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Garret N. Vanderplaats

Ames Research Center
Moffett Field, Calif. 94035

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16. Abstract

A method is presented for configuration optimization of finite element structures, given a reasonable initial geometry. The objective is to minimize weight or cost. Design variables include geometric as well as member sizing parameters. The number of elements and joints, and the element-joint relationships are prescribed and are not changed during the optimization process. However, the joint locations are changed. The structure is assumed to be linearly elastic and may be statically indeterminate. Multiple loading conditions are allowed. Constraints include limits on stiffness as well as strength. The method is demonstrated with application to truss design, subject to minimum size, strength, buckling, and displacement constraints. Major design improvements are achieved through configuration changes.

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DESIGN OF STRUCTURES FOR OPTIMUM GEOMETRY

Garret N. VANDERPLAATS, Research Scientist
NASA, Ames Research Center
Moffett Field, California, USA

1. Introduction

Optimization of finite element structures has received considerable attention in recent years. The majority of this work has dealt with structures of specified configuration, the design variables, the cross-sectional areas of bar elements and the thicknesses of membrane panels. Relatively little effort has been directed toward determination of the optimum configuration of the structure. However, that work which has been reported [1 - 10] is sufficient to demonstrate that major design improvements can be achieved by allowing for configuration changes during the automated design process.

A general method is presented here for geometric optimization of finite element structures. It is assumed that a reasonable initial geometry is specified. The number of finite elements, the number of joints and the element-joint relationships are specified and are not changed during the optimization process. The structure may be statically indeterminate and may support multiple loading conditions. Design variables include geometric and member sizing parameters. The design objective may be minimum weight or cost, and constraints include strength and stiffness limitations. The optimization procedure is a general mathematical programming approach.

The general optimization problem is formulated and the iterative design algorithm is outlined. This formulation is specialized for truss structures and design examples are presented. Conclusions based on this study are also presented.

2. General Formulation

The general design problem considered here is to minimize the weight or cost of the structure. Constraints on the design can include allowable stress, buckling, displacement, dynamic and aeroelastic response limitations. The geometric location of the joints, along with the element sizing variables, such as thickness or cross-sectional area, will be changed in order to optimize the structure. The general problem is stated mathematically as:

$$\text{Minimize} \quad F(\bar{X}) \tag{1}$$

Subject to:

$$G_j(\bar{X}) \leq 0 \qquad j = 1, m \tag{2}$$

where

$$\bar{X} = \left\{ \begin{array}{c} \bar{X}_G \\ \hline \bar{X}_M \end{array} \right\} \tag{3}$$

$F(\bar{X})$ is referred to as the objective function. The set of m constraints which are imposed on the design are defined by (2). The vector of design variables, $\bar{X}$, includes geometric design variables, $\bar{X}_G$, and member sizing design variables, $\bar{X}_M$ that will be changed during the optimization process. The geometric design variables may be the coordinates of the joints themselves or may be the coefficients of any functional relationship which describes the geometry. For example, the surface of an aircraft wing may be described by a polynomial representation in order to conveniently interface the required aerodynamic analysis with the structural analysis. In this case, the geometry design variables may be the coefficients of this polynomial. Similarly, the element member sizing variables, $\bar{X}_M$, may be the actual thicknesses, of membrane elements, or they may be the coefficients of a polynomial representation of the skin thickness distribution of a wing.

In developing a geometric optimization capability, it is desirable to devise a technique which (1) deals with as few design variables as is practicable at any point in the optimization process, (2) will reduce the ill-conditioning introduced by mixing member sizing and geometric design variables, and (3) take full advantage of state-of-the-art techniques in fixed-geometry design.

The approach used here is to treat the geometric design parameters as independent design variables. The member sizing parameters are treated as dependent variables which are determined as a sub-problem. This is essentially the same as the approach used in [6], but is extended here, using the techniques of [11], to deal with general finite element structures subject to generalized constraints. Beginning with an initial geometric design vector, $\bar{X}^0$, the design proceeds iteratively by the following relationship:

$$\bar{X}_G^{q+1} = \bar{X}_G^q + \alpha_G^* \bar{S}_G^q \tag{4}$$

where q is the iteration number and $\bar{S}^q$ is the search direction which is yet to be determined. α_G^* is a scalar parameter determining the distance of travel in the design space.

For each proposed geometric vector, $\bar{X}_G$, the structure is optimized with respect to the member sizing variables, $\bar{X}_M$, by solving the following suboptimization problem:

$$\text{Minimize} \quad F(\bar{X}_M) \tag{5}$$

Subject to:

$$G_j(\bar{X}_M) \leq 0 \qquad j = 1, m \tag{6}$$

Equations (5) and (6) are simply the standard form of the fixed-geometry optimization problem, and can be solved using any one of a variety of available algorithms.

It is now necessary to determine the search direction, $\bar{S}_G$. Assume that for the initial geometry the structure has been optimized with respect to the member sizing variables, and that for this sub-optimum design there are ℓ active constraints:

$$G_k(\bar{X}) = 0 \qquad k = 1, \ell \tag{7}$$

In practice, constraint $G_k(\bar{X})$ will be defined as active if its value is near zero, since precise zero is seldom meaningful on a digital computer. It is now necessary to find a search direction, $\bar{S}_G$, so that by moving in this direction in the geometric design space, the objective function will be reduced. This direction may be found by solving the following subproblem:

$$\text{Minimize } \nabla F(\bar{X}) \cdot \bar{S} \tag{8}$$

Subject to:

$$\nabla G_k(\bar{X}) \cdot \bar{S} \leq 0 \qquad k = 1, \ell \tag{9}$$

$$\bar{S} \cdot \bar{S} \leq 1 \tag{10}$$

where

$$\nabla \equiv \begin{Bmatrix} \frac{\partial}{\partial X_1} \\ \frac{\partial}{\partial X_2} \\ \\ \frac{\partial}{\partial X_n} \end{Bmatrix}, \qquad \bar{S} = \begin{Bmatrix} S_G \\ ------ \\ S_M \end{Bmatrix} \tag{11}$$

Equations (8 - 10) themselves define an optimization problem in which the design variables are the components of $\bar{S}$. This has the same form as the direction finding problem in the method of feasible directions, and the details for its solution are described in [12] and [13]. The $\bar{S}_G$ portion of $\bar{S}$ is now substituted into (4) and a one-dimensional search on α_G^* is performed to update the geometry.

The geometric optimization problem can now be summarized in the following algorithm:

1. Choose an initial set of design variables, $\bar{X}$. Set q = 0.
2. For the current set of geometric variables, $\bar{X}_G^q$, optimize the structure with respect to the member sizing variables, $\bar{X}_M$, using (5) and (6).
3. Determine the number of active constraints (7) and obtain the gradients of the objective and active constraints (11).
4. Determine the search direction, $\bar{S}_G^q$, using (8 - 10).
5. Perform a one-dimensional search in accordance with (4) to determine the value of α_G^* which will minimize the objective function subject to the constraints on geometry. For each proposed geometry, X_G, update the member sizes using step 2.
6. Check to see if the design has converged to the optimum. If no improvement has been achieved, terminate. If the design has been

improved, increase q by 1 and return to step 3.

3. Application to Truss Structures

The design algorithm is demonstrated here with application to three-dimensional statically indeterminate truss structures. The structure is assumed to be linearly elastic and is subject to multiple loading conditions. The objective is to minimize the total weight of the structure. Constraints include limits on member sizing, element stresses, joint displacements and Euler buckling of the elements. The weight objective function is

$$W = \sum_{i=1}^{NE} \rho_i A_i L_i , \qquad (12)$$

where ρ_i is the material density, A_i the cross-sectional area and L_i the length of member i. NE is the total number of elements in the structure. Constraints on the design are defined as follows:

$$\text{Side constraints,} \qquad A_i^{\ell} - A_i \leq 0 \qquad (13)$$

$$\text{Stress,} \qquad \sigma_{ij}/\bar{\sigma}_{ci} - 1 \leq 0 \qquad (14)$$

$$\sigma_{ij}/\bar{\sigma}_{ti} - 1 \leq 0 \qquad (15)$$

$$\text{Displacement,} \qquad \delta_{\ell kj}/\bar{\delta}_{\ell kj} - 1 \leq 0 \qquad (16)$$

$$\text{Euler buckling,} \qquad \sigma_{ij}/\bar{\sigma}_{bi} - 1 \leq 0 \qquad (17)$$

where

A_i^{ℓ} = lower bound on member area i,

σ_{ij} = calculated stress in member i under load condition j,

$\bar{\sigma}_{ci}$, $\bar{\sigma}_{ti}$ = compressive and tensile stress allowable, respectively, in member i,

$\bar{\sigma}_{bi}$ = stress at which Euler buckling occurs in member i,

$\delta_{\ell kj}$ = displacement at joint ℓ in direction k (k = 1, 2, 3) under load condition j,

$\bar{\delta}_{\ell kj}$ = allowable displacement at joint ℓ in directions k under load condition j.

The stress at which Euler buckling occurs is defined by the relationship:

$$\bar{\sigma}_{bi} = \frac{-K_i A_i E_i}{L_i^2} , \qquad (18)$$

where E_i is Young's modulus for element number i and K_i is a predetermined constant which depends on the cross section of the bar element.

The fixed geometry design utilized the constrained function minimization program described in [14]. This program is based on Zoutendijk's method of feasible directions [12] with modifications to deal with initially infeasible designs [13]. Recent developments in approximation concepts by Schmit et al. [15 - 17] were utilized to gain maximum efficiency in the suboptimization problem.

Three basic techniques were incorporated here for suboptimization. The first is referred to as constraint deletion, where at each stage in the fixed geometry optimization problem only those active or near-active constraints are included in the computations. The second is the use of the reciprocal design variables to transform the constraint functions into approximately linear form. The objective is nonlinear in reciprocal space, but is still explicit and is easily evaluated. Because of this approximate linearity of the constraints, it is logical to include as the third technique a Taylor series expansion on the constraint functions which have not been deleted. This yields a linearized explicit form of the constraints. However, this is complicated somewhat by dealing with Euler buckling constraints as defined by (17). The stress at which Euler buckling occurs is a function of the design variables yielding a nonlinear constraint, even in reciprocal space. In this case, the Taylor series expansion is performed on the stress in member i under load condition j to yield an explicit linear form of the stress. The Euler buckling stress can then be reevaluated for each new proposed design, so that (17) is now explicit, but nonlinear, in the new design variables.

The fixed geometry design proceeds by first analyzing the structure and determining which constraints are active or near active. The appropriate Taylor series expansion is then performed on these constraints and this explicit form of the problem is used to minimize the weight. If the structure is statically determinate, the solution is now complete. In the case of indeterminate structures, the structure is reanalyzed and the process is repeated until convergence is obtained. Even in highly indeterminate structures, this iterative procedure seldom requires more than 10 analyses, and a near-optimum design is usually obtained with only 5.

For geometric optimization, the joint coordinates of the structure were chosen as the design variables.

4. Design Examples

A computer program using these techniques was written for truss optimization. The finite element displacement method is used for structural analysis and all gradient information is computed directly in closed form. The programming language is FORTRAN IV and the examples were run on a CDC 7600 computer.

Case 1—*25-bar space truss with stress and buckling constraints.* Consider the 25-bar truss shown in figure 1. This structure has been used elsewhere as an example for fixed geometry design [18] and configuration optimization [6].

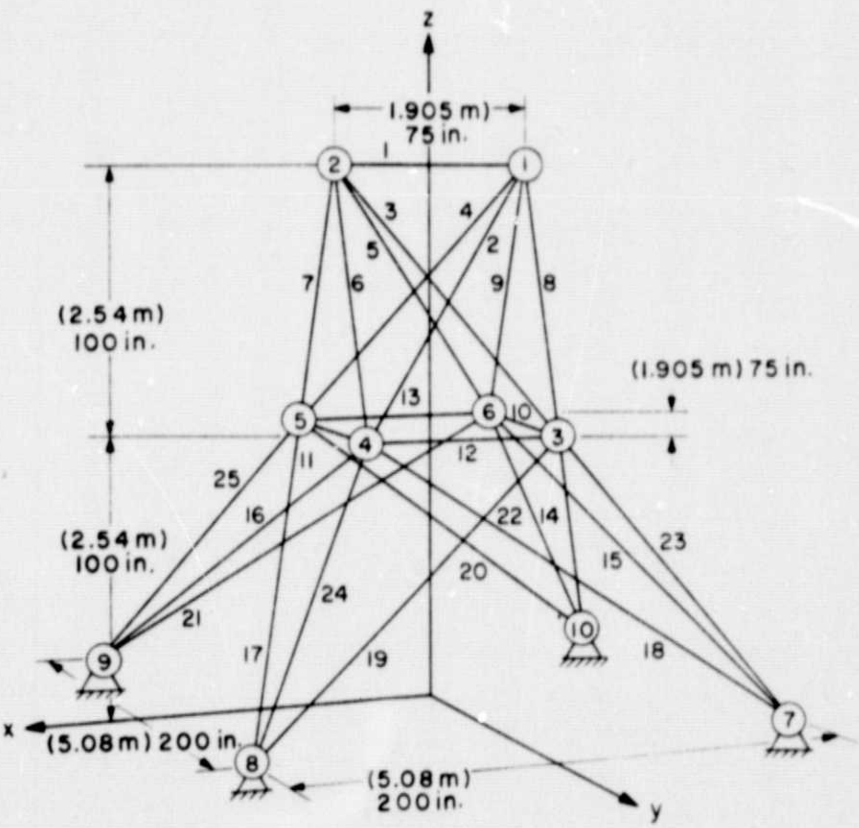

Figure 1 - Twenty-Five-Bar Space Truss

The truss was required to support two load conditions and was designed subject to constraints on member stresses and on Euler buckling. The loading is the same as that used in [6].

A minimum allowable cross-sectional area of 6.45×10^{-6} m^2 (0.01 in.^{2}) was specified. The allowable stresses for all members were specified as 27.58×10^7 N/m^2 (40,000 psi) in both tension and compression. Young's modulus was taken as 6.89×10^7 N/m^2 (10^7 psi), and the material density $\rho = 2767$ kg/m^3 (0.1 lb/in.^{3}). The members were considered to be tubular with the nominal diameter-to-thickness ratio of $D/t = 100$ to give an Euler buckling constant (18) of K_i = 39.274 for all members. The truss was required to remain symmetric with respect to both the x-z and the y-z planes. The independent coordinate variables were taken as x_4, y_4, z_4, x_8 and y_8. The remaining coordinates were linked to these design variables to maintain symmetry. The coordinates of joints 1 and 2 were held constant, and joints 7 through 10 were required to lie in the x-z plane. Member areas were linked in the following groups: A_1, A_2-A_5, A_6-A_9, A_{10}-A_{11}, A_{12}-A_{13}, A_{14}-A_{17}, A_{18}-A_{21} and A_{22}-A_{25}. There were then a total of five independent coordinate variables and eight independent area variables.

The resulting optimum geometry is given in Table I. For the final design, member 1 was constrained by its allowable tensile

TABLE I. - 25-BAR TRUSS GEOMETRY

Joint	Coordinates in m (in.) Initial	Final - Case 1	Final - Case 2
4	x = 0.952 (37.5)	0.544 (21.4)	0.831 (32.7)
	y = 0.952 (37.5)	1.176 (46.3)	1.519 (59.8)
	z = 2.54 (100.0)	2.53 (99.7)	2.939 (115.7)
8	x = 2.54 (100.0)	0.366 (14.4)	1.255 (49.4)
	y = 2.54 (100.0	2.123 (83.6)	3.576 (140.8)

stress. All other members were constrained by their respective buckling stresses in at least one member of each group. The weight of the truss was reduced by 48% from an optimum of 104.3 kg (229.9 lb) for the initial geometry to 54.4 kg (119.9 lb) for the final geometry. A total of 8 iterations (4) on geometry were required to reach the optimum. A graph of weight versus iteration is given in figure 2. The optimization required 27 fixed-geometry designs using 171 analyses and 16.5 CPU seconds of computer time. A near-optimum design of

61.0 kg (134.5 lb) was achieved in 4 geometry iterations requiring 15 fixed-geometry designs with 81 analyses.

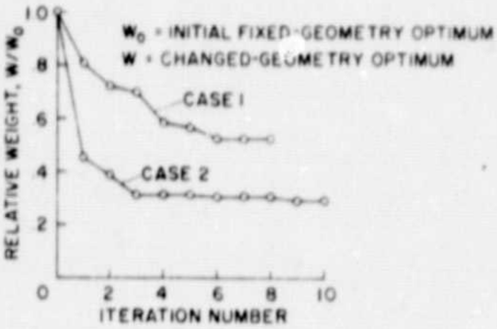

Figure 2. - Weight Versus Geometry Iteration Number

<u>Case 2</u>—*25-bar space truss with stress, buckling and displacement constraints.* The structure designed as Case 1 was reoptimized here with the additional requirement that the joint displacements not exceed 0.00889 m (0.35 in.) in any of the coordinate directions under any load condition. All other design conditions were the same as Case 1. The optimum design for the initial configuration weighed 255.6 kg (563.5 lb) and was constrained by the displacement limits at joints 1 and 2. The optimum configuration weighed 74.3 kg (163.8 lb) and was achieved in 10 geometry iterations. The resulting geometry is given in Table I and the iteration history in figure 2. For this configuration, members 1, 10 and 11 were minimum size. The remaining cross-sectional areas were constrained by the Euler buckling limitation in at least one member of each group, with the exception that no stress or buckling constraints were active on members 6 - 9. Also, displacement limits were active on joints 1 and 2. The design required 35 fixed-geometry optimizations using 204 analyses. The computer time was 20.9 CPU seconds. A near-optimum geometry weighing 79.7 kg (175.7 lb) was achieved in 3 geometry iterations requiring 14 fixed-geometry optimizations and 72 analyses.

5. <u>Conclusions</u>

A general design algorithm has been presented for the optimum geometry design of finite element structures where a reasonable initial geometry has been specified.

The basic conclusions of this study are:

(1) Structural geometry can efficiently be treated as a design

parameter and major weight reductions can often be achieved as a result of geometric changes.

(2) By considering two separate design spaces, geometry and member sizing, the ill conditioning problems usually associated with combining member sizing and coordinate variables are seldom encountered. Also, this approach allows for the application of very efficient fixed geometry optimization techniques currently available.

(3) The technique maintains the generality of mathematical programming, thereby allowing for the simultaneous consideration of a wide variety of constraints on the design.

(4) The method converges rapidly to a near-optimum geometry (see fig. 2).

(5) The computational technique is in no way limited to trusses and applies directly to general finite element structures.

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