(NASA-CR-143353) SOME LONG, RATE ONE-HALF, BINARY CONVOLUTIONAL CODES WITH AN OPTIMUM DISTANCE PROFILE AND THE SYSTEMATIC VERSUS NONSYSTEMATIC CODE QUESTION (Notre Dame Univ.) 8 p HC $3.25 CSCL 09B G3/63

N75-30828

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Department of

ELECTRICAL ENGINEERING

UNIVERSITY OF NOTRE DAME, NOTRE DAME, INDIANA

SOME LONG, RATE ONE-HALF, BINARY CONVOLUTIONAL CODES WITH AN OPTIMUM DISTANCE PROFILE AND THE SYSTEMATIC VERSUS NONSYSTEMATIC CODE QUESTION

Technical Rpt. No. EE-756

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August 19, 1975

Abstract This paper gives a tabulation of long systematic and long quick-look-in (QLI) nonsystematic rate R = 1/2 binary convolutional codes with an optimum distance profile (ODP). These codes appear attractive for use with sequential decoders. Simulations for two of the new codes are reported and confirm Massey's conjecture that systematic and non-systematic codes of the same rate yield nearly identical computational and error probability performance with sequential decoding when the number of digits transmitted in the "tail" of the encoded frame is the same for both codes.

This research was supported in part by the National Aeronautics and Space Administration under NASA Grant NSG 5025 at the University of Notre Dame in liaison with the Communications and Navigation Division of the Goddard Space Flight Center and in part by the Swedish Board for Technical Development under Grant 75-4165.

In this paper we report the results of computer searches for long, rate $R = 1/2$, fixed convolutional encoders (FCEs) with an optimum distance profile (ODP codes), i.e. with a distance profile equal to or superior to that of any other code with the same memory M. In a recent paper [1] we introduced the (M+1)-tuple $\underline{d} = [d_0, d_1, \ldots, d_M]$ and called it the distance profile of the FCE, where d_j is the j-th order column distance [2], i.e. the minimum Hamming distance between two encoded paths of length (j+1) branches, in the infinitely long trellis defined by the FCE, resulting from information sequences with a differing first branch. In particular, d_M is called the minimum distance and d_∞ is called the free distance of the FCE. When comparing two codes of the same memory and rate, we say that a distance profile $\underline{d}$ is superior to a distance profile $\underline{d}'$ if $d_i > d_i'$ for the smallest index i, $0 \leq i \leq M$, where $d_i \neq d_i'$.

Systematic ODP codes are already known for $M \leq 35$ [1]. Newly found systematic ODP codes are listed in Table I for $36 \leq M \leq 60$. The code generators are given in an octal form according to the convention introduced in [1]. In cases where the optimum code is not unique, ties were resolved using the number of low-weight paths as a further optimality criterion.

Massey and Costello [3] introduced a class of quick-look-in (QLI) nonsystematic codes in which the two code generators differ only in the second position. In Table II we list newly found ODP QLI codes for $24 \leq M \leq 50$. For $M \leq 23$ such codes are already known [1].

The excellence as regards d_M for the ODP codes can be seen from Figure 1 in which we have plotted d_M for these codes, the best of the systematic codes found by Bussgang [4], Lin-Lyne [5] and Forney [6], Costello's Algorithm A1 systematic codes [2], and Massey-Costello's QLI codes [3], [2]. The codes are also compared with the Gilbert bound [2,4]. We notice that the newly found codes have d_M equal to or superior to that of any previously known code with the same memory.

Recently Massey [7] has conjectured, in opposition to the presumed superiority of nonsystematic codes over systematic codes [3], that a sequential decoder will perform about as well with a systematic $R = 1/2$ code of memory 2M as with a nonsystematic $R = 1/2$ code of memory M. Since the longer code is systematic every other channel symbol in the tail [8], which is used to terminate an encoded information sequence, is a beforehand known zero that can be omitted before transmission. Hence the two codes require the same allotted space for transmission of their corresponding tails, which is the practical consequence of Massey's conjecture. To test the conjecture we have simulated the performance of a stack sequential decoder [9] on a binary symmetric channel (BSC) for (1) the $M = 46$ systematic ODP code with $d_M = 17$ and $17 \leq d_\infty \leq 30$ of Table I; and (2) the $M = 23$ nonsystematic ODP QLI code with $d_M = 11$ and $d_\infty = 19$ given in [1]. In Table III, we give the results of decoding 10,000 frames of 256 information bits for both codes for the BSC with crossover probability $p = 0.057$, which corresponds to transmission at rate $R = 1.1\ R_0$. where $R_0 = R_{comp}$.

This table gives striking confirmation of Massey's conjecture; the computational performances and the decoding error probability are virtually identical for the two codes. The import of Massey's conjecture, to which our simulations lend credence, is that a systematic $R = 1/2$ FCE can be used instead of a nonsystematic one without any sacrifice in the effective transmission rate, error probability or computational performance of a sequential decoder, provided that the memory of the systematic code is chosen as twice the allotted tail length in information symbols. Thus, the very long systematic FCE's of Table I appear very attractive for use with sequential decoders.

ACKNOWLEDGEMENT

My debt to Professor James L. Massey is obvious and gratefully acknowledged.

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M	$G^{(2)}$	d_M	# paths
36	671145454704	14	5
37	671145454676	14	2
38	671145457556 4	15	31
39	71446165734534	15	12
40	67114545755712	15	3
41	71446165734537	15	1
42	6711454575564 64	16	31
43	714461626554012	16	14
44	714461626554427	16	5
45	7144616265544274	16	1
46	6711454575564666	17	39
47	6711454575564667	17	13
48	67114545755646674	17	4
49	67114545755646676	17	1
50	67114545755646676	18	38
51	671145457556466760	18	16
52	671145457556466760	18	7
53	714461626553260462	18	2
54	7144616265556137204	19	43
55	7144616265556137206	19	20
56	7144616265556137206	19	7
57	71446162655561372064	19	2
58	71446162655561372064	20	60
59	67114545755646670367	20	25
60	671145457556466703670	20	10

Table I: ODP Systematic Convolutional Codes with Rate R = 1/2.

M	$G^{(1)}$	$G^{(2)}$	d_M	# paths
24	740424174	540424174	11	11
25	740415562	540415562	11	5
26	740424173	540424173	11	1
27	7404241724	5404241724	12	23
28	7404241712	5404241712	12	8
29	7404241713	5404241713	12	2
30	74042402074	54042402074	13	43
31	74042402072	54042402072	13	15
32	74042402071	54042402071	13	4
33	740424020714	540424020714	13	1
34	740424020712	540424020712	14	34
35	740424026637	540424026637	14	14
36	7404240266364	5404240266364	14	5
37	7404240266362	5404240266362	14	2
38	7404240207121	5404240207121	15	31
39	74042417136114	54042417136114	15	12
40	74042402071132	54042402071132	15	3
41	74042417136111	54042417136111	15	1
42	740424020712164	540424020712164	16	31
43	740424020712166	540424020712166	16	14
44	740424020713351	540424020713351	16	5
45	7404240207133514	5404240207133514	16	1
46	7404240207121636	5404240207121636	17	39
47	7404240207121635	5404240207121635	17	13
48	74042402071216354	54042402071216354	17	4
49	74042402071216356	54042402071216356	17	1
50	74042402071216357	54042402071216357	18	38

Table II: ODP QLI Convolutional Codes with R = 1/2.

Fraction of Frames with Computation More than N.

N	Systematic ODP code $M = 46$	ODP QLI code $M = 23$
278	1.0000	1.0000
330	0.8463	0.8416
360	0.7230	0.7185
450	0.4957	0.5024
600	0.3345	0.3378
1100	0.1662	0.1679
1700	0.1123	0.1176
2700	0.0743	0.0737

Fraction of Frames Decoded in Error

	0.0002	0.0002

Table III: Simulation Results for Decoding 10,000 Frames of 256 Bits Each for the BSC with $p = 0.057$ ($R = 1.1\ R_0 = 0.50$).

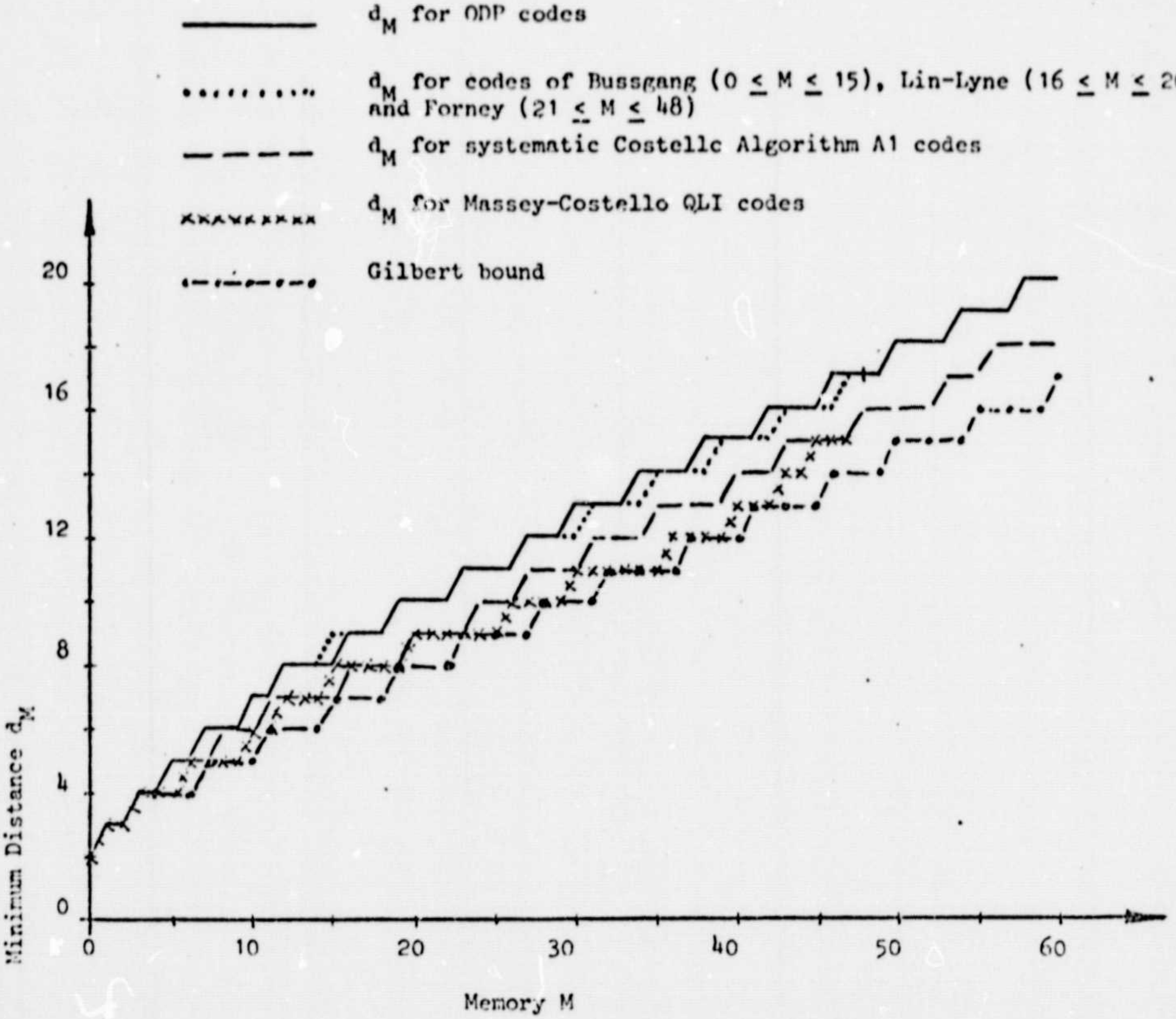

Figure 1: Minimum Distance d_M for Some Rate 1/2 Convolutional Codes.