Astronautics Research Report No. 74-5

SPACECRAFT CONTROL/FLEXIBLE STRUCTURES INTERACTION STUDY

(Final Report on JPL Contract No. 953807)

prepared by

Marshall H. Kaplan
(Principal Investigator)

and

Stanley E. Hillard

Department of Aerospace Engineering

The Pennsylvania State University

University Park, Pennsylvania 16802

September, 1974

"This work was performed for the Jet Propulsion Laboratory, California Institute of Technology sponsored by the National Aeronautics and Space Administration under Contract NAS7-100."

TECHNICAL CONTENT STATEMENT

"This report contains information prepared by the Department of Aerospace Engineering of the Pennsylvania State University under a JPL subcontract. Its content is not necessarily endorsed by the Jet Propulsion Laboratory, California Institute of Technology, or the National Aeronautics and Space Administration."

ABSTRACT

An initial study has been completed to begin development of a flight experiment to measure spacecraft control/flexible structure interaction. The work reported consists of two phases: identification of appropriate structural parameters which can be associated with flexibility phenomena, and suggestions for the development of an experiment for a satellite configuration typical of near future vehicles which are sensitive to such effects. Recommendations are made with respect to the type of data to be collected and instrumentation associated with these data. The approach consists of developing the equations of motion for a vehicle possessing a flexible solar array, then linearizing about some nominal motion of the craft. A set of solutions are assumed for array deflection using a continuous normal mode method and important parameters are exposed. Inflight and ground based measurements are distinguished. Interrelationships between these parameters, measurement techniques, and input requirements are discussed which assure minimization of special vehicle maneuvers and optimization of data to be obtained during the normal flight sequence. Limited consideration is given to flight data retrieval and processing techniques as correlated with the requirements imposed by the measurement system. Results indicate that inflight measurement of the bending and torsional mode shapes and respective frequencies, and damping ratios is necessary. Other parameters may be measured from design data. Areas of proposed investigation include examination of specific array configurations, computer simulation of array dynamics, interaction properties of the attitude control system, and design of the measurement system.

TABLE OF CONTENTS

4

TABLE OF CONTENTS (Cont'd)

LIST OF FIGURES

6

NOMENCLATURE

$\{\vec{a}\}$	array frame of reference
$\{\vec{b}\}$	vehicle frame of reference
$\{\vec{i}\}$	inertial frame of reference
a_j, b_{ij}, c_{ij}, d_j	
e_{ij}, g_j, h_j, m_j	mode dependent integral expressions defined in Appendix B
n_j, p_j, t_j, u_j	
f	frequency of hi... retained harmonic mode
m_1, m_2	mass dependent integral expressions defined in Appendix B
m^s	mass distribution of array (mass per unit area)
$\vec{q}_1$	angular velocity of craft in inertial frame
$\vec{q}_2$	angular velocity of $\{\vec{a}\}$ frame with respect to $\{\vec{b}\}$ frame
q_{10}^k	disturbance values for q_1^k, angular velocity of vehicle
$\vec{r}_i$	position vector ($i = 1,\ldots 6$)
t	time
C^1	transformation matrix between $\{\vec{b}\}$ and $\{\vec{a}\}$
C^2	transformation matrix between $\{\vec{b}\}$ and $\{\vec{i}\}$
C_{ij}^1, C_{ij}^2	transformation matrix components
$d\vec{F}^s$	force distribution on array differential element
F^s	force distribution on array
$\bar{F}^s$	force magnitude on appendage element
$\bar{F}_3^s$	component of F^s in a_3 direction
F_i	$\int_{r_3} \int_{r_4} \bar{F}_3^s \, \phi_i(r_3) dr_3 dr_4$

$\vec{H}$	angular momentum vector
I^s	moment of inertia of array element
I_p	vehicle moment of inertia ($p = 1,2,3$)
M	vehicle total mass
N	degree of highest harmonic mode retained in analysis
R	$\int_{r_4} dr_4$
R'	$\int_{r_4} r_4 dr_4$
dT^s	torque acting on array differential element
T_2^s	component in a_2 direction of T^s
T_i	$\int_{r_3} \int_{r_4} \bar{T}_s \psi_i(r_3) dr_4 dr_3$
$(Q_{10}^k(\lambda))$	angular velocity amplitude, frequency dependent
$(R_{10}^k(\lambda))_j$	bending amplitude of j^{th} mode
$(\Lambda(\lambda))_j$	torsional amplitude of j^{th} mode
μ	mass ratio
α	array torsional dependent variable
$\vec{\beta}^s$	array element rotation vector
δ_{ij}	Kronecker delta
λ	frequency parameter
ϕ_i	bending mode shape of i^{th} mode
ψ_i	torsional mode shape of i^{th} mode

I. INTRODUCTION

1. PURPOSE OF THE INVESTIGATION

Adverse effects of flexibility on satellite attitude motion were manifested early in the U.S. space program and have remained important factors in spacecraft designs. Mathematical analysis supported by ground experimentation have been relatively reliable for the present generation of satellites. However, future and presently developing spacecraft are progressing toward larger and highly flexible antennas and solar panels. In such cases ground experiments are impractical because of size and environmental restrictions.

The overwhelming importance of the flexibility problem coupled with possible inadequacies of present analytical approximations and ground testing procedures indicate the introduction of a program and apparatus capable of yielding inflight data on appendage dynamics and their effects on craft control. Only by implementation of a monitoring system on these new satellite configurations can continued progress be made on the control of these effects. In addition, further refinements on design procedures can be derived from this data.

It is the purpose of this investigation to identify the structural and control system parameters which can be associated with the flexibility-control interaction phenomenon, and develop an experiment for a satellite configuration typical of near future vehicles.

2. CURRENT AND RECENT FLEXIBILITY PROGRAMS

Many proposed vehicles have appendages which are very large and highly flexible. Examples are the radiotelescope satellite (Ref. 1,2), composed of flexible aluminum ribbons having a diameter of fifteen hundred meters. Other equally illustrative examples are abundant, i.e., the proposed direct T.V. broadcast satellite (Ref. 1) which possesses solar arrays with an area of one thousand square meters. The size of these vehicles indicates the importance of accurately anticipating flexibility interactions.

Programs designated for flexibility experiments have generally been among the first to suffer from cost cuts, weight decreases, and other considerations leading to system abolishment. Minor flexibility control interaction tests were performed in conjunction with the Gemini, Apollo programs (Ref. 3,4), and a major attempt at flexibility measurement was planned in conjunction with the Canadian Communications Technology satellite (Ref. 5,6). However, this effort has been stripped to an optical system which will measure only static deformations. The Flexible Rolled Up Solar Array (Ref. 7) has been flown successfully and is equipped with six accelerometers and three strain gages and temperature sensors for the detection of array dynamic response. Interaction effects were not observable and peak accelerations have been only of the order of a few milli-g's.

Several significant analytical studies have been undertaken to determine appendage dynamics and control interaction phenomenon. Studies have been completed which include the contribution of large flexible appendages to control difficulties arising from continuous and flexibly connected bodies. One of the more promising analytical methods uses hybrid coordinates (Ref. 8,9). This method formulates the flexible space vehicle control as a combination of discrete and modal coordinates. The analysis involves three distinct steps. First, preliminary design is based on root locus plots for single axis response of linearized systems. Second, modifications are made as required by eigenvalue analysis of the coupled linear systems. Third, nonlinear differential equation simulation is accomplished using numerical integration. The third step is used for design confirmation.

In particular cases, several other methods of analysis are adequate depending on vehicle geometry and appendage characteristics. These are the energy sink method, usually applicable to the compact, near rigid body type of vehicle; the discrete parameter method, which is best suited to a satellite with compact bodies and flexibly attached appendages; and the modal coordinate method, which is adaptable to highly flexible craft with larger overall dimensions.

II. FLEXIBLE STRUCTURE

1. DEFINITION

An adequate definition for flexibility may be evaluated only with regard to the context in which it is to be used. For applications to satellite flexibility control interaction, it is not necessary to define flexibility in the strictest physical sense but rather to define those aspects of flexibility necessary in predicting important coordinate values and parameters of the interaction. It is also necessary that the chosen mathematical model generate equations of motion which are computationally treatable.

Two realms of flexibility-control interaction immediately distinguish themselves as important. First, attention should be directed toward determining attitude control sensitivity to the elastic mode frequencies of appendages and flexibly connected subsystems of rigid bodies. Secondly, sensitivity of attitude control sensors to changes in vehicle inertial properties arising from flexibilities must be investigated. Restricting the definition of "critical flexibility" to those considerations critical to the interaction problem leads to the following definition:

> Critical flexibility with respect to the attitude control-structural dynamic interaction problem are those modes of flexibility which create elastic deformations of continuous and discrete masses with modal frequencies within the frequency sensitivity range of the control system, and also those flexibility states which cause redistribution of vehicle inertia to an extent that control system activation is generated.

III. DEVELOPMENT OF EQUATIONS OF MOTION

1. ASSUMPTIONS AND MODEL CONFIGURATION

The spacecraft shall be modelled using a relatively simple configuration. It is found that this greatly reduces mathematical complexity of the model without significantly compromising the utility of resulting equations. The configuration chosen consists of a rigid central body with one continuous flexible solar array. The second array may be considered stiff. The appendage axis in its undeformed position is assumed to act through the vehicle center of mass. It is assumed that no external forces or torques are applied to the array and that the control system will react with a pure torque only. The basic configuration of the craft, earth, sun system is shown in Fig. 1.

Three coordinate systems are used in the formulation of the problem. An inertial reference $\{\vec{i}\}$ is fixed at the earth's center. A second system $\{\vec{b}\}$ is fixed to the vehicle center of mass and is allowed to rotate with

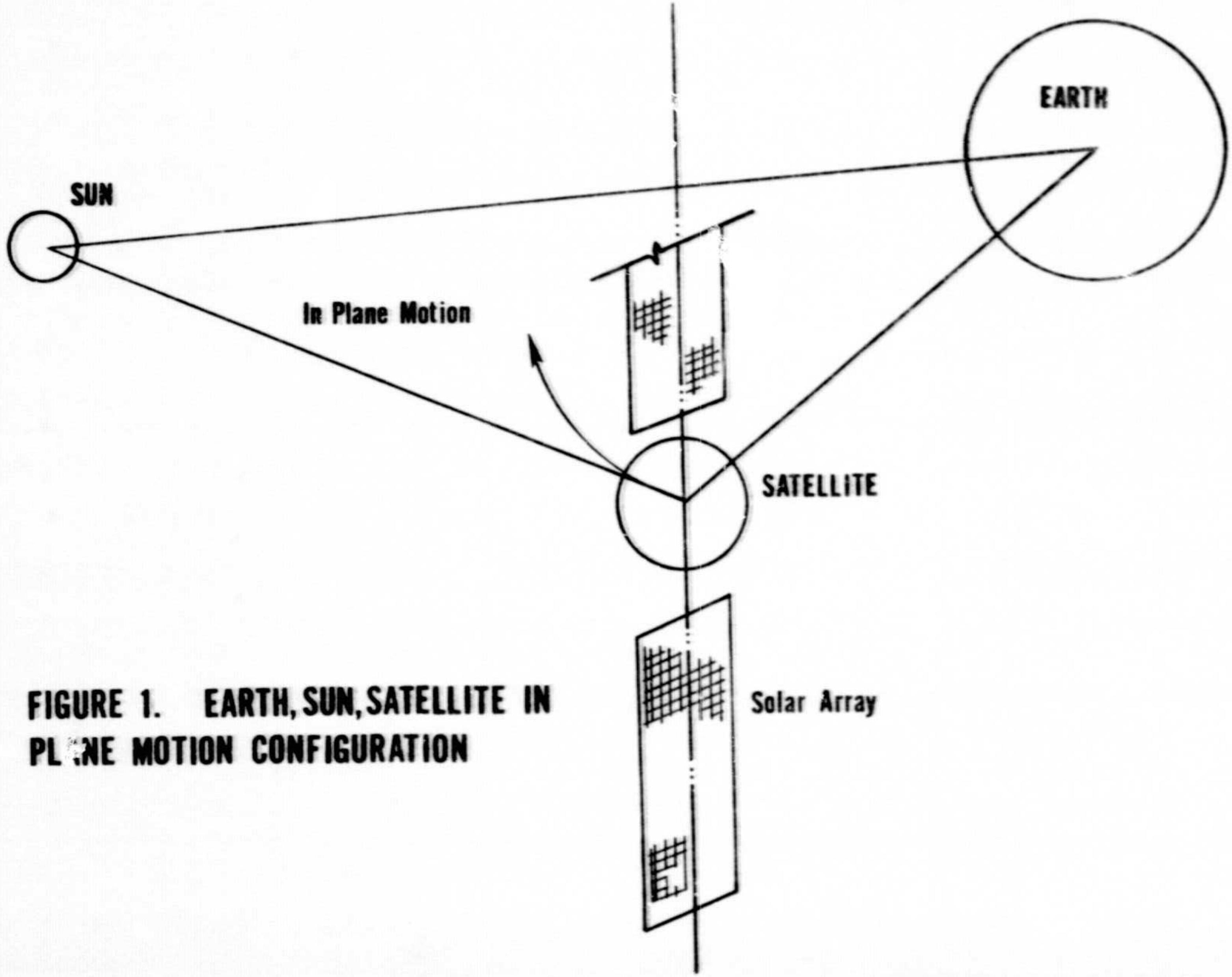

FIGURE 1. EARTH, SUN, SATELLITE IN PLANE MOTION CONFIGURATION

the rigid central body. The third coordinate system $\{\vec{a}\}$ is fixed in the solar array and allowed to rotate with the array as it maintains proper solar orientation. The origin of the $\{\vec{a}\}$ system will coincide to the origin of the $\{\vec{b}\}$ system when the array is undeflected, as will the a_2 and b_2 axes. For further simplification, the array is assumed flat and of homogeneous construction with no mass discontinuities. This assumption greatly reduces the analytical complexity while retaining the presence of important parameters. Array position vectors are shown in Fig. 2.

2. ARRAY EQUATIONS OF MOTION

If an element of the array is considered as shown in Fig. 2, the position vector is the sum of those vectors which may be referred to the previously defined reference frames. The equation of translational motion for the element may be derived from Newton's second law.

$$d\vec{F}^s = \sum_{i=1}^{6} \ddot{\vec{r}}_i dm^s \tag{1}$$

The frames of reference may be related through the use of appropriate transformation matrices.

$$\{\vec{a}\} = C^1\{\vec{b}\}$$

and

$$\{\vec{b}\} = C^2\{\vec{i}\}$$

The motion of the $\{\vec{a}\}$ frame with respect to the $\{\vec{b}\}$ frame will consist of a rotational motion known in time and a translational motion which is a function of the array deformation. For small time intervals, the motion of $\{\vec{b}\}$ with respect to $\{\vec{i}\}$ may be expressed in terms of the rigid

19

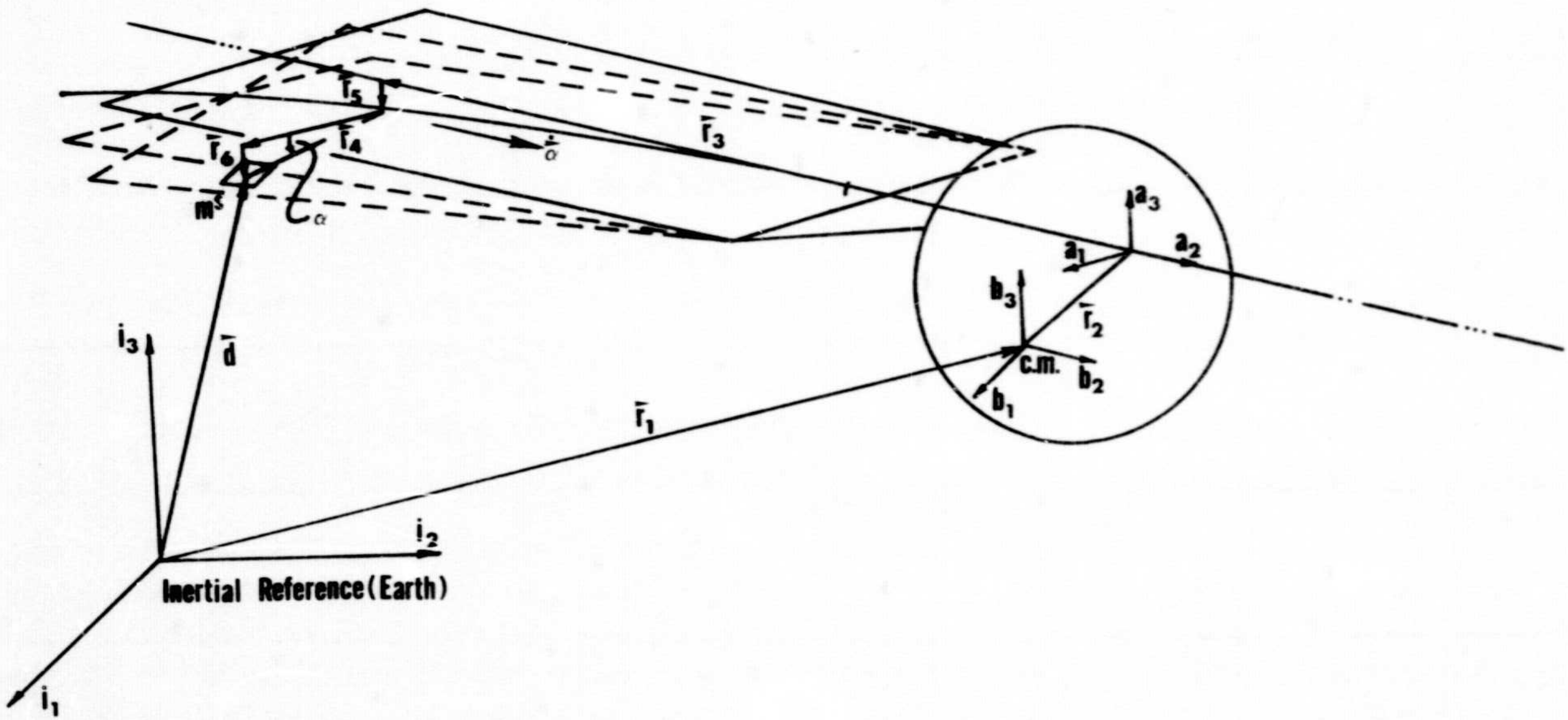

Figure 2. SATELLITE COORDINATE AND POSITION VECTORS

body angular velocity variables q_1. Transformation components of C^1 and C^2 shall be designated C^1_{ij} and C^2_{ij}.

Disturbance quantities shall be expressed as follows. Each component of a vector $\vec{A}$, shall consist of a nominal steady state value (e.g. A^1_1) plus a disturbance quantity (e.g. A^1_{10}). In reference frame $\{\vec{a}\}$, for example, vector $\vec{A}_1$ will appear as,

$$\vec{A}_1 = [a_1\ a_2\ a_3] \begin{bmatrix} (A^1_1 + A^1_{10}) \\ (A^2_1 + A^2_{10}) \\ (A^3_1 + A^3_{10}) \end{bmatrix}$$

where the superscript specifies the vector component. Position vectors are specified in Appendix A.

Using Eq. (1), a matrix relation extracted from work by Likin's Ref. 8), gives for the array element;

$$F^s = m^s \{C^1[C^2\ddot{r}_1 + \ddot{r}_2 + 2\tilde{q}_1\dot{r}_2 + \tilde{q}_1\tilde{q}_1 r_2 + \dot{\tilde{q}}_1 r_2]$$

$$- (\sum_{i=3}^{6} r_i)^{\sim}(C^1\dot{q}_1) + [(\widetilde{C^1 q_1})(\widetilde{C^1 q_1}) + 2(\widetilde{C^1 q_1})\tilde{q}_2 + \tilde{q}_2\tilde{q}_2](\sum_{i=3}^{6} r_i)$$

$$+ 2[\tilde{q}_2 + (\widetilde{C^1 q_1})](\dot{r}_5 + \dot{r}_6)^s + (\ddot{r}_5 + \ddot{r}_6)^s\} \qquad (2)$$

where the coordinate frame is $\{\vec{a}\}$.

The vector $\vec{r}_2$ exists only due to the array deformation. Therefore, it may be expressed in terms of the deflection variables $\vec{r}_5$ and $\vec{r}_6$. Defining the terms,

$$\mu = \frac{m^s}{M} \tag{3a}$$

$$R = \int_{r_4} dr_4 \tag{3b}$$

$$R' = \int_{r_4} r_4 dr_4 \tag{3c}$$

allows expression of r_2 as,

$$r_2 = -\mu \int_{r_3} C^2 [r_5 R + \alpha R'] dr_3 \tag{4a}$$

$$\dot{r}_2 = -\mu \int_{r_3} C^2 [\dot{r}_5 R + \dot{\alpha} R'] dr_3 \tag{4b}$$

$$\ddot{r}_2 = -\mu \int_{r_3} C^2 [\ddot{r}_5 R + \ddot{\alpha} R'] dr_3 \tag{4c}$$

Eqs. (4) must include the contribution of all dynamic components to the motion of the mass center. Since only the array is considered flexible, all contributions emanate from the array and are included in these integral expressions. The additional term "e" which occurs in Likin's work is zero under the assumptions here. The integral terms result in higher order terms and will be deleted upon linearizing the following equations. The orbital rate is assumed to be very small thus allowing the deletion of terms containing time derivatives of the transformation matrix C^2.

17

Thus, Eq. (2) may be expressed,

$$F^s = m^s \Bigg\{ C^1C^2 \Bigg[\ddot{r}_1 - \mu \int_{r_3} [\ddot{r}_5 R + \ddot{\alpha} R']dr_3 - 2\tilde{q}_1 \mu \int_{r_3} [\dot{r}_5 R + \dot{\alpha} R']dr_3$$

$$- (\tilde{q}_1\tilde{q}_1 + \dot{\tilde{q}}_1)\mu \int_{r_3} [r_5 R + \alpha R']dr_3 \Bigg] - (\sum_{i=3}^{6} r_i)^{\sim}(C^1\dot{q}_1)$$

$$+ [(\widetilde{C^1 q_1})(\widetilde{C^1 q_1}) + 2(\widetilde{C^1 q_1})\tilde{q}_2 + \tilde{q}_2\tilde{q}_2](\sum_{i=3}^{6} r_i)$$

$$+ 2[\tilde{q}_2 + (\widetilde{C^1 q_1})]\ (\dot{r}_5 + r_4\dot{\alpha})^s + (\ddot{r}_5 + r_4\ddot{\alpha})^s \Bigg\} \tag{5}$$

Out of Plane Bending

Bending in the array is considered normal to the array. Therefore, the third component equation is of interest here. Excluding higher order terms, the desired linearized equation becomes upon extracting the third component equation from Eq. (5),

$$F_3^s = m^s \Bigg\{ -(r_3[C^1_{11}\dot{q}^1_{10} + C^1_{12}\dot{q}^2_{10} + C^1_{13}\dot{q}^3_{10}]$$

$$+ r_4[C^1_{21}\dot{q}^1_{10} + C^1_{22}\dot{q}^2_{10} + C^1_{23}\dot{q}^3_{10}]) + (\ddot{r}^3_{50} + r_4\ddot{\alpha}) \Bigg\} \tag{6}$$

It should be noted again that the transformation matrix component's C^1_{ij} will contain a steady contribution and also a contribution which is a function of the vehicle angular velocity disturbance quantities. The latter will lead to higher order terms which may be deleted upon expansion of the transformation expression thus preserving the linear nature of the equation of motion.

The dependent variables of Eq. (6) may be expressed in frequency dependent form which is useful in analyzing array frequencies as correlated to those of the attitude control system. For a linearized system it may be assumed that the motion of the craft and array will be harmonic. The solutions may be expressed in complex form as a frequency dependent magnitude or summation of mode magnitudes times a time dependent exponential function.

The following solutions shall, therefore, be assumed.

$$(q_{10}^{k}) = [Q_{10}^{k}(\lambda)]e^{\lambda t} \tag{7a}$$

$$(r_{50}^{3}) = \sum_{j=1}^{n} [R_{50}^{3}(\lambda)]_j \phi_j(r_3) e^{\lambda t} \tag{7b}$$

$$(\alpha) = \sum_{j=1}^{n'} [\Lambda(\lambda)]_j \psi_j(r_3) e^{\lambda t} \tag{7c}$$

$$(F_3^s) = \bar{F}_3^s e^{\lambda t} \tag{7d}$$

In the model chosen for this investigation, no external forcing is assumed to exist. Therefore, the term F_3^s consists of the elastic restoring and damping forces produced by the array. Such forces are functions of the deformation variables and have generally the same form as those quantities found on the right hand side of Eq. (6). Thus, further specification of these terms will produce no additional information for our purposes.

It is possible to derive the desired set of equations by integrating Eq. (6) over the array width, then multiplying by the bending mode shape $\phi_i(r_3)$ and integrating over the array length. The resulting relation is a set of equations, one for each mode considered, upon which may be applied the orthogonality principle of normal modes (Ref. 10). The set, therefore, produces relations which are uncoupled in the bending modes.

Thus, integrating Eq. (6) over r_4 yields,

$$\int_{r_4} \bar{F}_3^s dr_4 = \lambda \Bigg\{ m_1 \Big[-r_3 \Big(C_{11}^1 [Q_{10}^1(\lambda)] + C_{12}^1 [Q_{10}^2(\lambda)] + C_{13}^1 [Q_{10}^3(\lambda)] \Big)$$

$$+ \lambda \sum_{j=1}^{n} [R_{50}^3(\lambda)]_j \phi_j(r_3) \Big] + m_2 \Big[C_{21}^1 [Q_{10}^1(\lambda)] + C_{22}^1 [Q_{10}^2(\lambda)]$$

$$+ C_{23}^1 [Q_{10}^3(\lambda)] + \lambda \sum_{j=1}^{n^1} [A(\lambda)]_j \psi_j(r_3) \Big] \Bigg\} \tag{8}$$

where the terms m_1 and m_2 are defined in Appendix B.

Multiplying Eq. (8) by $\phi_i(r_3)$ and integrating over r_3 gives the following set of relations.

$$F_i = \lambda \Bigg[\lambda \left(m_1 \sum_{j=1}^{n} [R_{50}^3(\lambda)]_j b_{ij} + m_2 \sum_{j=1}^{n^1} [A(\lambda)]_j C_{ij} \right)$$

$$+ m_1 g_i \left(- \left(C_{11}^1 [Q_{10}^1(\lambda)] + C_{12}^1 [Q_{10}^2(\lambda)] + C_{13}^1 [Q_{10}^3(\lambda)] \right) \right)$$

$$+ m_2 a_i \left(C_{21}^1 [Q_{10}^1(\lambda)] + C_{22}^1 [Q_{10}^2(\lambda)] + C_{23}^1 [Q_{10}^3(\lambda)] \right) \Bigg] \tag{9}$$

The integral expressions a_i, g_i, b_{ij}, and c_{ij} are found in Appendix B. It should be noted that

$$b_{ij} = \int_{r_3} \phi_i(r_3) \phi_j(r_3) dr_3 = \begin{cases} 0 \text{ for } i \neq j \\ 1 \text{ for } i = j \end{cases}$$

20

Array Torsion

An additional set of n' equations will now be derived for the elastic torsional motion of the array. Subsequently, the six additional relations required for analysis will be derived from rigid body considerations. The entire set of equations represents the equations of motion for the spacecraft and contain those vehicle parameters critical to the dynamic behavior of the craft.

The rotational motion of the array element may be examined by considering the Euler equation.

$$\vec{T}^s = \dot{\vec{H}}^s \tag{10}$$

Expanding this relation in terms of the variables previously presented and defining the "small" element rotation as,

$$\vec{\beta}^s = \left(\frac{\partial(r_5+r_6)}{\partial r_3} \right)\vec{a}_1 + \alpha\vec{a}_2 \tag{11}$$

allows the expression for Eq. (10) as,

$$\begin{aligned} T^s = \Big[& I^s[C^1\dot{q}_1 + \ddot{\beta}^s] + [I^s(\widetilde{C^1q_1}) + (\widetilde{C^1q_1})I^s + I^s\tilde{q}_2 \\ & + \tilde{q}_2 I^s - (I^s C^1 q_1 + I^s q_2)^\sim]\dot{\beta}^s + \Big[-[I^s(C^1\dot{q}_1 - \tilde{q}_2 C^1 q_1)]^\sim \\ & + I^s(C^1\dot{q}_1 - \tilde{q}_2 C^1 q_1)^\sim - (C^1 q_1 + q_2)^\sim[I^s(C^1 q_1 + q_2)]^\sim \\ & + (C^1 q_1 + q_2)^\sim I^s(C^1 q_1 + q_2)^\sim\Big]\, \beta^s - I^s\tilde{q}_2 C^1 q_1 \end{aligned}$$

$$+ [(\widetilde{C^1 q_1}) + \tilde{q}_2]\ I^s\ [(C^1 q_1) + q_2]\Big] \tag{12}$$

The term I^s is the moment of inertia matrix of the element and β^s is the rotation which is assumed to be small.

Extracting the component equation about the a_2 axis and deleting higher order terms yields,

$$T_2^s = \left[I_{22}^s (C_{21}^1 \dot{q}_{10}^1 + C_{22}^1 \dot{q}_{10}^2 + C_{23}^1 \dot{q}_{10}^3 + \ddot{\alpha})\right] \tag{13}$$

For the linear equation, the solutions may again be assumed to be of the form,

$$(\dot{q}_{10}^k) = \lambda [Q_{10}^k(\lambda)] e^{\lambda t} \tag{14a}$$

$$(\ddot{\alpha}) = \lambda^2 \sum_{j=1}^{n^1} [A(\lambda)]_j \psi_j(r_3) e^{\lambda t} \tag{14b}$$

$$(T_2^s) = \bar{T}_2^s\ e^{\lambda t} \tag{14c}$$

which yields upon substitution into Eq. (13),

$$\bar{T}_2^s = \Big[\lambda I_{22}^s \Big(C_{21}^1 [Q_{10}^1(\lambda)] + C_{22}^1 [Q_{10}^2(\lambda)]$$

$$+ C_{23}^1 [Q_{10}^3(\lambda)] + \lambda \sum_{j=1}^{n^1} [A(\lambda)]_j \psi_j(r_3)\Big)\Big] \tag{15}$$

Eq. (15) may be multiplied by the mode shape $\psi_i(r_3)$ and then successively integrated over r_4 and then r_3 to give,

22

$$T_i = \left[\lambda \left(C^1_{21}[Q^1_{10}(\lambda)] + C^1_{22}[Q^2_{10}(\lambda)] + C^1_{23}[Q^3_{10}(\lambda)]\right) d_i \right.$$
$$\left. + \lambda^2 \sum_{j=1}^{n^1} [A(\lambda)]_j e_{ij}\right] \tag{16}$$

which is a set of n' equations. In order to complete the equations of motion, the rigid body equations must now be developed.

3. VEHICLE EQUATIONS OF MOTION

The translational equations for the vehicle may be derived from Newton's second law.

$$\vec{F} = M\{\vec{I}\}^T\ddot{\vec{r}}_{10} \tag{17}$$

which may be expressed in the $\{\vec{b}\}$ frame as

$$F = MC^1\ddot{r}_{10} \tag{18}$$

It has been assumed that no external forces are applied on the vehicle or array, only pure torques. Therefore, both sides of Eq. (18) are identically zero. There will be no translational motion of the mass center of the vehicle.

The rotational equation for the vehicle may be derived from the Euler relation,

$$\vec{T} = \frac{d\vec{H}}{dt} \tag{19}$$

Expanded, Eq. (19) gives the matrix relation

$$T = \left[I\dot{q}_1 + \int_{r_3}\int_{r_4} (r_3 + r_4)(m^s)(\ddot{r}_5 + \ddot{r}_6)dr_4dr_3 \right.$$

$$\left. + \int_{r_3}\int_{r_4} I^s\ddot{\beta}^s dr_4dr_3 \right] \tag{20}$$

Expressing Eq. (20) in component form yields the relations,

$$T_1 = I_1\dot{q}_1^1 + \int_{r_3}\int_{r_4} m^s r_3(\ddot{r}_{50}^3)dr_4dr_3$$

$$+ \int_{r_3}\int_{r_4} m^s r_4 r_3 \ddot{\alpha} dr_4 dr_3 + \int_{r_3}\int_{r_4} I_{11}^s \frac{d^2}{dt^2}\left(\frac{\partial(r_5+r_6)}{\partial r_3}\right)dr_4dr_3 \tag{21}$$

$$T_2 = I_2\dot{q}_1^2 + \int_{r_3}\int_{r_4} m^s r_4 \ddot{r}_{50}^3 dr_4 dr_3$$

$$+ \int_{r_3}\int_{r_4} m^s (r_4^1)^2 \ddot{\alpha} dr_4 dr_3 + \int_{r_3}\int_{r_4} I_{22}^s \ddot{\alpha} dr_4 dr_3 \tag{22}$$

$$T_3 = I_3\dot{q}_1^3 \tag{23}$$

Solutions may be substituted analogous to those presented in previous sections and the integrals evaluated to give,

$$T_1 = \left(I_1\lambda[Q_{10}^1(\lambda)] + \lambda^2 \left[\sum_{j=1}^{n} [R_{50}^3(\lambda)]_j (g_j + m_j) \right. \right.$$

$$+ \sum_{j=1}^{n^1} [A(\lambda)]_j (h_j + n_j)\Big]\Big) \tag{24}$$

$$T_2 = \left(I_2 \lambda [Q_{10}^2(\lambda)] + \lambda^2 \left[\sum_{j=1}^{n} [R_{50}^3(\lambda)]_j p_j \right.\right.$$

$$+ \sum_{j=1}^{n^1} [A(\lambda)]_j (t_j + u_j)\Big]\Big) \tag{25}$$

$$T_3 = \left(I_3 \lambda [Q_{10}^3(\lambda)]\right) \tag{26}$$

These equations along with those derived in previous sections constitute the necessary set of equations for description of the appendage and craft motion.

IV. PARAMETERS OF IMPORTANCE

1. PARAMETER CLASSIFICATION

The craft's equations of motion may now be collected and are expressed as;

One set of n array bending equations ($i = 1$ to n),

$$F_i = \lambda \left[m_1 \lambda (\mu R C_{33}^1 + 1) \sum_{j=1}^{n} [R_{50}^3(\lambda)]_j b_{ij} \right.$$

$$+ \lambda(\mu m_1 C_{33}^1 R^1 + m_2) \sum_{j=1}^{n^1} [A(\lambda)]_j c_{ij} + m_2 a_i \left(C_{21}^1 [Q_{10}^1(\lambda)] \right.$$

$$\left. + C_{22}^1 [Q_{10}^2(\lambda)] + C_{23}^1 [Q_{10}^3(\lambda)] \right) - m_1 g_i \left(C_{11}^1 [Q_{10}^1(\lambda)] \right.$$

25

$$+ C^1_{12}[Q^2_{10}(\lambda)] + C^1_{13}[Q^3_{10}(\lambda)]\Big)\Big] \tag{27}$$

One set of n' array torsional equations (i = 1 to n'),

$$T_i = \Big[\lambda\Big(C^1_{21}[Q^1_{10}(\lambda)] + C^1_{22}[Q^2_{10}(\lambda)] + C^1_{23}[Q^3_{10}(\lambda)]\Big)\ d_i$$

$$+ \lambda^2 \sum_{j=1}^{n^1} [A(\lambda)]_j e_{ij}\Big] \tag{28}$$

One set of three component vehicle rotational equations (p = 1 to 3),

$$T_p = I_p\lambda[Q^p_{10}(\lambda)] + \lambda^2\Big[\sum_{j=1}^{n} [R^3_{50}(\lambda)]_j[(g_j + m_j)\delta_{1p}$$

$$+ p_j\delta_{2p}] + \sum_{j=1}^{n^1} [A(\lambda)]_j[(h_j + n_j)\delta_{1p}$$

$$+ (t_j + u_j)\delta_{2p}]\Big] \tag{29}$$

Examination of equations (27) through (29) show that the following classification of parameters may be made.

a.) Mass dependent parameters m_1, m_2, μ, and various inertia terms and integrals containing such terms.

b.) Geometry dependent parameters such as the integrals R and R', and the array length and width.

c.) Damping factors and natural mode frequencies. Frequency parameter λ.

d.) Mode shape dependent parameters and integrals such as a_i, b_{ij}, c_{ij}, d_i, e_{ij}, g_i, etc. defined in Appendix B.

Of these parameters the mass and geometry dependent and possibly natural frequencies may be computed prior to the flight. The terms to be measured inflight are then damping frequencies, and corresponding mode shapes.

V. DEVELOPMENT OF MEASUREMENT SYSTEM

1. SYSTEM INSTRUMENTATION

It is now necessary to qualitatively establish those methods by which the inflight measurement may be accomplished. Three methods of measurement are immediately apparent; direct video recording of appendage dynamics, strain gages, and mechanical or piezoelectric accelerometers. Methods have been demonstrated which allow extraction of mode shapes and frequencies for the simple bending of beams using a video recording of the vibration. However, the complexity of the onboard system, and complexity introduced by the bending-torsional combined vibration eliminate this technique from further consideration.

The interpretation of data for the multiple bending and torsional modes favors the use of the accelerometer type of instrument as opposed to the strain gage. Mode frequencies and array displacements may be found from the direct output of the accelerometers by passing the signal through a frequency filtering device.

Mode shape determination will require subsequent analysis of the apparent node conditions and amplitudes at each accelerometer station. It is essential that a strict time history be established for accurate measurement of phase differences between the motions at all accelerometer locations, in addition to securing general amplitude and frequency data.

This will assure appropriate data for the subsequent determination of mode shapes.

2. BENDING TORSIONAL MODE CONFIGURATIONS

A preliminary knowledge of the basic mode shape configurations is essential in the determination of accelerometer locations. Although precise specification of these modes is impossible, and in fact what we desire to measure, preliminary estimates as to their form may promote increased efficiency of the system and aid in the design and positioning of each accelerometer. Typical first four bending mode shapes are shown in Fig. 3, and the first four torsional mode shapes in Fig. 4.

3. SYSTEM CONFIGURATION

Although it is unclear at present how many modes will have frequencies within the sensitivity range of the attitude control system (which would permit an immediate truncation thus deleting all other modes from further consideration), it may be assumed that the system of accelerometers will have the general configuration as shown in Fig. 5, on the array surface itself. Contrary to the simplified configuration used in the determination of the important parameters, most arrays consist of a supporting structure (i.e. pantograph, etc.) plus the array proper. It is, therefore, believed that monitoring should include data from the array and also the supporting structure for proper analysis of the dynamic characteristics of the appendage. The spacing and number of such devices will be dependent on the number of modes retained after truncation. It is necessary to establish an adequate representation of each of the retained modes from data received from the system. This dictates certain restraints on accelerometer positioning. If possible, accelerometers should not be

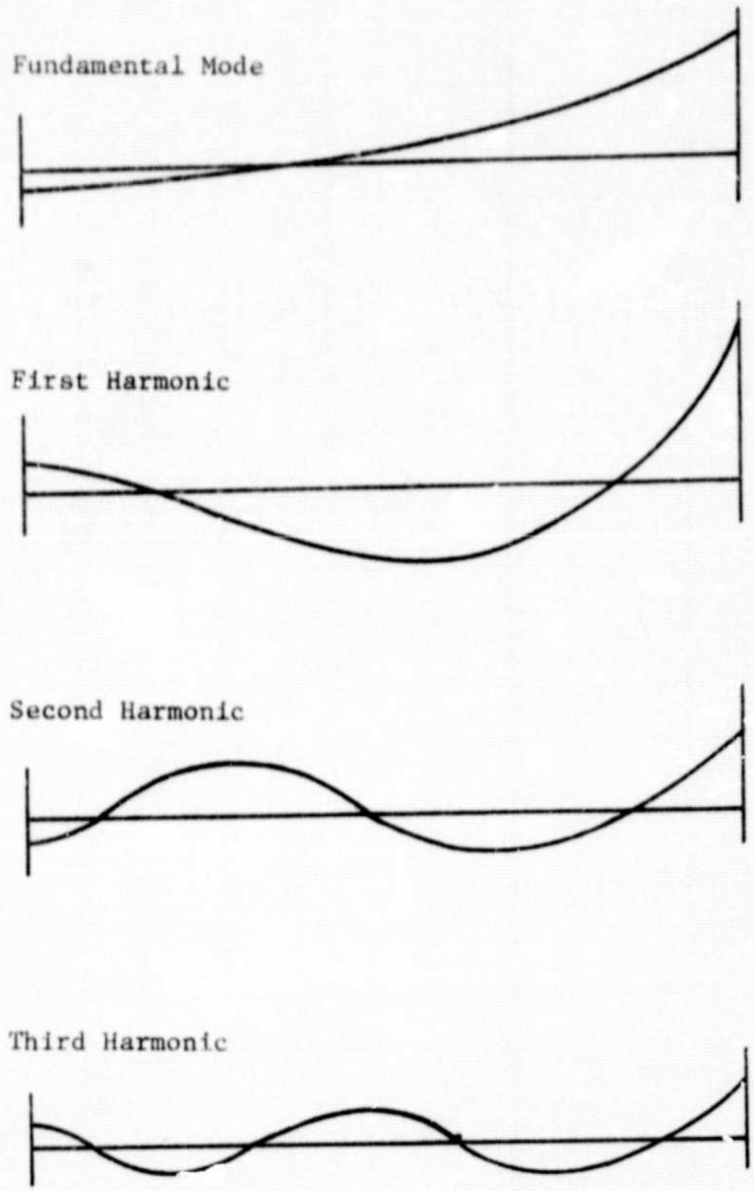

Figure 3. ARRAY BENDING MODE SHAPES

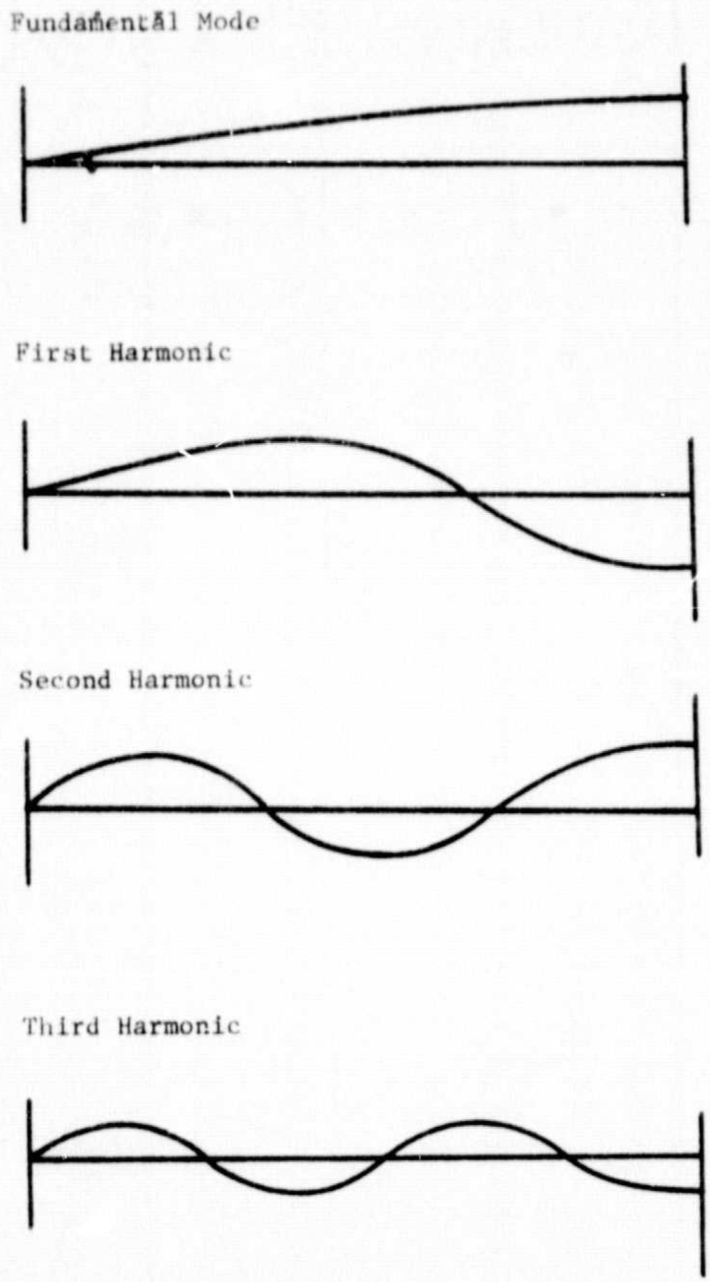

Figure 4. Array Torsional Mode Shapes

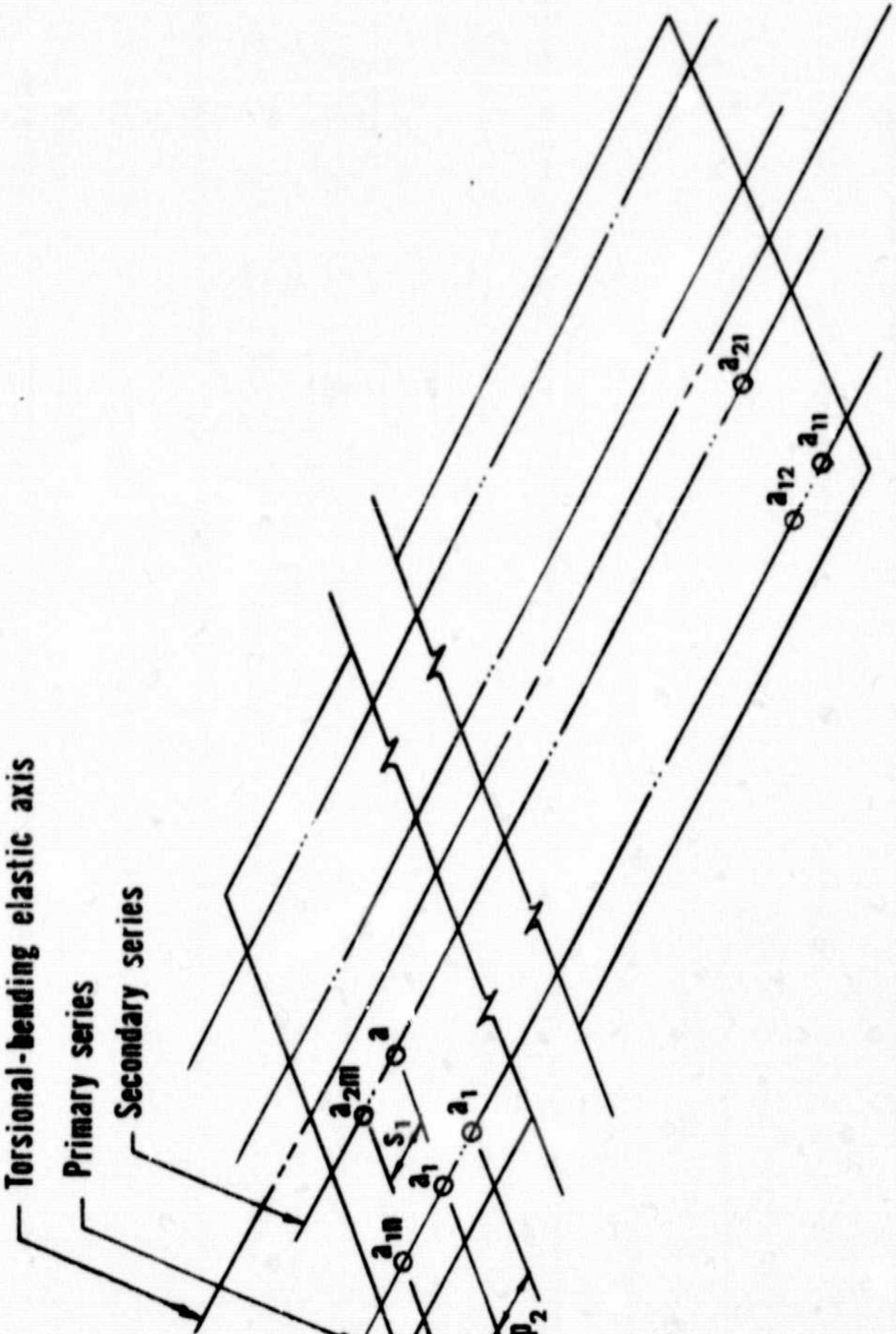

Figure 5. Accelerometer Measurement System

positioned on any nodal line of the retained modes of either the array or the support. This decreases their utility by restricting the number of modes from which data is obtained. The positioning must take into account preliminary estimates of node locations in the complex bending and torsional mode system. Fig. 5 suggests a primary series of accelerometers on each side of the elastic-torsional axis and a secondary series which along with data from the primary series may allow extraction of the complex mode locations for the array. Those distances s_1 and p_1 are, as previously indicated, functions of the specific modes retained. Instrument locations on the supporting structure will be determined by requirements similar to those for the array.

In considering the number of accelerometer positions to be used in the primary series for the array a reasonable representation of the highest harmonic mode shape may be acquired from data given by 2(N + 1) or 3(N + 1) accelerometer locations depending on the accuracy desired. Here N is the value of the highest harmonic mode retained. The secondary series which is to be used as a complimentary system to the primary one is estimated to require only about fifty percent of this number of positions.

4. ACCELEROMETER SAMPLING RATE

The sampling rate is a function of the frequency of the highest mode and should have a value for each accelerometer of about 4f where f is the frequency of the mode in cycles per second. The total data rate for the array system is then of the order,

$$\text{DATA RATE} = 8f(N + 1) \text{ to } 12f(N + 1)$$

As an example, let's assume that the fourth harmonic is the highest mode to be retained, with an estimated frequency of about 100 H_z.

This would mean that the data rate from the system would need to be,

DATA RATE = 4000 to 6000 bits per sec.

It would be advantageous to reduce this value. However, a detailed analysis of expected mode shapes and frequencies would be required to properly effect such a reduction or even investigate the feasibility of such a reduction. Such an investigation will be included in a later study.

5. INFLIGHT MANEUVERS

It is highly desirable to minimize any special inflight maneuvers or deviations from the normal flight sequence. The forcing of the array system necessary to produce vibrations in the required modes may be accomplished by a properly timed firing sequence of the attitude control thrusters. This would consist of an impulse torque applied about a given control axis with a subsequent correction torque applied at a specified time which would be determined by a modal frequency analysis. Depending upon the orientation of the thrusters, the required torques may be applied by any appropriate combination of thrusters. No other maneuver is necessary.

For data interpretations, the accelerometer system allows reasonably straightforward analysis of output. The desired mode frequencies may be extracted directly by analysis of the signal from each accelerometer location and the determination of those frequencies to which the given accelerometer is responding. The mode shapes will require geometrical analysis of the motion at each accelerometer position and the phase relationship between these motions. General guidance in this effort will be extended by the numerical results of a computer simulation not yet completed.

VI. CONCLUSIONS

It is possible to state several interim conclusions concerning the various aspects of the flexibility-control interaction phenomenon. Generally, these interim conclusions represent the nature of important parameters, measurement techniques, processing, and data retrieval and have not been in all cases rigorously substantiated. However, this preliminary work has served to indicate more clearly those areas of investigation which will produce necessary data for complete design of the inflight system which is to be accomplished in follow-on work. It is in this context that the following conclusions are presented.

a.) Any experiment of this type is very complex and data is not unique.

b.) The important structural parameters to the flexibility-control interaction problem are appendage and vehicle mass and inertia characteristics, geometrical properties, vibration damping and frequency values, forcing frequencies, and bending and torsional mode shapes.

c.) Those parameters which must be measured inflight are the damping modal frequencies and respective mode shapes.

d.) The nature of inflight systems and analysis of system output favors the use of accelerometers for the determination of frequencies and mode shapes.

e.) For the structure flexibility properties, the number of accelerometers necessary to create a useful measurement system is dictated by the degree of the highest harmonic mode retained in the analysis after truncation. It is felt that the number should be of the order 2(N + 1) to 3(N + 1) for the primary series of accelerometers and about half of that value for the secondary series. Such speculation requires further substantiation using a computer analysis of the combined bending-torsion node locations.

f.) The data rate for each accelerometer position in a function of the frequency of the highest harmonic mode retained. It is felt for accurate representation of the modes, a data rate of 4f is required where f is the frequency of the highest mode in cycles per second. This would indicate a system capability requirement of at least 12f(N + 1) data bits per second. It is felt that this value can be made feasible in most cases of solar array vibration. However, this will impose certain design restrictions on the array to be used in conjunction with the system design.

g.) Special inflight maneuvers will be a sequenced activation of the attitude control thrusters for excitation of the vibratory modes of the array, both torsional and bending.

h.) Data interpretation will consist of a frequency filtering of accelerometer output and a geometrical and phase analysis of such output for mode shape determination. Specific schemes for such interpretation will need to be presented at a later time.

VII. RECOMMENDATIONS

It is recommended that the following tasks be carried out in a follow-on study:

a.) A comparison of important parameters for the continuous roll out versus the flexibly connected rigid fold out array should be performed. Experimental aspects will be fitted to either type to the extent possible.

b.) The spacecraft attitude control system must be modelled to study interaction properties. This will be integrated into previous work to develop flexibility/control interaction phenomenon.

c.) Computer simulations will be carried out to establish a correlation matrix for the important parameters to be measured. Techniques for independent measurement will then be considered.

d.) An analysis of dynamic response characteristics for combinations of antisymmetric and symmetric bending and torsional modes will be developed.

e.) Candidate measurement devices and system options will be considered to obtain a "best" set up for collecting data. Position and implementation requirements will be included. Criteria for evaluation of expected data will also be established.

Several other tasks may also be considered which include the effects of mass discontinuities, a nondimensional study of the equations of motion to obtain a general interpretation of results through the use of the laws of similitude, and interface and system integration effects on vehicle power, structure, TT&C, and propulsion.

REFERENCES

1. Noll, R. B., Zvara, J., and Deyst, J. J., "Effects of Structural Flexibility on Spacecraft Control Systems," NASA Space Vehicle Design Criteria, NASA SP-8016- April 1969.

2. Noll, R. B., Deyst, J. J., and Spenny, C. H., "A Survey of Structural Flexibility Effects on Spacecraft Control Systems," AIAA Paper No. 69-116, January 1969.

3. Dotts, H. W. et al.; Operational Characteristics of the Docked Configuration. Gemini Summary Conference, NASA SP-138, February 1967, pp. 41-54.

4. Apollo Guidance and Navigation; Guidance System Operations Plan for Manual CM Earth Orbital Missions Using Program Sundisk, sec. 3 CSM Digital Autopilot. MIT Instrumentation Laboratory R-503, October 1965.

5. Franklin, C. A., and Davison, E. H., "A High Power Communications Technology Satellite for the 12 and 14 GHZ Bands," AIAA Paper No. 72-588, April 1972.

6. Cherchas, D. B., "SPAR Dynamics Analysis Capability," SPAR-R 484, SPAR Aerospace Products Ltd., Toronto, Ontario, April 1972.

7. Wolff, G., "The Flight of the FRUSA," IEEE Photovoltaic Specialists Conference, Silver Spring, Md., 1972.

8. Likins, P. W., Marsh, E. L., and Fleischer, G. E., "Flexible Space Vehicles," Technical Report No. 32-1329, Jet Propulsion Labs, Pasadena, Calif., January 1970.

9. Likins, P. W., "Dynamics and Control of Flexible Space Vehicles, Technical Report No. 32-1329, Jet Propulsion Labs, Pasadena, Calif., January 1970.

10. Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., "Aeroelasticity," Addison Wesley Publishing Co., Inc., Reading, Mass., 1955, pp. 67-114.

NEW TECHNOLOGY STATEMENT

There are no reportable items of new technology.

APPENDIX A

POSITION VECTORS

The following vectors are necessary to describe the position of an arbitrary element on the solar array.

$$\vec{r}_1 = \{\vec{i}\}^T \begin{bmatrix} (r_1^1 + r_{10}^1) \\ (r_1^2 + r_{10}^2) \\ (r_{10}^3) \end{bmatrix} \tag{A.1}$$

$$\vec{r}_2 = \{\vec{b}\}^T \begin{bmatrix} r_{20}^1 \\ r_{20}^2 \\ r_{20}^3 \end{bmatrix} \tag{A.2}$$

Note: The vector $\vec{r}_2$ contains no steady components as $\{\vec{a}\}$ and $\{\vec{b}\}$ are assumed to coincide in the undeformed configuration.

$$\vec{r}_3 = \{\vec{a}\}^T \begin{bmatrix} o \\ -r_3^2 \\ o \end{bmatrix} \tag{A.3}$$

$$\vec{r}_4 = \{\vec{a}\}^T \begin{bmatrix} r_4^1 \\ o \\ o \end{bmatrix} \tag{A.4}$$

$$\vec{r}_5 = \{\vec{a}\}^T \begin{bmatrix} o \\ o \\ -r_{50}^3 \end{bmatrix} \tag{A.5}$$

$$\vec{r}_6 = \{\vec{a}\}^T \begin{bmatrix} o \\ o \\ -r_{60}^3 \end{bmatrix} \tag{A.6}$$

The angular velocity of the center of mass with respect to inertial space is,

$$\vec{q}_1 = \{\vec{b}\}^T \begin{bmatrix} q_{10}^1 \\ q_{10}^2 \\ (q_1^3 + q_{10}^3) \end{bmatrix} \tag{A.7}$$

while the rotation of the array to maintain proper solar orientation gives,

$$\vec{q}_2 = \{\vec{a}\}^T \begin{bmatrix} o \\ q_2^2 \\ o \end{bmatrix} \tag{A.8}$$

One additional matrix form is useful in expressing matrix equations.

$$\tilde{A}_1 = \begin{bmatrix} o & -A_1^3 & A_1^2 \\ A_1^3 & o & -A_1^1 \\ -A_1^2 & A_1^1 & o \end{bmatrix} \tag{A.9}$$

This form is required in expressing cross products such as,

$$A_1 \times A_2 = \{\vec{e}\}^T A_1 \times \{\vec{e}\}^T A_2 = \{\vec{e}\}^T \tilde{A}_1 A_2 \tag{A.10}$$

41

APPENDIX B

INTEGRAL EXPRESSIONS

The following integral expressions occur in the derivation of the equations for array bending and torsion, and vehicle rigid body motion.

$$m_1 = \int_{r_4} m^s dr_4 \tag{B.1}$$

$$m_2 = \int_{r_4} r_4 m^s dr_4 \tag{B.2}$$

$$m_3 = \int_{r_4} (r_4)^2 m^s dr_4 \tag{B.3}$$

$$s_1 = \int_{r_4} I^s_{11} dr_4 \tag{B.4}$$

$$s_2 = \int_{r_4} r_4 I^s_{11} dr_4 \tag{B.5}$$

$$s_3 = \int_{r_4} I^s_{22} dr_4 \tag{B.6}$$

$$a_i = \int_{r_3} \phi_i(r_3) dr_3 \tag{B.7}$$

42

$$b_{ij} = \int_{r_3} \phi_i(r_3)\phi_j(r_3)dr_3 \quad \text{(B.8)}$$

$$c_{ij} = \int_{r_3} \psi_j(r_3)\phi_i(r_3)dr_3 \quad \text{(B.9)}$$

$$d_i = \int_{r_3} I_{22}^{1}\psi_i(r_3)dr_3 \quad \text{(B.10)}$$

$$e_{ij} = \int_{r_3} I_{22}^{1}\psi_j(r_3)\psi_i(r_3)dr_3 \quad \text{(B.11)}$$

$$g_j = \int_{r_3} r_3 m_1 \phi_j(r_3)dr_3 \quad \text{(B.12)}$$

$$h_j = \int_{r_3} r_3 m_2 \psi_j(r_3)dr_3 \quad \text{(B.13)}$$

$$m_j = \int_{r_3} s_1\left(\frac{\partial\phi_j(r_3)}{\partial r_3}\right)dr_3 \quad \text{(B.14)}$$

$$n_j = \int_{r_3} s_2\left(\frac{\partial\psi_j(r_3)}{\partial r_3}\right)dr_3 \quad \text{(B.15)}$$

$$p_j = \int_{r_3} m_2 \phi_j(r_3)dr_3 \quad \text{(B.16)}$$

$$t_j = \int_{r_3} m_3 \psi_j(r_3) dr_3 \qquad \text{(B.17)}$$

$$u_j = \int_{r_3} s_3 \psi_j(r_3) dr_3 \qquad \text{(B.18)}$$