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SOME RESULTS IN THE FUNDAMENTAL GEOMETRIC THEORY OF ONBOARD DIRECTIONAL SENSORS

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Earlier in the symposium, Dr. Velez mentioned the need for standardizing the many different kinds of attitude sensors. The primary attitude sensors now in use are directional sensors, which means that they sense or measure certain external reference directions relative to the spacecraft body. Interferometers, magnetometers, horizon scanners, and star trackers are examples of such sensors, and this paper will take a very elementary look at some of their fundamental geometric properties, with the hope that, by studying the basic geometry, we can derive a set of equations which are applicable to different kinds of sensors.

First we will look at the basic ingredients of directional information, the manner in which the observation equations govern these basic ingredients and convey attitude and orbit information, and also the manner in which errors enter into these equations. The second topic to be discussed is attitude observability; in other words, what combinations of these direction measurements will resolve attitude unambiguously. Lastly, we will consider some of the concepts developed to study horizon sensors. Horizon scanners are of interest because they also present orbit information and are somewhat unique in that the measurement is not actually a particular vector, but rather a scanning vector, which is tangent to the spherical earth.

In analyzing these different directional sensors very carefully, it is found that, although there are many different kinds of sensors, the directional information can be divided into two very simple types, the first of which is given by the scalar product of two vectors as shown in figure 1.

For this measurement, we have the direction R, which is the radiation from the transmitting station, and also a spaceborne antenna, which is a direction fixed onto the spacecraft. The measurement is the phase difference of the radiation arriving at the two ends of the antenna baseline and is given by the dot product of the reference direction R, which is a unit vector from the transmitting station to the spacecraft, and the unit vector K, which is the spacecraft-fixed direction. This measurement may be represented as the following observation equation:

$$y_1 = \cos\theta = \left(\frac{\lambda}{L}\right)\left(\frac{\Delta\phi}{2\pi}\right) = \bar{R} \cdot \bar{K}$$

$$= [R^I]^T [K^I] = [R^I]^T [A_{I/B}] [K^B]$$

The superscripts I and B refer to the inertial and spacecraft axes, respectively, and $A_{I/B}$ is the direction cosine matrix relating the spacecraft and inertial axes and containing all of the attitude information. On the left-hand side of the equation, y_1 is the measured quantity; K^B is a spacecraft-fixed unit vector and is therefore a known quantity. Therefore, the attitude matrix, $A_{I/B}$, is the only unknown quantity in this equation.

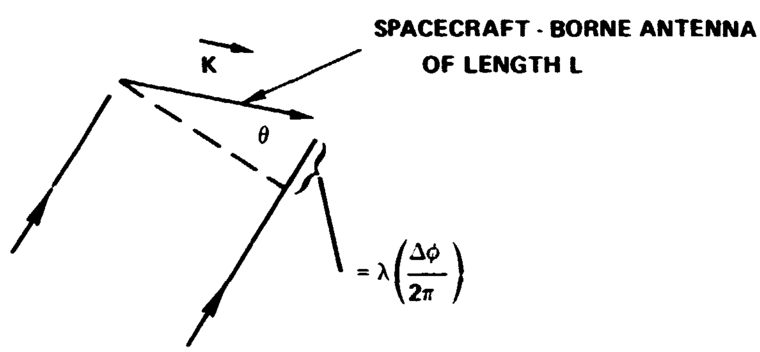


Figure 1. Attitude determination measurement as made by a short-baseline interferometer on the ATS-6 spacecraft.

Figure 2 shows the second type of direction measurement, made by the digital solar aspect sensor used on the Atmospheric Explorer Spacecraft. The vector R represents the line-of-sight from the spacecraft to the sun; i, j, and k are orthogonal unit vectors along spacecraft-fixed directions; and solar aspect angle E_S is specified by the cross-product of R and K. This measurement may be represented by the following equations:

$$y_2 = \sin E_S = \frac{\bar{R} \times \bar{K}}{|R \times K|} \cdot \bar{i} = \frac{\bar{R} \cdot \bar{j}}{\sqrt{1 - (R \cdot K)^2}} = \frac{[R^I]^T [A_{I/B}] [J^B]}{\sqrt{1 - ([R^I]^T [A_{I/B}] [K^B])^2}}$$

and

$$y_3 = \cos E_S = \frac{[R^I]^T [A_{I/B}] [I^B]}{\sqrt{1 - ([R^I]^T [A_{I/B}] [K^B])^2}}$$

Being scalar equations, this second class of measurement contains more information than the first class, although it is necessary that

$$y_2^2 + y_3^2 = 1.$$

It should be noted that y_2 and y_3 are related in a more complicated nonlinear way to the attitude matrix, $A_{I/B}$.

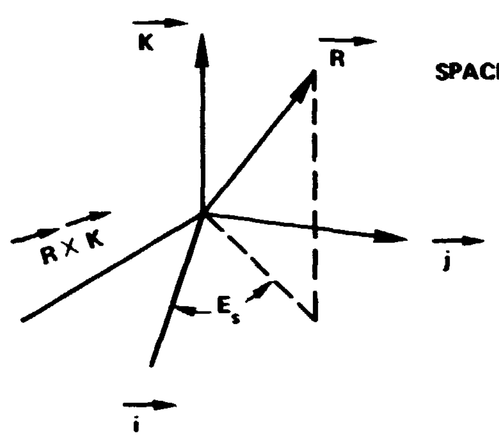


Figure 2. Orbit determination measurement as made by the digital solar aspect sensor on the AE spacecraft.

The various problems of interest may be classified as follows by referring to the measurement equations: In the usual attitude determination problem, only the attitude matrix, $A_{I/B}$, is considered an unknown. In the orbit determination problem, only the reference vector, R, is assumed unknown. In coupled attitude/orbit determination, both of these quantities are unknown. It is known that there are three unknown parameters for the attitude and three unknown parameters for the orbital position; therefore, in theory, at least six equations are needed to determine orbit as well as attitude. One good point about this type of equation is that it is applicable in general to different kinds of sensors; whatever errors exist must enter into appropriate quantities in these equations.

The first type of error to be considered is timing error; that is, instead of making the measurement at time t, the actual measurement is made at time $(t + \Delta t)$. With the timing error considered, the first measurement equation assumes the following form:

$$
\begin{aligned}
y_1 (t + \Delta t) &= [R^I (t + \Delta t)] \, [A_{I/B} (t + \Delta t)] \, [K^B] \\
&\cong [R^I (t)] \, [A_{I/B} (t + \Delta t)] \, [K^B] \\
&\cong [R^I (t)] \, [A_{I/B} (t)] \, \Big([K^B] + \Delta t \, [\Omega^B] \times [K^B] \Big)
\end{aligned}
$$

It can be seen that, generally, timing error does not have much effect on the reference direction sensed. The main effect is that, instead of measuring the component of R along the K direction, the measurement equation is measuring the component along a rotated K direction as a result of the spacecraft angular velocity, e^B. The following equation shows how errors in given quantities enter into the measurement equation:

$$
y_1 + \Delta y_1 = \Big([R^I] + [\Delta R^I] \Big)^T [A^I_{B/I}] \Big([K]^B + [\Delta K]^B \Big).
$$

On the left-hand side of the equation, Δy_1 could stand for instrument reading errors, biases, and so on. It is independent of particular instruments: that is, for different instruments, we may simply have a different bias, and so on. The second quantity, ΔR^I, represents the uncertainty about the reference direction. There may also be uncertainties because the instrument is not sensing the true direction; for instance, ionospheric refractions will result in terms like this. The last term, ΔK^B, represents the instrument alignment errors. So it can be seen that equations of this sort will be applicable to all different sensors, with any errors entering only into the ∂ quantities.

The next topic to be considered is attitude observability. The first basic principle we are concerned with here is that attitude is a relative notion, for although we generally refer to the attitude of spacecraft in inertial space, we could also refer to the attitude of the inertial space relative to the spacecraft. The information contained in one description is, of course, readily transformed to the other, but, conceptually, it often might be clearer to look at the problem one way rather than the other.

Another basic fact is that an attitude has three degrees of freedom and requires at least three independent measurements for determination. In addition, if all three independent measurements are related to only a single reference direction, the attitude cannot be resolved without ambiguity. This should be obvious, since, for a rigid body, the position of any three noncolinear points (or equivalently, two noncolinear vectors) are required to determine its attitude. So if two reference directions in inertial space are known, the attitude of inertial space relative to the spacecraft, and, therefore, the attitude of the spacecraft itself, is known. For directional sensors, of course, attitude observability is equivalent to the observability of two reference directions.

The following equation is an example of the analytical formulation of observability. The unit vectors e_1, e_2, and e_3 represent three reference directions.

$$\bar{e}_1 \cdot \bar{e}_1 = \bar{e}_2 \cdot \bar{e}_2 = \bar{e}_3 \cdot \bar{e}_3 = 1$$

$$\bar{e}_1 \cdot \bar{e}_2 = \alpha, \bar{e}_2 \cdot \bar{e}_3 = \beta, \bar{e}_1 \cdot \bar{e}_3 = \gamma$$

Assume that they are unit vectors and are known in inertial space. The components of these vectors along three spacecraft-fixed directions, r_1, r_2, and r_3, are measured, that is,

$$\bar{e}_1 \cdot \bar{r}_1 = m, \bar{e}_2 \cdot \bar{r}_2 = n, \bar{e}_3 \cdot \bar{r}_3 = p.$$

Taken together, these represent nine nonlinear equations for the nine components of e_1, e_2, and e_3. The attitude observability is equivalent to the uniqueness of the real solutions of these equations, which are not very easy. Since we are not concerned with error at this time, any measurement would correspond to an attitude. Thus there is no inconsistency, and the only concern is whether there is no uniqueness of solution.

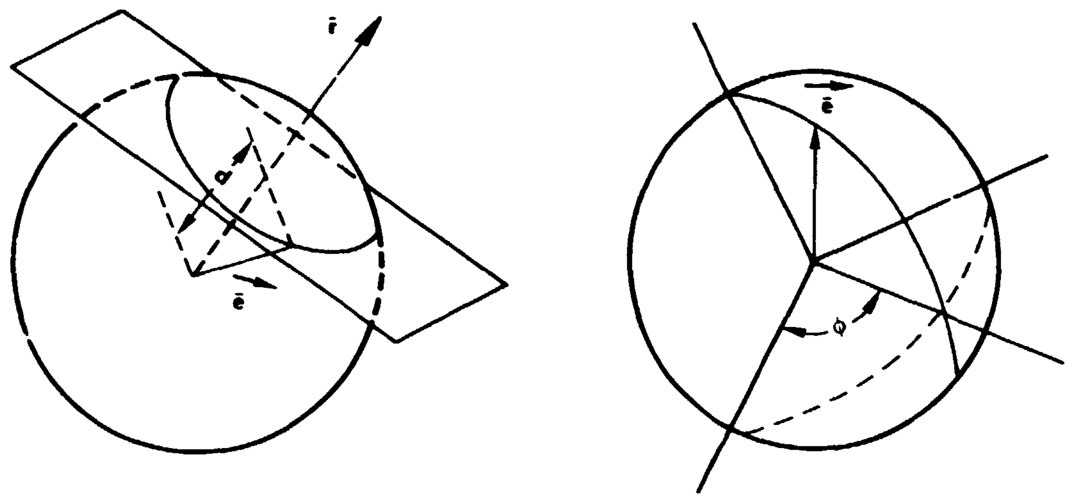
In general, the analytical determination of observability is difficult. Figure 3a shows a graphical construction which can help a great deal in providing insight into these problems. In the figure, vector e is a reference direction, and vector r is a spacecraft-fixed direction. A measurement of the component of e along the direction r will limit the reference direction to lie on a small circle, which is the intersection of a plane with the sphere. It would be very convenient to represent directions as well as attitudes on a unit sphere.

Figure 3b shows the second type of measurement, mentioned before, which gives the angle ϕ. In terms of geometry on the unit sphere, it is a half great circle, which represents all possible directions with the same meridional angle ϕ. Since this is a half great circle, it does provide a little more information than the first type of measurement.

Figure 4 shows the result of multiple measurements. It is assumed that two small circle measurements have been made, or that two components of a reference vector have been measured, which can be represented as the intersection of two small circles. In general, there is a twofold ambiguity, because after two components of a unit vector have been measured, the third component is determined up to an ambiguity in its sign. This is reflected by the two points of intersection as indicated in the figure.

Figure 5 shows what happens with half great circle measurements. In general, the intersection of two half great circles would completely define a direction. But these figures show that a combination of the second type of measurement, which is a half great circle, with the first type of measurement, which is a small circle, may still leave some ambiguity.

When two reference directions are available, the results are as seen in figure 6. There are three small circle measurements which give two possible positions of the direction e_1, as shown by the intersection of two circles. The reference direction e_2 must lie on a small circle centered at e_2 as shown in the figure. In addition, A and B intersect, A and C



(a) (b)

Figure 3. Graphical construction on a unit sphere. (a) Small circle representing all possible $\bar{e}$ which has component d along $\bar{r}$; (b) half great circle representing all possible directions with the same meridional angle ϕ.

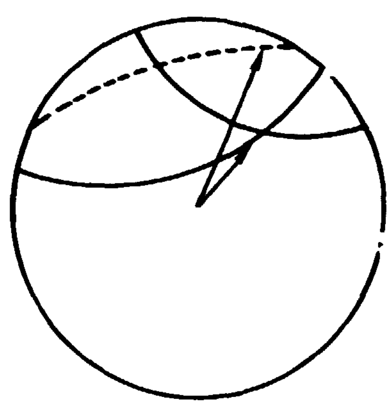
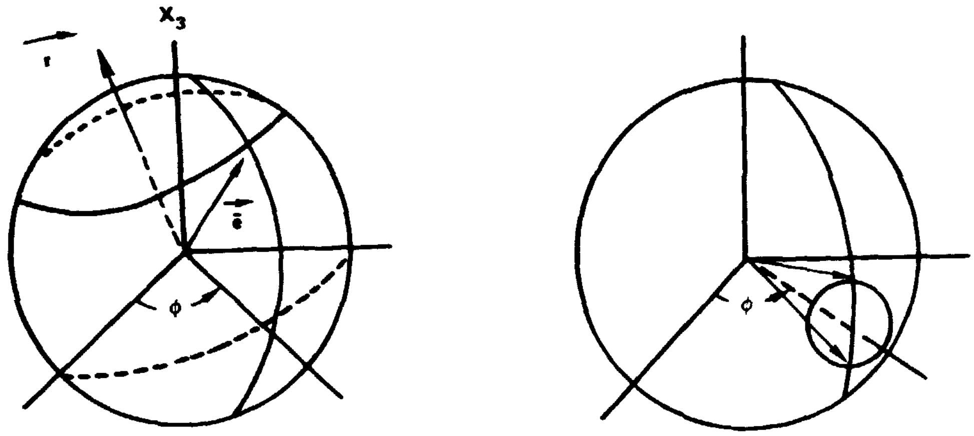


Figure 4. Intersection of two small circles gives two possible directions for $\bar{e}$; only components along linearly independent directions are independent measurements.

do not intersect, and e_1 is known. Also, e_2 has two possible positions, which means there is still some ambiguity in the attitude.

The first conclusion arrived at through these arguments is that, generally, five independent small circle measurements are required to determine attitude. Three of the measurements define one vector completely, and the other measurement, together with the known angle between the two reference directions, determines the other reference direction. Secondly, if we want to obtain attitudes from three measurements, then at least two of these measurements must be great circle measurements. A third conclusion is that, in general, three



(a) (b)

Figure 5. Half great circle measurements. (a) The X_3-r plane is perpendicular to the great circle plane, and $\bar{e}$ is determined uniquely; (b) two possible directions for $\bar{e}$.

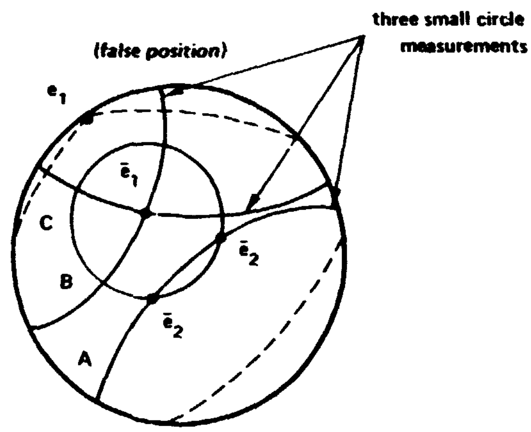


Figure 6. Three small circle measurements concerning two reference directions $\bar{e}_1$ and $\bar{e}_2$.

half great circle measurements plus any additional measurement would result in attitude without ambiguity.

It can also be concluded that attitude measurement based on three small circle measurements of three reference directions requires that the sensors not be on the same plane. This is easy to understand if we invert the role of the attitude of the spacecraft and the attitude of the inertial space; it can then be seen that this case is really the same as that covered by the first conclusion.

Other conclusions are as follows: In general, the symmetrical placement of sensors increases the chance of attitude ambiguity. Although attitude observability is very difficult to determine analytically, a mechanical model of a unit sphere can be constructed, which will readily resolve attitude observability without difficulty. Lastly, although sometimes more than three measurements are required to determine the three attitude parameters, four or five measurements available may also provide some redundancy for data smoothing.

Figure 7 introduces the topic of horizon scanners. The reference direction for the horizon scanner is the local vertical, and when a horizon scanner measurement is made, it means that the scanning ray is tangent to the spherical earth. The information available is the angle between the scanning ray and the local vertical. The measurement equation is given by

$$\cos\theta = \sqrt{1 - \left(\frac{r_e}{r_e + h(\tau)}\right)^2} = -\,\overline{R(\tau)} \cdot \overline{s(\tau)}$$

$$= \cos a \sin\phi(\tau) - \sin a \cos\phi(\tau) \cos\left\{\psi(\tau) + \lambda(\tau)\right\}.$$

where

a	=	the half cone angle of the scanner,
τ	=	time of horizon crossing,
ϕ	=	pitch angle of the scanning axle,
ψ	=	roll angle of the spacecraft,
λ	=	scanner roll angle, and
h	=	spacecraft altitude.

It is evident from the above equation and from what has been said previously that the horizon measurement does not convey any information concerning the yaw about local vertical.

There is a measurement equation like the above for each horizon transit. Since the scanning speed is fast, it may be assumed in a first approximation that neither the spacecraft altitude nor the attitude has changed from horizon entry to horizon exit. It may then be deduced immediately from the two measurement equations at entry and exit that the spacecraft has a roll angle,

$$\psi = -\left(\lambda + \frac{\Delta\lambda}{\alpha}\right)$$

and a pitch angle,

$$\phi = \mu + \alpha$$

or

$$\phi = \pi - \mu + \alpha,$$

where

$$\sin \mu = \left\{ \frac{1 - \left(\frac{r_e}{r_e + h} \right)^2}{1 - \sin^2 a \sin^2 \frac{\Delta\lambda}{\alpha}} \right\}^{1/2},$$

$$\mathrm{tau}\ \alpha = \mathrm{tau}\ a \cos \frac{\Delta\lambda}{\alpha},$$

and $\Delta\lambda$ is the earth width angle as seen by the scanner.

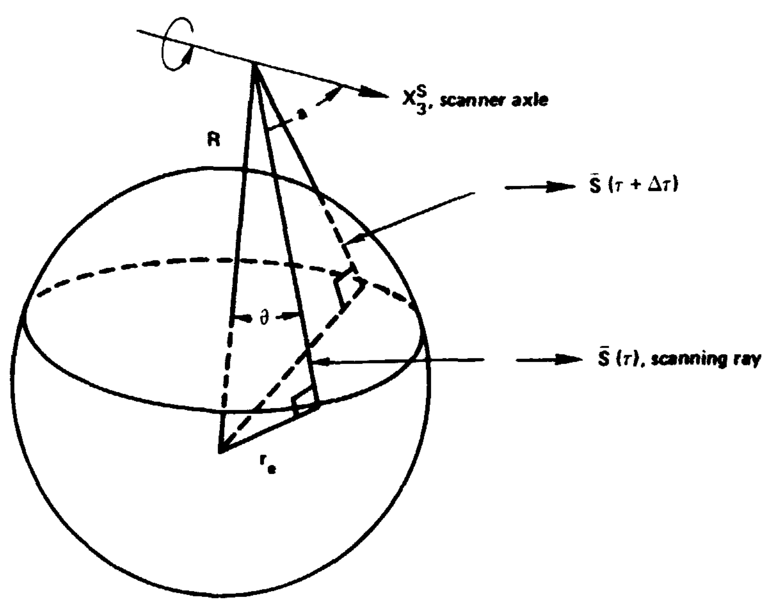


Figure 7. Horizon scanner geometry.

The existence of two possible pitch angles is easy to understand if it is realized that the two horizon measurements are now nothing but two small circle measurements about the local vertical. The extension of this result to higher approximations with the consideration of small attitude changes is straightforward and, in that case, the horizon measurements will relate to the spacecraft attitude as well as attitude rate.

Sometimes two scanners with different half cone angles a′ and a″ are mounted on the same axle. In that case, the first approximation pitch angle becomes

$$\phi = \tan^{-1}\left(\frac{\sin a' \cos \dfrac{\Delta\lambda'}{\alpha} - \sin a'' \cos \dfrac{\Delta\lambda''}{\alpha}}{\cos a' - \cos a''}\right)$$

The redundancy eliminates the pitch angle ambiguity and the need for altitude information and also provides smoothing for the roll information.

Actual measurements are often transit times rather than scanning angles and earth width. The time information is transformed to angular information using the scanning rate, and any bias in scanning rate will amplify with time. Therefore, without periodic reinitializations, the spacecraft roll, which is directly related to the scanning angle, cannot be determined accurately. On the other hand, the pitch angle is related to the earth width and is more susceptible to triggering errors.

DESCRIPTION OF THE ROTATIONAL MOTION OF A NONSYMMETRIC RIGID BODY IN TERMS OF EULER ANGLES, DIRECTION COSINES, AND EULER PARAMETERS

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The rotational motion of a nonsymmetric rigid body can be described by a set of three time-dependent Euler angles. In the torque-free case, the natural angles are the 1-2-1 or the 1-3-1 Euler angles if the angular momentum axis coincides with the space 1-axis, the 2-3-2 or 2-1-2 angles if it coincides with the space 2-axis, or the 3-1-3 or 3-2-3 angles if it coincides with the space 3-axis. In each case, the first angle satisfies a differential equation whose solution can be simply expressed in terms of theta functions, which can be readily computed with the aid of rapidly-converging series. One can then determine the nine direction cosines and/or four Euler parameters, whose transformation properties are more convenient than the Euler angles. The 2-3-2 or 2-1-2 angles offer certain advantages if the body 2-axis is the principal axis of intermediate inertia. The analytic torque-free solutions provide a good base for studying the motion of a nonsymmetric rigid body in the presence of perturbing torques.

ATTITUDE CAPTURE PROCEDURES FOR GEOS-C

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The scientific objective of the GEOS-C mission is to perform experiments to apply geodetic satellite techniques to solid-earth physics and oceanography. A spaceborne radar altimeter will map the topography of the ocean surface with a relative accuracy of ±1 meter.

To meet the objectives of the radar altimeter experiment, GEOS-C will be gravity-gradient stabilized in a nearly circular orbit at an altitude of 843 km. The orbital inclination will be 115° to maximize experimental coverage in the North Atlantic Ocean. A 21.5-ft, extendable scissors-type boom with a 100-lb end mass will provide passive pitch and roll stabilization and a momentum which will provide yaw stabilization. An eddy-current damper and magnetic coil are also provided.

Because no active attitude control hardware is provided and damper time constants are large, a detailed capture strategy has been developed at Computer Sciences Corporation (CSC) in coordination with AVCO and the Johns Hopkins Applied Physics Laboratory. This strategy has evolved from many detailed simulations and requires real-time attitude determination support for the initiation of critical commands.

POTENTIAL GEOSYNCHRONOUS ORBIT/ATTITUDE RESOLUTION USING LANDMARK DATA

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The information content in data other than conventional radar tracking is gaining increased popularity. This paper presents a covariance analysis of reducing the orbit and attitude state from landmarks (ground-truth) data extracted from images taken at geosynchronous altitude.

It is shown that comparable accuracy can be achieved with landmark data alone when compared with current optimistic reductions using conventional types of data.

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DETERMINATION OF ORBITAL POSITION FROM EARTH AND SUN SENSOR DATA ON SMS-A AND IMP-J

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Attitude data from earth and sun sensors on SMS-A and IMP-J were processed in an attempt to refine the orbital elements as determined from tracking data. The results were checked against additional tracking data and the discrepancies were investigated. The results have implications for the accuracy obtainable with any orbit-dependent attitude sensor.

ON-LINE ORBIT DETERMINATION AND ESTIMATION FOR ATS-6 FROM REDUNDANT ATTITUDE SENSORS

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ATS-6 is equipped with an onboard, two-axis interferometer which can provide direction cosines to earth-based transmitters. In addition, the spacecraft carries an earth scanner which can be thought of as an additional two-axis interferometer with transmitter at the earth's center. From the six-direction cosines thus available, both position and attitude determination can be performed. This paper describes a procedure proposed for use both in the SAPPSAC experiment on ATS-6 and in the ATS-6 on-line attitude determination program at GSFC which decouples position determination from spacecraft attitude. The resulting position pseudo-measurement is used in a constant gain Kalman filter for estimation of orbit state. Propagation of the state estimate is accomplished with circular orbit perturbation equations.