NASA TECHNICAL NOTE

NASA TN D-8011

NASA TN D-8011

AN IMPROVED TECHNIQUE FOR DETERMINING REFLECTION FROM SEMI-INFINITE ATMOSPHERES WITH LINEARLY ANISOTROPIC PHASE FUNCTIONS

Clifford L. Fricke

Langley Research Center
Hampton, Va. 23665

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • SEPTEMBER 1975

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-8011		

4. Title and Subtitle	5. Report Date
AN IMPROVED TECHNIQUE FOR DETERMINING REFLECTION FROM SEMI-INFINITE ATMOSPHERES WITH LINEARLY ANISOTROPIC PHASE FUNCTIONS	September 1975
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Clifford L. Fricke	L-10153
	10. Work Unit No.
9. Performing Organization Name and Address	176-11-31-04
NASA Langley Research Center Hampton, Va. 23665	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Note
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A solution to the problem of reflection from a semi-infinite atmosphere is presented, based upon Chandrasekhar's H-function method for linearly anisotropic phase functions. A modification to the Gauss quadrature formula which gives about the same accuracy with 10 points as the conventional Gauss quadrature does with 100 points is developed. A computer program achieving this solution is described and results are presented for several illustrative cases.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Chandrasekhar's H-function method Radiative transfer Reflection	Unclassified – Unlimited Subject Category 70

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	40	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

AN IMPROVED TECHNIQUE FOR DETERMINING REFLECTION FROM SEMI-INFINITE ATMOSPHERES WITH LINEARLY ANISOTROPIC PHASE FUNCTIONS

Clifford L. Fricke
Langley Research Center

SUMMARY

A solution to the problem of reflection from a semi-infinite atmosphere is presented, based upon Chandrasekhar's H-function method for linearly anisotropic phase functions. A modification to the Gauss quadrature formula which gives about the same accuracy with 10 points as the conventional Gauss quadrature does with 100 points is developed. A computer program achieving this solution is described and results are presented for several illustrative cases.

INTRODUCTION

In 1949 Chandrasekhar (ref. 1) published solutions to the radiative transfer problem for certain special phase functions. The solutions involved his H-functions. These functions depend on several parameters and cannot be simply defined, but can be computed by using methods described in reference 1. These procedures were quite cumbersome with the computing methods available in that day, especially when it is realized that the Gauss quadrature requires so many points. A modification of the Gauss quadrature introduced herein achieves better accuracy than that obtained by Chandrasekhar and with far fewer points. This is an important result since it can then be applied to more complex problems where the number of Gaussian points required for acceptable accuracy might exceed the capacity of even the most modern computers.

The computer program presented obtains solutions using the linearly anisotropic phase function $\omega_0(1 + x \cos \Theta)$ for the problem of reflection in any direction from a semi-infinite atmosphere for radiation incident in any direction. The modified or composite-Gauss quadrature is used and achieves the same accuracy with about one-tenth the number of points as the usual Gauss quadrature.

SYMBOLS

Algebraic	FORTRAN	
a_i	A(I)	weight at discrete values of μ_i
c	C	quantity which is a function of moments of H-function, defined in equation (10)
	DT	increment given to X1
E	E	exponent for division of intervals
F	F	$1/\pi$ times net flux of incident radiation or $1/\pi$ times rate of flow of radiant energy across a surface element per unit area
$H(\mu_i)$	H(I)	H-function at discrete values of μ_i
$H^{(0)}(\mu_i)$	HM0(I)	zeroth order H-function at discrete values of μ_i
$H^{(1)}(\mu_i)$	HM1(I)	first order H-function at discrete values of μ_i
I	ET	intensity of radiation in a certain direction or radiant energy per unit time transported across an element of area normal to radiation in an element of solid angle
	E0	that part of intensity independent of ϕ
	E1	that part of intensity dependent on ϕ
	ETX0	ratio of reflected to incident radiation or I/μ_0
k		independent variable of $S^{(\ell)}(k)$, defined in equation (6)
k_α	XK0(I)	characteristic roots for zeroth order Ψ
	XK1(I)	characteristic roots for first order Ψ

Algebraic	FORTRAN	
ℓ	L	order of term in Legendre polynomial expansion
m	M	number of subintervals and equal to $n/2$
n	N	number of discrete directions (positive μ only) in quadrature or integration sum
p		phase function
R	R	total reflectance
S	S	a function of k which gives characteristic equation when set equal to zero
SN	SN	quantity showing sign of S on left side of a pole
	SS	derivative of S with respect to k
x	X	anisotropic parameter
μ_j	X(J)	boundaries of subintervals
α_o	ALPH0	zeroth moment of $H^{(0)}$-function
α_1	APLH1	first moment of $H^{(0)}$-function
τ		optical depth
θ	THET	angle relative to vertical
Θ		angle between incident and scattered ray for a single scattering
ϕ	PHI	azimuthal angle
Ψ	PSI	characteristic function

Algebraic	FORTRAN	
μ	X1	$= \cos\theta$
μ_i	XM(I)	discrete value of μ
μ_o	X10	direction of incident radiation
ω_o	WO	albedo

Subscripts:

i	I,M	= 1, 2, . . ., n
j	J	= 1, 2, . . ., m
o	0	= direction of incident beam

THE EQUATION OF RADIATIVE TRANSFER

Figure 1 illustrates a beam of light I_o incident on a plane atmospheric surface at a polar angle of θ_o. In general, the intensity of the scattered radiation is a function of τ the optical depth in the negative z direction, μ the cosine of the polar angle θ, and ϕ the azimuth angle with respect to the X-axis, as illustrated in figure 1 which

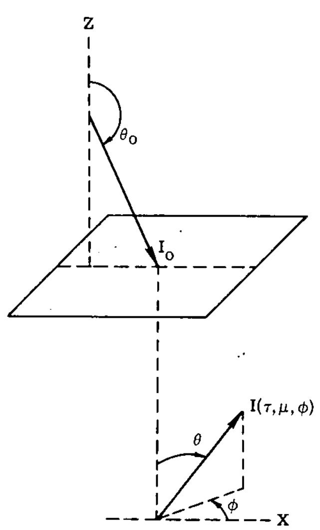


Figure 1.- Illustration of incident beam I_o and a scattered ray $I(\tau,\mu,\phi)$.

shows the ray $I(\tau,\mu,\phi)$. Rays in the downward direction have negative μ, so that the incident beam has a direction cosine $-\mu_0$ where μ_0 is a positive number. The equation of transport appropriate for the diffuse reflection and transmission of radiation (ref. 1, p. 22) is given by

$$\mu \frac{dI(\tau,\mu,\phi)}{d\tau} = I(\tau,\mu,\phi) - \frac{1}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi} p(\mu,\phi;\mu',\phi')\, I(\tau,\mu',\phi')\, d\mu'\, d\phi'$$

$$- \frac{F}{4} e^{-\tau/\mu_0} p(\mu,\phi; -\mu_0,\phi_0) \tag{1}$$

The term on the left is associated with the change of intensity as the depth is penetrated. The first term on the right is associated with the absorption. The second term is the contribution from light scattered in the direction (μ,ϕ) from all directions (μ',ϕ'). The last term can be identified with the incident beam which acts as a forcing function and drives the diffuse radiation. Solutions are required which satisfy the following boundary conditions at $\tau = 0$ and $\tau = \infty$ (semi-infinite atmosphere):

$$I(0,-\mu,\phi) = 0 \qquad (0 \leqq \mu \leqq 1) \tag{2}$$

$$\lim_{\tau\to\infty} I(\tau,\pm\mu,\phi) = 0 \tag{3}$$

The phase function of present interest is the linearly anisotropic expression

$$p(\cos\Theta) = \omega_0(1 + x\cos\Theta) \tag{4}$$

where

$$\cos\Theta = \mu\mu' + \sqrt{1-\mu^2}\,\sqrt{1-\mu'^2}\,\cos(\phi' - \phi)$$

When $x = 0$, the isotropic case is obtained. Other phase functions have been considered by Chandrasekhar but will not be considered herein.

For phase functions which can be expanded as a series of Legendre polynomials, the solution can be expressed in the form

$$I(\tau,\mu,\phi) = \sum_{\ell} I^{(\ell)}(\tau,\mu)\cos\ell(\phi - \phi_0)$$

where ℓ corresponds to the order of the Legendre polynomial and has the values 0 and 1 for the phase function of present interest.

THE METHOD OF SOLUTION

Chandrasekhar's method of solution assumes the radiant field to be divided into 2n streams in the directions μ_i where $i = \pm 1, \pm 2, \ldots, \pm n$ and is often referred to as the discrete-ordinate method. The integral equation is then replaced by a system of 2n linear equations. Associated with each direction μ_i is a weight a_i appropriate for an integration formula based on the division of the interval (-1,+1). For example, in this approximation, the integral

$$\int_{-1}^{1} I(\mu') \, d\mu'$$

can be expressed as the summation

$$\sum_{i=\pm 1}^{\pm n} a_i \, I(\mu_i)$$

By increasing the number of directions, one would expect to approach the exact solution as a limit. A division according to the Gauss method is used by Chandrasekhar; however, it will be shown that another division is better suited to this particular problem.

Chandrasekhar (ref. 1, p. 127) expresses the solution to the radiation equation in terms of H-functions indicated by the following equation:

$$H^{(\ell)}(\mu) = \frac{\Pi_{i=1}^{n} \left(1 + \frac{\mu}{\mu_i}\right)}{\Pi_{\alpha=1}^{n} \left(1 + k_\alpha \mu\right)} \tag{5}$$

where ℓ refers to the order of the H-function (which corresponds to the order of the Lengendre polynomial expansion of the phase function) and k_α are the roots of the characteristic equation (when $S = 0$) given by

$$S^{(\ell)}(k) = -1 + 2 \sum_{i=1}^{n} \frac{a_i \psi^{(\ell)}(\mu_i)}{1 - k^2 \mu_i^{\,2}} \tag{6}$$

The characteristic function $\psi^{(\ell)}(\mu)$ depends on the particular phase function. For the phase function in equation (4)

$$\Psi^{(0)} = \frac{1}{2}\,\omega_0\left[1 + x(1 - \omega_0)\mu^2\right] \tag{7}$$

and

$$\Psi^{(1)} = \frac{1}{2}\,x\omega_0\left[1 - \mu^2\right] \tag{8}$$

Note that when $x = 0$, this case reduces to isotropic scattering and $H^{(1)}$ is not needed.

REFLECTION IN TERMS OF THE H-FUNCTIONS

The solution to the equation for the phase function $\omega_0(1 + x\cos\Theta)$ in the case of the semi-infinite atmosphere (ref. 1, p. 138) is given by

$$I(0,\mu,\phi;\mu_0\phi_0) = \frac{\omega_0\mu_0 F}{4(\mu + \mu_0)}\Big\{H^{(0)}(\mu)\,H^{(0)}(\mu_0)\left[1 - c(\mu + \mu_0) - x(1 - \omega_0)\mu\mu_0\right]$$

$$+ x\sqrt{1 - \mu^2}\,\sqrt{1 - \mu_0{}^2}\,H^{(1)}(\mu)\,H^{(1)}(\mu_0)\cos(\phi_0 - \phi)\Big\} \tag{9}$$

where

$$c = x\omega_0(1 - \omega_0)\,\frac{\alpha_1}{2 - \omega_0\alpha_0} \tag{10}$$

$$\alpha_0 = \int_0^1 H^{(0)}(\mu)\,d\mu \tag{11}$$

and

$$\alpha_1 = \int_0^1 \mu\,H^{(0)}(\mu)\,d\mu \tag{12}$$

Another quantity of interest is the total reflectance in the vertical direction which is defined by

$$R(\omega_0,x,\mu_0) = \frac{1}{\mu_0 \pi F}\int_0^1 \int_0^{2\pi} I(0,\mu,\phi)\mu \, d\mu \, d\phi \tag{13}$$

Thus,

$$R(\omega_0,x,\mu_0) = \frac{\omega_0}{2} H^{(0)}(\mu_0) \int_0^1 H^{(0)}(\mu) \left[\frac{1 - x(1 - \omega_0)\mu\mu_0}{\mu + \mu_0} - c\right]\mu \, d\mu \tag{14}$$

This integration and those for the moments will be computed by using the summation at the same discrete values of μ and the same weighting as used in the calculation of the H-functions (eq. 5).

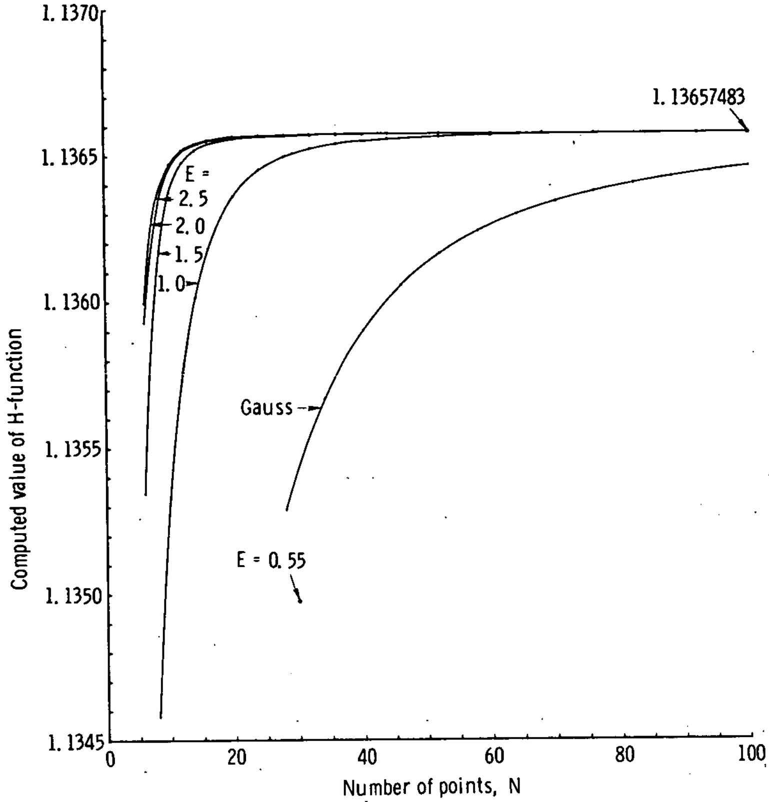
PRELIMINARY DISCUSSION OF COMPUTATIONAL METHODS

The main difficulties encountered in Chandrasekhar's H-function method are due to the large number of discrete ordinates required to give acceptable accuracy and the corresponding large number of roots of S obtained in equation (6). These difficulties are greatly diminished by a modification of the Gauss quadrature. Discussions of this modification and properties of the S-equation used in the computer program to find the roots follow.

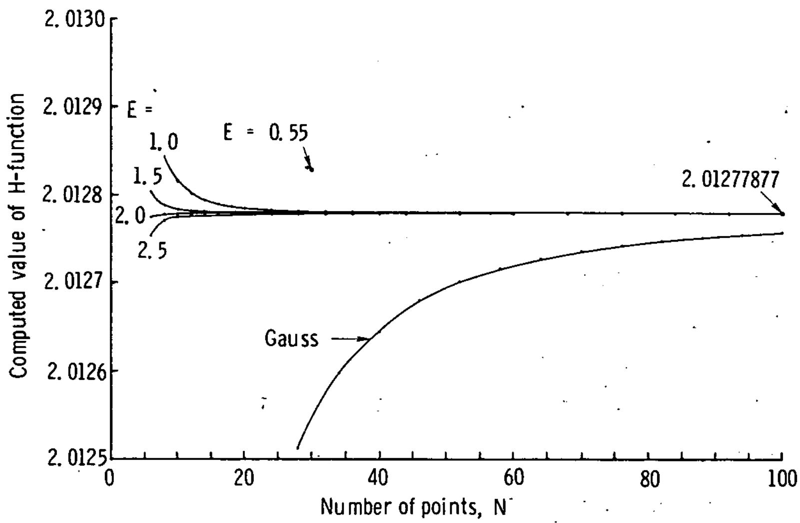
Integration Intervals

With modern computers it is possible to calculate to much higher orders of discrete ordinates than did Chandrasekhar who used an iterative scheme and interpolation to calculate the higher orders (ref. 1, p. 123). The computer program presented herein was used in order to test the variation of the H-function for the Gaussian integration as a function of the number of points up to 100 for isotropic scattering (i.e., $\omega_0 = 1$) corresponding to table XI, p. 125, of reference 1. Figure 2 shows plots of $H^{(0)}$ for $\mu = 0.05$, 0.5, and 1.0 as a function of the number of Gaussian points. The slowest convergence occurs for $\mu = 0.05$ where it is noted that even with 100 points an error of about 0.0001 occurs.

The Gauss method will exactly integrate polynomials of degree less than $2n$ where n is the number of Gaussian points. The difficulty is that the polynomial expansion of the integrand over the entire interval is of a higher order than that which can be practically used. Thus, it seems feasible to obtain a better approximation by breaking up the

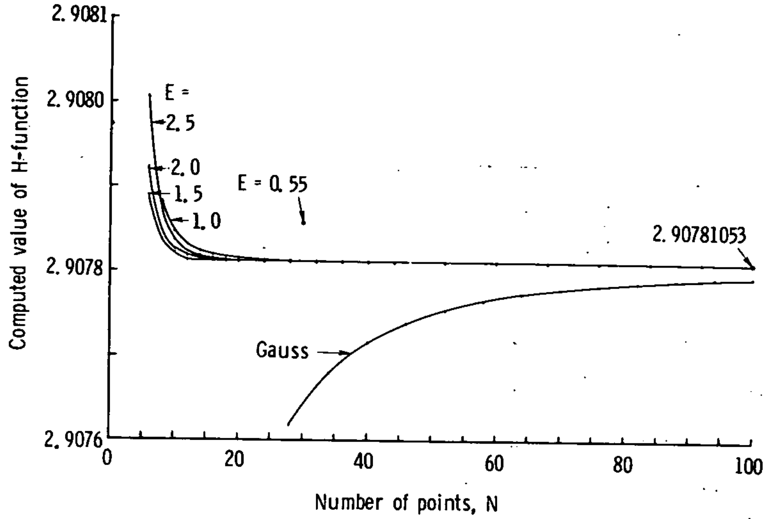


(a) $\mu = 0.05$.



(b) $\mu = 0.5$.

Figure 2.- Variation of computed values of H-function with number and distribution of points in quadrature (ref. 1).



(c) $\mu = 1.0$.

Figure 2.- Concluded.

entire interval into several subintervals, each of which can conceivably be expressed very accurately by relatively low-order polynomials. This procedure has been called a composite quadrature (ref. 2). The Gauss method places more points near the $\mu = 1$ end of the integration interval, but it will be shown that better accuracy is obtained if more points are placed near the $\mu = 0$ end. A scheme which gives excellent accuracy with few points is to break the interval (0,1) into a number of subintervals placing more points where needed and then within each subinterval use a two-point Gauss quadrature. A convenient way of doing this is to space the intervals according to the formula

$$\mu_j = \left(\frac{j}{m}\right)^E \qquad (j = 1, 2, \ldots, m) \tag{15}$$

where μ_j is a boundary between subintervals and $m = n/2$.

The weight for each point in the subinterval is equal to half the width of that subinterval (ref. 3, pp. 887 and 916):

$$a_{2j} = a_{2j-1} = \frac{\mu_j - \mu_{j-1}}{2}$$

The corresponding two discrete values of μ in the subinterval between μ_j and μ_{j-1} are given by the following equations (ref. 3, p. 916):

$$\mu_{2j-1} = \frac{\mu_j + \mu_{j-1}}{2} - \frac{\mu_j - \mu_{j-1}}{2\sqrt{3}}$$

and

$$\mu_{2j} = \frac{\mu_j + \mu_{j-1}}{2} + \frac{\mu_j - \mu_{j-1}}{2\sqrt{3}}$$

Equal subintervals correspond to $E = 1$ and would be expected to be superior to a Simpson Rule. For $E = 2$ the intervals are bunched at the $\mu = 0$ end. This scheme is used in subroutine SNTRVL presented later.

The program was run for various values of the number of points N. Figure 2 shows the convergence of $H^{(0)}$ as a function of N for the cases $E = 1.0$, 1.5, 2.0, and 2.5. An isolated point for $E = 0.55$ is also shown. With $E = 2.0$ and $N = 10$, the largest error is about 0.0001 for the worst case ($\mu = 0.05$) and is comparable with the 100-point Gauss method. With 20 terms the error is less than 1×10^{-5}. It may be noted that the error in $H^{(0)}$ (0.05) in reference 1 is 0.000225. With 100 terms and $E = 2$, the following values are obtained with eight decimal accuracy:

$$H^{(0)}(0.05) = 1.136\ 574\ 83$$

$$H^{(0)}(0.5) = 2.012\ 778\ 77$$

$$H^{(0)}(1.0) = 2.907\ 810\ 53$$

Roots of the S-Equation

Since equation (6) for S must be solved for the n values of k which give $S = 0$, it is instructive to examine the properties of S as a function of k. It may be noted that $\Psi^{(0)} > 0$ for all values of x, μ, and ω_o, but $\Psi^{(1)}$ always has the same sign as x. Thus,

$$\lim_{k\to 0} S^{(\ell)}(k) = \lim_{k\to 0}\left[-1 + 2\pi \sum a_j\, \Psi^{(\ell)}(\mu_j)\right] \left\{\begin{matrix} = 0 & (\omega_o = 1, \quad \ell = 0) \\ < 0 & (\omega_o < 1, \quad \ell = 0) \\ < 0 & (\ell = 1) \end{matrix}\right\} \tag{16}$$

and

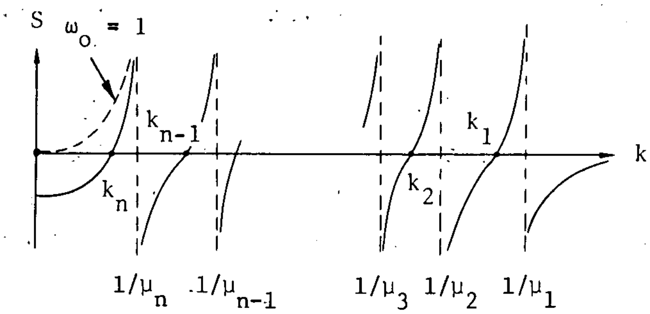
$$\lim_{k \to 1/\mu_i} S^{(\ell)}(k) = \pm\infty \tag{17}$$

Thus, there are n poles located at $k_i = 1/\mu_i$. In addition,

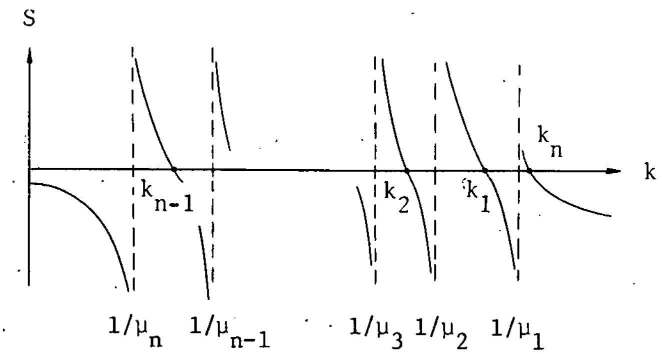
$$\lim_{k \to \infty} S = -1 \tag{18}$$

Consider also the quantity $SN = \frac{(-x)^\ell}{|x|}$ where $|x| > 0$; this quantity always gives the sign of $\Psi^{(\ell)}$ and indicates the sign of S just to the left of a pole.

These parameters are shown schematically in figures 3(a) and (b). These properties will be used in the computer search for roots.



(a) Locations when SN = +1 and either $\ell = 0$ for any x or $\ell = 1$ for $x > 0$.



(b) Locations when SN = -1 and $\ell = 1$ for $x < 0$.

Figure 3.- Schematic locations of roots and poles of S as a function of k.

DESCRIPTION OF COMPUTER PROGRAM

The main program CHSKR calls subroutine INTRVL and subroutine HFUNC (which in turn calls subroutine ROOT). The programs and subprograms are listed in appendix A. Flow charts of the main program and subprograms are shown in appendix B. The subroutines will be discussed first.

Subroutine INTRVL(N,XM,A)

This subroutine computes the n discrete values of μ_i storing them in XM and the weights a_i storing them in A(I).

Subroutine ROOT(X1,DT,SN,N,XM,A,WO,L,X)

This subroutine finds a root of the S-function which is known to lie in the interval between the two points X1 and X1-2*DT. The first trial root is X1 incremented by DT (where DT is equal to half the interval). The values of S and SS (the derivative of S) are computed first; then DT is halved. If the new value of S has changed sign, then the sign of DT is changed.

By Newton's method, the next increment would be S/SS. However, if the Newton increment is smaller than DT, then DT is set equal to S/SS. This prevents having an increment which is so large that the next trial value is completely out of the interval; such an increment can occur in the Newton method if the trial point is too far from a root.

When the increment DT is small compared with the value of the root X1, then the search is over and the root X1 is returned.

The expression for PSI in the subroutine gives the correct expression for $\Psi^{(\ell)}$ whether L is 0 or 1.

Subroutine HFUNC(WO,X,K,N,XM,A,XK,H)

This subroutine computes the H-function for either L = 0 or 1 and stores the value in H(M) corresponding to each μ_i or XM(M) value. The roots k_i of the function S are stored in XK(M).

The first statement sends the computer to statement 30 if X = 0 and L = 1, because then $H^{(1)}$ does not exist. In order to indicate this condition in a convenient manner, DO-loop 35 sets each root k_i equal to $1/\mu_i$ so that subsequent calculation will always give H = 1. (The actual value given to H is unimportant because x = 0 in eq. (9) will always make that term zero.)

The sign of SN indicates the sign of S just to the left of a pole. Each root except the last one (when J = N) lies between the poles 1./XM(J) or X1 and

1/XM(J+1) so the increment DT is set equal to $(X1-1./XM(J+1))/2$. Subroutine ROOT then finds this root and it is then stored in XK(J).

The last root (for J = N) can be equal to 0, less than 1/XM(M), or greater than 1/XM(1). If S0, the value of S at k = 0, is greater than or equal to 0, the root must be located at 0 (slight numerical errors may cause S0 to be slightly positive). However, if S0 differs in sign from SN, then the root lies to the right of 1/XM(1), but will always be less than 2/XM(1). Otherwise, the root lies between 0 and 1/XM(N).

Program CHSKR

The main program calculates all quantities for x = 1, 0, and -1, and for $\omega_0 = 1.0$ and 0.8.

Subroutine HFUNC is called first with $\ell = 0$ and then with $\ell = 1$ in order to compute the zeroth and first-order H-functions. Roots for the zeroth order characteristic equation are stored in XKO(I), while the H-function at discrete values of μ is stored in HMO(I); the first-order quantities are stored in XK1(I) and HM1(I).

The variable μ_0 is X10. The quantities $H^{(0)}(\mu_0)$ and $H^{(1)}(\mu_0)$ are called H00 and H10 in the program.

The reflected intensities ET are broken into two parts: E0 is the part of the intensity which is dependent on θ and E1 is the part which is dependent on ϕ.

ILLUSTRATIVE EXAMPLES

Appendix C shows the results of running the program with N = 30 for the cases x = 1.0, 0.0, and -1.0; $\omega_0 = 1.0$ and 0.8; and $\mu_0 = 1.0$, 0.8, and 0.6. The total running time on a Control Data 6400 computer system was about 21 seconds. Other cases can easily be obtained by slight changes in the program. The $H^{(0)}$-functions for the cases x = 0.0 with $\omega_0 = 1.0$ and 0.8 may be compared directly with reference 1, table XI, page 125; the case x = 1.0 with $\omega_0 = 0.8$ may be compared with reference 1, table XVI, page 139. The $H^{(1)}$-function for the cases $x\omega_0 = 1.0$, 0.8, -0.8, and -1.0 may be compared with reference 1, table XVIII, page 141. These comparisons are shown in tables I and II for selected values of μ. The agreement is excellent in all cases, except for small values of μ where a difference of 1 or 2 exists in the fourth decimal.

The reflection is shown for angles of θ from 0^o to 180^o in steps of 10^o and for angles of $\phi = 45^o$, 90^o, 135^o, and 180^o where appropriate. The reflection for the cases x = +1.0 and -1.0 with $\omega_0 = 1.0$ and $\mu_0 = 0.8$ agrees with reference 1, figure 11, page 148. The case for x = 1.0, $\omega_0 = 0.8$, and $\mu_0 = 0.6$ agrees with reference 1, figure 12, page 149.

CONCLUDING REMARKS

A computer program has been presented which computes the reflection from a semi-infinite atmosphere for the linearly anisotropic phase function. This program uses the method of Chandrasekhar, but with a new modification to the conventional Gauss quadrature formula. This departure from the traditional quadrature reduces the number of points so that answers for any set of parameters can be easily obtained with high accuracy and very short computer running time. The computer program presented can be easily modified for other phase functions which are capable of being solved by this method. This composite Gauss quadrature should also be valuable in more complex problems where the number of points must be limited.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
July 14, 1975

APPENDIX A

LISTING OF COMPUTER PROGRAM

This appendix contains a computer program for computing the reflection from a semi-infinite atmosphere for the linearly anisotropic phase function. This program uses the method of Chandrasekhar, but with a modification to the conventional Gauss quadrature formula.

```
      PROGRAM CHSKR(        OUTPUT)
******THIS PROGRAM COMPUTES THE REFLECTION ET IN ANY DIRECTION (THETA,PHI)
*     FROM A SEMI-INFINITE ATMOSPHERE HAVING A PHASE FUNCTION WO(1+XCOSTHETA)
******THE INCIDENT BEAM HAS THE DIRECTION COSINE X10
******CHANDRASEKHARS METHOD IS USED, BUT WITH A COMPOSITE GAUSS QUADRATURE
******THE DISCRETE ORDINATES ARE STORED IN XM, WITH WEIGHTS IN A
******THE ZEROTH AND FIRST ORDER CHARACTERISTIC ROOTS ARE STORED IN XK0 AND XK1
******THE ZEROTH AND FIRST ORDER H-FUNCTIONS ARE STORED IN HM0 AND HM1
******THE TOTAL REFLECTANCE R IS ALSO COMPUTED
      DIMENSION A( 30),XM( 30),XK0( 30),XK1( 30),HM0(30),HM1(30)
      PI=3.1415926536
      N=30
******CALCULATE THE N DISCRETE ORDINATES XM, AND WEIGHTS A
      CALL INTRVL(N,XM,A)
******COMPUTE FOR VARIOUS VALUES OF ANISOTROPY PARAMETER X, AND ALBEDO WO
      DO 20 J=1,3               $  X=2.0-1.0*J
      DO 20  K=1,2              $  WO=1.20-.20*K
******CALCULATE THE ZEROTH ORDER H-FUNCTION HM0, AND THE CHARACTERISTIC ROOTS
*     XK0 AT THE DISCRETE ORDINATES XM
      L=0
      CALL HFUNC (WO,X,L,N,XM,A,XK0,HM0)
******COMPUTE THE ZEROTH AND FIRST-ORDER MOMENTS OF H
      ALPH0=0.
      ALPH1=0.
      DO 5 I=1,N
      ALPH0=ALPH0+HM0(I)*A(I)
      ALPH1=ALPH1+HM0(I)*A(I)*XM(I)
5     CONTINUE
      C=X*WO*(1.-WO)*ALPH1/(2.-WO*ALPH0)
      PRINT 151,N,X,WO
151   FORMAT(*1  N=*I3*    X=*F5.2*    WO=*F5.2)
      PRINT 33
33    FORMAT(*0  MU            H0(MU)            H1(MU)*)
******CALCULATE THE FIRST-ORDER H-FUNCTION HM1 AND CHARACTERISTIC ROOTS
*     XK1 AT DISCRETE ORDINATES XM
      L=1
      CALL HFUNC (WO,X,L,N,XM,A,XK1,HM1)
******CALCULATE H-FUNCTIONS AT ARBITRARY X1 USING CHARACTERISTIC ROOTS
      DO 30 M=1,21              $  X1=.05*(M-1)
      H0=1.
      H1=1.
      DO 35 I=1,N
```

APPENDIX A

```
      H0=H0*(X1+XM(I))/XM(I)/(1.+XK0(I)*X1)
      H1=H1*(X1+XM(I))/XM(I)/(1.+XK1(I)*X1)
35    CONTINUE
      PRINT 34,X1,H0,H1
34    FORMAT(F5.2,2F15.8)
30    CONTINUE
      PRINT 36,ALPH0,ALPH1,C
36    FORMAT(*0    ALPHA0=*F10.8*    ALPHA1=*F10.8*    C=*F10.8)
******COMPUTE REFLECTION FOR VARIOUS VALUES OF DIRECTION OF INCIDENT LIGHT, X10
      DO 13 IX0=1,3              $  X10=1.2-.2*IX0
      PRINT 15
15    FORMAT(*1       X       W0      MU0      THET     PHI        I0          I1
     $        I        I/MU0*)
      H00=1.
      H10=1.
      DO 11 I=1,N
      H00 =H00 *(X10+XM(I))/XM(I)/(1.+XK0(I)*X10)
11    H10 =H10 *(X10+XM(I))/XM(I)/(1.+XK1(I)*X10)
      DO 10 IX=1,10              $  X1=COS(PI*(IX-1)/18.)
      THET=10.*(IX-1)
******COMPUTE MU-DEPENDENT PART OF REFLECTION, E0
      H0 =1.
      H1 =1.
      DO 12 I=1,N
      H0=H0*(X1+XM(I))/XM(I)/(1.+XK0(I)*X1)
12    H1=H1*(X1+XM(I))/XM(I)/(1.+XK1(I)*X1)
      E0=W0*X10/(X1+X10)/4.*H0*H00*(1.-C*(X1+X10)-X*(1.-W0)*X1*X10)
******COMPUTE PHI-DEPENDENT PART OF REFLECTION, E1
      DO 16 IPH=1,5              $  PHI=PI*(IPH-1)/4.
      PHID=PHI*180./PI
      E1=X*SQRT((1.-X1*X1)*(1.-X10*X10))*H1*H10*COS(PHI)*W0*X10/(X1+X10)
     $/4.
      ET=E0+E1
      ETX0=ET/X10
      PRINT 21,X,W0,X10,THET,PHID,E0,E1,ET,ETX0
21    FORMAT(3F7.2,F9.1,F7.1,2F10.5,F10.6,F10.4)
      IF(ABS(E1).LT.1.E-10.AND.IPH.EQ.1)  GO TO 19
16    CONTINUE
19    CONTINUE
10    CONTINUE
******CALCULATE TOTAL REFLECTANCE R
      R=0.
      DO 17 I=1,N
17    R=R+HM0(I)*((1-X*(1-W0)*XM(I)*X10)/(X10+XM(I))-C)*XM(I)*A(I)
      R=W0*H00*R/2.
      PRINT 18,R
18    FORMAT(*0  R=*F9.6)
13    CONTINUE
20    CONTINUE
      STOP            $  END
```

APPENDIX A

```
      SUBROUTINE HFUNC (WO,X,L,N,XM,A,XK,H)
***** THIS SUBROUTINE COMPUTES CHANDRASEKHARS H FUNCTIONS FOR THE CASE
*****    WO(1+XCOSTHETA).  L IS THE ORDER OF THE H FUNCTION.
      DIMENSION XM(1),A(1),XK(1),H(1)
*****   IF X=0 WE HAVE THE ISOTROPIC CASE.
      IF(X.EQ.0..AND.L.EQ.1)  GO TO 30
******SN IS THE SIGN OF S ON THE LEFT SIDE OF EVERY POLE.
      SN= (X/ABS(X))**L
******FIND THE N ROOTS.
      DO 19 J=1,N
****** THE LAST ROOT IS TREATED AS A SPECIAL CASE
      IF(J.EQ.N) GO TO 13
******THE ROOT IS LOCATED TO THE LEFT OF 1/XM(J) AND TO THE RIGHT OF 1/XM(J+1)
      X1=1./XM(J)
      DT=(X1-1./XM(J+1))/2.
      CALL ROOT(X1,DT,SN,N,XM,A,WO,L,X)
******STORE THE ROOT IN XK
      XK(J)=X1
      GO TO 19
13    CONTINUE
******NOW FIND THE LAST ROOT.  FIRST SET THE ROOT EQUAL TO ZERO AND CHANGE
*     LATER IF NECESSARY
      XK(J)=0.
******COMPUTE SO, THE VALUE OF S AT K=0.
      SO=-1.
      DO 14 I=1,N
      XMS=XM(I)*XM(I)
      PSI=WO/2./(L+1)*(1-L+(1-2*L)*X*(XMS*(1.-WO*(1-L))-L))
14    SO=SO+A(I)*2.*PSI
******IF SO IS EQUAL TO ZERO OR GREATER THAN ZERO, THEN THE ROOT LIES VERY
*     NEAR K=0., SO LET IT STAY ZERO
      IF(SO.GE.0.) GO TO 19
**    IF NOW SO IS LESS THAN ZERO WHEN SN IS -1, THE ROOT MUST
*     LIE TO THE RIGHT OF 1/XM(1)
      IF(SN*SO.GT.0.) GO TO 15
**    IF SO IS LESS THAN ZERO WHEN SN IS +1, THE ROOT MUST
*     LIE TO THE LEFT OF 1/XM(N)
      X1=1./XM(N)
      DT=X1/2.
      CALL ROOT(X1,DT,SN,N,XM,A,WO,L,X)
      XK(J)=X1
      GO TO 19
15    X1=1./XM(1)
      DT=-X1/2.
      CALL ROOT(X1,DT,-SN,N,XM,A,WO,L,X)
      XK(J)=X1
19    CONTINUE
******COMPUTE THE H-FUNCTION
      DO 20 M=1,N    $  X1=XM(M)
      H(M)=1.
      DO 25 I=1,N
      H(M)=H(M)*(X1+XM(I))/XM(I)/(1.+XK(I)*X1)
25    CONTINUE
20    CONTINUE
      RETURN
30    CONTINUE
******SET H=1 FOR THE ISOTROPIC CASE WHEN L=1
      DO 35 I=1,N
      XK(I)=1./XM(I)
      H(I)=1.0
35    CONTINUE
      RETURN
      END
```

APPENDIX A

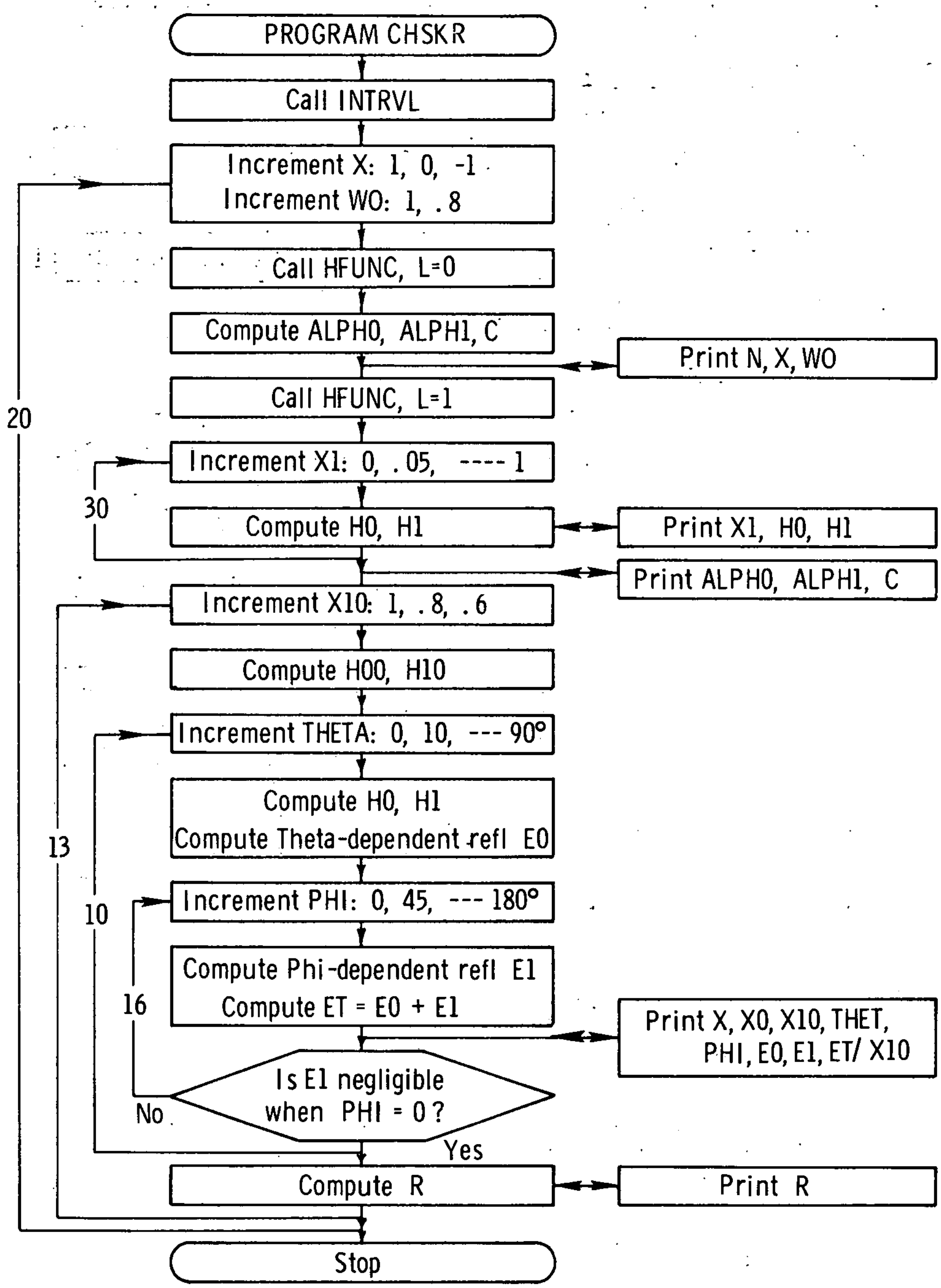
```
      SUBROUTINE INTRVL(N,XM,A)
******THIS SUBROUTINE COMPUTES THE N DISCRETE VALUES OF XM, AND WEIGHTS A BY
*     BREAKING UP THE INTERVAL BETWEEN 0 AND 1, INTO N/2 SUB-INTERVALS
*     AND USING A 2-POINT GAUSSIAN DIVISION WITHIN EACH SUB-INTERVAL
      DIMENSION XM(1),A(1),X(50)
      E=2.0
      M=N/2
      N=2*M
      SQR3=SQRT(3.)
******CALCULATE X, THE BOUNDARIES OF THE SUBINTERVALS
      DO 10 J=1,M
10    X(J)=(FLOAT(J)/FLOAT(M))**E
******CALCULATE THE DISCRETE VALUES XM, AND WEIGHTS A
      DO 20 J=2,M
      A(2*J)=A(2*J-1)=(X(J)-X(J-1))/2.
      XM(2*J-1)=(X(J)+X(J-1))/2.-A(2*J)/SQR3
20    XM(2*J)=(X(J)+X(J-1))/2.+A(2*J)/SQR3
      A(1)=A(2)=X(1)/2.
      XM(1)=X(1)/2.-A(1)/SQR3
      XM(2)=X(1)/2.+A(1)/SQR3
      RETURN
      END


      SUBROUTINE ROOT(X1,DT,SN,N,XM,A,WO,L,X)
      DIMENSION XM(1),A(1)
******THIS SUBROUTINE FINDS THE ROOT OF TH S-FUNCTION WHICH IS KNOWN TO
**    LIE BETWEEN X1 AND X1-2*DT
******THE SIGN OF SN IS EQUAL TO THE SIGN OF S NEAR X1
      SO=SN
      IT=0
10    X1=X1-DT
      IT=IT+1
***   INITIALIZE S AND THE DERIVATIVE SS
      S=-1.
      SS=0.
***   COMPUTE THE S-FUNCTION AND THE DERIVATIVE OF THE S-FINCTION, SS
      DO 11 I=1,N
      XMS=XM(I)*XM(I)
      D=1.-XMS*X1*X1
***   THE FUNCTION PSI DEPENDS ON THE ORDER L
      PSI=WO/2./(L+1)*(1-L+(1-2*L)*X*(XMS*(1.-WO*(1-L))-L))
      S1=A(I)*2.*PSI/D
      SS=SS+S1/D*2.*XMS*X1
11    S=S+S1
******HALVE THE INCREMENT
      DT=DT/2.
******IF S HAS CHANGED SIGN THEN ZERO HAS BEEN CROSSED--CHANGE SIGN OF INCREMENT
      IF(S*SO.LT.0.) DT=-DT
      SO=S
******IF THE NEWTON INCREMENT S/SS IS SMALLER THAN DT, THEN THE NEW
*     INCREMENT IS THE NEWTON INCREMENT
      IF(ABS(S/SS).LT.ABS(DT)) DT=S/SS
**    IF THE INCREMENT IS NOT SMALL COMPARED TO THE ROOT THEN RE-ITERATE
      IF(ABS(DT/X1).GT.1.E-13) GO TO 10
**    THE ROOT IS NOW RETURNED IN X1
      X1=X1-DT
      RETURN
      END
```

APPENDIX B

FLOW DIAGRAMS OF COMPUTER PROGRAMS

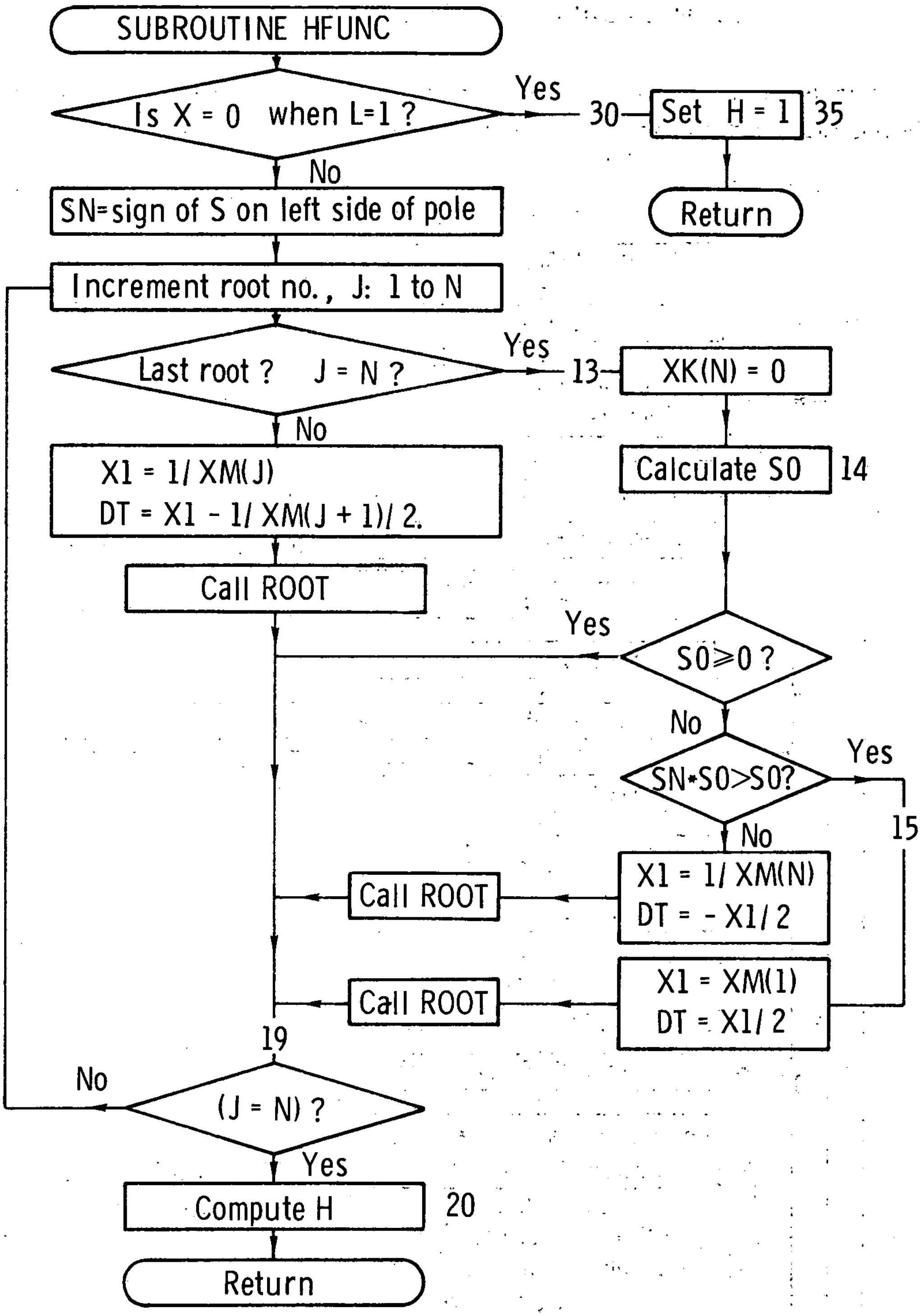
This appendix shows flow charts of the main computer program (program CHSKR) and subprograms (subroutine HFUNC and subroutine ROOT) given in appendix A.

Flow Diagram of Program CHSKR



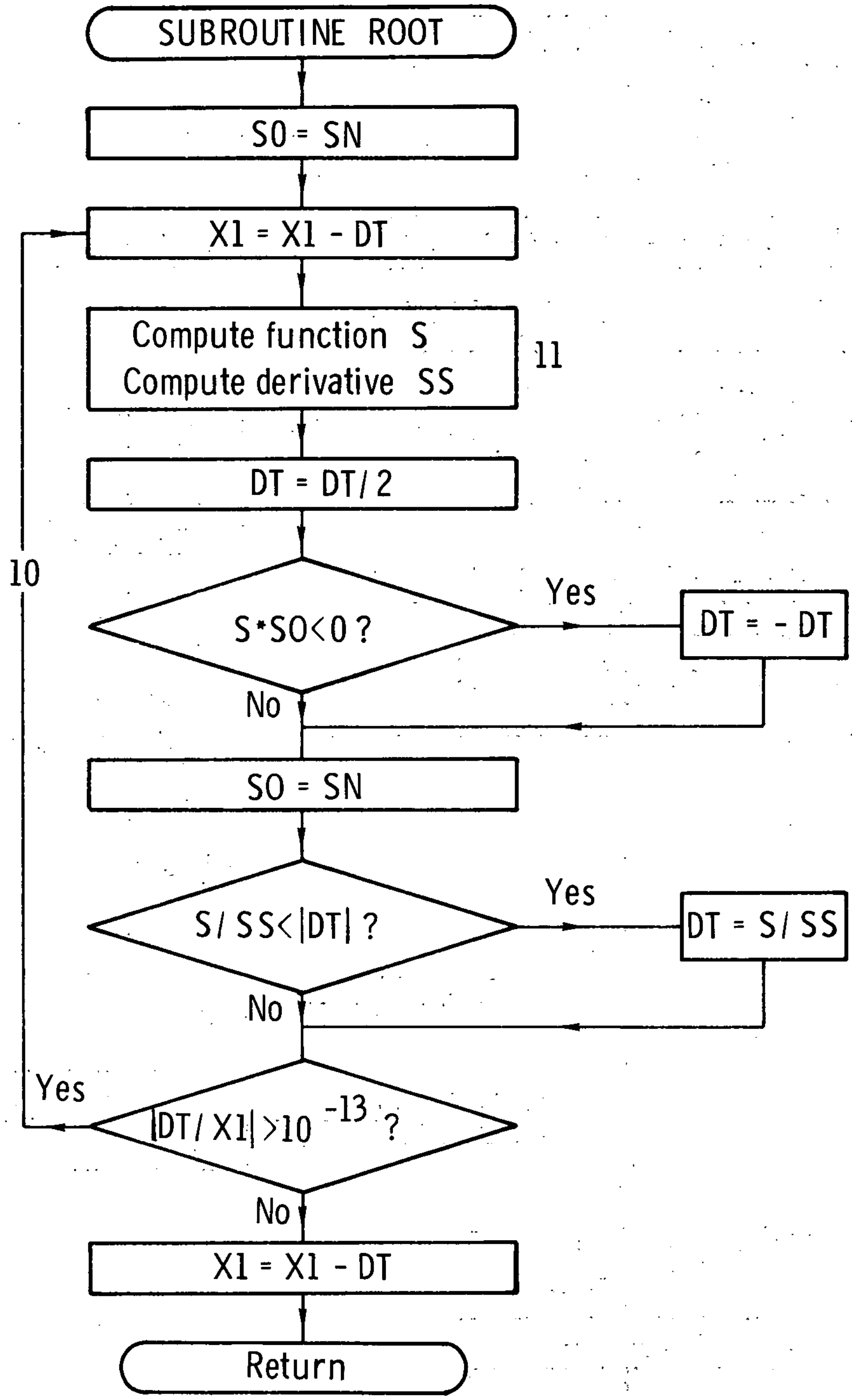
APPENDIX B

Flow Diagram of Subroutine HFUNC



APPENDIX B

Flow Diagram of Subroutine ROOT



APPENDIX C

DATA OUTPUT FOR COMPUTER PROGRAM

This appendix shows the results of the program for various parameters.

N= 30   X= 1.00   WU= 1.00

MU	H0(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.13657325	1.03582155
.10	1.24734962	1.05609911
.15	1.35083305	1.07122049
.20	1.45035104	1.08330765
.25	1.54732597	1.09333557
.30	1.64252208	1.10185853
.35	1.73640360	1.10922922
.40	1.82927553	1.11568861
.45	1.92134957	1.12140966
.50	2.01277878	1.12652112
.55	2.10367745	1.13112161
.60	2.19413309	1.13528837
.65	2.28421413	1.13908303
.70	2.37397503	1.14255555
.75	2.46345981	1.14574693
.80	2.55270447	1.14869126
.85	2.64173884	1.15141713
.90	2.73058785	1.15394873
.95	2.81927252	1.15630670
1.00	2.90781073	1.15850880

ALPHA0=2.00000000   ALPHA1=1.15470054   C=0.00000000

X	WU	MU0	THET	PHI	I0	I1	I	I/MU0
1.00	1.00	1.00	0.0	0.0	1.05692	0.00000	1.056920	1.0569
1.00	1.00	1.00	10.0	0.0	1.05516	0.00000	1.055163	1.0552
1.00	1.00	1.00	20.0	0.0	1.04975	0.00000	1.049752	1.0498
1.00	1.00	1.00	30.0	0.0	1.04025	0.00000	1.040251	1.0403
1.00	1.00	1.00	40.0	0.0	1.02583	0.00000	1.025826	1.0258
1.00	1.00	1.00	50.0	0.0	1.00505	0.00000	1.005051	1.0051
1.00	1.00	1.00	60.0	0.0	.97546	0.00000	.975463	.9755
1.00	1.00	1.00	70.0	0.0	.93251	0.00000	.932508	.9325
1.00	1.00	1.00	80.0	0.0	.86609	0.00000	.866092	.8661
1.00	1.00	1.00	90.0	0.0	.72695	0.00000	.726953	.7270

R= 1.000000

APPENDIX C

X	WO	MUO	THET	PHI	I0	I1	I	I/MUO
1.00	1.00	.80	0.0	0.0	.82475	0.00000	.824753	1.0309
1.00	1.00	.80	10.0	0.0	.82408	.01553	.839611	1.0495
1.00	1.00	.80	10.0	45.0	.82408	.01098	.835063	1.0438
1.00	1.00	.80	10.0	90.0	.82408	-.00000	.824083	1.0301
1.00	1.00	.80	10.0	135.0	.82408	-.01098	.813103	1.0164
1.00	1.00	.80	10.0	180.0	.82408	-.01553	.808555	1.0107
1.00	1.00	.80	20.0	0.0	.82200	.03132	.853322	1.0667
1.00	1.00	.80	20.0	45.0	.82200	.02215	.844148	1.0552
1.00	1.00	.80	20.0	90.0	.82200	-.00000	.822000	1.0275
1.00	1.00	.80	20.0	135.0	.82200	-.02215	.799851	.9998
1.00	1.00	.80	20.0	180.0	.82200	-.03132	.790677	.9883
1.00	1.00	.80	30.0	0.0	.81827	.04767	.865940	1.0824
1.00	1.00	.80	30.0	45.0	.81827	.03371	.851979	1.0650
1.00	1.00	.80	30.0	90.0	.81827	-.00000	.818273	1.0228
1.00	1.00	.80	30.0	135.0	.81827	-.03371	.784568	.9807
1.00	1.00	.80	30.0	180.0	.81827	-.04767	.770606	.9633
1.00	1.00	.80	40.0	0.0	.81245	.06488	.877327	1.0967
1.00	1.00	.80	40.0	45.0	.81245	.04588	.858325	1.0729
1.00	1.00	.80	40.0	90.0	.81245	-.00000	.812448	1.0156
1.00	1.00	.80	40.0	135.0	.81245	-.04588	.766572	.9582
1.00	1.00	.80	40.0	180.0	.81245	-.06488	.747569	.9345
1.00	1.00	.80	50.0	0.0	.80370	.08333	.887023	1.1088
1.00	1.00	.80	50.0	45.0	.80370	.05892	.862617	1.0783
1.00	1.00	.80	50.0	90.0	.80370	-.00000	.803695	1.0046
1.00	1.00	.80	50.0	135.0	.80370	-.05892	.744773	.9310
1.00	1.00	.80	50.0	180.0	.80370	-.08333	.720367	.9005
1.00	1.00	.80	60.0	0.0	.79047	.10345	.893911	1.1174
1.00	1.00	.80	60.0	45.0	.79047	.07315	.863613	1.0795
1.00	1.00	.80	60.0	90.0	.79047	-.00000	.790466	.9881
1.00	1.00	.80	60.0	135.0	.79047	-.07315	.717319	.8966
1.00	1.00	.80	60.0	180.0	.79047	-.10345	.687021	.8588
1.00	1.00	.80	70.0	0.0	.76960	.12568	.895280	1.1191
1.00	1.00	.80	70.0	45.0	.76960	.08887	.858468	1.0731
1.00	1.00	.80	70.0	90.0	.76960	-.00000	.769595	.9620
1.00	1.00	.80	70.0	135.0	.76960	-.08887	.680723	.8509
1.00	1.00	.80	70.0	180.0	.76960	-.12568	.643910	.8049
1.00	1.00	.80	80.0	0.0	.73320	.15019	.883396	1.1042
1.00	1.00	.80	80.0	45.0	.73320	.10620	.839405	1.0493
1.00	1.00	.80	80.0	90.0	.73320	-.00000	.733204	.9165
1.00	1.00	.80	80.0	135.0	.73320	-.10620	.627002	.7838
1.00	1.00	.80	80.0	180.0	.73320	-.15019	.583011	.7288
1.00	1.00	.80	90.0	0.0	.63818	.17230	.810480	1.0131
1.00	1.00	.80	90.0	45.0	.63818	.12184	.760013	.9500
1.00	1.00	.80	90.0	90.0	.63818	-.00000	.638176	.7977
1.00	1.00	.80	90.0	135.0	.63818	-.12184	.516339	.6454
1.00	1.00	.80	90.0	180.0	.63818	-.17230	.465872	.5823

R= 1.000000

APPENDIX C

X	WO	MUO	THET	PHI	I0	I1	I	I/MUO
1.00	1.00	.60	0.0	0.0	.59814	0.00000	.598137	.9969
1.00	1.00	.60	10.0	0.0	.59829	.01728	.615570	1.0260
1.00	1.00	.60	10.0	45.0	.59829	.01222	.610508	1.0175
1.00	1.00	.60	10.0	90.0	.59829	-.00000	.598287	.9971
1.00	1.00	.60	10.0	135.0	.59829	-.01222	.586065	.9768
1.00	1.00	.60	10.0	180.0	.59829	-.01728	.581003	.9683
1.00	1.00	.60	20.0	0.0	.59873	.03498	.633712	1.0562
1.00	1.00	.60	20.0	45.0	.59873	.02473	.623467	1.0391
1.00	1.00	.60	20.0	90.0	.59873	-.00000	.598734	.9979
1.00	1.00	.60	20.0	135.0	.59873	-.02473	.574000	.9567
1.00	1.00	.60	20.0	180.0	.59873	-.03498	.563755	.9396
1.00	1.00	.60	30.0	0.0	.59946	.05354	.653001	1.0883
1.00	1.00	.60	30.0	45.0	.59946	.03786	.637320	1.0622
1.00	1.00	.60	30.0	90.0	.59946	-.00000	.599463	.9991
1.00	1.00	.60	30.0	135.0	.59946	-.03786	.561606	.9360
1.00	1.00	.60	30.0	180.0	.59946	-.05354	.545925	.9099
1.00	1.00	.60	40.0	0.0	.60042	.07351	.673935	1.1232
1.00	1.00	.60	40.0	45.0	.60042	.05198	.652404	1.0873
1.00	1.00	.60	40.0	90.0	.60042	-.00000	.600425	1.0007
1.00	1.00	.60	40.0	135.0	.60042	-.05198	.548445	.9141
1.00	1.00	.60	40.0	180.0	.60042	-.07351	.526915	.8782
1.00	1.00	.60	50.0	0.0	.60148	.09561	.697088	1.1618
1.00	1.00	.60	50.0	45.0	.60148	.06761	.669085	1.1151
1.00	1.00	.60	50.0	90.0	.60148	-.00000	.601479	1.0025
1.00	1.00	.60	50.0	135.0	.60148	-.06761	.533873	.8898
1.00	1.00	.60	50.0	180.0	.60148	-.09561	.505870	.8431
1.00	1.00	.60	60.0	0.0	.60222	.12083	.723051	1.2051
1.00	1.00	.60	60.0	45.0	.60222	.08544	.687661	1.1461
1.00	1.00	.60	60.0	90.0	.60222	-.00000	.602223	1.0037
1.00	1.00	.60	60.0	135.0	.60222	-.08544	.516786	.8613
1.00	1.00	.60	60.0	180.0	.60222	-.12083	.481396	.8023
1.00	1.00	.60	70.0	0.0	.60145	.15059	.752041	1.2534
1.00	1.00	.60	70.0	45.0	.60145	.10648	.707934	1.1799
1.00	1.00	.60	70.0	90.0	.60145	-.00000	.601450	1.0024
1.00	1.00	.60	70.0	135.0	.60145	-.10648	.494966	.8249
1.00	1.00	.60	70.0	180.0	.60145	-.15059	.450859	.7514
1.00	1.00	.60	80.0	0.0	.59485	.18681	.781663	1.3028
1.00	1.00	.60	80.0	45.0	.59485	.13210	.726946	1.2116
1.00	1.00	.60	80.0	90.0	.59485	-.00000	.594849	.9914
1.00	1.00	.60	80.0	135.0	.59485	-.13210	.462752	.7713
1.00	1.00	.60	80.0	180.0	.59485	-.18681	.408035	.6801
1.00	1.00	.60	90.0	0.0	.54853	.22706	.775591	1.2927
1.00	1.00	.60	90.0	45.0	.54853	.16055	.709087	1.1818
1.00	1.00	.60	90.0	90.0	.54853	-.00000	.548533	.9142
1.00	1.00	.60	90.0	135.0	.54853	-.16055	.387979	.6466
1.00	1.00	.60	90.0	180.0	.54853	-.22706	.321476	.5358

R= 1.000000

APPENDIX C

N= 30   X= 1.00   WO= .80

MU	HO(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.08746267	1.02801646
.10	1.15012254	1.04362264
.15	1.20379348	1.05515674
.20	1.25163311	1.06431520
.25	1.29511719	1.07187305
.30	1.33511407	1.07826838
.35	1.37220207	1.08377841
.40	1.40679757	1.08859157
.45	1.43921685	1.09284248
.50	1.46970959	1.09663092
.55	1.49847867	1.10003301
.60	1.52569269	1.10310813
.65	1.55149420	1.10590353
.70	1.57600548	1.10845735
.75	1.59933263	1.11080084
.80	1.62156859	1.11295989
.85	1.64279546	1.11495616
.90	1.66308624	1.11680794
.95	1.68250626	1.11853081
1.00	1.70111427	1.12013812

ALPHAO=1.43657702   ALPHA1= .77178873   C= .14515179

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
1.00	.80	1.00	0.0	0.0	.14750	0.00000	.147495	.1475
1.00	.80	1.00	10.0	0.0	.14966	0.00000	.149662	.1497
1.00	.80	1.00	20.0	0.0	.15619	0.00000	.156194	.1562
1.00	.80	1.00	30.0	0.0	.16719	0.00000	.167185	.1672
1.00	.80	1.00	40.0	0.0	.18275	0.00000	.182745	.1827
1.00	.80	1.00	50.0	0.0	.20291	0.00000	.202912	.2029
1.00	.80	1.00	60.0	0.0	.22744	0.00000	.227437	.2274
1.00	.80	1.00	70.0	0.0	.25524	0.00000	.255241	.2552
1.00	.80	1.00	80.0	0.0	.28275	0.00000	.282750	.2827
1.00	.80	1.00	90.0	0.0	.29084	0.00000	.290839	.2908

R= .200170

APPENDIX C

X	W0	MU0	THET	PHI	I0	I1	I	I/MU0
1.00	.80	.80	0.0	0.0	.14190	0.00000	.141902	.1774
1.00	.80	.80	10.0	0.0	.14378	.01164	.155423	.1943
1.00	.80	.80	10.0	45.0	.14378	.00823	.152014	.1900
1.00	.80	.80	10.0	90.0	.14378	-.00000	.143784	.1797
1.00	.80	.80	10.0	135.0	.14378	-.00823	.135554	.1694
1.00	.80	.80	10.0	180.0	.14378	-.01164	.132145	.1652
1.00	.80	.80	20.0	0.0	.14948	.02349	.172971	.2162
1.00	.80	.80	20.0	45.0	.14948	.01661	.166091	.2076
1.00	.80	.80	20.0	90.0	.14948	-.00000	.149483	.1869
1.00	.80	.80	20.0	135.0	.14948	-.01661	.132874	.1661
1.00	.80	.80	20.0	180.0	.14948	-.02349	.125995	.1575
1.00	.80	.80	30.0	0.0	.15915	.03577	.194925	.2437
1.00	.80	.80	30.0	45.0	.15915	.02529	.184448	.2306
1.00	.80	.80	30.0	90.0	.15915	-.00000	.159154	.1989
1.00	.80	.80	30.0	135.0	.15915	-.02529	.133860	.1673
1.00	.80	.80	30.0	180.0	.15915	-.03577	.123383	.1542
1.00	.80	.80	40.0	0.0	.17304	.04874	.221785	.2772
1.00	.80	.80	40.0	45.0	.17304	.03447	.207508	.2594
1.00	.80	.80	40.0	90.0	.17304	-.00000	.173040	.2163
1.00	.80	.80	40.0	135.0	.17304	-.03447	.138572	.1732
1.00	.80	.80	40.0	180.0	.17304	-.04874	.124295	.1554
1.00	.80	.80	50.0	0.0	.19143	.06271	.254141	.3177
1.00	.80	.80	50.0	45.0	.19143	.04435	.235772	.2947
1.00	.80	.80	50.0	90.0	.19143	-.00000	.191426	.2393
1.00	.80	.80	50.0	135.0	.19143	-.04435	.147080	.1839
1.00	.80	.80	50.0	180.0	.19143	-.06271	.128712	.1609
1.00	.80	.80	60.0	0.0	.21451	.07605	.292561	.3657
1.00	.80	.80	60.0	45.0	.21451	.05519	.269700	.3371
1.00	.80	.80	60.0	90.0	.21451	-.00000	.214507	.2681
1.00	.80	.80	60.0	135.0	.21451	-.05519	.159314	.1991
1.00	.80	.80	60.0	180.0	.21451	-.07605	.136452	.1706
1.00	.80	.80	70.0	0.0	.24199	.09521	.337199	.4215
1.00	.80	.80	70.0	45.0	.24199	.06732	.309313	.3866
1.00	.80	.80	70.0	90.0	.24199	-.00000	.241992	.3025
1.00	.80	.80	70.0	135.0	.24199	-.06732	.174670	.2183
1.00	.80	.80	70.0	180.0	.24199	-.09521	.146784	.1835
1.00	.80	.80	80.0	0.0	.27168	.11452	.386201	.4828
1.00	.80	.80	80.0	45.0	.27168	.08098	.352658	.4408
1.00	.80	.80	80.0	90.0	.27168	-.00000	.271678	.3396
1.00	.80	.80	80.0	135.0	.27168	-.08098	.190697	.2384
1.00	.80	.80	80.0	180.0	.27168	-.11452	.157154	.1964
1.00	.80	.80	90.0	0.0	.28665	.13356	.420209	.5253
1.00	.80	.80	90.0	45.0	.28665	.09444	.381092	.4764
1.00	.80	.80	90.0	90.0	.28665	-.00000	.286654	.3583
1.00	.80	.80	90.0	135.0	.28665	-.09444	.192216	.2403
1.00	.80	.80	90.0	180.0	.28665	-.13356	.153099	.1914

R= .237571

APPENDIX C

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
1.00	.80	.60	0.0	0.0	.12609	0.00000	.126088	.2101
1.00	.80	.60	10.0	0.0	.12767	.01299	.140661	.2344
1.00	.80	.60	10.0	45.0	.12767	.00919	.136856	.2281
1.00	.80	.60	10.0	90.0	.12767	-.00000	.127669	.2128
1.00	.80	.60	10.0	135.0	.12767	-.00919	.118482	.1975
1.00	.80	.60	10.0	180.0	.12767	-.01299	.114677	.1911
1.00	.80	.60	20.0	0.0	.13248	.02630	.158786	.2646
1.00	.80	.60	20.0	45.0	.13248	.01860	.151082	.2518
1.00	.80	.60	20.0	90.0	.13248	-.00000	.132482	.2208
1.00	.80	.60	20.0	135.0	.13248	-.01860	.113882	.1898
1.00	.80	.60	20.0	180.0	.13248	-.02630	.106178	.1770
1.00	.80	.60	30.0	0.0	.14074	.04029	.181036	.3017
1.00	.80	.60	30.0	45.0	.14074	.02849	.169235	.2821
1.00	.80	.60	30.0	90.0	.14074	-.00000	.140744	.2346
1.00	.80	.60	30.0	135.0	.14074	-.02849	.112254	.1871
1.00	.80	.60	30.0	180.0	.14074	-.04029	.100453	.1674
1.00	.80	.60	40.0	0.0	.15283	.05539	.208220	.3470
1.00	.80	.60	40.0	45.0	.15283	.03916	.191997	.3200
1.00	.80	.60	40.0	90.0	.15283	-.00000	.152833	.2547
1.00	.80	.60	40.0	135.0	.15283	-.03916	.113669	.1894
1.00	.80	.60	40.0	180.0	.15283	-.05539	.097446	.1624
1.00	.80	.60	50.0	0.0	.16930	.07216	.241464	.4024
1.00	.80	.60	50.0	45.0	.16930	.05103	.220329	.3672
1.00	.80	.60	50.0	90.0	.16930	-.00000	.169302	.2822
1.00	.80	.60	50.0	135.0	.16930	-.05103	.118275	.1971
1.00	.80	.60	50.0	180.0	.16930	-.07216	.097140	.1619
1.00	.80	.60	60.0	0.0	.19088	.09143	.282313	.4705
1.00	.80	.60	60.0	45.0	.19088	.06465	.255534	.4259
1.00	.80	.60	60.0	90.0	.19088	-.00000	.190883	.3181
1.00	.80	.60	60.0	135.0	.19088	-.06465	.126232	.2104
1.00	.80	.60	60.0	180.0	.19088	-.09143	.099453	.1658
1.00	.80	.60	70.0	0.0	.21836	.11440	.332760	.5546
1.00	.80	.60	70.0	45.0	.21836	.08089	.299253	.4988
1.00	.80	.60	70.0	90.0	.21836	-.00000	.218360	.3639
1.00	.80	.60	70.0	135.0	.21836	-.08089	.137468	.2291
1.00	.80	.60	70.0	180.0	.21836	-.11440	.103961	.1733
1.00	.80	.60	80.0	0.0	.25172	.14285	.394571	.6576
1.00	.80	.60	80.0	45.0	.25172	.10101	.352730	.5879
1.00	.80	.60	80.0	90.0	.25172	-.00000	.251718	.4195
1.00	.80	.60	80.0	135.0	.25172	-.10101	.150705	.2512
1.00	.80	.60	80.0	180.0	.25172	-.14285	.108864	.1814
1.00	.80	.60	90.0	0.0	.27856	.17650	.455061	.7584
1.00	.80	.60	90.0	45.0	.27856	.12480	.403366	.6723
1.00	.80	.60	90.0	90.0	.27856	-.00000	.278564	.4643
1.00	.80	.60	90.0	135.0	.27856	-.12480	.153761	.2563
1.00	.80	.60	90.0	180.0	.27856	-.17650	.102066	.1701

R= .282650

APPENDIX C

N= 30   X= 0.00   WO= 1.00

MU	H0(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.13657325	1.00000000
.10	1.24734962	1.00000000
.15	1.35063305	1.00000000
.20	1.45035104	1.00000000
.25	1.54732597	1.00000000
.30	1.64252208	1.00000000
.35	1.73640360	1.00000000
.40	1.82927553	1.00000000
.45	1.92134957	1.00000000
.50	2.01277878	1.00000000
.55	2.10367745	1.00000000
.60	2.19413309	1.00000000
.65	2.28421413	1.00000000
.70	2.37397503	1.00000000
.75	2.46345981	1.00000000
.80	2.55270447	1.00000000
.85	2.64173884	1.00000000
.90	2.73058785	1.00000000
.95	2.81927252	1.00000000
1.00	2.90781073	1.00000000

ALPHA0=2.00000000   ALPHA1=1.15470054   C=0.00000000

X	W0	MU0	THET	PHI	I0	I1	I	I/MU0
0.00	1.00	1.00	0.0	0.0	1.05692	0.00000	1.056920	1.0569
0.00	1.00	1.00	10.0	0.0	1.05516	0.00000	1.055163	1.0552
0.00	1.00	1.00	20.0	0.0	1.04975	0.00000	1.049752	1.0498
0.00	1.00	1.00	30.0	0.0	1.04025	0.00000	1.040251	1.0403
0.00	1.00	1.00	40.0	0.0	1.02583	0.00000	1.025826	1.0258
0.00	1.00	1.00	50.0	0.0	1.00505	0.00000	1.005051	1.0051
0.00	1.00	1.00	60.0	0.0	.97546	0.00000	.975463	.9755
0.00	1.00	1.00	70.0	0.0	.93251	0.00000	.932508	.9325
0.00	1.00	1.00	80.0	0.0	.86609	0.00000	.866092	.8661
0.00	1.00	1.00	90.0	0.0	.72695	0.00000	.726953	.7270

R= 1.000000

APPENDIX C

λ	WO	MUO	THET	PHI	IO	I1	I	I/MUO
0.00	1.00	.80	0.0	0.0	.82475	0.00000	.824753	1.0309
0.00	1.00	.80	10.0	0.0	.82408	0.00000	.824083	1.0301
0.00	1.00	.80	20.0	0.0	.82200	0.00000	.822000	1.0275
0.00	1.00	.80	30.0	0.0	.81827	0.00000	.818273	1.0228
0.00	1.00	.80	40.0	0.0	.81245	0.00000	.812448	1.0156
0.00	1.00	.80	50.0	0.0	.80370	0.00000	.803695	1.0046
0.00	1.00	.80	60.0	0.0	.79047	0.00000	.790466	.9881
0.00	1.00	.80	70.0	0.0	.76960	0.00000	.769595	.9620
0.00	1.00	.80	80.0	0.0	.73320	0.00000	.733204	.9165
0.00	1.00	.80	90.0	0.0	.63818	0.00000	.638176	.7977

R= 1.000000

λ	WO	MUO	THET	PHI	IO	I1	I	I/MUO
0.00	1.00	.60	0.0	0.0	.59814	0.00000	.598137	.9969
0.00	1.00	.60	10.0	0.0	.59829	0.00000	.598287	.9971
0.00	1.00	.60	20.0	0.0	.59873	0.00000	.598734	.9979
0.00	1.00	.60	30.0	0.0	.59946	0.00000	.599463	.9991
0.00	1.00	.60	40.0	0.0	.60042	0.00000	.600425	1.0007
0.00	1.00	.60	50.0	0.0	.60148	0.00000	.601479	1.0025
0.00	1.00	.60	60.0	0.0	.60222	0.00000	.602223	1.0037
0.00	1.00	.60	70.0	0.0	.60145	0.00000	.601450	1.0024
0.00	1.00	.60	80.0	0.0	.59485	0.00000	.594849	.9914
0.00	1.00	.60	90.0	0.0	.54853	0.00000	.548533	.9142

R= 1.000000

APPENDIX C

N= 30 X= 0.00 WU= .80

MU	H0(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.08191330	1.00000000
.10	1.13880705	1.00000000
.15	1.18663969	1.00000000
.20	1.22863849	1.00000000
.25	1.26632228	1.00000000
.30	1.30058813	1.00000000
.35	1.33203394	1.00000000
.40	1.36108962	1.00000000
.45	1.38808059	1.00000000
.50	1.41326253	1.00000000
.55	1.43684200	1.00000000
.60	1.45898948	1.00000000
.65	1.47984811	1.00000000
.70	1.49953974	1.00000000
.75	1.51816929	1.00000000
.80	1.53582793	1.00000000
.85	1.55259560	1.00000000
.90	1.56854280	1.00000000
.95	1.58373216	1.00000000
1.00	1.59821955	1.00000000

ALPHA0=1.38196601 ALPHA1= .73581516 C=0.00000000

X	W0	MU0	THET	PHI	I0	I1	I	I/MU0
0.00	.80	1.00	0.0	0.0	.25543	0.00000	.255431	.2554
0.00	.80	1.00	10.0	0.0	.25669	0.00000	.256688	.2567
0.00	.80	1.00	20.0	0.0	.26048	0.00000	.260479	.2605
0.00	.80	1.00	30.0	0.0	.26684	0.00000	.266845	.2668
0.00	.80	1.00	40.0	0.0	.27582	0.00000	.275824	.2758
0.00	.80	1.00	50.0	0.0	.28737	0.00000	.287369	.2874
0.00	.80	1.00	60.0	0.0	.30116	0.00000	.301161	.3012
0.00	.80	1.00	70.0	0.0	.31611	0.00000	.316111	.3161
0.00	.80	1.00	80.0	0.0	.32876	0.00000	.328759	.3288
0.00	.80	1.00	90.0	0.0	.31964	0.00000	.319644	.3196

R= .285254

APPENDIX C

X	WU	MUO	THET	PHI	IO	I1	I	I/MUO
0.00	.80	.80	0.0	0.0	.21819	0.00000	.218186	.2727
0.00	.80	.80	10.0	0.0	.21945	0.00000	.219447	.2743
0.00	.80	.80	20.0	0.0	.22327	0.00000	.223269	.2791
0.00	.80	.80	30.0	0.0	.22977	0.00000	.229769	.2872
0.00	.80	.80	40.0	0.0	.23913	0.00000	.239125	.2989
0.00	.80	.80	50.0	0.0	.25154	0.00000	.251545	.3144
0.00	.80	.80	60.0	0.0	.26714	0.00000	.267142	.3339
0.00	.80	.80	70.0	0.0	.28558	0.00000	.285575	.3570
0.00	.80	.80	80.0	0.0	.30466	0.00000	.304656	.3808
0.00	.80	.80	90.0	0.0	.30717	0.00000	.307166	.3840

R= .313157

X	WU	MUO	THET	PHI	IO	I1	I	I/MUO
0.00	.80	.60	0.0	0.0	.17488	0.00000	.174884	.2915
0.00	.80	.60	10.0	0.0	.17608	0.00000	.176082	.2935
0.00	.80	.60	20.0	0.0	.17974	0.00000	.179737	.2996
0.00	.80	.60	30.0	0.0	.18604	0.00000	.186038	.3101
0.00	.80	.60	40.0	0.0	.19532	0.00000	.195315	.3255
0.00	.80	.60	50.0	0.0	.20806	0.00000	.208061	.3468
0.00	.80	.60	60.0	0.0	.22494	0.00000	.224938	.3749
0.00	.80	.60	70.0	0.0	.24666	0.00000	.246663	.4111
0.00	.80	.60	80.0	0.0	.27317	0.00000	.273174	.4553
0.00	.80	.60	90.0	0.0	.29180	0.00000	.291798	.4863

R= .347520

APPENDIX C

N= 30   X=-1.00   WO= 1.00

MU	HO(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.13657325	.97024002
.10	1.24734962	.95549387
.15	1.35083305	.94524536
.20	1.45035104	.93746059
.25	1.54732597	.93125460
.30	1.64252208	.92614881
.35	1.73640360	.92185205
.40	1.82927553	.91817326
.45	1.92134957	.91498020
.50	2.01277876	.91217756
.55	2.10367745	.90969448
.60	2.19413309	.90747693
.65	2.28421413	.90548284
.70	2.37397503	.90367886
.75	2.46345981	.90203816
.80	2.55270447	.90053888
.85	2.64173884	.89916298
.90	2.73058785	.89789546
.95	2.81927252	.89672369
1.00	2.90781073	.89563698

ALPHAO=2.00000000   ALPHA1=1.15470054   C=0.00000000

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
-1.00	1.00	1.00	0.0	0.0	1.05692	0.00000	1.056920	1.0569
-1.00	1.00	1.00	10.0	0.0	1.05516	0.00000	1.055163	1.0552
-1.00	1.00	1.00	20.0	0.0	1.04975	0.00000	1.049752	1.0498
-1.00	1.00	1.00	30.0	0.0	1.04025	0.00000	1.040251	1.0403
-1.00	1.00	1.00	40.0	0.0	1.02583	0.00000	1.025826	1.0258
-1.00	1.00	1.00	50.0	0.0	1.00505	0.00000	1.005051	1.0051
-1.00	1.00	1.00	60.0	0.0	.97546	0.00000	.975463	.9755
-1.00	1.00	1.00	70.0	0.0	.93251	0.00000	.932508	.9325
-1.00	1.00	1.00	80.0	0.0	.86609	0.00000	.866092	.8661
-1.00	1.00	1.00	90.0	0.0	.72695	0.00000	.726953	.7270

R= 1.000000

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
-1.00	1.00	.80	0.0	0.0	.82475	0.00000	.824753	1.0309
-1.00	1.00	.80	10.0	0.0	.82408	-.00942	.814663	1.0183
-1.00	1.00	.80	10.0	45.0	.82408	-.00666	.817422	1.0218
-1.00	1.00	.80	10.0	90.0	.82408	.00000	.824083	1.0301
-1.00	1.00	.80	10.0	135.0	.82408	.00666	.830744	1.0384
-1.00	1.00	.80	10.0	180.0	.82408	.00942	.833503	1.0419
-1.00	1.00	.80	20.0	0.0	.82200	-.01906	.802943	1.0037
-1.00	1.00	.80	20.0	45.0	.82200	-.01347	.808525	1.0107
-1.00	1.00	.80	20.0	90.0	.82200	.00000	.822000	1.0275
-1.00	1.00	.80	20.0	135.0	.82200	.01347	.835474	1.0443
-1.00	1.00	.80	20.0	180.0	.82200	.01906	.841056	1.0513
-1.00	1.00	.80	30.0	0.0	.81827	-.02915	.789125	.9864
-1.00	1.00	.80	30.0	45.0	.81827	-.02061	.797663	.9971
-1.00	1.00	.80	30.0	90.0	.81827	.00000	.818273	1.0228
-1.00	1.00	.80	30.0	135.0	.81827	.02061	.838884	1.0486
-1.00	1.00	.80	30.0	180.0	.81827	.02915	.847421	1.0593
-1.00	1.00	.80	40.0	0.0	.81245	-.03999	.772460	.9656
-1.00	1.00	.80	40.0	45.0	.81245	-.02828	.784172	.9802
-1.00	1.00	.80	40.0	90.0	.81245	.00000	.812448	1.0156
-1.00	1.00	.80	40.0	135.0	.81245	.02828	.840724	1.0509
-1.00	1.00	.80	40.0	180.0	.81245	.03999	.852437	1.0655
-1.00	1.00	.80	50.0	0.0	.80370	-.05197	.751726	.9397
-1.00	1.00	.80	50.0	45.0	.80370	-.03675	.766947	.9587
-1.00	1.00	.80	50.0	90.0	.80370	.00000	.803695	1.0046
-1.00	1.00	.80	50.0	135.0	.80370	.03675	.840443	1.0506
-1.00	1.00	.80	50.0	180.0	.80370	.05197	.855665	1.0696
-1.00	1.00	.80	60.0	0.0	.79047	-.06567	.724799	.9060
-1.00	1.00	.80	60.0	45.0	.79047	-.04643	.744032	.9300
-1.00	1.00	.80	60.0	90.0	.79047	.00000	.790466	.9881
-1.00	1.00	.80	60.0	135.0	.79047	.04643	.836900	1.0461
-1.00	1.00	.80	60.0	180.0	.79047	.06567	.856134	1.0702
-1.00	1.00	.80	70.0	0.0	.76960	-.08203	.687568	.8595
-1.00	1.00	.80	70.0	45.0	.76960	-.05800	.711593	.8895
-1.00	1.00	.80	70.0	90.0	.76960	.00000	.769595	.9620
-1.00	1.00	.80	70.0	135.0	.76960	.05800	.827597	1.0345
-1.00	1.00	.80	70.0	180.0	.76960	.08203	.851623	1.0645
-1.00	1.00	.80	80.0	0.0	.73320	-.10289	.630314	.7879
-1.00	1.00	.80	80.0	45.0	.73320	-.07275	.660449	.8256
-1.00	1.00	.80	80.0	90.0	.73320	.00000	.733204	.9165
-1.00	1.00	.80	80.0	135.0	.73320	.07275	.805958	1.0074
-1.00	1.00	.80	80.0	180.0	.73320	.10289	.836093	1.0451
-1.00	1.00	.80	90.0	0.0	.63818	-.13508	.503095	.6289
-1.00	1.00	.80	90.0	45.0	.63818	-.09552	.542660	.6783
-1.00	1.00	.80	90.0	90.0	.63818	.00000	.638176	.7977
-1.00	1.00	.80	90.0	135.0	.63818	.09552	.733693	.9171
-1.00	1.00	.80	90.0	180.0	.63818	.13508	.773257	.9666

R= 1.000000

APPENDIX C

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
-1.00	1.00	.60	0.0	0.0	.59814	0.00000	.598137	.9969
-1.00	1.00	.60	10.0	0.0	.59829	-.01069	.587596	.9793
-1.00	1.00	.60	10.0	45.0	.59829	-.00756	.590727	.9845
-1.00	1.00	.60	10.0	90.0	.59829	.00000	.598287	.9971
-1.00	1.00	.60	10.0	135.0	.59829	.00756	.605846	1.0097
-1.00	1.00	.60	10.0	180.0	.59829	.01069	.608977	1.0150
-1.00	1.00	.60	20.0	0.0	.59873	-.02170	.577036	.9617
-1.00	1.00	.60	20.0	45.0	.59873	-.01534	.583391	.9723
-1.00	1.00	.60	20.0	90.0	.59873	.00000	.598734	.9979
-1.00	1.00	.60	20.0	135.0	.59873	.01534	.614076	1.0235
-1.00	1.00	.60	20.0	180.0	.59873	.02170	.620431	1.0341
-1.00	1.00	.60	30.0	0.0	.59946	-.03338	.566083	.9435
-1.00	1.00	.60	30.0	45.0	.59946	-.02360	.575860	.9598
-1.00	1.00	.60	30.0	90.0	.59946	.00000	.599463	.9991
-1.00	1.00	.60	30.0	135.0	.59946	.02360	.623066	1.0384
-1.00	1.00	.60	30.0	180.0	.59946	.03338	.632842	1.0547
-1.00	1.00	.60	40.0	0.0	.60042	-.04620	.554229	.9237
-1.00	1.00	.60	40.0	45.0	.60042	-.03267	.567759	.9463
-1.00	1.00	.60	40.0	90.0	.60042	.00000	.600425	1.0007
-1.00	1.00	.60	40.0	135.0	.60042	.03267	.633091	1.0552
-1.00	1.00	.60	40.0	180.0	.60042	.04620	.646621	1.0777
-1.00	1.00	.60	50.0	0.0	.60148	-.06080	.540682	.9011
-1.00	1.00	.60	50.0	45.0	.60148	-.04299	.558489	.9308
-1.00	1.00	.60	50.0	90.0	.60148	.00000	.601479	1.0025
-1.00	1.00	.60	50.0	135.0	.60148	.04299	.644469	1.0741
-1.00	1.00	.60	50.0	180.0	.60148	.06080	.662277	1.1038
-1.00	1.00	.60	60.0	0.0	.60222	-.07820	.524018	.8734
-1.00	1.00	.60	60.0	45.0	.60222	-.05530	.546924	.9115
-1.00	1.00	.60	60.0	90.0	.60222	.00000	.602223	1.0037
-1.00	1.00	.60	60.0	135.0	.60222	.05530	.657523	1.0959
-1.00	1.00	.60	60.0	180.0	.60222	.07820	.680428	1.1340
-1.00	1.00	.60	70.0	0.0	.60145	-.10021	.501242	.8354
-1.00	1.00	.60	70.0	45.0	.60145	-.07086	.530592	.8843
-1.00	1.00	.60	70.0	90.0	.60145	.00000	.601450	1.0024
-1.00	1.00	.60	70.0	135.0	.60145	.07086	.672308	1.1205
-1.00	1.00	.60	70.0	180.0	.60145	.10021	.701659	1.1694
-1.00	1.00	.60	80.0	0.0	.59485	-.13049	.464363	.7739
-1.00	1.00	.60	80.0	45.0	.59485	-.09227	.502581	.8376
-1.00	1.00	.60	80.0	90.0	.59485	.00000	.594849	.9914
-1.00	1.00	.60	80.0	135.0	.59485	.09227	.687117	1.1452
-1.00	1.00	.60	80.0	180.0	.59485	.13049	.725335	1.2089
-1.00	1.00	.60	90.0	0.0	.54853	-.18150	.367038	.6117
-1.00	1.00	.60	90.0	45.0	.54853	-.12834	.420197	.7003
-1.00	1.00	.60	90.0	90.0	.54853	.00000	.548533	.9142
-1.00	1.00	.60	90.0	135.0	.54853	.12834	.676870	1.1281
-1.00	1.00	.60	90.0	180.0	.54853	.18150	.730029	1.2167

R= 1.000000

APPENDIX C

N= 30 X=-1.00 WU= .80

MU	H0(MU)	H1(MU)
0.00	1.00000000	1.00000000
.05	1.07710292	.97582090
.10	1.12908719	.96370523
.15	1.17202865	.95523751
.20	1.20920603	.94878001
.25	1.24216672	.94361658
.30	1.27182443	.93935823
.35	1.29878662	.93576746
.40	1.32348815	.93268795
.45	1.34625638	.93001116
.50	1.36734683	.92765870
.55	1.38696449	.92557216
.60	1.40527733	.92370692
.65	1.42242530	.92202815
.70	1.43852664	.92050823
.75	1.45368239	.91912489
.80	1.46797974	.91785996
.85	1.48149453	.91669843
.90	1.49429322	.91562781
.95	1.50643439	.91463756
1.00	1.51797001	.91371876

ALPHA0=1.33809785 ALPHA1= .70716547 C=-.12172548

X	WU	MU0	THET	PHI	I0	I1	I	I/MU0
-1.00	.80	1.00	0.0	0.0	.33260	0.00000	.332605	.3326
-1.00	.80	1.00	10.0	0.0	.33326	0.00000	.333258	.3333
-1.00	.80	1.00	20.0	0.0	.33522	0.00000	.335218	.3352
-1.00	.80	1.00	30.0	0.0	.33848	0.00000	.338481	.3385
-1.00	.80	1.00	40.0	0.0	.34300	0.00000	.343005	.3430
-1.00	.80	1.00	50.0	0.0	.34864	0.00000	.348639	.3486
-1.00	.80	1.00	60.0	0.0	.35495	0.00000	.354951	.3550
-1.00	.80	1.00	70.0	0.0	.36075	0.00000	.360754	.3608
-1.00	.80	1.00	80.0	0.0	.36256	0.00000	.362556	.3626
-1.00	.80	1.00	90.0	0.0	.34055	0.00000	.340549	.3405

R= .346774

APPENDIX C

X	WO	MUO	THET	PHI	IO	I1	I	I/MUO
-1.00	.80	.80	0.0	0.0	.27317	0.00000	.273167	.3415
-1.00	.80	.80	10.0	0.0	.27401	-.00784	.266179	.3327
-1.00	.80	.80	10.0	45.0	.27401	-.00554	.268474	.3356
-1.00	.80	.80	10.0	90.0	.27401	.00000	.274014	.3425
-1.00	.80	.80	10.0	135.0	.27401	.00554	.279555	.3494
-1.00	.80	.80	10.0	180.0	.27401	.00784	.281850	.3523
-1.00	.80	.80	20.0	0.0	.27658	-.01585	.260734	.3259
-1.00	.80	.80	20.0	45.0	.27658	-.01121	.265376	.3317
-1.00	.80	.80	20.0	90.0	.27658	.00000	.276582	.3457
-1.00	.80	.80	20.0	135.0	.27658	.01121	.287788	.3597
-1.00	.80	.80	20.0	180.0	.27658	.01585	.292430	.3655
-1.00	.80	.80	30.0	0.0	.28095	-.02423	.256716	.3209
-1.00	.80	.80	30.0	45.0	.28095	-.01713	.263814	.3298
-1.00	.80	.80	30.0	90.0	.28095	.00000	.280949	.3512
-1.00	.80	.80	30.0	135.0	.28095	.01713	.298084	.3726
-1.00	.80	.80	30.0	180.0	.28095	.02423	.305181	.3815
-1.00	.80	.80	40.0	0.0	.28723	-.03323	.254003	.3175
-1.00	.80	.80	40.0	45.0	.28723	-.02349	.263735	.3297
-1.00	.80	.80	40.0	90.0	.28723	.00000	.287230	.3590
-1.00	.80	.80	40.0	135.0	.28723	.02349	.310725	.3884
-1.00	.80	.80	40.0	180.0	.28723	.03323	.320457	.4006
-1.00	.80	.80	50.0	0.0	.29554	-.04315	.252397	.3155
-1.00	.80	.80	50.0	45.0	.29554	-.03051	.265034	.3313
-1.00	.80	.80	50.0	90.0	.29554	.00000	.295544	.3694
-1.00	.80	.80	50.0	135.0	.29554	.03051	.326054	.4076
-1.00	.80	.80	50.0	180.0	.29554	.04315	.338691	.4234
-1.00	.80	.80	60.0	0.0	.30590	-.05445	.251448	.3143
-1.00	.80	.80	60.0	45.0	.30590	-.03850	.267397	.3342
-1.00	.80	.80	60.0	90.0	.30590	.00000	.305901	.3824
-1.00	.80	.80	60.0	135.0	.30590	.03850	.344405	.4305
-1.00	.80	.80	60.0	180.0	.30590	.05445	.360354	.4504
-1.00	.80	.80	70.0	0.0	.31785	-.06789	.249967	.3125
-1.00	.80	.80	70.0	45.0	.31785	-.04800	.269850	.3373
-1.00	.80	.80	70.0	90.0	.31785	.00000	.317852	.3973
-1.00	.80	.80	70.0	135.0	.31785	.04800	.365854	.4573
-1.00	.80	.80	70.0	180.0	.31785	.06789	.385738	.4822
-1.00	.80	.80	80.0	0.0	.32913	-.08485	.244281	.3054
-1.00	.80	.80	80.0	45.0	.32913	-.05999	.269131	.3364
-1.00	.80	.80	80.0	90.0	.32913	.00000	.329126	.4114
-1.00	.80	.80	80.0	135.0	.32913	.05999	.389121	.4864
-1.00	.80	.80	80.0	180.0	.32913	.08485	.413971	.5175
-1.00	.80	.80	90.0	0.0	.32219	-.11014	.212043	.2651
-1.00	.80	.80	90.0	45.0	.32219	-.07788	.244303	.3054
-1.00	.80	.80	90.0	90.0	.32219	.00000	.322186	.4027
-1.00	.80	.80	90.0	135.0	.32219	.07788	.400069	.5001
-1.00	.80	.80	90.0	180.0	.32219	.11014	.432330	.5404

R= .368286

APPENDIX C

X	MO	MUO	THET	PHI	I0	I1	I	I/MUO
-1.00	.80	.60	0.0	0.0	.21035	0.00000	.210346	.3506
-1.00	.80	.60	10.0	0.0	.21129	-.00888	.202408	.3373
-1.00	.80	.60	10.0	45.0	.21129	-.00628	.205009	.3417
-1.00	.80	.60	10.0	90.0	.21129	.00000	.211288	.3521
-1.00	.80	.60	10.0	135.0	.21129	.00628	.217568	.3626
-1.00	.80	.60	10.0	180.0	.21129	.00888	.220169	.3669
-1.00	.80	.60	20.0	0.0	.21417	-.01802	.196149	.3269
-1.00	.80	.60	20.0	45.0	.21417	-.01274	.201427	.3357
-1.00	.80	.60	20.0	90.0	.21417	.00000	.214169	.3569
-1.00	.80	.60	20.0	135.0	.21417	.01274	.226911	.3782
-1.00	.80	.60	20.0	180.0	.21417	.01802	.232189	.3870
-1.00	.80	.60	30.0	0.0	.21915	-.02771	.191436	.3191
-1.00	.80	.60	30.0	45.0	.21915	-.01960	.199553	.3326
-1.00	.80	.60	30.0	90.0	.21915	.00000	.219149	.3652
-1.00	.80	.60	30.0	135.0	.21915	.01960	.238746	.3979
-1.00	.80	.60	30.0	180.0	.21915	.02771	.246863	.4114
-1.00	.80	.60	40.0	0.0	.22651	-.03833	.188180	.3136
-1.00	.80	.60	40.0	45.0	.22651	-.02711	.199408	.3323
-1.00	.80	.60	40.0	90.0	.22651	.00000	.226514	.3775
-1.00	.80	.60	40.0	135.0	.22651	.02711	.253621	.4227
-1.00	.80	.60	40.0	180.0	.22651	.03833	.264848	.4414
-1.00	.80	.60	50.0	0.0	.23669	-.05041	.186283	.3105
-1.00	.80	.60	50.0	45.0	.23669	-.03565	.201048	.3351
-1.00	.80	.60	50.0	90.0	.23669	.00000	.236693	.3945
-1.00	.80	.60	50.0	135.0	.23669	.03565	.272338	.4539
-1.00	.80	.60	50.0	180.0	.23669	.05041	.287103	.4785
-1.00	.80	.60	60.0	0.0	.25026	-.06476	.185499	.3092
-1.00	.80	.60	60.0	45.0	.25026	-.04579	.204468	.3408
-1.00	.80	.60	60.0	90.0	.25026	.00000	.250263	.4171
-1.00	.80	.60	60.0	135.0	.25026	.04579	.296058	.4934
-1.00	.80	.60	60.0	180.0	.25026	.06476	.315027	.5250
-1.00	.80	.60	70.0	0.0	.26784	-.08282	.185022	.3084
-1.00	.80	.60	70.0	45.0	.26784	-.05856	.209280	.3488
-1.00	.80	.60	70.0	90.0	.26784	.00000	.267844	.4464
-1.00	.80	.60	70.0	135.0	.26784	.05856	.326409	.5440
-1.00	.80	.60	70.0	180.0	.26784	.08282	.350667	.5844
-1.00	.80	.60	80.0	0.0	.28927	-.10746	.181811	.3030
-1.00	.80	.60	80.0	45.0	.28927	-.07599	.213285	.3555
-1.00	.80	.60	80.0	90.0	.28927	.00000	.289270	.4821
-1.00	.80	.60	80.0	135.0	.28927	.07599	.365256	.6088
-1.00	.80	.60	80.0	180.0	.28927	.10746	.396730	.6612
-1.00	.80	.60	90.0	0.0	.30158	-.14779	.153789	.2563
-1.00	.80	.60	90.0	45.0	.30158	-.10451	.197077	.3285
-1.00	.80	.60	90.0	90.0	.30158	.00000	.301582	.5026
-1.00	.80	.60	90.0	135.0	.30158	.10451	.406088	.6768
-1.00	.80	.60	90.0	180.0	.30158	.14779	.449376	.7490

R= .395269

REFERENCES

1. Chandrasekhar, S.: Radiative Transfer. Dover Publ., Inc., c.1960.

2. Ralston, Anthony: A First Course in Numerical Analysis. McGraw-Hill Book Co., Inc., c.1965.

3. Abramowitz, Milton; and Stegun, Irene A., eds.: Handbook of Mathematical Functions. Dover Publ., Inc., 1965.

TABLE I. - $H^{(0)}$ FUNCTIONS OBTAINED BY THE METHOD OF REFERENCE 1 AND PRESENT METHOD

μ	x = 0				x = 1	
	$\omega_0 = 0.8$		$\omega_0 = 1.0$		$\omega_0 = 0.8$	
	Method of reference 1	Present method	Method of reference 1	Present method	Method of reference 1	Present method
0.05	1.0820	1.081 91	1.1368	1.136 57	1.0876	1.087 46
.10	1.1388	1.138 81	1.2474	1.247 35	1.1501	1.150 12
.20	1.2286	1.228 64	1.4503	1.450 35	1.2516	1.251 63
.50	1.4132	1.413 27	2.0128	2.012 78	1.4697	1.469 71
1.00	1.5982	1.598 22	2.9078	2.907 81	1.7011	1.701 11

TABLE II. - $H^{(1)}$ FUNCTIONS OBTAINED BY THE METHOD OF REFERENCE 1 AND PRESENT METHOD

μ	$x\omega_0 = 1.0$		$x\omega_0 = 0.8$		$x\omega_0 = -0.8$		$x\omega_0 = -1.0$	
	Method of reference 1	Present method	Method of reference 1	Present method	Method of reference 1	Present method	Method of reference 1	Present method
0.05	1.0359	1.035 82	1.0281	1.028 02	0.9758	0.975 82	0.9702	0.970 24
.10	1.0561	1.056 10	1.0436	1.043 62	.9637	.963 71	.9555	.955 49
.20	1.0832	1.083 31	1.0642	1.064 32	.9488	.948 78	.9375	.937 46
.50	1.1265	1.126 52	1.0966	1.096 63	.9277	.927 66	.9122	.912 18
1.00	1.1586	1.158 52	1.1201	1.120 14	.9137	.913 72	.8956	.895 64

NASA-Langley, 1975 L-10153

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546