NSG 8001

(NASA-CR-143381) TRANSITION RADIATIONS IN X-RAY REGION Summary Report (Alabama A & M Univ., Normal.) 28 p HC $3.75 CSCL 20H

N75-30863

Unclas
G3/70 34664

SUMMARY REPORT

TRANSITION RADIATIONS IN X-RAY REGION

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JANUARY 1975

Prepared for the National Aeronautics
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Abstract

The theory of production of radiations in the transoptical region by the passage of high energy charged particles through the interface of two media is discussed. Based on the theoretical model calculations are made for electrons of selected energy range passing through mylar.

1. INTRODUCTION

The prediction, by Ginzberg and Frank (Ref. 1) that transition radiations will be produced in the optical as well as in the x-ray frequency range when charged particles of extremely high energy pass through the interface of two media, has opened up a new technique for experimental detection of charged particles occuring in upper atmosphere and also those produced in artificial laboratory conditions. The theoretical model to explain the mechanism and to evaluate the quantity of radiation produced has been developed by Garibyan (Ref. 2) and also is extensively discussed by Ter-Mikaelian (Ref. 3)

The present work is primarily concerned with quantitatively evaluating the x-ray production cross sections based on the theoretical method developed by Garibyan. A computer program is developed where varying parameters of the particle and detectors can be introduced to evaluate the x-ray production under varying conditions.

Transition radiations are reported to be emitted in an extremely narrow forward cone which is dependent on the energy of the incoming particle. Further it is also reported that the intensity of the radiation in the x-ray region has log γ dependence where γ is the Lorentz factor. Also it is of interest to investigate the increase in yield by using multilayer detectors. Therefore the code is designed to vary parameters such as energy of the incoming particle and the number of layers in the detector array.

A brief summary of the development of the theoretical model adopted is also discussed.

2. THEORETICAL DEVELOPMENT

Radiation yield by the passage of charged particle through a laminar medium is dependent on various parameters of the particle, such as its energy usually expressed in terms of the Lorentz factor γ where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, the charge e and detector parameters such as the number of layers N, the absorption coefficient of the medium, the spacing between detectors and other similar quantities discussed below. A brief summary of the model used in this investigation is discussed below.

When a particle with a velocity v enters from one medium described by a dielectric constant ϵ_1 and magnetic permeability μ_1 to another for which similar quantities are described by ϵ_2 and μ_2, it is reasonable to neglect the enrgy loss per unit path length compared to its kinetic energy. The field associated with the particle can be described by Maxwell's equations.

$$\nabla \times \bar{H} = \frac{1}{c}\frac{\partial \bar{D}}{\partial t} + \frac{4\pi}{c} V e \delta\,(\bar{r} - vt) \tag{1}$$

$$\nabla \cdot \bar{B} = 0 \tag{2}$$

$$\nabla \times \bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t}\;, \tag{3}$$

$$\nabla \cdot \bar{D} = 4\pi e \delta\,(\bar{r} - vt)\;. \tag{4}$$

Resolving into triple Fourier integrals, one can write

$$\bar{E}(\bar{r},t) = \int \bar{E}(k)\, e^{-i(kr-\omega t)} d\bar{k}\;, \tag{5}$$

Where

$$\bar{\omega} = k\bar{v},$$

$$\bar{D}(k) = \epsilon \bar{k},$$

$$\bar{B} = \mu \bar{H}\;. \tag{6}$$

Then the Fourier component of the fields can be written as

$$\bar{E}\ (k) = \frac{ei}{2\pi^2}\,\frac{1}{\epsilon}\,\frac{(\omega/c^2)\,\chi\bar{v} - \bar{k}}{k^2 - (\omega/c)^2}\quad , \tag{7}$$

$$\bar{H}\ (\bar{k}) = \frac{c}{c}\,\bar{v} \times \bar{E} \tag{8}$$

Here $\chi = \epsilon\mu$

From 7 and 8 $\bar{D}$ and $\bar{B}$ are easily evaluated. The quantities which refer to both media one and two may be expressed by appropriate suffix 1 or 2 in equations 6 through 8 so as to get the relevant equations for the first or second medium. It is convenient to assume that the particle is moving in the positive Z direction and also to set the interface at Z = 0. The solutions for the fields must always satisfy the physical boundary conditions to be acceptable. In this case the obvious boundary conditions are the requirement that the tangential component of $\bar{E}$ and $\bar{H}$ and the normal components of $\bar{D}$ and $\bar{B}$ be continious at the interface (Z = 0). Clearly the solutions for $\bar{E}$ and $\bar{H}$ as given in 7 and 8 do not satisfy these conditions. It is necessary to take into consideration the homogenous Maxwell's equations also and add the solutions of these to the solutions of the inhomogeneous case as given in 7 and 8 and and thus obtain a general solution and thus satisfy the required physical boundary conditions.

The solutions to the homogeneous Maxwell's equations are written in the form (Ref. 4)

$$E_h\ (\bar{r},t) = \int E_h(k)\ \exp\left\{i(\kappa\rho + \lambda\bar{Z} - \bar{\omega}t)\right\}\,d\bar{k} \tag{9}$$

Similar expression can be written for the H field also. In equation 9, p is the component of the vector $\bar{r}$ in the x-y plane and

$$\lambda^2 = (\frac{\omega}{c})^2 \chi - \kappa^2 \tag{10}$$

$$\lambda = \lambda' + \lambda'' \tag{11}$$

where λ' is the real part and λ'' is the imaginary part of λ. Obviously the first medium extends from $Z \leq 0$ to $-\infty$ and therefore the solution for the homogeneous Maxwell's equation as given in 9 will diverge as $Z \to -\infty$. This can be averted by placing the restriction $\lambda'' < 0$ for the first medium. Also from the physical fact that the radiation propogates only in the negative Z direction in the first medium it can be concluded that $\lambda'' < 0$ for the first medium. From identical consideration it is seen that $\lambda' > 0$ and $\lambda'' > 0$ for the second medium. Similar to expressions in equations 7 and 8 one can write corresponding quantities for the homogeneous case. Thus for example

$$H_h(\vec{k}) = (\frac{c}{\omega\mu}) \left[\kappa + \hat{n}\lambda) \times \vec{E}' \ (k) \right] . \tag{12}$$

Adopting the following notations

$$\eta = \frac{(\frac{\varepsilon_1}{\varepsilon_2} - \frac{v}{\omega} \lambda_1)}{(\kappa^2 - \frac{\omega^2}{C^2} \chi_1)} + \frac{(-1 + \frac{v}{\omega} \lambda_2)}{(\kappa^2 - \frac{\omega^2}{C^2} \chi_2)} \tag{13}$$

and $$\xi = \varepsilon_2 \lambda_1 - \varepsilon_1 \lambda_2 \tag{14}$$

Garibyan (Ref. 6) has shown that

$$\vec{E}'_{1_t} (\vec{k}) = \frac{ei}{2\pi^2} \kappa \frac{\lambda_1}{\xi} \eta \quad , \tag{15}$$

$$\vec{E}'_{1_n} (\vec{k}) = \frac{-ei}{2\pi^2} \frac{\kappa^2}{\xi} \eta \quad , \tag{16}$$

$$\vec{H}_1 (k) = -\kappa \frac{ei}{2\pi^2} \times \frac{\varepsilon_1 (\kappa \vec{v})}{\xi} \eta \quad . \tag{17}$$

Here t and n denote the transverse and normal components respectively. Identical expressions are obtained for medium 2 by replacing suffix 1 by 2 in equations 15 to 17.

II. Radiation from a charged particle passing through a layer of thin plates

When a thin plate of thickness a, is introduced at $Z = 0$, the region of space through which the charged particle is traversing can be considered to be divided into three sections. As before, assuming the particle to be moving in the positive Z direction, in the region of space behind the plate specified by $Z < 0$ there will be only reflected waves. In the region $0 \leq Z \leq a$ which is the region occupied by the plate there will be both reflected and forward moving waves. The region infront of the plate which is specified by $Z \geq a$, there will be waves only in the positive Z direction. Let there be N plates of thickness a, each placed parallel with a distance of b between each plate. This arrangement can also be classified into three regions. Clearly the region behind the plates where there are only reflected waves can be specified by $Z < 0$. The region ahead of the plates where $Z > Na + (Na - 1)b$, there will be only forward moving waves propogating in the positive Z direction.

The region of the plates satisfy the condition $0 \leq Z \leq Na + (N-1)b$ and in this region there will be both reflected and forward moving waves. For particles of interest here it is reasonable to assume that $\beta = \frac{v}{c} \to 1$. Further is the frequency region of interest the dielectric constant of the medium is given by

$$\varepsilon(\omega) = 1 - \frac{\sigma}{\omega^2} \quad ,$$

where

$$\sigma = \frac{4\pi n e^2}{m}$$

In this case Garibyan (Ref. 2) has given the following expression for the tangential component of the E field emitted in the forward direction for the case of N plates

$$E_{Nt}(k) = \frac{ei\varkappa\omega^2 c^{-2}\ (1-\varepsilon)}{2\pi^2\ \Lambda\Lambda_0} \quad \exp\ \left\{ i\phi'_o\ (a+b)(N-1) + i\phi'_o a \right\}$$

$$\times \left(1 - \left\{\exp\ - i\left(\frac{\omega}{v} - \lambda\right)\ a\right\}\right) \frac{1 - \{\exp\ - i\ (\phi-\chi\ )N\}}{1 - \exp\left\{-i\ (\phi-\chi\ )\right\}} \qquad (18)$$

In equation 18 the following additional notations are used

$$\Lambda = k^2 - \frac{\omega^2}{v^2}\ \varepsilon$$

$$\Lambda_o = k^2 - \frac{\omega^2}{c^2}$$

$$k^2 = \varkappa^2 + \frac{\omega^2}{c^2}$$

$$\phi'_o = \left(\frac{\omega}{v} - \lambda_o\right)$$

$$\phi = \frac{\omega}{v}\ (a + b)$$

$$\chi = \lambda_a + \lambda_o b$$

$$\lambda_o^2 = \frac{\omega^2}{c^2} - \varkappa^2$$

$$\lambda^2 = \frac{\omega^2}{c^2}\ \varepsilon - \varkappa^2$$

The radiation angle θ is related to the dielectric constant of the plates of the media through the inequality

$$\sin^2\theta + (1-\beta^2) \ll \left|\sqrt{\varepsilon} - 1\right| \qquad (19)$$

This often written in the approximation $\left|\sqrt{\varepsilon} - 1\right| \sim 1$

So that the inequality is expressed as

$$\sin^2\theta + (1 - \beta^2) \ll 1 \quad .$$

Obviously this will be satisfied in the extreme relativistic case if θ is extremely small as is expected. It remains to write down the flux of the

Poynting vector S beyond the stack of plates. This is written as

$$S_N = \frac{4\pi^2}{v^2 c} \int_0^\infty \int_0^{\pi/2} E_{N,t} (\bar{k})^2 \omega^2 d\omega \, 2\pi \sin\theta d\theta ; \tag{20}$$

Introducing the expression for $E_{N,t}$ from equation 18 and rearranging terms one obtains,

$$S_N = \frac{8e^2}{\pi c} \int_0^\infty \int_0^{\pi/2} \frac{(1-\varepsilon)^2 \sin^3 \theta d\theta d\omega}{(1-\beta^2 \cos^2\theta) [1-\beta^2(\varepsilon - \sin^2\theta)]^2}$$

$$\times \sin^2 \left[\frac{a}{2} \left(\frac{\omega}{v} - \lambda\right)\right] \left[\frac{\sin NX}{\sin X}\right]^2 \tag{21}$$

Where

$$X = \left(\frac{\omega}{v} - \lambda\right) \frac{a}{2} + \left(\frac{\omega}{v} - \lambda \frac{b}{2}\right) \tag{22}$$

Clearly, the term $\frac{\sin^2 NX}{\sin^2 X}$ can be replaced δ functions. The condition to be satisfied for this substitution is as follows.

$$\lim_{N\to\infty} \frac{\sin^2 NX}{\sin^2 X} = \pi N \quad \delta(X - \pi n) \tag{23}$$

Clearly for a finite number N of the order 10^2 or 10^3 which would be the case in an experimental situation, this condition is not strictly satisfied. However, this substitution is valid as long as that part of the integrand which does not have a finite range is a smooth function of θ. Further one can assume that in the case of transition radiations, θ will be small. Obviously reflection takes place at each boundary of between the successive plates and the gap betweeen them and this produces reflected waves in the region of the plates. However the stipulation that

$$r = (\sqrt{\varepsilon} - 1) \quad (\sqrt{\varepsilon} + 1)$$

$$= \frac{\sigma}{4\omega^2} << 1 \tag{24}$$

requires that the contribution due to reflected waves at a single plate is small compared to the forward moving wave and hence can be neglected. Even in the case of an array of N plates, N being a finite number of the order 10^2 and therefore condition

$$(rN)^2 << 1 \tag{25}$$

is clearly satisfied. Thus it is safe to assume that the contribution will be mostly due to the transmitted waves. It is also a reasonable to assume that the transmitted radiation will be confined to a narrow forward cone so that in the small angle approximation one can write

$$\sin\theta \to \theta \quad . \tag{26}$$

In addition to these assumptions, the following notations are introduced.

$$\omega_a' = \frac{4\pi v}{a(1-\beta^2)} , \tag{27}$$

$$\omega_p' = \frac{4\pi v}{p(1-\beta^2)} ,$$

$$\omega_a'' = \frac{a\sigma}{4\pi v} ,$$

$$\omega_p'' = \frac{p\sigma}{4\pi v} ,$$

$$p = a + b ,$$

$$\xi = 1 - \beta^2 ,$$

$$\eta = 1 - \beta^2 + \frac{\sigma}{\omega^2} ,$$

and

$$y = \theta^2 \quad .$$

Now equation 21 for the Poynting vector can be rewritten as

$$S_N = \frac{4\, e^2\sigma^2}{\pi c}\int_0^\infty \frac{d\omega}{\omega^4}\int_0^\infty \frac{y\, \sin^2\pi\left(\frac{\omega_a'}{\omega} + \frac{\omega}{\omega_a''} + \frac{a\omega}{4\pi v}\, y\right)}{(\xi + y)^2\,(\eta + y)^2}\, \Psi\, dy \quad . \tag{28}$$

$$\Psi = \left[\frac{\operatorname{Sin} N\pi\left(\dfrac{\omega''_a}{\omega} + \dfrac{\omega}{\omega'_p} + \dfrac{p\omega}{4\pi v}\,y\right)}{\operatorname{Sin} \pi\left(\dfrac{\omega''_a}{\omega} + \dfrac{\omega}{\omega'_p} + \dfrac{p\omega}{4\pi v}\,y\right)}\right]^2 \quad . \tag{28}$$

This can be written in a more compact form by making use of the following definitions.

$$Y = \pi\left(\frac{\omega''_a}{\omega} + \frac{\omega}{\omega'_a} + \frac{a\omega}{4\pi v}\,y\right) \tag{29}$$

and

$$X = \pi\left(\frac{\omega''_a}{\omega} + \frac{\omega}{\omega'_p} + \frac{p\omega}{4\pi v}\,y\right) \quad . \tag{30}$$

Equation 28 will now take the form

$$S_N = \frac{4e^2\sigma^2}{\pi c}\int_0^\infty \frac{d\omega}{\omega^4}\int_0^\infty \frac{y \operatorname{Sin}^2 Y}{(\xi + y)^2\,(\eta + y)^2}\;\frac{\operatorname{Sin}^2 NX}{\operatorname{Sin}^2 X}\,dy \quad . \tag{31}$$

The part of the integrand in 31 which depends on y can be written as

$$\Gamma = \frac{4e^2}{\pi c}\,\Gamma_1\,\Gamma_2 \quad , \tag{32}$$

where

$$\Gamma_1 = \frac{\sigma^2}{\omega^4}\,\frac{y}{(\xi + y)^2\,(\eta + y)^2} \tag{33}$$

$$\Gamma_2 = \operatorname{Sin}^2 Y \quad . \tag{34}$$

The remaining part of the integrand should lead to a δ function in the light of previous discussion. It is of interest here to investigate the maxima of Γ_1 and Γ_2. Treating Γ_1 as a function of y and equating the first derivative of Γ_1 to zero it is seen that the maximum of Γ_1 occurs for

$$y \sim (\eta + \xi) + \sqrt{(\eta + \xi) + 12\eta\xi}$$

Noting the definitions of η and ξ it is easily concluded that the maximum of Γ_1 occurs for

$$y \sim (1 - \beta^2) \tag{35}$$

On the other hand a similar investigation of the maximum of Γ_2 seen to occur when

$$\frac{\partial \Gamma_2}{\partial y} = 0$$

$$\text{ie,} \quad 2 \sin Y \cos Y \, d\, Y = 0$$

This obviously leads to the condition that

$$\cos Y = 0$$

$$\text{ie,} \quad \cos \left[\pi\left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega_o'} + \frac{a\omega}{4\pi v}\, y\right)\right] = 0$$

$$\text{or} \quad \pi\left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega_o} + \frac{a\omega}{4\pi v}\, y\right) = (2m + 1)\tfrac{1}{2} \tag{36}$$

Where m can only take integral values 0, 1 ....

From 36 it is easily seen that,

$$\bar{y} = \frac{4\pi v}{a\omega}\left[(2m + 1)\tfrac{1}{2} - \left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega_o'}\right)\right] \tag{37}$$

From the definition of ω_a' as

$$\omega_a' = \frac{4\pi v}{a(1 - \beta^2)} ,$$

$$\text{or} \quad \frac{4\pi v}{a} = (1 - \beta^2)\omega_a' , \tag{38}$$

and using 38 in 37 one obtains for

$$\bar{y} = \frac{\omega_a'}{\omega}(1 - \beta^2)\left[m - \frac{\omega_a''}{\omega} + \frac{\omega}{\omega_o} - \tfrac{1}{2}\right] . \tag{39}$$

From the definition of y it is seen that negative values are excluded and therefore m should start with integral values larger than

$$\frac{\omega_a''}{\omega} + \frac{\omega}{\omega_o'} - \frac{1}{2} \quad .$$

It is necessary to evaluate the width of Γ_1 and Γ_2 to assess the composite effect of these two functions which effectively enter into the calculation of Poynting Vector. The definition of Γ_1 as given in equation 33 is used to obtain the width of the function. The required condition is that at the two nearest minima

$$\Gamma_1 \ (y) = \Gamma_1 \ (y + L) \quad . \tag{40}$$

Further the first drivatives of each be zero and the second derivatives be positive. These conditions enable one to evaluate the value of L after some algebraic manipulations. It is found that

$$L \sim \ (1 - \beta^2) \tag{41}$$

A similar investigation of the zeroes of Γ_2 can be easily carried out by making use of equation 39. The zeroes should be between successive values the variable integer expressed within the square bracket. Clearly then the width of this function will be

$$\Delta \ \Gamma_2 = \frac{\omega_a'}{\omega} \ (1 - \beta^2) \quad . \tag{42}$$

When $\omega_a' > \omega$ clearly the contribution from Γ_2 and Γ_1 will over lap when this condition is satisfied. On the other hand for the region $\omega > \omega_a'$, from equation 39 it is readily seen that $\tilde{y}$, the maximum of Γ_2 will rapidly fall off and there will not be a sharp peak in the contribution from Γ. Thus the frequency restriction is stipulated through the condition

$$\omega_a' >> \omega \quad . \tag{43}$$

It is only the region which satisfies condition 43 which contributes to the integrand in equation 31. The frequency ω' which satisfies condition 43 can be denoted by $\bar{\omega}_s$ where $\bar{\omega}_s$ can be made to satisfy the required inequality by defining

$$\bar{\omega}_s = \frac{\omega_a''}{(m+\frac{1}{2})}, \qquad 44$$

and from the definition of ω_a'' and ω_a' in equation 27 clearly

$$\bar{\omega}_s << \omega_a' . \qquad 45$$

From 27 one can then write

$$\frac{\omega_a''}{\omega_a'} = \frac{a^2\sigma(1-\beta^2)}{(4\pi v)^2}. \qquad 46$$

Then from 44

$$\bar{\omega}_s' = \frac{\omega_a'}{(m+\frac{1}{2})} \frac{a^2\sigma(1-\beta^2)}{(4\pi v)^2} . \qquad 47$$

$$\text{Since, } \bar{\omega}_s' < \omega_a',$$

in order that the equality in 46 holds true clearly,

$$1 > \frac{a^2\sigma(1-\beta^2)}{(m+\ )\ (4\pi v)^2},$$

$$\text{or } \frac{(m+\frac{1}{2})}{(1-\beta^2)} > \frac{a^2\sigma}{(4\pi v)^2}. \qquad 48$$

When this inequality is satisfied the intensity of the radiation will be a maximum.

Thus one is left with the evaluation of the contribution of the term $\frac{\sin^2 NX}{\sin^2 X}$ in equation 31 for the Poynting Vector. Clearly the maximum of this function will occur at

$$X = \pi n .$$

Let the corresponding value of y be denoted by y_n.

Then from equation 30.

$$\pi n = \pi\left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega' p} + \frac{p\omega}{\pi} y_n\right)$$

and then

$$y_n = \frac{4\pi v}{p\omega}\left(n - \frac{\omega_a''}{\omega} - \frac{\omega}{\omega' p}\right) \quad . \qquad 49$$

It is possible that $\left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega' p}\right)$ may not be an integer. This can be made into the nearest larger integer by defining the complement $d(\omega)$ where,

$$0 \leq d(\omega) \leq 1 \quad .$$

Then the nearest integer can be written

$$n_{min} = \frac{\omega_a''}{\omega} + \frac{\omega}{\omega' p} + d(\omega) \quad . \qquad 50$$

When $n = n_{min}$ in equation 50, the corresponding value of y_n will be zero. In order to obtain successive maxima, then n must be written as,

$$n = n_{min} + k \ , \qquad 51$$

$$\text{where } k = 0, 1, 2......,$$

or equivalently from 50 and 51

$$n = \left(\frac{\omega_a''}{\omega} + \frac{\omega}{\omega' p}\right) + d(\omega) + k. \qquad 52$$

Therefore it is necessary to rewrite equation 48 which gives for maximum due to this part of the integral for S_N as

$$y_k = \frac{4\pi v}{p\omega}\left[\left(\frac{\omega}{\omega' p} + \frac{\omega_a''}{\omega}\right) + d(\omega) + k - \frac{\omega_a''}{\omega} - \frac{\omega}{\omega' p}\right]$$

$$= \frac{4\pi v}{p\omega}\left(d(\omega) + k\right) \qquad (53)$$

From 27

$$\frac{4\pi v}{p} = (1-\beta^2)\ \omega' p \qquad (54)$$

Thus

$$y_k = \frac{\omega' p}{\omega} \quad (1-\beta^2) \quad \left[d(\omega) + k\right] \quad . \tag{55}$$

The distance between successive maxima are given by

$$\Delta = y_{k+1} - y_k = \frac{\omega' p}{\omega} \quad (1-\beta^2) \left\{ \left[d(\omega) + k + 1\right] - \left[d(\omega) + k\right] \right\}$$

$$= \frac{\omega' p}{\omega} \quad (1-\beta^2) \quad . \tag{56}$$

The width Δs of a particular maxima of this part of the function the value of $y = y_k + \Delta s$ is used in equation 27 for X.

$$X = \pi \left[\frac{\omega a''}{\omega} + \frac{\omega}{\omega p'} + \frac{p\omega}{4\pi v} \left(y_k + \Delta s\right) \right]$$

$$\text{and } \Delta s = \left[\frac{X}{\pi} - \frac{\omega a''}{\omega} - \frac{\omega}{\omega_p} \right] \left[\frac{4\pi v}{p\omega} \right] - y_k$$

$$= \left[\frac{X}{\pi} - \frac{\omega a''}{\omega} - \frac{\omega}{\omega_p} \right] \frac{4\pi v}{p} - \frac{4\pi v}{p\omega} \left[d \; (\omega) + k \right] .$$

$$= \frac{4\pi v}{p\omega} \left[\frac{X}{\pi} - n \right] \quad , \tag{57}$$

where the defenition of y_k in terms of $d(\omega)$ and k as given in equation 53 is used. From 57 it is readily seen that

$$\frac{p\omega}{4\pi v} \Delta s = \left[\frac{X}{\pi} - n \right] . \tag{58}$$

In order to evaluate Δs the stipulation is made that equation 58 has an order of magnitude of $\frac{1}{N}$.

Thus,

$$\frac{p\omega}{4\pi v} \Delta s \sim \frac{1}{N} \quad . \tag{59}$$

From the defenition of ω'_p as given in 27 it is readily seen

$$\frac{\omega'_p}{\omega} = \frac{4\pi v}{p\omega(1-\beta^2)} \quad . \tag{60}$$

Then from 59 and 60

$$\Delta s = \frac{1}{N} \frac{4\pi v}{p\omega}$$

$$= \frac{\omega' p}{\omega} \frac{(1-\beta^2)}{N} \quad . \tag{61}$$

In order to transform $\frac{\mathrm{Sin}^2 NX}{\mathrm{Sin}^2 X}$, it is necessary to require that the width of the maxima due to this part be less than the width of the maxima obtained from the analysis of Γ_1 and Γ_2. Thus the conditon

$$\Delta S << \Delta \Gamma_2 \tag{62}$$

or

$$\frac{\Delta S}{\Delta \Gamma_2} << 1 \quad , \tag{63}$$

is imposed. It is already shown in 42 that

$$\Delta\Gamma_2 = \frac{\omega' a}{\omega} (1-\beta^2)$$

$$\Delta S = \frac{\omega' p}{\omega} (1 - \beta^2) \frac{1}{N}$$

Dividing one by the other

$$\frac{\Delta S}{\Delta \Gamma_2} = \frac{\omega' p}{\omega\ a} \frac{1}{N} \tag{64}$$

Combining 63 and 64

$$\frac{\omega' p}{\omega' a} \frac{1}{N} << 1 \quad ,$$

or

$$N >> \frac{\omega' p}{\omega' a} \quad . \tag{65}$$

Depending upon the frequency region chosen by the londitions $\omega >> \omega' a$ and $\omega' a > \omega$ as set in equation following 42 and in 43, it is possible to investigage the restrictions on N. Condition 62 leads to 65 which is no restrictrion on N at all. On the otherhand if the frequency region of interest is such that condition 43 is to be satisfied, N has to satisfy the inequality

$$N >> \frac{\omega' a}{\omega} \tag{66}$$

In addition to this restriction on N as given by equation 66, N should also satisfy the condition laid out in equation 25,

$$(rN)^2 << 1.$$

This earlier restriction has come about from the fact that in an actual experimental situation N, the number detector plates will be a finite quantity. These two limiting factors lead to an interesting restriction on ω'_p as discussed below.

From equation 24,

$$r = \frac{\sigma}{4\omega^2} << 1 \tag{67}$$

From 25

$$N^2 << \frac{1}{r^2},$$

$$N << \frac{1}{r} \tag{68}$$

Combining 65 and 66 it is seen

$$N << \frac{4\omega^2}{\sigma}. \tag{69}$$

However, from the requirement

$$\omega \leq \omega'_a \tag{70}$$

when combined with 66 leads to

$$\frac{\omega'_p}{\omega'_a} << \frac{\omega_p'}{\omega}. \tag{71}$$

Combining 66, 68 and 71

$$\frac{\omega'_p}{\omega'_a} << \frac{\omega_p'}{\omega} << \frac{4\omega^2}{\sigma}, \tag{72}$$

or

$$\frac{\omega'_p}{\omega'_a}\,\omega << \omega'_p << \frac{4\omega^3}{\sigma}. \tag{73}$$

From definitions in 27

$$\frac{\omega'_p}{\omega'_a} = \frac{a}{p} \quad . \tag{74}$$

Further from 24

$$\frac{\sigma}{4\omega^2} \quad << 1 \ .$$

or

$$\frac{\sqrt{\sigma}}{2} \quad << \omega \tag{75}$$

When 74 and 75 are used in 71 the inequality becomes

$$\frac{a}{p}\frac{\sqrt{\sigma}}{2} \quad << \omega'_p << \frac{4\omega^3}{\sigma} \quad . \tag{76}$$

The indicated sum of δ functions from the integrand for the Poynting Vector is now carried out as follows

$$\frac{\mathrm{Sin}^2\ NX}{\mathrm{Sin}^2\ X} = N \sum_{k=0}^{\infty} \Delta . \delta(y-y_k) \tag{77}$$

Where y_k is given by 55 and Δ by 56.

Now it remains to calculate Poynting vector as given in equation 31 which can be written as (78)

$$S_N = \frac{4e^2\sigma^2}{\pi c} \int \frac{d\omega}{\omega 4} \int_0^{\infty} \frac{\mathrm{Sin}^2\ (R + Ty)}{(\xi + 4)^2\ \ (\eta + y)^2} \quad dy \quad N \sum_{k=o}^{\infty} \Delta . \delta\ (y-y_k) \ ,$$

where

$$R = \frac{\sigma}{(1-\beta^2)\omega\omega''_a} + \omega \ ,$$

$$T = \frac{a\omega}{4\pi v}$$

and Δ and y_k are already defined. The integration on y is rather tedious, however when carried out leads to

$$S_N = N \frac{4e^2}{\pi c} \quad \omega''_p \int \frac{d\omega}{\omega^2} \sum \quad (k+d) \frac{\mathrm{Sin}^2 \left[\frac{\pi a}{p} \left(k+d+ \frac{\omega p''}{\omega} + \frac{\omega}{\omega p'}\right)\right]}{\left(k+d+ \frac{\omega p''}{\omega} + \frac{\omega}{\omega p'}\right)^2 \ \left(k+d+ \frac{\omega}{\omega p'}\right)^2} \tag{79}$$

Garibyan (Ref. 5) has shown that the number of quanta beyond a stack of N plates is given by

$$\frac{dN}{d\omega} = N(\omega)\ \frac{4e^2}{\pi c}\ \frac{\omega''}{\omega^2}p^2 \sum_{k=0}^{\infty} \cdot \frac{(k+d)\ \sin^2\left[\frac{\pi a}{p}\left(k+d+\frac{\omega''}{\omega}p+\frac{\omega}{\omega' p}\right)\right]}{\left(k+d+\frac{\omega''p}{\omega}+\frac{\omega}{\omega' p}\right)^2\left(k+d+\frac{\omega}{\omega' p}\right)^2} \tag{80}$$

where

$$N(\omega) = \frac{1-e^{-\mu a N}}{1-e^{-\mu a}} \tag{81}$$

In the above equation μ is the x-ray absorption coefficient for the detector material.

Computer Code Trax uses equation 80 to determine the x-ray photons emitted. The various quantities occuring in the equation are defined in the previous discussion. The listing of code is given in the appendix.

A sample problem is run, for a parallel plate geometry using 100 mylar detectors. The x-ray absorption coefficient for Mylar was taken to be

$$\mu = \frac{3062}{E^3} - \frac{934}{E} + 0.29\ (1-392.1^{-3}\ E + 2.1^{-5}\ E^2)$$

where E is the x-ray energy in keV. In the sample problem energies for 3 to 9 GeV were used for incoming electrons. The results are shown in Fig. 1 and 2.

Higashi etal have measured the x-ray production and have given the results for 3 and 8 GeV electrons. They have observed a transition radiation peak at about 2.2 keV in the case of mylar. They have reported that though theory predicts emission of transition radiations to much lower energies such radiations were not observed probably due to absorption in the detector. Further it is also reported that they have not been able to observe a linear increase in intensity with increasing γ. On the other hand Prince etal (Ref. 6) have reported that they have observed the yield of transition photon to rise steeply with energy up to about 5 GeV and that it reaches saturation between 5 and 10 GeV.

E_γ in keV	$\frac{dN}{d\omega}$
0.32	.0149
0.34	10.088
0.36	97.73
0.38	268.61
0.40	371.3
0.42	349.43
0.44	243.91
0.46	132.76
0.48	83.5
0.50	42.18
0.52	20.52
0.54	10.26
0.56	4.89

TABLE - 1. X-ray Photon Production for 3 GeV electrons in a parallel stack of 100 Mylar detectors.

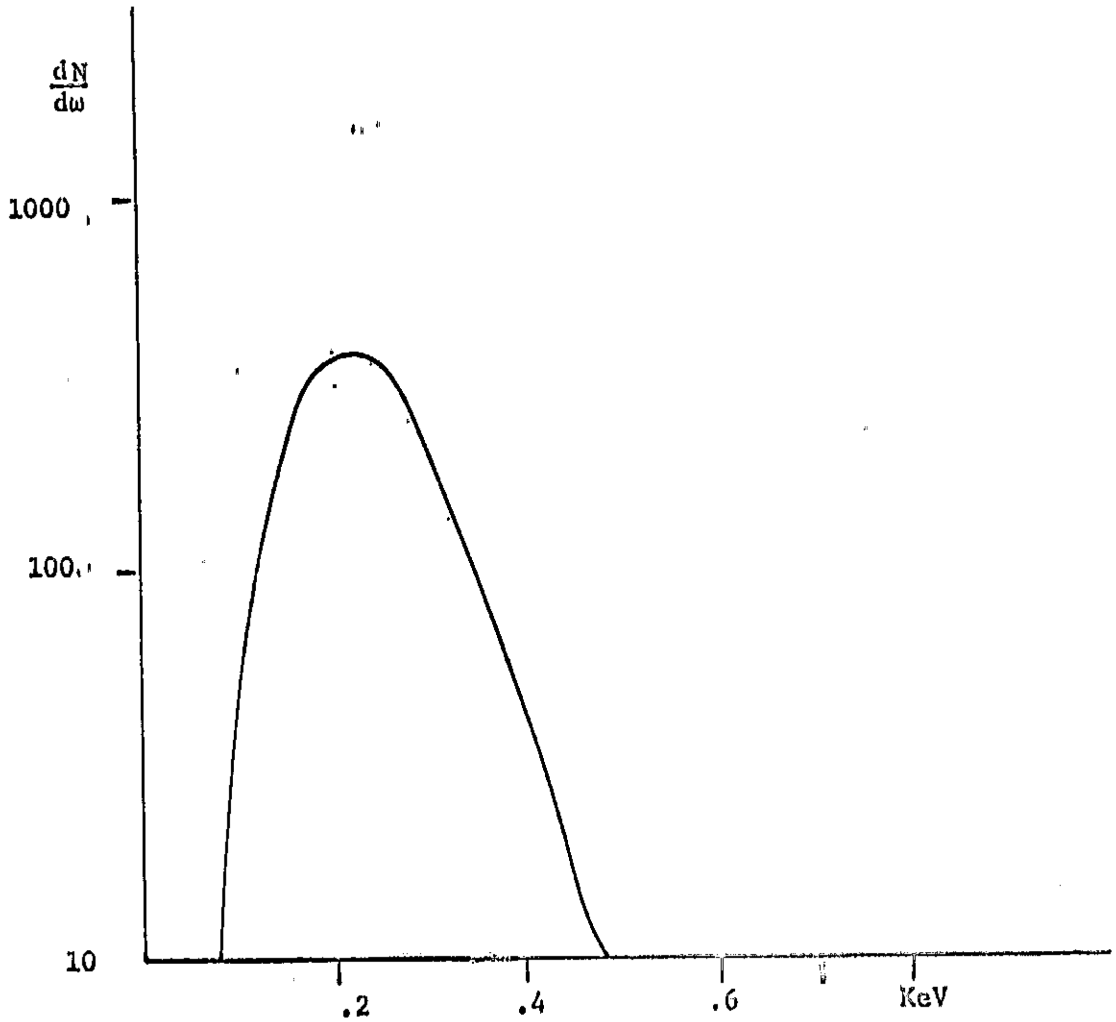

Fig. 1

Photon energy spectrum for a stack of 100 mylar detectors for 3 GeV Electrons.

E_γ in keV	$\frac{dN}{d\omega}$
0.2	2500
0.25	890
0.3	200
0.35	54.4
0.4	420.2
0.5	53.6
0.55	8.6
0.6	1.8
0.65	1.3
0.70	0.6
1.0	0.3
1.2	0.04
1.6	0.02
2	0.01
2.2	0.08
2.4	0.1
2.6	0.12
2.8	0.09
3	0.04

TABLE - 2. X-Ray Photon Production for 5 GeV electrons (Photon range from .2 to .3 keV) using a stack of 100 mylar dectectors

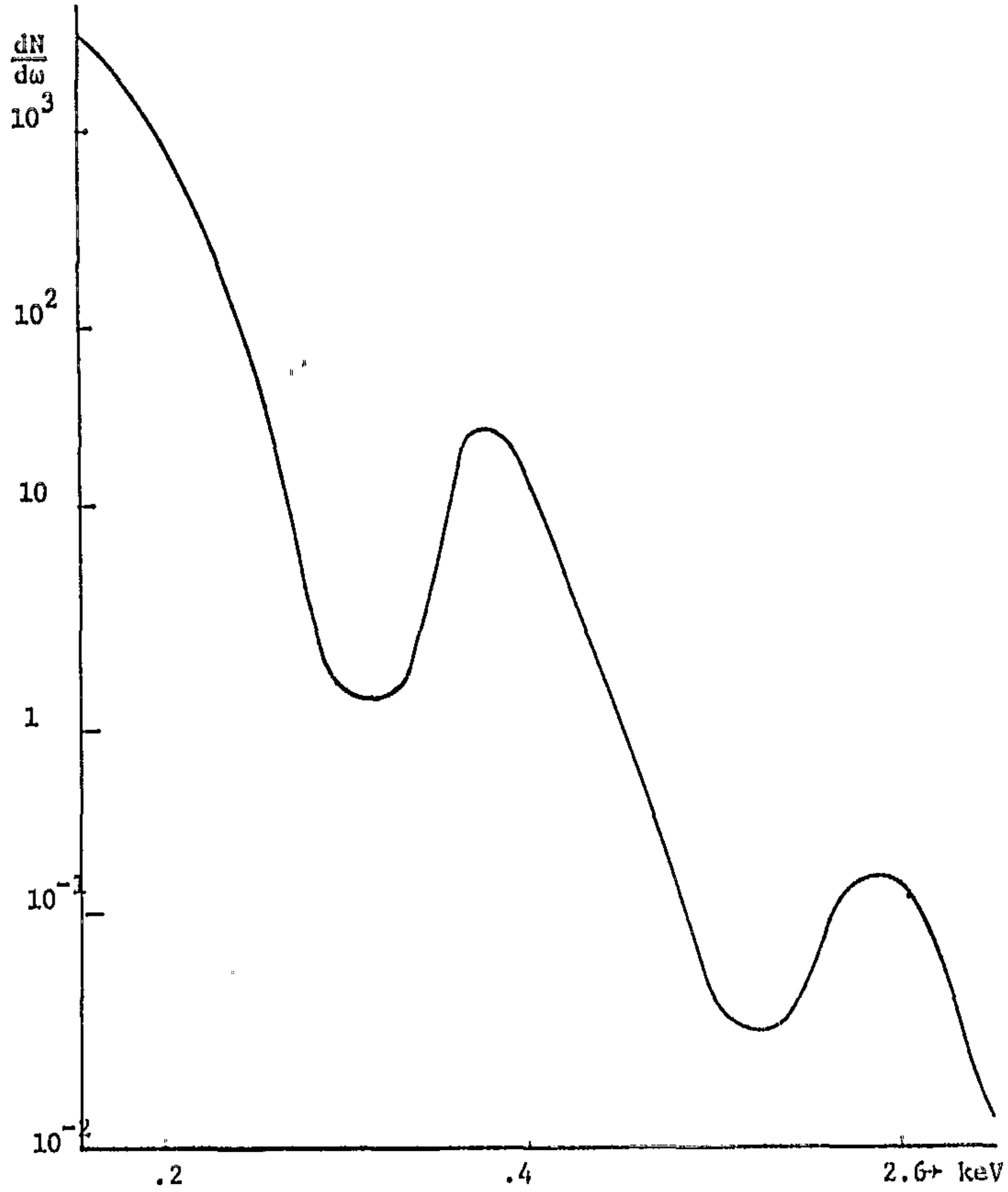

Fig .2

Photon energy spectrum for a stack of 100 mylar dectectors

for 5 GeV electrons

APPENDIX

COMPUTER CODE TRAX

```
      REAL N,M,EV,E1,NW,MU
      DOUBLE PRECISION OMEGA,SIGMA,OME2A,OME2P,OMEGAP,D,Y,X,S,T,SUM,DNDW
     1,Z
      PI=3.1415927                                                    1
      EV=0.1
      ETA=100
      K=1                                                             2
      READ(1,3) N,M,F,A,B,P
    3 FORMAT (F7.1/F8.2/F10.4/F8.2/F9.3/F9.3)
    2 FORMAT (' ',F12.6)
      EP=3
    5 E1=EP*10.0**3
    6 GAMMA=E1/.51
      V=(3.0*10.0**8.0)*((1-(.51/E1)**2)**.5)
   14 MU=(3062.0/EV**3.0)-(934.0/EV**4.0)+0.29*(1.0-3.92*10.0**(-3.0)*EV
     1+2.0*10.0**(-6.0)*EV**2.0)
      J=0
      R=0
      NW=0
      IF (J) 19,19,17
   19 A1=1
      B1=1
   10 IF(J-2) 15,18,18
   15 R=1
      IF  (J) 39,39,38
   17 J=J+1
      GO TO 10
   18 R=J*R
      GO TO 38
   38 A2=(-1)**J*((MU*A*ETA)**J)/R
      A1=A1+A2
      B2=(-1)**J*((MU*A)**J)/R
      B1=B1+B2
   39 NW=NW+(A1/B1)
   40 IF (J-10) 17,4,4
    4 OMEGA=EV*(1.6/1.0545)*10.0**18
      SIGMA=(4.0*PI*N*(F**2))/M
      OME2A=(A*SIGMA)/(4.0*PI*V)
      OME2P=(P*SIGMA)/(4.0*PI*V)
      OMEGAP=(4.0*PI*V*GAMMA**2.0)/P
      D=((OME2A/OMEGA)+(OMEGA/OMEGAP)) - ((OME2A/OMEGA)+(OMEGA/OMEGAP))  10
      Y=OME2P/OMEGA
      Z=OMEGA/OMEGAP
      X=SIN(PI*(A/P)*(K+D+(OME2P/OMEGA)+(OMEGA/OMEGAP)))**2
      S=(K+D+(OME2P/OMEGA)+(OMEGA/OMEGAP))**2
      T=(K+D+(OMEGA/OMEGAP))**2
      SUM=0.0                                                         11
      DO 25 K=1,200
      SUM=SUM+((K+D)*(SIN(PI*(A/P)*(K+D+Y+Z)))**2)/
     1((K+D+Y+Z)**2)*((K+D+Z)**2)
   25 CONTINUE                                                        14
      DNDW=(OME2P**2/OMEGA**3)*SUM
      DNDW2=NW*DNDW
      WRITE (3,30) E1,EV,DNDW2
```

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```
 30 FORMAT(' ',3(F12.6,3X))
    EV=EV+0.01
    IF(1.0-EV)27,27,14
 27 EV=0
    EV=EV+0.1
    IF (EP-12.0) 52,55,55
 52 EP=EP+1
    GO TO 5
 55 STOP
    END
```

VARIABLES

M - Mass of Particle
EV - Energy of Photon in keV
E1 - Energy of Particle in GeV
N - Electron density per cubic centimeter
MU - Photon absorption coefficient in detector material
ETA - Number of detectors

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