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MODULATION LINEARIZATION OF A FREQUENCY-MODULATED VOLTAGE-CONTROLLED OSCILLATOR

Prepared by

ELECTRONICS RESEARCH LABORATORY

M. A. HONNELL, PROJECT LEADER

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APPROVED BY:

J. D. Irwin, Head
Electrical Engineering

SUBMITTED BY:

M. A. Honnell, Professor
Electrical Engineering

FOREWORD

This report is a technical summary presenting the final results of a study by the Electrical Engineering Department, Auburn University, under the auspices of the Engineering Experiment Station, toward fulfillment of the requirements in NASA Contract NAS8-26193. The report describes studies made concerning the task order entitled "Frequency Stabilization and Modulation Techniques for High Frequency and High Data Rate Telecommunications."

Part III of the report presented herein describes an analytical method for determining frequency deviation and frequency deviation linearity for the stable FM oscillator previously described in the FINAL REPORT, Part I, dated September 1974.

MODULATION LINEARIZATION OF A

FREQUENCY-MODULATED VOLTAGE-CONTROLLED

OSCILLATOR

M. A. Honnell and R. E. Lee

An analysis is presented for the voltage-versus-frequency characteristics of a varactor-modulated VHF voltage-controlled oscillator in which the frequency deviation is linearized by using the nonlinear characteristic of a field-effect transistor as a signal amplifier. Equations developed are used to calculate the oscillator output frequency in terms of pertinent circuit parameters. It is shown that the nonlinearity exponent of the FET has a pronounced influence on frequency deviation linearity whereas the junction exponent of the varactor controls total frequency deviation for a given input signal. A design example for a 250-MHz frequency-modulated oscillator is presented.

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I. INTRODUCTION

Prior work conducted under NASA contract NAS8-26193 presented in Part I of the 1974 FINAL REPORT established an operating model of a linear stable FM oscillator operating at 115 MHz, with a deviation in excess of ± 10 MHz. In the basic circuit shown in Fig. 1, the nonlinear characteristic of the FET amplifier is used to linearize the frequency deviation of the push-pull oscillator.

The objective of the present study was to examine, in detail, the factors affecting frequency deviation and frequency deviation linearity of this oscillator as determined by the varactor and FET amplifier characteristics. Circuit defining equations were written which predict the oscillator output frequency for any given amplitude of input modulating signal.

A BASIC computer program was written for the HP2000E time-shared computer. The program outputs both frequency and normalized frequency for any given input signal between zero volts and V_p, the pinch-off voltage of the FET in the signal amplifier.

Using the BASIC program, the modulation circuit was modelled to study the effects of circuit and device parameters on linearity and deviation.

The parameters studied include: oscillator and signal FET driver supply voltages; drain resistor value; tuned-circuit inductance and stray capacitance; FET pinch-off voltage, drain saturation current

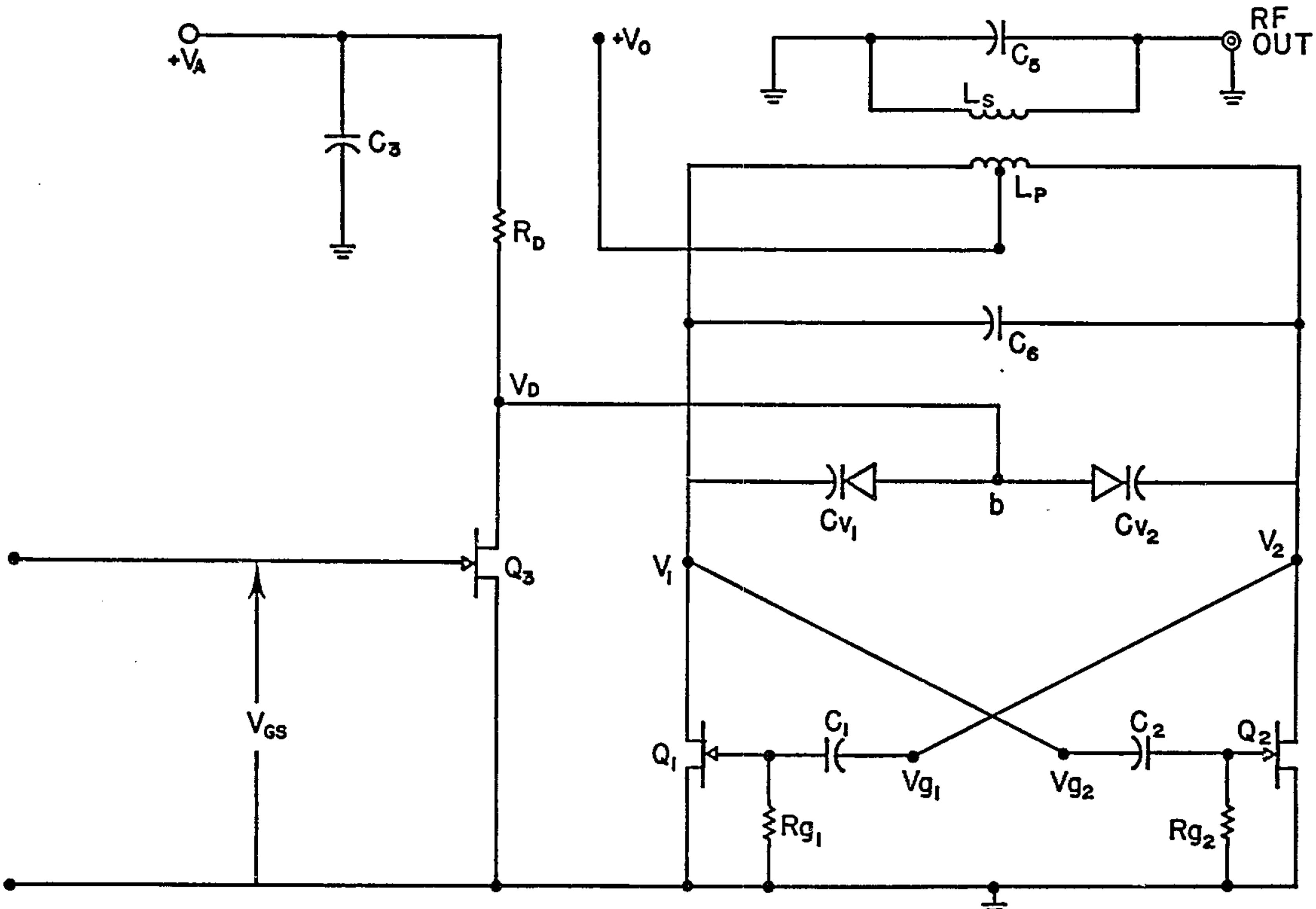

Figure 1. Basic oscillator circuit

(I_{DSS}) and FET exponent; VCO varactor maximum and minimum capacitance and junction exponent.

For a given FET and varactor, frequency linearity and deviation may be adjusted over a wide range by selecting an appropriate operating point on the varactor C-V curve. This is accomplished by independently adjusting the signal amplifier FET drain voltage (V_A), the oscillator voltage (V_o), and the FET drain resistor (R_D) of Figure 1.

The FET exponent was found to have a pronounced effect on deviation linearity, whereas the varactor exponent had control over total deviation. Computations showed that varactors with exponent values between 1 and 2 produce less deviation for a given set of conditions than varactors with exponent values of 0.5.

A detailed design for a 250-MHz oscillator is presented using techniques developed in this work.

II. FACTORS AFFECTING LINEARITY AND DEVIATION

Previous work [1] with the push-pull frequency-modulated voltage-controlled oscillator indicated that the nonlinearity of frequency deviation could be minimized by empirically using the FET nonlinear characteristic to compensate for the nonlinear characteristic of the varactor diodes and the inverse square-root relationship of the frequency equation. In order to more fully understand the factors affecting linearization, circuit equations were developed which relate the various circuit parameters.

With reference to Figure 1, Q_3 is the input compensating amplifier FET and C_{V1} and C_{V2} are the varactors. The voltage $V_o - V_D$ determines the varactor reverse bias, hence the oscillator frequency, since L_p, C_6 and the varactor capacitance form a frequency-determining LC network.

For purposes of this study, only dc input signals are considered at the gate of Q_3. The use of dc permits static point-by-point frequency measurements to be taken as the gate voltage V_{GS} is varied.

Figure 2 is a simplified version of the frequency-determining components of Figure 1. Figure 2 also shows the dc test setup used for measuring static deviation. For each increment of V_{GS}, the frequency is read on the frequency counter.

A typical linearized experimental plot of frequency as a function of V_{GS} is shown in Figure 3.

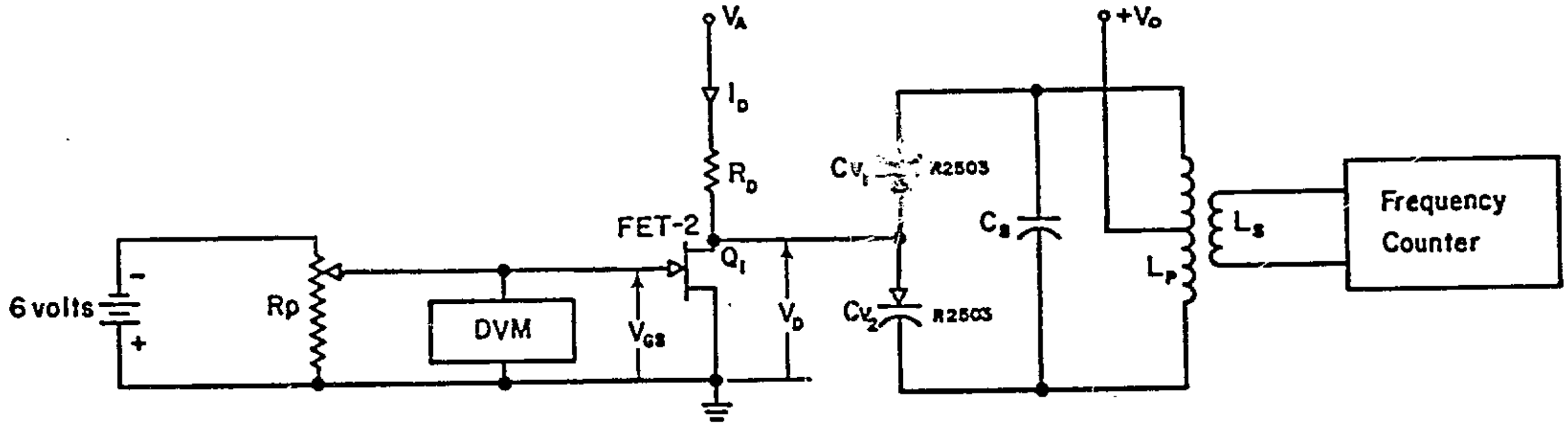

Figure 2. Test setup for measuring static frequency deviation

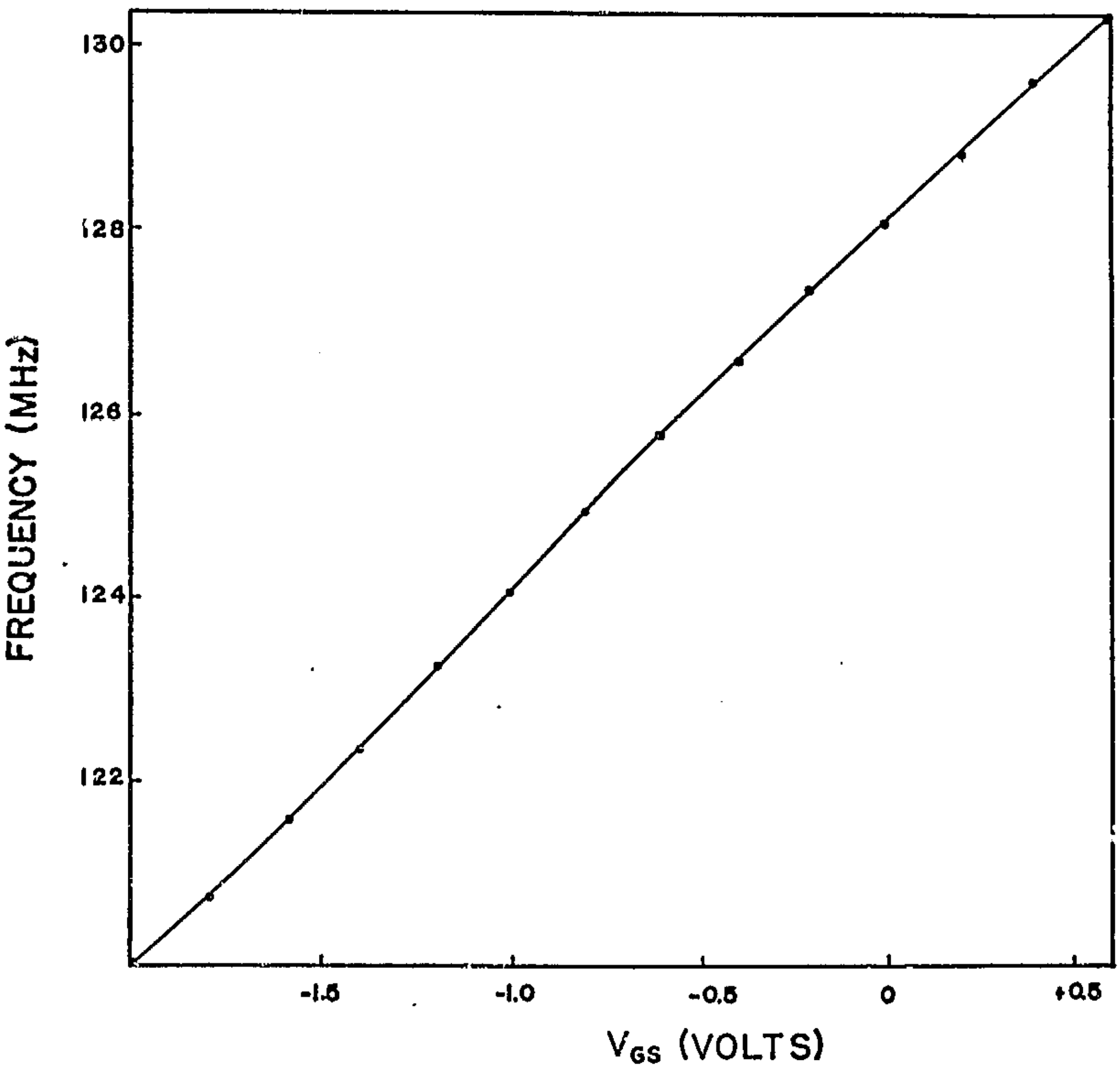

Figure 3. A typical linearized frequency deviation plot

Many such experimental plots were made for various conditions of supply voltages and circuit parameters in an attempt to optimize deviation linearity. It is very difficult to empirically optimize a given circuit variable, since the interaction of the linear and non-linear circuit elements is very complex. However, since the circuit equations for the FET amplifier and the varactor capacitors were known, a mathematical analysis of the circuit was easily carried out.

With reference to Figure 2, the following may be written:

$$\omega^2 = 1/(L_p C_T) \tag{1}$$

$$C_T = C_s + C(V)/2 = \text{total circuit capacitance} \tag{2}$$

It is assumed that the varactors follow the relationship

$$C(V) = C_{pk} + C_o(1 + V_R/V_T)^{-n} \tag{3}$$

Where C_{pk} is the varactor package capacitance, V_R is the reverse voltage, V_T is a constant (0.7V for silicon) and C_o and n are constants to be evaluated. (2) may now be written:

$$C_T = C_s + C_{pk}/2 + (C_o/2)(1 + V_R/V_T)^{-n} \tag{4}$$

$$V_R = V_o - V_A + I_D R_D \tag{5}$$

$$I_D = I_{DSS}(1 - V_{GS}/V_p)^m \tag{6}$$

I_{DSS}, V_p and m are constants to be determined. Equation (1) is solved for frequency in terms of the circuit parameters as follows:

$$f = (1/2\pi)\sqrt{2A/[L_p(AC_p + C_o)]} \tag{7}$$

$$A = \left\{1 + (1/V_T)[V_o - V_A + R_D I_{DSS}(1 - V_{GS}/V_p)^m]\right\}^n \tag{8}$$

$$= (1 + V_R/V_T)^n \tag{8a}$$

$$C_p = C_{pk} + 2C_s \tag{9}$$

A derivation of (7) and other related equations is presented in the Appendix. (7) and (8) were programmed in BASIC. The program prints out the frequency for incremental increases in V_{GS}. Circuit and device parameters may be easily adjusted by modifying the program. Typical output data for a 130-MHz oscillator is shown in Table 1. Table 2 is a copy of the BASIC program.

```
RUN
FETLEE

VO-VA              1/VP               FET EXP            VAR EXP
 10                 .36                1.61               .531

     L                 CP              RD
 8.80000E-08        1.80000E-11        1000

  IDSS                 CO
 .0099              9.43000E-11
```

V(VOLTS)	A(DIM'LESS)	F(MHZ)	F(NORMAL)
0	6.02753	130.808	100
.2	5.85088	129.9	99.3054
.4	5.6779	128.974	98.598
.6	5.50916	128.037	97.8816
.8	5.3453	127.091	97.1582
1	5.18711	126.142	96.4332
1.2	5.03548	125.199	95.7117
1.4	4.89146	124.27	95.0015
1.6	4.75632	123.366	94.3107
1.8	4.63154	122.505	93.6521
2	4.51895	121.702	93.0388
2.2	4.42082	120.984	92.4894
2.4	4.34018	120.379	92.0269
2.6	4.28164	119.931	91.6849

DONE

Table 1. Typical computer output data for a 130-MHz oscillator.

```
LIST
FETLEE

10  DIM A[50],F[50]
20  READ B,C,D,E,F,G,H,J,K
30  DATA 10
40  DATA .36
50  DATA 1.61
60  DATA .531
70  DATA 8.8E-08
80  DATA 1.8E-11
90  DATA 1000
100  DATA .0099
110  DATA 9.43E-11
120  PRINT "VO-VA","1/VP","FET EXP","VAR EXP"
130  PRINT B,C,D,E
140  PRINT
150  PRINT "      L","      CP","  RD"
160  PRINT F,G,H
170  PRINT
180  PRINT "  IDSS","       CO"
190  PRINT J,K
200  PRINT
210  PRINT
220  FOR I=0 TO 26
230  A[I+2]=(1+1.43*(B+H*J*(1-C*I/10)^D))^E
240  Z=A[I+2]
250  F[I+2]=1/6.2832*(2*Z/(F*G*Z+F*K))^.5
260  NEXT I
270  PRINT "V(VOLTS)","A(DIM'LESS)","F(MHZ)","F(NORMAL)"
280  FOR I=1 TO 14
290  PRINT 2*(I-1)/10,A[2*I],F[2*I]/1.E+06,F[2*I]/F[2]*100
300  NEXT I
310  END
```

Table 2. BASIC computer program for 130-MHz oscillator.

III. DETERMINATION OF FET PARAMETERS

For the FET driver, from the previous section

$$I_D = I_{DSS}(1 - V_{GS}/V_p)^m \qquad (6)$$

Where the value for m may be determined from experimental data.

Differentiating (6) with respect to V_{GS} and evaluating at $V_{GS} = 0$, we obtain

$$(dI_D/dV_{GS})\Big|_{V_{GS} = 0} = g_{mo} = mI_{DSS}/|V_p| \qquad (10)$$

Solving for m we obtain

$$m = g_{mo}|V_p|/I_{DSS} \qquad (11)$$

The values for g_{mo}, V_p and I_{DSS} may be taken from the manufacturers' data sheets or obtained experimentally.

The exponent m may also be obtained from (6) directly to yield

$$m = [\mathrm{Log}(I_D/I_{DSS})]/\mathrm{Log}[(1 - V_{GS})/V_p] \qquad (12)$$

Experimental plots of I_D as a function of V_{GS} for a randomly selected FET-2 and 2N4416 are shown in Figure 4. With reference to Figure 4, I_{DSS} is the value of I_D for $V_{GS} = 0$, while V_p is that value of V_{GS} for which $I_D = 0$. Values for m, the FET exponent, may now be calculated from either (11) or (12).

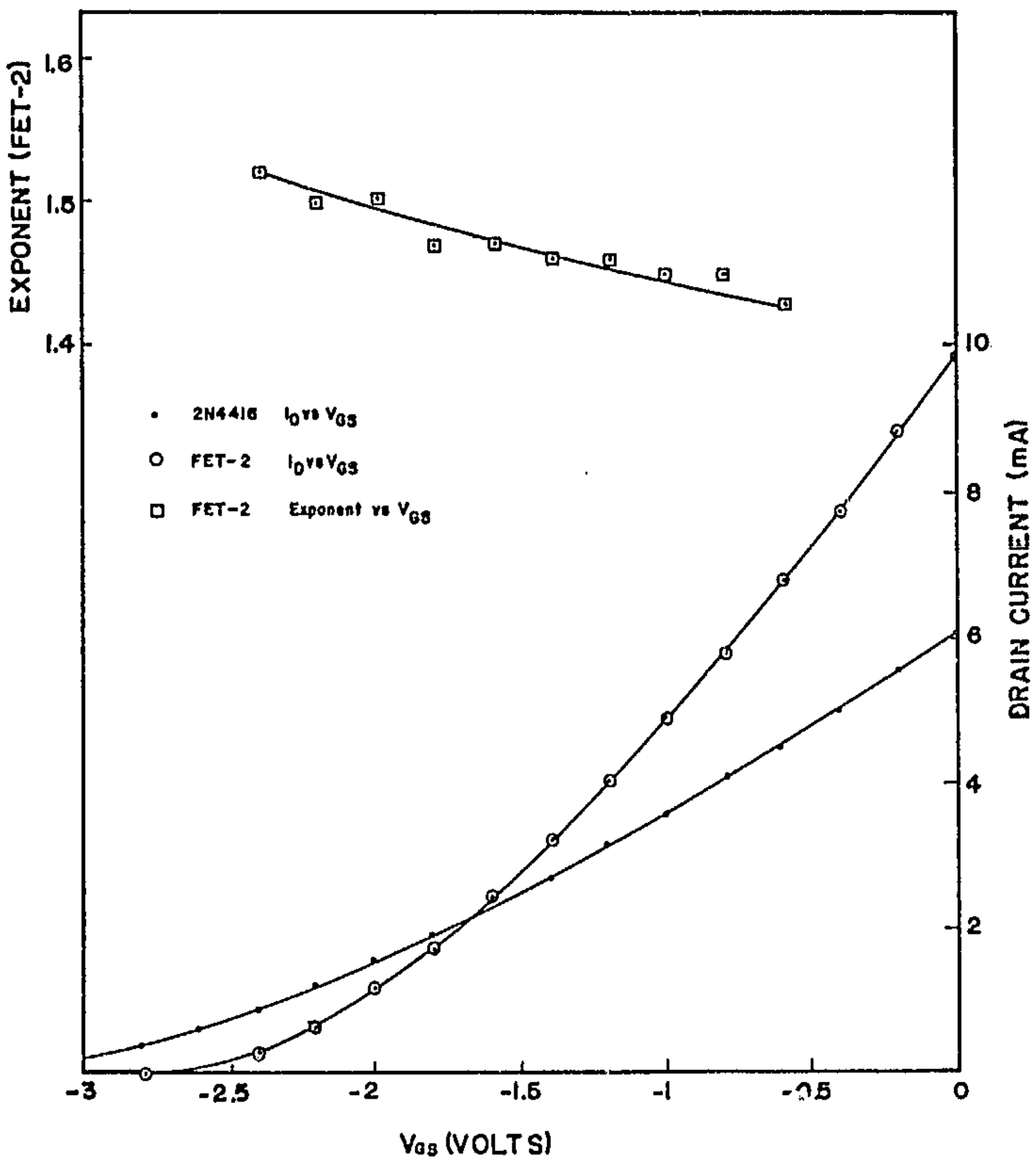

Figure 4. Experimental drain current plots for 2 randomly selected FETs. Variations in the exponent m for the FET-2 are also shown.

Using (11), the exponent m was calculated for both transistors and found to be 1.61 for the FET-2 and 1.60 for the 2N4416. This is rather close agreement in view of the large differences in I_{DSS}, V_p and g_{mo} for the two units.

Using (12), m was calculated again for the FET-2. This time V_{GS} was varied over a range of several volts. For each value of V_{GS}, I_D was measured and used to calculate m from (12). The results are shown in Figure 4.

IV. DETERMINATION OF VARACTOR PARAMETERS

The varactor capacitors used in the FM oscillator were assumed to follow the relation

$$C(V) = C_{pk} + C_o(1 + V_R/V_T)^{-n} \tag{13}$$

Where C_{pk} is the varactor package capacitance, C_o is the capacitance for $V_R = 0$, V_T is the diode junction potential, n is the junction exponent and V_R is the magnitude of the back bias. The diode exponent, n, is determined by the nature of the diode junction. Normally, n varies from approximately 0.33 for graded junctions to 1.0 or higher, for hyper-abrupt junctions. Step junctions have intermediate values for n on the order of 0.5.

By measuring C(V) as a function of the reverse voltage V_R at several points on the C-V curve, it is possible to evaluate accurately the parameters V_T, C_o, C_{pk} and n using graphical techniques [3], [4].

In the alternate method used, C_o and n may be conveniently estimated from the above C-V data. If we ignore C_{pk}, the package capacitance, and take $V_T = 0.7$, Equation (13) may be solved for n

$$n \doteq [\mathrm{Log}(C_o/C(V))]/\mathrm{Log}(1 + V_R/V_T) \tag{14}$$

Using this method, the R2503 varactors used in the 130-MHz oscillator were found to have $C_o = 94.3$ pF and n values ranging from 0.483 to 0.547, the mean being 0.531. Figure 5 shows the experimental C-V curve used to

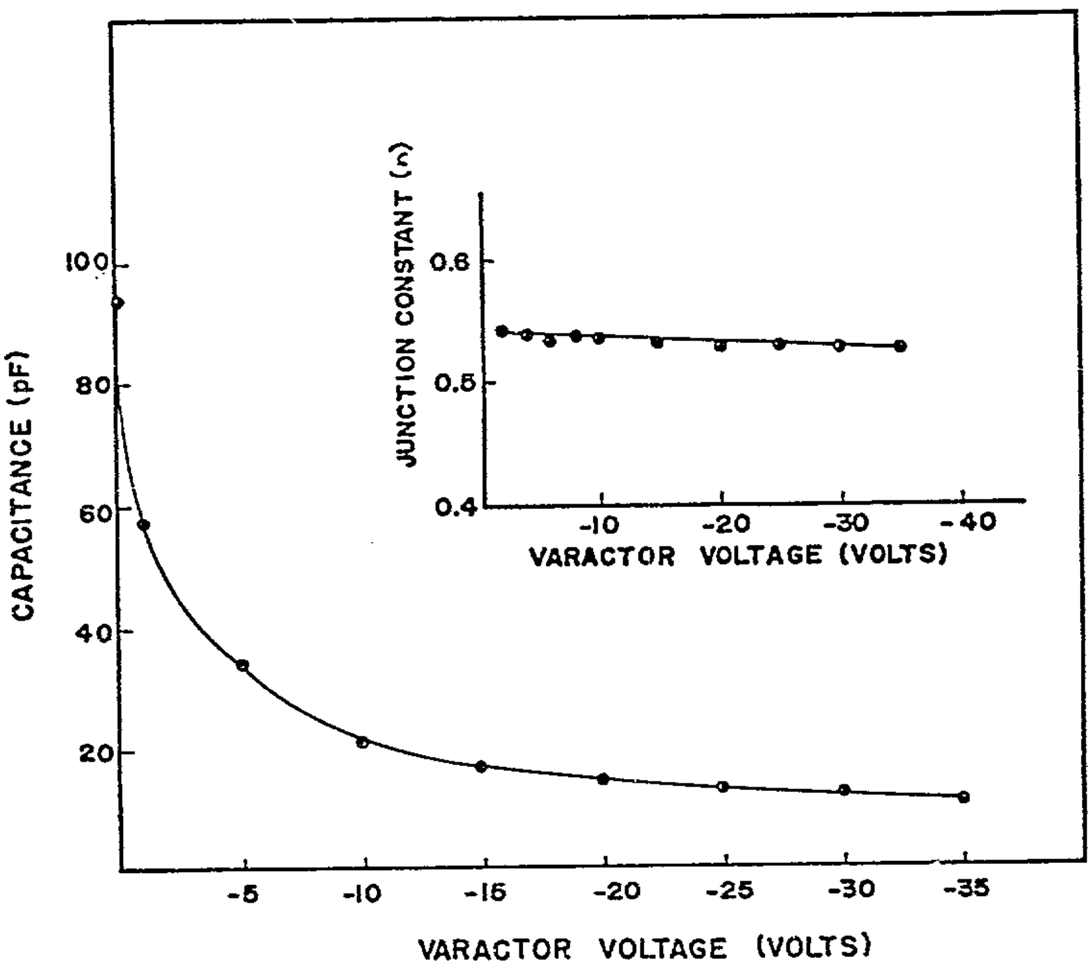

Figure 5. Experimental C-V plot for the R2503 varactor showing variations in the value of the diffusion exponent n.

obtain values for C_o and n in the analysis of the 130-MHz oscillator. The junction exponent n is relatively constant over a wide range of values of reverse bias. This data is also plotted in Figure 5.

V. ANALYSIS OF CIRCUIT PARAMETERS

Once the FET and varactor parameters are known, (7) and (8) may be used to calculate the frequency of oscillation in terms of the circuit parameters and driver FET input voltage, V_{GS}.

For a given set of conditions, the output frequency is plotted as a function of V_{GS}. Such frequency plots yield linearity and deviation information. It is possible to optimize circuit performance by varying one parameter at a time and noting the effect on linearity, deviation and center frequency. As a computational convenience, much of the work was done by computer.

Equations (7) and (8) may also be used to calculate the value of one or more unknown circuit constants for a specific design.

When using (7) and (8) for design purposes, it is helpful to find the range of the variable A by evaluating (8) at $V_{GS} = 0$ and $V_{GS} = V_p$. Equation (7) may then be solved for frequency at these two endpoints. Solving (8) for $V_{GS} = 0$

$$A = [1 + (1/V_T)(V_o - V_A + R_D I_{DSS})]^n \tag{15a}$$

and for $V_{GS} = V_p$

$$A = [1 + (1/V_T)(V_o - V_A)]^n \tag{15b}$$

Where V_p is the FET pinch-off voltage, n is the varactor exponent and V_T is the varactor junction potential taken as 0.7 volts. It should be noted that these equations are independent of m, the FET exponent.

Equation (7) may be rewritten as

$$\omega^2 = 2/[L_p(C_p + C_o/A)] \tag{16}$$

where C_o is the varactor capacitance at $V_R = 0$. The role of A in determining the frequency of oscillation now becomes more apparent. That is, ω increases with increasing values for A, and the effective value of C_o in the tuned output circuit is reduced by the factor 1/A. Rewriting (8a)

$$A = (1 + V_R/V_T)^n \tag{8a}$$

V_R is the varactor reverse-bias voltage. The minimum value for A is unity at $V_R = 0$. This condition represents the <u>minimum</u> frequency of oscillation. An examination of (15b) reveals this condition occurs when $V_o = V_A$. Likewise, the <u>maximum</u> frequency will occur when (15a) is a maximum.

An examination of (16) reveals that as A increases, C_o has progressively less effect on determining the frequency of oscillation. In the limit, L_p and C_p set the upper frequency bound.

To calculate the frequency sensitivity of the circuit for small changes in V_{GS} near the origin ($V_{GS} \doteq 0$) we may write $d\omega/dV_{GS} = (d\omega/dA)(dA/dV_{GS})$ radians per volt. By taking the FET exponent m in (8) to be 2, and writing $(1 - V_{GS}/V_p)^2 \doteq 1 - 2V_{GS}/V_p$ for $V_{GS} << V_p$, we may obtain dA/dV_{GS} from (8). The quantity $d\omega/dA$ is obtained by differentiating (16). By multiplying the derivatives we obtain

$$d\omega/dV_{GS} = -\beta L_p C_o n\omega^3/[4(\alpha - \beta V_{GS})^{n+1}] \qquad \text{rad/volt} \tag{17}$$

where

$$\alpha = 1 + (1/V_T)(V_o - V_A + R_D I_{DSS}) \tag{17a}$$

$$\beta = 2R_D I_{DSS}/(V_T V_p) \text{ volts}^{-1} \tag{17b}$$

$$A = (\alpha - \beta V_{GS})^n \tag{17c}$$

Evaluating (17) at the origin gives the initial slope of the frequency versus V_{GS} curve

$$(d\omega_o/dV_{GS})_{V_{GS}=0} = -\beta L_p C_o n \omega_o^3/(4\alpha^{n+1}) \quad \text{rad/volt} \tag{18}$$

where ω_o is the frequency corresponding to $V_{GS} = 0$. These derivations are shown in the Appendix.

VI. EXPERIMENTAL RESULTS

The 130-MHz oscillator constructed has the following parameters:

V_p = 2.8 volts (measured)

FET exponent (m) = 1.61 (calculated)

varactor exponent (n) = 0.531 (calculated)

L_p = 0.088 μH (measured)

C_p = 18 pF (measured)

C_o = 94.3 pF (measured)

R_D = 1000 ohms

V_o = 31.5 volts

V_A = 21.5 volts

I_{DSS} = 9.9 mA

The above parameters were selected experimentally for optimum circuit performance. Typical frequency plots are shown in Figures 3 and 6.

Values for the experimental circuit parameters were substituted into (7) and (8) and the equations solved for frequency as a function of V_{GS}. The data is plotted in Figure 6. The experimental data and computed data differ by approximately 0.5% at the highest frequency and 0.2% at the lowest frequency. Slight variations in C_o, C_p and L_p can account for these differences. In each case, the deviation is approximately 4 MHz per volt and the linearity is better than ± 1% for a 2-volt variation in V_{GS} [2].

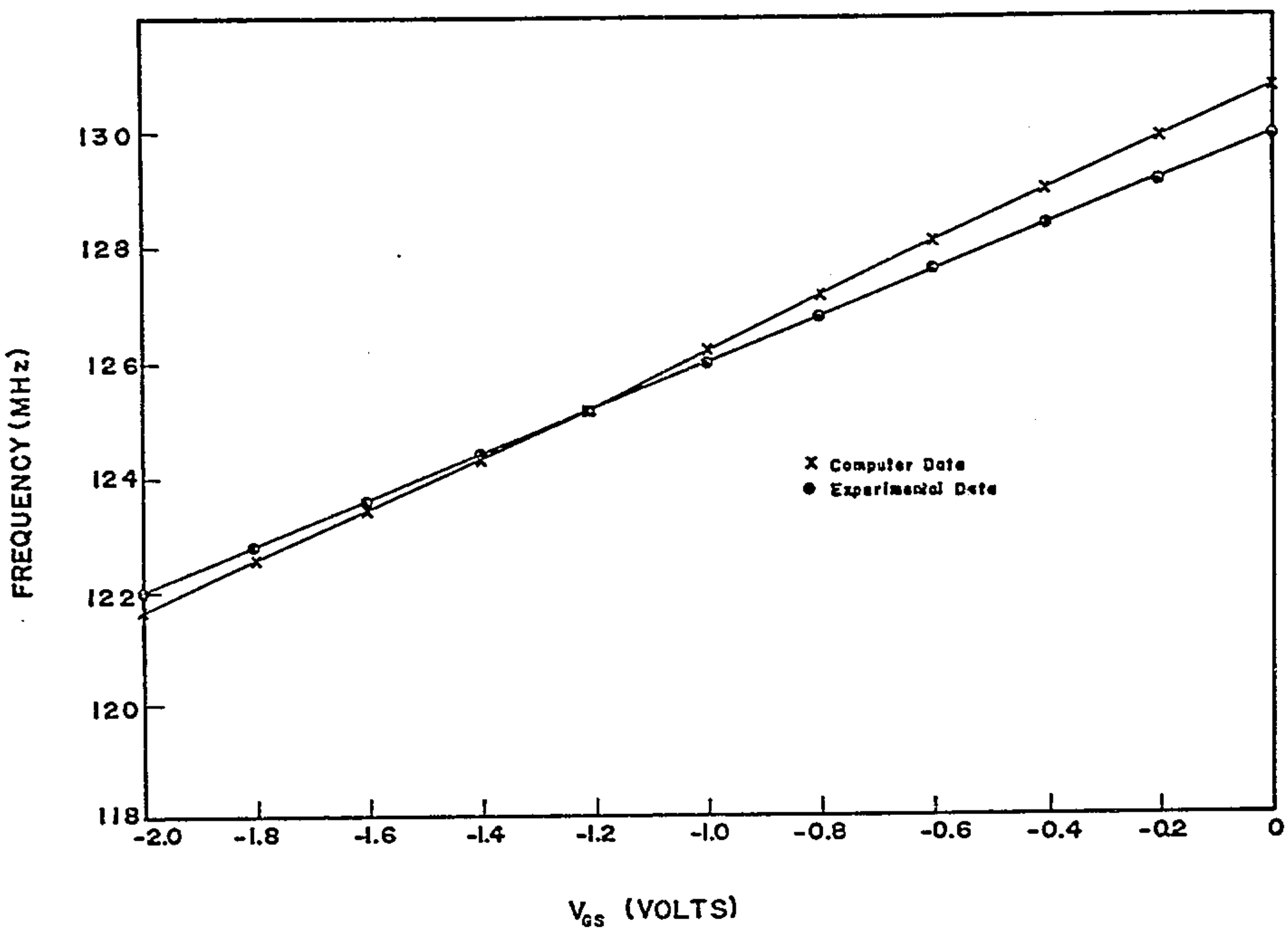

Figure 6. Frequency versus V_{GS} for the 130 MHz experimental oscillator. Experimental and calculated data.

Using (7) and (8) with the circuit parameters for the experimental oscillator, it is possible to predict f_{max}, f_{min} and deviation sensitivity for the experimental model with a high degree of accuracy.

For convenience (7) and (8) were programmed in BASIC using an HP2000E computer. The program for the experimental oscillator is shown in Table 2, the frequency output data in Table 1. The program prints out values for the variables A and frequency for each data point, although only frequency data is used for plotting purposes.

The following shows how the previously developed relations are used to calculate the experimental oscillator performance. From (15a)

$$A = [1 + 1.43(10 + 9.9)]^{0.531} = 6.027 \qquad (V_{GS} = 0)$$

From (15b) we get

$$A = [1 + 1.43(10)]^{0.531} = 4.257 \qquad (V_{GS} = V_p)$$

From (16)

$$f_{max} = (1/2\pi)\sqrt{2/[L_p(C_p + C_o/6.027)]} = 130.81 \text{ MHz}$$

and

$$f_{min} = (1/2\pi)\sqrt{2/[L_p(C_p + C_o/4.257)]} = 119.74 \text{ MHz}$$

from (18)

$$(df_o/dV_{GS})\Big|_{V_{GS} = 0} = -(1/2\pi)\beta L_p C_o n\omega_o^{3}/(4\alpha^{n+1}) \text{ MHz/volt} \qquad (19)$$

$\alpha = 1 + 1.43(10 + 9.9) = 29.46$

$\beta = 2(9.9)/[(0.7)(-2.78)] = -10.17 \quad \text{volt}^{-1}$

$\omega_o^{\ 3} = [2\pi(130.81)]^3 = 5.55 \times 10^{26} \quad \text{rad}^3/\text{sec}^3$

From (19), $df_o/dV_{GS} = 5.57$ MHz/volt at $V_{GS} = 0$. This is a higher value of df_o/dV_{GS} then observed in the experimental curve. This is due to the approximation, m = 2, in the derivation of (17) and (18) whereas Figure 6 is based on m = 1.61. In a later computer plot for m = 2 and other parameters as in the experimental oscillator, the slope was found to be 5.5 MHz/volt as predicted.

Taking $df/dV_{GS} \doteq (f_{high} - f_{low})/\Delta V_{GS}$ gives a sensitivity value of 4 MHz/volt, which is consistent with the experimental value obtained from Figure 6.

VII. EFFECTS OF CIRCUIT PARAMETER VARIATIONS ON DEVIATION AND LINEARITY

A detailed study of the effects of circuit parameter variations on deviation, linearity and center frequency was conducted. The parameter values used initially were those of the 130-MHz experimental oscillator.

Using the BASIC program of Table 2, one parameter at a time was varied in the program and a corresponding frequency versus V_{GS} plot constructed. The effects of these parameter variations on oscillator circuit performance were investigated for the following parameters:

FET exponent (m)

Varactor exponent (n)

Supply voltage difference (V_o - V_A)

FET drain resistor (R_D)

FET pinchoff voltage (V_p)

Circuit capacitance (C_p)

Tuned-circuit inductance (L_p)

In each case, the parameter of interest was varied over a sufficiently wide range to give a clear indication of its effect on frequency, deviation and linearity. Frequency plots for variations in the above parameters are shown in Figures 7-17. For convenience, much of the frequency data, for a given parameter variation, was normalized so that f_{max} = 100.

A. FET EXPONENT (m)

Figure 7 is a plot of frequency as a function of V_{GS} for m values ranging from 1.0 to 2.0. This data did not require normalization since the oscillator frequencies for $V_{GS} = 0$ and $V_{GS} = V_p$ are independent of the FET exponent. In other words, changes in m have no effect on f_{high}, f_{low} or total deviation. The exponent m, however, has a pronounced influence on linearity. For example, m = 1.5 shows better deviation linearity than the remaining curves, while m = 2 has the steepest slope. It is to be noted that m = 1.61 for the FET-2.

The curve m = 1 is included to show the effect of having a "linear" FET in the driver stage. The frequency linearity is poor since the FET nonlinear characteristic is needed in order to correct for the nonlinear capacitance variation of the varactors.

B. VARACTOR EXPONENT (n)

Figure 8 shows the effect of variations in n, the varactor exponent, for n equal to 0.33, 0.5 and 1.0. As n increases, f_{max} also increases. Over the range V_{GS} = 0 to -2 volts the deviation is quite linear for n = 0.33 and n = 0.5. As stated earlier, $n \doteq 0.33$ corresponds to a graded type junction for the varactor. Step junctions have exponents around 0.5 while n values between 1 and 2 correspond to hyper-abrupt junctions.

Figure 9 is a normalized plot of the same data. This plot reveals the rather unexpected result that the frequency deviation increases for n values between 0.33 and 0.50, then rapidly decreases as n approaches 1.0.

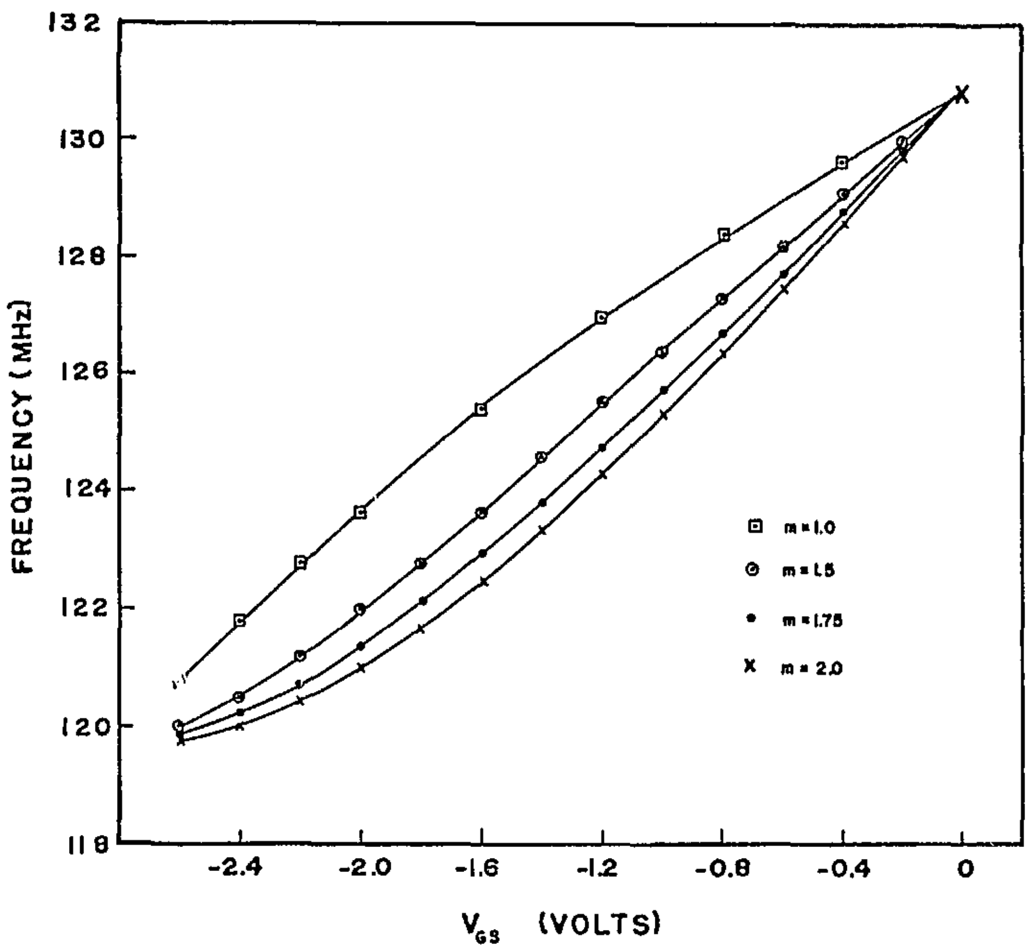

Figure 7. Effect of variations in m, the FET exponent, on frequency deviation. (Calculated data.)

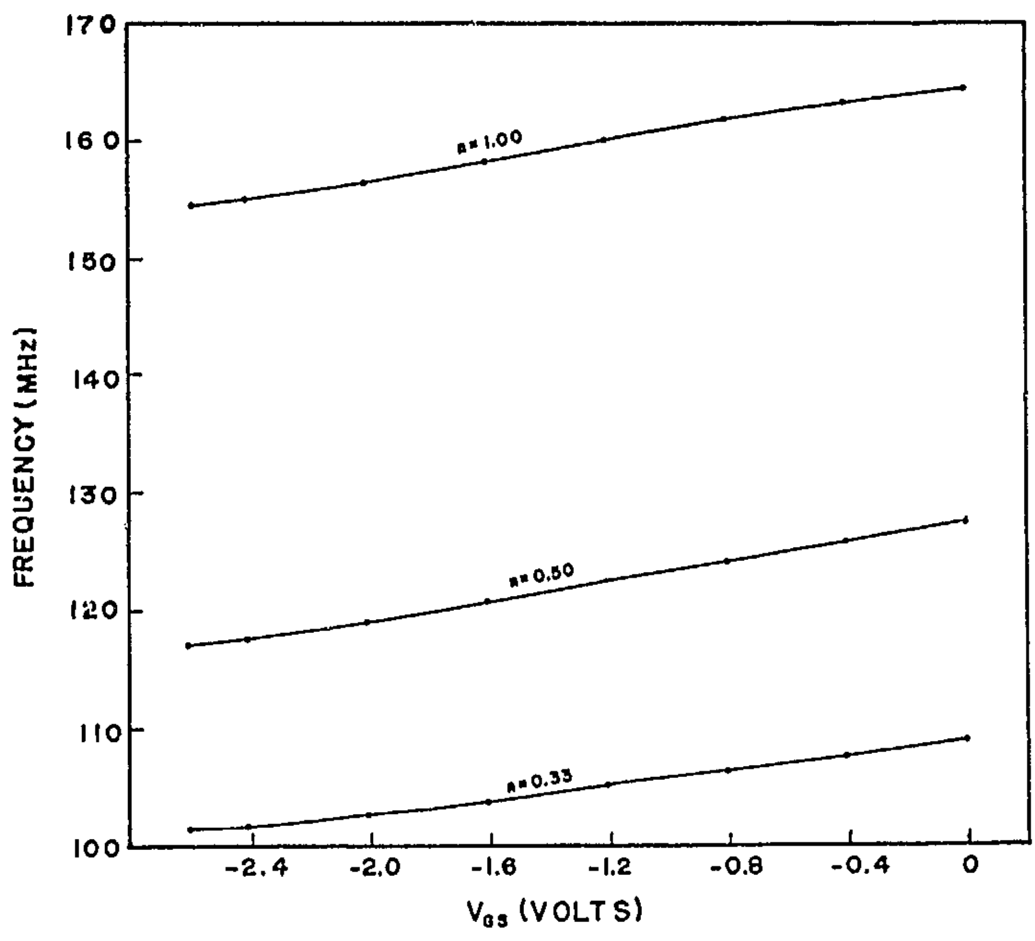

Figure 8. Effect of variations in n, the varactor junction exponent, on frequency deviation. (Calculated data.)

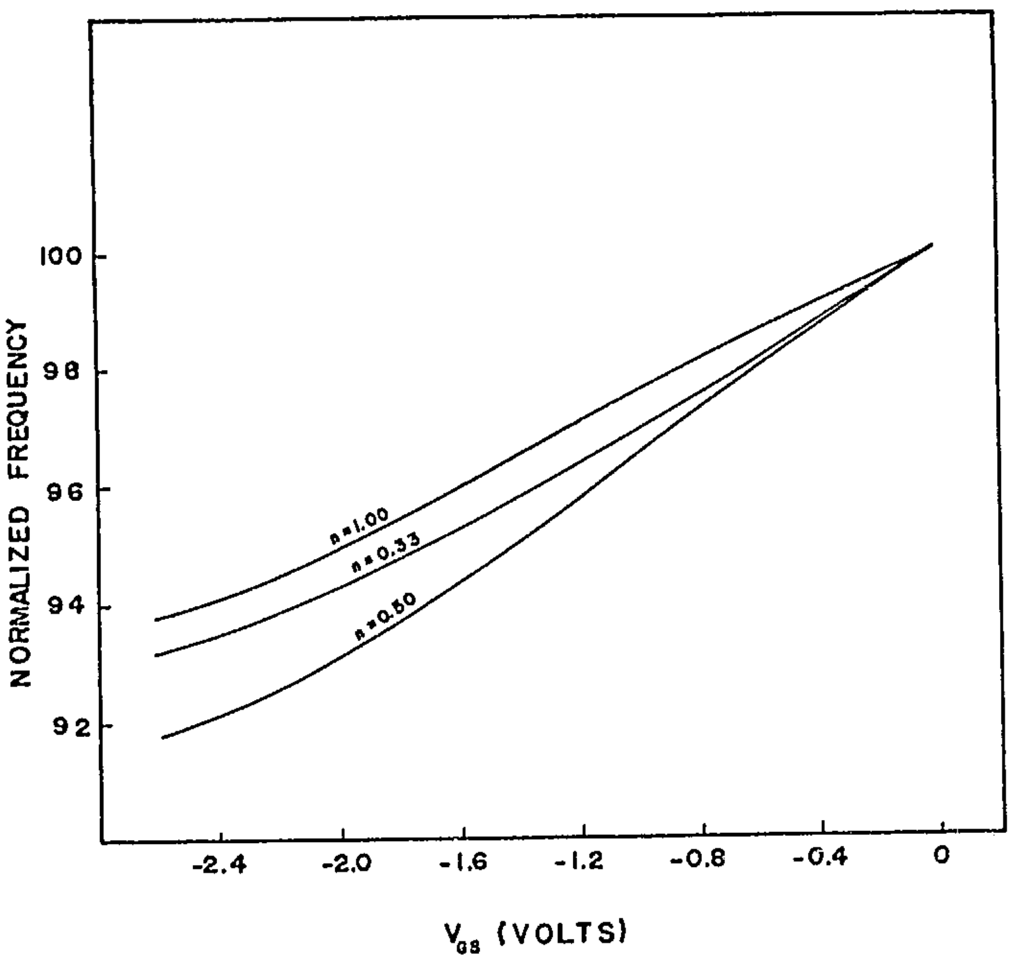

Figure 9. Normalized plot of Figure 8.

Further data was taken for n = 0.50 to 1.0 in increments of 0.1 volt. The results are plotted in Figure 10. From this plot, it appears that for n values around 0.6, the frequency deviation goes through a maximum. This type of diode junction may be approximated by an abrupt, or step, junction. The varactor used in the experimental circuit had an average calculated n value of 0.531. It appears that for this particular circuit, abrupt junction varactors with n values of 0.5 to 0.6 will produce both the highest linearity and greatest frequency deviation.

C. SUPPLY VOLTAGE DIFFERENCE ($V_o - V_A$)

With reference to Figure 2, $V_o - V_A$ is a major factor in determining the varactor operating point since from Equation (5) we see $V_R = V_o - V_A + I_D R_D$.

Figure 11 is a plot of frequency versus V_{GS} for $V_o - V_A$ = 0, 5, 10, and 15 volts. Figure 12 was obtained from Figure 11 by normalizing f_{max} to 100 for each of the 4 curves. Relative deviation is now more apparent. With reference to Figure 12, the following observations are made:

1. $V_o - V_A = 0$ produced a large deviation with poor linearity.
2. $V_o - V_A = 15$ produced fair linearity but little deviation.
3. $V_o - V_A = 5$ produced good linearity over a 2-volt change in V_{GS}, while maintaining a higher deviation (7 MHz/volt) than the value of 10 volts used in the experimental oscillator.

The value of $V_o - V_A$ = 5 volts is, therefore, the recommended value to be used in the circuit.

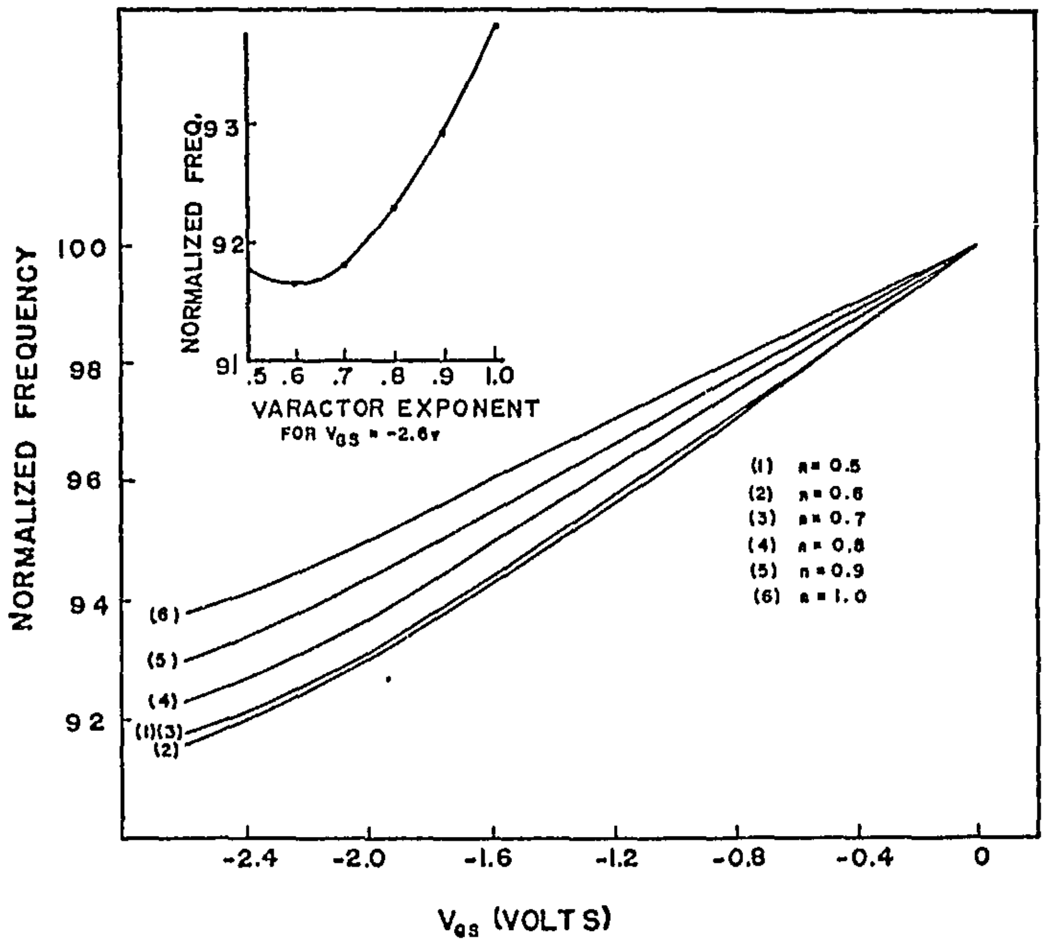

Figure 10. Additional normalized data showing effect of variations in n, the varactor junction exponent, on frequency deviation. (Calculated data.)

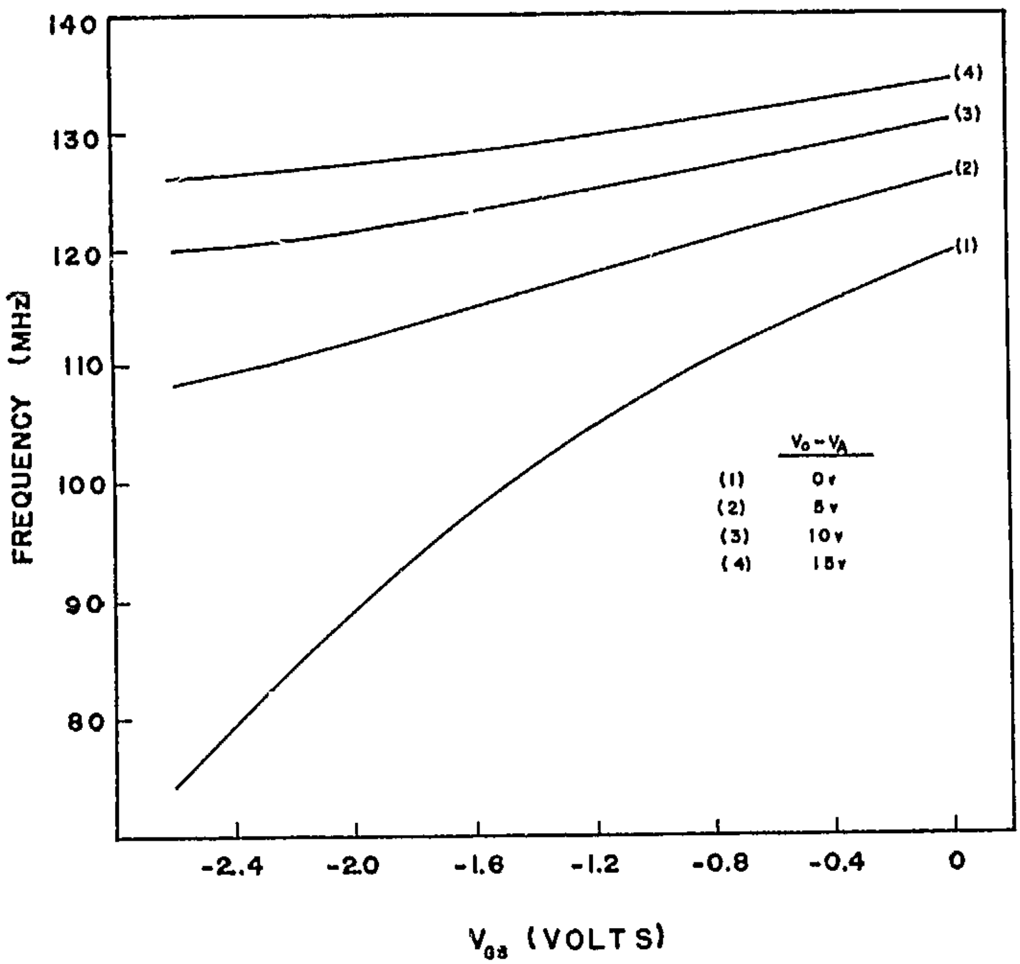

Figure 11. Effect of variations in V_0-V_A, the oscillator-FET driver supply voltage difference, on frequency deviation. (Calculated data.)

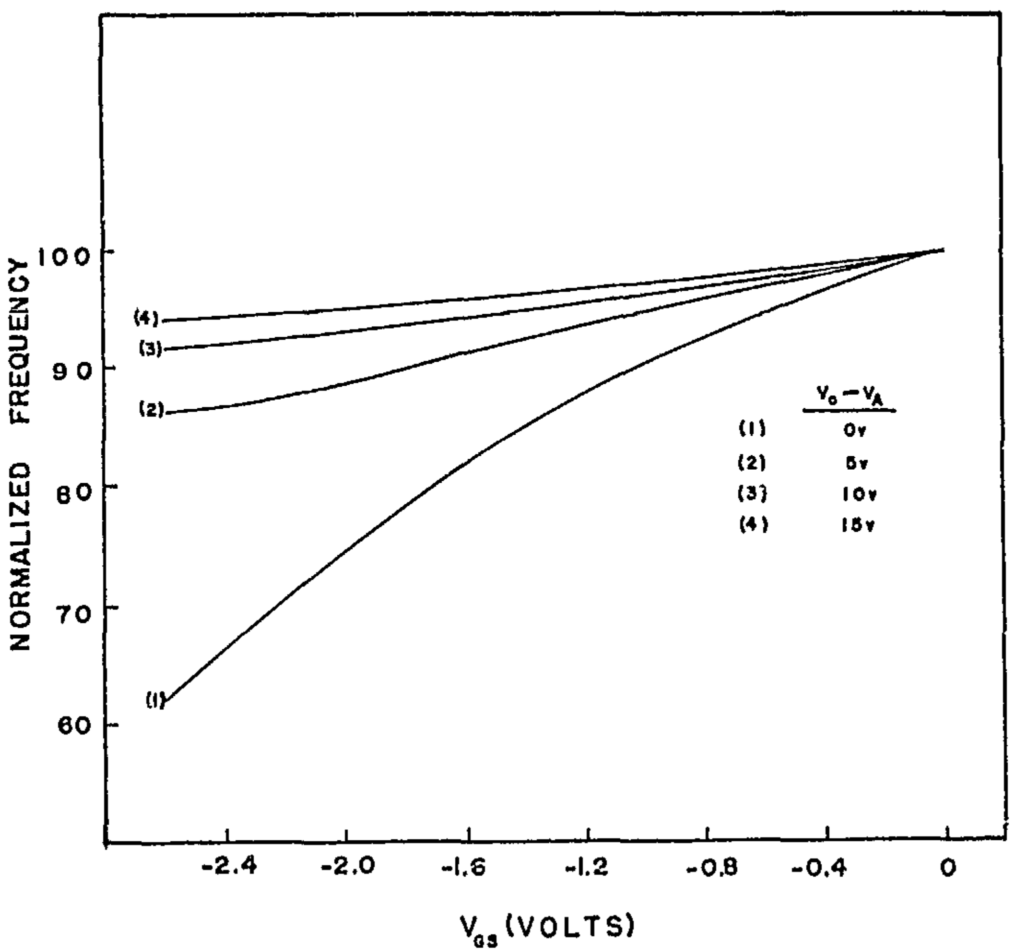

Figure 12. Normalized plot of Figure 11.

D. LOAD RESISTOR (R_D)

The program was modified to run with R_D = 500 ohms, 1000 ohms and 2000 ohms. The results are shown in Figure 13. The curve R_D = 1000 ohms appears to be the best compromise between linearity and deviation. This is the value for R_D used in the experimental oscillator. Higher values for R_D can greatly increase the maximum deviation, however, signal frequency response will suffer since R_D and $C_{V1} + C_{V2}$ determine the high-frequency response of Q_3.

It should be pointed out that R_D, the FET drain resistor, and the $g_{mo}V_p$ product play a similar role in determining the deviation and linearity. From (10) we see that $I_{DSS} = g_{mo}\ V_p/m$. Therefore, for a fixed m, I_{DSS} and the $g_{mo}V_p$ product are proportional.

From (8) we see that the value for A depends on the $R_D I_{DSS}$ product. For this reason, changes in the $g_{mo}V_p$ product have the same effect on deviation and linearity as do proportional changes in R_D. However, changes in R_D also affect the driver frequency response.

E. FET PINCH-OFF VOLTAGE (V_p)

Figure 14 shows the effect of changing the FET pinch-off voltage (V_p) from the experimental value of 2.8 volts to a value of 4.0 volts, while making no other changes in the FET parameters.

The curve V_p = 4.0 volts is more linear over the range shown, however, the frequency deviation is decreased. Note that f_{max} is independent of V_p and the two curves do not require normalization.

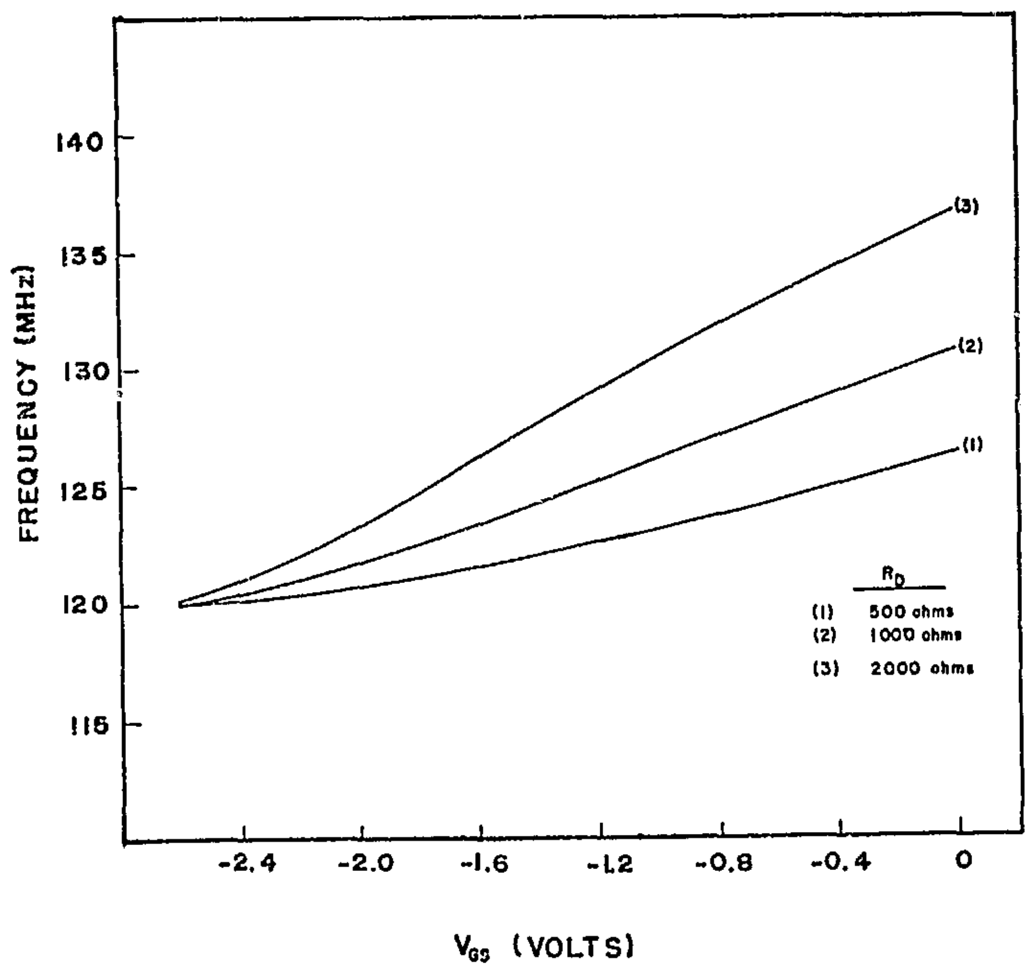

Figure 13. Effect of variations in R_D, the FET load resistor, on frequency deviation. (Calculated data.)

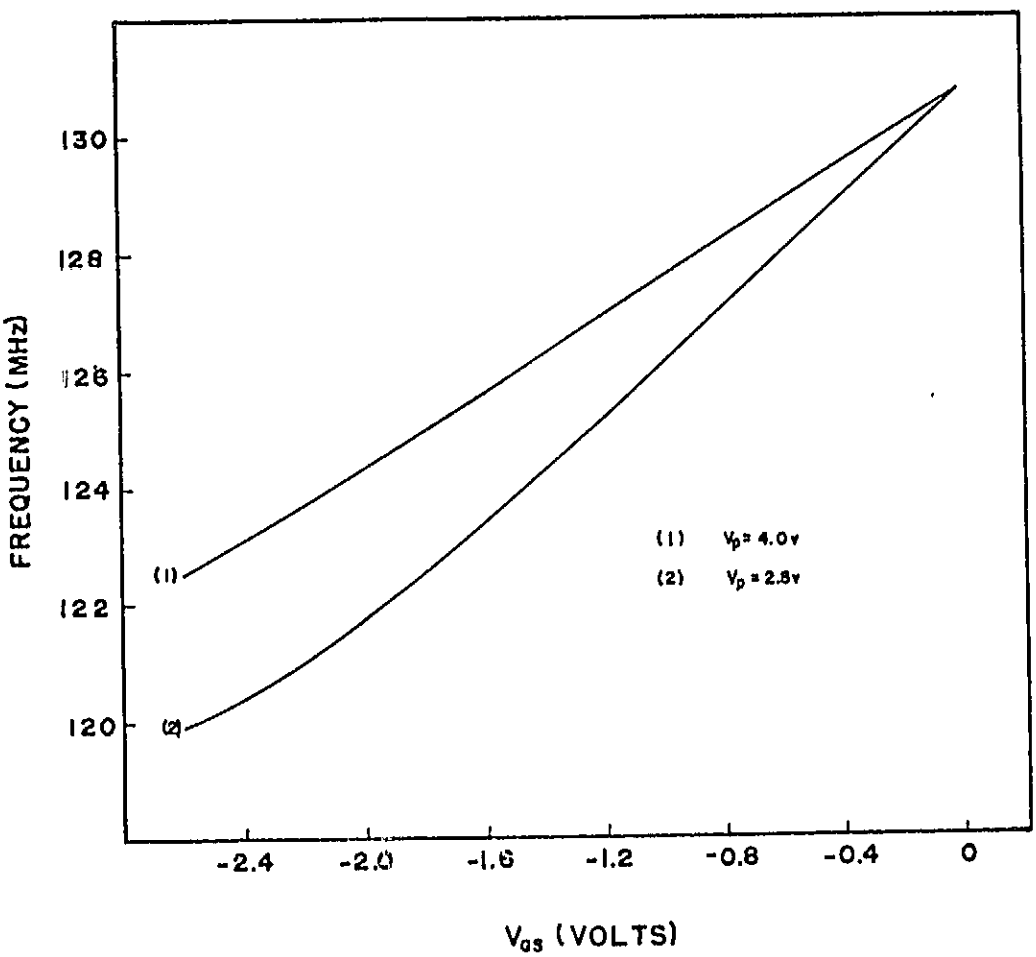

Figure 14. Effect of variations in V_p, the FET pinch-off voltage on frequency deviation. (Calculated data.)

F. CIRCUIT CAPACITANCE (C_p)

From (9) we note C_p is made up of C_{pk}, the varactor package capacitance and C_s, the stray capacitance.

Figure 15 is a plot of frequency versus V_{GS} for C_p values of 13, 18 and 23 pF. Figure 16 is a normalized plot of the same data. The plots clearly show that for small values of C_p, f_{max} and deviation increase, while deviation linearity improves with decreasing C_p.

As a result of the above, it appears that a reduction in C_p would improve the operation of the experimental oscillator. The present circuit has a C_p value of 18 pF. The temperature compensating capacitor C_6 in Figure 1 accounts for approximately 6 pF of this total and is the only component of C_p which may be easily reduced.

G. INDUCTANCE (L_p)

Figure 17 is a plot of the effect of circuit inductance on frequency. An examination of (7) reveals that L_p is a convenient frequency scaling factor. For example, multiplying L_p by a constant K is equivalent to dividing the frequency by $K^{\frac{1}{2}}$. Therefore, changing L_p has the effect of multiplying the frequency by a constant, but has no effect on linearity or deviation.

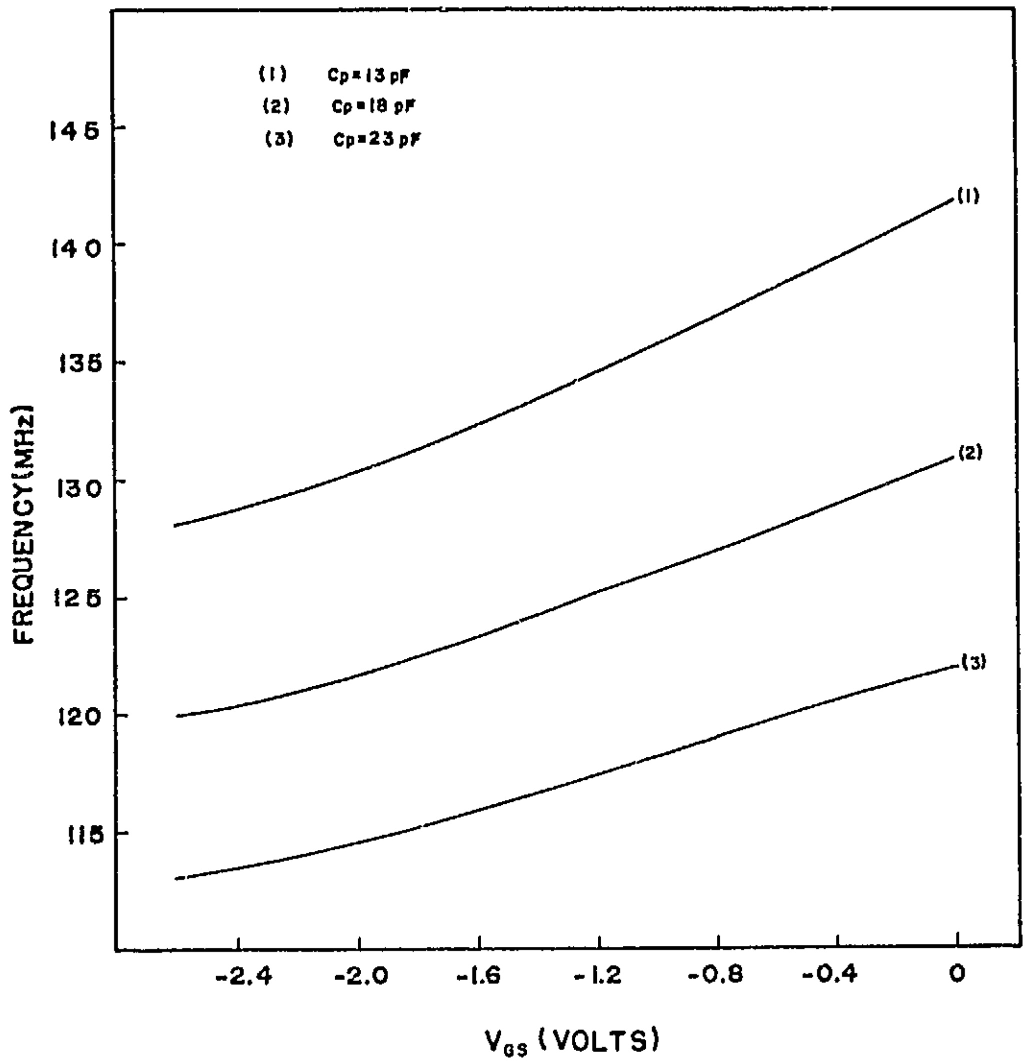

Figure 15. Effect of variations in C_p, the effective circuit capacitance, on frequency deviation. (Calculated data.)

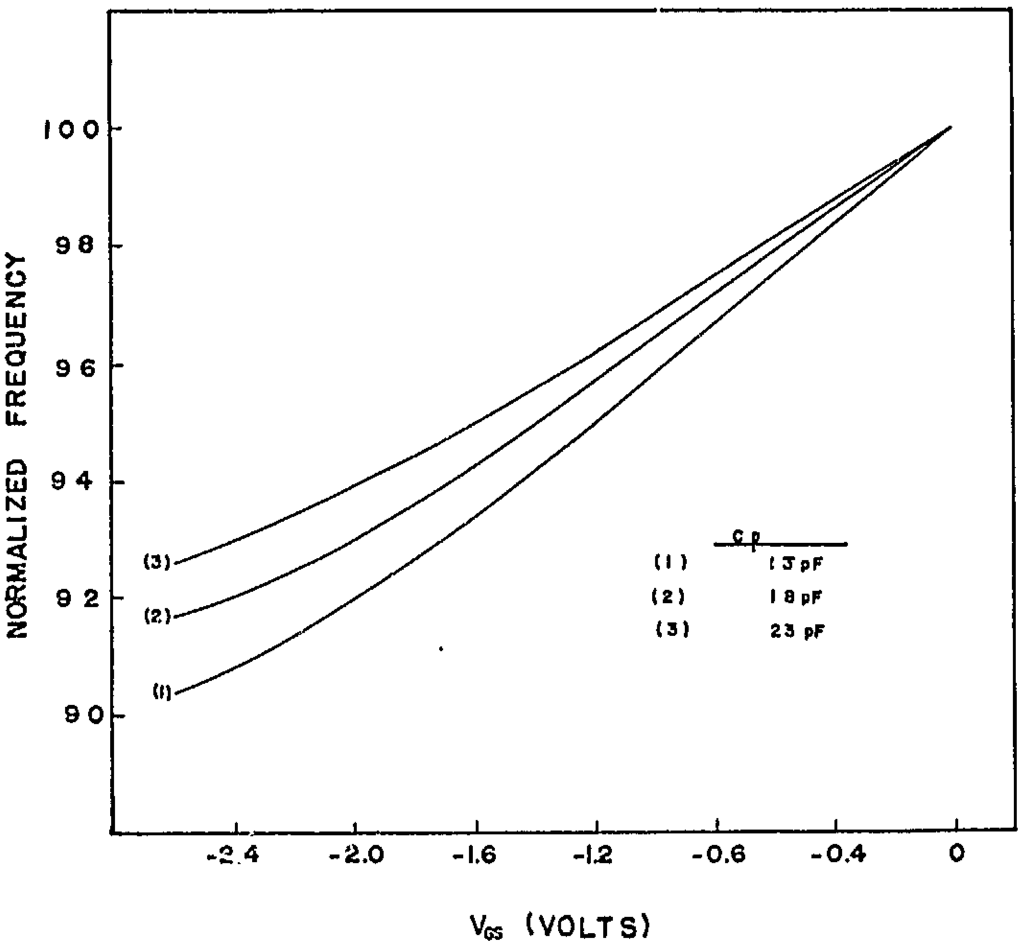

Figure 16. Normalized plot of Figure 15.

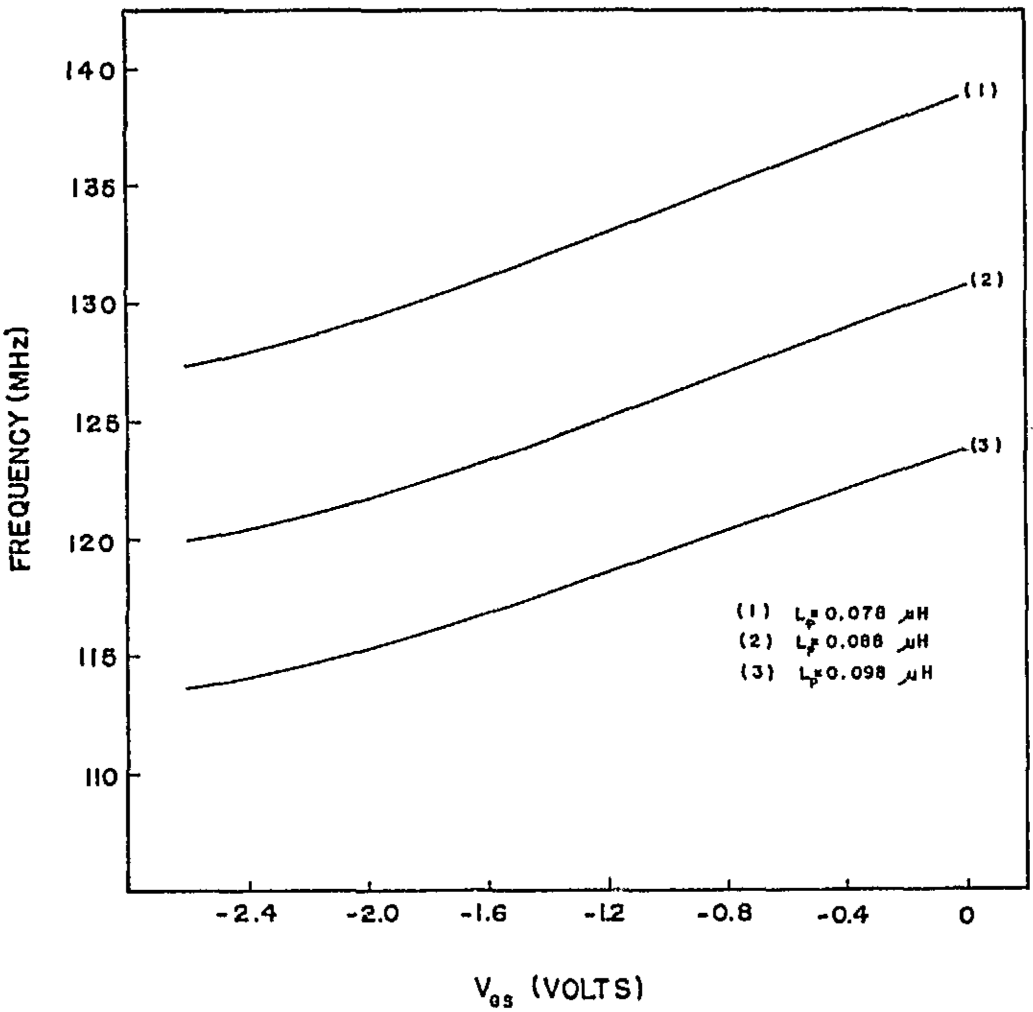

Figure 17. Effect of variations in L_p, the effective circuit inductance, on frequency deviation. (Calculated data.)

VIII. DESIGN EXAMPLE

As an example of how the previously developed methods are used for oscillator design, consider the following requirements for an FM oscillator:

Center frequency	250 MHz
Deviation	± 10 MHz
Nonlinearity	< ± 1%

Assume that the driver FET selected is the 2N4416 with g_{mo} = 5000 mMhos, V_p = -4V, I_{DSS} = 10 mA and cut-off frequency 400 MHz. From (11), the FET exponent (m) is found to be,

$$m = g_{mo}V_p/I_{DSS} = 2.0$$

The same transistors may also be used in the push-pull oscillator circuit.

The varactor selected is assumed to have the following characteristics:

C_o = 45 pF

C_{pk} = 5 pF

V_T = 0.7 volts

n = 0.5

The Motorola HEP R2502 has similar characteristics. C_p is taken to be 10 pF which includes the effect of C_{pk} plus stray circuit capacitance. Refer to (13) and (8b) for the definition of the above terms.

With reference to Figure 1, the drain resistor R_D and the supply voltages V_o and V_A are selected as follows:

R_D is selected to be 2-Kilohms and for numerical convenience, $V_o = V_A$. Equation (8) is now solved for

$A = 1$ for $V_{GS} = V_p = -4$ volts.

and

$A = 5.44$ for $V_{GS} = 0$.

From (7) with $f_{max} = 260$ MHz the required value for L_p is found to be 0.041 μH. Equations (7) and (8) are solved for A and f for values of V_{GS} between 0 volts and -4 volts.

A normalized plot of frequency as a function of V_{GS} is shown in Figure 18. The plotted data is obtained from the BASIC printout shown in Table 3. Figure 18 shows that for $V_o = V_A$, the curve has ample frequency deviation, but poor linearity. Additional data is shown for $V_o - V_A = 5$, 10 and 15 volts.

The process is then repeated for $R_D = 1$K ohms and $V_o - V_A = 0$, 2, 4, 5, 10 volts. The results are shown in Figure 19. An examination of this figure reveals that several of the curves show good linearity with ample deviation to satisfy the original requirements. The $V_o - V_A = 4$ volts curve is selected as adequate and L_p is adjusted to bring the high-frequency end of the curve back up to a little over 260 MHz to allow for circuit changes. The new value for L_p is 3.7×10^{-8}H.

```
RUN
FET250

VO-VA               1/VP               FET EXP            VAR EXP
 0                   .25                2                  .5

     L                 CP                RD
 4.10000E-08         1.00000E-11        2000

  IDSS                  CO
 .01                 4.50000E-11


V(VOLTS)            A(DIM'LESS)        F(MHZ)             F(NORMAL)
 0                   5.44059            260.051            100
 .4                  4.91589            253.988            97.6683
 .8                  4.39363            247.067            95.007
 1.2                 3.87479            239.1              91.9434
 1.6                 3.36095            229.846            88.3847
 2                   2.85482            219.001            84.2145
 2.4                 2.36136            206.214            79.2975
 2.8                 1.8905             191.19             73.52
 3.2                 1.46424            174.169            66.9749
 3.6                 1.13402            157.704            60.6436
 4                   1.                 149.886            57.6371

DONE
```

Table 3. Preliminary computer output data for the 250-MHz oscillator design.

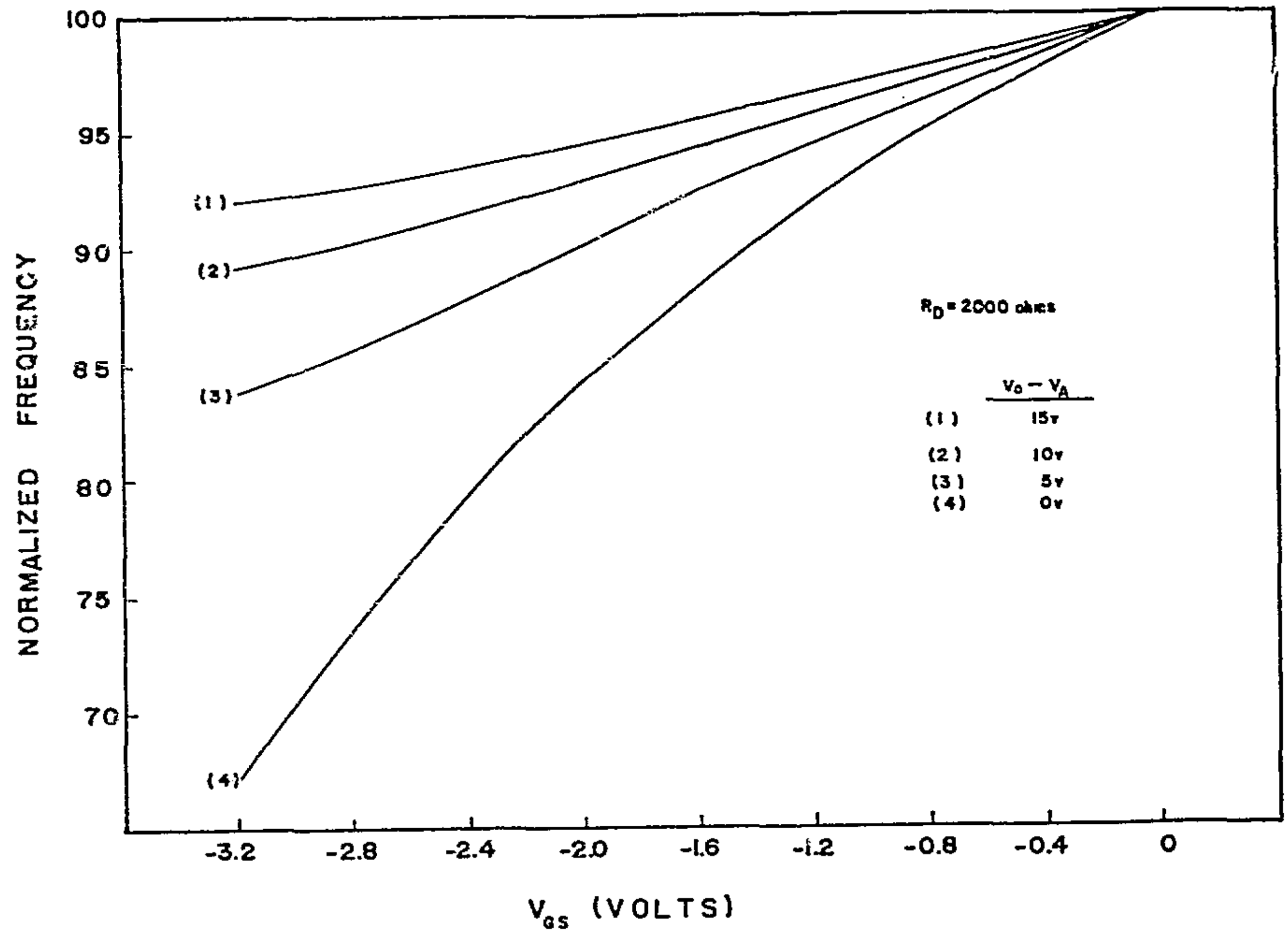

Figure 18. 250-MHz oscillator design showing effect of variations in V_0-V_A, the oscillator-FET driver supply voltage difference, on frequency deviation for R_D = 2000 ohms. (Calculated data.)

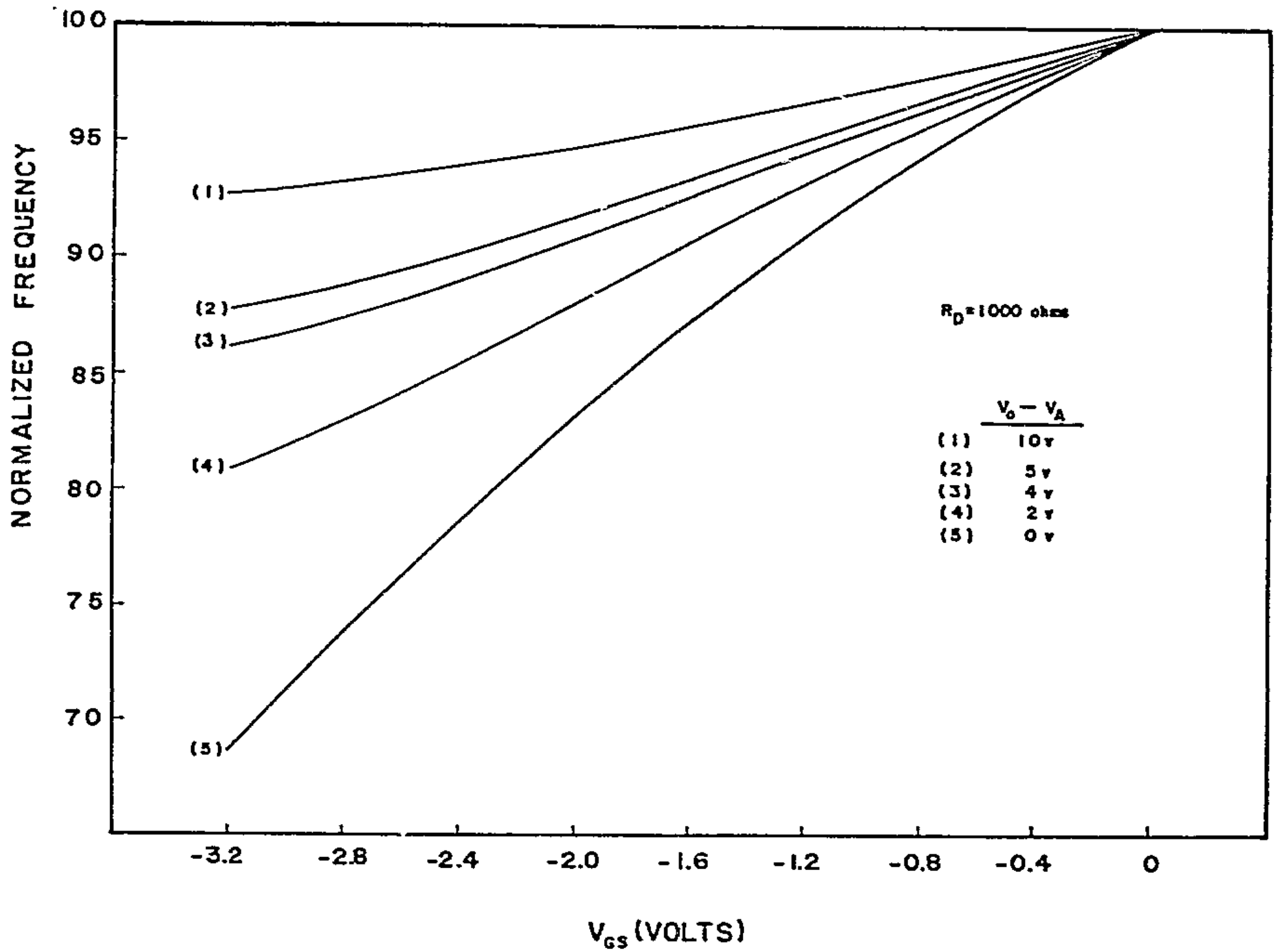

Figure 19. Same as Figure 18, with R_D = 1000 ohms.

The design having the lower value of R_D is preferable since the high-frequency response of the driver FET is improved under these conditions. The parameters used in the final design are summarized below:

FET

g_{mo} = 5000 μMhos

V_p = -4 volts

I_{DSS} = 10 mA

m = 2

Varactor

C_o = 45 pF

C_{pk} = 5 pF

V_T = 0.7 volts

n = 0.5

Circuit

C_p = 10 pF

L_p = 3.7×10^{-8} H

$V_o - V_A$ = 4 volts

R_D = 1000 ohms

Figure 20 is an expanded-scale plot of the oscillator frequency as a function of V_{GS}. An examination of (20) reveals that the non-linearity is less than 1% and the deviation is approximately 12 MHz per volt. The maximum frequency is found to be 262.9 MHz for V_{GS} = 0 and 223.7 MHz for V_{GS} = -4 volts. A gate-to-source off-set bias of

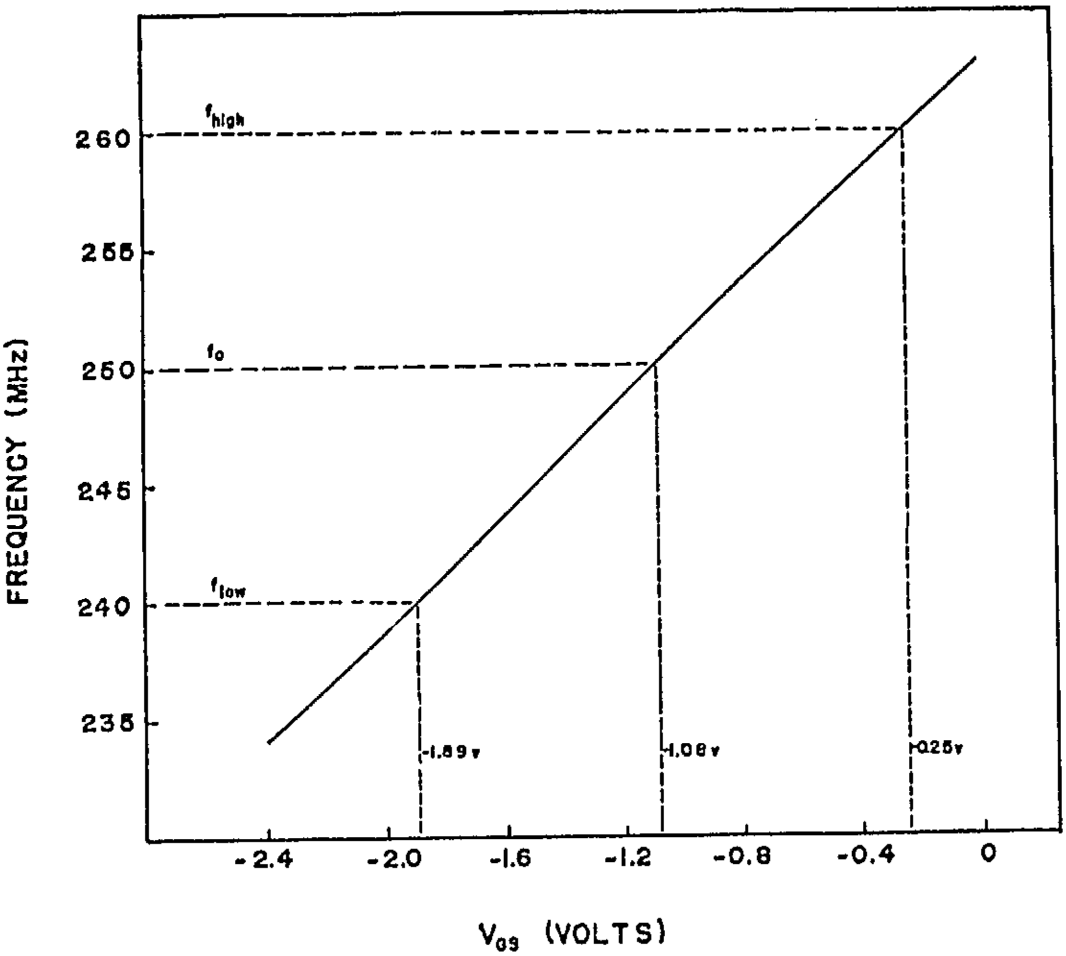

Figure 20. 250-MHz oscillator design, final frequency deviation curve for R_D = 1000 ohms, V_0-V_A = 4 volts. (Calculated data.)

approximately 1 volt is required to adjust the operating point to the most linear region of the f versus V_{GS} curve. Precise values for L_p are not required for the preliminary deviation and linearity plots. This is true since multiplying L_p in (5) by a constant K, is equivalent to dividing the frequency by $K^{1/2}$. That is, changing L_p has the effect of changing the frequency of oscillation but has no effect on linearity or relative deviation as mentioned earlier.

From (19) the slope

$$(df_o/dV_{GS})\Big|_{V_{GS}=0} = L_p C_o \beta_o \omega_o^3/(16\pi\alpha_o^{1.5})$$

$$= 11.1 \text{ MHz/volt}$$

close to the graphically measured value of 12 MHz/volt.

The BASIC program used for the final design of the 250-MHz oscillator is shown in Table 4. Table 5 is a frequency printout for the final design.

```
LIST
FET250

10  DIM A[50],F[50]
20  READ B,C,D,E,F,G,H,J,K
30  DATA 4
40  DATA .25
50  DATA 2
60  DATA .5
70  DATA 3.7E-08
80  DATA 1.E-11
90  DATA 1000
100  DATA .01
110  DATA 4.5E-11
120  PRINT "VO-VA","1/VP","FET EXP","VAR EXP"
130  PRINT B,C,D,E
140  PRINT
150  PRINT "      L","      CP","   RD"
160  PRINT F,G,H
170  PRINT
180  PRINT "  IDSS","       CO"
190  PRINT J,K
200  PRINT
210  PRINT
220  FOR I=0 TO 40
230  A[I+4]=(1+1.43*(B+H*J*(1-C*I/10)^D))^E
240  Z=A[I+4]
250  F[I+4]=1/6.2832*(2*Z/(F*G*Z+F*K))^.5
260  NEXT I
270  PRINT "V(VOLTS)","A(DIM'LESS)","F(MHZ)","F(NORMAL)"
280  FOR I=1 TO 11
290  PRINT 4*(I-1)/10,A[4*I],F[4*I]/1.E+06,F[4*I]/F[4]*100
300  NEXT I
310  END
```

Table 4. Computer program for final design of 250-MHz oscillator.

```
RUN
FET250

VO-VA              1/VP               FET EXP          VAR EXP
 4                  .25                2                 .5

     L                 CP              RD
 3.70000E-08       1.00000E-11        1000

  IDSS                 CO
 .01               4.50000E-11

V(VOLTS)           A(DIM'LESS)        F(MHZ)           F(NORMAL)
 0                  4.58476            262.867          100
 .4                 4.2782             258.322          98.271
 .8                 3.98397            253.566          96.4617
 1.2                3.705              248.65           94.5914
 1.6                3.445              243.659          92.6928
 2                  3.20858            238.727          90.8168
 2.4                3.00133            234.056          89.0399
 2.8                2.82966            229.911          87.4627
 3.2                2.70037            226.605          86.205
 3.6                2.61973            224.455          85.3874
 4                  2.5923             223.708          85.1031

DONE
```

Table 5. Computer printout of frequency as a function of V_{GS} for the 250-MHz oscillator. (Final design.)

IX. CONCLUSIONS

The frequency-versus-voltage characteristic of a varactor-modulated VHF oscillator was linearized by utilizing the nonlinear characteristic of a field-effect transistor amplifier to pre-distort the modulating signal. Values of m between 1.5 and 1.6 in the FET equation $I_D = I_{DSS}(1 - V_{GS}/V_p)^m$ produced optimum linearity.

It was determined that the total frequency deviation for a given change in input signal amplitude is a function of the varactor junction exponent and of the difference between the oscillator and amplifier supply voltage. This voltage difference which determines the varactor operating point also affects the frequency deviation linearity.

Deviation linearity measurements made on a 130-MHz experimental oscillator agreed within 1% with computed results programmed in BASIC for a mathematical model of the oscillator. The BASIC program was also utilized to determine the effect of pertinent parameters on the linearity of frequency deviation as a function of input signal voltage amplitude.

The 130-MHz experimental oscillator constructed exhibited a linearity of better than $\pm$ 1% for a total frequency deviation of $\pm$ 10 MHz.

Suggested areas for additional investigation are: (1) techniques for maintaining constant output power as a function of frequency deviation; and (2) methods for stabilizing the average frequency of the oscillator as a function of temperature. Both of these problems should be investigated experimentally and analytically.

REFERENCES

1. M. A. Honnell and A. W. Wales Jr., "A Stable, Linear Frequency-Modulated Oscillator", Final Report, Part I, NAS8-26193, Auburn University Engineering Experiment Station, Auburn, Alabama, September 19, 1974.

2. Leonard Strauss, Wave Generation and Shaping, New York: McGraw Hill, Inc., 1970, pp. 273-275.

3. F. J. Hyde, S. Deval, and C. Toker, "Varactor diode measurements", Radio Electronic Eng., Vol. 31, pp. 67-75, 1966.

4. R. B. Smith, B. Bramer, and D. G. Croft, "Determination of the constants of the capacitance/voltage law of a varactor diode", Electron. Lett., Vol. 4, pp. 274-275, 1968.

APPENDIX

DERIVATIONS OF CIRCUIT EQUATIONS

For the circuit of Figure 1, we note that C_{V1} and C_{V2} are effectively in series. Parallel resonance occurs when

$$\omega L_p = 1/\omega C_T \tag{A-1}$$

where L_p is the net effective inductance of the resonant circuit and C_T is the net effective capacitance.

From (2), (4) and (5) we get

$$\begin{aligned} C_T &= C_s + C_{pk}/2 \\ &\quad + (C_o/2)\left\{1 + (1/V_T)[V_o - V_A + R_D\ I_{DSS}(1 - V_{GS}/V_p)^m]\right\}^{-n} \end{aligned} \tag{A-2}$$

Equation (A-2) may now be simplified as follows

$$C_T = C_s + C_{pk}/2 + C_o/(2A) \tag{A-3}$$

where

$$A = \left\{1 + (1/V_T)[V_o - V_A + R_D\ I_{DSS}(1 - V_{GS}/V_p)^m]\right\}^n \tag{8}$$

The expression for C_T from (A-3) is now substituted into (A-1) and (A-1) is solved for f to yield

$$f = (1/2\pi)\sqrt{2A/[L_p A(2C_s + C_{pk} + C_o/A)]} \tag{A-4}$$

$$= (1/2\pi)\sqrt{2A/[L_p(AC_p + C_o)]} \tag{7}$$

where

$$C_p = C_{pk} + 2C_s \tag{9}$$

Equation (17) is now derived by inverting (16) and differentiating with respect to the variable A.

$$1/\omega^2 = L_pC_p/2 + L_pC_o/(2A) \tag{A-4}$$

$$2\omega d\omega/\omega^4 = 2L_pC_o dA/(4A^2)$$

$$d\omega/dA = L_pC_o\omega^3/(4A^2) \tag{A-5}$$

The quantity dA/dV_{GS} is now obtained from (8) for the case m = 2 and $V_{GS} << V_p$. This corresponds to an "ideal" square-law FET with an exponent of 2, operating in the region $V_{GS} \doteq 0$ volts.

Applying these conditions to (8), we may write $(1 - V_{GS}/V_p)^2 \doteq 1 - 2V_{GS}/V_p$ by dropping the higher-order term. By expanding and collecting terms, (8) becomes

$$A = [1 + (V_o - V_A + R_D\, I_{DSS})/V_T - 2R_D\, I_{DSS}\, V_{GS}/(V_TV_p)]^n \tag{A-6}$$

$$= (\alpha - \beta V_{GS})^n \tag{17C}$$

where

$$\alpha = 1 + (V_o - V_A + R_D\, I_{DSS})/V_T \tag{17a}$$

$$\beta = 2R_D\, I_{DSS}/(V_T\, V_p) \quad \text{volts}^{-1} \tag{17b}$$

It should be noted that β, as defined, is a negative quantity since V_p is negative for the n-channel FET.

From (17c) we obtain the derivative

$$dA/dV_{GS} = -n\beta(\alpha - \beta V_{GS})^{n-1} \tag{A-7}$$

Multiplying (A-5) and (A-7) we obtain

$$d\omega/dV_{GS} = -n\beta(\alpha - \beta V_{GS})^{n-1} L_p C_o \omega^3/(4A^2)$$

$$= -n\beta(\alpha - \beta V_{GS})^{n-1} L_p C_o \omega^3/[4(\alpha - \beta V_{GS})^{2n}]$$

$$= -\beta L_p C_o n\omega^3/[4(\alpha - \beta V_{GS})^{n+1}] \quad \text{rad/volt} \tag{17}$$