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A COST ASSESSMENT OF RELIABILITY REQUIREMENTS FOR SHUTTLE-RECOVERABLE EXPERIMENTS

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16 Abstract

The relaunching of unsuccessful experiments or satellites will become a real option with the advent of the space shuttle. This study examines the cost effectiveness of relaxing reliability requirements for experiment hardware by allowing more than one flight of an experiment in the event of its failure. Any desired overall reliability or probability of mission success can be acquired by launching an experiment with less reliability two or more times if necessary. Although this procedure leads to uncertainty in total cost projections, because the number of flights is not known in advance, a considerable cost reduction can sometimes be achieved. In cases where reflight costs are low relative to the experiment's cost, three flights with overall reliability 0.9 can be made for less than half the cost of one flight with a reliability of 0.9. An example typical of shuttle payload cost projections is cited where three low-reliability flights would cost less than $50 million and a single high-reliability flight would cost over $100 million.

The ratio of reflight cost to experiment cost is varied and its effect on the range in total cost is observed. An optimum design reliability selection criterion to minimize expected cost is proposed, and a simple graphical method of determining this reliability is demonstrated.

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A COST ASSESSMENT OF RELIABILITY REQUIREMENTS FOR SHUTTLE-RECOVERABLE EXPERIMENTS

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SUMMARY

With the advent of the space shuttle, the opportunity to refurbish and relaunch an unsuccessful experiment or satellite will become a reality. This study examines the cost effectiveness of relaxing reliability requirements for experiment hardware by allowing more than one flight of an experiment in the event of its failure. Any desired overall reliability or probability of mission success can be acquired by launching an experiment with less reliability two or more times if necessary. The maximum number of times that an experiment may have to be flown to achieve a desired overall reliability is derived as a function of its single-flight reliability. The actual number of flights will, therefore, be unknown in advance of the mission and, consequently, funding projections will have to allow for a range in total cost. It is shown, however, that the disadvantages associated with cost uncertainties may be offset by significant cost reductions. Furthermore, where this approach is applied to a number of experiments, uncertainties in total program cost become negligible.

The ratio of reflight cost to experiment cost is varied and its effect on the range in total cost is observed. It is shown that the curve of expected cost as a function of payload reliability exhibits a minimum at a value of payload reliability, and that this reliability increases as the ratio of reflight cost to experiment cost increases.

An optimum design reliability selection criterion to minimize expected cost is proposed, and a simple graphical method of determining this reliability is demonstrated. This method is then applied to a situation typical of projected shuttle payload costs. An example is cited where reflight cost is especially low compared with the cost of buying high-reliability experiments. In this case, three flights with overall reliability 0.9 can be flown for less than half the cost of making one flight with reliability 0.9.

INTRODUCTION

A primary contributor to the cost of space experiments has been their high reliability requirement. Because of their one-shot nature and the high cost of their deployment, experiments on space missions have necessarily been designed to have high probabilities of success. The term "reliability," as applied to experiments in this study, is intended to mean the probability that the experiment's objectives are met. Thus, any experiment can be considered as having only two outcomes, success or failure, as determined by the principal investigator. Since this subjective definition of reliability is somewhat vague from the standpoint of the manufacturer, it will be assumed that success is equivalent to the hardware's functioning correctly in the space environment for the duration of the mission. The fact that an experiment may be considered a success even though the hardware has malfunctioned after some period of successful performance would tend to reduce costs even below those estimated here.

The term "design reliability" is used to refer to that minimum level of reliability established as a goal for the manufacturer. Although, in general, the actual (unknown) reliability of a system will be higher than this minimum "design reliability," it is assumed here that the two are equal. Again, in terms of expected costs, this is a conservative assumption.

In examining the cost of a piece of experiment hardware, it is often difficult, if not impossible, to identify costs which are directly attributable to reliability requirements or specifications. Costs associated with acquiring high-reliability parts, redundancy features in the design, documentation, and testing can all be associated in part with reliability requirements, but there is considerable debate as to the degree of association.

It is assumed here that by some equitable means, a cost-reliability relationship can be determined for a piece of experiment hardware, and that this relationship has the following characteristic features: (1) cost becomes infinite as reliability approaches one, and (2) cost becomes relatively insensitive to reliability as reliability requirements are decreased. Results and methodology presented in this paper are dependent solely on the existence of such a relationship and not on any specific cost-reliability curve.

A relationship used by some manufacturers of space hardware to assign costs to reliability requirements is "Halving the probability of failure doubles the cost." Figure 1 is a graphical representation of this rule where the unit cost factor is selected to be the cost of 65 percent reliability. The mathematical representation of this rule is used in the remainder of this paper to illustrate the proposed method of selecting optimum design reliabilities. Figure 2, obtained from a Lockheed contractual study (ref. 1), shows a curve for subsystem cost compared with reliability which is based on empirical data for typical research and development subsystems. The similarity between this curve and that in figure 1 indicates that the latter is not atypical.

Design reliabilities are determined by weighing, either formally or informally, the cost of reliability against the cost of a failure. When a payload was deployed in a one-shot situation, placed into Earth orbit permanently without any chance for adjustment or repair other than by automated means, failure of that system meant complete loss or cancellation of the mission and generally a total waste of the associated deployment hardware (that is, the launching rocket, its fuel, etc.). Thus, the high penalty associated with failure drove the design reliability up and its resultant cost climbed along the right-hand part of its cost-reliability curve.

With the space shuttle providing the opportunity to check out satellites in orbit prior to deployment, make in-orbit repairs and provide maintenance, recover malfunctioning satellites, and, if necessary, refly an experiment, the cost of failure can be reassessed. In such cases the cost of failure may simply be the cost of retrieving and refurbishing the experiment for a second launch plus the cost of the fuel and ground-support operations required to make that launch. Thus, the failure cost may be greatly reduced and this, in turn, would have a significant impact on the selection of a design reliability.

This paper examines the effect on an experiment's total cost of relaxing reliability requirements at the risk of having to refly an unsuccessful experiment. A set of ground rules is established which prescribes a maximum number of attempts as a function of the experiment's reliability. Then the behavior of maximum cost and that of expected cost are examined as they depend on experiment reliability and on the reflight cost. A systematic procedure is outlined for establishing the design reliability which minimizes expected cost while simultaneously constraining potential maximum cost, if desired. An example related to shuttle payloads is discussed which uses costs typical of those projected for sortie Spacelab missions. In this example, it will be shown that by relaxing reliability requirements, total costs can be reduced by as much as 50 percent or more without reducing the probability of mission success.

The intent of this paper is to illustrate a method of establishing design reliabilities where one is not necessarily restricted to a single launch. Dramatic cost savings shown in the development of the method are largely a result of the assumed relationship between cost and reliability. (See fig. 1.) Actual cost savings in a particular application may be more or less dramatic depending on the cost of acquiring high reliability for that specific system.

SYMBOLS

A	constant in cost-reliability function
$C_e(p_e)$	cost of developing, manufacturing, and testing a system of experiment hardware with reliability p_e
C_o	value of $C_e(p_e)$ for the high reliability $p_e = p_o$

C_R experiment reflight cost; total additional cost incurred when an experiment is reflown

$C_{R,o}$ cost of flying an experiment on its first launch

C_T total cost of an experiment or group of experiments

E( ) expected value function

m number of experiments in a shuttle payload program

N maximum number of flights

n actual number of flights

p probability of success on each flight $\left(=p_e p_s\right)$

p_e single flight design reliability of an experiment

p_e^* value of p_e which minimizes expected cost

p_k probability that $n = k$, $k = 1, . . ., N$

p_o value of p_e which makes the probability of success on a single flight equal to desired probability of mission success S

p_s combined reliability of all support systems, independent of experiment, which must function correctly for mission success

q probability of failure on each flight, $1 - p$

S probability of mission success in N or fewer flights

x $= C_R/C_o$

σ_n standard deviation of n

Subscripts:

max maximum

min minimum

GROUND RULES

To study the effect of shuttle recoverability on experiment reliability requirements and their associated cost, a number of generalized ground rules were established as follows:

(1) An experiment which fails on its first flight may be reflown one or more times.

(2) The maximum number of times the experiment may be flown, N, is determined so that the probability of mission success in N or fewer attempts is not less than a constant S.

(3) The cost to refly the experiment is a fixed quantity C_R.

(4) The cost of developing, manufacturing, and testing the experiment hardware, $C_e(p_e)$, is an increasing function of its reliability p_e, where $C_e(1) = \infty$.

(5) The probability of the experiment's success on any single flight is $p = p_e p_s$ where p_s is the combined reliability of all supporting systems which are independent of the experiment (for example, launch vehicle, Spacelab, etc.). The necessarily high probability of crew safety is included in p_s and is not affected by varying p_e.

Subject to these ground rules, the experiment's design reliability was varied to determine its effect on the expected mission cost and on the maximum cost which would be incurred if all N attempts have to be made.

CALCULATION OF THE MAXIMUM NUMBER OF FLIGHTS N

According to the second ground rule, N is chosen such that the probability of mission success occurring in at most N attempts is not less than a preselected value S. Because total failure occurs if all N attempts fail, the probability of total failure $1 - S$ is equal to the probability of N successive individual failures $(1 - p)^N$. Equating these probabilities and solving for N as a continuous function of p, one obtains

$$N = \frac{\log(1 - S)}{\log(1 - p)} \tag{1}$$

However, since N can only assume integer values, N must be the smallest integer satisfying

$$N \geqq \frac{\log(1 - S)}{\log(1 - p)} \tag{2}$$

Figures 3 and 4 show N as a function of p for $S = 0.9$ and 0.95, respectively. In each of these figures, the dashed line is N from equation (1) and the solid line is N from equation (2).

ACTUAL NUMBER OF FLIGHTS

Although N is the maximum number of flights which may be necessary, the actual number n will be a random integer between 1 and N. Even if p is as low as 0.5, it is unlikely that all N flights will have to be made (for example, as unlikely as getting heads on four or five successive tosses of a coin).

For each integer k, $1 \leqq k \leqq N$, let p_k be the probability that k flights are made. The probability that only one flight is made is $p_1 = p$ where p is defined. For $1 < k < N$, k flights are made if the first $k - 1$ flights fail and the kth flight is successful. Thus,

$$p_k = p(1 - p)^{k-1} \tag{3}$$

which also holds for $k = 1$. For $k = N$, however, the probability that N flights are made is equal to the probability that the first $N - 1$ flights fail; that is,

$$p_N = (1 - p)^{N-1} \tag{4}$$

In other words, if the first $N - 1$ attempts fail, then n will be N regardless of the outcome of the Nth attempt. Table I lists values of p_k $(k = 1, \ldots, N)$ for several reliabilities p.

The expected number of flights E(n) is by definition

$$E(n) = \sum_{k=1}^{N} kp_k \tag{5}$$

Substituting for p_k from equations (3) and (4) gives

$$E(n) = \sum_{k=1}^{N-1} kp(1 - p)^{k-1} + N(1 - p)^{N-1} \tag{6}$$

By letting $q = 1 - p$, equation (6) can be written as

$$E(n) = (1 - q) \frac{d}{dq}\left(\sum_{k=1}^{N-1} q^k\right) + Nq^{N-1} \tag{7}$$

The sum over k is now just a finite geometric series or

$$\sum_{k=1}^{N-1} q^k = \frac{q - q^N}{1 - q} \tag{8}$$

Taking its derivative with respect to q as indicated in equation (7) and reducing terms gives

$$E(n) = \frac{1 - q^N}{1 - q} \tag{9}$$

or

$$E(n) = \frac{1 - (1 - p)^N}{p} \tag{10}$$

Recall that N was selected to be the smallest integer satisfying

$$(1 - p)^N \leqq 1 - S \tag{11}$$

If N did not have to be an integer, then equation (11) would be an equality. As an approximation, $(1 - S)$ can be substituted for $(1 - p)^N$ in equation (10). Thus,

$$E(n) \approx \frac{S}{p} = \frac{S}{p_e p_s} \tag{12}$$

Plots of $E(n)$, both exact (eq. (10)) and approximate (eq. (12)), and the range of n as functions of p are shown in figures 5 and 6 for $S = 0.9$ and 0.95, respectively. Although the approximation tends to underestimate $E(n)$, the difference between exact and approximate values is very small and is considered to be insignificant.

ESTIMATING COSTS

The total cost of the experiment, if it takes only one launch, is the cost of acquiring the hardware $C_e(p_e)$ plus the initial flight cost $C_{R,o}$. Subsequent flights will cost an additional C_R per flight. In general, C_R includes all the expenses included in $C_{R,o}$ plus refurbishment costs and, in the case of satellites, retrieval costs which may not be included in $C_{R,o}$. In the remainder of this paper, to simplify notation, it will be assumed that $C_{R,o} = C_R$. This assumption has the effect of elevating all cost curves by a constant amount $C_R - C_{R,o}$ and again adding conservatism to cost estimates. Cost reduction trends are not affected.

If n is the actual number of times the experiment is flown, the total cost of the experiment becomes

$$C_T = C_e(p_e) + nC_R \tag{13}$$

The minimum cost is

$$C_{T,min} = C_e(p_e) + C_R \tag{14}$$

the maximum cost is

$$C_{T,max} = C_e(p_e) + NC_R \tag{15}$$

and the expected cost is

$$E(C_T) = C_e(p_e) + E(n)C_R \tag{16}$$

where $E(n)$ from equation (12) will be used in the following development.

If the cost-reliability curve in figure 1 is adopted, then

$$C_e(p_e) = \frac{A}{1 - p_e} \tag{17}$$

where A is a constant which is determined by knowing one point on the curve.

Let p_o be the experiment reliability required to have the desired probability of success S on the first flight (that is, $p_o p_s = S$), and let C_o denote the cost of a one-flight experiment. That is,

$$C_o = C_e(p_o) = \frac{A}{1 - p_o} \tag{18}$$

The experimental reliability p_o and cost C_o would correspond to past design reliabilities and costs when payloads were deployed only once.

To depict total costs as functions of p_e without having to assume specific cost values, it is convenient to use C_o as a unit cost or nondimensionalizing factor. Figure 7 shows a total cost envelope where C_o is the unit cost, $p_s = 0.95$, $S = 0.9$, and reflight cost $C_R = 0.5C_o$. The minimum and maximum costs correspond to $n = 1$ and N, respectively, and the intermediate solid curves correspond to $n = 2, 3, \ldots, N - 1$. The dashed curve is the expected cost $E(C_T)$ from equation (16). Because expected cost is the sum of $C_e(p_e)$ which increases with p_e and $E(n)C_R$ which decreases with p_e, expected cost exhibits a minimum at the reliability p_e^* as shown in figure 7.

MINIMIZING EXPECTED COST

Substituting equations (17) and (12) into equation (16) gives $E(C_T)$ as a function of p_e

$$E\left(C_T\right) = \frac{A}{1 - p_e} + \frac{SC_R}{p_e p_s} \tag{19}$$

By taking the derivative of $E(C_T)$ with respect to p_e and setting this equal to zero, one finds that the value of p_e which minimizes expected cost is

$$p_e^* = \left(1 + \sqrt{\frac{Ap_s}{SC_R}}\right)^{-1} \tag{20}$$

(The solution for p_e^* given here is actually one of a pair of quadratic roots. It is possible to show that the other solution does not lie between 0 and 1, a basic requirement for any probability.)

The experimental reliability p_e^* would appear to be a logical choice for a design reliability because it minimizes expected cost. Although the actual cost of one experiment will not, in general, be the expected cost, the consistent choice of p_e^* as the design reliability for a whole program of experiments would minimize the total program cost

Solving for A in equation (18) and substituting this value into equation (20) gives

$$p_e^* = \left(1 + \sqrt{\frac{1 - p_o}{p_o x}}\right)^{-1} \tag{21}$$

where x is the ratio of C_R to C_o. This ratio is an attractive parameter because it eliminates the actual costs from the equation for p_e^*. Inflationary factors, for example, can enlarge C_R and C_o without significantly affecting x so that the value of x will remain almost constant for any given experiment and launch system.

It is possible to set p_e^* in equation (21) equal to the various experimental reliabilities which divide the regions in figures 3 and 4 corresponding to different values of N, and thereby solve for the ranges of x for which two, three, or more flights are cost effective. For example, by using figure 3, all values of x less than or equal to 18 result in a p_e^* which allows two flights. A third flight is permitted if $x \leqq 0.37$ and a fourth flight if $x \leqq 0.09$. Figure 8 shows a plot of p_e^* as a function of x by using $S = 0.9$ and $p_s = 0.95$ (that is, $p_o = 0.947$), and the values of x separating the different values of N are indicated.

A CONSTRAINT ON MAXIMUM TOTAL COST

From the preceding discussion, it would appear to be inefficient in virtually every case to buy the reliability p_o required to make overall reliability equal to S on the first launch. Clearly, the situation is not always so simple. For example, if C_R is 10 times greater than C_o (that is, $x = 10$), the minimum expected cost occurs in the reliability range where a second launch is possible with probability 0.11, but the cost of making two flights would be almost twice the cost of making one flight with overall reliability 0.9. Thus, under such circumstances one might wish to increase the initial experiment reliability to p_o so as not to risk an expensive second launch.

Such a decision is recommended when the selection of design reliabilities is made for a small number of experiments. Expected or average cost per experiment would still be minimized, theoretically, by the selection of p_e^*. However, because the averaging process is not as effective for small numbers, the actual average cost for such a small number of experiments might be very different from the theoretical expected cost.

In this situation, one should consider both maximum cost and expected cost when selecting a design reliability. A reasonable constraint might be that $C_{T,max}$ not be allowed to exceed the cost of a one-flight mission, $C_o + C_R$. The value of $C_{T,max}$ can be expressed in terms of p_e and x as

$$C_{T,max} = \frac{C_o(1 - p_o)}{(1 - p_e)} + NC_o x \tag{22}$$

Thus, the constraint that maximum total cost not exceed $C_o + C_R = C_o(1 + x)$ can be written as

$$\frac{1 - p_o}{1 - p_e} + Nx \leqq 1 + x \tag{23}$$

or $x \leqq x_{max}$ where

$$x_{max} = \frac{p_o - p_e}{(N - 1)\left(1 - p_e\right)} \tag{24}$$

Figure 9 shows a plot of x_{max} against p_e where $S = 0.9$ and $p_s = 0.95$. (The quantity x_{max} is discontinuous because N is discontinuous.)

If one wishes to require that maximum cost not exceed the cost of a one-flight mission, then any value for p_e may be selected as the design reliability provided that the ratio $x = \frac{C_R}{C_o}$ does not exceed x_{max} as given in equation (24). For example, if $x = 0.6$, then the selection of any design reliability between 0.72 and 0.87 would satisfy the constraint. A general procedure would be to select the value of p_e which minimized total expected cost subject to the constraint that $x \leqq x_{max}$. This selection could be done graphically with little difficulty if p_s and S are determined, and if the relationship of cost against reliability is known. The latter relationship may be an empirical one rather than the functional relationship given in equation (17). To illustrate this selection procedure, figure 10 shows a cost envelope similar to figure 7 where $x = 0.8$ instead of 0.5. Here one sees that when $p_e = p_e^*$, maximum cost exceeds the cost of a one-flight mission so that the constraint is not satisfied. The hatched area indicates the region where the constraint on maximum cost is satisfied (that is, where $C_{T,max} \leqq C_o + C_R$). By use of the suggested procedure for selecting a design reliability, one would select the value of p_e within this hatched region which minimizes $E\left(C_T\right)$. The choice in this case would be $p_e = 0.75$.

In figure 11, x_{max} is superimposed on the plot of p_e^* against x. Note that if $x \leqq 0.754$, then p_e^* is the optimum design reliability by the suggested criterion; that is, the constraint on maximum cost is automatically satisfied by $p_e = p_e^*$. Thus, the only occasion where the constraint may be necessary is if $x > 0.754$. In this case one would want to impose the constraint only when selecting design reliabilities for a relatively small number of experiments. To help in making this decision, a convenient rule of thumb is that the actual average cost of m experiments should be within $\pm 2C_R\sigma_n/\sqrt{m}$ of the theoretical expected cost $E\left(C_T\right)$, where σ_n, the standard deviation of n, is given by the formula

$$\sigma_n = \frac{\left[1 - p - p(2N - 1)(1 - p)^N - (1 - p)^{2N}\right]^{1/2}}{p} \tag{25}$$

Thus, if m is large enough to insure that $E\left(C_T\right) + 2C_R\sigma_n/\sqrt{m}$ is less than $C_o + C_R$, one should ignore the constraint on maximum cost altogether and select p_e^* as the optimum design reliability.

EXAMPLE

The Shuttle Experiments Office at the NASA Langley Research Center estimates that on an Advanced Technology Laboratory (ATL) mission utilizing the European Spacelab on shuttle sortie flights (ref. 2), the total cost of 15 typical experiments would be approximately \$35 million if each experiment has a reliability $p_e = 0.85$ They also estimate that the cost of reflying the shuttle for one 7-day sortie mission, including refurbishment of experiments and ATL/Spacelab ground operations, is approximately \$12 million. The shuttle itself is expected to have a reliability of $p_s = 0.95$, and the desired probability of mission success is $S = 0.9$.

"Mission success" is no longer well-defined since the collective outcome of 15 experiments is not a dichotomous (success or failure) situation. Instead, a percentage of the experiments will be successful, where "success" of a particular experiment is defined by the principal investigator responsible for that experiment. If one assumes that the total reflight cost, \$12 million, can be divided among the experiments according to their respective resource demands (fuel, volume, operator hours, etc.), then each experiment would have a different x value dependent on its associated cost-reliability curve $C_e(p_e)$ and pro rata reflight cost C_R. The procedure for selecting a design reliability as outlined would be appropriate for each experiment.

To simplify this illustration, however, it will be assumed that all the ATL experiments have the same x value and that the cost-reliability curve of figure 1 is appropriate for all experiments. Since the total cost $C_e(p_e)$ for 85 percent reliability is \$35 million, the constant A in equation (17) would be 5.25×10^6 and the cost associated with payload reliability $p_o = 0.947$ (that is, $S = 0.9$) would be $C_o = \$100$ million. Thus,

$$x = \frac{12}{100} = 0.12 \tag{26}$$

Figure 12 shows a cost envelope for this situation. The reliability which minimizes expected cost is $p_e{}^* = 0.6$. At this value, minimum total cost is \$25 million, expected total cost is \$32 million, and the maximum total cost is \$49 million. Since the maximum cost of \$49 million is well below the cost of a one-flight mission (\$112 million), $p_e{}^* = 0.6$ would be the optimum design reliability.

The minimum cost will be realized if all 15 experiments succeed on the first launch and the maximum cost will be realized if all 15 experiments fail on the first two launches. The probabilities of these two outcomes are about 0.5×10^{-3} and 1×10^{-12}, respectively. The actual total cost of the 15 experiments should be reasonably close to the expected cost of \$32 million. Computing the standard deviation of n from equation (25) by using $p \approx 0.57$ $(=0.6 \times 0.95)$ and $N = 3$ yields $\sigma_n = 0.78$. Thus, the actual total cost of $m = 15$ experiments should be within $\pm\$4.8$ million of the expected total cost.

In this example the cost of the mission would be less than half that of a one-flight mission, and yet the probability of mission success would still be 0.9 or higher. It appears reasonable to assume that this result would also be valid if the design reliabilities of individual experiments are selected on the basis of their own x values. Compared with smaller, simpler experiments, the large complicated experiments tend to cost more and, at the same time, tend to have higher reflight costs. Thus, the ratio of C_R to C_o for each experiment might remain fairly close to 0.12 so that p_e^* for each experiment would be approximately 0.6.

The dramatic cost savings shown in this illustration result largely from the premise that "halving the probability of failure doubles the cost." If one assumes instead that halving the probability of failure increases the cost by only 10 percent, the resultant cost envelope is that shown in figure 13. (Assumptions which are alike in figs. 12 and 13 are that $C_e(0.85)$ = \$35 million and C_R = \$12 million.) In this example, the cost of a one-flight approach with 90 percent overall reliability is \$52 million, and the payload reliability which minimizes expected cost is $p_e^* = 0.77$. At this reliability, if all 15 experiments succeed the first time, the minimum cost would be \$45 million, and, if all experiments fail the first time, the maximum cost would be \$57 million. The expected cost is \$48 million, and the actual total cost should be within ±\$2.8 million of this value (that is, between \$45 million and \$51 million). In this example the number of experiments ($m = 15$) is large enough so that the actual total cost at payload reliability p_e^* should be well below the cost of a one-flight payload. Thus, the constraint on maximum cost need not be imposed.

Figure 13 illustrates the generality of the proposed method. Although the potential savings are less impressive using this cost model, the method is appropriate nonetheless. The expected cost is still minimized by reducing reliability requirements and allowing a second flight when necessary. In general, regardless of the cost-reliability relationship used, there will be a value of reliability which minimizes expected cost and this value can be ascertained graphically without much difficulty.

CONCLUDING REMARKS

With the advent of the space shuttle, the opportunity to refurbish and relaunch an unsuccessful experiment, payload, or satellite will become a reality. It has been shown that in some circumstances, it costs less to make several launches using less reliable hardware than to buy the hardware with sufficient reliability to insure a high probability of success on the first launch because reflight cost may be considerably less than the cost of acquiring high reliability.

The maximum number of flights can be fixed so that the probability of mission success is greater than or equal to any desired constant. A relationship was derived relating expected total cost to the experiment's reliability, and the value of experiment reliability which minimizes expected total cost was derived. The ratio of reflight cost to experiment cost was varied and its effect on the expected cost was observed. By so doing, it was shown that reflight costs could be extremely large relative to the experiment's cost and yet the expected total cost is still minimized by allowing more than one flight.

Because it may be considered unwise, in some circumstances, to risk incurring huge reflight costs, a constraint on maximum cost can be imposed. It was suggested that a reasonable constraint might be to restrict maximum cost not to exceed the cost of building all the reliability into a single flight. This constraint would be appropriate when selecting design reliabilities for a small number of experiments whose reflight costs are high.

A criterion was suggested for selecting the design reliability, or that minimum level of reliability established as a goal for the hardware manufacturer. Simple graphical methods can be used to select this design reliability to be that value which minimizes expected cost either with or without the constraint on maximum cost.

An example typical of projected shuttle payload costs was discussed where the cost of making three flights whose collective reliability exceeds 0.9 was less than half the cost of making one flight for which the individual reliability is 0.9. In this example, the design reliability was 0.6; its associated expected total cost was $32 million, minimum total cost was $25 million, and maximum total cost was $49 million. The probability of mission success, allowing at most three attempts, was 0.9. In order to build the same amount of reliability into a single payload, the cost of the experiment hardware alone would be $100 million, and the total cost (maximum, minimum, and expected) would be $112 million. Clearly, the consequences of having to refly an experiment are not always so severe as to warrant extremely high reliability requirements. A reassessment of these requirements might significantly reduce the overall costs of space programs involving shuttle-recoverable experiments and satellites.

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TABLE I.- PROBABILITY p_k OF MAKING k FLIGHTS, $k = 1, \ldots, N$, FOR DIFFERENT SINGLE-FLIGHT RELIABILITIES p WHERE $S = 0.9$

p	N	p_1	p_2	p_3	p_4	p_5	p_6	p_7
1.00	1	1.00						
.95	1	1.00						
.90	1	1.00						
.85	2	.85	0.15					
.80	2	.80	.20					
.75	2	.75	.25					
.70	2	.70	.30					
.65	3	.65	.23	0.12				
.60	3	.60	.24	.16				
.55	3	.55	.25	.20				
.50	4	.50	.25	.125	0.125			
.45	4	.45	.25	.14	.16			
.40	5	.40	.24	.14	.09	0.13		
.35	6	.35	.23	.15	.10	.06	0.11	
.30	7	.30	.21	.15	.10	.07	.05	0.12

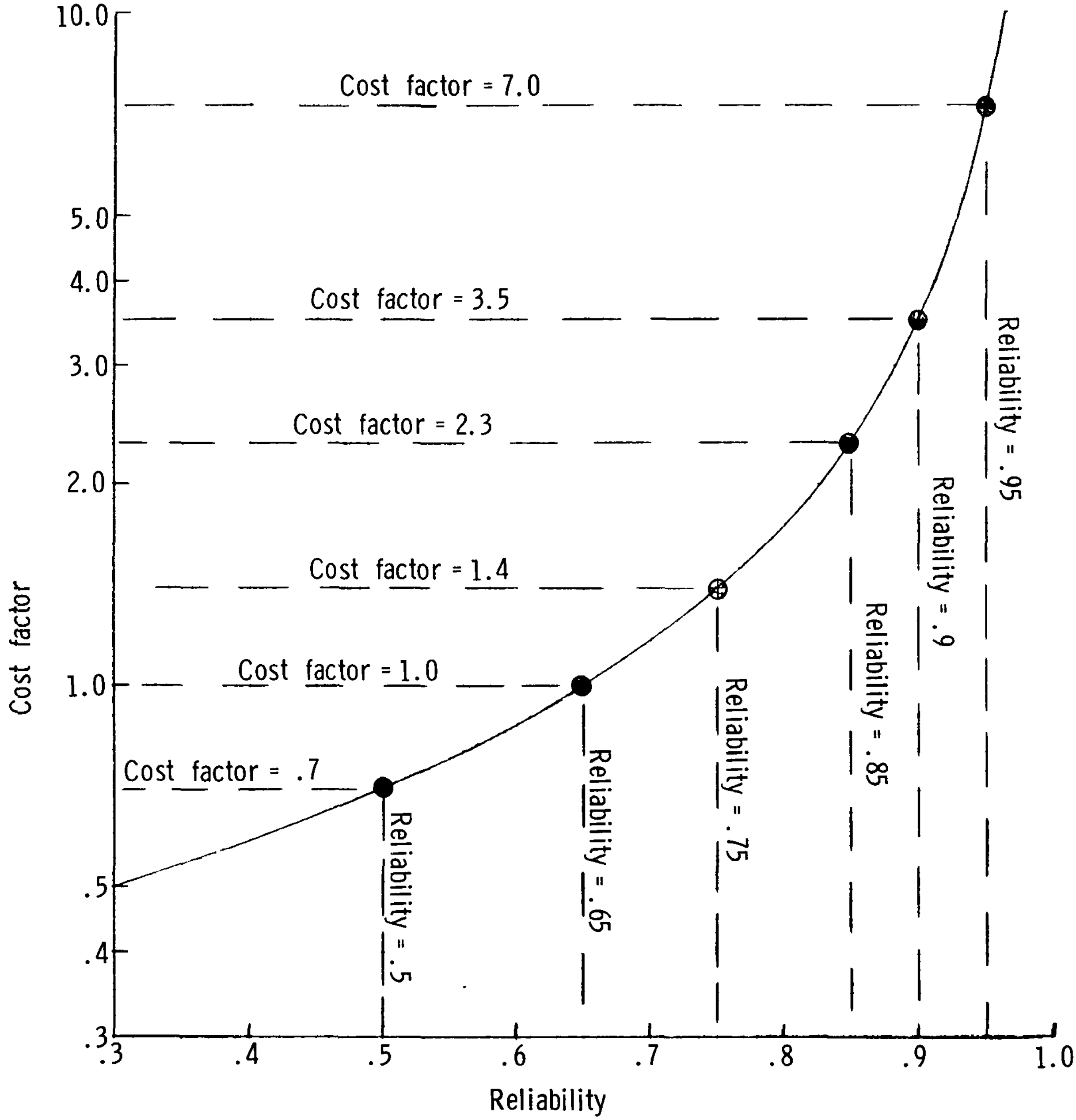

Figure 1.- Cost as a function of reliability. (Rule: Halving probability of failure doubles cost.)

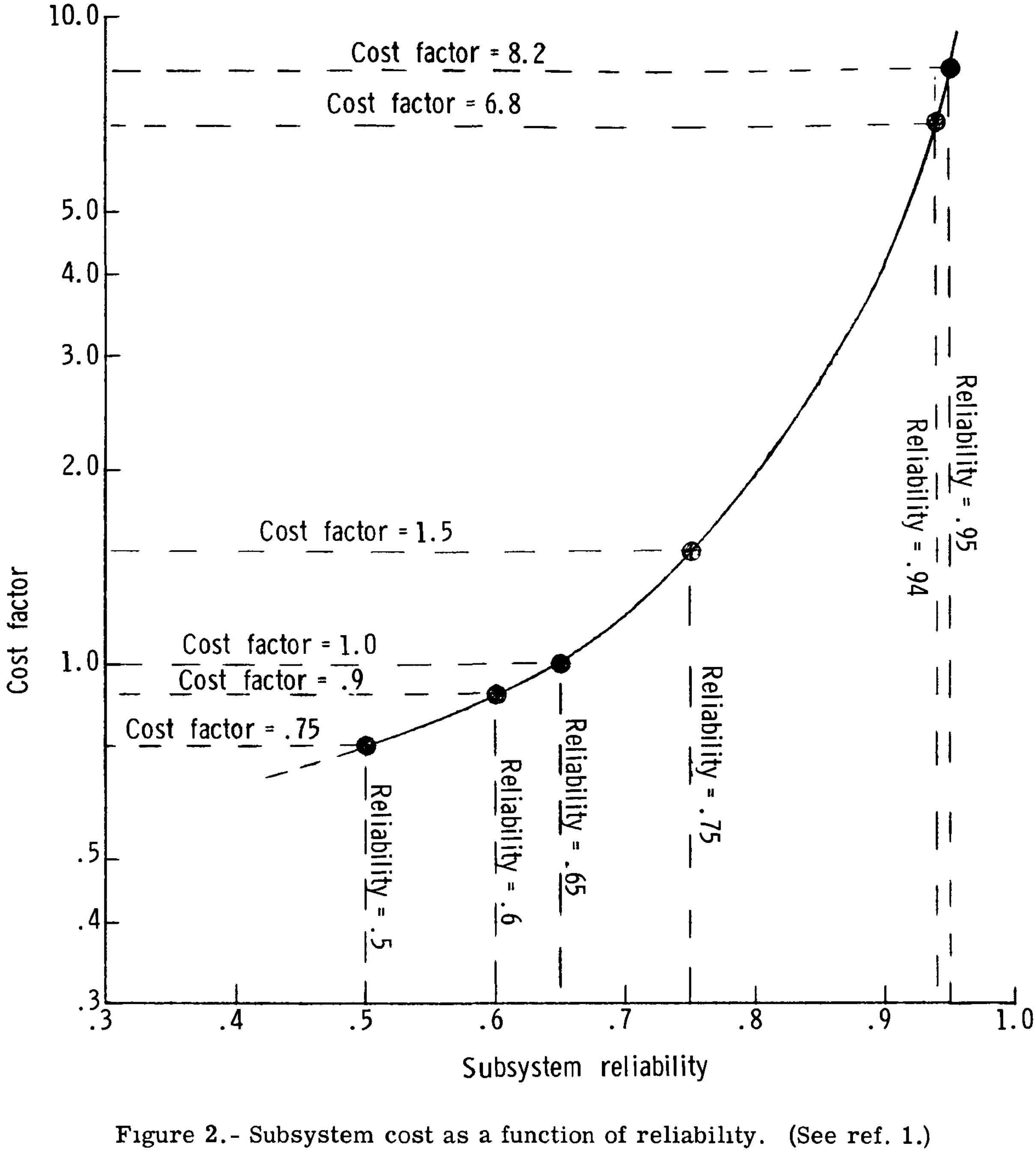

Figure 2.- Subsystem cost as a function of reliability. (See ref. 1.)

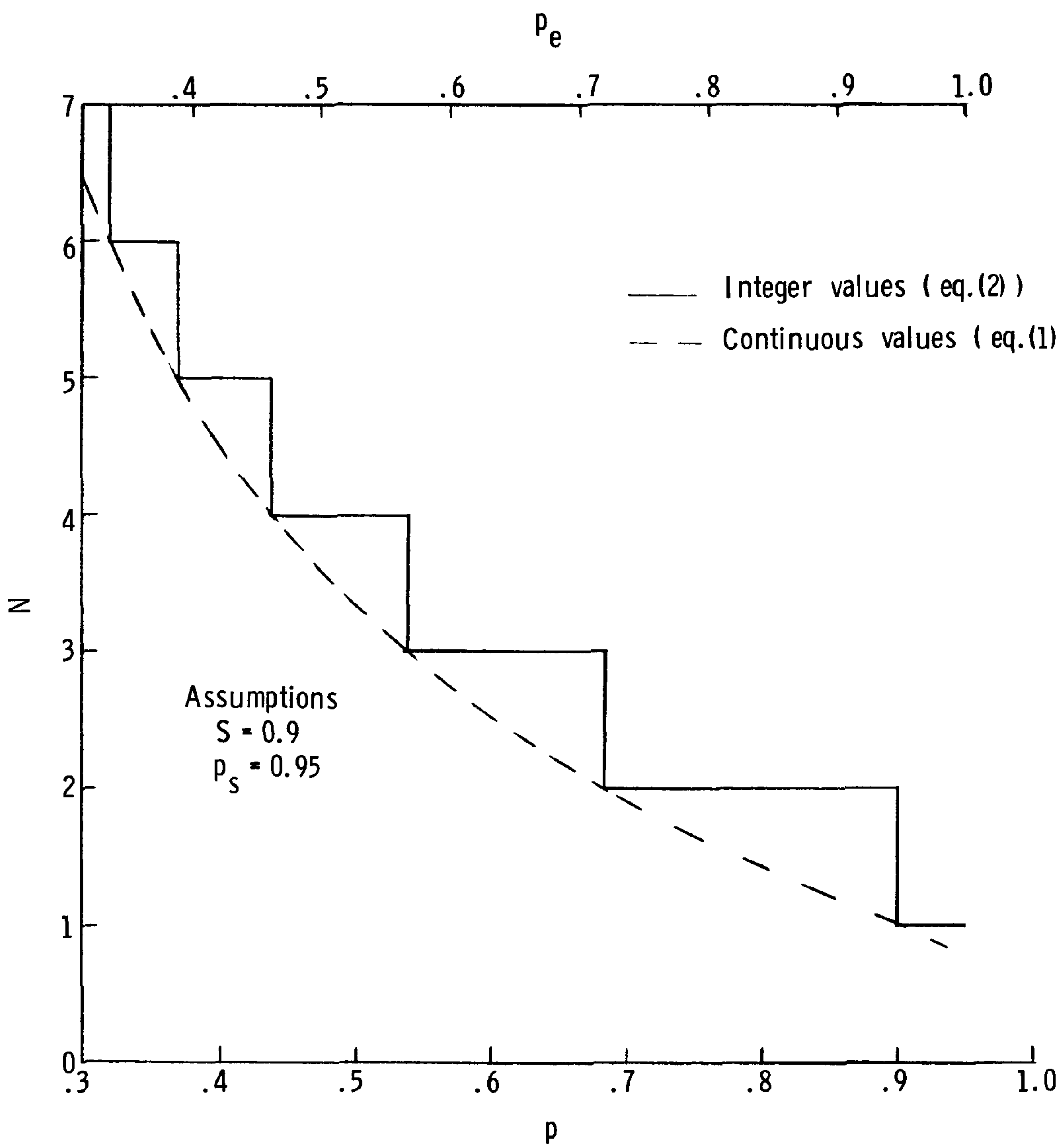

Figure 3.- Maximum number of flights to have 0.9 probability of mission success as a function of single-flight reliability and experimental reliability.

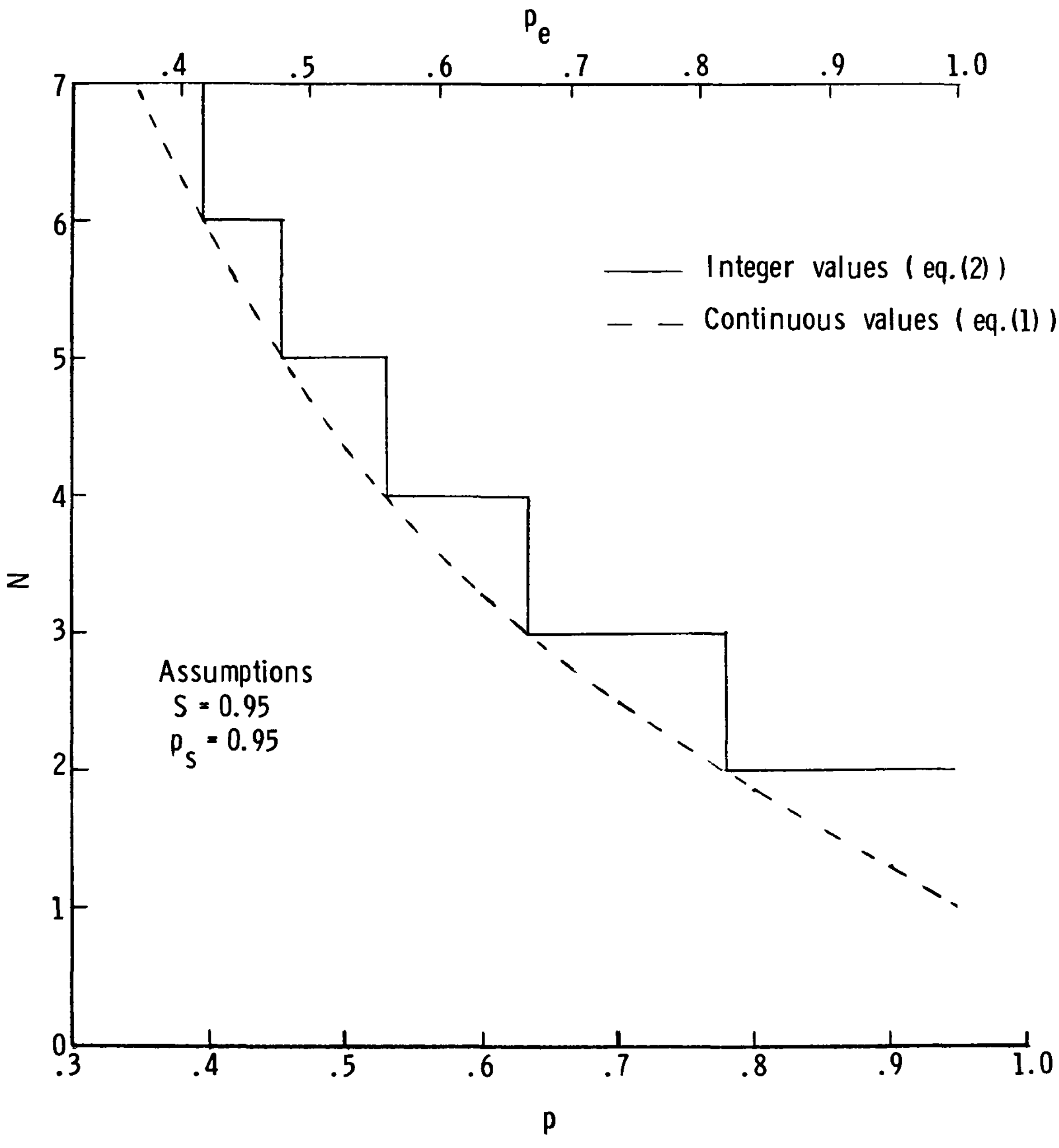

Figure 4.- Maximum number of flights to have 0.95 probability of mission success as a function of single-flight reliability and experimental reliability.

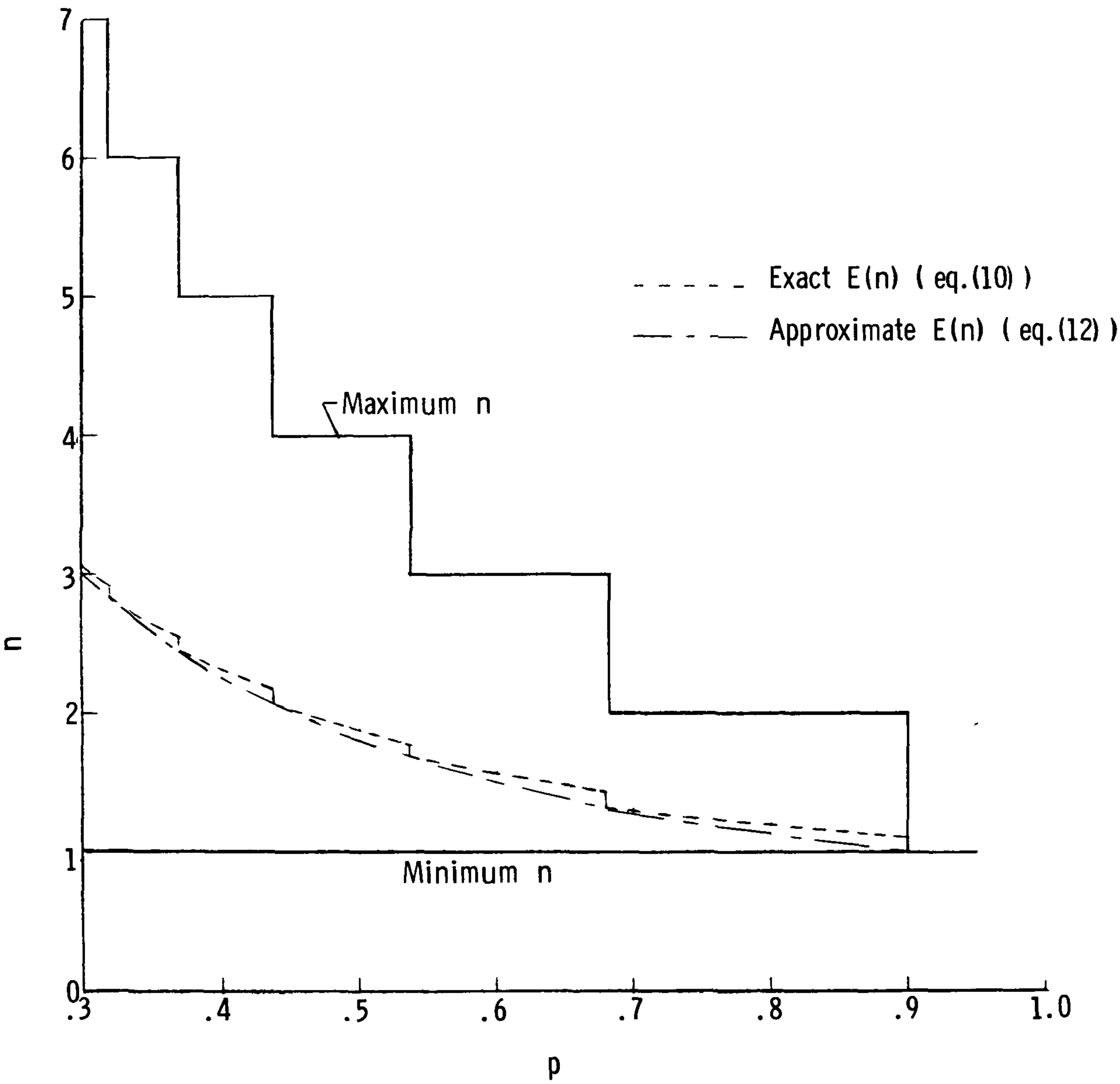

Figure 5.- Expected number of flights and the range in number of flights as a function of single-flight reliability for S = 0.9.

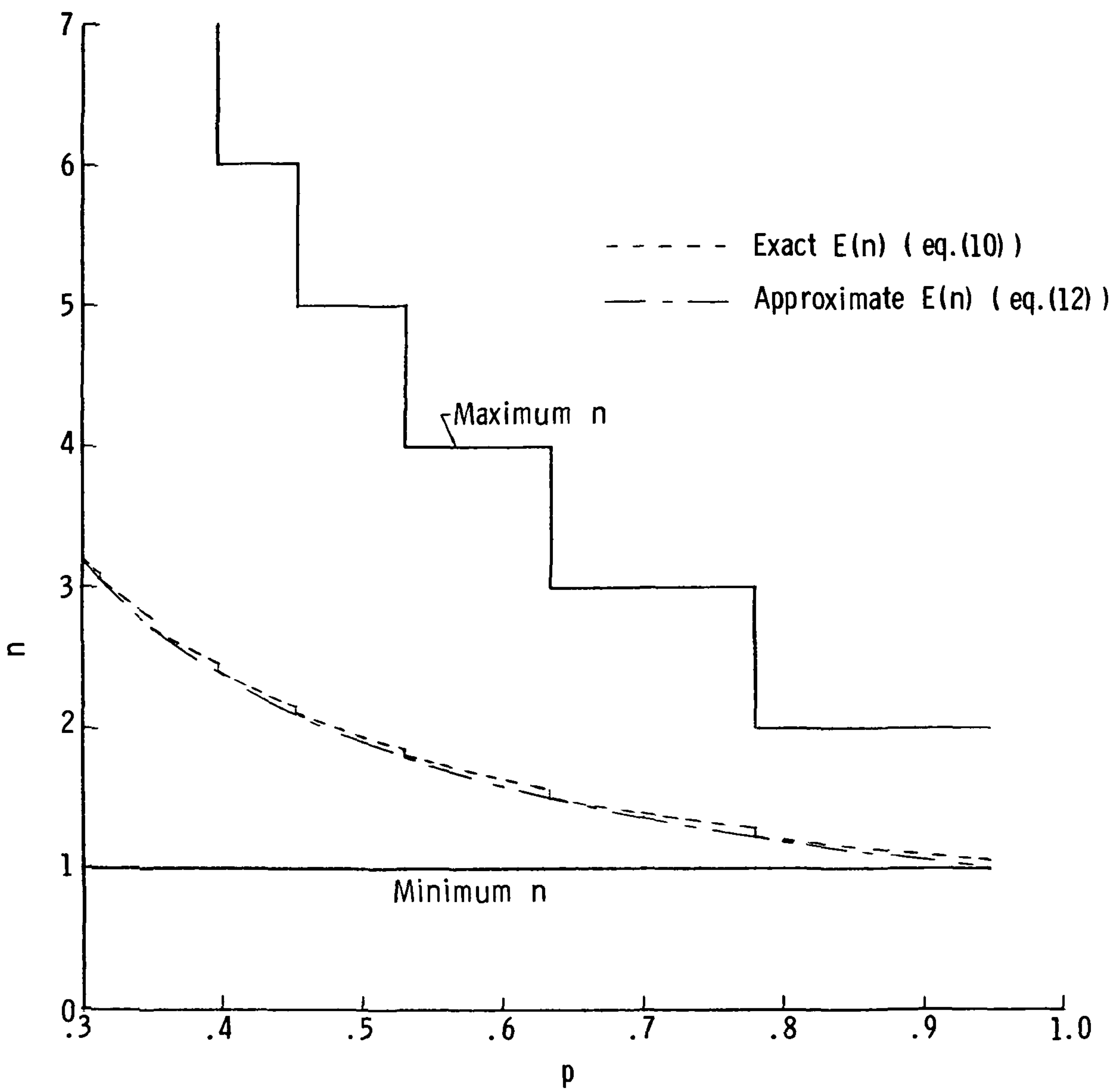

Figure 6.- Expected number of flights and the range in number of flights as a function of single-flight reliability for $S = 0.95$.

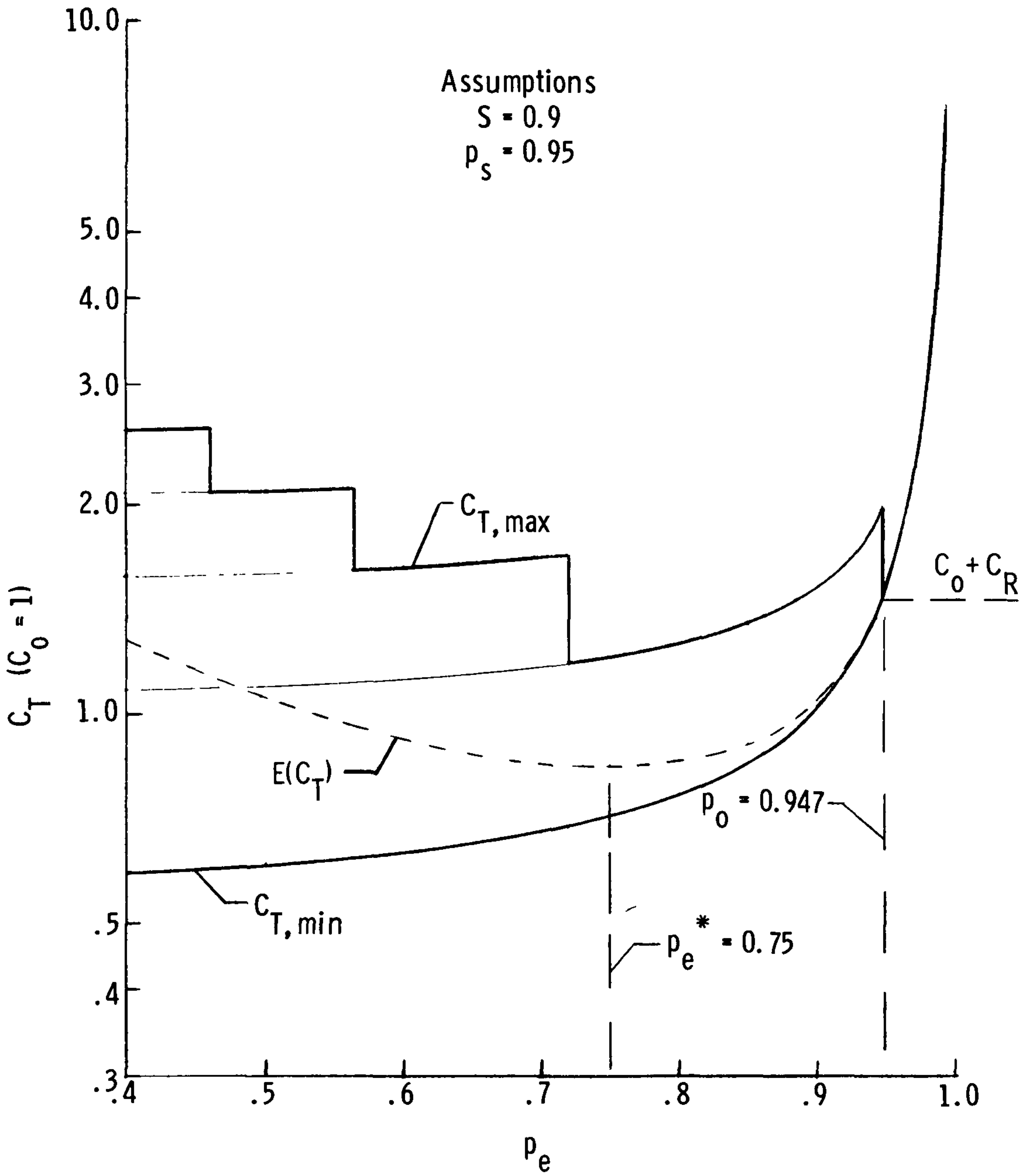

Figure 7.- Cost envelope for mission where $C_R = 0.5C_o$.

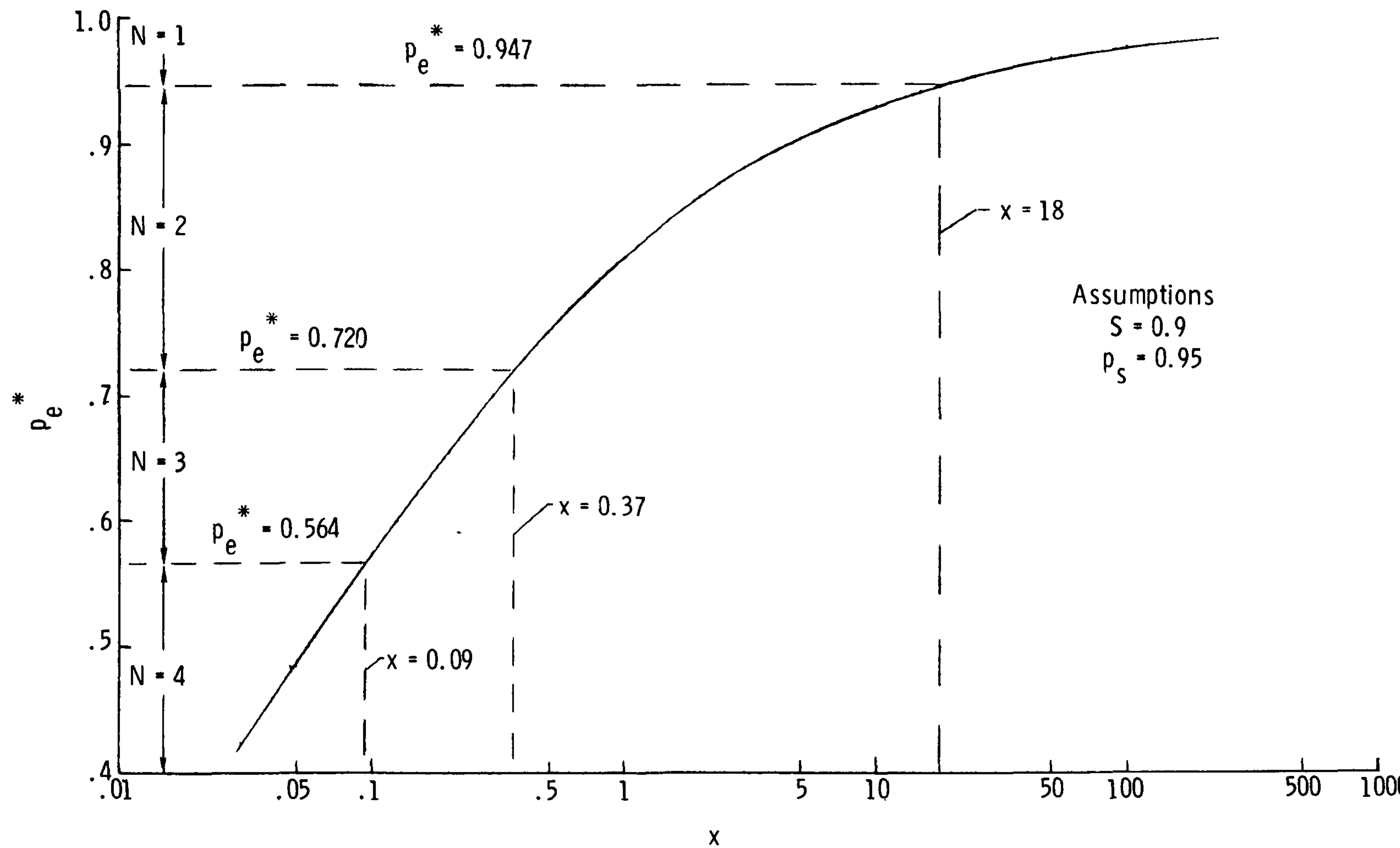

Figure 8.- Experiment reliability which minimizes $E(C_T)$ as a function of the ratio $x = C_R/C_O$.

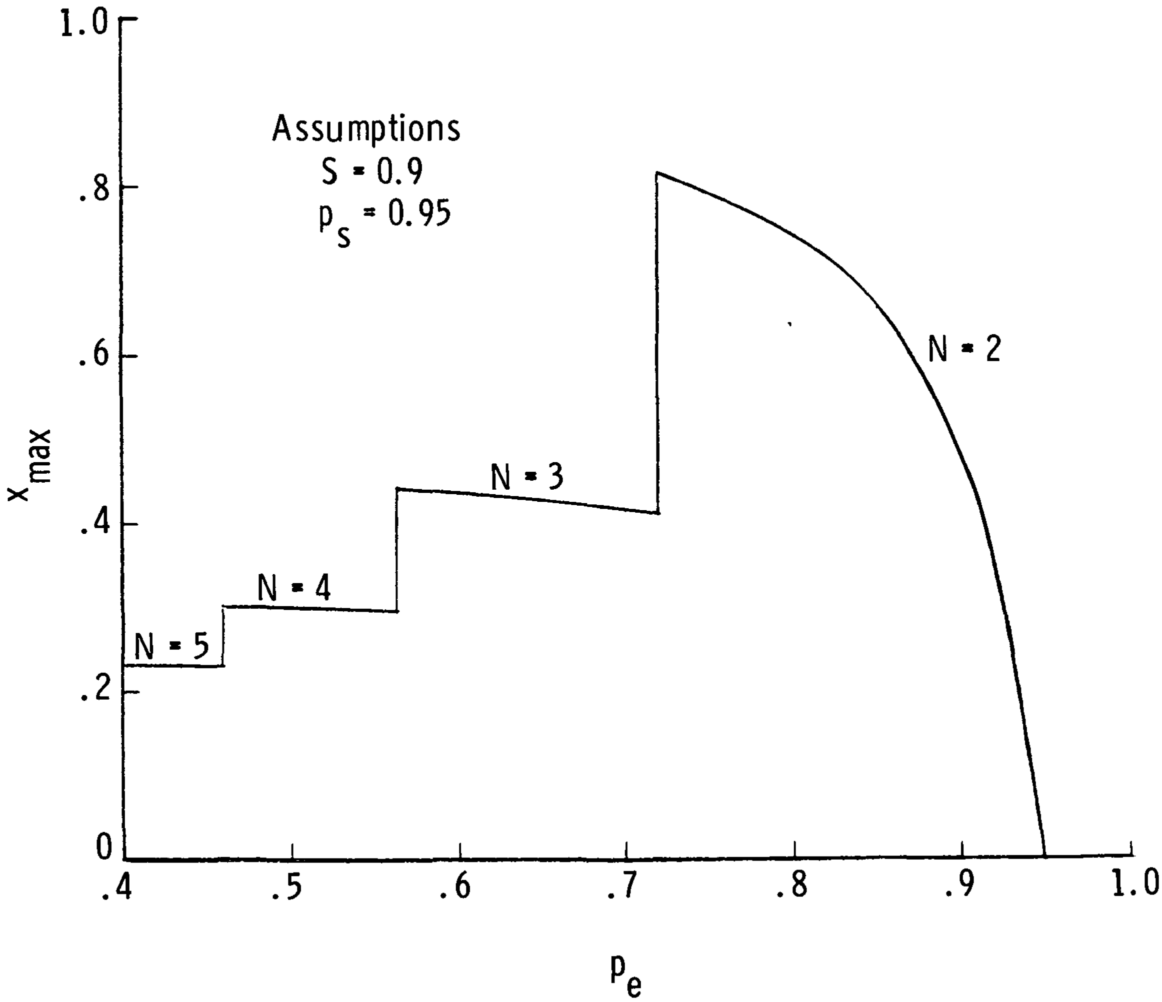

Figure 9.- Maximum x as a function of experiment reliability.

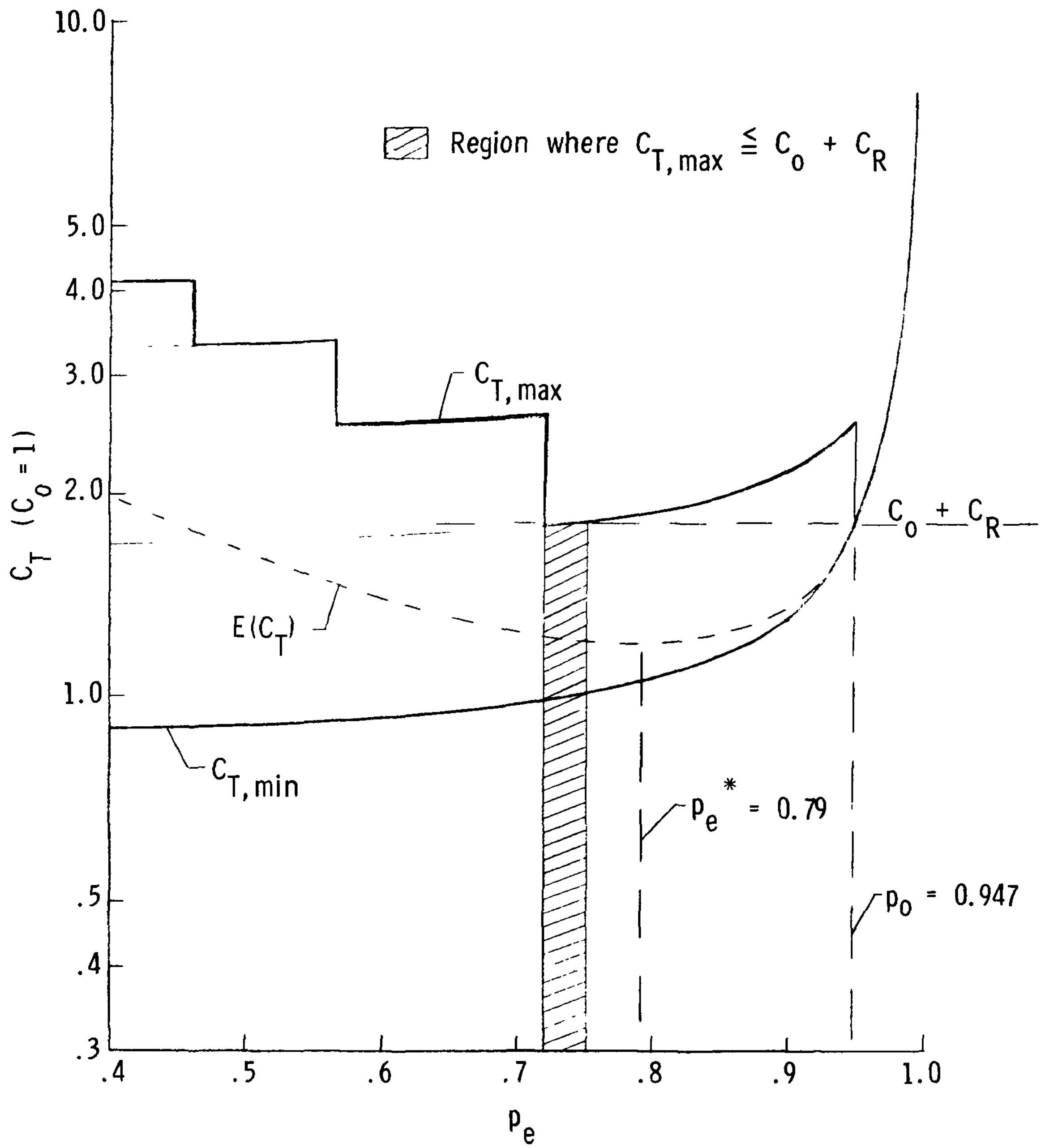

Figure 10.- Cost envelope for mission where $C_R = 0.8C_o$.

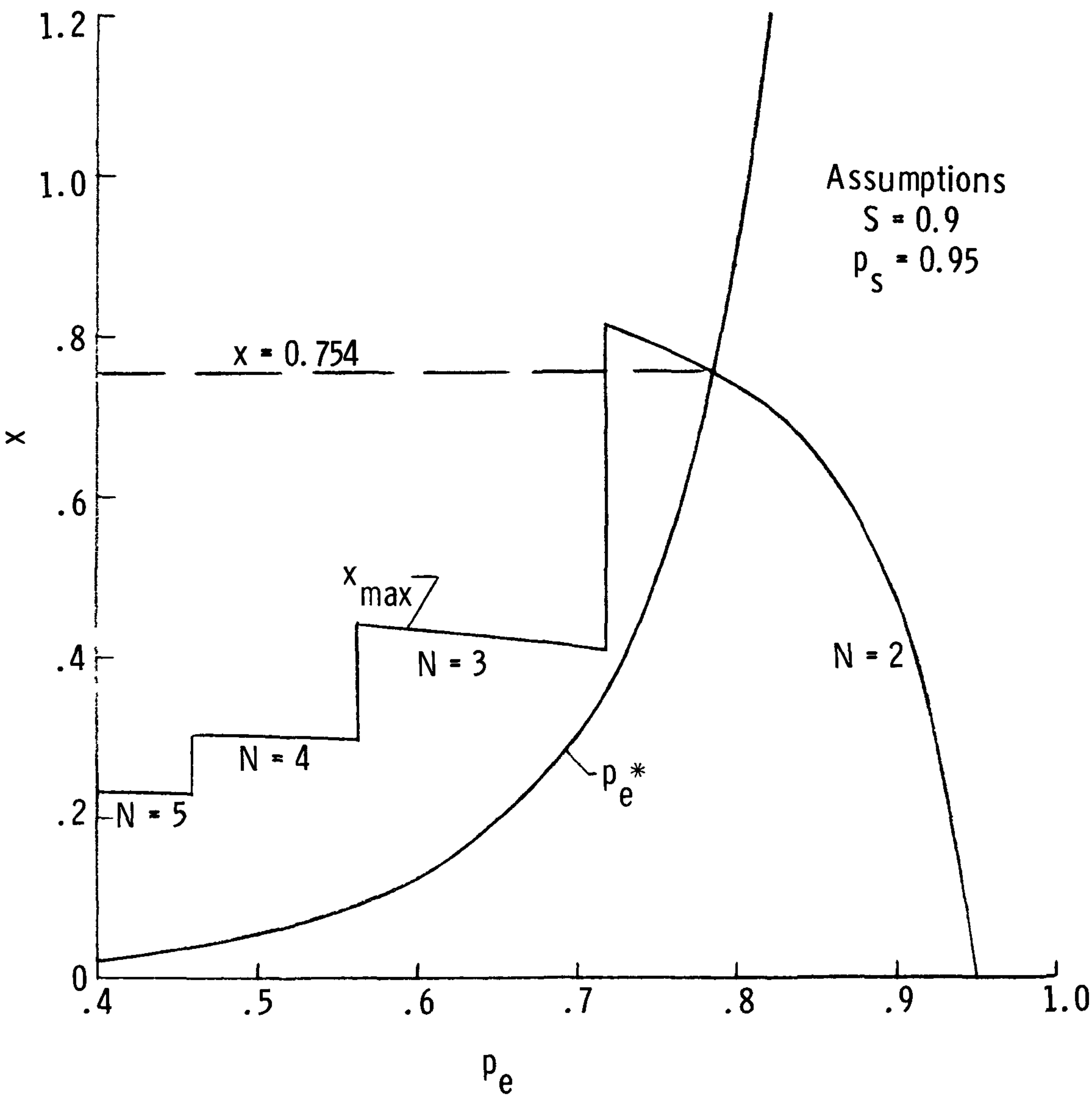

Figure 11.- Overlap of regions where $p_e = p_e^*$ and $x \leqq x_{max}$.

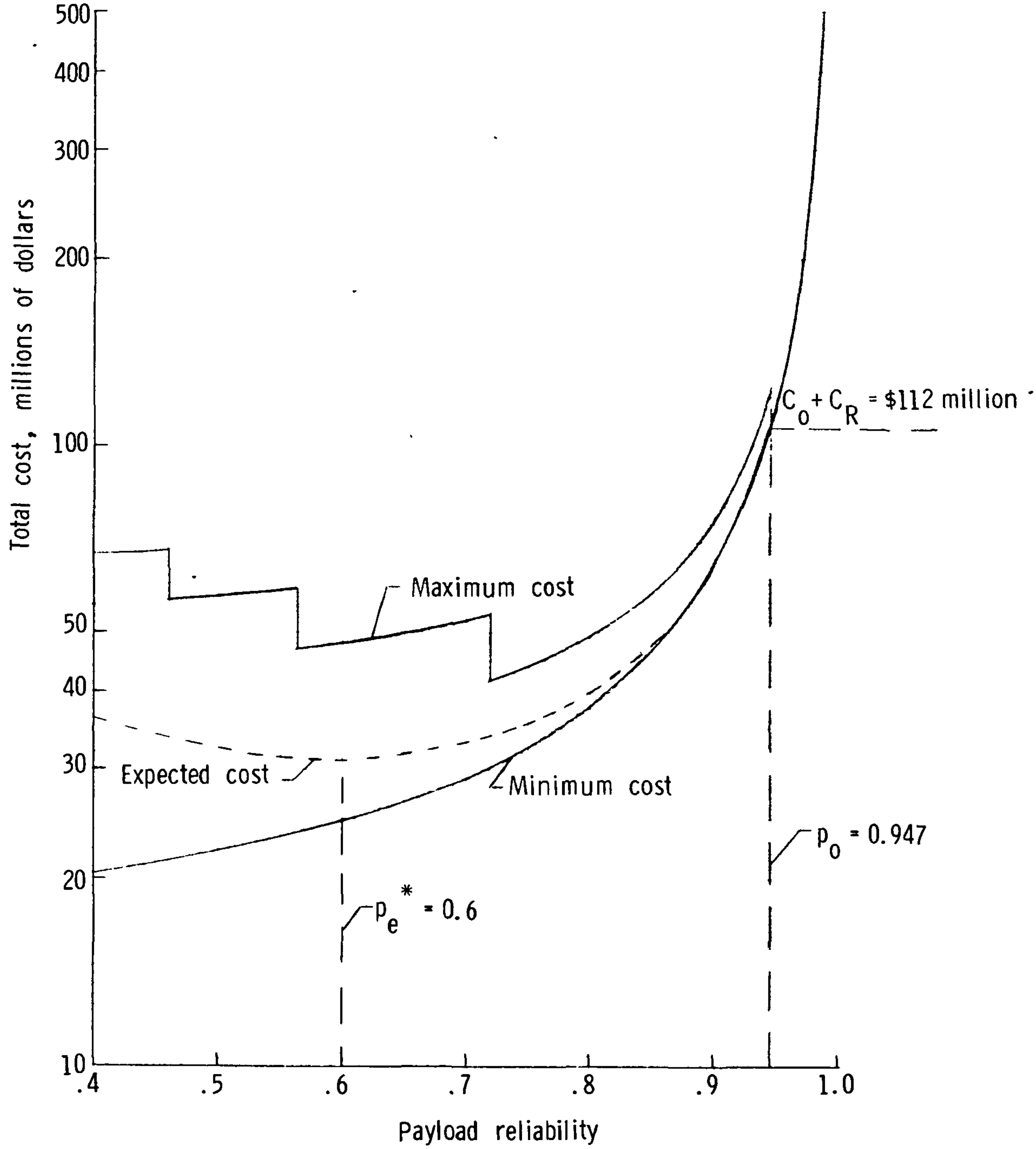

Figure 12.- Total cost envelope for Advanced Technology Laboratory as a function of payload reliability.

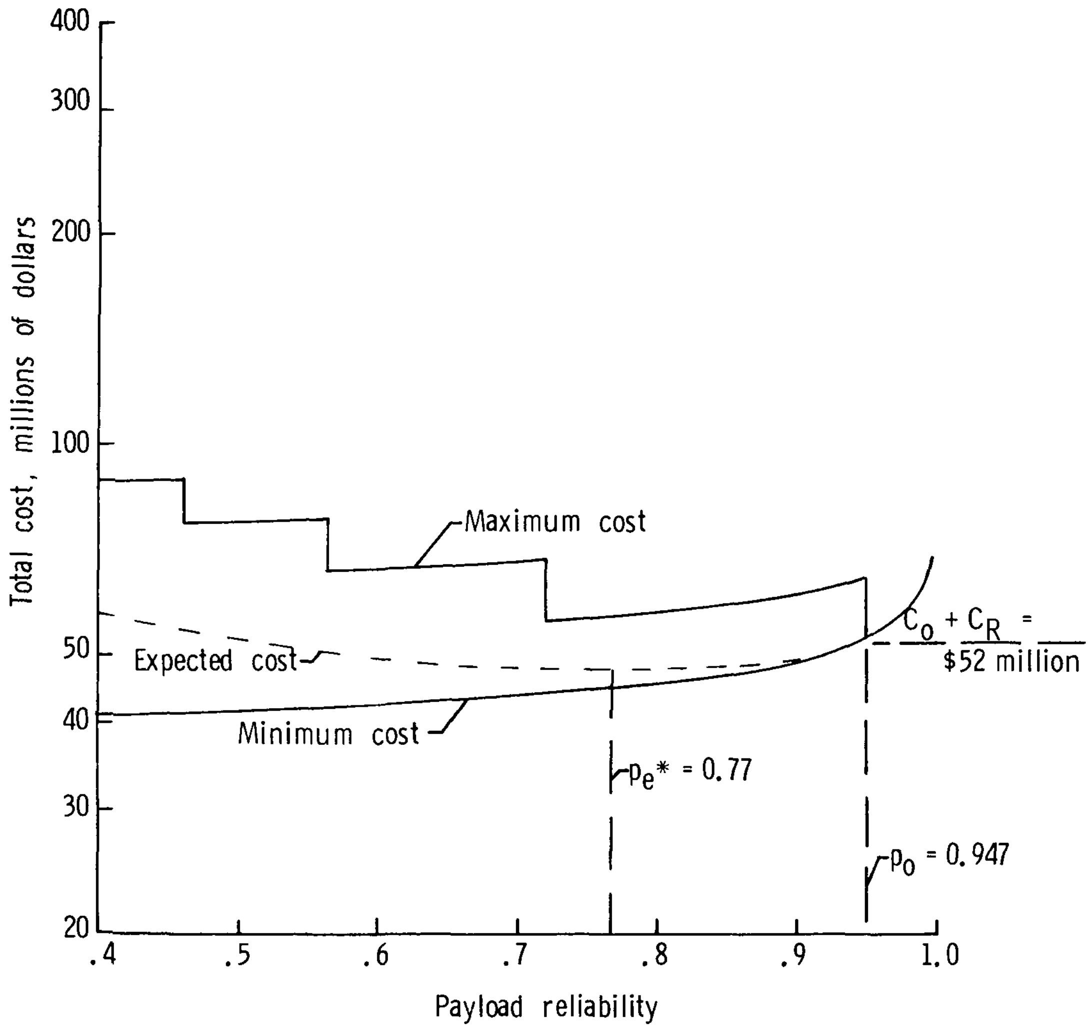

Figure 13.- Total cost envelope for Advanced Technology Laboratory based on conservative reliability-cost model.

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