

**N76-10175**

**LONG PERIOD NODAL MOTION OF SUN SYNCHRONOUS ORBITS**

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The sun synchronous orbit has been used since the early 1960s for almost every meteorological satellite launched as well as for the two Earth Resources Technology Satellites (Landsat-1 and Landsat-2). This well known orbit concept makes use of the earth's oblateness to induce a precession of the orbit line-of-nodes in order to maintain a fixed angular orientation of the orbit plane relative to the mean sun. The sun synchronous orbit, even though it has been used for many years, will continue to be used for most earth observation and remote sensing applications because:

- The satellite passes through each latitude point at the same local time thus ensuring similar ground lighting conditions on each pass and consequently facilitates data comparisons.
- The average spacecraft solar array angle of incidence to the sun remains within a fixed boundary thus ensuring the availability of electric power.
- The orbits (altitude and inclination) can be chosen such that the majority of the earth's surface can be mapped with near north-south contiguous swaths in a fixed period with repeatability.

The long-period perturbations which disturb sun synchronous orbits have not been the subject of detailed investigations in comparison to geosynchronous orbits where the literature abounds with material. The disturbances acting on sun synchronous orbits has not been examined in detail for several reasons:

- With the exception of the Landsat satellite, previous sun synchronous vehicles have not had orbit adjustment capability to fight the perturbations.
- The spacecraft hardware design lifetimes (1 to 2 years) have been sufficiently short that the perturbations could be neglected without significantly degrading mission performance.
- Prior to the use of inertial guidance on the Delta launch vehicle, injection uncertainties were sufficiently large so as to mask any perturbation effects present.

The assessment of the perturbations acting on sun synchronous orbits becomes more significant when longer lifetime spacecraft are developed as anticipated over the next 10 to 20 years.

With improvements in hardware technology and reliability and the on-orbit refurbishment capability associated with the advent of Space Shuttle, operational satellite lifetimes could easily exceed 5 years. This increase in lifetime is almost a reality today as most earth orbiting spacecraft launched in recent years have exceeded their design life.

The object of the study documented here was to determine which perturbations significantly affected the long term nodal motion of sun synchronous orbits and then construct an approximate model which described the phenomena observed. Many computer simulations were made with several independent computer programs to assess the relative effect of various combinations of perturbations. Typical of the perturbations included in the simulations were zonal and tesseral gravitational harmonics, third-body gravitational disturbances induced by the sun and moon, and atmospheric drag. It was observed that a model consisting of even-zonal harmonics through order 4 and solar gravity dominated the nodal motion. It was further observed that for long runs the orbit inclination and orientation of the line-of-nodes exhibited an oscillating behavior each having the same period. For all the cases run, the inclination amplitude was very small (always less than 1 degree); however, the nodal motion could be quite large. Due to these observations, it was felt that a resonance existed between the inclination and the nodal motion. The mean daily rate of change in inclination due to solar gravity (in radians/mean solar day) was found by analytic averaging to be

$$\frac{di}{dT} = 16200 \frac{\eta_s^2}{\eta} (1 + \cos i_s)^2 \sin i \sin 2 \Omega_{oc}$$

where

$\eta_s$  = mean motion of earth about the sun

$\eta$  = mean motion of satellite about the earth

$i_s$  = solar obliquity

$\Omega_{oc}$  = o'clock angle, the angle between the longitude of the ascending node and the mean sun used computing local time

$i$  = orbit inclination

The precession rate of the line-of-nodes due to zonal harmonics through order 4 (assuming a circular orbit) is

$$\dot{\Omega} = A \cos i + B \cos^3 i$$

where

$$A = -\frac{3}{2} n J_2 \left( \frac{Rc}{a} \right)^2 \left[ 1 - J_2 \left( \frac{Rc}{a} \right)^2 \left( 1 - \frac{15}{8} \frac{J_4}{J_2^2} \right) \right]$$

$$B = -\frac{3}{8} n J_2^2 \left( \frac{Rc}{a} \right)^4 \left( 19 - \frac{35}{2} \frac{J_4}{J_2^2} \right)$$

and

$$J_2, J_4 = \text{zonal gravitational coefficients}$$

$$Rc = \text{mean equatorial radius of the earth}$$

$$a = \text{orbit semimajor axis}$$

Differentiating  $\dot{\Omega}$ , substituting  $di/dt$ , and noting that

$$\dot{\Omega}_{oc} = \dot{\Omega} - \dot{\phi}_s$$

where  $\dot{\phi}_s \approx 0.9856^\circ/\text{day}$ , then

$$\ddot{\Omega}_{oc} = -16200 \frac{n^2}{n} (1 + \cos i_s)^2 \sin^2 i (\bar{A} - 3\bar{B} \cos^2 i) \sin 2\Omega_{oc}$$

where  $\bar{A}, \bar{B}$  are  $A$  and  $B$  in radians/mean solar day.

Examination of  $\ddot{\Omega}_{oc}$  shows that for altitude regions where drag can be neglected, and assuming that  $\sin^2 i$  is approximately constant,

$$\ddot{\Omega}_{oc} \approx k \sin 2\Omega_{oc}$$

a form of the familiar pendulum equation. It is known that systems which are characterized by the above equation can exhibit libration or circulatory characteristics. Both characteristics can be observed on the phase plane plot (figure 1) made for ITOS-type orbits. It is observed that this system has stable equilibria which correspond to orbits whose line-of-nodes lie through the 6:00 a.m. and 6:00 p.m. points. The pendulum equation can be solved analytically using elliptic functions with a typical solution for a quarter cycle oscillation being

$$\Omega_{oc} = \frac{1}{2} \cos^{-1} \left[ -1 + \frac{2m \operatorname{cn}^2(\sqrt{2k} T)}{1 - m \operatorname{sn}^2(\sqrt{2k} T)} \right]$$

where

$$m = 1/2 (1 + \cos 2 \Omega_{oc0})$$

$$0 \leq T \leq P_L/4$$

$$0 \leq \Omega_{oc} \leq 90^\circ$$

Note that  $P_L$  denotes the libration period and  $\Omega_{oc0}$  denotes the value of o'clock angle where  $\dot{\Omega}_{oc} = 0$ . The libration period,  $P_L$  is

$$P_L = \frac{4 K(m)}{\sqrt{2k}}$$

where  $K(m)$  denotes a complete elliptic integral of the first kind. Figure 2 shows the o'clock angle libration period for ITOS-type orbits ( $h = 1489$  km,  $i = 101.9^\circ$ ). It is seen that the libration period increases from 26 years as the reference o'clock angle moves from one of the stable equilibria toward one of the unstable ones. Investigation of the libration period has shown that the minimum libration period lies between 22 to 30 years for sun synchronous orbits between 200 and 2000 km.

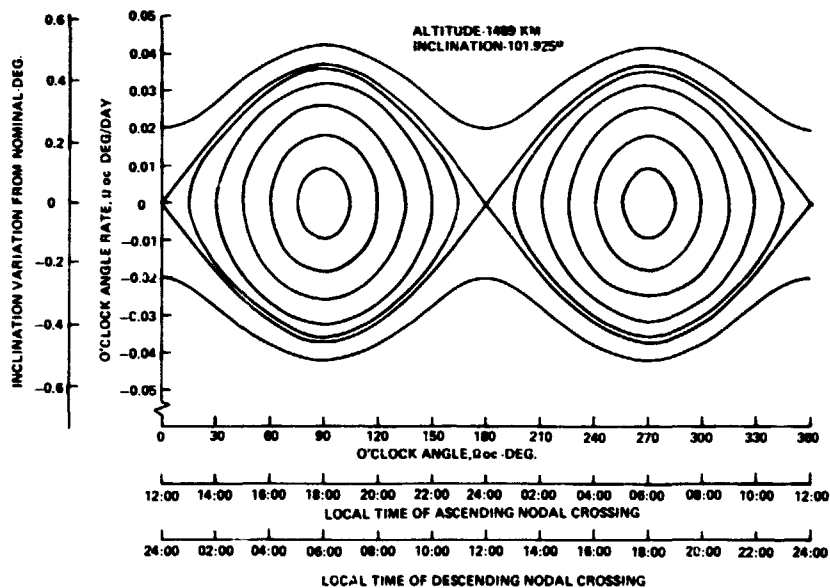


Figure 1. O'clock angle motion in phase space for ITOS-type orbit.

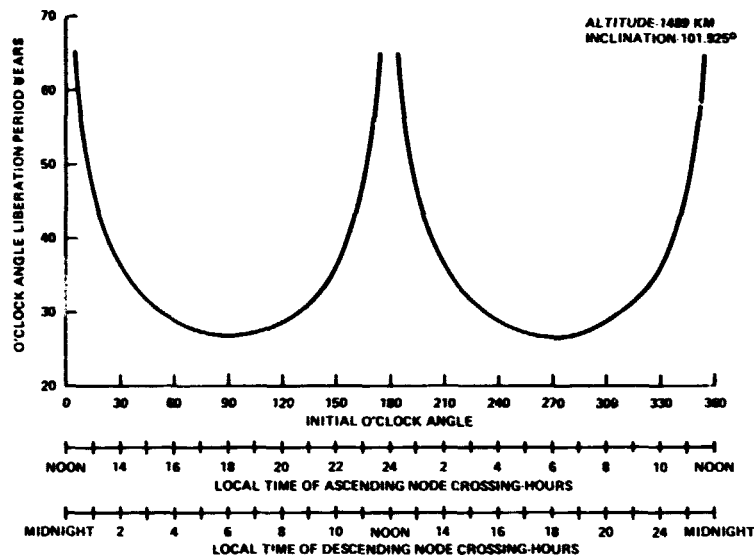


Figure 2. Libration period for ITOS-type orbit.

The pendulum analogy has been compared with both simulation and flight spacecraft data. Figure 3 compares the approximate solution with several simulations having various combinations of perturbations. The reference orbit in this comparison is the ITOS-type ( $h = 1489$  km,  $i = 101.9^\circ$ ) having an ascending nodal crossing at 3:00 p.m. local time. It is seen that there is excellent agreement between the pendulum analogy and those simulations which include zonal harmonics and solar gravity.

The remaining figures (4 through 9) compare the o'clock angle time history generated using the pendulum model with that obtained from Brouwer mean elements for several flight spacecraft. In examining these comparisons one observes that there are two curves representing approximate nodal drift propagation. The dashed curve is the propagation calculated using a set of orbit elements at the initial epoch for the particular satellite. The solid curve uses an iterated value of inclination to improve the agreement. This approach was taken because of the large relative uncertainty in measuring the inclination. It is seen in all cases that excellent agreement is obtained. In examining these data, note that the ESSA-2 solution (figure 4) is a case where the motion lies in the circulatory region in phase space while the other solutions lie in the libration region. The ESSA-8 solution (figure 7) exhibits the turnaround expected for the libration motion. The final observation to be noted is for Nimbus-5 (figure 9) where the node is located near one of the unstable equilibria. There is still good agreement for this case where the disturbing acceleration is near zero.

In conclusion, the nodal motion of sun synchronous orbits has been investigated and found to exhibit the characteristics of a pendulum. This pendulum motion results from solar

gravity inducing an inclination oscillation which couples into the nodal precession induced by the earth's oblateness. The pendulum model has been compared with simulations and flight data with excellent correlation observed.

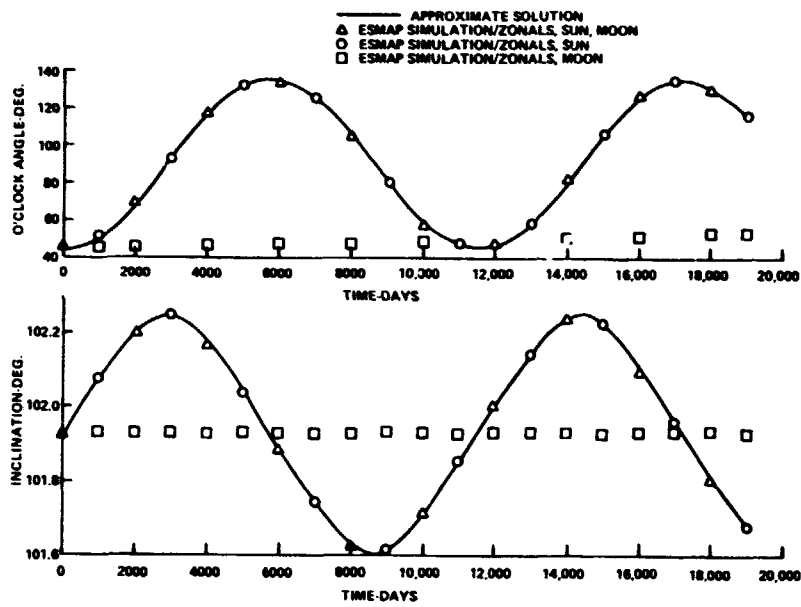


Figure 3. Approximate solution.

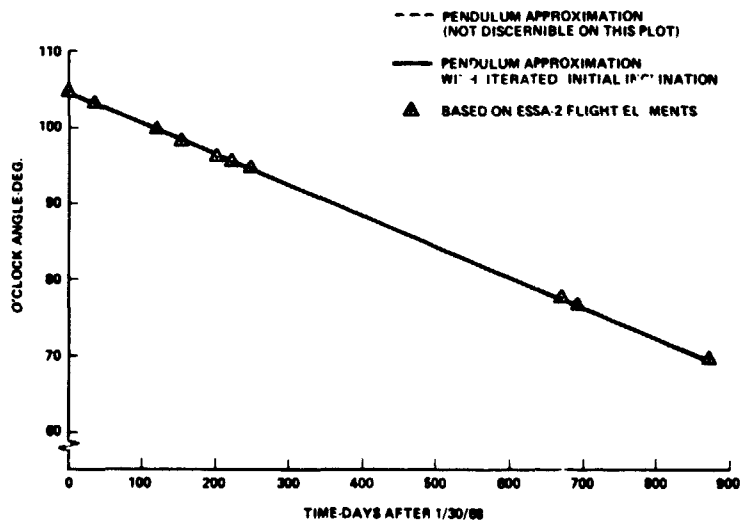


Figure 4. ESSA-2 pendulum solution comparison.

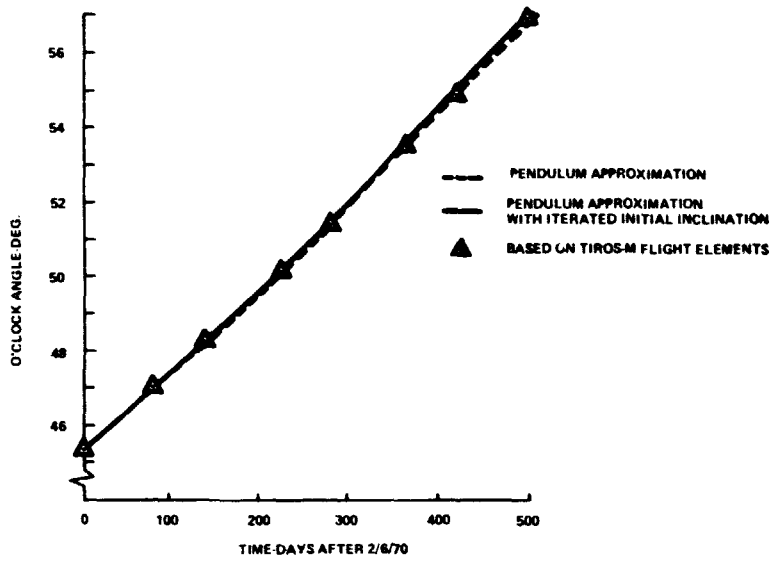


Figure 5. Tiros-M pendulum solution comparison.

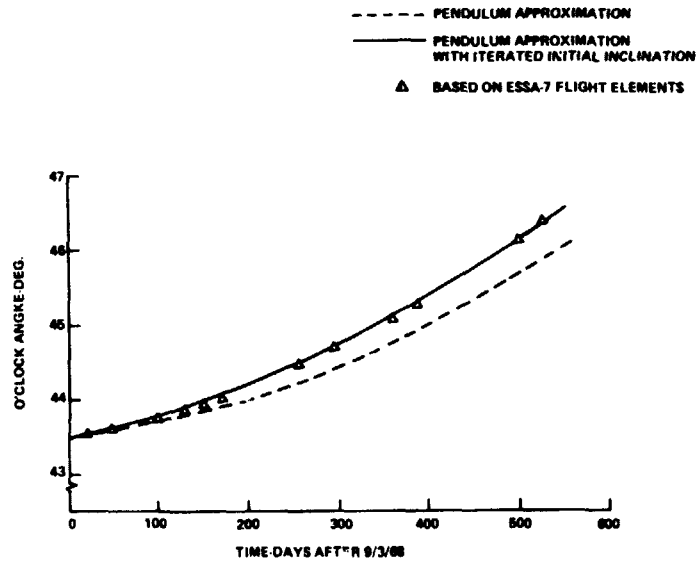


Figure 6. ESSA-7 pendulum solution comparison.

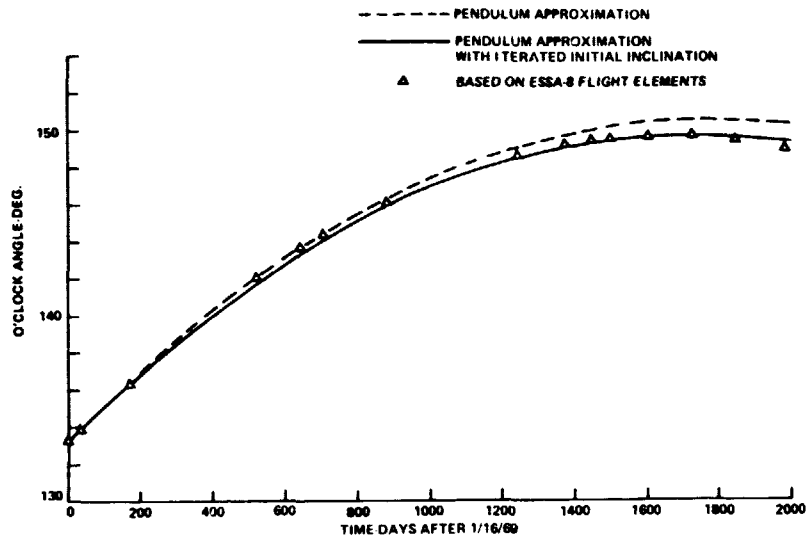


Figure 7. ESSA-8 pendulum solution comparison.

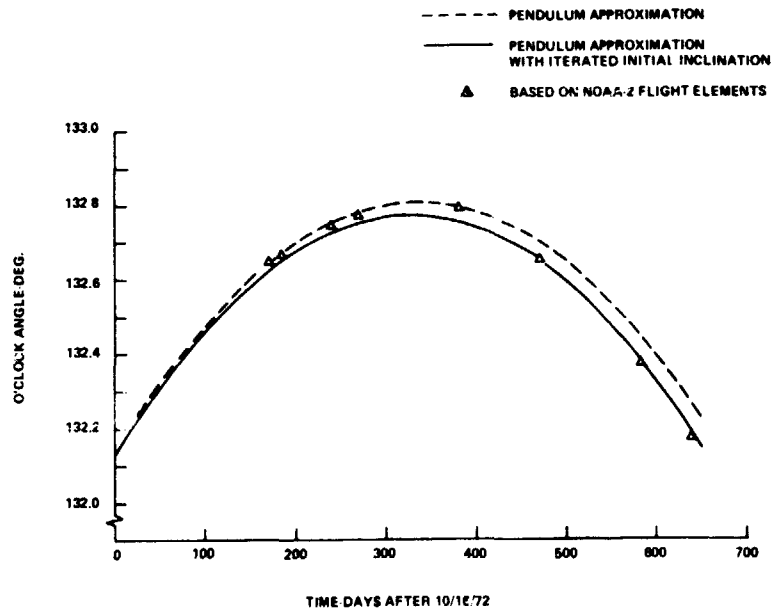


Figure 8. NOAA-2 pendulum solution comparison.



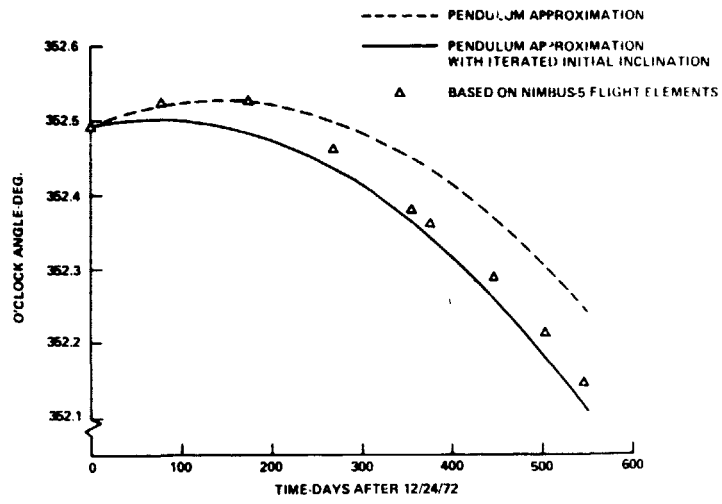


Figure 9. Nimbus-5 pendulum solution comparison.