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THE APPLICATION OF IMAGE PROCESSING TO SATELLITE NAVIGATION

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ABSTRACT

Given the locations of several landmarks on a satellite acquired image and their true geographic coordinates, the position and orientation of the satellite can be determined. Two methods for automatically locating the image coordinates of specified landmarks are described. The first, a particularly fast sequential similarity detection algorithm for template matching was originally described by Nagel and Rosenfeld. The second method involves iteratively resampling the picture function in the vicinity of the anticipated landmark. A variety of other speedup methods is also described. An application to SMS imagery is envisioned.

INTRODUCTION

An important role in satellite navigation is played by the recognition and location of landmarks on images of the earth's surface as viewed by the satellite. Given the locations of several landmarks on the image, and their true geographical locations, the position and orientation of the satellite can be determined. The mathematics of this process, and its error analysis, will not be recapitulated here. (See, for example, Phillips and Smith, 1972.)

We shall assume here that the approximate position and orientation of the satellite are already known, and that we want to use the landmark data to obtain more accurate estimates. This implies that we know approximately where, on the image, landmarks should appear—perhaps to within a few dozen picture elements (pixels). We shall ignore here any errors in our estimation of the orientations and sizes of the features; it is reasonable to assume that if the error in estimated position is small, then the errors in estimated orientation and size are negligible.

If we regard the orientations and sizes of the landmarks on the image as shown, and their positions as approximately known, then the problem of landmark identification and location can be solved by a template matching process. This basically involves comparing a template or small image of the landmark with the picture, in the range of positions where the landmark could be located. If a good match is obtained, the landmark has been detected, and the position of this good match gives the location of the landmark.

There are many different template matching processes that could be used for this purpose, and the matching can be implemented in various ways (digitally, optically, or by a human operator). We shall consider here only automated digital techniques.

The objective of this proposed effort is to make the template matching as computationally efficient as possible, while keeping machine storage requirements at the minicomputer level.

The standard digital template matching technique, which is based on cross-correlation, is relatively slow. Substantially faster results can be obtained by using an absolute difference measure of mismatch (sometimes known as the sequential similarity detection algorithm (SSDA). This approach is not only cheaper computationally, but also lends itself to the use of efficient search techniques for speeding up the detection of mismatches, as shown by Nagel and Rosenfeld, 1972. It can be combined with various pre- and post-processing techniques that should increase the sharpness of the matches obtained, thus increasing the accuracy, as well as the speed, with which landmarks can be located.

TEMPLATE MATCHING

Use of the Absolute Difference Mismatch Measure

The most commonly used measure of the match between a template, t (x, y), and a picture p (x, y), is the correlation coefficient (or normalized cross-correction)

$$\frac{\iint_T t(x, y)\, p(x + u, y + v)\, dxdy}{\sqrt{\iint_T t^2(x, y)\, dxdy \iint_T p^2(x + u, y + v)\, dxdy}}$$

where (u, v) is the displacement of the template relative to the picture. Here the area of integration, T, is the area occupied by the template. This coefficient takes on values between 0 and 1, and has value 1 only when t (x, y) and p (x + u, y + v) are identical (over the area T) except for a multiplicative constant. In the digital case, the expression for the correlation coefficient is identical, except that the integrals are replaced by sums over picture element arrays.

Computation of the correlation coefficient is relatively slow, because it involves a large number of multiplications of template values by picture values for each position of the template relative to the picture. The process can be speeded up by using Fourier transform techniques to compute the cross-correlation; however, this approach is more costly in computer memory requirements, and will not be discussed further here.

A more quickly computable measure of the mismatch between a template and a picture is the integrated (or summed) absolute difference.

$$\iint_T \left| t(x, y) - p(x + u, y + v) \right| \, dxdy$$

This measure is zero when t and p are identical (over the area T). Computation of this measure involves no multiplications; many fewer arithmetic operations are required than in the case of the correlation measure (for a quantitative comparison see Barnea and Silverman, 1972). In addition, use of the absolute difference measure makes it possible to take advantage of significant shortcuts in the matching process.

Stopping Criteria

A major advantage of the sum of the absolute differences as a mismatch measure is that this measure grows monotonically with the number of points whose absolute differences have been added into the sum. This means that if we are looking for a point of best match in a given region, we can stop adding up absolute differences in a given position (u, v) as soon as the sum has grown larger than the smallest sum so far obtained in any position (u_0, v_0), since we now know that (u, v) cannot possibly be the position of best match.

We can do even better if we set up an absolute threshold, such that if the mismatch measure exceeds θ at a given point we will not accept that point as a point of match. It is now sufficient to add up terms of the sum until θ is exceeded: when this happens, we can reject the given point and move on to the next point.

By using both mismatch comparison and absolute thresholding, the process of finding a best match (below the mismatch threshold) can be speeded up considerably. For points where the match is poor, the threshold should be exceeded rapidly so that only a fraction of the terms in the sum need be computed for such points. Even for points where the match is good, we need not always compute all the terms of the sum, since we can stop as soon as we exceed the smallest previously obtained sum.

Optimum Matching Sequences

It has been pointed out by Nagel and Rosenfeld (1972) that the expected amount of time required to exceed a mismatch threshold can be reduced by matching template pixels against the corresponding image pixels in a preselected order. To illustrate this idea, let us consider a simple example. Suppose that the template and image contain only three gray levels, e.g., 0, 1, and 2, and that, in the given region of the image, these levels occur with relative frequencies of 1/2, 1/3, and 1/6, respectively. If the template is not at or near the correct match position, we can assume that the image pixel that coincides with a given point of the template is uncorrelated with the template gray level at that point. Thus, we can compute the expected absolute gray level difference between template and image at a point as follows:

Template Gray Level	Probability of Difference					Probability of Absolute Difference			Expected Absolute Difference
	-2	-1	0	1	2	0	1	2	
0	1/6	1/3	1/2	0	0	1/2	1/3	1/6	2/3
1	0	1/6	1/3	1/2	0	1/3	2/3	0	2/3
2	0	0	1/6	1/3	1/2	1/6	1/3	1/2	4/3

In other words, the expected absolute difference between template and image at a template point having gray level 0 or 1 is 2/3; but, at a template point of gray level 2, the expected absolute difference is 4/3. Thus we can expect, on the average, a faster rate of growth of the sum of absolute differences if we compute the difference first for template points that have gray level 2. Thus, the mismatch threshold (or the lowest level of mismatch previously found) is likely to be exceeded sooner if we compare template points with image points in a special order–namely, first using template points of gray level 2–rather than using the template points in an arbitrary order.*

Generalizing the example just given, it is easily seen that the mismatch threshold (absolute or relative) will be exceeded faster on the average if the template points are compared with the corresponding image points in decreasing order of expected absolute gray level difference. The degree of speed-up achievable in this way depends, of course, on the probability distribution of gray levels in the given region of the image; if all gray levels are equally likely, no speedup is possible by this method.

The method just described should be even more effective when we are matching multispectral, rather than gray scale, images. This is because a probability distribution of multispectral vector values should be even more nonuniform than a distribution of gray levels would be. Thus it should be possible to select template points whose expected absolute differences from the corresponding image points should be quite large. Here the differences must, of course, be defined in vector terms, e.g., as sums of absolute differences of the individual vector components.

The speedup obtainable by this method will not be the same for all landmarks. But the best results should be obtained for landmarks in whose vicinity the distribution of gray levels, or multispectral values, is highly nonuniform. Given a set of available landmarks, one can determine, for each of them, how much speedup can be expected, and one can select preferred landmarks for which the expected speedup is greatest.

* It may take more time to locate the desired template points than it would if we tested them, perhaps in a simple raster sequence; but Nagel and Rosenfeld (1972) have shown that their method is faster than raster matching in spite of this.

The position of best match, determined by any method, may not be sharply defined, since there may be a set of nearby positions for which the match is nearly as good. It has been known for at least 20 years that match sharpness can be improved by differentiating or by high-pass filtering the image and the template before matching them. In other words, outlines of regions give sharper matches than do solid regions.* It would be of interest to investigate whether the matching speedup process described above is improved by using differentiated or outlined images and templates. This will certainly be the case if the differentiated image has a less uniform distribution of gray levels (or spectral values) than the original image, as we would normally expect.

ITERATIVE RESAMPLING

Match positions can be determined to within closer than a pixel by resampling the image (at a grid of points that have fractional coordinates, in terms of the original sampling grid) and assigning gray levels to the new pixels by interpolation. The match at such an interpolated position may be better than the match at any of the original (intercoordinate) position. The investigation of interpolated match positions is probably best done as a fine tuning of a coarse match position found without use of interpolation. The interpolated images will tend to have more uniform gray level or spectral distributions than the original image, so that the matching speedup process will probably not be as effective during this fine tuning stage as at the coarse search stage.

A refinement allowing more accurate (part pixel) registration, when such accuracy is warranted, may be described as follows:

Let F (x, y) be the cross-correlation function in terms of picture displacements (x, y) from the assumed best match point, x = 0, y = 0. Fit a quadratic to F (x, y) of the form

$$F(x, y) = ax^2 + 2h + by^2 + 2gx + 2fy$$

We may determine the coefficients from the values of F at a few points. At x = n, y = 0, for example:

$$F(n, 0) = an^2 + 2gn$$

Similarly,

$$F(-n, 0) = an^2 - 2gn$$

hence

$$a = [F(n, 0) + F(-n, 0)]/2n^2$$
$$g = [F(n, 0) - F(-n, 0)]/4n$$

* It may be of further advantage to use a thresholded outline image so as to reduce the effects of overall gray level differences due to seasonal changes, and so forth.

We can also find

$$b = [F(0, n) + F(0, -n)]/2n^2$$

$$f = [F(0, n) - F(0, -n)]/4n$$

$$h = [F(n, n) - F(n, 0) - F(0, n)]/2n^2$$

Differentiating, F (x, y) is a maximum when*

$$ax_0 + hy_0 + g = 0$$

and

$$by_0 + hx_0 + f = 0$$

so that,

$$x_0 = (hf - bg)/(ab - h^2)$$

$$y_0 = (gh - af)/(ab - h^2)$$

Then shift origin to (x_0, y_0) and repeat with n/2 area size. Since only 5 values of F need to be obtained per iteration, the speed of such a procedure may prove to be adequate.

As an example in one dimension, consider the following template:

3 1 9 1 3

and the image line gray values:

6 6 4 8 5 6 6

Take $(F(x)) = ax^2 + 2gx$.

$$F(-1) = \frac{(3 \times 6) + (1 \times 6) + (9 \times 4) + (1 \times 8) + (3 \times 5)}{\sqrt{(3^2 + 1^2 + 9^2 + 1^2 + 3^2)(6^2 + 6^2 + 4^2 + 8^2 + 5^2)}}$$

$$= 0.6208 = a - 2g$$

$$F(1) = \frac{(3 \times 4) + (1 \times 8) + (9 \times 5) + (1 \times 6) + (3 \times 6)}{\sqrt{(3^2 + 1^2 + 9^2 + 1^2 + 3^2)(4^2 + 8^2 + 5^2 + 6^2 + 6^2)}}$$

$$= 0.6656 = a + 2g$$

$$a = 1/2\,(F(1) + F(-1)) = 0.6432$$

$$g = 1/4\,(F(1) - F(-1)) = 0.0112$$

* After determining coefficients, we require that F be positive definite, i.e., $ab > h^2$, to guarantee a true maximum of F.

Differentiating, F (x) is a maximum when $ax_0 + g = 0$, i.e., for $x_0 = -g/a = -0.0174$ we see:

Gray Scale	Old Coordinate	New Coordinate
6	-3	-2.9826
6	-2	-1.9826
4	-1	-0.9826
8	0	0.0174
5	1	1.0174
6	2	2.0174
6	3	3.0174

In this new coordinate system we will need F(-0.5), hence by interpolation we will need the gray level values at X = -2.5, -1.5, -0.5, 0.5, 1.5, 2.5.

If linear interpolation is used, then

$$G(x) = G(x_1) + \frac{(G(x_2) - G(x_1))}{(x_2 - x_1)} (x - x_1)$$

thus,

$$G(-2.5) = 6.0000$$

$$G(-1.5) = 5.0348$$

$$G(-0.5) = 5.9304$$

$$G(\ 0.5) = 6.5522$$

and so forth.

Then a new iteration is undertaken and the process continues until the origin shift indicated is satisfactorily small.

OTHER SPEEDUPS

A possibility for still further improvement in matching efficiency, not discussed here up to now, is to identify locations in the image where the match can be expected to be good. This is typically done by making some simple local measurements on the image, and finding positions where the values of these measurements are close to their values for the template. In the present application, the positional uncertainty of the landmarks is not expected to be very large, so that it seems likely that much will be gained by the use of this approach.

There is, however, an important possibility for reducing the number of match positions that need be tried, once a match has been found for the first landmark. If this match is correct, the positional uncertainty of the remaining landmarks has now been reduced, since there are now fewer degrees of freedom. This is true even if the match is regarded as only approximate. Thus, after the first match is found, the next match can be searched for over a relatively smaller region. After enough matches have been found, additional matches have very little positional uncertainty, so that we can search for them over very small regions. Of course, if the subsequent matches are not found in the expected places, they must be sought for over wider regions, and the original matches must be reevaluated. Thus a feedback process can be used to zero in on a best combination of match positions.

TECHNICAL DISCUSSION

The specific application to which this research is directed is the navigation of the Synchronous Meteorological Satellite (SMS). In time it is hoped that the processing of recognized landmarks for navigation (both for location error correction and relocation) can take place in quasi-real-time, which in this context is approximately 5 minutes. Before this can come about, there is a need to determine landmark processing algorithms that are rapid, accurate, reliable, and economical in storage.

It is recognized that several operational questions influence the utility of the algorithm development results. In particular, eventual implementation may be effected on a minicomputer with a hardware dot product. The availability of such a feature would have some significance to the efficiency of algorithms in the multispectral domain.

Also, error budgeting will play a central role. It may not be useful, for example, to process the image data with extreme accuracy since the errors in the landmark surveys may well predominate (current landmark accuracies are believed to be about 1/100th of a degree in latitude and longitude, and may have a systematic bias). Hence it may be useful to concentrate on using the same landmarks repetitively for relative image-to-image geometric transformations rather than absolute geodetic location determination.

Thirdly, cloudcover distribution probability functions will affect the ability to use the landmarks. The geometric relationships require a well distributed set of control points if orientation and orbit parameters are to be determined accurately. Inadequate or ill-distributed control point data give rise to ill-conditioned matrices and the calculations become unreliable.

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CLOSED FORM SATELLITE THEORY WITH EQUINOCTIAL ELEMENTS

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A computer software system has been developed to generate the closed-form literal first-order perturbations due to any harmonic in the potential. In a first approach, classical elements and the true anomaly are used. In another approach, equinoctial elements and the true longitude are the basic variables. The solution in equinoctial elements does not have any zero eccentricity or zero inclination singularities.

In the case of tesseral and zonal harmonics, the rotation of the central body is neglected. The expansion of the potential is done with simple recurrence formulas.

FORMULATION OF AN ARBITRARY GEOPOTENTIAL TERM (TESSERAL) IN EQUINOCTIAL VARIABLES

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Previously, general formulas for the averaged disturbing potential were obtained (AIAA preprint 75-9) in equinoctial elements for the zonal and third-body harmonics. In addition, methods were given for recursive computation of these potentials. The current paper extends the applicability of this model by giving an explicit expression in equinoctial elements for the arbitrary geopotential term:

$$U = \frac{\mu}{r}\left(\frac{R_e}{r}\right)^n P_{nm}(\sin\phi)\,[C_{nm}\cos m\lambda + S_{nm}\sin m\lambda]$$

Expressions for the spherical harmonics $P_{n,m,}(\sin\phi)\cos m\lambda$ and $P_{n,m}(\sin\phi)\sin m\lambda$ are obtained in terms of Jacobi polynomials with the argument $(1 - p^2 - q^2)/(1 + p^2 + q^2)$ (p and q are equinoctial elements), polynomial functions which are straightforward generalizations of the C_s and S_s polynomials appearing in the zonal potential, the true longitude, and the Greenwich sidereal time, θ is fixed during the averaging period. However, an expansion of the true longitude in terms of the mean longitude and the equinoctial elements k and h would allow the consideration of resonant cases. Finally, consideration is given to recursive computation of the averaged potential for tesserals.

AN ANALYTIC METHOD TO ACCOUNT FOR ATMOSPHERIC DRAG WITH AN ANALYTIC SATELLITE THEORY

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The motion of an artificial earth satellite in the presence of air drag and the earth's gravitational potential is considered. In contrast to the classical methods of numerical integration, this approach presents a quadrature algorithm employing analytical expressions for the variation of orbital elements produced by the air drag. These expressions are well-defined over expanded subintervals of the solution, and produce accurate agreement with profiles of tabular density. This procedure then allows a flexibility in the selection of end points of the subintervals, which in turn ensures a minimum error bound on the required analytical function.

In this method the effect of oblateness is accounted for by either the Vinti spheroidal theory, the Brouwer orbit theory, or the Brouwer-Lydanne theory. The changes due to atmospheric resistance for a nonrotating sphere are accounted for by the solutions of the variational equations, evaluated with the appropriate theory.