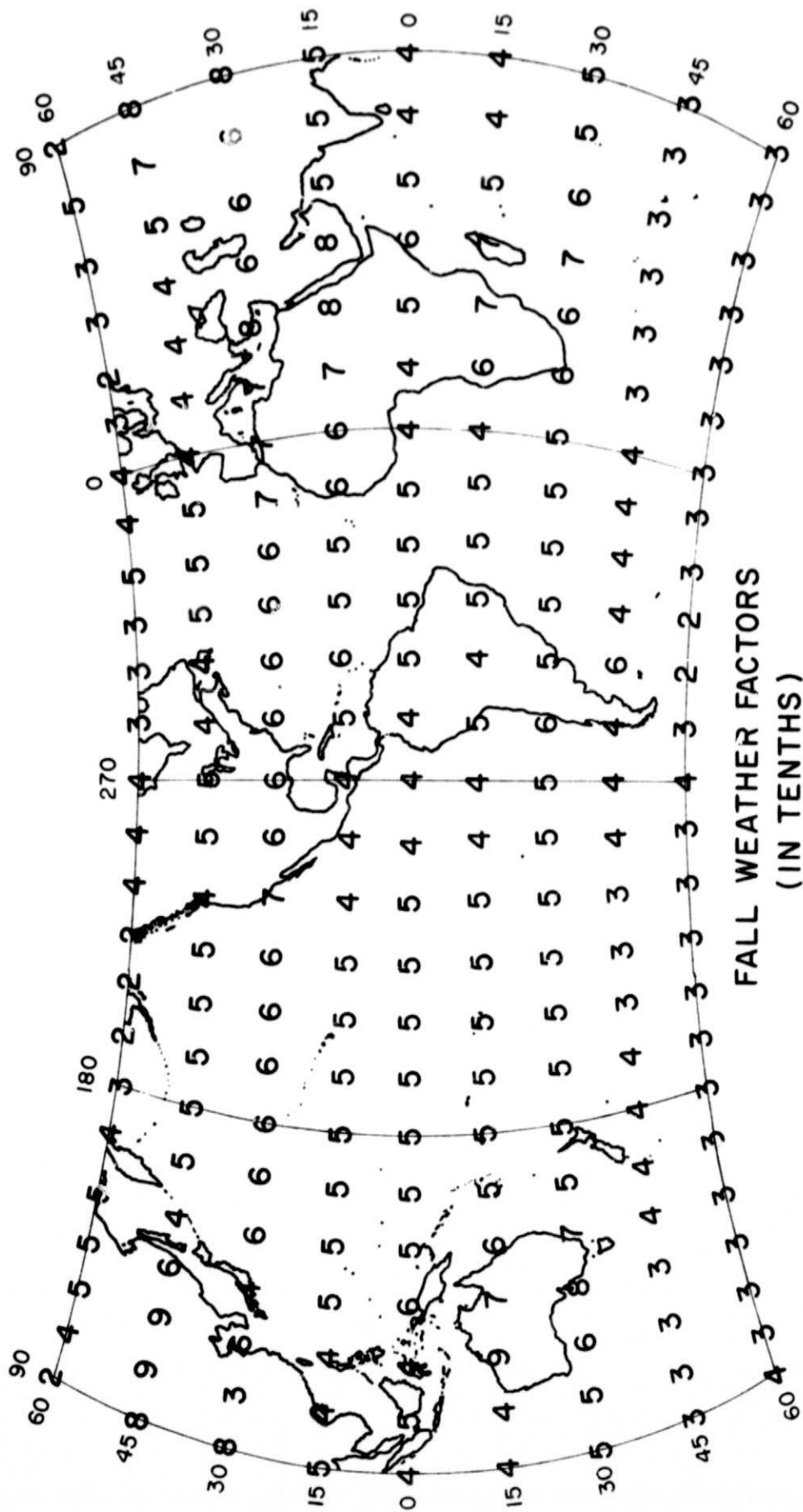


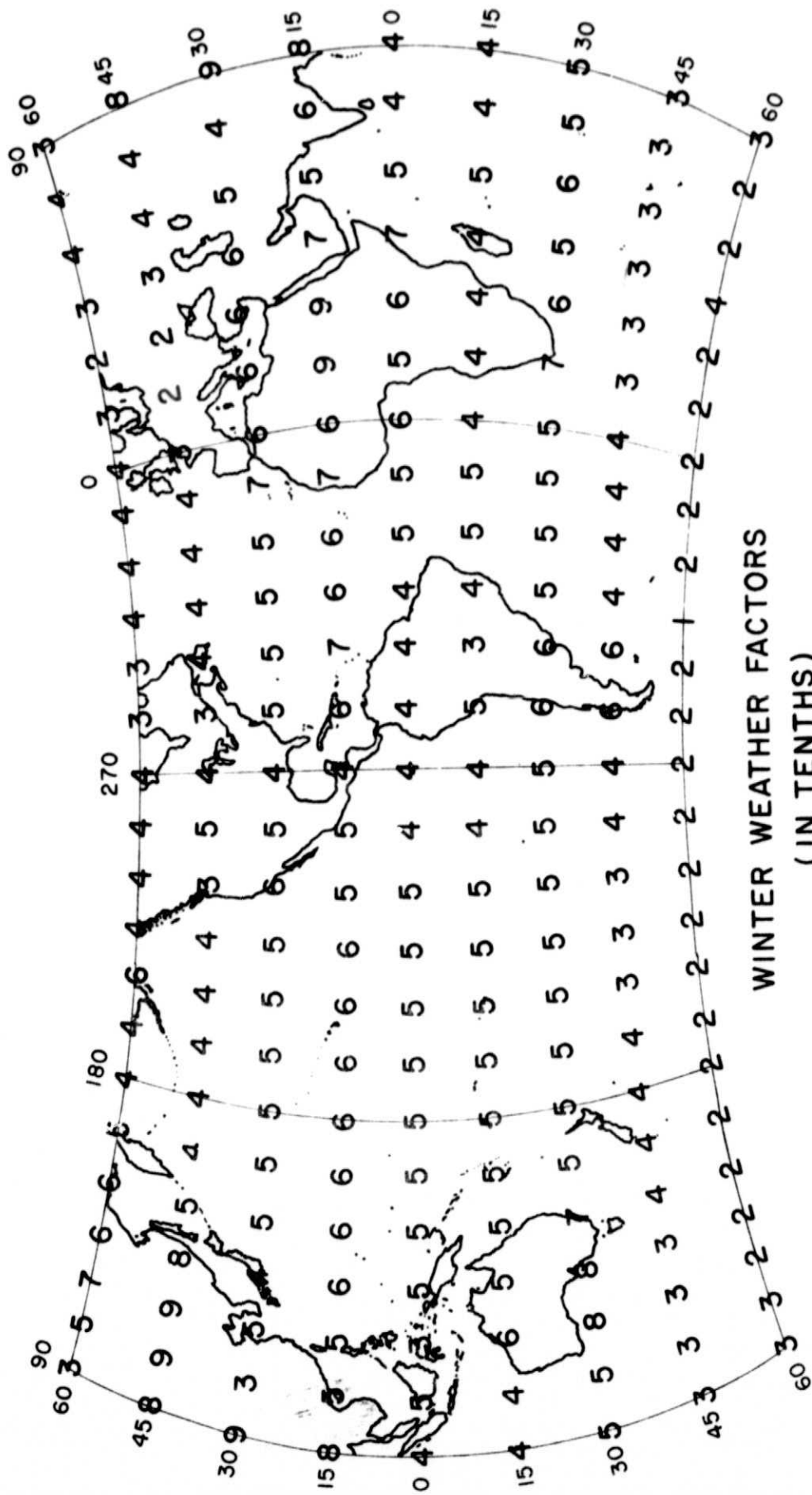
SPRING WEATHER FACTORS
(IN TENTHS)



SUMMER WEATHER FACTORS

(IN TENTHS)





WINTER WEATHER FACTORS
(IN TENTHS)

