BENEFIT-COST METHODOLOGY STUDY WITH EXAMPLE APPLICATION OF THE USE OF WIND GENERATORS

By
R. P. Zimmer
C. G. Justus
R. M. Mason
S. L. Robinette
P. G. Sassone
W. A. Schaffer

July 1975

Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia


## U.S. Department of Energy

## NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

This report has been reproduced directly from the best available copy.

Available from the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22161.

Price: Paper Copy $\$ 13.25$
Microfiche $\$ 3.00$

# BENEFIT-COST METHODOLOGY STUDY WITH EXAMPLE APPLICATION OF THE USE OF WIND GENERATORS 

R. P. Zimmer, Project Director
C. G. Justus, R. M. Mason, S. L. Robinette, P. G. Sassone, and W. A. Schaffer

Engineering Experiment Station
Georgia Institute of Technology
Atlanta, Georgia 30332

July 1975

Prepared for the
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135
Contract NAS 3-17827
and the
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION
Division of Solar Energy
Federal Wind Energy Program


[^0]
## FOREWORD

The "Benefit-Cost Methodology with Example Application of the Use of Wind Generators" under Contract NAS $3-17827$ was conducted by the Engineering Experiment Station (EES) at Georgia Tech in coniunction with the Schools of Industrial Management (IM) and Aerospace Engineering (AE). The program was administered under Georgia Tech Project A-1632 by the Systems Analysis Technical Area within the Systems and Techniques Department.

This report describes the work performed during the period June 1974 through May 1975. The program was managed by the NASA/Lewis Research Center Space Flight Systems Study Office and directed to be responsive in its example application to areas of interest to the Solar Energy Branch. The NASA Program Manager was Gerald F. Hein.

The Georgia Tech Project Director was Robert P. Zimmer and the project team was comprised of the following key personnel:

| C. G. Justus (AE) | Wind Data Analysis |
| :--- | :--- |
| R. M. Mason (EES) | Applications |
| S. L. Robinette (EES) | Applications |
| P. G. Sassone (IM) | Cost-Benefit Methodology |
| W. A. Schaffer (IM) | Cost-Benefit Methodology |

Special acknowledgment is due Gerald Hein, NASA Program Manager, whose timely guidance and assistance were so important in accomplishing the program ob jectives.

Many techniques associated with cost-benefit analysis (CBA) exist in the literature. Which technique(s) to use in evaluating competing technological alternatives is not always obvious and given the appropriate technique it is usually not clear how to use it or even to the extent to which it can be used. This program emphasized the assessment of the status of cost-benefit analysis methodology, the compilation of cost-benefit methodology, and the application of cost-benefit methodology to wind power systems.

A thorough review of literature dealing with cost-benefit methodologies or analyses was made; journal articles, books, and other sources were reviewed and cost-benefit analysis concepts were assessed. The most pertinent concepts are identified and explained herein and a logically consistent, explicit, yet flexible methodology developed for the performance of cost-benefit analyses is presented. A major consideration throughout the program was the desirability that any methodology developed as a decision tool should achieve a high level of acceptance among policy makers and analysts.

The cost-benefit methodology investigation described herein deals with the following topics description, origin, and use of CBA; basis of CBA in value theory and welfare economics; survey and critique of decision criteria, structure of decision problems, and matching the criteria to the structure; identifying, classifying, and measuring costs and benefits; shadow pricing, discount rate, social opportunity cost of capital, incommensurables and intangibles, and social impact analysis; sensitivity analysis; and organizing and evaluating a costbenefit study.

The example application of the cost-benefit methodology was to the use of wind generators. The approach adopted for the example application consisted of the following activities: (1) surveying of the available wind data and wind power system information, (2) developing models which quantitatively described wind distributions, wind power systems, and cost-benefit differences between conventional systems and wind power systems, (3) applying the costbenefit methodology to compare a conventional electrical energy generation system with systems which included wind power generators.

Wind speed distribution data were obtained from sites throughout the contigious United States and were used to compute plant factor contours shown on an annual and seasonal basis. Plant factor values (ratio of average out put power to rated power) were found to be as high as 0.6 (on an annual average basis) in portions of the central U.S. and in sections of the New England coastal area.

Analysis of tower data from several locations were used to develop methods for projecting observed wind parameters to a uniform height level for plotting of the plant factor contour map. Although the plant factor maps presented here are only for selected aerogenerator systems (for example, cut in speed of $3.6 \mathrm{~m} / \mathrm{s}$ and rated speed of $8.0 \mathrm{~m} / \mathrm{s}$ ) the general method developed could be used to evaluate plant factors for aerogenerators with other operating characteristics. Results of a parametric study of effect of aerogenerator operating characteristics on plant factor are presented.

Categories of wind power systems which might have significant power potential were identified with emphasis on possible large scale utilization. Alternative technologies and alternative end-uses of the system product were
used as two classification dimensions. Two types of wind power systems were selected for the application of the cost-benefit methodolgy; the first system was the basis for a macro analysis of a wind system which is linked into a utility power grid having no storage capability. The second system was used as a basis for a micro analysis from the viewpoint of a firm that utilizes an energy intensive process. The first system was considered to be used as a fuel saver and the second was considered to be an electricity saver resulting in reduced operating costs. A cost-benefit model was designed and implemented on the computer to establish a practical tool for studying the relative costs and benefits of wind power systems under a variety of conditions and to efficiently and effectively perform associated sensitivity analyses.

Results associated with the first system were found to be sensitive to the operational strategy (the most expensive fuels replaced first or all fuels replaced proportionately), installation costs, plant factor, fuel price increases, and discount rates. Based on both wind potential and fuel prices, it was found that wind systems appear to have the greatest benefits in the New England and mid-Atlantic regions. The study showed that even for systems which act only as ruel savers and do not contribute to a firm system capacity, aerogenerators in utility grid can be economically advantageous.

For the second system it was shown that the results are most sensitive to storage efficiency, wind plant factor, and installed costs. A range of parameter values exist that result in a positive net present value of the project scenario over the status quo scenario and that there is potential for the use of windmill generators to produce electricity for energy intensive processes. The alvergence between the social discount rate and the higher internal rate of return demanded by firms suggests that a socially optimal use of wind power can be
achieved only by appropriate government inducements to firms to adopt this new technology.

As a general conclusion it might be stated that the cost-benefit methodology described in this report can be utilized in forming a cost-benefit analysis of a variety of projects. Although the general methodology would apply, the specific models must be tailored to some extent for the particular problems analyzed. The utility of the methdology should be widely accepted by decision makers providing there is the appropriate interaction between the analyst and the decision maker.

## TABLE OF CONTENTS

Page
PART I INTRODUCTION ..... 1
PART II COST-BENEFIT METHODOLOGY STUDY ..... 7
Section 1. Background ..... 9
Section 2. The Structure of Decision Problems and the Choice of Criteria ..... 23
Section 3. Identifying Costs and Benefits ..... 41
Section 4. Quantifying Costs and Benefits ..... 55
Section 5. Special Problems in Measurement ..... 73
Section 6. Sensitivity Analysis ..... 93
Section 7. Performing a Cost-Benefit Analysis ..... 107
Section 8. Glossary of Cost-Benefit Analysis Terms ..... 127
PART III EXAMPLE APPLICATIONS TO THE USE OF WIND GENERATORS ..... 133
Section 9. Introduction ..... 135
Section 10. Approach ..... 141
Section 11. Categorization of Wind Power Systems ..... 147
Section 12. Wind Data Collection and Assessment ..... 163
Section 13. The Cost-Benefit Model ..... 191
Section 14. Assessment of System I - Macro Application ..... 205
Section 15. A Micro Analysis of the Potential of Wind Energy Systems ..... 237
PART IV CONCLUSIONS AND RECOMMENDATIONS ..... 279
Section 16. Conclusions ..... 281
Section 17. Recommendations ..... 289
Page
APPENDICES ..... 293
A. Social Values and Elasticity Considerations ..... 295
B. A Method for Evaluating Output Wind Power Characteristics from Input Wind Statistics ..... 337
C. The Wind Distribution Functions Examined, and the Methods for Evaluating Their Parameters. . . . ..... 345
D. Weibull Distribution Parameters by Season and Annual Average ..... 353
E. Regional Power Generation Data ..... 385

## LIST OF FIGURES

Figure Page
Figure 2.1 Illustration of minimum average cost criterion. ..... 31
Figure 2.2 A Formal Decision Tree. ..... 39
Figure 4.1 Salient economic features of the ideal firm ..... 70
Figure 4.2 Salient economic features of the hypothetical "newsprint" firm ..... 71
Figure 6.1 Illustrative demand curve for oil. ..... 99
Figure 6.2 Illustrative demand and supply curve for gas, given various oil prices ..... 100
Figure 6.3 Illustrative cumulative probability density function. ..... 104
Figure 7.1 Flow diagram depicting major steps in performing a cost-benefit analysis. Numbers in parenthesis refer to section in text dealing with the corresponding step. ..... 108
Figure 7.2 Factors affecting quantification of costs and benefits ..... 121
Figure 7.3 CBA Accounting Work Sheet ..... 124
Figure 10.1 Flow diagram illustrating overall approach to the application of the cost-benefit methodology to wind power systems ..... 142
Figure 12.1 Observed power law height variation of the Weibull scale factor $c$, which corresponds closely to the mean wind speed. Numbers beside each cunve are power law exponent values ..... 168
Figure 12.2 Observed height variation of the Weibull shape factor $k$, which is a measure of variance (high $k$ meaning low variance) ..... 170
Figure 12.3 The "universal" curve for $k / k_{\text {max }}$ versus height, where $k_{\text {max }}$ is the maximum $k$ value (near 60 or 70 m ). ..... 171
Figure 12.4 Observed height variation of the plant factor (ratio of average output power to rated power) using the cut- in and rated speed of NASA's Plumbrook unit (see Appendix B) ..... 172
Figure 12.5 Map of the sites used for the wind potential study. See site listing in Appendix $D$ ..... 174
Figure 12.6 Contours of annual average wind speed at a height of 30.5 m ( 100 ft. ). Wind speed values are in $\mathrm{m} / \mathrm{s}$. See tabular values in Tables $D-2$ and $D-3$ of Appendix $D$ ..... 176
Figure PageFigure 12.7 Annual map of contours of constant plant factor (inpercent) at a height of $30.5 \mathrm{~m}(100 \mathrm{ft}$.) for a windgenerator with the NASA Plumbrook characteristics,cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $8.0 \mathrm{~m} / \mathrm{s}$( 18 mph ). Contours were drawn from data shown in TableD-4 of Appendix D. . . . . . . . . . . . . . . . . . . .177Figure 12.8 Annual map of contours of constant plant factor (inpercent) at a height of 30.5 m ( 100 ft .) for a windgenerator with the NASA 1 MW design characteristics,cut-in speed $6.2 \mathrm{~m} / \mathrm{s}(15 \mathrm{mph})$, rated speed $13.4 \mathrm{~m} / \mathrm{s}$( 30 mph ). Contours were drawn from data shown inTable D-5 of Appendix D. . . . . . . . . . . . .178
Figure 12.9 Annual map of contours of constant plant factor (in percent) at a height of $61 \mathrm{~m}(200 \mathrm{ft}$.) for a wind generator with the NASA 1 MW design characteristics, cut-in speed $6.7 \mathrm{~m} / \mathrm{s}(15 \mathrm{mph})$, rated speed $13.4 \mathrm{~m} / \mathrm{s}$ ( 30 mph ). Contours were drawn from data shown in Table D-5 of Appendix $D$. ..... 179
Figure 12.10 Annual map of contours of constant plant factor (inpercent) at a height of $61 \mathrm{~m}(200 \mathrm{ft}$.) for a windgenerator with the NASA Plumbrook characteristics,cut-in speed $3.6 \mathrm{~m} / \mathrm{s}$ ( 8 mph ), rated speed $8.0 \mathrm{~m} / \mathrm{s}$( 18 mph ). Contours were drawn from data shown inTable D-4 of Appendix D180
Figure 12.11 Winter seasonal map of contours of constant plantfactor (in percent) at a height of 61 m ( 200 ft. )for a wind generator with the NASA Plumbrook char-acteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}$ ( 8 mph ), ratedspeed $8.0 \mathrm{~m} / \mathrm{s}$ ( 18 mph ). Contours were drawn fromdata shown in Table D-4 of Appendix D. . . . . . . . . . . 181
Figure 12.12 Spring seasonal map of contours of constant plant factor (in percent) at a height of 61 m ( 200 ft .) for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}$ ( 8 mph ), rated speed $8.0 \mathrm{~m} / \mathrm{s}$ ( 18 mph ). Contours were drawn from data shown in Table D-4 of Appendix D182
Figure 12.13 Summer seasonal map of contours of constant plant factor (in percent) at a height of 61 m ( 200 ft. ) for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}$ ( 8 mph ), rated speed $8.0 \mathrm{~m} / \mathrm{s}$ ( 18 mph ). Contours were drawn from data shown in Table D-4 of Appendix D.. . . . . . . . . . . 183
Figure Page
Figure 12.14 Fall seasonal map of contours of constant plant factor (in percent) at a height of 61 m ( $200 \mathrm{ft}$. ) for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}$ ( 8 mph ), rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph}$ ). Contours were drawn from data shown in Table D-4 of Appendix $D$. ..... 184
Figure 12.15 Parametric variation of plant factor versus generator rated speed $V_{1}$ and cut-in speed $V_{0}$ for three representative high, medium, and low average wind speed cities ..... 186
Figure 12.16 Parametric variation of the average output power per unit area $\bar{P} / A$ versus the generator rated speed for the three cities of Figure 12.15. Rated power per unit area $P_{r} / A$ is shown on top axis. ..... 187
Figure 13.1 Block diagram illustrating the comparative analysis model for use in assessing the potential of wind power systems. ..... 194
Figure 14.1 Nine power commission regions ..... 206
Figure 14.2 Effects of changes in time horizon, discount rate, and installation costs. ..... 217
Figure 14.3 NPV vs. plant factor. ..... 218
Figure 14.4 Effect of investment cost and operating factor $P_{f}=.5$, East North Central (5/74 fuel prices) ..... 219
Figure 14.5 NPV vs. discount rate and fuel cost ( $45^{\circ}$ from horizontal) ..... 220
Figure 14.6 NPV vs. discount rate and fuel cost ( $45^{\circ}$ from horizontal) ..... 221
Figure 14.7 NPV vs. fuel cost and discount rate ( $20^{\circ}$ from horizontal) ..... 222
Figure 14.8 NPV vs. investment cost - New England Region ..... 224
Figure 14.9 NPV vs. investment cost - Mid Atlantic Region ..... 225
Figure 14.10 NPV vs. investment cost - South Atlantic Region. ..... 226
Figure 14.11 NPV vs. investment cost - East South Central Region ..... 227
Figure 14.12 NPV vs. investment cost - West South Central Region ..... 228
Figure 14.13 NPV vs. investment cost - West North Central Region ..... 229
Figure 14.14 NPV vs. investment cost - East North Central Region ..... 230
Figure 14.15 NPV vs. investment cost - Mountain Region ..... 231
Figure 14.16 NPV vs. investment cost - Pacific Region. ..... 232
Figure 15.1 Stack Dust Processing Plant Flow Diagram. ..... 243
Figure Page
Figure 15.2 Net present value as a function of storage conversion efficiency. ..... 260
Figure 15.3 Net present value as a function of wind plant factor. ..... 261
Figure 15.4 Net present value as a function of cost per installed unit ..... 263
Figure 15.5 Net present value as a function of maintenance and operating cost factor as percent of the installation cost. ..... 264
Figure 15.6 Net present value as a function of time to recover investment ..... 265
Figure 15.7 Net present value as a function of discount rate. ..... 266
Figure 15.8 Investment Incentive ..... 267
Figure 15.9 Tax rate. ..... 269
Figure 15.10 Cost of electricity if purchased. ..... 270
Figure 15.11 Accelerated Depreciation. ..... 272
Figure 15.12 Interest paid on bonds ..... 273
Figure 15.13 Years to maturity of bonds ..... 274

## LIST OF TABLES

Table Page
TABLE 2.1 BENEFITS ASSOCIATED WITH SITE A IN HYPOTHETICAL BEACH DEVELOPMENT . ..... 36
TABLE 2.2 CALCULATION OF NET BENEFITS ASSOCIATED WITH ALTERNATIVE PROJECT COMBINATIONS IN HYPOTHETICAL BEACH DEVELOPMENT . ..... 37
TABLE 6.1 ILLUSTRATIVE OCCURRENCE PROBABILITIES ..... 99
TABLE 10.1 SUMMARY OF MODELS DEVELOPED. ..... 144
TABLE.11.1 MATRIX CATEGORIZATION OF WIND POWER SYSTEMS ..... 148
TABLE 11.2 KEY TO WIND SYSTEM CATEGORIZATION MATRIX OF TABLE 11.1 ..... 156
TABLE 14.1 LIST OF CONVENTIONAL SCENARIO PARAMETERS ..... 207
TABLE 14.2 COMBINATIONS OF FUELS AND GENERATION PROCESSES ..... 207
TABLE 14.3 POSTULATED WIND POWER SYSTEM ..... 209
TABLE 14.4 POTENTIAL SCENARIO DIFFERENCES ..... 209
TABLE 14.5 BASELINE VALUES OF MODEL PARAMETERS ..... 214
TABLE 14.6 NET PRESENT VALUE, BY REGION, OF NET DIFFERENCES IN SCENARIOS ..... 233
TABLE 14.7 REGIONAL SUMMARY - BREAKEVEN VALUES OF COST PER KILOWATT ..... 234
TABLE 15.1 BASELINE VALUES FOR SENSITIVITY ANALYSIS ..... 257
TABLE 15.2 SENSITIVITY OF NET PRESENT VALUE TO CASH FLOW FACTORS ..... 276

PART I

INTRODUCTION

## INTRODUCTION

In order to evaluate some technological alternatives competing for resources, the National Aeronautics and Space Administration (NASA) often must use the techniques of cost-benefit analysis.* Scientists, as well as managers, who have little experience in cost-benefit analysis are often required to submit program requests supported or justified by a cost-benefit analysis. It is usually not apparent what should be included in such an analysis; nor is it obvious to what depth the analysis should be conducted. On the other hand, higher level management receives for evaluation program requests that are supported by cost-benefit analyses in a variety of formats and utilizing a variety of methodologies. It is difficult for such decision makers to compare the relative merits of the alternative programs.

Many techniques associated with engineering economic analysis and social impact analysis as well as technology assessment exist in the literature. Which technique(s) to use in evaluating competing alternatives is not always obvious, and given the appropriate technique, it is usually not clear how to use it or even the extent to which it can be used. Thus, there is a definite need to assess the status of benefit/cost analysis methodologies that might be applicable to a wide range of technological alternatives of interest to NASA and to provide detailed guidelines on the use of these techniques. Results of such an assessment would facilitate the evaluation of competing technological alternatives so that appropriate decisions could be realized with a high degree of accuracy and credibility.

[^1]Like any methodology, its worth lies in the applicability to specific situations or problems and generally the broader the applicability the greater the value of the methodology. Thus, to demonstrate the application of the methodology it is necessary to select a technology or problem to address and wind power systems was the area that NASA believed would provide a suitable framework for the demonstration of the cost-benefit methodology.

Although windmills have been used more than a dozen centuries for grinding grain and pumping water, interest in large scale electric power generation has developed only over the past 50 years and is currently at a high level due to the recent energy crisis. Although concepts and technologies associated with wind power generation exist today, there is an apparent lack of economic justification for the use of wind as a source of energy. It is therefore highly desirable to determine the potential of wind power systems with emphasis on systems that supply reliable energy at a cost that is competitive with other energy systems.

There are many alternative wind power systems each having its own relative benefits and costs compared to existing power systems. For example, wind power systems can be operated autonomously, in tandem, in conjunction with existing conventional power systems, or with other applications. However, the potential of a particular wind power system cannot be estimated without determining favorable regions and sites for the operation of such a system. Although considerable data exist on climatological parameters, there is need to systematically compile and
analyze climatological data with the objective of determining the potential for wind power systems. Once the alternative wind power systems have been identified, appropriate data collected and analyzed, and suitable regions identified, then benefit-cost analyses can be made using appropriate benefit-cost methodologies.

Although estimates of the potential of wind power systems might be made based on benefit/cost ratios, a recommendation or decision based strictly on benefit/cost ratios might lead to an incorrect decision if intangible factors associated with the economic and social impact of implementing a particular alternative are not considered. Qualitative as well as quantitative aspects of each alternative must be included in the evaluation in order to arrive at the true relative merits of various competing alternatives. Also, in evaluating the alternatives, criteria must be used that are equally applicable to each alternative without having inherent biases.

In view of the above needs, the work on the program described herein was oriented toward answering the following questions:

1. Can a widely accepted methodology be established to evaluate the relative merits of alternative projects in todays complex economic political-social environment?
2. Can high level decision makers be convinced of the utility of such a methodology?
3. Can wind power systems be evaluated using such a methodology?
4. What is the potential for wind power systems from operational, benefit and cost, and social and political viewpoints?

In view of the above questions to be answered and the various disciplines involved, the program consisted of three major tasks; (1) assessment of the status of cost-benefit analysis methdology, (2) compilation of costbenefit methodology, and (3) application of cost-benefit methodology to wind power systems. Part II of this report is an exposition of the assessment and compilation of cost-benefit methodology. Part III of this report is devoted to an application of the methodology. The technology selected for this demonstration is the use of wind energy. Conclusions and Recommendations are presented in Part IV.

PART II
benefit cost methodology study

## SECTION 1

## BACKGROUND

### 1.1 Introduction

Extensive government expenditures characterize today's economics. The governments of advanced economies spend for national defense and for diverse social projects such as water resource development, transportation networks, manpower training and technology transfer and assessment. The members of these economies, enjoying a relatively high consumption of goods provided by private enterprise (food, shelter, clothing, etc.), have turned largely to the public sector for further want satisfaction. Not enjoying high standards of living, the less developed nations have determined that a way to achieve prosperity quickly is to develop their "social infrastructures" (communication and transportation systems, pools of skilled labor, education and cultural facilities, etc.). The governments of these countries take the lead in sponsoring projects to meet these ends. Thus, for varying reasons, public spending is becoming increasingly more important around the world. And with resource scarcity becoming more severe every year, governments are compelled to choose wisely the projects they wish to undertake. Of a large number of competing projects, only a few can be chosen for implementation. Public projects are commonly large-scale in nature and frequently have irreversible consequences. The need for careful analysis is apparent.

The process of identifying acceptable public projects has become identified with the term "cost-benfit analysis" (CBA).* The purpose of Part II, Section I of this report is to develop the fundamentals of CBA in terms that

[^2]would be useful to the engineer-scientist as well as to engineering and administrative managers. Sections 2 through 7 deal with various decision criteria used in CBA, means of identifying the costs and benefits related to public projects, various means of measuring costs and benefits, special measurement problems, measurement techniques used in previous studies, sensitivity analysis as applied to CBA, and suggestions for organizing and performing a cost-benefit analysis. The remainder of Section 1 is devoted to defining cost-benefit analysis and to establishing its economic basis.

### 1.2 Definition

Among non-economists, "cost-benefit analysis" and "cost-effective analysis" are often erroneously considered to be "techniques" for appraising public projects. If CBA is to be considered "a technique," it is at best a loosely defined one. A "cost-effectiveness analysis" is considered to be a special form or subset of CBA distinguished by the difficulty with which project benefits can be identified in terms of dollars.

Cost-benefit analysis is defined as an estimation and evaluation of net benefits associated with alternatives for achieving specific public goals. The meaning or implications of the words in this definition will unfold throughout the reading of this report.

CBA is a generic term embracing a wide range of evaluative procedures leading to a statement assessing costs and benefits relevant to project alternatives. The variety of problems addressed and the ingenuity which must be exercised in estimating costs and benefits make it particularly difficult if not impossible to design an all-purpose CBA procedure. Several general
principles may be stated and a number of guidelines have been established over the years, but public projects differ so much in character that an all-encompassing procedure cannot be defined.

### 1.3 A Brief History

Although evaluations of public projects have doubtless occurred throughout history, the modern literature on CBA normally dates from 1844 with the publication of an essay "On the Measurement of the Utility of Public Works" by Jules Dupuit [1]. A French engineer, Dupuit opened his discussion as follows:

Legislators have prescribed the formalities necessary for certain works to be declared of public utility; political economy has not yet defined in any precise manner the conditions which these works must fulfill in order to be really useful; at least, the ideas which have been put about on this subject appear to us to be vague, incomplete, and often inaccurate [2].

When confronted with the task of actually producing a cost-benefit analysis, the analyst today feels that he faces, at least initially, the same vagueness, incompleteness, and inaccuracies that Dupuit experienced.

Dupuit's most important contribution to economic literature was the idea of consumer's surplus [3] which he presented along with a graphical interpretation. He pointed out that the output of a project multiplied by its price was equal to the minimum social benefit of the project; some consumers might be willing to pay more than the market price and so would enjoy excess utility, or consumer's surplus. This idea led directly to the concept of net social benefit which is now basic to CBA.

While Dupuit's work was the beginning of a stream of thought, we normally consider the application of CBA to have started much later, with the United States Flood Control Act of 1936. By this Act, the Congress declared that
benefits of Federal projects "to whomsoever they may accrue" should exceed costs. But, as observed by Dupuit much before the fact, no consistent methods were developed by which to examine these benefits and costs. The Corps of Engineers, the Soil Conservation Service, the Bureau of Reclamation, and other agencies all used different approaches. With such accumulation of analytic experience, the Federal Government has attempted to standardize its projectappraisal procedures.

In 1950, the Subcommittee on Benefits and Costs of the Federal InterAgency River Basin Committee issued its Proposed Practices for Economic Analysis of River Basin Projects [4]. Known as the "Green Book," this document attempted to merge the language of project appraisal and welfare economics. Although it never achieved official standing, it formed a base for further work. The document was revised in 1958.

In 1952, the Bureau of the Budget issued its Budget Circular A-47 [5] formally setting forth considerations which would guide the Bureau in evaluating proposed projects. Although criticized along with the Green Book for its emphasis on gains as measured by changes in Gross National Product and for its ignoring of income-distribution issues and of gains and losses not measured in terms of national income, it remained the official guide for project evaluation into the $1960^{\prime} \mathrm{s}$.

In 1962, Budget Circular A-47 was replaced by Senate Document 97, "Policies, Standards, and Procedures in the Formulation, Evaluation, and Review of Plans for Use and Development of Water and Related Land Resources '[6]. And after extended review, this document was replaced by "Principles and Standards for Planning Water and Related Land Resources" in 1973 [7]. "Principles
and Standards . . ." represents a substantial revision of Federal practices as established in the $1950^{\prime}$ s. For example, much more than gains and losses in GNP are now considered. Four accounts are used in displaying beneficial and adverse effects and for analyzing tradeoffs among plans: national economic development, environmental quality, regional development, and social wellbeing. While "plans . . . will be directed to improvement in the quality of life through contributions to the objectives of national economic development and environmental quality," separate accounts are also prepared on regional development and social well-being [8].

While Federal efforts were directed toward these revisions in practice, a firm theoretical base was being constructed in scholarly circles. Otto Eckstein's Water Resources Development [9] came out of the Harvard University Water Program in 1958 followed by a book of case studies edited by Eckstein and John Krutilla [10]. At the same time Roland N. McKean's Efficiency in Government through Systems Analysis [11] appeared from RAND. These books were quickly followed by others in public expenditure analysis by such scholars as Jack Hirshleifer, J. C. DeHaven, and Jerome W. Milliman, Charles J. Hitch and Roland N. McKean, Arthur Maass, and Robert Dorfman [12].

In addition to these major critical works, numerous other studies have appeared in almost every field of public expenditure. Several excellent tests are now available on cost-benefit analysis including those by E. J. Mishan, Ajit K. Dasgupta and D. W. Pearce, and Leonard Merewitz and Stephen H. Sosnick [13].

### 1.4 The Economic Basis of Cost-Benefit Analysis

Economics is a social science dealing, with human behavior. Consequently, economics is far less precise than the physical sciences, but
perhaps slightly more precise than sister social sciences. Paul Samuelson has defined economics as "the study of how men and society end up choosing, with or without the use of money, to employ scarce productive resources that could have alternative uses, to produce various commodities and distribute them for consumption, now or in the future, among various people and groups in society. It analyzes the costs and benefits of improving patterns of resource allocation [11].

The field of economics may be partitioned into positive and normative areas. Positive economics describes, explains, and predicts actual economic phenomena and is devoid of value judgement. It says nothing about whether given economic states of affairs are good or bad. Normative economics, on the other hand, explicitly introduces value judgements, or norms. Its purpose is to assess the relative desirability of different economic states, or conditions. Abstractly at least, it follows a well-aćcepted two-step paradigm: first, the stipulation of one or several criteria by which to judge states and, second, analyses of the states according to the criteria. Since the decision to implement a public project leads to a change from one economic state to another, and since our desire is certainly to determine which state is "better" (a value judgement), CBA falls directly into the province of normative economics.

The term "normative" is not in common use among economists. Rather, its synonym, "welfare," is the usual term. The reader has doubtless run across allusions to welfare conomics. As happens all too often, the same word has achieved a connotation in the vernacular different from its meaning in economics. The common misunderstanding is to equate welfare economics with so-called
government welfare programs, such as the school lunch program or food stamps. Welfare programs are those which, in some manner, transfer real income from the well-to-do to the less-well-to-do members of society. Welfare economics is used to analyze such programs, but welfare economics does not espouse such programs. Welfare economics is politically neutral; it is not an apology for political liberalism, and in itself espouses nothing. As a method of analysis (the two-step paradigm mentioned above), it is merely a tool in the hands of the practitioner. If the reader feels that CBA, being related to welfare economics, is somehow politically biased to the left, he should know that one of the most common criticisms of CBA is that it ignores income redistribution. That is, the "whomsoever" receiving the benefits of a public project often turns out to be the well-to-do, while the costs often accrue to the less well-to-do, A good CBA will circumvent this pitfall; but if there's any bias at all, it's probably counter to espousing welfare programs. In brief, welfare economics, a neutral analytic method, provides the theoretical basis for CBA. Thus, it is sometimes said that CBA is simply applied welfare economics.

If making value judgements about the desirability of economic states is the thrust of welfare economics, its cutting edge is the decision criterion adopted. A welfare economic analysis has merit only insofar as the criterion meets general acceptance. For example, the authors of this manual might suggest the following criterion: economic state one is better for society than economic state two if the authors get more income in one than in two. Economic analysis of alternative states based on this criterion is likely to achieve little general acceptance for the simple reason that the criterion ignores the preferences of the other members of society. At least in Western
soclety, a guiding rule in formulating criteria is that each individual's preferences must (somehow) count in the evaluation of alternative economic states. This rule has given rise to four popular criteria; which may be labelled unanimity, Pareto superiority, majority rule, and potential Pareto superiority.

Unanimity. Economic state one is to be judged socially superior to economic state two if each member of society* individually judges one superior to two.

This criterion provokes virtually no dissent. Who can argue that it is not ethical or moral or just? Unfortunately, the criterion is useless. In real life, one will never find a substantive policy issue-a policy which moves the economy from one state to another-on which unanimous agreement can be elicited. In the absence of unanimous agreement, this criterion gives no guidance as to which state is socially better. Thus it has no value as a guide to policy making.

Pareto superiority. Economic state one is to be judged socially superior to economic state two if at least one person individually judges one superior to two, and no one judges two superior to one. (Vilfredo Pareto was a European social scientist writing around the beginning of this century. This criterion is based on his work.)

This criterion amounts to a slight weakening of the previous one. It amounts to allowing indifference by some individuals in choosing between two states not to affect what is otherwise unanimity. In other words, state one is socially superior to state two if one or more persons prefer one to two and

[^3]everyone else is indifferent. If only one person prefers two to one, the criterion breaks down. It then says nothing about which state is socially preferable. While economists have long favored this criterion in theoretical discussions, it should be obvious to the reader that it is a useless policy guide since it will never be applicable. In a real policy choice betwen two states, preferences on both sides of the issue are bound to exist.

Majority Rule. Economic state one is to be judged socially superior to economic state two if the majority of the members of society prefer two to one.

The democratic flavor of this criterion suggests that it might be widely acceptable. In fact, of course, it is not employed. Although, we may vote for our representatives in government, we do not usually vote directly on policy issues (local referenda would be the exception). Our representatives (those elected or those responsible to elected officials) generally make the policy decisions. Why is this the case? On one level, the answer is simply that this is what government constitutions provide for. On another level, the better answer would be that one could not expect voters to be completely knowledgeable about the hundreds of issues which arise every year. Thus, a policy decision based directly on voting may not be a wellinformed decision.

Potential Pareto superiority. Economic state one is to be judged socially superior to economic state two if those who gain by the choice of one over two could compensate those who lose, so that if compensation were paid, the final result would be that no one would be worse off in state one than he would be in state two.

This criterion is more complicated than the previous ones. An example will serve to clarffy it. Suppose only two persons, $A$ and $B$, are affected one way or the other by the movement of the economy from the status quo to either state one or two. If the change is to state one, $A$ gains $\$ 20$ and $B$ loses $\$ 10$. If the change is to state two, $B$ gains $\$ 5$ and $A$ loses nothing. These effects are summarized as follows:

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Person | State One | Net Potential <br> A | State Two <br> State One over Two |
|  | +20 | 0 | +4 |
| S | -10 | +5 | +6 |

According to this criterion, state one is socially superior to state two because $A$ can give $B$ an amount of money between $\$ 15$ and $\$ 20$, say $\$ 16$, so that both end up better off than in state two. In this case, A would end up with $\$ 4$ (after giving $B \$ 16$ of the $\$ 20$ he gets in state one) and $B$ ends up with a net gain of $\$ 6$ (after subtracting $B ' s \$ 10$ loss in state one from his $\$ 16$ transfer from A). This net potential result is shown in the last column of the above table. Since no one is worse off than he would be in state two, state one is socially superior to state two by the Potential Pareto Criterion. Note that if the movement were from the status quo to state two, there is no compensation which $B$ could pay A to make them both no worse off than in state one. Thus, the criterion would state, as we would hope, that state two is not socially superior to state one.

The great advantage that this criterion has over the first two criteria is that it is always applicable. It is always the case that the Potential Pareto Criterion, in comparing any two states, will find one superior to the other, or will find them equal (equality would have occurred in the example
if $B$ 's gain from state two were +10 instead of +5 , making the aftercompensation result of choosing either state identical to the other state).

A disadvantage of this criterion is that it does not command the universal acceptance that the first two criteria are accorded. This is because the superiority of one state over another is based on a potential, rather than actual, compensation of the losers by the gainers. The criterion does not demand that the compensation actually be padi, only that it is possible that suitable compensation exists to leave no one worse off. There are two defenses for this argument with neither being completely convincing. First, the progressive tax structure tends to force compensation from gainers to losers. Second, when a large number of policy decisions are made, losers from one policy will be gainers from another, i.e.. differences tend to wash out. Empirical evidence in support of these contentions has never been presented, and probably never will be.

The Potential Pareto Criterion forms the basis for the quantitative part of CBA. As will be discussed at length later, the qualitative aspects of CBA are attempts to circumvent the lack of universal acceptance of the criterion.

Looking back at the table, the reader will appreciate how the Potential Pareto Criterion translates directly into CBA. CBA attempts to ascertain the net benefit (total benefit less total cost) of a policy or project. The net additional benefit of state one is +10 , while the net additional benefit of state two is +5 ( 20 less 10 , and 5 less 0 , respectively). A little
reflection on the analysis accompanying the example will lead to the appreciation that the difference of $\$ 5$ in net benefits of state one over state two was the critical factor in the Potential Pareto Criterion's choice of state one over state two.

In review, cost-benefit analysis is applied welfare economics. Resulting value judgements are based on the Potential Pareto Criterion. In effect, this criterion amounts to choosing the state with the greatest net benefits. Since the Potential Pareto Criterion is not universally accepted as the one and only welfare norm, CBA's quantitative aspects must be supplemented by qualitative analysis designed to ferret out any socially unacceptable implications the application of the criterion might entail in any specific circumstance. 1.5 Summary

A cost-benefit analysis identifies and evaluates net benefits associated with alternatives for achieving defined public goals. Techniques used in identifying and comparing cost and benefits are almost as numerous as existing analyses. Nevertheless, some principles and guidelines can be stated.

As applied welfare economics, cost-benefit analysis uses a decision criterion identified as the Potential Pareto Superiority criterion which labels a project as superior if those who gain from the project could compensate those who lose so that none would be worse off with the project. This criterion identifies net benefits to whomsoever they might accrue and forms the basis for a more detailed review of decision criteria in the following section.

1. Jules Dupuit, "On the Measurement of the Utility of Public Works" (1844), International Economic Papers, II (1952), 83-110, translated from the French by R. H. Barback.
2. Ibid, 83.
3. Alfred Marshall later gave this name to Dupuit's concept. For a discussion of earlier treatment, see Joseph A. Schumpeter, History of Economic Analysis (New York: Oxford University Press, 1954), 1061n.
4. Inter-Agency Committee on Water Resources, Subcommittee on Benefits and Costs, Proposed Practices for Economic Analysis of River Basin Projects (Washington: U. S. Government Printing Office, 1950).
5. U.S. Bureau of the Budget, "Reports and Budget Estimates Relating to Federal Programs and Projects for Conversation, Development on Use of Water and Related Land Resources," Circular A-47, (Washington, 1952).
6. Water Resources Council, Policies, Standards and Criteria for Formulation, Evaluation and Review of Plans for Use and Development of Water and Related Land Resources, Senate Document 97, 87th Congress (Washington: U.S. Government Printing office, 1962).
7. Water Resources Council, "Principles, Standards and Procedures for Water and Related Land Resources Planning," Federal Register, IIIVIIII-174 (September 10, 1973), Part III.
8. Ibid, 5.
9. Otto Eckstein, Water Resources Development (Cambridge: Harvard University Press, 1958).
10. John Krutilla and Otto Eckstein, Multiple Purpose River Development (Baltimore: The Johns Hopkins Press, 1958).
11. Roland N. McKean, Efficiency in Government through Systems Analysis (New York: John W. Wiley and Sons, 1958).
12. Jack Hirshleifer, J. C. DeHaven, and Jerome W. Milliman, Water Supply: Economics, Technology and Policy (Chicago: University of Chicago Press, 1960) ; Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Nuclear Age (Cambridge: Harvard University Press, 1960); Arthur Mass, et al., Design of Water Resource Systems (Cambridge: Harvard University Press, 1962); and Robert Dorfman, Measuring Benefits of Government Investments (Washington: The Brookings Institution, 1965).

## REFERENCES (cont.)

13. E. J. Mishan, Cost-Benefit Analysis (London: George Allen and Unwin Ltd. 1971); Ajit K. Dasgupta and D. W. Pearce, Cost-Benefit Analysis (New York: Barver and Noble, 1972); and Leonard Merewitz and Stephen H. Sosnick, The Budget's New Clothes (Chicago: Markham Publishing Company, 1971).
14. Paul A. Samuelson, Economics, Ninth Edition (New York: McGraw-Hil1, Incs., 1973), 3.

## THE STRUCTURE OF DECISION PROBLEMS AND THE CHOICE OF CRITERIA

### 2.1 Introduction

A sound public decision based on a cost-benefit analysis requires that the analysis be formulated with the appropriate decision criterion in mind. This section is a survey of the criteria which might be used in making decisions and structuring decision problems.

### 2.2 A Survey of Decision Criteria

Many criteria have been suggested as appropriate for evaluating alternative investment projects. Some, such as benefit-cost ratios, have a long history of use in cost-benefit analysis and some, such as cut-off and payback periods, have been employed only occasionally in public expenditure evaluations. One, however, is considered appropriate for many applications: net present value. A brief critical review of these criteria will be presented beginning with the one recommended for most applications.

### 2.2.1 Net Present Value

The net present value (NPV) method reduces a stream of costs and benefits to a single number in which costs or benefits which are projected to occur in the future are "discounted." For example, if a project is expected to yield a benefit worth $\$ 100$ next year, we might value that $\$ 100$ next year, as $\$ 95$ today. There are several reasons for discounting and a number of competing arguments as to how the discount rate ought to be determined. These are discussed elsewhere in this work. The formula is

$$
N P V=-C_{0}+\frac{B_{1}-C_{1}}{(1+d)^{1}}+\ldots+\frac{B_{n}-C_{n}}{(1+d)^{n}}=\sum_{t=0}^{n} \frac{B_{t}-C_{t}}{(1+d)^{t}}
$$

where $C_{t}$ is the dollar yalue of costs incurred at time $t$, $B_{t}$ is the dollar value of benefits incurred at time $t$,
c is the discount rate, and
$n$ is the life of the project, in years.
The principal problems associated with using the NPV method are the determination of the appropriate discount rate and the fact that NPV does not discriminate magnitudes of benefits and costs. However, as we shall see, the consideration of a range of reasonable values is often sufficient in a CBA.
2.2.2 Cut-Off Period

Here, a specific time in the future is chosen. A project is acceptable only if it will cover all its costs by that time. Clearly, this method discriminates against projects whose benefits occur some time after the date of inception, even if these benefits are quite substantial. While this method might have its place in a firm's profit calculus, especially in risky ventures, it appears to be unsuitable for the evaluation of public projects.

### 2.2.3 Pay-Back Period

According to this criterion, that project which recovers its costs in the shortest period of time is considered best. Its myopia is easily demonstrated. Consider the following comparison of projects $A$ and $B$ :

| Project | $C_{0}$ | $\frac{B_{1}-C_{1}}{A}$ | $\frac{B_{2}-C_{2}}{10}$ |
| :---: | :---: | :---: | :---: |
| $B$ | 100 | 110 | 1 |
|  | 100 | 0 | 1000 |

Both A and B involve an initial outlay of 100 and both last two years, Since A returns 110 while $B$ returns nothing after one year, $A$ is judged superior to B. However, considering the second year payoffs, the "pay-back period" criterion appears somewhat faulty.

### 2.3.4 Net Average Rate of Return

The net average rate of return (NARR) is defined as the sum of the net benefits over the life of the project divided by the number of years over which such benefits are incurred. While overtly reasonable, it has its shortcomings. They are illustrated below:

| Project | $C_{0}$ <br> A | $\frac{\mathrm{B}_{1}-\mathrm{C}_{1}}{}$ |  | $\mathrm{~B}_{2}-\mathrm{C}_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| B | 100 | 115 |  | NARR |
|  | 100 | 114 |  | 114 |

Project A lasts one year while B lasts two years. Using the definition given above, the net average rate of return for each project is easily computed. These are presented in the last column of the table. Note that while the criterion chooses A over B, B is superior. The problem, of course, is that NARR does not adequately consider the length of a project's life. Put in another way, NARR implicitly assumes that any project can be done $\underline{n}$ times and will result in an $n$-fold increase in the original net benefits. This assumption is usually unwarranted in the public sector.
2.2.5 Internal Rate of Return The internal rate of return (IRR) is a measure popularized by John Maynard Keynes and has received a good deal of attention. The IRR
of a project is defined to be that rate of discounting the future that equates the initial cost and the sum of the future discounted net benefits. That is, the IRR is some $\underline{r}$ such that

$$
C_{0}=\frac{B_{1}-C_{1}}{(1+r)^{1}}+\ldots+\frac{B_{t}-C_{t}}{(1+r)^{t}}+\ldots+\frac{B_{n}-C_{n}}{(1+r)^{n}}
$$

Alternatively, it is that rate $r$ which would make the NPV of the project equal to zero. A project exceeding some predetermined level (the social discount rate) is deemed acceptable. The problem encountered with this criterion is
the $r$ which solves the above equation is not necessarily unique. Since the equation is of degree $n$, it has $n$ roots. Thus, if the social discount rate is $5 \%$, and roots $3 \%$ and $7 \%$ are calculated, the interpretation of the IRR is not at all clear.

Multiple solutions for the IRR occur when there is more than one change of sign in the flow of funds. To prevent this occurrence a conversion to a single change of sign may be performed using the various interest factors and some prespecified rate of interest. However, this transformation results in ambiguity with respect to the solved value of the $I R R$ and the prespecified discount rate mentioned above.

### 2.2.6 Annual Value

This criterion is formally equivalent to the NPV method. Essentially, it transforms a generally fluctuating actual time stream of net benefits into an NPV-equivalent constant stream. That is, let the actual time stream of net benefits $\left(\mathrm{NB}_{1}=\mathrm{B}_{\mathrm{i}}-\mathrm{C}_{\mathrm{i}}\right)$ be

$$
\mathrm{NB}_{0}, \mathrm{NB}_{1}, \ldots, \mathrm{NB}_{t}, \ldots, \mathrm{NB}_{\mathrm{n}} .
$$

The corresponding NPV is

$$
\sum_{t=0}^{n} \frac{N B_{t}}{(1+d)^{t}}
$$

Then the annual value, $A$, is such that

$$
\sum_{t=0}^{n} \frac{A}{(1+d)^{t}}=\sum_{t=0}^{n} \frac{N B_{t}}{(1+d)^{t}}
$$

In other words, if $\underline{A}$ were received every year for $\underline{n}$ years, the NPV would be the same as when the variable $N B_{t}$ is received in each of the $\underline{n}$ years.
2.2.7 Benefit-Cost Ratio

The benefit-cost ratio (B/C) is normally defined in terms of discounted values. The formula for computing the $B / C$ ratio is

$$
\frac{B}{C}=\frac{\sum_{t=0} \frac{{ }^{B_{t}}}{(1+d)^{t}}}{\sum_{t=0}^{n} \frac{C_{t}}{(1+d)^{t}}}
$$

While this has been a traditionally popular criterion, it has a flaw when being used to compare two or more projects. Specifically, the benefitcost ratio gives the (discounted) benefits per dollar of (discounted) cost. Thus the smaller of two projects may have a higher $B / C$, yet yield a smaller total net benefit. An example will clarify this. Two projects, $x$ and $y$, are being considered for adoption, but only one can be chosen. Each has a life span of 1 year. Let the discount rate, $d$, be $5 \%$. The values required
for comparing these two projects are as follows:

| Project | B ${ }_{\text {O }}$ | Co | $\mathrm{B}_{1}$ | $\mathrm{C}_{1}$ | B/C | NPV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x | 0 | 1 | 2 | 0 | 1.9 | 0.9 |
| y | 0 | 5 | 8 | 0 | 1.5 | 2.6 |

As can be seen, $\underline{x}$ is judged superior to $y$ on the $B / C$ criterion, while the situation is reversed by NPV.

Another difficulty in using $B / C$ is its sensitivity to the definition of benefits and the definition of costs. While it would seem that a positive benefit should be identical to a negative cost (of the same magnitude), it clearly makes a difference in the calculation of a ratio whether a sum-is added to the numerator or subtracted from the denominator. An application where this difficulty is likely to surface is in the assessment of external effects, e.g., pollution. Is a reduction of pollution a positive benefit to society or a reduction in cost?

The benfit-cost ratio should be used as a ranking technique when several independent projects are to be chosen, and there is a given capital constraint. It is then appropriate to rank the projects by their respective benefit-cost ratios, implementing successively lower projects until the capital budget is exhausted or until the $B / C$ of the marginal project reaches unity. To see the logic of this approach, consider the following example. There are seven possible projects, A through G. Each has a lifespan of one year, and each incurs only initial costs (i.e.. no operating costs). Assume a discount rate of $5 \%$, and a capital budget
of 5 . The relevant information is summarized below:

| Project |  | $C_{0}$ |  | $B_{1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | NPV |  | B/C |
| A |  | 10.5 | 5 | 2 |  |
| B |  | 1 | 3.15 | 2 | 3 |
| C |  | 1 | 4.20 | 3 | 4 |
| D | 1 | 2.63 | 1.5 | 2.5 |  |
| E | 1 | 3.15 | 2 | 3 |  |
| F | 1 | 2.63 | 1.5 | 2.5 |  |
| G | 1 | 3.15 | 2 | 3 |  |

NPV and B/C are calculated for each project using their respective formulae as presented above. Based on this information, the projects may be ranked by the alternative criteria, NPV and $B / C$, as follows:

| Project ranking <br> by NPV | Project ranking <br> by B/C |
| :--- | :--- |
| A | C |
| C | B, E, G |
| B, E, G | D, F |
| D, F | A |

Projects are ranked top to bottom for each criterion--those on the same level are equal according to the criterion. Looking first at the NPV ranking, the budget of 5 dictates that only the top-ranked project, A, could be implemented since A exhausts the capital budget. The net benefits accruing
to society from $A$ are given by $A$ 's NPV, which is 5 . Turning to the $B / C$ ranking, projects $C, B, E, B$, and $D$ or $F$, all of which are smaller, would be implemented. These exhaust the capital budget and the sum of their NPV's is 10.5. Thus we see that in order to maximize the total (or sum of) NPV over several independent projects, subject to a capital constraint, the rule is to adopt projects based on the $B / C$ ranking.

### 2.2.8 Minimum Average Unit Cost

This criterion addresses the scale question. It purports that the optimum scale of a project is that scale which minimizes average cost. The criterion is unequivocally incorrect for the simple reason that by focusing exclusively on costs it takes no account of benefits. In fact, the proper scale criterion is to set the scale so that marginal cost equals marginal benefit. This criterion and its relation to the minimum average cost criterion are illustrated graphically in Figure 2.1. It is assumed that scale is a continuous variable. The upper graph depicts total cost and total benefit as functions of scale; the lower graph presents corresponding marginal cost and benefit and average unit cost.

Looking first at the "total" graph, minimum average unit cost occurs at $S$. (Graphically, the average unit cost at any point on a curve is the slope from the origin to that point. The segment OA represents the minimum average unit cost in this case.) However, the net benefits (TB-TC) are not a maximum at $S$, but rather at $S *$ which is reflected by the intersection of $M B$ and $M V$ at $S^{*}$. This is a graphic portrayal of the fact that the first derivative of net benefits must be equated to zero to maximize net benefits (a function of scale).


Figure 2.1 Illustration of minimum average cost criterion.
(Note: MARGINAL FUNCTIONS ARE FIRST DERIVATIVES OF THE CORRESPONDING COST AND BENEFIT FUNCTIONS.)

### 2.2.9 Equity

This criterion addresses the impact of the benefits and costs of a project on the individual members (or groups or classes) of society. On the most abstract level, the equity issue can never be adequately resolved by economic reasoning because at its foundation is the moral or ethical issue of how to compare the relative importance (or value) of different persons. Is "social welfare" unchanged if we take $\$ 100$ from one person and give it to another? The answer might be that it depends on the individuals--their incomes, their wealth, their expenses, etc. No rule can be developed which adequately covers every circumstance. However, economists are in virtual accord that a project which benefits only the rich and costs only the poor should be judged inferior to the converse situation. Of course, almost any project will benefit some rich and poor alike and cost some rich and poor alike. Thus, the above criterion hardly provides a complete equity-decision criterion.

With regard to the equity issue, then, the way to proceed is not to incorporate a formal "equity function" into the NPV maximand (since any such function is highly arbitrary). Rather, any cost-benefit analysis should include a separate, detailed statement as to how the costs and benefits of the project will be distributed among the members of society. In the final analysis, the decision maker must subjectively weigh the NPV of a project against any adverse equity consequences. Such subjective weighing will necessarily reflect the decision maker's own ethical standards, and possibly political realities as well.

The argument is made by many economists that mildly adverse equity consequences should not necessarily preclude the acceptance of a project because diverse projects are probably being undertaken by a variety of government agencies at any given time. Distributional (equity) consequences are thus likely to be self-canceling. (It should be noted that there is no empirical substantiation for this argument.)

Another argument in support of the contention that mildly adverse equity consequences not preclude a project is that there are far better means of achieving a given distribution of benefits and costs among society ${ }^{\text {' }}$ than by choosing public projects to that end. Adjustment of income tax rates is a far more flexible and accurate tool. Thus, in principle, an identifiable group of individuals who are repeatedly hurt by public projects could be compensated by more favorable tax treatment. However, there is no assurance that this would actually occur. 2.3 The Structure of Decision Problems
2.3.1 Alternative Decision Forms

Once a social objective and the alternative means (projects) by which it might be achieved are precisely and explicitly defined (no mean task), it will be found that the structure of the decision problem takes one of three mutually exclusive forms. These are:

1) one project is to be accepted or rejected;
2) one of several candidate projects is to be accepted;
3) several of many candidate projects are to be accepted. The first two forms are relatively simple and require little discussion. But for the last form, it is important to determine whether the projects
are independent or dependent and whether there is an effective capital constraint limiting the sum of initial expenditures on the group of selected projects.

### 2.3.2 Project Interdependence

The independent-dependent issue demands some clarifications.
A project is independent of other projects if the net present value (NPV) of that project is invariant with respect to whether or not any of the other projects are implemented and with respect to the scale of those projects. Projects are independent of each other so long as the above criterion is satisfied for each project. This definition implies that the NPV of a project is unambiguously given by a scalar number (not by conditional relations) and the NPV of any subgroup of projects is simply the sum of the NPV scalars of each of those projects if the projects are independent. A project is dependent on other projects if the above underscored criterion is not satisfied.

By way of example, suppose public beaches are being considered for development along a stretch of coastline. The stretch is 60 miles long, and 6 million persons live evenly distributed along its entire length. There are three possible sites (A, B, G) for the beaches (the 10,30 and 50 mile points along its expanse), and $0,1,2$ or 3 sites are to be developed as beaches. The NPV of a beach is $\$ 50$ to an individual if he doesn't have to travel to reach it. Travelling reduces its value by $\$ 1$ per mile. Thus, an individual values a beach 8 miles away at $\$ 42$. The same individual values a beach 50 miles away at 0 since he'd never use it. Each site, if developed, will be equal in every way. The cost of development is $\$ 50$ million per site.

The question addressed by the cost-benefit analysis is which site(s) to develop, if any.

Let us now determine the benefits associated with a beach at Site A. All persons west of A will use that beach; the net present value for those individuals is $\$ 45$ million (average benefit of $\$ 45$ per person times number of persons). All persons east of Site A will not necessarily use that facility. If Site B is developed, only persons west of the 20 -mile point will use A; B is closer for all others. If only C and A are developed, persons living west of Site B will use A. Therefore, the benefits associated with $A$ are not invariant with respect to whether or not the other projects are implemented. The calculations for $A$ must be presented in tabular form. Benefits cannot be expressed unambiguously by a single scalar. Such a table may take the form shown in Table 2.1.

By continuing the example, another crucial point can be illustrated. When projects are dependent, the only proper way to proceed is to form all possible (or economically feasible if there is an effective capital constraint) combinations of projects, and to evaluate the NPV of each combination. For this example, there are 7 possible combinations. The relevant decision table is shown in Table 2.2 Note that the dependence of the projects is clearly apparent in the table. The NPV of any combination of projects is not equal to the sum of NPV for the individual projects.
2.3.3 Capital Constraints

One point frequently omitted from superficial discussions of public expenditures is the relative scarcity of funds. This is especially true of situations in which positive decisions are based on a benefit-cost

TABLE 2.1

BENEFITS ASSOCIATED WITH SITE A IN HYPOTHETICAL BEACH DEVELOPMENT

| Site (s) Developed | Benefits from $A$ |
| :---: | :---: |
| A Alone | \$170 Million |
| A and B | $\$ 90$ Million |
| A and C | $\$ 125$ Million |
| A, $B$, |  |

## TABLE 2.2

CALCULATION OF NET BENEFITS ASSOCIATED WITH ALTERNATIVE PROJECT COMBINATIONS IN HYPOTHETICAL BEACH DEVELOPMENT

| Projects | Present value <br> of benefits | Present Value <br> of Costs | Net Present <br> value |
| :---: | :---: | :---: | :---: |
| None | $\$ 0$ | $\$ 0$ | 0 |
| A | $\$ 170 \mathrm{M}$ | $\$ 50 \mathrm{M}$ | $\$ 120 \mathrm{M}$ |
| B | $\$ 240 \mathrm{M}$ | $\$ 50 \mathrm{M}$ | $\$ 190 \mathrm{M}$ |
| C | $\$ 170 \mathrm{M}$ | $\$ 50 \mathrm{M}$ | $\$ 120 \mathrm{M}$ |
| A,B | $\$ 255 \mathrm{M}$ | $\$ 100 \mathrm{M}$ | $\$ 155 \mathrm{M}$ |
| A, C | $\$ 250 \mathrm{M}$ | $\$ 100 \mathrm{M}$ | $\$ 155 \mathrm{M}$ |
| A,B,C | $\$ 270 \mathrm{M}$ | $\$ 120 \mathrm{M}$ |  |

ratio (B/C) greater than one. But, whether implicit or explicit, capital constraints always exist and this decision criterion is of very limited value.*

### 2.3.4 A Formal Decision Tree

We are now prepared to present the formal decision tree matching each problem structure with the appropriate decision criterion. This tree is presented in Figure 2.2

To illustrate use of the tree, consider the beach development example which involved the choice of a few projects; they were clearly dependent, and there was no capital constraint. Each possible combination was listed; the one with maximum NPV was chosen. Had there been a capital constraint of, say, $\$ 100 \mathrm{M}$, the set $(A, B, C)$ would have been excluded from the feasible set. In this case, the final choice would not have been altered. It is not difficult to envision circumstances where the imposition of a financial constraint would alter the project choice.

[^4]

Figure 2.2 A Formal Decision Tree.

1. The criteria surveyed are so common that no references are cited in this section. For further reading see texts listed in note 13, Section 1.
2. The usefulness of an acceptance criterion of $B / C$ greater than one is so small that the term "benefit-cost analysis" is not commonly used primarily to avoid the restrictive implications of $B / C$ ratios.

IDENTIFYING COSTS AND BENEFITS

### 3.1 The Identification Problem

Once the problem structure has been defined, the next major step in performing a cost-benefit analysis is properly identifying costs and benefits. The choice of a decision criterion is simple since it is normally dictated by the problem structure. Sensitivity analysis becomes relatively easy once the researcher has a clear grasp of the procedures involved, and such questions as the choice of a proper discount rate are often resolved by a political decree. But identifying the detailed categories of costs and benefits in the course of a study is a major task which demands clear thought and careful planning.

The major stumbling block in identifying costs and benefits is the double-counting problem. Much of the criticism levied against early CBA's and much of the nontechnical controversy in the literature has concerned the counting of benefits more than once, usually in an attempt to cover all possible objectives for a project.

In assessing the benefits and costs of a project, benefits and costs may be classified in several ways and classification schemes can be both useful and harmful. They are harmful in that various categories overlap with others and may frequently lead to confusion and double counting. On the other hand, they are useful in that classification is an aid to identifying effects, and knowledge of the various classification schemes can eliminate such problems as double-counting. One obvious example of a
scheme is that of "benefit and costs" which divides the effects of a project into positive effects (benefits) and negative ones (costs). This section deals with the various classifications of benefits and costs which appear in the literature and is concluded with some general remarks on developing scenarios for CBA. Throughout the section it is important to realize that although it may be desirable to place a benefit in one category or another, the important thing is that benefits (costs) be additions (deletions) to the real product of an economy.

### 3.2 Classification Schemes

3.2.1 Internal vs. External Effects

Internal benefits accrue directly or indirectly to the entity under study. In the simplest case, the benefits returned by a private investment would be the revenues produced. For social investment, internal benefits might properly be construed as those increases in values produced directly by the project itself as well as secondary increases in welfare occurring in other parts of the social entity. The domain of a project is commonly restricted to the project itself, and internal benefits are those which are "captured" by the project.

External effects are much more complex in definition. External benefits "escape" the project and fall into the hands of others. Although these benefits may be valued, they cannot be priced. The quantity of external effects may vary with the size of the decision unit. For example, a private hydroelectric dam may render flood control benefits to outsiders living downstream; these are external benefits. But a dam constructed by the Corps
of Engineers on behalf of the United States may render flood control benefits to citizens of the United States. To avoid undue controversy, externalities should be defined with reference to the project itself and the proper definitional question should be whether or not the benefits can be captured, priced, and sold by the project entity.

External benefits may thus be defined as benefits involuntarily received by others for which they pay nothing. External costs are similarly defined as costs imposed on others without compensation. Collectively, these external effects are often called externalities. They are neither deliberately produced nor deliberately consumed.

Externalities may be classified as either technological or pecuniary [1]. Technological externalities involve changes in real consumption or production opportunities for outsiders. Thus increased recreational opportunities and flood controls associated with a private hydroelectric dam are technological externalities. These externalities represent increased social welfare, cannot easily be priced, and are produced incidental to the purpose of the dam. Most frequently, technological externalities result from joint products.

Pecuniary externalities are associated with the financial effects of the project on others, as felt through price changes for outputs or inputs. Thus decreases in the price of a product itself, increases in the price of a complement, decreases in the price of a substitute, decreases in the price of a joint product, or increases in the price of a resource used in production are all pecuniary externalities.

Technological externalities clearly should be accounted for in a costbenefit analysis--they are real and they increase or decrease social welfare. Pecuniary externalities should normally be excluded: they most likely represent redistribution of income and their inclusion would represent double counting. For example, a rapid-transit station may increase the mobility of a nearby resident, yielding to him great time-savings. The value of his time saving is real and should be counted; the increased value of his house is pecuniary and should not be counted since it is derived from the real time-saving gain.

### 3.2.2 Incommensurables and Intangibles

Incommensurables are effects which "cannot readily be translated into the common denominator or denominators that are being used" [2]. Intangibles are incommensurables which are not measurable in even their own terms.

Use of the term "incommensurable" has been questioned by Dasgupta and Pierce, who point out that "logically, there can be no such things as an 'incommensurable' good. By definition of the concept of a shadow price*..., every outcome has a social opportunity cost, and hence a shadow price" [3]. These authors prefer the term "intangible" as descriptive of effects "... in which there is no market, or in which there is reason to suppose that existing markets do not value an effect completely" [4].

The distinction between incommensurables and intangibles is important. Although incommensurables might technically not exist if "cannot readily be translated" is emphasized in the definition, the set of effects to which the shadow pricing question applies has been isolated. The real problem

[^5]in CBA is developing adequate measures for this category of effects. "Intangible" can be reserved for the really unmeasurable effects.

A useful distinction of these "extramarket" effects might be between "those of a material or economic nature and those involving values beyond the economic. Thus the provision of recreation facilities is obviously economic in nature in that additional commodities or services are made available to the public; it is ...[incommensurable] solely because of difficulties of measurement which are not, as a matter of fact, completely intractable. The preservation of human life or of democratic processes, on the other hand, brings into account values beyond the economic" [5]. With these remarks in mind, we prefer to use the term "incommensurable" to refer to all extramarket effects, reserving "intangible" to describe qualitative terms which are non-economic in nature.

The analyst may be tempted to ignore incommensurables in an effort to compile a single dollar-value number for net benefits or a single benefit-cost ratio. This could very well be a mistake, for the effects of incommensurables could be just as important as others. When a decision maker chooses between alternatives, he implicitly values the incommensurables; the analyst simply faces the problem of having no generally accepted procedure for quantitatively integrating these terms into his analysis and of presenting an analysis with marred neatness.

An example might be seen in comparing two projects for orbiting manned spaceships. One is more expensive in dollars but includes multiple backup and recovery systems which minimize possible loss of life while the other is less expensive in dollars but has a higher probability associated
with loss of 1 ife. These alternatives cannot readily be expressed in common terms yet both must be considered by the decision maker.

Other examples of incommensurables include human life, air pollution, noise, national defense, scenic or historic sites, public recreation facilities, public transportation benefits, prestige, social institutions, redistribution of production or consumption (net of efficiency), etc.

Incommensurables can be treated in cost-benefit studies in several ways. Although considered more thoroughly in the discussion of shadow pricing, several alternatives deserve mention. One approach is simply to ignore the values of incommensurables, but this approach is obviously hazardous, and incorrect. The decision maker has a very inadequate notion of alternatives and, in fact, must know the values of effects not counted to know the importance of values included. At the very least, the analyst should list or describe all effects which are not quantitatively evaluated in his analysis. Another approach may be to identify effects in terms of physical (or other) units. If the number of measures is small enough, the decision maker may then have adequate information to properly weigh alternatives. This approach may also be sufficient to suggest an alternative valuation scheme to the analyst.

In the case of public goods for which shadow prices cannot be constructed, the cost-benefit analysis may become a cost-effectiveness analysis (CEA), which, in essence, is a CBA with benefits not defined in the same terms as costs. Thus the objective may be maximizing physical benefits subject to a cost constraint, or it may be minimizing costs for a given level of physical benefits, or, in the case of an intangible, for a given benefit.

For example, consider the provision of equal access to public facilities for all citizens, including the handicapped and aged. "Equality of access" is an intangible benefit which cannot be quantified. The CBA, or CEA, would then compare alternatives for achieving this goal.

The value of an incommensurable may also be estimated in terms of alternatives. For example, consider that alternative $A$ is associated with considerable incomensurables while $B$ is not. Alternative $A$ might be a domed stadium to house numerous major-league activities lending substantial prestige to a city while $B$ might be park and recreation facilities estimated to yield equivalent recreational benefits to city residents. For the city to prefer the stadium to the parks it must value the prestige at least as much as the difference in the costs of the two projects. By casting a decision to build or not to build a stadium in terms of alternatives, the analyst permits the decision maker to see the values which must be placed on incommensurables for a positive decision.

Shadow prices may be assigned in several ways. Values of similar goods in private markets, the results of consumer surveys, prices implicit in historic governmental decisions, etc., may be used as proxies for a market price for incommensurables. These problems are discussed in detail in Appendix D.

The important point is that incommensurables should be displayed and discussed. Even if valued, they might best be considered separately to emphasize their non-market nature.

### 3.2.3 Direct vs. Indirect Effects

A direct benefit of a project is defined as an increased real value of output associated with the project. The most common direct benefit would be greater physical production such as more grain from an irrigation project, more power from a hydroelectric dam, etc. Direct benefits may also arise from changes in quality (e.g., development of a higher grade turkey), in temporal value (e.g., from storage facilities), in spatial value (e.g., from transportation facilities), or in form (e.g., from sorting fruit).

Secondary or indirect benefits "reflect the impact of the project on the rest of the economy" [6]. The term is normally applied to "the increased incomes of various producers . . . that stem from . . . projects" [7]. Its use has been severely criticized because of the doubtful applicability of the concept [8].

Secondary benefits are a form of external benefits. The term has been used primarily to identify incomes "stemming from" or "induced by" a project. Benefits stemming from a project include the net incomes of processors between the primary product and consumers (e.g., the merchants, haulers, millers, bakers, etc. lying between grain producers and consumers). This notion is much akin to the concept of "forward linkages" used in development economics. Benefits induced by a project are related in the same vein to the concept of "backward linkages" and represent a counting of incomes of firms which supply inputs to primary producers. Because of the nature of these trackings, economic multipliers have been occasionally used to estimate secondary benefits.

But the validity of tracking such benefits has been severely questioned and it is important to understand the criticisms levied against users of the category, primarily the Corps of Engineers. In an economic analysis of agricultural projects, Gittinger has summarized "...the conditions under which the full multiple-effects . . . constitute a real net change in welfare are specific and operationally very limiting." These conditions include the following:
(1) the public expenditure is not financed out of tax revenues so that the multiplier-creating expenditures are not drawn away from the private sector;
(2) the conditions of supply for all factors stimulated to employment by the investment are perfectly elastic at prevailing prices;
(3) the opportunity costs of those factors in the absence of the investment are zero; and
(4) the outputs which result do not simply substitute for other products in the market place and, thus, do not result in unemployment for other factors of production [9].

It is obvious that these conditions could seldom apply, especially in the long period over which most cost-benefit analyses normally apply.

Eckstein summarizes the arguments from a national point of view against considering indirect benefits as follows:
. . . Stemming benefits are very unlikely in depression, are a possibility during inflation if the specific commodities are in particularly short supply, and can only be granted for periods of economic balance in those instances where the premise of mobility can be denied because of extraordinary circumstances. The routine calculation of stemming benefits, therefore, is not warranted.

- . Induced benefits, on the other hand, are largely confined to the construction of the project. They are large in times of depression, nonexistent in times of economic balance, and negative during inflation.
- . it can . . . only be concluded that the use of indirect benefits in benefit-cost analysis must be confined to cases where it can be shown that there are unemployed and immobile resources or that there is underutilized capacity in associated economic activities [10].

Perhaps the most violent attack on the counting of secondary benefits has come from McKean [11]. In an exhausting presentation, McKean points out the clear arguments against counting secondary effects in a fully employed economy, as above, and goes on to question the usefulness of the concept under conditions of unemployment. Not only must involuntary unemployment or underemployment exist for secondary employment benefits to be counted, but the condition must have otherwise existed for the entire project period. The hazards and uncertainties associated with projecting long-term resource unemployment are such that measurement of secondary benefits in a national cost-benefit analysis is not warranted.

From a regional point of view, the estimation of secondary benefits is less risky since the "openness" of a regional economy lessens the constraints on resource use. But the important question here is not in counting benefits but in defining objectives. If the objective of the project is regional development, or spatial redistribution of economic activity, then secondary benefits may be real and important and should at least be identified and listed (although inclusion in a formal summation of benefits is still open to question). Maass succinctly summarizes this point as follows:

[^6]legitimate reason for holding that the efficiency benefits are primary and should be included in the benefit-cost analysis whereas benefits in support of other objectives are secondary and should be mentioned, if at all, in separate subsidiary paragraphs of the survey report. Using the current language and current standards, most of the benefits to the Indians in the Indian irrigation project are secondary benefits.. How silly [12]!

### 3.3 Developing a Scenario

With some of the problems associated with identifying costs and benefits in mind, it is now worthwhile to introduce the concept of a scenario. How does one go about developing a scenario for analysis? This question will be briefly commented on in this subsection and will be discussed in more detail in Section 7.

The first step in developing a scenario is to identify the objective of a public expenditure. If a dam is to be built, is the real objective to increase recreation alternatives, control flooding, and produce electric power in a river basin, or is it justified by contributions to social welfare through serving several objectives? Can the objective(s) be separated from the means (the dam, itself)? The analyst may work under a restrictive mandate from a public authority and so be limited in his pursuit of alternative means; in this case, alternatives may be investigated by other interest groups. Or the analyst may be charged with examining all possibilities. So the first question to be settled is: What are the objectives of and alternatives to a project?

Once this question is settled, a set of accounts must be devised through which to organize the analysis. This process is based on experience and observation and, to some extent, public law.* Federal projects, for example,

[^7]require both national economic development and environmental accounts, with distributional accounts (such as regional development or income-class) displayed for information. After the summary accounts are established, then the analyst must identify the benefits and costs appearing under each account and carefully check for double-counting problems.

## REFERENCES

1. This distinction was made in Tibor Scitovsky, "Two Concepts of External Economies," Journal of Political Economy LXII (Apri1 1954), 143-151.
2. Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Defense in the Nuclear Age (Cambridge: The Harvard University Press, 1960), 182.
3. Ajit K. Dasgupta and D. W. Pearce, Cost-Benefit Analysis (New York: Barnes and Noble, 1972), 113.
4. Ibid, 112.
5. Jack Hirshleifer, et al., Water Supply: Economics Technology and Policy (Chicago: University of Chicago Press, 1960) 132. Note that these authors use the term "intangible," probably in keeping with the practices of the Corps of Engineers. We have substituted the term in square brackets.
6. Otto Eckstein, Water Resources Development (Cambridge: Harvard University Press, 1958), 202.
7. Roland N. McKean, Efficiency in Government through Systems Analysis (New York: John W. Wiley and Sons, 1958), 154.
8. See Eckstein, op. cit., 202-14 and McKean, op. cit., 151-67.
9. J. P. Gittinger, Economic Analysis of Agricultural Projects (Baltimore: The Johns Hopkins University Press, 1972) 27.
10. Eckstein, op. cit., 211-2.
11. See McKean, op. cit., Chapter 8.
12. Arthur Maass, "Benefit-Cost Analysis: Its Relevance to Public Investment Decisions," The Quarterly Journal of Economics, LXXX (1966), 211.
13. Water Resources Council, "Principles, Standards, and Procedures for Water and Related Land Resources Planning," Federal Register, Vol. 38, No. 174 , (September 10, 1973), Part III.

## SECTION 4

QUANTIFYING COSTS AND BENEFITS

### 4.1 Introduction

This section provides an introduction to some of the principles of quantifying costs and benefits. First, an outline is given of the costbenefit framework to establish the analytic environment in which shadow prices arise. Then, a discussion is given on the ethical basis for CBA as seen in the principles of consumer sovereignty and in the assumptions regarding the distribution of income. Next, the concept of shadow pricing is illustrated with a linear program and shadow prices are derived with the theory of pure competition. Lastly, an example is presented to show the proper place of marginal social benefits and marginal social costs. This section thus establishes the logic of quantification. Later sections become more specific regarding actual measurement problems.
4. 2 The Cost-Benefit Framework

The most important aspect of a cost-benefit analysis is the identification of all the relevant costs and benefits. Second only to this in importance is the quantification of such costs and benefits. The raison d'etre of quantification is to facilitate the analysis of trade-offs. Any CBA will involve considerations of both losses and gains to society. It's obvious that the magnitudes of such losses and gains are a crucial input to the decision maker.

A1though a good deal of controversy has been generated by this topic of quantification or measurement in CBA, it should be noted that the controversy
centers on the specifics of application rather than on general principle. In fact, it will not be difficult to state the widely accepted principle of measurement in CBA. First however, the formal framework and terminology of CBA must be introduced. A state (of the world), $S$, is a specific distribution of utility ${ }^{*}$ among the members of society. That is,

$$
\mathrm{S}=\left(\mathrm{U}^{1}, \mathrm{U}^{2}, \ldots, \mathrm{U}^{\mathrm{j}}, \ldots \mathrm{U}^{\mathrm{N}}\right)
$$

for a society of $N$ members. A project is a well-defined, intentional action or set of actions which will lead society from the status guo (current state), $S^{0}$, to alternative state $S^{\prime}$. The value of a project to individual $\mathcal{j}$, $V_{j}$, is the maximum amount he would be willing to pay to have the project adopted when he favors the project and is the negative of the minimum amount he would accept as payment to leave him just as well off in $S^{\prime}$ as in $S^{\circ}$ when he does not favor the project. When the project does not alter j's utility, $V_{j}=0$. (Economists have a special term for $V_{j}$ : compensating variation.) The social value of a project, $V$, is

$$
\sum_{j=1}^{n} v_{j}
$$

That is, social value is based on willingness to pay of individuals. This is the basic and straightforward principle of measurement in CBA.

How is $V$ to be inferred? Asking every individual for his $V_{j}$ is clearly a hopeless task for two reasons. First, each person would have to be apprised of every detail of the project and its consequences so that he could come to some conclusion about its value to himself. This

[^8]alone would appear to be a practical impossibility. Second, there is no reason to suppose every person would convey his true $V_{j}$, particularly if it were negative. The tendency would be to overstate one's opinion of the project in the hope of influencing the final decision.

Thus, the $V_{j}$ 's are best determined without recourse to interviews or questionnaires. Subject to a number of qualifications discussed later, market prices reflect the $V_{j}$ 's.

Suppose the only effects of a project are an increase in the production of good $X$ by $\Delta X$ and a decrease in the production of good $Y$ by $\Delta Y$. The prices of the goods, $P_{x}$ and $P_{y}$, remain unchanged. The value of the project to individual $\underset{j}{ }$ is

$$
V_{j}=P_{x} \cdot \Delta X_{j}-P_{y} \cdot \Delta Y_{j}
$$

where $\Delta X_{j}$ and $\Delta Y_{j}$ are $j$ 's changes in consumption of $X$ and $Y$. Clearly

$$
\Delta X=\sum_{j=1}^{n} \Delta X_{j} \text { and } \Delta Y=\sum_{j=1}^{n} \Delta Y_{j}
$$

To find the social value of the project directly--by summing the $V_{j}$ 's-- is an onerous task. It involves knowing each $\Delta X_{j}$ and each $\Delta Y_{j}$. The job is greatly simplified by noting that

$$
\begin{aligned}
V=\sum_{j=1}^{n} V_{j} & =\sum_{j=1}^{n}\left(P_{x} \Delta X_{j}-P_{y} \Delta Y_{j}\right) \\
& =P_{x} \sum_{j=1}^{n} \Delta X_{j}-P_{y} \sum_{j=1}^{n} \Delta Y_{j} \\
& =P_{x} \Delta X-P_{y} \Delta Y
\end{aligned}
$$

Thus, CBA conclusions can be reached from knowledge of the gross physical effects (the $\Delta X$ and $\Delta Y$ ) and the corresponding market prices of those goods. Clearly, these market prices must have some normative significance--that is, they must say something about the value of the goods--if the evaluations of projects intended to improve the welfare of society are based on them. In economics, the notion of a shadow price arises from the recognition that market prices do not always reflect social value. That adjusted price, which does reflect social value, is a good's shadow price.

### 4.3 Ethical Foundations of Cost-Benefit Analysis

Before entering into a more detailed discussion of shadow prices, a final point must be made about the CBA general principle of measurement: willingness-to-pay. It should be noted that this principle is based, at least implicitly, on the acceptance of two ethical postulates. The first is that of consumer sovereignty. By this is meant that the individual is the best judge of his own welfare, or state of well-being. Thus, when we speak of the "value of a project to individual $j$ " in a CBA, value is computed with specific reference to the consumer's own judgements as to the worth of a good. The acceptance of consumer sovereignty is to be contrasted with a "dictatorial regime" wherein the decision maker is allowed to base his decision on how he feels one should value a good. By way of example, suppose the market price of a "baked Alaska" is \$3, and a project will reduce their production by 100 units. Consumer sovereignty demands we value that decrease as a cost of $\$ 300$ to society however much the decision maker feels that we are all too fat and less baked Alaska is not a loss at all, but rather a gain.

There is a possible point of confusion here, and it deserves some mention. Often a project will involve the gain or loss of some commodity which is not exchanged through a market. Such a commodity might be a public park, and suppose a project is to provide one. Since there does not exist an observable price which reflects consumer evaluation of the park, the costbenefit analyst must somehow inpute an evaluation to the relevant consumers. The imputation, per se, is not a violation of consumer sovereignty. For the analyst's task is to determine how much consumers would be willing to pay, not how much they should be willing. He must base his calculation on his best assessment of the consumers' own evaluations.

The second ethical postulate is that the existing distribution of income is acceptable. Distribution of income is an ethical issue because a judgement is being made, for example, that those in poverty deserve it in some sense. The fact is that the distribution of income influences consumer evaluations as they are reflected in market prices. A corollary of this statement is that it is possible that a project accepted under one income distribution would be rejected under another, and conversely. This attitude casts an unwholesome air of capriciousness about CBA unless one is willing to sanction the given distributions of income.

To see that income distribution affects prices, thereby potentially affecting the results of a CBA, consider the following example. A project, being evaluated by CBA, has a social opportunity cost of $\$ 5,000$. Its only benefit will be an increase in the output of walking canes, the kind used exclusively by retired persons (whose average income is notoriously low). The current market price of these canes is $\$ 7$. The projected increase in
output is 600 canes. The market value of the benefits is $\$ 4200$. The net social benefits are $-\$ 800$. If retired persons had higher incomes, the market demand curve for canes would be shifted to the right. Assuming the long-run supply curve of canes is upward-sloping, the result would be a higher market price for canes--say $\$ 9$ each. Net social benefits are now $+\$ 400$. If the decision maker bases his judgement entirely on the CBA, the distribution of income is the critical factor. Paradoxically, higher income qualifies the retired persons to get more benefits. The point here, then, is that for a decision maker to reject the project because it lost $\$ 800$, he must implicitly sanction the existing income distribution. 4.4 Shadow Pricing

The term "shadow price" has received a good deal of attention in recent years. Two circumstances account for this. First, the so-called "dual" variables of linear programming attain significant import when they are interpreted as shadow prices. Second, the widespread interest in CBA has placed the term "shadow pricing" in the vocabulary of most policy analysts, independent of its linear programming heritage, as some form of costmeasurement technique.

A shadow price may be defined as a value associated with a unit of some good which indicates by how much some specified index of performance can be increased (or decreased) by the use (or loss) of the marginal unit of that commodity. This definition applies equally to the linear programming and CBA uses of the term, suggesting that an understanding of the term in one of its uses will facilitate its appreciation in the other. Specifically, a simple linear model of an economy is constructed to show how shadow prices
arise quite naturally. Then considering the standard economic model of perfect competition, it is found that competitive market prices correspond exactly to shadow prices. Finally, considering the complexities of the real world, economic theory enables some progress toward deriving shadow prices from observed, but not perfectly competitive, prices.

What is the relation between the terms "shadow price" and "opportunity cost"? Both claim to measure "true" social cost. The shadow price is the per-unit opportunity cost of some good. The term opportunity cost is usually reserved for the aggregate values of a set of resources. Thus, if a project is to use 5 units of $X$ and 10 units of $Y$, whose shadow prices are 2 and 3, respectively, the opportunity cost of the project is

$$
5 \cdot 2+10 \cdot 3=40
$$

To clearly appreciate the concept of shadow prices, it is useful to begin with a linear programning framework. Imagine a simple economy whose main features are:
A) There are two types of goods, final consumption goods and raw materials. Call these $X$ and $Y$, respectively.
B) The members of society have valued the final goods by attaching prices to them. The higher the price, the more valuable that good is to them. The higher the price, the more valuable that good is to each individual. These prices are $P_{i}(i=1,2, \ldots, N)$.
C) There is a linear technology through which society transforms the
raw materials into final consumption goods.

$$
\begin{aligned}
& X_{1}=a_{11} Y_{11}+a_{12} Y_{12}+\ldots+a_{1 M} Y_{1 M} \\
& X_{2}=a_{21} Y_{21}+a_{22} Y_{22}+\ldots+a_{2 M} Y_{2 M} \\
& \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots+a_{N M}{ }^{Y} N M \\
& X_{N}=a_{N 1} Y_{N 1}+a_{N 2} Y_{N 2}+\ldots \ldots+\ldots
\end{aligned}
$$

$X_{i}$ is the number of units of consumption good $\underline{i}$ produced, and $\underset{i}{i}$ goes from 1 through $N . \quad Y_{i j}$ is the number of units of raw material $j$ used in the production of final good $\underset{i}{ }$. The $a_{i j}$ are non-negative parameters.
D) The total amount of any raw material available for use in any time period is limited. The maximum available amounts are $\bar{Y}_{j}(j=1,2, \ldots, M)$. Thus, society's production process must also satisfy

$$
\begin{aligned}
& \mathrm{Y}_{11}+\mathrm{Y}_{21}+\ldots+\mathrm{Y}_{\mathrm{N} 1} \leq \overline{\mathrm{Y}}_{1} \\
& \mathrm{Y}_{12}+\mathrm{Y}_{22}+\ldots+\mathrm{Y}_{\mathrm{N} 2} \leq \overline{\mathrm{Y}}_{2} \\
& \mathrm{Y}_{1 \mathrm{M}}+\mathrm{Y}_{2 \mathrm{M}}+\ldots+\mathrm{Y}_{\mathrm{NM}} \leq \overline{\mathrm{Y}}_{\mathrm{M}}
\end{aligned}
$$

E) Society's goal is to maximize the value of its total production. That is, of all feasible sets of final goods ( $\mathrm{X}_{1}, \mathrm{X}_{2}, \ldots, \mathrm{X}_{\mathrm{N}}$ ), society wants to produce that set which maximizes

$$
V=P_{1} X_{1}+P_{2} X_{2}+\ldots+P_{N} X_{N}
$$

When society's production planning agency solves the problem stated in E, it finds that it has also found values for the linear programming dual variables $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{M}$. Dual variable $\lambda_{j}$ corresponds to the constraint
on $\bar{Y}_{j}$ in $D$. These $\lambda$ 's would not be very interesting except for the remarkable fact that, if society were somehow able to find one more unit of $Y_{j}$ to use in production, the value of the final output would increase by $\lambda_{j}$. That is, $\lambda_{j}$ tells the production planning agency how much the marginal unit of $Y_{j}$ is worth to society. Briefly,

$$
\lambda_{j} \approx \frac{\Delta V}{\Delta Y_{j}}
$$

Thus, $\lambda_{j}$ is the shadow price of $Y_{j}$.
Now let's consider the performance of CBA in this simple economy. Two projects have been identified as possibly worth undertaking. Each involves a new method of "extracting" raw materials from the earth. Because of political considerations, only one project can be chosen for implementation. (A maximum of one will be chosen since neither may actually be worthwhile.) Project Alpha would involve taking one unit of $Y_{1}$ and two units of $Y_{2}$ out of the production of final goods, but would use them to increase the extraction of $Y_{3}$ by two units. The new $Y_{3}$ could then be used in the production of consumer goods. Project Beta would use one $Y_{1}$ and one $Y_{3}$ to get three more $Y_{2}$. Suppose the shadow prices of $Y_{1}, Y_{2}$, and $Y_{3}$ are 1,2 , and 3 respectively. How can we evaluate these projects? By assumption $E$, society is interested only in the output of final consumer goods--the X's. Society's interest in raw materials extends only to how they affect production; raw materials have no social value in themselves. Thus, the appropriate way to attack this problem is to ask how Alpha and Beta would affect $\underline{V}$, the value of final output. Clearly it would be useful to have some way to relate changes in the Y's to $V$. But this is precisely the role of the shadow price! The cost-benefit
analysis of Alpha and Beta must be performed in terms of the relevant shadow prices. The following table summarizes the analyses:

| Project Alpha |  | Project Beta |  |
| :--- | :--- | :--- | :--- |
| Social Benefit | Social Cost | Social Benefit | Social Cost |
| $2 Y_{3} @ 3=6$ | $1 Y_{1} @ 1+$ | $3 Y_{2} @ 3=6$ | $1 Y_{1} @ 1+$ |
|  | $2 Y_{2} @ 2=5$ |  | $1 Y_{3} @ 3=4$ |

Project Beta turns out to be the preferred alternative, since it would increase $\underline{V}$ by 2 while Alpha would only increase $\underline{V}$ by 1 . Cost-benefit analysis is an almost trivial task in this simple economy.

A final crucial point remains to be made in the context of this model. Suppose that the $Y$ 's had prices associated with them which, for some reason, did not correspond in direct proportion to the shadow prices or the $\lambda$ 's. However tempting, it would be incorrect to use those prices in CBA. They would be completely irrelevant to the problem. Only if there were some systematic deviation of those prices from the shadow prices would the former be useful in inferring the values of the latter. This constitutes the foundation of actual attempts to derive shadow prices, as will be explained in more detail.

### 4.5 The Ideal Economic Model: Perfect Competition

A linear programming approach to determining shadow prices for an actual economy might seem a good way to proceed. The model of the previous section, after all, does capture many of the salient aspects of reality: consumption goods and raw materials (physical resources, capital, labor) a limited technology which transforms one type of good into the other, an objective of
maximizing the value of production, limited resources, etc. Why can't we just construct the actual problem, solve it, and thereby determine the dual variables, or shadow prices? One reason is that an actual economy is far too complex for a sufficiently detailed model to be constructed. In addition to inherent non-linearities and even non-convexities (which are major obstacles to analysis), there would be many thousands of technical relationships to be estimated, and hundreds of institutional and other non-economic types of constraints. However, the primary reason is that there is a far better method--better in the sense that the expected accuracy of the modeling approach can be achieved at much less cost. This method is economic theory.

The theme which has attracted the most attention from modern economists is construction and analysis of the model of a perfectly competitive economy. This model is based on the following assumptions:
A) In the market for each good, there are a large number of relatively small buyers and sellers.
B) A11 firms in the same industry produce homogeneous goods. Thus no buyer has any a priori reason to prefer the output of one firm over that of another. Another way of saying the same thing is that products are completely standardized among firms and there is no brand loyalty among consumers.
C) Resources are completely mobile. Owners of productive resources (land, labor, capital) are free to put them to whatever use they please. Anyone can work in, or sell his physical resources to, any industry he pleases. There are no barriers to establishing a firm in any industry.
D) Each economic agent is an optimizer. Each individual acts to maximize his satisfaction, each firm acts to maximize its profits.
E) Each economic agent has perfect knowledge. He knows, with certainty, all present and future prices.
F) There are no price rigidities. Prices may move up or down subject to market pressures.

If the above conditions hold, it is easy to prove that: 1) prices are determined by the market equilibration of supply and demand; and 2) in the long run, all goods, are produced and sold at the lowest possible price.

If, in addition to $A-F$, some other conditions hold, then another result very useful to CBA can be established. Among these conditions are:
G) Individual utility functions are "selfish." Each person's feeling of well-being is determined exclusively by his own consumption. He is free of both sympathy and envy in the sense that others' misfortune or fortune do not affect his feeling of satisfaction.
H) Individuals are "greedy," or "more is better." A person never reaches the satiation point. He always feels better off by consuming more.
I) Individual preferences are such that diminishing marginal rates of substitution between goods exist. That is, indifference curves are convex to the origin.
J) There are no production processes which exhibit increasing returns to scale.
K) There are no externalities.
L) There are no public goods.
M) There is no government taxation (except, perhaps, a head or poll tax) and no government subsidy of any good.
N) All goods are exchanged in markets.
0) All markets are in equilibrium.

If conditions A-O are satisfied, then economic theory has established this very important result: all goods have market prices, and the market prices are exactly equal to the corresponding shadow prices (true social values).

Once again, CBA would reduce to a relatively trivial matter. But this time we did not have to resort to a programming model of the economy. Instead, by employing the results of economic theory, we have arrived at the same point--an environment in which foolproof CBA can be accomplished.

But what form of chicanery is this? For the conditions of this model seem as unrealistic as the construction of the programming model was impossible. The key is in the last paragraph of the previous section-systematic deviation. Even though conditions A-0 may not all hold, economic theory can often predict, by determining which assumptions are violated, the direction in which the observed price deviates from the shadow price. Sometimes, but less often, it is possible to make a reasonably good estimate of the magnitude of the deviation. When prices do not exist, i.e. when there are not markets for the goods (1ike public parks), economic theory can at least suggest the principles and problems of measurement to guide the analyst in making his approximations.

This, then, is the forte of attacking shadow pricing with economic theory rather than mathematical programming. While neither can be directly
applied to reality, the former takes account of the systematic deviations of reality from the idealized model, the latter cannot. In fact, it is probably fair to state that the topic which has attracted the most attention from economists, second only to the development and analysis of the ideal model. is the analysis of the implications of deviations from that ideal. But make no mistake, economic theory is far from a panacea. A great deal of information is needed to determine shadow prices, information which is often not readily available. However, even this quantity of information does not approach the amount required for the construction of a full-scale model of the economy--a model from which meaningful dual variables could be elicited.
4.6 An Example

To illustrate the use of economic theory in deriving shadow prices through the analysis of systematic deviations from the ideal model (assumptions A-0 above), consider the case of a hypothetical producer of newprint who occupies a monopolistic position in his market. Newsprint involves externalities in both the production and sales ends of operations. The production process pollutes the water source, causing an external diseconomy. The final product, the newprint, provides an external benefit (economy) to society by virtue of its role as a medium of vast amounts of information. Such information flows are the foundation of political democracy (informing readers about issues, world events, government actions) and economic competition (advertisements about new products, prices, new stores, etc.). Presumably, the value to society of newsprint exceeds its relatively low final cost to consumers. For example, an individual may be willing to pay $\$ 50$
per year for a given quantity (and quality) of printed news, but society may feel that the overall value of keeping a citizen well informed is $\$ 55$ or $\$ 60$ or even more.

Suppose that a government policy is being considered which will have the effect of increasing newsprint production by some marginal amount, perhaps through more favorable tax treatment. How does shadow pricing apply to this cost-benefit decision?

Before answering this question, let's consider the economics of the "ideal" firm. This will provide a base for the systematic deviations of the newsprint firm from ideal characteristics. Among other things, the ideal firm is a price-taker (meaning its relatively small size forces it to accept the prevailing market price as the price at which it will sell its output) and does not induce any external economies or diseconomies. Note, then, that our hypothetical newsprint firm is in violation of assumptions $A$ and $K$ above. Figure 4.1 presents the salient features of the ideal firm. The market price is $O B$. Since the firm feels it can sell all it wishes at $O B$, the demand curve it faces is $B C$. The constant selling price means $B C$ is the marginal revenue curve. Since demand reflects willingness-to-pay (the measure of benefits) $B C$ is the marginal private benefit curve. Since there are no externalities, $B C$ is also the marginal social benefit curve. DE is the marginal cost of production for the firm. Again, because of no externalities, $D E$ is also the marginal social cost of production. The firm's output, assuming it maximizes profit, is $O A$. Thus, with respect to marginal changes in output, $O B$ is simultaneously the market price of the output, the shadow price of output (true value to consumers), the market


Figure 4.1 Salient economic features of the ideal firm.
price of inputs (true measure of the cost to society of using those inputs). Since $O B$ is directly observable, $C B A$ involving gains or losses of that good is a simple matter. To reiterate the point made earlier, CBA is an almost trivial matter in the "ideal" economy.

Turning now to the economics of the newsprint firm, the situation becomes more complex as we consider how it deviates from the ideal firm. As a monopolist, it faces the entire market demand for its output. Such demand, of course, is downward sloping to the right--only at lower prices can more be sold. The demand curve, identical to the marginal private benefits curve, is $A B$ in Figure 4.2. Since there are external economies associated with newsprint production, the marginal social benefit curve lies above $A B$ at $C D$.


Figure 4.2 Salient economic features of the hypothetical "newsprint" firm.

The firm will produce 0 K units of output, since marginal revenue equals the firm's marginal cost at that output level. Then, with respect to marginal changes in output, $O L$ is the firm's unit production cost, $O M$ is the unit production cost to society ( $O M>$ OL because of the pollution by-product of the production process), ON is the price the firm will charge per unit, and $O P$ is the value to society of each unit. Now note that only $O N$ is direct observable. OL may usually be estimated reasonably well using accounting data and knowledge of the firm's production operations. But for $C B A, O M$ and $O P$ are the crucial magnitudes. In contrast to the case of the ideal model, these magnitutes are not directly observable. However, and
here is the solid contribution of economic theory, the requisite information is now neatly circumscribed. What is needed is the local (in the neighborhood $K$ ) properties of $H J$ and CB. The use of a programming model would have required more information.

## 4. 7 Summary

In summary, it has been shown that shadow prices can be thought of either as dual variables arising from mathematical programming or as true economic valuations. In principle, the two meanings are identical because the programming approach explicitly optimizes over a set of constraints and the results of economic theory have implicitly accounted for the same optimizing behavior. The market demand curve, for example, is based on consumer utility maximization. Economic theory views each economic agent (consumer or firm) as an optimizer. The equilibrium conditions in economics are nothing more than first- and second-order optimization conditions. In a sense, this is nothing more than Adam Smith's notorious "invisible hand." Relying on economic theory rather than programming for shadow-pricing guidance exploits what order there is in economic behavior. In a sense, the programming approach forces one to reestablish already well known results, and to collect excess data. In deriving shadow prices, then, economic theory's main contribution is the specification of the minimum requisite information, accomplished by exploiting the systematic deviations of the real world from the "ideal" competitive model. In the final analysis, however, the situation at hand will dictate whether to use the programming approach or economic theory or both.

## SECTION 5

## SPECIAL PROBLEMS IN MEASUREMENT

### 5.1 Introduction

This section deals with two specific problem areas in CBA, the discount rate and the social opportunity cost of capital, and is concluded with a few comments on "social impact analysis," a term recently introduced into the literature to describe cost-benefit analyses subject to some broad definitional and measurement problems. There are no completely satisfactory solutions to any of these problems, but an attempt is made to explain each problem area, to analyze its significance, and to make broad recommendations to guide the analyst.

### 5.2 The Discount Rate

Invariably, a project CBA will have costs and benefits spread over a number of years. To compare one project to another, or to determine the economic viability of a particular project, the time stream of costs and benefits must be reduced to a single number. This number may be the Net Present Value (NPV) of a project. The reader will recall that NPV and related terms were discussed in Section 2.2. The NPV approach discounts future values to their present value. Cleary, the rate of discount is a crucial parameter in the NPV calculation. In the evaluation of a single project, the discount rate will affect whether the NPV is greater or less than zero. In comparison of projects, the discount rate will affect their NPV ranking. This last statement may not be obvious, for it may be thought that while the chosen discount rate affects the magnitude of NPV, it does not affect the ranking of projects. This fallacious notion is
easily dispelled by an example. Projects $A$ and $B$ each last three years. Their annual net benefits (i.e. each entry is total annual benefits less total annual cost) are as follows:

|  | NET BENEFITS <br>  <br>  <br> Project A |  |  |  | -100 |
| :--- | :---: | :---: | :---: | :---: | :---: |

Project $A$ has a large initial return which tapers off over time. Project $B$ has net benefits occurring only in the terminal year. Now let us calculate NPV for each project at discount rates of $1 \%$ and $10 \%$. The results are as follows:

|  | NPV at $1 \%$ | NPV at $10 \%$ |
| :--- | :---: | :---: |
| Project A | 143 | 120 |
| Project B | 158 | 100 |

Note that $B$ is superior to $A$ at a discount rate of $1 \%$, but $A$ is superior to $B$ at a discount rate of $10 \%$. Thus, the discount rate obviously can affect the ranking of projects. High discount rates penalize projects whose benefits occur further in the future.

In CBA, one of the problems which has attracted a considerable amount of attention is the discount rate to be used. Through much research, debate, and soul-searching, economists have generally concluded that this question is not amenable to strictly economic analysis. This is because the social rate of time preference--the rate at which society as a whole is willing to give up present consumption for future consumption (the correct
discount rate for $C B A$ )--is not reflected by an individual's rate of time preference and only the latter can be observed in economic data. Even if it were possible to determine each individual's rate of time preference, and even if an appropriate "averaging" technique could be agreed upon, the resultant rate would not necessarily be the social rate of time preference. In general, it would be too high. To see this, consider the following decision (or game) matrix. It is assumed an individual obtains satisfaction from knowing that future society will "inherit" goods the present society has provided through investment--i.e. through devoting current resources to projects which will pay benefits in the future, even though depriving the present of some consumption opportunities. The individual may feel this way for any number of reasons, among which are
-he may be part of the future society,
-his children and other relatives may be part of that society, or -a general feeling of altruism toward mankind.

For simplicity, assume the representative individual has two choices: invest for the future (I), or don't invest for the future (DI). Likewise, he perceives that society as a whole has those same choices. Suppose his "payoff" (its value to him) when both he and the rest of society invests for the future is 100. If the rest of society invests and he doesn't, he is even better off. For then he has the satisfaction of knowing the future is being prepared for, but he personally has not had to forego any present consumption. Say this value is 150 . The converse situation--he invests and society does not--is the worst possible. Not only does the future get nothing (since his lone contribution has a negligible impact in the future), but he has foregone present consumption. Let the value of this outcome be -50 . If society chooses

DI and the individual does also, at least he hasn't given up present consumption. He is clearly better off than -50 . However, since hs loses the satisfaction of knowing the future is being provided for, he gets less than 100 from this outcome. Say the (DI, DI) payoff is 25 . These choices and their values to the individual are summarized in the following payoff matrix:

## SOCIETY'S CHOICES



Now let us analyze the investment decision in two contrasting contexts: a private investment decision and a public investment decision. Suppose, for concreteness, that both investments involve the purchase of land which is currently unusable but will be a beautiful natural area twenty-five years in the future, e.g. swamp land, where drainage is expected with certainty. The individual would like to dedicate the small tract he purchases as a wildife refuge. He realizes, however, that as an individual he has no control over what society does with all the adjoining land twenty-five years hence. Thus, his "wildife refuge" of several acres may turn out to be surrounded by heavy industry (i.e. he invests for the future and society does not). On the other hand, if everyone else buys the land to keep it natural, and he does not, his marginal acres will not make much difference. The above decision matrix has obvious application. The individual's rational choice must be DI, since the payoff is higher for $D I$, no matter what the rest of society does. That is, the individual will decide not to purchase the land for dedication as a natural
area. Such private decisions about provision for the future seem to imply that individuals have a "high" discount rate for future goods. This is what private investment decisions would seem to imply, and this would be reflected in the data. For instance, the data would show that at current market interest rates, individuals do not feel that this public investment is worthwile.

Suppose the individual can affect society's choice. That is, suppose the matter of what to do with all the land is to be decided by a public vote. If the mafority decided to purchase it for a natural state, the purchase will be financed by taxes. Now each individual will consider the decision from society's point of view. The rational social choice is I--to invest. By a public decision to invest, each individual forces the rest of society to choose I in return for being forced to choose himself. Thus each person can assure himself a payoff of 100. Society's rational choice to invest in the land means its discount rate, the social rate of time preference, must be lower than the discount rate reflected by private decisions. The example shows that in the case of this type of investment for the future, everyone is made better off by having the decision made with reference to the social rate of time preference rather than the individual rate of time preference. The social rate of time preference is lower than the individual rate, and it is reflected only in public decisions. At the risk of being repetitive, we must stress that this social rate cannot be derived from data on individual investment decisions. Operationally, what all this means is that market interest rates, which reflect individual investment decisions, are no guide to the correct discount rate for CBA, except as an upper bound.*

[^9]We must look then, at public investment decisions themselves to determine the proper discount rate for public projects. The reasoning here is not circular in a democratic political system. While every public investment decision is not thrown open to a public vote, the public officials reponsible for those decisions are subject to periodic public review via the election process. Administrations which implement projects inconsistent with the true social rate of time preference tend to lose votes. The democratic political process ensures the tendency of public decisions to conform to public desires. Thus, the correct discount rate for CBA might be inferred from recent popular public investment decisions.

Finally, there is the basic question: what is the cost-benefit analyst to do about the discount rate? While the above discussion hopefully illuminated some of the problems in choosing a rate, it admittedly circumvented a direct answer. Fortunately, a direct answer may readily be given. Since the choice of a discount rate is really a policy decision, the analyst should not choose the figure. Often, in CBA for a particular government agency, the agency itself will specify a discount rate. Failing that, the analyst should parameterize the rate. CBA results should be presented for a number of discount rates, e.g., $3 \%, 5 \%, 8 \%$. Again, the analyst's goal is to present the decision maker with all the relevant information in a convenient format. The analyst should clearly explain how, if at all, the discount rate affects the results of the CBA. Another useful calculation by the analyst is the determination of a critical value for the discount rate. For example, in the comparisons of two projects,
there will generally exist a discount rate below which one project has greater expected value than the other and above which vice versa. If such a critical value turns out to be high, e.g. 14\%, the decision maker need not agonize over choosing a specific rate; rather, on the knowledge that the proper rate is definitely below $14 \%$, his choice of projects is greatly facilitated. This is but one example of how the analyst properly aids the decision maker.

In actual practice, how have decision makers determined the discount rate? It seems that, the foregoing objections not withstanding, the rate has been tied to the federal government's cost of borrowing. The following is taken from the Water Resources Council's relatively recently established Principles and Standards for Planning for Water and Related Land Resources.*

The discount rate will be established in accordance with the concept that the Government's investment decisions are reläted to the cost of Federal borrowing.
(a) The interest rate to be used in plan formulation and evaluation for discounting future benefits and costs, or otherwise converting benefits and costs to a common time basis, shall be based upon the estimated average cost of Federal borrowing as determined by the Secretary of the Treasury taking into consideration the average yield during the twelve months preceding his determination on interestbearing marketable securities of the United States with remaining periods to maturity comparable to a 50 -year period of investment: Provided, however, that the rate shall be raised or lowered by no more than or less than one-half percentage point for any year.

When the average cost of Federal borrowing as determined by the Secretary of the Treasury exceeds the established discount rate by more than 0.25 percentage points, the rate shall be raised 0.5 percentage points. When the average cost is less than the established rate by more than 0.25 percentage points, the rate shall be lowered 0.5 percentage points.
(b) The Water Resources Council shall determine, as of July 1, the discount rate to be used during the fiscal year. The Director of the Water Resources Council shall annually request the Secretary of

[^10]the Treasury during the month of June to advise the Water Resources Council of his determination of the average cost of Federal borrowing during the preceding twelve months.
(c) Notwithstanding the provisions of paragraphs (a) and (b) of this section, the discount rate to be used in plan formulation and evaluation during the remainder of the fiscal year 1974 shall be 6-7/8 percent.

The following table will enable the reader to get a "feel" for alternative discount rates. The entries in the table are the "weights" which the discounting process assigns to costs or benefits incurred in the corresponding year and at the corresponding discount rate.

EFFECTIVE DISCOUNTING WEIGHTS

| Discount <br> Rate | 5 | Weight in Nth year,$\mathrm{N}=$ <br> 10 |  |  |  | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 1 | 1 | 1 | 1 |  |
| 1 | .95 | .90 | .82 | .74 | .67 |  |
| 3 | .86 | .74 | .55 | .41 | .30 |  |
| 5 | .78 | .61 | .38 | .23 | .14 |  |
| 7 | .71 | .51 | .26 | .13 | .07 |  |
| 10 | .62 | .39 | .15 | .06 | .02 |  |
| 15 | .50 | .25 | .06 | .02 | .00 |  |

For example, at a discount rate of $7 \%$, a benefit incurred 20 years into the future is worth only $26 \%$ of what that same benefit would be worth if incurred in the present. Clearly, the adoption of a higher discount rate implies less concern with the future than does the adoption of a lower discount rate.

This discussion can now be briefly summarized. The discount rate to be used in CBA is the social rate of time preference--the rate at which society is willing to forego present consumption in return for future consumption.

This rate, a crucial figure in CBA, cannot be determined by strictly economic analysis. Rather, it reveals itself in the political process. The cost-benefit analyst should not choose the discount rate, since this is essentially a policy decision. If the rate to be used is not specified by the agency sponsoring the CBA, the analyst should parameterize the discount rate and/or compute critical values. This information and its implications should then be communicated to the decision maker. The final choice of the appropriate discount rate rests with the decision maker. 5.3 The Social Opportunity Cost of Capital

The literature dealing with this topic raises the issue that funds transferred from the private to the public sector to finance a project represent a greater real cost to society than conventional CBA would imply. The reasoning is that the transfer of funds reduces both consumption and investment in the initial period. The reduced investment causes a lower-than-otherwise level of national income in succeeding years, and consequently a lower level of consumption in those years. The quite legitimate argument is made that this lower-than-otherwise consumption is the real social cost of the project.

In more formal terms, the argument starts with the time stream of consumption which society would enjoy in the absence of the project. Denote this consumption stream by

$$
C_{0}, C_{1}, C_{2}, \ldots, C_{t}, \ldots, C_{T} .
$$

This consumption stream is associated with a corresponding stream of savings which are invested to maintain the consumption stream in future periods.

By drawing resources from the private sector, a public project reduces private investment. This private investment would have had a positive effect over the years. That is, investment today would produce a continuous stream of returns (income) in the future. In each of these years, investment is further increased because of the higher income level. Thus, each year gives rise to its own stream of extra consumption in the future. For example, in the fifth year, consumption includes that generated by investments in the first, second, third, and fourth years. Thus, the argument states that all these consumption streams are foregone by the transfer of funds to the government. Denote the resultant time stream of consumption by

$$
\mathrm{C}_{0}^{\prime}, \mathrm{C}_{1}^{\prime}, \mathrm{C}_{2}^{\prime}, \ldots, \mathrm{C}_{\mathrm{t}}^{\prime}, \ldots, \mathrm{C}_{\mathrm{T}}^{\prime}
$$

This time stream is net of the consumption which the government project would give rise to, since that is to be added to the benefit side of the calculation. Here our only interest is the cost of the project. Of course, in general,

$$
C_{t}^{\prime}<C_{t}, t=0,1, \ldots, T
$$

The real cost to society, then is the difference in the two time streams. The present value of this difference is the social opportunity cost (SOC) of the capital transferred to the public sector.

$$
S O C=\sum_{t=0}^{T} \frac{\left(C_{t}-C_{t}^{\prime}\right)}{(1+d)^{t}}
$$

It is maintained, then, that in computing the social value of a project, the SOC (as defined above) should be subtracted from the present value of the benefit stream.

Implied by this analysis is the existence of a multiplier, M. Suppose that the project in question initially removes $\underline{R}$ dollars of resources from the private sector. In general, SOC will exceed $\underline{R}$. The exact relation can be expressed as

$$
\mathrm{SOC}=\mathrm{M} \cdot \mathrm{R}
$$

where $M$ will exceed unity. For example, if $M=2.5$, this means that every dollar transferred out of the private sector (in some marginal range) would have increased the present value of future consumption by $\$ 2.50$. By making some fairly restrictive assumptions about economic behavior, it is possible to construct an economic model which permits $M$ to be specified as a function of observable economic statistics, such as the interest rate and the marginal propensity to consume [1]. Operationally, $\underline{M}$ would be calculated rather than the alternative consumption time streams.

Now, what can be said about the validity of this approach, and should it be adopted in CBA? First, it must be stated that the approach to the measurement of costs--the value of foregone consumption--is unassailable. This is the approach stressed throughout this volume. A social cost is a benefit foregone. The real (and fatal) flaw of this approach is that it is built on highly questionable assumptions. When the assumptions are satisfied, the approach is valid. However, it cannot be accepted that the assumptions are generally satisfied to a degree sufficient to warrant the
use of this approach. There are three basic assumptions involved. The first is the most unrealistic. It alone would be sufficient reason to preclude the general use of a SOC multiplier in CBA.

Assumption 1: A government decision to implement a project causes a transfer of funds from the private sector to the public sector equal to the cost of the project.

The government budgetary process is an immensely complicated affair, and this is not the place to delve into how projects are financed. However, it is very definitely the case that it does not work such that the approval of a $\$ 1$-million project at the agency level causes a memo to be sent to the Treasury specifying that an additional \$1 million must be raised from the private sector. The $\$ 1$ million comes out of the agency's budget. There is no reason to believe the budget is any higher because of that specific project. If that project did not exist, another one would. Each agency vies for a portion of the federal budget. Each agency's current appropriation is determined in large measure by its previous appropriation, current government fiscal and monetary policy, and the Administration's goals. Thus, a specific project may have little or no effect on the size of an agency's budget. Furthermore, since the size of the total government budget is dictated, at the margin, by broad economic policy objectives, more for one agency necessarily means less for another. What all this amounts to is that a project competes for funds directly with alternative projects of the same agency and indirectly with projects of other agencies. Thus, the base of reference for the analysis of a project should be the alternative use of funds within the government sector, not the private sector.

Assumption 2: The transfer of funds from the private sector to the public sector reduces private investment.

Although there is no quarrel with this assumption taken literally, one must ask whether the transfer of funds causes a "significant" reduction in investment. If the true reduction is small, errors of measurement are likely to overwhelm the object of the measurement. - It seems this assumption ignores the "liquidity preference" aspects of economic behavior. For example, an increase in government taxation or borrowing could cause individuals to draw on their financial assets rather than forego investment opportunities. Even in a full-employment economy, consumption may decline to compensate for the transfer of purchasing power, and investment may not be affected.

Assumption 3: The foregone private investment has a time stream of impacts whose present value is positive.

This is a reasonable assumption; the objection is simply a way of introducing mitigating factors. First, private investment often has some negative side effects (external diseconomies) such as pollution. This tends to reduce its "face value." Second, the number of business failures indicates that much private investment is done in error. When a firm invests in an already saturated market, for example, it draws capital from other uses (a social cost), but may not survive to expand the amount of consumption goods available to the economy (no social benefit). However, since the effects discussed here are marginal changes in total private investment, and businesses which fail are very likely well represented at this margin, a marginal decrease in total private investment may well more than proportionately reduce the number of "wrong" private investments. Again,
the marginal social value of marginal private investment may be well below the average social value of private investments. This is an important point because, in practice, average values are often taken as proxies for marginal values.

The lack of faith in these three premises, particularly the first, has spurred an apparent concensus among economists that the social cost of a project ought not be calculated using the multiplier approach (except, of course, in the unusual circumstance when the assumptions do appear satisfied).

### 5.4 Social Impact Analysis

"Social impact analysis" is an alternative term used to describe costbenefit analysis in the broad sense presented here. It is sometimes used to ensure that a CBA presenting only the costs and benefits which can be given a dollar valuation not be taken as encompassing all effects of a project. To a perceptive analyst or to a person accustomed to reviewing good analyses, introduction of a new term was unnecessary. What was and is needed is simply a correct interpretation of the analyst's task to begin with. Nevertheless, the term has taken its place in the vocabulary in response to shortcomings appearing in existing analyses. Although it is not the purpose here to pursue these shortcomings in detail, a few of them should be mentioned.

Most of the problems leading to the term "social impact analysis" are measurement problems. As discussed in Section 3.2, some effects, called "incommensurables and intangibles," are difficult both to identify and to measure. The former are effects which are economic in nature, yet not readily measured in monetary terms. With those, the problem is not so much determining
whether the effect is good or bad, but what magnitude ought to be attached to it. The latter are noneconomic effects and so are not only not measurable in dollars but defy any measurement whatsoever. In general, these effects must be judged by values beyond economic ones.

Examples of incommensurables are recreation, nonrenewable resources, and changes in technology. Intangibles may relate to politics, some demographic effects, social justice, individual liberty, aesthetics, and social harmony. The reader is referred back to Section 3.2.3 for further discussion.

A second problem leading to the use of the term "social impact analysis" is associated with the regional effects of a project. The problem is really one of definition.

If the economy under study is the region itself, the regional effects of the projects are the economic effects--the benefits and costs--themselves and no problem exists. If the economy under study is the nation, then the regional development effects may or may not be subject to inclusion in project evaluation. Under conditions of full employment, resources employed in a project in one region have been diverted from use in others. In this case, regional benefits and costs should clearly be counted and the problem is no different from the usual CBA problem. Under conditions of severe resource unemployment, the unemployed resources may be used at no cost to society. But accounting for these resources as costless over any length of time requires such strong assumptions about the future that the general concensus of opinion is to account for resources at normal market costs and to avoid "full employment" as a potential benefit.

Other problems leading to the use of the term "social impact analysis" are concerned primarily with the defining of objectives. If the objective of a project is solely national efficiency, then the distributional changes pursued in a social impact analysis are superfluous. As noted in Section 3.2.4, the indirect effects of a project on regional development, or on a regional redistribution of resource use under conditions of full employment, and the effects of a project on the distribution of income among classes are effects which are to be included in a calculation of costs and benefits subject to some specific assumptions. If, as is traditional, the net gain "to whomsoever it may accrue" is a primary concern, redistributions may represent double counting. But if the objective of a project is to effect a redistribution of the society's total project (broadly defined) across income classes, regions, or other groupings of the population, then the objective function of the CBA for this project must be similarly defined. As mentioned above, in any cost-benefit analysis certain of the identified effects of a project will be incommensurable or intangible. That is, they will not readily lend themselves to quantification or monetary valuation. The problem, of course, is that such effects must nonetheless be somehow incorporated in the cost-benefit analysis. Social effects are no less costs or benefits because of their inherent intractability. As a decision aid, CBA must present all relevant information to the decision maker.

A fruitful approach to social impact analysis is the iterative interactive decision mode. This approach combines objective data analysis by the analyst and subjective problem analysis by the decision maker.

The merit of the method is based on quantifying and evaluating the minimum number of social effects necessary for the decision maker to act. $A$ number of variations on the basic theme may be conceived, but essentially proceeds as follows.

Step 1. List the sets of all valid project costs, $C$, and all valid project benefits, B (Section 7 - "Performing a Cost-Benefit Analysis" details the procedures by which such a list may be generated.)

Step 2. Taking into account such factors as data availability, the goals of the CBA, the relative importance of the various costs and benefits, time and budgetary restrictions, and the needs of the decision maker, choose a subset of the cost set, $C_{1}$, and a subset of the benefit set, $B_{1}$.

Step 3. Find the Present Value (PV) of the costs and the Present Value of the benefits listed in Step 2.

Step 4. Develop reasonably detailed qualitative descriptions of the costs and benefits omitted in Step 3 , i.e., complements of $B_{1}$ and $C_{1}$ : $B_{1}^{\prime}$ and $C_{1}{ }^{\prime}$.

Step 5. Present the decision maker with the current state of the analysis:

$$
\operatorname{NPV}(\operatorname{Project})=\left[P V\left(B_{1}\right)+P V\left(B_{1}^{\prime}\right)\right]-\left[P V\left(C_{1}\right)+P V\left(C_{1}^{\prime}\right)\right]
$$

Example:

$$
\begin{aligned}
& \text { NPV (Project })=\left[\$ 100 M+\text { Qualitative Description of } B_{1}^{\prime}\right] \\
& -\left[\$ 90 M+\text { Qualitative Description of } C_{1}^{\prime}\right]
\end{aligned}
$$

Step 6. The decision maker determines whether the present state of the analysis is sufficient for a decision to be made, or whether further quantification of effects is necessary.

Here, quite obviously, the subjective element is formalized. With reference to the above example, the decision maker must subjectively determine whether the net value of the unquantified effects is more or less than $-\$ 10 M$, that is, whether the value of the unquantified costs exceeds the value of the unquantified benefits by $\$ 10 \mathrm{M}$. If so, the NPV (Project) is negative; if not, NPV(Project) is positive.

Step 7. If the decision maker determines that the current information content of the analysis is insufficient, return to Step 2 and enlarge the benefit and cost sets to be quantified.

### 5.5 Summary

This section has focused on problems of which the cost-benefit analyst must have a sharp awareness although he has little control over their eventual resolution. The discount rate used in a CBA is the social rate of time preference as revealed in the political process. Although the analyst may explore the sensitivity of his results (see Section 6) to variation in the discount rate and present these explorations in his report, the final choice between present and future consumption as expressed in the discount rate rests with the decision maker.

Although the social opportunity cost of capital appears to be greater than the costs implied in a conventional CBA, the consensus among economists is to avoid on practical grounds the complex estimations
associated with this concept even though its basis-the value of foregone consumption--1s flawless. Government decisions are most frequently in allocation of existing budgets--in the weighing of alternative projects against themselves rather than against private consumption foregone-and so do not involve social opportunity cost calculations vis-a-vis the consumer.

Social impact analysis is a term summarizing a variety of problems associated with the proper identification of project objectives and the measurement of project effects. It arose from a feeling that cost-benefit analyses were incomplete, focusing only on effects measurable in dollars. As defined in this work, however, social impact analysis and cost-benefit analysis are synonymous terms; they both attempt to ferret out the true costs and benefits of public projects.

SECTION 6

SENSITIVITY ANALYSIS

### 6.1 Introduction

Up to this point, emphasis has been placed on the identification and measurement of specific costs and benefits as they relate to CBA. Of course, any CBA will involve the consideration of a number of different costs and a number of different benefits, each spanning a number of years. We know, from Section 2 that the best way to aggregate these figures into a single number useful for decision making is to compute the net present value of a project.

If $b_{i t}$ is the value of the ith benefit in year $t$, and $c_{i t}$ the value of the ith cost in year $t$, the expression for net present value is

$$
N P V=\sum_{t} \frac{\sum_{i}\left(b_{i t}-c_{i t}\right)}{(1+d)^{t}}
$$

For example, $b_{i t}$ might be the value to all consumers of a drop in the price of some good for some year, and $c_{\text {it }}$ might be the value to all consumers of an increase in pollution for that year. In previous sections it was pointed out how the $b_{i t}$ 's and the $c_{i t}$ 's should be estimated, with the correct measurement approach depending on the specific circumstances.

Of course, each $b_{i t}$ and $c_{i t}$ that is estimated will be just that--an estimate. Clearly, the reliability of the final NPV figure will depend on the accuracy of these estimates. Admonishing the analyst to be accurate does not resolve the issue of NPV reliability, for there is always some degree of error inherent to the measurement process itself. Even if one attempted to
measure a physical phenomenon which occurred in the past, say the change in wheat production from 1973 to 1974 , the estimate cannot be $100 \%$ reliable. In CBA, many of the measurements deal with the nonphysical, such as willingness to pay, and all deal with the future and thus are predictions. No rational being is likely to accept that, under these circumstances, the calculated NPV of a project is to be interpreted as a precise figure. Given the intrinsic uncertainty surrounding the computed NPV, is there any way the analyst can aid the decision maker, beyond the perfunctory caveat that the computations are subject to error? The answer is yes. The analyst should provide the decision maker with some idea of the degree of error that the estimates are subject to. Then, for example, in a CBA of two alternative projects, if the NPV estimate of one is "much" larger than the NPV estimate of the other, and the analyst finds that the degree of error is "small," the decision maker can feel confident about the choice of the former over the latter. On the other hand, if the difference in the estimates of NPV were small relative to the degree of error, the decision maker might well choose the other, or neither, or comission a CBA of a third project, a further study of the original two, etc.

The important point is that the analyst should present to the decision maker as much information as possible in a format useful to the decision maker. The analyst must be careful to imply neither a greater nor lesser degree of confidence in this estimate than the data permit.

The variance of an estimate should supplement the mean as input to the decision making process. In contrast, the argument is often made that for public projects the mean alone is a sufficient decision input. This argument
rests on the premise that, at any given time, the government is engaged in a large number of similarly risky projects. Those that fail to meet expectations are balanced out by those which more than do so. Thus, the deviations net out, and society ends up with the mean values. The argument is theoretically sound, given the premise. However, there is no evidence that this premise is usually satisfied. Quite the contrary, experience would suggest that each project has a number of unique characteristics. The acceptance of the mean alone as a decision guideline is not appropriate as a general rule. In the discussion below, it is assumed that some measure of dispersion is relevant to the decision.

The analyst's attempts to gauge the degree of error in his estimates fall under the general term "sensitivity analysis." Conceptually, one can distinguish among three levels of sensitivity analysis: subjective estimate, selective sensitivity analysis, and general sensitivity analysis. The following sections discuss these levels in detail.
6. 2 Subjective Estimates

This is the least rigorous and quickest approach. Calling on his previous experience, intuition, "gut feelings," etc., the analyst determines some estimate of the actual degree of error. For example, after calculating the NPV of a project, he might state that this figure is subject to an error of plus or minus $10 \%$. Or he might state that the chance of the true NPV being more than $10 \%$ different from his estimate is less than one in twenty. There are many ways the analyst can state his error estimate. The point here is that the error estimate is obtained subjectively without recourse to formal calculation.

The advantages of subjective estimates are their ability to account for variability not reflected in objective measures, and (ordinarily) the ease with which they can be formulated. The drawbacks of the subjective approach are that the decision maker may place less confidence in such an estimate and that he may have difficulty defending his decision to critics. Further, the absence of a well-defined approach to error estimation, which necessarily occurs in subjective estimates, makes it impossible for anyone to trace the analyst's approach and to assess its reasonableness.

### 6.3 Selective Sensitivity Analysis

This method is an objective approach to error estimation in the sense that it is arrived at via an explicit series of calculations. The most common variant of selective sensitivity analysis goes as follows. The analyst selects a parameter in the NPV calculation which he feels is both subject to error and capable of significantly affecting the NPV calculation. He selects likely high and low (or best and worst) values for this parameter and computes the NPV with each. The decision maker is then presented with three NPV estimates for each project--high, medium and low--and for each parameter selected for sensitivity analysis.

For example, in a project to determine the economic viability of a wind energy system, the price of oil for the period $1980-1985$ may be an important parameter. The NPV for the project would be initially computed using all the "best" estimates for each parameter. Then, NPV would be computed using the high and low prices of oil, but with the same "best" estimate of other parameters. Thus, the decision maker will have information on how sensitive NPV is to the 1980-1985 price of oil. The same procedure, for example, could be carried out for the 1980-1985 demand for electricity.

The advantages of selective sensitivity analysis are derived from the objective nature and relative ease of computation. Its objectivity ensures that defenders and critics alike argue the merits of the analysis on wellspecified data and assumptions. The major difficulty with this approach is that it is usually unsuited for the analysis of anything more than a few parameters. This difficulty can be appreciated from the following.

For concreteness, let us suppose that the calculations for each of two alternative projects involve ten parameters, each a candidate for sensitivity analysis. A selective sensitivity analysis of the ten parameters would produce twenty NPVs for each project, in addition to the initial "best" estimate. The analyst must present to the decision maker a total of forty-two NPVs. Such a large number of figures may not aid the decision maker at all. In fact, the presentation of all NPV estimates might even violate the analyst's charge to present the decision maker with results in a format convenient for his use.

Even more important than format convenience, the twenty-one NPV's presented to the decision maker for each project omit a great deal of important information. For instance, the decision maker may wish to know the worst outcome he can reasonably expect. He might associate this outcome with the simultaneous realization of, say, seven worst outcomes and three medium outcomes on the parameters. (Recall that this information is not computed under selective sensitivity analysis. Each parameter is evaluated at its worst while every other is set at medium. No simultaneous "worsts" are calculated.) Furthermore, the decision maker would undoubtedly like to know the chance of such a worst outcome.

The reader may object that it is not difficult, in principle, to calculate all the combinations of worst, medium, and best for each parameter.

Then, for this relatively simple case of ten parameters, the decision maker would be presented with $3^{10}$ or 59,049 NPVs for each project! And one still has not incorporated information such as the chance of one of the bottom 1000 outcomes actually happening. These objections are all answered by the next approach.

### 6.4 General Sensitivity Analysis

This approach hinges on the derivations of a probability distribution of NPV outcomes. In this way, all of the information contained in the 59,049 individual possible NPV outcomes of the previous paragraph is captured in a format very convenient for the decision maker. At a glance he can tell for each project the chances of breaking even, of complete disaster or of overwhelming triumph. Since this approach is least likely to be familiar to the reader, it shall be sketched in greater detail than the previous approaches.

The $b_{i t}$ and the $c_{i t}$ which constitute the heart of the NPV calculation depend on a number of factors or parameters. Call these parameters the set

$$
\alpha=\left\{\alpha_{1}, \alpha_{2}, \ldots, \alpha_{K}\right\}
$$

For each specification of $\alpha$, a particular NPV will result.
In general, the members of $\underline{\alpha}$ will not all be independent of each other. For example, suppose $\underline{\alpha}$ contains the three parameters:

1985 price of oil, $\mathrm{P}_{\mathrm{o}}$,
1985 quantity of oil consumed, $Q_{o}$, and
1985 price of natural gas, $\mathrm{P}_{\mathrm{G}}$.
Suppose that high, medium, and low estimates are available for each. These high, medium, and low estimates are projected to occur with certain probabilities for each parameter. Table 6.1 summarizes the nature of the raw data. Since these are three parameters, each with three possible values, it might be

Table 6.1 Illustrative Occurence Probabilities

|  | $\frac{3}{c}$ Probability of Occurence |  |  |
| :---: | :---: | :---: | :---: |
| Farameter | High | Medium | Low |
| $\mathrm{P}_{\mathrm{o}}$ | $1 / 3$ | $1 / 2$ | $1 / 6$ |
| $\mathrm{Q}_{\mathrm{o}}$ | $1 / 6$ | $1 / 2$ | $1 / 3$ |
| $\mathrm{P}_{\mathrm{G}}$ | $1 / 3$ | $1 / 2$ | $1 / 6$ |



Figure 6.1 Illustrative demand curve for oil
thought that these parameters alone would give rise to $3^{3}$ or 27 NPV figures (for each specification of the remaining parameters in $\alpha$ ). This would not be correct. Since these three parameters are related, only certain of the 27 possibilities can really occur. The relation is clear from elementary economic reasoning. The price of oil and the quantity consumed are related by the demand curve for oil. The higher the price, the lower the quantity, and conversely. Figure 6.1 depicts the necessary relation. It is clear that rather than nine possible combinations of values for $P_{o}$ and $Q_{o}$, there are only three. These are

$$
\left(P_{0}, Q_{0}\right)=(H, L),(M, M), \text { and }(L, H) .
$$

In many ways, natural gas is a substitute for oil. Thus, if the price of oil were high, some users would switch to gas, increasing the demand for that product. The increased demand, of course, drives up the price of gas. The relation between the prices of oil and gas is illustrated in Figure 6.2. $\quad D_{L}$,


Figure 6.2 Illustrative demand and supply curves for gas, given various oil prices.
$D_{M}$, and $D_{H}$ are the demand curves for gas when the price of oil is low, medium, and high, respectively. Note that the market price of natural gas will tend to be high, medium, or low as the market price of oil is high, medium, or low, respectively, Thus, there are only three, not nine, possible relations between $P_{0}$ and $P_{G}$. These are

$$
\left(P_{0}, P_{G}\right)=(H, H),(M, M),(L, L)
$$

Finally, it is clear that instead of twenty-seven there are only three possible sets of values for all three parameters:

$$
\left(P_{0}, Q_{n}, P_{G}\right)=(H, L, H),(M, M, M),(L, H, L)
$$

The associated probabilities are $1 / 3,1 / 2$, and $1 / 6$, respectively, for these sets to occur. Note how the proper consideration of dependencies among parameters does affect the final NPV probability distributions. Proceeding as though the three parameters were not related, the outcome ( $H, M, H$ ) has a probability of $1 / 3 \times 1 / 2 \times 1 / 3=1 / 18$, and a corresponding NPV would be calculated. In fact, ( $H, M, H$ ) will not occur, so its correct probability is zero; no corresponding NPV is to be figured.

In general, the parameters in the set $\alpha$ must be separated into subsets on the following bases:

1) If any two parameters $\alpha_{i}$ and $\alpha_{j}$ are related, they must be in the
same subset.
2) If any two parameters $\alpha_{i}$ and $\alpha_{j}$ are not related, they cannot be in
the same subset.

It follows that each $\alpha_{i}$ must be a member of one, and only one, subsei.
It is likely that some subsets have only one $\alpha_{i}$ in them. Denote the subsets as $A_{1}, A_{2}, \ldots, A_{j}, \ldots, A_{j}$, where

$$
A_{j}=\left\{\alpha_{i}, \ldots\right\}
$$

and $J \leqq K$ (the number of original parameters).
Since the $\alpha_{i}$ 's in each $A_{j}$ are related, there are only certain combinations of values each $A_{j}$ can assume. The analyst must determine these combinations and the corresponding probabilities. Suppose the set $A_{j}$ can assume $\theta_{j}$ different configurations. Denote these configurations as

$$
\begin{aligned}
& A_{j 1}, A_{j 2}, \cdots, A_{j \theta_{j}} ; \text { and the corresponding probabilities as } P\left(A_{j 1}\right), \\
& P\left(A_{j 2}\right), \ldots, P\left(A_{j \theta_{j}}\right)
\end{aligned}
$$

Naturally, the probabilities over the values of any $A_{j}$ must sum to unity.
It might be useful at this point to summarize the discussion and clarify the notation via an example. Suppose

$$
\begin{aligned}
& \alpha=\left\{\begin{array}{lll}
\left.\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{6}\right\} \quad \text { and } \\
A_{1} & =\left\{\alpha_{1}, \alpha_{3}, \alpha_{4}\right\} \\
A_{2} & =\left\{\begin{array}{l}
\alpha_{2}, \\
\alpha_{5}
\end{array}\right\} \\
A_{3}=\left\{\alpha_{6}\right\}
\end{array} .\right.
\end{aligned}
$$

A1so,

$$
\begin{array}{lll}
A_{11}=(H, L, H) & A_{21}=(L, L) & A_{31}=(L) \\
A_{12}=(M, M, M) & A_{22}=(L, M) & A_{32}=(M) \\
A_{13}=(L, H, L) & A_{23}=(M, M) & A_{33}=(H) \\
& A_{24}=(M, H) &
\end{array}
$$

and

$$
\begin{array}{lll}
\mathrm{P}\left(\mathrm{~A}_{11}\right)=1 / 3 & \mathrm{P}\left(\mathrm{~A}_{21}\right)=1 / 10 & \mathrm{P}\left(\mathrm{~A}_{31}\right)=1 / 3 \\
\mathrm{P}\left(\mathrm{~A}_{12}\right)=1 / 2 & \mathrm{P}\left(\mathrm{~A}_{22}\right)=3 / 10 & \mathrm{P}\left(\mathrm{~A}_{32}\right)=1 / 3 \\
\mathrm{P}\left(\mathrm{~A}_{13}\right)=1 / 6 & \mathrm{P}\left(\mathrm{~A}_{23}\right)=4 / 10 & \mathrm{P}\left(\mathrm{~A}_{33}\right)=1 / 3 \\
& \mathrm{P}\left(\mathrm{~A}_{24}\right)=2 / 10 &
\end{array}
$$

In this example, $K=6, J=3, \theta_{1}=3, \theta_{2}=4, \theta_{3}=3$.
Returning to the development of general sensitivity analysis, the distribution of NPV can be done in either of two ways: complete enumeration or random sampling.

### 6.4.1 Complete Enumeration

When the total number of parameter combinations ( $\theta_{1} \times \theta_{2} \times \ldots x \theta_{J}$ ) is small, say less than 100 , then all the possible NPV's can be computed with their corresponding probabilities. (The use of an electronic computer would certainly ease matters.) It should be clear to the reader that the number of NPV calculations equals the number of parameter combinations. Each calculation will yield a NPV and a probability of observing it. Of course, the probability is calculated separately from the NPV. In terms of the foregoing example, one might begin by choosing $A_{11}, A_{21}$, and $A_{31}$. The parameter values are

$$
\left(\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{6}\right)=(H, L, L, H, L, L)
$$

In practice numbers replace the $H^{\prime} s, M^{\prime} s$, and L's. With all the parameters specified, the NPV is calculated--say NPV $=100$. Then

$$
P\left(A_{11}\right) \times P\left(A_{21}\right) \times P\left(A_{31}\right)=1 / 90
$$

For the example, 36 ( $3 \times 4 \times 3$ ) such calculations must be performed to yield each NPV and its corresponding probability. From these pairs, it is an easy matter to construct the cumulative probability density function of NPV for the project at hand. To do this, choose a number of arbitrary NPV figures, and add up the probabilities of the calculated NPV's which fall below each of those arbitrarily chosen figures. The results may be plotted to yield


Figure 6.3 Illustrative cumulative probability density function
a graphical display. It would look something like Figure 6.3. The interpretation is straightforward. The decision maker can tell at a glance that if he chooses this project

- the chance that its NPV turns out less than zero is about . 05 or one in twenty,
- there is no chance that NPV can be less than -1000 ,
- the chance of a positive NPV is 95 percent,
- the chance of an NPV over 5000 is zero,
- the chance of an NPV between 0 and 3000 is about 80 percent,
- the expected (or best single estimate of) NPV is 1500 , etc.

The NPV cumulative probability distribution is a powerful tool for the decision maker. It presents him with all the relevant information in a very convenient format. It explicitly shows what risks a decision entails.

### 6.4.2 Random Sampling

When the total number of parameter combinations ( $\theta_{1} \times \theta_{2} \times \ldots \times \theta_{J}$ ) is large, it is both impractical and unnecessary to compute the NPV and associated
probability for each combination. Instead, a random sample of, say, 100 combinations can be drawn from the total population of combinations. The random process is assurance that these 100 will be representative. The NPV cumulative probability function is then derived just as above. The interpretation is also the same as above.

The discussion of general sensitivity analysis has assumed that the analyst is able to assign meaningful probabilities to the sets of $\alpha_{i}^{\prime}$, i.e. to $A_{1}$, $A_{2}$, etc. Suppose that the analyst determines that for some subset of the $\alpha_{i}^{\prime}$, say $A_{j 1}$, $\ldots, A_{j} \theta_{j}$, he simply cannot assign probabilities which are anything more than totally arbitrary. Formally, this situation--where meaningful probabilities cannot be assigned--is called a situation of uncertainty, while the situation where probabilities are assignable is called a situation of risk. Previous discussion has dealt with risk, not uncertainty. It is difficult to say why probabilities can be assigned in one circumstance and not in another. To some extent, it is a matter of the analyst's judgement. However, it is probably fair to say that uncertain situations are usually relatively unique and/or involve guessing about conscious human choice. For example, an analyst might consider that the state of East-West relations over Berlin in 1980 is an uncertain situation. On the other hand, the state of midwest rainfall in 1980 is a risk environment. Meaningful probabilities can be assigned to the possible outcomes using past data.

The uncertainty of a parameter--the inability or unwillingness of the analyst to assign probabilities to the possible values of that parameter--does not destroy general sensitivity analysis, but it does complicate matters somewhat. The analyst must compute a NPV cumulative probability function for each value (say $H, M, L$ ) of the uncertain parameter or set of parameters. In terms of the previous example, suppose the analyst has uncertainty about the possible values
of $A_{1}=\left(\alpha_{1}, \alpha_{3}, \alpha_{4}\right)$. That is, he knows the possible values are (H, L, H), ( $M, M, M$ ), and ( $L, H, L$ ), but he cannot assign probabilities to these outcomes. Three NPV cumulative probability functions can be computed, each considering one of the uncertain values as given. This information is then presented to the decision maker who must subjectively determine the likelihood of the occurrence of the uncertain states and act accordingly. The analyst has provided the decision maker with as much information as possible in a digestable format. By admitting uncertainty, the analyst has not implied his calculations are more precise than they really are.

### 6.5 Choice of Sensitivity Analysis

The three levels at which sensitivity analysis can be performed has been discussed in this section. Which one should the analyst use? If the costbenefit analyst were not constrained by time and resources in conducting his CBA, general sensitivity analysis would be the recommendation for all but the simplest cases. It provides the most complete and reliable information in a digestable format for the decision maker. However, the time and resources available for a CBA are usually limited. In the absence of a specific charge by the decision maker, the analyst must determine the proper level of sensitivity analysis by an exercise of judgement.

## SECTION 7

## PERFORMING A COST-BENEFIT ANALYSIS

### 7.1 Introduction

The purpose of this section is to present an overall design for CBA and integrate the material of earlier sections into that design. A central theme of this section is the importance of planning the design, or charting the course of a CBA. Too often, the tendency is to plunge directly into gathering data and estimating benefits and costs with the hope that it will all fit together. In an undertaking as complex as CBA, this is not a wise course. Much effort is wasted and much remains undone when precise plans do not guide the analysis.

Another theme of this section is the analyst's interaction with the decision maker. The decision maker is the beginning and the end of the CBA cycle. Initially, the decision maker must communicate to the analyst a detailed description of the problem to be addressed and the nature of the information he desires, e.g., the scope of the sensitivity analysis or the emphasis of the social impact analysis. The analyst's design of the CBA will reflect, in large measure, the requirements of the decision maker. The completed CBA is finally used by the decision maker as an aid in making the requisite decision. CBA is an information-processing "machine." The decision maker's input to the analyst will affect the analyst's output to the decision maker. The better the problem is specified the more useful will be the final report to the decision maker.

Figure 7.1 presents a schematic representation of the major steps in CBA. Some of the steps have already been discussed in detail, the others are the primary subject matter of this chapter. The numbers adjacent to each block refer to that part of this section dealing with that topic.


Figure 7.1 Flow diagram depicting major steps in performing a cost-benefit analysis. Numbers in parenthesis refer to section in text dealing with the corresponding step.

Although defining the problem to be analyzed may appear to be an almost trivial task, any CBA veteran will testify otherwise. This first step gives direction to the remainder of the analysis. It is here that the decision maker plays a crucial role, communcating to the analyst precisely what he wishes to be done. It is the analyst's task to record these desires, and elicit whatever information is needed to exactly define the problem. While each project has its own unique features, many aspects of problem definition are common to most and though such a listing can never be complete, it forms a basic checklist for both the analyst and decision maker. A discussion of these aspects is given below.

### 7.2.1 Project and Scenario

A technical description and a detailed scenario definition of the projects to be analyzed are obviously important initial steps. The main point here is that explicit recognition should be given to all resource inputs and final outputs of the projects, and the calendar time in which they will occur. On the input side, these descriptions must include the types and amounts of resources (e.g., numbers of scientists, managers, clerical staff; various types of capital components for initiation, operation, and maintenance of the projects; amount and nature of land needed to site the facilities, etc.). On the output side, the time streams of each final good of the projects (e.g. electrical energy, miles of highway, retrained manpower, etc.) are equally important. The nature and physical dimensions of "externalities," (e.g., smoke, noise, water pollutants, etc.) must also be communicated to the analyst.

Often, some of this information will not be available. This lack of information is not detrimental to the analysis as long as this lack is
recognized and dealt with, not ignored. The usual ways of solving this information problem are either to perform a simultaneous "engineering" study to determine unknown technical values or to parameterize the unknown values in recognition that the final results will be conditional on the assumed parameter values.

### 7.2.2 Status-Quo Scenario

Similarly, a technical description and detailed scenario of the universal alternative--the status quo--should be constructed. Every project has an alternative, even if it is to "do nothing." For to "do nothing" implies a time stream of costs and benefits to society just as a project does. Of course, it's exactly this "do-nothing" or status quo scenario with which each project is compared. CBA focuses on how a project will change the status quo time stream of social well-being. Thus, only the differences between the status quo time stream and the with-project time stream are considered in CBA. The "good" differences are the benefits of the project, the "bad" differences are the costs. Since the difference that the project will make is of primal importance, it is essential to have the status-quo scenario with which to compare the project scenario. An example will clarify this need for a status-quo scenario.

Consider a project to provide electric energy using wind, i.e., "windmill" construction. Suppose the social cost of a windmill--the value of the resources used to build a windmill--is known. Are the benefits the value of the electricity produced? Not necessarily. It depends on the status-quo scenario. If, in the absence of windmills, conventional means of producing electricity would be expanded so that society will get the same amount of electric energy without as with windmills, the benefits would be in the value of fuel saved by conven-
tional power generators, not in the extra electricity. There would be no difference in electricity generated, but there would be a difference in the amount of oil, for example, that society could put to alternative uses. On the other hand, if the status-quo scenario provided less electricity than did the windmill project, at least part of the benefits of the project would be in the value of electricity produced by windmills.

### 7.2.3 Definition of Society

CBA attempts to assess social costs and social benefits; that is, CBA takes the public point of view. As the reader will recall from Section 4, the value of a project is the sum of its value to each member of society. Clearly, then costs and benefits depend on who is included in society. For projects at the national level, the usual definition is that society consists of all U.S. citizens. At the regional, state, and local levels, the operational definition of society is not so easily made. There are often benefit and cost spillovers (externalities) beyond the stipulated geographical bounds of the project. For example, a state-level manpower-training program has obvious spillover benefits--some persons who receive training will eventually migrate out of that state. Benefits will accrue to both residents and non-residents of the state. Which benefits are to be counted in the CBA? The most appealing (to economists) normative answer is that all benefits ought to be counted. However, there are any number of circumstances in which this will not be very palatable. If the training program were financed entirely by taxes on state residents, political realities might dictate that the benefits to the residents outweigh the costs, irrespective of whoever else gains. The point here is that the decision maker must define the "society" which the analyst is to employ. Almost inevitably, some uncounted effects will occur and spill over onto persons
not included in the CBA's society. When this spillover is apparent, the analyst should point it out to the decision maker. To reiterate, the decision maker is the final authority on the bounds of "society" for the purposes of the CBA.

### 7.2.4 Constraints on the Problem

It may be necessary that to be chosen, a project must satisfy a number of diverse constraints. Such constraints may be budgetary, legal, social, political, or institutional. These, of course, must be communicated to the analyst at the start of the CBA. This early communication will enable the analyst to quickly exclude alternative projects which obviously are not feasible. It is impossible to completely explore the scope of each type of constraint; however, an example of each will convey their spirit:
budgetary: The initial cost of the project cannot exceed $\$ X$ and annual operating costs cannot exceed $\$ Y$.
legal: Pollution caused by the project cannot exceed some set standards.
social: Benefits and costs of the project cannot be divided along racial lines.
political: Benefits and costs of the project cannot be inequitably divided among different political jurisdictions, e.g. states.
institutional: The project cannot usurp the powers of institution $X$ in favor of institution $Y$, e.g. place matters pertaining to the Department of Agriculture in the domain of the AEC.

Although the placement of a particular constraint in a particular category may be somewhat arbitrary, the important point is that each constraint be explicitly recognized to the extent possible and incorporated into the analysis. It is the decision maker's task to inform the analyst of all such constraints.

### 7.2.5 Direction of Social Impact Analysis

One can argue that, in principle, the analyst should have free rein
over the social impact analysis. After all, he must carefully describe all relevant non-quantifiable effects of the projects in an objective manner. However, the harsh realities of time and budgetary restrictions will of ten impede a completely thorough approach. Thus, when the analyst is forced to trade off one area of investigation against another, it is useful to be aware of the decision maker's preferences and needs.

Accepting the decision maker's direction in the social impact analysis should not undermine the integrity of the analyst's report. The previous paragraph may cause alarm in those who feel the decision maker often has biases and his influence will alter the neutrality of the CBA. Although the existence of bias is, of course, a possibility, the analyst must flatly state in his report to the extent desirable which areas have not been investigated, and also state his opinion as to whether such an investigation would affect the overall assessment of a project. In addition, he should state to what extent his choice of areas for social impact investigation was influenced by the decision maker. In this way, the decision maker may be accommodated without a sacrifice of CBA integrity.

### 7.2.6 Control Variables

Often, all the technical details of a project will not be initially specified by the decision maker. Rather, the analyst will be charged with choosing optimal values for some variables, such as scale, location, start-up time, number of installations, etc. In a strict sense, optimization falls outside the domain of CBA and generally into the domain of optimization methods. The variables to be optimized, if any, should be clearly distinguished from those to be parameterized. Ordinarily, the latter are outside the control of the decision maker (sometimes called "state variables") and the former are not.

However, the distinction is not always so clearcut and the decision maker and analyst should agree on which non-specified variables are to be optimized and which merely parameterized.
7.2.7 Discount Rate

As mentioned previously, the discount rate is best considered a policy variable, to be set by the decision maker. He may desire that a single rate be used, or he may request that several values be considered. Alternatively, he may wish critical values to be computed. The analyst must get this direction from the decision maker.

### 7.2.8 Time Horizon

The time horizon is also a policy variable, though it is not as volatile an issue as the discount rate. The decision maker must decide how far into the future that costs and benefits are to be projected and thus counted into the net present value of the project. Ordinarily, most costs of a public project are incurred in its early years, so a truncated time horizon has the effect of excluding more benefits than costs from consideration. Thus, a time horizon places a conservative bias on the NPV calculation but it should be realized that with time horizons of fifty years or more, the bias is very slight. The discounting process is such that values occurring fifty years or more in the future add little to present value. Clearly, the higher the discount rate chosen, the shorter the time horizon that need be considered.

### 7.2.9 Data Sources

Although source identification and data gathering are responsibilities of the analyst, it will often be the case that the decision maker, through his own investigations prior to commissioning the CBA, will have come across relevant data sources. The analyst, in the interest of saving time, should explore such possibilities before initiating his own searches.

### 7.2.10 Format of Results

Throughout these discussions, it has been stressed that the analyst's task is to present the decision maker with all the relevant information in a convenient format. Although it may not seem like an important point, the convenience of the format may well affect the extent to which the decision maker utilizes the CBA as a decision aid. Thus, the analyst should elicit from the decision maker his preferences regarding the scope of the sensitivity analysis, use of critical values, and what general level of "technical language" should be used in the report proper.

In summary, defining the problem is the first step in a CBA. It requires close cooperation and communication between decision maker and analyst. Insofar as it gives direction to the rest of the study, it should be treated as a major part of a CBA. Failure to invest time in problem definition almost invariably results in confusion and wasted efforts in the remainder of the study.

### 7.3 Designing the Analysis

Formally designing the cost-benefit analysis should be done during the early stages of a CBA before plunging into data collection and cost and benefit estimation. Six basic points are involved in carrying out the design and are discussed below.

### 7.3.1 The Problem Structure

Determining the analytic structure of the problem follows directly from defining the problem. The purpose here is to determine which measure (e.g. net present value on benefit-cost ratio) to employ in comparing alternatives. In Section 2.3 the relation between the structure of a problem and the decision measure to employ was discussed. The main aspects of structure
are the dependence or independence of projects, the type of constraints, and the variables to be optimized. At this stage of the design, the analytic structure of the problem should be written out as carefully as possible and all vagaries should be uncovered.

### 7.3.2 Preliminary Identification of Costs and Benefits

The identification problem was discussed at length in Section 3. Basically, there are two ways of discovering costs and benefits: searching for affected goods and services or searching for affected persons. In practice, it is useful to employ both of these approaches, remembering, however, that each is a different way of arriving at the same costs and benefits. That is, either the commodities or the persons approach is a good way to discover effects, but only one can be used to count a cost or benefit. Using both results in double counting. How are the affected commodities and persons to be discovered? A number of complementary ways can be used to suggest what interrelationships exist between the project and the rest of the economy:
a) Economic theory
b) Professional literature dealing with previous similar projects
c) The scenarios developed in defining the problem
d) Introspection
e) Brainstorming with colleagues
f) Interviews with interested persons, including the decision-maker.

Thus, the result of this step is a list of costs and benefits which are likely to be incurred by each project under consideration.
7.3.3 Assessment of the Listed Costs and Benefits

This assessment is with respect to validity and quantifiability. With regard to the former, the analyst must be wary of including transfer
payments or sunk costs as social benefits or costs. He must also be sure that true values are not being double counted. It must then be determined whether, to what extent, and in what dimensions each valid cost and benefit can be quantified. This determination requires a cursory survey both of data availability and of the potential of gathering new data.

### 7.3.4 Scope and Dimensions of the Quantitative Analysis

In principle, a CBA should deal with all. the costs and benefits of a project. Some of these will be quantified, the others treated in a qualitative fashion. It is not too great a departure from conventional usage to bring all the qualitative analysis under the umbrella term of social impact anlaysis. Of necessity, some costs and benefits such as intangibles can be treated only qualitatively. Among the quantifiable costs and benefits, some may not be quantified in the CBA because of time and budgetary restrictions. Of those which are quantified, some will be put in money terms and others in their own dimensions (incommensurables). However, by no means is there a well-defined boundary between incommensurables and the costs and benefits which have ready dollar values. It is probably best to consider the costs and benefits of a project as lying along a spectrum of "quantifiability," ranging from intangibles through incommensurables to market goods. Intangibles would include the project's effects on such things as social justice, social harmony, personal freedom, democracy, aesthetics, etc. These all involve values beyond the economic and do not exhibit even likely dimensions for measurement, much less actual numerical values. Incommensurables would include lives lost, injuries and illnesses sustained, national defense, other public goods such as recreation facilities, and some externalities. Evidently, incommensurables may involve economic or non-economic values. Their distinguishing characteristic
is that they may be readily quantified, but not in money terms. For example, measurements can easily be made of number of lives lost, number of work days lost due to illness or number of user-days of a recreation facility. Measurements can even be made of national defense as a probability of forestalling pre-emptive nuclear attacks, or as a percentage of population survival after an enemy's first strike. Of course, to a greater or lesser extent, these measurements are not easily converted into dollar values.

Market goods are agricultural products, textiles, electricity, auto servicing, etc.--any good or service exchanged through a market. The most importanc feature of a market good is the existence of a corresponding market price which, subject to the qualifications outlined in Section 4, directly measures social value in money terms.

Thus, with regard to a spectrum of "quantifiability," all non-quantifiable costs and benefits fall into the intangibles range and all quantifiable effects are in the incommensurable-market goods range. Only effects in the market goods range, however, are readily measured in money terms. There is no clearcut boundary between any of the ranges in the spectrum and it often happens that some cost or benefit will appear to lie somewhere between incommensurables and market goods. Such a cost will be readily measurable in non-monetary terms but will also appear convertible into a meaningful dollar value. As an example, such costs may be associated with recreation benefits, or losses due to illnesses or injuries. One of the major problems faced by the analyst is determining how far to go in converting apparent incommensurables into dollar values. Some observers would argue that the analyst should convert all effects into dollar values, even intangibles. The idea is simply that the NPV thus computed captures everything. This complete conversion virtually
obviates the role of the decision maker, since he could easily be replaced by a $3 \times 5$ file card containing such immutable rules as: If NPV $>0$, accept the project. This notion--total conversion into dollar values--has probably been the greatest source of criticism for CBA. Fortunately, the advocates of that notion seem to be waning in strength.

On the other hand, a CBA which fails to convert enough effects into dollars will not be a successful decision aid. For the decision maker will then be forced to compare projects on the basis of two- or three-dozen dimensions, a situation not too far removed from eyeballing raw data. Once again, then, how far is the analyst to go in converting seeming incommensurables into dollar values? Although there is no categorical answer, the decision maker can specify to the analyst those apparent incommensurables for which he can accept dollar conversions and those for which he cannot. The decision maker and the analyst can jointly determine the dimensionality of the results. In effect, with the technical aid of the analyst in elucidating relevant tradeoffs, the decision maker determines the cut-off point in the cost-benefit spectrum between effects usefully measured in dollars and those better measured in their own dimensions. This process would appear to be the only way the analyst can ensure that his approach to quantification will be acceptable to the decision maker in the sense that the results are credible and thus useful as a decision aid.

In brief, this discussion has centered on determining the scope and dimensionality of the quantitative part of the CBA. Implicitly, then, the breadth of the social impact analysis (the qualitative part of the CBA) is determined simultaneously. For whatever effects are not quantitatively analyzed must be qualitatively analyzed, at least cursorily. The factors affecting this
determination are portrayed in Figure 7.2. There is no denying that the analyst must exercise his own judgement in allowing each of these factors to influence his determination.

### 7.3.5 Choice of Sensitivity Analysis

The three broad levels of sensitivity analysis discussed in Section 6 were subjective, partial, and general. There it was pointed out that the choice of which level of sensitivity analysis to employ depends on the inevitable trade off between time and resources spent on one part of the CBA versus time and resources spent on another, and how this relates to the quality of the overall CBA. It was also mentioned that in certain circumstances mean values alone are sufficient to guide the decision maker, obviating the need for extensive sensitivity analysis. Finally, the desires of the decision maker must be considered. There is no point in developing extensive probability distributions of net present value if the decision maker will not use the information. Of course, the analyst should perform that analysis when the decision maker expresses the desire for such information. Once again, this issue must be decided by the good judgement of the analyst.
7.3.6 Determination of Data to be Collected

This flows directly from the discussions of sub-sections 7.3.4 and 7.3.5 above. Once the nature of the quantitative analysis is set and the type of sensitivity analysis which will be employed is known, the necessary data to accomplish these tasks is manifest. Essentially, the process of sub-section 7.3.4 determines the category of data needed (e.g., price of electricity in 1985) and that of 7.3 .5 determines whether point estimates are needed, or bounding estimates should be used like high and low values in addition to a medium "best" estimate, or whether corresponding probabilities of occurrence


ALL EFFECTS OF THE PROJECT


Figure 7.2 Factors affecting quantification of costs and benefits
need be sought out.

### 7.4 Collecting the Data

Although it isn't necessary to go into a detailed discussion on collecting data, a few common sense considerations deserve mention. Planning the format of the collected data is extremely important. The format should specify the number of significant figures for each entry and should allow easy access to any part of the data, and should be capable of quick updating. The data should be gathered from original sources when possible. Using original sources minimizes the risk of recording errors which creep into transcribed data. All the qualifications to the data should be accurately recorded. Finally, the sources of all data should be recorded for eventual reference in preparing footnotes and bibliography.

### 7.5 Performing the Analysis

Quantitative analysis was treated at length in Sections 4, 5 and 6. The essence of this task is the use of raw data and the economic theory of Section 4 to make good estimates of social costs and benefits. The identification of such costs and benefits was discussed in Section 3. If a thorough job of designing the analysis (discussed above) has been done, the analyst hopefully will encounter no major problems at this state. Performing a thorough job is not to say that every estimate will be precise, only that any lack of precision will be acknowledged either verbally, or in formal sensitivity analysis. The quantitative analysis includes finding "best" point estimates of the social value of a project along with a sensitivity analysis.

Performing the social impact analysis, defined to include an examination of non-economic effects, was discussed in Section 5. In this part of the analysis all non-quantified effects are brought out as clearly as possible.

As mentioned previously, some aspects may receive more extensive treatment at the expense of other aspects. There is no objection to this type of treatment so long as the relative importance of each effect is not obscured, and the analyst holds fast to a completely scientific (i.e. neutral) viewpoint. 7.6 Preparing the Results

Throughout Part I, three key points emerge time and again. These are
(1) that CBA depends on the proper identification and measurement of all project effects,
(2) that incommensurables and intangibles, which are those effects which are not susceptible to quantification or monetization, must be acknowledged and displayed as accurately as possible, and
(3) that CBA, ultimately, is an aid to the decision maker.

These three points provide, in a sense, the critical test of a CBA accounting scheme. Such a scheme must permit the comprehensive itemization of project effects and their corresponding quantification, the qualitative assessment of intangibles, and all in a format useful to the decision maker.

A CBA accounting scheme should also lend itself to the special demands which are often made on project analyses. These special demands include analyses of project impacts on regional development, income redistribution among income classes, the environment, and social values in general.

Figure 7.3 presents an accounting format which is designed to fulfill the foregoing requirements. All project effects with which the analyst has associated dollar values are listed under monetized effects. Here, the entries are generally descriptive. However, quantitative information can also be presented, as when the particular effect is an "incommensurable." For both benefits and costs, the national entries are analyzed into regional and income

note: Line 3 - Line 1 + Line 2
LINE 4 - LINE $1+$ LINE 2
Figure 7.3 CBA Accounting Work Sheet
class components. Line 1 summarizes the real direct effects of the project. Line 2 allows whatever income transfers are present to be displayed. Line 3 summarizes the monetary effects on a regional basis, and line 4 summarizes the effects by income class.

A general summary table, less detailed than the foregoing accounting format, is often very useful. The following figure serves the dual purpose of suggesting a format for a general summary table and succinctly reviewing for the reader what information the analyst must eventually provide the decision maker. The illustration assumes two projects are being compared (each of course, to the status-quo). Suppose there are two uncertain parameters which affect the results, $\alpha_{1}$ and $\alpha_{2}$. Suppose they are dependent and can jointly assume only the values (high, low) and (medium, medium). Recall that uncertainty is to be distinguished from risk. Under risk, even though a correct value is not known for sure, meaningful probabilities can be attached to the various possibilities. Under uncertainty, the analyst is unable to assign such probabilities. Thus, a "risky" parameter can be incorporated directly into a sensitivity analysis by weighting its various values by the probabilities. On the other hand, an uncertain parameter can only be used "conditionally" in a sensitivity analysis. These ideals were discussed more fully in the previous chapter.

## SECTION 8

## GLOSSARY OF COST-BENEFIT ANALYSIS TERMS

This glossary defines major terms frequently employed in cost-benefit analysis. It is not intended to be comprehensive and many of the less important terms used in this report have not been included. The number in brackets after each definition refers to the text page on which the term is discussed.

COMPETITIVE MODEL: An abstract model of a market economy which satisfies certain well-defined assumptions. It is the basic model of economic analysis. Economists have shown that an actual economy patterned after the competitive model will make the most efficient use of resources and make society as well off as possible (according to the Pareto criterion). If certain assumptions are made in addition to those characterizing the competitive model, it can be shown that observable market prices equal the shadow prices of the economy's goods and services

COMPENSATING VARIATION: In considering movement from one economic state to another, the maximum amount of money the individual would be willing to pay to make the move (if he favors the move), or the minimum amount he would accept as compensation for making the move (if he does not favor the move)

CONSUMER'S SURPLUS: The difference between what a consumer would be willing to pay for some good and the price of that good. Measures of consumer's surplus are derived from the consumer's demand curve, and are widely used in cost-benefit analysis when the project, being investigated will cause a significant price change in some good. Consumer's surplus is an approximation to the more technically proper compensating variation. In general consumer's surplus is not a proper measure of benefits.

COST: What must be given up to acquire or achieve something. Costs to individuals are often different than costs to society. This occurs when transfer payments or externalities are involved. Examples: Buying a used car is a cost to an individual but is not cost to society, since the transaction represents a transfer payment.
Operating a car is a greater cost to society than to the individual, since pollution is created. This is an external diseconomy.

COST-BENEFIT ANALYSIS (CBA): A systematic evaluation of a project to determine whether, and to what extent, its social benefits outweigh its social costs. Also, the various techniques used to perform the evaluation, such as shadow pricing and discounting. CBA draws heavily on the concepts and methods of economics

COST-EFFECTIVENESS ANALYSIS (CEA): A systematic evaluation of alternative approaches to achieving a specified goal. The object is to select the least cost approach. CEA is most useful when the benefits are not readily and meaningfully translated into dollar amounts

DEMAND: The schedule of the various quantities of some good which will be purchased at various prices during a specified period of time. The concept may refer to an individual or to the sum of all individuals--the market. Demand schedules may be represented in tabular, graphic, or equation form

DIRECT EFFECTS: Increased real value of output or real cost associated with a project

DISCOUNT RATE: Given some benefit (or loss) which will be incurred at some specified date in the future, the number which, when the future benefit (or loss) is discounted by that amount, makes that benefit (or loss) comparable to one incurred in the present. The number is usually specified as an annual rate. Example: Suppose $\$ 100$ is expected to be received immediately. If the discount rate is $10 \%, 10 \% \times \$ 100=\$ 10$ means the $\$ 100$ now is comparable to $\$ 110$ one year from now

ECONOMIC STATE (OF AFFAIRS): The distribution of utility, or satisfaction, among the members of society

EFFICIENCY: A characteristic of a part, or the whole, of an economic system. Efficiency prevails when, for a given amount of input, the greatest possible output is produced. Alternatively, efficiency prevails when, for a given amount of output, the least possible input is used to produce it

ELASTICITY: A measure of the responsiveness of quantity to price along demand or supply curves. It is defined as the percentage change in quantity divided by percentage change in price

EQUILIBRIUM: A state of balance between opposing forces. An economic equilibrium is a situation which is gravitated towards and, once achieved, remains. The most common application is market equilibrium, wherein the forces of supply and demand drive the market price to an equilibrium. At equilibrium, the price tends to remain constant unless disturbed by new forces

EQUITY: The "fairness" of the distribution of income, or utility, in an economic system. Since the concept inherently involves value judgements, there are no acceptable universal quantitative measures. Ordinarily, in CBA, the decision maker, when presented all the evidence, must subjectively determine whether reasonable equity standards are satisfied

EXTERNAL EFFECTS: See "externality."

EXTERNALITY: A factor which causes an individual or firm to become better or worse off, but over which that individual or firm has no control, and for which that individual or firm can be charged no fee (in the case of an external economy) or can exact no compensation (in the case of an external diseconomy). Pollution is an often cited external diseconomy

IMPERFECT COMPETITION: A term characterizing a market which is not perfectly competitive, such as monopoly, oligopoly, or monopolistic competition

INCOMMENSURABLE: A gain or loss which, while easily quantified in its own dimensions, is not readily translated into monetary terms. The classic example is the loss of human life. Number of lives lost is (usually) easily determined, but the associated monetary value is elusive

INDIRECT EFFECTS: The impact of a project on the rest of the economy. Indirect or secondary benefits are a form of external benefits. Their inclusion in cost-benefit analyses has been subject to violent attack in recent years. The logic of counting these benefits should be carefully constructed and justified in terms of the objectives of a project

INTANGIBLE: A gain or loss for which there are not apparent dimensions in whicl: to quantify the value of the gain or loss. Examples would include gains or losses in fields of aesthetics, personal freedom, soctal justice, international peace, or changes in the distribution of income

INTERNAL EFFECTS: The effects of a project which accrue directly or indirectly to the entity under study. They are the benefits (costs) which are "captured" ("suffered") by a project and clearly are included in a cost-benefit analysis.

MARGINALISM: A characterization applying to most forms of economic analysis in recognition of the fact that economic decisions are rarely "all or none" but rather "more or less." Thus, economic decisions are most often made "at the margin"

MONOPOLY: A market situation in which there is only one firm selling a product with no close substitutes. Also, the firm itself

NET PRESENT VALUE: A single number representation the value of a future stream of benefits and costs discounted to the present

NORMATIVE ECONOMICS: See "welfare economics"
OPPORTUNITY COST: Sometimes called "alternative cost." The value of the benefits foregone by choosing one course of action over another. As an aggregate measure, it is composed of individual shadow price valuations.

PARETO CRITERION: This is a criterion for judging an economic state which has achieved a high degree of acceptance among economists. It states that State One is (Pareto) superior to State Two if, in State One, no one is worse off than he would be in State Two and at least one person is better off. The problem with the Pareto Criterion is that it fails to be applicable to real situations wherein some persons are worse off, and some better off, in going from one state to another

PECUNIARY EXTERNALITIES: The financial effects of a project on other parts of an economy as felt through price changes for outputs or inputs. They are not generally included among the effects of a project because they do not reflect changes in the real production of goods and services and often would lead to the double counting of project benefits on costs

POSITIVE ECONOMICS: That branch of economics which describes, explains, and predicts actual economic phenomena. It is devoid of value judgements, saying nothing about whether or not given economic states of affairs are good or bad

POTENTIAL PARETO CRITERION: This is a decision criterion used in judging the superiority of an economic state. By this criterion, State One is judged socially superior to State Two if those who gain by the choice of one over two could compensate those who lose such that, if compensation were paid, the final result would be that no one would be worse off. This is the criterion most frequently used in cost-benefit analysis

PUBLIC GOOD: A good with two characteristics:
i) Non-Rivalry in Consumption
ii) Non-Excludability

The first means that, at least up to some point, the consumption of the good by one person does not diminish the amount available to another person. The second means that, once provided, it is impractical, or impossible, to exclude anyone from consuming the good. Examples of public goods include bridges, parks, national defense, and disease control

SCENARIO: An outline or synopsis indicating scenes, characters, plot, etc. This term has been adopted from theater use to dramatize the need for establishing and visualizing clearly the detailed nature of a project alternative

SECONDARY EFFECTS: See "indirect effects."
SENSITIVITY ANALYSIS: Given some relation $A=F\left(P_{1}, P_{2}, \ldots, P_{n}\right)$, where the $P^{\prime} s$ are parameters, the determination of the responsiveness in $Q$ to changes in the parameters. This is an important aspect of cost-benefit analysis, since values for some parameters must often be crudely estimated. This allows the analyst to determine how sensitive his conclusions are to his choices of parameter values.

SHADOW PRICE: The true economic value of a good, as measured by its ability to contribute to social well-being. The shadow price in economics is analogous to the dual variables of linear programoing. In a perfectly competitive economy, market prices would accurately reflect shadow prices. Shadow prices are the proper valuations to employ in cost-benefit analysis

SOCIAL IMPACT ANALYSIS: The attempt to identify all the significant direct and indirect effects of a proposed action on man's economic, social, cultural, political, and physical environment. The analysis attempts to assess the magnitude of each impact and its value. Through the process of valuation, an attempt is made to determine, as far as possible, whether the overall effect of the proposed action is socially favorable or not. S.I.A. also attempts to determine how detrimental effects can be circumvented. The analysis is an aid to the decision maker and should present as much information as possible in a digestable and useful format. Care must always be exercised to accurately convey the reliability limits of the analysis

SOCIAL OPPORTUNITY COST: What society must give up in order to accomplish some goal or achieve some end. It represents the true cost of a project

SOCIAL RATE OF TIME PREFERENCE: The discount rate at which society as a whole is willing to give up present consumption for future consumption. Although it cannot be observed in economic data and must be approximated, it is generally considered the correct discount rate for use in costbenefit analysis

SUPPLY: The schedule of the various quantities of some good which will be offered for sale at various prices during a specified period of time. The concept may refer to a single firm or the sum of all firms--the market. Supply schedules may be represented in graphic, tabular, or equation form

TECHNOLOGICAL EXTERNALITIES: Real consumption or production opportunity changes for other units in an economy which are due to a project. They represent changed social welfare, cannot easily be priced and are frequently incidental joint products. They are normally included in a cost-benefit analysis

TRANSFER PAYMENT: A shift in income from one person to another or from government to some person for which there is no corresponding increase in current production. Thus, transfer payments are financial transactions which are not reflected in national income or national product accounting statements

VALUE THEORY: That branch of economics which deals with explaining and predicting the values of goods, as such values are revealed in economic transactions. Value theory is associated with supply-demand analysis and marginalism

WELFARE ECONOMICS: That branch of economics concerned with measuring and improving individual and social well-being. It is based on explicitly stated value judgements, or criteria, by which economic states may be compared. The Pareto criterion is a widely used value judgement in welfare economics

WILLINGNESS-TO-PAY: The widely accepted measure of the value of some good to some individual. It is used for estimating the value of certain types of benefits, especially when market prices are not available

## PART III

## EXAMPLE APPLICATIONS OF

## THE USE OF WIND GENERATORS

## SECTION 9

INTRODUCTION

A small fraction of the solar energy falling on the earth each day is converted into surface winds, which in some areas are quite strong and provide a useful source of energy for performing mechanical work and generating electric power. Although windmills have been used more than a dozen centuries for grinding grain and pumping water, interest in large scale electric power generation has developed only over the past 50 years and is currently at high level due to the recent energy crisis [1].

### 9.1 Work to Date on Wind Power Systems

In order to estimate the potential of wind power systems it is necessary to identify candidate systems and obtain as much information as possible on their overall system performance characteristics, mechanical and electrical characteristics, estimated costs, etc. Following is a brief description of some of the perhaps more important works in the area of wind power systems. References to authors and titles of these works are listed at the end of this section.

In 1939 work was begun on a 1.25 NW wind power plant on Grandpa's Knob near Rutland, Vermont. Electricity was generated and delivered to the utility transmission grid in October, 1941, the first synchronous generation of power from the wind. The rotors and electric generator were mounted on a 110 foot tower and turned in any direction to face the wind. The two stainless steel blades weighed 7.5 tons each and swept a circle 175 feet in diameter with a rated speed of 28.7 rpm . Full power operation was achieved for wind velocities in excess of 30 miles per hour, which occurred about 70 percent of the time. Icing of the blades was not a problem since the
ice would break up during rotation. The total weight of the wind power generator was 250 tons, and the cost was slightly over one million dollars [2].

Partly because the project was rushed to completion in the days preceeding World War II, it was plagued with component failures. Replacements were especially difficult to obtain during the war. Finally, on March 26, 1945, the blade broke during a storm. Because of the limited financial resources of the company operating the plant, the generator was not repaired, but dismantled and removed from the site [3].

Based on the experience at Grandpa's Knob, the Federal Power Commission conducted a study of wind electric power generation for use with interconnected utility networks, and concluded that a power plant capacity between 5 and 10 Megawatts could make wind power economical. A 7.5 MW unit was designed using two-bladed propellers, similar to the propellers used on small airplanes. A separate design for a 6.5 MW plant used three-bladed propellers. A wind-driven d.c. generator provided power to a converter which produced synchronous a.c. power. The projected costs of these plants, in 1945 dollars, were $\$ 68$ per kilowatt of capacity for the 7.5 MW unit and $\$ 75$ per kilowatt for the 6.5 MW unit [4].

In addition to the Grandpa's Knob experiment, a variety of similar projects have been undertaken around the world. A 100 kW direct current wind turbine with a 30 meter propeller diameter was operated in the Soviet Union in 1931. In 1942 a three bladed propeller 50 kW a.c. plant was operated in Germany, and the next year a 20 kW generator using two 6 bladed 9 meter diamter propellers was tested in Berlin. In Denmark a 200 kW generator with a single 3 bladed propeller of 24 meter diameter has been operated,
and a 100 kW 15 meter three bladed turbine has been tested in England. Between 1961 and 1966 a 35 meter diameter, 100 kW double bladed wind turbine operated in West Germany, following tests with a 10 kW model. The power output of the 100 kW unit increased linearly from 10 kW at a wind velocity of 4 meters/sec to 90 kW at 9 meters/sec. The power out put was usually held to 90 kW for wind velocities greater than 9 meters/sec. The most spectacular European wind generator has been the 31 meter diameter, 800 kW generator built in France in 1958. This generator used a single threebladed propeller [5].

Another type of windmill known as the sail wing uses cloth sails on a wooden or tubular metal framework. These windmills are lightweight and cheap to construct, but require periodic maintenance of the cloth sails. Sherman [6] described work on an 8 meter diameter sail wing windmill erected on a small peanut and sesame farm in South India to lift soil and rock from the well being hand dug below the windmill. Sweeney and Nixon [7] have reported on the current developments of the sail wing concept at Princeton University.

Clews [8] reported on his home power system which uses a windmill to provide all his power for lights, appliances, TV, tools, etc. He uses a 2 kW generator for use as an emergency backup system in case of prolonged calm periods. The complete installation costs $\$ 2800$.

A 100 kW windmill generator was constructed,by the NASA Lewis Research Center with a 125 foot diameter rotor blade mounted on a 100 foot tower [9]. It is located at the Plumbrook test area at Sandusky, Ohio and began operation in mid 1975. The project is designed to determine the performance, operating characteristics and economics of windmills for the future generation of commercial
electric power. The rotor blades are located on the downwind side of the tower and the alternator and transmission are housed in the enclosure on top. The three-phase generator is expected to reach 100 kW output from an 18 mph wind. This is the first large wind energy system constructed in the United States in 30 years.

### 9.2 Scope of the Present Study

Although concepts and technologies associated with wind power generation exist today, there is a lack of economic justification for the use of wind as a source of energy and the utility of wind power systems is not well established. It is therefore highly desirable to determine the potential of wind power systems with emphasis on systems that supply reliable energy at a cost that is competitive with other energy systems.

The present study is not intended to be a complete cost-benefit evaluation for wind power systems. Cost-benefit analysis is inherently a project related decision making tool (i.e., accept or reject a proposed project, or select one or more of several alternate proposed projects). It is certain that there will be some wind energy projects which will be cost effective (e.g. on-site power generators at remote facilities), whereas certain applications of wind power may never be cost effective (e.g. wind powered transportation systems).

The following sections give example applications of the economic techniques developed in Part II to cost-benefit evaluations for two example wind energy projects. Both examples are of the type where wind energy is used only as a fuel saver, not to augment electric generating capacity. These two cases are examples of the type of projects for which wind energy may find application.

The scope of this program did not include the design of a windmill or a wind generator system. Emphasis was placed on determining not how much does a windmill system cost but rather what ranges of cost and scenario characteristics correspond to positive net present values.

With the economic tools developed in Part II, the wind assessment data in Section 12, and the examples developed in the following sections, the decision maker should be able to conduct a cost-benefit evaluation of any actual wind energy project under consideration.

## REFERENCES

1. Williams, J. R., Solar Energy Technology and Applications, Ann Arbor Publishers, May 1974.
2. Wilcox, Carl, "Motion Picture History of the Erection and Operation of the Smith-Putnam Wind Generator," Wind Energy Conversion Systems Workshop Proceedings, NSF/RA/W-73-006, pp. 8-10, December 1973.
3. Smith, B. E., "Smith-Putnam Wind Turbine Experiment," Wind Energy Conversion Systems Workshop Proceedings, NSF/RA/W-73-006, pp. 5-7, December 1973.
4. Lines, C.W., "Percy Thomas Wind Generator Designs," NSF/RA/W-73-006, pp. 11-18.
5. Noel, J.M., "French Wind Generator Systems," Proc. Wind Energy Conversion System Workshop, NSF/RA/W-73-006, December 1973.
6. Sherman, M.M. "The Sail Wing Windmill and Its Adaptation for Use in Rural India," Proc. Wind Energy Conversion System Workshop, NSF/RA/W-73-006, December 1973.
7. Sweeney, T.D. and W. B. Nixon, "An Introduction to Princeton Sailwing Windmil1," Proc. Wind Energy Conversion Systems Workshop, NSF/RA/W-73-006, pp. 70-73, December 1973.
8. Clews, H.M., "Wind Power Systems for Individual Applications," Proc. Wind Energy Conversion System Workshop, NSF/RA/W-73-006, December 1973.
9. Puthoff, R.L., and P. J. Sirocky, "Preliminary Design of a 100 kW Wind Turbine Generator," NASA-TM-X-71585, 1974.

## APPROACH

This section describes the overall approach utilized in applying the cost-benefit methodology to the analysis of wind power systems. The approach adopted for the example application of the methodology consisted of the following activities:
A. Surveying of the available wind data and wind power system information;
B. Developing models which quantitatively described
(1) wind distributions,
(2) wind power systems,
(3) conventional electric power generation systems, and
(4) the cost and benefit differences between the conventional systems and wind power systems;
C. Applying the cost-benefit methodology to compare a conventional (a specified baseline) electrical energy generation system with systems which included wind power generators.

Figure 10.1 illustrates the overall approach to the application of the costbenefit method to wind power systems.

Surveying the available wind data and information of wind power systems established a data base from which subsequent decisions could be made regarding system definitions, the structure of quantitative models, and the values of model parameters. This survey identified wind data which were available and accessible, and it permitted the setting of realistic objectives for the collection and analysis of the data.


Models were developed which quantitatively described wind statistics, wind-wind power system relationships, and benefit-cost differences between a scenario of conventional electric power generation (a baseline scenario) and a scenario which included electric power generated by the wind. Table 10.1 summarizes the models and model parameters. These models were necessary in order to precisely state the assumptions made, and to provide a consistent framework for analyzing alternative systems and scenarios. Additionally, the models permit iterative calculations to determine the sensitivity of the results to variations or changes in the values of parameter and variable. This sensitivity analysis is necessary because many of the quantities are not known with certainty. Some parameters, such as maintenance costs and investment costs, have no values established because of the lack of large scale production and operational data on wind power systems.

The selected approach specified two different wind power systems. The detailed specification is necessary in order to illustrate the application of cost-benefit analysis methodology and two systems were selected in order to illustrate different wind power applications:

System I, the "Macro" system application, consists of wind power units in a regional network which is linked with the electric transmission grid. No energy storage is postulated, thus the wind power network can furnish power to the utility grid only when the wind speed exceeds the cut-in speed of the units. The purpose of this system is to act as a "fuel saver," reducing the amount of fuel consumed by conventional electric power plants. The detailed description of System $I$ and cost-benefit assessment of its application are given in Section 14.

TABLE 10.1
SUMMARY OF MODELS DEVELOPED

| NAME | PRIMARY INPUTS | PRIMARY OUTPUTS |
| :--- | :--- | :--- |
| Wind Speed <br> Model | Wind Data | 2 parameter descriptive model <br> of probability distribution of <br> wind speed at particular <br> heights and locations |
| Wind Power | Cut-in Speed, rated <br> speed, feathering speed, <br> height, and rated power <br> of wind generator; two <br> parameters of the wind <br> speed distribution | Plant Factor, Power Output |

System II, the "Micro" system application, consists of wind powered units which generate electricity for an industrial application, a chemical process which is energy intensive and requires large amounts of electrical energy in one of the process steps. The purpose of this system is to reduce the cost of energy to the industrial plant and possibly to reduce its dependence on the public utility for electric power. The system, detailed scenarios of potential applications, and the cost-benefit assessment of system use are described in Section 15.

## CATEGORIZATION OF WIND POWER SYSTEMS

The purpose of this task is to identify the categories of wind power systems which might have significant positive potential especially for largescale utilization. The objective is a list of system options, categorized in a matrix classification scheme using (1) alternative technologies and (2) alternative end uses of the system product as the two classification dimensions. This classification scheme permits a comprehensive overview of alternative systems which either have been developed or might be developed. The matrix format simply furnishes a convenient framework for presenting the background material on wind systems.

### 11.1 Applications

As shown on the vertical, left side of the matrix (Table 11.1), wind system applications were classified under three major headings. Electric power generation has received considerable emphasis recently and may be economically feasible now or in the near future. Mechanical power is probably the oldest application of wind energy systems and was used extensively by the early Chinese and Persian civilizations. Special applications such as the production of hydrogen gas or oxygen gas, probably through the intermediate steps of electrolysis and electrical energy generation, are not necessarily independent of the other categories, but this is included as a separate category because of the relatively distinct possible applications of the system outputs.

Electrical power generation may be further categorized as
(1) an isolated system, furnishing power to a particular location;
TABLE 11.1

## MATRIX CATEGORIZATION OF WIND POWER SYSTEMS

(a) Extractor/Momentum Exchange

| APPLICATION |  | EXTRACTOR/MOMENTUM EXCHANGE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Vertical <br> Axis Rotor | Propeller-type Horizontal Axis Rotor | Sail Wing Horizontal Axis Rotor | Turbine-type Horizontal Axis Rotor | Other |
| Electric <br> Power Generation | Isolated, Individual System | 5, 29, 32 | $\begin{aligned} & 10,16,21,24, \\ & 15,25,27,30, \\ & 31 \end{aligned}$ | 6, 7 | 22 |  |
|  | DC or Synchronous AC Power into Network, No Storage/Firm Capacity | 5 | $\begin{aligned} & 1,2,3,14, \\ & 23,29 \end{aligned}$ | 7 | 8 | 19 |
|  | DC or Synchronous AC Power into Network, with Storage/Firm Capacity |  | 12, 13 |  |  | 19 |
| Mechanical | Pumping Water - <br> Irrigation |  | 9, 11 | 9 |  |  |
|  | Pumping Water - <br> Energy Storage |  |  |  |  |  |
|  | Compressing Gas |  |  |  |  |  |
|  | Process Power - e.g., Grinding Wheel Power |  | 9 |  |  |  |
| Special <br> Applications | $\mathrm{H}_{2}$ Production | 32 | 20 |  |  |  |
|  | $0_{2}$ Production |  | 20 |  |  |  |

TABLE 11.1 (cont.)
(b) Support Structure

| APPLICATION |  | SUPPORT STRUCTURE |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Single Pole | Tower <br> Frame | Billboard Frame | Tower and Cable Suspension |
| Electric <br> Power <br> Generation | Isolated, Individual System | $\begin{aligned} & 3,6, \\ & 31 \end{aligned}$ | $\begin{aligned} & 3,10,16,21, \\ & 22,24,25,27, \\ & 30,15 \end{aligned}$ | 3, 26 | 5, 29, 32 |
|  | DC or Synchronous AC Power into Network, No Storage/Firm Capacity | 3, 8 | $\begin{aligned} & 1,3,14,23, \\ & 28,29 \end{aligned}$ |  | 5 |
|  | DC or Synchronous AC Power into Network, with Storage/Firm Capacity |  | 12 |  | 13 |
| Mechanical | Pumping Water Irrigation |  | 9 |  | 11 |
|  | Pumping Water - <br> Energy Storage |  |  |  |  |
|  | Compressing Gas |  |  |  |  |
|  | Process Power - e.g., Grinding Wheel Power |  | 9 |  |  |
| Special <br> Applications | $\mathrm{H}_{2}$ Production |  | 20 |  | 32 |
|  | $\mathrm{O}_{2}$ Production |  | 20 |  |  |

TABLE 11.1 (cont.)
(c) Convertor

| APPLICATION |  | CONVERTOR |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $\overline{\mathrm{DC}}$ Generator | AC Generator | Mechanical |
| Electric <br> Power <br> Generation | Isolated, Individual System | $\begin{aligned} & 10,15,16, \\ & 22,31,32 \end{aligned}$ | $\begin{aligned} & 10,21,24, \\ & 26,27,30 \end{aligned}$ |  |
|  | DC or Synchronous AC Power into Network, No Storage/Firm Capacity | 4, 28, 29 | $\begin{aligned} & 1,4,14,8, \\ & 19,23,29 \end{aligned}$ |  |
|  | DC or Synchronous AC Power into Network, with Storage/Firm Capacity | 12, 13, 19 |  |  |
| Mechanical | Pumping Water Irrigation |  |  | 9, 11 |
|  | Pumping Water - <br> Energy Storage |  |  |  |
|  | Compressing Gas |  |  |  |
|  | Process Power - e.g., Grinding Wheel Power |  |  | 9 |
| Special Applications | $\mathrm{H}_{2}$ Production | 20, 32 | 20 |  |
|  | $\mathrm{O}_{2}$ Production | 20 | 20 |  |

TABLE 11.1 (cont.)
(d) Storage Mechanism

| APPLICATION |  | STORAGE MECHANISM |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Batteries | Compressed Gas | Pumped Hydro | Hydrogen Gas |
| Electric <br> Power Generation | Isolated, Individual System | 10, 31 |  |  | 20 |
|  | DC or Synchronous AC Power into Network, No Storage/Firm Capacity | NA | NA | NA | NA |
|  | DC or Synchronous AC Power into Network, with Storage/Firm Capacity | 12, 13, 19 |  |  |  |
| Mechanical | Pumping Water Irrigation |  |  |  |  |
|  | ```Pumping Water - Storage``` |  |  |  |  |
|  | Compressing Gas |  |  |  |  |
|  | Process Power - e.g., Grinding Wheel Power |  |  |  |  |
| Spectal Applications | $\mathrm{H}_{2}$ Production |  |  |  |  |
|  | $\mathrm{O}_{2}$ Production |  |  |  |  |

TABLE 11.1 (cont.)
(e) Delivery

| APPLICATION |  | DELIVERY |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Electrical Transmission <br> Lines, Cables | Pipes | Channels |
| Electric <br> Power Generation | Isolated, Individual System | $\begin{aligned} & 3,5,15,16,20,21,22, \\ & 24,26,27,29,30,31, \\ & 32 \end{aligned}$ |  |  |
|  | DC or Synchronous AC Power into Network, No Storage/Firm Capacity | $\begin{aligned} & 1,2,3,4,5,8,14,19, \\ & 23,28,29 \end{aligned}$ |  |  |
|  | DC or Synchronous AC Power into Network, with Storage/Firm Capacity | 12, 13, 19 |  |  |
| Mechanical | Pumping Water Irrigation |  | 11 | 9 |
|  | $\begin{aligned} & \text { Pumping Water - } \\ & \text { Storage } \end{aligned}$ |  |  |  |
|  | Compressing Gas |  |  |  |
|  | Process Power - e.g., Grinding Wheel Power |  |  |  |
| Special Applications | $\mathrm{H}_{2}$ Production |  | 20, 32 |  |
|  | $\mathrm{O}_{2}$ Production |  | 20 |  |

(2) a DC or synchronous AC generation system which furnishes power into the electrical network on an interruptible basis. This application is strictly a "fuel saver," and no energy storage is included; and
(3) a DC or synchronous AC power generation system which furnishes power into the electrical network on a firm basis. This application increases the firm capacity of the network system and includes energy storage.

Mechanical power generation may be further categorized as
(1) pumping of water for irrigation purposes:
(2) pumping of water for energy storage, either for hydroelectric power generation as a later step or as potential energy for mechanical power systems;
(3) compressing gas for stored mechanical energy; and
(4) process power, such as power for grinding wheels, cane crushers, mechanical threshers and winnowers, etc.

### 11.2 Technology

The horizontal dimension of the categorization matrix represents the alternative technologies which might be utilized in a wind power system. A complete wind power system consists of the following subsystems and major components:
an extractor/momentum exchange mechanism (Table 11.1a),
a support structure (Table 11.1b),
a convertor; e.g., mechanical to electrical energy (Table 11.1c)
an (optional) energy storage mechanism (Table 11.1d), and
a delivery system (Table 11.le).
These major components and subsystems fulfill essential functions (energy storage is an optional function), and alternative technologies may be utilized to fulfill these functions.

The "momentum exchange" function is to extract power from the wind. Typically, this function is accomplished using some form of mechanical mechanism rotating about a vertical or horizontal axis: vertical axis rotors (Savonius "S" rotor, Darrieu rotor), propeller-type horizontal axis rotors, sail wing type horizontal axis rotors, turbine-type horizontal axis rotor, and other types of mechanisms (e.g., the University of Montana design of cars "sailing" on an oval track).

The "support structure" function is to support the momentum exchange mechanism (and perhaps the convertor mechanism). Support structures may be a single pole (guyed or free-standing), a space-frame tower, a billboardtype frame (usually with multiple rotors), or a tower-cable suspension arrangement.

The "convertor" function is to transform the mechanical motion of the momentum exchange mechanism into electrical energy or into other forms of mechanical energy. The convertor typically is a mechanical (e.g., rotary-to-1inear convertor) or an $A C$ or $D C$ electrical generator.

The "storage mechanism" function is to store the energy obtained from the wind for later use. Electrical storage batteries, compressed gas, pumped water (hydroelectric) and hydrogen gas production are alternative methods for storing wind-derived energy.

The "delivery system" function is to convey the wind-derived power from the generation site to the utilization site. Electrical energy will typically be conveyed via electrical cables or transmission lines, pipes or channels (e.g., canals) may be used to transport pumped water.

### 11.3 Approach

The project team contacted researchers known to be investigating wind energy systems, contacted known supporters of wind energy research, and utilized literature on wind energy identified through manual and computer searches. No attempt was made to perform a comprehensive 1iterature survey, since wind energy systems have been in existence for many years and the literature on windmills and wind power is extensive. However, the approach taken resulted in a meaningful overview of wind power systems, particularly of recent developments and current concepts of wind power.

Each distinguishable and separate wind power concept identified through the personal contacts and through the literature was examined to determine (1) the application of the system and (2) the technology used by the system. Each concept was identified by a number and brief reference to a complete bibliographic citation (Table 11.2, "Key to Categorization"). The numbers, each representing a different wind energy concept, were placed in the matrix (Table 11.1) whose elements represent different application--technology combinations. The resulting matrix displays the relative concentration of interest, as evidenced by recent research and literature, in different wind power system concepts. The matrix also can be used as a reference guide to research and literature for a particular application--technological concept; by looking up the numbers found in the matrix element of interest, appropriate descriptive reports (or personal contacts) are identified for use in further investigations.

The project team investigated wind system costs and performance through discussions with knowledgeable researchers and through literature scanning.

TABLE 11.2

> KEY TO WIND SYSTEM CATEGORIZATION MATRIX OF TABLE 11.1 (Complete Citations given in "References")

| Matrix Number | Brief Identification | Reference Number |
| :---: | :---: | :---: |
| 1 | Smith-Putnam (Grandpa's Knob, Rutland, Vt.) NSF p. 5, 0. 8; lecture on alternatives p. 14 | 10 |
| 2 | Various designs by Thomas $2-10 \mathrm{MW}$ units NSF pp. 11-18 | 10 |
| 3 | Past European installations NSF Pp. 19-22 | 10 |
| 4 | Cyclone D-30 NSF p. 23, 33 | 10 |
| 5 | Aerogenerator Arrays NSF p. 53 (Hewson) | 10 |
| 6 | Princeton Sail Wing Sweeney NSF p. 70; personal communication | 10,11 |
| 7 | Grumman Sail Wing System (based on Princeton Sall Wing design) William Carl, personal communication, Lindsley | 2, 8 |
| 8 | Hughes, William (Oklahoma State) personal communication, Lindsley (Pop. Sci.) | 4, 8 |
| 9 | Windmills in India NSF pp. 75-77 | 10 |
| 10 | Jacobs Generating Plants NSF pp. 155-157 | 10 |
| 11 | Barbadox irrigation NSF p. 160, 205 | 10 |
| 12 | Quick wind generator NSF pp. 166-169 | 10 |
| 13 | Electro wind generator NSF pp. 167-169 | 10 |
| 14 | French systems NSF Pp. 186-196 | 10 |
| 15 | 1-kW Quick-Barbados NSF p. 159 | 10 |
| 16 | 9-kW Andreau NSF p. 159 | 10 |
| ** 17 | Haiti pump NSF p. 160 | 10 |

```
TABLE 11.2 (cont.)
```

| Matrix Number | Brief Identification | Reference Number |
| :---: | :---: | :---: |
| ** 18 | Montreal-powers house NSF p. 160 | 10 |
| 19 | Cars on track NSF p. 177; Lindsley (Pop. Sci.) | 8, 10 |
| 20 | ```OWPS Electrolyzer Plant Heronemus, 1972; Wolf, 1973``` | 14 |
| 21 | Design of 100 kW Turbine Generator Puthoff and Sirocky | 7 |
| 22 | Honeff's power plant Juchem (NASA-TT-F-15860) | 6 |
| 23 | ```Stoetten power plant Gross (NASA- TT-F-15855)``` | 3 |
| 24 | J. Jowl power plant ( 45 kW ) Gross (NASA-TT-F-15855) | 3 |
| 25 | Gedser power plant ( 200 kW ) Gross (NASA-TT-F-15855) | 3 |
| 26 | Orkney Islands power plant ( 100 kW ) Gross (NASA-TT-F-15855) | 3 |
| 27 | Cherbourg power plant ( 130 kW ) Gross (NASA-TT-F-15855) | 3 |
| 28 | ```Various windmills in Denmark (NASA-TT-F 15868)``` | 12 |
| 29 | Bogoe power plant Rangi, et al | 9 |
| 30 | ```Vester-Egesborg experimental (NASA- TT-F-15439)``` | 5 |
| 31 | $\begin{aligned} & \text { Shell offshore platforms } 0 i 1 \& \text { Gas J.v. } \\ & 72 \text { N } 36,9 / 9 / 74, \text { p. } 96 \end{aligned}$ | 1 |
| 32 | NASA 15 ft . dia. windmill Walters-Mech. Eng. V. 96 N. 4, 4/19/74, pp. 55-65 | 13 | Eng. V. 96 N. 4, 4/19/74, pp. 55-65

No detailed, original research was undertaken on either wind system design, system costs, or system technical performance.

### 11.4 Results

Table 11. 1 illustrates the results of the categorization of wind power systems in the application--technology matrix. Thirty-two separate wind system concepts were identified and categorized. As shown in the table, most of the categorized concepts are intended to be applied for the generation of eletrical energy, either as an independent system or as a part of an electrical utility system. Particular technologies for the system components also exhibited significantly higher interest than other technologies; propeller-type, horizontal axis rotors; tower frame support; both $A C$ and $D C$ generators; and electrical cables/transmission lines for transporting the energy.

As will be seen in Section 12, wind system performance depends on several factors, and the following scheme generally appears to be adequate for describing performance/wind/system design relationships for electrical power wind generators. A design typically will specify a cut-in speed ( $\mathrm{V}_{\mathrm{o}}$ ), a rated speed $\left(V_{1}\right)$ and a rated power $\left(P_{R}\right)$, and a feathering speed $\left(V_{2}\right)$. The methodology described in Section 12 on wind analysis can be used with wind data and these four system parameters $\left(V_{0}, V_{1}, V_{2}\right.$, and $\left.P_{R}\right)$, as inputs to determine the annual energy output and the plant factor (ratio of average power output over the year to the rated power of the system). Because of the power available in the wind is proportional to the cube of the velocity and the power output of an aerogenerator typically is designed to be constant between the rated speed and the feathering speed, the output of a wind system
is more sensitive to the value of the rated speed than to the cut-in speed. (These issues are discussed in more detail in the next section on wind analysis.)

From the overview of the literature, it was determined that there $\otimes$ were no current values for wind system and component costs, with the exception of relatively small ( $\leq 10 \mathrm{~kW}$ capacity) units. Costs of large aerogenerator systems and components are not well established for current technologies, and it was judged that costs of previous designs (e.g., the Grandpa's Knob generator) could not be meaningfully extrapolated to current dollars.

## REFERENCES

1. Anon., "Shell Tests Windmill Generators Offshore." Oil Gas J. V72 N36 Sept. 9, 1974, p. 92 (31).
2. Car1, William (Grumman). personal communication with R. M. Mason July 22, 1974 (7).
3. Gross, A. T. H.; "Wind Power Usage in Europe," Techtran Corp., Glen Burnie, Md., Aug. 74, Report No. 18, NASA-TT-F-15855, p. 19 (23-27).
4. Hughes, William L. (Oklahoma State University), personal communication with R. M. Mason July 26, 1974 (8).
5. Jewel, J., SEAS "Examination of Windpower in 1952. Change of a Wind Power Mill from DC to AC Production," Linguistic Systems, Inc., Cambridge, Mass., July 1974. Report No. 18, NASA-TT-F-15439, p. 19 (30) .
6. Juchem, P., "Are Wind Driven Power Plants Possible?" Techtran Corp., Glen Burnie, Md., Aug. 1974, Report No. 18, NASA-TT-F-15860; 10 p. (22).
7. Puthoff, A. L., Sirocky, P. J., "Preliminary Design of a 100 kW Turbine Generator." Vational Aeronautics and Space Administration, Lewis Research Cel.cer, Cleveland, Ohio, 1974. Report No. 18, NASA-TM-X-71585, E-8037, 22 p. (21).
8. Lindsley, E. F., "Wind Power," Popular Science July 1974, pp. 54-59, 124, 125 (7, 8, 19).
9. Rangi, R. S., South P., Templin, P. J., Nat1. Aeronaut Estab1, Ottawa, Ont., "Wind Power and the Vertical Axis Wind Turbine Developed at the National Research Council," Natl Res. Conne Can, Div. Mech. Eng., 2 Bull N2 1974 p 1-14 (29).
10. Savino, Joseph M. (Ed.) Wind Energy Conversion Systems - Workshop Proceedings (June 11-13, 1973), Washington, D.C.; National Science Foundation/National Aeronautics and Space Administration, NSF/RA/ W-73-006, December 1974 (1-6, 9-19).
11. Sweeny, Thomas E. (Princeton University), personal communication with R. M. Mason July 22, 1974 (6).
12. "Utilization of Wind Energy in Denmark," Scientific Translation Service, Santa Barbara, California, Sept. 1974, Report No. 18, NASA-TT-F-15868, 16 p. (28).
13. Walters, Samuel, "Power from Wind." Mech Eng V96 N4 April 1974, pp 55-56 (32).
14. Wolf, Martin, Univ, of Pennsylvania, Philadelphia, "Potential Impacts of Solar Energy," IEEE Electron and Aerospace System Convenction, Rec, Washington, D.C., Sept. 17-19, 1973, pp. 95-106. Published by IEEE (73 CHO 783-1 AES), New York, 1973 (20).

## WIND DATA COLLECTION AND ASSESSMENT

Several attempts have been made previously to estimate the average wind power potential for the United States. Two examples of these estimates are those of Thomas [1] and Reed, et al. [2]. Thomas presented a map of isopleths (constant value contours) of mean wind speed $\overline{\mathrm{V}}=:\langle\mathrm{V}\rangle$ for the continental U.S., while Reed, et al. presented isopleths of mean wind energy density $\left(P_{m}=1 / 2 \rho\left\langle V^{3}\right\rangle\right.$, where $\rho$ is the air density). Although these types of presentation are good for determining which areas of the country are relatively good or bad for wind power applications, they are not sufficient for making detailed estimates of power output from specific wind generators. However, the desired information can be obtained easily if the probability distribution (or probability density function) of wind speed $p(V)$ is known. With $p(V)$ known, the mean wind speed can be calculated from

$$
\begin{equation*}
\bar{V}=\langle V\rangle=\int^{\infty} V p(V) d V \tag{12.1}
\end{equation*}
$$

and the mean wind energy density can be evaluated by

$$
\begin{equation*}
P_{m}=1 / 2 \rho\left\langle V^{3}\right\rangle=1 / 2 \rho \int_{0}^{\infty} V^{3} p(V) d V \tag{12.2}
\end{equation*}
$$

More importantly, however, the average power output $\overline{\mathrm{P}}$ of a wind generator, whose power output as a function of wind speed is some known function $P(V)$, can be compiled by

$$
\begin{equation*}
\bar{P}=\int^{\infty} P(V) p(V) d V \tag{12.3}
\end{equation*}
$$

The output power function $P(V)$ for most wind generator systems can be characterized as zero up to some speed $\mathrm{V}_{0}$, known as the cut-in speed, then increasing linearly (or with some curvature, which may easily be approximated as parabolic) up to the rated power output $P_{r}$ at some speed $V_{1}$, known as the rated speed. Above the rated speed the power output remains level at the value $P_{r}$ until some speed $V_{2}$, known as the feathering speed, at which point the system is shut down (e.g., by feathering the blades) to avoid damage under high wind loads (See equation ( $B-1$ ) and Figure $B-1$ in Appendix $B$ ).

Routine wind data measured at a number of National Weather Service stations throughout the U.S. is adequate to evaluate details of the wind speed probability distribution function at these sites. However, for ease in data manipulation for the large number of sites required on a nationwide wind potential assessment, and for simple means of comparing and relating data from different sites or times, it is convenient to parameterize the probability distribution function as some analytical function, with a limited number of arbitrary parameters to be evaluated in determining the probability distribution at each site. Both analytical and practical considerations indicate that a one-parameter wind speed probability distribution function is inadequate. However, at least two different forms of two-parameter analytic functions could be used to characterize the probability distribution function. After careful examination of the wind data records from a number of sites, the Weibull distribution was selected as the better of the two different two-parameter functions studied (the other being the log-normal). The Weibull distribution function is given by

$$
\begin{equation*}
p(V) d V=(k / c)(V / c)^{k-1} \exp \left[-(V / c)^{k}\right] d V \tag{12.4}
\end{equation*}
$$

where the scale parameter $c$ has units of speed and is related very closely to the mean wind speed $\overline{\mathrm{V}}$, and the shape parameter k (dimensionless) is a measure of the variance of the distribution (the larger $k$, the smaller the variance). See Appendix $C$ for a discussion of the choice of the Weibull distribution and the method used for determination of its parameters $c$ and $k$. 12.1 Evaluation of the Wind Distribution Parameters

Frequency of wind speed occurance data were obtained from several different sources in order to be used in the evaluation of Weibull distribution parameters and eventually in the evaluation of wind energy potential across the U.S. Data from several sites were obtained in the form of monthly and annual wind speed summaries as listed in 1951-1960 summaries of hourly observations, published by the National Oceanic and Atmospheric Administration (NOAA) and available from the National Climatic Center (NCC) in Asheville, N.C. The other primary source of wind summary data was obtained on a special tape prepared by NCC and containing seasonal and annual summaries of wind speed distributions from "STAR" (STAbility Rose) wind summaries. A large number of stations were represented by the data from these two sources. However, only sites with constant anemometer heights over the period of the wind speed summary were usable in the distribution analysis. Anemometer heights for the National Weather Service (NWS) sites were determined from station histories published as part of NOAA's Local Climatological Data series. After elimination of sites whose anemometer height changed during the period of the summary, there were 134 stations with usable data from these two data sources. Additional wind summary data were obtained in the form of a copy of a tape originally prepared by NCC for the Sandia Corporation. This tape contained data from a large number of NWS sites and also numerous military stations.

Many of the NWS sites on the Sandia tape were duplications or were otherwise unusable, and all of the military sites were not usable at the present time because anemometer heights could not be obtained for them (these height data are available at NCC, but each military station would probably have to be looked up in separate station history logs). On the Sandia tape there were only nine usable new data sets not available from the other two data sources. Since these 9 data sets had seven different speed category breakdowns, it was decided not to use any of the Sandia tape data at the present time, but to await further information on the anemometer heights from the military sites.

Weibull distribution scale parameter values ( $c$, in $\mathrm{m} / \mathrm{s}$ ) and shape parameter values ( $k$, dimensionless) were evaluated from the 1951-1960 and "STAR" format wind summaries, by the method discussed in Appendix C. These results, along with the anemometer height values, are given for each of the sites in Appendix D. A11 values in Appendix D were evaluated from 5 or more years of wind summaries.

### 12.2 Height Variation of the Wind Distribution Parameters

The anemometer heights of the sites listed in Appendix $D$ show a wide range of variation and most are lower than heights at which large wind generators would be operated. Therefore, in order to facilitate intercomparison between sites and to evaluate output power characteristics of wind generators at realistic heights, a method had to be determined for estimating the height variation of the $c$ and $k$ values from measured values at one given height. Two basic approaches were used to devise such a height variation method: analysis of wind summary data from several levels on
meteorological towers, and differences in $c$ and $k$ values between two different anemometer heights (at two different times) at the same station (see, e.g., Mobile, Ala., Tallahassee, Fla., and Atlanta, Ga. listings in Appendix D). Wind data from NASA meteorological towers at Kennedy SC (1967-1969) and Wallops Island (1961-1965) were obtained from the National Climatic Center (NCC) in Asheville. Additional data were also obtained from the Battelle tower [3] at Hanford, Washington (1955-1970) and NOAA's instrumented tower [4] WKY-TV at Oklahoma

City (June 1966-May 1967). Wind distribution parameters were calculated at each height level for these tower data, by the methods discussed in Appendix $C$. Figure 12.1 shows the observed height variation, for Kennedy SC 1967-1969, of the Weibull scale parameter $c$ (which corresponds closely to the mean wind speed). As seen in Figure 12.1, $c$ increases as a power law with height in each season and for the annual mean. The Kennedy SC power law exponents vary from 0.17 for spring to 0.23 in the winter, with an annual average of 0.20. The average exponents for the other tower data were found to be 0.27 for Wallops and 0.23 for both Hanford and WKY. Thus the overall average exponent for all four sets of tower data is 0.23 , with a standard deviation of 0.03 . This power law exponent at 0.23 agrees well with the power law exponent for the height variation of the mean wind ( 0.24 ) for these four sets of tower data. Such a value of exponent is indicative of surface roughness lengths corresponding to terrain between smooth flat terrain and forest [5]. The average surface roughness length for the four data sets was found to be 24 cm , in agreement with the exponent in indicating slightly rough surfaces, and in general agreement with previous estimates of 32 cm for the surface roughness at Kennedy $\mathrm{SC}[6]$ and 6 cm at the WKY facility [7].


Figure 12.1 Observed power law height variation of the Weibull scale factor $c$, which corresponds closely to the mean wind speed. Numbers beside each curve are power law exponent values.

Good data were available from 24 stations with anemometers at two different heights during two separate time periods (see Appendix D). In all of these cases the scale factor $c$ increased with height at a rate corresponding to a power law with average exponent of 0.15 with 0.08 standard deviation. Data from three sites (St. Louis, Mo., Asheville, N.C., and Harrisburg, Pa., ) were rejected because the $c$ values showed no increase or showed a decrease with height. For two reasons it was decided to use the tower data exponent of 0.23 for the height variation and to ignore the two-anemometer height analysis results: 1) the larger variability of the exponent values for the two-anemometer height analysis ( $0.08 \sigma$ vs. $0.03 \sigma$ for the tower data), and 2 ) the smaller height differences in the two-anemometer height analysis (only $6 \Delta Z$ values greater than $12.2 m$ (40 ft.), compared to an average $\Delta \mathrm{Z}$ (top to bottom) of 187 m ( $613 \mathrm{ft}$. ) for the tower data).

Figure 12.2 shows the shape parameter values $k$ determined from the Kennedy SC data. The increase up to a maximum near a height of 60 m ( 200 ft .) was found in all four sets of tower data. For this reason, a normalized curve of $k / k_{\text {max }}$, shown in Figure 12.3 could be constructed. With a single value for $k$ determined at any height for a given location, the complete height variation of $k$ at that location can be found by application of the $k / k_{\text {max }}$ curve in Figure 12.3.

The height variation of $p$ lant factor $F_{p}$, the ratio of the average output power to the rated power (see Appendix B), was also evaluated from the tower data. Figure 12.4 shows the computed height variation of $F_{p}$ for the Kennedy SC data. Frequently a $1 / 7$ power law is used for the height variation of the


Figure 12.2 Observed height variation of the Weibull shape factor $k$, which is a measure of variance (high $k$ meaning low variance).



Figure 12.4 Observed height variation of the plant factor (ratio of average output power to rated power) using the cut-in and rated speed of NASA's Plumbrook unit (see Appendix B).
mean wind and the output power ( $F_{p}$ times the rated power) is assumed to follow a $\mathrm{V}^{3}$ relation with the mean wind, so that a $3 / 7$ ( 0.43 ) power law of output power (or $F_{p}$ ) is assumed [8]. Figure 12.4 shows that $F_{p}$ does not increase with height as the cube of the velocity increase (i.e., not with an exponent of $3 \times 0.23-0.69$. Nor does it increase as a $3 / 7$ (0.43) power law. Instead the actual exponent for the power law of $F_{p}$ versus height is found to be somewhere in between these two values. Operating characteristics of the NASA Plumbrook unit [9] i.e., cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$ and rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$ were used for the calculation of the $F_{p}$ values in Figure 12.4 .

### 12.3 Evalaution of Nationwide Wind Potential

The seasonal and annual values of the Weibull parameters for each of the sites 11 sted in Appendix $D$ were projected to a height of 61 m ( $200 \mathrm{ft}$. ) by using a power law exponent of 0.23 for the scale parameter $c$ and the "Universial" $k / k_{\max }$ curve of Figure 12.3. A height of 61m (200 ft.) was chosen because it represents a reasonable height for operating large wind generators, and, from the results of Figure 12.4 , it appears that little added output power would be gained by going above about this height (the optimum height would depend somewhat on the cut-in and rated speeds used to evaluate $F_{p}$ in Figure 12.4). The lower height of 30.5 (100 ft.) corresponds to the height of the present NASA 100 kW test unit at Plumbrook, and represents a more easily achieved height for smaller generator systems where tower costs are a more important part of the total costs. Figure 12.5 shows the locations of all of the sites listed in the tables of Appendix D. Height projected values of Weibull $c$ and $k$ parameters and mean wind speed $\bar{u}$


Figure 12.5 Map of the sites used for the wind potential study. See site listing in Appendix D.
at heights of 30.5 m and 61 m are shown in Tables $\mathrm{D}-2$ and $\mathrm{D}-3$ of Appendix D , and annual average wind speed contours at the 30.5 m height are given in Figure 12.6. The Weibull $c$ and $k$ values at 30.5 m and 61 m were used in the method described in Appendix B to evaluate plant factors for two sizes of aerogenerator systems: 100 kW and 1 MW . Numerical plant factor results are given in Tables D-4 and D-5 of Appendix D. Contour maps of the plant factors are shown in Figures 12.7 through 12.14. Annual average plant factors for the 100 kW and 1 MW units at heights of 30.5 m and 61 m are shown in Figure 12.7 through 12.10 , while Figures 12.11 through 12.14 show plant factor contours on a seasonal basis for the 100 kW unit at a height of 61m. For the 100 kW (NASA Plumbrook site) unit the following generator characteristics were used: cut-in speed $\mathrm{V}_{\mathrm{o}}=3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $\mathrm{V}_{1}=8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$, feathering speed $\mathrm{V}_{2}=26.8 \mathrm{~m} / \mathrm{s}(60 \mathrm{mph})$. For the 1 MW system, generating characteristics were used as follows: $\mathrm{V}_{\mathrm{o}}=6.7 \mathrm{~m} / \mathrm{s}$ $(15 \mathrm{mph}), \mathrm{V}_{1}=13.4 \mathrm{~m} / \mathrm{s}(30 \mathrm{mph}), \mathrm{V}_{2}=26.8 \mathrm{~m} / \mathrm{s}(60 \mathrm{mph})$. Figures 12.7 through 12.14 show that (not surprisingly) the central states of Nebraska, Kansas, Oklahoma, and northern Texas are the states with the highest wind potential. For the 1 KW generator, plant factors in this central U.S. region average about $15 \%$ at 30.5 m or $25 \%$ at 6 Im , while for the 100 kW generator the plant factors average about $50 \%$ and $60 \%$ at these two respective heights. The areas of the country with the least potential for wind energy application are central California (plant factors right on the coast may be higher than indicated by the maps, however), and the mountain areas of east Tennessee, west North Carolina, western Virginia and eastern West Virginia. Spring is found to be the best season and summer the worst. One

Figure 12.6 Contours of annual average wind speed at a height of 30.5 m ( 100 ft ). Wind speed values are in $\mathrm{m} / \mathrm{s}$. See tabular values in Tables $\mathrm{D}-2$ and $\mathrm{D}-3$ of Appendix $D$.

Figure 12.7 Annual map of contours of constant plant factor (in percent) at a height of 30.5 m ( 100 ft ) for a wind generator with the NASA Plumbrook characteristics, cut-in speed 3.6 in of Appendix $D$.

Figure 12.8 Annual map of contours of constant plant factor (in percent) at a height of 30.5 m ( 100 ft )

Figure 12.9 Annual map of contours of constant plant factor (in percent) at a height of 61 m (200 ft )


Figure 12.11 Winter seasonal map of contours of constant plant factor (in percent) at a height of
$61 \mathrm{~m}(200 \mathrm{ft})$ for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$. Contours were drawn from data shown in Table $D-4$ of Appendix $D$. Figure 12.11

Figure 12.12 Spring seasonal map of contours of constant plant factor (in percent) at a height of 61 m (200 ft) for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$. Contours were drawn from data shown in Table D-4 of Appendix D.

Figure 12.13 Summer seasonal map of contours of constant plant factor (in percent) at a height of $61 \mathrm{~m}(200 \mathrm{ft})$ for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$. Contours were drawn from data shown in Table D-4 of Appendix D.


[^11]problem for the concept of augmenting electric generating capacity by wind generator systems is that, in most sections of the country, peak capacity is needed in the summer, while the winds are lowest in this season.

In order to facilitate evaluation of plant factors for wind generators with operating characteristics other than those used here, a parametric study was done for a range of rated speed values. Figure 12.15 shows a plot of plant factor versus rated speed $V_{1}$ and cut-in speed $V_{0}$ for three typical locations with low, medium, and high values of scale parameter $c$. The Brunswick (microsystem application, see Section 15) was evaluated by averaging results from Savannah, Ga. and Jacksonville, Fla. Figure 12.15 indicates that maximum plant factor is achieved by the lowest possible rated speed and lowest possible cut-in speed, or, in other words, the highest efficiency (a rather obvious statement when put that way). The importance of Figure 12.15 is in applying it to evaluate the average output power of various systems. As shown by Noel [10], the rated power per unit area $P_{r} / A$ for various generator systems is proportional to the third power of their rated speed. Since the design rated power per unit area for the NASA Plumbrook unit is $0.088 \mathrm{~kW} / \mathrm{m}^{2}$ at a rated speed of $8.0 \mathrm{~m} / \mathrm{s}$, the rated power per unit area of other systems with high rated speeds can be evaluated by a simple $\mathrm{V}_{1}{ }^{3}$ scaling. The average output power per unit area would be the product $F_{p}\left(P_{r} / A\right)$, where $F_{p}$ versus rated speed is taken from Figure 12.15. A plot of average output power per unit area versus rated speed is shown in Figure 12.16, which also shows the rated power per unit area on the top scale. This figure shows that considerable increase in average output power with an increase in rated speed can be achieved at sites with high scale parameters


Figure 12.15 Parametric variation of plant factor versus generator rated speed $V_{1}$ and cut-in speed $V_{o}$ for three representative high, medium, and low average wind speed cities.


Figure 12.16 Parametric variation of the average output power per unit area $\bar{P} / \mathrm{A}$ versus the generator rated speed for the three cities of Figure 12.15. Rated power per unit area $\mathrm{P}_{\mathrm{r}} / \mathrm{A}$ is shown on top axis.
(high mean winds). However the increase in power per unit area is never as great as the simple cubic, with which rated power per unit area increases. At sites with low $c$ values (low wind speeds) there is little increase in average output power per unit area by increasing the rated wind speed.

### 12.4 Accuracy of the Results

There were three cities which had more than one airport whose results could be compared, in order to assess the accuracy of the results: Milwaukee (Mitchell Field, MKE; and Timmerman, MWC), Chicago (O'Hare, ORD; and Midway, MDW), and New York City (Laguardia, LGA; and Kennedy JFK). The Weibull parameters for these sites may be compared by consulting Appendix $D$. Comparison of their annual plant factor values showed, for the three cities, a root mean square difference of $11 \%$ between the two airport estimates. Thus the contours in Figures 12.7 through 12.14 should be interpreted as accurate only to within about $10 \%$ for the 100 kW unit and $5 \%$ for the 1 MW unit (i.e., one contour separation). In certain areas of the country, especially those having 100 kW unit plant factors below about $40 \%$, the consistency was better between the various cities. Hence, in regions where $\mathrm{F}_{\mathrm{p}} \leq 40 \%$, the accuracy is probably about 5 percent for the 100 kW unit (i.e., half of one contour separation).

Another interesting comparison was between the results for New York Central Park which has a projected 61 m level 100 kW unit plant factor of only $20 \%$ whereas the two New York airports (LGA and JFK) averaged $70 \%$ 100 kW unit plant factors. This difference is apparently due to wind blockage effects of the city buildings reducing the available wind power at the Central Park site. For this reason the Central Park results were
not incorporated into Figures 12.7 through 12.14. A similar effect may be responsible for low values of plant factor estimates for Brunswick, Ga. Although the data were not adequate (too few speed categories) for an accurate computation of plant factor at Brunswick, the estimated 100 kW unit plant factor at 61 m was only $13 \%$, whereas the nearby cities of Savannah, Ga. and Jacksonville, Fla. had $25 \%$ and $29 \% 100 \mathrm{~kW}$ unit plant factors at 61m. The low Brunswick values, may be due to a combination of wind blockage by tall pines which surround the airport and multi-story blimp hangar buildings on the airport grounds. The Brunswick station (Glynco Naval Air Station) has now closed, and it was not possible to confirm the location of the anemometer relative to the buildings and trees. Instead, the Brunswick values used in the micro system analysis were taken as averages of the Jacksonvi11e and Savannah results.

With regard to application of the results plotted in Figures 12.7 through 12.14 , it should be noted that the values shown should be interpreted as plant factors for wind generators at a height of either 30.5 m (100 ft.) or $61 \mathrm{~m}(200 \mathrm{ft}$.$) above relatively smooth terrain (roughness$ lengths ~ 5 to 50 cm ) with relatively flat topography. This is because all values in these maps were generated from wind data measured at airport locations. Although some approximations could possibly be made in order to estimate the differences which would occur for hilly or mountainous terrain, these effects should be estimated almost on a case-by-case basis only. A generalized method for accounting for hilly and mountainous terrain effects must await considerably more extensive research than has been done to date in this field.

## REFERENCES

1. Thomas, P. H. (1945): "Electric Power from the Wind," Federal Power Commission, March.
2. Reed, J. W., R. C. Maydew, and B. F. Blackwell (1974): "Wind Energy Potential in New Mexico," Sandia SAND74-0071, July.
3. Stone, W. A., D. E. Jenne, and J. M. Thorp (1972): "Climatography of the Hanford Area," Battelle, BNWL-1604, June.
4. Crawford, K. C., and H. R. Hudson (1970): "Behavior of Winds in the Lowest 1500 Feet in Central Oklahoma: June 1966 - May 1967," NOAA, ERLTM-NSSC-48, August.
5. Davenport, A. G: (1965): "The Relationship of Wind Structure to Wind Loading," Proc. Conf. on Wind Effects on Buildings and Structures, London, 54-102.
6. Fichtl, G. H. (1968): "Characteristics of Turbulence Observed at the NASA 150m Meteorological Tower," J. App1. Meteorol., 7, 838-844.
7. Sanders, L. D., and A. H. Weber (1970): "Evaluation of Roughness Lengths at NSSL-WKY Meteorological Tower," NOAA, ERLTM-NSSL-47, August.
8. Reed et.a1., op cit.
9. Puthoff, R. L., and Paul J. Sirocky (1974): "Preliminary Design of a 100 kW Wind Turbine Generator," NASA-TM-X-71585.
10. Noe1, John M. (1973): "French Wind Generator Systems," Proceedings of the Workshop on Wind Energy Conversion Systems, Washington, D.C., Inc. NSF/RA/W-73-006.

## SECTION 13

THE COST-BENEFIT MODEL

### 13.1 Basis for Model

The purpose of the cost-benefit model is the efficient and effective application of the cost-benefit method of analysis to wind power systems. The objective of the development of the model was to establish a practical tool for studying the relative costs and benefits of wind power systems under a variety of conditions.

The model development effort included (a) the definition of the problem to be analyzed, (b) the selection of the appropriate measure and criterion to use in the cost-benefit analysis, (c) the formulation of equations relating the costs, benefits, and measures, and (d) the encoding of a computer program to solve the model equations. The model development assumed that the wind power system would be used to generate electric power, therefore the model was based on comparing conventional electric power generation with an electrical system which includes wind-powered generators. The model was developed primarily for use in analyzing System I, the Macro Application, but the basic approach and concepts apply to the analysis of System II, the Micro Application.

The problem to be analyzed was assumed to be a comparison between two scenarios: a baseline scenario, which assumes the continuation of conventional electrical power generation techniques, and an alternative scenario, which assumes that wind-powered generators are used along with conventional generators in the electric power system. The problem could thus be stated as, "what are the benefits and costs (compared with 'doing nothing') of adding to an electrical power system the capability for wind-generated electric power?"

Because the problem is stated as a choice between two alternatives, the appropriate measure for comparison (as described in Part II, Section 2) is the Net Present Value (NPV) of the difference in net benefits (benefits minus costs) between the alternative scenario (i.e., with wind power units) and the baseline scenario (i.e., with no wind power units). Moreover, as discussed in Section 2, the appropriate decision criterion is to choose the alternative (wind power) scenario over the baseline scenario if the NPV of the difference in net benefits is positive.

The equation to calculate the NPV of the difference in net benefit is

$$
\begin{equation*}
\text { NPV }=\sum_{k=0}^{H} \frac{B_{k}-C_{k}}{(1+i)^{k}} \text {, where } \tag{13.1}
\end{equation*}
$$

$H$ is the time horizion,
$i$ is the discount rate,
$B_{k}$ is the value, in year $k$, of the additional benefits of the alternative scenario over the baseline scenario, and
$C_{k}$ is the value, in year $k$, of the additional costs of the alternative scenario compared with the baseline scenario.

The additional benefit of the alternative scenario is primarily the electricity generated by the wind power units, and this may be evaluated in several ways. However, the cost of fuel which would have been used to generate the electrictiy by conventional plants is one measure of the value of this electricity, and fuel cost is an important input to the calculation of benefits.

Construction, operation, and maintenance of the wind power units are the primary cost differences between conventional systems and a system
which utilizes wind powered generators. The details of the assumptions regarding these costs are discussed in Section 14.

Figure 13.1 illustrates the overall model used to compare the differences in net benefits between the baseline scenario and the alternative scenario. Although the diagram indicates three "modules" to the model, the completed computer model consisted of only two programs: a power calculation program and a net present value calculation program. The dotted line in Figure 13.1 indicates the separation between the two programs, showing that the calculation of net benefit and cost differences between the baseline and alternative scenarios is performed in the same program as the net present value calculation. The power calculation model was described above in Section 12 (Wind Data), and the net present value calculation model is described below. The description below illustrates the program as used for analyzing System I; similar analyses are appropriate for System II and are described in more detail in Section 15.

For System I, the costs and benefits are estimated by three major categories of parameters: fundamental parameters which describe the wind systems, economic parameters of the wind system, and conventional system parameters (including fuel costs). In addition, certain parameters and decision variables, such as the construction/installation schedule for the wind power system and the choice of discount rate, will affect the net present value of the differences in net benefits.

### 13.2 Fundamental System Parameters

The fundamental wind system parameters include the size or capacity (e.g., in kilowatts) of the individual wind units and the number of units in the system. The maximum output of the wind power system is the product

Figure 13.1 Block diagram illustrating the comparative analysis model for use in assessing the potential of wind power systems.
of these two parameters. If the wind blew continuously (at a speed above the rated speed and less than the feathering speed), then the annual production of electricty would be 8764 (the number of hours in a year) times this product. However, with wind speeds varying according to the distribution fitted to the actual wind data, the annual output will be scaled by the plant factors; that is, if $G_{W}$ represents the annual electric power (in kilowatt-hours) generated by the wind system,

$$
\begin{equation*}
G_{w}=\left(P_{f}\right)(N)\left(P_{r}\right)(8764) \quad \text { where } \tag{13.2}
\end{equation*}
$$

$P_{f}=p l a n t$ factor (calculated using the wind power model), $\mathrm{N}=$ total number of wind units (postulated in the system definition),
$\begin{aligned} P_{r}= & \text { rated power, or capacity, of each wind unit (postulated in } \\ & \text { the system definition), and }\end{aligned}$ $8764=$ the number of hours in a year.

### 13.3 Economic Parameters

The economic parameters of the wind system include investment costs (including land acquisition, site preparation, installation and testing of the units) and operational and maintenance costs. For this model, $R \& D$ costs were not specifically considered, although such costs could be assumed to be included on a prorated basis as a component of the investment cost. Later studies, and other studies which are more concerned with details of wind power unit designs, should address the $R \& D$ costs in greater depth.

Investment costs for wind systems are not well established (production costs depend on design details and market size, and currently neither is known with any certainty), and so a single value for this parameter could not
be assumed for the model. However, discussions of wind power unit design typically use assumed values or ranges of a cost per unit capacity, usually dollars per installed kilowatt, and this is the parameter included in the model. The values for this parameter considered for System I ranged from a low of $\$ 200 / \mathrm{kW}$ upwards to $\$ 1600 / \mathrm{kW}$. This range compares with fossil fueled generating plant costs of approximately $\$ 400-500 / \mathrm{kW}$ and with nuclear fueled generating plant costs of approximately $\$ 800 / \mathrm{kW}$. Values within this range should include economically feasible values for wind power system costs.

Current information is not available to project accurate estimates of operational and maintenance costs of wind power units; the model includes these annual costs as a percentage of the total investment. Most reported wind system designs appear to have low operational costs and virtually no maintenance costs. The NASA design is intended to be suitable for unmanned, remotely monitored (if at all) operation. Periodic blade inspections and replacements and perhaps minimum attention required for lubrication servicing are the only operational efforts anticipated. Consequently, annual operational and maintenance costs are expected to be very low, and values of 1 to 4 percent of total investment costs should provide an adequate, though perhaps pessimistic, estimate for this cost component.

The cost component of the basic NPV equation (13.1) is then

$$
\begin{equation*}
C_{k}=I_{k}+O_{k} \tag{13.3}
\end{equation*}
$$

where $I_{k}$ is the investment cost in year $k$ and $O_{k}$ is the operational and maintenance cost in year $k$. In terms of $A$, the cost per installed kilowatt,

$$
\begin{equation*}
I_{k}=n_{k} P_{r} A, \text { where } \tag{13.4}
\end{equation*}
$$

$\mathrm{n}_{k}$ is the number of units installed in year $k$ and $P_{r}$ is the rated power of each unit. Because the annual operational and maintenance cost is assumed to be a fraction of the total investment to date, this cost increases with cumulative investment. In particular, if the wind power system consists of a total of $\mathrm{N}_{t}$ units and is installed at a uniform rate over a period of $Y_{1}$ years, then

$$
O_{k}= \begin{cases}k\left(N_{t} / Y_{1}\right) A P_{r} m & \text { for } 0 \leq k \leq(Y-1)  \tag{13.5}\\ N_{r} A P_{r} m & \text { for } Y_{1} \leq k \leq H\end{cases}
$$

where $m$ is the fraction corresponding to the assumed percentage of total investment which is postulated for annual operation and maintenance costs.

The primary benefit components depend on the systems assumed for the baseline and alternative (wind system) scenarios. As described in more detail in Section 14, System I (Macro application) includes no energy storage facilities; consequently, the primary function of this system is that of a "fuel saver." Under these conditions, the principal benefit differences between the baseline scenario and the wind system scenario are reflected by the value of the fuel which would have been consumed if the wind system had not been in existence. That is, if $G_{k}$ represents the amount of electricty (kWh) generated by the wind system in year $k$, then the principal difference in benefits between the baseline scenario and the alternative scenario is the difference in fuel consumed under the two scenarios. The value of this difference in benefits is then the value of the fuel saved. This value is at least equal to the cost (market value) of the fuel saved, and thus the model uses fuel costs as the measure of the benefits of the wind power system.

Two conditions may be assumed in calculating the value of the benefit. Under one condition, the savings in fuel is distributed proportionally among all the fuel-consuming processes used to generate electricity within the region. (Hydroelectric-generated power is excluded from the base of the calculation, since no fuel is consumed by this process.) Under the second condition, the wind-generated electricity is assumed to replace the electricity generated by the process having the highest fuel costs per kilowatthour output.

At any particular time, if wind-generated electricty is available, the rational strategy would be for the utility to reduce the power output from conventional plants by cutting back the most expensive process. Thus condition 2 represents a rational choice at any particular time, but over a long period (e.g., a year), the most expensive processes (for example, oil-fired turbines) may not be in use at times when wind-generated power is available. For example, the most costly electricity in terms of fuel cost/unit output is typically that generated by oil fired turbines or internal combustion generators which are used for peak power generation. The peaks of power demand in the south typically occur on summer afternoons and evenings (due to cooking plus air conditioner loads), and such times are unlikely to be the best times for high winds. Consequently, although condition 2 represents a rational strategy to follow at any particular time, benefit calculations made under this condition should be considered as optimistic or "best case" estimations.

Conversely, condition 1 (proportional distribution of electricity among all fuel processes) does not represent a rational strategy at a particular time,
and represents a pessimistic estimation of the benefits of the wind powered electricity generation. This condition does not define the "worst case," which would be represented by the allocation of all of the wind-generated electricity to the least expensive fuel/process generated electricity. However, the worst case does not represent a realistic condition, and the proportional distribution of fuel savings is judged to be adequately pessimistic to provide a lower bound to the estimated benefits of the wind power system.

For both conditions 1 and 2, eight different fuel-process combinations may be considered for generating electricity:

1. coal-fired steam
2. oil-fired steam
3. gas-fired steam
4. nuclear-fueled steam
5. oil-fired turbine
6. gas-fired turbine
7. oil-fueled internal combustion
8. gas-fueled internal combustion

If $G_{k}$ is the total electric power ( $k W h$ ) generated during year $k$ in a particular region and $G_{\ell k}$ is the power generated by the $\ell$ th fuel-process combination (letting $G_{o} k$ correspond to hydroelectric-generated power), then

$$
\begin{equation*}
G_{k}=\sum_{\ell=0}^{8} G_{\ell k} \tag{13.6}
\end{equation*}
$$

If $f_{\ell k}$ the fraction of total power generated by the lth fuel-process combination, is defined as

$$
\begin{equation*}
\mathrm{f}_{\ell k}=\frac{\mathrm{G}_{\ell \mathrm{k}}}{\mathrm{G}_{\mathrm{k}}} \tag{13.7}
\end{equation*}
$$

and the subscript $k$ is dropped (assuming, for example, that the fraction remains constant from year to year), then the fraction of fuel-generated electricity generated by the $\ell t h$ combination is given by

$$
\begin{equation*}
\mathrm{f}_{\ell}^{\prime}=\frac{\mathrm{f}_{\ell}}{1-\mathrm{f}_{0}} \tag{13.8}
\end{equation*}
$$

If a system were 100 percent efficient, one million BTU's (MBTU's) of heat energy would be converted into 293.1 kilowatt-hrs. of electrical energy. If the $\ell$ th process efficiency is $E_{\ell}$ and the fuel cost is $F_{\ell}$ (dollars per MBTU), then the fuel cost per kilowatt-hour for this fuel-process combination is

$$
\begin{equation*}
\mathrm{F}_{\mathrm{C}}=\frac{\mathrm{F}_{\ell}}{(293.1) \mathrm{E}_{\ell}} . \tag{13.9}
\end{equation*}
$$

The fuel cost per kilowatt hour, weighted by the fraction of power generated by each combination, and averaged over the different fuel-process combinations is therefore

$$
\begin{equation*}
\mathrm{F}_{\mathrm{CAVG}}=\frac{1}{293.1} \sum_{\ell=1}^{8}\left(\frac{\mathrm{f}_{\ell}}{1-\mathrm{f}_{o}}\right)\left(\frac{\mathrm{F}_{\ell}}{\mathrm{E}_{\ell}}\right) \tag{13.10}
\end{equation*}
$$

Under the "proportional distribution of fuel savings" condition (condition 1 ), then the value of the benefit in year $k$ is given by

$$
\begin{equation*}
B_{k}=\frac{G_{k}}{293.1} \sum_{\ell=1}^{8}\left(\frac{f_{\ell}}{1-f_{o}}\right)\left(\frac{F_{\ell}}{E_{\ell}}\right) . \tag{13.11}
\end{equation*}
$$

For the purposes of this model, the efficiencies of coal-, oil-, and gas-fired steam plants were assumed to be equal within the individual regions and therefore $E_{1}=E_{2}=E_{3}$. Similarly, the efficiencies of natural gas and petroleum-fueled turbines are assumed to be equal, and the efficiencies of petroleum-and natural gas fueled internal combustion generators are assumed to be equal within a particular region; that is, $E_{5}=E_{6}$ and $E_{7}=E_{8}$. Fuel costs (dollars per MBTU) for a particular fuel were assumed to be the same regardless of how the fuel was utilized.

The quantity of electricity generated by the wind power system in year $K$ depends on the number of wind power units in operation, the capacity of each unit, and the plant factor. If $N$ is the total number of units in the system after it has reached full capacity, and $Y_{1}$ represents the number of years to rearb full capacity, then

$$
G_{k}= \begin{cases}0 & k=0  \tag{13.12}\\ \sum_{j=0}^{k} n_{j} P_{R} P_{f} & k=1,2, \ldots ., Y_{1} \\ N P_{P} P_{f} & Y_{1}<k \leq H\end{cases}
$$

For condition 2 (electricity generated by the most expensive fuel-process combination is replaced by the wind-generated electricity), the expression for the benefits in year $k$ are

$$
\begin{equation*}
B_{k}=\frac{1}{293.1} G_{k} \frac{F_{\ell^{*}}}{E_{\ell^{*}}} \quad G_{k} \leq G_{\ell *}, \tag{13.13}
\end{equation*}
$$

where $\ell^{*}$ is the $\ell$ which maximizes $F_{\ell} / E_{\ell}$. For values of $G_{k}>G_{\ell *}$, similar expressions which indicate the replacement of the electricity generated by the next most expensive fuel-process are appropriate.

## Conventional System Parameters

The conventional electric power generation system is defined by a limited set of parameters, chosen to permit the detection of the significant differences between the baseline scenario, costs and benefits and the scenario which includes wind power system. Because the wind power system is basically a "fuel saver," the cost of fuel is an important parameter for the cost benefit model. Because of the above assumption alternatives (condition 1 and condition 2, above), the amount of electricity generated using the different types of fuels is an important parameter. Finally, the efficiency of each of the fuel/conversion process combinations is an important parameter, since this determines the actual fuel cost per kilowatt hour of electricity. In order to establish the region-to-region differences in benefits and costs, values of these conventional system parameters were established for each of the nine Federal Power Commission (FPC) regions. Tables of the parameter values are given in Appendix E.

Periodic $F P C$ reports include the quantities of fuels consumed in each region, the most recent fuel costs, and the amount of electricity produced by the different generation processes. Additional data on the generation of electricity by nuclear power plants were obtained from economic analysts in the FPC, and the assumption was made that the type of process completely determined the efficiency of the generation process. (This assumption means, for example, that the efficiencies of coal-fired, gas-fired, and oil-fired
steam plants are equal.) The error introduced by this assumption is judged to be relatively small and insignificant for the purposes of the model; this is discussed further in Appendix E. This assumption and the FPC data permit the estimation of the quantities and parameters, described in the above paragraphs, used in calculating the benefits of the wind power system in each of the nine regions.

## SECTION 14

## ASSESSMENT OF SYSTEM I - MACRO APPLICATION

System I, a Macro Application of wind power, was chosen for study after considering the results of the wind system categorization effort. The overview of the literature suggested that a system which is linked into the utility power grid, having no storage capability, would be of substantial interest. Such a system might be technically feasible in the near time frame, and widespread implementation could significantly reduce the consumption of energy for electrical power generation. Consequently, the cost-benefit analysis was designed to compare two scenarios: the status quo scenario and an alternative scenario which included the installation of aerogenerators in the power grid.

### 14.1 Scenario Specifications

The status quo scenario is defined to be a "business as usual" scenario, with electrical power being generated by fossil- and nuclear-fueled and hydroelectric power generation plants. Nine different conventional, scenarios were postulated, one for each of the nine Federal Power Commission (FPC) regions shown in Figure 14.1. Each scenario consisted of the specification of the values of the parameters 1isted in Table 14.1. Values for these parameters were determined from FPC data on fuel usage and power generation and from personal communications (FPC releases and reports, 1972 and 1974 , Collier, and Raymond). For each region, nine different combinations of fuels and generation processes were included. Table 14.2 lists these nine combinations.

Few data on nuclear fuel costs and nuclear plant efficiencies were collected, but this was not expected to be a significant constraint on the study. Nuclear generation of electricity accounts for only a small


## LIST OF CONVENTIONAL SCENARIO PARAMETERS <br> (Values for Each of Nine Regions)

Total Electricity Generated in year (G) [kWhrs]
Fraction of $G$ from each of nine generation processes ( $f_{\ell}, 0 \leq \ell \leq 8$ )
Conversion Efficiency of each process ( $E_{\ell}$ )
Fuel cost of each fuel ( $F_{\ell}$ ) [\$/MBtu]
Annual rate of increase of fuel costs (r) [\%]

TABLE 14.2
COMBINATIONS OF FUELS AND GENERATION PROCESSES

| Fuel | Generation Process |
| :--- | :--- |
| -- | Hydro |
| Coal | Steam Turbine |
| Oil | Steam Turbine |
| Natural Gas | Steam Turbine |
| Nuclear | Steam Turbine |
| Oil | Turbine |
| Natural Gas | Turbine |
| Oil | Internal Combustion |
| Natural Gas | Internal Combustion |

percentage of the total electrical energy generated in a region. Because the cost of nuclear fuel is relatively low (estimated at $\$ .22$ per million BUT's), it is reasonable to assume that the more expensive fuels would be saved by the wind-generated power before reducing the output of a nuclear plant. Consequently, the overall impact of variations in these data was judged to be relatively slight.

The alternative scenario is defined as the same configuration as the status quo scenario plus the addition of aerogenerators dispersed throughout the network. Each aerogenerator provides electrical power to the utility grid whenever the wind speed is above its cut-in speed (a parameter of the wind power unit). Table 14.3 sumarizes the basic addition which transforms the status quo system into the alternative system.

The capacity of the wind power system, 500 MW , was selected to be comparable to the capacity of some of the modern conventional power plants. The installation rate (100 each year for five consecutive years) was chosen as a reasonable rate of building the wind system from initial implementation to full capacity. The units were assumed to be dispersed because, with imperfect spatial correlation of wind statistics, such a dispersed system may provide a higher average power output than a system having all its capacity at a single location. (One member of the project team, Dr. Justus, currently is investigating this hypothesis empirically under an NSF grant.)

The approach to estimating wind system costs was chosen on the basis of the following considerations: data availability, the accuracy of available data, the questions to be answered by the analyses, and the availability of analytical tools for the study. The approach chosen was an overall cost

TABLE 14.3
POSTULATED WIND POWER SYSTEM

- 500 l-megawatt (MW) units, dispersed throughout region
- Installed at rate of 100 each year for five years
- No energy storage capability

TABLE 14.4
POTENTIAL SCENARIO DIFFERENCES

| Benefits | Costs |
| :--- | :--- |
| Reduced Air Pollution | Visual Pollution |
| Tourist Attraction | R\&D Costs |
| Focus for National Spirit | Noise Pollution |
| R\&D Stimulus | Increased Air Pollution |
| Reduced Maintenance of Conventional P1ants | Initial Investment |
| Increased Availability of Fuels | Operational and Maintenance |
| Decreased Investment Required for <br> Building Conventional System Capacity | Ecological Damage |

treatment rather than a detailed breakdown of costs by component or subsystem, and this choice reflected the relative paucity of up-to-date, valid estimates of component and system costs and the fact that the study objectives did not require a more detailed treatment of costs.

The two scenarios are compared on the basis of a 40 year time horizon (comparable to the planning horizon for conventional power generation plants) and using an annual discount rate of five percent. The chosen discount rate is high compared with older cost benefit analysis practices but low compared with some recent guidelines. The choice of discount rate reflects the analysts' and decision makers' subjective judgment about the future value of costs and benefits. The rate of five percent was judged to adequately represent both the recognition that future resource flows are less valuable than current flows and the recognition that future energy production is the primary motivation for performing the study.

### 14.2 Identifying Scenario Differences

The project team, through surveys of the literature, discussions with the project sponsor, brainstorming, assuming the roles of special interest groups, examination of economic and social theory, and the definitions of the scenarios themselves, identified the potential scenario differences listed in Table 14.4. This list was reduced by considering each potential difference in view of the following criteria:

- the objectives of the analysis,
- values of the decision maker,
- conservative assumptions,
- preliminary judgment of relative importance,
- data availability,
- resource constraints, and
- analytic tools.
"Reduced air pollution" might be either a benefit or a cost, depending on how the fuel which is saved by the wind generated power is consumed by other uses. The fuel saved, to be a benefit, must be utilized in some way and not simply left unproductive (as, for example, coal being left in the ground). If the other use of the fuel produces more pollution than the use of the fuel in an electrical power generating plant, then the net result of more pollution is a cost. On the other hand, if the other use for the fuel results in a cleaner transformation of energy and less pollution, the net result of reduced pollution is a benefit. Because of the ambiguity of this difference and what is judged to be its second order significance, this difference was not utilized in the cost-benefit analysis.
"Tourist attraction" and "focus for national spirit" are parallel to the Dutch experience. However, values of these benefits of the wind system (alternative) scenario are judged to be relatively difficult to estimate and were assumed to be zero for this study.
"R\&D stimulus" is a potential benefit, but difficult to assess and measure. Its contribution to the analysis was judged to be inconsequential for the purposes of this analysis.
"Reduced maintenance for conventional plants" might result from reduced operation of these plants due to operation of the wind generators. However, this potential benefit is judged to be small compared with the primary benefit of reduced fuel consumption.
"Increased availability of fuels" is judged to be the most important benefit of the postulated system. The value of the fuel saved therefore is the most valuable difference in benefits between the two scenarios.

Because the system is postulated to be a fuel saver (no energy storage), the concept of increased firm capacity of the power system by adding the wind units is not consistent with the defined scenario. The conventional power plants will continue to be built in both the status quo and the alternative scenario; consequently, the benefit "decreased investment required for building conventional system capacity" does not exist.
"Visual and noise pollution" are intangible and consequently were not examined in the study. Similarly, "ecological damage," although potentially measurable, is not well-defined, and evaluation of this potential cost was beyond the scope of this study.
"R\&D costs" may be considered either (A) as sunk costs, and thus inappropriate as cost elements in the analysis, or (B) as a portion of the investment costs assumed for the wind system. These costs were not examined explicitly in the study.

The major cost difference between the two scenarios is the "initial investment" required in the alternative scenario for the installation of the wind units. A smaller, though recurring, cost difference is the additional "operating and maintenance" cost of the wind units. The chosen approach to examining the wind system cost was that of assuming a cost parameter, investment cost in dollars per unit capacity in kilowatts. Operational and maintenance cost was represented as an annual percentage of the total wind system investment cost. Thus the difference in costs between the two
scenarios was specified by two parameters: (1) investment cost per unit capacity and (2) an annual operational/maintenance percentage rate.

In summary, the differences between the two scenarios which were examined in the study include both a cost difference and a benefit difference in resource flows over time. The cost difference is the additional investment and operational/maintenance costs of the wind system. The benefit difference is the value of the fuel saved by the operation of the wind power units in the alternative scenario. The amount of electricity generated was assumed to be the same in both scenarios.

### 14.3 NPV Calculations

A computer program was used to perform the calculations described in Section 13. The model was utilized to perform calculations for baseline values of the model parameters, for sensitivity and parametric studies, and for regional comparisons.
14.3.1 Choice of Baseline Values

The baseline parameter values are shown in Table 14.5. The values for plant factor, total electrical energy generated in the region, the fraction of energy generated by the different processes, and the fuel costs are representative of the East North Central (Eastern Great Lakes) Region. The installation cost of $\$ 800 / \mathrm{kW}$ was chosen because this value is comparable to installation costs of nuclear and modern conventional plants. The other baseline value, $\$ 350 / \mathrm{kW}$, was used because an early calculation (using relatively low fuel costs) showed this to be the breakeven cost for the Great Lakes area: costs above $\$ 350$ resulted in negative net present

TABLE 14.5
baSEline values of model parameters
Basic Parameters Value
Plant Factor .....  5
Rated Power (each unit) ..... 1 MW
Number of Units ..... 500
Installation Cost ..... \$800/kW\$350/kW
Operational/Maintenance Rate ..... 1\%
Time Horizon40 Years
Installation Rate ..... $100 / \mathrm{yr}$ for 5 yrs.
Discount Rate5\%
Annual Rate of Fuel Cost Increase ..... 0\%
Energy Generation (East North Central Region, 1973 Data)
Total Electrical Energy ..... $3.45 \times 10^{\prime \prime} \mathrm{kWh}$
Fraction by:
Hydroelectric ..... 011
Coal-Fired Steam ..... 827
Oil-Fired Steam ..... 037
Gas-Fired Steam .....  021
Nuclear .....  084
Oil-Fired Turbine .....  008
Gas-Fired Turbine .....  008
Oil-Fired Internal Combustion .....  002
Gas-Fired Internal Combustion .....  002
Fuel Costs (East North Central Region, ..... (\$/MBtu)September 1974 Data)
Coal .....  78
Oil ..... 1.78
Natural Gas ..... 82
Nuclear Fuel .....  22

## Conditions

1. Pessimistic Operation - fuels replaced proportionately
2. Optimistic Operation - most expensive fuel replaced first
values of the difference between the conventional and alternative (wind system) scenarios.

Wind system designs are projected to require unmanned sites and to be virtually maintenance-free; consequently, the choice of an annual cost of $1 \%$ of the total wind system investment was judged to be adequate for periodic blade replacement and preventive maintenance on the units.

FPC data (1973) were used to calculate the fraction of electrical energy generated by the different generation processes, and later monthly FEA data on fuel prices and heating values were used to calculate the fuel costs.
14.3.2 Parametric/Sensitivity Calculations

Calculations of NPV were made using different values for the plant factor, installation cost, operational and maintenance rate, time horizon, discount rate, and annual rate of fuel cost increase. Calculations were made by assuming, in turn, each of the two operating conditions.
14.3.3 Regional Comparisons

The model was used to compute the NPV of the scenario differences assuming that the same wind systems were installed in each of the nine FPC regions. Values of three types of parameters changed from region to region: (1) the plant factor (which depends on the wind speed distribution), (2) the energy generated and the fraction generated by each process, and (3) the fuel costs. In order to summarize the potential for utililzing aerogenerators for electrical power production in the different regions, the breakeven value of $A$ (cost per installed kilowatt) was calculated for each region under each of the two assumed operating conditions.
14.4.1 Parametric/Sensitivity Study

Figure 14.2 illustrates the effect of changes in discount rate on the NPV of the differences between the scenarios. From these curves, it is clear that the lower value of the investment parameter A (dollars per installed kilowatt) yields a positive NPV. Negative NPV does not itself imply a loss, but rather the Present value with respect to a particular interest rate. The effect of increasing the discount rate is to reduce the NPV; this occurs because the benefits of the wind system accrue in the future, and thus higher discount rates reduce the contribution of the future benefits to the NPV. From this figure, it is evident that changes in time horizon (from 40 to 50 years) do not affect the NPV significantly.

Figure 14.3 illustrates the effects of changes in investment cost, operating condition, and plant factor on NPV. Note that the assumed operation condition not only affects NPV, it also influences the sensitivity of the results to changes in plant factor.

Figure 14.4 illustrates the effect of investment cost and operational/ maintenance factor on NPV. As one might anticipate, because the contribution of maintenance cost to total costs is small, changes in the operational/maintenance factor do not yield substantial changes in NPV.

Figures 14.5, 14.6, and 14.7 illustrate the relationships of discount rate and fuel price rate of increase on NPV. These three dimensional plots may be used to show the range of values of interest from different angles and perspectives. As is evident from the figures, very high NPV's may result for particular parameter values.








Figure 14.7 NPV vs fuel cost and discount rate ( $20^{\circ}$ from horizontal)

### 14.4.2 Regional Comparisons

Figures 14.8 through 14.16 illustrate the calculations for each of the nine FPC regions of NPV as a function of investment cost per unit of installed capacity, with operational condition and two values of plant factor as parameters. The higher curves in these figures correspond to values of plant factor calculated at 200 feet for the 100 kW Plumbrook operating characteristics. The lower curves correspond to calculations made by assuming one-half the plant factors calculated for the Plumbrook unit, reflecting the higher cut-in and rated speeds anticipated for a 1 MW aerogenerator design. These curves determine, for the assumptions specified, ranges of parameters and conditions which may correspond to economic feasibility for wind systems in this application. Note that for some regions, the difference between condition 1 and condition 2 is quite large, indicating a large difference between average fuel cost per unit of electricity and most expensive fuel cost per unit of electricity.

Table 14.6 summarizes the NPV of the baseline system, using plant factors, fuel prices, and electrical power generation process fractions appropriate for each region. The values in this table illustrate that both high average winds (high plant factor) and high fuel prices contribute to high NPV's. For example, the Mountain region has reasonably high winds ( $P_{f}=.5$ ) but, because of inexpensive fuel, the NPV is negative for a wide range of investment values.

Table 14.7 summarizes the regional comparison by listing the breakeven values of the investment parameter--i.e., the largest value of $A$ for which the NPV will be positive or zero. These values may be considered as "design goals" for aerogenerator costs. The left-hand side of the table shows










TABLE 14.6
NET PRESENT VALUE, BY REGION, OF NET DIFFERENCES IN SCENARIOS (BASE VALUES OF PARAMETERS, $9 / 74$ FUEL PRICES)

|  | NPV (\$MILLION) |  |
| :--- | :---: | :---: |
| REGION | CONDITION 1 | CONDITON 2 |
| New England | 480 | 780 |
| Mid Atlantic | 320 | 900 |
| South Atlantic | 130 | 580 |
| East South Central | 0 | 420 |
| West South Central | 40 | 780 |
| West North Central | 40 | 720 |
| East North Central | 80 | 690 |
| Mountain | -20 | 550 |
| Pacific | 160 | 490 |

TABLE 14.7
regional summary - breakeven values of cost per kilowatt

calculations made from the plant factors shown in the left-hand column. These values were taken from the contour map showing plant factors for 200 foot units having the P1umbrook 100 kW unit's operating characteristics. The right hand side values were calculated by assuming one-half these plant factors, reflecting the anticipated lower plant factors which would result from higher cut-in and rated speeds expected for a 1 MW wind unit.

### 14.5 Conclusions

This study examined the economics of using aerogenerators in a utility power grid in order to save fossil and nuclear fuel. The study used available data to establish cost goals for different technical and operating conditions and compared these goals for different regions.

The results indicate that, under certain conditions, the potential exists for economically benefiting from wind power systems as fuel savers in a utility grid. Although many regions might benefit, the highest potential appears to exist for the New England states, which have both relatively high winds and relatively costly fuels used for electricity power generation.

Several conditions assumed for the study may change in the future, and, consequently, the results and conclusions are subject to several caveats. If fuel prices continue to increase, the wind power system scenario will become more attractive than this study indicates. Similarly, experience with a network of aerogenerators may indicate the feasibility of a new scenario, that of using a portion of the wind system capacity to reduce the firm capacity of the conventional system. If this is feasible, the investment in conventional systems can be reduced.

On the other hand, there is no assurance that wind systems can be built which (1) fall within the ranges of costs used in this study, and (2) perform
as assumed in the calculations. In particular, the 1 MW units postulated for this study were assumed to have the same cut-in and rated speeds as the 100 kW NASA P1umbrook unit. Design trade-offs appear to indicate that it is less expensive to increase these speeds for larger unit, lowering the plant factor but also lowering the investment per unit of energy output. In summary, the study results indicate that, even for systems which act only as fuel savers and do not contribute to firm system capacity, aerogenerators in a utility grid can be economically advantageous.

A MICRO ANALYSIS OF THE
POTENTIAL OF WIND ENERGY SYSTEMS

### 15.1 Introduction

The objectives of this micro analysis are to (1) illustrate the principles of engineering economic analysis, (2) assess the potential of wind energy systems in private sectors, and (3) explore the behavioral or motivating forces, that is profits, that would induce the entrepreneur to utilize a wind power system. Unlike the macro analysis which reflects a viewpoint from society as a whole, this micro analysis emphasizes costs and benefits to a firm. Although "cost" and "benefits" will be addressed, the micro analysis is not a cost-benefit analysis in the true meaning of the words, but rather it would be more properly called an engineering economic impact analysis. As will be seen, whereas in the macro analysis benefits were in terms of fuel saved, the micro analysis benefits are in terms of reduced costs to the firm and assoclated profit. The approach or methodology used in this analysis is based in general on that described in Section 7; that is, the problem will be defined, the analysis designed, the format of the results established, and the results calculated and presented to the decision maker. The problem definition is in terms of the status quo scenario and the project scenario with emphasis on the relevant differences between scenarios. Society in this analysis is considered simply to be the firm and consequently there will be no social impact analysis.

As background for the motivation of this analysis, there has been a recent breakthrough in the chemical processing of stack dust and there are obvious potential benefits with the one of interest being the extraction of
magnesium. However, the energy requirement for the steps of the chemical process associated with producing magnesium is intensive and the windmill offers the potential of saving electricity. This particular process for extracting secondary metals from stack dust was selected as a target application to evaluate the potential of wind power systems in the private sector. The project chosen for analysis is associated with a pilot plant located near Brunswick, Georgia and enters the unit of the plant that would produce magnesium metal by electrolysis of magnesium chloride. For this unit, electric power could be purchased from a public utility company or it could alternatively be produced by a windmill generating facility.

The engineering economic analysis, which though not totally appropriate will be called a cost-benefit analysis, and will in essence be a capital investment analysis from the point of view of the plant owners. Their question is postulated as follows: Can a plant-owned windmill generating facility be constructed which will result in a return on investment with reasonable earnings after taxes? The source of the earnings would be the production cost saved by producing instead of purchasing electric power. After adjustments for taxes and maintenance the savings would become yearly positive cash flows. Although the project "yield" and the amount of money the plant owner can afford to invest in a windmill generating facility are of concern [1], the analysis will delineate elements of the plant owners' decision about constructing a generating facility and also attempt to determine the ranges of cost that would make the purchase of a windmill attractive to the plant owner.

### 15.1.1 Electric Power Requirements for Magnesium Production

In 1971 U.S. production of primary magnesium totalled 123,500 tons, all of it from two manufacturers, Dow Chemical Co. at Freeport, Texas, and American Magnesium Co. at Snyder, Texas. N.L. Industries planned the start-up of a plant which would process 45,000 tons/year of magnesium out of brine from Utah's Great Salt Lake. Late in the year, American Magnesium Co. shut down its Snyder, Texas, production, but planned to resume by 1974. Dow slowed down the construction of its new 25,000 ton/year plant in Dallesport, Washington, at a cost of $\$ 3,000,000$ in contract cancellation fees. Alcoa, at about the same time, announced plans for a new 20,000 tons/year, $\$ 50,000,000$ plant at Addy, Washington. Alcoa's plant would employ 300 to 400 persons with an annual payroll of about $\$ 3$ million $[2,3]$.

Metal extraction processes are by nature energy intensive. The new Alcoa plant would employ a "silicothermic" process; but the bulk of magnesium is produced by electrolysis of molten magnesium salts. Older electrolysis methods required 10 kWh per pound of magnesium; but recent improvements have reduced energy consumption.

For a requirement of only 6.5 kWh per pound [4] for the electrolytic production of primary ingots, the electric energy used in 1971 to produce magnesium would have been at least $1.6 \times 10^{9} \mathrm{kWh}$, and the average continuous power demand would have been 190 megawatts. That amount of electric energy for magnesium extraction would have been less than 0.1 percent of the total electric energy generated in the U.S., but at a rate of 2 cents $/ \mathrm{kWh}$, it would have cost $\$ 32$ million. Equivalent fuels required would have been 2.5 million barrels of oil ( $\$ 28$ million at $\$ 11 / \mathrm{bb} 1$ ), 740,000 tons of coal
( $\$ 19$ million at $\$ 25 /$ ton) assuming conversion efficiencies of 33 percent. But it would have been equivalent to only one-half day's oil consumption by U.S. automobiles.

These fragmentary statistics on U.S. magnesium production give perspective to the engineering economic analysis of wind generated electric power for the plant that has been proposed for location at Colonel's Island, near Brunswick, Georgia.
15.1.2 Economics of Magnesium Production

The electrolytic extraction of magnesium is costly. At 5 mills per kWh ( 1968 costs), electricity cost $\$ 1.46$ per million BTU, compared with 40 cents for natural gas and 20 cents for coal. But in 1973 an estimated $200 \times 10^{9} \mathrm{kWh}$ of electric energy (about $11 \%$ of the U.S. total) was used by steel, aluminum, magnesium, and other plants for "metal recovery and purification by electrowinning, electrorefining, and electric furnace smelting" [3,5,6]. The reasons for using electric energy instead of fossil fuel energy include the ability to fractionate metals directly and selectively from mixtures of metallic salts, and the ability to free a metal from even small quantities of impurities. At today's cost of 2 cents per kWh [7] the electrowinning industry pays in excess of $\$ 4$ billion for electricity.

Magnesium and aluminum are light metals with similar characteristics, but raw materials for magnesium are practically unlimited in the U.S., whereas almost all aluminum ore is now imported. Yet the U.S. consumption of aluminum is some 50 times greater than the consumption of magnesium. Part of the reason is in the comparative desirability of the chemical and physical features of aluminum; in addition, however, the costs of producing
aluminum have been some 20 percent lower than the costs of producing magnesium. Research in metallurgy and changes in market conditions (such as the recent increase in price of foreign bauxite) could reduce aluminum's advantages and increase the demand for magnesium. This is one of the market forces behind the proposal to build the Brunswick plant.

A 1968 study by the Bureau of Mines examined the costs of producing magnesium by seven distinct final processes, and by several intermediate routes from feed materials such as dolomite, $\mathrm{M}_{\mathrm{g}} \mathrm{CO}_{3}$, sea water, and oyster shells [6]. In an electrolysis cell, magnesium is extracted from a molten solution of chlorjides of magnesium, sodium, calcium and potassium. Optimum concentration of magnesium in the mix is from 10 to 25 percent. The magnesium metal separates by floating in the melt; and as magnesium is removed, the other salts accumulate and must be removed by periodic cleaning. (A cell using a mix of pure anhydrous chlorides of magnesium and lithium would be somewhat more efficient in that the magnesium metal would sink and its recombination with evolving chlorine gas would be prevented.)

The process step that involves extraction of anhydrous magnesium chloride (which is used to charge the electrolytic cell) from an aqeous solution requires very close control of temperature and drying conditions, to keep the dehydrated chloride from hydrolyzing to magnesium oxide, hydroch1oric acid and water, thus undoing earlier process steps. Early Dow processes avoided this difficulty by charging the electrolytic cells with magnesium chloride dihydrate: $\mathrm{MgCl}_{2} \cdot 11 / 2 \mathrm{H}_{2} \mathbf{0}$, rather than anhydrous $\mathrm{MgCl}_{2}$; but the Bureau of Mines study found production cost to be lower using the anhydrous magnesium chloride route rather than the Dow process, partly
because less electrical energy is consumed in side reactions, and partly because the electrical resistivity of the anhydrous salt is high, so that resistive heating of the charge keeps the salt molten without requiring additional external heating.

### 15.2 Scenario for Analysis (Problem Definition)

A plant to process stack dust, recovering zinc oxide and "electronic" or pigment grade iron oxide, has been proposed for location at Colonel's Island, near Brunswick, Georgia. The process, which is diagrammed in Figure 15.1, offers an economically attractive method for handling the pollutants produced by a nearby plant $[4,6]$.

The precipitation of the pigment grade iron oxide is accomplished by additions of hydrochloric acid and magnesium oxide, and an additional set of process steps would extract magnesium metal from the solution of magnesium chloride which remains when the ferric oxide precipitates. Distillation of the magnesium chloride solute is indicated in the diagram as a possible avenue to anhydrous magnesium chloride, which is used to charge the electrolyte cell.

Cost in 1968 for the extraction of anhydrous magnesium chloride from a 35 percent solution was 5.1 cents per pound of magnesium metal, with 3.1 cents per pound allotted to production costs and 2.0 cents for a 20 percent return on investment. For the electrolytic extraction of magnesium metal from the anhydrous magnesium chloride, production cost in 1968 was 15.7 cents and the 20 percent return on investment was 8.3 cents per pound of metal. Parameters for the 1968 analysis were:
(1) 24,000 tons of magnesium per year
(2) 1968 fixed capital costs

Figure 15.1 Stack Dust Processing Plant Flow Diagram
(3) 350-day operating year
(4) 20 year linear depreciation
(5) 5 mills per kWh for electricity
(6) $\$ 3.25$ per hour, average labor cost

At 5 mills per kWh and 6.5 kWh per pound, the 1968 cost for electrlu energy was 3.25 cents per pound of magnesium metal. The cost for electrical energy, at today's estimated cost of 20 mills per kWh , would be 13 cents per pound. This rate for electric energy corresponds roughly to a fuel-plusoverhead cost of $\$ 2.00 /$ million BTU (thermal) for coal. It is this portion of production cost which, after making adjustments for taxes, would have to "buy" a windmill generating facility for the Brunswick plant owners. Since it is posturated that the brunswick plant would produce 20,000 tons per year of magnesium, the yearly increase in equivalent earnings (through reduced production costs) arising from a plant-installed generating facility would be a cash flow of $\$ 5.2$ million, before taxes and before deducting maintenance and operating costs.

Postulating 20,000 tons per year of magnesium production, and assuming that 6.5 kWh electric energy per pound of ingot is required, the annual demand for electric energy would be $2.6 \times 10^{8} \mathrm{kWh}$. If this energy is used in 350 working days, or 8400 hours, the windmill generating facility would have to generate 31 megawatts of continuous power.

The peak power required of the generating facility or total rated power would depend in part on the wind characteristics in the vicinity of the proposed site on Colonel's Island. Data taken at a nearby military installation indicate a plant factor of only 0.15 at a low altitude, but at a
higher altitude the plant factor should approach 0.3 . It will be assumed that one-megawatt or larger units would be installed, on 200 to 300 feet towers or cable arrays, and that the wind plant factor would be 0.3. If it is desired that the plant-owned windmill generating facility be independent of commercial power, there would have to be means for storage of energy in order to maintain constant delivery of 31 megawatts of power., The efficiency of the storage and reconversion of energy would have to be as high as possible--certainly it would have to be higher than the 50 percent cited [9,10] for hydrogen electrolysis systems. A super flywheel driven by each windmill is an attractive high-efficiency, energy storage system. Or it might be possible for the public utility power grid to serve as an energy storage pool. Power produced during periods of strong wind could be more than the requirements for electrolysis; the excess energy could be fed into the power grid. Then, during periods of calm, the power required for electrolysis would be drawn from the grid. Net exchange of energy with the grid would be zero over a year if the windmill system is sized properly. Using either the super flywheel or the power grid for energy storage could minimize the number of windmills required.
15.3 Comparative Analyses (Analysis Design)
15.3.1 Investment A1ternatives

If a windmill generating facility is included in the proposed magnesium plant, the yearly price of utility company electricity would constitute a saving of production cost. That saving, when reduced by operating expenses for the windmill facility and by net tax paid on the
increased earnings, would constitute a benefit to the plant owners. Factors which affect the benefit include the yearly electric energy used, the electric utility rate, the income tax rate, the investment tax credit, depreciation of the windmill facility, operations and maintenance expenses. method of financing, and the expected rate of return on the investment. The choice by the potential investor of an expected rate of return will reflect his estimate of the investment risk and will be influenced by available alternatives. A plant owner traditionally expects 15 to 20 percent rate of return. The alternatives facing the plant owner could be (1) whether to purchase or produce the electric energy for electrolysis of magnesium, (2) whether to make a cash investment in the windmill plant or invest the cash in bonds, and (3) how to finance the windmill plant. As was explained in Section 2, there are various criteria that can be used in making a decision. These criteria include net present value, cut off period, pay back period, net average rate of return, internal rate of return and benefit-cost ratio. It is assumed that the structure of the decision is really to accept or reject the use of a windmill so that the criterion to be used in making the decision is whether or not the net present value is greater than zero (see Figure 2).

The net present value can be expressed simply for the status quo and its alternative; that is,
(1) Purchase electricity (Status Quo):

$$
\begin{equation*}
(B-C)_{P U}=N P V\left[B-C_{e}-C_{o}\right] \tag{15.1}
\end{equation*}
$$

(2) Produce electricity:

$$
\begin{equation*}
(B-C)_{P R}=N P V\left[B-C_{w p 1}-C_{o}\right] \tag{15.2}
\end{equation*}
$$

where

$$
\begin{aligned}
B & =\text { all benefits accruing to production of magnesium } \\
C_{e} & =\text { cost of purchased electricity } \\
C_{0} & =\text { other costs associated with production of magnesium } \\
C_{\text {wpl }} & =\text { costs of the windmill plant. }
\end{aligned}
$$

It is assumed that the gross income (sales) will not be affected by the use of a wind power system and that costs, other than those associated with electricity for the energy intensive process, will be the same with or without the windmill. Thus, the advantage of producing rather than purchasing electricity is then the difference in cash flow between the alternative and the status quo, or

$$
\begin{equation*}
(B-C)_{P R}-(B-C)_{P U}=N P V\left[C_{e}-C_{w p 1}\right] \tag{15.3}
\end{equation*}
$$

The analytical formulations for these two terms, $C_{e}$ and $C_{w p 1}$, will be developed in the following section. However, before proceeding, the various cash flow factors should be identified. As might be readily arrived at, the cash flow factors include yearly energy used, electric utility rate, income tax rate, investment tax credit, depreciation of the windmill facility, operations and maintenance costs, method of financing and expected rate of return on investment. Unlike the macro analysis, taxes, depreciation, investment credit and method of financing enter into the micro analysis. If society as a whole were being considered these factors would not be included
in the analysis and an analysis equivalent to that of the macro analysis would be performed. Let us now consider how these factors enter into the calculation of the two costs, $C_{e}$ and $C_{w p 1}$.
15.3.2 Net Present Value of Producing Electricity

The savings of production cost, realized by producing instead of purchasing electricty, would be a positive, annual cash flow:

$$
\begin{equation*}
C_{e}=\operatorname{NPV}\left[\left(2.6 \times 10^{8}\right) R_{p}\right] \tag{15.4}
\end{equation*}
$$

where

$$
\begin{aligned}
2.6 \times 10^{8}= & \text { electricity required to produce } 20,000 \text { tons of } \\
& \text { magnesium, } \mathrm{kWH} / \text { year } \\
R_{p}= & \text { cost for electricity, if it were purchased, } \$ / \mathrm{kWH}
\end{aligned}
$$

The windmill costs would be negative cash flows:

$$
\begin{equation*}
C_{w p l}=N P V[N A+0+T] \tag{15.5}
\end{equation*}
$$

where

$$
\begin{aligned}
0= & \text { operating costs for windmill-generators, } \$ / \text { year } \\
\mathrm{N}= & \text { number of windmill generator units } \\
\mathrm{A}= & \text { installed cost per windmill-generator, \$/unit } \\
\mathrm{T}= & \text { tax costs, on the savings realized by producing rather than } \\
& \text { purchasing electricity, \$/year }
\end{aligned}
$$

The number of one-megawatt windmill-generator units, if no energy storage is required, is:

$$
\begin{equation*}
N=\frac{2.6 \times 10^{8}}{(8400)\left(P_{f}\right) 10^{3}}=\frac{31}{P_{f}} \tag{15.6}
\end{equation*}
$$

where

$$
\begin{aligned}
2.6 \times 10^{8} & =\text { electric energy used per year, } \mathrm{kWH} \\
8400 & =\text { plant operating year in hours ( } 350 \text { production days) } \\
\mathrm{P}_{\mathrm{f}} & =\text { wind plant factor at chosen site. }
\end{aligned}
$$

For general analysis purposes, it can be assumed initially that the electric generating facility is independent of the commercial power company and consequently a means for storing energy must be provided. Not only would this independence add the cost of storage equipment to the cost of windmill-generators, it would also require an increased number of windmillgenerator units. Although the development of storage units for windmill usage is:in its early stages and not much cost-technology tradeoffinformation exists, it inas been pointed out that the efficiency of storing and reconverting energy by the hydrogen route would be 50 percent or less [9,10]. If energy is stored and reconverted the number of generators required would be

$$
\begin{align*}
N^{\prime} & =\frac{N}{e_{c}}  \tag{11.7}\\
& =\frac{31}{P_{f} e_{c}}
\end{align*}
$$

where

$$
\begin{aligned}
e_{c} & =\text { conversion efficiency for storing and reconverting energy } \\
& =\text { unity if no reconversion is used. }
\end{aligned}
$$

If $e_{c}$ is 50 percent, $N^{\prime}$ would be two times $N$.

The net present value of producing windmill power can be written:

$$
\begin{equation*}
\operatorname{NPV}_{P R}=\operatorname{NPV}\left[2.6 \times 10^{8} R_{p}\right]-N P V\left[\frac{31 A}{P_{f} e_{c}}+0+T\right] \tag{15.8}
\end{equation*}
$$

The net present value of the cost of incremental taxes on the amount of money saved by producing instead of purchasing electricty would be:

$$
\begin{equation*}
\operatorname{NPV}(T)=N P V\left[t\left(c_{e}-0\right)-t(D)-R_{i} N^{\prime} A\right] \tag{15.9}
\end{equation*}
$$

where

$$
\begin{aligned}
t & =\text { tax rate } \\
& =\text { zero only if the entire plant does not show a profit } \\
D & =\text { yearly depreciation of windmill/generator facility } \\
R_{i} & =\text { investment incentive rate }
\end{aligned}
$$

The annual operating cost can be estimated as $m$ percent of the windmill plant cost:

$$
\begin{equation*}
0=\left(N^{\prime} A\right)(m) \tag{15.10}
\end{equation*}
$$

The effect of depreciation is to defer taxes and over the lifetime of a project, the investment decision is somewhat insensitive to the depreciation method used (straight line or accelerated). The widely accepted method used in decision making is the conventional straight-line method which was considered suitable for this analysis. Use of the straight-1ine method results in the net present value of depreciation,

$$
\begin{equation*}
N P V(D)=N P V_{n}\left[\frac{N^{\prime} A}{n}\right] \tag{15.11}
\end{equation*}
$$

where

$$
\mathrm{n}=\text { years to zero salvage value }
$$

A further tax savings, or tax shield, is possible if the purchase of the windmill plant is financed by bonds. For analysis bond interest paid can be considered production cost and subtracted from $C_{e}$. Thie debt fi nancing tax shield is treated in the next section.

The cost of purchased electricity is rising. To take account of this, 1et

$$
\begin{equation*}
R_{p}=0.02+(s) \tag{15.12}
\end{equation*}
$$

where

$$
\begin{aligned}
0.02 & =\text { early } 1975 \text { industríal rate, } \$ / \mathrm{kWH} \\
\mathrm{~s} & =\text { increase in rate per year, } \$ / \mathrm{kWh} / \mathrm{YR}, \\
\mathrm{k} & =\text { year number, counting from } 1975 .
\end{aligned}
$$

The present forecast [8] is that the industrial rate will double in five or ten years. The base parameter value chosen for $s$ (for the sensitivity analysis below) is $\$ 0.002 / \mathrm{kWH} / \mathrm{YR}$ which reflects a doubling of the rate in 10 years; i.e., a 10 percent per year growth rate.

The expression for net present value of the benefit of the windmill plant can now be formulated as follows:

$$
\begin{align*}
N P V_{P R} & =N P V\left[C_{e}-C_{W P 1}\right] \\
& =N P V\left[2.6 \times 10^{8} R_{p}-\frac{31 A}{P_{f} e_{c}}-0-T\right] \\
& =N P V\left[2.6 \times 10^{8} R_{p}-\frac{31 A}{P_{f} e_{c}}-m \frac{31 A}{P_{f} e_{c}}\right. \\
& -\left(2.6 \times 10^{8} R_{p}\right)(t)+(m) \frac{31 A}{P_{f}^{e} e_{c}} \\
& \left.+\frac{31 A}{P_{f}^{e} e_{c}}\left(\frac{t}{n}\right)+R_{i} \frac{31 A}{P_{f} e_{c}}\right] \tag{15.13}
\end{align*}
$$

Gathering terms which are computed over the same periods yields:

$$
\begin{align*}
N P V_{P R} & =N P V_{H}\left[\left(2.6 \times 10^{8} R_{p}-\frac{31 \mathrm{~mA}}{P_{f} e_{c}}\right)(1-t)\right] \\
& +N P V_{n}\left[\left(\frac{31 A}{P_{f} e_{c}}\right)\left(\frac{t}{n}\right)\right]+N P V_{1}\left[\frac{31 R_{i} A}{P_{f}{ }^{e}}\right] \\
& -N P V_{F}\left[\frac{31 A}{P_{f}{ }^{e}}\right] \tag{15.14}
\end{align*}
$$

Here the subscripts denote

$$
\begin{aligned}
& H= \text { horizon for return of investment } \\
& \mathrm{n}= \text { years to depreciate to zero salvage value } \\
& 1= \text { one year for the investment credit } \\
& \mathrm{F}= \text { years to finance the investment }(\mathrm{F}=0 \text { implies cash payment } \\
& \text { for the plant before the startup of the plant.) } \\
& 15.3 .3 \text { Financing the Investment }
\end{aligned}
$$

$$
\begin{equation*}
\frac{31 A}{e_{c}{ }^{P_{f}}} \tag{15.15}
\end{equation*}
$$

includes a potential tax shield. The windmill plant could be paid for in cash, as an expenditure by the plant owners made in anticipation that a specified rate of return, say 15 percent, would be satisfied over a 40 year span, or it could be financed by the sale of bonds, or the owner/ managers could finance it through sale of stock. Even before making the decision about how to finance the windmill plant, however, the plant owner needs to examine alternative investments of available funds.

If the plant owner invested in 10 percent bonds or other paper instead of in windmill plants, the difference in profits over a period of 40 years, assuming
the rate of return on plant investment to be 15 percent, would be:

$$
\begin{align*}
L & =F\left[(1.15)^{40}-(1.10)^{40}\right] \frac{31 \mathrm{~A}}{P_{f} e_{c}} \\
& =\left[\frac{31 \mathrm{~A}}{\mathrm{P}_{f} e_{c}}\right][223] \tag{15.16}
\end{align*}
$$

It would be advantageous to invest in the windmills, if a rate of return higher than 10 percent can be realized.

A second alternative to cash purchase of the windmill plant would be debt financing through the sale of bonds. The cost for the windmills can be written using the appropriate "net present value factors" as

$$
\begin{align*}
& \operatorname{NPV}_{F}\left[\frac{31 A}{P_{f} e_{c}}\right]=\left[\frac{31 A}{P_{f} e_{c}}\right]\left[\frac{1}{(1+i)^{y}}\right] \\
& +\left[\left(\frac{31 A}{P_{f} e_{c}}\right)\left(R_{b}\right)(1-t)\right]\left[\frac{(1+i)^{y}-1}{i(1+i)^{y}}\right] \tag{15.17}
\end{align*}
$$

where the factor on the right of each term is the appropriate net present value factor and

$$
\begin{aligned}
& R_{b}=\text { interest paid on bonds } \\
& y=\text { years to maturity of bonds. }
\end{aligned}
$$

The second term reflects the tax shield generated by the interest cost. When $y$ is zero, the second term vanishes, and the expression reduces to the cash outlay term,

$$
\begin{equation*}
\frac{31 \mathrm{~A}}{e_{c} P_{f}} \tag{15.18}
\end{equation*}
$$

- 

The third alternative, financing through the sale of additional stock to outsiders is in reality a dilution of plant ownership. The choice between
the status quo and buying windmills by selling bonds might require the use of a higher discount rate (because of the higher risk inherent in the obligation to pay annual interest and to repay the bonds) than would be used to evaluate financing through sale of additional stock. Since the sale of stock simply changes the complement of plant ownership, nowever, it does not require a different determination of costs and benefits. It can be viewed simply as a method of raising funds for the cash purchase of plant. It will be shown that the cash purchase of windmills would not be cost beneficial, whereas debt financing would. The decision about the degree of debt financing of the total plant is outside the scope of this analysis.
15.3.4 Benefit-Cost Model

The net present value of the benefit-cost of producing electricity can be formulated for calculation in the form:

$$
\begin{align*}
N P V_{P R} & =\left[\left(5.2 \times 10^{6}-\frac{31 \mathrm{~mA}}{P_{f} e_{c}}\right)(1-t)\right]\left[\frac{(1+i)^{H}-1}{i(1-i)^{H}}\right] \\
& +\left[\left(\frac{31 A}{P_{f} e_{c}}\right)\left(\frac{t}{n}\right)\right]\left[\frac{(1+i)^{n}-1}{i(1+i)^{n}}\right]{ }_{n \leq H} \\
& +\left(\frac{31 R_{i} A}{P_{f} e_{c}}\right)\left(\frac{1}{1+i}\right) \\
& +(1-t)\left(2.6 \times 10^{8}\right)(s) \sum_{k=1}^{H} \frac{k}{(1+i)^{k}} \\
& -\left[\left(\frac{31 A}{P_{f} e_{c}}\right)\left(R_{b}\right)(1-t)\right]\left[\frac{(1+i)^{y}-1}{i(1+i)^{y}}\right] \\
& -\left(\frac{31 A}{P_{f} e_{c}}\right)\left[\frac{1}{(1+i)^{y}}\right] \tag{15.19}
\end{align*}
$$

In summary, the six terms in the final expression of Equation 15.19 in the order in which they are written, represent the following:

1. The first term represents that part of the equivalent income (derived by producing, instead of buying, electricity) which would be realized if the 1974 millage rate held constant over the assumed horizon. The amount saved (\$5.2 million) each year is reduced by the assumed operating expenses for the windmills,
$m\left(\frac{31 A}{P_{f}{ }^{e}}\right)$
and the net "income" is shown reduced by an amount that could be called "tax before tax credit."
2. One reduction of taxes paid out each year is shown as the second term, which represents the tax credit for depreciation of the windmill property.
3. The third term represents the investment incentive "bonus," which is currently allowed as a one-time-only tax credit.
4. The fourth term represents the effect of the expected future increases in the cost of electricity if it were purchased. By producing the electricity, the windmills save the current purchase price each year, which is assumed to increase by "S" cents per kilowatt hour. In the kth year, the increase over the 1975 millage rate is thus (k) (S).
5. The fifth term represents the effect of paying yearly interest on bonds, the sale of which is assumed to finance the windmills, all installed by the beginning of the first plant year. The tax shield
of the interest payments is included. Note that if the term $y$, which indicates the life of the bond issue, is zero, the fifth term correctly disappears because that condition would indicate cash outlay for the windmills.
6. The sixth and last term represents the redemption of the bonds. If $y$ is zero, it represents cash outlay for the windmills, $\frac{31 \mathrm{~A}}{\mathrm{P}_{\mathrm{f}} \mathrm{e}_{\mathrm{c}}}$. 15.3.5 Sensitivity Analyses

As was described in Section 7, there are three types of sensitivity analyses that could be performed: subjective, partial and general. Which one to use strongly depends upon the resources available (money and time) as well as the type of questions to be answered. For this analysis a partial sensitivity analysis was performed that was consistent with the data available and the anticipated type of results that might be useful to a decision maker.

Before performing the partial sensitivity analysis, it was necessary to establish baseline values for the various parameters that are associated with the project scenario. In establishing these baseline values, a subjective analysis was performed to determine what might be typical values for the parameters as well as the range of values over which to perform the sensitivity analyses. Selected baseline values for the sensitivity analyses are shown in Table 15.1. The partial sensitivity analysis was performed by investigating the sensitivity of net present value to changes in values for a selected parameter, with all other parameter values held equal to their baseline values as listed in the table. The results of the analysis are given in Figures 15.2 through 15.13. Most of the basic assumptions for

TABLE 15.1
BASELINE VALUES FOR SENSITIVITY ANALYSIS

| Parameter | Symbol | Base Values |
| :---: | :---: | :---: |
| Wind Plant Factor | $\mathrm{P}_{\mathrm{f}}$ | 0.3 |
| Storage/Conversion Efficiency Factor | $e_{c}$ | 0.9 |
| Number of 1 MW Units | $\mathrm{N}^{\prime}$ | $31 / \mathrm{P}_{\mathrm{f}} \mathrm{e}_{\mathrm{c}}$ |
| Cost/Installed Unit | A | $\$ 5 \times 10^{5}$ |
| Operations/Maintenance Cost Factor | m | 0.01 |
| Time Horizon | H | 40 |
| Discount Rate | i | 0.15 |
| Investment Tax Credit Rate | $\mathrm{R}_{\mathbf{i}}$ | 0.07 |
| Tax Rate | t | 0.5 |
| Electric Power Rate | $\mathrm{R}_{\mathrm{p}}$ | (0.02 + sk) |
| Slope of Average Projected Rate Increase | $s$ | 0.002 |
| Depreciation Period | n | 40 |
| Bond Maturity Period | y | 10 |
| Rate of Interest on Bonds | $\mathrm{R}_{\mathrm{b}}$ | 0.1 |

the sensitivity analysis are listed as base parameters in Table 15.1; other underlying assumptions are:

1. The boundaries for the benefits and costs are of a business nature only.
2. Debt financing of the windmill facility is possible, and will not unduly risk the total plant investment.
3. A site can be found where the wind plant factor is 30 percent or higher. The site will accommodate on the order of 120 one-megawatt windmill-generators.
4. High efficiency energy storage and reconversion are included. An arrangement to exchange energy with the public utility power grid would be one possibility.
5. The plant owners are willing to accept a 15 to 20 percent rate of return on the investment they make in the generating facility.

The selection of the baseline values are based on interaction with NASA/Lewis Research Center personnel, discussions with the Georgia Power Company, and input from the wind data analysis performed for the Brunswick, Georgia area. The plant factor of 0.3 is based on the point design of the 100 kW , Plumbrook windmill and a windmill height of 61 meters. This is an optimistic value for the plant factor corresponding to the Brunswick, Georgia wind data statistics. However, it is believed that higher winds are available nearer the ocean where the windmills could possibly be located. No attempt was made to optimize site locations. The cost per installed unit of $\$ 500 / \mathrm{kW}$ is interpreted to include all start-up costs associated with the production of electricity, including possible storage costs. This baseline
cost is considered simply to be a value about which the range of unit costs is varied. The time horizon is taken to be 40 years which corresponds to typical life times associated with windmill towers. On each of the figures the baseline value for the parameter of interest is indicated.

Figure 15, 2 illustrates that there is a double penalty in an isolated generating system independent of the utility company. The cost of required energy storage equipment increases the investment burden per windmill, and the number of windmills also increases as the energy conversion efficiency decreases. As mentioned previously there is little cost-performance data available in the area of energy storage but two attractive approaches to low-cost energy storage might be the super flywheel and the exchange of electric energy with the public utility grid. The cost for a zero net exchange of power with the utility grid should be quite small. The base value assumed for $e_{c}, 0.9$, is equivalent to assuming a means of energy storage 80 percent more efficient than the hydrogen route suggested by Hieronymous. The cost for energy storage, whether by super flywheel, utility exchange, or other means, is assumed to be included in the 10 percent penalty in the base parameter.

As shown in Figure 15.3, the wind plant factor, $P_{f}$, has the same influence on $N P V_{P R}$ as the conversion factor, $e_{c}$. To obtain a larger value for $P_{f}$ than the value of 0.3 reported for the vicinity of Brunswick, the windmills might be erected offshore. The continental shelf extends some 75 miles to the east of Brunswick, the depth of water is only about 120 feet. The wind factor offshore should approach 0.5. The decision about siting would require a close look at actual wind plant factor, and also a

tradeoff analysis of costs per windmill for seaflow and erection versus a larger number of windmills near Brunswick.

Figure 15.4 shows the effect of windmill cost on the net present value of windmill generation. The cost per windmill should be as small as possible, consistent with reliability. A worthwhile target might be $\$ 500,000$ for each one-megawatt unit, including windmill, generator, tower, energy storage equipment, power conditioning equipment, and per-windmill wiring that is in excess of what would be required if the electricity was purchased. As indicated in the figure breakeven cost is $\$ 600 / \mathrm{kW}$ or the maximum windmill cost that would result in a positive net present value under the given assumptions.

The maintenance and operating cost factor, becuase it is a small percentage of the cost of the installation, has only a weak influence on $N_{P R}$, as shown in Figure 15.5. The time horizon that is chosen would perhaps be expected to have a strong influence on the $N_{P V} V_{P R}$ but because a relatively high discount rate (compared to the social discount rate) is used, NPV as suggested by Figure 15.6 , is not very sensitive to the time horizon.

The discount rate assumed for basis in the analysis is 15 percent. This value is lower than the 20 percent usually assumed for investment decision analysis. Figure 15.7 indicates the sensitivity of the $N P V_{P R}$ to the discount rate.

Figure 15.8 shows the sensitivity of $N P V_{P R}$ to changes in the investment tax credit rate. The effects of an increase in the investment tax credit rate to 12 percent or more to encourage fuel saving can be seen from this figure. Increasing the rate from 7 percent to 12 percent would raise the value of $\mathrm{NPV}_{\mathrm{PR}}$ \$3 million.


Figure 15.5 Net present value as a function of maintenance and operating cost factor as percent of the installation cost.
(\$ uoṭlcth) $\Lambda \mathrm{dN}$

(\$ wottitme) MdN


Figure 15.8 Investment incentive.

Lowering the income tax rate to 40 percent has also been suggested as an incentive to save fuel. The values of $N P V_{P R}$ for tax rates from zero to 50 percent have been plotted in Figure 15.9. The discount rate of 5 percent is near the basis generally accepted for cost-benefit analyses of public and government projects. The rate of 10 percent is near that permitted to regulated public utility companies. The base value assumed for the Brunswick plant is on the low side of the value generally used to justify alternative business investments. Note from the Figure 15,9 that reducing the tax rate from 50 percent to 40 percent would benefit the plant owner \$5 million, but it would benefit a public utility company $\$ 9$ million. Note also that if the windmill plant is constructed by the government, which pays no taxes, the $\mathrm{NPV}_{P R}$ is $\$ 170$ million higher than the plant owner's $N P V_{P R}$. This is a measure of the social value of windmill production facilities.

The industiral electric power rate presently quoted by a Southeastern electric power company [8] is 2 cents per kWH . The projection of future rates anticipates that the rate will climb to 4 cents in 5 or 10 years. The base value assumed for this analysis is an increase of 10 percent of today's price per year. Thus the rate will be 4 cents in 1985, 6 cents in 1995, 8 cents in 2005 , and 10 cents in 2015. Figure 15.10 shows the influence of the electric power rate on $N P V_{P R}$. If the average increase of the rate is 5 percent per year ( $\$ 0.001 /$ year) the $N_{P V}{ }_{P R}$ will be about $\$ 6.5$ million less than the $N P V_{P R}$ for the assumed base value of 10 percent per year increase.



Figure 15.11 shows the benefit of accelerated depreciation, with a gain in $N P V_{P R}$ of $\$ 4$ million by depreciating to zero salvage value in 20 years rather than 40 years. The amount of the gain depends on the size of the investment.

Figures 15.12 and 15.13 show the power of debt financing. The left end of the curve in Figure 15.13 (zero years for financing) represents cash outlay by the owners for the plant. It is obvious that the lower the interest rate and the longer the maturity term, the greater the advantage to the plant owners. Figure 15.12 indicates that interest at 7 percent instead of 10 percent would be worth $\$ 4.5$ million, net present value, for 10 -year bonds. Extension of the term of the 10 percent bonds to 15 years would likewise increase the net present value about $\$ 4.5 \mathrm{million}$. One must ask how an increase in the discount rate from the base value of 15 percent to 20 percent (to compensate for the increased risk of debt financing) would affect the net present value. Figure 15.7 indicates that the net present value would be decreased about $\$ 10$ million. Tradeoffs and adjustments that should be made in the parameters are readily apparent in the sensitivity curves.

### 15.4 Baseline Sensitivity Values

To obtain an estimate of the relative importance of the various parameters and how sensitive the net present value is to changes in the parameter values, point calculations of sensitivity were made. Sensitivity at a point can be defined by the following equation:

$$
\begin{equation*}
S=\frac{\frac{\text { change in NPV }}{\text { NPV (at Baseline) }}}{\text { parameter value (at Baseline) }}=\frac{\frac{\Delta N P V}{N P V}}{\frac{\Delta X}{X}} \tag{15.20}
\end{equation*}
$$





The above definition of sensitivity simply gives the percentage change in net present value in response to a percentage change in the parameter value, More simply,

$$
\begin{equation*}
S=(\text { Slope })\left(\frac{X}{N P V}\right) \tag{15.21}
\end{equation*}
$$

The sensitivity was calculated using the above equation for the various parameters of Figure 15.2 to 15.11 . The calculated results are given in Table 15.2. As can be seen from the table, Net Present Value is most sensitive to storage efficiency, installed cost, discount rate, wind plant factor and rate of fuel increase. For example, a 10 percent increase in plant factor will result in a 50 percent increase in net present value and a 10 percent decrease in installed cost will result in a 60 percent increase in net present value.

### 15.5 Conclusions

Techniques of cost-benefit analysis, more appropriately called engineering economic analysis for the micro analysis, permit detailed examination of the various parameters which bear upon investment decisions. It has been shown that for the stated assumptions a range of parameter values exist that result in a positive net present value of the project scenario over the status quo scenario and that there is a definite potential for the magnesium plant at Brunswick, Georgia, to produce electricity with windmill generators, instead of buying electricity.

It is interesting to extend the results of this analysis from that of a single firm within the electrowinning industry to that of the industry and consider the total amount of fuel that might be saved. Now the potential

TABLE 15.2
SENSITIVITY OF NET PRESENT VALUE TO CASH FLOW FACTORS

$$
\text { Sensitivity }=\frac{\frac{\Delta N P V}{N P V}}{\frac{\Delta X}{X}}
$$

| Wind Plant Factor | +5 |
| :--- | :--- |
| Storage Efficiency | +7 |
| Installed Cost | -6 |
| Rate of Fuel Increase | +2 |
| Operating Cost Factor | -0.8 |
| Time Horizon | -0.1 |
| Depreciation | -0.1 |
| Discount Rate | -6 |
| Bond Maturity | +0.8 |
| Bond Interest Rate | -2 |

of fuel saving by windmill generation of electric energy for the electrowinning industry can be estimated as follows. The annual power consumption was estimated to be about $2 \times 10^{8}$ megawatt hours, or about 25,000 megawatts average. Assuming an industry wide average wind plant factor of 0.4 and a storage efficiency of 0.9 results in a rated power required of about 70,000 megawatts. Plant investment in windmills at $\$ 500 / \mathrm{kW}$ would be $\$ 35$ billion; for a 20 -year schedule of installation, the investment would average $\$ 1.75$ billion a year. Equivalent fuel saved would be about 16 million barrels of oil the first year, 32 million, the second, 48 million the third, etc. At $\$ 10$ a barrel, the savings would be $\$ 160$ million the first year. Assuming a ten percent per year rise in the price of oil, the second year saving would be $\$ 362$ million, the third year $\$ 576$ million, etc.

From the viewpoint of the economy, reducing oil consumption is desirable; therefore, a discount rate of 5 percent seems reasonable when estimating the net present value to the economy of the oil saved. The net present value of oil saved by the 20 -year schedule of installed windmill generators would be $\$ 38$ billion. The net present value of the expenditure of $\$ 1.75$ billion each year, for 20 years, is $\$ 22$ billion so that the difference in net present values alone would be $\$ 16$ billion. Other benefits would accrue from reducing the deleterious effect on our balance of trade caused by importing oil, reducing the drain on nonreplenishing fossil fuels, and reducing pollution from fossil fuels.

## REFERENCES

1. Michael J. Frost, Values for Money, Gower Press, London, 1971
2. Minerals Yearbook, U. S. Department of Interior, Vol. 1, 1971.
3. 1974 Commodity Yearbook.
4. Personal communication with entrepreneur of Brunswick plant.
5. Joe B. Rosenbaum, "Applications of Electrometallurgy in Processing of Minerals," Proc. Sympos. Electrometallurgy, Ed. T. A. Henrie and D. H. Baker, Jr., Pp. 43-51, AIMMPE, 1969.
6. D. A. Elkin, K. L. Dean and J. B. Rosenbaum, "Economic Aspects of Magnesium Production," op. cit.
7. James Hightower, "Your Power Bill Goes up Again," page 1, The Atlanta Journal, December 24, 1974.
8. Personal telephone communcation with Georgia Power Company engineer.
9. W. O. Hughes "Energy Storage Using High Pressue Electrolysis and Methods for Reconversion," Wind Energy Conversion Systems, Workshop Proceedings, 1973.
10. W. E. Heronemus, "Windpower: Near-Term Partial Solution to Energy Crisis," 1973 EASCON, IEEE, 1973.

## PART IV

CONCLUSIONS AND RECOMMENDATIONS

### 16.1 Cost-Benefit Methdology Study

At the outset of this program, questions posed included the following: "Can a widely accepted methodology be established to evaluate the relative merits of alternative projects in todays complex economic-political-social environment?" and if so, "Can high level decision makers be convinced of the utility of such a methodology?" To address these questions, a thorough review of literature dealing with cost-benefit methodolgies or analyses was made; journal articles, books, and other sources were reviewed and cost-benefit analysis concepts were assessed. The most pertinent concepts were identified and explained. This work resulted in a logically consistent, explicit, yet flexible methodology (summarized in Section 7) developed for the performance of costbenefit analyses for NASA type problems. The following are some specific conclusions that can be made based on the cost-benefit methodology study.
(1) A cost-benefit analysis identifies and evaluates the benefits and costs associated with alternatives for achieving defined public goals. Techniques used in identifying and comparing costs and benefits are almost as numerous as existing analyses.
(2) As applied welfare economics, cost-benefit analysis uses a decision criterion identified as "Potential Pareto Superiority" which labels a project as superior if those who gain from the project could compensate those who lose so that none would be worse off with the project.
(3) The criterion for use in decision making on the selection of alternative projects depends upon the problem structure, i.e., type of decision to be made, independency of project, and the type of capital constraint. Criteria include the net present value criterion and the benefit/cost ratio. It should be emphasized that performing a cost-benefit analysis does not and should not imply using a benefit/cost ratio as the criterion to rank projects.
(4) The most important aspect of a cost-benefit analysis is the identification of all relevant costs and benefits.
(5) The major stumbling block in identifying costs and benefits is the double-counting problem relative to the several ways and classification schemes that benefits and costs may be classified.
(6) Classification schemes include internal and external effects, incommensurables, intangibles, and direct and indirect effects. Although it may be desirable to place a benefit in one category or another, the important thing is that benefits (costs) be additions (deletions) to the real product of an economy or to the real welfare of its members.
(7) Quantification of costs and benefits is usually based on market prices of the goods under consideration as long as these prices indicate the value of the goods; however, because market prices do not always reflect social value, consideration must be given to utilizing a shadow price, that is, an adjusted price which does reflect social value.
(8) Shadow prices can be thought of either as dual variables arising from mathematical programing or as true economic valuations. Although these two meanings are identical, the situation at hand will dictate whether to use the programming approach or economic theory or both.
(9) To compare one project to another or to determine the economic viability of a particular project, the time stream of appropriate costs and benefits must be reduced to a single number and in such calculations the rate of discount is a crucial parameter. The "social rate of time preference" is the rate at which society as a whole is willing to give up present consumption for future consumption. It is not reflected by an individual's rate of time preference and only the latter can be observed in economic data. It is important to realize that the use of a low discount rate (say 3 to 5 percent) values long term benefits much more than if a higher discount rate (of, say 10 percent) were used. "Social Impact Analysis" is an alternative term used to describe cost-benefit analysis in the broad sense. A fruitful approach to social impact analysis is the iterative-interactive decision mode which combines objective data analysis by the analyst and subjective problem analysis by the decision maker.
(11) Although the social opportunity cost of capital appears to be greater than the cost implied in a conventional cost-benefit analysis, the consensus among economists is to avoid on practical grounds the complex estimations associated with this concept even though its technical basis is flawless.
(12) An important step in a cost-benefit analysis is the performance of a sensitivity analysis which is also a means of presenting to the decision maker as much information as possible in a format useful to the decision maker. Ranges of estimates provide the decision maker with information on the identification of the most critical paramscers as well as the accuracy with which the parameter values have to be known. Three levels of sensitivity analysis are subjective, selective, and general.
(13) In performing a cost-benefit analysis, perhaps the most important ingredient. is the analyst's interaction with the decision maker who is the beginning and the end of the cost-benefit analysis cycle.

### 16.2 Wind Data Collection and Assessment

From the wind assessment of the present study several conclusions can be made with regard to aerogenerator plant factor (ratio of average output power to rated power):
(1) Plant factors can be accurately computed from wind speed distributions, taken to be Weibull distribution functions. The plant factor is a nonlinear function of the Weibull distribution parameters (as shown in Figure B-2). Highest plant factors are obtained with high mean wind speeds and low variance of the wind speed.
(2) Although plant factor increases with height, there is a level at which further increase in height would not be effective in increasing aerogenerator output (as shown by Figure 12.2). For aerogenerators with operating characteristics near those for NASA's Plum Brook
unit, it would not be necessary to go above a height of 60 m (200 ft) for maximum system performance.
(3) As shown in Figures 12.6 through 12.10, on a seasonal and annual basis the average plant factor, at a height of $60 \mathrm{~m}(200 \mathrm{ft})$, is greater than 0.6 for a large section of the middle portion of the U.S. and certain parts of the New England coast.
(4) Although aerogenerator output may be increased by designing improvements in the rated power per unit area of the blades, actual output power per unit area does not increase proportionally. As shown in Figure 13.12, the best gains would be achieved at high wind speed sites, but at best a doubling of output power per unit area is achieved by almost a quadrupling of rated power per unit area. In contrast, at low wind speed sites, very little increase in output power per unit area is realized by increasing rated power per unit area.

### 16.3 Application of the Cost-Benefit Methodology

Categories of windmill generator systems which might have significant positive potential were identified especially in view of possible large scale utilization. A list of system options was categorized in a matrix classification scheme using alternative technologies and alternative end uses of the system product as the two classification dimensions. Cost of large aerogenerator systems and components are not well established for current technologies, and it was judged that costs of previous designs could hot be meaningfully extrapolated to current dollars. Two types of wind power systems were selected for the application of the cost-benefit methodology;
the first system was a macro analysis of a wind system which is linked into an utility power grid having no storage capability. The second system was a micro analysis of a wind power system from the viewpoint of a firm that utilizes an energy intensive process. System $I$ was considered to be used as a fuel saver and System II was considered to be an electricity saver resulting in reduced operating cost. A cost-benefit model was designed and implemented on the computer to establish a practical tool for studying the relative costs and benefits of wind power systems under a variety of conditions to efficiently and effectively perform associated sensitivity analyses. Based on the wind data analysis results, the cost of fuels in the various regions throughout the United States, the efficiency of fuel conversion processes and on the baseline values for the status quo scenario and the alternative scenario (project), the conclusions that may be drawn about the System I and System II applications are described below.
(1) The analyses of System I and II demonstrated the cost-benefit analysis and engineering economic analyses concept in social and private frameworks.
(2) System I results were found to be sensitive to the operational strategy (the most expensive fuel replaced first or all fuels replaced proportionately), installation costs, plant factor, fuel price increases, and discount rates.
(3) Based on both wind potential and fuel prices, wind systems appear to have greatest benefits in the New England and Mid Atlantic regions.
(4) The study results indicate that even for systems which act only as fuel savers and do not contribute to a firm system capacity, aerogenerators in a utility grid can be economically advantageous.
(5) For System II, a range of parameter values exist that results in a positive net present value of the project scenario over the status quo scenario and that there is potential for the use of windmill generators to produce electricity for energy intensive processes.
(6) System II results are most sensitive to storage efficiency, wind plant factor, and installed cost.
(7) The divergence between the social discount rate and the higher internal rate of return demanded by firms suggest that a socially optimal use of wind power can be achieved only by appropriate government inducements to firms to adopt this new technology.

As a general conclusion, it might be stated that the cost-benefit methodology that was described in Part II of this report can be utilized in performing a cost-benefit analysis of a variecy of projects. Although the general methodology would apply, the specific models must be tailored to some extent for the particular problems being analyzed. A method has been demonstrated to determine and present results in a format suitable for a decision maker even when the cost of the project components are known with great uncertainty or not known at all. Thus the utility of the methodology should be widely accepted by decision makers providing that there is the appropriate interaction between the analyst and the decision maker.

## RECOMMENDATIONS

### 17.1 Cost-Benefit Methodology

The following recommendations are made with regard to use of the methodology described in Sections 1 through 8.
(1) The results of any cost-benefit analysis should be used as a decision aia and there should be considerable interaction with the decision maker, especially in the problem definition phase,
(2) Although the methodology or procedures are presented as a unidirectional approach to cost-benefit analysis, iterations should be made as appropriate to reflect higher order effects.
(3) All "social" effects, which are typically incommensurables and intangibles, should be considered and at least described in the anatysis.
(4) The iterative-interactive approach, described in Section 5.4, should be considered as a possible methodology for incorporating the social effects into a quantitative analysis.
(5) During the early stages of a cost-benefit analysis, the analyst should convey to the decision maker for approval the format of the results and this format should allow the results to be presented in a clear and understandable manner.

With regard to extending the cost-benefit methodology or "manual," the following recommendations are made.
(6) An example application other than wind would enhance the utility of the methodology since not all concepts discussed in this program were
applicable to the wind generator cost-benefit analysis.
(7) Additional investigations should be made so that a section on costbenefit analysis of research and development programs could be included in the manual format.
(8) An empirical study should be made of the discount rate relative to the appropriate one to be used in cost-benefit analyses.
(9) A compendium of social impact assessment methodologies should be developed which would include an identification of the various methodologies and a description of data requirements and usefullness of results. Each methodology would be assessed for validity and applicability to NASA problems. New approaches should be synthesized and described and relations between social impact assessment and cost-benefit analysis should be explored further.

### 17.2 Wind Data Collection and Assessment

The wind data collection and assessment performed in this study is adequate for evaluation of wind generators operated as fuel savers. (rather than to augment generating capacity) and operated as single units (rather than in large interconnected arrays). For fuel saver operations the plant factor (fraction of rated output power actually realizable) is the only parameter which needs to be evaluated from the wind data. This can be done adequately from the data in Appendix $D$ by the method discussion in Appendix $B$. Results are shown in Figures 12.6 through 12.10. However, for wind generator systems which are not utilized to augment electrical system capacity it is recommended that the following additional wind data analyses be made.
(1) Analysis of frequency distributions of return times (time required for out put power to return above a certain level once it goes under that value) would be necessary in order to properly evaluate storage requirements of systems when a certain generated power capacity is necessary.
(2) Arrays of aerogenerators will also be able to add system capacity, if they cover sufficient area that even if one part of the array suffers caims, other portions of the array will still be generating. Therefore, statistics of array output power as a function of spatial size of the array would be necessary for evaluating the potential of the method of generating capacity augmentation.
(3) Both of these parameters (frequency of return times, and statistics of array output) cannot be evaluated with the single site wind speed frequency distributions used in this study. Instead, the hourly (or 3 hourly) wind values at sites (or arrays of sites) must be used.
(4) This wind data collection and assessment has been limited to the contiguous United States. It is recommended that :consideration be given to performing similar assessments of winds of non-contiguous U.S. regions.

### 17.3 Cost-Benefit Analysis of Wind Generators

The cost-benefit analysis of wind generators described herein was performed following the methodology outlined in Sections 1 through 8 and is considered to be a first iteration. The results presented are preliminary in nature and a second iteration should be done to achieve a more complete costbenefit analysis. Additional investigations should include the following recommendations.
(1) Sensitivity analyses were made for a set of baseline values and critical parameters were identified. Additional analyses should be made with emphasis on investigating the sensitivity of net present value to changes in the values of the critical parameters. For these analyses, baseline values should be used that reflect the most updated information available on wind generator systems.
(2) As part of the second iteration, a social impact analysis should be made in view of the possible implementation of windmills in the most promising regions.
(3) The analyses carried out were for large windmill systems conceptually employed as a fuel saver and as an alternative source of electricity. The second iteration should also include other applications that incorporate smaller units associated with residences or industrial complexes.
(4) Other aspects of assessing the potential of wind power systems should be considered although they may not be the prime responsibility of NASA. Such items include (a) government inducement of firms to adopt wind generator systems, (b) property rights to wind, (c) impact on zoning restrictions, and (d) land-use planning.
(5) In addition to the above recommendations, it is suggested that a methodology be developed for determining the "optima1" energy source development for a region; sources would include wind and solar energy in addition to combinations of fossil, hydroelectric, nuclear, and waste-conversion sources.

APPENDICES

## APPENDIX A

## SOCIAL VALUES AND ELASTICITY CONS IDERATIONS

As discussed previously, CBA reduces to a simple task in an "IDEAL" economy characterized by agreement with a large number of restrictive assumptions. These assumptions were listed in the previous section. Now let us examine specific circumstances which cause the actual economy to deviate from the ideal, and how economic theory can aid in determining correct social values. In CBA, knowing and accepting the limitations of one's tools is crucial. Thus, the prupose here:is also to frankly assess the merit of the suggested approaches.

## A. 1 Shadow Pricing Under Imperfect Competition and Production Hierarchies

The question addressed in this section is: What can be said about the true social value (shadow price) of a good produced under imperfect competition and (possibly) used as an input to other production processes, when these other processes are also in the context of imperfect competition?

The importance of this topic to CBA stems from the recognition that inputs to public projects are, alternatively, inputs to private production processes; and as such these inputs would otherwise add to the production of final consumer goods. Furthermore, when there is a production hierarchy, and it is not characterized by perfect competition all the way through, the market price of a good produced within the hierarchy is not indicative of its true social value. Likewise, the benefits of a project may be in the saving of some good (as in a wind energy project where fossil fuel is saved),
and if this good is part of an imperfectly competitive production hierarchy, its market value may not adequately reflect its social value.

## A.1.1 The Analytic Framework*

The objective here is to develop an operational approach to estimating the social value of a good produced and used in an imperfectly competitive production hierarchy. By "operational" is meant that the "formula" developed should depend on only observational data. "Imperfectly competitive" means the firms selling the goods are not price-takers, i.e., they have some control over the market price of the good. This means that the demand curve faced by a representative firm slopes down to the right, it is not horizontal at the market price as in the case of a perfectly competitive firm. It should be noted that this situation--imperfect competition--undoubtedly characterizes the majority of U.S. markets. A "production hierarchy" is the set of industries and their input-output relation to one another, where no good is a direct or indirect input to its own production. To achieve reasonable generality, the hierarchy considered here is one wherein each good is an input to each "later" good. To make the analysis tractable, we initially limit our consideration to a four-tier hirarchy. This initial analysis is then readily generalized to a production hierarchy of an arbitrary number of tiers.

The analytic framework is as follows. There are four goods in the model, $X_{0}, X_{1}, X_{2}, X_{3}, X_{0}$ is a "primary" good (relative to the model) and

$$
\begin{aligned}
& x_{1}=F^{1}\left(x_{01}\right) \\
& x_{2}=F^{2}\left(x_{02}, x_{12}\right) \\
& x_{3}=F^{3}\left(x_{03}, x_{13}, x_{23}\right)
\end{aligned}
$$

[^12]where $F^{1}, F^{2}, F^{3}$ are production functions, $X_{i}$ is the total output of the ith good, and $X_{i j}$ is the amount of $X_{i}$ used in $X_{j} . X_{3}$ is a final consumer good, so all of $X_{3}$ produced goes toward final consumption. However, $X_{0}, X_{1}$, and $X_{2}$ are, in addition to inputs, also final consumer goods. Thus, part of the production of these goods is diverted to consumers, and the remainder is passed forward for use in production. The representative firms in each of these four industries are imperfect competitors in their product markets, but perfect competitors in their factor markets. For example, the representative firm producing $X_{2}$ faces a downward sloping demand curve for his output, thus he can jary the price he receives by varying his output. However, the representative $X_{2}$ firm, in purchasing inputs $X_{0}$ and $X_{1}$, simply pays the established market price. He has no control over his factor prices. The concern will be the determination of the shadow price of $\mathrm{X}_{0}$. For concreteness, it is assumed that a proposed public project will use up (or release) a relatively small amount of $X_{0}$, and its social value is required as an input to a CBA.* The project's use of $X_{0}$ is one-time only, the supply of $X_{0}$ is assumed perfectly inelastic.

The social value of a decrease in the quantity of $X_{0}$ avallable for non-project uses is the value of consumption foregone by society because of the decreased availability of $X_{0}$, where the value of consumption is regarded as willingness-to-pay. Before proceeding, some additional notation must be introduced, and for convenience, that already introduced is reviewed.
$X_{i j} \quad$ the quantity of $X_{i}$ used in the production of $X_{j}$ $i<j$; and $i=0,1,2 ; j=1,2,3$

[^13]$X_{i}{ }^{C} \quad$ the quantity of $X_{i}$ devoted to final consumption
S.P. $\left(X_{0}\right)$ the shadow price of $X_{0}$
$M_{i j} \quad$ the marginal physical product of $X_{i j}$
$M R P_{i j}$ the marginal revenue product of $X_{i j}$
$M_{i} \quad$ the marginal revenue from $X_{i}$
$P_{i} \quad$ the market price of $X_{i}$
From the foregoing discussion, we have the definition
\[

$$
\begin{equation*}
\text { S.P. }\left(X_{0}\right)=P_{3} \frac{\partial X_{3}^{C}}{\partial X_{0}}+P_{2} \frac{\partial X_{2}^{C}}{\partial X_{0}}+P_{1} \frac{\partial X_{1}^{C}}{\partial X_{0}}+P_{0} \frac{\partial X_{0}^{C}}{\partial X_{0}} \tag{1}
\end{equation*}
$$

\]

Equation (1) can be expanded to

$$
\begin{equation*}
\text { S.P. }\left(X_{0}\right)=P_{3} \frac{\partial X_{3}^{C}}{\partial X_{3}} \frac{\partial X_{3}}{\partial X_{0}}+P_{2} \frac{\partial X_{2}^{C}}{\partial X_{2}} \frac{\partial X_{2}}{\partial X_{0}}+{ }^{P} 1 \frac{\partial X_{1}^{C}}{\partial X_{1}} \cdot \frac{\partial X_{1}}{\partial X_{0}}+P_{0} \frac{\partial X_{0}^{C}}{\partial X_{0}} \tag{2}
\end{equation*}
$$

Now consider the expressions of the form $\partial X_{i} / \partial X_{o}$, in particular consider $\partial X_{3} / \partial X_{0}$. From the hierarchial production relations, we have

$$
\begin{equation*}
X_{3}=F^{3}\left\{X_{03}, F^{1}\left(X_{01}\right), F^{2}\left[X_{02}, F^{1}\left(X_{01}\right)\right]\right\} \tag{3}
\end{equation*}
$$

Thus,

$$
\begin{align*}
\frac{\partial X_{3}}{\partial X_{0}} & =\frac{\partial X_{3}}{\partial X_{03}} \frac{\partial X_{03}}{\partial X_{0}}+\frac{\partial X_{3}}{\partial X_{13}} \frac{\partial X_{13}}{\partial X_{1}} \frac{\partial X_{1}}{\partial X_{01}} \frac{\partial X_{01}}{\partial X_{0}} \\
& +\frac{\partial X_{3}}{\partial X_{23}} \frac{\partial X_{23}}{\partial X_{2}} \frac{\partial X_{2}}{\partial X_{12}} \frac{\partial X_{12}}{\partial X_{1}} \frac{\partial X_{1}}{\partial X_{01}} \frac{\partial X_{01}}{\partial X_{0}}  \tag{4}\\
& +\frac{\partial X_{3}}{\partial X_{23}} \frac{\partial X_{23}}{\partial X_{2}} \frac{\partial X_{2}}{\partial X_{02}} \frac{\partial X_{02}}{\partial X_{0}}
\end{align*}
$$

and likewise

$$
\begin{equation*}
\frac{\partial X_{2}}{\partial x_{0}}=\frac{\partial X_{2}}{\partial X_{12}} \frac{\partial X_{12}}{\partial x_{1}} \frac{\partial x_{1}}{\partial X_{01}} \frac{\partial x_{01}}{\partial X_{0}}+\frac{\partial x_{2}}{\partial x_{02}} \frac{\partial X_{02}}{\partial x_{0}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial X_{1}}{\partial X_{0}}=\frac{\partial X_{1}}{\partial X_{01}} \frac{\partial X_{0.1}}{\partial X_{0}} \tag{6}
\end{equation*}
$$

Employing the notation $M P P_{i j}$ for $\frac{\partial X_{j}}{\partial X_{i j}}, R_{i c}$ for $\frac{\partial X_{i}^{C}}{\partial X_{i}}$, and $R_{i j}$ for $\frac{\partial X_{i j}}{\partial X_{i}}$, and substituting (4), (5), and (6) into (2), we have

$$
\begin{align*}
S . P .\left(X_{0}\right) & =: P_{3} \cdot R_{3 C}\left[M P P_{23} \cdot R_{23} \cdot M P P_{12} \cdot R_{12} \cdot M P P_{01} \cdot R_{01}\right. \\
& +M P P_{23} \cdot R_{23} \cdot M P P_{02} \cdot R_{02}+M P P_{13} \cdot R_{13} \cdot M P P_{01} \cdot R_{01}  \tag{7}\\
& \left.+M P P_{03} \cdot R_{03}\right]+P_{2} \cdot R_{2 C}\left[M P P_{12} \cdot R_{12} \cdot M P P_{01} \cdot R_{01}\right. \\
& \left.+M P P_{02} R_{02}\right]+P_{1} \cdot R_{1 C}\left[M P P_{01} \cdot R_{01}\right]+P_{0} \cdot R_{0 C}
\end{align*}
$$

By definition, $\mathrm{MRP}_{\mathrm{ij}}=\mathrm{MPP}_{\mathrm{ij}} \cdot \mathrm{MR}_{\mathrm{j}}$;
and assuming profit maximizing behavior by the firms, $X_{i j}$ will be such that

$$
\begin{equation*}
P_{i}=M R P_{i j} \tag{9}
\end{equation*}
$$

Thus, from (8) and (9),

$$
\begin{equation*}
M P P_{i j}=\frac{P_{i}}{M R_{j}} \tag{10}
\end{equation*}
$$

Using (10) in (7), and rearranging terms,

$$
\begin{aligned}
& +\frac{P_{3} P_{1} P_{0}}{M R_{3} M R_{1}} \cdot{ }^{R_{3}} C^{R_{1}}{ }^{R} R_{01}+\frac{P_{3} P_{0}}{M R_{3}} \cdot{ }^{R_{3}} C^{R_{03}}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{\mathrm{P}_{1} \mathrm{P}_{0}}{\mathrm{MR}_{1}} \quad \mathrm{R}_{1} \mathrm{C}^{\mathrm{R}} 01+\mathrm{P}_{0} \mathrm{R}_{0 \mathrm{C}}
\end{aligned}
$$

The question arises, at this point, as to whether (11) is in any sense operational. That is, does (11) depend only on observational data? To answer the question in the affirmative, the MR's and R's must be related to observational data. Recall $R_{i j}=\frac{\partial X_{i j}}{\partial X_{i j}}$ is the change in the amount of $X_{i}$ used in the production of $\mathrm{X}_{\mathrm{j}}$ as the total production of $\mathrm{X}_{\mathrm{i}}$ changes. In other words, $\mathrm{R}_{\mathrm{ij}}$ has to do with the demand for $X_{i}$. For concreteness, consider the demand for $X_{2}$. There are two groups of demanders of $X_{2}$ : consumers of $X_{2}$ and producers of $X_{3}$. Write these demand functions as

$$
\begin{equation*}
\mathrm{X}_{2}^{\mathrm{C}}=\mathrm{D}_{2 \mathrm{C}}\left(\mathrm{P}_{2}\right) \text { and } \mathrm{X}_{23}=\mathrm{D}_{23}\left(\mathrm{P}_{2}\right) \tag{12}
\end{equation*}
$$

Assuming the market for $X_{2}$ is in short-run equilibrium,

$$
x_{2}=x_{2}^{C}+x_{23} .
$$

It follows

$$
\frac{\partial x_{2}}{\partial P_{2}}=\frac{\partial x_{2}^{\mathrm{C}}}{\partial \mathrm{P}_{2}}+\frac{\partial \mathrm{x}_{23}}{\partial \mathrm{P}_{2}}=\mathrm{D}_{2 \mathrm{C}}^{\prime}+\mathrm{D}_{23}^{\prime}
$$

So, for example,

$$
R_{23}=\frac{\partial X_{23}}{\partial X_{2}}=\frac{\partial X_{23}}{\partial P_{2}} \quad \frac{\partial P_{2}}{\partial X_{2}}=\frac{D_{23}^{\prime}}{D_{2 C}^{\prime}+D_{23}^{\prime}}
$$

and

$$
R_{2 C}=\frac{D^{\prime}}{D_{2 C}^{\prime}}{ }_{2 C}+D_{23}^{\prime}
$$

In general,

$$
\begin{equation*}
R_{i k}=\frac{D^{\prime} i k}{\sum_{j} D^{\prime} i j} \tag{13}
\end{equation*}
$$

Note that $\mathrm{R}_{23}+\mathrm{r}_{2 \mathrm{C}}=1$ and in general,

$$
\begin{equation*}
\sum_{j} R_{i j}=1 \text { for each } i \tag{14}
\end{equation*}
$$

Note further that the elasticity of demand for $X_{i j}$ is

$$
\begin{align*}
& E_{i j}=-\frac{P_{i}}{X_{i j}} \frac{\partial X_{i j}}{\partial P_{i}}=-\frac{P_{i}}{X_{i j}} D_{i j}^{\prime} \text {, so } \\
& D_{i j}^{\prime}=-\frac{X_{i j}}{P_{i}} E_{i j} \tag{15}
\end{align*}
$$

We find, from (13) and (15), that the $R^{\prime}$ 's depend on the ( $D^{*}$ )'s and the ( $D^{\prime}$ )'s depend on price, quantity, and the sector* elasticity of demand. These are all, in general, observational variables.

Turning now to $M R_{i}$, note it is defined as

$$
\frac{\partial\left(P_{i} X_{i}\right)}{\partial X_{i}}=P_{i}+X_{i} \frac{\partial P_{i}}{\partial X_{i}}=P_{i}\left(1+\frac{X_{i}}{P_{i}} \frac{\partial P_{i}}{\partial X_{i}}\right)=P_{i}\left(1-\frac{1}{E_{1}}\right)
$$

* The two sectors here are consumers and $X_{3}$ producers.

Thus, $M R$ depends on the elasticity of market demand,
Summing up the results of this subsection, a computational formula (11) was developed for finding the shadow price of a good in a production hierarchy, and the formula was shown to be operational.

> A.1.2 Generalization to a Production Hierarchy with Arbitrary Number of Tiers
> In what follows, consider $X_{0}$ as the basic commodity whose shadow price is to be determined. The greater the number of tiers, the "more basic" $X_{0}$ is. In a one-tier hierarchy (if such a term is applicable), $X_{0}$ is the only good, it is not an input to "less basic" goods, and is, itself, a strictly consumption good. In a two-tier hierarchy, $\mathrm{X}_{0}$ is an input to $\mathrm{X}_{1}, \mathrm{X}_{1}$ is strictly for final consumption, but part of $\mathrm{X}_{0}$ may also be used for consumption. The following table is useful for representing input-output relations in a hierarchy. In any column, the

| One-Tier | Two-Tier | Three-Tier | Four-Tier |
| :--- | :--- | :--- | :--- |

sequence of $X^{\prime}$ s show how the production of the final good is affected by the basic good, $X_{0}$. For example, in column four, $X_{3}$ is affected by $X_{2}$,
which is (directly) affected by $X_{0}$, and so on. These relations follow from the specification of the hierarchial relations in the preceding subsection. Now note, in (11), that the sequence of subscripts on the $P^{\prime} s$ in each term correspond to one of the subscript sequences in the first four columns of the table. There are eight terms in the RHS of (11), and the subscript sequences correspond to the first four columns. Likewise, had (11) corresponded to a five-tier hierarchy, the RHS would contain sixteen terms, each corresponding to one of the sequencies in all five columns of the table. Thus, to construct the shadow price formula for a hierarchy of $N$ tiers, an $N$ column table; similar to the one presented, can be generated. The term corresponding to sequence

$$
X_{r_{M}} X_{r_{M-1}} \ldots X_{r_{K}} X_{0}
$$

is

$$
\frac{{ }^{P_{r_{M}}}{ }^{P_{r_{M-1}}} \cdots{ }^{P_{r_{K}}}{ }^{P_{0}}}{{ }^{R_{r_{M}}}{ }^{M R_{r_{M-1}}} \cdots{ }^{M R_{r_{K}}}} \cdot{ }^{R_{r_{M} C}}{ }^{R_{r_{r_{M-1}}}}{ }^{I M} \cdots{ }^{R_{r_{K}}}{ }^{r_{K+1}}{ }^{R_{0 r_{k}}}
$$

For example, the term corresponding to

$$
x_{4} x_{3} x_{1} x_{0}
$$

is

$$
\frac{P_{4} P_{3} P_{1} P_{0}}{M_{4} M_{3} M_{1}} \cdot R_{4 C} R_{34} R_{13} R_{01}
$$

It is obvious that the number of terms in the general forumula for an $N$-tier hierarchy is $2^{\mathrm{N}-1}$. In practice, of course, this number will be pared down when certain possible input sequences do not, in fact, occur.

## A.1.3 The Rules of Shadow Pricing

Using the analysis developed thus far it is possible to state
the basic rules which apply to shadow pricing. The assumptions underlying the analysis, and hence underlying the derivative rules, are

- the supply of the good whose shadow price is to be determined is perfectly inelastic
- the decrease in the availability of the basic good, and the resulting decreases throughout the hierarchy, are "relatively small." Thus significant price changes do not occur, and there is no need to consider losses of consumer surplus
- there are no externalities associated with the goods in the hierarchy
- markets are fn short-run equilibrium.

Rule 1: If the production hierarchy is perfectly competitive (each industry is composed of price-taking firms), the shadow price of a good is its market price.

To see this, recall that for a perfect competitor, price equals marginal revenue. Thus, (11) becomes

$$
\begin{align*}
\operatorname{SP}\left(X_{0}\right) & =P_{0}\left(R_{3 C} R_{23} R_{12} R_{01}+R_{3 C} R_{23} R_{02}+R_{3 C} R_{13} R_{01}\right. \\
& \left.+R_{3 C} R_{03}+R_{2 C} R_{12} R_{01}+R_{2 C} R_{02}+R_{1 C} R_{01}+R_{0 C}\right) \tag{16}
\end{align*}
$$

From (14), $\mathrm{R}_{3 \mathrm{C}}=1$, so the parenthesized portion of the RHS of (16) can be written as

$$
\begin{aligned}
& R_{23}\left(R_{12} \cdot R_{01}+R_{02}\right)+R_{01}\left(R_{13}+R_{1 C}\right)+R_{2 C}\left(R_{12} R_{01}+R_{02}\right) \\
& +R_{03}+R_{0 C}
\end{aligned}
$$

Factoring further,

$$
\begin{aligned}
& \left(R_{12} \cdot R_{01}+R_{02}\right)\left(R_{23}+R_{2 C}\right)+R_{01}\left(R_{13}+R_{1 C}\right)+R_{03}+R_{0 C} . \\
& \text { By }(14), R_{23}+R_{2 C}=1, \text { and so } \\
& R_{12} \cdot R_{01}+R_{02}+R_{01}\left(R_{13}+R_{1 C}\right)+R_{03}+R_{0 C} \text {, or } \\
& R_{01}\left(R_{12}+R_{13}+R_{1 C}\right)+R_{02}+R_{03}+R_{0 C}
\end{aligned}
$$

Again, by (14), the parenthesized term equals, 1 , leaving

$$
R_{01}+R_{02}+R_{03}+R_{0 C}
$$

which, by (14), also equals 1 . Thus, Rule 1 holds. Rule 2: The shadow price of a good is never less than its market price, under normal demand conditians.*

In Rule 1 , when $P=M R$ for each good in the hierarchy (except $X_{0}$ ), S.P. $\left(X_{0}\right)=P_{0}$. Under normal demand conditions, firms face downward sloping (to a greater or lesser extent) demand curves. In this case, price exceeds marginal revenue, so each term $P_{i} / M R_{i}$ will exceed one. Thus, the shadow price of $X_{0}$ will never be less than $P_{0}$. This is easily verified from (11).
Rule 3: The more imperfectly competitive the production hierarchy, the greater the excess of S.P. $\left(X_{0}\right)$ over $P_{0}$.

Since the usual measure of the degree of competition faced by a firm is the ratio of its price to its marginal cost, and since a profit maximizing firm equates marginal revenue with marginal cost, the $P_{i} / M R_{i}$ rations in (11) enables one to directly incorporate the degree of competition into shadow price analysis. Thus, it is clear from (11) that S.P. ( $X_{0}$ ) rises as the degree of competition deteriorates.
" By "normal" is meant downward-sloping-to-the-right demand curves.

Rule 4: Under normal demand conditions, the "more basic" a good, the more (percentage-wise) its shadow price deviates from its market price.

Here, "more basic" refers to the number of tiers overlying the good in question. Thus, in terms of the foregoing analysis, $X_{0}$ is more basic than $X_{1}, X_{1}$ is more basic than $X_{2}$, etc.

Compare the shadow price of $\mathrm{X}_{1}$ with the shadow price of $\mathrm{X}_{0}$. S.P. ( $\mathrm{X}_{0}$ ) was computed in (11) above. It is easy to see that

$$
\begin{align*}
\text { S.P. }\left(X_{1}\right) & =\frac{P_{3} \cdot P_{2} \cdot P_{1}}{M R_{3} \cdot M R_{2}} \cdot R_{23} R_{12} R_{3 C}+\frac{P_{3} \cdot P_{1}}{M R_{3}} R_{13} R_{3 C} \\
& +\frac{P_{2} \cdot P_{1}}{M R_{2}} \cdot R_{12} R_{2 C}+P_{1} \cdot R_{1 C} \tag{17}
\end{align*}
$$

A formal proof of Rule 4 is tedious. It may be. demostrated by example. Suppose the price of each good is 2, marginal revenue from each good is 1 , and each good is evenly divided among alternative uses, that is, $R_{01}=R_{02}=R_{03}=R_{0 C}=1 / 4, R_{12}=R_{13}=R_{1 C}=1 / 3, R_{23}=R_{2 C}=1 / 2$, and $R_{3 C}=1$.
From (17),

$$
\begin{aligned}
\text { S.P. }\left(X_{1}\right) & =\frac{2 \cdot 2 \cdot 2}{1 \cdot 1} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot 1+\frac{2 \cdot 2}{1} \cdot \frac{1}{3} \cdot 1 \\
& +\frac{2 \cdot 2}{1} \cdot \frac{1}{3} \cdot \frac{1}{2}+2 \cdot \frac{1}{3} \\
& =\frac{8}{6}+\frac{8}{6}+\frac{4}{6}+\frac{4}{6} \\
& =4
\end{aligned}
$$

And, from (11),

$$
\begin{aligned}
\text { S.P. }\left(X_{0}\right) & =\frac{2 \cdot 2 \cdot 2 \cdot 2}{1 \cdot 1 \cdot 1} \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}+\frac{2 \cdot 2 \cdot 2}{1 \cdot 1} \cdot 1 \cdot \frac{1}{2} \cdot \frac{1}{4} \\
& +\frac{2 \cdot 2 \cdot 2}{1} \cdot \frac{1}{1} \cdot \frac{1}{3} \cdot \frac{1}{4}+\frac{2 \cdot 2}{1} \cdot 1 \cdot \frac{1}{4}
\end{aligned}
$$

$$
\begin{aligned}
\text { S.P. }\left(X_{0}\right) & =\frac{2 \cdot 2 \cdot 2}{1 \cdot 1} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}+\frac{2 \cdot 2}{1} \cdot \frac{1}{2} \cdot \frac{1}{4}+\frac{2 \cdot 2}{1} \cdot \frac{1}{3} \cdot \frac{1}{4} \\
& +2 \cdot \frac{1}{4}=\frac{4}{6}+\frac{6}{6}+\frac{4}{6}+\frac{6}{6}+\frac{2}{6}+\frac{3}{6}+\frac{2}{6}+\frac{3}{6} \\
& =5
\end{aligned}
$$

Thus, as predicted, S.P. $\left(\mathrm{X}_{0}\right)$ exceeds the S.P. $\left(\mathrm{X}_{1}\right)$.

## A. 2 Unemployment of Resources

Economists refer to the factors of production--1abor, capital, and land--as the resources of the economy. The more restrictive notions of resources as minerals, oil deposits, or timber stands, etc. are subsumed in the classification "land." Whenever any of these factors are available but not being used to the fullest possible extent, there is resource unemployment. Capital means tools, machines, ana other manufactured means of production.

In a perfectly functioning competitive market economy, unemployment of any resource is a strictly temporary phenomenon. For unemployment simply means that at the current market price, the quantity supplied exceeds the quantity demanded. Market forces will then tend to lower the market price (which increases quantity demanded and/or decreases quantity supplied) until a full employment market equilibrium is achieved. If it is the case that quantity supplied always exceeds quantity demanded, the price falls to zero. Such is the case for the good, air.

It is clear that ours is not a perfectly functioning competitive market economy. Especially in the labor market, unemployment does not drive the price of the good (wages are the price labor) down * thus unemployment can

[^14]be more than strictly temporary. When price does not respond to supply and demand, and consequently does not reflect social values (insofar as it is agreed that a perfect market does reflect social values), market price is not a useful measure for CBA. We now examine each factor category to see how unemployment affects CBA measurements.

## A.2.1 Labor

Labor which would be otherwise unemployed should be valued at a zero social cost when employed in a project. This is obviously in spite of the fact that the use of that labor has a dollar cost. To see why this rule is valid, let us illustrate its derivation. Recall that a project moves society from one state to another. In the initial state, $\mathrm{S}^{0}$, there is one unemployed worker, $\alpha$, who is receiving $\$ 50$ per week unemployment compensation. For simplicity, suppose taxpayer B alone is taxed $\$ 50$ to pay $\alpha$. The new state, $S^{\prime}$, has $\alpha$ employed by the government, earning $\$ 150$ per week. $\beta^{\prime}$ 's taxes are increased to cover this new government expenditure. Assume $\alpha^{\prime}$ s output has a social vlaue of $\$ 150$ per week. The table summarizes the social accounting (weekly basis). The government's unemployment payments to $\alpha$, and $\beta^{\prime}$ 's tax payments to the government are transfer payments. Transfer payments do not reflect the production of goods or services.

| STATE | COST |  |
| :---: | :---: | :---: |
| $S^{0}$ | \$50 | (Value of consumption foregone by $\beta$ ) |
| $S^{\prime}$ | \$150 | (Value of consumption foregone by $\beta$ ) |

BENEFITS
\$50 (Value of consumption enjoyed by $\alpha$ )
$\$ 150$ (Value of consumption enjoyed by $\alpha$ )
$\$ 150$ (Value to society of of $\alpha^{\prime}$ s production)

Note. first, that Net Social Value in $S^{0}$ must be zero if $S^{0}$ is to be the base of reference.

The change from $S^{\circ}$ to $S^{\prime}$ entails a change in Social Costs of $\$ 100=\$ 150-\$ 50$. The corresponding change in Social Benefits is $\$ 250=\$ 150+\$ 150-\$ 50$.

Therefore, the Net Social Value of the change from $S^{\circ}$ to $S^{\prime}$ equals the change in benefits less the change in costs, or

Net Social Value $\left(S^{\circ}\right.$ to $\left.S^{\prime}\right)=\$ 150=\$ 250-\$ 100$.
Once such an exercise has been performed, its simpler equivalent may be employed. And that is the rule stated above. To reiterate, the rule states that $\alpha^{\prime}$ s employment costs society nothing if $\alpha$ would be otherwise unemployed. Of course, the social benefit is the value of $\alpha$ 's production. The shorthand accounting procedure implied by the rule is presented in the table. of course, Net Social Value of $S^{\circ}$ to $S^{\prime}$ again equals $\$ 150$. Ordinarily, the marginal social benefit resulting from $\alpha$ 's employment would not be easily isolated. Rather, that benefit would be aggregated into some broader category. His employment cost, on the other hand, will be specifically recorded. For that reason, the rule is more concerned with the cost of employment rather than benefit.

| CHANGE OF STATES | COSTS |  |
| :---: | :---: | :--- |
| $S^{\circ}$ BENEFITS |  |  |

Properly interpreted, the rule means one must be very wary of counting jobs created as a benefit of a project, for this is correct only if the job holders would be otherwise unemployed. Two pitfalls must be avoided. First, a job created by a project will exist over a number of years. Costs
and benefits must be tallied for each year of the project. Even if the job goes to an unemployed individual in the first year of the project, this is no reason to suppose he would remain unemployed over the time span the job exists. Unemployment is cyclical. The proper approach, given current unemployment, would be to value the social cost of employment at zero for the first year (maybe two years) and then at the wage rate thereafter.

The second pitfall is for the analyst to suppose his is the truly marginal project. It is tempting to look at unemployment statistics and, noting that there is always some unemployment (even beyond transitory unemployment--people between jobs), conclude that this project will draw from that unemployed labor pool. The difficulty is that if this approach is taken in each project, the "margin" may well become ten million workers wide. Again, the proper approach is to value labor costs at the wage rate, except when the analyst is reasonably sure the jobs will be filled by the unemployed. Such certainty, given the current state of economic forecasting, cannot extend beyond one or two years for the general labor market. However, for certain classes of jobs, it may be possible to construct an employment probability distribution over future years. This approach, if warranted within the context of the analysis, could provide the decision maker with a better grade of information.

## A.2.2 Capital

Capital which would be otherwise unemployed should be valued at zero social cost when employed in a project. The original or replacement costs, or the depreciated value are simply not relevant. Society incurs the cost of producing capital when it is produced--for the social cost
is the consumption opportunities currently prec1uded by devotịng resources to capital construction instead of consumer goods. Only present and future costs are relevant to decisions, past (sunk) costs are not. For example, if rotors to capture wind energy could be placed on unmodified electric transmission towers, the social cost of using those towers for the project is zero. Their construction costs were incurred in the past--their social cost has al ready been paid. However, if the same project requires new towers, their construction is a social cost of the project.

## A.2.3 Land

Land which would be otherwise unemployed should be valued at zero social cost when employed in a project. Land is a good which provides a time flow of service. The price of a tract of land is usefully considered to be an approximation to the Net Present Value of the flow of rental receipts. (This would be exactly true in a land market characterized by perfect knowledge of future land uses and demands). This point of view explains why idle land is valuable. The higher the price of a tract, the higher the anticipated rental receipts. Thus, the market price of a tract is a good measure of its soeial value because rental receipts are based on a tract's productive capacity. When a project will employ a tract, to the exclusion of other uses, for a significant length of time, the market price should be taken as the social cost of the land. For shorter periods, a rental value should be taken as social cost. If the analyst can be reasonably certain the land would remain idle for some years, zero social cost should be charged for those years.

## A. 3 Non-Marginal Price Changes

> A.3.1 Consumer Surplus
> So far attention has been restricted to situations in which the production of goods has increased or decreased without a concomitant change in the price of the good. Then, social benefit or cost is measured by the price times the change in quantity. Since reasonable approximations are the best CBA can hope to achieve, even a small (marginal) price change need not alter this approach. For then it may not make much difference to the final result whether the change in quantity is multiplied by the new price or the old price.

However, when there is a substantial (non-marginal) change in price, greater precision is demanded. It is achieved through the use of the con-cept--consumer's surplus. In the Figure A.1, let $A D$ represent a consumer's demand for $Q$, and let the price of $Q$ be $P^{o}$. The demand curve shows that if the price of $Q$ were 50 , the consumer would purchase one unit. In other words, the consumer's WTP for the first unit of $Q$ is 50 . However, since the price is $P^{0}$, he pays only $P^{0}$ for that unit. Thus, he receives a surplus from that first unit of $50-\mathrm{P}^{\circ}$. Likewise, it could be argued that his surplus on the second unit is $48-\mathrm{P}^{\circ}$, on the third unit is $46-\mathrm{P}^{\circ}$, and so on. His surplus on the last unit he purchases, the fifteenth in the figure, is zero. How much is the consumer's total surplus from consuming fifteen units? Clearly a very good approximation to this figure is the area $A B P^{\circ} . A B P^{0}$ is the surplus from the consumption of fifteen units of $Q$ since his total WTP for those units is the area $A B Q^{\circ} 0^{0}$, his cost is $P^{\circ} B Q^{\circ} O^{\prime}$, and

$$
A B P^{O}=A B Q{ }^{\circ} 0-P^{O_{B Q}}{ }^{\circ} 0 .
$$

Now suppose that a project has the effect of lowering the price of $Q$ to $P^{\prime}$, resulting in an increase in consumption from $Q^{\circ}$ to $Q^{\prime}$. The net surplus to the consumer of this change in states (from $\mathrm{S}^{0}$ characterized by $P^{0}$ and $Q^{0}$ to $S^{1}$ characterized by $P^{\prime}$ and $Q^{\prime}$ ) must be equal to the increase in WTP less the increase in expenditure.

$$
\begin{aligned}
& \text { In } S^{0}, W T P=A B Q^{0} 0 \\
& \text { In } S^{1}, W T P=A C Q^{1} 0 \\
& \Delta W T P\left(S^{0} \text { to } S^{1}\right)=A C Q^{0} 0=B C Q^{1} Q^{0}, \text { or } \\
& \Delta W T P=I I+I I I \\
& \text { In } S^{0} \text {, expenditure }=P^{0} B_{B} Q^{0}=I+I V \\
& \quad \text { In } S^{1}, \text { expenditure }=P^{1} C Q^{1} 0=I I I+I V \\
& \text { Net change of movement } S^{0} \text { to } S^{1}=\Delta W T P \text { plus } \\
& \Delta \text { expenditure }=(I I+I I I)+(I-I I I) \\
& \quad=I+I I
\end{aligned}
$$

There area $P^{0}{ }_{B C P}{ }^{1}=I+I I$ is the value to the consumer of the drop in price for this particular commodity. It is called the consumer's surplus due to the price change.

## A.3.2 Compensating Variation

In the beginning of this section, it was agreed that WTP determines the value of a good to an individual for CBA. Directly above, a certain area under a demand curve was equated with WTP. For all practical purposes, this equation is unassailable. However, from a strict theoretic point of view, it is not quite correct. The following discussion is not meant to modify the use of consumer surplus as described above, it is intended only to elucidate a technical point which might otherwise trouble some readers.


Figure A. 1 Illustration of consumers' surplus

The proper way to interpret a demand curve, such as that illustrated in Figure $A .2$, is: when $D$ is the demand curve for $Q$ for a specified time


Figure A. 2 Interpreting a demand curve.
interval (i.e., per week, per month, per year), then if the price is constant at $P^{0}$ for that interval, the consumer will purchase $Q^{0}$ over that interval. If the price is constant at $P^{\prime}$ for that interval, the consumer will purchase Q' for that interval. If the price is constant at $P^{\prime \prime} .$. and so forth. The demand curve does not mean: if the price is initially at $\mathrm{P}^{\mathrm{o}}$, the consumer will purchase $Q^{\circ}$. If the price drops to $P^{1}$ during the time interval for which the demand curve is drawn (e.g. at the beginning of the second week for a monthly demand curve), the consumer will purchase an additional $Q^{\prime}-Q^{0}$, if it drops again to $P^{\prime \prime}$ during the interval, he will purchase an additional $Q^{\prime \prime}-Q^{\prime}, \ldots$

While this is strictly not a correct interpretation, a little reflection will indicate it is the one the consumer surplus approach is based on.

To derive an exact measure of the value of a price drop to an individual, it is convenient to phrase the question this way: what is the maximum amount of money the consumer would be willing to pay to be able to buy (all he wants of) the good at the lower price rather than the higher price?

The answer to the question is that amount of money which leaves him at the same level of utility at the lower price with less money to spend as at the higher price with more money to spend. That amount of money is called his compensating variation (of income).

To determine a compensating variation, the analyst must either know the consumer's utility function (a practical impossibility) or be able to subject him to some elaborate experiment designed to reveal the compensating variation (very costly).

Fortunately, there are two theorems which obviate the need to attempt a computation of compensating variation. Loosely stated, they are:

Theorem 1: The smaller the price change in question, the closer is the consumer's surplus measurement to the compensating variation.

Theorem 2: The smaller the proportion of income spent on the good in question, the closer is the consumer's surplus measurement to the compensating variation.

Actually, theorem 1 doesn't really help very much, since in practice attention is restricted to significant price changes. However, the second theorem is quite useful since, at least in advanced economies almost every good consumed, with the exception of housing, accounts for a small proportion of total expenditures: Thus, for practical purposes, consumer's surplus is a close enough measure of the value of a price change, the technical difficulty notwithst anding.
A. 4 Increasing Returns (to Scale)

In an "ideal" perfectly competitive economy, firms set their prices equal to their marginal costs. Thus market price is a measure of both WTP for the good and the SOC (or shadow price) of that good. When a firm's production is characterized by increasing returns to scale (alternatively, diminishing average cost), the firm will actually lose money-make negative profits-if it sets its price equal to its marginal cost of production. Naturally, the firm will not accept negative profits, and sets its price in excess of marginal cost, as does the non-perfectly competitive firm. Thus, the market price may differ from the shadow price.

Increasing returns characterizes many public utilities, and this is the basis for the importance of this case. Public utilities are regulated by
government agencies. Such regulation is designed, at least in theory, to protect the consuming public against indiscriminently high prices (excess profits) and insufficient quantities supplied. The formula which accomplishes both these goals is average cost pricing. This is illustrated in Figure A. 4 . $D, A C, M C$ represent the market demand, average cost, and marginal cost curves,


Figure A. 4 Public utility pricing
respectively. A policy of marginal cost pricing would imply a price of $0 C$ (at which MC crosses D). However, at a price of $O C$ and production of $O G$ (which equates the quantity supplied to the quantity demanded at price $O C$ ), the firm's total revenue (price times quantity) is CFGO. The firm's total cost (average cost times quantity) is BEGO. The firm's loss is therefore BEFC. On the other hand, pricing at OA and producing OH simultaneously
satisfies all demand (at that price) and covers all costs without economic profits. This is the type of outcome regulatory commissions seek to achieve.

Relevant to CBA is that the social benefit of a marginal unit of output is equal to the WTP, or OA; while the social cost (assuming competition elsewhere in the economy) is HJ.
A. 5 Government Taxation and Subsidization

These tools of government cause observed prices to deviate from the corresponding shadow prices. While taxes and subsidies ought to be left in the price when computing WTP for the marginal unit, they should be netted out in calculating the $S O C$, or shadow price, of producing the marginal unit. Figure A. 5 shows the supply and demand for good $Q$ in a perfectly competitive economy. Government levies a per unit tax on $Q$ in the amount of $t$. The tax is collected from the manufacturers. This shifts


Figure A. 5 Shadow pricing with taxation.
the original supply curve, $S^{0}$, upward by $t$ to $S^{\prime}$. The new equilibrium price and quantity are $P^{\prime}$ and $Q^{\prime}$. Since, at the margin, consumers are willing to pay $P^{\prime}$ for $Q, P^{\prime}$ is the social benefit of increased output. However, $C$ is the social cost of producing the additional units. This can be inferred from the supply curve because no economic profits are being made by the firms. All receipts are used to cover the tax and the costs of production. A government subsidy for $Q$ may be analyzed analogously.

## A. 6 External Effects

External effects (or externalities or spillovers) are usually, although not necessarily, identifled with some form of pollution. Such effects may be said to exist when the actions of one economic agent affect the welfare of another, and the former is neither compensated (in the case of a good spillover) nor charged a fee (in the case of a bad spillover) by the latter. Thus, a bad (or negative) externality either reduces utility or increases production costs, depending on whether it impinges on a consumer or a firm. In either case, the externality has a SOC. Therefore, externalities must be valued and incorporated into CBA. Public projects can give rise to external effects in two ways--through inputs to the project and through the outputs of the project.

## A.6.1 External Effects from Inputs

Suppose good $Q$ is an input to a public project, the project will employ $\bar{Q}$ units of $Q$, and each unit of $Q$ produced has an external cost of $e$ dollars. For concreteness, we can think of $Q$ as electric power from a thermal plant, and $Q$ 's production involves sulphur emissions. Should the project be assessed a social cost of e $\bar{Q}$ ? Not necessarily. How
much to assess the project depends on the supply conditions of Q. Put another way, it is necessary to determine how much more $Q$ is actually produced because of the project. To illustrate, consider three supply situations: perfectly inelastic, perfectly elastic, and the "normal" case.
a) With perfectly inelastic supply, the quantity supplied does not respond to price changes. For the relevant time period, it is rigidly fixed. Let $D^{\circ}$ be the original demand for $Q$, and $D^{\prime}$ the demand after the project's demand of $\bar{Q}$ has been incorporated. Figure A.6a shows that the only effect of the increased demand is to raise the price of $Q$. Therefore, the $\bar{Q}$ units of $Q$ will be purchased by the project, but the higher price will decrease others' purchase of $Q$ by $\bar{Q}$. There is no increase in external effects due to the project. No externality charge should be levied against the project in this case.
b) When the supply is perfectly elastic, the increase in quantity supplied exactly matches the increased quantity demanded. This is the case where the project does subject society to increased externality costs of $e \bar{Q}$.
c) In the normal case--where $S$ is neither vertical nor horizontal-the increased demand causes both a price rise and increase in output. While the project again gets its $\bar{Q}$ units, some of these are given up by purchasers who do not wish to pay the higher price. The remainder constitute new production. Only the externality costs attributable to the new production should be charged against the project. Clearly, for this case, the costs are between 0 and e $Q$.

Figure A. 6 a


Figure A.6b


Figure A. 6c


Figure A. 6 Externalities and shadow pricing

## A.6.2 External Effects from Outputs

Any external effects arising from the output of a project are to be charged against the project. For example, in the evaluation of a proposed airport, the daily aircraft noise is a cost to individuals nearby. This cost should be accounted for in the CBA.

It is often easy to determine the existence of a cost due to an external effect, but it is usually quite difficult to determine the magnitude of that cost. This is because, by definition, the effects are uncompensated, or unpriced. There is no market determination of value to guide the analyst. The best approximations are necessarily quite crude. Note that the principle of value determination--WTP or SOC or shadow pricing--still applies. The problem lies in making the principle operative. With regard to the airport noise problem, for example, one could conceive of asking each individual how much he would be willing to pay to avoid the noise. While such a complete survey is often not feasible, that is not even the main problem. The main problem is getting individuals to reveal their true valuations. Each person who opposes the construction of the airport is motivated to exagerate the noise costs, each person who favors it is motivated to understate it.

In CBA, the analyst can deal with externalities in a number of ways, none of which is completely satisfactory.
a) Conduct a survey of WTP among the affected individuals and hope that true preferences are revealed, or that the exagerations cancel out the understatements.
b) As an estimate of WTP, compute the costs of avoiding the externality, such as the costs of sound insulation for homes and autos, ear plugs
for being outdoors, etc. This is neither an upper bound nor a lower bound on the true cost. For example, avoidance costs may be $\$ 3000$, but one person may value silence at $\$ 10,000$ while another values it at $\$ 100$. The fact that it would cost the airport administration only $\$ 3000$ to give the first individual the silence he values at $\$ 10,000$ does not diminish the fact that the lack of silence is a cost of $\$ 10,000$ to that person. Thus, the avoidance cost has no special significance to a CBA beyond its intuitive appeal as a reasonable number to look at, and perhaps as an indicator of the order of magnitude of some individual valuations.
c) Compute the critical value of the externality. All the social benefits and all other social costs can be computed to yield'a Net Social Value of the project before the inclusion of the value of the externality. Thus, if the value is already negative, or lower than some alternative project, the project is definitely not worth the undertaking, and an exact computation of the loss due to the externality need not be attempted. If the value turns out positive, then a critical value can be computed for the externality, and the judgement left to the decision maker as to whether the actual social cost of the externality exceeds the critical value. For example, in the airport noise problem, suppose the Net Social Value of the airport, exclusive of noise considerations, is $\$ 10$ million. The decision maker must then judge whether that figure outweighs, or is outweighed by, the social cost of the noise.
d) As a last resort, only a qualitative description of the impact of the externality can be presented to the decision maker. This effectively shifts the burden of analysis from the analyst to the decision maker. It is not recommended except when all quantitive methods fail.

## A. 7 Public Goods

Public goods are goods which are consumed jointly by individuals. Formally, a public good has the following characteristics:
a) Consumption of such a good is non-rival in the sense that one person's consumption does not diminish the amount available to any other person.
b) It is not feasible (and sometimes not even possible) to exclude any individual from consuming the good, once the, good is provided.

Examples of public goods include national defense, mosquito control activities, light houses, and certain governmentally preserved "natural" areas. Many other goods exhibit "publicness" to lesser, but still significant degrees. These goods include police and fire services, and July 4th fireworks displays.

Since it is not feasible--in the sense of at reasonable cost--to exclude any person from consuming a public good, it follows no firm is likely to find it profitable to supply public goods to the market since it would have to rely on voluntary contributions for its revenues. Payments for private (non-public) goods are not voluntary insofar as one must pay to receive the good. Here, due to non-excludability, one receives the good whether or not he pays. Therefore, it is possible to 'simultaneously have a
demand for a good and no firm willing to supply it. In such situations, governments undertake to provide the goods and finance this provision through taxes. In general, there is no precise relation between the taxes one pays and the public goods one consumes. In addition, one tax payment to a government unit will go towards the payment for a variety of government services. For example, a local property tax may finance local education, police and fire services, street lighting, road maintenance, etc. The point of all this is that there is no meaningful per unit "price" to the individual for the consumption of public goods. This lack of price means a lack of an objective yardstick of value for public goods. Therefore, when a public project affects the quantity of some public good, the cost-benefit analyst is faced with the difficult problem of determining the value of that good without any guidance from objective measures of value such as market prices.

The modern economic theory of public goods was formulated by Paul Samuelson in a series of articles in the mid-1950's. The theory is interesting because it simultaneously provides a specific formula for determining the value of a public good to an individual and then explains why this formula can never be actually applied. (This, by the way, is not a shortcoming of the theory. It results from an appreciation of human avarice.) It is important that the analyst, charged with valuing public goods, be familiar with this theory, if only so he knows what his approximations should be approximations to.

For expositional ease, the theory is developed in the context of an economy with one public good, and three consumers. Once the general
principles are discovered, extension to more realistic cases is straightforward. First, assume that one can meaningfully express the units of measurement for the public good, e.g. number of acres in a park, soldiers in an army, or mosquitos killed. In Figure $A .7$, let $S$ represent the economy's supply curve for the public good, and assume the prices correspond to shadow prices. $D^{\alpha}, D^{\beta}, D^{\delta}$ are the demands for the public good by the three consumers.


Figure A. 7 Supply and demand for public goods.

D is the market demand curve derived by adding the individual demands vertically. This is in contrast to the horizontal addition of demand curves for private goods. The optimal amount of the public good the government should provide is $Q^{*}$, determined by the intersection of $S$ and $D$. To see this, consider the alternatives. If some amount less than $Q^{*}$ were provided, say $\bar{Q}$, the total $W T P$ for one extra unit would be $\bar{P}$, while the
cost to society (the three consumers) of providing that extra unit would be $\mathrm{P}^{\mathrm{o}}$. Since $\overline{\mathrm{P}}$ exceeds $\mathrm{P}^{0}$, total social welfare is increased by providing that marginal unit. This argument applies to any value of $Q$ less than $Q^{*}$. That is. whenever the amount provided is less than $Q^{*}$, society's welfare can be increased by increasing $Q$. A similar argument shows that if more than $Q^{*}$ were provided, society could increase its welfare by decreasing that amount. Therefore, $Q^{*}$ is the optimal amount of the public good since any deviation from $Q^{*}$ causes a drop in welfare.

How should $Q$ * be financed? A reasonable criterion is that each individual should pay in proportion to the benefits he receives, which are expressed by his WTP. Thus, from the individual demand curves, when $Q^{*}$ units are provided, $\alpha, \beta$, and $\delta$ are willing to pay $P^{\alpha}+P^{\beta}$ $+P^{\delta}=P$. The result is that the optimal amount of the public good can be determined, and it can be financed by assessing individuals on the basis of the WTP reflected in their demand curves.

With regard to $C B A$, it would appear there is a clearcut method for determining the value of a public good to an individual--simply refer to his demand curve for a measure of WTP.

But that is the catch! In general, there is no way to get an individual to reveal his true WTP for a public good. For private goods, true WTP is revealed by consumers' purchases of various quantities at various prices, i.e. through the market. But public goods are not traded in a market (due to their non-excludability property, discussed above), so WTP is not revealed by behavior. Why not simply approach the consumer and put the matter before him--"The government is considering providing a 50 acre park in this area, how much would you be willing to pay to get such a park? The park, incidentally, will be financed by general tax revenues."

If he favors the idea, he would try to increase the chance of its being adopted. Since he knows he is not going to be assessed the amount he states, he will likely respond with an exaggerated figure. If he's against the idea, even though the park would yield him some benefits, he will likely understate his WTP.

At this point, the reader might object that the consumer should be told that he will be assessed in proportion to the amount he claims. That should force him to be more truthful, particularly if he favors the project. Let us analyze how a consumer might respond in that situation, assuming he does favor the project. First of all, he definitely wouldn't exagerate the value he would get from the park. If he did, and the park were provided, he'd suffer a net loss. The remaining alternatives are to tell the truth or to understate true WTP. Recognizing these alternatives, the individual will assume that everyone else will also consider them when responding. It is convenient to use a "game" matrix to represent the individual's decision problem. He has two strategies--tell the truth or understate. He assumes "everyone else" has the same options. As the matrix indicates, there are four possible outcomes, or "payoffs" to the individual. Suppose that the value to the individual is 100 if he and everyone tells the true WTP.*

Everyone Else's
Strategies
Truth Understate

Individual's
Strategies

Truth 100 140

80
100

[^15]On the other hand, suppose he understates and everyone else tells the truth. Then he still gets a park, and in addition saves some money. This outcome is clearly superior to the former, say its valued at 140 . If he tells the truth, and everyone else understates, his assessment will be closer to his stated WTP. Certainly, the value of this outcome must be less than when everyone, including himself, tells the truth. Suppose it's worth 80. Finaliy, suppose everyone, including himself, understates. Then the proportional assessments will tend to be the same as when everyone tells the truth. Value it also at 100.

Let us now determine the rational strategy for the individual. Clearly it is to understate. For no matter what everyone else does, understating always yields the higher payoff. If everyone perceives the situation as does this individual, everyone is motivated to understate his true WTP. Thus, a straightforward inquiry addressed to individuals does not hold much promise in eliciting true responses concerning the value of public goods for CBA.*

To summarize, the theory of public goods outlined above suggests that assessing individuals on the basis of their WTP can lead to an optimal provision of public goods. Unfortunately, there is no foolproof method to get individuals to reveal their WTP. Their retorts, it is reasonably feared, may easily be biased by strategic considerations.

What is the cost-benefit analyst to do? Once again, there is no completely satisfactory approach. However, approaches usually followed include
a) Surveys, where the questions are asked in a manner disguising

[^16]their purpose. The problem still remains that responses to survey questions may not be based on careful consideration of one's own values.
b) Analogy to private goods. Where the public good is related to some marketed good, the price of the latter may be a guide to the value of the former.
c) Experiments. Individuals might be asked to participate in "realistic" games designed to reveal their true preferences. Such information is costly, and usually of questionable reliability.
d) Public referenda which provide a number of output-cost levels to vote on, and where the means of financing the project can reasonably be claimed to be currently funknown. Since no one knows how the costs will be shared, it is hoped the votes do not reflect strategic behavior. While this method might be useful for determing the most preferred level of output, it does not ascertain whether the Net Social Value, even at that level, is positive.

Note: In certain types of economic studies, it is frequenly assumed that the cost of providing a good is equal to the value of consuming the good. Such an assumption must never be made in CBA, for it clearly sidesteps the whole problem of determining whether or not benefits exceed costs.

## A. 8 Fundamental Elasticity Concepts

Elasticity is best thought of as simply meaning "responsiveness." Elasticity of demand is a measure of how responsive the quantity demanded is to price. Elasticity of supply is a measure of how responsive the quantity supplied is to price. In what follows, demand elasticity will be treated explicitly. Supply elasticity is entirely analogous. Elasticity of demand is defined as

$$
E_{D}=\frac{\text { Percentage Change in Quantity }}{\text { Percentage Change in Price }}
$$

The reliance on percentages frees the measure of elasticity from dependance on the units of measurement. Otherwise, elasticity could be manipulated by changing price measurements from dollars to cents or quantity measurements from pounds to tons, watts to kilowatts, etc.

Elasticity of demand is a measurement taken along a demand curve. In general, elasticity will be different at different points on the curve. Thus, one must always speak of the elasticity of demand at a certain point on the curve, or in a certain small neighborhood on the curve. There are two approaches to measuring elasticity: arc elasticity and point elasticity. The former is useful when demand is not represented by a function, but rather by a table or a graph. Arc elasticicy is computed between two points on a demand curve; while point elasticity, as the term suggests, is computed at a single point.

The equation for arc elasticity is

$$
\frac{Q^{0}-Q^{\prime}}{Q^{\circ}+Q^{\prime}}
$$

$$
E_{D}=-\frac{2}{\frac{P^{0}-P^{1}}{\frac{P^{0}+P^{1}}{}}}
$$

where ( $P^{0}, Q^{0}$ ) is one price-quantity combination on the demand curve and ( $P^{\prime}, Q^{\prime}$ ) is the other. Two remarks need be made about the formula. First, the minus sign preceding the RHS insures that $E_{D}$ turns out to be a nonnegative number. Since demand curves slope downward to the right, the direction of the change in quantity is always opposite to the direction of the change in price. Without the minus sign, then, the RHS would always be negative. Simply for the convenience of having $E_{D}$ non-negative, the minus sign is included. Second, note that $\frac{Q^{\circ}+Q^{\prime}}{2}$ and $\frac{P^{\circ}+P^{\prime}}{2}$ are the averages of the quantities and prices, respectively. This alleviates the problem of having to choose one or the other as the "base" from which to compute the percentage change. In general, a different base will lead to different values for $E_{D}$ between the same two point.

The following example illustrates the use of the arc elasticity formula. Suppose it is known that at the price of 10 , quantity demanded is 25 ; and at the price of 15 , quantity demanded is 20 . Arbitrarily let $\left(P^{\circ}, Q^{\circ}\right)=(10,25)$ and $\left(P^{\prime}, Q^{\prime}\right)=(15,20)$. Then,

$$
E_{D}=-\frac{\frac{25-20}{25+20}}{2}-\frac{\frac{5}{22.5}}{\frac{\frac{10-15}{10+15}}{2}}=.55
$$

Point elasticity is used when the demand relation is a function, such as would result from estimation via regression analysis. The formula for point elasticity is derived from the formula for arc elasticity by a limiting process, i.e., letting $Q^{\circ}$ approach $Q^{\prime}$. The formula becomes

$$
E_{D}=-\frac{P}{Q} \frac{d Q}{d P}
$$

By way of example, suppose the demand relation is

$$
Q=-: 01 P^{2}-.1 P+100
$$

and suppose we wish to find the elasticity of demand when $P=50$.
When $P=50$,

$$
\begin{aligned}
Q & =-.01\left(50^{2}\right)-.1(50)+100 \\
& =-25-5+100 \\
& =70
\end{aligned}
$$

Also

$$
\frac{\mathrm{dQ}}{\mathrm{dP}}=-.02 \mathrm{P}-.1=-.02(50)-.1=-1.1
$$

Hence

$$
E_{D}=-\frac{50}{70}(-1.1)=.79
$$

We say that demand is elastic (at some point or in some neighborhood) if $E_{D}>1$.

Demand is inelastic if $E_{D}<1$.
Demand is unitary elastic if $E_{D}=1$.
The significance of the elasticity concept is its relation to the effect on total expenditures as price changes. In particular, the following results hold:

If demand is elastic, raising the price will lower total expenditures on that good.

If demand is elastic, lowering the price will increase total expenditures on that good.

If demand is inelastic, raising the price will increase total expenditures on that good.

If demand is inelastic, lowering the price will decrease total expenditures on that good.

If demand is unitary elastic, any change in price will leave total expenditure the same.

The proof of these results is quite simple. Each addresses the change in expenditure as price changes or $\frac{d E}{d P}$, where $E$ is expenditure.

Since total expenditure on a good is simply price per unit times number of units sold,
$\mathrm{E}=\mathrm{P} \cdot \mathrm{Q}$.
$\frac{d E}{d P}=P \frac{d Q}{d P}+Q$
$=\left(\frac{P}{Q} \frac{d Q}{d P}+\frac{Q}{Q}\right) Q$
$=\left(1-E_{D}\right) Q$
Since $Q$ is always positive,
$\begin{aligned} \mathrm{dE} & > \\ \frac{\mathrm{dP}}{} & <0 \text { as } \mathrm{E}_{\mathrm{D}} \\ & =1 .\end{aligned}$
It is useful to relate elasticity to graphical representations of demand curves. Caution: while elasticity is related to the slope of a demand curve, the slope alone does not indicate elasticity values. The best example of this is a linear demand: its slope is constant throughout, yet elasticity varies point to point.

A vertical demand curve is perfectly inelastic ( $E_{D}=0$ ): changes in price do not affect quantity demanded.


A horizontal demand curve is perfectly elastic ( $E_{D}=\infty$ ): the good can only be sold at the given price. Any attempt to raise it causes sales to plunge to 0 .


A demand curve shaped like a rectangular hyperbola (given by a demand equation like $Q=K / P$, where $K$ is any positive constant) is unitary elastic everywhere ( $E_{D}=1$ )


Linear demand curves are elastic at high prices, inelastic at low prices, and unitary elastic at their midpoint.


## APPENDIX B

## A METHOD FOR EVALUATING OUTPUT WIND POWER

 CHARACTERISTICS FROM INPUT WIND STATISTICSThe method discussed here is based on certain assumptions with regard to the wind frequency distribution and the wind generator operating characteristics. These assumptions are: 1) The wind frequency distribution $p(V)$, which gives the fraction of time over the year (or other time interval) during which the wind speed has a value $V$ (within certain small limits $d V$ ), is a Weibull distribution (see Appendix C), characterized by two parameters, the scale parameter $c$ and the shape parameter $k$. 2) The wind generator has a given value of cut-in speed $V_{0}$, rated speed $V_{1}$, and feathering speed $V_{2}$ and rated power $P_{r}$, which uniquely determine the output power as described by equation ( $B-1$ ). given below.

The variable (instantaneous) output power $P(V)$ as a function of the instantaneous wind speed $V$ is, from assumption 2, given by

$$
P(V)=\begin{array}{lr}
0 & V<V_{0}  \tag{B-1}\\
A+B V+C V^{2} & V_{0} \leq V<V_{1} \\
P_{r} & V_{1} \leq V<V_{2} \\
0 & V>V_{2}
\end{array}
$$

where the coefficients $A, B$, and $C$ are determined by the two conditions

$$
\begin{align*}
& P\left(V_{0}\right)=A+B V_{0}+D V_{0}^{2}=0  \tag{B-2}\\
& P\left(V_{1}\right)=A+B V_{1}+C V_{1}^{2}=P_{r} \tag{B-3}
\end{align*}
$$

and the assumption that

$$
\begin{equation*}
P\left(V_{c}\right)=A+B V_{c}+C V_{o}^{2}=P_{c} \tag{B-4}
\end{equation*}
$$

where the wind speed $V_{c}$ is defined as $V_{c}=\left(V_{o}+V_{1}\right) / 2$ and the power value $\mathrm{P}_{\mathrm{c}}$ is taken to be

$$
\begin{equation*}
P_{c}=P_{r}\left(V_{c} / V_{1}\right)^{3} \tag{B-5}
\end{equation*}
$$

From the conditions ( $B-2$ ) through ( $B-4$ ), the values of $A, B$, and $C$ are determined to be

$$
\begin{align*}
& B=\left[P_{r}\left(V_{c}^{2}-V_{o}^{2}\right)-P_{c}\left(V_{1}^{2}-V_{o}^{2}\right)\right] / D  \tag{B-6}\\
& C=\left[P_{c}\left(V_{1}-V_{o}\right)-P_{r}\left(V_{c}-V_{o}\right)\right] / D  \tag{B-7}\\
& A=-B V_{o}-C V_{o}^{2} \tag{B-8}
\end{align*}
$$

where $D$ is given by

$$
\begin{equation*}
D=\left(v_{1}-v_{o}\right)\left(v_{c}^{2}-v_{o}^{2}\right)-\left(v_{c}-v_{o}\right)\left(v_{1}^{2}-v_{o}^{2}\right) \tag{B-9}
\end{equation*}
$$

Figure B. 1 shows the design turbine power curve for the NASA Plumbrook unit [1] which has the characteristics $V_{0}=3.6 \mathrm{~m} / \mathrm{s}$ $(8 \mathrm{mph}), \mathrm{V}_{1}=8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph}), \mathrm{V}_{2}=26.8 \mathrm{~m} / \mathrm{s}(60 \mathrm{mph}), \mathrm{P}_{\mathrm{r}}=100 \mathrm{~kW}$ (generator output). The dashed curve in Figure $\mathbf{B . 1}$ is the approximation to the actual curve, by equation $(B-1)$. The approximate curve is seen to be identical with the actual curve above a speed $V$ of about 12 mph . This form of output curve is obviously a simplified parameterization of an actual output curve from a real wind generator system, but should be accurate for the present design purposes.


Figure B. 1 Operating characteristics (turbine power versus wind speed) for NASA's Plumbrook unit. Cut-in speed $\because$ is $3.6 \mathrm{~m} / \mathrm{s}$ ( 3 mph ). Rated speed $\mathrm{V}_{1}$ is $3.0 \mathrm{n} / \mathrm{s}^{\mathrm{o}}$ (13 mph). Rated power (at the generator output) is 100 ll.

The annual average output power $\bar{P}$ can be evaluated by integrating $P(V)$, from equation ( $B-1$ ), weighted by the probability $p(V)$ for observing the speed v. Thus $\overline{\mathrm{P}}$ is given by

$$
\begin{align*}
\bar{P}=\int_{0}^{\infty} P(V) p(V) d V & =\dot{f}_{v_{0}}^{v_{1}}\left(A+B V+C V^{2}\right) p(V) d B  \tag{B-10}\\
& +\dot{\delta}_{v_{1}}^{v_{2}} p(V) d V
\end{align*}
$$

where, from assumption 1 , above, the probability distribution $p(V)$ is given by

$$
\begin{equation*}
p(V) d V=(k / c)(V / c)^{k-1} \exp \left[-(V / C)^{k}\right] d V \tag{B-11}
\end{equation*}
$$

Through a change of variables $x=(V / c)^{k}, x_{0}=\left(V_{0} / c\right)^{k}, x_{1}=\left(V_{1} / c\right)^{k}$, and $x_{2}=\left(V_{2} / c\right)^{k}$, equation $(B-10)$ becomes

$$
\begin{equation*}
\stackrel{\rightharpoonup}{P}=A p_{0}+B c \int_{x_{0}}^{x_{1}} x^{1 / k} e^{-x} d x+C c^{2} \int_{x_{0}}^{x_{1}} x^{2 / k} e^{-x} d x+p_{12} \tag{B-12}
\end{equation*}
$$

where $\mathrm{P}_{01}$ and $\mathrm{P}_{12}$ are the cummulative probabilities

$$
\begin{align*}
& p_{01}=\int_{V_{0}}^{v_{1}} p(V) d V=e^{-x_{0}}-e^{-x_{1}}  \tag{B-13}\\
& p_{12}=\int_{v_{1}}^{v_{2}} p(v) d V=e^{-x_{1}}-e^{-x_{2}} \tag{B-14}
\end{align*}
$$

The integrals in (B-12) must be evaluated by series approximation

$$
\begin{align*}
& \int_{x_{0}}^{x_{1}} x^{n / k} e^{-x} d x=\left\{x ^ { n / k } \left[x(1+n / k)^{-1} / 0!-x^{2}(2+n / k)^{-1} / 1!\right.\right. \\
& \left.\left.\quad+x^{3}(3+n / k)^{-1} / 2!-x^{4}(4+n / k)^{-1} / 3!+\ldots\right]\right\}\left.\right|_{x_{0}} ^{x_{1}} \tag{B-15}
\end{align*}
$$

where n takes on the values 1 or 2 for the integrals appearing in ( $B-12$ ).
From the average output power $\overline{\mathrm{P}}$, evaluated by (B-12), various factors of interest can be easily evaluated. The plant factor $F_{p}$, the ratio of the average output power to the rated power is given by

$$
\begin{equation*}
\mathrm{F}_{\mathrm{p}}=\overline{\mathrm{P}} / \mathrm{P}_{\mathrm{r}} \tag{B-16}
\end{equation*}
$$

The energy pattern factor $f$ is defined as the ratio of the average available power to the power available in the mean wind, or numerically, in terms of the plant factor $F$, the energy pattern factor is given by

$$
\begin{equation*}
f=\left(v_{1}^{3} / \bar{v}^{3}\right) F_{p} \tag{B-17}
\end{equation*}
$$

where $\overline{\mathrm{V}}$ is the mean wind speed, either evaluated directly from the wind data, or estimated from the previously determined Weibull parameters by the relation

$$
\begin{equation*}
\overline{\mathrm{V}}=\int_{0}^{\infty} \mathrm{V}_{\mathrm{p}}(\mathrm{~V}) \mathrm{dv}=\mathrm{c} \int_{0}^{\infty} \mathrm{x}^{1 / k} \mathrm{e}^{-\mathrm{x}} \mathrm{dx}=(\mathrm{c} / \mathrm{k}) \Gamma(1 / \mathrm{k}) \tag{B-18}
\end{equation*}
$$

The available power $P_{m}$ in the mean wind is defined by

$$
\begin{equation*}
P_{m}=P_{r}\left(\bar{V} / V_{1}\right)^{3} \tag{B-19}
\end{equation*}
$$

Thus $P_{m}$ is the available power in the mean wind assuming the same system efficiency at $\vec{V}$ as it has at the rated speed $V_{1}$. The annual specific energy output in $k W$ hours per rated $k W, \bar{E}$, is given by

$$
\begin{equation*}
\overline{\mathrm{E}}=8766 \overline{\mathrm{P}} / \mathrm{P}_{\mathbf{r}}=8766 \mathrm{~F}_{\mathrm{p}} \tag{B-20}
\end{equation*}
$$

and, of course, the actual annual energy output $E$, in $k W$ hours would be

$$
\begin{equation*}
E=8766 \overline{\mathrm{P}} \tag{B-21}
\end{equation*}
$$

A computer program has been written which takes input values of cut-in wind speed $\left(V_{o}\right)$, rated wind speed $\left(V_{1}\right)$, feathering wind speed $\left(V_{2}\right)$, rated power ( $P_{r}$ ), and for each input value of wind frequency scale parameter (c) and shape parameter (k) calculates, via the above equations, output values for the following: plant factor ( $F_{p}$ ), arithmetic mean wind speed ( $\overline{\mathrm{V}}$ ), available power in the mean wind $\left(P_{m}\right)$, average available power $(\bar{P})$, energy pattern factor ( f ), the annual specific energy output in $k W$ hours per rated $k W$ ( $\overline{\mathrm{E}}$ ), and the annual energy output in $k W$ hours (E). The program uses an efficient method for computing an accurate numerical approximation to the series expansion of the integral in equation ( $B-15$ ).

Figure B. 2 shows a plot of plant factor values computed by this program for a wind generator having the operating characteristics of the NASA Plumbrook unit ( $V_{0}=8 \mathrm{mph}, V_{1}=18 \mathrm{mph}$ ). Qualitatively the results in Figure B. 2 may be explained as follows: at high mean wind speeds (high c values) a small variance (high $k$ value) is desirable to limit the wind speeds to only high values near the (high) mean (where the output will be at or near the rated output); at low mean wind speeds (low c values) a large variance (low $k$ value) is desirable to allow some possibility of high wind speeds occuring.


Figure B. 2 Parametric variation of plant factor $F$ (ratio of average output power to rated power) versus Weibull scale factor $c$ and shape factor $k$.

APPENDIX C<br>THE WIND DISTRIBUTION FUNCTIONS EXAMINED, AND<br>THE METHODS FOR EVALUATING THEIR PARAMETERS

Empirically it is found that the probability distributions of the wind components $u$ and $v$ are approximately Gaussian. If the $u$ component is Gaussian with mean $u$ and standard deviation $\sigma_{u}$, and the $v$ component is Gaussian with mean $\bar{v}$ and standard deviation $\sigma_{v}$, then the joint probability distribution for simultaneous occurrence of components $u$ and $v$ is given (in terms of normalized variables $x=(u-\bar{u}) / \sigma_{u}$ and $\left.y=(v-\bar{v}) / \sigma_{v}\right)$ by

$$
\begin{align*}
p(u, v) d u d v= & (2 \pi)^{-1}\left(1-\rho^{2}\right)^{-1 / 2} \exp \left[-\left(x^{2}+y^{2}\right.\right. \\
& \left.-2 \rho x y) / 2\left(1-\rho^{2}\right)\right] \mathrm{dx} d y \tag{C-1}
\end{align*}
$$

where $\rho$ is the correlatiun wefficient for the $u-v$ cross correlation. The probability distribution of wind speeds $p(V) d V$ can, in principle, be evaluated by transforming from Cartesian to polar representation ( $p(u, v) d u d v$ to $p(V, \theta) V d V d \theta$ where $\theta$ is the wind direction) and integrating out the angular dependence

$$
p(V) d V=\int_{0}^{2 \pi} p(V, \theta) V d V d \theta
$$

A closed form integration of (C-2) using $p(u, v)$ from ( $C-1$ ) cannot be performed without resorting to special assumptions. If it is assumed that $\bar{u}=\bar{v}=0, \rho=0$, and $\sigma_{u}=\sigma_{v}=\sigma$, then $(C-2)$ can be evaluated using $p(u, v)$ from (C-1), as follows

$$
\begin{align*}
p(V) d V & =\left(2 \pi \sigma^{2}\right)^{-1} \int_{0}^{2 \pi} \exp \left[-\left(v^{2} / 2 \sigma^{2}\right)\right] V d V d \theta \\
& =\left(V / \sigma^{2}\right) \exp \left[-\left(V^{2} / 2 \sigma^{2}\right)\right] d V \tag{c-3}
\end{align*}
$$

which is the Rayleigh distribution. The Rayleigh distribution has only one adjustable parameter, $\sigma$, which can be adjusted to find the best fit to an empirically observed set of observed frequencies of occurrence at wind speed From an analytical standpoint; the Rayleigh distribution is not expected to fit observed wind frequency distributions very well because of the severely limiting assumptions used in its derivation. Non zero values for $\bar{u}, \bar{v}$ and $\rho$ and inequality of $\sigma_{u}$ and $\sigma_{v}$ would add additional parameters (and considerable complexity) to (C-3). From a practical standpoint, the single adjustable parameter $\sigma$ in the Rayleigh distribution does not give enough adjustability to provide good fits to observed distributions.

In an attempt to provide better comparison between the analytical and the observed distributions, but yet remain with fairly simple analytical expressions (e.g. two parameter distributions), two empirical wind speed distribution functions have been used: the Weibull, and the log-normal. The Weibull distribution

$$
\begin{equation*}
p(V) d V=(k / c)(V / c)^{k-1} \exp \left[-(V / c)^{k}\right] d V \tag{C-4}
\end{equation*}
$$

has found application in the study of wind loads on buildings [1] and the log-normal distribution

$$
\begin{equation*}
p(V) d V=(2 \pi)^{-1 / 2} s^{-1} \exp \left[-\ell n(V / V g) / 2 s^{2}\right] d V / V \tag{C-5}
\end{equation*}
$$

has been used in air pollution studies, for example by Larsen and Church [2], primarily because of the observations that air pollution concentrations are distributed in a log normal fashion [3]. The Weibull parameters and $k$ are respectively known as the scale factor and the shape factor. The log normal parameters are $V_{g}$, the geometric mean wind speed, and $s$, where $\sigma_{g}=e^{s}$ is known as the geometric standard deviation.

For empirical evaluation of these various distribution parameters, we have sets of data which consist of number of observations $N_{i}(i=1$ to $n)$ of wind speeds in $n$ different speed class intervals, each interval covering a speed range $V_{i-1}$ to $V_{i}$. The frequencies of occurrence $f_{i}$ of speeds in each interval is evaluated by $f_{i}=N_{i} / N_{t}$, where $N_{t}=\Sigma N_{i}$ ( $\Sigma$ implies summation from 1 to $n$ ). The cummulative frequencies $F_{j}$ are computed by summing the individual frequencies

$$
\begin{equation*}
F_{j}=\sum_{i=1}^{j} f_{i} \tag{C-6}
\end{equation*}
$$

These cummulative frequencies can then be compared to the expected cummulative probabilities $P\left(V_{j}\right)$

$$
\begin{equation*}
P\left(V_{j}\right)=\int_{0}^{V_{j}} p(V) d V \tag{C-7}
\end{equation*}
$$

for each of the distribution functions considered.

## Rayleigh

For the Rayleigh distribution the cummulative probability is given by

$$
\begin{align*}
P\left(V_{i}\right) & =\sigma^{-2} \int_{0}^{V_{i}} \exp \left(-V^{2} / 2 \sigma^{2}\right) V d V \\
& =1-\exp \left(-V_{i}^{2} / 2 \sigma^{2}\right) \tag{C-8}
\end{align*}
$$

Through a change of variables $y_{i}=\ln \left(1-P\left(V_{i}\right)\right) x_{i}=V_{i}^{2}(C-8)$ becomes

$$
\begin{equation*}
y=a x \tag{C-9}
\end{equation*}
$$

where the constant $a=-1 / 2 \sigma^{2}$. The parameter $\sigma$ can then be evaluated by a least squares straight line fit to ( $C-9$ ), i.e. by finding the value of a which minimizes

$$
\begin{equation*}
\varepsilon=\Sigma\left[\ln \left(1-F_{i}\right)-a V_{i}^{2}\right]^{2} \tag{C-10}
\end{equation*}
$$

## Weibull

The Weibull cummulative distribution is given by

$$
\begin{align*}
P\left(V_{i}\right) & =\int_{0}^{V_{i}}(k / c)(V / c)^{k-1} \exp \left[-(V / c)^{k}\right] d V \\
& =1-\exp \left[-(V / C)^{k}\right] \tag{C-11}
\end{align*}
$$

which, through a change of variables $Y_{i}=\ln \left\{-\ell n\left[1-P\left(V_{i}\right)\right]\right\}$ and $X_{i}=\ln V_{i}$, becomes

$$
\begin{equation*}
Y_{i}=a+b X_{i} \tag{C-12}
\end{equation*}
$$

where $c$ and $k$ are related to $a$ and $b$ by

$$
\begin{align*}
& c=\exp (-a / b) \\
& k=b \tag{C-13}
\end{align*}
$$

The $c$ and $k$ parameters can be evaluated by a least squares straight line
fit to (C-12), i.e. by finding the values of $a$ and $b$ which minimize

$$
\begin{equation*}
\varepsilon=\Sigma\left\{\ln \left[-\ln \left(1-F_{i}\right)\right]-a-b \ln V_{i}\right\}^{2} \tag{C-14}
\end{equation*}
$$

## Log-Normal

The log-normal cummulative distribution is given by

$$
\begin{align*}
P\left(V_{i}\right) & =\int_{o}^{\ln V_{i}}(2 \pi)^{-1 / 2} \mathrm{~s}^{-1} \exp \left[-\ln \left(V / V_{g}\right) / 2 s^{2}\right] d(\ln V) \\
& =0.5+\operatorname{erf}\left[\ln \left(V_{i} / V_{g}\right) / s\right] \tag{C-15}
\end{align*}
$$

where erf is the error function

$$
\begin{equation*}
\operatorname{erf}\left(Z_{f}\right)=(2 \pi)^{-1 / 2} \int_{0}^{Z_{i}} e^{-Z^{2} / 2} d Z \tag{C-16}
\end{equation*}
$$

Through the change of variables $y_{i}=\operatorname{erf}^{-1}\left[P\left(V_{i}\right)-0.5\right], x_{i}=\ln V_{i}$, where $\mathrm{erf}^{-1}$ is the mathematical inverse of ( $C-16$ ), i.e.

$$
\begin{equation*}
\operatorname{erf}^{-1}[\operatorname{erf}(Z)]=Z \tag{C-17}
\end{equation*}
$$

equation (C-15) becomes

$$
\begin{equation*}
y_{i}=a+b x_{i} \tag{C-18}
\end{equation*}
$$

when $V_{g}$ and $\sigma_{g}$ are related to $a$ and $b$ by

$$
\begin{align*}
& v_{g}=\exp (-a / b) \\
& \sigma_{g}=\exp (1 / b) \tag{C-19}
\end{align*}
$$

The $V_{g}$ and $\sigma_{g}$ parameters can be evaluated by a least squares straight line fit to ( $C-18$ ), i.e. by finding the values of $a$ and $b$ which minimize

$$
\varepsilon=\varepsilon\left[\operatorname{erf}^{-1}\left(F_{i}-0.5\right)-a-b \ln V_{i}\right]^{2}
$$

Comparison of the Distribution Functions
As expected, when goodness-of-fit of these distribution functions to observed data was tested, both the log-normal and the Weibull distributions performed better than the Rayleigh distribution. Observed frequencies of low wind speeds ( $V<2 \mathrm{~m} / \mathrm{s}$ ) did not fit the log-normal distribution well, but the Weibull distribution fit well over the entire speed range. This presented no real problem, however, since the distribution function need only fit the observations accurately between the cut-in speed $\mathrm{V}_{\mathrm{o}}$ and the rated speed $\mathrm{V}_{1}$ (see Appendix $B$, equation ( $B-12$ ) through ( $B-14$ ). In order to get the best possible agreement between the analytical distribution functions and the observed data, the distributions were fit only between the limits of $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$ and 11.2 $\mathrm{m} / \mathrm{s}(25 \mathrm{mph})$ because it was assumed that this range would include the cut-in and rated speeds of most generator units. Figure C. 1 shows examples of observed wind frequencies for Atlanta, Ga. The straight lines in Figure C-1 represent Weibull distributions fit to all the data. The $c$ and $k$ values listed in Figure C. 1 are for the Weibull fits only the 3.6 to $11.2 \mathrm{~m} / \mathrm{s}$ range (see Appendix D).


Figure C. 1 Average distribution of wind speed in Atlanta, Ga. from anemometers at 72 ft . (1951-1960) and at 20 ft . (1967-1971). The straight lines represent Weibull distributions. Weibull statistics were evaluated from best fit between $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$ and 11.2 ( 25 mph ), see listing in Appendix D.

## REFERENCES

1. Davenport, A. G. (1967): "The Dependence of Wind Lands on Meteorological Parameters," Proc. Conf. on Wind Effects on Buildings and Structures, Ottawa, Canada, September, pp. 19-82.
2. Larsen, R. E. and H. W. Church (1974): "Estimation of Long-Term Concentrations Using a "Universal" Wind Speed Distribution," J. Appl. Meteorol., 13, 910.
3. Larsen, R. J., C. D. Zimmer, D. A. Lynn, and K. G. Blemel (1967): "Analyzing Air Pollutant Concentration and Dosage," J. Air Pollution Contro1 Assoc., 17, 85-93.

APPENDIX D<br>WEIBULL DISTRIBUTION PARAMETERS BY SEASON AND ANNUAL AVERAGE

Seasonal and annual Weibull distribution scale parameters ( $\mathrm{c}, \mathrm{m} / \mathrm{s}$ ) and shape parameters ( $k$, dimensionless) for various sites in the U.S., arranged alphabetically by state and city. Codes are the three letter airport codes identifying the airport at which the data were taken. A11 data had constant anemometer heights (as listed) over the period of record. All data sets contained five or more years of wind data except the Kennedy SC (1967-1969) and WKY-TV (1966-1967) tower data. Table D. 1 shows $c$ and $k$ values for wind distributions measured at the indicated heights. Tables D. 2 and D. 3 show values of $c, k$ and mean wind speed $\bar{u}$ at heights of 30.5 m and 61 m , respectively. All c and k values were determined by methods described in Appendix $C$. The average wind speeds $\bar{u}$ were evaluated from $c$ and $k$ by equation $B-18$. A contour map of $\bar{u}$ at 30.5 m height is shown in Figure 12.6.

Plant factors (ratio of average power output to rated power output) were evaluated for the $c$ and $k$ values and are shown in Tables $D-4$ and $D-5$. These plant factor values were evaluated by the method described in Appendix B. Contour maps of plant factors are also given in Figures 12.7 through 12.14.
TABLE D. 1 WEIBULL c AND $k$ VALUES AT "SURFACE" ANEMOMETER HEIGHTS

| CITY | CODE | ANEMOMETER <br> HEIGHT <br> $\mathrm{m}(\mathrm{ft})$. | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{c}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ |
| Alabama |  |  |  |  |  |  |  |  |  |  |  |  |
| Birmingham | BHM | 19.2(63) | 4.74 | 1.91 | 4.59 | 1.82 | 2.97 | 1.52 | 3.90 | 1.73 | 4.09 | 1.71 |
| Mobile | MOB | 17.1(56) | 5.47 | 1.95 | 5.21 | 2.06 | 3.37 | 1.54 | 4.42 | 1.81 | 4.64 | 1.81 |
|  | MOB | 6.7 (22) | 4.59 | 2.14 | 4.50 | 1.95 | 2.43 | 1.31 | 3.56 | 1.77 | 3.89 | 1.79 |
| Arizona |  |  |  |  |  |  |  |  |  |  |  |  |
| Phoenix | PHX | 5.5(18) | 1.95 | 1.09 | 2.91 | 1.41 | 2.76 | 1.32 | 2.62 | 1.35 | 2.58 | 1.29 |
| Tucson | TUS | 6.1 (20) | 3.43 | 1.31 | 4.11 | 1.53 | 3.87 | 1.57 | 3.73 | 1.42 | 3.80 | 1.46 |
| Yuma | YUM | $6.1(20)$ | 3.23 | 1.48 | 3.86 | 1.80 | 3.98 | 1.79 | 3.14 | 1.47 | 3.58 | 1.64 |
| Arkansas |  |  |  |  |  |  |  |  |  |  |  |  |
| Ft. Smith | FSM | 7.0 (23) | 4.00 | 1.93 | 4.11 | 2.09 | - | - | - | - | 3.62 | 1.86 |
| Little Rock | LIT | 6.1(20) | 4.13 | 2.01 | 4.23 | 2.07 | - | - | - | - | 3.71 | 1.88 |
| California |  |  |  |  |  |  |  |  |  |  |  |  |
| Bakersfield | BFL | 61. (20) | 1.93 | 1.11 | 2.91 | 1.42 | - | - | 2.06 | 1.13 | 2.67 | 1.38 |
| Fresno | FAT | 12.8(42) | 2.30 | 1.27 | 3.46 | 1.72 | 3.04 | 1.72 | 2.30 | 1.46 | 2.68 | 1.42 |
| Los Angeles | LAX | 6.1(20) | 3.09 | 1.52 | 4.26 | 1.92 | - | - | 3.73 | 1.90 | 3.92 | 1.92 |
| Oakland | OAK | 14.9(49) | 2.75 | 1.12 | 4.54 | 1.91 | 4.53 | 2.17 | 3.15 | 1.45 | 3.85 | 1.63 |
| Sacramento | SMF | 6.1(20) | 3.22 | 1.35 | 4.04 | 1.79 | 3.82 | 1.93 | 3.18 | 1.48 | 3.57 | 1.60 |
| Colorado |  |  |  |  |  |  |  |  |  |  |  |  |
| Colorado Springs | COS | $6.7(22)$ | 3.97 | 1.27 | 5.15 | 1.51 | 4.15 | 1.52 | 4.14 | 1.39 | 4.35 | 1.40 |
| Denver | DEN | 21.9(72) | 4.80 | 1.71 | 5.11 | 1.62 | 4.46 | 1.75 | 4.25 | 1.67 | 4.63 | 1.65 |
| Pueblo | PUB | $6.7(22)$ | 3.09 | 1.03 | 4.57 | 1.30 | 4.28 | 1.43 | 3.20 | 1.11 | 3.82 | 1.21 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER HEIGHT | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | m(ft.) | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K |
| Connecticut |  |  |  |  |  |  |  |  |  |  |  |  |
| Bridgeport | BDR | 25.6(84) | 6.87 | 2.04 | 6.24 | 2.06 | 4.95 | 2.21 | 5.83 | 1.93 | 5.92 | 1.94 |
| Hartford | BDL | 30.2 (99) | 5.32 | 1.96 | 5.46 | 2.11 | 4.26 | 2.03 | 4.58 | 2.01 | 4.89 | 2.01 |
| F1orida |  |  |  |  |  |  |  |  |  |  |  |  |
| Kennedy SC | XMR | 60.0(197) | 6.57 | 2.50 | 6.69 | 2.92 | 4.82 | 2.26 | 6.45 | 2.42 | 6.14 | 2.41 |
| Dayton | DAB | 7.0 (23) | 4.29 | 1.98 | 4.83 | 2.27 | - | - | 4.39 | 2.14 | 4.32 | 2.03 |
| Jacksonville | JAX | 19.2(63) | 4.15 | 1.71 | 4.56 | 2.04 | 3.98 | 2.01 | 4.30 | 1.89 | 4.08 | 1.62 |
| Miami | MIA | 7.0 (23) | 4.66 | 2.10 | 5.02 | 2.34 | 3.71 | 1.86 | 4.37 | 2.09 | 4.47 | 2.08 |
| Orlando | ORL | 16.2(53) | 4.63 | 1.83 | 4.65 | 1.94 | 3.97 | 1.76 | 4.19 | 1.78 | 4.27 | 1.78 |
| Tallahassee | TLH | 16.5(54) | 3.50 | 1.91 | 3.83 | 2.32 | 3.61 | 1.78 | 3.97 | 1.76 | 4.19 | 1.78 |
|  | TLH | 7.6 (25) | 3.63 | 1.88 | - | - | 2.18 | 1.24 | - | - | 3.18 | 1.70 |
| Tampa | TPA | $6.7(22)$ | 3.87 | 1.75 | 4.41 | 2.26 | 2.58 | 1.25 | 3.57 | 1.70 | 3.69 | 1.73 |
| Georgia |  |  |  |  |  |  |  |  |  |  |  |  |
| Atlanta | ATL | 21.9(72) | 5.51 | 2.05 | 5.11 | 1.94 | 3.47 | 1.66 | 4.43 | 1.41 | 4.65 | 1.82 |
|  | ATL | 6.1 (20) | 4.22 | 2.14 | 3.83 | 1.94 | - | - | - | - | 3.55 | 1.86 |
| Augusta | AGS | 7.6 (25) | 3.55 | 1.62 | 3.46 | 1.51 | 3.00 | 1.78 | 2.69 | 1.39 | 3.09 | 1.48 |
|  | AGS | 6.1 (20) | 3.58 | 1.80 | 3.40 | 1.56 | - | - | 2.36 | 1.38 | 2.94 | 1.50 |
| Columbus | CSG | 6.1 (20) | 3.83 | 2.02 | 3.83 | 1.97 | - | - | - | - | 3.52 | 1.90 |
| Macon | MCN | 7.0(23) | 3.74 | 1.85 | 3.83 | 1.86 | - | - | - | - | 3.41 | 1.77 |
| Savannah | SAV | 18.0(59) | 4.30 | 1.80 | 4.20 | 1.84 | 3.21 | 1.66 | 3.83 | 1.87 | 3.79 | 1.65 |
|  | SAV | 6.1 (20) | 3.68 | 1.67 | 4.07 | 1.89 | - | - | - | - | 3.45 | 1.69 |
| Idaho |  |  |  |  |  |  |  |  |  |  |  |  |
| Pocatello | PIH | 10.4(34) | 5.48 | 1.41 | 5.32 | 1.42 | 4.39 | 1.47 | 4.17 | 1.32 | 4.80 | 1.38 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER$\frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\underset{\mathrm{m} / \mathrm{s}}{\mathrm{C}}$ | K | $\frac{C}{m / s}$ | K |
| Illinois |  |  |  |  |  |  |  |  |  |  |  |  |
| Chicago | MDW | 6.1 (20) | 5.47 | 2.05 | 5.65 | 2.14 | 4.21 | 1.96 | 4.80 | 1.96 | 5.02 |  |
|  | ORD | 6.1 (20) | 4.66 | 1.86 | 4.87 | 1.91 | . 21 | 1.96 | 3.94 | 1.92 | 5.02 4.10 | 1.96 1.72 |
| Rockford <br> Springfield | RFD | 6.1(20) | 5.30 | 2.29 | 5.49 | 2.31 | 4.32 | 2.20 | 4.80 | 2.13 | 4.95 | 1.72 2.16 |
| Springfield | SPI | 14.9(49) | 6.70 | 2.29 | 6.66 | 2.11 | 4.36 | 1.83 | 5.64 | 1.92 | 5.85 | 1.94 |
|  | SPI | 6.1 (20) | 5.96 | 2.10 | 6.09 | 2.30 | 4.07 | 1.93 | 4.96 | 1.92 2.07 | 5.26 | 1.94 1.98 |
| Indiana |  |  |  |  |  |  |  |  |  |  |  |  |
| Evansville | EVV | 19.5(64) | 5.29 | 2.17 | 5.27 | 1.98 | 3.36 | 1.60 | 4.36 |  |  |  |
|  | EVV | 6.1 (20) | 4.54 | 2.04 | 4.49 | 2.09 | 3.36 | 1.60 | 4.36 3.82 | 1.85 | 4.63 | 1.88 |
| Indianapolis | IND | 6.1 (20) | 4.52 | 1.84 | 4.51 | 1.98 | - | - | 3.82 | 1.86 | 3.97 3.90 | 1.87 1.73 |
| South Bend | SBN | 17.7(58) | 5.88 | 2.63 | 5.95 | 2.27 | 4.47 | 2.12 | 3.52 5.25 | 1.69 2.10 | 3.90 5.39 | $\begin{aligned} & 1.73 \\ & 2.20 \end{aligned}$ |
| Iowa |  |  |  |  |  |  |  |  |  |  |  |  |
| Burlington | BRL | 10.1(33) | 5.60 | 2.14 | 5.60 | 2.14 | 3.51 | 1.60 |  |  |  |  |
| Des Moines | DSM | 20.1(66) | 5.98 | 2.07 | 6.55 | 2.16 | 4.80 | 1.60 2.05 | 4.77 5.55 | 1.99 1.88 | $\begin{aligned} & 4.92 \\ & 5.71 \end{aligned}$ | $\begin{aligned} & 1.91 \\ & 1.97 \end{aligned}$ |
| Kansas |  |  |  |  |  |  |  |  |  |  |  |  |
| Wichita | ICT | 8.8(29) | 6.58 | 2.08 | 7.26 | 2.19 | 6.29 | 2.27 | 6.49 | 2.28 | 6.64 | 2.18 |
| Kentucky |  |  |  |  |  |  |  |  |  |  |  |  |
| Lexington Louisville | LEX | 18.6(61) | 5.61 | 2.20 | 5.31 | 2.02 | 3.88 | 1.94 |  |  |  |  |
|  | SDF | 21.6(71) | 4.99 | 2.14 | 4.93 | 1.98 | 3.88 3.44 | 1.94 1.77 | 4.33 4.13 | 1.73 1.87 | 4.77 4.35 | 1.87 1.84 |
|  | SDF | 6.1 (20) | 4.48 | 2.12 | 4.44 | 2.12 | 3.4 | $\cdots$ | - 13 | 1.87 | 4.98 | 1.84 1.97 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER$\frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\underset{\mathrm{m} / \mathrm{s}}{\mathrm{C}}$ | $\underline{\mathrm{K}}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{\mathrm{K}}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K |
| Louisiana |  |  |  |  |  |  |  |  |  |  |  |  |
| New Orleans | MSY | 16.2(53) | 5.08 | 1.84 | 4.87 | 1.87 | 3.57 | 1.99 | 4.50 | 1.85 | 4.43 | 1.73 |
|  | MSY | 6.1 (20) | 4.49 | 2.05 | 4.51 | 1.97 | 2.63 | 1.40 | 3.62 | 1.73 | 3.91 | 1.79 |
| Shreveport | SHV | 16.8(55) | 5.15 | 2.06 | 5.06 | 2.00 | 3.72 | 1.79 | 4.17 | 1.84 | 4.53 | 1.88 |
|  | SHV | 6.1 (20) | 3.89 | 1.77 | 4.04 | 1.98 | 2.52 | 1.42 | 3.19 | 1.63 | 3.48 | 1.70 |
| Maine |  |  |  |  |  |  |  |  |  |  |  |  |
| Portland | PWM | 16.8(55) | 4.83 | 1.71 | 5.14 | 1.92 | 3.96 | 1.83 | 4.27 | 1.66 | 4.53 | 1.73 |
| Maryland |  |  |  |  |  |  |  |  |  |  |  |  |
| Baltimore | BAL | $40.5(133)$ | 5.08 | 1.64 | 5.54 | 2.08 | 4.09 | 1.84 | 4.45 | 1.75 | 4.77 | 1.71 |
| Massachusetts |  |  |  |  |  |  |  |  |  |  |  |  |
| Boston | BOS | $6.7(22)$ | 6.86 | 2.31 | 6.71 | 2.41 | 5.53 | 2.71 | 5.75 | 2.34 | 6.19 | 2.30 |
| Michigan |  |  |  |  |  |  |  |  |  |  |  |  |
| Detroit | DET | 24.7(81) | 5.60 | 2.20 | 5.21 | 2.08 | 3.95 | 1.78 | 4.89 | 1.93 | 4.97 |  |
| Flint | FNT | $6.4(21)$ | 5.41 | 2.18 | 5.19 | 2.11 | 3.94 | 1.92 | 4.77 | 2.07 | 4.83 | 2.01 |
| Grand Rapids | GRR | 19.5(64) | 5.31 | 2.19 | 5.53 | 2.11 | 4.05 | 1.92 | 4.75 | 1.90 | 4.92 | 2.00 |
| Lansing | LAN | 15.8(52) | 7.11 | 2.54 | 6.91 | 2.27 | 4.53 | 1.93 | 6.02 | 1.28 | 6.16 | 2.13 |
| Muskegon | MKG | 6.1(20) | 5.95 | 2.20 | 5.54 | 2.21 | 4.47 | 2.18 | 5.18 | 2.13 | 5.26 | 2.08 |
| Minnesota |  |  |  |  |  |  |  |  |  |  |  |  |
| Duluth | DLH | 6.4 (21) | 5.29 | 2.17 | 5.38 | 2.15 | 4.42 | 1.91 | 4.68 | 2.01 | 4.95 | 2.04 |
| Minneapolis | MSP | 6.4 (21) | 4.92 | 2.11 | 5.44 | 2.22 | 4.46 | 2.02 | 4.83 | 2.10 | 4.91 | 2.09 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER HEIGHT | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | m(ft.) | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ |
| Mississippi |  |  |  |  |  |  |  |  |  |  |  |  |
| Jackson | JAN | 6.1 (20) | 4.28 | 1.91 | 4.12 | 2.03 | 2.27 | 1.33 | 3.22 | 1.60 | 3.59 | 1.72 |
| Missouri |  |  |  |  |  |  |  |  |  |  |  |  |
| Columbia | COU | 14.6(48) | 3.57 | 1.77 | 3.75 | 1.78 | - | - | - | - | 3.16 | 1.62 |
| Springfield | SGF | 18.0(59) | 6.76 | 2.36 | 6.93 | 2.28 | 5.29 | 2.21 | 5.92 | 2.19 | 6.21 | 2.17 |
|  | SGF | 6.1(20) | 5.13 | 2.33 | 5.16 | 2.24 | - | - | 4.35 | 2.07 | 4.60 | 2.08 |
| St. Louis | STL | 25.0(82) | 4.91 | 2.09 | 5.18 | 2.00 | 3.50 | 1.89 | 4.14 | 1.93 | 4.38 | 1.83 |
|  | STL | 6.1 (20) | 4.93 | 1.92 | 4.85 | 1.95 | 3.29 | 1.70 | 4.03 | 1.88 | 4.27 | 1.78 |
| Montanna |  |  |  |  |  |  |  |  |  |  |  |  |
| Billings | BIL | 11.9(39) | 6.11 | 2.15 | 5.52 | 1.66 | 4.56 | 1.53 | 5.15 | 1.73 | 5.37 | 1.75 |
|  | BIL | 7.6 (25) | 6.00 | 2.14 | 5.11 | 1.96 | 3.97 | 1.69 | 5.45 | 2.14 | 5.16 | 1.94 |
| Glasgow | GGW | 6.1 (20) | 5.13 | 1.73 | 5.80 | 1.82 | 5.07 | 1.78 | 5.12 | 1.78 | 5.27 | 1.77 |
| Harve | HVR | 6.1 (20) | 4.93 | 1.56 | 5.48 | 1.76 | 4.74 | 1.73 | 5.07 | 1.68 | 5.05 | 1.67 |
| Helena | HLN | 13.4(44) | 3.24 | 1.12 | 4.15 | 1.54 | 3.54 | 1.37 | 3.41 | 1.50 | 3.57 | 1.39 |
| Kalispell | FCA | 6.1 (20) | 3.20 | 1.15 | 3.90 | 1.67 | 3.25 | 1.57 | 2.88 | 1.36 | 3.25 | 1.36 |
| Miles City | MLS | 12.2(40) | 4.47 | 1.65 | 5.23 | 1.74 | 4.12 | 1.61 | 4.45 | 1.47 | 4.56 | 1.59 |
| Nebraska |  |  |  |  |  |  |  |  |  |  |  |  |
| Scottsbluff | BFF | 6.1 (20) | 5.38 | 1.70 | 5.60 | 1.79 | 3.96 | 1.66 | 4.39 | 1.52 | 4.81 | 1.61 |
| Nevada |  |  |  |  |  |  |  |  |  |  |  |  |
| E1y | ELY | 6.1 (20) | 4.58 | 1.58 | 4.44 | 1.52 | 3.99 | 1.69 | 4.23 | 1.65 | 4.27 | 1.57 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER$\frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K |
| New Jersey |  |  |  |  |  |  |  |  |  |  |  |  |
| Atlantic City | ACY | 6.1 (20) | 5.62 | 1.71 | 5.68 | 2.00 | 4.17 | 1.90 | 4.36 | 1.77 | 4.91 | 1.73 |
| Newark | EWR | 6.1 (20) | 5.57 | 2.14 | 5.39 | 2.07 | 4.20 | 2.13 | 4.52 | 1.96 | 4.93 | 1.96 |
| New Mexico |  |  |  |  |  |  |  |  |  |  |  |  |
| Albuquerque | $A B Q$ | 14.6(48) | 3.09 | 1.10 | 4.72 | 1.48 | 3.77 | 1.39 | 3.30 | 1.19 | 3.76 | 1.28 |
| New York |  |  |  |  |  |  |  |  |  |  |  |  |
| Albany | ALB | 12.2(40) | 4.94 | 1.89 | 4.99 | 2.01 | 3.97 | 1.97 | 4.25 | 1.92 | 4.50 | 1.87 |
|  | ALB | 6.1 (20) | 5.74 | 1.96 | 4.02 | 1.71 | - | - | - | - | 3.61 | 1.65 |
| Buffalo | BUF | 6.1(20) | 6.11 | 1.92 | 5.43 | 1.86 | 4.46 | 1.75 | 4.72 | 1.79 | 5.15 | 1.77 |
| New York City | JFK | 6.1 (20) | 6.22 | 2.08 | 5.86 | 2.14 | 4.76 | 2.21 | 5.12 | 1.89 | 5.45 | 1.97 |
|  | LGA | 6.1 (20) | 6.32 | 2.39 | 5.70 | 2.37 | 4.57 | 2.33 | 5.08 | 2.18 | 5.42 | 2.13 |
| Syracuse | SYR | 21.9(72) | 5.14 | 1.87 | 5.10 | 1.89 | 4.16 | 1.82 | 4.71 | 1.88 | 4.77 | 1.86 |
| North Carolina |  |  |  |  |  |  |  |  |  |  |  |  |
| Asheville | AVL | 28.0(92) | 3.95 | 1.57 | 4.23 | 1.85 | 2.46 | 1.47 | 3.00 | 1.37 | 2.85 | 1.20 |
|  | AVL | 6.1 (20) | 4.84 | 1.70 | 4.09 | 1.57 | 2.44 | 1.41 | 3.43 | 1.48 | 3.69 | 1.45 |
| Cape Hatteras | HAT | 9.8 (32) | 6.33 | 2.22 | 5.82 | 2.32 | 4.99 | 2.37 | 5.61 | 2.20 | 5.67 | 2.19 |
| Charlotte | CLT | 6.1 (20) | 3.27 | 1.57 | 3.58 | 1.73 | - | - | - | - | 2.97 | 1.54 |
| Greensboro | GSO | 17.1(56) | 4.04 | 1.65 | 4.33 | 1.81 | 2.96 | 1.46 | 3.60 | 1.62 | 3.73 | 1.60 |
|  | GSO | 6.1 (20) | 3.68 | 1.74 | 3.69 | 1.87 | - | - | - | - | 3.16 | 1.64 |
| Raleigh | RDU | 6.1 (20) | 4.35 | 1.85 | 4.32 | 1.78 | - | - | - | - | 3.76 | 1.68 |
| North Dakota |  |  |  |  |  |  |  |  |  |  |  |  |
| Bismark | BIS | 13.1(43) | 5.03 | 1.68 | 6.26 | 1.95 | 5.44 | 1.98 | 5.58 | 1.67 | 5.58 | 1.80 |
|  | BIS | 6.1 (20) | 4.96 | 1.64 | 5.58 | 1.82 | 4.79 | 1.91 | 4.95 | 1.67 | 5.06 | 1.73 |
| Williston | ISN | 6.1 (20) | 4.61 | 1.72 | 5.15 | 1.84 | 4.41 | 1.76 | 4.76 | 1.77 | 4.73 | 1.76 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER$\frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{C}{m / s}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{\mathrm{K}}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ |
| Ohio |  |  |  |  |  |  |  |  |  |  |  |  |
| Akron | CAK | 11.3(37) | 5.88 | 2.35 | 5.44 | 2.07 | 3.76 |  |  |  |  |  |
| Cincinnati | CVG | 19.5(64) | 5.21 | 2.11 | 5.16 | 2.02 | 3.76 3.53 | 1.79 1.77 | 4.74 4.50 | 1.94 2.00 | 4.98 4.59 | 1.96 |
| Cleveland | CLE | 6.1 (20) | 5.79 | 2.23 | 4.97 | 2.01 | 3.53 | 1.77 | 4.41 | 1.90 | 4.59 | 1.89 |
| Dayton | DAY | 16.8 (55) | 5.83 | 2.16 | 5.38 | 1.88 | 3.58 | 1.66 | 4.41 4.43 | 1.90 1.67 | 4.71 4.82 | 1.90 |
| Toledo | DAY | $6.1(20)$ $14.3(47)$ | 5.14 | 2.14 | 4.86 | 2.05 | 3.59 | 1.81 | 4.35 | 1.67 2.02 | 4.82 4.49 | 1.75 1.95 |
| Youngstown | TOL | $14.3(47)$ $18.9(62)$ | 6.14 5.85 | 2.20 | 6.23 | 2.09 | 4.05 | 1.69 | 5.02 | 1.88 | 5.38 | 1.89 |
| Youngstown |  | 18.9(62) | 5.85 | 2.38 | 5.19 | 1.95 | 3.74 | 1.77 | 4.74 | 1.93 | 4.91 | 1.94 |
|  | YNG | 6.1 (20) | 5.50 | 2.44 | 4.91 | 2.13 | 3.21 | 1.74 | 4.26 | 2.10 | 4.53 | 2.03 |
| Ok1 ${ }^{\text {ahoma }}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| Tulsa | TUL | $7.0(23)$ | 5.02 | 1.89 | 5.80 | 2.11 | 4.57 | 2.01 |  |  |  |  |
| Okl. City TV Tower | WKY | $44.5(146)$ | 5.02 | 1.89 | 5.80 | 2.11 | 4.57 | 2.01 | 4.71 | 1.90 | $\begin{aligned} & 5.02 \\ & 6.57 \end{aligned}$ | $\begin{aligned} & 1.93 \\ & 2.07 \end{aligned}$ |
| Oregon |  |  |  |  |  |  |  |  |  |  |  |  |
| Eugene | EUG | 6.1 (20) | 3.68 | 1.57 | 3.34 | 1.51 |  |  |  |  |  |  |
| Portland | PDX | $6.1(20)$ | 4.66 | 1.78 | 3.17 | 1.44 | - | - | 3.16 3.40 | 1.48 1.45 | 3.35 3.49 | $\begin{aligned} & 1.54 \\ & 1.49 \end{aligned}$ |
| Salem | SLE | 6.1 (20) | 4.27 | 1.53 | 3.44 | 1.44 1.52 | - | - | 3.40 3.65 | 1.45 1.64 | 3.49 3.49 | $\begin{aligned} & 1.49 \\ & 1.47 \end{aligned}$ |
| Pennsylvania |  |  |  |  |  |  |  |  |  |  |  |  |
| Allentown Harrisburg | ABE | 6.1(20) | 5.18 | 1.58 | 4.96 | 1.76 | 3.39 | 1.58 | 3.94 | 1.51 |  |  |
|  | HAR | 14.0(46) | 4.16 | 1.54 | 4.33 | 1.77 | 2.55 | 1.58 1.37 | 3.94 3.15 | 1.51 1.45 | 4.31 3.53 | 1.52 1.46 |
|  | HAR | $6.7(22)$ | 4.52 | 1.51 | 4.11 | 1.60 | 2.70 | 1.52 | 3.23 | 1.39 | 3.53 3.58 | 1.46 1.40 |
| Philadelphia | PHL | 6.1 (20) | 5.24 | 1.97 | 5.31 | 2.15 | 3.51 | 1.77 | 4.22 | 1.88 | 3.58 4.58 | 1.40 |
| Pittsburgh | PIT | 6.1(20) | 5.14 | 2.06 | 4.69 | 1.85 | 3.51 | 1.7 | 4.20 | 1.88 1.81 | 4.58 4.37 | 1.85 1.81 |
| Scranton | AVP | 28.0(92) | 4.74 | 2.17 | 4.57 | 2.09 | 3.70 | 2.05 | 4.09 | 1.81 2.05 | 4.37 4.15 | 1.81 2.01 |


| TABLE D. 1 (cont.) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CITY | CODE | $\begin{aligned} & \text { ANEMOMET } \\ & \frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)} \end{aligned}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ |
| Rhode Island |  |  |  |  |  |  |  |  |  |  |  |  |
| Providence | PVD | 6.1 (20) | 5.46 | 1.78 | 5.48 | 2.05 | 4.16 | 2.15 | 4.14 | 1.76 | 4.73 | 1.76 |
| South Carolina |  |  |  |  |  |  |  |  |  |  |  |  |
| Charleston | CHS | 6.1 (20) | 3.98 | 1.74 | 4.30 | 1.92 | - | - | 3.43 | 1.77 | 3.79 | 1.77 |
| Columbia | CAE | 11.0 (36) | 3.81 | 1.69 | 4.09 | 1.70 | 3.09 | 1.63 | 3.17 | 1.61 | 3.41 | 1.51 |
|  | CAE | 6.1 (20) | 3.61 | 1.78 | 3.81 | 1.76 | 3.09 | 1.63 | 2.69 | 1.45 | 3.13 | 1.51 |
| Greenville | GSP | $7.0(23)$ | - | - | 3.62 | 1.83 | - | _ | 2.69 | 1.45 | 3.13 | 1.61 |
| South Dakota |  |  |  |  |  |  |  |  |  |  |  |  |
| Huron | HON | 12.5(41) | 5.56 | 1.91 | 6.47 | 2.08 | 5.38 | 2.03 | 5.84 | 1.91 | 5.81 | 1.95 |
| Rapid City | RAP | 9.8(32) | 4.80 | 1.29 | 5.82 | 1.70 | 5.04 | 1.79 | 5.84 | 1.91 1.47 | 5.81 5.27 | 1.95 1.54 |
|  | RAP | 6.4(21) | 4.81 | 1.22 | 5.97 | 1.47 | 4.53 | 1.63 | 5.09 | 1.33 | 5.06 | 1.54 1.36 |
| Sioux Falls | FSD | 5.2 (17) | 5.51 | 1.97 | 6.30 | 2.36 | 5.11 | 2.28 | 5.40 | 2.16 | 5.57 | 2.14 |
| Tennessee |  |  |  |  |  |  |  |  |  |  |  |  |
| Bristol | TRI | 6.1 (20) | 3.28 | 1.62 | 3.46 | 1.76 | - | - | - | - | 2.78 | 1.49 |
| Chattanooga | CHA | 17.4 (57) | 3.93 | 1.81 | 3.95 | 1.76 | 2.34 | 1.33 | 3.35 | 1.83 | 2.78 3.38 | 1.49 1.60 |
|  | CHA | 6.1 (20) | 3.43 | 1.75 | 3.70 | 1.88 | 1.86 | 1.18 | 3.35 | 1.83 | 3.16 | 1.69 |
| Knoxville Memphis | TYS | $6.7(22)$ | 3.14 | 1.50 | 3.44 | 1.56 | - | 1.18 | - | _ | 2.83 | 1.43 |
| Memphis Nashville | MEM | $6.7(22)$ | 4.74 | 1.93 | 4.84 | 2.27 | 3.16 | 1.67 | 3.93 | 1.93 | 4.18 | 1.87 |
| Nashville | BNA | 7.6(25) | 4.55 | 1.88 | 4.68 | 2.01 | - | - | 3.61 | 1.68 | 3.99 | 1.75 |
| Texas |  |  |  |  |  |  |  |  |  |  |  |  |
| Abilene <br> Amarillo <br> Austin | ABI | 6.1 (20) | 5.50 | 2.09 | 6.05 | 2.42 | 5.08 | 2.37 | 5.27 | 2.14 | 5.47 | 2.20 |
|  | AMA | 10.1(33) | 6.12 | 1.96 | 6.96 | 2.24 | 5.85 | 2.34 | 5.88 | 2.14 | 5.47 6.20 | 2.20 2.12 |
|  | AUS | 9.8(32) | 5.03 | 2.06 | 5.41 | 2.27 | 4.57 | 2.27 | 4.17 | 1.74 | 4.81 | 2.04 |
|  | AUS | 6.1 (20) | 4.73 | 1.84 | 4.99 | 2.07 | . | 2.27 | 3.96 | 1.70 | 4.39 | 1.84 |

TABLE D. 1 (cont.)

| CITY | CODE | ANEMOMETER$\frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{\mathrm{K}}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | ${\underset{\mathrm{m} / \mathrm{s}}{\mathrm{C}}}^{\text {( }}$ | $\underline{K}$ |
| Texas (continued) |  |  |  |  |  |  |  |  |  |  |  |  |
| Brownsville | BRO | 17.1(56) | 6.10 | 2.12 | 7.12 | 2.40 | 6.09 | 2.59 | 5.33 | 2.09 | 6.21 | 2.13 |
| Dallas | DAL | 6.1 (20) | 5.20 | 2.15 | 5.81 | 2.58 | 4.49 | 2.26 | 4.71 | 2.08 | 5.06 | 2.21 |
| E1 Paso | ELP | 25.9(85) | 5.06 | 1.42 | 6.32 | 1.70 | 5.05 | 1.81 | 4.70 | 1.64 | 5.25 | 1.60 |
| Fort Worth | FTW | 25.9(85) | 6.21 | 2.27 | 6.71 | 2.35 | 5.67 | 2.44 | 5.64 | 2.14 | 6.05 | 2.29 |
|  | FTW | $6.7(22)$ | 4.83 | 1.78 | 5.22 | 1.93 | 3.78 | 1.73 | 9.17 | 1.77 | 4.48 | 1.74 |
| Galveston | GLS | 43.6 (143) | 6.33 | 2.09 | 6.25 | 2.48 | 5.47 | 2.51 | 5.45 | 1.86 | 5.86 | 2.18 |
| Houston | HOU | 26.5(87) | 6.14 | 2.23 | 6.55 | 2.21 | 4.64 | 1.96 | 5.21 | 2.12 | 5.60 | 2.09 |
|  | HOU | 6.1 (20) | 5.40 | 2.18 | 5.68 | 2.34 | 4.11 | 1.96 | 4.23 | 1.81 | 4.88 | 2.02 |
| Lubbock | LBB | 20.7(68) | 6.65 | 2.03 | 7.49 | 2.34 | 6.06 | 2.35 | 6.03 | 2.16 | 6.54 | 2.15 |
| Port Arthur | BPT | 6.1(20) | 5.21 | 2.22 | 5.66 | 2.58 | 4.24 | 2.23 | 4.29 | 1.92 | 4.87 | 2.16 |
| Waco | ACT | 23.8(78) | 6.16 | 2.24 | 6.58 | 2.41 | 5.87 | 2.59 | 5.39 | 2.06 | 6.01 | 2.28 |
| Wichita Falls | SPS | 9.4 (31) | 4.94 | 2.12 | 5.66 | 2.28 | 4.72 | 2.29 | 4.65 | 2.00 | 5.00 | 2.14 |
| Utah |  |  |  |  |  |  |  |  |  |  |  |  |
| Salt Lake City | SLC | 6.1 (20) | 2.86 | 1.08 | 3.64 | 1.33 | 3.33 | 1.45 | 3.13 | 1.25 | 3.23 | 1.26 |
| Vermont |  |  |  |  |  |  |  |  |  |  |  |  |
| Burlington | BTV | 6.1 (20) | 4.44 | 2.02 | 4.38 | 1.95 | - | - | 4.17 | 1.88 | 4.18 | 1.92 |
| Virginia |  |  |  |  |  |  |  |  |  |  |  |  |
| Lynchburg | LYH | $6.1(20)$ | 3.67 | 1.74 | 4.09 | 1.96 | - | - | , | 5 | 3.38 | 1.71 |
| Richmond | RIC | 20.4(67) | 3.76 | 1.85 | 4.06 | 1.98 | 2.66 | 1.46 | 3.08 | 1.57 | 3.31 | 1.59 |
| Roanoke | ROA | 9.1(30) | 4.58 | 1.55 | 4.46 | 1.74 | - | - | 3.35 | 1.45 | 3.75 | 1.46 |
| Wallops | WAL | 61.0(200) | 8.03 | 2.06 | 8.28 | 2.33 | 6.49 | 2.14 | 7.41 | 2.31 | 7.57 | 2.17 |

TABLE D. 1 (cont.)

| CITY | CODE | $\begin{aligned} & \text { ANEMOMETER } \\ & \frac{\text { HEIGHT }}{\mathrm{m}(\mathrm{ft} .)} \end{aligned}$ | WINTER |  | SPRING |  | SUMMER |  | FALL |  | ANNUAL |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{\mathrm{K}}$ | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | K | $\frac{\mathrm{C}}{\mathrm{~m} / \mathrm{s}}$ | $\underline{K}$ | $\frac{C}{m / s}$ | K |
| Washington |  |  |  |  |  |  |  |  |  |  |  |  |
| Seattle | SEA | 6.1 (20) | 4.44 | 1.78 | 3.69 | 1.64 | - | - | 3.59 | 1.62 | 3.36 | 1.60 |
| Spokane | SFF | 6.1 (20) | 4.36 | 1.44 | 4.42 | 1.75 | 3.92 | 1.80 | 4.00 | 1.55 | 4.12 | 1.57 |
| Hanford | HAN | 61.0 (200) | 4.28 | 1.27 | 5.58 | 1.55 | 5.47 | 1.53 | 4.43 | 1.35 | 4.95 | 1.41 |
| West Virginia |  |  |  |  |  |  |  |  |  |  |  |  |
| Charleston | CRW | 9.8(32) | 3.92 | 2.19 | 3.38 | 1.64 | 2.13 | 1.31 | 2.92 | 1.69 | 2.98 | 1.54 |
| Huntington | HTS | 6.1(20) | 3.32 | 1.53 | 3.14 | 1.53 | - | - | - | - | 2.71 | 1.41 |
| Wisconsin |  |  |  |  |  |  |  |  |  |  |  |  |
| Green Bay | GRB | 14.3(47) | 5.89 | 2.21 | 5.81 | 2.09 | 4.49 | 1.83 | 5.65 | 1.92 | 5.47 | 1.97 |
|  | GRB | 6.1 (20) | 5.29 | 2.10 | 5.17 | 2.17 | 3.94 | 1.76 | 4.66 | 1.97 | 4.78 | 1.98 |
| La Crosse | LSE | 6.4 (21) | 4.21 | 1.85 | 4.46 | 1.94 | 3.72 | 1.88 | 4.11 | 1.90 | 4.11 | 1.86 |
| Madison | MSN | 6.4 (21) | 5.01 | 2.07 | 5.06 | 2.06 | 3.99 | 2.03 | 4.60 | 2.07 | 4.62 | 1.98 |
| Milwaukee | MKE | 6.1 (20) | 5.89 | 2.32 | 5.52 | 2.06 | 4.54 | 2.04 | 5.20 | 2.22 | 5.78 | 2.10 |
|  | MWC | 6.1 (20) | 5.29 | 2.25 | 5.18 | 2.36 | 4.21 | 2.11 | 4.81 | 2.25 | 4.87 | 2.20 |
| Wyoming |  |  |  |  |  |  |  |  |  |  |  |  |
| Casper Cheyenne | CPR | 6.1 (20) | 7.54 | 1.95 | 6.06 | 1.89 | 4.27 | 1.54 | 5.90 | 1.80 | 5.93 | 1.70 |
|  | CYS | 12.2(40) | 7.69 | 1.63 | 6.76 | 1.64 | 4.88 | 1.74 | 5.60 | 1.64 | 6.12 | 1.55 |
|  | CYS | 10.1(33) | 7.28 | 1.76 | 6.48 | 1.84 | 4.76 | 1.85 | 5.94 | 1.81 | 6.05 | 1.71 |

TABLE D. 2
WEIbuLL c AND K VALUES AND MEAN WIND

$$
\begin{aligned}
& \stackrel{\rightharpoonup}{0} \\
& \stackrel{0}{4} \\
& \stackrel{\rightharpoonup}{u}
\end{aligned}
$$


0
0
$\vdots$
$\vdots$
$\vdots$
$\vdots$
in
$\vdots$

$$
\begin{array}{ll}
n & \vec{~} \\
\stackrel{y}{*} & \ddot{1} \\
\dot{N} & \dot{\sim}
\end{array}
$$

$$
\begin{aligned}
& \circ \\
& \dot{\circ}
\end{aligned}
$$

$$
8 \pi \cdot 2
$$

$$
\begin{gathered}
\infty \\
\stackrel{\infty}{0} \\
\vdots \\
\hdashline
\end{gathered}
$$

$$
\begin{aligned}
& \ddot{\sigma} \\
& \vdots \\
& \square
\end{aligned}
$$

$$
\begin{array}{ll}
0 & \ddot{0} \\
\infty & 0 \\
-1 & \grave{N}
\end{array}
$$

$$
\begin{array}{ll}
\ddot{0} \\
\stackrel{y}{0} \\
\dot{N}
\end{array}
$$

$$
\begin{aligned}
& \text { n } \\
& \vdots \\
& \vdots
\end{aligned}
$$

$$
\begin{array}{ll}
n & \hat{0} \\
0 & \vdots \\
- & 0
\end{array}
$$

$$
\begin{aligned}
& \dot{0} \\
& \dot{\sim}
\end{aligned}
$$

$$
\begin{aligned}
& \infty \\
& \dot{0} \\
& \dot{N}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{m} \\
& \dot{w}
\end{aligned}
$$

$$
\begin{gathered}
0 \\
\dot{\sim}
\end{gathered}
$$

$$
\begin{aligned}
& \text { on } \\
& \vdots \\
& \dot{N}
\end{aligned}
$$

$$
\begin{aligned}
& \underset{\sim}{m} \\
& \dot{\sim}
\end{aligned}
$$

$$
\begin{array}{ll}
0 & \infty \\
\stackrel{\infty}{0} & 0 \\
\dot{\sim} & \dot{\sim}
\end{array}
$$

$$
\begin{array}{lcc} 
\pm & M \\
\stackrel{N}{N} & \underset{\sim}{N} \\
\dot{N} & \dot{N}
\end{array}
$$

$$
\begin{array}{ll}
\hat{n} & 0 \\
0 & \text { n } \\
\dot{-} & \dot{\sim}
\end{array}
$$

$$
\begin{array}{ll}
m & \Xi \\
\infty & \vdots \\
- & -
\end{array}
$$

$$
\begin{array}{ll}
\stackrel{\infty}{0} & \stackrel{\infty}{\sigma} \\
\vdots & \vdots \\
\hdashline & \vdots
\end{array}
$$

$$
\begin{aligned}
& r \\
& a \\
& \vdots
\end{aligned}
$$

$$
\stackrel{\wedge}{\wedge}
$$

$$
\begin{array}{ll}
0 & 0 \\
0 & 1 \\
\therefore & 0
\end{array}
$$

$$
\begin{array}{ll}
v \\
\vdots \\
\vdots \\
\vdots
\end{array}
$$

$$
\begin{aligned}
& \ddot{3} \\
& \dot{\sim}
\end{aligned}
$$

$$
\begin{array}{ll}
0 & n \\
0 & \dot{\sim} \\
\dot{\sim} & \dot{n}
\end{array}
$$

$$
\begin{aligned}
& \stackrel{a}{\sim} \\
& \stackrel{y}{\sim} \\
& \stackrel{y}{\sim}
\end{aligned}
$$

$$
\begin{aligned}
& \stackrel{\propto}{\sim} \\
& \dot{j} \\
& \sim \\
& \hdashline \\
& \hdashline
\end{aligned}
$$

$$
\begin{array}{llll}
0 & \exists & m & 1 \\
\vdots & \vdots & 0 & 0 \\
0 & 0 & \infty & f
\end{array}
$$

$$
\begin{array}{ll}
n & へ \\
0 & \Xi \\
\vdots & \vdots
\end{array}
$$

$$
\begin{array}{ll}
\infty & \infty \\
& \circ \\
\dot{j} & \dot{j}
\end{array}
$$

$$
\begin{array}{ll}
\underset{~}{7} & \stackrel{n}{7} \\
\dot{=} & \infty
\end{array}
$$

$$
\begin{array}{cccc}
M & m & 0 & 0 \\
\cdots & \infty & N & 0 \\
\dot{n} & 0 & \dot{n} & \dot{N}
\end{array}
$$

$$
\begin{array}{ll}
N & 0 \\
\therefore & \dot{r}
\end{array}
$$

$$
\begin{array}{ll}
m & n \\
= & i
\end{array}
$$

TABLE D． 2 （cont．）

|  | $\stackrel{N}{\sim}$ | $\begin{aligned} & \stackrel{\bullet}{\circ} \\ & \stackrel{\circ}{*} \end{aligned}$ | $0$ | © | $\stackrel{\leftarrow}{\approx}$ | $\begin{aligned} & 5 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\circ}{\sim} \\ & \stackrel{1}{*} \end{aligned}$ | C | $\begin{aligned} & \hat{n} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \mathscr{\alpha} \\ & \stackrel{-}{\circ} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\alpha}{0} \\ & -1 \end{aligned}$ | 응 | $\begin{aligned} & \stackrel{0}{\mathrm{~N}} \\ & \stackrel{1}{2} \end{aligned}$ | $\begin{aligned} & c \\ & \infty \\ & \dot{-} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \stackrel{0}{\sim} \end{aligned}$ | $\begin{gathered} \stackrel{\alpha}{n} \\ \stackrel{1}{n} \end{gathered}$ | $\begin{aligned} & \stackrel{n}{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \ddagger \\ & \infty \\ & \hdashline-1 \end{aligned}$ | $\underset{\dot{\sim}}{\underset{\sim}{m}}$ | $\begin{aligned} & \text { à } \\ & \vdots \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \hdashline \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\boldsymbol{\sim}} \\ & \stackrel{-1}{2} \end{aligned}$ | $\underset{\dot{n}}{\underset{\sim}{n}}$ | $\begin{aligned} & \text { N } \\ & \hdashline-1 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \stackrel{y}{n} \end{aligned}$ | $\begin{aligned} & \hat{\imath} \\ & \dot{\sim} \end{aligned}$ | $\begin{array}{ll}\text { N } & \text { n } \\ 0 \\ \stackrel{n}{\sim} & \dot{\sim}\end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { 픙 } \\ & \hline \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{*} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & m \\ & \vdots \\ & \vdots \end{aligned}$ | 응 | － | $\begin{aligned} & \text { N } \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \dot{0} \end{aligned}$ | $\stackrel{0}{\square}$ | 응 | $\stackrel{\sim}{\stackrel{0}{\mathrm{o}}}$ | $\begin{aligned} & 7 \\ & \stackrel{7}{4} \end{aligned}$ | $\underset{\sim}{\sim}$ | ㅇ | $\begin{aligned} & \vec{~} \\ & \stackrel{0}{n} \end{aligned}$ | $\begin{aligned} & \text { in } \\ & \stackrel{3}{3} \end{aligned}$ | $\begin{gathered} \stackrel{\infty}{\underset{~}{n}} \end{gathered}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\infty} \\ & \infty \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \mathbf{j} \end{aligned}$ | $\underset{\sim}{N}$ | $\begin{aligned} & \mathrm{m} \\ & \dot{+} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \underset{~}{\prime} \end{aligned}$ | $\begin{aligned} & \text { 士 } \\ & \text { 4 } \end{aligned}$ | $\begin{aligned} & m \\ & \infty \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{\infty}{n} \end{aligned}$ | $\begin{array}{r} n \\ \vdots \\ \vdots \end{array}$ | － | a $\pm$ $=$ | $\begin{array}{ll}\text { N } \\ \sim \\ i & 0\end{array}$ |
| $\mathbf{N}$ | $\dot{m}$ | $\stackrel{\imath}{n}$ | $\subsetneq$ | $?$ | $\stackrel{\sim}{\dot{m}}$ | $\begin{aligned} & \therefore \\ & \dot{n} \end{aligned}$ | $\stackrel{\rightharpoonup}{n}$ | ？ |  | $8$ | $\stackrel{?}{\dot{n}}$ | ? | $\begin{aligned} & \because \\ & \vdots \end{aligned}$ | $\stackrel{5}{5}$ | $\stackrel{\rightharpoonup}{\square}$ | $\stackrel{\underset{\sim}{*}}{\stackrel{1}{2}}$ | $\begin{aligned} & \dot{\sigma} \\ & \dot{m} \end{aligned}$ | $\begin{aligned} & m \\ & \dot{m} \end{aligned}$ | $\stackrel{\rightharpoonup}{\dot{m}}$ | $\stackrel{\infty}{\dot{m}}$ | $\begin{aligned} & 0 \\ & ; \end{aligned}$ | $\underset{\text { i }}{\stackrel{1}{2}}$ | $\stackrel{c}{\stackrel{C}{n}}$ | $\stackrel{\sim}{n}$ | \＃ | $\pm$ | $\because \underset{\sim}{\square}$ |
| $\underset{\sim}{\boldsymbol{\omega}}$ | $\begin{aligned} & \overrightarrow{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \text { A } \\ & \underset{~}{2} \end{aligned}$ | $\begin{aligned} & \dot{N} \\ & \dot{N} \end{aligned}$ | $\stackrel{n}{\dot{\sim}}$ | $\begin{aligned} & \stackrel{n}{\sigma} \\ & \therefore \end{aligned}$ | $\begin{aligned} & \text { os } \\ & \text { n } \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & i \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} \mathbf{m} \\ \stackrel{1}{\sim} \end{gathered}$ | $\stackrel{\circ}{\dot{\sim}}$ | $\begin{gathered} \infty \\ \stackrel{D}{\sim} \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & \infty \\ & \stackrel{0}{\dot{\sim}} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{N}{\underset{\sim}{n}} \end{aligned}$ | $\begin{aligned} & \text { or } \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{N} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} m \\ \dot{\sim} \end{gathered}$ | $\stackrel{0}{\stackrel{0}{\sim}}$ | $\begin{aligned} & \vec{S} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} \pm \\ \dot{\sim} \end{gathered}$ | $\begin{aligned} & n \\ & 0 \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \overrightarrow{0} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \dot{\infty} \\ & \dot{\sim} \end{aligned}$ | $\begin{gathered} M \\ \stackrel{m}{\sim} \end{gathered}$ | － | $\begin{aligned} & \stackrel{N}{N} \\ & \stackrel{\sim}{n} \end{aligned}$ |  |
|  | $\begin{aligned} & \vec{r} \\ & \dot{i} \end{aligned}$ | $\begin{aligned} & \stackrel{C}{9} \\ & \hdashline \end{aligned}$ | $\begin{aligned} & n \\ & \dot{n} \\ & i n \end{aligned}$ | $\begin{aligned} & \text { in } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \text { き } \\ & \text { ¿ } \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & 0 \end{aligned}$ | $\stackrel{\infty}{\infty}$ | $\begin{aligned} & n \\ & \stackrel{n}{\circ} \\ & \stackrel{n}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{\sigma} \\ & \stackrel{0}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\AA}{\otimes} \\ & \stackrel{\circ}{\circ} \end{aligned}$ | $\begin{aligned} & \mathbf{a} \\ & \text { in } \end{aligned}$ | $\begin{aligned} & m \\ & \stackrel{m}{0} \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{N} \\ & \stackrel{1}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \stackrel{\rightharpoonup}{n} \end{aligned}$ | $$ | $\begin{aligned} & \stackrel{n}{2} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \text { n } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \text { t } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \vec{\infty} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \circ \\ & \text { in } \end{aligned}$ | $\stackrel{\sigma}{\dot{n}}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \dot{\sigma} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { in } \end{aligned}$ | $\stackrel{m}{\square}$ | $\stackrel{m}{7}$ | $m$ $\stackrel{n}{\circ}$ $\dot{\infty}$ |
|  | $\dot{\sigma}$ | $\begin{aligned} & N \\ & \dot{J} \end{aligned}$ | $\begin{aligned} & 0 \\ & j \end{aligned}$ | $\underset{y}{\infty}$ | $\begin{aligned} & \cong \\ & \hdashline \end{aligned}$ | $\because$ | $\stackrel{M}{N}$ |  | $\ddot{\sim}$ | $\stackrel{\circ}{\circ}$ | N | $\begin{aligned} & \infty \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \dot{0} \end{aligned}$ | $0$ | $\stackrel{7}{6}$ | $0$ | $\stackrel{m}{n}$ | 굴 | $\stackrel{0}{0}$ | $\overrightarrow{~ B}$ | N | $\stackrel{0}{\dot{于}}$ | $\stackrel{n}{\infty}$ | $\stackrel{\infty}{*}$ | $\stackrel{0}{0}$ | $\pm$ | $\because \square$ |
|  | $\stackrel{7}{i}$ | $\stackrel{U}{\sim}$ | $\stackrel{\sim}{\sim}$ | $\stackrel{\#}{7}$ | $\stackrel{\rightharpoonup}{\square}$ | $\begin{aligned} & \infty \\ & n \\ & \vdots \end{aligned}$ | O i | $\stackrel{\infty}{\bullet}$ | $\begin{aligned} & \infty \\ & \stackrel{0}{0} \\ & i \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\dot{v}} \end{aligned}$ | $\stackrel{\underset{\sim}{\underset{u}{u}}}{\stackrel{1}{n}}$ | $\begin{aligned} & n \\ & \underset{v}{u} \end{aligned}$ | $\begin{aligned} & 0 \\ & \dot{\infty} \\ & \dot{~} \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \dot{\mathrm{v}} \end{aligned}$ | $\stackrel{\text { N }}{\text { i }}$ | $\begin{aligned} & \text { n } \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & m \\ & \dot{~} \end{aligned}$ | $\stackrel{N}{N}$ | $\stackrel{\sim}{0}$ | $\stackrel{\rightharpoonup}{+}$ $\dot{\text { v }}$ | $\stackrel{m}{\substack{c \\ \vdots \\-1}}$ | $\stackrel{\infty}{0}$ | $\infty$ $\stackrel{0}{0}$ $\dot{1}$ | $\stackrel{\sim}{N}$ | $\stackrel{ \pm}{\text { L }}$ | c $\stackrel{y}{*}$ $j$ | N $\stackrel{i}{*}$ $i$ |
|  | $\begin{aligned} & \stackrel{\rightharpoonup}{c} \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \infty \\ & \pm \\ & \vdots \\ & \vdots \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{N} \\ & \end{aligned}$ | $\begin{aligned} & n \\ & \underset{c}{n} \\ & \dot{5} \end{aligned}$ | $\stackrel{N}{\sim}$ | $\begin{aligned} & \underset{\sim}{N} \\ & \dot{N} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{7} \\ & \dot{0} \end{aligned}$ | $\stackrel{r}{\mathbf{r}}$ | $\begin{aligned} & o \\ & \infty \\ & \dot{\sim} \end{aligned}$ | $\begin{aligned} & \stackrel{c}{\infty} \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \vec{~} \\ & \dot{0} \\ & \dot{0} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{N} \\ & \dot{C} \end{aligned}$ | $\begin{aligned} & \stackrel{\sim}{\sim} \\ & \stackrel{\sim}{n} \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \dot{\infty} \end{aligned}$ | $\begin{aligned} & \dot{\sim} \\ & \underset{\sim}{0} \end{aligned}$ | $\begin{aligned} & 9 \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \vdots \\ \hline 0 \end{gathered}$ | $\begin{aligned} & \vec{\sigma} \\ & \dot{\sigma} \end{aligned}$ | $\begin{aligned} & \text { ज } \\ & \stackrel{n}{n} \end{aligned}$ | $\stackrel{0}{\sim}$ | N | $\infty$ 0 0 | $\stackrel{\sim}{\sim}$ | － | $\begin{array}{cc}0 & \sim \\ \sim \\ \sim & 0 \\ \sim\end{array}$ |
|  | ? | $\because$ | $\begin{aligned} & \pi \\ & \ddagger \end{aligned}$ | $\begin{aligned} & 0 \\ & \dot{f} \end{aligned}$ | $?$ | $\begin{aligned} & 7 \\ & 0 \end{aligned}$ | $\because$ | $0$ | $\dot{0}$ | $\stackrel{?}{i}$ | $\begin{array}{r} \sim \\ i n \end{array}$ | $\stackrel{\infty}{\infty}$ | $\begin{aligned} & 0 \\ & \dot{0} \end{aligned}$ | $\pm$ | $\infty$ | $\stackrel{\infty}{\sim}$ | $\underset{\sim}{\wedge}$ | $\stackrel{\square}{\square}$ | $\begin{aligned} & \sim \\ & \dot{n} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{n} \\ & \hline \end{aligned}$ | $\stackrel{\square}{9}$ | ？ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & n \\ & \end{aligned}$ | $\stackrel{0}{0}$ | $\begin{aligned} & N \\ & \vdots \end{aligned}$ | $\stackrel{\sim}{\square}$ |



|  |  |  |  |  | TABL | D. 2 | nt.) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.0 | 7.90 | 2.20 | 8.0 | 9.98 | 2.52 | 6.7 | 7.55 | 2.63 | 6.7 | 7.59 | 2.40 | 7.1 | 8.00 | 2.38 |
| 「. 4 | 6.54 | 2.32 | 6.2 | 7.03 | 2.56 | 5.3 | 5.94 | 2.56 | 4.8 | 5.42 | 1.96 | 5. | 6.25 | 2.30 |
| 6.2 | 6.97 | $2 \cdot 26$ | 7.2 | 8.14 | 2.56 | 6.2 | 6.96 | 2.76 | 5.4 | 6.09 | 2.23 | 6.3 | 7.10 | 2.27 |
| 0.7 | 7.53 | C. 52 | 7.5 | 3.41 | 3.02 | $5 \cdot 8$ | 6.50 | 2.64 | 6.0 | 6.82 | 2.43 | 6.5 | 7.33 | 2.59 |
| 4.8 | 5.25 | 1.45 | 5.9 | $6 \cdot 56$ | 1.73 | 4.7 | 5.24 | 1.84 | 4.4 | 4.88 | 1.67 | 4.9 | 5.45 | 1.63 |
| 5.7 | 6.45 | c. 31 | 6.2 | 6.97 | 2.39 | 5.2 | 5.89 | 2.49 | 5.2 | 5.85 | 2.18 | 5.6 | 6.28 | 2.29 |
| 5.2 | 5.83 | 2.00 | 5.1 | 5.76 | 2.38 | 4.5 | 5.04 | 2.40 | 4.5 | 5.02 | 1.78 | 4.8 | 5.40 | 2.09 |
| 5.6 | 6.34 | <.27 | 6.0 | 0.76 | 2.25 | 4.2 | 4.79 | 1.99 | 4.8 | 5.38 | 2.15 | 5.1 | 5.78 | 2.12 |
| 0.4 | 7.27 | c.12 | 7.3 | 8.18 | 2.44 | 5.9 | 6.62 | 2.45 | 5.8 | 6.59 | 2.26 | 6.3 | 7.15 | 2.25 |
| 6.7 | 7.54 | <.60 | 7.3 | 8.20 | 3.02 | $5 \cdot 5$ | 6.14 | 2.61 | 5.5 | 6.21 | 2.25 | 6.3 | 7.05 | 2.53 |
| 5.8 | 6:52 | 2.30 | 6.2 | 0.97 | 2.48 | $5 \cdot 5$ | 6.22 | 2.66 | 5.1 | 5.71 | 2.12 | 5.6 | 6.36 | 2.35 |
| 5.7 | 6.47 | 2.39 | 6.6 | 7.41 | 2.57 | $5 \cdot 5$ | 6.18 | 2.59 | 5.4 | 6.09 | 2.26 | 5.8 | 6.55 | 2.42 |
| 3.8 | +. 14 | 1.26 | 4.7 | 5.27 | 1.56 | $4 \cdot 3$ | 4.82 | 1.70 | 4.1 | 4.53 | 1.46 | 4.2 | 4.68 | 1.47 |
| 5.7 | 6.43 | c. 36 | 5.6 | 6.34 | 2.28 | -0 | . 00 | .00 | 5.3 | 6.04 | 2.20 | 5.4 | 6.05 | 2.25 |
| 4.7 | 5.31 | $<.04$ | 5.2 | 5.92 | 2.29 | -u | . 00 | -00 | - 0 | . 00 | . 00 | 4.3 | 4.89 | 2.00 |
| 3.7 | $4 \cdot 12$ | 1.94 | 3.9 | 4.45 | 2.07 | $2 \cdot 6$ | 2.92 | 1.53 | 3.0 | 3.38 | 1.64 | $3 . ?$ | 3.63 | 1.66 |
| 5.4 | 6.04 | 1.76 | 5.2 | 5.88 | 1.97 | - 0 | . 00 | .00 | 4.0 | 4.42 | 1.64 | 4.4 | 4.95 | 1.65 |
| 6.1 | 6.85 | 1.89 | 6.3 | 7.06 | 2.13 | 4.9 | 5.53 | 1.96 | 5.6 | 6.32 | 2.12 | 5.7 | 6.45 | 1.99 |
| 3.7 | 6.43 | c.08 | 4.7 | $5 \cdot 34$ | 1.92 | - 0 | . 00 | .00 | 4.6 | 5.20 | 1.90 | 4.7 | 5.26 | 1.87 |
| 5.6 | 6.31 | 1.68 | 5.7 | 6.40 | 2.05 | 5.0 | 5.68 | 2.11 | 5.1 | 5.79 | 1.81 | 5.3 | 5.97 | 1.84 |
| 3.5 | 3.65 | 1.17 | 4.3 | 4.76 | 1.42 | 4.2 | 4.66 | 1.40 | 3.5 | 3.78 | 1.24 | 3.9 | 4.22 | 1.29 |
| 4.5 | 5.09 | 2.47 | 3.9 | 4.37 | 1.85 | 2.上 | 2.17 | 1.47 | 3.4 | 3.79 | 1.90 | 3.5 | 3.87 | 1.73 |
| 4.3 | 4.81 | 1.79 | 4.0 | $4 \cdot 55$ | 1.79 | - 0 | . 00 | . 00 | - 0 | . 00 | . 00 | 3.5 | 3.92 | 1.65 |
| 6.2 | 7.11 | 2.40 | 6.1 | 6.91 | 2.27 | 4.7 | 5.34 | 1.99 | 6.0 | 6.72 | 2.08 | 5.A | 6.51 | 2.14 |
| 5.3 | 6.03 | <.16 | 5.7 | 6.39 | 2.26 | 4.7 | 5.33 | 2.19 | 5.2 | 5.88 | 2.21 | 5.2 | 5.88 | 2.17 |
| 6.4 | 7.17 | 2.41 | 6.4 | 7.25 | 2.40 | 5.1 | 5.71 | 2.37 | 5.8 | 6.59 | 2.41 | 5.9 | 6.63 | 2.31 |
| 7.5 | 8.53 | c. 71 | 7.1 | 7.99 | 2.41 | 5.8 | 6.57 | 2.39 | 6.7 | 7.53 | 2.60 | 7.4 | 8.37 | 2.46 |
| -. 3 | 7.06 | c.63 | 6.7 | 7-5! | 2.76 | $5 \cdot 4$ | 6.10 | 2.47 | 6.2 | 6.96 | 2.63 | 6.3 | 7.05 | 2.57 |
| 9.7 | 10.92 | 4.23 | 7.A | 3.77 | 2.21 | $5 \cdot 5$ | 6.18 | 1.80 | 7.6 | 8.54 | 2.11 | 7.6 | 8.59 | 1.99 |

$$
\stackrel{\square}{\dot{\sim}} \underset{\sim}{\underset{\sim}{n}} \underset{\sim}{\sim} \underset{\sim}{\dot{\sim}}
$$

$$
\stackrel{\propto}{\dot{\sim}} \stackrel{\infty}{\underset{\sim}{n}}
$$

$$
\begin{aligned}
& \text { ü } \\
& \stackrel{0}{0} \\
& \text { un }
\end{aligned}
$$



$$
\begin{aligned}
& \text { TABLE } 0.3 \text { (cont.) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { TABLE } 0.3 \text { (cont.) }
\end{aligned}
$$



$$
\begin{aligned}
& w \\
& c \\
& c \\
& w \\
& k \\
& v
\end{aligned}
$$

$$
\begin{aligned}
& \text { TABLE D. } 4 \text { (cont.) }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 「 }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 花 } \\
& \text { 㞔 } \\
& \text { 药 } \\
& \text { 㞔 } \\
& \text { 言 } \\
& \text { 药 } \\
& \text { 尾 } \\
& \text { 我 } \\
& \text { 范 } \\
& \text { 宽 }
\end{aligned}
$$

$$
\begin{aligned}
& \text { き, }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 色 }
\end{aligned}
$$

$$
\begin{aligned}
& \text { ¢ }
\end{aligned}
$$

$$
\begin{aligned}
& \text { 笑 }
\end{aligned}
$$

$$
\begin{aligned}
& \text { ¢ } \\
& \text { 莫 }
\end{aligned}
$$

TABLE

$$
\begin{gathered}
\text { en } \\
\\
\hline
\end{gathered}
$$

¢


¢


¢
岂
¢

足
بِ


$$
\begin{aligned}
& \text { ¢ }
\end{aligned}
$$

$$
\begin{aligned}
& \text { ¢ }
\end{aligned}
$$

## APPENDIX E

REGIONAL POWER GENERATION DATA

This appendix summarizes the data required for the model described in Section 13 and used in the analyses for Sections 14 and 15 . The model and analyses required regional data describing

- the amount of electric power generated by each type of conventional power generating system,
- the efficiency of each type of generating system, and
- the costs of fuels used in generating the electricity.

Figure 14.1 illustrates the nine regions for which FPC data were available; consequently, these boundaries were assumed for the analyses of regional wind power potential. Table E. 1 provides data for the total U.S. production of electric power by process, the quantities of fuels consumed, the process efficiencies, and the percentage of electrical energy produced by each type of fuel-system combination. Tables E. 2 through E. 10 display similar data for each of the nine FPC regions.

The values in column 2 (fuel quantity) in each of Tables E. 1 through E. 10 were determined from FPC 1973 data [1]. The numbers in column 3 were calculated using the quantities in column 2 and the mean heat values for the fuels in the different regions. The mean heating values of the different fuels in the individual regions, shown in Table E. 11, were determined from FPC 1974 data $[2,3]$.

The "energy produced" (column 4) values were determined from FPC data and information [1,4]. Working papers [4] were necessary to establish the amount of nuclear-fueled electrical energy produced in 1973. Column 5 has two parts:
the numbers on the right are the percentages of electrical energy produced by the different processes in column 1 and the numbers on the left are the percentages produced by the different fuel-process combinations. The process percentages reflect the relative percentages of each value of "energy produced" in column 4. The percentages for the fuel-process combinations were calculated by assuming that the process efficiencies (column 6), determined by dividing the kilowatt-hours produced by the Btu's consumed and the appropriate energy conversion factor, remain the same regardless of the fuel used in the process. By making this assumption, the percentage of each process can be divided proportionally among the different fuels according to the quantities of energy consumed by fuel type (column 3). This assumption is judged to be reasonably valid and should yield results which are accurate to within a few percent, particularly for those steam plants in which different fuels can be utilized. However, some of the newer, more efficient steam plants burn coal, and larger errors (e.g., five percent) may occur in such cases. The error caused by this assumption is judged to be greatest in the South Atlantic region. The average steam plant efficiency in this region is $34.7 \%$; however, the Marshall plant in this region has an average efficiency of greater than 39\% [5].

The last colum in the tables, efficiency, was calculated for the different processes by dividing the values in column 4 by the values in column 3 and comparing this quotient with the energy equivalent factor ( $1 \mathrm{MBTU}=3412 \mathrm{kWh}$ ).

Few data were on nuclear process factors were collected. Nuclear processes contribute only a small percentage to the total energy generated in any particular region; the fuel cost in nuclear-generated electric power is relatively low; and it may be assumed that nuclear plants would be the last
ones to be cut back as the wind system generates power. Consequently, precision and accuracy in the nuclear data were not required. On a national. basis, it is estimated that efficiencies of nuclear power plants average about 33\% [6].

Figures E. 1 through E. 10 illustrate the fossil fuel costs for the contiguous United States and for each of the nine FPC regions for the last half of 1973 and the first three quarters of 1974. These values were determined from FPC data on fuel costs for electric power generation (reference 1 and 2). Nuclear fuel costs were assumed to be approximately $\$ .22 / \mathrm{MB}$ tu for each of the regions. This value is approximately the recent purchase price; under older contracts, nuclear fuel may cost as little as $\$ .14 / \mathrm{MBtu}$ [6].
TABLE E. 1
TOTAL ELECTRICAL ENERGY PRODUCED AND FUEL CONSUMED FOR THE INDICATED

| PROCESS | FUEL QUANTITY | $\begin{aligned} & \text { ENERGY } \\ & \text { CONSUMED } \end{aligned}$ (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (PERCENT) } \end{aligned}$ | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $3.883 \times 10^{8}$ TONS OF OOAL $5.093 \times 10_{9}^{8}$ BBLS OF OIL $3.358 \times 10$ MCF OF GAS | $\begin{aligned} & 8.510 \times 10^{15} \\ & 3.124 \times 10^{15} \\ & 3.449 \times 10^{15} \end{aligned}$ | $1.459 \times 10^{12}$ | $\begin{array}{ll} 44.5 * & \\ 16.3^{*} & 78.9 \\ \text { 18.0* } & \end{array}$ | 33.0 |
| NUCLEAR |  |  | $8.329 \times 10^{10}$ | 4.54 .5 |  |
| TURBINE | $4.314 \times 10^{7}$ BBLS OF OIL $1.997 \times 10^{8}$ MCF OF GAS | $\begin{aligned} & 2.646 \times 10^{14} \\ & 2.051 \times 10^{14} \end{aligned}$ | $2.949 \times 10^{10}$ | $\begin{array}{ll} 0.9 * & 1.6 \\ 0.7 * & \end{array}$ | 21.4 |
| INTERNAL COMBUSTION | $4.439 \times 10^{6}$ BBLS OF OIL $4.681 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 2.772 \times 10^{13} \\ & 4.807 \times 10^{13} \end{aligned}$ | $6.202 \times 10^{9}$ | $\begin{array}{ll} 0.1 * & 0.3 \\ 0.2 * & \end{array}$ | 28.1 |
| HYDROELECTRIC |  |  | $2.710 \times 10^{11}$ | 14.714 .7 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energyfrom various fuels. |  |  |  |  |  |

TABLE E. 2

| PROCESS | FUEL QUANTITY | ENERGY CONSUMED <br> (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ |  |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $1.121 \times 10^{6}$ TONS OF COAL $8.377 \times 10^{7}$ BBLS OF OIL $5.368 \times 10^{6} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 2.794 \times 10^{13} \\ & 5.132 \times 10^{14} \\ & 5.379 \times 10^{12} \end{aligned}$ | $5.149 \times 10^{10}$ | $\begin{array}{r} 3.7 * \\ 67.0 \% \\ 0.7 \% \end{array}$ | 71.3 | 32.1 |
| NUCLEAR |  |  | $1.437 \times 10^{10}$ | 19.9 | 19.9 |  |
| TURBINE | $1.703 \times 10^{6}$ BBLS OF OIL $2.647 \times 10^{5} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 1.043 \times 10^{13} \\ & 2.652 \times 10^{11} \end{aligned}$ | $6.629 \times 10^{\text {6 }}$ | $\begin{aligned} & 0.9 * \\ & 0.0 \end{aligned}$ | 0.9 | 21.2 |
| INTERNAL COMBUSTION | $5.163 \times 10^{5}$ BBLS OF OIL $4.956 \times 10^{5} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 3.163 \times 10^{12} \\ & 4.966 \times 10^{11} \end{aligned}$ | $3.091 \times 10^{8}$ | $\begin{aligned} & 0.4 \\ & 0.1 \end{aligned}$ | 0.4 | 28.8 |
| HYDROELECTRIC |  |  | $5.353 \times 10^{9}$ | 7.4 | 7.4 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 3

| PROCESS | FUEL QUANTITY | ENERGY CONSUMED (BTU) | ENERGY PRODUCED (KWH) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCEI } \\ & \text { (PERCEN? } \end{aligned}$ |  | EFFICIENCY (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $4.697 \times 10^{7}$ TONS OF COAL $1.405 \times 10^{8}$ BBLS OF OIL $6.573 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 1.124 \times 10^{15} \\ & 8.569 \times 10^{14} \\ & 7.184 \times 10^{13} \end{aligned}$ | $1.935 \times 10^{11}$ | $\begin{gathered} 43.2^{*} \\ 32.9 * \\ 2.8^{*} \end{gathered}$ | 78.8 | 32.2 |
| NUCLEAR |  |  | $1.117 \times 10^{10}$ | 4.6 | 4.6 |  |
| TURBINE | $2.176 \times 10^{7}$ BBLS OF OHL <br> $3.064 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 1.327 \times 10^{14} \\ & 3.349 \times 10^{13} \end{aligned}$ | $1.028 \times 10^{10}$ | $\begin{aligned} & 3.4^{*} \\ & 0.8^{*} \end{aligned}$ | 4.2 | 21.1 |
| INTERNAL COMBUSTION | $5.034 \times 10^{5}$ BBLS OF OIL <br> $5.007 \times 10^{5} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 3.070 \times 10^{12} \\ & 5.473 \times 10^{11} \end{aligned}$ | $3.255 \times 10^{8}$ | $\begin{aligned} & 0.1 * \\ & 0.0 * \end{aligned}$ | 0.1 | 30.7 |
| HYDROĖLECTRIC |  |  | $3.019 \times 10^{10}$ | 12.3 | 12.3 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 4

| PROCESS | FUEL QUANTITY | $\begin{aligned} & \text { ENERGY } \\ & \text { CONSUMED } \\ & \text { (BTU) } \end{aligned}$ | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \end{aligned}$ (KWH) |  |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $1.334 \times 10^{8}$ TONS OF COAL $2.157 \times 10^{7}$ BBLS OF OLL $8.526 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 2.944 \times 10^{15} \\ & 1.315 \times 10^{14} \\ & 7.486 \times 10^{13} \end{aligned}$ | $3.051 \times 10^{11}$ | $\begin{gathered} 82.7 \% \\ 3.7 * \\ 2.1 * \end{gathered}$ | 88.5 | 33.0 |
| NUCLEAR |  |  | $2.884 \times 10^{10}$ | 8.4 | 8.4 |  |
| TURBINE | $6.624 \times 10^{6}$ BBLS OF OLL $4.830 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 4.039 \times 10^{13} \\ & 4.241 \times 10^{13} \end{aligned}$ | $5.497 \times 10^{9}$ | $\begin{aligned} & 0.8^{*} \\ & 0.8^{*} \end{aligned}$ | 1.6 | 22.7 |
| INTERNAL COMBUSTION | $1.043 \times 10^{6}$ BBLS OF OIL $8.641 \times 10^{6} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 6.360 \times 10^{12} \\ & 7.587 \times 10^{12} \end{aligned}$ | $1.316 \times 10^{9}$ | $\begin{aligned} & 0.2 * \\ & 0.2 * \end{aligned}$ | 0.4 | 32.2 |
| HYDROELECTRIC |  |  | $3.854 \times 10^{9}$ | 1.1 | 1.1 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 5

| PROCESS | FUEL QUANTITY | ENERGY. CONSUMED (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ |  |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $3.549 \times 10^{7}$ TONS OF COAL <br> $2.742 \times 10^{6}$ BBLS OF OIL <br> $3.694 \times 10^{8} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 7.016 \times 10^{14} \\ & 1.705 \times 10^{13} \\ & 3.664 \times 10^{14} \end{aligned}$ | $9.620 \times 10^{10}$ | $\begin{aligned} & 53.5 * \\ & 1.3 * \\ & 28.0 * \end{aligned}$ | 82.8 | 30.3 |
| NUCLEAR |  |  | $3.869 \times 10^{9}$ | 3.3 | 3.3 |  |
| TURBINE | $1,227 \times 10^{6}$ BBLS OF OIL $1.568 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 7.629 \times 10^{12} \\ & 1.555 \times 10^{13} \end{aligned}$ | $1.536 \times 10^{9}$ | $\begin{aligned} & 0.4 * \\ & 0.9 * \end{aligned}$ | 1.3 | 22.6 |
| INTERNAL COMBUSTION | $8.222 \times 10^{5}$ BBLS OF OIL $2.237 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 5.112 \times 10^{12} \\ & 2.219 \times 10^{13} \end{aligned}$ | $2.291 \times 10^{9}$ | $\begin{aligned} & 0.4 * \\ & 1.6 * \end{aligned}$ | 2.0 | 28.6 |
| HYDROELECTRIC |  |  | $1.234 \times 10^{10}$ | 10.6 | IU. 6 |  |
| from various fuels. <br> * An estimation assuming process efficiency remains constant for equivalent amounts of energy |  |  |  |  |  |  |

TABLE E. 6

| PROCESS | $\begin{gathered} \text { FUEL } \\ \text { QUANTITY } \end{gathered}$ | ENERGY CONSUMED (BTU) | ENERGY PRODUCED (KWH) | ENERGY PRODUCED <br> (PERCENT) | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $\begin{aligned} & 7.624 \times 10^{7} \text { TONS OF COA: } \\ & 1.423 \times 10^{8} \mathrm{BBLS} \text { OF OIL } \\ & 2.077 \times 10^{8} \mathrm{MCF} \text { OF GAS } \end{aligned}$ | $\begin{aligned} & 1.818 \times 10^{15} \\ & 8.815 \times 10^{14} \\ & 2.127 \times 10^{14} \end{aligned}$ | $2.961 \times 10^{11}$ | $\begin{aligned} & 54.9 * \\ & 26.6 * 87.9 \\ & 6.4 * \end{aligned}$ | 34.7 |
| NUCLEAR |  |  | $1.770 \times 10^{10}$ | 5.35 .3 |  |
| TURBINE | $8.997 \times 10^{6} \mathrm{BBLS}$ OF OIL $4.094 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 5.574 \times 10^{13} \\ & 4.192 \times 10^{13} \end{aligned}$ | $6.157 \times 10^{9}$ | $\begin{array}{ll} 1.0 \% & 1.8 \\ 0.8 * & \end{array}$ | 21.5 |
| INTERNAL COMBUSTION | $\begin{aligned} & 5.797 \times 10^{5} \mathrm{BBLS} \text { OF OIL } \\ & 2.688 \times 10^{6} \mathrm{MCF} \text { OF GAS } \end{aligned}$ | $\begin{aligned} & 3.591 \times 10^{12} \\ & 2.753 \times 10^{12} \end{aligned}$ | $5.359 \times 10^{8}$ | $\begin{array}{ll} 0.1 * & 0.2 \\ 0.1 * & \end{array}$ | 28.8 |
| HYDROEL ECTRIC |  |  | $1.931 \times 10^{10}$ | 5.75 |  |
| * An estimation from various | assuming process efficie fuels. | y remains con | nt for equival | amounts of | nergy |

TABLE E. 7

| PROCESS | FUEL QUANTITY | $\begin{aligned} & \text { ENERGY } \\ & \text { CONSUMED } \end{aligned}$ (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ | ENERGY (PERCEN |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $6.323 \times 10^{7}$ TONS OF COAL <br> $5.913 \times 10^{6}$ BBLS OF OIL <br> $7.428 \times 10^{7}$ MCF OF GAS | $\begin{aligned} & 1.408 \times 10^{15} \\ & 3.584 \times 10^{13} \\ & 8.854 \times 10^{13} \end{aligned}$ | $1.512 \times 10^{11}$ | $\begin{array}{r} 77.38 \\ 1.97 \end{array}$ | 84.2 | 33.66 |
| NUCLEAR |  |  | $2.727 \times 10^{8}$ | 0.2 | 0.2 |  |
| TURBINE | $1.164 \times 10^{6}$ BBLS OF OII <br> $7.987 \times 10^{6} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 7.056 \times 10^{12} \\ & 9.521 \times 10^{12} \end{aligned}$ | $1.000 \times 10^{9}$ | $\begin{aligned} & 0.3^{*} \\ & 0.3^{*} \end{aligned}$ | 0.6 | 20.6 |
| INTERNAL COMBUSTION | $\begin{aligned} & 0.0 \\ & 2.57 \times 10^{2} \end{aligned} \begin{aligned} & \text { BBLS OF OIL } \\ & \text { MCF OF GAS } \end{aligned}$ | $\begin{aligned} & 0.0 \\ & 3.063 \times 10^{8} \end{aligned}$ | 0.0 | $\begin{aligned} & 00.0 \\ & 00.0 \end{aligned}$ | 00.0 |  |
| HYDROELECTRIC |  |  | $2.705 \times 10^{10}$ | 15.1 | 15.1 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 8
TOTAL ELECTRICAL ENERGY PRODUCED AND FUEL CONSUMED FOR THE INDICATED
PROCESSES IN THE WEST SOUTH CENTRAL REGION DURING THE CALENDAR YEAR 1973

| PROCESS | FUEL QUANTITY | $\begin{aligned} & \text { ENERGY } \\ & \text { CONSUMED } \end{aligned}$ (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ | ENERGY PRODUCED (PERCEN' |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $4.733 \times 10^{6}$ TONS OF COAL <br> $2.094 \times 10^{7}$ BBLS OF OHL <br> $1.895 \times 10^{9}$ MCF OF GAS | $\begin{aligned} & 6.626 \times 10^{13} \\ & 1.278 \times 10^{14} \\ & 1.950 \times 10^{15} \end{aligned}$ | $2.063 \times 10^{11}$ | $\begin{gathered} 2.9 \% \\ 5.7 \% \\ 86.2 * \end{gathered}$ | 94.7 | 32.8 |
| NUCLEAR |  |  | 0.0 | 0.0 | 0.0 |  |
| TURBINE | $3.792 \times 10^{4}$ BBLS OF OIL <br> $1.213 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 2.313 \times 10^{11} \\ & 1.248 \times 10^{13} \end{aligned}$ | $8.575 \times 10^{8}$ | $\begin{aligned} & 0.0 \\ & 0.4^{\star} \end{aligned}$ | 0.4 | 23.0 |
| INTERNAL COMBUSTION | $2.142 \times 10^{5}$ BBLS OF OIL <br> $9.006 \times 10^{6} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 1.307 \times 10^{12} \\ & 9.267 \times 10^{12} \end{aligned}$ | $8.487 \times 10^{8}$ | $\begin{aligned} & 0.1 \\ & 0.3^{*} \end{aligned}$ | 0.4 | 27.4 |
| HYDROELECTRIC |  |  | $9.799 \times 10^{9}$ | 4.5 | 4.5 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 9

| PROCESS | FUEL QUANTITY | ENERGY CONSUMED (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \\ & \text { (KWH) } \end{aligned}$ | ENERG <br> PRODUC <br> (PERCE |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $2.363 \times 10^{7}$ TONS OF COAL $9.169 \times 10^{6}$ BBLS OF OHL $2.081 \times 10^{8} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 4.535 \times 10^{14} \\ & 5.583 \times 10^{13} \\ & 2.089 \times 10^{14} \end{aligned}$ | $6.623 \times 10^{10}$ | $\begin{array}{r} 43.2^{*} \\ 5.3^{*} \\ 19.9 * \end{array}$ | 68.9 | 31.2 |
| NUCLEAR |  |  | 0.0 | 0.0 | 0.0 |  |
| TURBINE | $7.300 \times 10^{5}$ BBLS OF OIL $1.699 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 4.445 \times 10^{12} \\ & 1.706 \times 10^{13} \end{aligned}$ | $1.582 \times 10^{9}$ | $\begin{aligned} & 0.3 * \\ & 1.3 * \end{aligned}$ | 1.7 | 25.1 |
| INTERNAL COMBUSTION | $1.013 \times 10^{5}$ BBLS OF OIL $3.136 \times 10^{6} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 1.017 \times 10^{8} \\ & 6.163 \times 10^{12} \end{aligned}$ | $3.040 \times 10^{8}$ | $\begin{aligned} & 0.0 \\ & 0.3 * \end{aligned}$ | 0.3 | 16.8 |
| HYDROELECTRIC |  |  | $2.794 \times 10^{10}$ | 29.1 | 29.1 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |


| PROCESS | FUEL QUANTITY | ENERGY CONSUMED <br> (BTU) | $\begin{aligned} & \text { ENERGY } \\ & \text { PRODUCED } \end{aligned}$ (KWH) | ENERG PRODUC (PERCE |  | EFFICIENCY <br> (PERCENT) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| STEAM | $3.177 \times 10^{6}$ TONS OF COAL $7.370 \times 10^{7}$ BBLS OF OIL $4.458 \times 10^{8} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 5.985 \times 10^{13} \\ & 4.519 \times 10^{14} \\ & 4.734 \times 10^{14} \end{aligned}$ | $9.696 \times 10^{10}$ | $\begin{gathered} 2.5 \% \\ 18.5 \% \\ 19.4 \% \end{gathered}$ | 40.4 | 33.6 |
| NUCLEAR |  |  | $7.063 \times 10^{9}$ | 2.9 | 2.9 |  |
| TURBINE | $7.368 \times 10^{5}$ BBLS OF OIL <br> $1.268 \times 10^{7} \mathrm{MCF}$ OF GAS | $\begin{aligned} & 4.517 \times 10^{12} \\ & 1.347 \times 10^{13} \end{aligned}$ | $9.666 \times 10^{8}$ | $\begin{aligned} & 0.1^{*} \\ & 0.3 \end{aligned}$ | 0.4 | 18.3 |
| INTERNAL COMBUSTION | $3.319 \times 10^{4}$ BBLS OF OIL <br> 0.0 MCF OF GAS | $\begin{aligned} & 2.035 \times 10^{11} \\ & 0.0 \end{aligned}$ | $1.468 \times 10^{7}$ | $\begin{aligned} & 0.0 \\ & 0.0 \end{aligned}$ | 0.0 | 24.6 |
| HYDROELECTRIC |  |  | $1.349 \times 10^{11}$ | 56.2 | 56.2 |  |
| * An estimation assuming process efficiency remains constant for equivalent amounts of energy from various fuels. |  |  |  |  |  |  |

TABLE E. 11

| REGION | MEAN FUEL HEATING VALUES |  |  |
| :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { COAL } \\ (\mathrm{BTU} / \mathrm{TON}) \end{gathered}$ | $\begin{gathered} \text { OII } \\ (\mathrm{BTU} / \mathrm{BBLS}) \end{gathered}$ | $\begin{gathered} \text { GAS } \\ (B T U / \mathrm{MCF}) \end{gathered}$ |
| TO'AL UNITED STATES | $2.191 \times 10^{7}$ | $6.133 \times 10^{6}$ | $1.027 \times 10^{6}$ |
| NEW ENGLAND REGION | $2.492 \times 10^{7}$ | $6.126 \times 10^{6}$ | $1.002 \times 10^{6}$ |
| MIDDLE ATLANTIC REGION | $2.393 \times 10^{7}$ | $6.099 \times 10^{6}$ | $1.093 \times 10^{6}$ |
| EAST NORTH CENTRAL REGION | $2.207 \times 10^{7}$ | $6.098 \times 10^{6}$ | $.878 \times 10^{6}$ |
| WEST NORTH CENTRAL REGION | $1.977 \times 10^{7}$ | $6.218 \times 10^{6}$ | $.992 \times 10^{6}$ |
| SOUTH ATLANTIC REGION | $2.385 \times 10^{7}$ | $6.195 \times 10^{6}$ | $1.024 \times 10^{6}$ |
| EAST SOUTH CENTRAL REGION | $2.227 \times 10^{7}$ | $6.062 \times 10^{6}$ | $1.192 \times 10^{6}$ |
| WEST SOUTH CENTRAL REGION | $1.400 \times 10^{7}$ | $6.101 \times 10^{6}$ | $1.029 \times 10^{6}$ |
| MOUNTAIN REGION | $1.919 \times 10^{7}$ | $6.089 \times 10^{6}$ | $1.004 \times 10^{6}$ |
| PACIFIC CONTIGUOUS REGION | $1.620 \times 10^{7}$ | $6.131 \times 10^{6}$ | $1.062 \times 10^{6}$ |



Figure E. 1 Mean fuel costs as a function of time for the indicated fuels consumed in the United States.


Figure E. 2 Mean fuel costs as a function of time for the indicated fuels in the New England region of the United States.


MONTHS

Figure E. 3 Mean fuel costs as a function of time for the indicated fuels in the Middle Atlantic region of the United States.


Figure E. 4 Mean fuel costs as a function of time for the indicated fuels in the East North Central region of the United States.


Figure E. 5 Mean fuel costs as a function of time for the indicated fuels in the West North Central region of the United States.


Figure E. 6 Mean fuel costs as a function of time for the indicated fuels in the South Atlantic region of the United States.


MONTHS

Figure E. 7 Mean fuel costs as a function of time for the indicated fuels in the East South Central region of the United States.


Figure E. 8 Mean fuel costs as a function of time for the indicated fuels in the West South Central region of the United States.


MONTHS

Figure E. 9 Mean fuel costs as a function of time for the indicated fuels in the Mountain region of the United States.


MONTHS

Figure E. 10 Mean fuel costs as a function of time for the indicated fuels in the Pacific region of the United States.

## REFERENCES

1. Federal Power Commission, "FPC Releases Preliminary 1973 Power Production, Fuel Consumption Data," News Release No: 20333; May 24, 1974.
2. Federal Power Commission, "FPC Issues April 1974 Report on Fuel Cost, Quality," New Release No. 20523; July 29, 1974.
3. Federal Power Commission, "FPC News," week ending December 27, 1974.
4. Raymond, F. L., FPC (OEC), personal communications; Data from internal FPC work papers.
5. Federal Power Commission, Steam-Electric Plant Construction Cost and Annual Production Expenses, Twenty-fifth Annual Supplement-1972; Washington, D.C., April 1974. Available from Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (Price \$2.10).
6. Collier, Dan, Nuclear Assurance Corporation, Atlanta, Georgia; personal communication.

## REFERENCES

1. Federal Power Commission, "FPC Releases Preliminary 1973 Power Production, Fuel Consumption Data," News Release No: 20333; May 24, 1974.
2. Federal Power Commission, "FPC Issues April 1974 Report on Fuel Cost, Quality," New Release No. 20523; July 29, 1974.
3. Federal Power Commission, "FPC News," week ending December 27, 1974.
4. Raymond, F. L., FPC (OEC), personal communications; Data from internal FPC work papers.
5. Federal Power Commission, Steam-Electric Plant Construction Cost and Annual Production Expenses, Twenty-fifth Annual Supplement-1972; Washington, D.C., April 1974. Available from Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402 (Price \$2.10).
6. Collier, Dan, Nuclear Assurance Corporation, Atlanta, Georgia; personal communication.

[^0]:    * For sale by the National Technical Information Service, Springfield, Virginia 22151

[^1]:    * The term "cost-benefit" is used rather than "benefit-cost" to avoid possible confusion with the "benefit-cost" ratio, to be discussed in Part II.

[^2]:    * The term "cost-benefit analysis (CBA)" is used rather than "benefitcost analysis" to avoid any implication of a benefit/cost ratio.

[^3]:    * In practice, the definition of society used in a cost-benefit analysis depends on the particular alternative projects under analysis and may be considered on a local level (e.g., a firm or town) or on a macroscopic level, the level used in these discussions.

[^4]:    * In fact, the usefulness of an acceptance criterion of $B / C$ greater than one is so small that the term (benefit/cost analysis) is not commonly used primarily to avoid the restrictive implications of $B / C$ ratios.

[^5]:    * A full discussion of shadow price is given in Section 4.

[^6]:    . . . it is interesting to examine the arguments over so-called secondary benefits and how they should be included, if at all, in project analyses. There is no such thing as a secondary benefit. A secondary benefit, as the phrase has been used in the benefit-cost literature, is in fact a benefit in support of an objective other than efficiency. The word benefit (and the word cost, too) has no meaning by itself, but only in association with an objective; there are efficiency benefits, income redistribution benefits, and others. Thus, if the objective function for a public program involves more than economic efficiency--and it will in most cases--there is no

[^7]:    $\star$ The best source of information on setting up accounts is the "Principles, Standards, and Procedures for Water and Related Land Resource Planning" and the critical 1iterature associated with this document.

[^8]:    $\star$ The utility function reflects all of the decision maker's feelings about various events and may well include factors other than monetary factors.

[^9]:    * The normative significance of market interest rates is further obscured by the realization that market rates are manipulated by federal policies (e.g. stabilization, employment, foreign exchange) for reasons far removed from long-term public investment decisions.

[^10]:    * Federal Register, Sept. 10, 1973, Vol. 38, No. 174, Part III.

[^11]:    Fall seasonal map of contours of constant plant factor (in percent) at a height of 61 m (200 ft) for a wind generator with the NASA Plumbrook characteristics, cut-in speed $3.6 \mathrm{~m} / \mathrm{s}(8 \mathrm{mph})$, rated speed $8.0 \mathrm{~m} / \mathrm{s}(18 \mathrm{mph})$. Contours were drawn from data shown in Table $\mathrm{D}-4$ of Appendix $D$.

    ๆT・てT วגnถิтฺ

[^12]:    For the reader not knowledgeable in basic elasticity concepts, it is suggested that he read Section A. 8 before continuing.

[^13]:    * Attention is restricted to a "relatively small" quantity of $X_{0}$
    to avoid the complicating issue of consumers" surplus.

[^14]:    * This is usually attributed to the presence of labor unions, which have a monopoly supply of labor. This is a key facet in Keynsian macroeconomics, where the theory asserts the likelihood of an unemployment equilibrium.

[^15]:    * 

    The values in the matrix are expected values, since total WTP may exceed actual cost, he may not be asked to contribute his entire stated WTP. Furthermore, the payoffs assume the park is built. If it is not built, there is neither loss nor gain.

[^16]:    * It should be obvious that the specific numbers used in the game matrix are irrelevant, so long as they bear the proper directional inequality relation to one another. Behavior of the sort predicted by this game is called the "free rider" problem.

