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~~A~~ PASSIVE GUST ALLEVIATION SYSTEM FOR A LIGHT AIRCRAFT, FINAL REPORT

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16. Abstract

A passive aeromechanical gust alleviation system has been examined for application to a Cessna 172. The system employs small auxiliary wings to sense changes in angle of attack and to drive the wing flaps to compensate the resulting incremental lift. The flaps also can be spring loaded to neutralize the effects of variations in dynamic pressure. Conditions for gust alleviation are developed and shown to introduce marginal stability if both vertical and horizontal gusts are compensated. Satisfactory behavior is realized if only vertical gusts are absorbed; however, elevator control is effectively negated by the system. Techniques to couple the elevator and flaps are demonstrated to restore full controllability without sacrifice of gust alleviation.

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NOMENCLATURE

Symbol	Description
a	Gust spectrum multiplicative constant
$\bar{A}_{\alpha_g}$	Vertical gust forcing coefficient vector
$\bar{A}_{\delta}$	Flap forcing coefficient vector
$\bar{A}_{u_g}$	Horizontal gust forcing coefficient vector
AR	Aspect ratio
b_{aw}	Span of one auxiliary wing (m)
c	Mean aerodynamic chord of wing (m)
c_f	Chord of flap (m)
C_L	Lift coefficient at equilibrium
C_{L_t}	Lift coefficient of horizontal stabilizer, based on wing area
C_{L_α}	Lift curve slope of basic aircraft
$C_{L_{\alpha_w}}$	Lift curve slope of wing
$C_{L_{\alpha_t}}$	Lift curve slope of horizontal stabilizer, based on wing area
$C_{L_\alpha}^{aw}$	Auxiliary wing lift curve slope
$C_{L_{\dot{\alpha}}}$	Lift coefficent derivative due to downwash lag for basic aircraft
C_{L_δ}	Lift coefficient derivative with flap deflection
$C_{L_{\delta_w}}$	Tail off lift coefficient derivative with flap deflection
$C_{L_{\dot{\delta}}}$	Lift coefficient derivative due to flap-induced downwash lag
C_{L_q}	Lift coefficient derivative with normalized pitching rate

$C_{L_{\dot{u}_g}}$	Lift coefficient derivative with normalized horizontal gust acceleration for basic aircraft
$C_{L_{\delta_e}}$	Lift coefficient derivative with elevator deflection
$C^{aw}_{L_{\delta_t}}$	Auxiliary wing lift coefficient derivative with auxiliary wing tab deflection
C_{m_α}	Pitching moment coefficient with angle of attack for basic aircraft
$C_{m_{\alpha_w}}$	Tail-off pitching moment coefficient derivative with angle of attack for basic aircraft, measured at center of gravity
$C_{m_{\dot{\alpha}}}$	Pitching moment coefficient derivative due to downwash lag for basic aircraft
C_{m_δ}	Pitching moment coefficient derivative with flap deflection, measured at center of gravity
$C_{m_{\delta_w}}$	Tail-off pitching moment coefficient derivative with flap deflection, measured at center of gravity
$C^a_{m_{\delta_w}}$	Tail-off pitching moment coefficient derivative with symmetric aileron deflection, measured at center of gravity
$C_{m_{\delta_e}}$	Pitching moment coefficient derivative with elevator deflection
C_{m_q}	Damping in pitch coefficient
$C_{m_{\dot{u}_g}}$	Pitching moment coefficient derivative with normalized horizontal gust acceleration for basic aircraft
$C_{m_{\dot{\delta}}}$	Pitching moment coefficient derivative due to flap downwash lag
C_{x_α}	Longitudinal force coefficient derivative with angle of attack for basic aircraft
C_{x_u}	Longitudinal force coefficient derivative with normalized speed perturbation
C_{x_δ}	Longitudinal force coefficient derivative with flap deflection
$C_{x_{\delta_e}}$	Longitudinal force coefficient derivative with elevator deflection
C_{H_r}	Flap hinge restraining moment coefficient required for horizontal gust alleviation

C_{H_α}	Flap hinge moment coefficient derivative with angle of attack. Includes effect of auxiliary wing.
$C^f_{H_\alpha}$	Hinge moment coefficient derivative with angle of attack for flap alone
C_{H_δ}	Flap hinge moment coefficient derivative with flap deflection
$C_{H_{\delta_t}}$	Flap hinge moment coefficient derivative with flap tab deflection.
$C^f_{H_{\dot\delta}}$	Hinge moment damping coefficient due to flap alone
C_{H_q}	Flap hinge moment coefficient derivative with pitching rate
C_{D_δ}	Drag coefficient derivative with flap deflection
C_{T_u}	Thrust coefficient derivative with speed perturbation
d	Measure of high frequency pitching rate response
D/Dx	Total derivative with respect to x
G	Aileron-flap coupling ratio for analytic model
G'	Aileron-flap coupling ratio for test data model
H_{eq}	Applied flap hinge moment for equilibrium(N-m)
H_r	Flap hinge restraining moment required for horizontal gust alleviation (N-m)
H_δ	Flap hinge moment derivative with flap deflection(N-m/rad)
I_y	Normalized moment of inertia about pitch axis
I_{aw}	Normalized moment of inertia of auxiliary wing about its longitudinal hinge line
I^{ref}_{aw}	Reference normalized auxiliary wing moment of inertia
I_f	Normalized flap moment of inertia
$I_{f,aw}$	Normalized flap-auxiliary wing system moment of inertia
i_{aw}	Incidence of auxiliary wing
K_a	Vertical gust alleviation gain constant
K^f_a	Vertical gust alleviation gain constant if no auxiliary wing is employed

K_v	Horizontal gust alleviation gain constant
L	Arbitrary size of auxiliary wing
$\mathcal{L}$	Scale length of atmospheric turbulence spectrum (m)
ℓ	Distance between wing and stabilizer aerodynamic centers in mean aerodynamic chords
m	Mass of aircraft (Kg)
P(0)	Zero degree coefficient of characteristic equation
q	Normalized pitching rate; also dynamic pressure (N/m^2)
r	Radius of spring linkage from hinge line (m)
S	Wing area (m^2)
S_t	Horizontal stabilizer area (m^2)
S_{aw}	Auxiliary wing area (one wing) (m^2)
S_f	Flap area (one flap) (m^2)
SM	Static margin
t	Thickness of auxiliary wing components (m)
u	Normalized speed perturbation
u_g	Normalized horizontal gust velocity
V	Aircraft velocity (m/sec)
W	Aircraft weight (N)
α	Angle of attack perturbation
α_g	Angle of attack perturbation due to vertical gust
β	Angle between flap linkage radius and flap datum
Γ	Auxiliary wing to flap gearing ratio, $\partial\delta_{aw}/\partial\delta$
γ	Flight path angle
$\hat{\gamma}$	Normalized rate of change of flight path angle
δ	Flap deflection
δ_{aw}	Auxiliary wing deflection about longitudinal hinge line
δ_e	Elevator deflection
δ_t	Flap or auxiliary wing tab deflection
Δn	Load factor (g)

$\partial\varepsilon/\partial\alpha$	Downwash derivative with angle of attack for basic aircraft
$\partial\varepsilon/\partial\delta$	Downwash derivative with flap deflection
ζ	Damping ratio
θ	Pitch attitude
μ	Normalized mass, $2m/\rho Sc$
ρ	Air density (Kg/m^3)
σ	Root mean square gust velocity (m/sec)
Φ_q	Power spectral density of pitching rate
Φ_{w_g}	Power spectral density of vertical gusts
Φ_n	Power spectral density of load factor
Ω	Spacial frequency (rad/m)
ω	Temporal frequency (rad/sec)
ω_n	Normalized natural frequency
ω_{n_f}	Normalized natural frequency of free-floating flap
ω_{n_p}	Normalized phugoid natural frequency
$\omega_{n_{sp}}$	Normalized short-period natural frequency

Superscripts

'	Pertains to alleviated aircraft

Subscripts

f	Applies to flap
aw	Applies to auxiliary wing

I. INTRODUCTION

The study has been undertaken to determine the feasibility of improving ride comfort in a light aircraft in cruise without having to resort to the use of relatively expensive servomechanisms. The system hopefully will be attractive to industry to offer as a low cost option and, perhaps, even as a retrofit package to present aircraft. The major design considerations for different aircraft center on modifying the flap design and ascertaining flap hinge moment characteristics, from which the size of the auxiliary wings can be prescribed.

When attempting gust alleviation, there are several basic problems encountered which lead to distinct design and performance compromizes. Alleviation implies maintaining a constant load factor which is normally influenced by vertical and longitudinal gust components. At low cruise lift coefficients, loads due to vertical gusts predominate by a significant margin and horizontal gusts can be neglected. This would not be the case, for instance, of a STOL aircraft taking off or landing. Compensating both gust components introduces some interesting stability problems which are discussed briefly later. Alleviating vertical gusts corresponds to maintaining a constant lift coefficient. Consequently, the lift curve slope is zero and the damping in pitch is reduced.

The most practical way to manipulate the lift coefficient is with trailing edge flaps. Although their linear range of operation is limited, they nevertheless can compensate gusts up to about 3 m/sec which covers most of the turbulence commonly encountered. However, unfavorable pitching moment and downwash characteristics are introduced by these devices. The increased camber produces a nose-down pitching moment and the modified downwash pattern has a strong influence on the downwash derivative with respect to angle of attack at the tail. Although for normal inboard flaps the latter effect somewhat offsets the pitching moment due to camber, the net result is a sharp reduction in pitch stiffness (C_{m_α}). Downwash lag also is affected, and results in lower damping in pitch and amplified pitching response to gusts. One cannot design a flap only system which will maintain a constant load factor and zero pitching rate response while providing adequate stability.

There is one more difficulty associated with the dynamic behavior of gust-alleviated aircraft which, fortunately, can be circumvented by proper design. Because elevator control is effective through the influence of the pitching moment on angle of attack, and because the lift curve slope of the alleviated aircraft is zero, the aircraft is not controllable in the normal sense. The flight path angle will not respond satisfactorily to elevator movement. Some means of overriding the system must be provided to restore maneuverability.

The basic concepts of gust alleviation are not new. In 1938 Rene Hirsch[1] of France conducted model tests of a flap-type alleviation system which employed the horizontal stabilizer as an angle of attack sensor. These surfaces were articulated on a longitudinal axis and coupled to the flaps. Hirsch was the first to recognize the need to concentrate on long wavelength absorption and was not concerned with the sensor lag of this configuration. His interest continued after World War II when he built a small twin-engined aircraft using these concepts. He attempted to absorb both vertical and horizontal gusts, but found it necessary to avoid the zero root attendant this system by employing altitude rate coupling through a variometer to the elevator. His choice of mechanization was rather ingeneous in that it automatically bypassed the alleviation system and provided the maneuverability associated with a conventional aircraft.

In the early 1950's, Phillips and Kraft[2] investigated the effects on the short period dynamics of introducing a vane-type angle of attack sensor mounted on the nose of a twin-engined aircraft and driving the flaps and elevator through a hydraulic servo. They developed the relationships identifying the flap characteristics required to provide vertical gust alleviation and zero pitching response and recognized the degeneration of the short period roots to a real pair, one being zero and the other having a very short time constant. In addition they pointed out the difficulties of designing a flap system to achieve the specified conditions and suggested the possibility of using a segmented flap to help meet the downwash requirements. The concept was applied to a full scale aircraft[3] which demonstrated the feasibility of adjusting the flap downwash to improve the pitching response of the alleviated aircraft. Maneuverability not only was recovered, but was improved by coupling the stick to the flap actuator, thereby overriding the alleviation system and providing direct lift control. In a recent report by Phillips[4], the early analysis was extended to include horizontal gust alleviation and its effect on the phugoid mode. He confirmed the stability problems introduced by such a system as first recognized by Hirsch. He further examined the horizontal accelerations resulting from variations in drag due to the flap as it responds to the gusts. In a numerical example of a STOL aircraft, these accelerations amounted to as much as sixty percent of the values for the vertical accelerations which would be experienced by the unalleviated aircraft. A method of coupling spoilers to the flaps was suggested as a means for reducing the net drag due to flap deflection, thereby diminishing the source of these accelerations.

The present system under consideration is forced to compromize the optimum alleviation conditions not only to maintain stability and handling parameters, but also to work within the framework of system objectives, namely, to make use of existing control surface regions. The flaps must be redesigned to permit upward deflection; a plain, mass-balanced flap with hingeline at twenty percent chord is substituted. Furthermore, while analysis for both vertical and horizontal gust alleviation is presented, only the former is proposed for a conventional light aircraft.

The system is analyzed for the Cessna 172 aircraft. The angle of attack sensors are chosen to be two auxiliary wings, large enough to drive the flaps directly and located ahead of the cargo compartment door so as not to impede entry to the cabin. This concept was originated by W. H. Phillips as an alternative to the Hirsch method of using the empennage as the gust sensor. These wings are mass balanced and are articulated in roll to deflect the flaps and in pitch for coupling to the elevator in order to regain basic aircraft maneuverability. Elevator coupling with the flaps is not considered necessary to reduce pitching motions (although it could be required for other aircraft); complication of the control stick - elevator interface is thereby avoided.

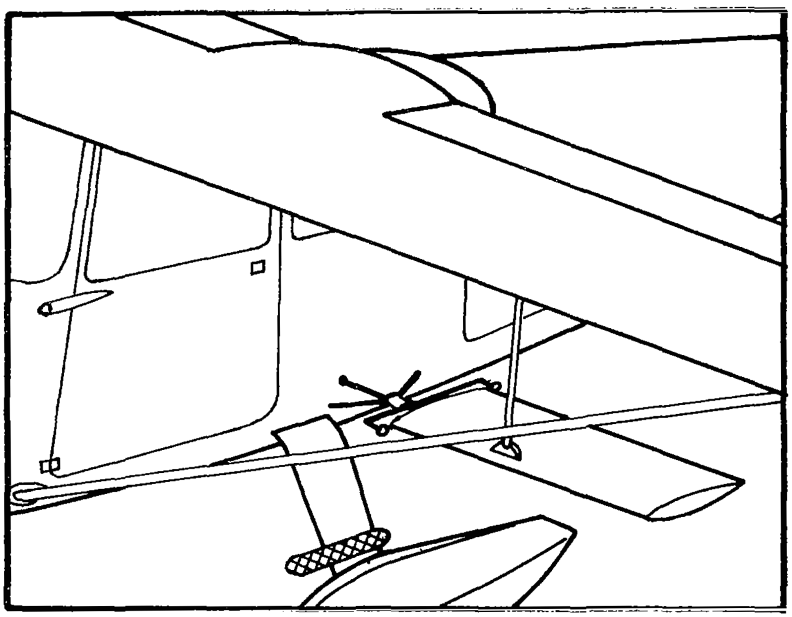


Figure 1. Auxiliary Wing Installation

The bulk of the analysis presented here is predicated on several fundamental assumptions regarding the behavior of the aircraft. These are:

1. The aircraft is a rigid body.
2. Unsteady aerodynamic effects can be neglected.
3. The flap operates only in its linear lift curve region.
4. The equations of motion are linear in the perturbation variables.
5. The auxiliary wing-flap system responds instantaneously compared to the rigid body motions.

The first two are justified by considering the spectra of gusts; the power is negligible at frequencies much higher than that of the short-period mode. The third and fourth merely impose limits on the region of validity for the analysis. The fifth is made to simplify the analysis and to identify the capability and limitations of an idealized flap-auxiliary wing system whose dynamic characteristics do not influence the gust response of the aircraft. Actual responses for a wide range of flap system dynamic parameters are presented to call attention to the importance of carefully selecting system components to achieve satisfactory response characteristics.

If the flap characteristics are considered to be linear over a range of fifteen degrees, they can nullify an incremental lift corresponding to about three degrees of change in angle of attack. At cruise speed, this value translates into a vertical gust of about three meters per second.

II. SYSTEM DESIGN

The auxiliary wings must be large enough to provide the power necessary to drive the flaps but care must be exercised not to overalleviate the gusts since this would lead to instability. While it is recognized that theoretical estimates of flap hinge moment parameters involve substantial uncertainty and whenever possible, should be replaced by experimental data from full-scale wind tunnel tests or actual flight information, they nevertheless permit sizing estimates.

The steady-state response of the flap to gusts can be described by a static hinge moment equation in which moments due to gusts are balanced by that developed by a flap deflection.

$$C_{H_\delta}\delta + C_{H_\alpha}\alpha_g - 2C_{H_r}u_g = 0 \qquad (1)$$

The last term results from spring loading the flaps which creates a torque independent of flap deflection and dynamic pressure. The required hinge moment coefficient derivative with angle of attack can be formulated from the first two terms of equation (1) and equation (E10) which defines the relation between flap deflection and gust induced angle of attack for optimal gust alleviation.

$$C_{H_\alpha} = C_{H_\delta}\frac{C_{L_\alpha}}{C_{L_\delta}} \qquad (2)$$

If a pushrod is used it is attached to the auxiliary wing quarter chord line at the point midway along the span to neutralize the aerodynamic loads sustained by the hinge supports. For small amplitude motions the auxiliary wing lift is independent of flapping angle. Consequently the only flap parameter to be substantially modified by the coupling is the hinge moment sensitivity to angle of attack. In terms of flap and auxiliary wing parameters

$$C_{H_\alpha} = C^{f}_{H_\alpha} - \frac{1}{2}\frac{\partial\delta_{aw}}{\partial\delta}\frac{(Sb)_{aw}}{(Sc)_f} C^{aw}_{L_\alpha} \tag{3}$$

In view of its small size the auxiliary wing contribution to C_{L_α} is assumed negligible.

Equations (1) and (2) can be combined and solved for the span of the auxiliary wing, if the aspect ratio and airfoil section characteristics are known. An aspect ratio of 2.0 and an NACA 0009 airfoil yield a lift curve slope of 2.54. (Ref. 9) The gearing ratio between the flap and auxiliary wing was selected to be unity. The resulting wing span is 1.13 m. A larger gearing ratio would lead to a smaller auxiliary wing size but larger deflections (δ_{aw}).

Horizontal gust alleviation is provided by loading the flaps to achieve the sensitivity specified in Equation (E-20).

$$H_r = -\frac{W}{S}(Sc)_f \frac{C_{H_\delta}}{C_{L_\delta}} \tag{4}$$

If a spring is used to realize this moment, it effectively must have zero stiffness to avoid changes in torque due to flap deflection, yet must be able to be set to the torque level specified by the weight of the aircraft, which varies during flight.

In equilibrium flight, the flap deflection can be trimmed to zero by adjusting the incidence of the auxiliary wing to counteract the normal floating tendency of the flap. Thus, engaging the gust alleviation system would not affect the aircraft trim condition. If horizontal gust alleviation is not employed, this incidence setting is independent of airspeed.

III. MODIFIED AIRCRAFT DYNAMICS

A relatively simple set of equations of motion can be used to investigate the dynamic behavior of a gust-alleviated aircraft. The usual longitudinal equations of motion are augmented with the flap-auxiliary wing system hinge moment equation.

$$\begin{bmatrix} C_{x_u} - 2\mu s & C_{x_\alpha} + C_L & -C_L & C_{x_\delta} \\ 2C_L & C_{L_\alpha} + (2\mu + C_{L_{\dot\alpha}})s & (C_{L_q} - 2\mu)s & C_{L_\delta} + C_{L_{\dot\delta}} s \\ 0 & C_{m_\alpha} + C_{m_{\dot\alpha}} s & (C_{m_q} - I_y s)s & C_{m_\delta} + C_{m_{\dot\delta}} s \\ -2C_{H_r} & C_{H_\alpha} & C_{H_q} s & -I_{f,aw} s^2 + C_{H_{\dot\delta}} s + C_{H_\delta} \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \theta \\ \delta \end{bmatrix} =$$

$$- \begin{bmatrix} C_{x_{\delta_e}} \\ C_{L_{\delta_e}} \\ C_{m_{\delta_e}} \\ 0 \end{bmatrix} \delta_e - \begin{bmatrix} C_{x_\alpha} + C_L \\ C_{L_\alpha} + (C_{L_{\dot\alpha}} - C_{L_q})s \\ C_{m_\alpha} + (C_{m_{\dot\alpha}} - C_{m_q})s \\ C_{H_\alpha} \end{bmatrix} \alpha_g - \begin{bmatrix} C_{x_u} \\ 2C_L + C_{L_{\dot u_g}} s \\ C_{m_{\dot u_g}} s \\ -2C_{H_r} \end{bmatrix} u_g \qquad (5)$$

To simplify the analysis and to focus attention on the modification of the rigid-body modes introduced by the gusts alleviation system, the flaps are assumed to operate fast compared with the rigid-body motions and the flap equation can be reduced to a static balance similar to that given in equation (1) but including the perturbation variables α and u. With this simplified equation, the conditions for gust alleviation (equations E-10 and E-20) are introduced. It is convenient to introduce gains representing degrees of alleviation, which are useful in examining the effects of the system on the aircraft dynamics. The flap equation becomes

$$\delta = -K_a\frac{C_{L_\alpha}}{C_{L_\delta}}(\alpha+\alpha_g)-K_v\frac{2C_L}{C_{L_\delta}}(u+u_g) \tag{6}$$

The common assumptions associated with small perturbation techniques for conventional aircraft apply.[5] In addition, for the low Mach number regime of the aircraft, C_{m_u} is assumed zero. Lift and pitching moment derivatives with $\dot{u}_g$ are included as forcing terms to examine the response of the aircraft at frequencies just above the short period mode.

$$\begin{bmatrix} C'_{x_u}-2\mu s & C'_{x_\alpha}+C_L & -C_L \\ 2C_L(1-K_v)-K_v\frac{2C_L}{C_{L_\delta}}C_{L_{\dot\delta}}s & C'_{L_\alpha}+(2\mu+C'_{L_{\dot\alpha}})s & (C_{L_q}-2\mu)s \\ -K_v\frac{2C_L}{C_{L_\delta}}(C_{m_\delta}+C_{m_{\dot\delta}}s) & C'_{m_\alpha}+C'_{m_{\dot\alpha}}s & (C_{m_q}-I_y s)s \end{bmatrix}\begin{bmatrix} u \\ \alpha \\ \theta \end{bmatrix} =$$

$$-\begin{bmatrix} C_{x_{\delta_e}} \\ C_{L_{\delta_e}} \\ C_{m_{\delta_e}} \end{bmatrix}\delta_e - \begin{bmatrix} C_{x_\alpha}+C_L \\ C'_{L_\alpha}+(C'_{L_{\dot\alpha}}-C_{L_q})s \\ C'_{m_\alpha}+(C'_{m_{\dot\alpha}}-C_{m_q})s \end{bmatrix}\alpha_g - \begin{bmatrix} C'_{x_u} \\ 2C_L(1-K_v)+C'_{L_{\dot u_g}}s \\ -K_v\frac{2C_L}{C_{L_\delta}}C_{m_\delta}+C'_{m_{\dot u_g}}s \end{bmatrix}u_g \tag{7}$$

The primed quantities are interpreted as follows:

$$C'_{x_\alpha} = C_{x_\alpha}-K_a\frac{C_{L_\alpha}}{C_{L_\delta}}C_{x_\delta} \tag{8}$$

$$C'_{x_u} = C_{x_u}-K_v\frac{2C_L}{C_{L_\delta}}C_{x_\delta} \tag{9}$$

For the lift and pitching moment terms, L and m are substituted for x in these expressions. The rate-dependent derivatives are similar except that the last parameter in each expression is a function of $\dot{\delta}$.

To comprehend the consequences of gust alleviation, these equations should be examined without imposing all of the optimal conditions simultaneously. The lift curve slope is reduced to zero for $K_a=1$, thereby eliminating the constant term in the angle of attack perturbation of lift. The corresponding velocity perturbation term also is eliminated if $K_v=1$. The combination of the two conditions leads to a zero root for the characteristic equation when the nominal flight path angle is zero.

The static stability of the aircraft is determined by the sign of the constant term P(0) in the expansion of the characteristic determinant.

$$P(0) = -\ 2C_L^{\ 2}\left[\left.\frac{DC_M}{D\alpha}\right|_{K_a=1}(1-K_v)+\frac{C_{L_\alpha}}{C_{L_\delta}}C_{M_\delta}(1-K_a)\right] \tag{10}$$

It is thus apparent that the fully alleviated aircraft is marginally stable since both of the alleviation constants are unity and P(0)=0. If full horizontal gust alleviation is only partially implemented ($K_a=1$, $K_v<1$)

$$P(0) = -2C_L^{\ 2}(1-K_v)\ \frac{DC_M}{D\alpha} \tag{11}$$

so that in this case, static stability requires negative total pitch stiffness.

$$\frac{DC_M}{D\alpha} < 0 \tag{12}$$

The following table illustrates the possible range of this parameter for the Cessna 172.

Configuration	$DC_m/D\alpha$
flap	-0.15
aileron	.93
flap-aileron	.40

Thus only the flap configuration can provide static stability, for the stability derivatives assumed. Test data recently supplied by NASA indicate the flap pitching moment derivative is smaller than estimated by this analysis, thus expanding the possibilities of stable operation.

If the flap pitching moment characteristics are developed, C'_{m_α} can be expressed by

$$C'_{m_\alpha} = C_{m_\alpha} - K\frac{C_{L_\alpha}}{aC_{L_\delta}}[C_{m_{\delta_w}} - C_{m_{\alpha_t}}\frac{\partial\varepsilon}{\partial\delta}] \tag{13}$$

For a positive flap downwash derivative and center of gravity behind the wing aerodynamic center, the large negative pitching moment due to increased camber is somewhat offset, but never is eliminated completely. If the flaps were designed to produce a negative downwash derivative, as specified in Appendix E, the aircraft would be unstable unless the elevator was coupled to the flap to provide a compensating pitching moment. The independence of C'_{m_α} on the center of gravity location is developed easily. The trimmed, alleviated aircraft exhibits no pitching moment about the center of gravity. If the center of gravity is moved, the resulting moment is equal to the lift multiplied by the distance moved. The corresponding change in C'_{m_α} is this distance multiplied by C'_{L_α}. However, the latter is zero, resulting in the independence cited. Hence, the static stability of the vertical gust alleviated aircraft cannot be improved by moving the center of gravity forward as is possible with a conventional configuration. A similar independence on center of gravity location is found for C'_{m_u} when horizontal gust alleviation is mechanized.

$C'_{m_{\dot{\alpha}}}$ depends on the downwash lag and is influenced strongly by $\partial\varepsilon/\partial\delta$. For inboard flaps the flap downwash derivative is large and positive, and reduces the short period damping. For the optimal case, it is negative and $C'_{m_{\dot{\alpha}}}$ is increased to the value of C_{m_q}.

As more of the optimal alleviation conditions are imposed, more of the coefficients of the gust forcing terms are eliminated. When they all are satisfied, no forcing terms remain. As this process proceeds, assessment of the dynamic behavior of the aircraft becomes complex and at this point a numerical example is introduced to show the results of a practical application. The locus of roots for a Cessna 172 is shown in Figure 2 where K_a is varied from 0 to 1.5 and K_v is zero or one. The short period mode is independent of K_v and decreases in frequency and damping with increasing K_a. If loci for a variety of static margins were superimposed, they all would intersect at the point where K_a=1, since C'_{L_α}=0 and C'_{m_α} is independent of static margin. The stability of the alleviated aircraft is demonstrated by the phugoid mode loci. For K_v=0, the aircraft is tolerant of minor over-alleviation and yields roots which can be accomodated by the pilot. However, for K_v=1, the system is extremely sensitive and diverges rapidly if over-alleviated by even the slightest amount. A companion locus, not presented, shows somewhat less sensitivity to over-alleviation of horizontal gusts when K_a=1. The loci for the aileron-configured aircraft (Figure 3) where outboard flaps reverse the sign of the downwash derivative,

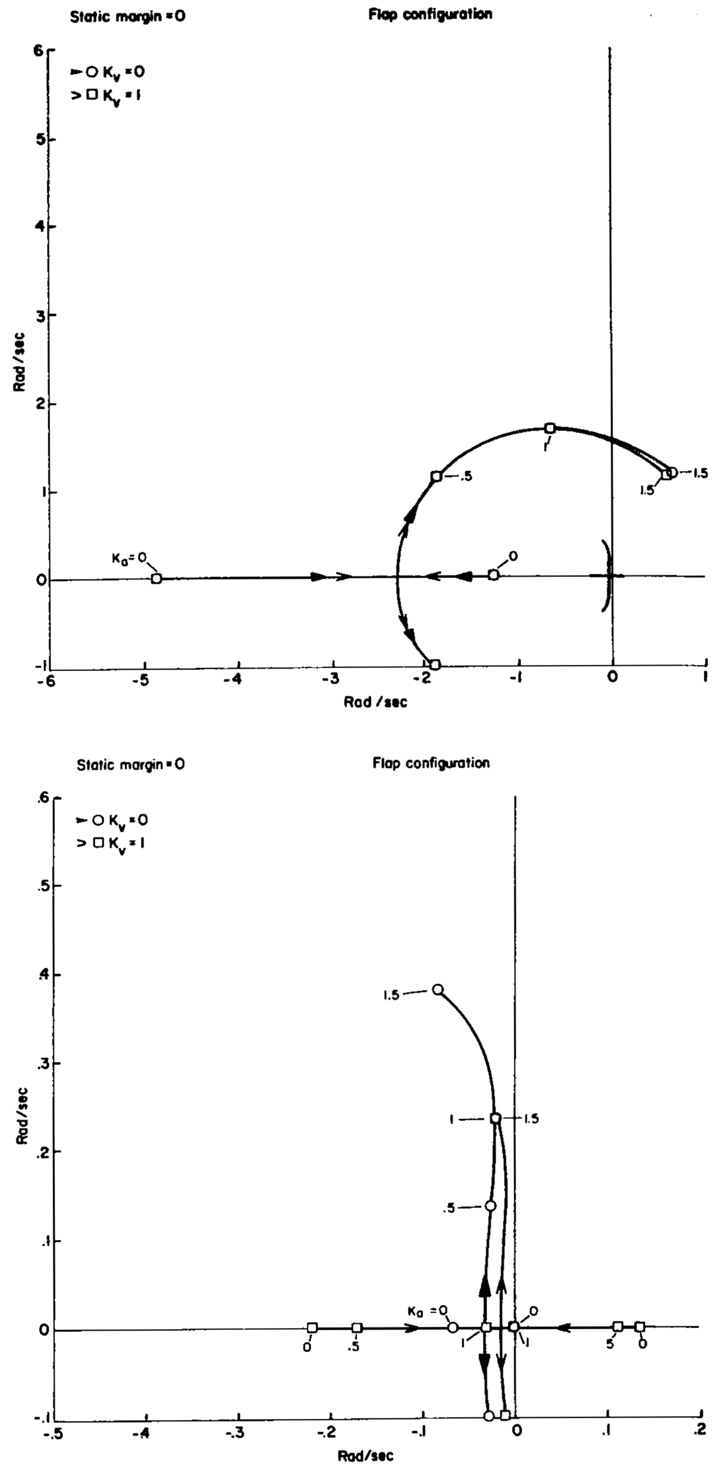
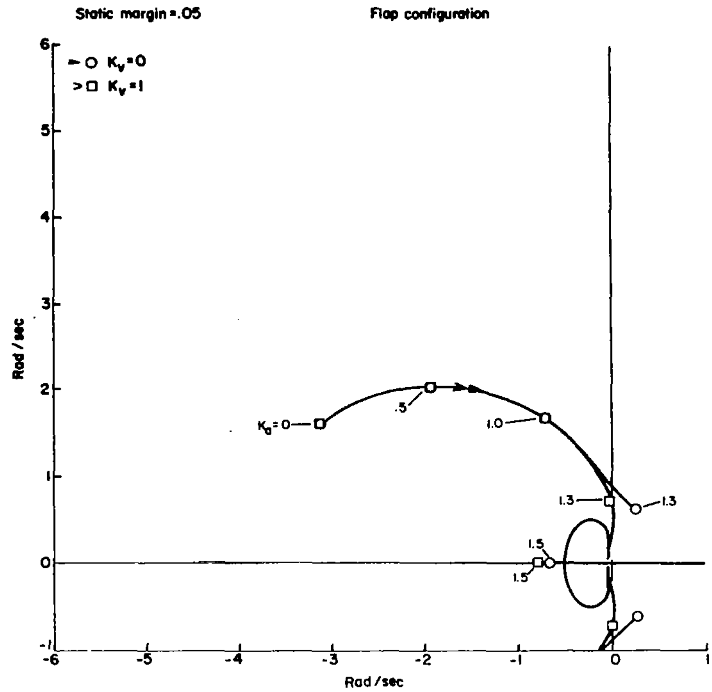
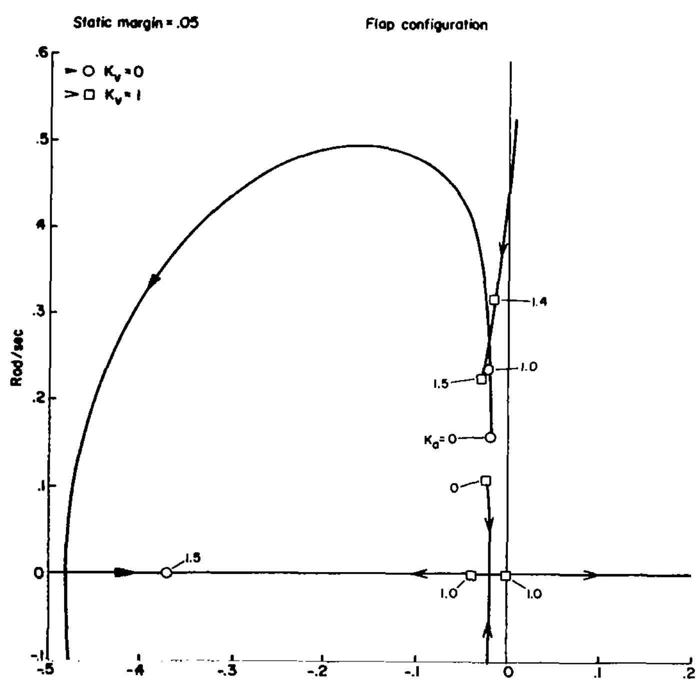


Figure 2 Root Locus of Flap-Configured Aircraft as a Function of Vertical Gust Alleviation Gain





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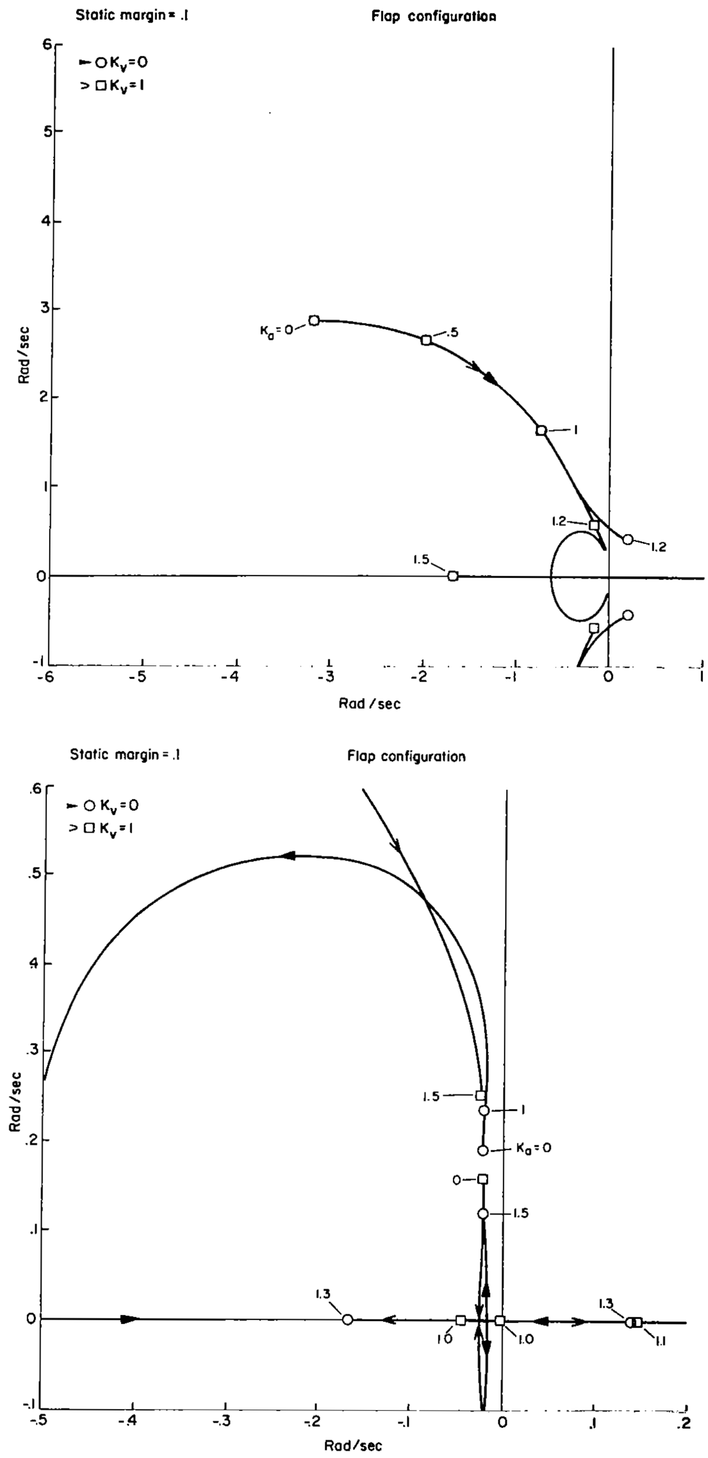

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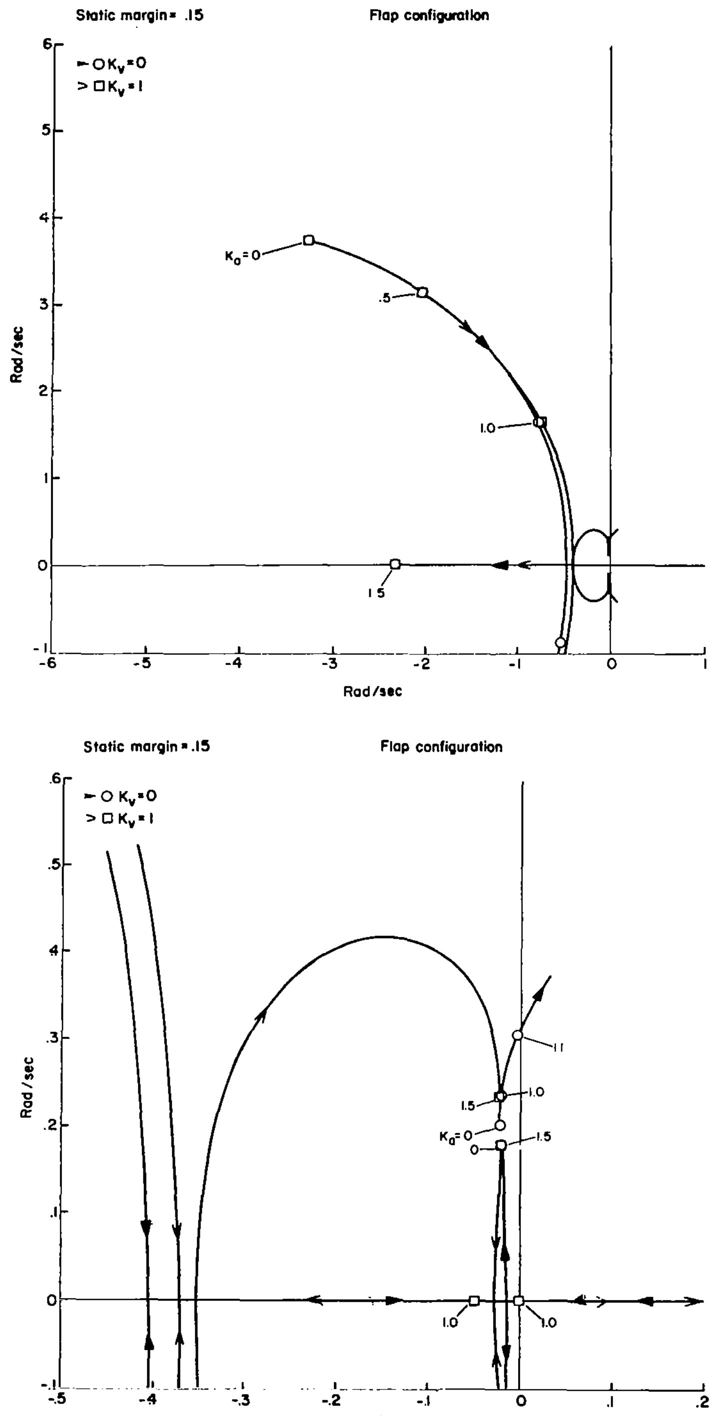

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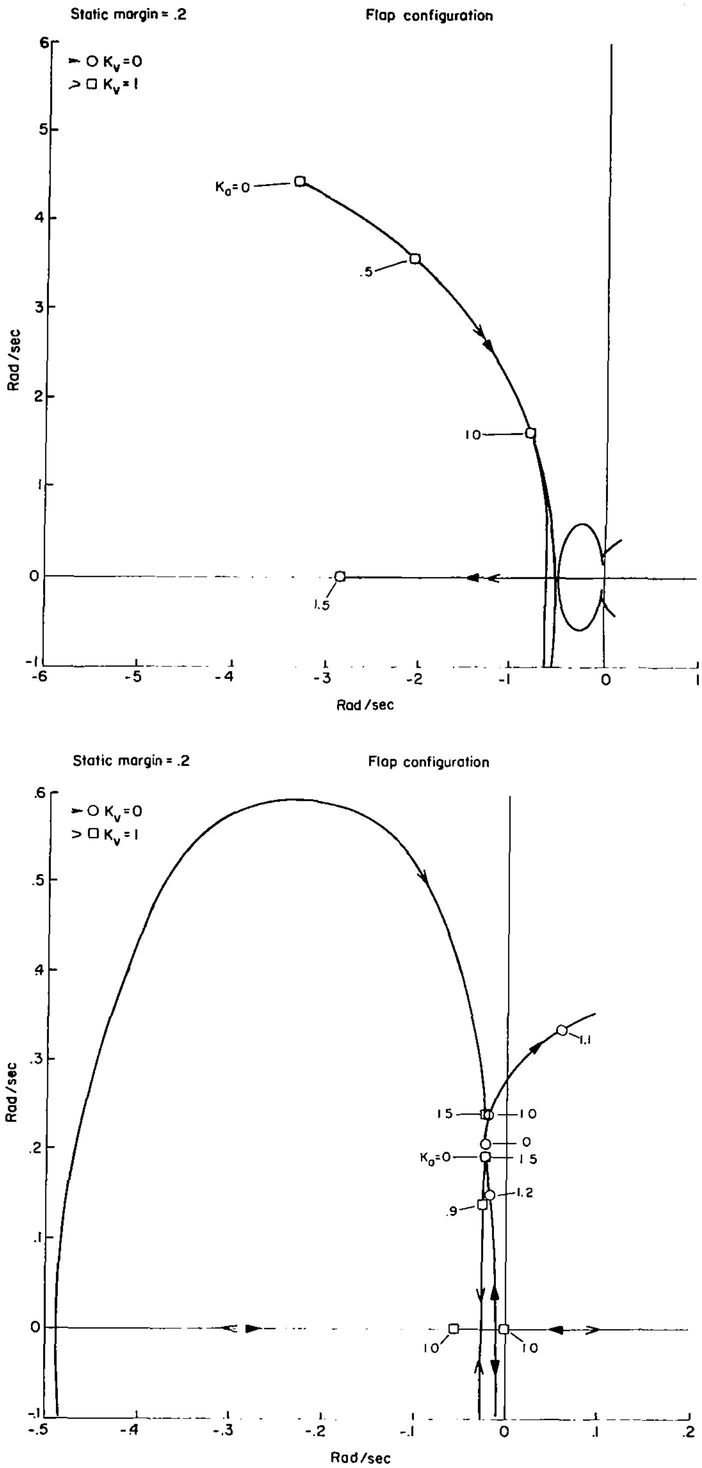

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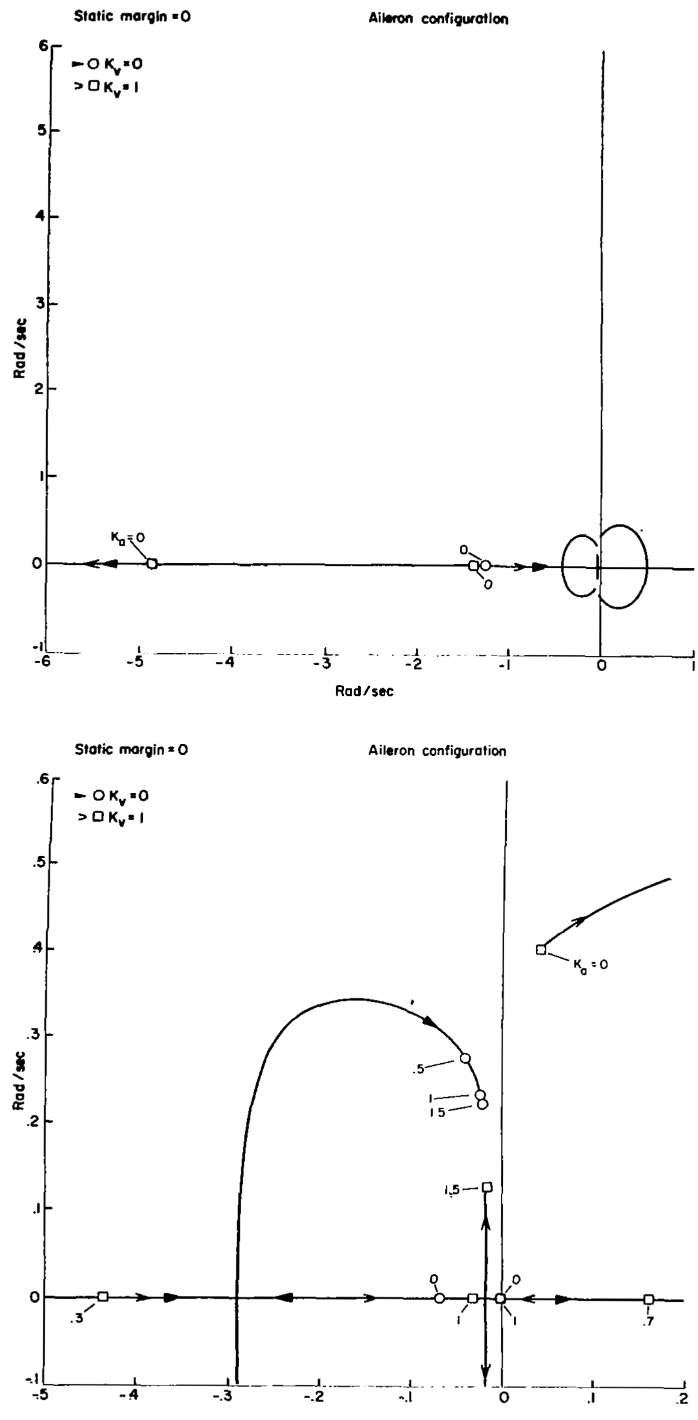
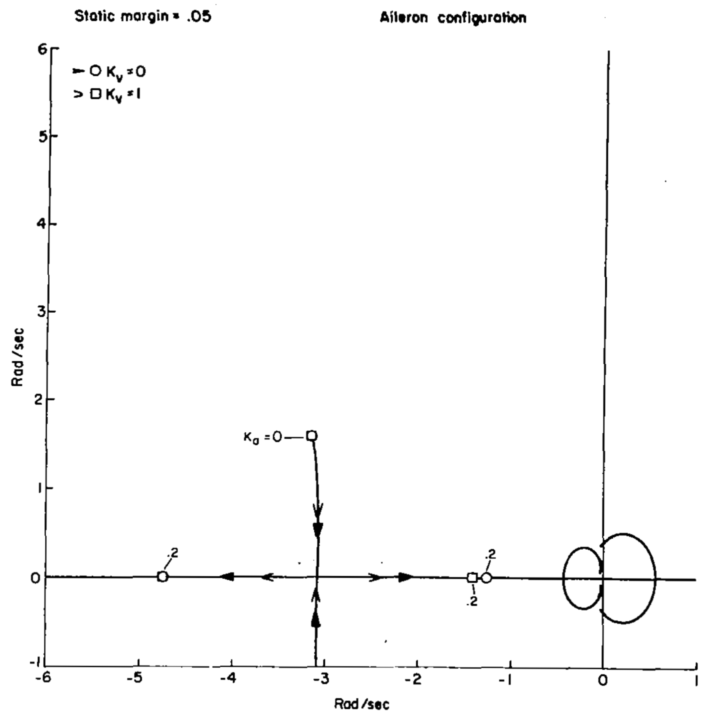
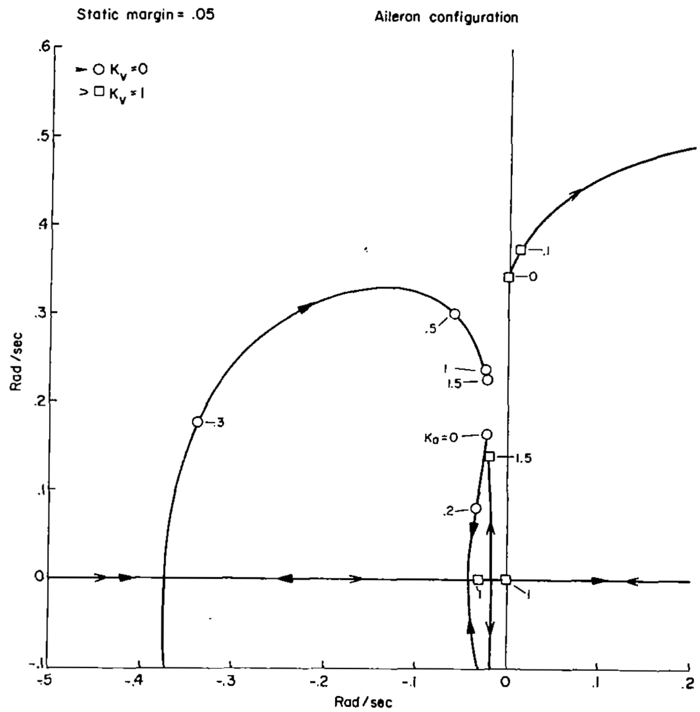


Figure 3 Root Locus of Aileron-Configured Aircraft as a Function of Vertical Gust Alleviation Gain





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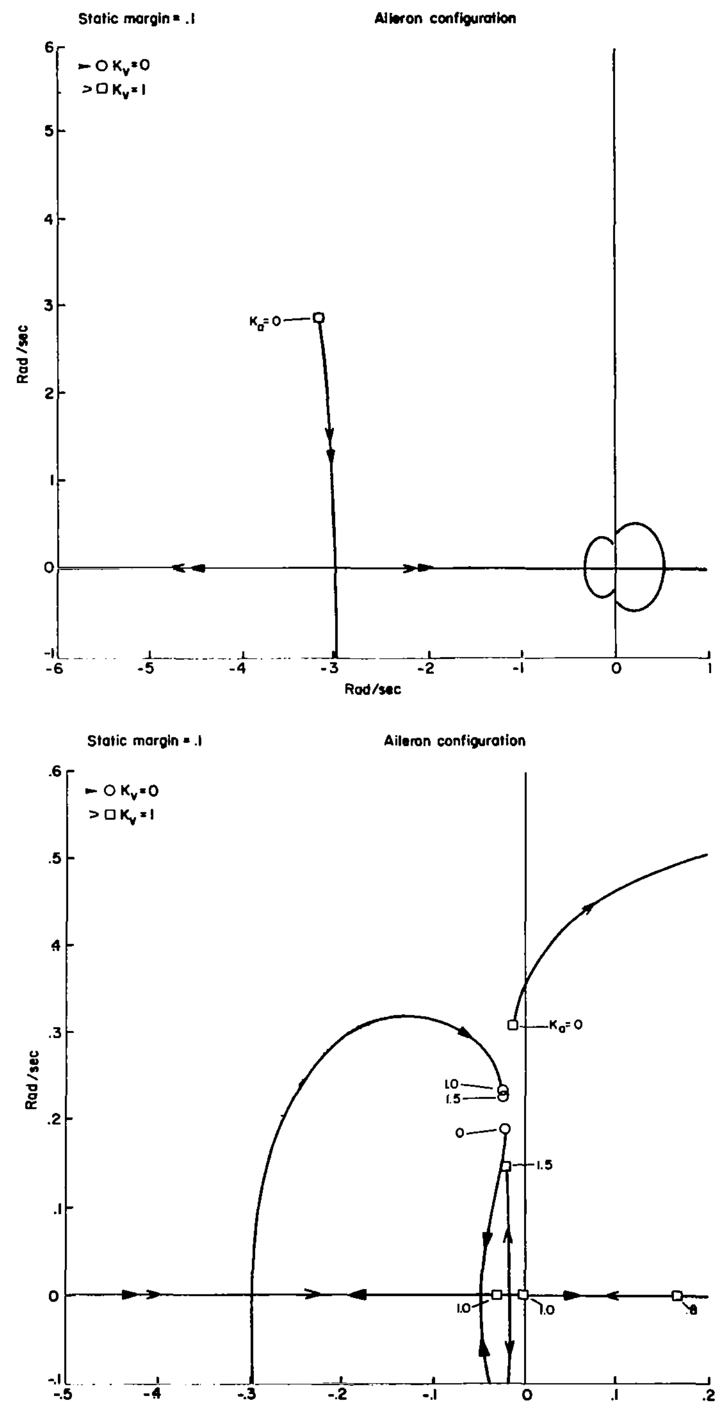

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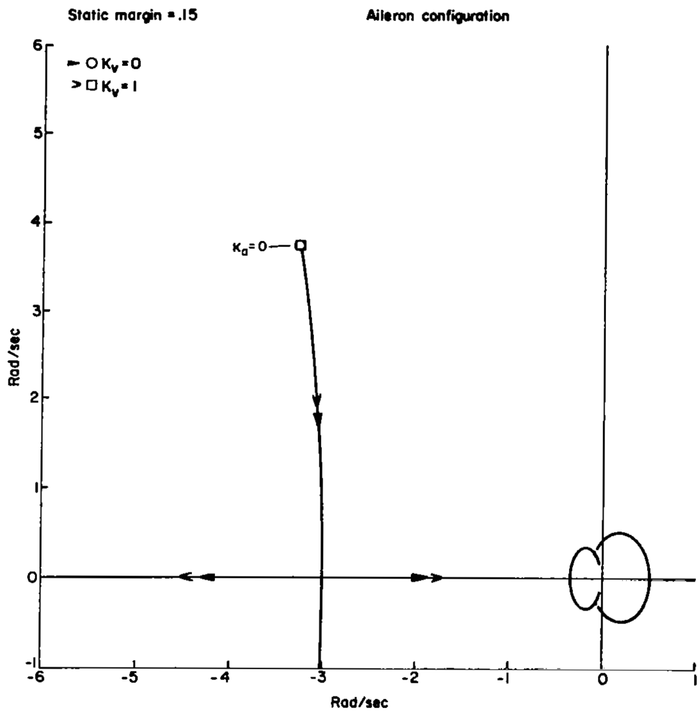

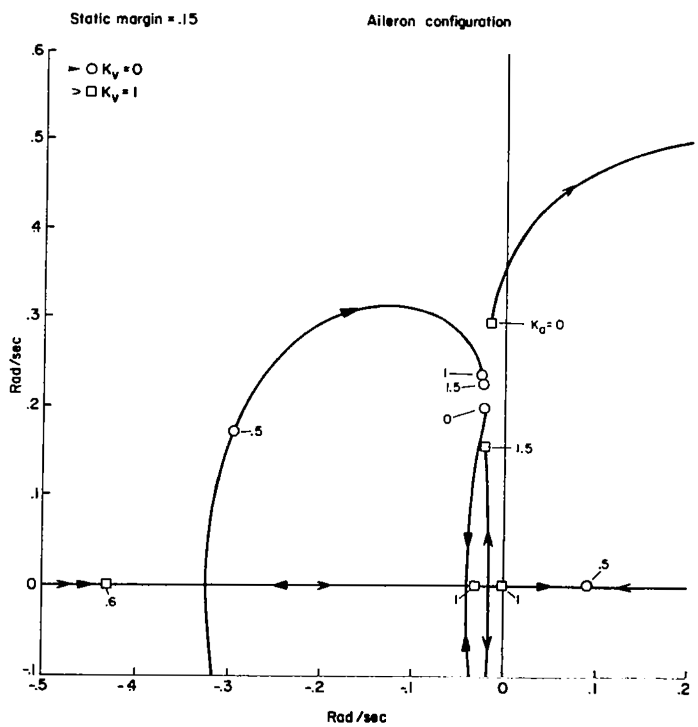

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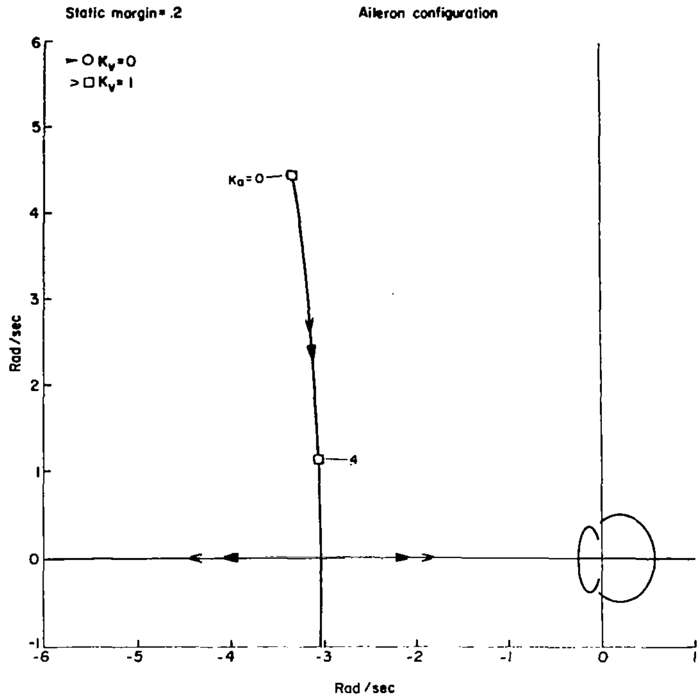


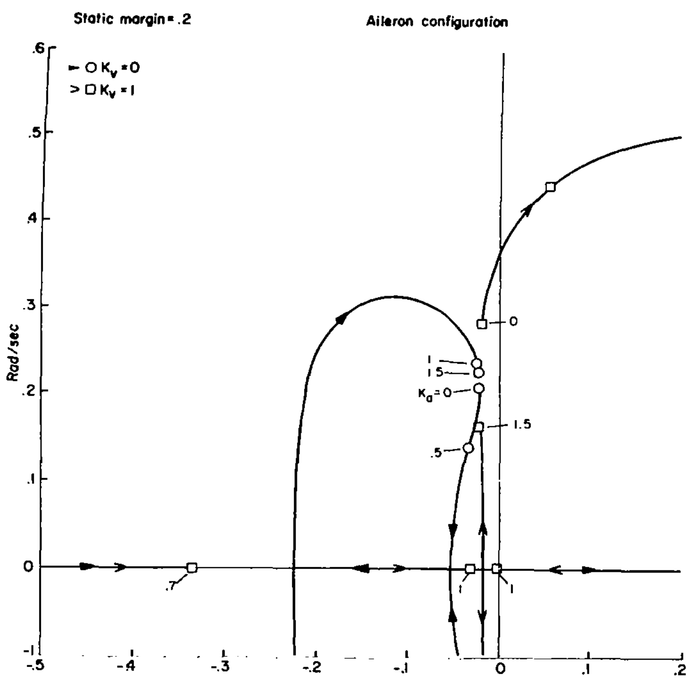


Figure 3 continued

show that very little vertical gust alleviation can be mechanized before the onset of instability. A similar, less stringent condition is imposed on an aircraft fitted with full-span flaps. (Figure 4). Recent test data generated by NASA indicates the tail-off flap pitching moment derivative with deflection has about half the computed value. An analysis of the short period mode is presented in Chapter VII and shows both the aileron and full-span flap configurations to be stable for $K_a=1$, $K_v=0$. This analysis clearly emphasizes the need for accurate parameter estimation before undertaking alleviation system design.

Because the stability of the aircraft is very sensitive to over-alleviation of either vertical or horizontal gusts if the other is fully implemented, it is appropriate to carefully assess the relative influence of these two components on ride comfort. For equal gust amplitudes the ratio of load factors due to verical and horizontal gusts is $C_{L_\alpha}/2C_L$ which suggests very large lift coefficients must be realized before alleviation of horizontal gusts becomes important. It is for this reason that only vertical components are considered for light aircraft in cruise.

For the Cessna 172, using inboard flaps as gust alleviators, C'_{m_α} remains negative. As shown in the next section, the pitching response to gusts is increased. This problem could be moderated by proportionately coupling the ailerons to the flaps to the extent that the aircraft remains stable. This technique provides the designer with a means of controlling the flap downwash at the tail. However, it introduces the additional complexity of developing linkages to permit both symmetric and antisymmetric deflections. Another degree of control can be exercised by coupling the elevator to the flaps as proposed by Phillips. In the case of all mechanical linkages, though, some thought must be given to the cockpit interface to prevent undue loading of the stick.

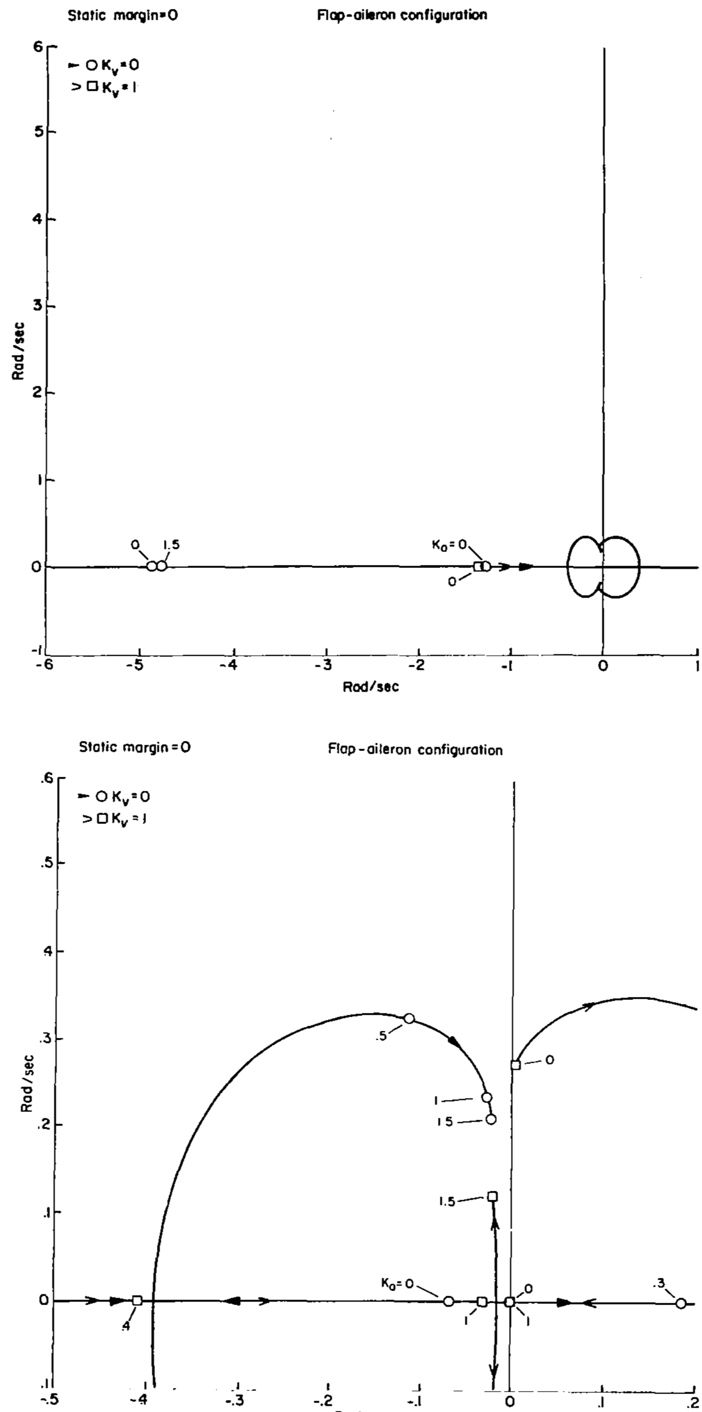
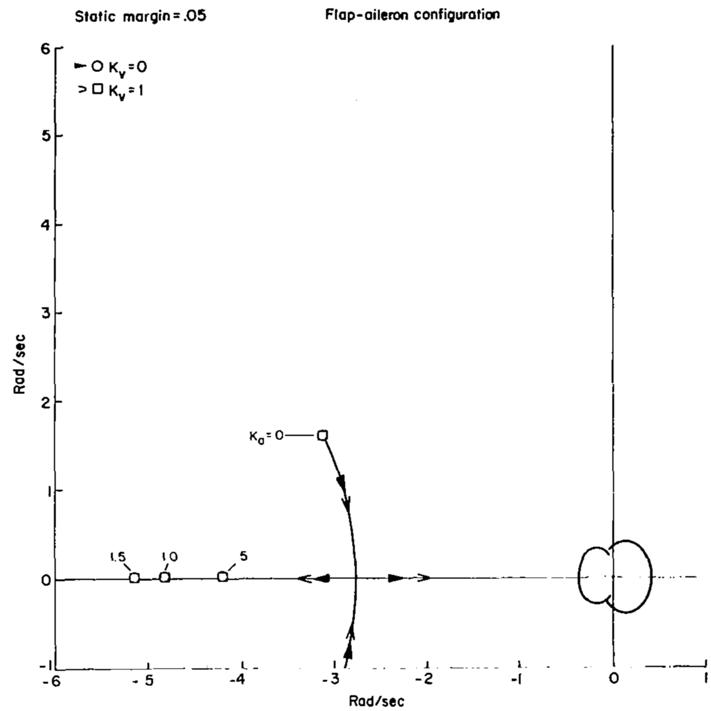
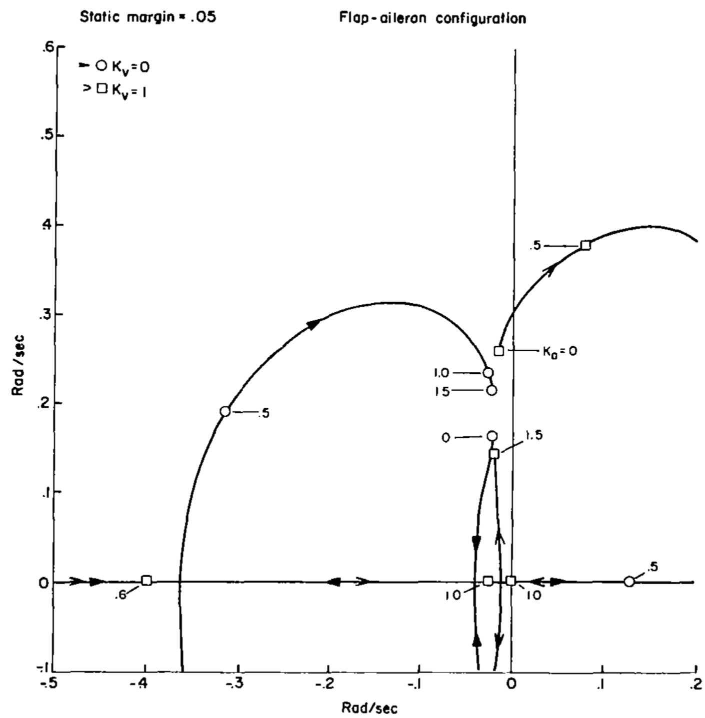


Figure 4 Root locus of Flap-Aileron-Configured aircraft as a Function of Vertical Gust Alleviation Gain





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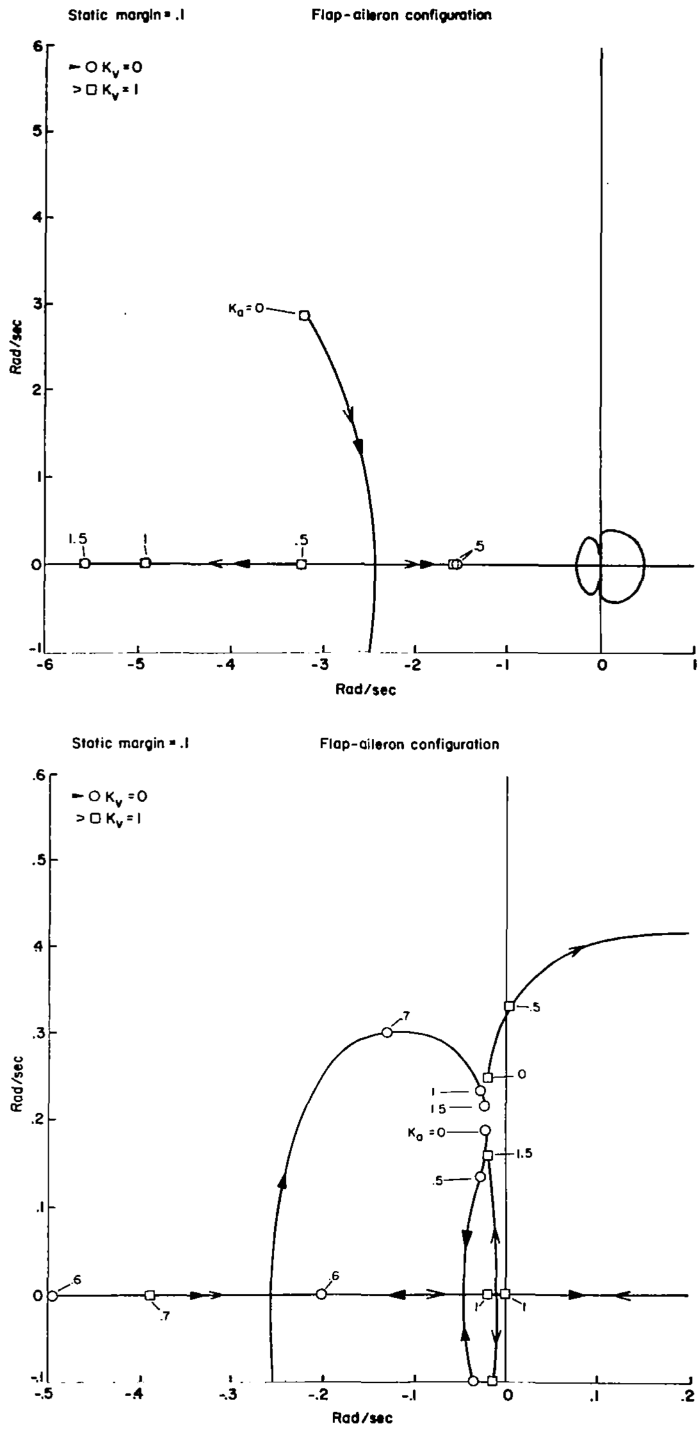

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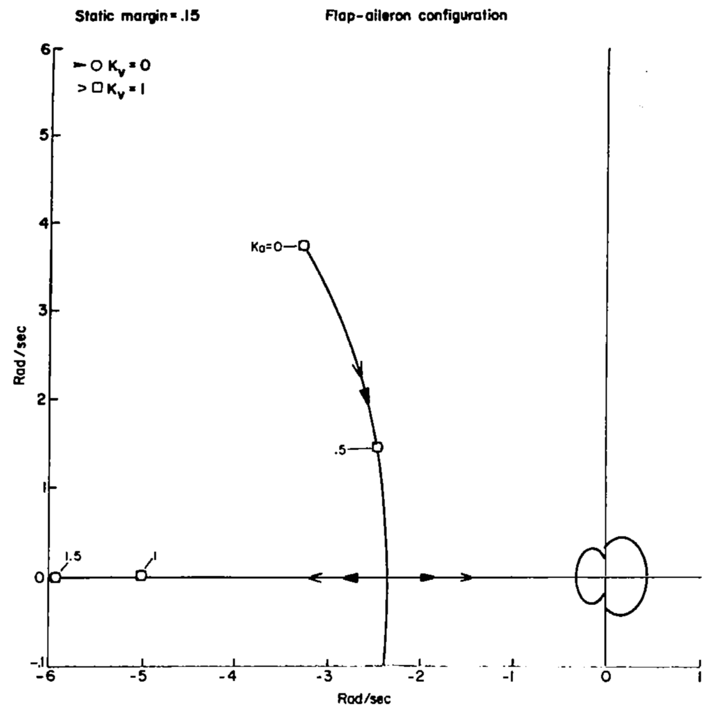

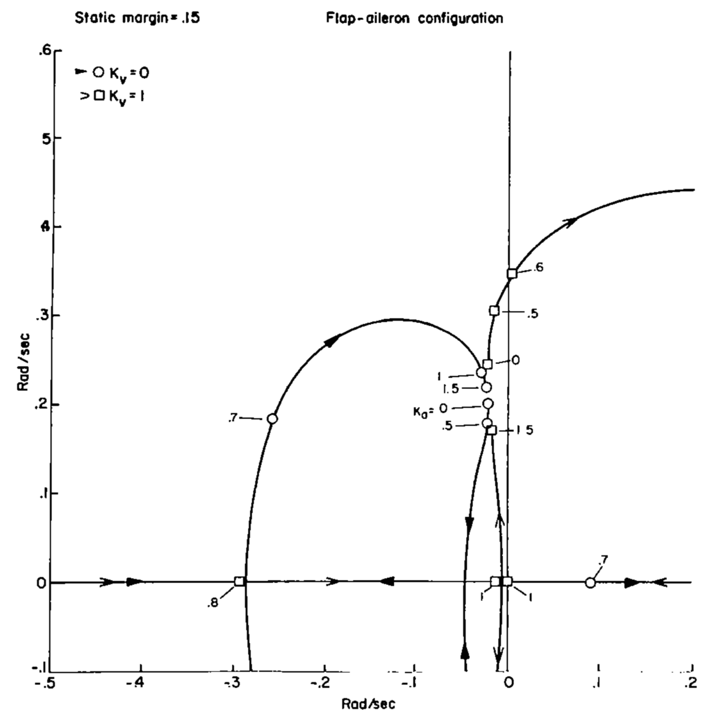

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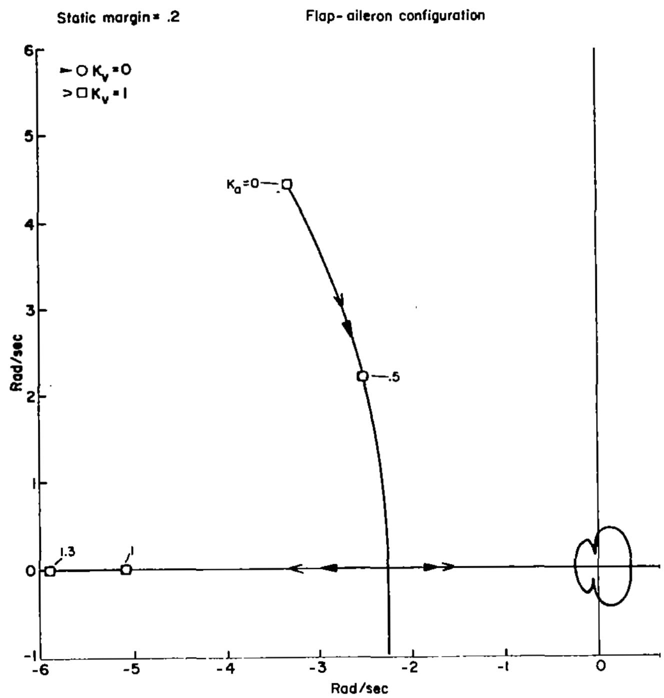


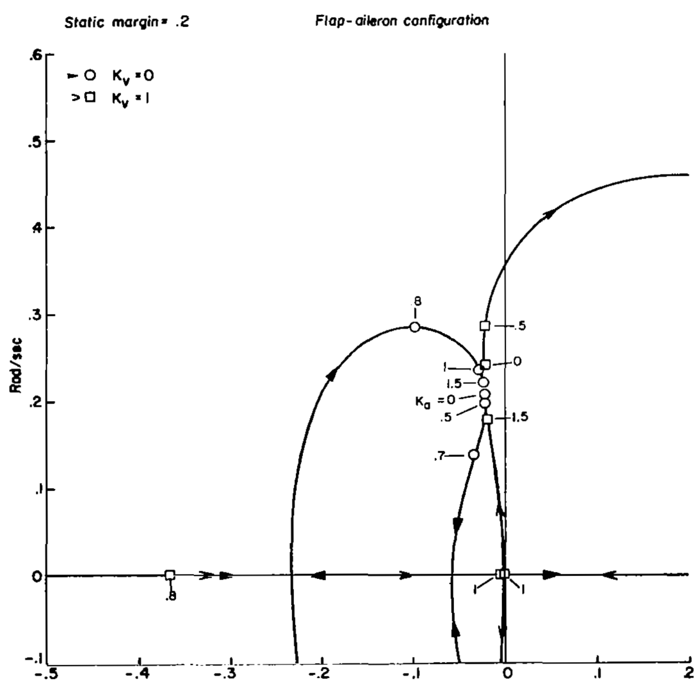


Figure 4 continued

IV. RESPONSES TO GUSTS

The Cessna 172 application is used to examine the frequency responses of basic and alleviated aircraft. It is important to recognize that these are not responses of an optimally alleviated aircraft; rather they are the result of working within the configuration constraints mentioned previously. However, the response levels and ramp characteristics cited parametrically on the graphs are general and can represent the optimal case. These relations can be developed by making use of the short period and phugoid approximations[5] and assuming the pitching rate, angle of attack rate, and longitudinal gust rate derivatives result only from tail loads. The first four figures represent an idealized configuration, with instantaneously responding flap; the last segment of this chapter presents curves for a wide range of flap system characteristics.

The load factor response (Figure 5) of the basic aircraft is greatest at frequencies above the short period. There are two real zeros, one near this frequency and the other above 100 rad/sec which produce a flat response in this band. The inertia of the aircraft dominates and prevents any significant buildup of vertical speed to offset the gust-induced angle of attack and complete loading, proportional to $dL/d\alpha$, is realized. At lower frequencies, aircraft motion does offset the gusts, except at the phugoid frequency where the very low damping produces a sharp peak in the response. At the lowest frequencies, the response is determined by the convection condition, whereby the aircraft moves with the air mass and the total angle of attack perturbation is zero, $\alpha+\alpha_g=0$.

For the alleviated aircraft, the zeros transform to an undamped pair just above the short period frequency, which introduces an upward slope in the response at high frequencies. This represents tail loading only and results from not satisfying the

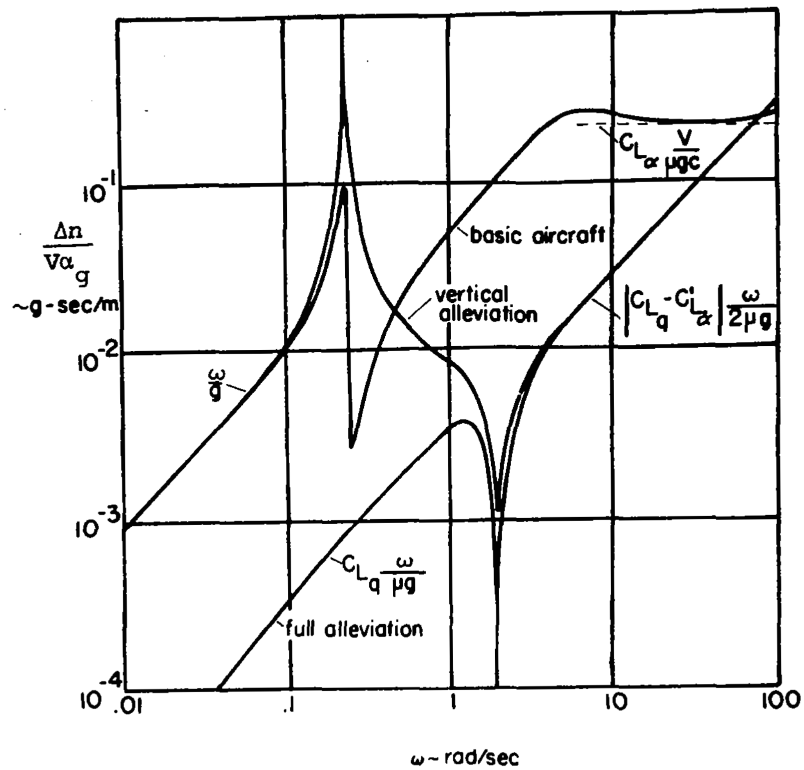


Figure 5 Frequency Response of Load Factor to Vertical Gusts

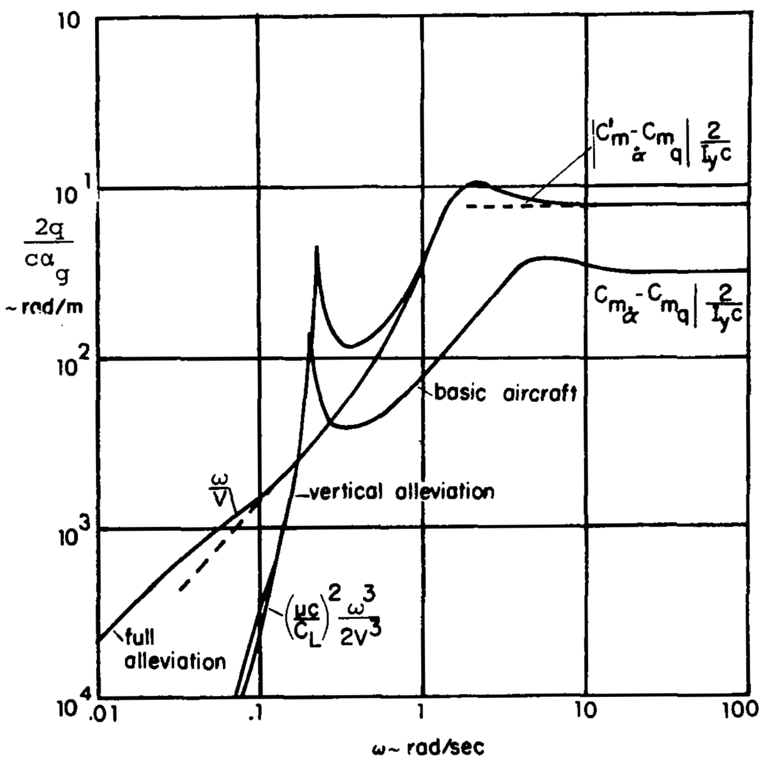


Figure 6 Frequency Response of Pitching Motion to Vertical Gusts

downwash criterion. The short period is longer because the lift curve slope is zero and C_{m_α} is diminished. At low frequencies, the phugoid persists when only vertical alleviation is implemented, but disappears with full alleviation. For the Cessna 172, gust alleviation is quite good in the region of interest, namely between the phugoid frequency and ten rad/sec. Above that the analysis is of limited value because unsteady lift effects, flap dynamics, and bending modes have been neglected. These frequencies are unimportant with regard to gust loads because the gust spectrum is very weak in this region. The effects on the aircraft are further diminished at the highest frequencies by the isotropic nature of these gusts which affords some spanwise averaging of the induced loads.

The significant pitching behavior (Figure 6) can be deduced from the response at very high frequencies. Inertia prevents any angular perturbation of the aircraft and the pitching moment results only from gust rate dependent tail loads. Consequently, the pitching rate is in phase with the gust angle of attack. For the alleviated Cessna 172, the forcing term increases in magnitude as a result of the change in sign of $C'_{m_{\dot{\alpha}}}$, which is due to the flap downwash lag. It is difficult to estimate the effect of this pitching motion on ride comfort. Although it is three times greater than for the basic aircraft, gusts are weak at these high frequencies. A flight test evaluation can best determine its significance.

The responses (Figures 7 and 8) to horizontal gusts for the basic aircraft are much smaller than their vertical gust counterparts except near the phugoid frequency. As would be expected, there are only minor differences between the responses for the vertical-gust-alleviated aircraft and those for the basic aircraft over the entire frequency range. These are due mainly to differences in mode shapes for the two aircraft. Implementation of horizontal gust alleviation eliminates the load factor forcing term dependent on u_g, leaving only the rate-dependent term which leads to the ramp response. The pitching rate response for this

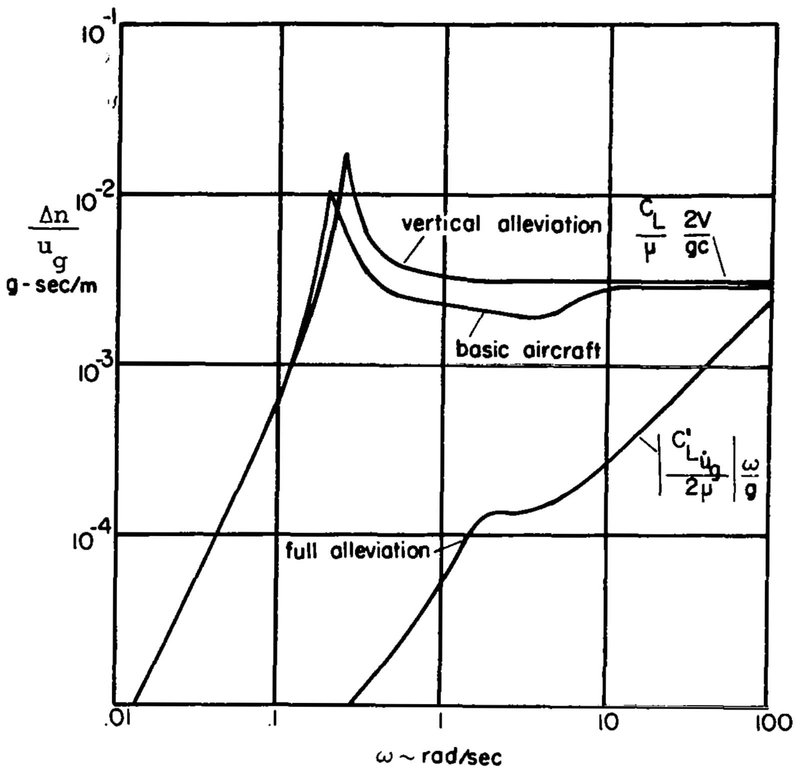


Figure 7 Frequency Response of Load Factor to Horizontal Gusts

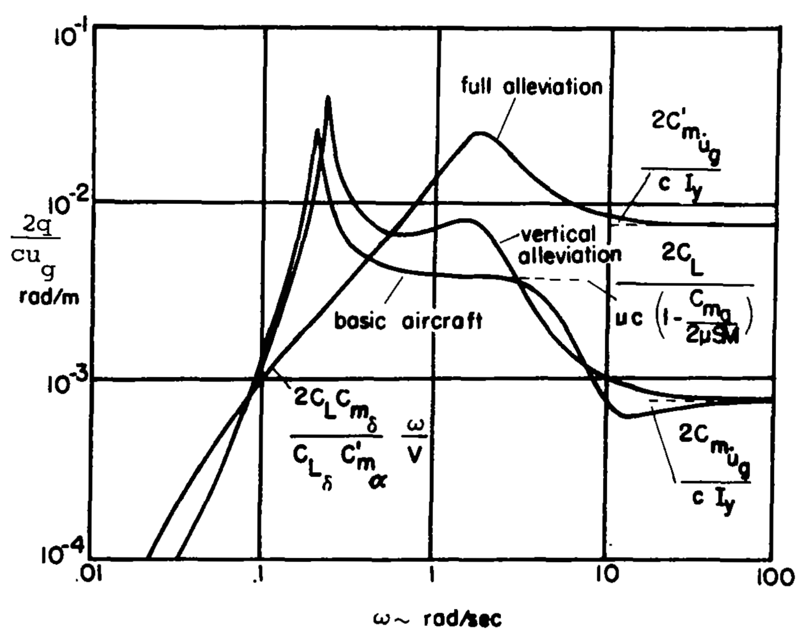


Figure 8 Frequency Response of Pitching Motion to Horizontal Gusts

case is much greater at high frequencies than for the basic aircraft since $C_{m_{\dot{\delta}}}$ is nearly an order of magnitude larger than $C_{m_{\dot{u}_g}}$. Despite this increase, the response is less than one third the pitching rate response of the alleviated aircraft to vertical gusts.

In order to redirect attention to the effect of flap dynamics and to investigate any possible interference with the short-period mode of the aircraft, the complete equations of motion were used to obtain the load factor and pitching responses to vertical gusts corresponding to a variety of flap-auxiliary wing system inertia and aerodynamic damping. The initial design, with a natural frequency of 13.2 rad/sec and damping ratio of .45 has a rather poor load factor attenuation (slightly in excess of 50%) at the basic aircraft short-period frequency. As the natural frequency of the system is increased while maintaining a constant damping coefficient ($C_{H_{\dot{\delta}}}=-2.5$), for example by reducing the structural weight of the system without changing the auxiliary wing shape or size, very little improvement in load factor attenuation is achieved, (Fig 9). This is due to the fact that the damping ratio of the flap mode is inversely proportional to the natural frequency of the system; thus, as the inertia is reduced, critical damping is reached and the degenerate mode becomes essentially damping limited. Therefore, in order to shorten the flap-auxiliary wing response time, the natural frequency of the system should be increased and the aerodynamic damping reduced correspondingly. Examination of the relations developed in Appendix A shows that the inertia and damping contributed by geometrically similar auxiliary wings are inversely proportional to their size and size squared respectively, if the gearing ratio is chosen to achieve full vertical gust alleviation ($K_a=1$, $\Gamma L^3=1$). Thus in general terms, the larger the auxiliary wings, the better the load factor attenuation.

The effect of the reduced damping is clearly seen on the load factor frequency response curves corresponding to a constant damping ratio ($\zeta=.45$) equal to that calculated for the initial flap-auxiliary wing design. Note that the zero inertia curve on ($\omega_n = \infty$) represents the idealized case of instantaneous flap deflection. Even better results are obtained by going towards smaller damping ratios as seen on Fig 11. Improved load factor attenuation in the short-period frequency range is achieved at the expense of an increased resonance at the flap system natural frequency where the gust energy is much smaller. A damping ratio of .2 and flap-auxiliary wing natural frequency of 20 rad/sec seems to be a satisfactory compromize. Good overall load factor attenuation with nearly one decade at the basic aircraft short-period shows the real utility of the passive gust alleviation system. Furthermore these dynamic characteristics are believed to be achievable in practice by proper optimization of auxiliary wing weight size and shape.

Since alleviation of normal accelerations is not optimal with the real flap system, the high frequency pitching response is not as severely aggravated as suggested by the simplified analysis. Fig 14 shows this response is only 1.6 times as large as that for the basic aircraft at the short-period for a flap system with ω_n=20 rad/sec. This should cause no hardship on passenger comfort.

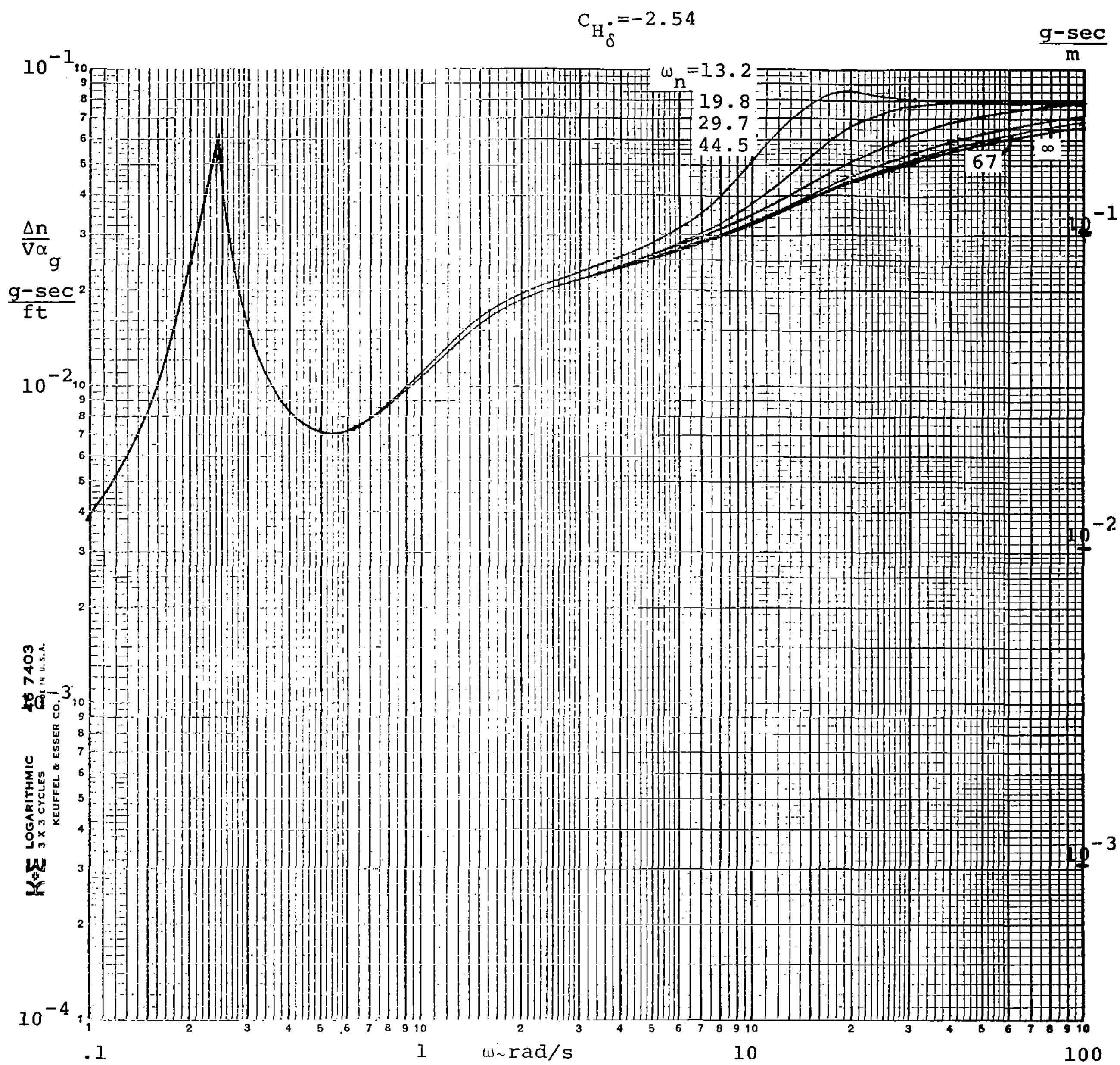


Figure 9 Effect of Flap Dynamics on Load Factor Response

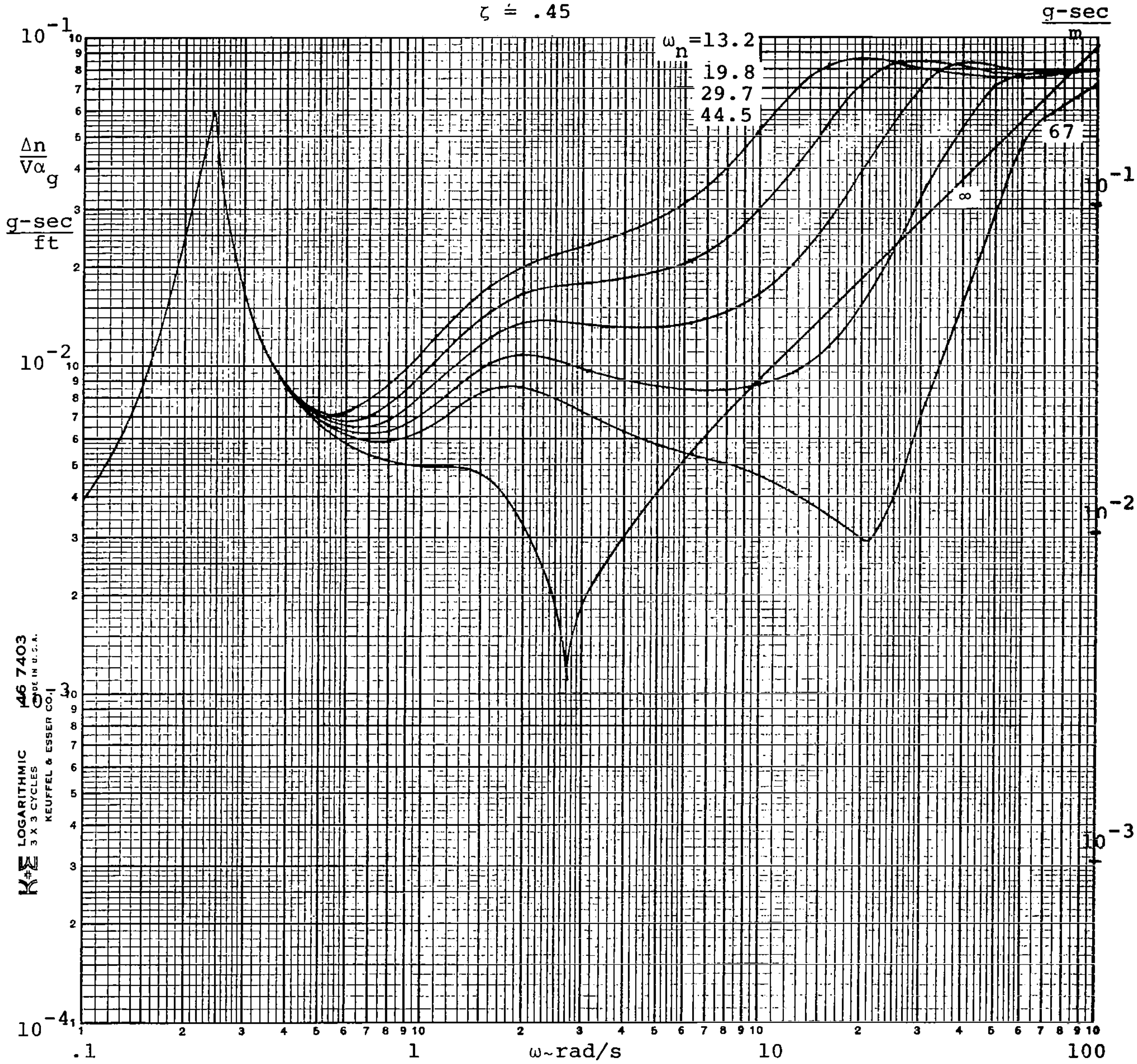


Figure 10 Effect of Flap Dynamics on Load Factor Response

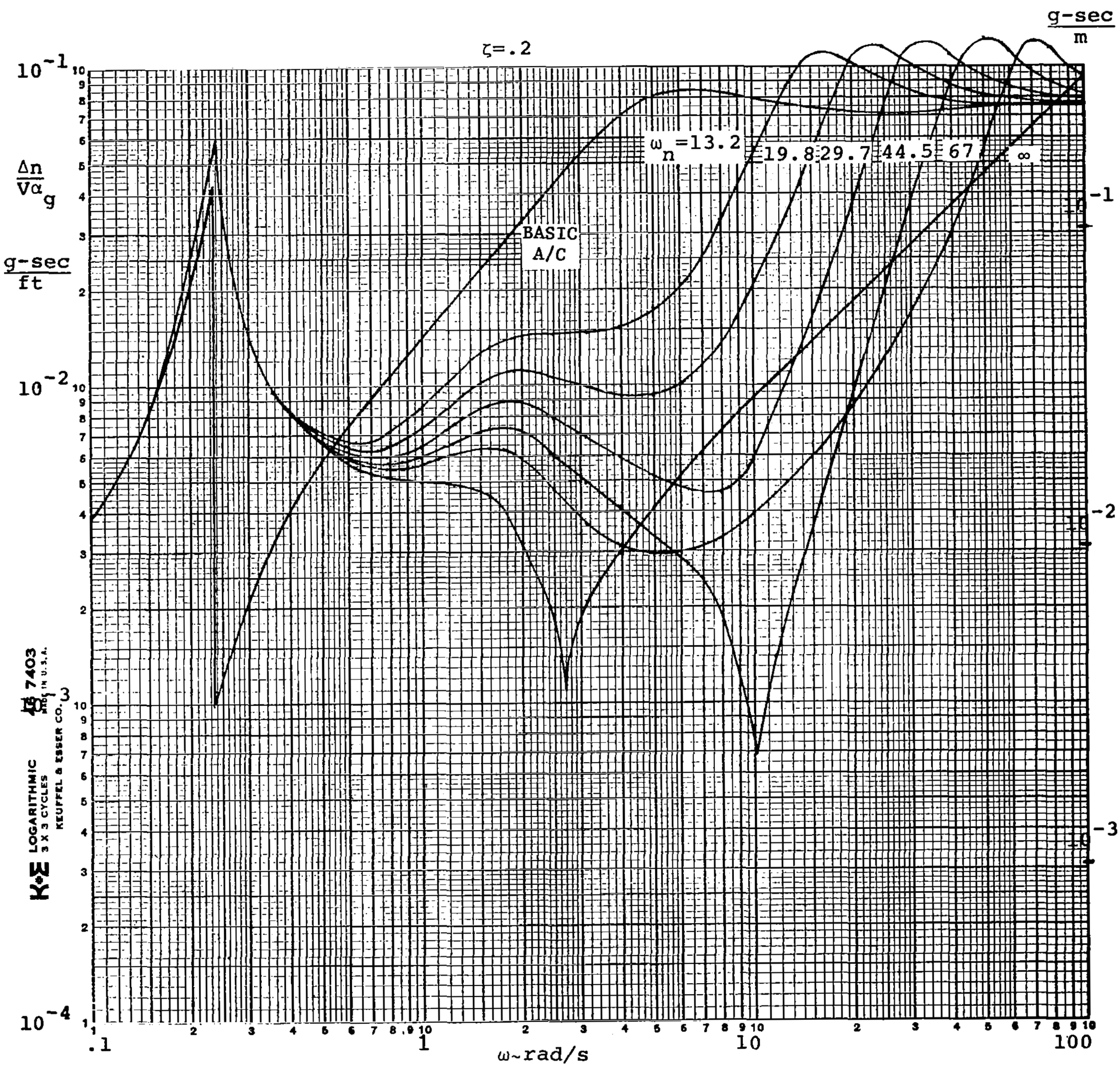


Figure 11 Effect of flap Dynamics on Load Factor Response

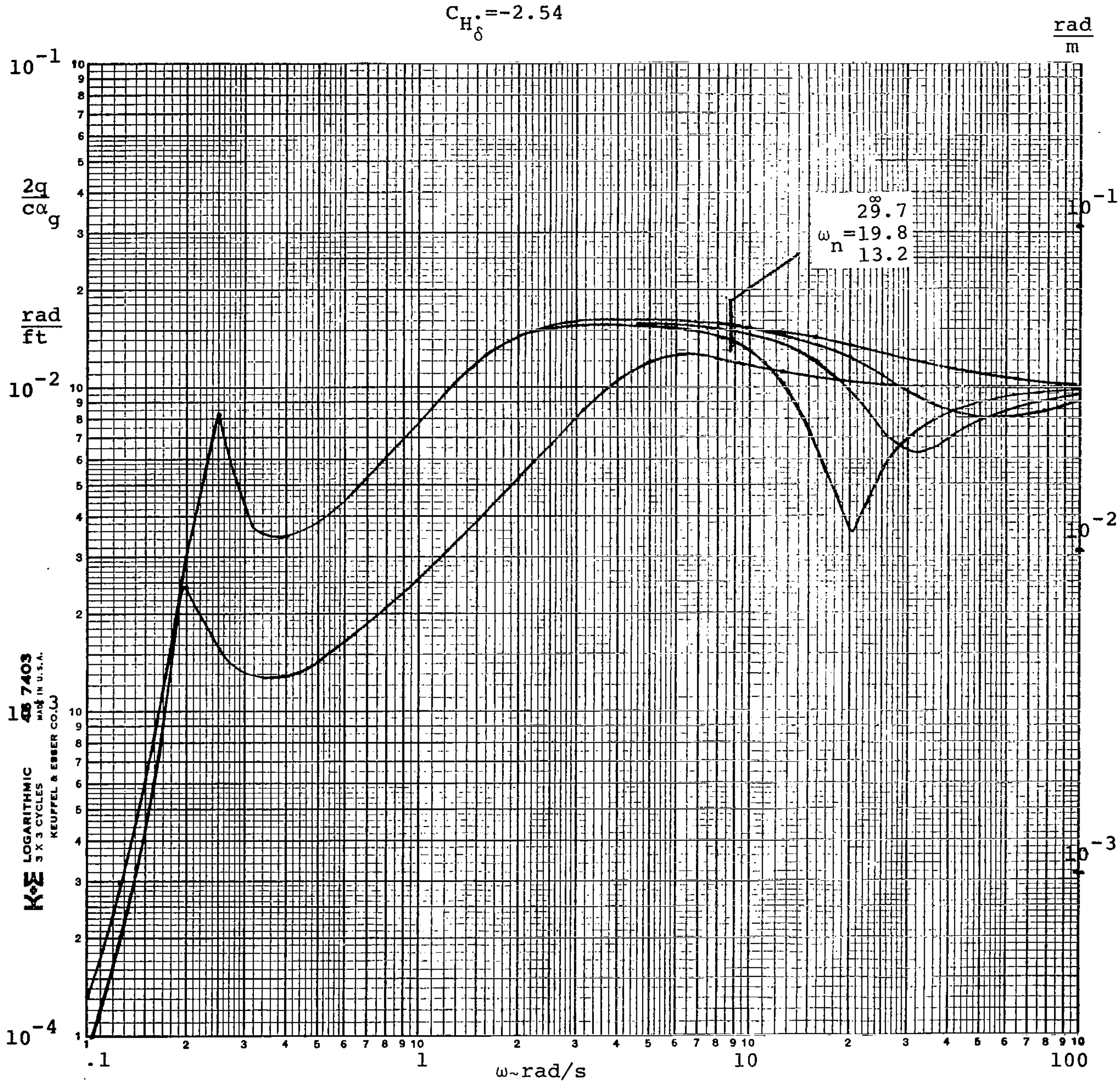


Figure 12 Effect of Flap Dynamics on Pitching Response

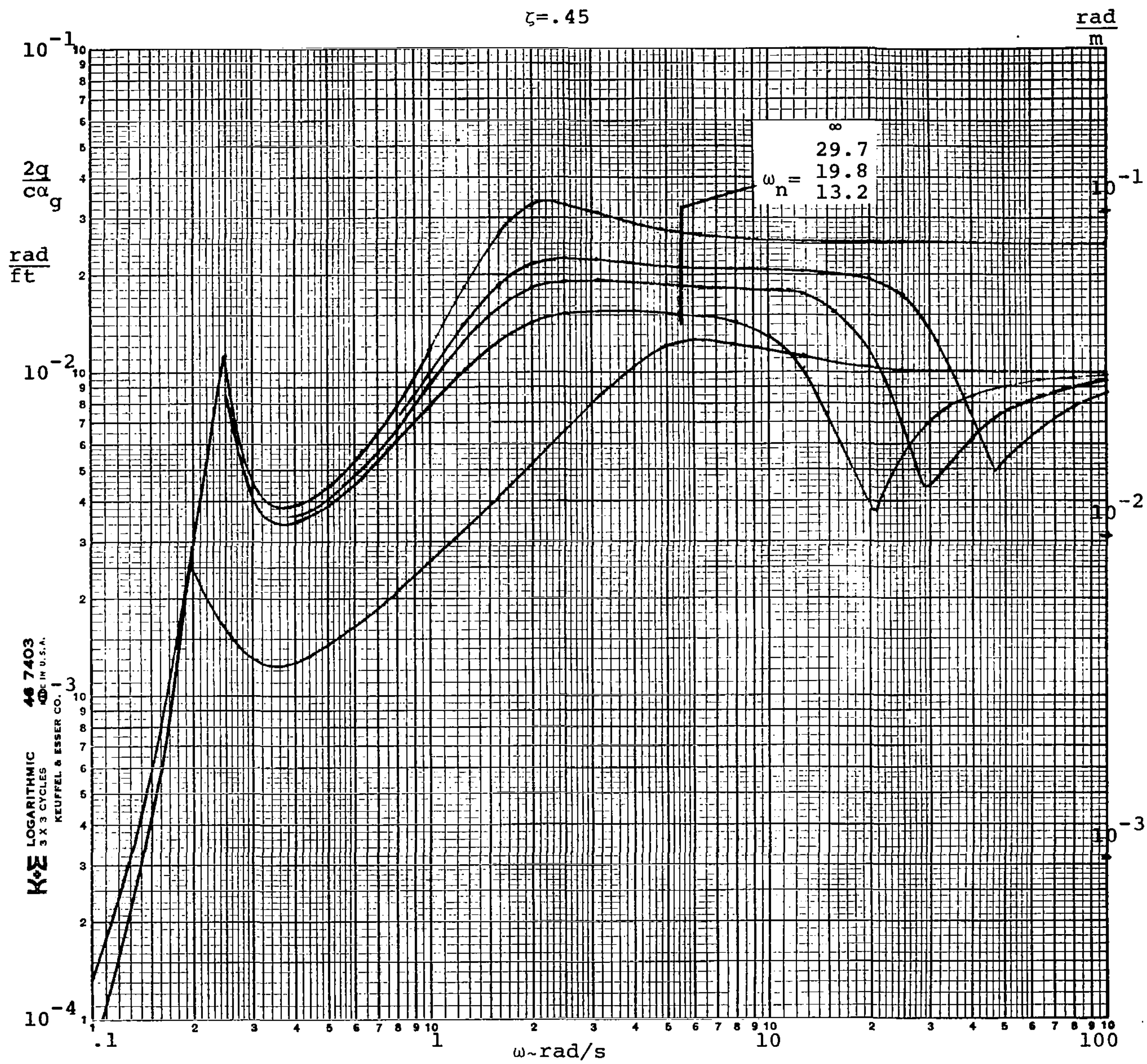


Figure 13 Effect of Flap Dynamics on Pitching Response

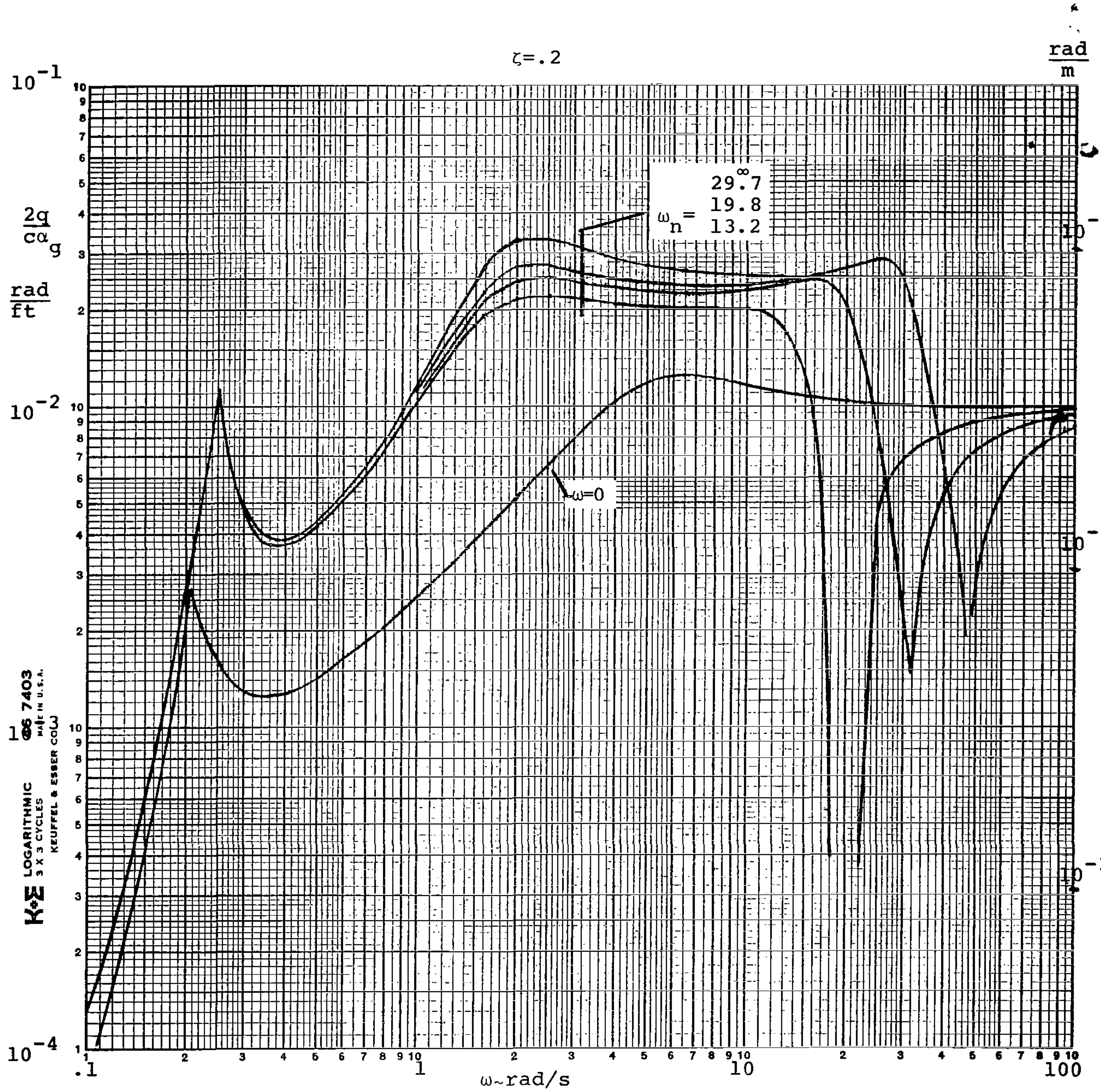


Figure 14 Effect of Flap Dynamics on Pitching Response

V. FLIGHT IN A TURBULENT ATMOSPHERE

The one dimensional Von Karman spectrum is chosen to model the vertical component of high altitude turbulence. The vertical gust power spectral density is

$$\Phi_{w_g}(\Omega) = \frac{\sigma^2 \mathcal{L}}{2\pi} \frac{1+\frac{8}{3}(a\mathcal{L}\Omega)^2}{[1+(a\mathcal{L}\Omega)^2]^{11/6}} \tag{14}$$

$$a=1.339$$

σ is the root mean square gust velocity and can be upwards of 3 m/s in rough weather. The scale of the turbulence is large, typically a few thousand feet and has a critical effect on the relative levels of excitation of the two rigid body modes. As the scale of the turbulence is increased, the spectrum shifts towards the lower frequencies while increasing in level below the break frequency. The power spectral density of the normal acceleration is expressed in terms of the gust spectrum,

$$\Phi_n(\omega) = \left|\frac{\Delta n}{\alpha_g V}(i\omega)\right|^2 \Phi_{w_g}(\omega) \tag{15}$$

The gust wave number Ω is related to the frequency ω through:

$$\Omega = \frac{\omega}{V} \tag{16}$$

Similarly for the pitching velocity power spectrum,

$$\phi_q(\omega) = \left|\frac{2q}{c\alpha_g}(i\omega)\right|^2 \Phi_{w_g}(\omega)$$

A convenient tool for easily evaluating the mean-square gust responses and for identifying the most significant frequency bands in these responses is a graph of $\Omega\Phi(\Omega)$ (on a linear scale) versus frequency (on a logarithmic scale). The resulting curve can be integrated linearly to yield the mean square value.

$$\overline{x^2} = \int_{-\infty}^{\infty} \Phi_x(\omega)\,d\omega = 2\cdot\text{Area}\cdot\ln 10 \tag{17}$$

By neglecting the contribution of gusts over 10 rad/sec an approximate value can be obtained for the basic aircraft's response to atmospheric turbulence with, for example, scale length of 305 meters.

$$\frac{\text{rms }\Delta n}{\sigma} \simeq 0.082 \text{ g-sec/m} \qquad \frac{\text{rms}\left(\frac{2qV}{c}\right)}{\sigma} \simeq 0.65^{\circ}/\text{m} \tag{18}$$

For the fully alleviated aircraft with instantaneously acting alleviation system (flap configuration, 15% SM),

$$\frac{\text{rms }\Delta n}{\sigma} \simeq 0 \qquad \frac{\text{rms}\left(\frac{2qV}{c}\right)}{\sigma} \simeq 2.3^{\circ}/\text{m} \tag{19}$$

Power spectral density plots of load factor and pitching motion versus frequency of the type previously discussed ($\Omega\Phi$ vs. Log ω) are presented for several degrees of alleviation. The aircraft is flap-configured and responses for three values of basic airplane static margins are examined. The scale of turbulence was chosen to be 305 m which sets the dominant gust wavelength in the phugoid range.

As the gain of the vertical gust alleviation system is increased, the contribution of the high frequency gusts to the r.m.s. load factor is progressively reduced to zero while at the same time the pitching motion in the short period frequency band is considerably increased. In order to get any significant acceleration alleviation, K_a must be near unity. On the other hand the introduction of even a little vertical gust alleviation results in a sizeable increase in pitching response.

The horizontal gust alleviation influences only the low frequency response and essentially eliminates the resonant peak at the phugoid frequency.

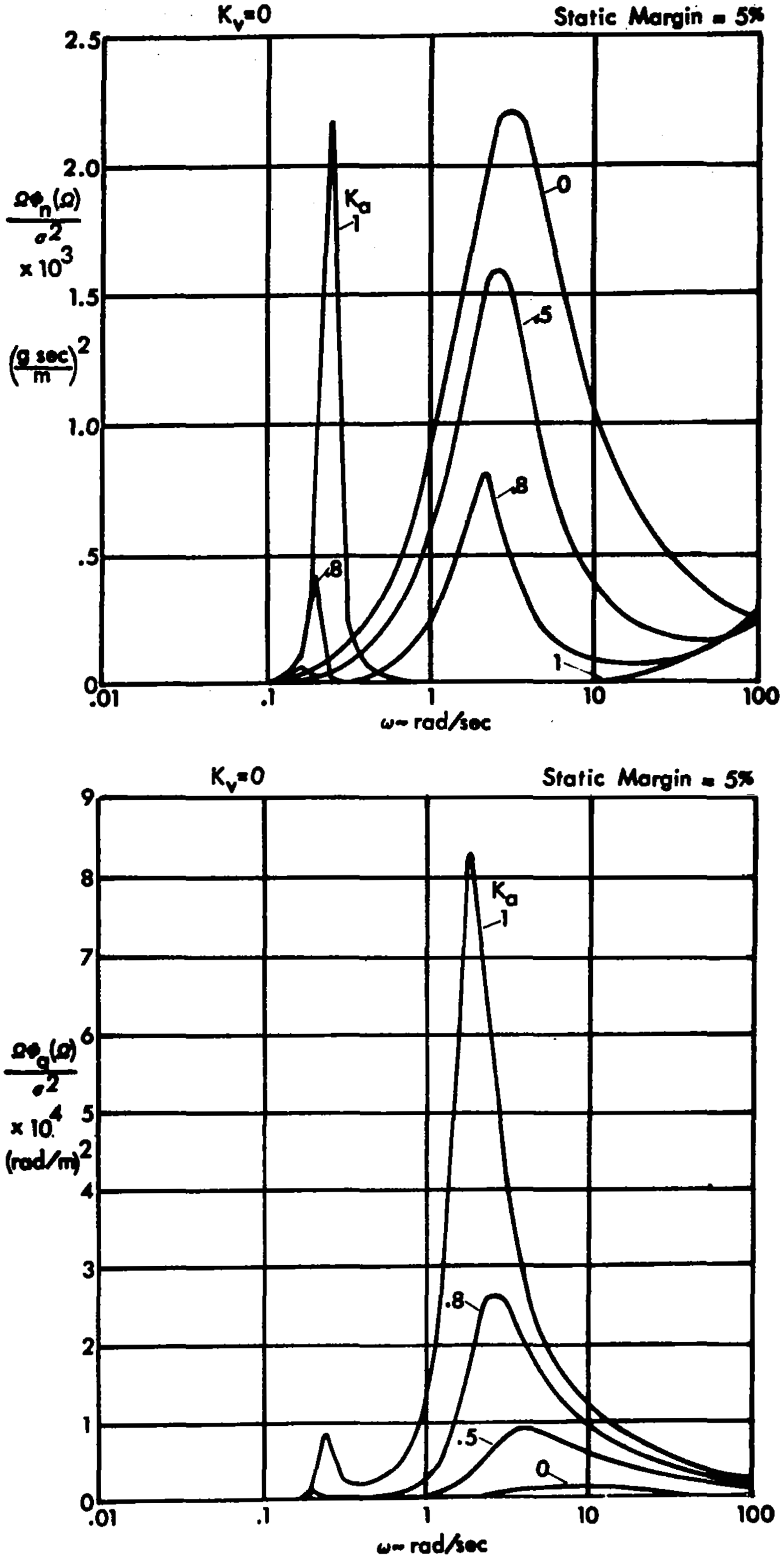
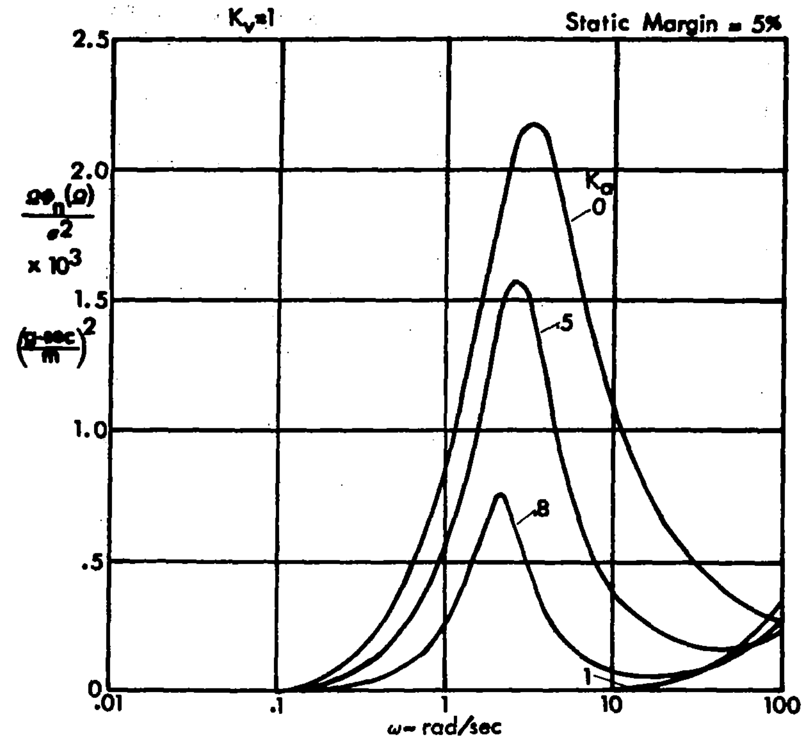
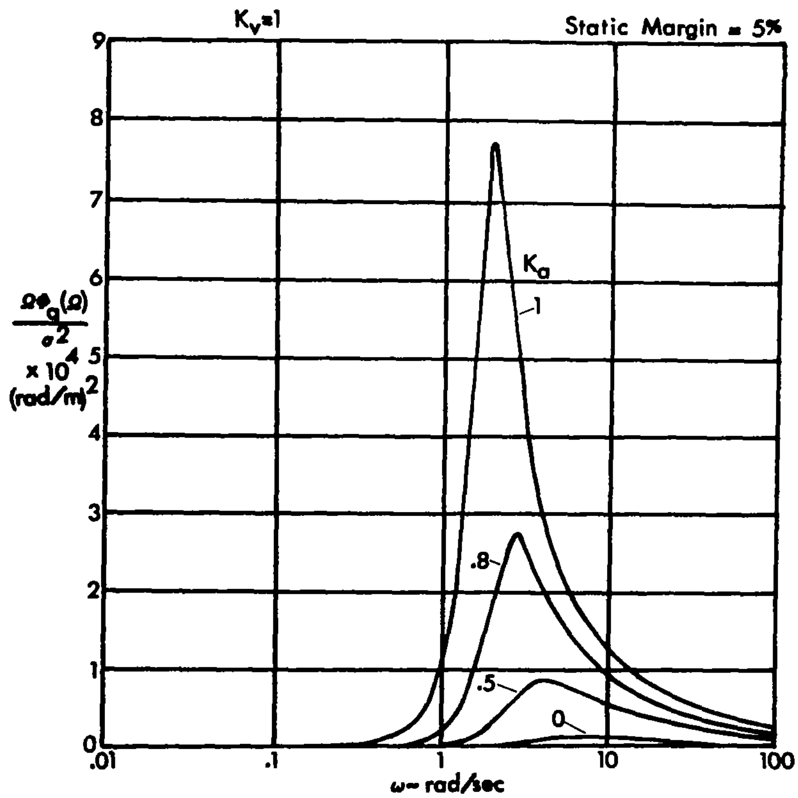


Figure 15 Gust Spectral Responses





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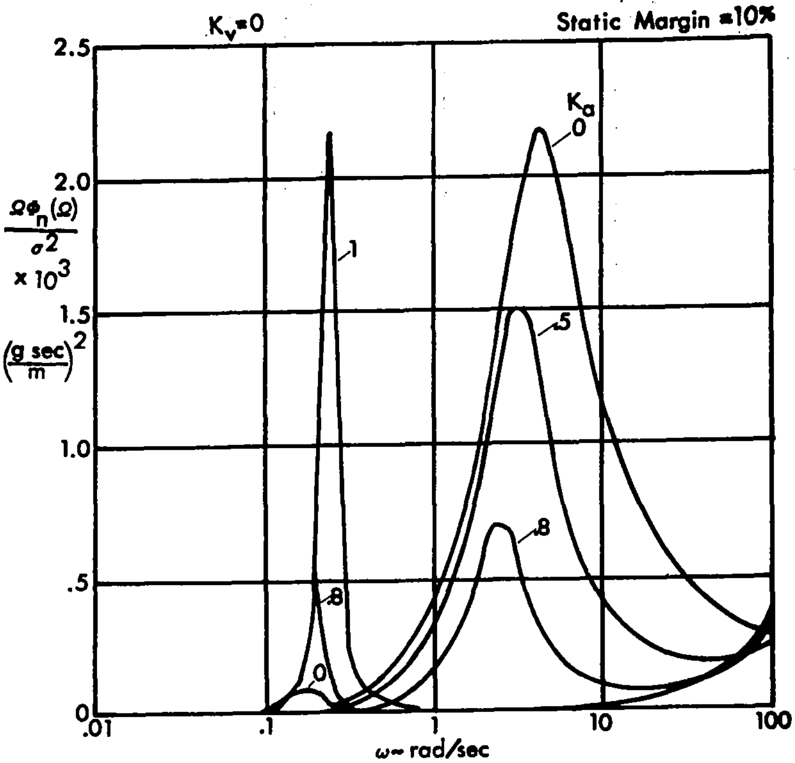

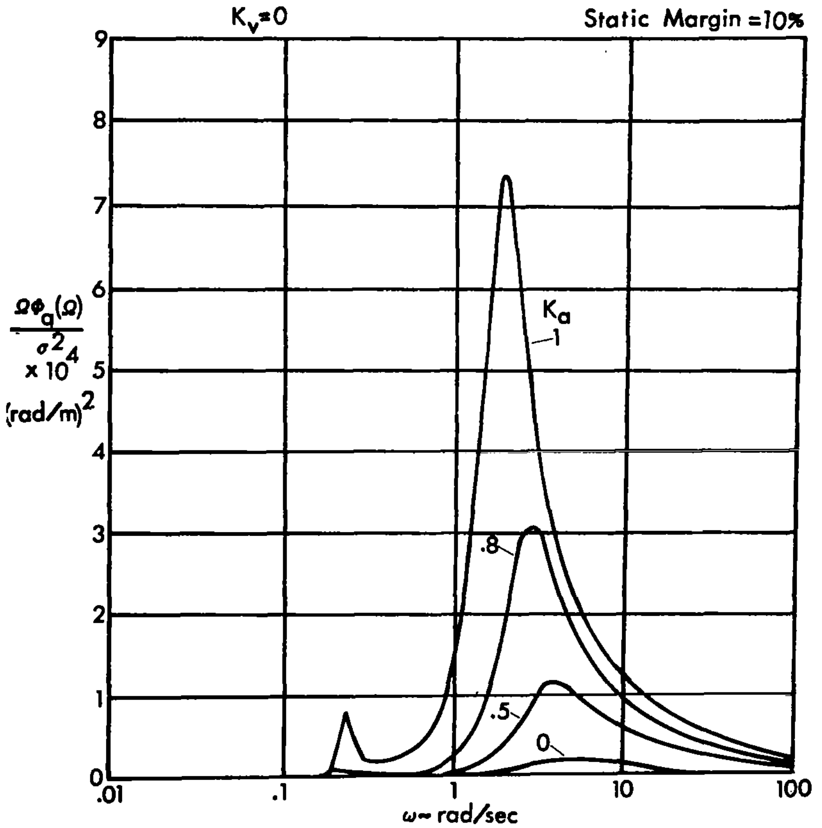

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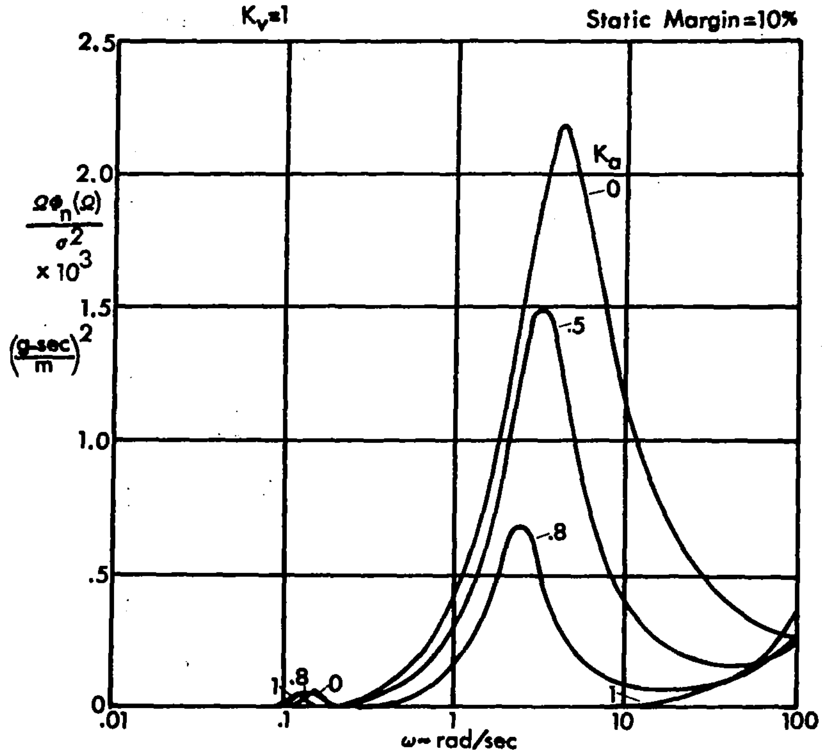

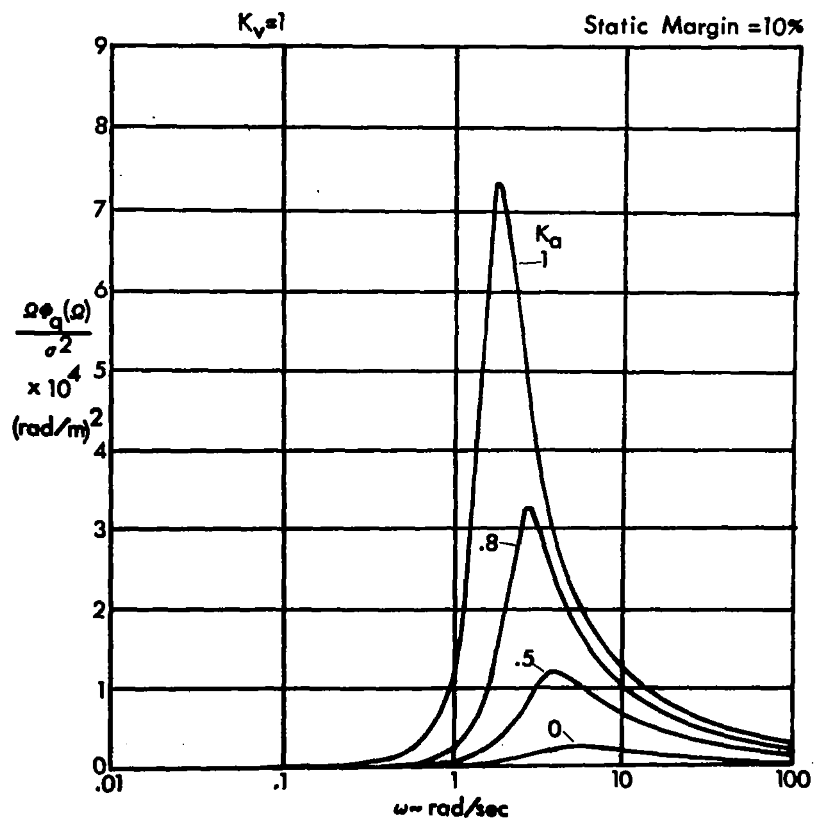

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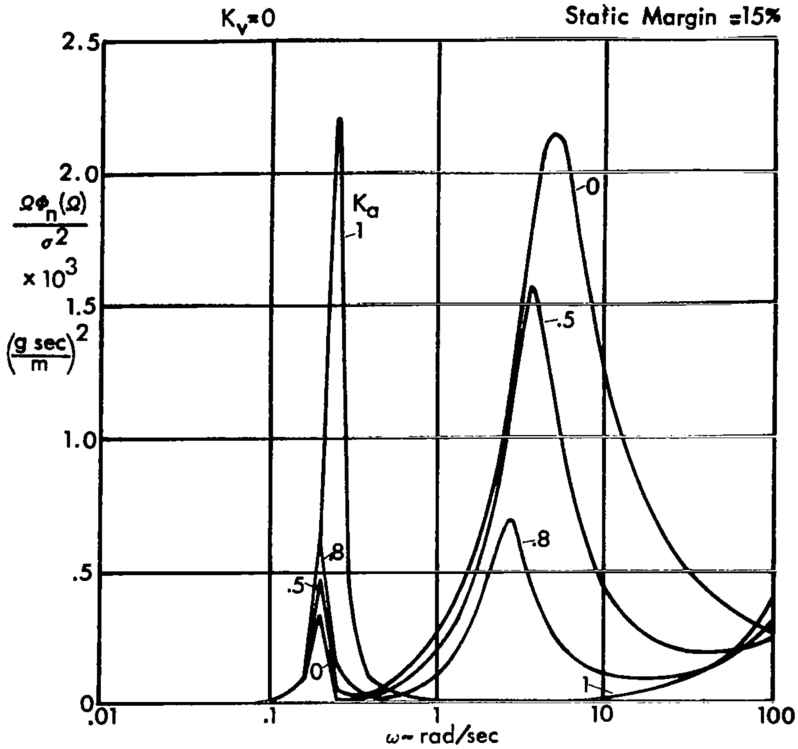

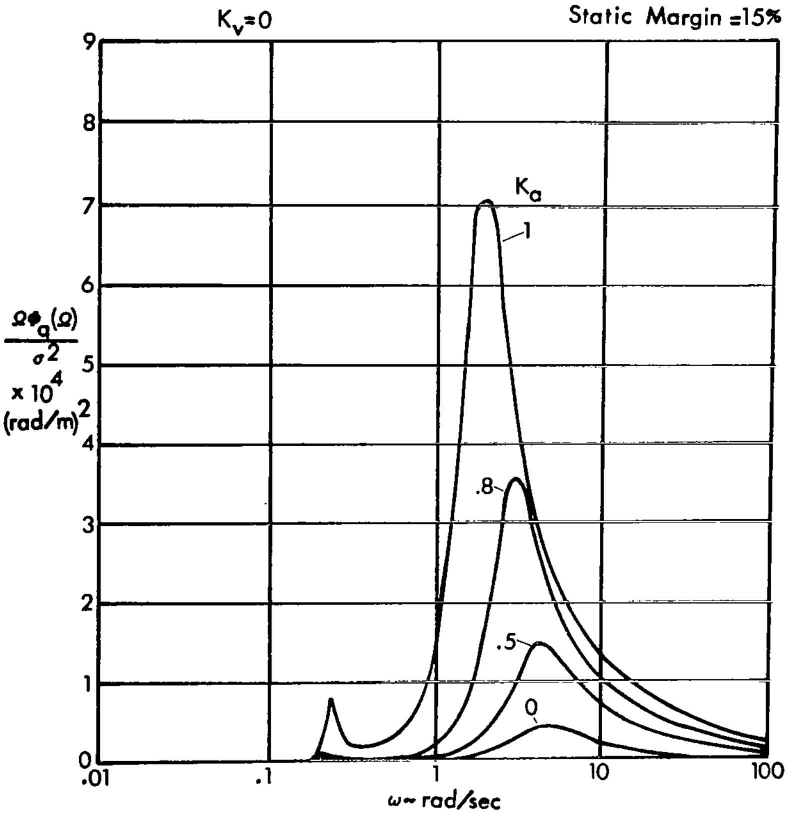

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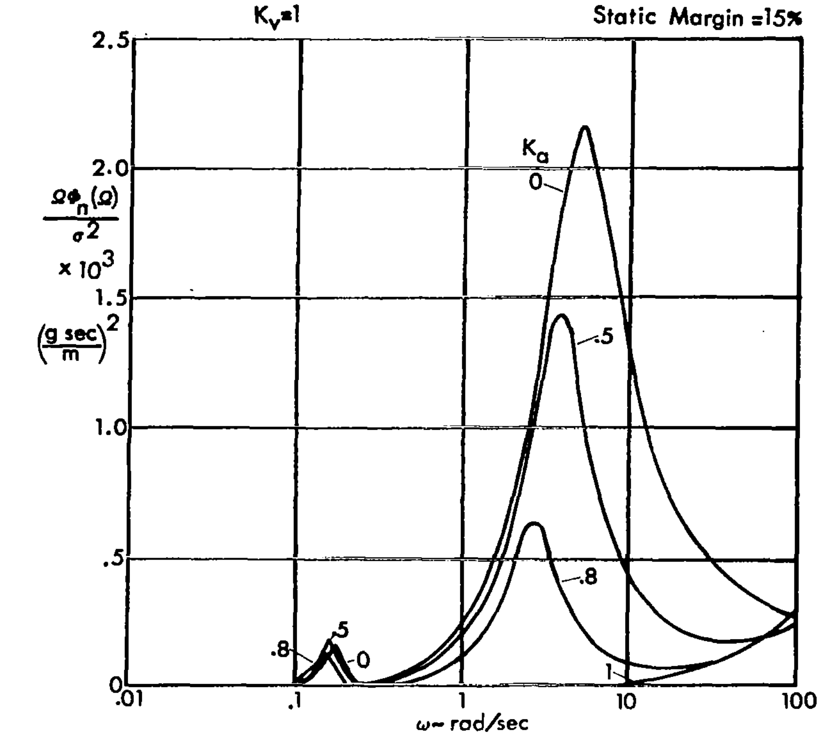

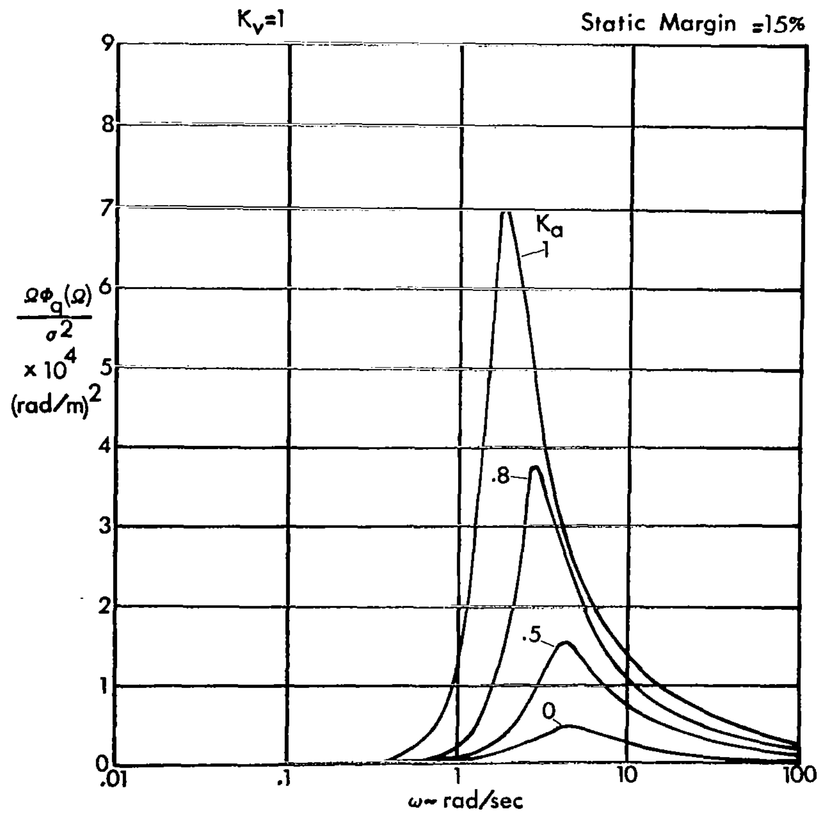

Figure 15 continued

VI. MANEUVERABILITY

Analysis of the effects of the gust alleviation system on the maneuverability of the aircraft shows that the elevator control effectiveness is almost completely lost. From a practical standpoint it is convenient to differentiate between short-term and long-term response to control deflections. The short-period mode is essentially a constant speed oscillation in which normal and pitching motions occur. Consequently, spring loading the flaps to attenuate normal accelerations produced by changes in dynamic pressure has no effect on the response to high frequency elevator deflections. Thus both the vertically-alleviated and the fully-alleviated aircraft have the same short term maneuverability.

The zero value of the effective lift curve slope is responsible for the loss of short-term control of flight path angle. Because of the constant wing lift, the flight path angle is governed by the tail loading. When the stick is pulled back, the reduced lift on the tail results in the aircraft sinking slowly. The reduced pitch stiffness produces a considerable increase in the attitude and angle-of-attack sensitivities to elevator deflections in the phugoid to short-period frequency band. For the Cessna 172 the angle of attack response is increased by a factor of eight so that even minor stick deflections might quickly lead to wing stall.

The long term response of the aircraft is determined by the phugoid mode where horizontal gust alleviation has an important effect. The constant static lift condition, around which the fully alleviated aircraft is designed, causes the phugoid to degenerate into a pair of real roots, one of which is zero, so that the flight path diverges with a slightly untrimmed elevator. On the other hand, if just vertical gust alleviation is implemented the phugoid is affected only moderately. When the elevator angle is varying slowly, the angle-of-attack perturbation is no longer small and is determined by the condition that pitch equilibrium is maintained. The steady-state angle of attack is multiplied by a factor of six and the response is flat up to the reduced short-period natural frequency. Another negative aspect of the gust alleviation system is the loss of the airspeed trimming capability of the elevator which is due to the zero lift curve slope. The steady-state velocity perturbation changes sign and is reduced by a factor of twenty.

The basic aircraft long term attitude response to a step of elevator occurs primarily at the phugoid frequency. A significant change in this response occurs when angle-of-attack gust alleviation is introduced. A pair of zeros in the attitude transfer function virtually eliminates the resonant peak of the phugoid so there is very little residual long-period oscillation in the transient response. The small pitch stiffness is responsible for the increased steady-state pitch angle.

The maneuverability of the vertical-gust-alleviated aircraft is recovered without significant change in stability characteristics or sacrifice of riding qualities if the elevator and flaps are coupled to offset the hinge moments produced by changes in angle-of-attack so that the flaps remain in static equilibrium when the elevator is operated. This might be accomplished, for instance, by installing a tab on the auxiliary wing or on the flap itself, or by articulating the whole auxiliary wing about its quarter-chord line. The feasibility of each depends on the coupling and hinge moment requirements. With the addition of a tab, the hinge moment equation for the flap system becomes:

$$C_{H_\delta}\delta + C_{H_\alpha}\alpha + C_{H_{\delta_t}}\delta_t = 0 \tag{20}$$

The tab-elevator gearing $\partial\delta_t/\partial\delta_e$ is determined by the condition that the flaps remain undeflected

$$\frac{\partial\delta_t}{\partial\delta_e} = -\frac{C_{H_\alpha}}{C_{H_{\delta_t}}}\left(\frac{\alpha}{\delta_e}\right)_0 \tag{21}$$

where $(\alpha/\delta_e)_0$ denotes the angle of attack transfer function of the basic aircraft. Since a mechanical linkage with a fixed gearing ratio is to be used, the control recovery condition cannot be satisfied at all elevator frequencies. This is of no great importance however, since $(\alpha/\delta_e)_0$ essentially is flat up to the short period frequency. At low frequencies,

$$(\frac{\alpha}{\delta_e})_0 = -\frac{C_{m_{\delta_e}}}{C_{m_\alpha}} = -1.52 \tag{22}$$

Between the phugoid and short period frequencies it is:

$$(\frac{\alpha}{\delta_e})_0 = -\frac{C_{m_{\delta_e}}}{C_{m_\alpha}} \frac{2\mu\mathcal{M}}{2\mu\mathcal{M} - C_{m_q}} = -1.11 \tag{23}$$

Therefore if the gearing is chosen to recover low frequency controllability it also should be satisfactory at high frequencies. This is confirmed by the frequency responses (Figures 16 to 19) and by analog simulation of the aircraft response to step elevator inputs (Figure 20). Control of airspeed and flight path angle are virtually fully recovered. Angle of attack and pitching response are increased at frequencies above the phugoid although this is somewhat offset by the reduced natural frequency of the short period motion. The main difference is to be found in the larger phugoid oscillation in angle of attack; however this is not perceived easily by the pilot.

In order to keep tab deflection small while allowing for full elevator travel the gearing ratio should be kept reasonably close to unity. When Equation (2) is applied to the tab-elevator gearing equation, the result is,

$$\frac{\partial\delta_t}{\partial\delta_e} = -\frac{C_{H_\delta}}{C_{H_{\delta_t}}} \frac{C_{L_\alpha}}{C_{L_\delta}} (\frac{\alpha}{\delta_e})_0 \simeq 9.0 \frac{C_{H_\delta}}{C_{H_{\delta_t}}} \tag{24}$$

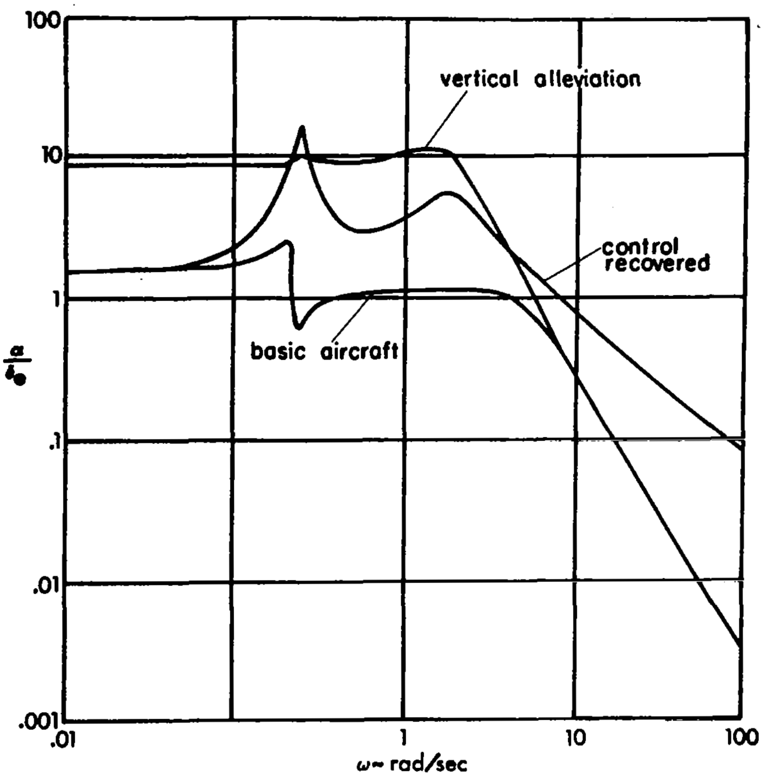

Figure 16 Angle of Attack Response to Elevator Deflection

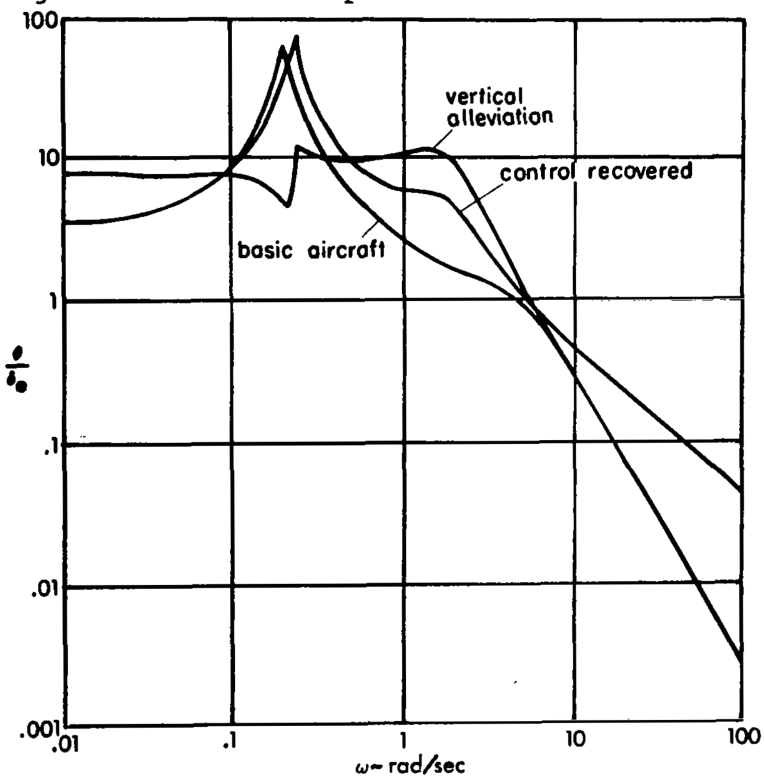

Figure 17 Pitch Response to Elevator Deflection

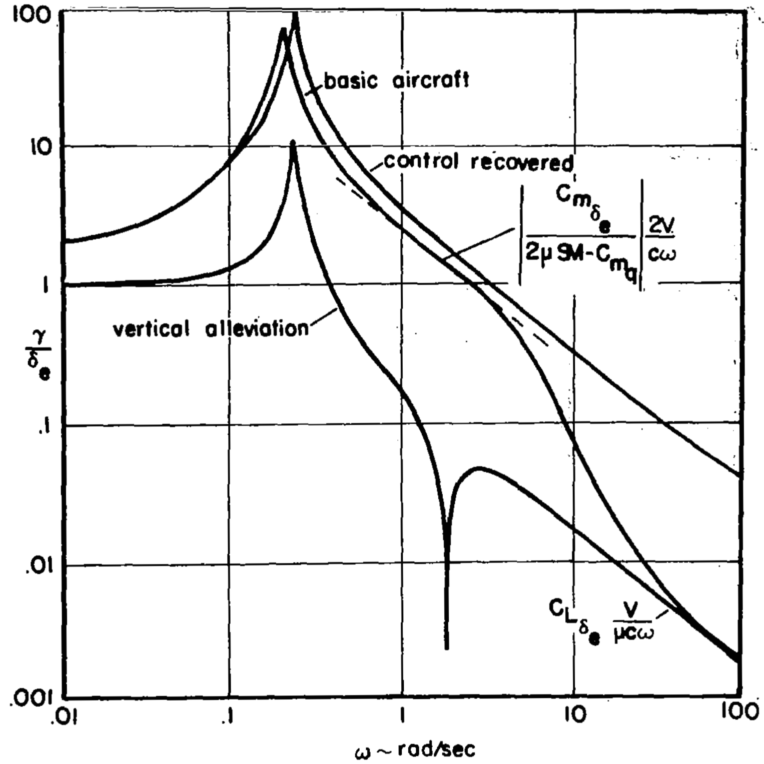

Figure 18 Flight Path Angle Response to Elevator Deflection

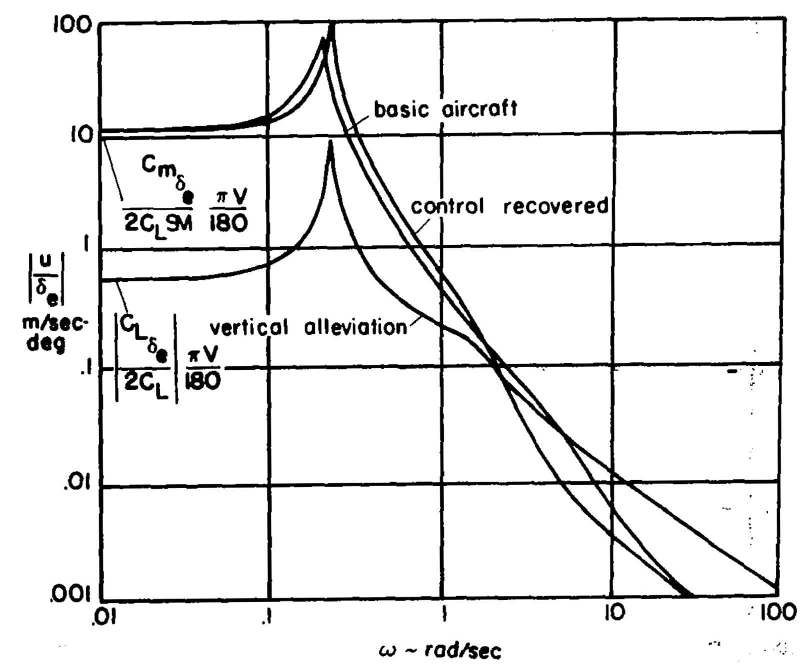

Figure 19 Velocity Perturbation Response to Elevator Deflection

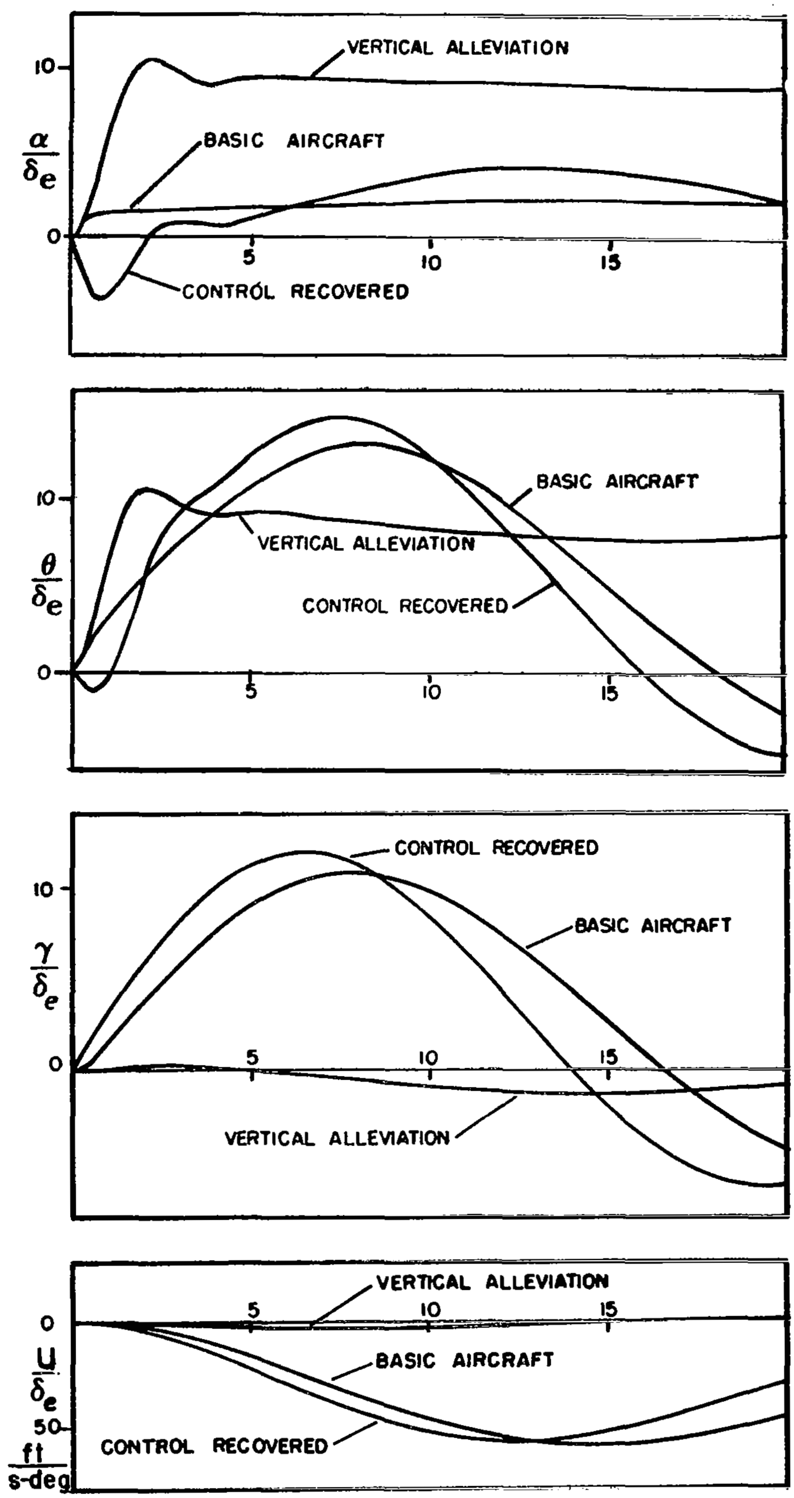

Figure 20 Time Responses to a Step Elevator Input

It is apparent that fitting tabs to the trailing edge of the flaps would require an excessively large gearing ratio between elevator and tabs for the present flap configuration which is not aerodynamically balanced. Inspection of the flap-auxiliary wing coupling relationships shows that fitting tabs to the auxiliary wings leads to a smaller gearing ratio,

$$\frac{\partial \delta_t}{\partial \delta_e} = - \frac{C^{aw}_{L_\alpha}}{C^{aw}_{L_{\delta_t}}} \left(\frac{\alpha}{\delta_e}\right)_0 \frac{C_{H_\alpha}}{C_{H_\alpha} - C^f_{H_\alpha}} \tag{25}$$

When the flap contribution to the total angle of attack derivative, $C^f_{H_\alpha}$, is small compared to that of the auxiliary wing, this relationship can be interpreted physically as meaning that stick deflections have no effect on the auxiliary wing position so that the flaps remain undeflected. The expression can be rewritten,

$$\frac{\partial \delta_t}{\partial \delta_e} \left(\frac{C_{L_{\delta_t}}}{C_{L_\alpha}}\right)_{aw} = \frac{C_{m_{\delta_e}}}{C_{m_\alpha}} = 1.52 \tag{26}$$

It is apparent from this relation that fitting a tab to the auxiliary wing might require too great a gearing ratio between tab and elevator to permit each to operate over the full extent of its linear range. The best solution is to tilt the entire auxiliary wing about its quarter chord line, thus changing its incidence, i_{aw}, and reducing the corresponding gearing ratio to the minimum value of

$$\frac{\partial i_{aw}}{\partial \delta_e} = \frac{C_{m_{\delta_e}}}{C_{m_\alpha}} = 1.52 \tag{27}$$

This effectively means tilting the auxiliary wing by an amount equal to the steady-state change in angle of attack due to elevator deflection. If $C^f_{H_\alpha}$ is not neglected in this expression, the result is a slight tilting past the free stream angle to offset this torque.

VII. SENSITIVITY ANALYSIS

Because of the large uncertainties in the estimation of the flap hinge moment parameters and the poor load factor attenuation of the underalleviated system, but more importantly, because of the generally unacceptable and possibly dangerous stability characteristics resulting from overalleviation, it is necessary to determine the relative sensitivities of the general alleviation constant K_a to the various system parameters and to use these to devise methods of safely implementing full alleviation. In addition, knowledge of the behavior of the short period roots to uncertainties in flap pitching moment and downwash derivatives permits the designer to make full use of various control surface geometries and to determine the need to couple elevator with the flap to modify the pitching tendency. Recall that

$$K_a = \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{C_{L_\delta}}{C_{L_\alpha}} \tag{28}$$

The augmented flap hinge moment derivative in the presence of the auxiliary wing is

$$C_{H_\alpha} = C^f_{H_\alpha} - \frac{1}{2}\Gamma \frac{(Sb)aw}{(Sc)_f} C^{aw}_{L_\alpha} \tag{29}$$

where $\Gamma = \partial\delta_{aw}/\partial\delta$. The relative sensitivity of K_a to C_{H_δ} can be computed by taking the ratio of logarithmic partial derivatives:

$$\frac{C_{H_\delta}}{K_a} \frac{\partial K_a}{\partial C_{H_\delta}} = \frac{\partial L_n K_a}{\partial L_n |C_{H_\delta}|} = - \frac{\partial L_n |C_{H_\delta}|}{\partial L_n |C_{H_\delta}|} = -1 \tag{30}$$

Similarly, the sensitivity to $C^f_{H_\alpha}$ is given by:

$$\frac{C^f_{H_\alpha}}{K_a}\frac{\partial K_a}{C^f_{H_\alpha}} = \frac{K^f_a}{K_a} \simeq .025 \tag{31}$$

Because of the very small floating tendency of the flaps, the aircraft is virtually insensitive to uncertainties in the flap hinge moment derivative with respect to changes in angle of attack. Thus, a 100% error in $C^f_{H_\alpha}$ only produces a 2.5 % uncertainty in K_a.

The same cannot be said, however, of the effect of uncertainties in the determination of the aerodynamic stiffness of the flap. Since the auxiliary wings do not contribute to the overall aerodynamic stiffness of the system, K_a is inversely proportional to C_{H_δ}. Thus, a 1% underestimation of C_{H_δ} results in the same percentage overalleviation. Although the uncertainty in the theoretical estimate of this derivative can be reduced to some extent by experimental measurement of the appropriate hinge moment coefficient, it must be known within 10% of its exact value if K_a is to remain within the acceptable range of 0.9 to 1.1.

However, the uncertainties in C_{H_δ} theoretically can be compensated by varying the auxiliary wing size L or the gearing ratio Γ. If the auxiliary wing aspect ratio is kept constant,

$$\frac{L}{K_a}\frac{\partial K_a}{\partial L} = 3(1-\frac{K^f_a}{K_a}) \simeq 3 \tag{32}$$

If only the span is changed, the dependence of the lift curve slope on the aspect ratio causes the relative sensitivity to be slightly in excess of 2.

$$\frac{L}{K_a}\frac{\partial K_a}{\partial L} = (2+\frac{\partial L_n C^{aw}_{L_\alpha}}{\partial L_n AR})(1-\frac{K^f_a}{K_a}) \tag{33}$$

It would appear that one could make use of the large sensitivities of K_a to variations in auxiliary wing size to overcome uncertainties in C_{H_δ}. However, it is rather impractical to trim the auxiliary wing dimensions and, fortunately, adjustment of the gearing ratio offers a more realistic compensation technique, and is fairly easy to implement.

$$\frac{\Gamma}{K_a} \frac{\partial K_a}{\partial \Gamma} = 1 - \frac{K_a^f}{K_a} \simeq 1 \qquad (34)$$

Thus, the absolute sensitivities of K_a to the gearing ratio and to C_{H_δ} are the same.

In order to trim the aircraft to the full vertical alleviation condition, it first should be flown with a small enough gearing ratio so that it is effectively in an underalleviated condition (with the pushrod attached fairly close to the flap hinge). The gearing ratio then should be increased progressively by increasing the flap moment arm until the time to halve the pitching response of the aircraft to a step of elevator increases to about 0.9 second, corresponding to that for the flap-configured Cessna 172. This procedure accommodates small uncertainties in C_{m_δ} and $\partial\varepsilon/\partial\delta$, which have a marked effect on the short-period natural frequency but relatively little effect on the time to halve amplitude. As a final check on the value of K_a, the phugoid oscillation should practically disappear from this response, the indicated airspeed should remain constant and the altimeter should show no short term perturbation. This entire test program, of course, should be undertaken without having the elevator coupled to the auxiliary wing. Finally, the incidence of the auxiliary wing should be adjusted so that the flap remains undeflected in the absence of gusts.

The following analysis investigates the effect of flap pitching moment and downwash derivatives on the short period dynamic stability of the vertically alleviated airplane. For $K_v=0$ and $K_a=1$ the short-period natural frequency and damping ratio are determined by:

$$\omega_{n_{sp}}^2 = - \frac{C'_{m_\alpha}}{I_y} \tag{35}$$

$$2\zeta\omega_{n_{sp}} = - \frac{C_{m_q}}{2I_y} \left(1+ \frac{D\varepsilon}{D\alpha}\right) \tag{36}$$

The augmented pitch stiffness, C'_{m_α} takes into account the modified lift distribution of the wing and the change in the downwash at the tail resulting from flap deflection.

$$C'_{m_\alpha} = C_{m_{\alpha_w}} - \frac{C_{L_\alpha}}{C_{L_\delta}} C_{m_{\delta_w}} + \left(1- \frac{D\varepsilon}{D\alpha}\right) C_{m_{\alpha_t}} \tag{37}$$

where $$\frac{D\varepsilon}{D\alpha} = \frac{\partial\varepsilon}{\partial\alpha} - \frac{C_{L_\alpha}}{C_{L_\delta}} \frac{\partial\varepsilon}{\partial\delta} \tag{38}$$

The short-period roots can be traced in the complex frequency plane as a function of $\frac{\partial\varepsilon}{\partial\delta}$ and $C_{m_{\delta_w}}$, yielding two sets of curves.

As shown in Figure 21, curves of constant flap downwash, $\frac{\partial\varepsilon}{\partial\delta}$, are simply straight lines parallel to the imaginary axis while those of constant flap-wing pitching moment are slightly curved. The intersection of two members of different families yields a pair of complex conjugate roots representing the oscillatory mode corresponding to a particular set of flap characteristics. If the two curves do not intersect, the short period mode is degenerated into a pair of real roots which are symmetrical about the corresponding $\frac{\partial\varepsilon}{\partial\delta}$ line since $\zeta\omega_{n_{sp}}$ remains the arithmetic mean of the roots. It should be noted that the amplitude of the pitching response to vertical gusts of high frequency is equal to twice the distance between the roots (or the mean of the roots if the mode is degenerated) and the line $\frac{D\varepsilon}{D\alpha} = +1$ which represents optimal vertical gust alleviation. This distance

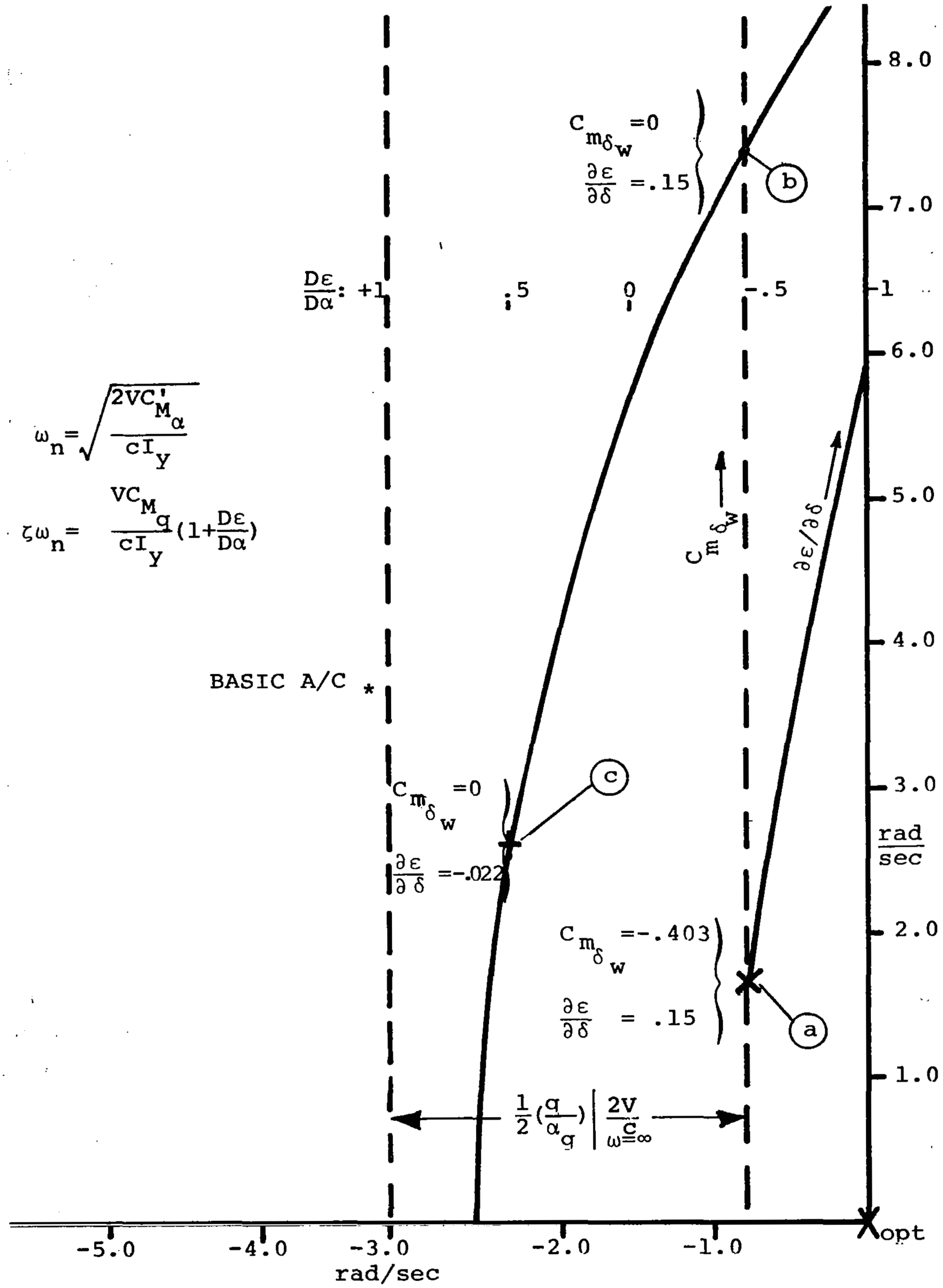

Figure 21 Effects of Flap Downwash and Pitching Moment on the Short-Period Roots of the Vertical-Gust Alleviated Aircraft

is simply:

$$d = \frac{1}{2}\left(\frac{q}{\alpha_g}\right)\left|\frac{2V}{c}\right._{\omega\to\infty} \tag{39}$$

Several comments are in order. By somehow decreasing the magnitude of the adverse nose-down pitching moment of the flaps, substantial increases in the natural frequency of the short-period mode can be achieved without changing the high frequency pitching response to gusts or time to halve amplitude of the damping exponentials. For example, reducing $C_{m_{\delta_w}}$ to zero shifts the roots from point (a) to point (b) in Figure 21 and increases the natural frequency by a factor of four. This reduction in flap pitching moment in principle could be obtained by proportional coupling of the elevator (or a portion of the elevator) to the flaps so that an upward gust would produce up flap and down elevator deflections. Then if the flap downwash derivative could be reduced until the damping of the mode increased to the desired level, substantial reductions in pitching response could be achieved while also improving the stability characteristics of the alleviated aircraft. For $C_{m_{\delta_w}}=0$ and $\frac{D\varepsilon}{D\alpha}=.5$ corresponding to $\frac{\partial\varepsilon}{\partial\delta}=-.022$ (point (c)), the pitching and load factor responses to high frequency gusts can be reduced by a factor of 3 compared with point (a) while the natural frequency is approximately doubled and the damping ratio is increased to .5, which is the same as the basic aircraft.

It should be pointed out that flaps with $\frac{\partial\varepsilon}{\partial\delta}< 0$, that is creating upwash on the stabilizer when lowered, produce an attenuation in the pitching response relative to the basic aircraft ($\frac{D\varepsilon}{D\alpha}$ = .37, which is to the right of point (c) in Figure 21).

If the flexibility of altering the flap geometry is not available to the designer and elevator coupling is too complex, he is left with the option to couple the ailerons to the flaps to reduce the magnitude of the downwash while maintaining a stable aircraft. The results of this process are given in Figure (22) which shows the locus of the short period roots of an alleviated aircraft in which the gearing between the flaps and symmetric mode ailerons is varied. As can be seen, reducing the downwash derivative $\partial\varepsilon/\partial\delta$ by positive coupling (flaps and ailerons in phase) reduces the pitching response slightly but quickly leads to degenerate and unstable modes. Because of the strong adverse pitching moment of the flap, the range of permissible aileron coupling is severly limited and offers little advantage.

However, test data for a model of the Cessna 172 generated too late in the program to be used in any but the sensitivity analysis are applied to Figure 22 to provide a second locus as a function of aileron coupling, which provokes considerably different conclusions. The very low tail-off pitching moment derivative with flap deflection and a slightly smaller value for $\partial\varepsilon/\partial\delta$ yields a curve which in essence is shifted upwards by about 4 1/2 units and to the right by one quarter unit. Because of the resulting higher frequencies, the sensitivity of the roots to aileron coupling is reduced substantially. Most importantly, aileron coupling not only becomes beneficial, but the ailerons should be considered the principal control surfaces and moderate flap coupling is helpful in avoiding the real root with long time constant introduced by using the ailerons and to yield short period roots that satisfy handling criteria.

With the use of ailerons, the downwash affords a more favorable high frequency pitching response. If the aileron-flap coupling ratio, G', is 5, there is no change in this portion of the response when the vertical gust alleviation system is implemented. Even with no coupling, the response is only doubled, compared to the tripled value for the analytical model.

It should be noted that test data are not available for the aileron parameters. These were calculated by methods presented in Reference 6, as were the flap parameters. Since the tail-off flap pitching moment derivative with deflection was twice

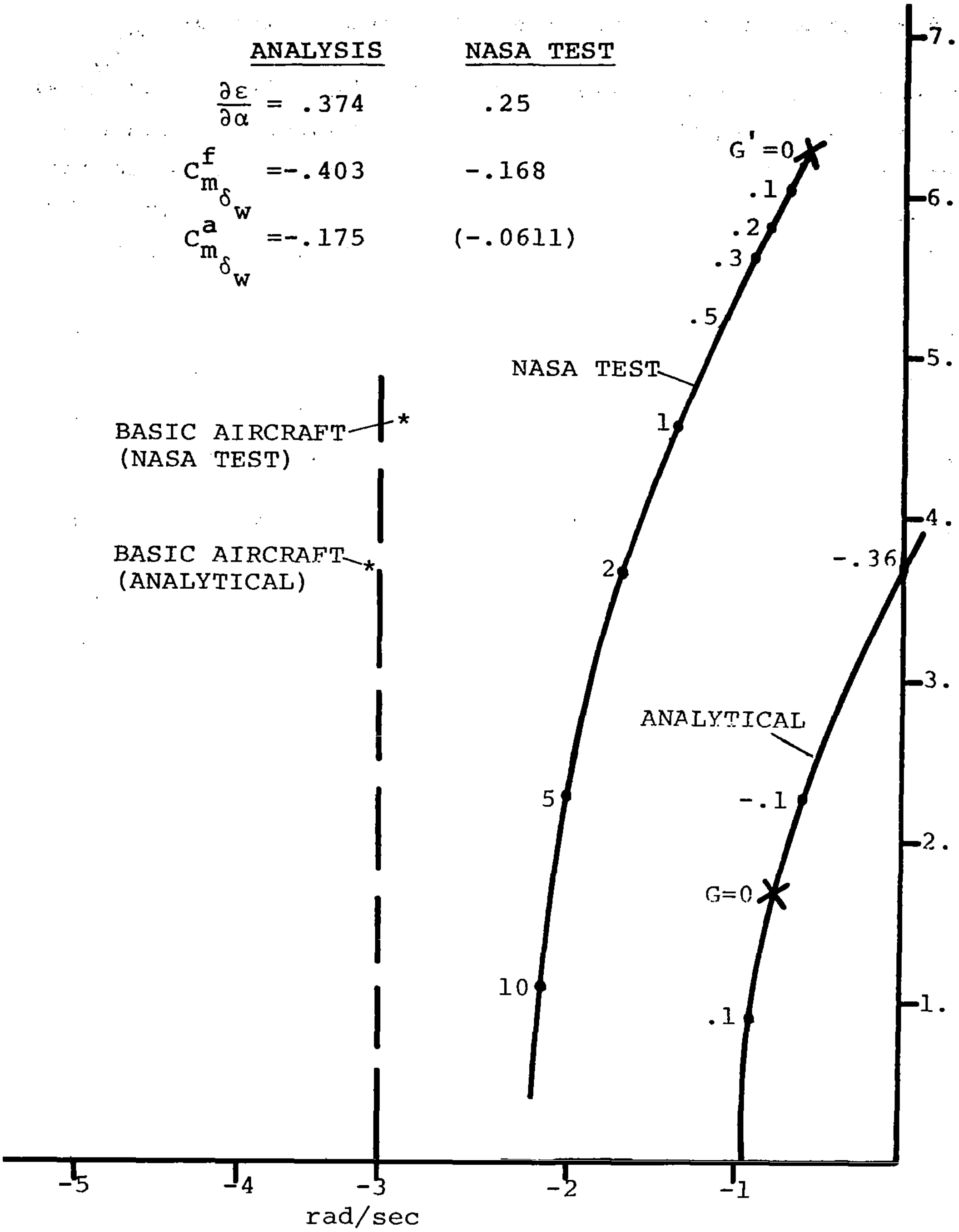

Figure 22 Effect of Aileron Coupling on Short Period Roots

the magnitude of the measured value, a similar relationship was assumed for the aileron derivative. The test value of $C^{a}_{m_{\delta_w}}$ given in Figure 22 was derived by scaling the value computed at the quarter m.a.c. (-.235) by the ratio of tested and computed flap derivatives at this point (.25/.485) and then referencing the derivative to the 33 percent center of gravity location by making use of $C_{L_{\delta_a}}$ and the eight percent m.a.c. moment arm.

In conclusion, introduction of the test data emphasizes the need for accurate measurement of the flap/aileron characteristics. Short-period roots are quite sensitive to uncertainties in these parameters, particularly to $C_{m_{\delta_w}}$. If the full scale aircraft reflects the model data, the short-period mode would be lightly damped if flaps are used to alleviate gusts. If ailerons are employed, the roots are real and stable. Use of the ailerons with modest coupling of the flaps would yield roots preferable with regard to handling qualities.

VIII. CONCLUSIONS AND RECOMMENDATIONS

A passive aeromechanical gust alleviation system is developed (specifically for a Cessna 172) using existing lift and control surface regions.

Conditions for optimal gust alleviation are developed and shown to be incompatible with normal stability requirements. A stability analysis in the form of root loci is conducted to identify acceptable flap characteristics.

The efficiency of the gust alleviation system is examined in terms of load factor and pitching responses to gusts over the broad frequency spectrum. The Von Karman model of atmospheric turbulence is then applied in order to investigate the relative importance of these responses at the short period and phugoid frequencies. As predicted, the considerable attenuation in normal acceleration is accompanied by a substantial increase in pitching response. The effects of the alleviation system on the maneuverability of the aircraft are then analyzed and the concept of coupling the pilot's stick to the flap system is developed to explore the possibility of recovering some of the controllability normally associated with elevator movement. Finally a sensitivity analysis is conducted to determine the designer's ability to compensate for uncertainties in knowledge of flap/aileron hinge moment characteristics.

The analysis has shown that passive gust alleviation is practical. Gust loads can be absorbed well at frequencies between the phugoid and short-period mode where the important gust energy is concentrated. The major task is to determine the flap/aileron characteristics on which to base the auxiliary wing design. In order to minimize the size and additional weight of the sensors, the lift control surfaces should be properly balanced about their hinge line in order to provide the largest possible aerodynamic floating tendency. Also a fast flap-auxiliary wing response is necessary to achieve good alleviation at higher frequencies (near the short-period and above). Thus the system should be made as light as structurally feasible while also maintaining the damping ratio within acceptable bounds. For a given alleviation constant (but variable gearing ratio) the inertia and aerodynamic damping (referred to the flap hinge line) contributed by geometrically similar auxiliary wings are inversely proportional to their size and size squared respectively. Thus the sensors should be made as large as possible subject to limitations of weight, drag and linkage flexibility.

The pitching moment and downwash characteristics of the lift control surfaces are also shown to have a large effect on the aircraft's response to gusts above the short-period frequency. The pitching motion of the aircraft can be aggravated by the alleviation system if the nose down pitching moment of the flaps due to increased camber of the wing is excessive. Alleviation of this motion as well as improved load factor attenuation can be achieved if the adverse flap pitching moment is small enough to allow substantial reduction in the downwash produced by lowering the trailing edge surfaces without jeopardizing the aircraft stability. This reduction in downwash can be achieved in practice by coupling symmetric mode ailerons to the flaps.

Maneuverability is virtually eliminated by the alleviation system but can be fully recovered by proper auxiliary wing-elevator coupling and even enhanced by the effective introduction of direct lift control.

Finally, residual uncertainties in flap hinge moment parameters can be accomodated by adjustments in the flap-auxiliary wing gearing. A safe and practical procedure for implementing the alleviation system is presented.

RECOMMENDATIONS

In view of the importance of flap-auxiliary wing parameters on the performance of the gust alleviation system a general parameter optimization program could further enhance the design process. Also the low speed handling characteristics of the aircraft at take-off and landing need to be examined in order to determine whether or not the alleviation system should be disengaged and if necessary to design a cockpit activated decoupling or locking device.

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APPENDIX A

Cessna 172 Paramters

The Cessna 172 parameters used in this study are presented in the table below. They were derived by the procedures given in the Air Force DATCOM, except for the rotary derivatives which were taken from NASA CR-1975 in which Cessna 182 parameters were developed. These aircraft are sufficiently alike to warrant the use of these data. Very near the end of the program, NASA furnished data based on wind tunnel tests of a six-foot span model of the Cessna 172. This provided measurements of flap characteristics which are difficult to estimate, and confirmed a number of the computed stability derivatives.

TABLE A-1

Cessna 172 Parameters

	Program	NASA Test
C_L	0.416	
C_{L_α}	5.50	5.50
$C_{L_{\alpha_w}}$	5.07	5.03
$C_{L_{\alpha_t}}$	0.66	0.62
$\frac{\partial \varepsilon}{\partial \alpha}$	0.37	0.25
$C_{L_{\dot{\alpha}}}$	1.49	
$*C_{L_q}$	3.88	
$*C_{m_\alpha}$	-0.83	
$*C_{m_{\dot{\alpha}}}$	-4.36	
$*C_{m_q}$	-11.40	

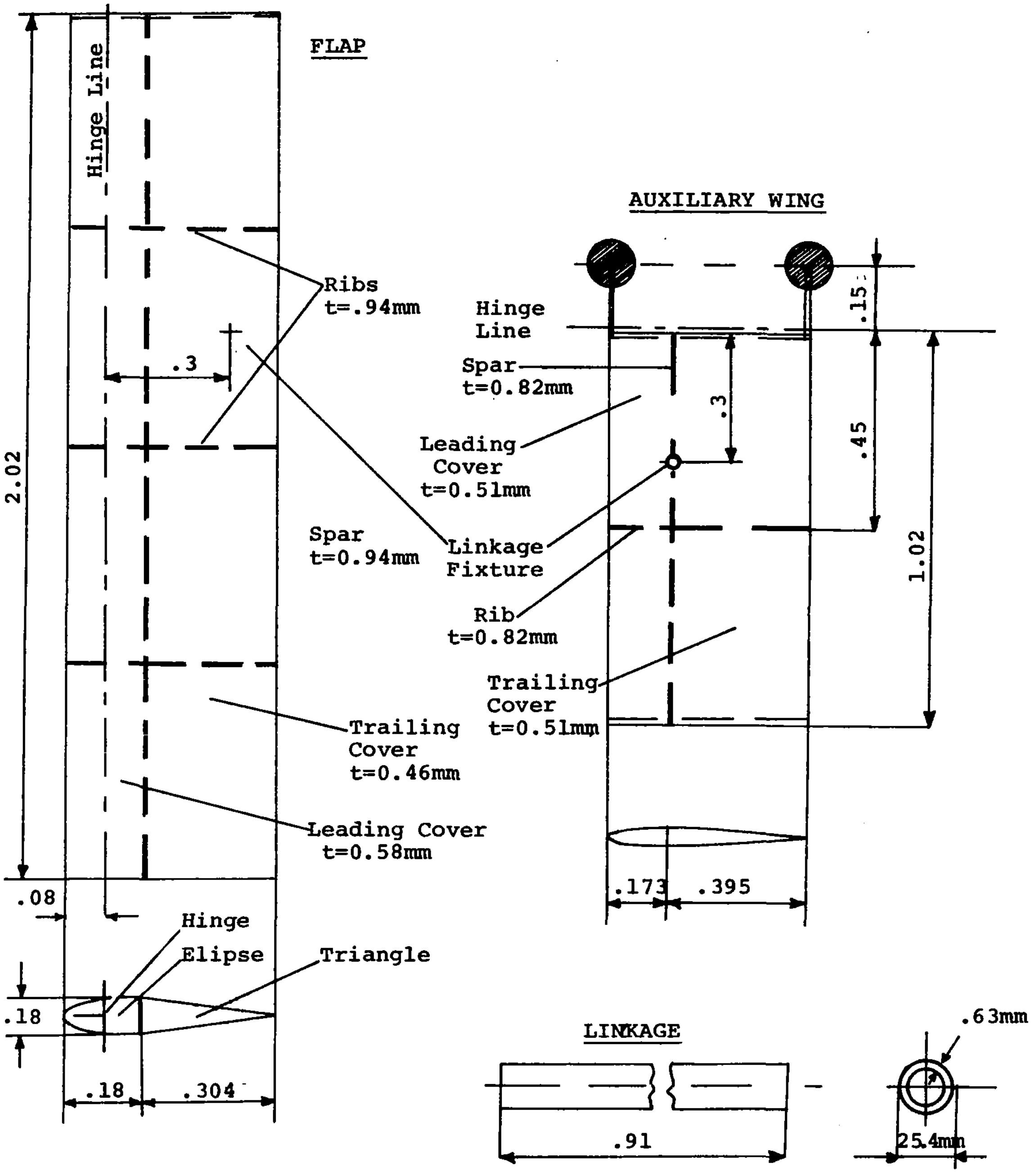


Figure A-1 Flap/Auxiliary Wing Geometry

TABLE A-1 cont.

	Program
$C_{m_{\delta_e}}$	-1.26
C_{D_α}	0.25
C_{T_u}	-0.116
μ	99.9
I_y	287 (1581 $Kg\text{-}m^2$)
V	59.13 m/sec
m	1000 kg
c	2.30 m

Flap Characteristics

	flap	aileron	flap-aileron	NASA Test (flap)
$C_{L_{\delta_w}}$	1.03	.75	1.78	1.10
$C_{L_{\dot{\delta}}}$	.598	-.140	.458	-
*$C_{m_{\delta_w}}$	- .403	-.175	- .578	-.168
*$C_{m_{\dot{\delta}}}$	-1.75	.408	-1.34	-
$\frac{\partial\varepsilon}{\partial\delta}$	.150	-.035	.115	.15
C_{D_δ}	.034	.034	.034	-
C_{H_δ}	-.46	-	-	-.40
C_{H_α}	-.068	-	-	-

* Values shown for CG at 33% MAC, static margin 15% MAC

The parameters $C_{L_{\delta_w}}$, $C_{m_{\delta_w}}$, and $\frac{\partial\varepsilon}{\partial\delta}$ have approximately linear relationships among the configurations; that is, flap values plus aileron values equal flap-aileron values. (Ref. 6, Sec.4.4 and 6.1). The flap downwash lag derivatives are assumed due only to tail loads and are found from the formulations given in Appendix E. As a comparison with the DATCOM, the flap induced downwash at the tail also was computed from NACA TR 648. The derivative $\partial\varepsilon/\partial\delta$ was found to be .25 for the inboard flap and .178 for full span flaps. Use of the DATCOM-derived figures assures conservative stability estimates.

The flaps and auxiliary wing physical parameters are given below, and are followed by the computation of the system natural frequency. The mass balance for the flap has been neglected in computing the flap moment of inertia. However, its contribution to the moment of inertia of the system is negligible.

MOMENT OF INERTIA OF FLAP ABOUT ITS HINGE LINE

1) Spar

Weight = .33 kg
M.I. = .0037 kg-m^2

2) Ribs

Weight = .31 kg
M.I. = .0079 kg-m^2

3) Leading Cover

Weight = 1.27 kg
M.I. = .0120 kg-m^2

4) Trailing Cover

Weight = 1.65 kg

M.I. = .120 kg-m^2

Weight of Flap = 3.57 kg

M.I. of FLap = .143 kg-m^2

MOMENT OF INERTIA OF THE AUXILIARY WING ABOUT ITS HINGE LINE

1) Spar

Weight = .12 kg

M.I. = .053 kg-m^2

2) Ribs

Weight = .11 kg

M.I. = .056 kg-m^2

3) Leading Cover

Weight = .55 kg

M.I. = .232 kg-m^2

4) Trailing Cover

Weight = 1.21 kg

M.I. = .51 kg-m^2

5) Linkage

Weight = .36 kg

M.I. = .034 kg-m^2

6) Counterbalances

Weight = 7.10 kg

M.I. = .25 kg-m^2

Weight of the Auxiliary Wing = 9.45 kg

Moment of Inertia about Hinge Line = 1.14 kg-m^2

The moment of inertia of the flap-auxiliary wing system about the flap hinge line depends upon the gearing ratio,

$$I_{f,aw} = I_f + \Gamma^2 I_{aw} \tag{A-1}$$

The damping term in the hinge moment equation is primarily a function of the "roll-damping" developed by the auxilіry wing. Simple strip theory can be used to estimate this coefficient. If x represents the spanwise ordinate along this wing,

$$H_{aw} = -\int_0^{b_{aw}} q\, c_{aw} c_{L_\alpha} \frac{\dot{\delta}_{aw}}{V} x^2 dx \tag{A-2}$$

$$= -\frac{1}{3} q (Sb^2)_{aw}\, c_{L_\alpha} \frac{\dot{\delta}_{aw}}{V} \tag{A-3}$$

where c_{L_α} is the section lift curve slope of the auxiliary wing. The moment and angle transmitted to the flap hinge are:

$$\delta_{aw} = \Gamma \delta_f \qquad H = H_f + \Gamma H_{aw}$$

Then,

$$C_{H_{\dot{\delta}}} = \frac{1}{q(Sc)_f} \frac{\partial H}{\partial \left(\frac{\dot{\delta}_f c}{2V}\right)} = C^f_{H_{\dot{\delta}}} - \frac{2}{3}\Gamma^2 \frac{(Sb)_{aw}}{(Sc)_f} \frac{b^{aw}}{c} c_{L_\alpha} \tag{A-4}$$

or,

$$C_{H_{\dot{\delta}}} \simeq -.6 - 6.2 = -6.8$$

A more accurate result is obtained particularly in the case of low aspect ratio vanes if the expression for the roll damping of a wing is used instead,

$$C_{H_{\dot{\delta}}} = C_{H_{\dot{\delta}}}^{f} + 4\Gamma^2 \frac{(Sb)_{aw}}{(Sc)_f} \frac{b^{aw}}{c} C_{\ell_p} \qquad \text{(A-5)}$$

where C_{ℓ_p} is the roll derivative based on twice the auxiliary wing span.

This method yields a much smaller value for the damping contributed by the vane,

$$C_{H_{\dot{\delta}}} = -.6 - 1.9 = -2.5$$

From the foregoing analysis and consideration of the flap deflection stiffness, the system characteristic equation can be written,

$$-I_{f,aw}s^2 + C_{H_{\dot{\delta}}}s + C_{H_\delta} = 0 \qquad \text{(A-6)}$$

The natural frequency of this system is given by:

$$\omega_n = \sqrt{\frac{C_{H_\delta}}{-I_{f,aw}}} = 0.166 \quad (13.2 \text{ rad/sec}) \quad \text{(A-7)}$$

where: $$I_{f,aw} = \frac{\text{Inertia}}{\rho(Sc)_f \cdot \frac{c}{8}^2} = \frac{1.28 \text{ kg-m}^2}{.0761 \text{ kg-m}^2} \simeq 16.8$$

APPENDIX B

Analog Simulation of Aircraft Longitudinal Dynamics

Simulation of basic, gust-alleviated and control-recovered aircraft longitudinal dynamics was conducted on a small, 10 volt analog computer in order to investigate the response in real time of the modified aircraft to gust and control inputs. The flap auxiliary-wing system dynamics also were included in the simulation in order to identify any adverse effect that might occur as a result of interference with the short-period mode of the aircraft. The high inertia of the initial design resulted in only 50% load factor attenuation at the basic aircraft short period. However, by increasing the natural frequency of the system and reducing the aerodynamic damping correspondingly, the idealized levels of attenuation calculated in the analytical treatment could be reached.

An attempt was made to simulate atmospheric turbulence by feeding the output of a "white" noise generator through a low pass filter in order to reproduce the Dryden spectrum. Unfortunately lack of a noise generator with good low freqency characteristics made it unsuccessful. Discrete gusts were simulated using a well damped second-order system. Gust shape and duration were adjusted by controlling the damping and natural frequency of the double integrator feedback system.

A schematic diagram identifying the scaling factors (s_α, s_θ, etc.) and corresponding analog quantities in brackets is shown in Figure B-1. Elevator deflection was scaled at 100 volts/radian. All other angular quantities were scaled at 10v/rad. The nondimensional velocity perturbation was scaled at 10v/unit. $\tau = \frac{c}{2V} \simeq .0125$ sec. designates the non-dimensionalizing time scaling factor.

Lift Equation

$$-\frac{2C_{L}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{s_\alpha}{\tau s_u}[u] = -.329[u]$$

$$-\frac{C_{L_\alpha}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{1}{\tau}[\alpha] = -2.18[\alpha]$$

$$+\frac{2\mu-C_{L_q}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{s_\alpha}{s_\theta}[\theta]' = +.973[\theta]'$$

$$-\frac{C_{L_\delta}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{s_\alpha}{\tau s_\delta}[\delta] = -.368[\delta]$$

$$-\frac{C_{L_{\dot{\delta}}}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{s_\alpha}{s_\delta}[\delta]' = -.00297[\delta]'$$

$$-\frac{C_{L_{\delta_e}}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{s_\alpha}{\tau s_{\delta_e}}[\delta_e] = -.017[\delta_e]$$

$$-\frac{C_{L_\alpha}}{2\mu+C_{L_{\dot{\alpha}}}}\frac{1}{\tau}[\alpha_g] = -2.18[\alpha_g]$$

$$-\frac{C_{L_{\dot{\alpha}}}-C_{L_q}}{2\mu+C_{L_{\dot{\alpha}}}}[\alpha_g]' = +.0119[\alpha_g]'$$

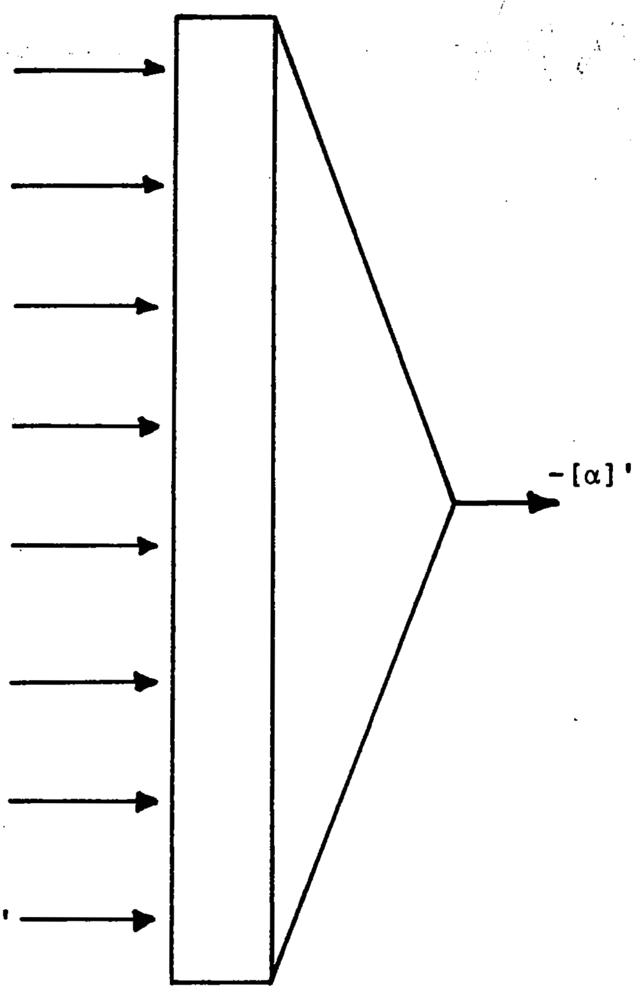


Figure B-1 Analog Computer Schematic

Pitch Equation

$$\frac{C_{M_\alpha}}{I_y\tau^2}\frac{s_\theta}{s_\alpha}[\alpha] = -18.2[\alpha]$$

$$\frac{C_{M_{\dot\alpha}}}{I_y\tau}\frac{s_\theta}{s_\alpha}[\alpha]' = -1.21[\alpha]'$$

$$\frac{C_{m_q}}{I_y\tau}[\theta]' = -3.15[\theta]'$$

$$\frac{C_{m_\delta}}{I_y\tau^2}\frac{s_\theta}{s_\delta}[\delta] = -2.54[\delta]$$

$$\frac{C_{m_{\dot\delta}}}{I_y\tau}\frac{s_\theta}{s_\delta}[\delta]' = -.485[\delta]'$$

$$\frac{C_{m_{\delta_e}}}{I_y\tau^2}\frac{s_\theta}{s_{\delta_e}}[\delta_e] = -2.79[\delta_e]$$

$$\frac{C_{m_\alpha}}{I_y\tau^2}\frac{s_\theta}{s_\alpha}[\alpha_g] = -18.2[\alpha_g]$$

$$\frac{C_{m_{\dot\alpha}}-C_{m_q}}{I_y\tau}\frac{s_\theta}{s_\alpha}[\alpha_g]' = 1.94[\alpha_g]'$$

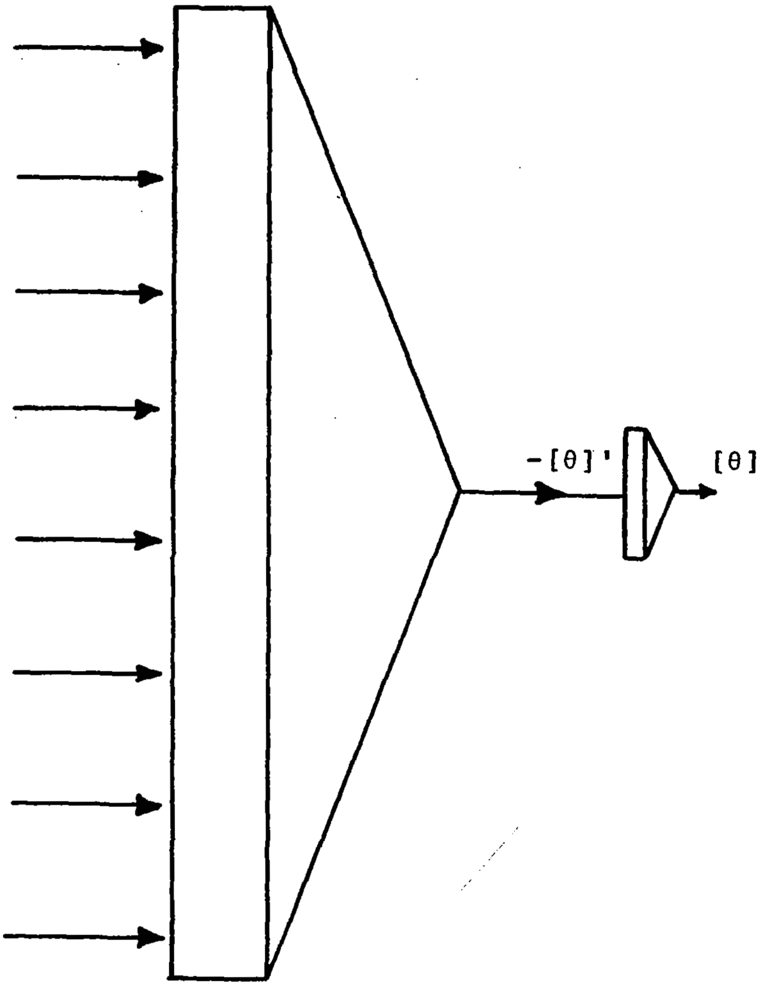


Figure B-1 cont.

Drag Equation

$$\frac{C_{x_u}}{2\mu\tau}[u] = -.0463\,[u]$$

$$\frac{C_{x_\alpha}+C_L}{2\mu\tau}\,\frac{s_u}{s_\alpha}[\alpha] = +.0662\,[\alpha]$$

$$\frac{-C_L}{2\mu\tau}\,\frac{s_u}{s_\theta}[\theta] = -.166\,[\theta]$$

$$\frac{C_{x_\delta}}{2\mu\tau}\,\frac{s_u}{s_\delta}[\delta] = -.0136\,[\delta]$$

$$\frac{C_{x_{\delta_e}}}{2\mu\tau}\,\frac{s_u}{s_{\delta_e}}[\delta_e] = -.00239\,[\delta_e]$$

$$\frac{C_{x_\alpha}+C_L}{2\mu\tau}\,\frac{s_u}{s_\alpha}[\alpha_g] = +.0662\,[\alpha_g]$$

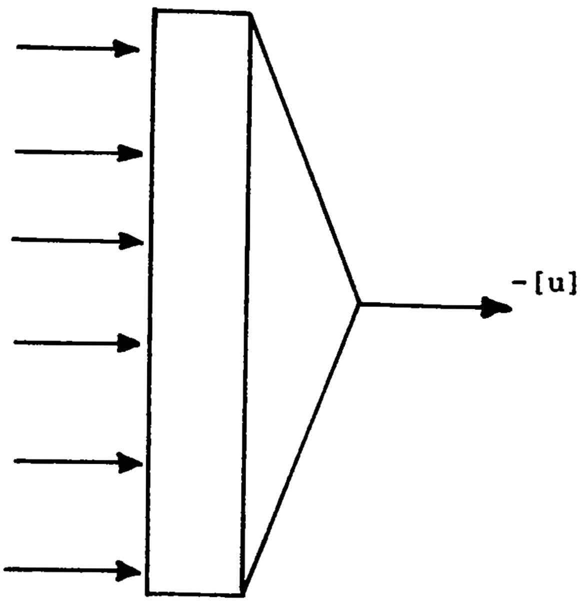


Figure B-1 cont.

Flap System Equation

$$\frac{C_{H_{\dot{\delta}}}}{I_f \tau}[\delta]' = -11.86[\delta]'$$

$$\frac{C_{H_\delta}}{I_f \tau^2}[\delta] = -173.9\ [\delta]$$

$$\frac{-2C_{H_r}}{I_f \tau^2}\frac{s_\delta}{s_u}[u] = -155.6\ K_v[u]$$

$$\frac{C_{H_\alpha}}{I_f \tau^2}\frac{s_\delta}{s_\alpha}[\alpha] = -1029K_a[\alpha]$$

$$\frac{C_{H_\alpha}}{I_f \tau^2}\frac{\partial i_{aw}}{\partial \delta_e}\frac{s_\delta}{s_{\delta_e}}[\delta_e] = -102.9\frac{\partial i_{aw}}{\partial \delta_e}[\delta_e]$$

$$\frac{C_{H_\alpha}}{I_f \tau^2}\frac{s_\delta}{s_\alpha}[\alpha_g] = -1029K_a[\alpha_g]$$

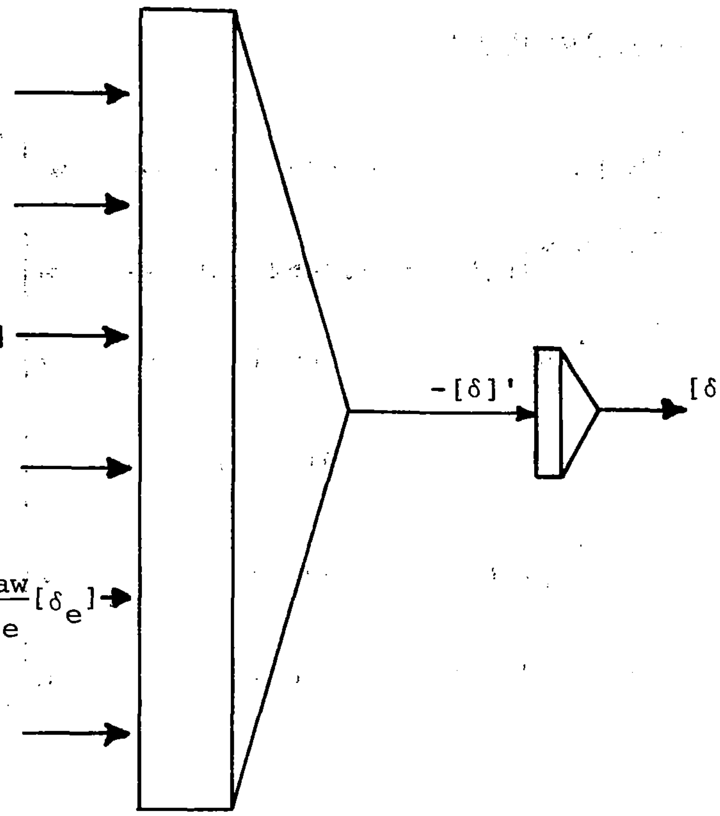


Figure B-1 cont.

APPENDIX C

On the Mechanization of the Gust Alleviation Device

The degree of vertical gust alleviation as measured by the general alleviation constant K_a is determined by:

$$K_a = \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{C_{L_\delta}}{C_{L_\alpha}} \qquad \text{(C-1)}$$

The free floating aerodynamically unbalanced flap does not contribute much to the improvement of riding qualities in turbulent air since it only yields a very small value of K_a which is due to its relative insensitivity to angle of attack gusts,

$$K_a^f = \left(\frac{C_{H_\alpha}}{C_{H_\delta}} \frac{C_{L_\delta}}{C_{L_\alpha}}\right)_f \simeq .025 \qquad \text{(C-2)}$$

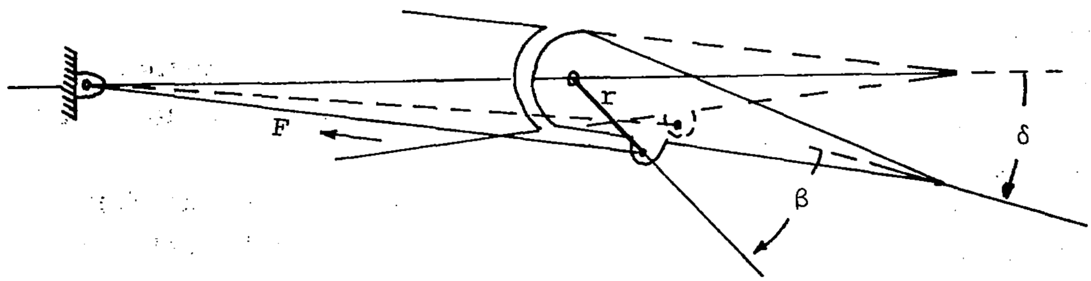
As demonstrated previously, full vertical gust alleviation requires K_a to be unity.

Means of increasing the flap response to angle of attack (without the use of servo-mechanisms) essentially fall into two categories:

- Increasing the flap hinge-moment sensitivity to angle of attack C_{H_α} (by a factor of 40) by coupling a pair of auxiliary wings to the lift control surfaces in order to provide enough power to drive them. This type of device was adopted for the present system.
- Reducing the coefficient C_{H_δ} (by the same factor) in order to increase the aerodynamic balance of the flaps to the required level. This could be done in two ways;

1. Mechanically, by providing a destabilizing spring moment about the flap hinge.
2. Aerodynamically, by installing a servo-tab at the trailing edge of the flaps.

The spring mechanism would appear as shown below:



The spring offset angle β is introduced to provide horizontal gust alleviation capability as measured by K_v. The hinge moment provided by the spring about the flap hinge is:

$$H = Fr \sin (\delta + \beta) \tag{C-3}$$

The tension F in the spring can be made constant by providing a large enough equilibrium extension. For zero flap deflection,

$$H_{eq} = Fr \sin \beta \tag{C-4}$$

and

$$H_{\delta_{eq}} = Fr \cos \beta \tag{C-5}$$

The resulting hinge moment coefficients for a particular flight condition become:

$$C_{H_\delta} = C^f_{H_\delta} + \frac{Fr \cos \beta}{q(Sc)_f} = \frac{1}{K_a} \frac{C_{L_\delta}}{C_{L_\alpha}} C_{H_\alpha} \tag{C-6}$$

$$C_{H_r} = \frac{Fr \sin \beta}{q(Sc)_f} = -K_v C_L \frac{C_{H_\delta}}{C_{L_\delta}} \tag{C-7}$$

or,

$$\tan \beta = \frac{C_L}{C_{L_\delta}} \left\{ \frac{K_v}{\frac{K_a}{K^f_a} - 1} \right\} \tag{C-8}$$

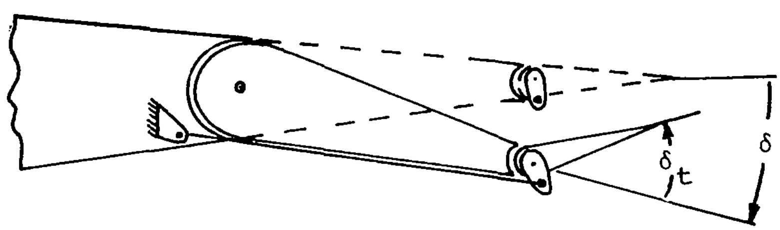
Thus if only vertical gust alleviation is to be implemented $K_v=0$ and the spring offset angle $\beta=0$. However, even if full vertical and horizontal gust alleviation are introduced, β is only .66 degree. The magnitude of the required spring loading is determined from equation (C-6),

$$Fr \cos \beta = -q(Sc)_f \, C^f_{H_\delta} \left\{ 1 - \frac{K^f_a}{K_a} \right\} \simeq 160 \text{ ft lb.} \tag{C-9}$$

Two comments are in order:

- At the most r is of the order of 1 ft with the consequence that a rather large transverse force is applied to the flap hinge and may cause structural problems.
- The spring tension must be trimmed to the correct value for the equilibrium dynamic pressure even if only vertical gust alleviation is implemented.

These problems are eliminated if a servo-tab is used to improve the aerodynamic balance of the flaps.



The tab is geared to the flap so as to deflect upward when the flap is lowered thus reducing the magnitude of $C_{H_{\delta_f}}$ to:

$$C_{H_\delta} = \left(1 + \frac{\partial \delta_t}{\partial \delta_f} \cdot \frac{C_{H_{\delta_t}}}{C_{H_{\delta_f}}}\right) C_{H_{\delta_f}} \tag{C-10}$$

in order to stay in the range of linear aerodynamics, the gearing ratio should be reasonably close to unity:

$$\frac{\partial \delta_t}{\partial \delta_f} \sim -1 \tag{C-11}$$

The required tab moment $C_{H_{\delta_t}}$ is determined by the condition:

$$1 + \frac{\partial \delta_t}{\partial \delta_f} \cdot \frac{C_{H_{\delta_t}}}{C_{H_{\delta_f}}} = \frac{K_a^f}{K_a} \tag{C-12}$$

Thus, for full vertical gust alleviation, the tab should be designed so that

$$\frac{C_{H_{\delta_t}}}{C_{H_{\delta_f}}} \simeq 1$$

If horizontal gust alleviation is to be included, the flaps must be spring loaded as previously considered or the tab can be spring loaded.

The elimination of the auxiliary wing results in a "cleaner" and probably less expensive alleviation device. Also the inertia of the system is greatly reduced when the auxiliary wings are removed (by a factor of 9). However the reduction in the restoring moment H_δ is even greater (1/40) so that the natural frequency of the system drops to:

$$\omega_n = \sqrt{\frac{\frac{K_a^f}{K_a} C_{H_\delta}^f}{I_f}} : 6.3 \text{ rad/s}$$

which is approximately half the present value. This value may interfere with the alleviated aircraft short period and introduce instability. However, further analysis shows that a combination of a spring and an auxiliary wing can raise the system natural frequency above that for a system with an auxiliary wing alone.

In the presence of the auxiliary wing

$$C_{H_\alpha} = C_{H_\alpha}^f - \tfrac{1}{2}\Gamma \frac{(Sb)_{aw}}{(Sc)_f} C_{L_\alpha}^{aw} \qquad \text{(C-13)}$$

$$\text{where } \Gamma = \frac{\partial \delta_{aw}}{\partial \delta_f}$$

In the presence of a spring or tab:

$$H_\delta = H_\delta^f + Fr \qquad \text{(C-14)}$$

For full vertical gust alleviation, $K_a=1$,

$$\frac{C_{H_\alpha}}{C_{H_\delta}} = \frac{C_{L_\alpha}}{C_{L_\delta}} \tag{C-15}$$

For geometrically similar auxiliary wings:

$$I_{aw} = I_{aw}^{ref} L^5 \tag{C-16}$$

where the reference auxiliary-wing characteristics are those already calculated: NACA 0009, AR=2, b^{ref}=1.13m , I_{aw}^{ref}=1.14kg-m^2 Γ^{ref}=1. (I_f=.143 kg-m^2)

After some manipulation, equations (C-13), (C-14), and (C-15) yield the following relation between parameters Γ, L, and $X= \frac{H_\delta}{H_{\delta_f}} = 1 + \frac{Fr}{H_{\delta_f}}$

$$\Gamma L^3 = \frac{X-K_a^f}{1-K_a^f} \tag{C-17}$$

The natural frequency of the alleviation device is given by:

$$\omega_n^2 = - \frac{C_{H_\delta}}{I_f+\Gamma^2 I_{aw}} \tag{C-18}$$

from (16), (17) and (18),

$$\left(\frac{\omega_n}{\omega_{n_f}}\right)^2 = \frac{X}{1 + \left(\frac{X-K_a^f}{1-K_a^f}\right)^2 \frac{I_{aw}^{ref}}{I_f L}}$$

where ω_{n_f} is the free floating flap natural frequency.

For small values of K_a^f:

$$\left(\frac{\omega_n}{\omega_{n_f}}\right)^2 = \frac{X}{1+X^2 \frac{I_{aw}^{ref}}{I_f L}}$$

Then $\frac{\omega_n}{\omega_{n_f}} L^{-1/4}$ is just a function of $\frac{X}{\sqrt{L}}$

The ratio $\left(\frac{\omega_n}{\omega_{n_f}}\right)^2$ has been plotted against the spring dependent parameter X yielding a family of curves corresponding to different auxiliary wing sizes. For a given auxiliary wing size there is a value of the spring moment corresponding to a maximum natural frequency of the system. For all practical auxiliary wing sizes this optimum moment is destabilizing. The limiting size corresponds to $L = I_{aw}^{ref}/I_f \simeq 8.4$ for which no improvement in natural frequency can be obtained through the use of a spring; but this case is rather academic since the auxiliary wing span would then be approximately twice that of the wing!

For a given spring moment (which may be zero) the larger the auxiliary wing size, the larger the natural frequency of the system. For the present auxiliary wing, b^{aw}=3.7ft (L=1), X_{max}=.345 corresponds to Fr $\simeq$ 107 ft lb and $\left(\frac{\omega_n}{\omega_{n_f}}\right)^2 = \frac{1}{2} X_{max}$

or: $\omega_{n_{max}} = 16.8$ rad/s since $\omega_{n_f} = 40.4$ rad/s

For $\begin{cases} X = L = 1 \\ \text{No spring} \end{cases}$ $\frac{\omega_n}{\omega_{n_f}} = \frac{1}{\sqrt{1+\frac{I_{aw}^{ref}}{I_f L}}} = .326$ $\omega_n = 13.2$ rad/s

For $\begin{cases} X = K_a^f \\ \text{No aux.wing} \end{cases}$ $\frac{\omega_n}{\omega_f} = \sqrt{K_{a_f}} = .16$ $\omega_n = 6.3$ rad/s

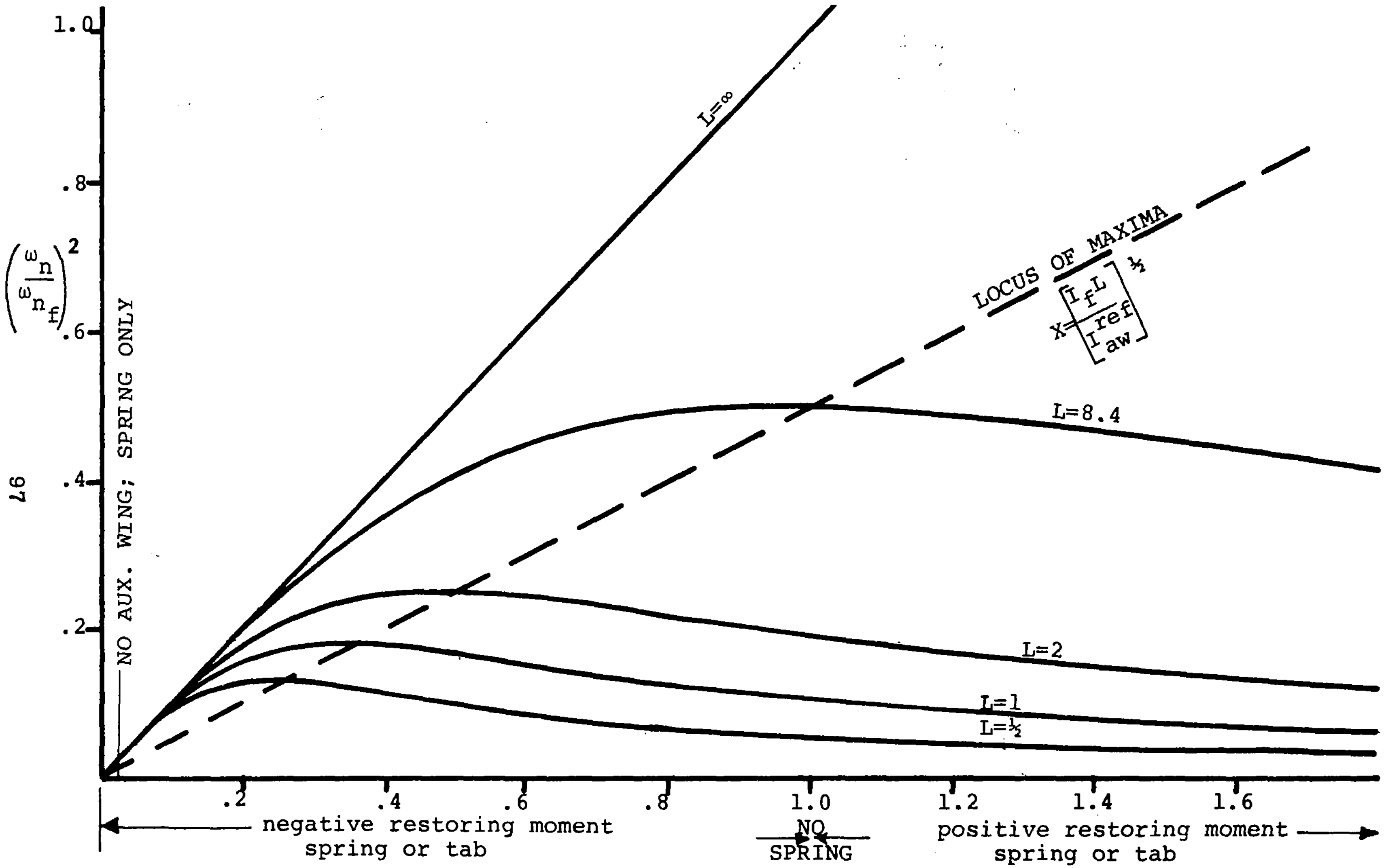


Figure C-1 Variation of Flap-Auxiliary Wing Natural Frequency as a Function of Restoring Moment

APPENDIX D

Simplified Frequency Analysis

I. Analysis of Frequency Responses to Gusts

The Short Period Approximation

The velocity perturbation is assumed to be negligible, with the result that the equations of motion simplify to,

$$\begin{bmatrix} C'_{L_\alpha} + s(C'_{L_{\dot\alpha}} + 2\mu) & s(C_{L_q} - 2\mu) \\ C'_{m_\alpha} + s\,C'_{m_{\dot\alpha}} & s(C_{m_q} - I_y s) \end{bmatrix} \begin{bmatrix} \alpha \\ \theta \end{bmatrix} + \begin{bmatrix} C'_{L_\alpha} + s(C'_{L_{\dot\alpha}} - C_{L_q}) \\ C'_{m_\alpha} + s(C'_{m_{\dot\alpha}} - C_{m_q}) \end{bmatrix} \alpha_g = 0 \tag{D-1}$$

If it is assumed that the tail is the only contributor to the pitching rate and angle of attack rate stability derivatives, the relationships given by Phillips and Kraft[1] yield the following identity,

$$C_{L_q} C_{m_{\dot\alpha}} - C_{m_q} C_{L_{\dot\alpha}} = 0 \tag{D-2}$$

The angle of attack transfer function then simplifies to:

$$\frac{\alpha}{\alpha_g} = \frac{C_{L_q} - C'_{L_{\dot\alpha}}}{2\mu + C'_{L_{\dot\alpha}}} \cdot \frac{s^2 + 2\bar\zeta\,\bar\omega_n s + \bar\omega_n^2}{s^2 + 2\zeta\,\omega_{n_{sp}} s + \omega_{n_{sp}}^2} \tag{D-3}$$

where

$$\omega_{n_{sp}}^2 = -\frac{C'_{L_\alpha} C_{m_q} + (2\mu - C_{L_q}) C'_{m_\alpha}}{I_y (2\mu + C'_{L_{\dot\alpha}})} \qquad 2\zeta\omega_{n_{sp}} = \frac{C'_{L_\alpha} I_y - 2\mu (C_{m_q} + C'_{m_{\dot\alpha}})}{I_y (2\mu + C'_{L_{\dot\alpha}})}$$

$$\bar\omega_n^2 = \frac{C'_{L_\alpha} C_{m_q} + (2\mu - C_{L_q}) C'_{m_\alpha}}{I_y (C_{L_q} - C'_{L_{\dot\alpha}})} \qquad 2\bar\zeta\bar\omega_{n_{sp}} = -\frac{C'_{L_\alpha} I_y + 2\mu (C_{m_q} - C'_{m_{\dot\alpha}})}{I_y (C_{L_q} - C'_{L_{\dot\alpha}})}$$

Similarly for the pitch angle,

$$\frac{\theta}{\alpha_g} = -\frac{2\mu(C'_{m_{\dot\alpha}} - C_{m_q})s + (2\mu + C_{L_q})C'_{m_\alpha} - C_{m_q}C'_{L_\alpha}}{I_y(2\mu + C'_{L_{\dot\alpha}})\,(s^2 + 2\zeta\,\omega_{n_{sp}}s + \omega_{n_{sp}}^2)} \tag{D-4}$$

The flight path angle response then becomes,

$$\frac{\gamma}{\alpha_g} = \frac{\theta - \alpha}{\alpha_g} = -\frac{C_{L_q} - C'_{L_{\dot\alpha}}}{2\mu + C'_{L_{\dot\alpha}}} \cdot \frac{s^2 + 2\tilde\zeta\,\tilde\omega_n s + \tilde\omega_n^2}{s^2 + 2\zeta\,\omega_{n_{sp}}s + \omega_{n_{sp}}^2} \tag{D-5}$$

where

$$\tilde\omega_n^{\,2} = \frac{2(C_{m_q}C'_{L_\alpha} - C_{L_q}C'_{m_\alpha})}{I_y(C_{L_q} - C'_{L_{\dot\alpha}})}, \quad 2\tilde\zeta\,\tilde\omega_n = -\frac{C'_{L_\alpha}}{C_{L_q} - C'_{L_{\dot\alpha}}}$$

It should be noted that terms of the form $(s^2+2\zeta\omega_n s+\omega_n^2)$ do not necessarily represent oscillatory modes of natural frequency ω_n and damping ratio ζ, but are general second-order terms that may very well be nonoscillatory. Such is the case with the zeros of the basic aircraft's flight path angle transfer function where $\tilde\omega_n^{\,2}$, is negative and the term has two first-order roots; one is large and positive at approximately

$$s = \frac{C_{L_\alpha}}{C_{L_q} - C'_{L_{\dot\alpha}}}$$

and the other small and negative

$$s \approx \frac{2C_{m_q}}{I_y}$$

The Phugoid Approximation

The phugoid approximation is based on the assumption that the total angle of attack perturbation $(\alpha+\alpha_g)$ is negligibly small and that the pitching moment equation is identically satisfied. With these assumptions the equations of motion reduce to:

$$\begin{bmatrix} C'_{x_u} - 2\mu s & -C_L \\ 2C_L(1-K_v) - K_v \frac{2C_L}{C_{L_\delta}} C_{L_{\dot\delta}} s & s(C_{L_q} - 2\mu) \end{bmatrix} \cdot \begin{bmatrix} u \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ -s(C_{L_q}+2\mu) \end{bmatrix} \alpha_g = 0 \tag{D-6}$$

The velocity perturbation transfer function is then:

$$\frac{u}{\alpha_g} \simeq \frac{C_L}{2\mu} \frac{s}{s^2+2\zeta_p \omega_{n_p} s + \omega_{n_p}^2} \tag{D-7}$$

Similarly for pitch angle,

$$\frac{\theta}{\alpha_g} \simeq - \frac{s-C'_{x_u}/2\mu}{s^2+2\zeta_p\omega_{n_p} s+\omega_{n_p}^2} \cdot s \tag{D-8}$$

The flight path angle response thus becomes,

$$\frac{\gamma}{\alpha_g} = \left(\frac{\theta}{\alpha_g}+1\right) \simeq -\frac{C_{L_q}}{\mu} \frac{s^2+2\zeta_p\omega_{n_p} s-\frac{\mu}{C_{L_q}} \omega_{n_p}^2}{s^2+2\zeta_p\omega_{n_p} s + \omega_{n_p}^2} \tag{D-9}$$

where:

$$\omega_{n_p}^2 \simeq \frac{C_L^2(1-K_v)}{2\mu^2}, \quad 2\zeta_p\omega_{n_p} \simeq -\frac{C'_{x_u}}{2\mu}$$

For the basic airplane, $\omega_n = \frac{C_L}{\mu\sqrt{2}}$, and in the phugoid frequency range the flight path angle reduces to:

$$\frac{\gamma}{\alpha_g} \simeq \frac{\omega_{n_p}^{\;2}}{s^2 + 2\zeta_p \omega_{n_p} s + \omega_{n_p}^{\;2}} \qquad \text{(D-10)}$$

For the airplane equipped with full horizontal gust alleviation capability, $K_v = 1$, $\omega_{n_p} = 0$, and

$$\frac{\gamma}{\alpha_g} \simeq -\frac{C_{L_q}}{\mu} \qquad \text{(D-11)}$$

The simplified frequency response analysis permits the construction of approximate Bode diagrams of various quantities. Attention is concentrated on the load factor and pitching rate which are of particular interest with regards to riding qualities of the aircraft in turbulent atmospheric conditions.

The non-dimensional rates of change of flight path angle and pitch attitude are related to the corresponding dimensional load factor and angular pitching velocity response:

$$\frac{\Delta n}{\alpha_g V} = \frac{2V}{gc}\left(\frac{\hat{\dot{\gamma}}}{\alpha_g}\right) \qquad \text{(g-sec/m)} \qquad \text{(D-12)}$$

$$\frac{2q}{c\alpha_g} = \frac{2}{c}\left(\frac{\hat{\dot{\theta}}}{\alpha_g}\right) \qquad \text{(rad/m)} \qquad \text{(D-13)}$$

Since the effectiveness of the gust alleviation system depends essentially on the level of attenuation of the load factor response in the short-period mode frequency band, it is important to be able to predict the response of both the basic and the alleviated airplane in that range of frequencies.

The basic airplane is seen to have a flat load factor response between the short-period frequency $\omega_{n_{sp}}$ and the upper break frequency at $\dfrac{C_{L_\alpha}}{C_{L_q} - C_{L_{\dot\alpha}}}$.

A simple extrapolation of the high frequency asymptotic response down to the upper break frequency enables us to determine its level.

$$\frac{\hat{\gamma}}{\alpha_g} \simeq -\frac{C_{L_\alpha}}{2\mu} \quad \text{or} \quad \left|\frac{\Delta n}{\alpha_g V}\right| \simeq \frac{2V}{gc} \cdot \left|\frac{C_{L_\alpha}}{2\mu}\right| \simeq .223 \ \text{(g-sec/m)} \tag{D-14}$$

The basic aircraft's pitching response also levels off beyond the short-period frequency:

$$\frac{q}{\alpha_g} \simeq \frac{C_{m_{\dot\alpha}} - C_{m_q}}{I_y} \quad \text{or} \quad \left|\frac{2q}{c\alpha_g}\right| \simeq \frac{2}{c} \left|\frac{C_{m_{\dot\alpha}} - C_{m_q}}{I_y}\right| \simeq .031 \ \text{rad/m} \tag{D-15}$$

In a similar way the high frequency response of the alleviated aircraft can be approximated. The load factor's asymptotic behavior is characterized by:

$$\frac{\hat{\gamma}}{\alpha_g} \simeq -\frac{C_{L_q} - C_{L_{\dot\alpha}}}{2\mu} s \quad \text{or} \quad \left|\frac{\Delta n}{\alpha_g V}\right| \simeq \left|\frac{C_{L_q} - C'_{L_{\dot\alpha}}}{2\mu}\right| \frac{\omega}{g} \simeq 3.02\ 10^{-3}\omega \ \text{(g-sec/m)} \tag{D-16}$$

where ω is the gust frequency in radians per second and is related to the nondimensional Laplace variable s through $s = i\omega\frac{c}{2V}$ with $i = \sqrt{-1}$.

The pitching motion response behaves similarly to that for the basic aircraft beyond the short period frequency.

$$\frac{q}{\alpha_g} \simeq \frac{C'_{m_{\dot\alpha}} - C_{m_q}}{I_y} \quad \text{or} \quad \left|\frac{2q}{c\alpha_g}\right| \simeq \frac{2}{c} \left|\frac{C'_{m_{\dot\alpha}} - C_{m_q}}{I_y}\right| \simeq .082 \ \text{rad/m} \tag{D-17}$$

A similar analysis has been made for frequencies below the phugoid. Resulting expressions are shown on the Bode plots in Chapter IV along with those presented above.

II. Responses to Elevator Deflection

Under the assumption of instantaneous flap deflection, the response of the airplane in horizontal cruising flight to elevator deflection is found by solving the following matrix equation:

$$\begin{bmatrix} C'_{x_u}-2\mu s & C'_{x_\alpha}+C_L & -C_L \\ 2C_L+\frac{\partial\delta}{\partial u}(C_{L_\delta}+sC_{L_{\dot\delta}}) & C'_{L_\alpha}+s(2\mu+C'_{L_{\dot\alpha}}) & s(C_{L_q}-2\mu) \\ \frac{\partial\delta}{\partial u}(C_{M_\delta}+sC_{M_{\dot\delta}}) & C'_{m_\alpha}+sC'_{m_{\dot\alpha}} & s(C_{m_q}-I_y s) \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ \theta \end{bmatrix} + \begin{bmatrix} C_{x_{\delta_e}} \\ C_{L_{\delta_e}} \\ C_{m_{\delta_e}} \end{bmatrix} \delta_e = 0$$

(D-18)

The preceeding equation was used directly for machine computation of frequency response functions which basically require only routine operations on matrices with complex coefficients. The resulting velocity, angle of attack, pitch and flight path angle perturbation amplitude Bode diagrams are presented in Figures 16 - 19.

However, for analysis one needs literal expressions for the various transfer functions which is impractical for the complete system especially if the inverse Laplace transforms are required to study the system in the time domain. The exact solutions, including the effect of flap dynamics were obtained using real time integration of the original differential equations on an analog computer.

For analytical work associated with control system design, approximate forms of the transfer functions are useful and are readily obtained by extension of the previously developed short period and phugoid approximations to the case of the elevator inputs.

The Short Period Approximation

The short period mode of the longitudinal motion is essentially a constant speed oscillation, in which normal and pitching motions occur. The characteristics of the mode, that is the frequency and damping, together with the control system dynamics and sensitivity to gusts, define the short term response to control inputs and to gusts, and therefore can have a large influence on a pilot's views of the handling qualities of the aircraft.

It is assumed that $u \simeq 0$ and that the speed equation of motion is identically satisfied.

The system of equations thus reduces to:

$$\begin{bmatrix} C'_{L_\alpha} + s(2\mu + C'_{L_{\dot\alpha}}) & s(C_{L_q} - 2\mu) \\ C'_{m_\alpha} + sC'_{m_{\dot\alpha}} & s(C_{m_q} - I_y s) \end{bmatrix} \begin{pmatrix} \alpha \\ \theta \end{pmatrix} + \begin{pmatrix} C_{L_{\delta_e}} \\ C_{m_{\delta_e}} \end{pmatrix} \delta_e = 0 \qquad \text{(D-19)}$$

Under the assumption that led to the determination of stability derivatives involving tail lag effects,

$$C_{m_q} C_{L_{\delta_e}} - C_{L_q} C_{m_{\delta_e}} = 0 \qquad \text{(D-20)}$$

The angle of attack transfer function reduces to:

$$\frac{\alpha}{\delta_e} = -\frac{C_{L_{\delta_e}}}{2\mu} \cdot \frac{s - 2\mu C_{m_{\delta_e}} / I_y C_{L_{\delta_e}}}{s^2 + 2\zeta\omega_{n_{sp}} s + \omega_{n_{sp}}^2} \qquad \text{(D-21)}$$

while the pitching angle reduces to:

$$\frac{\theta}{\delta_e} = \frac{C_{m_{\delta_e}}}{I_y} \frac{s+(C'_{L_\alpha} C_{m_{\delta_e}} - C'_{m_\alpha} C_{L_{\delta_e}})/2\mu C_{m_{\delta_e}}}{s(s^2+2\zeta\omega_{n_{sp}} s+\omega^2_{n_{sp}})} \tag{D-22}$$

The flight path angle then follows:

$$\frac{\gamma}{\delta_e} = \frac{C_{L_{\delta_e}}}{2\mu} \frac{s^2+(C'_{L_\alpha} C_{m_{\delta_e}} - C'_{m_\alpha} C_{L_{\delta_e}})/I_y C_{L_{\delta_e}}}{s(s^2+2\zeta\omega_{n_{sp}} s+\omega^2_{n_{sp}})} \tag{D-23}$$

The short period mode natural frequency and damping ratio were determined previously and are given by:

$$\omega^2_{n_{sp}} \simeq - \frac{C'_{L_\alpha} C_{m_q} + 2\mu C'_{m_\alpha}}{2\mu I_y} \; ; \quad 2\zeta\omega_{n_{sp}} = \frac{C'_{L_\alpha} I_y - 2\mu(C_{m_q} + C'_{m_{\dot{\alpha}}})}{2\mu I_y}$$

In view of the fact that the zero appearing in the angle of attack transfer function,

$$s_\alpha = \frac{2\mu C_{m_{\delta_e}}}{I_y C_{L_{\delta_e}}} : -162 \text{ rad/s}$$

is very large compared to the short period natural frequency, the angle of attack perturbation is well approximated in the frequency band of interest by the following transfer function:

$$\frac{\alpha}{\delta_e} \approx \frac{C_{m_{\delta_e}}/I_y}{s^2+2\zeta\omega_{n_{sp}} s+\omega^2_{n_{sp}}} \tag{D-24}$$

Between the phugoid and short period frequencies the angle of attack response is flat, the level of the response being strongly dependent on the degree of vertical gust alleviation as determined by K_a.

$$K_a=0 \text{ (Basic)} \quad \frac{\alpha}{\delta_e} = \frac{2\mu C_{m_{\delta_e}}}{C_{L_\alpha} C_{m_q} + 2\mu C_{m_\alpha}} = -1.11 \tag{D-25}$$

$$K_a=1 \text{ (Alleviated)} \quad \frac{\alpha}{\delta_e} = -\frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} = -8.67 \tag{D-26}$$

The pitching response in the short period frequency band is dominated by the presence of the finite zero,

$$s_\theta = -\frac{1}{2\mu}\left(C'_{L_\alpha} - C'_{m_\alpha}\frac{C_{L_{\delta_e}}}{C_{m_{\delta_e}}}\right)$$

the position of which is strongly dependent on K_a. Thus, for the basic aircraft, $K_a=0$

$$s_\theta \simeq -\frac{C_{L_\alpha}}{2\mu} : -2.2 \text{ rad/s}$$

$$\left(\frac{\theta}{\delta_e}\right)_{K_a=0} \simeq \frac{C_{m_{\delta_e}}}{I_y} \frac{s + C_{L_\alpha}/2\mu}{(s^2 + 2\zeta\omega_{n_{sp}} s + \omega_{n_{sp}}^2)} \tag{D-27}$$

The initial attitude response to a step input in elevator is therefore well represented by a ramp corresponding to the steady pitching rate,

$$\left(\frac{q}{\delta_e}\right)_{s\to 0} = -\frac{C_{m_{\delta_e}}}{C_{m_q} - 2\mu SM} : -2.42 \text{ sec}^{-1} \tag{D-28}$$

On the other hand, for the alleviated aircraft, $K_a=1$, the zero is negligible compared to the short period natural frequency,

$$s_\theta = \frac{C'_{m_\alpha} \, C_{L_{\delta_e}}}{2\mu \, C_{m_{\delta_e}}} : 0.02 \text{ rad/s}$$

The pole at the origin is therefore effectively cancelled and the pitch angle reduces to:

$$\frac{\theta}{\delta_e} \simeq \frac{C_{m_{\delta_e}}/I_y}{s^2+2\zeta\omega_{n_{sp}} s + \omega^2_{n_{sp}}} \tag{D-29}$$

The initial attitude response is that of a simple second order system of natural frequency ω_n:1.82 rad/s and damping ratio $\zeta_{sp}=.41$ with a steady-state gain

$$\left(\frac{\theta}{\delta_e}\right)_{s\to 0} \simeq -\frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} = -8.67 \tag{D-30}$$

It should be noted that within the limits of the approximations made, the aircraft equipped with full vertical gust alleviation has identical pitch and angle of attack transfer functions in the short period frequency band so that the flight path angle and consequently the altitude are not affected by elevator deflections at these frequencies.

If we return to the original expression of the flight path angle, the basic aircraft response is given by:

$$\frac{\gamma}{\delta_e} \simeq \frac{C_{L_{\delta_e}}}{2\mu} \; \frac{s^2+C_{L_\alpha} C_{m_{\delta_e}}/I_y C_{L_{\delta_e}}}{s(s^2+2\zeta\omega_{n_{sp}} s+\omega^2_{n_{sp}})} \tag{D-31}$$

whereas when full vertical gust alleviation is implemented, the response is strongly attenuated,

$$\frac{\gamma}{\delta_e} \simeq \frac{C_{L_{\delta_e}}}{2\mu} \cdot \frac{s^2+\omega_{n_{sp}}^2}{s(s^2+2\zeta\omega_{n_{sp}} s+\omega_{n_{sp}}^2)} \tag{D-32}$$

Thus, in the short period frequency band, the alleviated aircraft effectively "climbs on it's tail" since

$$\frac{\gamma}{\delta_e} \simeq \frac{C_{L_{\delta_e}}}{2\mu s} \tag{D-33}$$

except at the modified short period natural frequency where an undamped pair of oscillatory zeros cause the vertical response to drop sharply to zero. Note the reversal in control.

The Phugoid Approximation

The phugoid is a lightly damped low frequency oscillation in which changes of airspeed and height occur at almost constant incidence. When the elevator angle is varying however, the angle of attack perturbation can no longer be assumed small. The equivalent assumption in this case is that pitch equilibrium be maintained,

$$\frac{\partial\delta}{\partial u}(C_{m_\delta}+s\,C_{m_{\dot\delta}})u + (C'_{m_\alpha}+s\,C'_{m_{\dot\alpha}})\alpha + s(C_{m_q}-I_y s)\theta + C_{m_{\delta_e}}\delta_e = 0 \tag{D-34}$$

At low elevator frequencies, the above expression can be replaced by:

$$\frac{\partial\delta}{\partial u}\,C_{m_\delta}\,u + C'_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e = 0 \tag{D-35}$$

The spring loading of the flaps is designed to attenuate the load factor response to variations in dynamic pressure resulting primarily from horizontal gusts. It also will make the flaps sensitive to changes in airspeed induced by control deflections or angle of attack gusts exciting the phugoid mode and thus drastically modifies the basic aircraft's low frequency characteristics.

More specifically, implementation of the device virtually eliminates the load factor response to horizontal gusts over the broad frequency spectrum, up to 10 rad/s. In the cruising configuration, the sensitivity to horizontal gusts is relatively unimportant. However, when landing at low speed and high lift coefficient, horizontal gusts have a greater influence on the normal acceleration. Furthermore, the phugoid frequency is higher at this speed and may be more difficult for the pilot to control. Substantial attenuation of the normal acceleration in low frequency vertical gusts and in particular suppression of the resonant peak at the phugoid mode is another benefit of spring loaded flaps although of minor significance since, in general, the pilot should have little difficulty in controlling that mode. There is however a fundamental deficiency in the stability characteristics of an aircraft equipped with a gust alleviation system designed to satisfy the constant static lift condition which is immediately apparent from the lift equation and results in marginal stability of the airplane. This makes controllability unacceptable without adequate corrective measures.

With these facts in mind we will first concentrate on the phugoid approximation as applied to the basic aircraft and to the alleviated aircraft without a spring restrained flap system.

With $K_v=0$ and at low elevator frequencies, the pitch equation requires that the angle of attack be proportional to the elevator deflection:

$$\frac{\alpha}{\delta_e} = -\frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} = \begin{cases} -1.52 & K_a=0 \\ -8.67 & K_a=1 \end{cases} \tag{D-36}$$

So that the steady state angle of attack is approximately multiplied by a factor of 6. Substitution in the remaining lift and drag equations yields the pitch angle transfer function,

$$\frac{\theta}{\delta_e} = -\frac{\begin{vmatrix} C_{x_u}-2\mu s & C_{x_{\delta_e}} - \dfrac{C_{m_{\delta_e}}}{C'_{m_\alpha}}(C'_{x_\alpha}+C_L) \\ 2C_L & C_{L_{\delta_e}} - \dfrac{C_{m_{\delta_e}}}{C'_{m_\alpha}}(C'_{L_\alpha}+2\mu s) \end{vmatrix}}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2C_L{}^2} \tag{D-37}$$

while the velocity perturbation is given by

$$\frac{u}{\delta_e} = -\frac{\begin{vmatrix} C_{x_{\delta_e}} - \dfrac{C_{m_{\delta_e}}}{C'_{m_\alpha}}(C'_{x_\alpha}+C_L) & -C_L \\ C_{L_{\delta_e}} - \dfrac{C_{m_{\delta_e}}}{C'_{m_\alpha}}(C'_{L_\alpha}+2\mu s) & -2\mu s \end{vmatrix}}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2\ C_L{}^2} \tag{D-38}$$

Expanding and simplifying the numerator determinant yields the basic aircraft velocity transfer function:

$$\frac{u}{\delta_e} \simeq \frac{2\mu\left(C_{x_{\delta_e}} - \dfrac{C_{m_{\delta_e}}}{C_{m_\alpha}} C_{x_\alpha}\right)s + C_L\ C_{m_{\delta_e}}\ C_{L_\alpha}/C_{m_\alpha}}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2C_L{}^2} \tag{D-39}$$

The derivative term can be neglected at low elevator frequencies because the zero is large compared to the phugoid natural frequency,

$$s_u = -\frac{C_L\, C_{m_{\delta_e}}\, C_{L_\alpha}}{2\mu C_{m_\alpha}\left(C_{x_{\delta_e}} - \frac{C_{m_{\delta_e}}}{C_{m_\alpha}} C_{x_\alpha}\right)} : -4.33 \text{ rad/s}$$

The basic aircraft velocity response is then

$$\frac{u}{\delta_e} \approx -\frac{C_L\, C_{m_{\delta_e}}/SM}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2C_L^2} \tag{D-40}$$

At high elevator frequencies the velocity perturbation is essentially negligible so that the basic aircraft velocity response is essentially that of a slow, second-order system of natural frequency $\omega_{n_p} = C_L/\mu\sqrt{2}$, damping ratio $\zeta_p = -C_{x_u}/C_L\sqrt{2}$, and steady state gain:

$$\left(\frac{u}{\delta_e}\right)_{s\to 0} = -\frac{C_{m_{\delta_e}}}{2C_L SM} = 10.1 \text{ per rad.} \tag{D-41}$$

when full vertical gust alleviation is implemented ($K_a=1$) but without spring loading the flaps ($K_v=0$) the velocity transfer function reduces to:

$$\frac{u}{\delta_e} \approx \frac{2\mu\left(C_{x_{\delta_e}} - \frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} C'_{x_\alpha}\right) s - C_L\, C_{L_{\delta_e}}}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2C_L^2} \tag{D-42}$$

The zero is now shifted close to the origin of the complex frequency plane and the derivative term can no longer be neglected,

$$s_u = \frac{C_L \, C_{L_{\delta_e}}}{2\mu \left(C_{x_{\delta_e}} - \frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} C'_{x_\alpha}\right)} : .195 \text{ rad/s}$$

The essential difference however is the strong attenuation in the overall level of the velocity perturbation which is apparent from the frequency response plots and results in a steady state gain which is opposite in phase and reduced by a factor of 20,

$$\left(\frac{u}{\delta_e}\right)_{s\to 0} = - \frac{C_{L_{\delta_e}}}{2C_L} = -.516 \text{ per rad.} \tag{D-43}$$

The same technique can be applied to the phugoid approximation of the basic airplane's pitch angle transfer function which has two real zeros. One is large at approximately:

$$s_{\theta_1} = - \frac{C_{L_\alpha}}{2\mu} : -2.2 \text{ rad/s}$$

and one small at:

$$s_{\theta_2} = - K_\theta \frac{C_L{}^2 SM}{\mu C_{m_{\delta_e}}} : -.06 \text{ rad/s}$$

At low elevator frequencies, this expression can be reduced to:

$$\frac{\theta}{\delta_e} \approx \frac{C_{m_{\delta_e}}}{2\mu SM} \; \frac{s_\theta - s_{\theta_2}}{s^2 + 2\zeta\omega_{n_p} s + \omega_{n_p}{}^2} \tag{D-44}$$

The steady state gain K_θ is given by:

$$K_\theta = -\frac{C_{x_u} C_{m_{\delta_e}}}{2C_L^2 SM} + \frac{1}{C_L}\{C_{x_{\delta_e}} - \frac{C_{m_{\delta_e}}}{C_{m_\alpha}}(C_{x_\alpha} + C_L)\} \simeq -3.56 \qquad \text{(D-45)}$$

The frequency response shows a large resonant peak at the phugoid so that the pitch attitude response to a step elevator input is dominated by the slow, lightly damped oscillations of large amplitude and overshoot characteristic of the phugoid mode. A significant change in the attitude response to low frequency elevator deflections occurs when full angle of attack gust alleviation is introduced. The pitch angle transfer function becomes:

$$\frac{\theta}{\delta_e} \simeq -\frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} \; \frac{4\mu^2 s^2 - 2\mu\left(C_{x_u} + \frac{C_{L_{\delta_e}}}{C_{m_{\delta_e}}} C'_{m_\alpha}\right)s - 2C_L^2 \frac{C'_{m_\alpha}}{C_{m_{\delta_e}}} K'_\theta}{4\mu^2 s^2 - 2\mu C_{x_u} s + 2C_L^2} \qquad \text{(D-46)}$$

And the dc gain is now:

$$K'_\theta = -\frac{C_{x_u} C_{L_{\delta_e}}}{2C_L^2} + \frac{C_{x_{\delta_e}}}{C_L} - \frac{C_{m_{\delta_e}}}{C'_{m_\alpha}}\left(1 + \frac{C'_{x_\alpha}}{C_L}\right) \simeq -7.65 \qquad \text{(D-47)}$$

To a lesser degree of accuracy this gain can be approximated by:

$$K'_\theta \simeq -\frac{C_{m_{\delta_e}}}{C'_{m_\alpha}} = -8.67 \qquad \text{(D-48)}$$

The complex pair of zeros in the attitude transfer function have therefore approximately the same natural frequency but a smaller damping ratio than the characteristic phugoid roots. This imperfect pole - zero cancellation is the reason for the singularity observed on the pitch angle Bode plot at the phugoid frequency.

As can be checked on the results of the analog simulation there is very little residual phugoid oscillation in the transient attitude response to a step elevator input while the steady-state value of the pitch angle is more than doubled.

To summarize some of the most significant results of the preceeding analysis, the addition of the flap-auxiliary wing system, designed to alleviate vertical gusts,

- Reduces the natural frequency and the damping ratio of the short period oscillations,
- Considerably increases the angle of attack and pitch sensitivities to elevator deflections,
- Makes elevator control of airspeed ineffective,
- Virtually eliminates transient phugoid oscillations,
- Produces considerable attenuation of the flight path angle response in the short period frequency band thus losing short term elevator control of altitude.

APPENDIX E

Optimal Alleviation System Characteristics

Complete alleviation implies no perturbation of the aircraft's equilibrium state. If the coefficients of the gust forcing functions are eliminated, these conditions are satisfied. In this analysis, the required relations are developed without regard to the dynamic behavior of the resulting configuration. To obtain these relations, vertical and horizontal gust alleviation can be considered independently. Since the former is more important, it is treated first.

If an aircraft makes use of control surfaces to eliminate the gust forcing terms in the three equations of motion, these terms and those introduced by the control surfaces must offset one another. In particular, if wing flaps are employed, the following relations must hold:

$$\bar{A}_{\delta}\delta + \bar{A}_{\alpha_g}\alpha_g = 0 \tag{E-1}$$

where the elements of the three-dimensional $\bar{A}$ vectors are given by

$$\bar{A}_{\delta} = \begin{bmatrix} C_{x_\delta} \\ C_{L_\delta}+sC_{L_{\dot{\delta}}} \\ C_{m_\delta}+sC_{m_{\dot{\delta}}} \end{bmatrix} \qquad \bar{A}_{\alpha_g} = \begin{bmatrix} C_{x_\alpha}+C_L \\ C_{L_\alpha}+s(C_{L_{\dot{\alpha}}}-C_{L_q}) \\ C_{m_\alpha}+s(C_{m_{\dot{\alpha}}}-C_{m_q}) \end{bmatrix}$$

Except for drag, each of the forcing terms involves the gust-induced angle of attack and its derivative.

This set of equations is overdescribed to yield a solution for flap deflection as a function of gusts alone, and yields a set of conditions on the elements of the A vectors which also must be met to satisfy equation (E-1) if α_g is to remain arbitrary. For example, if the drag equation is solved for δ as a function of α_g and this solution is inserted in the lift equation, the relationship of the A vector elements can be expressed by the determinant given in equation (E-2). Similarly, if the lift and pitching moment equations are treated in this manner, equation (E-3) results.

$$\begin{vmatrix} C_{x_\delta} & C_{x_\alpha} + C_L \\ C_{L_\delta} + s\,C_{L_{\dot\delta}} & C_{L_\alpha} + s\,(C_{L_{\dot\alpha}} - C_{L_q}) \end{vmatrix} = 0 \qquad \text{(E-2)}$$

$$\begin{vmatrix} C_{L_\delta} + s\,C_{L_{\dot\delta}} & C_{L_\alpha} + s\,(C_{L_{\dot\alpha}} - C_{L_q}) \\ C_{m_\delta} + s\,C_{m_{\dot\delta}} & C_{m_{\dot\alpha}} + s\,(C_{m_{\dot\alpha}} - C_{m_q}) \end{vmatrix} = 0 \qquad \text{(E-3)}$$

The second of these determines the absorption of vertical gusts in the short period frequency band and as such has a major impact on the overall smoothness of flight in rough air.

If gust alleviation is to be achieved independent of frequency, it is seen that the required conditions can be satisfied by setting each of the resulting polynomial coefficients to zero. This is equivalent to satisfying the following three relations.

$$\begin{vmatrix} C_{L_{\dot\delta}} & C_{L_{\dot\alpha}} - C_{L_q} \\ C_{m_{\dot\delta}} & C_{m_{\dot\alpha}} - C_{m_q} \end{vmatrix} = 0 \qquad \text{(E-4)}$$

$$\begin{vmatrix} C_{L_\delta} & C_{L_{\dot\alpha}} - C_{L_q} \\ C_{m_\delta} & C_{m_{\dot\alpha}} - C_{m_q} \end{vmatrix} + \begin{vmatrix} C_{L_{\dot\delta}} & C_{L_\alpha} \\ C_{m_{\dot\delta}} & C_{m_\alpha} \end{vmatrix} = 0 \qquad \text{(E-5)}$$

$$\begin{vmatrix} C_{L_\delta} & C_{L_\alpha} \\ C_{m_\delta} & C_{m_\alpha} \end{vmatrix} = 0 \qquad \text{(E-6)}$$

Condition (E-4) is automatically satisfied if the contributions of the wing-fuselage combination to the rate derivatives are negligible relative to those of the tail. With this assumption the stability derivatives take the following forms:

$$C_{L_\alpha} = C_{L_{\alpha_w}} + \left(1-\frac{\partial\varepsilon}{\partial\alpha}\right)C_{L_{\alpha_t}} \qquad C_{L_{\dot\alpha}} = 2\ell\frac{\partial\varepsilon}{\partial\alpha}C_{L_{\alpha_t}}$$

$$C_{L_q} = 2\ell C_{L_{\alpha_t}}$$

$$C_{L_\delta} = C_{L_{\delta_w}} - \frac{\partial\varepsilon}{\partial\delta}C_{L_{\alpha_t}} \qquad C_{L_{\dot\delta}} = 2\ell\frac{\partial\varepsilon}{\partial\delta}C_{L_{\alpha_t}}$$

where $C_{L_{\alpha_t}}$ is the lift curve slope of the tail, nondimensionalized with respect to wing area and dynamic pressure. The moment derivatives have the same structure.

Conditions (E-5) and (E-6) reduce to

$$C_{m_{\delta_w}} = C_{m_{\alpha_w}}\frac{C_{L_{\delta_w}}}{C_{L_{\alpha_w}}} \tag{E-7}$$

$$\frac{\partial\varepsilon}{\partial\delta} = -\frac{C_{L_{\delta_w}}}{C_{L_{\alpha_w}}}\left(1-\frac{\partial\varepsilon}{\partial\alpha}\right) \tag{E-8}$$

These relations were originally developed by Phillips and Kraft[1].

The lift portion of Equation (E-1) is solved for the flap deflection transfer function:

$$\frac{\delta}{\alpha_g} = -\frac{C_{L_\alpha} + s\,(C_{L_{\dot\alpha}} - C_{L_q})}{C_{L_\delta} + s\;C_{L_{\dot\delta}}}$$

The definition of the stability derivatives and condition (E-8) can be used to show that:

$$\begin{vmatrix} C_{L_\alpha} & C_{L_{\dot\alpha}} - C_{L_q} \\ C_{L_\delta} & C_{L_{\dot\delta}} \end{vmatrix} = 0 \tag{E-9}$$

with the consequence that flap deflection becomes independent of frequency.

$$\frac{\delta}{\alpha_g} = -\frac{C_{L_\alpha}}{C_{L_\delta}} \tag{E-10}$$

When the conditions for optimality are satisfied, the lift and pitching moment of both wing and tail become independent of angle of attack. The flaps will then act as perfect acceleration alleviators in the short period frequency range, in the sense that operation of the flaps to offset the accelerations produced by high frequency vertical gusts will result in no pitching motions. Furthermore, the flap deflection required under these conditions can be implemented by a simple passive aeromechanical system of high natural frequency so that the flaps respond fast compared to gust perturbations.

Condition (E-2) determines the alleviation system characteristics necessary to filter out completely the low frequency component of vertical gusts. Elimination of the polynomial coefficient yields the following two relations:

$$\begin{vmatrix} C_{x_\delta} & C_{x_\alpha} + C_L \\ C_{L_\delta} & C_{L_\alpha} \end{vmatrix} = 0 \tag{E-11}$$

$$\begin{vmatrix} C_{x_\delta} & C_{x_\alpha} + C_L \\ C_{L_{\dot\delta}} & C_{L_{\dot\alpha}} - C_{L_q} \end{vmatrix} = 0 \tag{E-12}$$

These are equivalent for the system considered because of Equation (E-9) and both reduce to relation (E-13) which defines the optimal drag characteristics of the lift control surfaces:

$$C_{x_\alpha} - \frac{C_{L_\alpha}}{C_{L_\delta}} C_{x_\delta} = - C_L \tag{E-13}$$

The conditions (E-7), (E-8) and (E-13) for optimal vertical gust alleviation can be rewritten in simple form with the aid of the total derivative,

$$\frac{D}{D\alpha} = \frac{\partial}{\partial\alpha} - \frac{C_{L_\alpha}}{C_{L_\delta}} \frac{\partial}{\delta\partial}$$

The conditions become:

$$\frac{DC_m}{D\alpha} = 0, \qquad \frac{D\varepsilon}{D\alpha} = 1, \qquad \frac{DC_x}{D\alpha} + C_L = 0 \tag{E-14}$$

Examination of these relations shows that the chordwise distribution of incremental lift associated with control surface deflection should produce zero pitching moment about the aerodynamic center of the wing. This is impossible to realize in practice although coupling of the elevator with the flaps could be used to offset this natural pitching tendency. Also, in order for the angle of attack at the tail to be independent of that of the wing as required by the optimal downwash condition, the flaps should produce downwash on the stabilizer when sensing an upward gust which indicates that the spanwise distribution of the incremental lift should be concentrated towards the wing tips. Effectively this means that the lift control surfaces should be of the aileron type i.e. located outboard.

The drag condition implies the need for employing a device to counteract C_{x_δ} . Phillips[4] has recognized the impact of this parameter for a STOL aircraft. However, this aircraft exhibits much larger values for the lift curve slope and incremental drag due to flap deflection than does the Cessna 172. As a result the ratio of drag and lift increments due to a vertical gust is more than an order of magnitude smaller for the Cessna 172 than for the STOL aircraft and longitudinal accelerations introduced by the gust alleviation systems should be tolerated easily in the light aircraft.

A similar analysis can be performed with respect to horizontal gusts, yielding another set of optimal flap characteristics. Stability derivatives with respect to rate of change of gust velocity are introduced to measure the effects of the lag in dynamic pressure on the tail when the aircraft penetrates a region of horizontal gust gradients:

$$C_{L_{\dot{u}_g}} = -2\ell \; C_{L_t} \tag{E-15}$$

where C_{L_t} is the lift coefficient of the tail normalized with respect to wing area and dynamic pressure. $C_{m_{\dot{u}_g}}$ is the corresponding moment coefficient derivative about the center of gravity. Ideally the gust alleviation system should completely absorb the incremental forces and moments produced by the horizontal component of atmospheric turbulence in order to reduce the net forcing terms in the equation of motion to zero over the relevant gust frequency spectrum.

$$\bar{A}_\delta \delta + \bar{A}_{u_g} u_g = 0 \tag{E-16}$$

where

$$\bar{A}_{u_g} = \begin{bmatrix} C_{x_u} \\ 2C_L + s\, C_{L_{\dot{u}_g}} \\ sC_{m_{\dot{u}_g}} \end{bmatrix}$$

The imposed conditions on the coefficients of the A vectors can be expressed by,

$$\begin{vmatrix} C_{x_\delta} & C_{x_u} \\ C_{L_\delta}+sC_{L_{\dot\delta}} & 2C_L+sC_{L_{\dot u_g}} \end{vmatrix} \equiv 0 \qquad \text{(E-17)}$$

$$\begin{vmatrix} C_{L_\delta}+sC_{L_{\dot\delta}} & 2C_L+sC_{L_{\dot u_g}} \\ C_{m_\delta}+sC_{m_{\dot\delta}} & sC_{m_{\dot u_g}} \end{vmatrix} \equiv 0 \qquad \text{(E-18)}$$

The analysis developed previously in the case of vertical gusts can be followed step by step using the same assumptions concerning the rate derivative and yields the following three optimality conditions,

$$\frac{DC_m}{D\dot u_g} = 0, \qquad C_{M_\delta} = 0, \qquad \frac{DC_x}{Du} = 0 \qquad \text{(E-19)}$$

where the total derivatives are defined by:

$$\frac{D}{Du} = \frac{\partial}{\partial u} - \frac{2C_L}{C_{L_\delta}}\frac{\partial}{\partial \delta}$$

$$\frac{D}{D\dot u_g} = \frac{\partial}{\partial \dot u_g} - \frac{2C_L}{C_{L_\delta}}\frac{\partial}{\partial \dot\delta}$$

In this case, the optimal flap deflection also is found to be independent of gust frequency,

$$\frac{\delta}{u_g} = -\frac{2C_L}{C_{L_\delta}} \qquad \text{(E-20)}$$

This response can be achieved physically by loading the flaps with a spring in such a way that the added hinge moment does not vary with flap deflection or dynamic pressure.

The first of the optimality conditions might be satisfied by adjusting the loading on the tail so that:

$$C_{L_t} + \frac{2C_L}{C_{L_\delta}} \frac{\partial \varepsilon}{\partial \delta} C_{L_{\alpha_t}} = 0 \tag{E-21}$$

The second requires the flap downwash derivative to be:

$$\frac{\partial \varepsilon}{\partial \delta} = \frac{C_{m_{\delta_w}}}{C_{m_{\alpha_t}}} \tag{E-22}$$

which generally is positive. The third condition specifies the drag characteristics of the flaps required to eliminate the longitudinal accelerations produced by low frequency horizontal gust, and could in principle be satisfied through the use of spoilers coupled to the flaps.

A close examination of the conditions for optimal horizontal and vertical gust alleviation shows them to be incompatible. The zero flap pitching moment requirement for optimal horizontal gust alleviation cannot be reconciled with the zero effective pitch stiffness condition for optimal vertical gust alleviation unless the basic aircraft is neutrally stable. Furthermore optimal vertical gust alleviation requires that flaps produce upwash on the tail when deflected downward whereas they should produce downwash for optimal horizontal gust alleviation.

Since vertical gust alleviation is the more useful, the stability characteristics of the corresponding optimal configuration are examined. It is shown that if the flap-auxiliary wing system is designed for optimal vertical gust alleviation, as defined in the previous section, the augmented aircraft is marginally stable, and the short period mode of oscillation degenerates into a pair of real roots, one of which is at the origin of the complex frequency plane. Marginal stability is implied directly by one of the optimality conditions previously developed,

$$\frac{DC_m}{D\alpha} = 0 \qquad \text{(E-23)}$$

When all of the optimality conditions for vertical gust alleviation are met, and horizontal gust alleviation is neglected, the characteristic determinant reduces to:

$$P(s) \equiv |\bar{A}_u, \bar{A}'_\alpha, \bar{A}_\theta| = \begin{vmatrix} C_{x_u} - 2\mu s & 0 & -C_L \\ 2C_L & s(2\mu + C_{L_q}) & s(C_{L_q} - 2\mu) \\ 0 & s\, C_{m_q} & s(C_{m_q} - I_y s) \end{vmatrix} \qquad \text{(E-24)}$$

Expansion of the determinant neglecting small terms yields the fourth degree polynomial

$$P(s) = 4\mu^2 I_y s^4 - 8\mu^2 C_{m_q} s^3 + 4\mu C_{m_q} C_{x_u} s^2 - 2C_L{}^2 C_{m_q} s \qquad \text{(E-25)}$$

The rigid body modes of the augmented aircraft are clearly separated so that this characteristic equation can be factored approximately into:

$$4\mu^2 I_y s\left(s - \frac{2C_{m_q}}{I_y}\right)\left(s^2 - \frac{C_{x_u}}{2\mu}s + \frac{C_L{}^2}{4\mu^2}\right) = 0 \qquad \text{(E-26)}$$

The short period mode is thus degenerated into a pair of real roots, one of which is at the origin of the complex frequency plane (s=0) while the other is large and stable at $s = \frac{2C_{m_q}}{I_y}$: -6.30 rad/s.

As expected, the phugoid is relatively unaffected by the introduction of the angle of attack compensation system. The natural frequency of the mode is reduced by 30% ($\omega_{n_p} = C_L/2\mu$:.166 rad/s) while the damping ratio is increased, ($\zeta_p = -C_{x_u}/2C_L = 0.14$) so that the time to halve amplitude remains unchanged.

The degenerate short period mode associated with the loss of pitch stiffness resulting from the requirements for optimal vertical gust alleviation is obviously unacceptable from the point of view of stability. It should be emphasized here that applying gust alleviation conditions is contrary to normal stability requirements. In any practical system a satisfactory compromize must be reached.

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