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# NUMERICAL ANALYSIS AND PARAMETRIC STUDIES OF THE BUCKLING OF COMPOSITE ORTHOTROPIC COMPRESSION AND SHEAR PANELS

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16. Abstract

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# NUMERICAL ANALYSIS AND PARAMETRIC STUDIES OF THE BUCKLING OF COMPOSITE ORTHOTROPIC COMPRESSION AND SHEAR PANELS

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## SUMMARY

A computer program has been developed for the combined compression and shear of stiffened variable thickness orthotropic composite panels on discrete springs; boundary conditions are general and include elastic boundary restraints. Buckling solutions are obtained by using a newly developed trigonometric finite-difference procedure which improves the solution convergence rate over conventional finite-difference methods. The trigonometric finite-difference procedure introduces two new parameters into the solution. These parameters can be computed by the program or selected by the user. The validity of the program has been substantiated by comparisons with existing solutions, and a program listing, input description, and sample problem are provided.

The classical general shear-buckling results (in terms of universal orthotropic parameters), which exist only for simply supported panels over a limited range of orthotropic properties, have been extended to the complete range of these properties for simply supported panels and, in addition, to the complete range of orthotropic properties for clamped panels. The program has also been applied to parametric studies which examine the effect of filament orientation upon the buckling of graphite-epoxy panels. These studies included an examination of the filament orientations which yield maximum shear or compressive buckling strength for panels having all four edges simply supported or clamped over a wide range of aspect ratios. Panels with such orientations had higher buckling loads than comparable, equal-weight, thin-skinned aluminum panels. Also included among the parameter studies were examinations of combined axial compression and shear buckling and examinations of panels with rotational elastic-edge restraints.

## INTRODUCTION

The use of filamentary composite materials in aircraft and space structures offers a potential for weight savings over conventional (all metal) construction. Also, composites introduce added versatility into the design process by allowing the structure to be better tailored to meet the design criteria. One such design criterion is the prevention of compressive and shear buckling in panels of laminated construction. In laminated panels the stiffness properties can be tailored by controlling the filament orientation in each lamina.

A considerable amount of literature exists on the buckling of flat isotropic and orthotropic panels under various boundary conditions. (See refs. 1 to 6.) Few results exist, however, for finite aspect-ratio panels, especially for shear buckling of orthotropic panels. General results for shear buckling, in terms of universal orthotropic parameters, exist only for simply supported panels over a limited range of orthotropic parameters. (See ref. 6.) Several general-purpose computer programs exist which could be employed to obtain results for panels with general boundary conditions under general loading states (refs. 7 to 9). These programs, however, tend to be expensive to use in performing parameter studies; therefore, a program which is suitable for performing parametric buckling studies of orthotropic flat rectangular panels was developed and is employed in this paper.

The present computerized analysis is applicable to the combined compression and shear buckling of stiffened, variable-thickness, flat rectangular orthotropic panels on discrete springs; boundary conditions are general and include elastic boundary restraints. Calculation of the flexural stiffnesses of a laminate from the properties of filament-reinforced laminas is automatically performed. The analysis makes use of a newly developed trigonometric finite-difference procedure. In contrast to conventional (polynomial) finite differences, trigonometric differences take advantage of the sinusoidal form of the buckle pattern to achieve converged solutions with fewer degrees of freedom, hence reducing computer time. The analysis has been validated by many comparisons with solutions in the literature and has been used to produce a variety of additional orthotropic and some isotropic panel results.

The classical general results for the shear buckling of simply supported orthotropic panels are extended in this paper to cover the complete range of orthotropic parameters. Also, the general results for the shear buckling of clamped panels over the complete range of orthotropic parameters have been calculated and are presented herein. In addition, it is of practical interest to present results which consider the effects of filament orientation upon the buckling strength of laminated composite panels. Consequently, parameter studies are presented for graphite-epoxy panels of various aspect ratios, boundary conditions, and in-plane loadings over a wide range of filament orientations, and those orientations which led to maximum buckling loads are identified. Finally, results are presented for the shear buckling of simply supported isotropic panels, each with a central stiffener.

## SYMBOLS

| | |
|---|---|
| $a,b$ | dimensions of rectangular plate parallel to X- and Y-axes, respectively |
| $A^{(r)}$ | coefficients defined by equation (C3) |
| $C_x,C_{yx}$ | correction factors defined in equations (B4) and (B5) |
| $D$ | isotropic plate flexural stiffness |
| $D_3$ | $= D_{12} + 2D_{66}$ |
| $D_{11},D_{22},D_{12},D_{66}$ | orthotropic plate flexural stiffnesses |
| $e_{ij}^{(r)}$ | elements of matrix defined by equation (C2) |
| $EI$ | flexural stiffness of discrete stiffener |
| $E_1,E_2$ | Young's moduli of fibrous reinforced material parallel to fibers and transverse to fibers, respectively |
| $G_{12}$ | shear modulus of fibrous reinforced material |
| $h$ | core thickness of sandwich plate |
| $I_1,I_3$ | row designations of boundaries ① and ③ (see fig. 2(a)) |
| $J_2,J_4$ | column designations of boundaries ② and ④ (see fig. 2(a)) |
| $k_\ell$ | discrete lateral spring stiffness |
| $k_R$ | uniformly distributed rotational spring stiffness |
| $k_s$ | shear-buckling load coefficient $\dfrac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}}$ |
| $k_x,k_y$ | stiffness of rotational springs which resist moments acting about Y- and X-axes, respectively |

$K_{ij}$ plate stiffness terms defined by equation (A13)

$M,N$ total number of rows and columns of finite-difference stations, respectively

$M_e,N_e$ total number of rows and columns of finite-difference stations at which equilibrium is satisfied

$M_x,M_y,M_{xy}$ bending moments in plate (see fig. 1)

$N_x,N_y,N_{xy}$ in-plane loads (see fig. 1)

$\overline{N}_x,\overline{N}_y,\overline{N}_{xy}$ shear-buckling stress coefficients $\dfrac{b^2N_x}{\pi^2D_{11}}$, $\dfrac{b^2N_y}{\pi^2D_{11}}$, $\dfrac{b^2N_{xy}}{\pi^2D_{11}}$, respectively

$\hat{N}_x,\hat{N}_y,\hat{N}_{xy}$ buckling parameters $\dfrac{b^2N_x}{E_1t^3\left[1-\left(\frac{h}{t}\right)^3\right]}$, $\dfrac{b^2N_y}{E_1t^3\left[1-\left(\frac{h}{t}\right)^3\right]}$, $\dfrac{b^2N_{xy}}{E_1t^3\left[1-\left(\frac{h}{t}\right)^3\right]}$, respectively

$N_{x_o},N_{xy_o}$ buckling loads for pure axial compression and pure shear, respectively

$\overline{p}$ buckling eigenvalue (see eq. (13))

$r_x,r_y,r_{xy}$ change of $\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$ with $\overline{p}$, respectively (see eq. (13))

$R_x,R_{xy}$ ratio of $N_x/N_{x_o}$ and $N_{xy}/N_{xy_o}$, respectively

$S_{ij}$ spring-stiffness terms defined by equation (A17)

$t$ total thickness of sandwich plate

$\overline{t}_x,\overline{t}_y,\overline{t}_{xy}$ values of $N_x$, $N_y$, $N_{xy}$ when $\overline{p} = 0$

$w$ displacement of panel in positive z-direction

$x,y,z$ panel coordinates shown in figure 1

$\alpha_{ij}$ curvature terms defined in equation (A14)

$\beta$ ratio of panel width to buckle length in an infinitely long panel

$\gamma_1,\gamma_2,\gamma_3$ coefficients defined by equation (7) or (8)

$\delta U$ internal virtual work

$\delta V_N$ virtual work of in-plane loads

$\delta V_S$ virtual work of discrete springs

$\Delta_x,\Delta_y$ finite-difference mesh spacings in x- and y-directions, respectively

$\hat{\Delta}_x,\hat{\Delta}_y$ trigonometric finite-difference coefficients as defined in equation (10)

$\Delta_x^*,\Delta_y^*$ trigonometric finite-difference terms defined in equation (A24)

$\theta$ filament orientation (see fig. 2(a))

$\Theta,B$ universal orthotropic parameters defined in equations (15) and (16)

$\lambda_x,\lambda_y$ trigonometric parameters defined through equation (10)

$\nu_{12}$ major Poisson ratio relating contraction normal to filament direction to extension parallel to filament direction

$\xi_x,\xi_y,\eta_x,\eta_y$ functions defined by equations (A6) to (A9)

$\chi_{ij}$ twist terms defined in equation (A16)

$\psi_{ij}$ curvature terms defined in equation (A15)

Comma preceding a subscript denotes differentiation with respect to the subscript.

## ANALYSIS

### Assumptions

The buckling analysis of linear elastic orthotropic plates has been carried out under the following assumptions:

1. Coupling between bending and extensional deformation is neglected. (In practice this assumption implies a midplane symmetric laminated panel.)

2. Coupling between bending and twisting deformation is neglected. (In practice this assumption implies a balanced laminate.)

3. The deformations of the panel obey the Kirchhoff hypothesis (see ref. 10).

4. The nonlinear strain-displacement relationships used to obtain (linear) buckling equations are

$$e_x = u_{,x} + \frac{1}{2}\left(w_{,x}\right)^2$$

$$e_y = v_{,y} + \frac{1}{2}\left(w_{,y}\right)^2$$

$$\gamma_{xy} = u_{,y} + v_{,x} + w_{,x}w_{,y}$$

where $e_x$, $e_y$, and $\gamma_{xy}$ are the strains and $u$, $v$, and $w$ are the displacements in x-, y-, and z-directions, respectively.

5. The in-plane loads, $N_x$, $N_y$, and $N_{xy}$, are uniformly distributed along the appropriate edges of the plate.

6. Discrete stiffeners have no torsional stiffness and are symmetrically disposed with respect to the neutral surface of the panel.

### Governing Equations

The internal virtual work of the panel during buckling may be expressed as

$$\delta U = \int_0^b \int_0^a \left(M_x \delta w_{,xx} + M_y \delta w_{,yy} + 2M_{xy} \delta w_{,xy}\right) dx\, dy \tag{1}$$

where $a$ and $b$ are the dimensions of the panel parallel to the X- and Y-axes, respectively, and $\delta$ is the variational operator. Also,

$$
\left.
\begin{aligned}
M_x &= D_{11}w_{,xx} + D_{12}w_{,yy} \\
M_y &= D_{12}w_{,xx} + D_{22}w_{,yy} \\
M_{xy} &= 2D_{66}w_{,xy}
\end{aligned}
\right\} \tag{2}
$$

The sign conventions of the bending moments are given in figure 1, and the flexural stiffnesses, $D_{11}$, $D_{12}$, $D_{22}$, and $D_{66}$, given in reference 11, are about a unique neutral plane which has the property that matrix $[B]$, which represents coupling between bending and extension, is null with respect to this plane. As given by reference 12, the virtual work of the applied in-plane loads is given by

$$
\delta V_N = \int_0^a \int_0^b \left(N_x w_{,x} \delta w_{,x} + N_y w_{,y} \delta w_{,y} + N_{xy} w_{,y} \delta w_{,x} + N_{xy} w_{,x} \delta w_{,y}\right) dy\, dx \tag{3}
$$

where the sign conventions for $N_x$, $N_y$, and $N_{xy}$ are shown in figure 1.

In appendix A, equations (A1) to (A3) are expressed in trigonometric finite-difference form (see fig. 2 for finite-difference station layout) and are substituted into the statement of the principle of virtual work, that is,

$$
\delta U = \delta V_N + \delta V_S \tag{4}
$$

where $\delta V_S$ is the virtual work of the discrete springs. (See appendix A, eq. (A11).) Equation (4) yields the governing equations which are of the following form:

$$
K_{ij} + S_{ij} + N_x \alpha_{ij} + N_y \psi_{ij} + 2N_{xy} \chi_{ij} = 0 \qquad \begin{pmatrix} i = 1, \ldots, M \\ j = 1, \ldots, N \end{pmatrix} \tag{5}
$$

where $K_{ij}$, $S_{ij}$, $\alpha_{ij}$, $\psi_{ij}$, and $\chi_{ij}$ are defined by equations (A13) to (A17) in appendix A.

The numerical technique of trigonometric finite differences and the numerical extraction of the buckling loads $N_x$, $N_y$, and $N_{xy}$ from equation (5) are different from those conventionally used and therefore require further discussion.

## Numerical Techniques

Trigonometric finite differences.- Conventionally, the central difference approximation for the derivative of a function $f(x)$ at $x = x_0$ is approximated as

$$\frac{df}{dx}(x_0) \approx \frac{1}{\Delta_x}\left[f\left(x_0 + \frac{\Delta_x}{2}\right) - f\left(x_0 - \frac{\Delta_x}{2}\right)\right] \tag{6}$$

The right-hand side of equation (6) is denoted as the conventional finite-difference approximation for the derivative. In the limit as the finite-difference mesh spacing $\Delta_x$ approaches zero, the right-hand side of equation (6) expresses the definition of the derivative. If $f(x)$ is parabolic in the neighborhood of $x_0$,

$$f(x) = \gamma_1 + \gamma_2(x - x_0) + \gamma_3(x - x_0)^2 \tag{7}$$

and it may be readily shown that the approximate expression given by equation (6) becomes an equality. If, however, $f(x)$ is trigonometric about $x = x_0$,

$$f(x) = \gamma_1 + \gamma_2 \sin \frac{\pi(x - x_0)}{\lambda_x} + \gamma_3 \cos \frac{\pi(x - x_0)}{\lambda_x} \tag{8}$$

where $\lambda_x$ is a wavelength parameter. It may be readily shown that

$$\frac{df}{dx}(x_0) = \frac{1}{\hat{\Delta}_x}\left[f\left(x_0 + \frac{\Delta_x}{2}\right) - f\left(x_0 - \frac{\Delta_x}{2}\right)\right] \tag{9}$$

where

$$\frac{1}{\hat{\Delta}_x} = \frac{\pi}{2\lambda_x \sin\left(\dfrac{\pi\Delta_x}{2\lambda_x}\right)} \tag{10}$$

The right-hand side of equation (9) is denoted as the trigonometric finite-difference approximation for the derivative. (In a two-dimensional problem a similar set of relationships would be derived for the y-direction, introducing the quantities $\Delta_y$, $\hat{\Delta}_y$, and $\lambda_y$.)

The only difference between the right-hand side of equation (9) and that of equation (6) is that in the trigonometric expression $1/\hat{\Delta}_x$ replaces $1/\Delta_x$ of the conventional expression. As $\lambda_x$ approaches infinity, $\hat{\Delta}_x$ approaches $\Delta_x$ and, consequently, the trigonometric difference expression reduces to the conventional expression.

Convergence of trigonometric finite-difference solutions.- Inasmuch as the buckling mode shape is usually trigonometric in nature, the trigonometric finite-difference solution can be made to exhibit a much faster convergence rate than the conventional difference solution by appropriate selection of $\lambda_x$ and $\lambda_y$. This advantage is demonstrated with several isotropic plate examples discussed in appendix B. The convergence rate can also be degraded, however, by an inappropriate choice of $\lambda_x$ and $\lambda_y$. It should be emphasized though, that the selection of $\lambda_x$ and $\lambda_y$ does not constrain the buckle mode shape to have wavelengths given by $\lambda_x$ and $\lambda_y$. Rather, the trigonometric solution will always converge to the exact solution if enough degrees of freedom (finite-difference stations) are used.

Selection of trigonometric parameters $\lambda_x$ and $\lambda_y$.- Selecting appropriate values of $\lambda_x$ and $\lambda_y$ which improve the convergence rate of solutions is predominantly based upon engineering judgment and experience. One engineering approach which has proven useful is to select $\lambda_x$ and $\lambda_y$ based upon the buckle length of infinitely long panels; that is,

$$\frac{\lambda_x}{a} = \frac{b/a}{\beta} \tag{11}$$

$$\frac{\lambda_y}{b} = 1 \tag{12}$$

where $\beta$ is the wavelength parameter of an infinitely long panel, defined as the ratio of the panel width to the buckle length. The value of $\beta$ for the combined compression and shear buckling of simply supported and clamped infinite panels may be determined from equations (B2) and (B3) in appendix B. Additional suggestions for the selection of $\lambda_x$ and $\lambda_y$ are given in appendix B.

Stability determinant evaluation and eigenvalue extraction.- In this analysis the order of the stability determinant is kept to a manageable size by using the two-dimensional marching procedure outlined in appendix C. This procedure is basically an extension of the one-dimensional procedure used in reference 13. Briefly, the marching procedure successively operates on the equilibrium equations at each finite-difference station to achieve a relatively low-order stability determinant.

In searching for the combined load system which produces buckling, it is convenient to introduce dimensionless stress coefficients, $\overline{N}_x$, $\overline{N}_y$, and $\overline{N}_{xy}$, which may be determined from the dimensional quantities, $N_x$, $N_y$, and $N_{xy}$ (fig. 1), by multiplying by the factor $b^2\pi/D_{11}$. It is assumed that $\overline{N}_x$, $\overline{N}_y$, and $\overline{N}_{xy}$ are linear functions of an eigenvalue $\bar{p}$, that is,

$$\left.\begin{aligned} \bar{N}_x &= \bar{t}_x + \bar{p}r_x \\ \bar{N}_y &= \bar{t}_y + \bar{p}r_y \\ \bar{N}_{xy} &= \bar{t}_{xy} + \bar{p}r_{xy} \end{aligned}\right\} \tag{13}$$

This assumption allows some loads to be held constant while others are increased to buckling, or it allows the loads to increase with a fixed proportionality.

To find the lowest value of $\bar{p}$ which makes the stability determinant vanish, a determinant plotting technique is used. In order to increase the speed of the plotting technique, a variable step size is employed. This step size is based upon a numerical parabolic extrapolation of the stability determinant at each step of the determinant plotting procedure.

## COMPUTER PROGRAM

A computer program denoted BOP (Buckling of Orthotropic Panels) has been developed for the buckling of flat rectangular orthotropic laminated panels. The program is applicable to panels with compression and/or shear loading, discrete lateral deflection and rotational springs, discrete stiffeners, and general boundary conditions.

The program utilizes trigonometric finite differences to improve the problem convergence and thus requires the selection of $\lambda_x$ and $\lambda_y$. The user has the option of determining and supplying $\lambda_x$ and $\lambda_y$ (based upon the discussion in appendix B) or allowing the program to automatically calculate and use values based on equations (11) and (12).

In addition, the user has the option of either (1) supplying the bending stiffnesses of the panel or (2) supplying the elastic moduli, filament orientation, and thickness of each lamina in a laminated panel and allowing the program to calculate the bending stiffnesses. When the second option is chosen, the program prints the flexural stiffness matrix D, defined in reference 11, as well as the laminate Young's moduli, shear modulus, and Poisson's ratios. (The second option may be used independently of the buckling analysis.) A complete description of the program is provided in appendix D.

Results from the computer program have been compared with many classical results for unstiffened isotropic and orthotropic panels under various boundary conditions and with some classical results for stiffened isotropic panels. These comparisons which are discussed in subsequent sections were found to be excellent, thereby indicating the validity of the program.

## RESULTS AND DISCUSSION

### Shear Buckling of General Orthotropic Panels

From the general fourth-order equation for the shear buckling of orthotropic panels the buckling load coefficient may be expressed as

$$k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}} \tag{14}$$

This coefficient is a function of only two variables

$$\Theta = \frac{\sqrt{D_{11} D_{22}}}{D_3} \tag{15}$$

and

$$B = \frac{b}{a} \sqrt[4]{\frac{D_{11}}{D_{22}}} \tag{16}$$

where $D_3 = D_{12} + 2D_{66}$. (Note that an isotropic panel implies $\Theta = 1$.)

Classically, general shear-buckling results for simply supported finite aspect-ratio panels have been obtained only for values of $\Theta \geqq 1$ (see ref. 6). In figure 3 numerical results for $\Theta < 1$ have been presented. Also, for completeness and comparison purposes numerical results for $\Theta \geqq 1$ are presented. The good agreement between these curves and those of reference 6 indicates the validity of the numerical results from the computer program. General results for the shear buckling of clamped panels, furthermore, do not appear in the literature for any range of $\Theta$ with the exception of $\Theta = 1$ (the isotropic case); consequently, numerical results for clamped panels are presented in figure 4.

Both the results for simply supported and clamped panels indicate that the percentage decline in buckling load from $B = 1$ to $B = 0$ decreases as $\Theta$ increases. Also, a comparison of figures 3 and 4 shows that the percentage increase in buckling load of clamped panels over simply supported panels increases with increasing $\Theta$. The abrupt changes in slope appearing in these figures are due to changes in mode shape (from symmetric to antisymmetric modes). As anticipated from isotropic results (ref. 1), these abrupt changes are more predominant in clamped panels than in simply supported panels.

Tables 1 and 2 present the shear-buckling load coefficients used in obtaining the general orthotropic panel results of figures 3 and 4. Additionally, the trigonometric difference parameters (the mesh-spacing parameters $a/\Delta_x$ and $b/\Delta_y$ and the wavelength parameters $\lambda_x/a$ and $\lambda_y/b$) used in obtaining the buckling coefficients are presented in tables 1 and 2.

### Shear Buckling of a Simply Supported Panel With a Central Stiffener

Figure 5 presents results for the shear buckling of simply supported isotropic panels each of which contains one central flexural stiffener parallel to either the longer or shorter edges of the panel. As anticipated, the use of a central stiffener always provides an increase in the shear-buckling stress coefficient over that of the unstiffened panels $\left(\frac{EI}{bD} = 0\right)$. The percentage increase over unstiffened panels is greater in square panels than in rectangular panels. In rectangular panels of the same aspect ratio, the percentage increase over unstiffened panels is greater when the stiffeners are parallel to the longer direction than when they are parallel to the shorter direction. The central-stiffener results of figure 5, moreover, are in reasonably good agreement with similar results given in reference 14 for slightly curved panels. This agreement indicates the validity of the computer program for the solution of stiffened panels.

### Parametric Studies of Orthotropic Filament Reinforced Panels

Results are presented for the buckling of sandwich panels whose upper and lower skins are of laminated graphite-epoxy construction. Although some of the results in this section could be obtained from general orthotropic curves, such as those of figures 3 and 4, it is of interest to examine the effect of filament orientation upon the buckling load. (The material properties for the graphite-epoxy skins are given in table 3, with their equivalent general orthotropic parameter values $\Theta$ and $B$ at various filament orientations.)

In addition to the assumptions listed in the analysis section of this report, it is assumed in this section that

1. The panel is symmetric about the middle surface
2. Each lamina has the same filament orientation $\theta$ except for sign
3. The core carries no load and undergoes no transverse shear deformation

As a consequence of these assumptions, it may be shown that the buckling parameters $\hat{N}_x$, $\hat{N}_y$, and $\hat{N}_{xy}$ defined as

$$\hat{N}_x = \frac{b^2 N_x}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17a}$$

$$\hat{N}_y = \frac{b^2 N_y}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17b}$$

$$\hat{N}_{xy} = \frac{b^2 N_{xy}}{E_1 t^3 \left[1 - \left(\frac{h}{t}\right)^3\right]} \tag{17c}$$

depend only on the magnitude of $\theta$, the panel aspect ratio, and the boundary conditions. They do not depend on the thickness of each lamina, the number of laminas, or the core thickness. However, in order for assumption 2 of the analysis section to be reasonable – that is, neglect of bending-twisting coupling – it may be necessary that the ratio of core thickness to total thickness $h/t$ be nearly unity and that the amount of material in either cover oriented in the $+\theta$ and $-\theta$ directions be equal.

The variation of the buckling load with filament orientation for panels of various aspect ratios is presented in figure 6 for axial compression and in figure 7 for shear. The figures indicate that the buckling loads are highly dependent upon filament orientation and that optimum orientations (those which yield a maximum buckling load) may be determined for each aspect ratio. Also, the figures indicate that clamping has a greater effect on compressive buckling than on shear buckling.

An indication of the buckling strength of the epoxy panels as compared to equal-weight aluminum panels is provided by a comparison of the discrete buckling loads appearing on the right-hand ordinate of figures 6 and 7 with the curves in the same figures. These comparable values are valid for thin-skinned sandwich panels which have the same core, of thickness $h$, as the graphite-epoxy panels, but which have aluminum skins. For all the cases considered, a range of filament orientations exists for which the buckling strength of the graphite-epoxy panels exceeds that of the comparable aluminum panel with the same aspect ratio and boundary conditions. In the case of a clamped square panel in shear, the buckling strength of the graphite-epoxy panel exceeds that of the aluminum panel at all filament orientations.

It should be noted that, if the restriction that each lamina have the same filament orientation $\pm\theta$ is removed, isotropic skins can be produced from groups of three or more laminas (for example, 0, +60, and -60) which will have the same weight as the $\pm\theta$ skins but will yield a higher buckling load for each case shown in figures 6 and 7 and for many other shear and compression loadings. However, this is not necessarily true in all cases; for example, in the transverse compression of long panels ($a/b$ approaching zero), an orthotropic panel with filaments running transversely ($\theta = 0^{\circ}$) provides a higher buckling load than an equivalent isotropic panel. Furthermore, there are many applications where for various reasons (for example, strength or fabrication criteria) orthotropic panels are preferable to isotropic ones.

In figures 8 to 11 optimum filament orientations are shown for all aspect ratios. The curve of figure 8 was determined from the exact closed-form relationship for the compression of simply supported plates (ref. 6), while the curves of figures 9 to 11 were determined using program BOP. The abrupt changes in the slopes of these curves are caused by changes in the buckling mode shape associated with the optimum filament orientation. Except for figure 8, the location of these abrupt changes has been approximated since it is difficult to determine exactly where they occur.

In the compressive buckling curves (figs. 8 and 9) the optimum filament orientation for small aspect ratio $a/b$ is $0^{\circ}$ (parallel to the X-axis or to the direction of compression). This orientation angle rapidly increases at about $a/b = 0.56$ for simply supported panels and at about $a/b = 1.05$ for clamped panels. However, a comparison of the aspect-ratio 1 and 1.1 curves for a clamped panel as shown in figure 6 indicates that the optimum buckling load does not exhibit such a rapid change but decreases slightly as the aspect ratio goes from 1 to 1.1. For higher aspect ratios the optimum orientation oscillates with decreasing excursion about $\pm 45^{\circ}$ and, in general, a practical filament orientation for $a/b > 1$ is $\theta = \pm 45^{\circ}$.

In the case of shear buckling (figs. 10 and 11), the symmetry of the problem requires that the deviation of the optimum filament orientation from $45^{\circ}$ for a panel of aspect ratio $a/b$ be equal but opposite to that of a panel with aspect ratio $b/a$. Also, the peaks of figure 7 are quite flat; that is, they have a large radius of curvature associated with them. Consequently, it was difficult to determine precisely the optimum filament orientations in figures 10 and 11. However, it is reasonable to say from figures 10 and 11 that for large aspect ratios $a/b > 2$, $\theta = \pm 60^{\circ}$ to $\pm 62^{\circ}$ is a practical filament orientation.

Figures 12 and 13 present interaction curves for the buckling of simply supported and clamped panels in combined axial compression and shear for various filament orientations and aspect ratios. The optimum filament orientations (those that correspond to the highest values of the buckling parameters) change according to aspect ratio $a/b$ and the ratio of $N_{xy}/N_x$. For simply supported panels (fig. 12), when $a/b = 1$, the optimum orientation for all combinations of $N_x$ and $N_{xy}$ is $\theta = \pm 45^{\circ}$. When $a/b = 2$ or 5, the optimum filament orientation for predominantly shear loading is near $\pm 60^{\circ}$ and for predominantly compressive loading is near $\pm 45^{\circ}$. For clamped panels (fig. 13) when $a/b = 1$ the optimum orientation changes from $\theta = \pm 45^{\circ}$ for shear loading to $\theta = 0^{\circ}$ for compression. When $a/b = 2$ or 5, the optimum orientation changes from $\theta = \pm 60^{\circ}$ for pure shear to $\theta = \pm 45^{\circ}$ for pure compression. This behavior was the same as that exhibited by simply supported panels.

A summary of the data from figures 12 and 13 is shown in figure 14, which indicates the banded region in which all the results lie. For orthotropic panels it was found that the band is bounded from below by the following simple relationship given in reference 15 for isotropic panels:

$$R_x + R_{xy}^2 = 1 \tag{18}$$

where

$$\left.\begin{aligned} R_x &= \frac{N_x}{N_{x_o}} \\ R_{xy} &= \frac{N_{xy}}{N_{xy_o}} \end{aligned}\right\} \tag{19}$$

In equations (19), $N_{x_o}$ and $N_{xy_o}$ are the buckling loads for pure longitudinal compression and pure shear, respectively. Consequently, for the orthotropic cases considered, equation (18) is a reasonable conservative approximation for combined longitudinal compression and shear buckling of composite panels.

Figures 15 and 16 contain, respectively, compression and shear-buckling results for graphite-epoxy sandwich panels with nondeflecting edge supports and rotational edge springs for various filament orientations and aspect ratios. The associated boundary conditions are given by equations (A20) to (A22), and the rotational springs were assumed to be uniformly distributed about the panel edges. When the spring stiffness is zero, all four edges are simply supported and, when infinite, all four edges are clamped.

In general, the figures indicate that the buckling load increases sharply as the spring stiffness parameter $bk_R/E_1t^3$ increases from zero to one, the buckling loads obtaining at least 80 percent of their clamped value when the spring stiffness parameter is one. With further increase in the spring stiffness the buckling loads slowly approach the clamped value, increasing to within at least 10 percent of the clamped value when the spring stiffness parameter is three. Furthermore, the curves for the $\pm 45^\circ$ filament orientation generally approached the clamped values most rapidly.

## CONCLUDING REMARKS

A computerized analysis has been developed for the combined compression and shear buckling of stiffened orthotropic composite panels on discrete springs. Boundary conditions are general and include elastic boundary restraints. Buckling solutions are obtained by using a newly developed trigonometric finite-difference procedure which increases the solution convergence rate over conventional finite-difference methods, thus allowing problems to be solved with the same accuracy as with conventional differences but with fewer degrees of freedom. The trigonometric finite-difference procedure introduces two new parameters into the solution. These parameters can be internally selected by the program during problem execution or can be selected by the user. The validity of the program has been substantiated by comparisons with many existing known solutions. A program listing, input description, and sample problem are provided.

Using the program, the classical general shear-buckling results (in terms of universal orthotropic parameters), which are available only for simply supported panels over a limited range of orthotropic properties, have been extended to the complete range of these properties for simply supported panels and clamped panels. Results for the shear buckling of isotropic panels with a central stiffener have also been obtained.

The program has been applied to parametric studies which examine the effect of filament orientation upon the buckling of graphite-epoxy sandwich panels. From these studies optimum filament orientations (those which yield maximum buckling loads) were determined within a class of graphite-epoxy sandwich panels for all aspect ratios. In particular, it was found that for shear buckling of high-aspect-ratio panels (greater than two) reasonable filament orientations are between $\pm 60^{\circ}$ and $\pm 62^{\circ}$ while, for axial compression of panels with aspect ratio greater than one, a reasonable filament orientation is $\pm 45^{\circ}$. In addition, interaction curves were determined for the combined axial compression and shear buckling of panels with varying filament orientations. A parabolic interaction relationship previously developed for isotropic infinite strips in combined axial compression and shear provided a reasonably accurate and conservative estimate for the buckling loads of the orthotropic panels considered herein.

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## APPENDIX A

## DEVELOPMENT OF GOVERNING EQUATIONS

For completeness, equations (1) to (3) of the main text are repeated here:

$$\delta U = \int_0^b \int_0^a \left(M_x \delta w_{,xx} + M_y \delta w_{,yy} + 2M_{xy} \delta w_{,xy}\right) dx\, dy \tag{A1}$$

$$\left.\begin{aligned} M_x &= D_{11} w_{,xx} + D_{12} w_{,yy} \\ M_y &= D_{12} w_{,xx} + D_{22} w_{,yy} \\ M_{xy} &= 2D_{66} w_{,xy} \end{aligned}\right\} \tag{A2}$$

$$\delta V_N = \int_0^a \int_0^b \left(N_x w_{,x} \delta w_{,x} + N_y w_{,y} \delta w_{,y} + N_{xy} w_{,y} \delta w_{,x} + N_{xy} w_{,x} \delta w_{,y}\right) dy\, dx \tag{A3}$$

Then, replacing the derivatives in equations (A2) by trigonometric central differences yields

$$\left.\begin{aligned} \left(w_{,xx}\right)_{ij} &= \frac{1}{\hat{\Delta}_x^{\,2}}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right) \\ \left(w_{,yy}\right)_{ij} &= \frac{1}{\hat{\Delta}_y^{\,2}}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right) \\ \left(w_{,xy}\right)_{ij} &= \frac{1}{\hat{\Delta}_x \hat{\Delta}_y}\left(w_{i+1,j+1} - w_{i,j+1} - w_{i+1,j} + w_{ij}\right) \end{aligned}\right\} \tag{A4}$$

where $\hat{\Delta}_x$ and $\hat{\Delta}_y$ are the trigonometric difference coefficients defined by equation (10). The terms $\left(w_{,xx}\right)_{ij}$ and $\left(w_{,yy}\right)_{ij}$ are defined at the full stations denoted by the circles in figure 2(b), while $\left(w_{,xy}\right)_{ij}$ is defined at the half stations denoted by the squares in figure 2(b). Consequently, the indices (i,j) attached to a variable may refer to the variable being evaluated at either full or half stations, depending on the variable.

Introducing equations (A2) and (A4) into equation (A1) and replacing the double integral by a double sum yields

$$\delta U = \Delta_x \Delta_y \sum_{j=1}^{N} \sum_{i=1}^{M} \left\{ \xi_{x_i} \xi_{y_j} \left[ \frac{1}{\hat{\Delta}_x^{\,2}} M_{x_{ij}} \left( \delta w_{i+1,j} - 2\delta w_{ij} + \delta w_{i-1,j} \right) + \frac{1}{\hat{\Delta}_y^{\,2}} M_{y_{ij}} \left( \delta w_{i,j+1} - 2\delta w_{ij} + \delta w_{i,j-1} \right) \right] + 2\eta_{x_i} \eta_{y_j} \frac{M_{xy_{ij}}}{\hat{\Delta}_x \hat{\Delta}_y} \left( \delta w_{i+1,j+1} - \delta w_{i,j+1} - \delta w_{i+1,j} + \delta w_{ij} \right) \right\} \tag{A5}$$

where N and M are the total number of finite-difference stations in the x- and y-directions, respectively, and $\xi_{x_i}$, $\xi_{y_j}$, $\eta_{x_i}$, and $\eta_{y_j}$ have the following definitions:

$$\xi_{x_i} = \begin{cases} 0 & (i < I_1 \quad \text{or} \quad i > I_3) \\ 1/2 & (i = I_1 \quad \text{or} \quad i = I_3) \\ 1 & (I_1 < i < I_3) \end{cases} \tag{A6}$$

$$\xi_{y_j} = \begin{cases} 0 & (j < J_4 \quad \text{or} \quad j > J_2) \\ 1/2 & (j = J_4 \quad \text{or} \quad j = J_2) \\ 1 & (J_4 < j < J_2) \end{cases} \tag{A7}$$

$$\eta_{x_i} = \begin{cases} 0 & (i < I_1 \quad \text{or} \quad i \geqq I_3) \\ 1 & (I_1 \leqq i \leqq I_3) \end{cases} \tag{A8}$$

$$\eta_{y_j} = \begin{cases} 0 & (j < J_4 \quad \text{or} \quad j \geqq J_2) \\ 1 & (J_4 \leqq j < J_2) \end{cases} \tag{A9}$$

In equations (A6) to (A9), $I_1$ and $I_3$ are the row designations of boundaries ① and ③, respectively, and $J_2$ and $J_4$ are the column designations of boundaries ② and ④, respectively. (See fig. 2(a).)

Replacing the derivatives in equation (A3) by central trigonometric differences and the double integral by a double sum yields

$$\begin{aligned} \delta V_N = -\Delta_x \Delta_y \sum_{i=1}^{M} \sum_{j=1}^{N} \Bigg\{ & \xi_{y_j} \eta_{x_i} \frac{N_x}{\hat{\Delta}_x^2}\left(w_{i+1,j} - w_{ij}\right)\left(\delta w_{i+1,j} - \delta w_{ij}\right) + \xi_{x_i} \eta_{y_j} \frac{N_y}{\hat{\Delta}_y^2}\left(w_{i,j+1} \right. \\ & \left. - w_{ij}\right)\left(\delta w_{i,j+1} - \delta w_{ij}\right) + \eta_{x_i} \eta_{y_j} \frac{N_{xy}}{4\hat{\Delta}_x \hat{\Delta}_y}\Big[\left(w_{i+1,j} - w_{ij} + w_{i+1,j+1} - w_{i,j+1}\right)\left(\delta w_{i,j+1} \right. \\ & \left. - \delta w_{ij} + \delta w_{i+1,j+1} - \delta w_{i+1,j}\right) + \left(w_{i,j+1} - w_{ij} + w_{i+1,j+1} - w_{i+1,j}\right)\left(\delta w_{i+1,j} - \delta w_{ij} \right. \\ & \left. + \delta w_{i+1,j+1} - \delta w_{i,j+1}\right)\Big]\Bigg\} \end{aligned} \tag{A10}$$

In deriving equation (A10), the first and second terms in the integrand of equation (A3) have been replaced by trigonometric differences evaluated at stations indicated by "x" and "y," respectively, in figure 2(b), while the third and fourth terms have been evaluated at half stations, indicated by squares in figure 2(b), by averaging the derivatives.

The external forces and moments on the panel are those coming from discrete lateral deflection and rotational springs. The virtual work of these forces and moments may be expressed as

$$\begin{aligned} \delta V_S = & \sum_{i=1}^{M} \sum_{j=1}^{N} k_{\ell_{ij}} w_{ij} \delta w_{ij} + \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{k_{x_{ij}}}{\hat{\Delta}_x^2}\left(w_{i+1,j} - w_{ij}\right)\left(\delta w_{i+1,j} - \delta w_{ij}\right) \\ & + \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{k_{y_{ij}}}{\hat{\Delta}_y^2}\left(w_{i,j+1} - w_{ij}\right)\left(\delta w_{i,j+1} - \delta w_{ij}\right) \end{aligned} \tag{A11}$$

where $k_\ell$ is the spring stiffness associated with a lateral deflection spring and $k_x$ and $k_y$ are stiffnesses associated with rotational springs which resist moments acting about the Y- and X-axes, respectively. The $k_\ell$ type springs act at full stations, indicated by circles in figure 2(b), while the $k_x$ and $k_y$ type springs act at positions indicated by "x" and "y," respectively, in figure 2(b).

Substituting equations (A5), (A10), and (A11) into the statement of the principle of virtual work, equation (4) yields

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\left(K_{ij} + S_{ij} + N_x\alpha_{ij} + N_y\psi_{ij} + 2N_{xy}\chi_{ij}\right)\delta w_{ij} = 0 \tag{A12}$$

where

$$\begin{aligned} K_{ij} = {} & \xi_{y_j}\frac{1}{\hat{\Delta}_x^{\,2}}\left(\xi_{x_{i+1}}M_{x_{i+1,j}} - 2\xi_{x_i}M_{x_{ij}} + \xi_{x_{i-1}}M_{x_{i-1,j}}\right) + \xi_{x_i}\frac{1}{\hat{\Delta}_y^{\,2}}\left(\xi_{y_{j+1}}M_{y_{i,j+1}} - 2\xi_{y_j}M_{y_{ij}}\right. \\ & \left. + \xi_{y_{j-1}}M_{y_{i,j-1}}\right) + \frac{1}{\hat{\Delta}_x\,\hat{\Delta}_y}\left(\eta_{x_{i-1}}\eta_{y_{j-1}}M_{xy_{i-1,j-1}} - \eta_{x_{i-1}}\eta_{y_j}M_{xy_{i-1,j}}\right. \\ & \left. - \eta_{x_i}\eta_{y_{j-1}}M_{xy_{i,j-1}} + \eta_{x_i}\eta_{y_j}M_{xy_{ij}}\right) \end{aligned} \tag{A13}$$

$$\alpha_{ij} = \frac{1}{\hat{\Delta}_x^{\,2}}\left[\xi_{y_j}\eta_{x_i}\left(w_{i+1,j} - w_{ij}\right) - \xi_{y_j}\eta_{x_{i-1}}\left(w_{ij} - w_{i-1,j}\right)\right] \tag{A14}$$

$$\psi_{ij} = \frac{1}{\hat{\Delta}_y^{\,2}}\left[\xi_{x_i}\eta_{y_j}\left(w_{i,j+1} - w_{ij}\right) - \xi_{x_i}\eta_{y_{j-1}}\left(w_{ij} - w_{i,j-1}\right)\right] \tag{A15}$$

$$\begin{aligned} \chi_{ij} = \frac{1}{4\hat{\Delta}_x\,\hat{\Delta}_y}\Big[ & \left(w_{i+1,j+1} - w_{ij}\right)\eta_{x_i}\eta_{y_j} - \left(w_{i+1,j-1} - w_{ij}\right)\eta_{x_i}\eta_{y_{j-1}} \\ & - \left(w_{ij} - w_{i-1,j-1}\right)\eta_{x_{i-1}}\eta_{y_{j-1}} + \left(w_{ij} - w_{i-1,j+1}\right)\eta_{x_{i-1}}\eta_{y_j}\Big] \end{aligned} \tag{A16}$$

$$S_{ij} = \frac{1}{\Delta_x \Delta_y} k_{\ell_{ij}} w_{ij} + \frac{1}{\Delta_x \Delta_y \hat{\Delta}_x^2}\left[k_{x_{i-1,j}}\left(w_{ij} - w_{i-1,j}\right) - k_{x_{ij}}\left(w_{i+1,j} - w_{ij}\right)\right]$$

$$+ \frac{1}{\Delta_x \Delta_y \hat{\Delta}_y^2}\left[k_{y_{i,j-1}}\left(w_{ij} - w_{i,j-1}\right) - k_{y_{ij}}\left(w_{i,j+1} - w_{ij}\right)\right] \tag{A17}$$

From equations (A2) and (A4), the moments are related to the displacements as follows:

$$\left.\begin{aligned}
\left(M_x\right)_{ij} &= \left(D_{11}\right)_{ij}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right)\frac{1}{\hat{\Delta}_x^2} + \left(D_{12}\right)_{ij}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right)\frac{1}{\hat{\Delta}_y^2} \\
\left(M_y\right)_{ij} &= \left(D_{22}\right)_{ij}\left(w_{i,j+1} - 2w_{ij} + w_{i,j-1}\right)\frac{1}{\hat{\Delta}_y^2} + \left(D_{12}\right)_{ij}\left(w_{i+1,j} - 2w_{ij} + w_{i-1,j}\right)\frac{1}{\hat{\Delta}_x^2} \\
\left(M_{xy}\right)_{ij} &= 2\left(D_{66}\right)_{ij}\left(w_{i+1,j+1} - w_{i,j+1} - w_{i+1,j} + w_{ij}\right)\frac{1}{\hat{\Delta}_x \hat{\Delta}_y}
\end{aligned}\right\} \tag{A18}$$

where $\left(M_x\right)_{ij}$ and $\left(M_y\right)_{ij}$ act at the full stations, indicated by circles in figure 2(b), and $\left(M_{xy}\right)_{ij}$ acts at the half stations, indicated by squares in figure 2(b).

## Boundary Conditions

All four boundaries free or spring-supported. - If on the plate boundaries no constraints exist on $w$ or its derivatives normal to the boundary, equation (A12) must be valid for all virtual displacements $\delta w_{ij}$, thus yielding equation (5) which is repeated here:

$$K_{ij} + S_{ij} + N_x \alpha_{ij} + N_y \psi_{ij} + 2N_{xy} \chi_{ij} = 0 \qquad \begin{pmatrix} i = 1, \ldots, M \\ j = 1, \ldots, N \end{pmatrix} \tag{A19}$$

Equation (A19) represents equilibrium at each finite-difference station with each equilibrium equation containing an array of 13 values of $w$ as depicted in figure 2(b). In solving these equations by the procedure discussed in appendix C, the terms $w_{ij}$ represent the unknowns and equations (A18) are used to determine the moments appearing in the relationship for $K_{ij}$, equation (A13).

When a difference station lies on the boundary of the plate (that is, $i = I_1$ or $i = I_3$ or $j = J_4$ or $j = J_2$), the corresponding equilibrium equation reduces to the natural boundary condition on the Kirchhoff shear, reference 6. Also, when a difference station lies one finite difference interval off the plate (that is, $i = I_1 - 1$ or $i = I_3 + 1$ or $j = J_4 - 1$ or $j = J_2 + 1$), the corresponding equilibrium equation reduces to the natural boundary condition on the bending moment. Furthermore, when a difference station lies two or more finite-difference intervals off the plate (that is, $i < I_1 - 1$ or $i > I_3 + 1$ or $j < J_4 - 1$ or $j > J_2 + 1$), the corresponding equilibrium equations reduce to the trivial equation $0 = 0$. Consequently, no equilibrium equations exist for these stations.

Edges with nondeflecting lateral supports and rotational springs.- Equation (A19) may be used in approximating the solution of problems with nondeflecting edges; for example, if $w = 0$ on an edge, equation (A19) may be used in conjunction with extremely stiff lateral springs placed along the edge. Alternatively, an edge which is restrained from lateral motion may be handled as a special case, and in so doing the number of computations required for the problem solution is reduced.

The boundary condition for a nondeflecting edge is

$$w = 0 \qquad \text{(on the edge)} \tag{A20}$$

If, in addition, uniformly distributed rotational springs act along boundaries ① and ③ (see fig. 2(a)),

$$M_x = k_R w_{,x} \qquad \text{(on the edge)} \tag{A21}$$

or, if uniformly distributed rotational springs act along boundary ② or ④,

$$M_y = k_R w_{,y} \qquad \text{(on the edge)} \tag{A22}$$

As a result of the foregoing, equation (A20) replaces the boundary condition on the Kirchhoff shear, while the difference form of equation (A21) or (A22) replaces the boundary condition on the edge moment. Furthermore, as an example, equation (A21) on boundary ① becomes

$$\left(M_x\right)_{I_1,j} = \frac{k_R\left(w_{I_1+1,j} - w_{I_1-1,j}\right)}{\hat{\Delta}_x^*} \tag{A23}$$

where

$$\frac{1}{\hat{\Delta}_x^*} = \frac{\pi}{2\lambda_x \sin \frac{\pi \Delta_x}{\lambda_x}} \tag{A24}$$

Substituting for $M_x$ from equations (A18) and employing equation (A23) yields

$$\left(M_x\right)_{I_1,j} = \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}\left(w_{I_1+1,j} + w_{I_1-1,j}\right) = \frac{k_R}{\hat{\Delta}_x^*}\left(w_{I_1+1,j} - w_{I_1-1,j}\right) \tag{A25}$$

Then

$$w_{I_1-1,j} = \frac{\left[\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}\right] w_{I_1+1,j}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}} \tag{A26}$$

Substituting into the first of equation (A25) yields

$$\left(M_x\right)_{I_1,j} = \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}\left[1 + \frac{\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}}\right] w_{I_1+1,j} \tag{A27}$$

It is evident from an examination of the first of equation (A25) that equation (A23) is satisfied by setting $w_{I_1-1,j} = 0$ and $\left(D_{11}\right)_{I_1,j} = \left(D_{11}^*\right)_{I_1,j}$ where

$$\left(D_{11}^*\right)_{I_1,j} = \left[1 + \frac{\frac{k_R}{\hat{\Delta}_x^*} - \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}}{\frac{k_R}{\hat{\Delta}_x^*} + \frac{\left(D_{11}\right)_{I_1,j}}{\hat{\Delta}_x{}^2}}\right] \left(D_{11}\right)_{I_1,j} \tag{A28}$$

Similar relationships may be developed for boundaries ②, ③, and ④.

In summary, for a nondeflecting boundary with uniformly distributed rotational springs, equilibrium on the boundary and one station off the boundary are not used. Instead, in the remaining equilibrium equations, $w$ on the boundary and one station off the boundary are set equal to zero and $D_{11}$ on the boundary is set equal to $D_{11}^*$ if the boundary is number ① or ③, and $D_{22}$ on the boundary is set equal to $D_{22}^*$ if the boundary is number ② or ④.

The limiting cases of simply supported or clamped boundaries are readily provided by letting $k_R$ approach zero or infinity, respectively. Hence, for a simply supported boundary

$$D_{11}^* = 0 \quad \text{if the boundary is ① or ③}$$

$$D_{22}^* = 0 \quad \text{if the boundary is ② or ④}$$

and for a clamped boundary

$$D_{11}^* = 2D_{11} \quad \text{if the boundary is ① or ③}$$

$$D_{22}^* = 2D_{22} \quad \text{if the boundary is ② or ④}$$

## Flexural Stiffeners

The effects of flexural stiffeners are accounted for in a manner similar to that used for nondeflecting supports. At each finite-difference station along the stiffener, $(D_{11})_{ij}$ is replaced by $(\overline{D}_{11})_{ij}$ if the stiffener is parallel to the X-axis and $(D_{22})_{ij}$ is replaced by $(\overline{D}_{22})_{ij}$ if the stiffener is parallel to the Y-axis, where

$$\left.\begin{aligned} (\overline{D}_{11})_{ij} &= (D_{11})_{ij} + \frac{EI}{\Delta_y} \\ (\overline{D}_{22})_{ij} &= (D_{22})_{ij} + \frac{EI}{\Delta_x} \end{aligned}\right\} \tag{A29}$$

and $EI$ is the lateral bending stiffness of the stiffener about the neutral plane of the panel.

# APPENDIX A

## Summary of Finite-Difference Stations at Which Equilibrium Is Enforced

As a result of the foregoing discussions on free or spring-supported edges and nondeflecting edges, the rows i and columns j at which equilibrium is enforced are, respectively,

$$\left.\begin{aligned} M_e &= I_3 - I_1 + 3 - \text{Twice the number of nondeflecting edges parallel to the Y-axis} \\ N_e &= J_2 - J_4 + 3 - \text{Twice the number of nondeflecting edges parallel to the X-axis} \end{aligned}\right\} \quad \text{(A30)}$$

## APPENDIX B

## TRIGONOMETRIC FINITE DIFFERENCES

Trigonometric finite differences introduce the trigonometric parameters $\lambda_x$ and $\lambda_y$ which are not present in conventional finite differences. Consequently, the first purpose of this appendix is to present and demonstrate some effective procedures for selecting values of $\lambda_x$ and $\lambda_y$ which results in an improved convergence rate over conventional differences. The second purpose is to point out some of the limitations of trigonometric finite differences.

### Selection of $\lambda_x$ and $\lambda_y$

Selection of values of $\lambda_x$ and $\lambda_y$ which improve the convergence rate of trigonometric finite-difference solutions over those of conventional finite-difference solutions is predominantly based on engineering considerations and experience. Experience has shown that it is often advantageous to select trigonometric parameters whose ratio is determined on the basis of the infinitely long panel solution as is done in equations (11) and (12), that is,

$$\frac{\lambda_y}{\lambda_x} = \beta \tag{B1}$$

where $\beta$ is the wavelength parameter of an infinitely long panel, defined as the ratio of the panel width to the buckle length. Imposing equation (B1) on the parameter selection should be reasonable for panels which buckle with more than two half waves along their length.

The value of $\beta$ may be determined to any degree of accuracy by extending the isotropic results of reference 16. For a panel with its long dimension parallel to the X-axis, first approximations of the buckling eigenvalue $\overline{p}_\infty$ and wavelength parameter $\beta$ satisfy the following two simultaneous equations for panels whose long sides are simply supported:

$$\left.\begin{aligned} &\left(\overline{t}_{xy} + \overline{p}_\infty r_{xy}\right)^2 - \frac{9}{4} M_1 M_2 = 0 \\ &\frac{\partial}{\partial \beta}\left(M_1 M_2\right) = 0 \end{aligned}\right\} \tag{B2}$$

and, for panels whose long sides are clamped, $\bar{p}_\infty$ and $\beta$ satisfy the two simultaneous equations

$$\left.\begin{aligned} &\left(t_{xy} + \bar{p}_\infty r_{xy}\right)^2 - \frac{15}{32}\left(2M_0 + M_2\right)\left(M_1 + M_3\right) = 0 \\ &\frac{\partial}{\partial\beta}\left(M_0 + M_2\right)\left(M_1 + M_3\right) = 0 \end{aligned}\right\} \tag{B3}$$

where

$$M_n = \frac{\pi}{8\beta}\left[\frac{D_{22}}{D_{11}} n^4 + 2\,\frac{D_3}{D_{11}} n^2\beta^2 + \beta^4 - \beta^2\left(\bar{t}_x + \bar{p}_\infty r_x\right) - n^2\left(\bar{t}_y + \bar{p}_\infty r_y\right)\right] \qquad (n = 0,\ 1,\ 2,\ 3)$$

## Convergence Behavior

Figures 17(a) to 17(f) illustrate the convergence of trigonometric finite-difference solutions when $\lambda_y/\lambda_x$ is fixed on the basis of equation (B1). Results for both simply supported and clamped isotropic panels under either axial compression or shear are shown in these figures. In each case the panel was modeled using an equal number of finite-difference stations in the x- and y-directions. Exact and approximate values for these cases are given in references 1, 6, 16, and 17.

The dashed curve in each of figures 17(a) to 17(f) illustrates the convergence of the conventional difference solution – that is, $\lambda_x$ and $\lambda_y$ infinite – while the solid and dash-dot curves illustrate the convergence achieved with some finite values of $\lambda_x$. Comparison of the curves indicates that some values of $\lambda_x$ increase the convergence rate over the conventional rate while other values decrease it. (In those special cases where the buckle shape is exactly a double sine wave, the trigonometric difference solution is exact when $\lambda_x$ and $\lambda_y$ are equal to the buckle half wavelength.) Consider though the dash-dot curve of each figure. These curves show the convergence when $\lambda_y$ is simply taken equal to the panel width and $\lambda_x$ is taken equal to the buckle length of the infinitely long panel; that is, equations (11) and (12) are applied. Comparison of the dash-dot curves and the dashed curves indicates that equations (11) and (12) provide reasonable values of $\lambda_x$ and $\lambda_y$ which improve the solution convergence. As figures 17(a) to 17(f) indicate, however, other values of $\lambda_x/a$ could be selected which further improve the convergence rate. Such values may be found by making a condensed cross plot of each figure; for example, consider the case of the compression of a square isotropic clamped panel as shown in figure 17(c). For this case, equations (B3) predict $\beta = 1.5$. Then, using program BOP with $\lambda_y/\lambda_x = 1.5$, $\lambda_x/a$ is varied from 0.25 to 1 for mesh sizes of $a/\Delta_x = b/\Delta_y = 5$ and $a/\Delta_x = b/\Delta_y = 6$; these curves are shown in figure 18. As the mesh spacing is decreased, the curves will approach the exact solution at all values of $\lambda_x/a$. However, the two curves cross at $\lambda_x/a = 0.35$ and $\overline{N}_x = 9.75$, which implies that convergence is most rapid at this value of $\lambda_x/a$ since increasing the mesh size did not change the buckling stress coefficient. It is evident from figure 17(c) that, if such a choice of $\lambda_x$ were used, convergence would be improved beyond that achieved by selecting $\lambda_x$ from equation (11).

As further examples, consider the results in table 4 for the shear buckling of the orthotropic panels described in table 3. The values of $\lambda_x$ and $\lambda_y$ were determined by making the required cross plots. It is evident by comparing the conventional and trigonometric solutions given in the table that the selected values of $\lambda_x$ and $\lambda_y$ provided excellent results.

The additional effort involved in finding better values of $\lambda_x$ may be justified in problems where convergence would otherwise be extremely slow. It may also be justified in the performance of parameter studies. In such studies some typical problems within the problem class to be studied are chosen; for these, improved values of $\lambda_x$ are found and then interpolated to yield $\lambda_x$ for other problems within the study class.

## Correction Factors for Equations (11) and (12)

Equations (B2) and (B3) which provide $\beta$ for equations (11) and (12) do not cover every case; the boundary conditions may not be simply supported or clamped, or it may be inappropriate to use $\beta$ based on an infinitely long panel. Consequently, equations (11) and (12) must be used with engineering judgment. Some allowance is provided by introducing correction factors $C_x$ and $C_{yx}$ into equations (11) and (12), that is,

$$\frac{\lambda_y}{\lambda_x} = C_{yx}\beta \tag{B4}$$

$$\frac{\lambda_x}{a} = \frac{b}{a}\frac{C_x}{\beta} \tag{B5}$$

A numerical routine which calculates $\beta$ from equations (B2) or (B3), and then $\lambda_x$ and $\lambda_y$ from equations (11) and (12), is used in program BOP. This program is briefly discussed in the main text and is documented in appendix D.

## Limitations of Trigonometric Finite Differences

In figure 19 a sketch of the variation with $\lambda_x$ of the coefficient $1/\hat{\Delta}_x$ as defined by equation (10) is presented. The reader's attention is called to the singularities of

$1/\hat{\Delta}_x$ at $\lambda_x = \frac{\Delta_x}{2}, \frac{\Delta_x}{4}, \frac{\Delta_x}{8}$, etc. In order to avoid these singularities and the rapidly varying behavior of $1/\hat{\Delta}_x$ between them, $\lambda_x$ and similarly $\lambda_y$ must be chosen such that

$$\left.\begin{aligned} \lambda_x &> \frac{\Delta_x}{2} \\ \lambda_y &> \frac{\Delta_y}{2} \end{aligned}\right\} \tag{B6}$$

Moreover, if uniformly distributed rotational springs are prescribed on the boundaries in the manner presented in equations (A20) to (A24), then to avoid singularities in $\hat{\Delta}_x^*$ and $\hat{\Delta}_y^*$ choose

$$\left.\begin{aligned} \lambda_x &> \Delta_x \\ \lambda_y &> \Delta_y \end{aligned}\right\} \tag{B7}$$

## APPENDIX C

## STABILITY DETERMINANT EVALUATION

Since the total number of rows and columns at which equilibrium is enforced is $M_e$ and $N_e$, respectively, a stability determinant of order $M_eN_e \times M_eN_e$ would result. To produce a stability determinant of smaller size, a marching procedure is employed. This procedure, which is described herein, operates on the equilibrium equations to produce, by a process of successive elimination, a determinant of size $2M_e \times 2M_e$.

The marching procedure takes advantage of the fact that each of the difference equations of equilibrium, equations (5), is linear and homogeneous, with each one containing no more than 13 unknown deflections. For a station (i,j) away from the plate edges

$$I_f + 1 < i < I_\ell - 1$$

$$J_f + 1 < j < J_\ell - 1$$

where $I_f$ and $I_\ell$ are the first and last rows of finite-difference stations at which equilibrium is prescribed, and $J_f$ and $J_\ell$ are the first and last columns of finite-difference stations at which equilibrium is prescribed, the 13 unknown deflections form the geometric pattern shown in figure 2(b). It is evident from this pattern that the deflections at stations in column $j + 2$ can be determined by using equilibrium at stations in column $j$ if the deflections in columns $j - 2$, $j - 1$, $j$, and $j + 1$ are known or prescribed. For equilibrium at stations lying near the edges, however, the geometric pattern of figure 2(b) is reduced. Consequently, equilibrium at stations in the first column $J_f$ may be used to determine the deflections at stations in column $J_f + 2$ if the deflections only in columns $J_f$ and $J_f + 1$ are prescribed, since deflections in columns $J_f - 1$ and $J_f - 2$ do not appear in these equilibrium equations.

Having found the deflections in column $J_f + 2$ from prescribed values in column $J_f$ and $J_f + 1$, equilibrium at stations in column $J_f + 1$ can be used to obtain the deflections in column $J_f + 3$; likewise, equilibrium at stations in column $J_f + 2$ can provide deflections in column $J_f + 4$, etc. Thus, a marching routine is developed from column to column which determines the deflections throughout the panel from prescribed values in the first two columns. It should be noted that equilibrium at stations in the last two columns, $J_\ell - 1$ and $J_\ell$, is not used at this stage of the marching procedure.

The evaluation of the stability determinant can now be performed numerically for a given value of the eigenvalue by choosing $2M_e$ linearly independent sets of assumed deflections for the first two columns. These assumed sets are taken as

$$\left[w^{(1)}\right] = \begin{bmatrix}1\\0\\0\\0\\ \cdot\\ \cdot\\ \cdot\\0\end{bmatrix} \qquad \left[w^{(2)}\right] = \begin{bmatrix}0\\1\\0\\0\\ \cdot\\ \cdot\\ \cdot\\0\end{bmatrix} \qquad \left[w^{(3)}\right] = \begin{bmatrix}0\\0\\1\\0\\ \cdot\\ \cdot\\ \cdot\\0\end{bmatrix}, \ldots, \left[w^{(2M_e)}\right] = \begin{bmatrix}0\\0\\0\\0\\ \cdot\\ \cdot\\ \cdot\\1\end{bmatrix} \tag{C1}$$

where each column contains $2M_e$ values. By marching across the plate with the rth set of these assumed values, deflections throughout the plate $w_{ij}^{(r)}$ are determined. However, the equilibrium equation at stations in the last two columns will not, in general, be satisfied by any of these assumed sets. Therefore, consider the column matrix

$$\left\{e^{(r)}\right\} = \begin{bmatrix} e^{(r)}_{I_f,J_\ell-1} \\ \cdot \\ \cdot \\ \cdot \\ e^{(r)}_{I_\ell,J_\ell-1} \\ e^{(r)}_{I_f,J_\ell} \\ \cdot \\ \cdot \\ \cdot \\ e^{(r)}_{I_\ell,J_\ell} \end{bmatrix} \tag{C2}$$

where each element of the matrix represents the value of the left-hand side of an equilibrium equation at a station in columns $J_\ell - 1$ or $J_\ell$ for the rth assumed set and would be identically zero if the assumed deflections were exact. The total solution is a linear superposition of all the assumed sets, that is,

$$w_{ij} = \sum_{r=1}^{2M_e} A^{(r)} w_{ij}^{(r)} \qquad \begin{pmatrix} I_f \leqq i \leqq I_\ell \\ J_f \leqq j \leqq J_\ell \end{pmatrix} \tag{C3}$$

Correspondingly, the total contribution to equilibrium at columns $J_\ell - 1$ and $J_\ell$ for all assumed sets of deflections is

$$[e] = \sum_{r=1}^{2M_e} A^{(r)} \left\{e^{(r)}\right\} \tag{C4}$$

The coefficients $A^{(r)}$ are determined by enforcing equilibrium at stations in the last two columns which leads to

$$[e] = 0 \tag{C5}$$

or

$$\left[e^{(1)} \middle| e^{(2)} \middle| \; . \; . \; . \; \middle| e^{(r)} \middle| \; . \; . \; . \; \middle| e^{(2M_e)}\right] \begin{bmatrix} A^{(1)} \\ A^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ A^{(r)} \\ \cdot \\ \cdot \\ \cdot \\ A^{(2M_e)} \end{bmatrix} = 0 \tag{C6}$$

For a nontrivial solution of equation (C6) the determinant of the coefficients must vanish, resulting in

$$|e| = 0 \tag{C7}$$

and it is clear from equation (C6) that $|e|$ is of order $2M_e \times 2M_e$.

# APPENDIX D

## COMPUTER PROGRAM

The computer program BOP (Buckling of Orthotropic Panels) was written in FORTRAN IV on a SCOPE 3.1 system modified for Langley Research Center and executes and loads with a field length of 60000 octal locations. The program is applicable to the combined compression and shear of stiffened, variable-thickness, flat rectangular orthotropic panels on discrete springs; boundary conditions are general and include elastic boundary restraints. A description of the input, an example problem showing input and output, and a program listing are provided.

### Input Description

For each case the input consists of a single identification card and a Namelist BUCKLE as follows:

ISTIFF,ISTEP,IX,JX,MSHAPE,MA,NOMAT,TH,AT,MATYPE,E1,E2,U1,G12,IBC,AKR,D1, D2,D12,D66,DS1,XA,XB,AKL,AKX,AKY,NUPRIT,EI,IORIENT,LOC,TX,TY,TXY,RX,RY, RXY,P1,DELP,PFIN,TEST,MR,NC,X,Y,DS2,DS12,DS66

Many of the input variables have associated default values as will be indicated in the following descriptions:

Control parameters

| | |
|---|---|
| ISTIFF = 1 | no preprocessing of laminate properties – execute for buckling (only) |
| = 2 | preprocess and execute for buckling |
| = 3 | preprocess only – do not execute for buckling |

DEFAULT:ISTIFF = 2

| | |
|---|---|
| ISTEP = 1 | program automatically varies the input step size, DELP |
| = 2 | step size fixed and equal to DELP |

DEFAULT:ISTEP = 1

| | |
|---|---|
| IX = 1 | output of intermediate results |
| = 2 | output of intermediate results suppressed |
| JX = 1 | output of flexural stiffnesses at each finite-difference station |
| = 2 | output of flexural stiffnesses suppressed |

DEFAULT:IX = JX = 2

| | |
|---|---|
| MSHAPE = 1 | compute mode shape |
| = 2 | do not compute mode shape |

DEFAULT:MSHAPE = 2

Laminate and lamina properties (Required if ISTIFF = 2 or 3)

| | |
|---|---|
| MA | number of laminas in the laminate |
| NOMAT | number of different materials comprising the laminate |
| TH | a one-dimensional array in which the ith element of the array corresponds to the filament orientation (as measured from the X-axis in degrees) in the ith lamina |
| AT | a one-dimensional array in which the ith element of the array corresponds to the thickness of the ith lamina |
| MATYPE | a one-dimensional array in which the ith element is the number designation of the material in the ith lamina |
| E1 | a one-dimensional array in which the jth element of the array corresponds to the Young's modules parallel to the fibers in the jth material |
| E2 | a one-dimensional array specifying the Young's modulus transverse to the fibers |
| U1 | a one-dimensional array specifying Poisson's ratio $\nu_{12}$ in each lamina |
| G12 | a one-dimensional array specifying the shear modulus in each material |

Boundary conditions

| | |
|---|---|
| IBC | a one-dimensional array of four elements in which the ith element refers to the ith boundary (see fig. 2(a)); four options are available at each boundary |
| IBC(I) = 1 | nondeflecting lateral support with uniform rotational springs on edge I |
| = 2 | simple support on edge I |
| = 3 | clamped on edge I |
| = 4 | free on edge I |
| = 5 | other boundary conditions – set by user through appropriate input of D1, D2, D12, and D66 |

AKR — a one-dimensional array in which the ith element of the array corresponds to the uniformly distributed rotational spring stiffness per unit length of boundary on the ith boundary; required if any boundary has IBC = 1

Laminate flexural stiffnesses (Required if ISTIFF = 1)

D1 — a two-dimensional array in which the (i,j)th element of the array corresponds to the value of $(D_{11})_{ij}$

D2 — similar to D1, but specifying $(D_{22})_{ij}$

D12 — similar to D1, but specifying $(D_{12})_{ij}$

D66 — similar to D1, but specifying $(D_{66})_{ij}$

DS1 — reference value of $D_{11}$

Plate geometry

XA = a — dimension parallel to X-axis (fig. 2(a))

XB = b — dimension parallel to Y-axis (fig. 2(a))

Discrete springs

AKL — a two-dimensional array in which the (i,j)th element corresponds to $(k_{\ell})_{ij}$

AKX — similar to AKL but referring to $(k_x)_{ij}$

AKY — similar to AKL but referring to $(k_y)_{ij}$

Discrete flexural stiffeners

NUPRIT — number of stiffeners

EI — a one-dimensional array whose ith element specifies the flexural stiffness of the ith stiffener about the neutral plane of the panel

IORIENT — a one-dimensional array whose ith element specifies whether the stiffener is parallel to X- or Y-axis

= 1 stiffener parallel to X-axis

= 2 stiffener parallel to Y-axis

LOC — a one-dimensional array whose ith element gives the row or column location of the ith stiffener

DEFAULT:NUPRIT = 0; EI, IORIENT and LOC need not be input

## APPENDIX D

### Applied in-plane loads

In-plane loads are assumed to be uniform over the boundary to which they are applied and are increased to buckling according to the relationships prescribed by equations (13); therefore, the user inputs

TX = $\bar{t}_x$

TY = $\bar{t}_y$

TXY = $\bar{t}_{xy}$

RX = $r_x$

RY = $r_y$

RXY = $r_{xy}$

### Eigenvalue search parameters

P1 — starting value of $\bar{p}$. If P1 < 0., the program will calculate P1 from equation (B2) or (B3) according to the relation,

$$P1 = ABS(P1)*PBAR \tag{D1}$$

where PBAR is $\bar{p}_\infty$ from equation (B2) or (B3).

DEFAULT:P1 = 0.9*PBAR

DELP — increment of ($\bar{p}$); if P1 < 0., DELP = 0.1*PBAR; if ISTEP = 1, DELP is automatically varied during the eigenvalue search

PFIN — maximum value of $\bar{p}$ during the eigenvalue search

TEST — eigenvalue accuracy

DEFAULT:1. $\times 10^{-3}$

### Trigonometric finite-difference data

MR — number of rows of finite-difference stations interior to the plate – not including boundaries

NC — number of columns of finite-difference stations interior to the plate – not including boundaries

Note: The marching procedure requires NC ≧ 4

X = $\lambda_x/a$

Y = $\lambda_y/b$

Note: If the user inputs $X \leqq 0$, the program automatically calculates a new value of X and Y according to the relationship expressed by equations (B4) and (B5); that is,

$$X = ABS(X)*XB/BETA/XA \tag{D2}$$

$$Y = ABS(Y) \tag{D3}$$

where the input magnitudes of X and Y (that is, ABS(X) and ABS(Y)) replace $C_x$ and $C_{yx}$ in equations (B4) and (B5). Also, in equation (D1), BETA = $\beta$, and $\beta$ is calculated from equation (B2) or (B3).

When ISTIFF = 1 and the evaluation of X and Y is chosen, the user must also input

| | |
|---|---|
| DS2 | average or typical value of $D_{22}$ |
| DS12 | average or typical value of $D_{12}$ |
| DS66 | average or typical value of $D_{66}$ |

DEFAULT: Calculation of X and Y using equations (D2) and (D3) where ABS(X) and ABS(Y) are set equal to unity.

## Example Problem

Consider the shear buckling of a 12-inch by 3-inch clamped sandwich panel which has as its lay-up, 45/-45/45/-45/CORE/-45/45/-45/45. The core thickness is 0.0605 inch and each lamina of the skins is graphite-epoxy with a thickness of 0.0055 inch.

## Sample Input

```
THIS IS A FREE FIELD IDENTIFICATION CARD
$BUCKLE TX=.0,TY=.0,TXY=.0,RX=.0,RY=.0,RXY=1.,
XA=12,XB=3.,MR=12,NC=6,IBC=4*3,NOMAT=2,E1=2.10E7,1.,E2=2.39E6,1.,
U1=.31,.2,G12=6.5E5,1.,MA=9,MATYPE=4*1,2,4*1,
AT=4*.0055,.0605,4*.0055,TH=45.,-45.,45.,-45.,.0,-45.,45.,-45.,45.
$
```

## Sample Output

```
INPUT FOR CASE

 THIS IS A FREE FIELD INDENTIFICATION CARD

                                  111111111111***** Y
                                  4           2
                                  4           2
                                  4           2
                                  4           2
                                  4           2
                                  4           2
                                  333333333333
                                  *
                                  *
                                  *
                                  *
                                  *

                                  X

     ISTIFF=2    ISTEP=1           IX=2    JX=2          TEST=   1.00000000E-03

     XA=   1.20000000E+01        XB=   3.00000000E+00     ASPECT RATIO=   4.00000000E+00

                MR=12      NC= 6

     TX=   0.                    TY=   0.                     TXY=   0.

     PI= -9.00000000E-01         DELP=   1.00000000E-01       PFIN=   1.00000000E+02

     X= -1.00000000E+00          Y=  1.00000000E+00

     RX=   0.                    RY=   0.                     RXY=   1.00000000E+00
```

LAMINATED PLATE PROPERTIES

| MATERIAL KIND | E1 | E2 | U1 | GXY |
|---|---|---|---|---|
| 1 | 2.10000000E+07 | 2.39000000E+06 | 3.10000000E-01 | 6.50000000E+05 |
| 2 | 1.00000000E+00 | 1.00000000E+00 | 2.00000000E-01 | 1.00000000E+00 |

| LAYER NO. | MAT. KIND | THICK | THETA |
|---|---|---|---|
| 1 | 1 | 5.50000000E-03 | 4.50000000E+01 |
| 2 | 1 | 5.50000000E-03 | -4.50000000E+01 |
| 3 | 1 | 5.50000000E-03 | 4.50000000E+01 |
| 4 | 1 | 5.50000000E-03 | -4.50000000E+01 |
| 5 | 2 | 6.05000000E-02 | 0. |
| 6 | 1 | 5.50000000E-03 | -4.50000000E+01 |
| 7 | 1 | 5.50000000E-03 | 4.50000000E+01 |
| 8 | 1 | 5.50000000E-03 | -4.50000000E+01 |
| 9 | 1 | 5.50000000E-03 | 4.50000000E+01 |

******************************************************************************

CAUTION -- COUPLING BETWEEN EXTENSION AND BENDING MAY BE SIGNIFICANT
IF THE FOLLOWING FOUR VALUES ARE NOT ALL EQUAL.
IF THIS IS THE CASE, THE RESULTS SHOULD BE USED WITH DESCRETION

4.76775528E-17 7.15163293E-17 8.80102296E-17 5.97234077E-17

******************************************************************************

A MATRIX

| | | |
|---|---|---|
| 3.05215229E+05 | 2.48015179E+05 | 0. |
| 2.48015179E+05 | 3.05215229E+05 | 0. |
| 0. | 0. | 2.43655139E+05 |

B MATRIX

| | | |
|---|---|---|
| 1.45519152E-11 | 2.18278728E-11 | 8.00355338E-11 |
| 2.18278728E-11 | 2.18278728E-11 | 7.27595761E-11 |
| 7.27595761E-11 | 7.27595761E-11 | 1.45519152E-11 |

```
                    D MATRIX

   5.31653046E+02     4.32016589E+02     4.69571561E+01
   4.32016589E+02     5.31653046E+02     4.69571561E+01
   4.69571561E+01     4.69571561E+01     4.24421785E+02


              OVERALL LAMINATE PROPERTIES

 EX=  9.92156003E+05  EY=  9.92156003E+05  GXY=  2.33162813E+06
 NUXY=  8.12591100E-01  NUYX=  8.12591100E-01


 *********************************************************

     P=  1.30776204E+01  B=  9.57954000E-01  F= -1.83044904E-08G=  3.83577069E-03

     PROGRAM HAS COMPUTED AND USED X=  2.60972865E-01
     AND Y=  1.00000000E+00 BASED ON THE INFINITE PLATE WAVE LENGTH

     BASED ON AN INFINITELY LONG PLATE THE BUCKLING STRESS COEFFICIENTS ARE
     NXBAR=  0.
     NYBAR=  0.
     NXYBAR=  1.30776204E+01
     AND THE STRAINS ARE
     STRNX=  0.
     STRNY=  0.
     STRNXY=  3.12923870E-02

 PROGRAM WILL NOW CONTINUE WITH FINITE ASPECT RATIO SOLUTION


 *********************************************************


                BOUNDARY CONDITIONS

            BOUNDARY NO. 1 IS CLAMPED

            BOUNDARY NO. 2 IS CLAMPED

            BOUNDARY NO. 3 IS CLAMPED
```

BOUNDARY NO. 4 IS CLAMPED

| NXBAR | NYBAR | NXYBAR | DETERMINANT |
|---|---|---|---|
| 0. | 0. | 1.17698583E+01 | 6.54187653E+12 |
| 0. | 0. | 1.19006345E+01 | 2.18161022E+12 |
| 0. | 0. | 1.20314107E+01 | 2.37244168E+11 |
| 0. | 0. | 1.20473676E+01 | 1.33435159E+11 |
| 0. | 0. | 1.20687996E+01 | 3.35545245E+10 |
| 0. | 0. | 1.20761369E+01 | 9.37018457E+09 |
| 0. | 0. | 1.20793082E+01 | 0. |

BUCKLING LOADS PER UNIT OF LENGTH ALONG BOUNDARY EDGE

NX= 0.  NY= 0.  NXY= 7.04251216E+03

(XB**2)*NX/(PI**2)/(T**3) = 0.
(XB**2)*NY/(PI**2)/(T**3) = 0.
(XB**2)*NXY/(PI**2)/(T**3) = 5.62757767E+06

(XB**2)*EPSILONX/(T**2) = -0.
(XB**2)*EPSILONY/(T**2) = -0.
(XB**2)*EPSILONXY/(T**2) = -1.19105540E+01

## APPENDIX D

```
      PROGRAM BOP(INPUT=201,OUTPUT=201,TAPE5=INPUT,TAPE6=OUTPUT,          100000
     1TAPE7=201)                                                           200000
C                                                                          300000
C  NASA - LANGLEY RESEARCH CENTER - PROGRAM                                400000
C                                                                          500000
C     THE PROGRAM FINDS THE BUCKLING LOADS (COMPRESSIVE OR SHEAR) OF ORTHOTROPIC   600000
C     PANELS WHOSE BOUNDARY CONDITIONS ON EACH EDGE MAY BE EITHER SIMPLY   700000
C     SUPPORTED, CLAMPED OR ELASTICALLY CONSTRAINED BY ROTATIONAL SPRINGS.   800000
C     THE ORTHOTROPIC ELASTIC PROPERTIES OF THE PANEL MAY BE EITHER DIRECTLY   900000
C     SUPPLIED BY THE USER OR MAY BE INTERNALLY PREPROCESSED BY THE PROGRAM   1000000
C     FOR LAMINATED PANELS.                                               1100000
C                                                                          1200000
C     LAMINATE DEFINITIONS                                                 1300000
C     ********************                                                 1400000
C                                                                          1500000
C     MA=NUMBER OF LAMINAS IN THE LAMINATE                                 1600000
C     NOMAT=NUMBER OF DIFFERENT TYPES OF MATERIAL                          1700000
C     TH=FIBER ORIENTATION (IN DEGREES)  MEASURED FROM X AXIS              1800000
C     MATYPE=TYPE OF MATERIAL IN EACH LAMINA                               1900000
C                                                                          2000000
C     BOUNDARY CONDITIONS                                                  2100000
C     ********************                                                 2200000
C                                                                          2300000
C     IBC=1, ROTATIONAL SPRING SUPPORT                                     2400000
C     IBC=2, SIMPLE SUPPORT                                                2500000
C     IBC=3, CLAMPED SUPPORT                                               2500001
C     IBC=4, FREE BOUNDARY                                                 2600000
C     IBC=5, BOUNDARY CONDITION ENTERED THRU D MATRICES                    2700000
C     AKR=ROTATIONAL SPRING STIFFNESS PER UNIT LENGTH OF BOUNDARY          2800000
C                                                                          2900000
C     CONTROL OPTIONS                                                      3000000
C     ***************                                                      3100000
C                                                                          3200000
C     ISTIFF=1, DO NOT PREPROCESS, BUT EXECUTE FOR BUCKLING                3300000
C     ISTIFF=2, PREPROCESS AND EXECUTE FOR BUCKLING (DEFUALT)              3400000
C     ISTIFF=3, PREPROCESS ONLY. DO NOT EXECUTE FOR BUCKLING               3500000
C     MSHAPE=1, EXECUTE FOR BUCKLING AND EXECUTE MODE SHAPE OPTION         3600000
C     MSHAPE=2, DO NOT EXECUTE MODE SHAPE OPTION                           3700000
C     MSHAPE=3, DO NOT EXECUTE FOR BUCKLING, BUT DO EXECUTE MODE SHAPE OPTION.   3800000
C     ITSEP=1, PROGRAM AUTOMATICALLY VARIES THE INITIAL STEP SIZE,DELP. (DEFAULT   3900000
C     ISTEP=2, STEP SIZE FIXED AND EQUAL TO DELP                           4000000
C                                                                          4100000
```

```
          C                                                                                                  4200000
          C                                                                                                  4300000
000003          DIMENSION A(56,56),W(32,32),CASE(8),PP(200),DET(200)                                         4400000
000003          DIMENSION D1(32,32),D2(32,32),D12(32,32),D66(32,32) ,IBC(4),AKS(4)                           4500000
               1,TH(100),AT(100),MATYPE(100),E1(10),E2(10),U1(10),G12(10)                                   4600000
               2,EI(5),IORIENT(5),LOC(5),B(56,1),IPIVOT(56)                                                 4700000
               * ,AKR(4),WK(56)                                                                             4800001
000003          EQUIVALENCE (AKR,AKS)                                                                        4900000
          C                                                                                                  5000000
          C                                                                                                  5100000
          C                                                                                                  5200000
000003          COMMON/LAYER/NOMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                                              5300000
000003          COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),                                 5400000
               1ZIY(32),ETAX(32),ETAY(32)                                                                   5500000
000003          COMMON/SPRING/AKL(32,32),AKX(32,32),AKY(32,32)                                               5600000
000003          COMMON/XY/TX,TY,TXY,RX,RY,RXY,PI,ADB,DS1,DS2,DS12,DS66,                                      5700000
               1IBC2,AKS2,XB,EX,EY,EXY,GXY,TT                                                               5800000
000003          NAMELIST/BUCKLE/ISTIFF,IX,JX,TX,TY,TXY,RX,RY,RXY,                                            5900000
               1 XA,XB,MR,NC,X,Y,TEST,                                                                      6000001
               2 AKR,IBC,D1,D2,D12,D66,P1,DELP,PFIN,                                                        6100000
               3 NOMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                                                         6200000
               4,NUPRIT,EI,IORIENT,LOC,MSHAPE,ISTEP                                                         6300000
               5,AKL,AKX,AKY,DS1,DS2,DS12                                                                   6400000
000003          ABSCISA(X3,X2,X1,Y3,Y2,Y1)=X3+((Y1*Y3-Y2*Y3)*(X3-X1)*(X2-X3))/(Y1*00000026                   6600000
               1Y2*(X1-X2)+Y1*Y3*(X3-X1)+Y2*Y3*(X2-X3))                                     00000027        6700000
          C                                                                                                  6800000
          C     SET DEFAULTS                                                                                 7000000
000032          NUPRIT=0                                                                                     7100000
000033          X=-1.                                                                                        7200000
000035          Y=1.                                                                                         7300000
000036          MSHAPE=2                                                                                     7400000
000037          ISTIFF=2                                                                                     7500000
000040          IX=JX=2                                                                                      7600000
000042          ISTEP=1                                                                                      7700000
000043          AKS(2)=.0                                                                                    7800000
000044          P1=-.9                                                                                       7900001
000046          PFIN=100.                                                                                    8000000
000047          INCX=0                                                                                       8100000
000050          ICNT=1                                                                                       8200000
000051          DELP=.1                                                                                      8300000
000052          P=1.                                                                                         8300001
000053          TEST=1.E-3                                                                                   8300002
000055          DO 9 I=1,32                                                                                  8400000
```

```
000056              DO 9 J=1,32
000057            9 AKL(I,J)=AKX(I,J)=AKY(I,J)=.0
             C      END OF DEFUALT LIST
             C
000070            1 IF(ICNT .GT. 1)GO TO 2
000074            7 READ(5,5001) CASE
000102              IF(EOF,5) 4,5
000105            4 PRINT 8997
000111              STOP
000113            5 PRINT 5032
000117              WRITE(6,5000) CASE
000125              WRITE(7,5000) CASE
000133              PRINT 5025
000137              READ BUCKLE
000142              WRITE(7,8998) MR,NC,X,Y
000156         8998 FORMAT(5X,*MR=*,I2,* NC=*,I2,*   X=*,E16.8,*   Y=*,E16.8/)
000156              ADB=XA/XB
000160              WRITE(7,8999) TH(1),ADB
000170         8999 FORMAT(/,5X,*TH(1)=*,E16.8,5X,*ASPECT RATIO=*,E16.8/)
000170              ADB2=ADB*ADB
000172              EL=MR+1
000174              B=NC+1
000176              PIS=PI
000200              DELPS=DELP
000201              PFINS=PFIN
000203              XSS=X
000204              PRINT 5021,ISTIFF,ISTEP,IX,JX,TEST
000222              IF(ISTIFF .EQ. 3 .OR. ISTEP .EQ. 3)GO TO 98
000231              PRINT 5022,XA,XB,ADB,MR,NC
000246              PRINT 5023,TX,TY,TXY,P1,DELP,PFIN,X,Y,RX,RY,RXY
000300              GO TO 97
000301           98 PRINT 5038
000302           97 CONTINUE
000305              CALL PREP(D1,D2,D12,D66,DS1,DS2,DS12,DS66,IX,JX,MR,NC,TT,EX,EY,EXY,GXY
                   1,GXY,IBC,ISTIFF)
000330              MR1=MRTOT
000332              NC1=NCTOT
000333          105 CONTINUE
000333              PI=3.14159265359
000335              PI2=PI*PI
000336              IBC2=IBC(2)
000337              AKS2=AKS(2)
000341              IF(INCX .EQ. 0)GO TO 249
```

```
000342         2 X=XSTART+(ICNT-1)*XDELTA                                    12700000
000350           IF(X .GT. XSTOP)GO TO 7                                     12800000
000353           IF(ICNT .EQ. 1)CALL AUTOXY(XZ,YZ,BETA,INCX)                 12900000
000357           Y=A09*X*DELTA                                               13000000
000362           ICNT=ICNT+1                                                 13100000
000363           PRINT 8996,MR,NC,X,Y                                        13200000
000377           WRITE(7,8998) MR,NC,X,Y                                     13300000
000413           GO TO 168                                                   13400000
000414       249 IF(X .LE. 0. .OR. PI .LT. 0.)CALL AUTOXY(X,Y,BETA,INCX,P)   13500001
000427       168 CONTINUE                                                    13600000
000427           PRINT 5010                                                  13700000
000433           S2X=SIN(.5*PI/X/EL)                                         13800000
000441           SX=SIN(PI/X/EL)                                             13900000
000446           S2Y=SIN(.5*PI/Y/B)                                          14000000
000454           SY=SIN(PI/Y/B)                                              14100000
000461           IF(NUPRIT .EQ. 0)GO TO 113                                  14400000
000462           PRINT 5027                                                  14500000
000465           DO 112 I=1,NUPRIT                                           14600000
000467           EH=EI(I)                                                    14700000
000471           IF(IORIENT(I) .EQ. 2)GO TO 114                              14800000
000473           EPS=XA/EL                                                   14900000
000475           EI(I)=EH/EPS/DS1                                            15000000
000500           R=1.+EI(I)                                                  15100000
000502           CALL SET(D1,1,IBPOS1,IBPOS3,LOC(I),R)                       15200000
000506           GO TO 112                                                   15300000
000507       114 EPS=XB/B                                                    15400000
000511           EI(I)=EH/EPS/DS1                                            15500000
000515           R=DS2+EI(I)                                                 15600000
000517           CALL SET(D2,2,IBPOS4,IBPOS2,LOC(I),R)                       15700000
000523       112 PRINT 5026,I,IORIENT(I),LOC(I),EH,EI(I)                     15800000
000544       113 CONTINUE                                                    15900000
000544           CALL BC(X,Y,EL,B,AKS,DS1,DS2,DS12,D1,D2,D12,D66,MR,NC,IBC,  15900001
               * XA,XB)                                                      15900002
000565           IF(JX .EQ. 2)GO TO 125                                      16000000
000567           PRINT 5016                                                  16100000
000573           PRINT 5015, ((D1(I,J),J=1,NC1),I=1,MR1)                     16200000
000612           PRINT 5017                                                  16300000
000616           PRINT 5015, ((D2(I,J),J=1,NC1),I=1,MR1)                     16400000
000635           PRINT 5018                                                  16500000
000641           PRINT 5015, ((D12(I,J),J=1,NC1),I=1,MR1)                    16600000
000660           PRINT 5019                                                  16700000
000664           PRINT 5015, ((D66(I,J),J=1,NC1),I=1,MR1)                    16800000
000703       125 CONTINUE                                                    16900000
```

```
000703             IF(ISTIFF .EQ. 3)GO TO 84                                    17000000
             C                                                                  17100000
             C     BEGIN BUCKLING ANALYSIS                                      17200000
             C                                                                  17300000
000705             IF(X .GE. 100. .OR. Y .GE. 100.)GO TO 122                    17400000
             C     EDL=EHATX/EHATY*ADB                                          17500000
000716             SX=SIN(.5*PI/EL/X)                                           17600000
000724             XSX=2.*X*SX/PI                                               17700000
000727             EDL=X/Y*ADB*SX/SIN(.5*PI/B/Y)                                17800000
000737             EDL1=2.*X/Y*ADB*SX/SIN(PI/B/Y)                               17900000
000751             EDE=2.*SX/SIN(PI/EL/X)                                       18000000
000760             GO TO 123                                                    18100000
000760        122  EDE=1.                                                       18200000
000762             XSX=1./EL                                                    18300000
000764             EDL=EDL1=ADB*B/EL                                            18400000
000767        123  MR2=2*(MR1-4)                                                18500000
000771             PIEL=PI**2                                                   18600000
000773             TXSS=TX*PIEL                                                 18700000
000774             TYSS=TY*PIEL                                                 18800000
000776             TXYSS=TXY*PIEL                                               18900000
000777             IF(MSHAPE .EQ. 3)GO TO 84                                    19000000
001001        120  P1=P1S*PIEL                                                  19100000
001003             DELP=DELPS*PIEL                                              19200000
001005             PFIN=PFINS*PIEL                                              19300000
001006             IF(P1 .GT.0.)GO TO 129                                       19300001
001011             P1=ABS(P1)*P                                                 19300002
001012             DELP=.01*P*PIEL                                              19300003
001015        129  CONTINUE                                                     19300004
001015             P1B=P1                                                       19400000
001017             PRINT 61                                                     19500000
001022       7700  L=0                                                          19600000
001023             LC=1                                                         19700000
001024       7702  CONTINUE                                           00000108  19800000
001024             P1=P1B*ADB2                                                  19900000
001026             TXS=TXSS*ADB2+RX*P1                                          20000000
001032             TYS=TYSS*ADB2+RY*P1                                          20100000
001035             TXYS=TXYSS*ADB2+RXY*P1                                       20200000
001041         77  CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE,  20300000
                 1 2,IBC)                                                       20400000
001062       4002  L=L+1                                                        20500000
001064             CALL DEUPPS(A,MR2,CDET,56)                                   20600000
001067             DET(L)=CDET                                                  20700000
001071             PP(L)=P1B/PIEL                                               20800000
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001073        TXS=TXS/PIEL/ADB2                                              20900000
001076        TYS=TYS/PIEL/ADB2                                              21000000
001077        TXYS=TXYS/PIEL/ADB2                                            21100000
001101     80 PRINT 6,TXS,TYS,TXYS,DET(L)                                    21200000
001115        IF(L.LT.3)GO TO 90                                             21300000
001120        AD1=DET(L)*DET(L-1)                                            21400000
001122        AD2=DET(L)*DET(L-2)                                            21500000
001124        IF(ABS(DELP) .LT. PP(L)*TEST*PIEL)GO TO 155                    21600001
001131        IF(ISTEP .EQ. 2)GO TO 154                                      21700000
001133        IF(LC .EQ. 21)GO TO 155                                        21800000
001135        DELP=DET(L)*DELP/(DET(L-1)-DET(L))                             21900000
001141        IF(DELP .LT. 0. .AND. LC .GT. 5)GO TO 155                      22000000
001152        IF(DELP .GT. 10.*DELPS*PIEL)DELP=10.*DELPS*PIEL                22100000
001157        IF(ABS((PP(L-1)-PP(L))/PP(L)).GT. .01 .AND. LC .EQ. 1)GO TO 154 22200000
001174        IF(LC .GE. 15)GO TO 156                                        22300000
001176        DEL1=(PP(L)-PP(L-1))*PIEL                                      22400000
001202        DEL2=(PP(L-1)-PP(L-2))*PIEL                                    22500000
001204        DEL1P2=DEL1+DEL2                                               22600000
001206        FD=(DET(L)-DET(L-1))/(PP(L)-PP(L-1))/PIEL                      22700000
001213        FDD=(DEL1*DET(L-2)-DEL1P2*DET(L-1)+DEL2*DET(L))/(.5*DEL1*DEL2  22800000
             1 *DEL1P2)                                                      22900000
001225        DELP=-DET(L)/FD - FDD*(DET(L)**2)/(8.*FD**3)                   23000000
001235        IF(ABS(DELP) .LT. PP(L)*1.E-6)GO TO 155                        23100000
001241        LC=LC+1                                                        23200000
001243        GO TO 154                                                      23300000
001243    156 DELP=.03*PP(L)*PIEL                                            23400000
001246        LC=LC+1                                                        23500000
001250    154 IF(L .GT. 100)GO TO 1                                          23600000
001254        IF(AD1 .GE. .0 .AND. AD2 .GE. .0)GO TO 90                      23700000
001263    155 CONTINUE                                                       23800000
001263        CISSA=ABSCISA(PP(L),PP(L-1),PP(L-2),DET(L),DET(L-1),DET(L-2))  23900000
001270        P=CISSA                                                        24000000
001271        DD=0.                                                          24100000
001272        TXS=TXSS/PIEL+RX*P                                             24200000
001275        TYS=TYSS/PIEL+RY*P                                             24300000
001300        TXYS=TXYSS/PIEL+RXY*P                                          24400000
001304        PRINT 6,TXS,TYS,TXYS,DD                                        24500000
001317        WRITE(7,6)TXS,TYS,TXYS,DD                                      24600000
001333        WRITE(7,6) P,DD                                                24700000
001343        TXS=TXS*PI*PI*DS1/(XB*XB)                                      24800000
001347        TYS=TYS*PI*PI*DS1/(XB*XB)                                      24900000
001351        TXYS=TXYS*PI*PI*DS1/(XB*XB)                                    25000000
001353        PRINT 5014,TXS,TYS,TXYS                                        25100000
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001365          WRITE(7,5014)TXS,TYS,TXYS                                      25200000
           C                                                                   25300000
           C    COMPUTATION OF USEFUL BUCKLING PARAMETERS                      25400000
           C                                                                   25500000
001377          IF(ISTIFF .NE. 2)GO TO 130                                     25600000
001401          FA=XB*XB/P12/(TT**3)                                           25700000
001405          TXT=TXS*FA                                                     25800000
001406          TYT=TYS*FA                                                     25900000
001410          TXYT=TXYS*FA                                                   26000000
001411          PRINT 5030, TXT,TYT,TXYT                                       26100000
001423          FA=XB*XB/(TT**3)                                               26200000
001426          EXT=FA*(1./EX)*(TXS-EXY/EY*TYS)                                26300000
001434          EYT=FA*(1./EY)*(TYS-EXY/EX*TXS)                                26400000
001442          EXYT=.5*FA*TXYS/GXY                                            26500000
001446          EXT=-EXT                                                       26600000
001447          EYT=-EYT                                                       26700000
001450          EXYT=-EXYT                                                     26800000
001451          PRINT 5031, EXT,EYT,EXYT                                       26900000
001462          PRINT 5032                                                     27000000
001466      130 IF(MSHAPE .EQ. 1)GO TO 87                                      27100000
001470          GO TO 85                                                       27200000
001471       90 IF(P1B .GT. PFIN)GO TO 85                                      27300000
001475          P1B=P1B+DELP                                                   27400000
001476          GO TO 7702                                                     27500000
001477       85 P1=P1S                                                         27600000
001501          DELP=DELPS                                                     27700000
001502          PFIN=PFINS                                                     27800000
001504          X=XSS                                                          27900000
001505          GO TO 1                                                        28000000
           C                                                                   28100000
           C    COMPUTE MODE SHAPES                                            28200000
           C                                                                   28300000
001506       84 P=P1                                                           28400000
001510          IF(MSHAPE .EQ. 2)GO TO 1                                       28500000
001512       87 TXS=(TXSS+RX*PIEL*P)*ADB2                                      28600000
001517          TYS=(TYSS+RY*PIEL*P)*ADB2                                      28700000
001524          TXYS=(TXYSS+RXY*P*PIEL)*ADB2                                   28800000
001530          CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE, 28900000
              1 2,I6C,AKL,LOCX,LOCY,NK)                                        29000000
001554          DO 115 I=1,MR2                                                 29100000
001556      115 B(I,1)=-A(I,MR2)                                               29200000
001565          MR2M1=MR2-1                                                    29300000
001567          CALL GELIM(56,MR2M1,A,1,B,IPIVOT,0,WK,IERR)                    29400001
```

```
001577          B(MR2,1)=1.                                                    29500000
001601          DO 116 I=1,MR                                                  29600000
001603          W(I+2,3)=B(I)                                                  29700000
001605      116 W(I+2,4)=B(I+MR)                                               29800000
001611          CALL ARRAY(D1,D2,D12,D66,EDL,W,A,TXS,TYS,TXYS,MR,NC,XSX,EDL1,EDE,   29900000
               1 1,IBC)                                                        30000000
001631          PRINT 5028                                                     30100000
001635          WM=0.                                                          30200000
001636          DO 170 I=3,MRP2                                                30300000
001640          DO 170 J=3,NCP2                                                30400000
001641          W1=ABS(W(I,J))                                                 30500000
001645      170 WM=AMAX1(W1,WM)                                                30600000
001654          WMI=1./WM                                                      30700000
001656          DO 171 I=2,MRP3                                                30800000
001657          DO 172 J=2,NCP3                                                30900000
001660      172 W(I,J)=W(I,J)*WMI                                              31000000
001666          PRINT 5029, (W(I,J),J=2,NCP3)                                  31100000
001702      171 PRINT 5033                                                     31200000
001711          GO TO 85                                                       31300000
001711        6 FORMAT(2X,E16.8,3X,E16.8,4X,E16.8,10X,E16.8)                   31400000
001711       61 FORMAT(///,9X,*NXBAR*,15X,*NYBAR*,13X,*NXYBAR*,17X,*DETERMINANT*/)   31600001
001711     5000 FORMAT(////15H INPUT FOR CASE///1X8A10////)                    31700000
001711     5001 FORMAT(8A10)                                       00000087   31800000
001711     5002 FORMAT(1X8A10)                                                 31900000
001711     5010 FORMAT(//,20X,19HBOUNDARY CONDITIONS )                         32000000
001711     5011 FORMAT(/,5X,*BOUNDARY NO. *,I1,* HAS A ROTATIONAL SPRING SUPPORT O   32100000
               1F MAGNITUDE *,E16.8/)                                          32200000
001711     5012 FORMAT(/,13X,*BOUNDARY NO. *,I1,* IS SIMPLY SUPPORTED*,/)      32300000
001711     5013 FORMAT(/,13X,*BOUNDARY NO. *,I1,* IS CLAMPED*,/)               32400000
001711     5014 FORMAT(//,15X,*BUCKLING LOADS PER UNIT OF LENGTH ALONG BOUNDARY ED   32500000
               1GE*,/,5X,*NX=*,E16.8,5X,*NY=*,E16.8,5X,*NXY=*,E16.8/)          32600000
001711     5015 FORMAT(/,(3E20.8))                                             32700000
001711     5016 FORMAT(//,30X,*D1*)                                            32800000
001711     5017 FORMAT(//,30X,*D2*)                                            32900000
001711     5018 FORMAT(//,30X,*D12*)                                           33000000
001711     5019 FORMAT(//,30X,*D66*)                                           33100000
001711     5021 FORMAT(5X,*ISTIFF=*,I1,3X,*ISTEP=*,I1,10X,*IX=*,I1,3X,*JX=*,I1,   33300001
               * 9X,*TEST=*,F16.8//)                                           33300002
001711     5022 FORMAT(5X,*XA=*,E16.8,5X,*XB=*,E16.8,5X,*ASPECT RATIO=*,E16.8//   33400000
               1,15X,*MR=*,I2,5X,*NC=*,I2//)                                   33500000
001711     5023 FORMAT(5X,*TX=*,E16.8,5X,*TY=*,E16.8,7X,*TXY=*,E16.8//         33600001
               1,5X,*P1=*,E16.8,5X,*DELP=*,E16.8,5X,*PFIN=*,E16.8//            33700000
               * ,5X,*X=*,E16.8,6X,*Y=*,E16.8//,5X,*RX=*,E16.8,5X,*RY=*,E16.8,7X,   33800001
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```
                  3*RXY=*,E16.8//)                                                     33900000
001711      5024  FORMAT(/,1X,100(1H*))                                                34000000
001711      5025  FORMAT(//,30X,12(1H1),5(1H*),1X ,1HY,/,6(30X,1H4,10X,1H2,/),30X,     34100000
                  1 12(1H3),/,3(30X,1H*,/),/,30X,1HX,////)                             34200000
001711      5026  FORMAT(1X,I2,3X,I1,3X,I2,4X,E16.8,2X,E16.8)                          34300000
001711      5027  FORMAT(//,30X,*STIFFENER DATA*,/,*NO.*,2X,*IORIENT*,2X,*LOC*,10X,    34400001
                  1 *EI*,12X,*EI/EPS/D1*,/)                                            34500000
001711      5028  FORMAT(//,40X,*MODE SHAPE*,/)                                        34600000
001711      5029  FORMAT(1X,8(E15.7,1X))                                               34700000
001711      5030  FORMAT(//,2X,28H(XB**2)*NX/(PI**2)/(T**3) = ,E16.8/,                 34800000
                  1            2X,28H(XB**2)*NY/(PI**2)/(T**3) = ,E16.8/,              34900000
                  2            2X,29H(XB**2)*NXY/(PI**2)/(T**3) = ,E16.8//)            35000000
001711      5031  FORMAT(//,2X,26H(XB**2)*EPSILONX/(T**2) = ,E16.8/,                   35100000
                  1            2X,26H(XB**2)*EPSILONY/(T**2) = ,E16.8/,                35200000
                  2            2X,27H(XB**2)*EPSILONXY/(T**2) = ,E16.8/)               35300000
001711      5032  FORMAT(1H1)                                                          35400000
001711      5033  FORMAT(//)                                                           35500000
001711      5035  FORMAT(/,2X,*WARNING,  A BUCKLING VALUE MAY HAVE BEEN SKIPPED*,/)    35600000
001711      5038  FORMAT(//,10X,*ISTEP=3 OR ISTIFF=3 - PREPROCESS ONLY*,//)            35700000
001711      8997  FORMAT(//*                    STOPPED ON EOF,5          *)  00000032 35800000
001711            END                                                                  35900000
```

```
                 SUBROUTINE ARRAY(D1,D2,D12,D66,EDL,W,A,TX,TY,TXY,MR,NC,XSX,EDL1,
                1 EDE,MS,IFC)
000024           DIMENSION IBC(4)
000024           DIMENSION D1(32,32),D2(32,32),D12(32,32),D66(32,32)
000024           DIMENSION W(32,32),A(56,56)
000024           COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),
                1ZIY(32),ETAX(32),ETAY(32)
000024           COMMON/SPRING/AKL(32,32),AKX(32,32),AKY(32,32)
000024           WXX(K,L)=W(K+1,L)-2.*W(K,L)+W(K-1,L)
000042           WYY(K,L)=(W(K,L+1)-2.*W(K,L)+W(K,L-1))*EDLS
000057           WXY(K,L)=(W(K+1,L+1)-W(K+1,L)-W(K,L+1)+W(K,L))*EDL
000075           WX(K,L)=ZIY(L)*ETAX(K)*(W(K+1,L)-W(K,L))
000112           WY(K,L)=ZIY(L)*ETAX(K-1)*(W(K,L+1)-W(K,L))*EDLS
000127           WXY2(K,L)=((W(K+1,L+1)-W(K,L))*ETAX(K)*ETAY(L)-(W(K+1,L-1)-W(K,L))
                1*ETAX(K)*ETAY(L-1) -(W(K,L)-W(K-1,L-1))*ETAX(K-1)*ETAY(L-1)
                2 +(W(K,L)-W(K-1,L+1))*ETAX(K-1)*ETAY(L))*EDLP*.25
000173           TW(K,L)=(TX*(WX(K,L)-WX(K-1,L)) + TY*(WY(K,L)-WY(K,L-1))
                1 +2.*TXY*WXY2(K,L))*XSX**2
                2 +(AKL(K,L)*W(K,L)*XSX**2+AKX(K-1,L)*(W(K,L)-W(K-1,L))
                3 -AKX(K,L)*(W(K+1,L)-W(K,L)) +(AKY(K,L-1)*(W(K,L)-W(K,L-1))
                4-AKY(K,L)*(W(K,L+1)-W(K,L)))*EDLS)*EDL
000304           XM(K,L)=D1(K,L)*WXX(K,L)+D12(K,L)*WYY(K,L)
000327           YM(K,L)=D2(K,L)*WYY(K,L)+D12(K,L)*WXX(K,L)
000352           XYM(K,L)=2.*D66(K,L)*WXY(K,L)
000366           XXM(K,L)=XM(K+1,L)-2.*XM(K,L)+XM(K-1,L)
000413           YYM(K,L)=(YM(K,L+1)-2.*YM(K,L)+YM(K,L-1))*EDLS
000441           XYXYM(K,L)=(XYM(K,L)-XYM(K,L-1)-XYM(K-1,L)+XYM(K-1,L-1))*EDL
000474           EKE(K,L)=XXM(K,L)+2.*XYXYM(K,L)+YYM(K,L)+TW(K,L)
000525           EDLP=EDL1*EDE
000530           EDLS=EDL**2
000531           IROW=MRTOT-2
000533           ICOL=NCTOT-4
000535           IF(MS .EQ. 1)GO TO 35
000537           NNC=ICOL+1
000540           NCC=ICOL+2
000542           DO 10 J=1,NCTOT
000543           DO 10 I=1,MRTOT
000544           W(I,J)= 0.0
000550     10    CONTINUE
000555           MMN=0
000556           DO 40 N=3,4
000557           DO 40 M=3,IROW
```

```
000560          MN=0                                                    40200000
000561          MMN=MMN+1                                               40300000
000563          DO 39 I=3,IROW                                          40400000
000564       39 W(I,3)=W(I,4)=.0                                        40500000
000572          W(M,N)=1.0                                              40600000
          C                                                             40700000
          C     MARCHING PROCEDURE                                      40800000
          C                                                             40900000
000576       35 DO 30 J=3,ICOL                                          41000000
000600          DO 30 I=3,IROW                                          41100000
000601          YDM=-XXM(I,J)-2.*XYXYM(I,J)-TW(I,J)                     41200000
000621          YMOM=YDM/EDLS+2.*YM(I,J)-YM(I,J-1)                      41300000
000636          YK=(YMOM-D12(I,J+1)*WXX(I,J+1))/D2(I,J+1)               41400000
000653          IF(I .EQ. IBPOS1 .OR. I .EQ. IBPOS3) YK=YMOM/D2(I,J+1)  41500000
000665          W(I,J+2)=YK/EDLS+2.*W(I,J+1)-W(I,J)                     41600000
000676       30 CONTINUE                                                41700000
000703        1 FORMAT(/,2(1X,7E18.8/))                                 41800000
000703          IF(MS .EQ. 1)RETURN                                     41900000
000706          DO 20 J=NNC,NCC                                         42000000
000710          DO 20 I=3,IROW                                          42100000
000711          MN=MN+1                                                 42200000
000713          NUM=NC/2                                                42300000
000715          CF=10.**NUM                                             42400000
000720          A(MN,MMN)=EKE(I,J)/CF                                   42500000
000732       20 CONTINUE                                                42600000
000737       40 CONTINUE                                                42700000
000743        2 FORMAT(/,4(1X,5E20.8/))                                 42800000
000743          RETURN                                                  42900000
000744          END                                                     43000000
```

```
                  SUBROUTINE DEUPPS(A,N,DET,MAX)                 43100000
                                                                 43200000
000007            DIMENSION A(MAX,N)                             43300000
000007            DET=1.                                         43400000
000010            NN = N-1                                       43500000
                                                                 43600000
            C     PIVOT SEARCH                                   43700000
                                                                 43800000
000012            DO 560 I=1,NN                                  43900000
000013            CAVM = 0.                                      44000000
000014            II = I                                         44100000
000015            DO 105 J=II,N                                  44200000
000016            SWAP = A(J,II)                                 44300000
000022            CAVA=ABS(SWAP)                                 44400000
000024            IF(CAVM.GE.CAVA) GO TO 105                     44500000
000027            IROW = J                                       44600000
000030            CAVM = CAVA                                    44700000
000031        105 CONTINUE                                       44800000
000034            IF(CAVM.EQ.0.) GO TO 720                       44900000
                                                                 45000000
            C     ROW INTERCHANGE                                45100000
                                                                 45200000
000035            IF(IROW.EQ.II) GO TO 203                       45300000
000037            DET = -DET                                     45400000
000040            DO 200 L=II,N                                  45500000
000041            SWAP = A(IROW,L)                               45600000
000045            A(IROW,L) = A(II,L)                            45700000
000053            A(II,L) = SWAP                                 45800000
000056        200 CONTINUE                                       45900000
000060        203 SWAP = A(II,II)                                46000000
000064            DET = DET*SWAP                                 46100000
                                                                 46200000
            C     NORMALIZE PIVOT ROW                            46300000
                                                                 46400000
000065            K = II+1                                       46500000
000067            DO 350 L=K,N                                   46600000
000070        350 A(II,L) = A(II,L)/SWAP                         46700000
                                                                 46800000
            C     ELIMINATION                                    46900000
                                                                 47000000
000077            DO 550 LI=K,N                                  47100000
000101            SWAP = A(LI,II)                                47200000
```

```
000105              DO 520 L=K,N                                    47300000
000107              A(I1,L) = A(I1,L)-A(II,L)*SWAP                  47400000
000121         520 CONTINUE                                         47500000
000124         550 CONTINUE                                         47600000
000126         560 CONTINUE                                         47700000
000131              GO TO 730                                       47800000
000131         720 DET=0.                                           47900000
000132              GO TO 750                                       48000000
000132         730 DET = DET*A(N,N)                                 48100000
000136         750  RETURN                                          48200000
000137              END                                             48300000
```

```
                 SUBROUTINE BC(X,Y,EL,B,AKS,DS1,DS2,DS12,D1,D2,D12,D66,MR,NC,IBC,
                1 XA,XB)
             C
             C   COMPUTES VALUES OF PLATE STIFFNESSES SUCH THAT THE APPROPRIATE
             C   BOUNDARY CONTITIONS ARE SET
             C
             C   CPTIONS
             C   IBC=1  ROTATIONAL SPRING SUPPCRT
             C   IBC=2  SIMPLE SUPPORT
             C   IBC=3  CLAMPED SUPPORT
             C   IBC=4  FREE BOUNDARY
             C   IBC=5, BOUNDARY CONDITIONS NCT SET BY THIS SUBROUTINE
             C
000024           DIMENSION AKS(4),IBC(4),D1(32,32),D2(32,32),D12(32,32),D66(32,32)
000024           COMMON/SIZE/IBPOS1,IBPOS2,IBPCS3,IBPOS4,MRTOT,NCTOT,ZIX(32),
                1ZIY(32),ETAX(32),ETAY(32)
000024           PI=3.14159265359
000025           SX=SIN(PI/X/EL)
000034           SY=SIN(PI/Y/B)
000044           S2X=SIN(.5*PI/X/EL)
000055           S2Y=SIN(.5*PI/Y/B)
000066           IBP3M1=IBPOS3-1
000070           IBP2M1=IBPOS2-1
000072           IBP4P1=IBPOS4+1
000073           IBP2M1=IBPOS2-1
000074           DO 150 M=1,4
000076           L=IBC(M)
000101           GO TO(151,152,153,154,150)L
000112       151 PRINT 5011,M,AKS(M)
000126           IF(M .EQ. 2 .OR. M .EQ. 4)GO TO 160
000141           GAMMAX=AKS(M)*XA/DS1
000144           EHATX=4.*X*S2X**2/PI/SX
000150           CX=(1.-.5*GAMMAX*EHATX)/(1.+.5*GAMMAX*EHATX)
000160           IF(M .EQ. 3) GO TO 170
000162       181 R=DS1*(1.-CX)/DS1
000165           CALL SET(D1,2,IBPOS4,IBPOS2,IBPOS1,R)
000172           GO TO 150
000176       170 R=DS1*(1.-CX)/DS1
000201           CALL SET(D1,2,IBPOS4,IBPCS2,IBPOS3,R)
000206           GO TO 150
000212       160 GAMMAY=AKS(M)*XB/DS2
000216           EHATY=4.*Y*S2Y**2/PI/SY
```

```
000223        CY=(1.-.5*GAMMAY*EHATY)/(1.+.5*GAMMAY*EHATY)                 52600000
000233        IF(M .EQ. 4)GO TO 180                                        52700000
000235    183 R=DS2*(1.-CY)/DS1                                            52800000
000241        CALL SET(D2,1,IBPOS1,IBPOS3,IBPOS2,R)                        52900000
000245        GO TO 150                                                    53000000
000251    180 R=DS2*(1.-CY)/DS1                                            53100000
000255        CALL SET(D2,1,IBPOS1,IBPOS3,IBPOS4,R)                        53200000
000261        GO TO 150                                                    53300000
000265    152 PRINT 5012,M                                                 53400000
000273        CX=CY=1.                                                     53500000
000276        GO TO(181,183,170,180)M                                      53600000
000311    153 PRINT 5013,M                                                 53700000
000317        CX=CY=-1.                                                    53800000
000322        GO TO(181,183,170,180)M                                      53900000
000335    154 PRINT 5014,M                                                 54000000
000343        GO TO (201,202,203,204)M                                     54100000
000357    201 DO 221 J=IBPOS4,IBPOS2                                       54200000
000361        D2(IBPOS1,J)=.5*D2(IBPOS1,J)*(1.-D12(IBPOS1,J)/D1(IBPOS1,J)  54300000
             1 *D12(IBPOS1,J)/D1(IBPOS1,J))                                54400000
000377        D1(IBPOS1,J)=.0                                              54500000
000403    221 D12(IBPOS1,J)=.0                                             54600000
000407        GO TO 150                                                    54700000
000410    202 DO 223 I=IBPOS1,IBPOS3                                       54800000
000412        D1(I,IBPOS2)=.5*D1(I,IBPOS2)                                 54900000
000416        D2(I,IBPOS2)=.5*D2(I,IBPOS2)                                 55000000
000421    223 D12(I,IBPOS2)=D12(I,IBPOS2)*.5                               55100000
000427        GO TO 150                                                    55200000
000427    203 DO 225 J=IBP4P1,IBP2M1                                       55300000
000431        D2(IBPOS3,J)=.5*D2(IBPOS3,J)*(1.-D12(IBPOS3,J)/D2(IBPOS3,J)  55400000
             1 *D12(IBPOS3,J)/D1(IBPOS3,J))                                55500000
000447        D1(IBPOS3,J)=.0                                              55600000
000452    225 D12(IBPOS3,J)=.0                                             55700000
000456        GO TO 150                                                    55800000
000457    204 DO 227 I=IBPOS1,IBPOS3                                       55900000
000461        D1(I,IBPOS4)=.5*D1(I,IBPOS4)                                 56000000
000465        D2(I,IBPOS4)=.5*D2(I,IBPOS4)                                 56100000
000470    227 D12(I,IBPOS4)=.5*D12(I,IBPOS4)                               56200000
000475    150 CONTINUE                                                     56300000
000477        DO 260 I=1,MRTOT                                             56400000
000501        ZIX(I)=ETAX(I)=1.                                            56500000
000505    260 IF(I .LT. IBPOS1 .OR. I .GT. IBPOS3)ZIX(I)=ETAX(I)=.0        56600000
000525        ZIX(IBPOS1)=.5                                               56700000
000527        ZIX(IBPOS3)=.5                                               56800000
```

```
                                                                        56900000
000530          ETAX(IBPOS3)=.0                                         57000000
000531          DO 261 J=1,NCTOT                                        57100000
000533          ZIY(J)=ETAY(J)=1.                                       57200000
000537      261 IF(J .LT. IBPOS4 .OR. J .GT. IBPOS2)ZIY(J)=ETAY(J)=.0   57300000
000557          ZIY(IBPOS4)=.5                                          57400000
000561          ZIY(IBPOS2)=.5                                          57500000
000562          ETAY(IBPOS2)=.0                                         57600000
000563     5011 FORMAT(/,13X,*BOUNDARY NO. *I1,* HAS A ROTATIONAL SPRING SUPPORT O 57700000
               1F MAGNITUDE *,F16.8/)                                   57800000
000563     5012 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS SIMPLY SUPPORTED*,/) 57900000
000563     5013 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS CLAMPED*,/)         58000000
000563     5014 FORMAT(/,13X,*BOUNDARY NO. *I1,* IS FREE*,/)            58100000
000563          RETURN                                                  58200000
000564          END
```

```
                 SUBROUTINE PREP(D1,D2,D12,D66,DS1,DS2,DS12,DS66,IX,JX,MR,NC,TT,EX,        58300000
                1EY,EXY,GXY,IBC,ISTIFF)                                                    58400000
          C                                                                                58500000
          C      THIS SUBROUTINE PREPROCESSES THE ORTHOTROPIC PROPERTIES OF A LAMINATE     58600000
          C                                                                                58700000
000026           DIMENSION Q(10,4),E1(10),E2(10),U1(10),G12(10),DD(3,3),IBC(4),            58800000
                1AT(100),TH(100),MATYPE(100),D1(32,32),D2(32,32),D12(32,32),               58900000
                2D66(32,32)                                                                59000000
000026           COMMON/LAYER/NUMAT,E1,E2,U1,G12,MA,MATYPE,AT,TH                           59100000
000026           COMMON/SIZE/IBPOS1,IBPOS2,IBPOS3,IBPOS4,MRTOT,NCTOT,ZIX(32),              59200000
                1ZIY(32),ETAX(32),ETAY(32)                                                 59300000
000026           IBPOS1=2+IBC(1)/4                                                         59400000
000031           IBPOS2=NC+3+IBC(4)/4*2                                                    59500000
000040           IBPOS3=IBPOS1+MR+1                                                        59600000
000043           IBPOS4=2+IBC(4)/4*2                                                       59700000
000051           MRTOT=MR+4+IBC(1)/4   +IBC(3)/4                                           59800000
000055           NCTOT=NC+4+IBC(2)/4*2+IBC(4)/4*2                                          59900000
000067         1 FORMAT(//,10I5//)                                                         60200000
000067           IF(ISTIFF .EQ. 1)GO TO 160                                                60300000
000072           DO 165 I=1,MRTOT                                                          60400000
000073           DO 165 J=1,NCTOT                                                          60500000
000074       165 D1(I,J)=D2(I,J)=D12(I,J)=D66(I,J)=.0                                      60600000
000115           TT=0.                                                                     60700000
000116       100 PRINT 5003                                                                60800000
000122           PRINT 5006                                                                60900000
000126           DO 110 K=1,NOMAT                                                          61000000
000133           PRINT 5007,K,E1(K),E2(K),U1(K),G12(K)                                     61100000
000150           ANVXY=U1(K)                                                               61200000
000152           ANVYX=ANVXY*E2(K)/E1(K)                                                   61300000
000155           ANU=1./(1.-ANVXY*ANVYX)                                                   61400000
000160           Q(K,1)=E1(K)*ANU                                                          61500000
000163           Q(K,2)=E2(K)*ANU                                                          61600000
000165           Q(K,3)=ANVXY*E2(K)*ANU                                                    61700000
000170       110 Q(K,4)=G12(K)                                                             61800000
000200           PRINT 5008                                                                61900000
000203           DO 115 J=1,MA                                                             62000000
000210           PRINT 5009,J,MATYPE(J),AT(J),TH(J)                                        62100000
000223       115 TT=TT+AT(J)                                                               62200000
000234           ZL=.5*TT                                                                  62300000
000236           CALL EVAL(AT,TH,MATYPE,ZL,MA,Q,DD,IX,JX,EX,EY,EXY,GXY)                    62400000
000255           DS1=DD(1,1)                                                               62500000
000262           DS2=DD(2,2)                                                               62600000
```

```
000263        DS12=DD(1,2)
000264        DS66=DD(3,3)
000266        DO 140 KR=IBPOS1,IBPOS3
000270        DO 140 KC=IBPOS4,IBPOS2
000272        D1(KR,KC)=DD(1,1)/DS1
000276        D2(KR,KC)=DD(2,2)/DS1
000301    140 D12(KR,KC)=DD(1,2)/DS1
000310        IB2M1=IBPOS2-1
000312        IB3M1=IBPOS3-1
000313        DO 150 KR=IBPOS1,IB3M1
000315        DO 150 KC=IBPOS4,IB2M1
000317    150 D66(KR,KC)=DD(3,3)/DS1
000330        RETURN
000331    160 DS1I=1./DS1
000333        DO 170 I=IBPOS1,IBPOS3
000335        DO 170 J=IBPOS4,IBPOS2
000337        D1(I,J)=D1(I,J)*DS1I
000343        D2(I,J)=D2(I,J)*DS1I
000346        D12(I,J)=D12(I,J)*DS1I
000350    170 D66(I,J)=D66(I,J)*DS1I
000357        IB1M1=IBPOS1-1
000361        IB2P1=IBPOS2+1
000363        IB3P1=IBPOS3+1
000364        IB4M1=IBPOS4-1
000365        DO 180 I=1,IB1M1
000367        DO 180 J=1,NCTOT
000370        D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0
000405    180 CONTINUE
000411        DO 181 I=IB3P1,MRTOT
000413        DO 181 J=1,NCTOT
000414        D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0
000431    181 CONTINUE
000435        DO 182 J=1,IB4M1
000437        DO 182 I=IBPOS1,IBPOS3
000441        D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0
000456    182 CONTINUE
000463        DO 183 J=IB2P1,NCTOT
000465        DO 183 I=IBPOS1,IBPOS3
000467        D1(I,J)=D12(I,J)=D2(I,J)=D66(I,J)=.0
000504    183 CONTINUE
000511   5003 FORMAT(//,30X,26HLAMINATED PLATE PROPERTIES )
000511   5004 FORMAT(//,20X,35HMID-PLANE USED AS REFERENCE SURFACE )
000511   5005 FORMAT(//,20X,5HZREF=,E16.8)
```

```
000511      5006 FORMAT(//,1X,13HMATERIAL KIND,8X,3H E1,15X,3H E2,13X,5H U1   ,12X,    67000000
                13HGXY)                                                                67100000
000511      5007 FORMAT(7X,I2,7X,E16.8,3(1X,E16.8))                                    67200000
000511      5008 FORMAT(//,1X,9HLAYER NO.,3X,9HMAT. KIND,6X,5HTHICK,12X,5HTHETA)       67300000
000511      5009 FORMAT(5X,I2,9X,I2,5X,E16.8,1X,E16.8)                                 67400000
000511           RETURN                                                                67500000
000512           END                                                                   67600000
```

```
                SUBROUTINE EVAL(T,TH,IL,ZL,NL,Q,DD,IX,JX,EX,EY,EXY,GXY)           67700000
000020          DIMENSION T(100),TH(100),IL(100),DD(3,3),Q(10,4),AA(3,3),BB(3,3), 67800000
               2QB(3,3)                                                           67900000
000020          O3=1./3.                                                          68000000
000022          TT=0.                                                             68100000
000023          DO 6 IV=1,3                                                       68200000
000024          DO 6 JV=1,3                                                       68300000
000025          AA(IV,JV)=0.                                                      68400000
000030          BB(IV,JV)=0.                                                      68500000
000032        6 DD(IV,JV)=0.                                                      68600000
000040          DO 20  J=1,NL                                                     68700000
000042          K=IL(J)                                                           68800000
000044          THETA=TH(J)*3.14159265359/180.                                    68900000
000047          CALL TRANS(Q,THETA,QB,K,IX,JX)                                    69000000
000054          HJ=ZL-TT                                                          69100000
000062          HJP1=HJ-T(J)                                                      69200000
000065          HSQ=.5*(HJ**2-HJP1**2)                                            69300000
000070          TT=TT+T(J)                                                        69400000
000073          DO 20 IV=1,3                                                      69500000
000074          DO 20 JV=1,3                                                      69600000
000075          AA(IV,JV)=AA(IV,JV)+QB(IV,JV)*T(J)                                69700000
000104          BB(IV,JV)=BB(IV,JV)+QB(IV,JV)*HSQ                                 69800000
000111       20 CONTINUE                                                          69900000
000120          ZREF11=BB(1,1)/AA(1,1)                                            70000000
000122          ZREF22=BB(2,2)/AA(2,2)                                            70100000
000124          ZREF12=BB(1,2)/AA(1,2)                                            70200000
000126          ZREF33=BB(3,3)/AA(3,3)                                            70300000
000130          PRINT 9                                                           70400000
000133          PRINT 8, ZREF11,ZREF22,ZREF12,ZREF33                              70500000
000147          PRINT 9                                                           70600000
000153          ZL=ZL-ZREF11                                                      70700000
000161          TT=.0                                                             70800000
000161          DO 30 J=1,NL                                                      70900000
000163          K=IL(J)                                                           71000000
000165          THETA=TH(J)*3.14159265359/180.                                    71100000
000170          CALL TRANS(Q,THETA,QB,K,IX,JX)                                    71200000
000175          HJ=ZL-TT                                                          71300000
000203          HJP1=HJ-T(J)                                                      71400000
000206          HCUBE=O3*(HJ**3-HJP1**3)                                          71500000
000212          TT=TT+T(J)                                                        71600000
000215          DO 30 IV=1,3                                                      71700000
000216          DO 30 JV=1,3                                                      71800000
```

```
000217      33 DD(IV,JV)=DD(IV,JV)+QB(IV,JV)*HCUBE
000236         PRINT 3
000241         PRINT 2,((AA(IV,JV),IV=1,3),JV=1,3)
000257         PRINT 4
000263         PRINT 2,((BB(IV,JV),IV=1,3),JV=1,3)
000301         PRINT 5
000305         PRINT 2,((DD(IV,JV),IV=1,3),JV=1,3)
000324         ANUXY=AA(1,2)/AA(2,2)
000326         ANUYX=AA(1,2)/AA(1,1)
000333         EX=AA(1,1)*(1.-ANUXY*ANUYX)/TT
000335         EY=AA(2,2)*(1.-ANUXY*ANUYX)/TT
000342         EXY=ANUXY*EY
000344         GXY=AA(3,3)/TT
000346         PRINT 7,EX,EY,GXY,ANUXY,ANUYX
000364       2 FORMAT(//,3(1X,E16.8,2X,E16.8,2X,E16.8/))
000364       3 FORMAT(//,25X,8HA MATRIX)
000364       4 FORMAT(//,25X,8HB MATRIX)
000364       5 FORMAT(//,25X,3HD MATRIX)
000364       7 FORMAT(//,15X,*OVERALL LAMINATE PROPERTIES*,//,* EX=*,E16.8,
              1 *  EY=*,E16.8,*  GXY=*,E16.8,/,* NUXY=*,E16.8,*  NUYX=*,E16.8//)
000364       3 FORMAT(/,5X, *CAUTION -- COUPLING BETWEEN EXTENSION AND BENDING MA
              1Y BE SIGNIFICANT*,/,5X,*IF THE FOLLOWING FOUR VALUES ARE NOT ALL E
              2QUAL.*,/,5X,*IF THIS IS THE CASE, THE RESULTS SHOULD BE USED WITH
              3DESCRETION*,//,4(2X,E16.8),/)
000364       9 FORMAT(1X,76(1H*),/)
000364         RETURN
000365         END
```

```
              SUBROUTINE TRANS(Q,THETA,QB,K,IX,JX)
000011        DIMENSION T(3,3),TI(3,3),QS(3,3),ST(3,3),QB(3,3),Q(10,4)
000011        SN=SIN(THETA)
000016        CS=COS(THETA)
000024        T(1,1)=TI(1,1)=T(2,2)=TI(2,2)=CS**2
000036        T(1,2)=TI(1,2)=T(2,1)=TI(2,1)=SN**2
000050        T(1,3)=TI(2,3)=2.*CS*SN
000055        T(3,1)=TI(3,2)=-SN*CS
000061        T(2,3)=TI(1,3)=-T(1,3)
000065        T(3,2)=TI(3,1)=-T(3,1)
000071        T(3,3)=TI(3,3)=CS*CS-SN*SN
000076        QS(1,1)=Q(K,1)
000100        QS(2,2)=Q(K,2)
000102        QS(1,2)=QS(2,1)=Q(K,3)
000106        QS(3,3)=2.*Q(K,4)
000111        QS(1,3)=QS(3,1)=QS(2,3)=QS(3,2)=0.
000122        IF(IX .EQ. 1)PRINT 1,((QS(I,J),J=1,3),I=1,3)
000145        IF(IX .EQ. 1)WRITE(7,1) ((QS(I,J),J=1,3),I=1,3)
000170        DO 5 IM=1,3
000172        DO 5 IS=1,3
000173      5 ST(IM,IS)=QB(IM,IS)=0.
000204        DO 10 IM=1,3
000205        DO 10 IS=1,3
000206        DO 10 IR=1,3
000207     10 ST(IM,IS)=ST(IM,IS) + QS(IM,IR)*T(IR,IS)
000230        DO 20 IR=1,3
000231        DO 20 IS=1,3
000232        DO 20 IM=1,3
000233     20 QB(IR,IS)=QB(IR,IS)+TI(IR,IM)*ST(IM,IS)
000254        DO 30 IM=1,3
000255     30 QB(IM,3)=.5*QB(IM,3)
000262        IF(IX .EQ.1)PRINT 1,((T(I,J),J=1,3),I=1,3),((TI(I,J),J=1,3),I=1,3)
000317        IF(IX .EQ. 1)PRINT 1,((QB(I,J),J=1,3),I=1,3)
000346        IF(IX .EQ. 1)WRITE(7,1) ((QB(I,J),J=1,3),I=1,3)
000375      1 FORMAT(3(3X,E16.8))
000375        RETURN
000376        END
```

```
                     SUBROUTINE AUTOXY(X,Y,B,INCX,P)
000010               COMMON/XY/TX,TY,TXY,RX,RY,RXY,PI,ADB,DS1,DS2S,DS12,DS66,
                    1IBC2,AKS2,XB,EX,EY,EXY,GXY,TT
000010               PRINT 53
000013               D12S=(DS12+2.*DS66)/DS1
000017               DS2=DS2S/DS1
000020               IC=1
000021               P=.1
000026               B=1.
000027               DP=DB=1.
000031               PID8=PI/8.
               C
               C     SIMPLE SUPPORTS, IK=1
               C     CLAMPED SUPPOSTS, IK=2
               C
000032               IF(IBC2 .EQ. 3 .OR. AKS2*XB/DS2 .GT. 100.)IK=2
000047             1 CX=TX+P*RX
000052               CY=TY+P*RY
000055               CXY=TXY+P*RXY
000060               AM1=PID8*(DS2+2.*D12S*B*B+B**4-B*B*CX-CY)
000072               AM1B=PID8*(4.*D12S*B+4.*B*B*B-2.*CX*B)
000102               AM1BB=PID8*(12.*B*B-2.*CX+4.*D12S)
000110               AM1P=PID8*(-B*B*RX-RY)
000113               AM1BP=-PI*.25*RX*B
000117               AM2=PID8*(16.*DS2+8.*D12S*B*B+B**4-B*B*CX-4.*CY)
000132               AM2B=PID8*(16.*D12S*B+4.*B**3-2.*CX*B)
000142               AM2BB=PID8*(12.*B*B+16.*D12S-2.*CX)
000150               AM2P=PID8*(-B*B*RX-4.*RY)
000154               AM2BP=-PI*.25*RX*B
000160               IF(IK .EQ. 2)GO TO 20
000162               F=(CXY*B)**2-2.25*AM1*AM2
000167               G=2.*CXY*CXY*B-2.25*(AM1B*AM2+AM2B*AM1)
000175               FB=G
000177               FP=2.*CXY*RXY*B*B-2.25*(AM1P*AM2+AM2P*AM1)
000207               GB=2.*CXY*CXY-2.25*(AM1BB*AM2+AM2BB*AM1+2.*AM1B*AM2B)
000221               GP=-2.25*(AM1BP*AM2+AM2BP*AM1+AM1B*AM2P+AM2B*AM1P)+4.*CXY*B*RXY
000235               GO TO 25
000235             2 AM0=PID8*(B**4-B*B*CX)
000241               AM0B=PID8*(4.*B**3-2.*CX*B)
000246               AM0BB=PID8*(12.*B*B-2.*CX)
000252               AM0P=-PID8*B*B*RX
000255               AM0BP=-.25*PI*RX*B
```

```
000260          AM3=PID8*(81.*DS2+18.*D12S*B*B+B**4-B*B*CX-9.*CY)                   82400000
000273          AM3B=PID8*(36.*D12S*B+4.*B**3-2.*B*CX)                              82500000
000303          AM3BB=PID8*(12.*B*B+36.*D12S-2.*CX)                                 82600000
000311          AM3P=PID8*(-B*B*RX-9.*RY)                                           82700000
000316          AM3BP=-.25*PI*RX*B                                                  82800000
000321          F=(CXY*B)**2-(.21972656)*(2.*AM0+AM2)*(AM1+AM3)                     82900000
000333          G=2.*CXY*CXY*B-(.21972656)*((2.*AM0B+AM2B)*(AM1+AM3)                83000000
               1+(2.*AM0+AM2)*(AM1B+AM3B))                                          83100000
000352          FB=G                                                                83200000
000354          FP=2.*CXY*RXY*B*B-(.21972656)*((2.*AM0P+AM2P)*(AM1+AM3)             83300000
               1+(2.*AM0+AM2)*(AM1P+AM3P))                                          83400000
000374          GB=-(.21972656)*((2.*AM0BB+AM2BB)*(AM1+AM3)+2.*CXY*CXY              83500000
               1+(2.*AM0+AM2)*(AM1BB+AM3BB)+2.*(2.*AM0B+AM2B)*(AM1B+AM3B))          83600000
000420          GP=-(.21972656)*((2.*AM0BP+AM2BP)*(AM1+AM3)+(2.*AM0+AM2)            83700000
               1*(AM1BP+AM3BP)+(2.*AM0B+AM2B)*(AM1P+AM3P)+(2.*AM0P+AM2P)*(AM1B      83800000
               2 +AM3B))+4.*CXY*B*RXY                                               83900000
000455       25 IF(ABS(DP) .LT. 1.E-5 .AND. ABS(DB) .LT. 1.E-5)GO TO 10             84000000
000467          AJACOBI=FP*GB-FB*GP                                                 84100000
000472          DP=(-F*GB+G*FB)/AJACOBI                                             84200000
000476          DB=(-FP*G+F*GP)/AJACOBI                                             84300000
000502          P=P+DP                                                              84400000
000503          B=B+DB                                                              84500000
000505           LC=LC+1                                                            84600000
000506          IF(LC .GT. 50)RETURN                                                84700000
000512          GO TO 1                                                             84800000
000513       10 PRINT 101,P,B,F,G                                                   84900000
000527          WRITE(7,101) P,B,F,G                                                85000000
000546      101 FORMAT(    5X,*P=*,E16.8,2X,*B=*,E16.8,2X,*F=*,E16.8,*G=*,E16.8)    85100000
000546          IF(INCX .EQ. 1)RETURN                                               85200000
000554          IF(X .GT. 0.)GO TO 60                                               85200001
000557          X=1./B/ADB*ABS(X)                                                   85300000
000562          Y=ABS(Y)                                                            85400000
000563          PRINT 51,X,Y                                                        85500001
000573          WRITE(7,51) X,Y                                                     85500002
000606       60 COEF=PI*PI*DS1/(XB*XB)                                              85500003
000611          ANX=CX*COEF                                                         85600000
000613          ANY=CY*COEF                                                         85700000
000615          ANXY=CXY*COEF                                                       85800000
000616          STRNX=(ANX/EX-EXY*ANY/EY)/TT                                        85900000
000624          STRNY=(ANY/EY-EXY*ANX/EX)/TT                                        86000000
000630          STRNXY=ANXY/GXY/TT                                                  86100000
000632          WRITE(7,52) CX,CY,CXY,STRNX,STRNY,STRNXY                            86400000
000652          PRINT 52,CX,CY,CXY,STRNX,STRNY,STRNXY                               86500000
```

```
                 SUBROUTINE SET(S,N,LL,LU,IF,R)                87900000
000011           DIMENSION S(32,32)                            88000000
000011           IF(N .EQ.2)GO TO 10                           88100000
000013           DO 20 IR=LL,LU                                88200000
000014        20 S(IR,IF)=R                                    88300000
000022           RETURN                                        88400000
000023        10 DO 30 IC=LL,LU                                88500000
000025        30 S(IF,IC)=R                                    88600000
000033           RETURN                                        88700000
000034           END                                           88800000
```

## REFERENCES


1. Gerard, George; and Becker, Herbert: Handbook of Structural Stability. Part I – Buckling of Flat Plates. NACA TN 3781, 1957.
2. Becker, Herbert: Handbook of Structural Stability. Part II – Buckling of Composite Elements. NACA TN 3782, 1957.
3. Johns, D. J.: Shear Buckling of Isotropic and Orthotropic Plates – A Review. R. & M. No. 3677, British A.R.C., 1971.
4. Lekhnitskii, S. G. (S. W. Tsai and T. Cheron, transl.): Anisotropic Plates. Gordon and Breach Sci. Publ., c.1968.
5. Ashton, J. E.; and Whitney, J. M.: Theory of Laminated Plates. Progress in Materials Science Series, Volume IV, Technomic Pub. Co., Inc., c.1970.
6. Timoshenko, Stephen P.; and Gere, James M.: Theory of Elastic Stability. Second ed. McGraw-Hill Book Co., Inc., c.1961.
7. Almroth, B. O.; Brogan, F. A.; Meller, E.; and Petersen, H. T.: User's Manual for STAGS. LMSC-D358197, Lockheed Missiles & Space Co., Jan. 1974.
8. McCormick, Caleb W., ed.: The NASTRAN User's Manual (Level 15). NASA SP-222(01), 1972.
9. Reed, D. L.: Laminated Sandwich Panel Analysis. FZM-5590, Convair Aerospace Div., General Dynamics, Oct. 1, 1971.
10. Fung, Y. C.: Foundations of Solid Mechanics. Prentice-Hall, Inc., c.1965.
11. Ashton, J. E.; Halpin, J. C.; and Petit, P. H.: Primer on Composite Materials: Analysis. Technomic Pub. Co., Inc., c.1969.
12. Libove, Charles; and Batdorf, S. B.: A General Small-Deflection Theory for Flat Sandwich Plates. NACA Rep. 899, 1948. (Supersedes NACA TN 1526.)
13. Housner, J. M.; and Stein, Manuel: Flutter Analysis of Swept-Wing Subsonic Aircraft With Parameter Studies of Composite Wings. NASA TN D-7539, 1974.
14. Stein, Manuel; and Yaeger, David J.: Critical Shear Stress of a Curved Rectangular Panel With a Central Stiffener. NACA TN 1972, 1949.
15. Stowell, Elbridge Z.; and Schwartz, Edward B.: Critical Stress for an Infinitely Long Flat Plate With Elastically Restrained Edges Under Combined Shear and Direct Stress. NACA WR L-340, 1943. (Formerly NACA ARR 3K13.)

16. Batdorf, S. B.; Schildcrout, Murry; and Stein, Manuel: Critical Combinations of Shear and Longitudinal Direct Stress for Long Plates With Transverse Curvature. NACA TN 1347, 1947.

17. Budiansky, Bernard; and Hu, Pai C.: The Lagrangian Multiplier Method of Finding Upper and Lower Limits to Critical Stresses of Clamped Plates. NACA Rep. 848, 1946. (Supersedes NACA TN 1103.)

TABLE 1.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES SIMPLY SUPPORTED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED

| Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$ | Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$ | No. of mesh points in x- and y-directions | | Wavelength ratios used in trigonometric differences | | Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$ |
|---|---|---|---|---|---|---|
| | | $a/\Delta_x$ | $b/\Delta_y$ | $\lambda_x/a$ | $\lambda_y/b$ | |
| 0.2 | 1.0 | 9 | 9 | 0.56 | 0.56 | 26.28 |
| | .8 | 9 | 9 | .56 | .60 | 21.43 |
| | .6 | 9 | 9 | .56 | .80 | 17.33 |
| | .5 | 9 | 9 | .50 | .90 | 15.36 |
| | .4 | 11 | 11 | .50 | 1.00 | 13.77 |
| | .2 | 13 | 13 | .35 | 1.00 | 11.55 |
| | [a]0 | --- | --- | --- | --- | 10.87 |
| .4 | 1.0 | 9 | 9 | .56 | .56 | 15.78 |
| | .8 | 9 | 9 | .56 | .60 | 12.98 |
| | .6 | 9 | 9 | .56 | .80 | 10.86 |
| | .5 | 9 | 9 | .50 | .90 | 9.93 |
| | .4 | 11 | 11 | .50 | 1.00 | 9.29 |
| | .2 | 15 | 8 | .30 | 1.00 | 8.21 |
| | [a]0 | --- | --- | --- | --- | 7.72 |
| .6 | 1.0 | 9 | 9 | .56 | .56 | 12.21 |
| | .8 | 9 | 9 | .56 | .60 | 10.11 |
| | .6 | 9 | 9 | .56 | .80 | 8.67 |
| | .5 | 9 | 9 | .50 | .90 | 8.09 |
| | .4 | 11 | 11 | .50 | 1.00 | 7.73 |
| | .2 | 15 | 8 | .25 | 1.00 | 6.71 |
| | [a]0 | --- | --- | --- | --- | 6.53 |
| .8 | 1.0 | 9 | 9 | .56 | .56 | 10.40 |
| | .8 | 9 | 9 | .56 | .60 | 8.66 |
| | .6 | 9 | 9 | .56 | .80 | 7.57 |
| | .5 | 9 | 9 | .50 | .90 | 7.10 |
| | .4 | 11 | 11 | .50 | 1.00 | 6.80 |
| | .2 | 15 | 8 | .25 | 1.00 | 6.02 |
| | [a]0 | --- | --- | --- | --- | 5.79 |
| 1 | 1.0 | 9 | 9 | .56 | .56 | 9.31 |
| | .8 | 9 | 9 | .56 | .60 | 7.68 |
| | .6 | 9 | 9 | .56 | .80 | 6.91 |
| | .4 | 11 | 11 | .50 | 1.00 | 6.22 |
| | .2 | 15 | 8 | .23 | 1.00 | 5.49 |
| | [a]0 | --- | --- | --- | --- | 5.33 |

[a]For $B = 0$, $k_s$ was calculated by using equations (B2).

TABLE 1.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES SIMPLY SUPPORTED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED – Concluded

| Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$ | Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$ | No. of mesh points in x- and y-directions | | Wavelength ratios used in trigonometric differences | | Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$ |
|---|---|---|---|---|---|---|
| | | $a/\Delta_x$ | $b/\Delta_y$ | $\lambda_x/a$ | $\lambda_y/b$ | |
| 1.25 | 1.0 | 9 | 9 | 0.56 | 0.56 | 8.43 |
| | .8 | 9 | 9 | .56 | .60 | 7.08 |
| | .6 | 9 | 9 | .56 | .80 | 6.38 |
| | .4 | 11 | 11 | .50 | 1.00 | 5.75 |
| | .2 | 15 | 8 | .22 | 1.00 | 5.09 |
| | .1 | 25 | 9 | .13 | 1.00 | 5.05 |
| | [a] 0 | --- | --- | --- | --- | 4.96 |
| 1.667 | 1.0 | 9 | 9 | .56 | .56 | 7.54 |
| | .8 | 9 | 9 | .56 | .60 | 6.37 |
| | .6 | 9 | 9 | .56 | .80 | 5.85 |
| | .4 | 11 | 11 | .50 | 1.00 | 5.26 |
| | .2 | 15 | 8 | .22 | 1.00 | 4.72 |
| | .1 | 22 | 8 | .13 | 1.00 | 4.68 |
| | [a] 0 | --- | --- | --- | --- | 4.60 |
| 2.5 | 1.0 | 9 | 9 | .56 | .56 | 6.65 |
| | .8 | 9 | 9 | .56 | .60 | 5.66 |
| | .6 | 9 | 9 | .56 | .80 | 5.32 |
| | .4 | 11 | 11 | .50 | 1.00 | 4.77 |
| | .2 | 15 | 8 | .22 | 1.00 | 4.32 |
| | .1 | 22 | 8 | .13 | 1.00 | 4.33 |
| | [a] 0 | --- | --- | --- | --- | 4.17 |
| 5 | 1.0 | 9 | 9 | .56 | .56 | 5.74 |
| | .8 | 9 | 9 | .56 | .60 | 4.94 |
| | .6 | 9 | 9 | .56 | .80 | 4.78 |
| | .4 | 11 | 11 | .50 | 1.00 | 4.27 |
| | .2 | 15 | 8 | .22 | 1.00 | 3.90 |
| | .1 | 22 | 8 | .13 | 1.00 | 3.86 |
| | [a] 0 | --- | --- | --- | --- | 3.75 |
| ∞ | 1.0 | 9 | 9 | .56 | .56 | 4.83 |
| | .8 | 9 | 9 | .56 | .60 | 4.22 |
| | .6 | 9 | 9 | .56 | .80 | 4.25 |
| | .4 | 11 | 11 | .50 | 1.00 | 3.76 |
| | .2 | 15 | 8 | .22 | 1.00 | 3.47 |
| | [a] 0 | --- | --- | --- | --- | 3.30 |

[a] For $B = 0$, $k_s$ was calculated by using equations (B2).

TABLE 2.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES CLAMPED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED

| Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$ | Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$ | No. of mesh points in x- and y-directions | | Wavelength ratios used in trigonometric differences | | Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11}D_{22}^3}}$ |
|---|---|---|---|---|---|---|
| | | $a/\Delta x$ | $b/\Delta y$ | $\lambda_x/a$ | $\lambda_y/b$ | |
| 0.2 | 1.0 | 9 | 9 | 1.00 | 1.0 | 32.56 |
| | .8 | 9 | 9 | .80 | 1.0 | 26.31 |
| | .6 | 9 | 9 | .60 | 1.0 | 22.21 |
| | .4 | 11 | 11 | .40 | 1.0 | 18.91 |
| | .2 | 17 | 9 | .31 | 1.0 | 17.34 |
| | .1 | 25 | 9 | .15 | 1.0 | 17.31 |
| | [a]0 | --- | --- | --- | --- | 17.13 |
| .4 | 1.0 | 9 | 9 | 1.10 | 1.1 | 21.63 |
| | .8 | 9 | 9 | .90 | 1.0 | 17.92 |
| | .6 | 9 | 9 | .60 | 1.0 | 15.43 |
| | .4 | 11 | 11 | .40 | 1.0 | 13.62 |
| | .2 | 17 | 9 | .25 | 1.0 | 12.64 |
| | .1 | 25 | 9 | .13 | 1.0 | 12.89 |
| | [a]0 | --- | --- | --- | --- | 12.51 |
| .6 | 1.0 | 9 | 9 | 1.10 | 1.1 | 17.86 |
| | .8 | 9 | 9 | .90 | 1.0 | 14.89 |
| | .6 | 9 | 9 | .60 | 1.0 | 13.06 |
| | .4 | 11 | 11 | .40 | 1.0 | 11.60 |
| | .2 | 15 | 8 | .22 | 1.0 | 10.64 |
| | .1 | 25 | 9 | .13 | 1.0 | 10.95 |
| | [a]0 | --- | --- | --- | --- | 10.69 |
| .8 | 1.0 | 9 | 9 | 1.10 | 1.1 | 15.94 |
| | .8 | 9 | 9 | .90 | 1.0 | 13.34 |
| | .6 | 9 | 9 | .60 | 1.0 | 11.84 |
| | .4 | 11 | 11 | .40 | 1.0 | 10.55 |
| | .2 | 17 | 9 | .24 | 1.0 | 9.99 |
| | .1 | 25 | 9 | .13 | 1.0 | 10.16 |
| | [a]0 | --- | --- | --- | --- | 9.63 |
| 1 | 1.0 | 9 | 9 | 1.20 | 1.2 | 14.81 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 12.44 |
| | .6 | 9 | 9 | .60 | 1.0 | 11.08 |
| | .4 | 11 | 11 | .40 | 1.0 | 9.89 |
| | .2 | 17 | 9 | .22 | 1.0 | 9.27 |
| | .1 | 25 | 9 | .12 | 1.0 | 9.11 |
| | [a]0 | --- | --- | --- | --- | 8.99 |

[a]For $B = 0$, $k_s$ was calculated by using equations (B3).

TABLE 2.- SHEAR-BUCKLING LOAD COEFFICIENTS FOR RECTANGULAR ORTHOTROPIC PANELS WITH ALL EDGES CLAMPED AND THE TRIGONOMETRIC DIFFERENCE PARAMETERS ON WHICH THEY ARE BASED – Concluded

| Stiffness parameter, $\Theta = \frac{\sqrt{D_{11}D_{22}}}{D_3}$ | Aspect-ratio parameter, $B = \frac{b}{a}\sqrt[4]{\frac{D_{11}}{D_{22}}}$ | No. of mesh points in x- and y-directions | | Wavelength ratios used in trigonometric differences | | Shear-buckling load coefficient, $k_s = \frac{b^2 N_{xy}}{\pi^2 \sqrt[4]{D_{11} D_{22}^3}}$ |
|---|---|---|---|---|---|---|
| | | $a/\Delta_x$ | $b/\Delta_y$ | $\lambda_x/a$ | $\lambda_y/b$ | |
| 1.25 | 1.0 | 9 | 9 | 1.20 | 1.2 | 13.87 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 11.68 |
| | .6 | 9 | 9 | .60 | 1.0 | 10.46 |
| | .4 | 9 | 9 | .40 | 1.0 | 9.39 |
| | .2 | 15 | 8 | .22 | 1.0 | 8.80 |
| | .1 | 22 | 8 | .12 | 1.0 | 8.98 |
| | [a]0 | --- | --- | --- | --- | 8.45 |
| 1.667 | 1.0 | 9 | 9 | 1.20 | 1.2 | 12.91 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 10.90 |
| | .6 | 9 | 9 | .60 | 1.0 | 9.80 |
| | .4 | 9 | 9 | .40 | 1.0 | 8.86 |
| | .2 | 15 | 8 | .22 | 1.0 | 8.34 |
| | .1 | 22 | 8 | .12 | 1.0 | 8.58 |
| | [a]0 | --- | --- | --- | --- | 7.93 |
| 2.5 | 1.0 | 9 | 9 | 1.20 | 1.2 | 11.93 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 10.11 |
| | .6 | 9 | 9 | .60 | 1.0 | 9.07 |
| | .4 | 9 | 9 | .40 | 1.0 | 8.31 |
| | .2 | 15 | 8 | .22 | 1.0 | 7.84 |
| | .1 | 25 | 9 | .12 | 1.0 | 8.12 |
| | [a]0 | --- | --- | --- | --- | 7.32 |
| 5 | 1.0 | 9 | 9 | 1.20 | 1.2 | 10.94 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 9.31 |
| | .6 | 9 | 9 | .60 | 1.0 | 8.33 |
| | .4 | 9 | 9 | .40 | 1.0 | 7.74 |
| | .2 | 15 | 8 | .22 | 1.0 | 7.33 |
| | .1 | 25 | 9 | .12 | 1.0 | 7.66 |
| | [a]0 | --- | --- | --- | --- | 6.72 |
| $\infty$ | 1.0 | 9 | 9 | 1.20 | 1.2 | 9.92 |
| | .8 | 9 | 9 | 1.00 | 1.0 | 8.48 |
| | .6 | 9 | 9 | .60 | 1.0 | 7.57 |
| | .4 | 11 | 11 | .40 | 1.0 | 6.97 |
| | .2 | 15 | 8 | .22 | 1.0 | 6.79 |
| | .1 | 25 | 9 | .12 | 1.0 | 7.17 |
| | [a]0 | --- | --- | --- | --- | 6.11 |

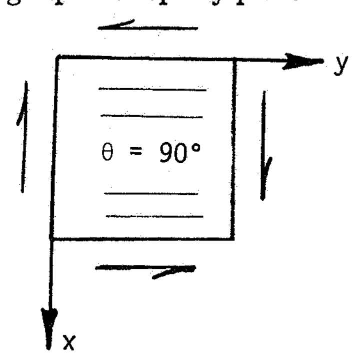
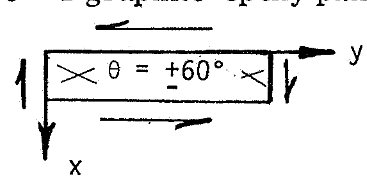
[a] For $B = 0$, $k_s$ was calculated by using equations (B3).

TABLE 3.- MATERIAL PROPERTIES OF GRAPHITE-EPOXY SKINS WITH THEIR EQUIVALENT ORTHOTROPIC PARAMETERS AT VARIOUS FILAMENT ORIENTATIONS

$\left[E_1 = 145\ \text{GN/m}^2\ (21 \times 10^6\ \text{psi});\quad E_2/E_1 = 0.1138;\quad G_{12}/E_1 = 0.03095;\quad \nu_{12} = 0.31\right]$

| Filament orientation, $\pm\theta$, deg | $\Theta = \dfrac{\sqrt{D_{11}D_{22}}}{D_3}$ | $\dfrac{a}{b} B = \sqrt[4]{\dfrac{D_{11}}{D_{22}}}$ |
|---|---|---|
| 0 | 3.50 | 1.722 |
| 30 | .511 | 1.389 |
| 45 | .415 | 1.000 |
| 60 | .511 | .720 |
| 90 | 3.50 | .581 |

TABLE 4.- COMPARISON OF CONVENTIONAL AND TRIGONOMETRIC FINITE DIFFERENCES FOR ORTHOTROPIC PANELS

| Problem description | Degrees of freedom | | $\overline{N}_{xy}$ | | $\lambda_x/a$ | $\lambda_y/\lambda_x$ |
|---|---|---|---|---|---|---|
| | $M_e$ | $N_e$ | Conventional | Trigonometric | | |
| Shear buckling of a clamped, square, graphite-epoxy panel   | 4 | 4 | 56.03 | ---- | --- | --- |
| | 6 | 6 | 48.43 | 42.84 | 0.55 | 1 |
| | 8 | 8 | 45.65 | ---- | --- | --- |
| | 12 | 12 | 43.79 | ---- | --- | --- |
| | 20 | 20 | 42.90 | ---- | --- | --- |
| Shear buckling of a simply supported $5 \times 1$ graphite-epoxy panel   | 20 | 10 | 20.40 | 19.17 | 0.21 | 1 |
| | 29 | 13 | 19.70 | ---- | --- | --- |
| | 40 | 15 | 19.39 | ---- | --- | --- |
| | 50 | 20 | 19.22 | ---- | --- | --- |

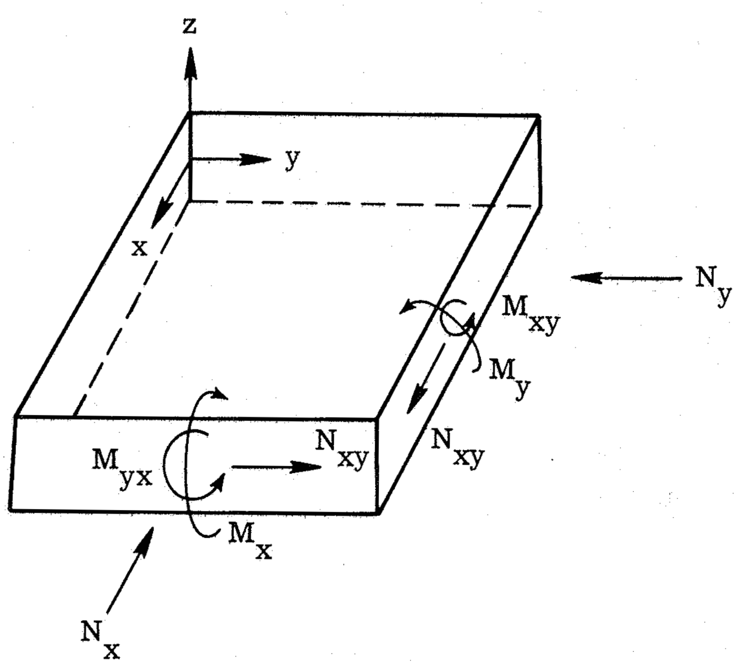
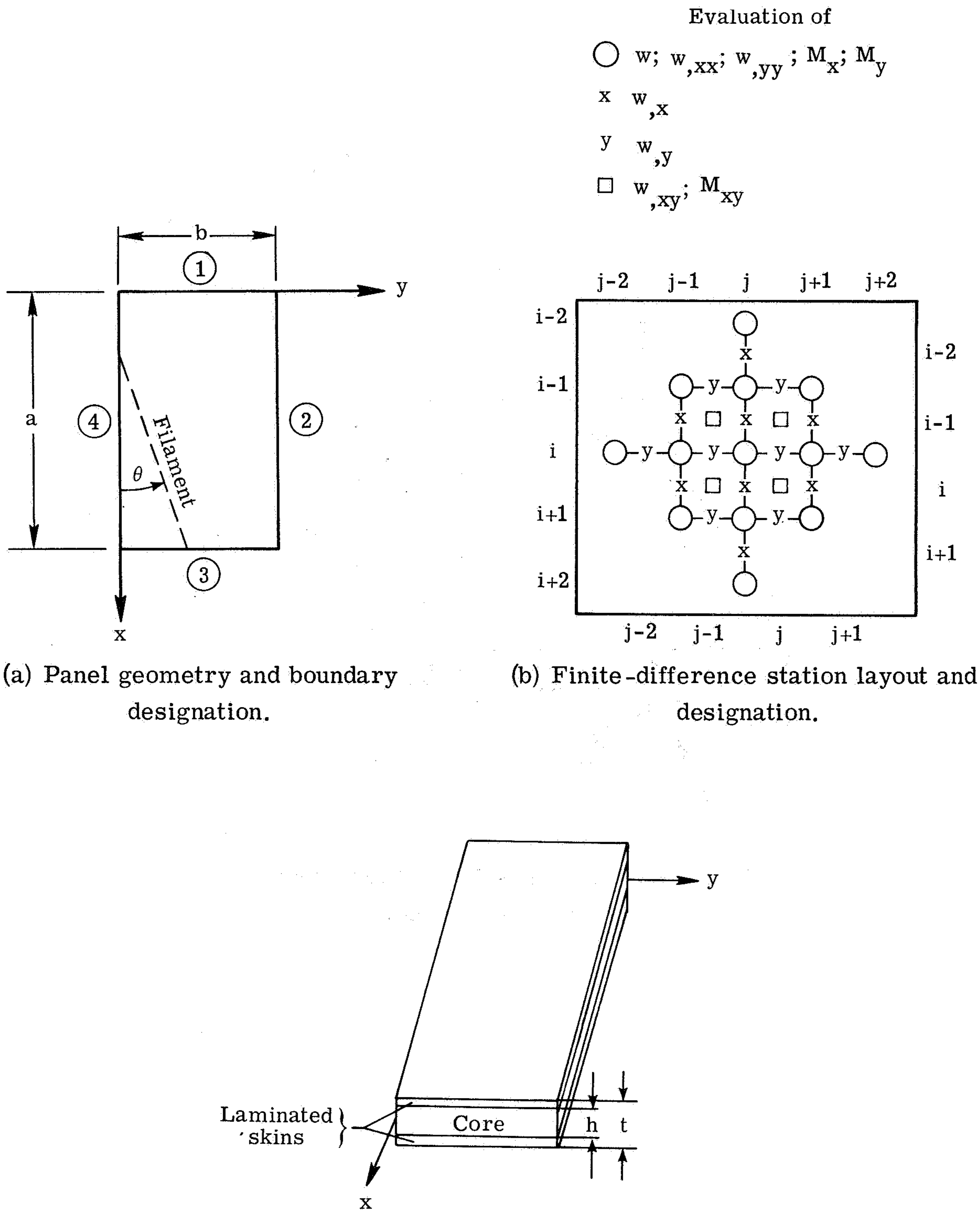



Figure 1.- Stress resultants acting upon an element of the plate.




(a) Panel geometry and boundary designation.

(b) Finite-difference station layout and designation.

(c) Sandwich panel.

Figure 2.- Geometrical and numerical configurations.

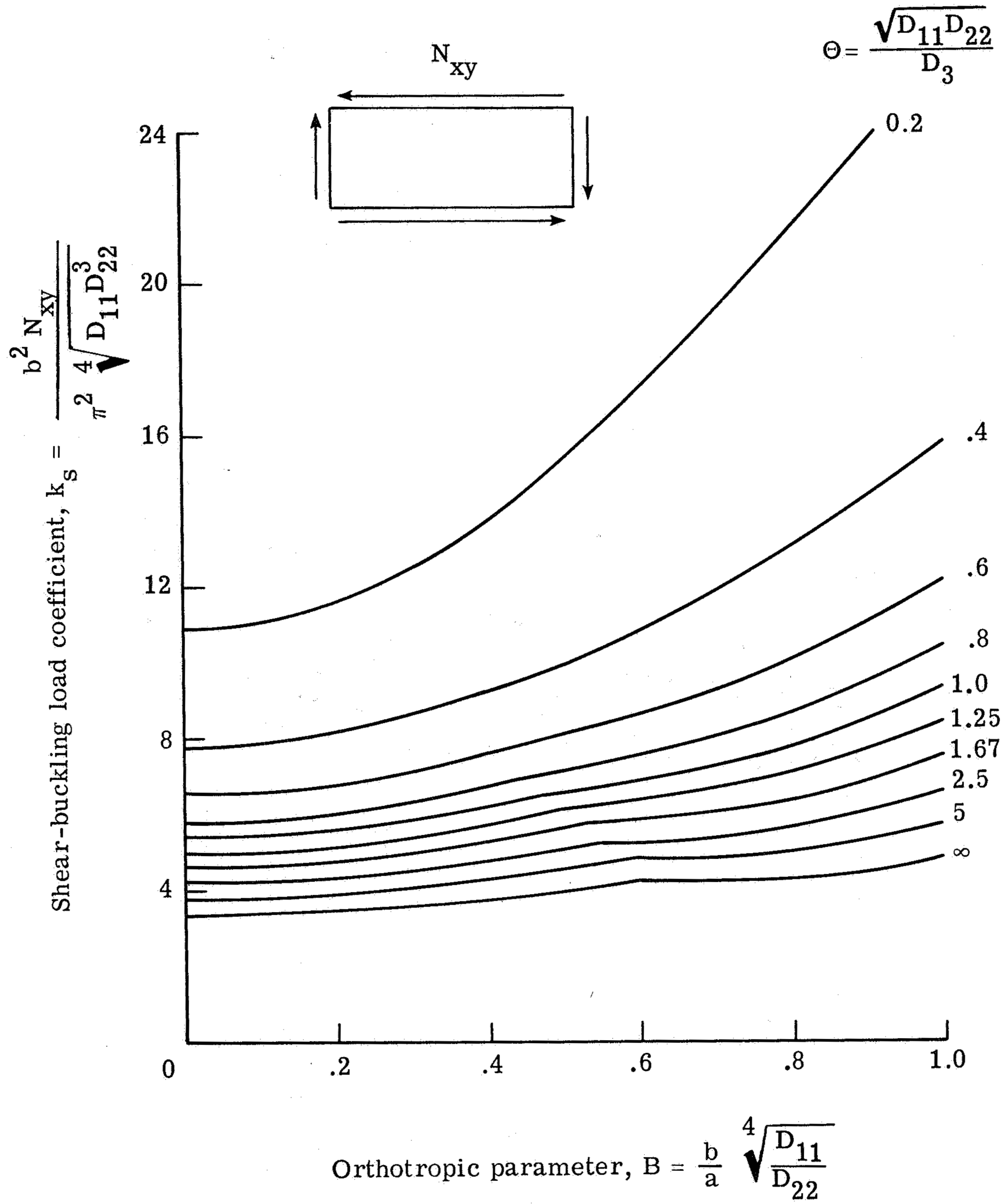



Figure 3. - Shear-buckling load coefficients for rectangular orthotropic plates with all edges simply supported.

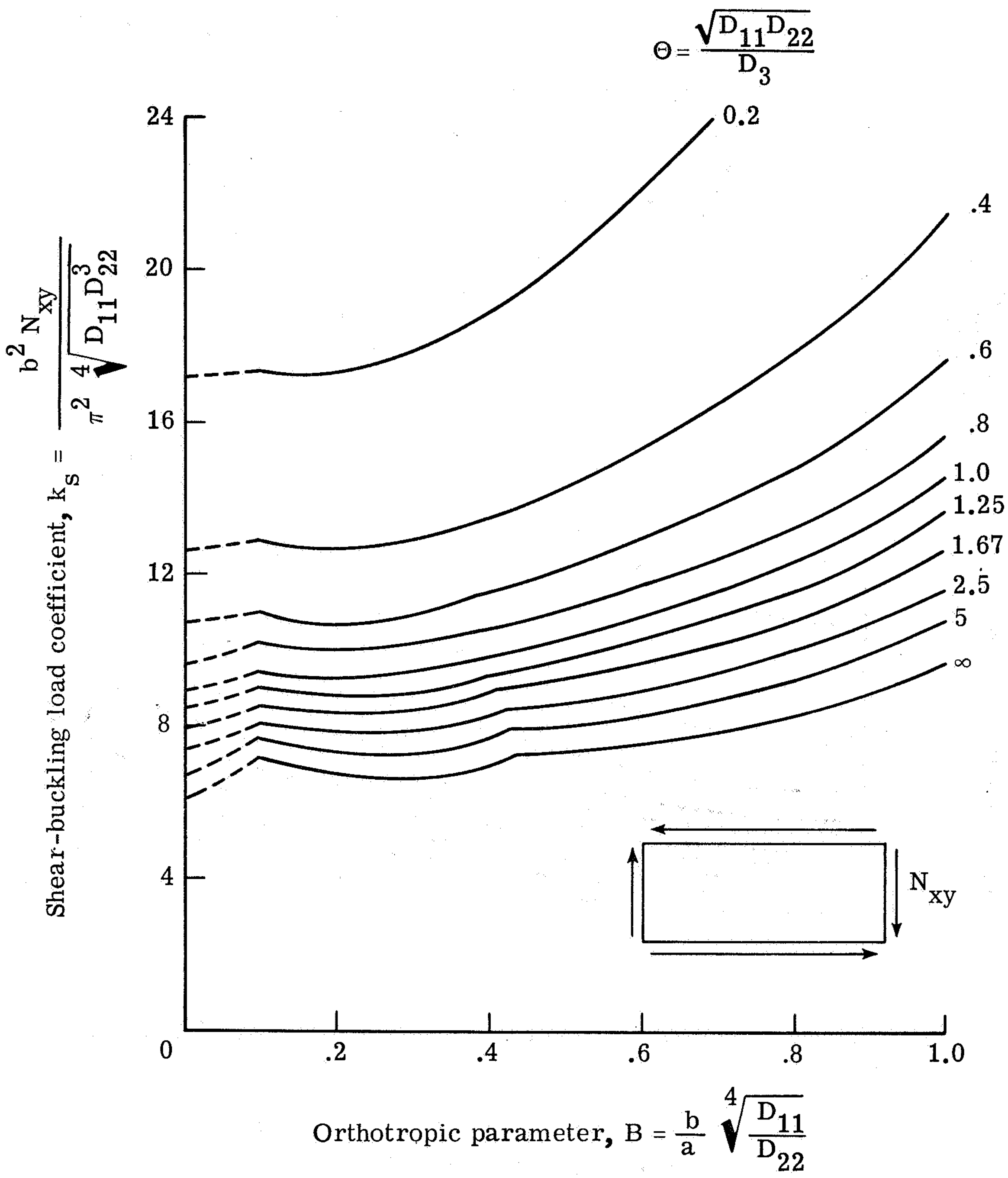



Figure 4. - Shear-buckling load coefficients for rectangular orthotropic plates with all edges clamped.

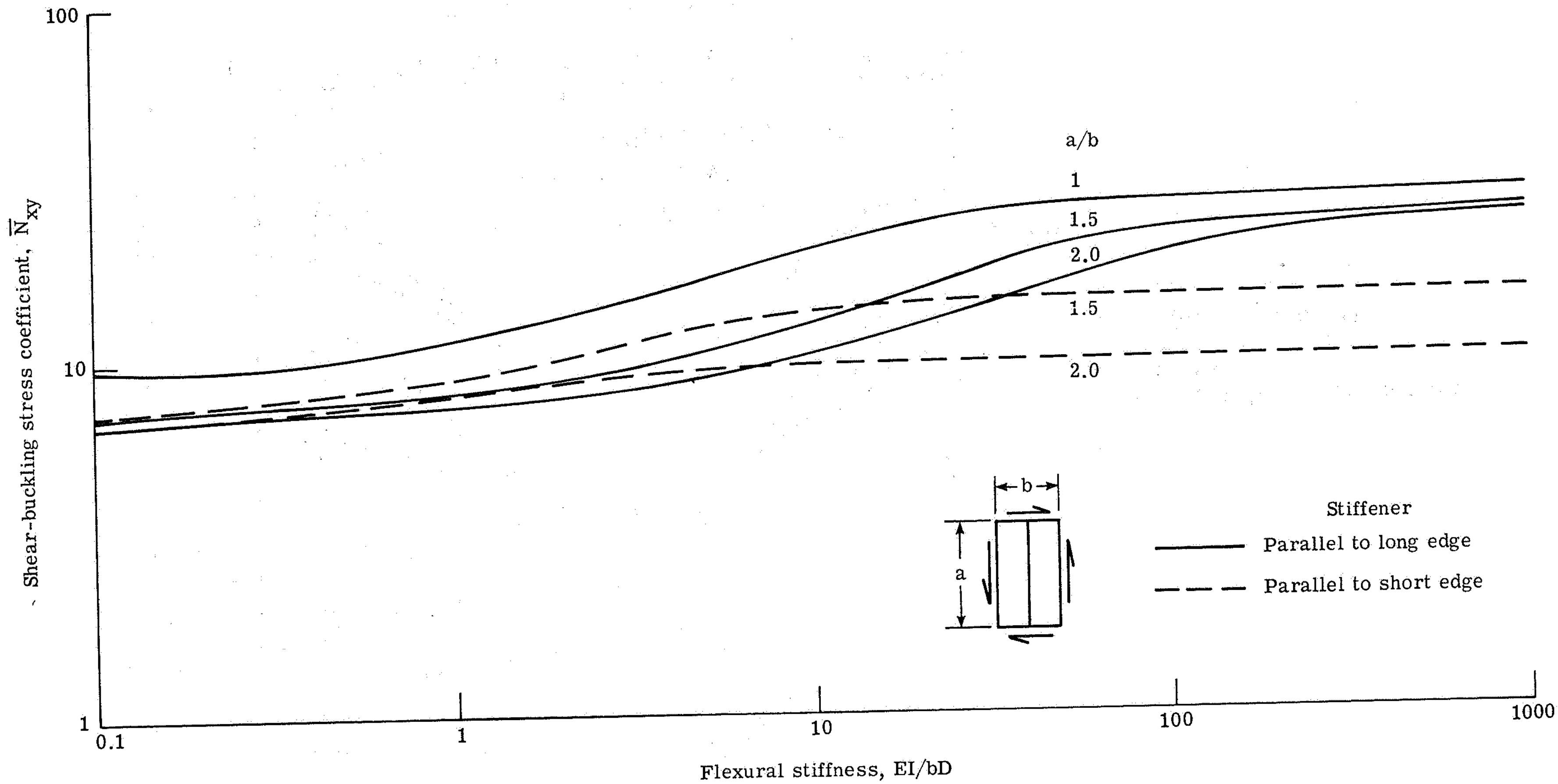



Figure 5. - Shear buckling of simply supported isotropic panels each with a central stiffener.

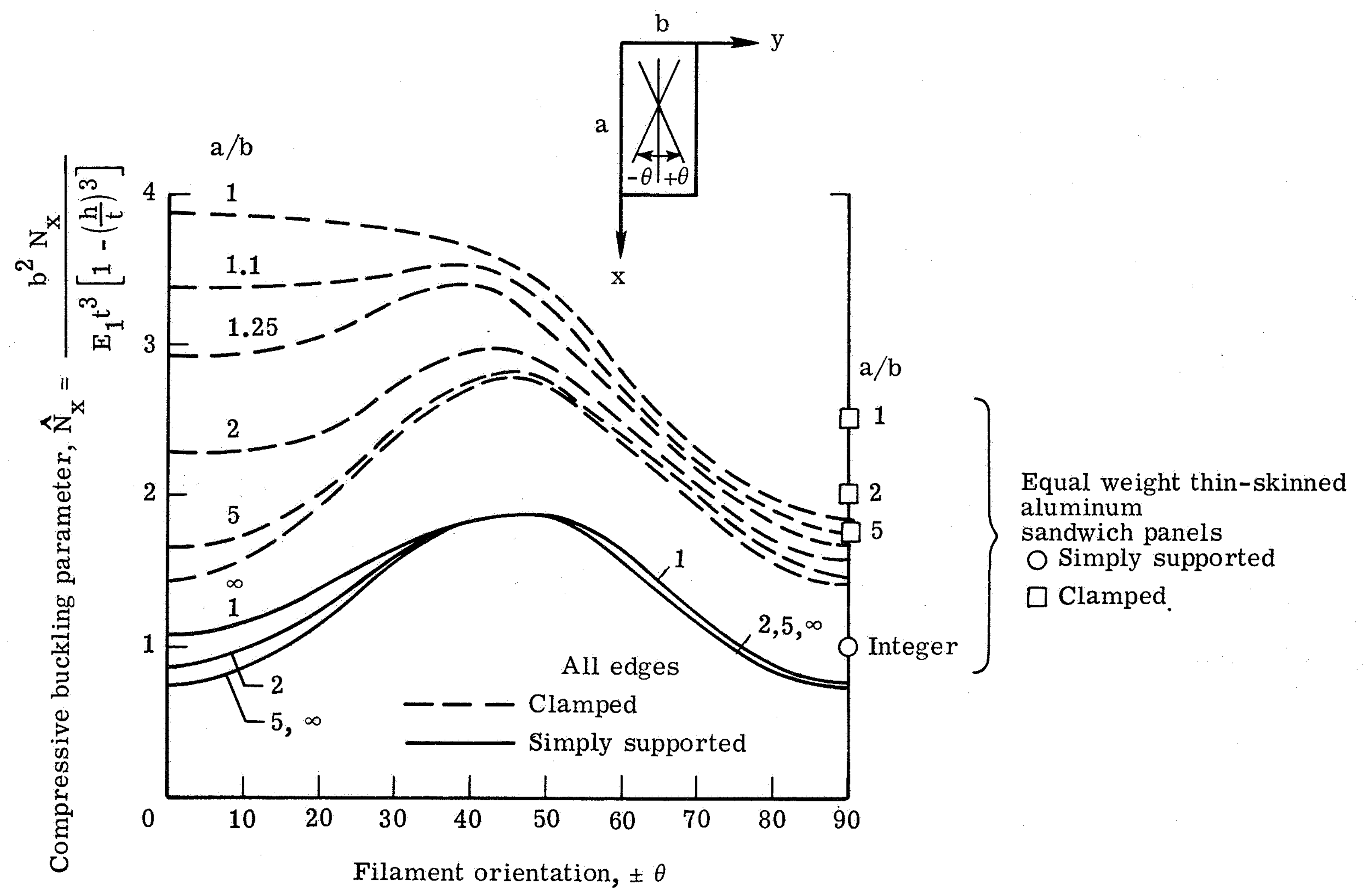



Figure 6.- Variation of compressive buckling load with filament orientation for panels of various aspect ratios.

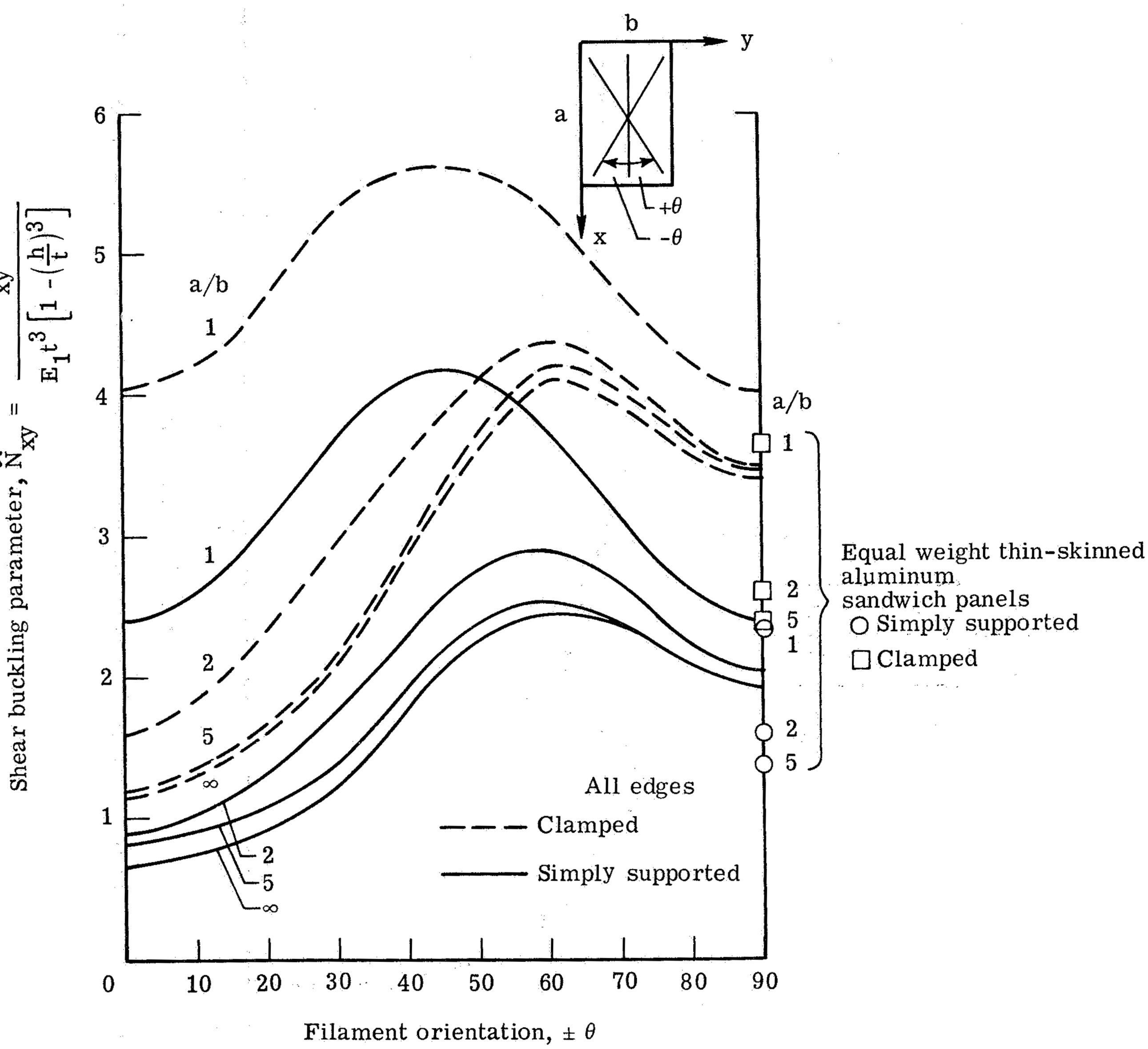



Figure 7.- Variation of shear buckling load with filament orientation for panels of various aspect ratios.

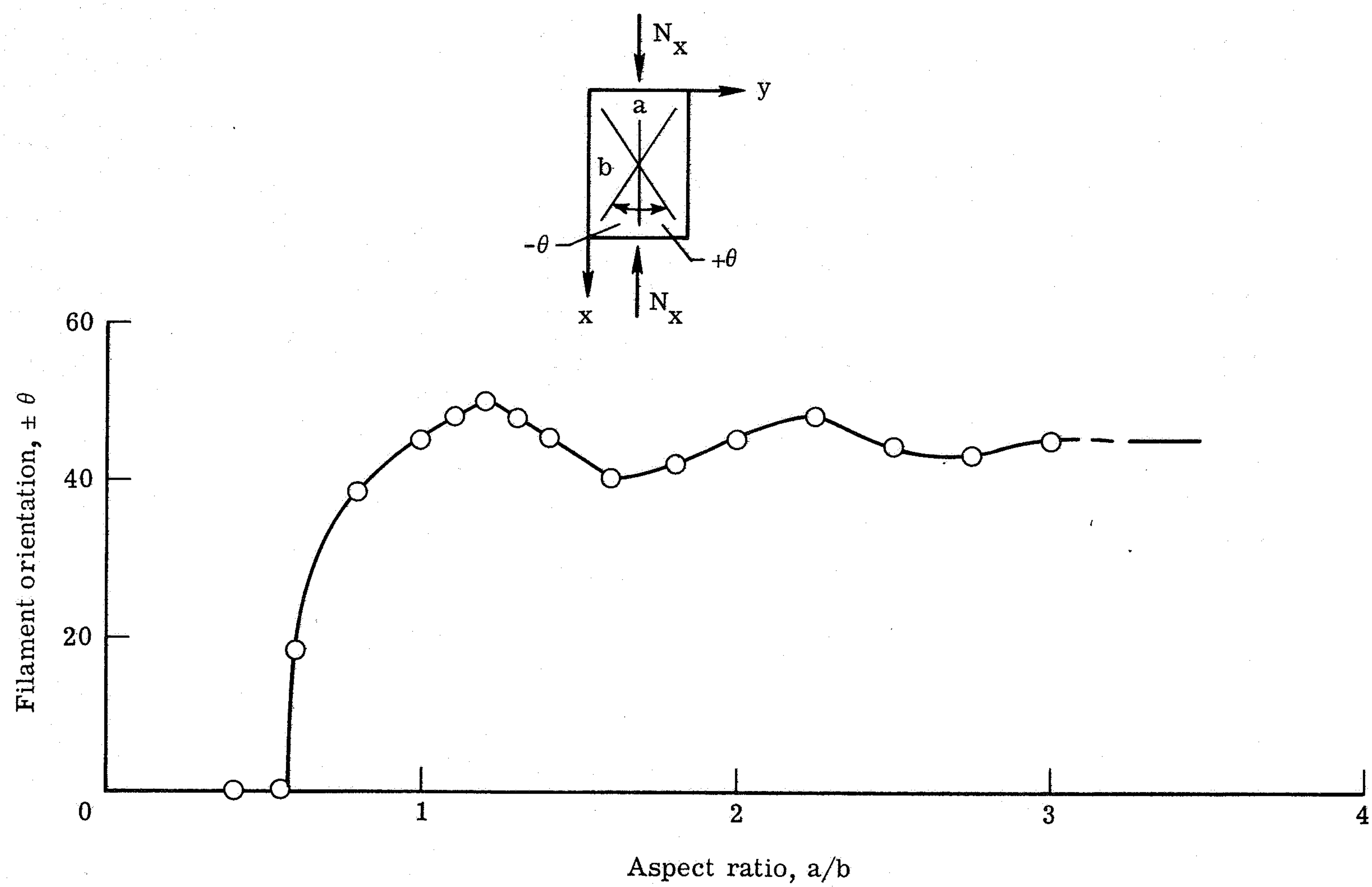



Figure 8.- Optimum filament orientation for the compressive buckling of a simply supported panel.

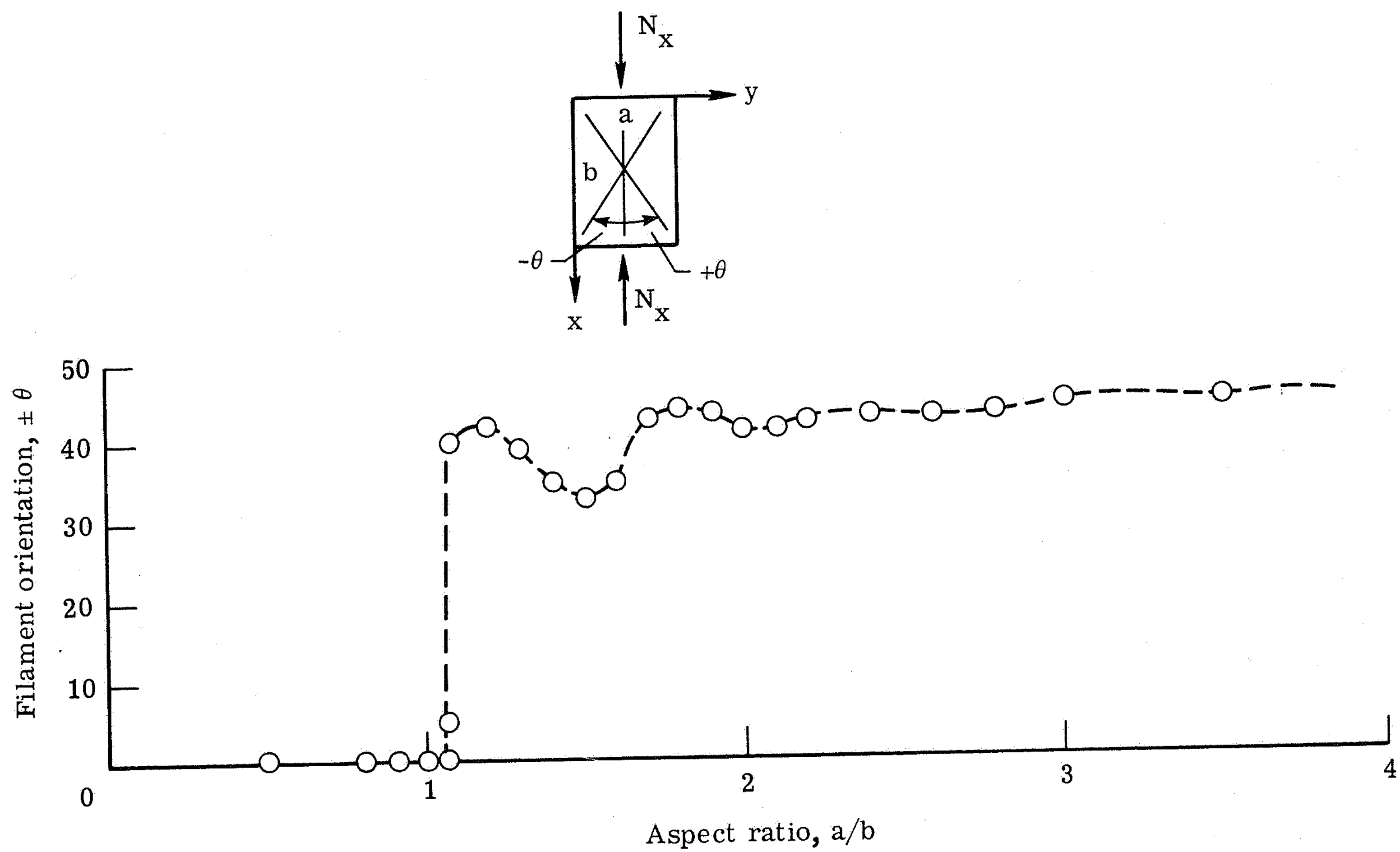



Figure 9.- Optimum filament orientation for the compressive buckling of a clamped panel.

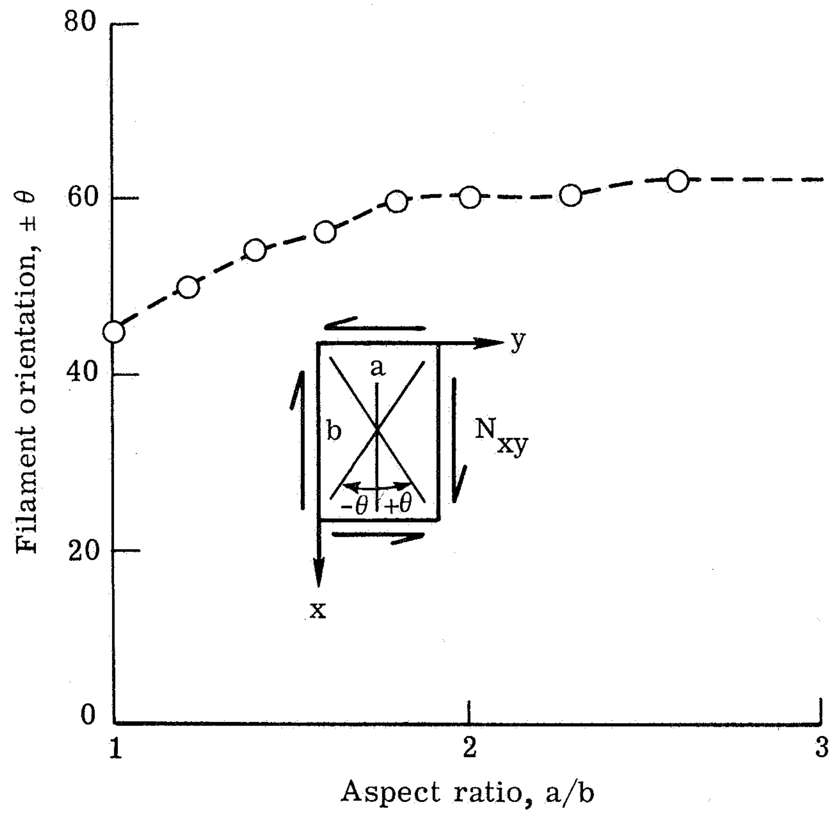



Figure 10. - Optimum filament orientation for the shear buckling of a simply supported panel.

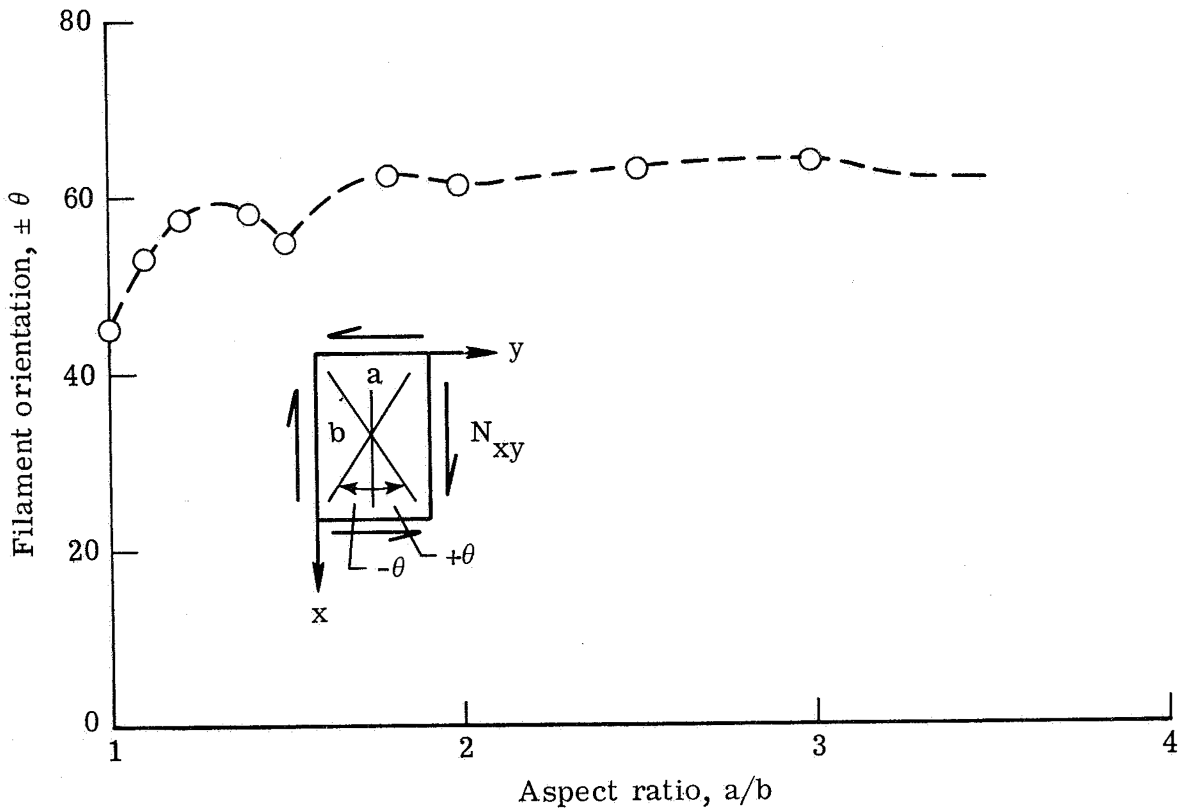
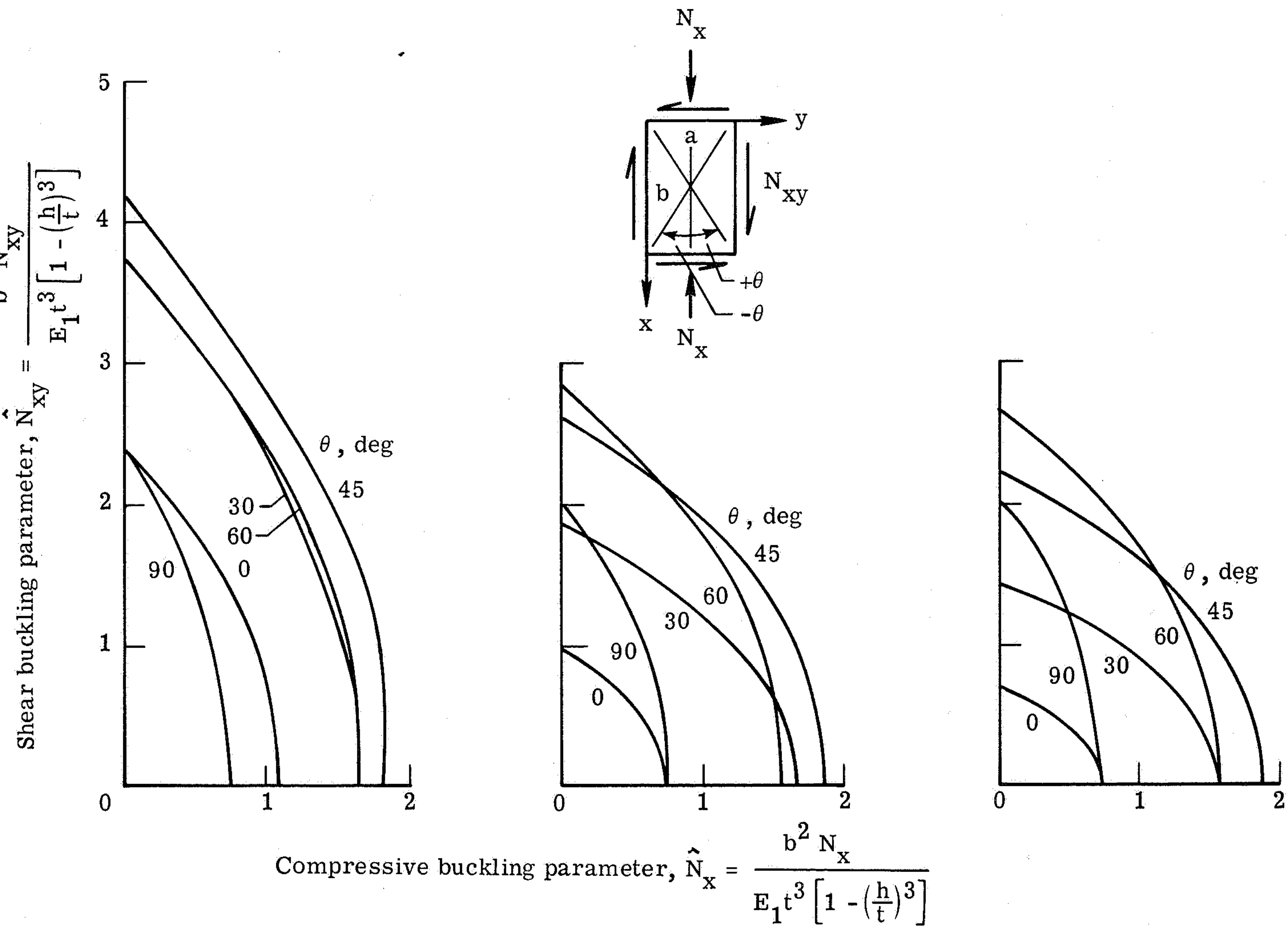
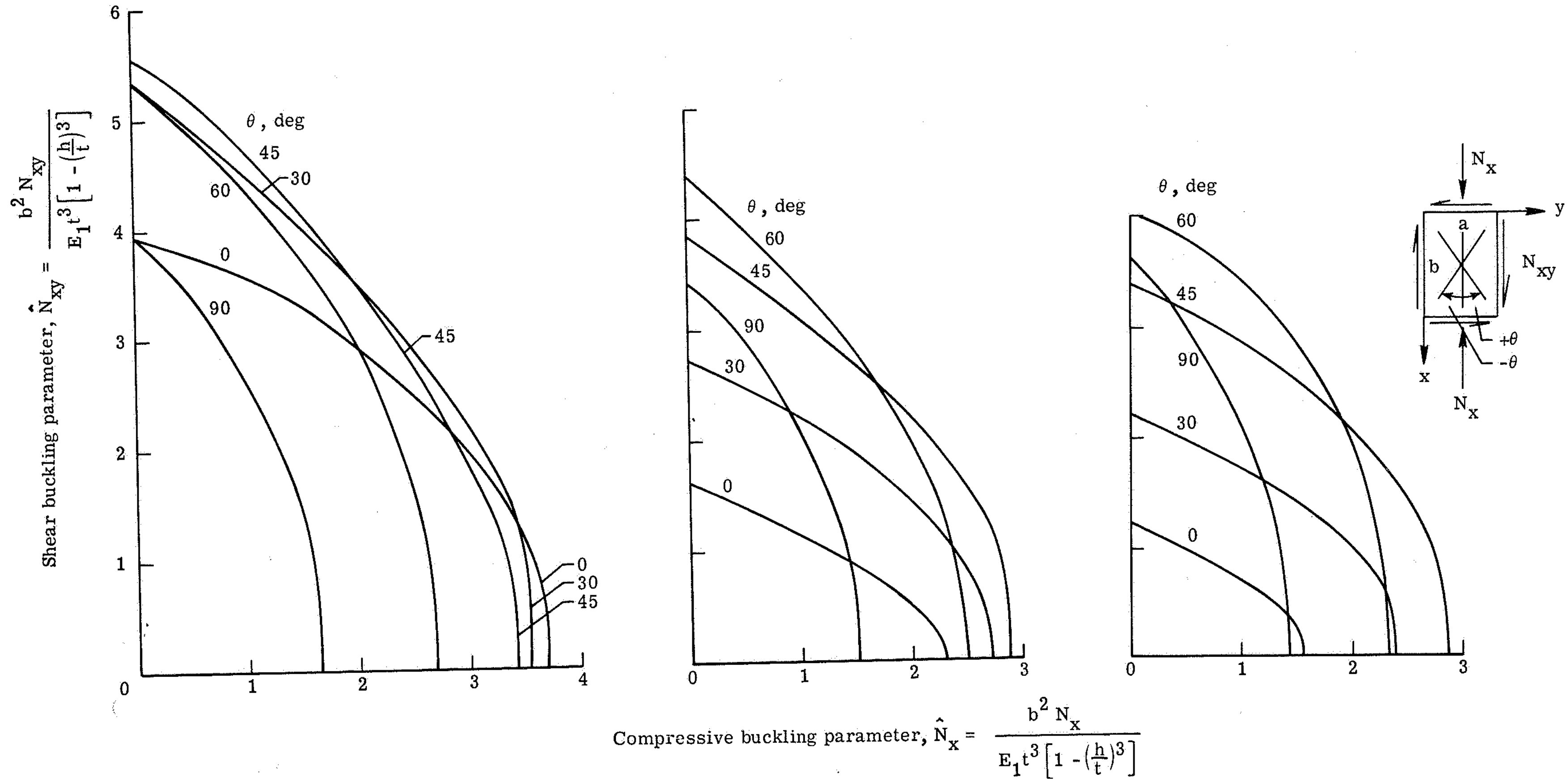



Figure 11. - Optimum filament orientation for the shear buckling of a clamped panel.




(a) a/b = 1. (b) a/b = 2. (c) a/b = 5.

Figure 12. - Combined axial compression and shear of simply supported panels.




(a) a/b = 1.

(b) a/b = 2.

(c) a/b = 5.

Figure 13. - Combined axial compression and shear of clamped panels.

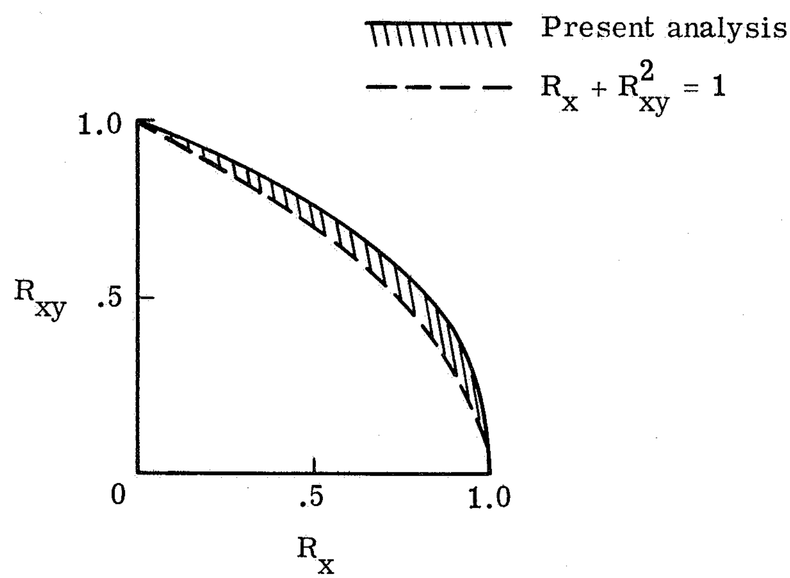
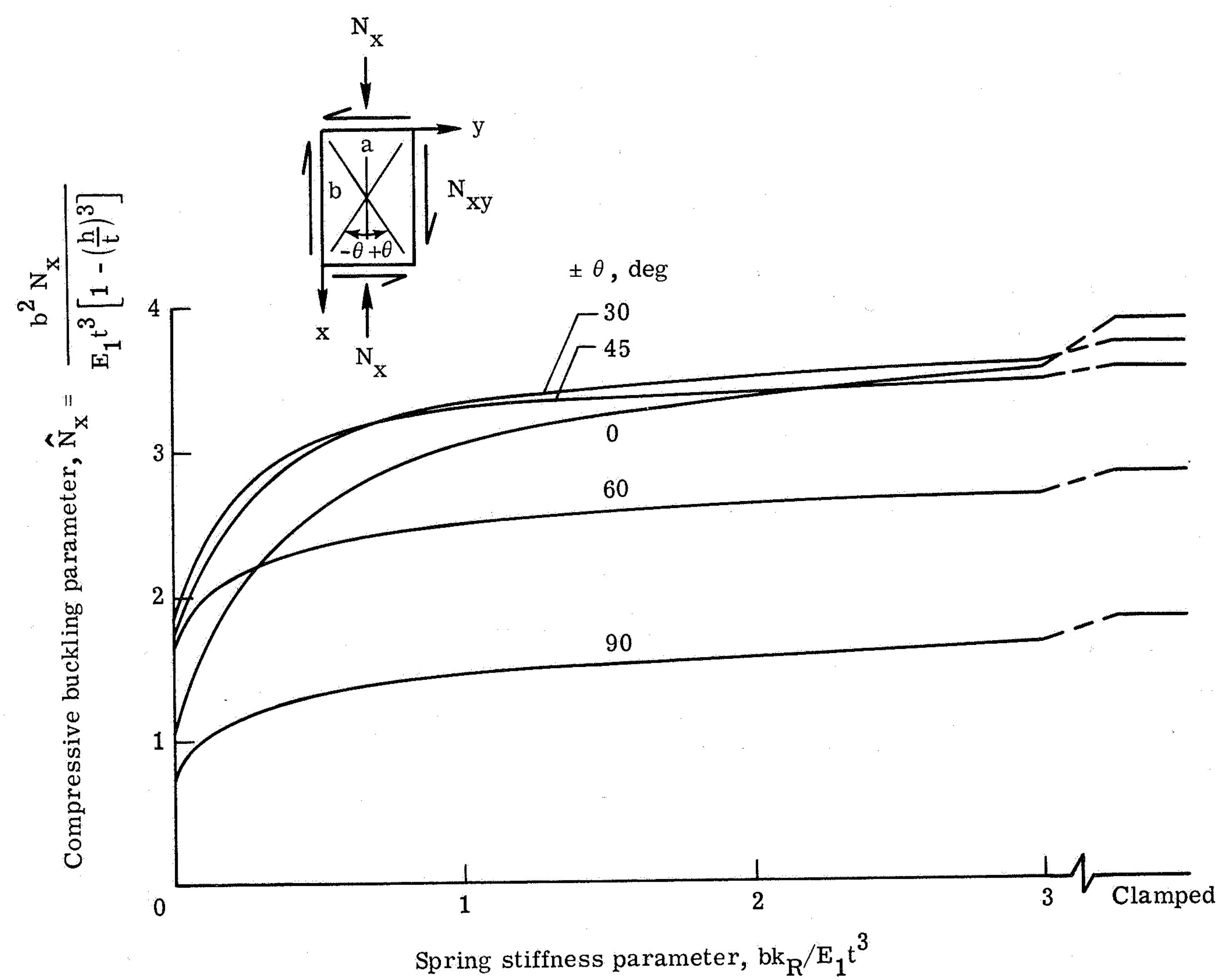
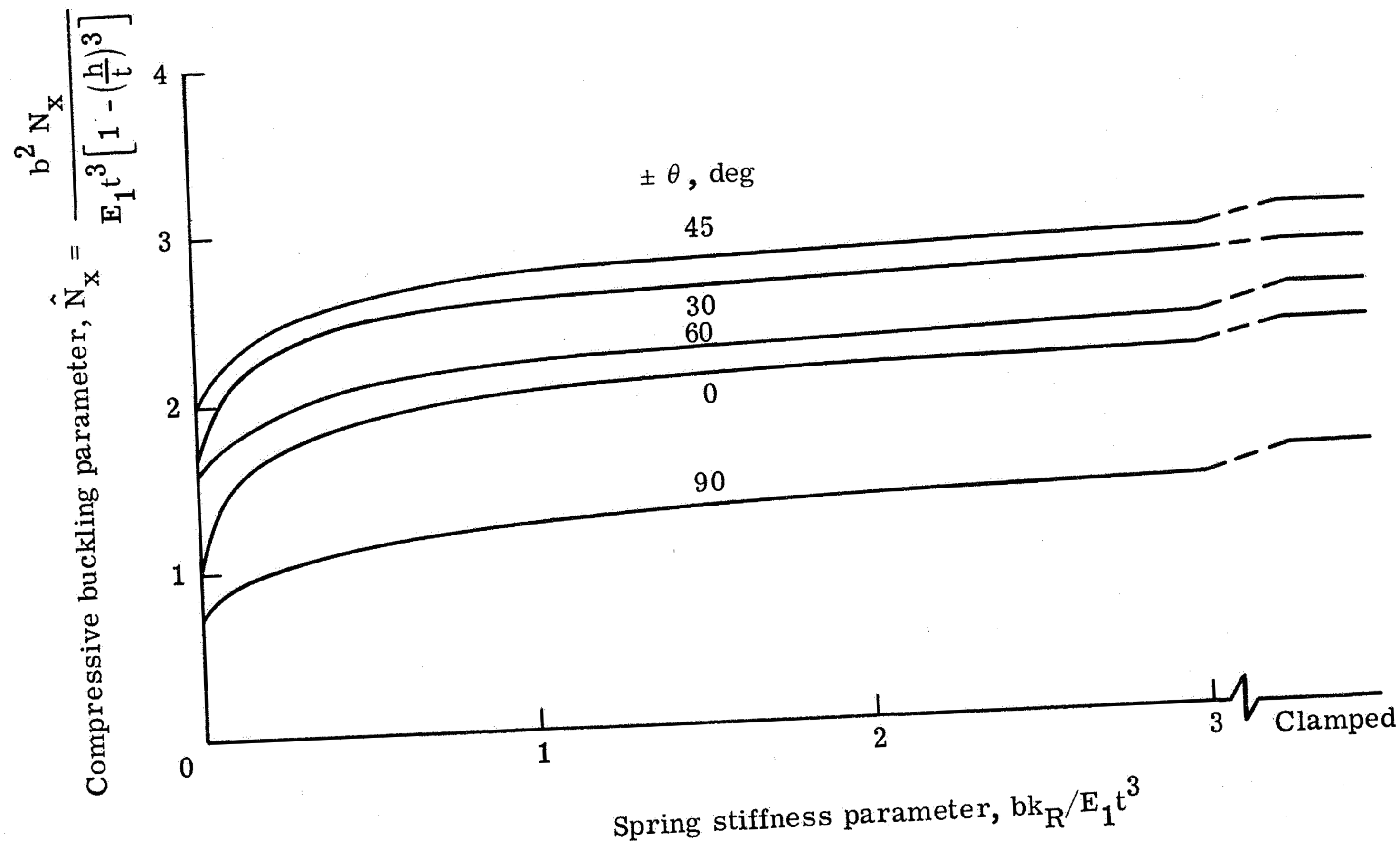



Figure 14.- Summary of combined axial compression and shear-buckling results for simply supported and clamped panels.



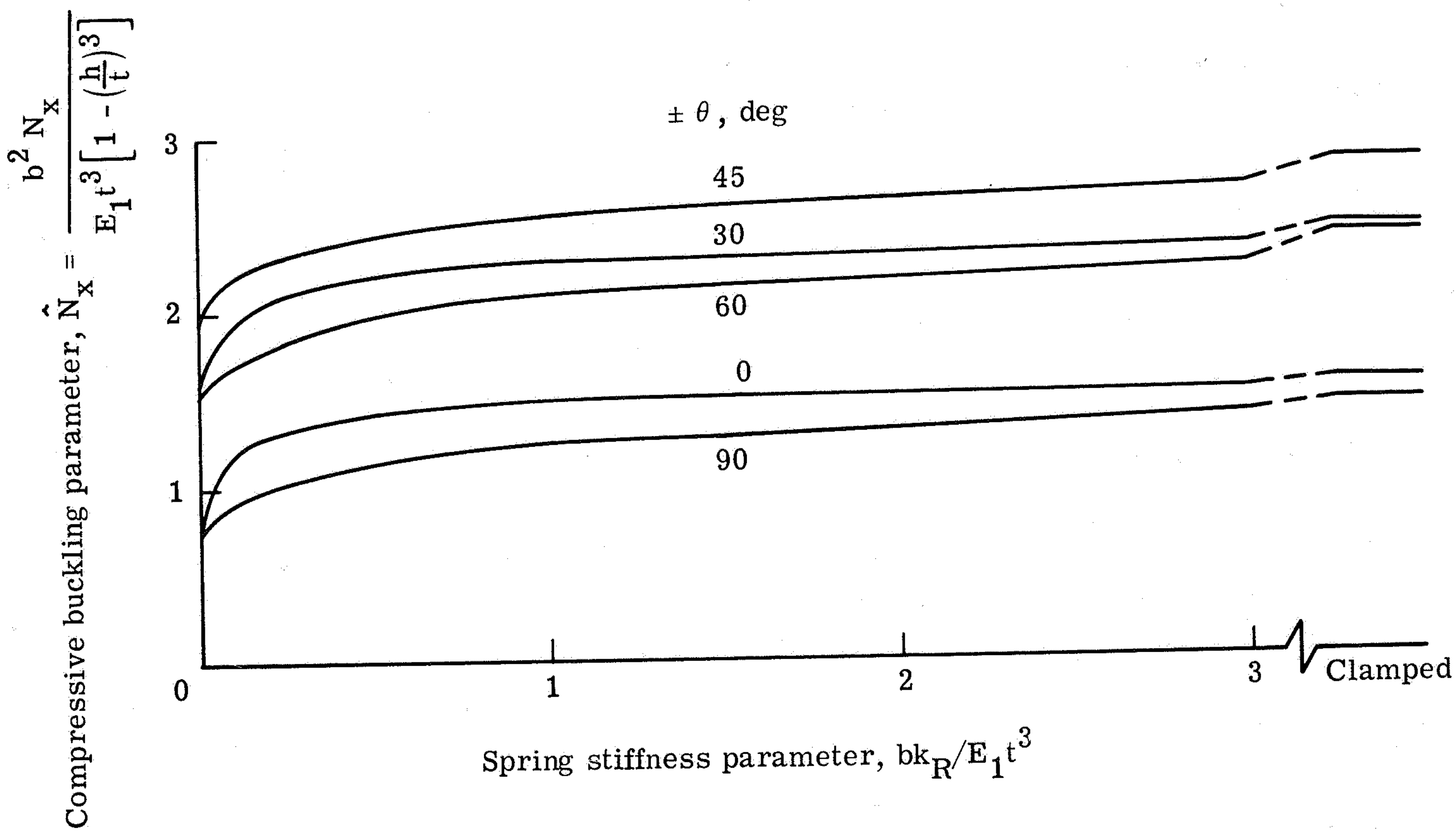

(a) a/b = 1.

**Figure 15.- Variation of compressive buckling parameter with rotational edge spring stiffness.**



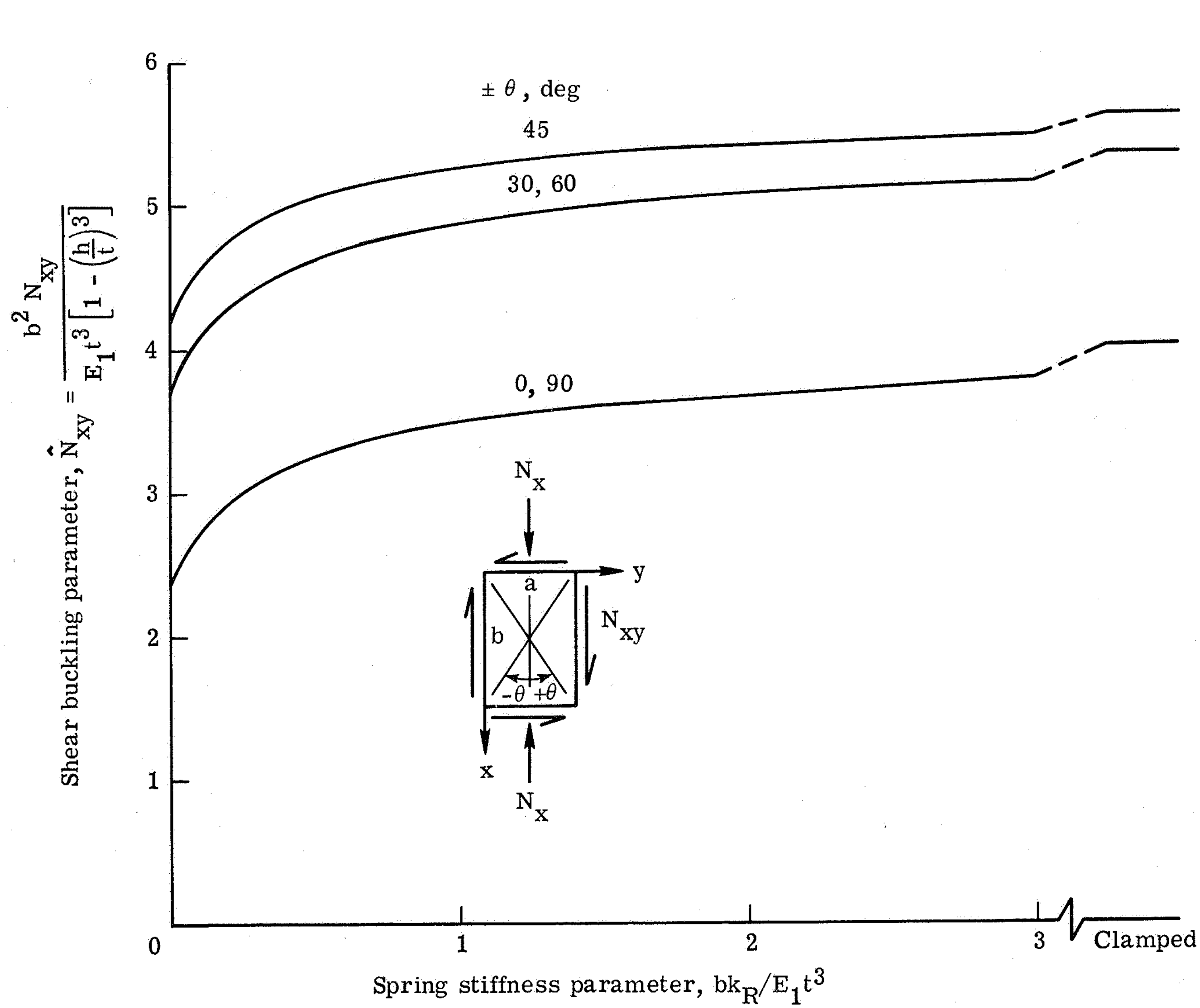

(b) a/b = 2.

Figure 15.- Continued.



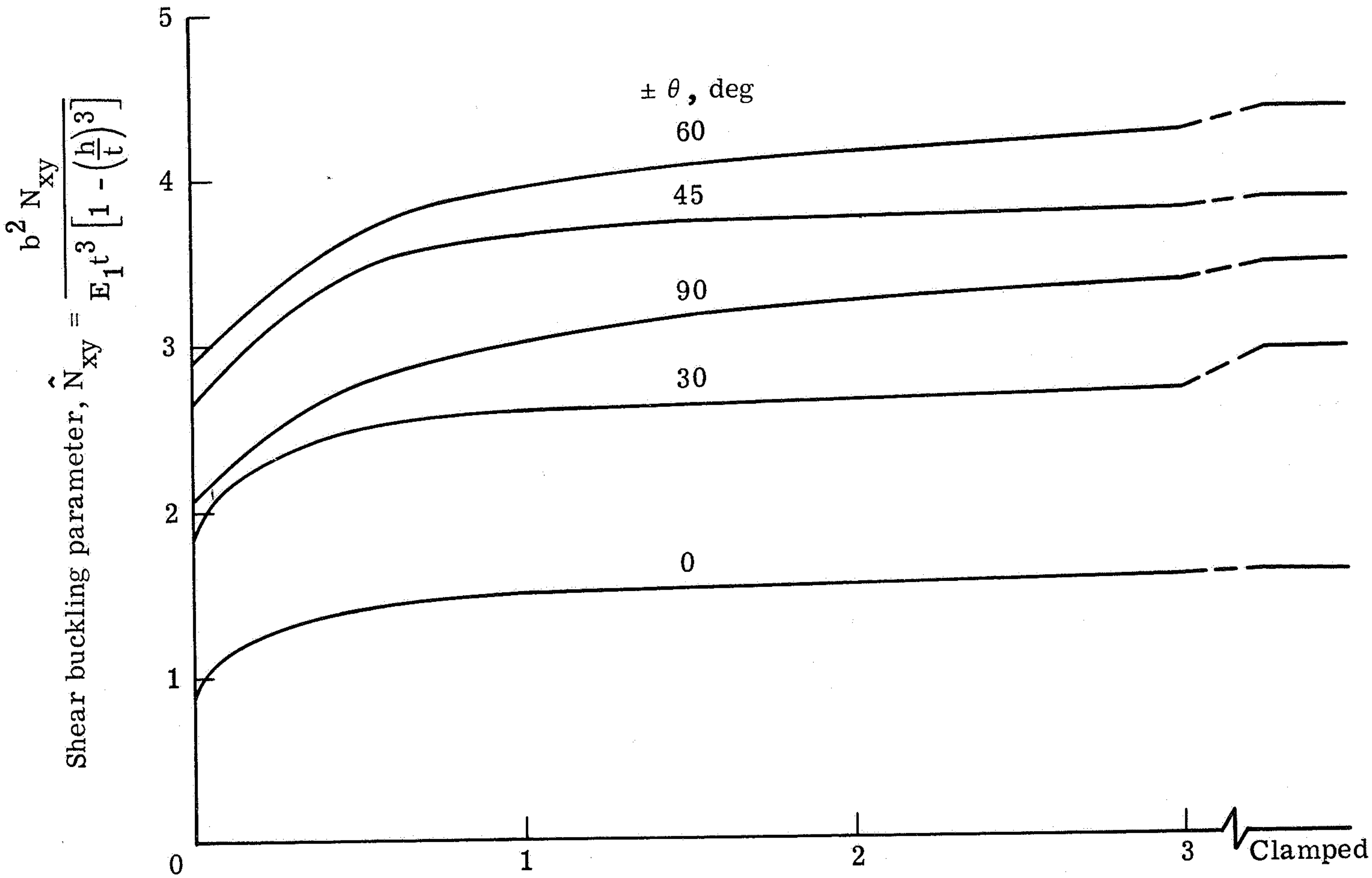

(c) a/b = 5.

Figure 15.- Concluded.



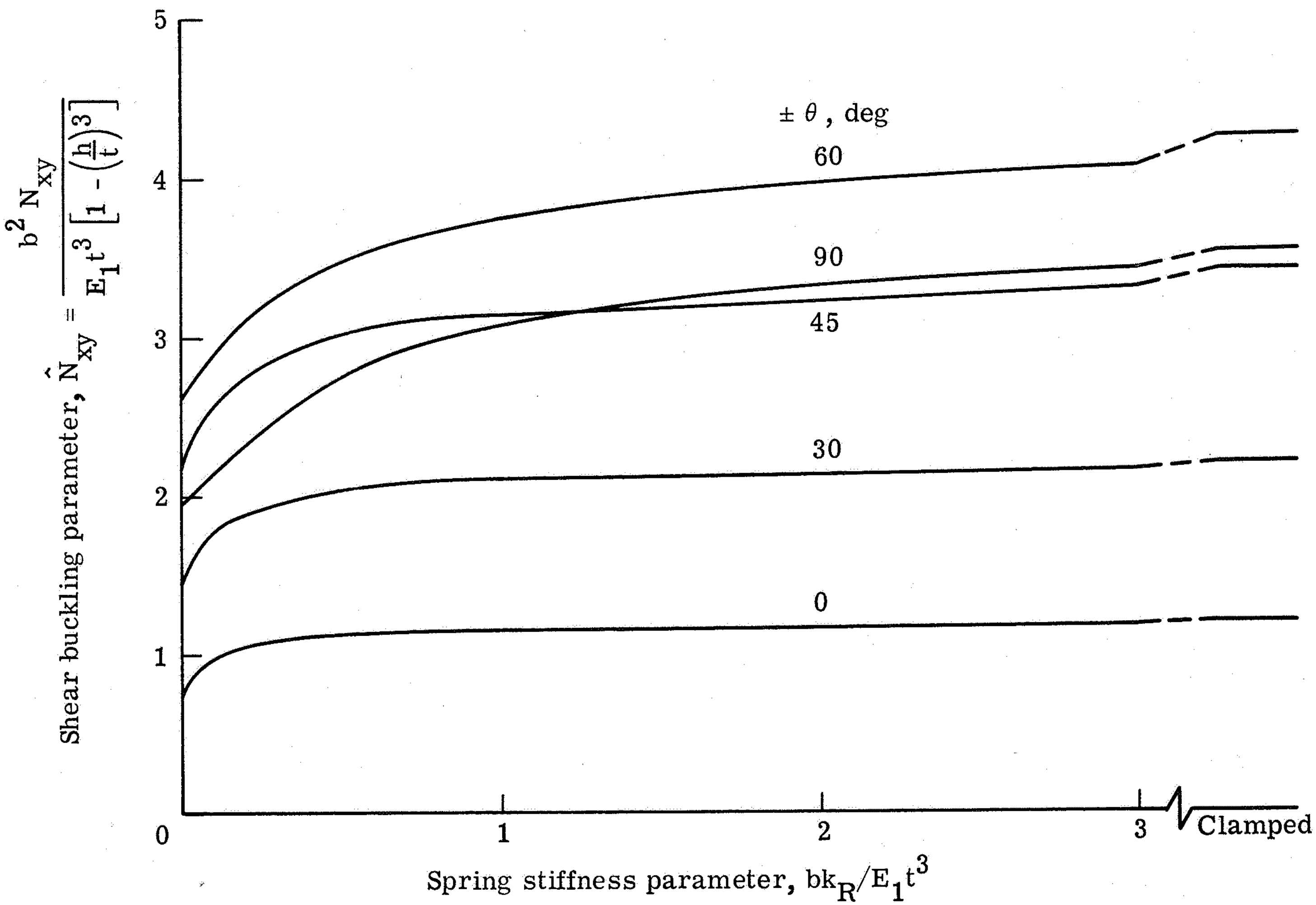

(a) a/b = 1.

Figure 16.- Variation of shear buckling parameter with rotational edge spring stiffness.



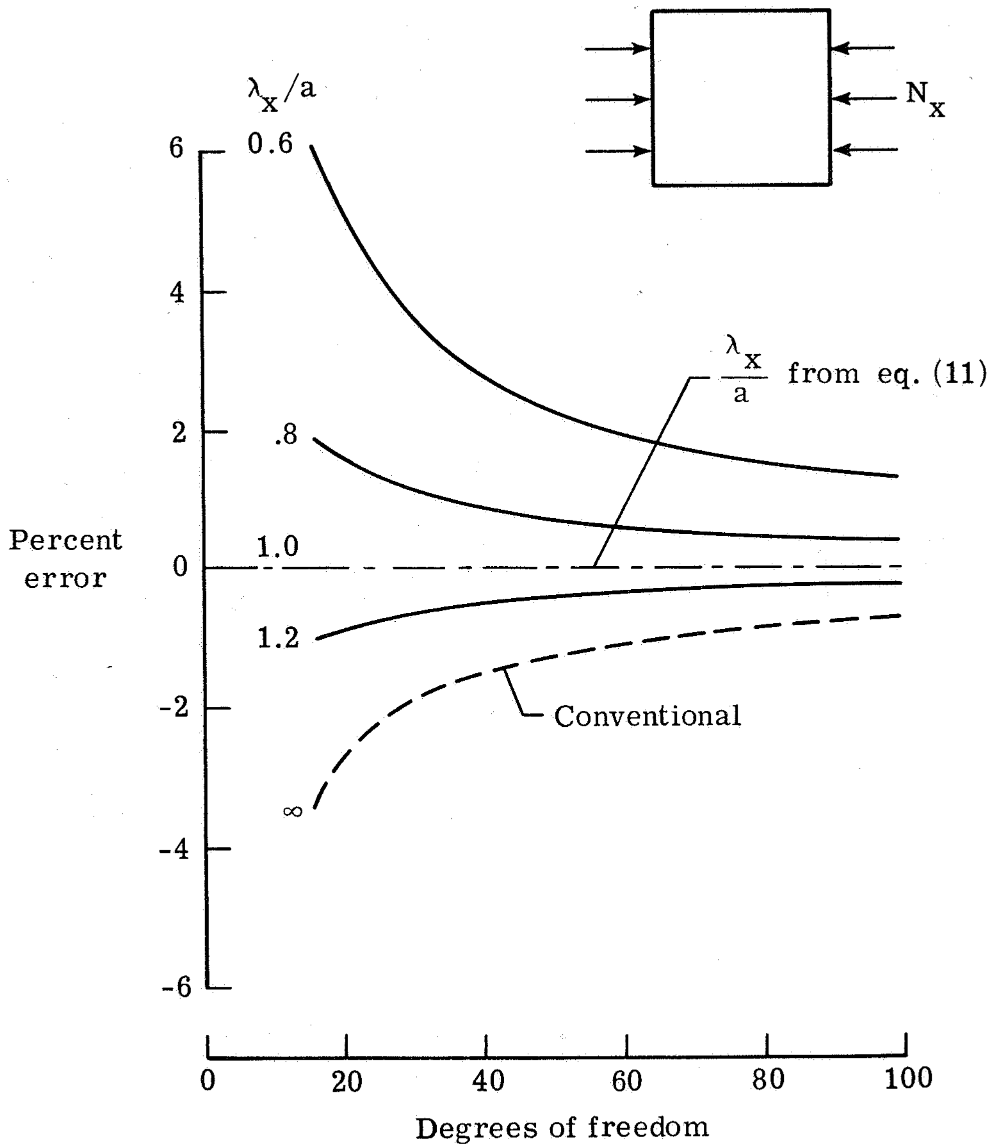

(b) a/b = 2.

Figure 16.- Continued.



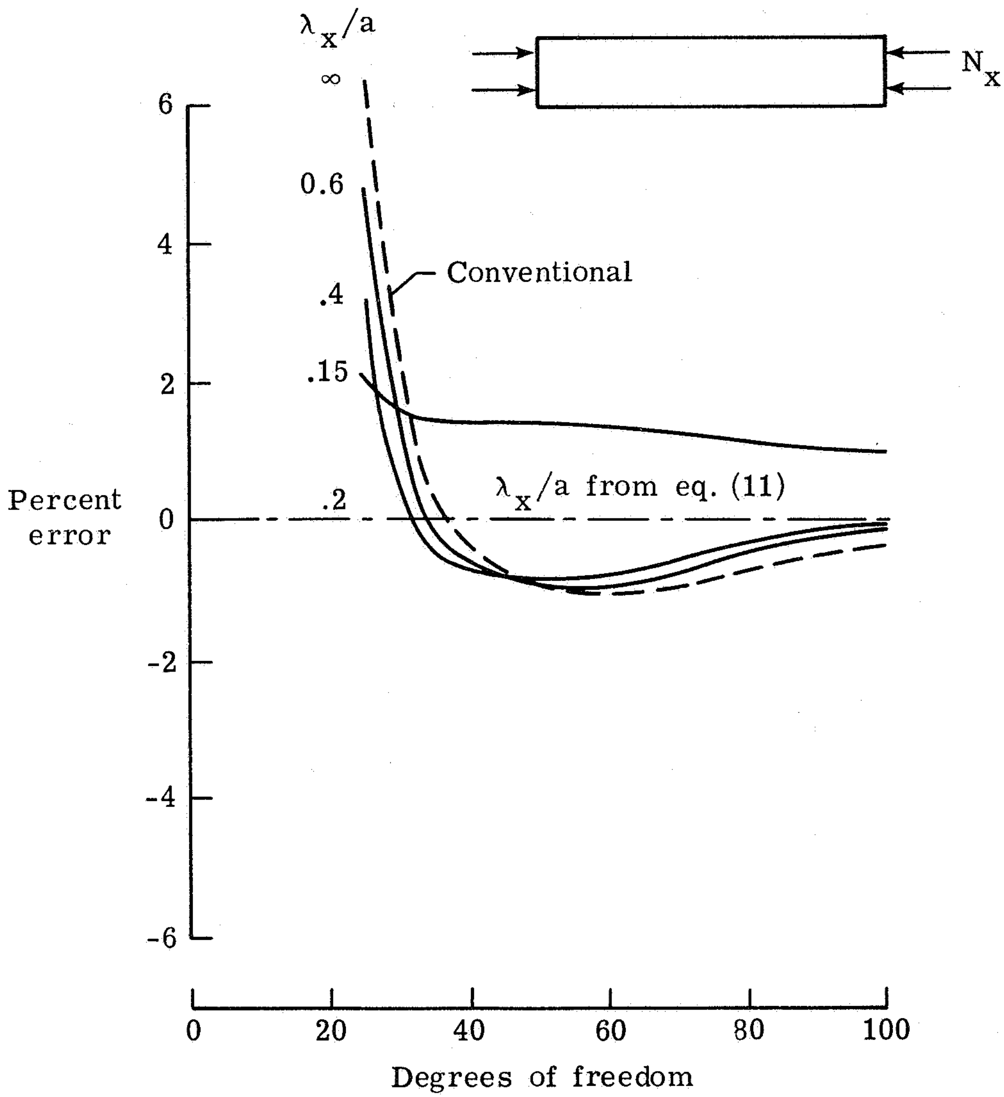

(c) a/b = 5.

Figure 16.- Concluded.



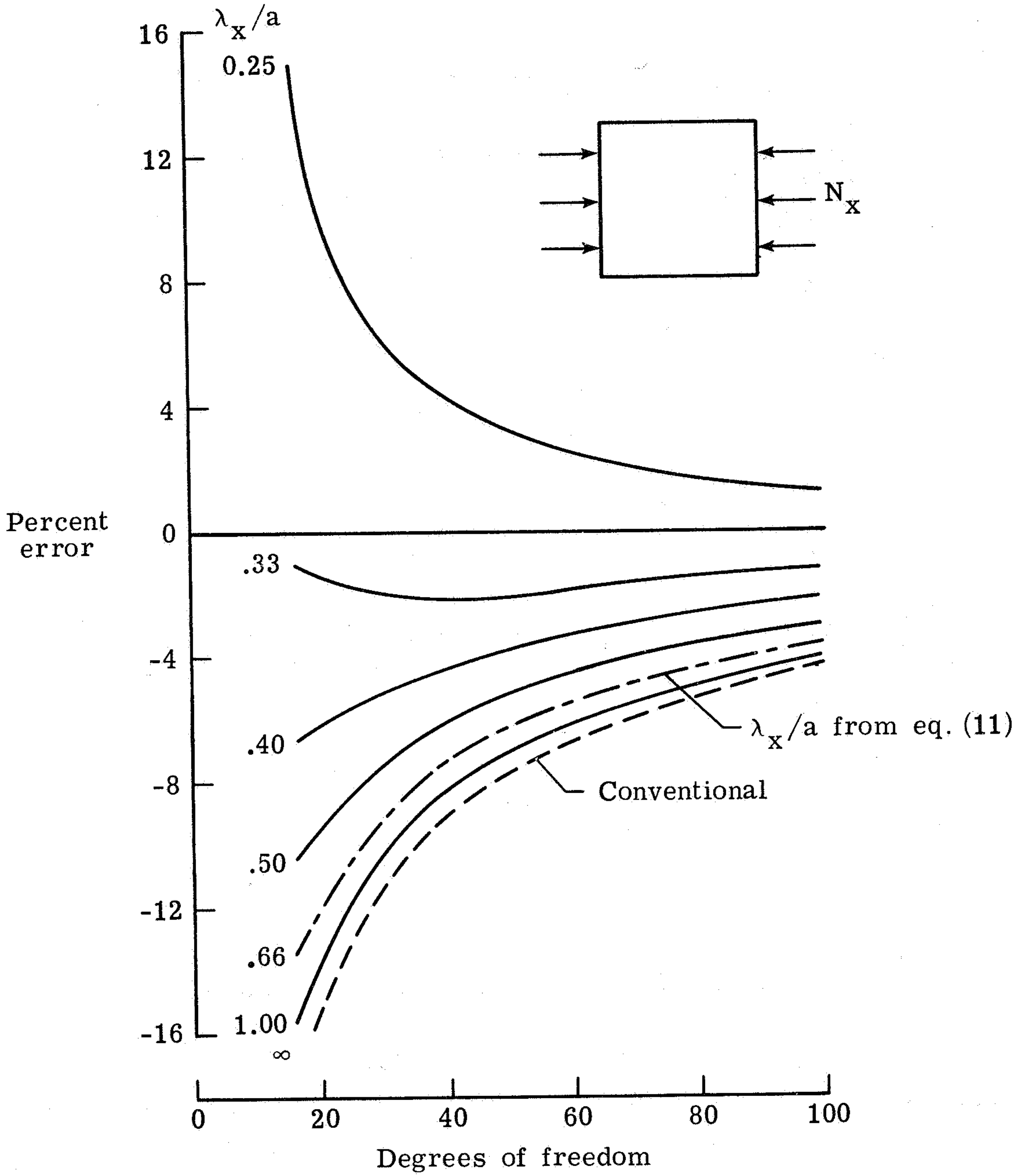

(a) Convergence for the compressive buckling of simply supported isotropic square panels. $\lambda_y/\lambda_x = \beta = 1$. From reference 6, $\overline{N}_x = 4.0$.

Figure 17.- Convergence characteristics of trigonometric finite differences.



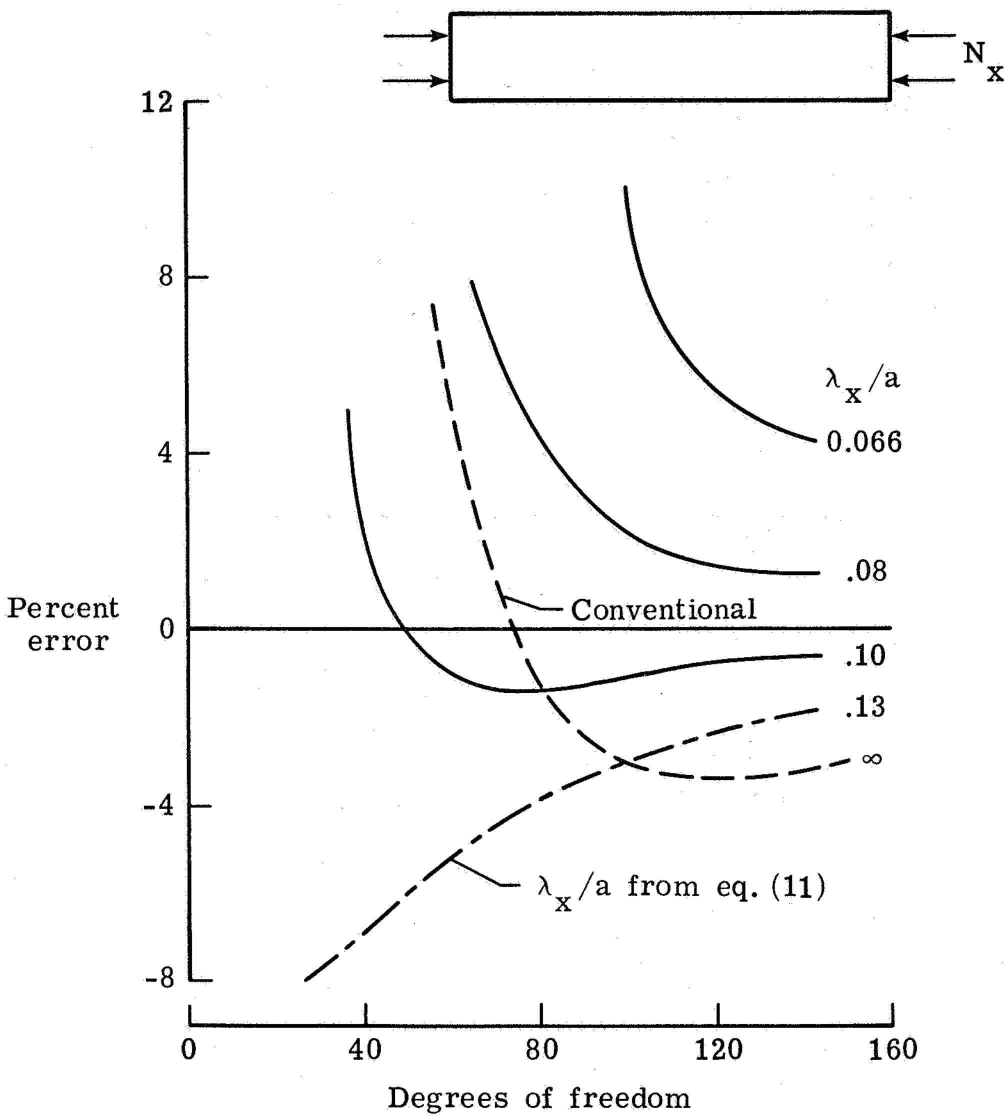

(b) Convergence for the compressive buckling of 5 × 1 simply supported isotropic panels. $\lambda_y/\lambda_x = \beta = 5$. From reference 6, $\overline{N}_x = 4.0$.

Figure 17.- Continued.



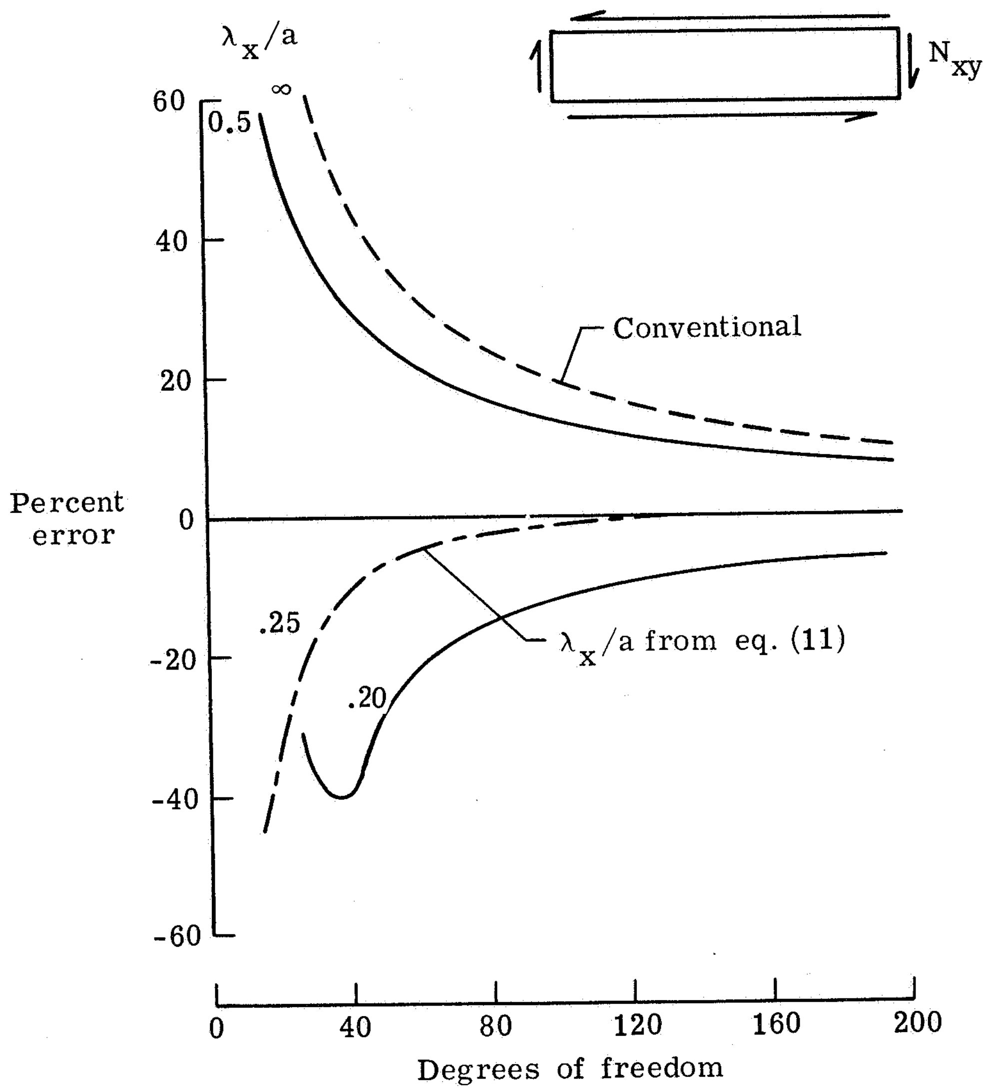

(c) Convergence for the compressive buckling of clamped square isotropic panels. $\lambda_y/\lambda_x = \beta = 1.5$. From reference 17, $\overline{N}_x \approx 10.074$.

Figure 17.- Continued.



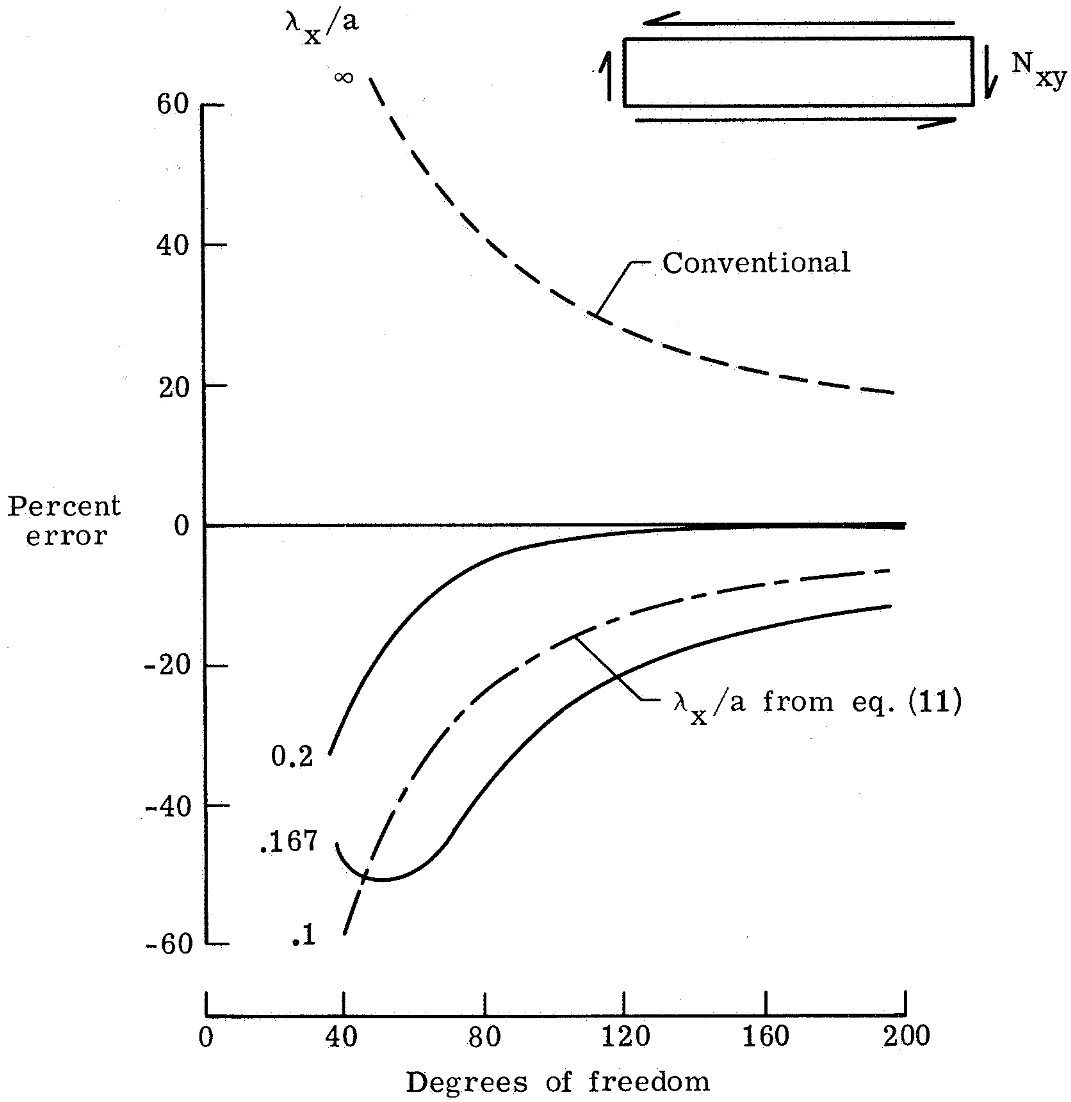

(d) Convergence for the compressive buckling of isotropic clamped 5 × 1 panels. $\lambda_y/\lambda_x = \beta = 1.5$. From reference 16, $\overline{N}_x \approx 7.0$.

Figure 17.- Continued.




(e) Convergence for the shear buckling of isotropic simply supported 5 × 1 panels.
$\lambda_y/\lambda_x = \beta = 0.8$. From reference 1, $\overline{N}_{xy} \approx 5.55$.

Figure 17.- Continued.




(f) Convergence for the shear buckling of isotropic clamped $5 \times 1$ panels. $\lambda_y/\lambda_x = \beta = 1.2$. From reference 1, $\overline{N}_{xy} \approx 9.3$.

Figure 17.- Concluded.

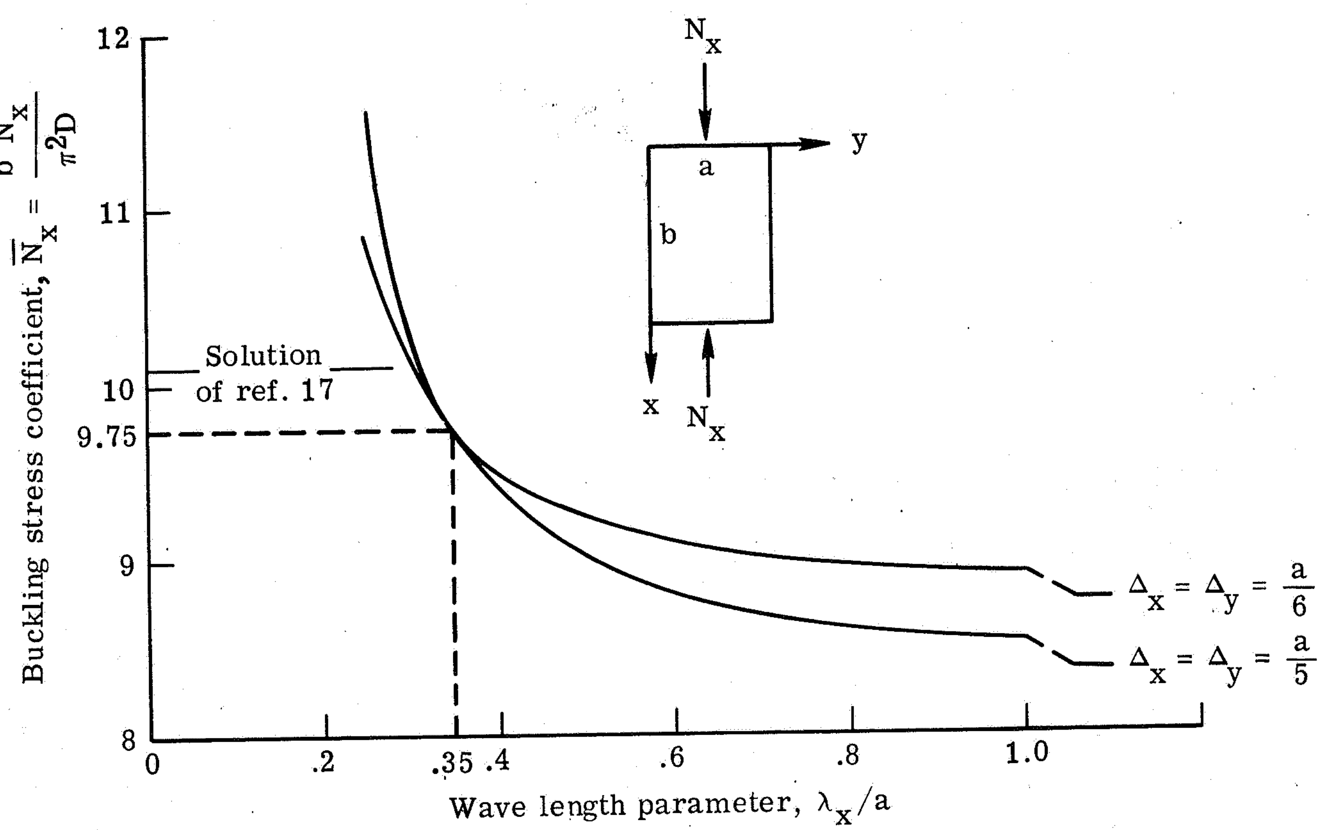



Figure 18.- Variation of buckling stress coefficient $\overline{N}_x$ with trigonometric wavelength parameter $\lambda_x/a$ for the compression buckling of a clamped square isotropic panel.

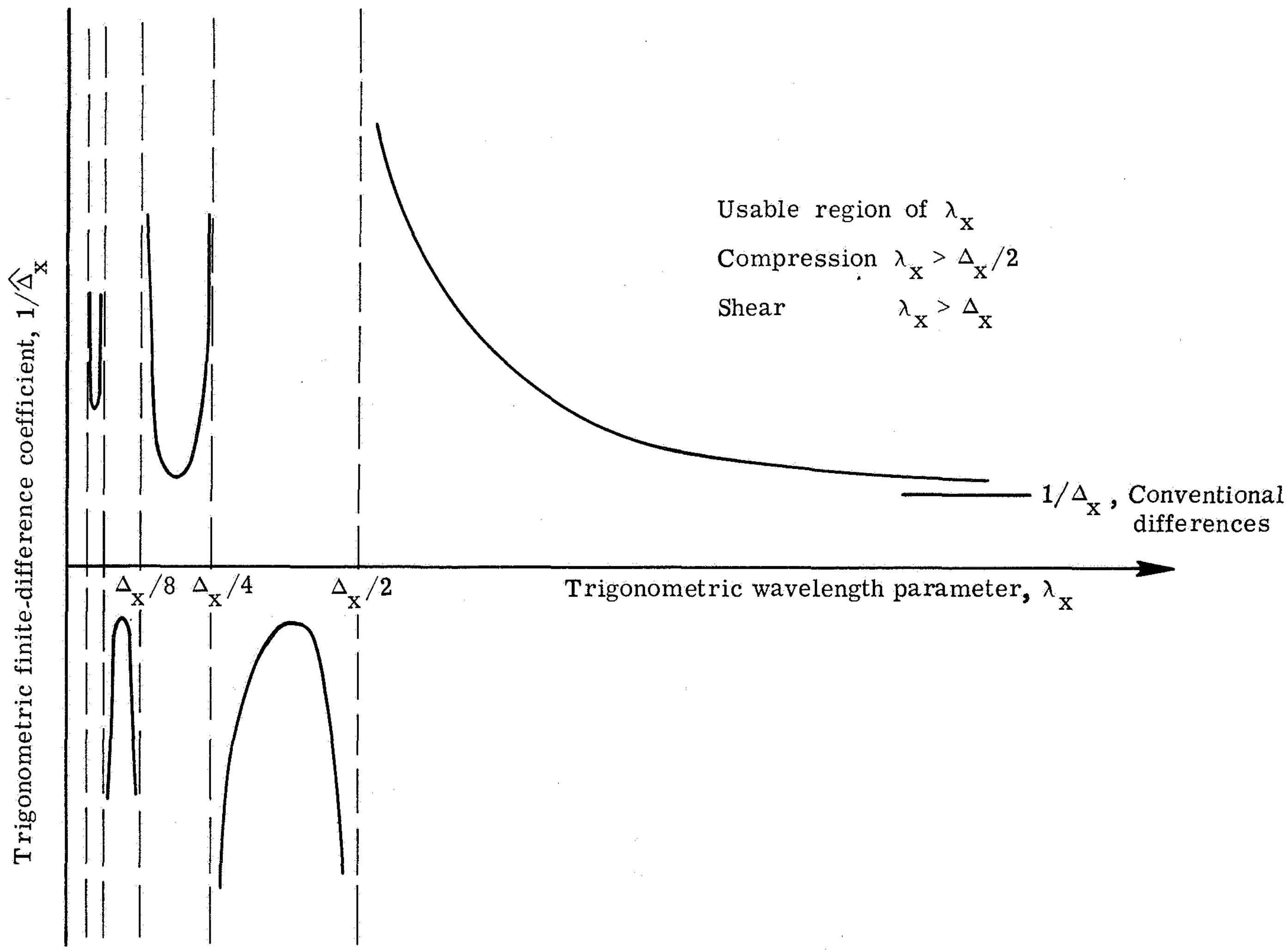



Figure 19.- Variation of $1/\hat{\Delta}_x$ with trigonometric wavelength parameter $\lambda_x$.



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