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Annual Progress Report

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Prepared by

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During this reporting period under the step funding provision of the Grant, manpower was reduced and reached a level of 25% of the principal investigator in February 1975. There has been a corresponding reduction in output. The main emphasis of the activity has been placed upon evaluating and testing the most recent method of calculation of the performance index, application to various example designs, and programming for the discrete data control system case. The specific items are summarized in the paragraphs that follow. The detail descriptions are to be contained in a Contractor Report, the writing of which will form a major part of the activity under the Grant in the coming year.

1. Computation of the performance index:

Due to the inaccuracies that were introduced by digital word length round-off and by the matrix inversion routines, an alternative method of computing the performance index was developed. This involves the direct integration of the performance index and makes use of the partial fraction expansion of the Laplace transform of the excitation input to the error state. This technique requires accurate knowledge of the poles and zeroes of that transform so that accurate values of residues can be obtained. This in turn made it necessary to spend effort modifying that portion of the program, and finally a program package for finding the eigenvalues of matrices prepared at the Argonne Laboratory was adopted. In the examples investigated to this date, the performance index calculation has exhibited acceptable accuracy. A brief development of the new computational technique is presented in Appendix A.

2. Corresponding changes were made to the computer program for the discrete design case. That program has not been tried out to the extent that the program for the continous systems has. There remain several areas that need to be modified. It is not anticipated that these involve major programming efforts, but rather involve changes that are needed to expedite the manner in which data is presented in the input description. Since both continuous and discrete sub-systems need to receive input data description, the input requirements should be simple and easy to implement to avoid input data errors.

The discrete program has been checked out using the same configuration that was evolved for an analog C^* system. Figure 1 presents a mathematical block diagram for the system. The airplane and elevator servo are analog, and the rest of the control

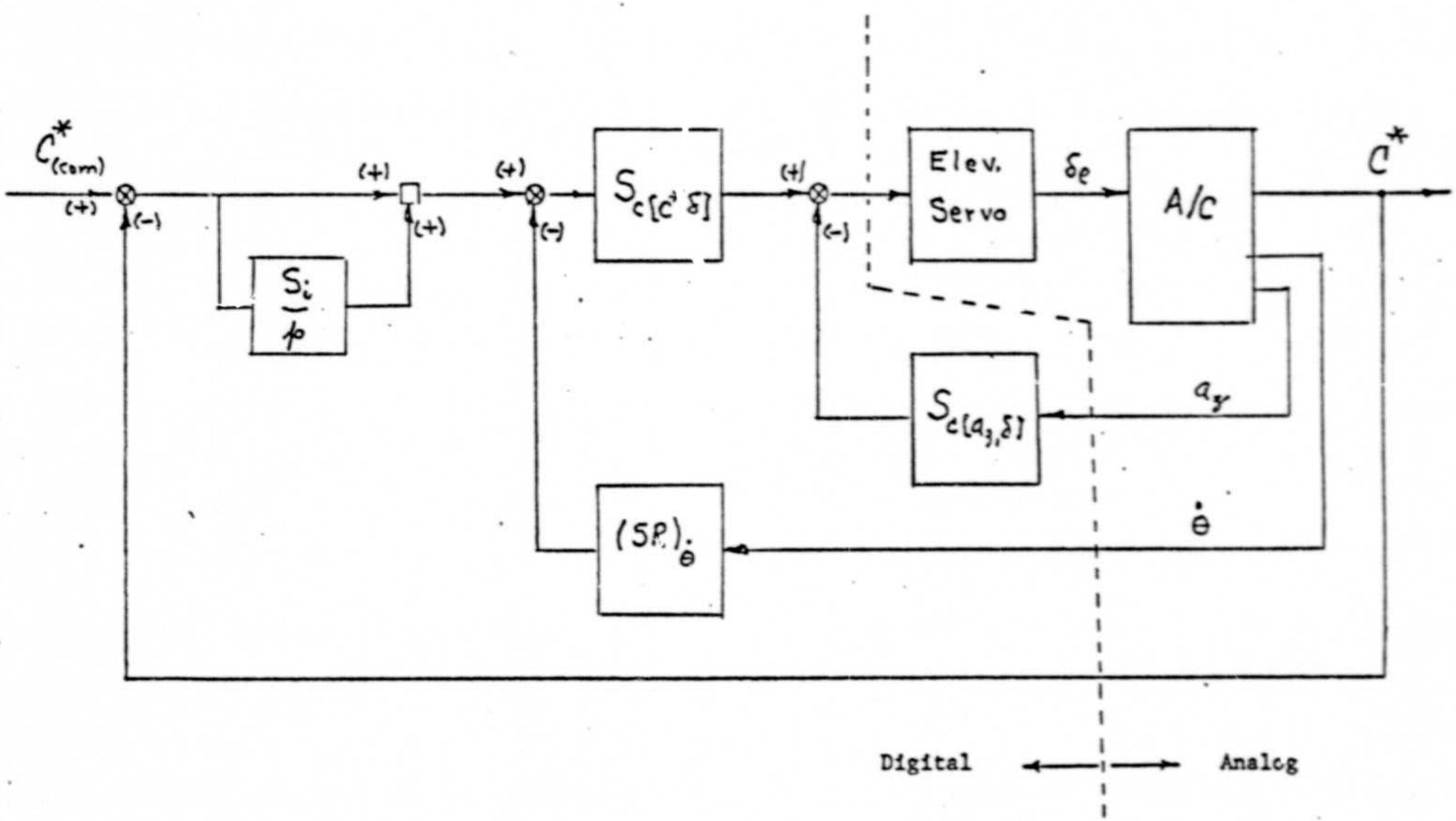
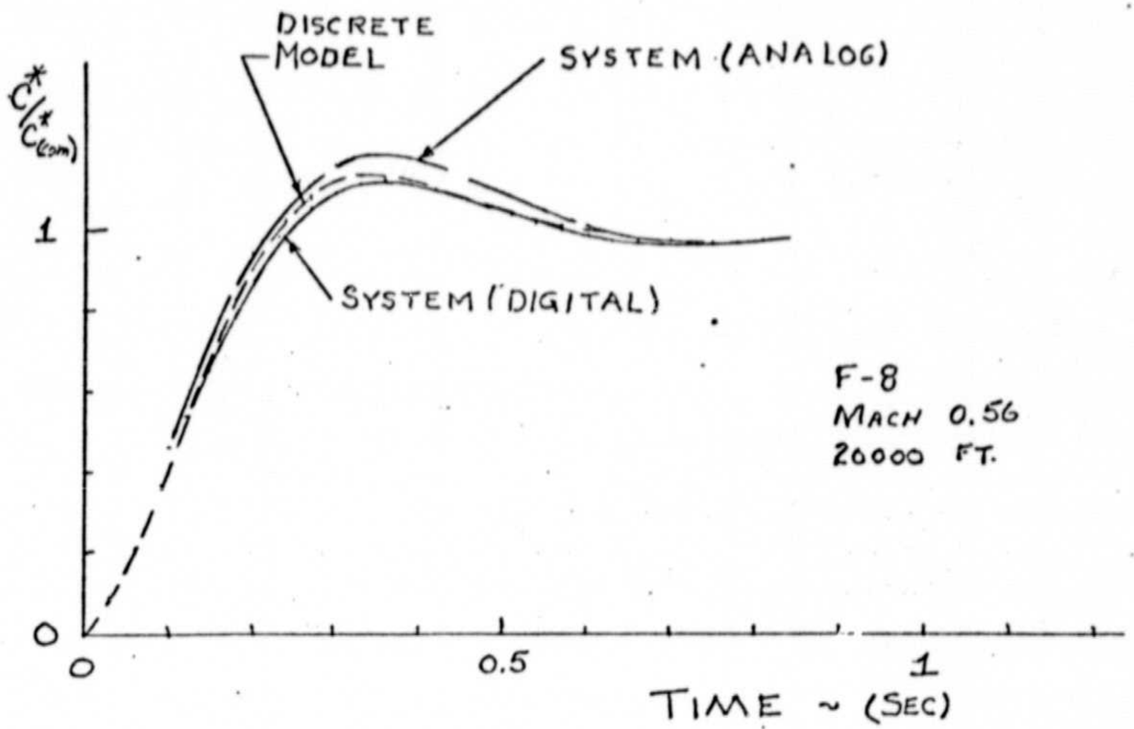


Fig. 1. Mathematical Block Diagram of an Aircraft C* Control System



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$$G_m(z) = \frac{0.08130\, z}{(z - p_1)(z - \bar{p}_1)}$$

$$p_1 = 0.8168 + 0.2185j$$

$$C^* = -0.03106\, a_z + 10.062\, \dot{\theta}$$

Fig. 2. Step C* Command Response

system configuration is provided by the digital section. The C* error signal is formed and fed to the elevator. Inner loop feedbacks of pitch rate, $\dot{\theta}$, and normal acceleration, a_z, are provided to give a closed loop static sensitivity of unity. Figure 2 presents the step function C* response. The optimization of the continuous system indicated that the pitch rate feedback sensitivity ratio was insensitive, and for this preliminary check of the digital program that gain was kept constant. The variable parameters were chosen to be the forward loop static sensitivity, $S_{c[C^*,\delta]}$, the acceleration feedback, $S_{c[a_z,\delta]}$, and the integrator sensitivity, S_i. The model response shown in Figure 2 is that of the discrete model which differed from the continuous model by less than 5% due to the difference between an analog and digital representation. The model corresponds to a second order lag with ω_n = 9.5 rad/sec and ζ = 0.5. Table 1 summarizes the parameter values that resulted. The optimization was terminated

Table 1

Parameter	Analog System	Discrete System
$S_{c[C^*,\delta]}$ (Deg/g)	2.02	1.84
$S_{c[a_z,\delta]}$ (Deg/g)	-0.89	-1.13
S_i $\sec^{-1}$	0.504	0.504

when the parameters become insensitive. It may be that even closer agreement would be found at the mathematical optimum.

Figure 2 indicates that such variation is insignificant however in a practical design.

3. Search algorithm: The algorithm for finding the minimum of the performance index is a straightforward gradient search. Some control of step size is incorporated, and effort was expended in improving the convergence characteristics as more design examples have been investigated. No algorithm seems to be optimum for all cases, but the present version appears to be a practical compromise. Appendix B presents a summary of the algorithm.

4. Lateral control system studies: As an example of a more complicated control system configuration, an investigation of a rudder coordination system for the F-8 was undertaken. One can select several different indications of desired coordination: (a) the minimization of the excitation of the lateral oscillation for aileron inputs, (b) minimizing sideslip by causing the yaw stability axis component of angular velocity to be proportional to roll angle during the entry into a constant bank angle turn, or (c) minimizing the average squared sideslip during turn entries. Cases (a) and (b) involve different model choices, and case (c) involves a slightly different performance index. Since the presently used version of the performance index computation evaluates an expression of the form

$$(PI) = \int [i(t)]^2 dt$$

where the Laplace transform of i(t) is known in pole-zero form, the use of

$$(PI)_\beta = \int [\beta(t)]^2 dt$$

is a minor modification requiring only that β rather than i be taken as the output quantity. When case (b) was examined, the closed-loop zeroes of the yaw rate transfer function apparently cause a large initial transient effect that leads to numerically high values of the minimum performance index. This seems to be unsatisfactory and needs to be investigated more thoroughly. Otherwise all three examples gave satisfactory results.

Planning

The next year of this Grant will be devoted to writing the final report and to more thorough examination of the discrete design case. During the latter, emphasis will be placed upon the effect of sampling frequency, and preliminary work on this has already begun.

Appendix A. Computation of the Performance Index

Reference:

Palsson, T., Parameter Uncertainties in Control System Design, Measurement Systems Laboratory Report TE-46, M.I.T., May 1971.

Section 3.5 of the referenced report shows that the Model Performance Index is directly equivalent to

$$(PI) = \int [i(t)]^2 dt$$

where i(t) is the excitation input to the dynamic model of the error states. Referring to figure A-1, the error state can be obtained in two ways, giving the two expressions

$$\Delta y = y - y_m = (G - G_m)u$$

$$\Delta y = G_m i \qquad\qquad u(p) = \frac{1}{p}$$

Solving for i

$$i = \left[\frac{G}{G_m} - 1\right] \cdot u, \qquad\qquad i(p) = \sum_{i=1}^{n+k} R_i/(p-p_i)$$

where p_i are the $(n + k)$ poles obtained from the system poles and the model zeroes; and R_i are the corresponding residues. Then

$$i(t) = \sum_{i=1}^{n+k} R_i e^{p_i t}$$

$$(PI) = \int_0^{\infty} \sum_{j=1}^{n+k} \sum_{i=1}^{n+k} R_i e^{p_i t} R_j e^{p_j t}$$

$$(PI) = - \sum_{j=1}^{n+k} \sum_{i=1}^{n+k} \frac{R_i R_j}{p_i + p_j}$$

$$\text{Real } (p_i) < 0$$

Thus by obtaining the partial fraction expansion of i(p) one can evaluate the performance index explicitly. This also requires having the eigenvalues of the augmented system.

Appendix B. Currently Used Search Algorithm for the Parameter Optimization Program 7 August 1975

This memo summarizes the algorithm used to search for the minimum of the performance index as of the modification made up to 22 July 1975.

1. The changes made to the parameters at the end of an iteration are proportional to the gradient of the performance index. The proportionality factor, SGRD, is initially taken to be 10.0. The predicted change to the performance index is calculated using the parameter increments, and a correction factor, CORR, is calculated which is the ratio of the desired fractional change in the performance index to the predicted fractional change. In effect the proportionality factor is then corrected by multiplying by CORR. If the first order prediction were accurate, the actual fractional change in the performance index on the next iteration would be equal to the desired change.

2. If a parameter has reached a point at which its gradient changes sign, its contribution to the reduction of the performance index is reduced by the step factor, STF, which is generally equal to or less than one. The gradient step change is multiplied by STF. When a parameter's gradient changes sign, its step factor is reduced to 0.6 of its previous value. Thus if a parameter is alternately stepping across its optimum value, its permitted change decreases rapidly. To allow for the case for which the

gradient changes sign as a result of other parameters reaching a more favorable point in parameter space, the step factors are allowed to recover by increasing them by 15% when successive iterations result in the same sign of the gradient. The increase is not applied if the step factor is 1.0 or greater.

3. When the gradients are small, the predicted change in parameters needed to obtain a desired change in the performance index may be large. The correction factor is further modified so that no parameter will change by more than 200% in any given iteration.

4. After the resulting parameter changes have been calculated, the parameters are updated, and any parameter constraints applied.

5. The search algorithm also provides for automatic adjustment of the called for change in the performance index on any iteration. This takes place by modifying the parameter DELTA whose initial value is read as part of the input data. DELTA is the desired fractional change in (PI) in a given iteration. Since (PI) seldom varies linearly with a parameter value, the linear prediction is good only far from the optimum point where the slope changes slowly. By modifying DELTA one can slow the parameter change as the minimum is approached and cut down the tendency to overshoot the minimum. The success ratio, SR, is defined as the ratio of the actual change in (PI) to the predicted change. Typically a success ratio of 0.5 is good. The program increments

DELTA by ±20% whenever SR is outside the range 0.4 to 0.8. This is done in the OPT1 subroutine.

6. When the minimum is approached, large changes in (PI) are not possible. If one has specified a value other than zero for the reference (PI) value, DELTA is further changed so that the called for change in (PI) would not be greater than one that changes (PI) to the reference value. This again prevents excessive parameter changes near the optimum point. It may not be necessary however.

7. Further logic affecting the search is provided in the OPT1 subroutine. If the parameter change causes the system to become unstable, all parameter changes are decreased to 40% and the iteration re-started. If the parameter changes cause the performance index to increase over the value on the previous iteration, the changes are reduced by 70%. If this still results in increased (PI), the maximum parameter change is reduced to 5%. If this still results in passing the minimum, the search terminates since a 5% change in the parameters should be within practical tolerance bounds on the design parameters. One is cautioned however that if one parameter (or more perhaps) is very insensitive in this region of parameter space at which the search terminates, a large change in the insensitive parameter may take the process to a region at which further optimization of the other parameter can result in a significantly lower value of the (PI). The current program leaves it to the engineer to use his judgement to restart the parameter

if he feels further optimization is necessary rather than letting the gradient search proceed in very small increments using excessive computation time.

8. Various stopping conditions are used in the OPT1 subroutine. One can specify (PIREF) in the input data as a value of (PI) that is acceptably small and stop the search when it is reached. If the change in (PI) is less than 5% on three neccessive iterations, the search terminates on the assumption that the DELTA modifications are indicating that the minimum point has been reached. It also terminates on reaching the specified maximum number of iterations. This can be used to calculate (PI) for given parameters values by inputting ITMAX = 0. If the normalized slope is less than 10% of the specified reference (PI), it would take a ten-fold parameter change to make a change in (PI) equal to the reference (PI). That is an indication of acceptably small gradients and hence that the minimum has been found, so the search is terminated.