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SPECIAL PERTURBATIONS USING BACK-CORRECTION METHODS OF NUMERICAL INTEGRATION

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Previous speakers have discussed how to change the analytical formulation of various problems in order to introduce stabilizing effects or to improve the accuracy and thereby increase the efficiency of numerical integration. The present topic is concerned with how to improve the process of numerical integration itself, in order to increase efficiency in the development of accurate ephemerides for earth satellites. In particular, the subject of this discussion is a new class of linear multistep methods for the numerical integration of ordinary differential equations. These methods are distinguished from the classical methods in that they permit the solution to be corrected at certain "back" points. That is, in the case of satellite computations, the solution is corrected at certain points in the past as the integration advances in time. Algorithms have been developed for the solution of both first- and second-order differential equations, although only the second-order case is considered here.

There are two reasons for correcting the solution at back points: First, when a polynomial obtained from interpolating evenly spaced data is used for approximating a function at a point, the coefficient of the error term is smaller when the point is nearer the middle of the range of data. Therefore, by performing the last correction at an internal point of the grid, a more accurate solution (that is, smaller truncation error for a given order) is obtained. The second (and not so intuitive) reason is that the introduction of back corrections induces numerical stability. It is well known that the general stability boundaries for the predictor-corrector methods of the Stormer-Cowell type decrease geometrically with increasing order. That is, when the higher order Stormer-Cowell methods are used on a practical problem, the step size is severely constrained by stability considerations. This is not the case with the methods using back corrections. In fact, in some cases, these methods possess general stability boundaries which are 30 to 40 times larger than those of the Stormer-Cowell methods of the same order. Moreover, the stability regions of these methods do not exhibit geometrical decay but remain relatively unchanged up to the eighteenth or nineteenth order.

The back-correction methods are given by:

$$x_{n+1} = (m+2)\,x_{n-m} - (m+1)\,x_{n-m-1} + h^2 \sum_{j=0}^{k} \alpha_j f_{n-j} \qquad (1)$$

and

$$x_{n+1-\ell} = (m+2-\ell)\,x_{n-m} - (m+1-\ell)\,x_{n-m-1} + h^2 \sum_{j=0}^{k} \beta_{j\ell} f_{n+1-j} \qquad (2)$$

for $\ell = 0, 1, \ldots, m$. The position vector of the satellite at time t_n is denoted by x_n, and the acceleration vector, by f_n. The number of back corrections is m. The coefficients $\{\alpha_j\}$ and $\{\beta_{j\ell}\}$ are determined in such a way that the highest possible order is obtained for a given k. It should be noted that for m = 0, the equations reduce to those of the Stormer-Cowell method. Therefore, the back-correction methods are simply a generalization of the classical Stormer-Cowell method.

These methods can be used with pseudo-evaluations in the following algorithm, provided the acceleration can be separated according to dominant and perturbing terms:

a. Predict a value, x^0_{n+1}, for x_{n+1} using equation 1.

b. Evaluate the dominant and perturbing accelerations using this value for x_{n+1}, saving the perturbing acceleration for subsequent calculations.

c. Set $\ell = 0$.

d. Using equation 2, obtain a corrected value, $x^{\ell+1}_{n+1-\ell}$, for $x_{n+1-\ell}$.

e. Reevaluate only the dominant acceleration using $x^{\ell+1}_{n+1-\ell}$ and obtain $f_{n+1-\ell}$ by adding the previously calculated perturbing acceleration at $t_{n+1-\ell}$.

f. If $\ell = m$, proceed to the next step of the integration. Otherwise, set $\ell = \ell + 1$ and go to (d) above.

In problems involving earth satellites, the forces can readily be separated according to dominant and perturbing terms. In such problems, these algorithms are especially efficient, because all evaluations of the forces after the first are simply pseudo-evaluations requiring only the reevaluation of the dominant forces (in this case, the two-body force).

The methods using back corrections have been tested on several problems. A numerical integration of the orbit of the Applications Technology Satellite-F (ATS-F) has been performed in which case the errors were smaller by two or three orders of magnitude when compared to the classical Stormer-Cowell method using pseudo-evaluations.

The results shown in table 1 are obtained when the seventeenth-order back-correction algorithm is applied to the to the Geodetic Earth Orbiting Satellite-C (GEOS-C) orbit.

Also shown are the results obtained using the twelfth-order Stormer-Cowell algorithm presently available in the Goddard Trajectory Determination System (GTDS). The errors are obtained by comparing with the results obtained using a step size of 40 seconds. It should be noted that for reasonable step sizes, the seventeenth-order back-correction method provides greater accuracy than the twelfth-order Stormer-Cowell method. Since pseudo-evaluations are used in both methods, the computer time required per step is the same (within a few percent) for both methods.

Table 1
Numerical Results for GEOS-C Satellite

Step Size (s)	Errors in Position after 24 Hours (km)	
	12*th* Order Stormer-Cowell	17*th* Order Back-Correction Method
60	5.7×10^{-5}	1.9×10^{-7}
80	3.2×10^{-4}	8.8×10^{-7}
100	1.1×10^{-2}	1.5×10^{-4}
200	6.9×10^{-1}	2.2×10^{-1}

The stability region of the twelfth-order Stormer-Cowell method is, in fact, slightly smaller than that of the seventeenth-order back-correction method. A higher order Stormer-Cowell method would no doubt have exhibited greater accuracy than the twelfth-order Stormer-Cowell method, but the stability region would have been considerably reduced. Consequently, the largest meaningful step size attainable with such a higher order method would have been much smaller.

The methods using back corrections appear to be more efficient than the classical methods for problems in which the dominant and perturbing forces can be readily separated and in which the evaluation of the perturbing terms requires much more computer time than the evaluation of the dominant terms.

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