

7.3 Reduction of Trimmed Drag

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It is important at the outset to distinguish between "trim drag" and "trimmed drag." According to the USAF Stability and Control Handbook,⁽¹⁾ the trim drag coefficient is "the drag coefficient increment between the drag coefficient of the complete vehicle in pitch equilibrium and the drag coefficient of the wing-body-vertical tail configuration." The trimmed drag coefficient, on the otherhand, is the drag coefficient of the complete vehicle in pitch equilibrium. It is clear that our interest should be focused on reducing the trimmed drag and not on the nebulous problem of reducing the trim drag penalty. Consequently, emphasis will be placed on the complete configuration and the associated trimmed lift and drag with particular attention paid to the load distribution between the wing-body and the tail surfaces.

Aircraft Equations for Equilibrium, Balance, and Drag

The equations for the total aircraft lift and pitching moment coefficient are given by (for small downwash, ϵ)^{(2), (6)}

$$C_L = a_{wb} (\alpha - \alpha_{owb}) + C_{L_t} n_t S_t/S \quad (1)$$

and

$$C_m = C_{m_{owb}} + C_{m_{\alpha wb}} (\alpha - \alpha_{owb}) - C_{L_t} n_t S_t/S l_t/\bar{c} \quad (2)$$

For balance in equilibrium flight, $C_m = 0$, allowing equations (1) and (2) to be solved for the tail lift coefficient and the aircraft angle of attack:

$$C_{L_t} = \frac{C_{m_{\alpha wb}} C_L + a_{wb} C_{m_{owb}}}{n_t S_t/S (a_{wb} \frac{l_t}{\bar{c}} + C_{m_{\alpha wb}})} \quad (3)$$

$$\alpha - \alpha_{owb} = \frac{\frac{l_t}{c} C_L - C_{m_{owb}}}{a_{wb} \frac{l_t}{c} + C_{m_{\alpha wb}}} \quad (4)$$

Equations (3) and (4) govern the distribution of the required lift force between the wing and the tail and insure a zero pitching moment. Several observations can be made concerning these equations:

- (i) The tail contribution to the aircraft lift coefficient, $(C_L)_t = C_{L_t} \eta_t S_t/S$, is a function of wing-body properties, c.g. position, and lift coefficient. Consequently for a given speed and weight, $(C_L)_t$, the tail load can be adjusted by shifting the c.g. position or by changing the wing-body aerodynamic characteristics.
- (ii) The expression in the denominator common to both equations is independent of the c.g. position.
- (iii) The magnitude of C_{L_t} determined by equation (3) must be less than $C_{L_{tmax}}$.

The key to the selection of wing-body parameters and c.g. position is the introduction of the aircraft drag coefficient. We would like to select these parameters to reduce or minimize the drag coefficient for a given lift coefficient. The drag coefficient for the aircraft is given by (for small downwash ϵ):⁽⁶⁾

$$C_D = C_{D_{owb}} + K_{wb} C_{L_{wb}}^2 + [(C_{D_{ot}} + K_t C_{L_t}^2 + C_{L_t} \epsilon) \eta_t S_t/S] \quad (5)$$

where

$$C_{L_{wb}} = a_{wb} (\alpha - \alpha_{owb}) \quad (6)$$

$$\epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} \alpha \equiv \frac{\partial \epsilon}{\partial \alpha} (\alpha - \alpha_{owb}) + \bar{\epsilon}_0$$

The bracketed term in equation (5) is the trim drag coefficients as indicated by the definition at the beginning of the paper.

The problem of the aircraft designer then is to select the wing-body aerodynamic parameters and c.g. position such that the trimmed drag coefficient given

by equation (5) is minimized in some sense, subjected to the equilibrium lift and zero pitching moment constraints given by equations (3) and (4). Conventional design practices also require that certain inherent stability specifications be satisfied. Consequently in current design practices the stability and performance characteristics of an aircraft are virtually determined independent of each other in the sense that one aspect (stability) is considered and then the other (performance).⁽³⁾

The continuous improvement of digital computational equipment with respect to size, speed and reliability has led to the consideration of utilizing digital control systems to maintain stability, reducing the number of constraints on the selection of c.g. position and wing-body aerodynamic parameters to reduce the aircraft drag coefficient.^{(4), (5)} These increased degrees of freedom present a considerable challenge to the aircraft designer leading to some of the new concepts of design associated with controlled configured vehicles. Although it is anticipated that such sophisticated control systems will not be available for general aircraft for a considerable period of time the advantages of such systems should not be completely ignored.

In what follows the concept of reducing the aircraft drag coefficient by appropriate selection of the wing-body aerodynamic parameters and c.g. position, will be examined. This approach is equivalent to finding the minimum drag for a given speed as opposed to maximum L/D for the aircraft.

C. G. Position for Minimum Drag

If we ignore stability requirements it is possible to determine the c.g. position which minimizes the drag coefficient for a given lift coefficient. In order to accomplish this, the appropriate terms in (5) are replaced by the expressions given in (3), (4) and (6). Furthermore the c.g. position can be introduced by noting the following relations:

$$\frac{l_t}{\bar{c}} = h_t - h_0 \tag{7}$$

$$C_{m_{awb}} = a_{wb} (h_0 - h_{nwb})$$

where h_x is position of x in chord lengths behind the leading edge of wing and

$x = 0$ c.g. position

= t tail aerodynamic center

= nwb wing-body aerodynamic center

The derivative of the drag coefficient with respect to c.g. location can be evaluated and set equal to zero. The resulting expression can then be solved for the c.g. position, h_o , which provides minimum drag coefficients. The result is

$$h_o = \frac{[(2k_{wb} a_{wb} - \frac{\partial \epsilon}{\partial \alpha})h_t + (2k'_t a_{wb} - \frac{\partial \epsilon}{\partial \alpha})h_{n_{wb}}] C_L + 2[\frac{\partial \epsilon}{\partial \alpha} - a_{wb}(k_{wb} + k'_t)] C_{m_{owb}} + \bar{\epsilon}_o a_{wb}(h_{nwb} - h_t)}{2[a_{wb}(k_{wb} + k'_t) - \frac{\partial \epsilon}{\partial \alpha}] C_L} \quad (8)$$

where $k'_t = k_t / (n_t S_t / S)$

Equation (8) gives the c.g. position for given lift coefficient for minimum drag coefficient in terms of wing-body aerodynamic parameters, tail parameters and geometry.

Several observations concerning equation (8) can be made:

- (i) The c.g. position for minimum drag coefficient changes with lift coefficient (speed)
- (ii) The c.g. position dictated by (8) is not restricted by stability constraints allowing the possibility of inherent static stability
- (iii) The c.g. location given by (8) is a function of wing-body and tail aerodynamic parameters and geometry. Consequently the c.g. location for minimum drag coefficient can be changed by judicious selection of these parameters.

Design Characteristics

As indicated earlier it is undesirable to have an inherently unstable (or overly stable) aircraft when sophisticated control systems are not available for compensation purposes. Consequently it would be desirable to take advantage of observation (iii) and adjust the aerodynamic and geometric parameters in such a manner so that the c.g. position for minimum drag provides the desired static margin. It is possible to approach this problem several ways, two of which will be outlined below.

One method of approach is to treat the drag coefficient as a function of several aerodynamic and geometric parameters, including c.g. position and attempt to find a minimum with respect to all these parameters subject to certain specified constraints (static margin, etc.). The drawback with such a method is that a large

number of constraints may have to be applied to obtain "optimal" parameters which are realistic.

Another approach takes advantage of equation (8) and the related observation (iii). Here the optimal c.g. position as a function the aerodynamic and geometric parameters is determined by (8). Furthermore, the drag coefficient and the neutral point position can be determined in terms of the same set of parameters. For small changes in the parameters we can approximate the changes in drag coefficient, c.g. position for minimum drag coefficient, and neutral point by:

$$\Delta h_o \approx \frac{\partial h_o}{\partial p_1} \Delta p_1 + \frac{\partial h_o}{\partial p_2} \Delta p_2 + \dots \quad \text{from (8)}$$

$$\Delta C_D \approx \frac{\partial C_D}{\partial p_1} \Delta p_1 + \frac{\partial C_D}{\partial p_2} \Delta p_2 + \dots \quad \text{from (5)} \quad (9)$$

$$\Delta h_n \approx \frac{\partial h_n}{\partial p_1} \Delta p_1 + \frac{\partial h_n}{\partial p_2} \Delta p_2 + \dots$$

Consequently for a given aircraft, the "optimal" c.g. position can be selected from equation (8). Then equations (9) can be used to find the changes in the parameters p_i required to move the c.g. and neutral point to satisfy static margin requirements and at the same time keep $\Delta C_D \leq 0$. In other words Δh_o , Δh_n and ΔC_D are specified and (9) solved for p_i . If there are more or less than three parameters the solution is either nonunique or not possible. In such a case a minimum norm. type solution is proposed.

The changes in the parameters can be incorporated by appropriate changes in the wing-body and tail geometry, another area which needs development. Again several observations can be made:

- (i) Clearly a necessary assumption is that the parameters can be changed independently. This assumption is better for small changes in parameters and decreases in its validity as the magnitude of the changes increases.
- (ii) The calculation of sensitivities in the above method allows an evaluation of the importance of each parameter in achieving a desired goal.

Concluding Remarks

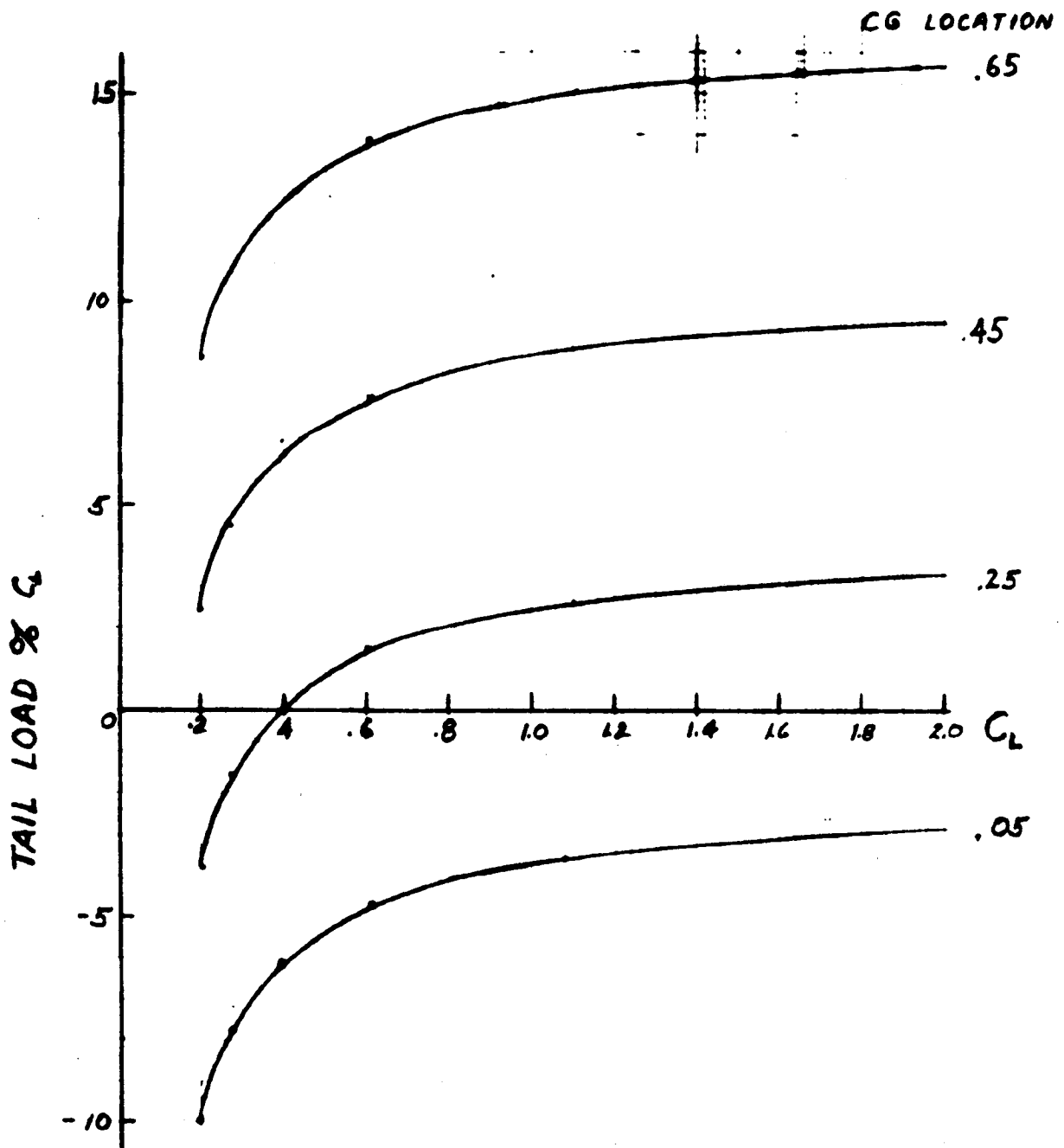
The above remarks were aimed at examining methods for reducing the aircraft drag coefficient for a given aircraft lift coefficient, or speed. The emphasis was placed in determining the load distribution between the wing-body combination and the tail which would reduce the overall drag coefficient. Furthermore a technique was presented which would allow the determination of various aerodynamic and geometric parameters which would permit the 'best' c.g. location to satisfy inherent stability requirements. Included in the method was the calculation of sensitivity coefficients which indicates the importance of various parameters in achieving specified goals ie. c.g. movement, drag coefficient change, etc.

Preliminary results indicate that such an approach is feasible. For given aircraft parameters c.g. movement alone yields drag coefficient reduction of the order of 1% over the nominal case for a conventionally designed aircraft. Tentative results indicate that if the downwash angle at the tail is large enough (at zero lift) then a down load on the tail at the expense of the same additional load on the wing is desirable in reducing the overall drag coefficient. The reason is that the tail lift vector is tilted rearward by the downwash angle. If the tail lift is negative the contribution to the aircraft drag is negative. Under these circumstances the optimum c.g. is forward of the nominal. The amount and direction of movement is sensitive to this downwash parameter.

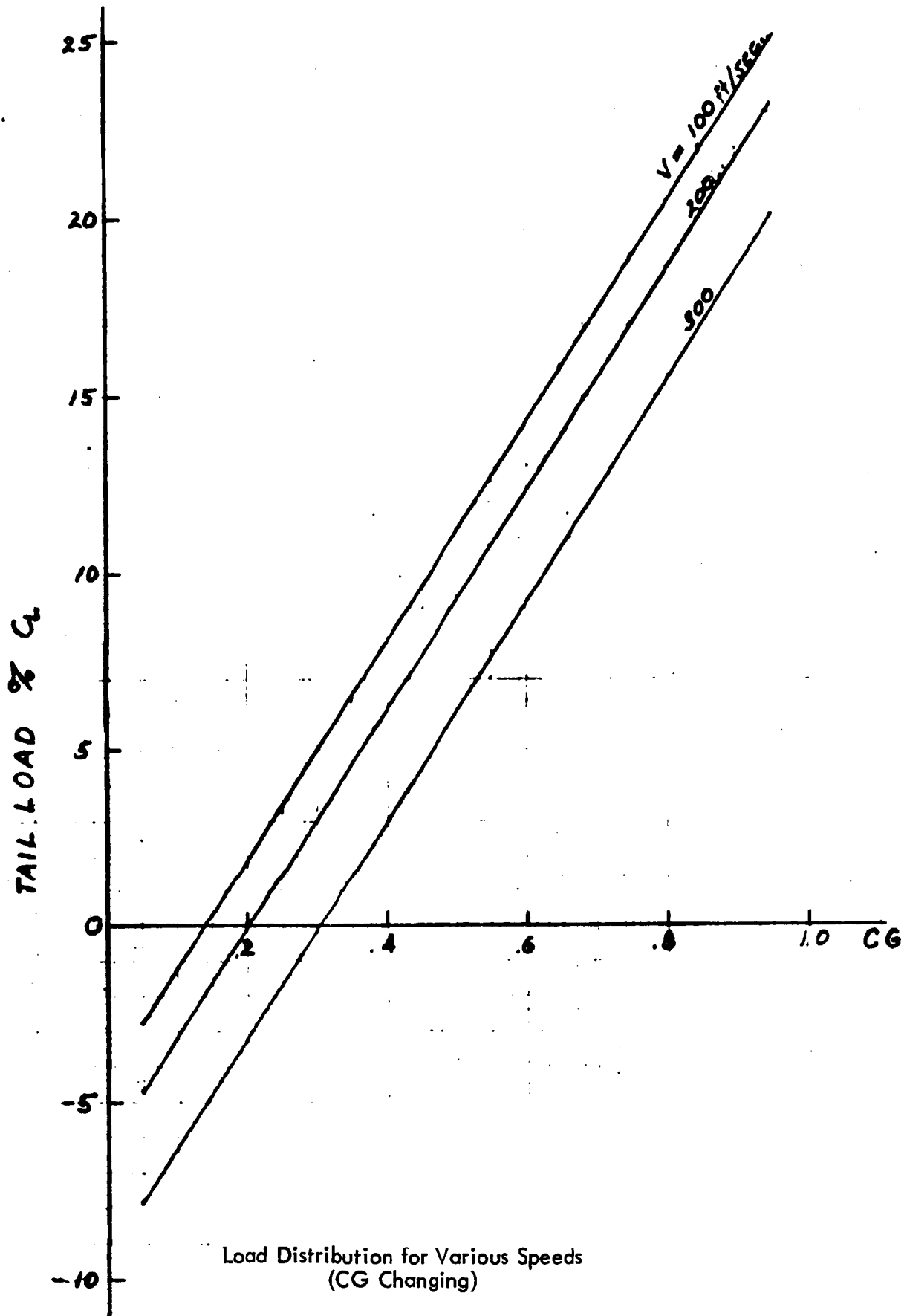
Although the drag reduction due to c.g. movement alone is small, the inclusion of other parameter changes can improve this drag reduction significantly. How these desired parameter changes can be obtained through wing-body and tail geometry changes still needs to be investigated. Clearly all drag reduction methods should be examined together. (7)

References

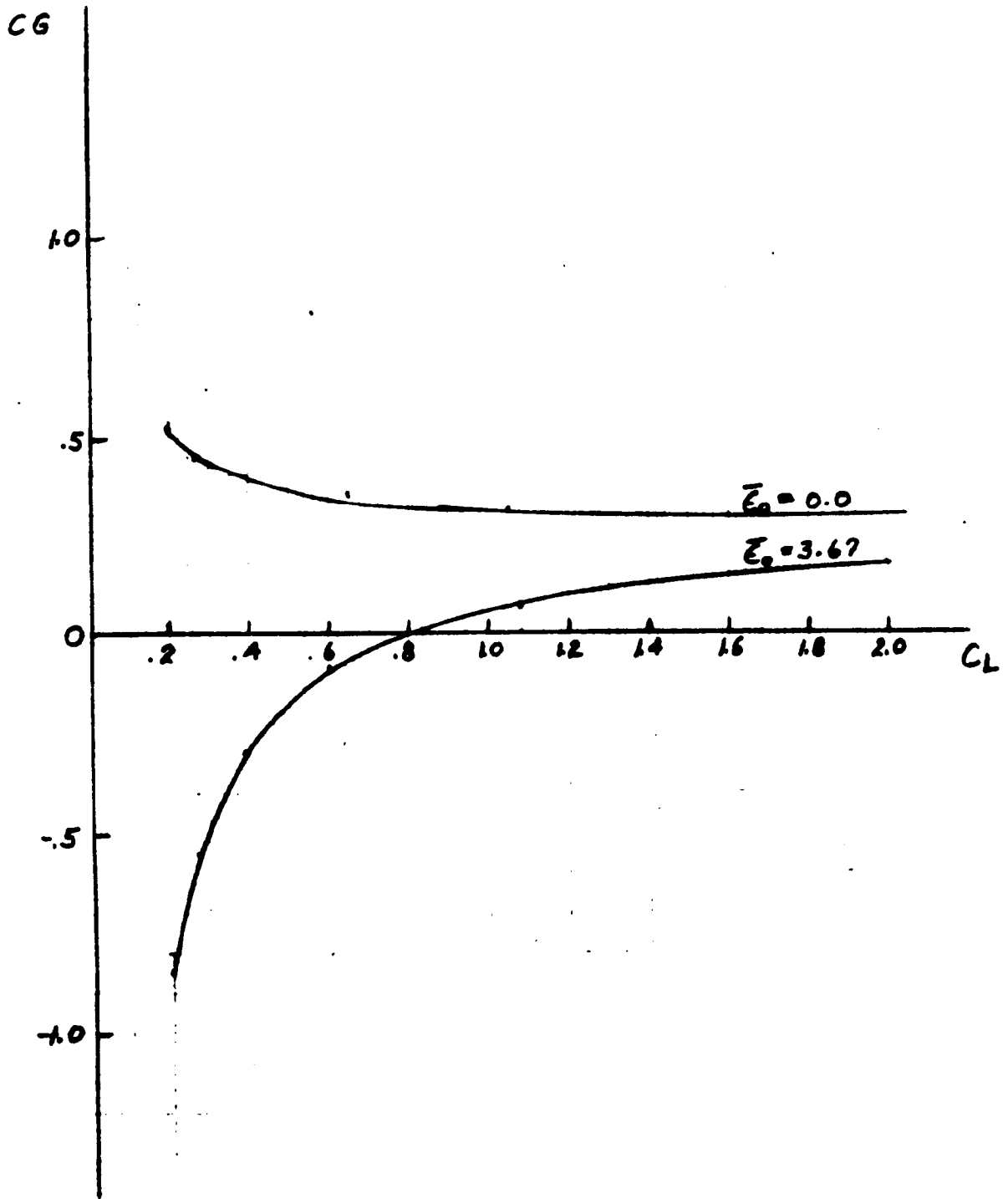
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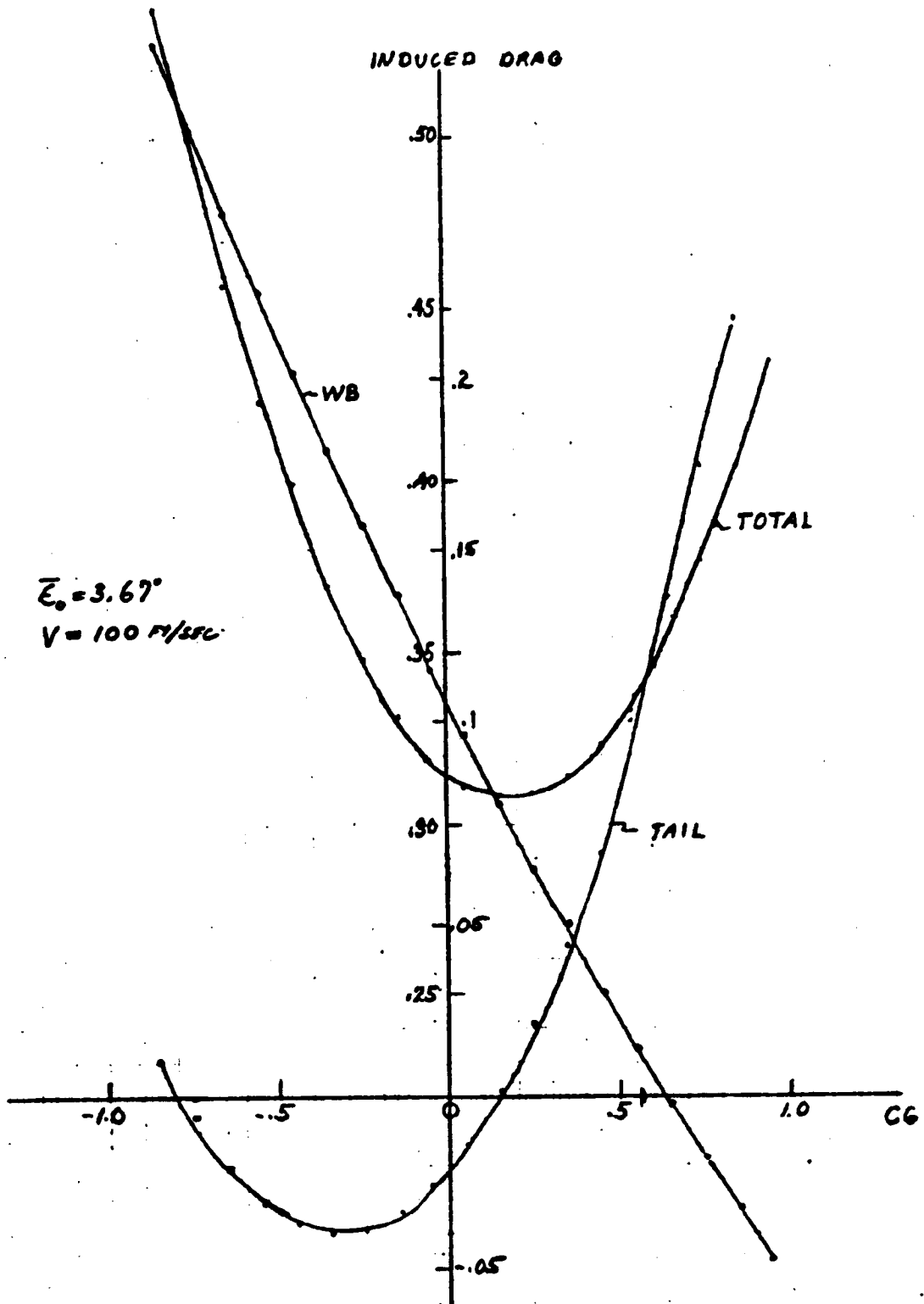
Load Distribution for Various CG Positions
(Speed Changing)



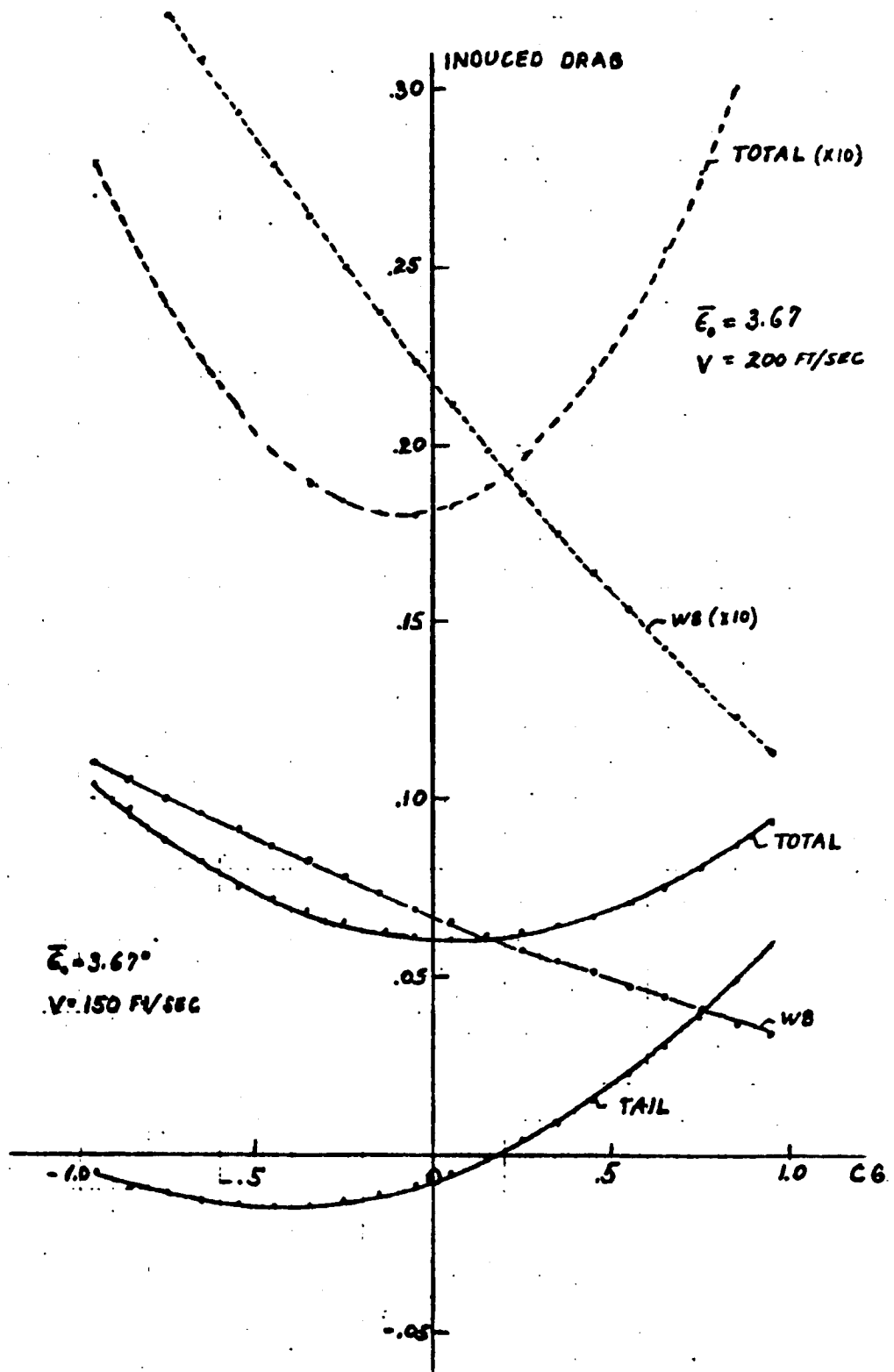
Load Distribution for Various Speeds
(CG Changing)



CG Position for Minimum Drag Coefficient



Induced Drag vs CG Position



Induced Drag vs CG Position