FINAL TECHNICAL REPORT

FURTHER TESTS OF PLANT CANOPY REFLECTANCE MODELS AND INVESTIGATION OF NON-LAMBERTIAN PROPERTIES OF PLANT CANOPIES

by

E. W. LeMaster, Principal Investigator
Pan American University
Edinburg, Texas 78539

(NASA-CR-145615) FURTHER TESTS OF PLANT CANOPY REFLECTANCE MODELS AND INVESTIGATION OF NON-LAMBERTIAN PROPERTIES OF PLANT CANOPIES Final Technical Report, 1 Sep. 1974 - 31 Aug. 1975 (Pan American Univ., N76-11548 Unclas G3/43 03083

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Headquarters, Washington, D. C. 20546

November, 1975

Funding September 1, 1974, to August 31, 1975

Contract NSG 9015

TABLE OF CONTENTS

ABSTRACT

A theoretical study by J. E. Chance and J. M. Cantu is included in the Appendix of this paper on the vegetation model of G. H. Suits[1]. The experimental bidirectional reflectance of cotton is presented and compared to the Suits model. Some wheat reflectance data is presented for a Mexican dwarf wheat. The general results are that the exchange of source position and detector position gives the same reflectance measurement if the irradiance is purely specular. This agrees with Suits. The reflectance versus sun angle and reflectance versus detector angle do not agree with the Suits predictions. There is qualitative agreement between the Suits model and reflectance versus wavelength, but quantitative agreement has not been observed. Reflectance of a vegetation canopy with detector azimuth shows a change of 10 to 40% for even sun angles near zenith, so it seems advisable to include azimuthal angles into models of vegetation.

[1] G. H. Suits, Remote Sensing of Environment 2,117 (1972).

FINAL REPORT

INTRODUCTION

The task of identifying varieties of agricultural crops from satellite altitude has proven to be a very challenging problem. An empirical approach using statistical techniques has been used predominantly, but in recent years a physical model approach which includes both models of the atmosphere and of the plants themselves has been used to try to establish cause-effect relationships.

The number of temporal variables involved in the problem renders the empirical approach useless if a training field is used to categorize crops over a period as long as 1 or 2 hours as was shown by Malila, et. al.(1) At the same time these time-dependent parameters make physical models so complicated that they rapidly become formidable.

The Suits(2) model was an attempt to use only the predominant physical parameters of a plant canopy that affect the reflectance. This report is a study of the properties of the Suits model on cotton and, to a limited extent, wheat to determine if the dominant characteristics have, indeed, been modeled. The principal investigator aroused the interest of a professor from the Mathematics Department at Pan American University, Dr. J. E. Chance, and a Master's Degree Candidate, Mr. Juan Manuel Cantu, to further study the mathematical properties of the Suits model. As far as was practicable the same questions that were asked of the model were asked of the field measurements with comparisons being made in this report. The main factors that are of interest are the following:

(a) Will the reflectance measurement be affected by interchange of source and detector?

(b) In what regime of source position and observer position is the bidirectional reflectance function the most sensitive? The least sensitive?

(c) Does the top surface reflection of leaves, which in many plants is admittedly non-Lambertian, as observed in single-leaf experiments of Breece and Holmes[3] account for the non-Lambertian properties of vegetative canopies.

(d) In general characteristics, how is the qualitative agreement between the model and field measurements? Is the agreement about the same for low leaf area index (L.A.I.) crops and for high L.A.I. crops?

(e) Is the azimuthal variation negligible as implied by Suits' assumption of azimuthal symmetry?

Most of the data were collected on cotton, sampled from two different fields where the L.A.I. was 5.3 and 5.5. The wheat was a Mexican dwarf variety grown in Eagle Pass, Texas. The variety was Penjamo and is nearly identical to the high-yield varieties introduced 10 years ago into India and Pakistan and now called Kalyan, Kalyan-sona, and Sona-lika. The L.A.I. of the wheat reported here was 2.1 and was in a mature green stage with well-developed heads.

THEORY OF MEASUREMENTS

The reflectance is defined as $\rho = \frac{|\vec{E}_r{}^2|}{|\vec{E}_i{}^2|}$, where $|\vec{E}_r|$ or $|\vec{E}_i|$ are the magnitudes of the electric field vectors of the reflected and incident radiation, respectively. This can easily be written as the ratio of the total time-averaged "outward" moving radiant power relative to the surface being considered. In a laboratory spectrometer single leaf reflectance is normally measured by irradiating the leaf with white light from a diffuse source and measuring the light reflecting in a particular direction. In order to apply the definition of the reflectance, it appears to this investigator that one should specify the type of source and type of detector as: "hemispherical - hemispherical reflectance" or "hemispherical - angular reflectance", etc. This would also dictate that the reflectance function should be defined as

$$\rho = \frac{\text{radiance}}{\text{irradiance}}$$

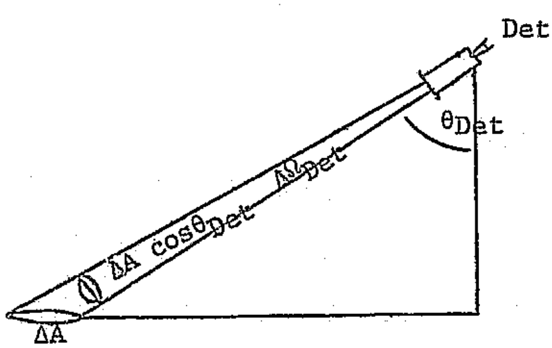
as done by Breece and Holmes[3].

In the case of field studies, the source of irradiance is a combination specular from direct sunlight and diffuse from skylight. In any field studies one should separately measure both components.

The reflectance of the canopy as determined in this report is the bi-angular or angular-angular reflectance. The ratio of specular to diffuse solar irradiance varied from ~95% to a worst case of 67%. If one assumes pure specular flux from the sun, then the reflectance can be determined as follows:

$$\rho_c = \frac{P_\lambda \ (\text{canopy})}{\Delta\Omega_{Det} \Delta A \ \text{Det} \ \cos\theta_{Det}} \Bigg/ \frac{P_\lambda \ (\text{sun})}{\Delta\Omega_{sun} \Delta A \ \cos\theta_{sun}}$$

where the crop is viewed with a detector looking through a solid angle $\Delta\Omega_{Det}$ at a projected area $\Delta A \cos\theta_{Det}$ and $\frac{P_\lambda \ (\text{sun})}{\Delta\Omega_{sun} \Delta A \ \cos\theta_{sun}}$ is measured irradiance of the sun falling on an area ΔA. P_λ is the power in watts arriving at the detector in the wavelength interval centered on λ. This assumes a completely black sky and a sun subtending a solid angle $\Delta\Omega_{sun}$ at the position of the observer.



Since for field measurements, a technique must be utilized that integrates the total irradiance of direct sunlight plus diffuse light. This can be accomplished in the following manner: Use a diffusely reflecting panel having a known reflectance, ρ_λ (panel), placed in the same position as the canopy whose reflectance is to be determined. One then obtains for the panel

$$\rho_\lambda \text{ (panel)} = \frac{P_\lambda \text{ (panel}}{\Delta\Omega_{Det}\ \Delta A \cos\theta_{Det}} \Bigg/ \frac{P_\lambda \text{ (sun)}}{\Delta\Omega_{sun}\ \Delta A \cos\theta_{sun}}$$

where ΔA is the area seen by the detector. This then yields an integrated value for the total solar irradiance; the canopy reflectance becomes

$$\rho \text{ (canopy)} = \frac{P_\lambda \text{ (canopy)}}{\Delta\Omega_{Det}\ \Delta A_{Det} \cos\theta_{Det}} \Bigg/ \frac{P_\lambda \text{ (panel)}}{\rho\lambda \text{ (panel)} \Delta\Omega_{Det}\ \Delta A_{Det} \cos\theta_{Det}} .$$

Because of the experimental difficulty of having a reflectance standard large enough to be observed at the same position as the agricultural canopy, a horizontal panel is placed about 1 meter below the detector head and observed from a vertical position of the head. This requires that in the denominator $\theta_{Det} = 0^\circ$, and the equation thus yields

$$\rho \text{ (canopy)} = \frac{P_\lambda \text{ (canopy)}}{A_{Det} \cos\theta_{Det}} \Bigg/ \frac{P_\lambda \text{ (panel)}}{\Delta A_{Det}\ \ \rho\lambda \text{ (panel)}}$$

$$\rho \text{ (canopy)} = \left[\left(\frac{P_\lambda}{\Delta\ A_{Det}} \right) \text{(canopy)} \Bigg/ \frac{P_\lambda \text{ (panel)}}{\Delta A_{Det}} \right] \frac{\rho\lambda \text{ (panel)}}{\cos\theta_{Det}} .$$

One thus has a measuring technique that does not involve the geometry of the detector.

EXPERIMENTAL METHODS:

Reflectance measurements taken in the field are made from the top of portable construction platform using a wedge-filter type radiometer (Isco Model SR) equipped with a 1.8 meter fiber optics probe. The field of view to half maximum is 13° and full field is 19°. The spectral band width is 15 nanometers (nm) in the visible and 30 nm in the infrared with a range of 380 nm to 1550 nm.

Spectral intensity was measured in the field by viewing the canopy from 4.9 m above the top of the plants at some azimuth angle measured from 0° as magnetic north and a polar angle measured from 0° straight down along a plumb line. The radiometer readings were combined with the absolute reflectance of the reflectance panel to give the absolute bidirectional reflectance function for the canopy as detailed in the previous section.

The reflectance panel was spray painted with at least 3 layers of Eastman Kodak White Reflectance Coating #6080 made of barium sulfate and having a reflectance >97%. The absolute reflectance was determined periodically from the United States Department of Agriculture, Agricultural Research Service (USDA, ARS) at Weslaco, Texas, standard vitrolite sample using a Beckman DK-2A[5].

The variations in radiance from the canopy as a function of azimuthal and of polar angle were recorded on an X-Y recorder as follows: A drafting machine was disassembled and modified to allow the radiometer fiber optics probe to be clamped at any polar angle. A 0.1% precision potentiometer was placed above the rotating head as shown in Fig. 1 to give a voltage read-out proportional to the angle displayed on the X-axis of the recorder while simultaneously the Y-axis is the radiometer output. This 180° azimuth is scanned in about 1 minute and reversed to give some indication of the noise and transient fluctuations.

RESULTS AND DISCUSSION:

(a) The first result is the effect of interchanging the source and detector positions. The altitude or polar angles were measured from zenith and the azimuth angles are magnetic compass measurements.

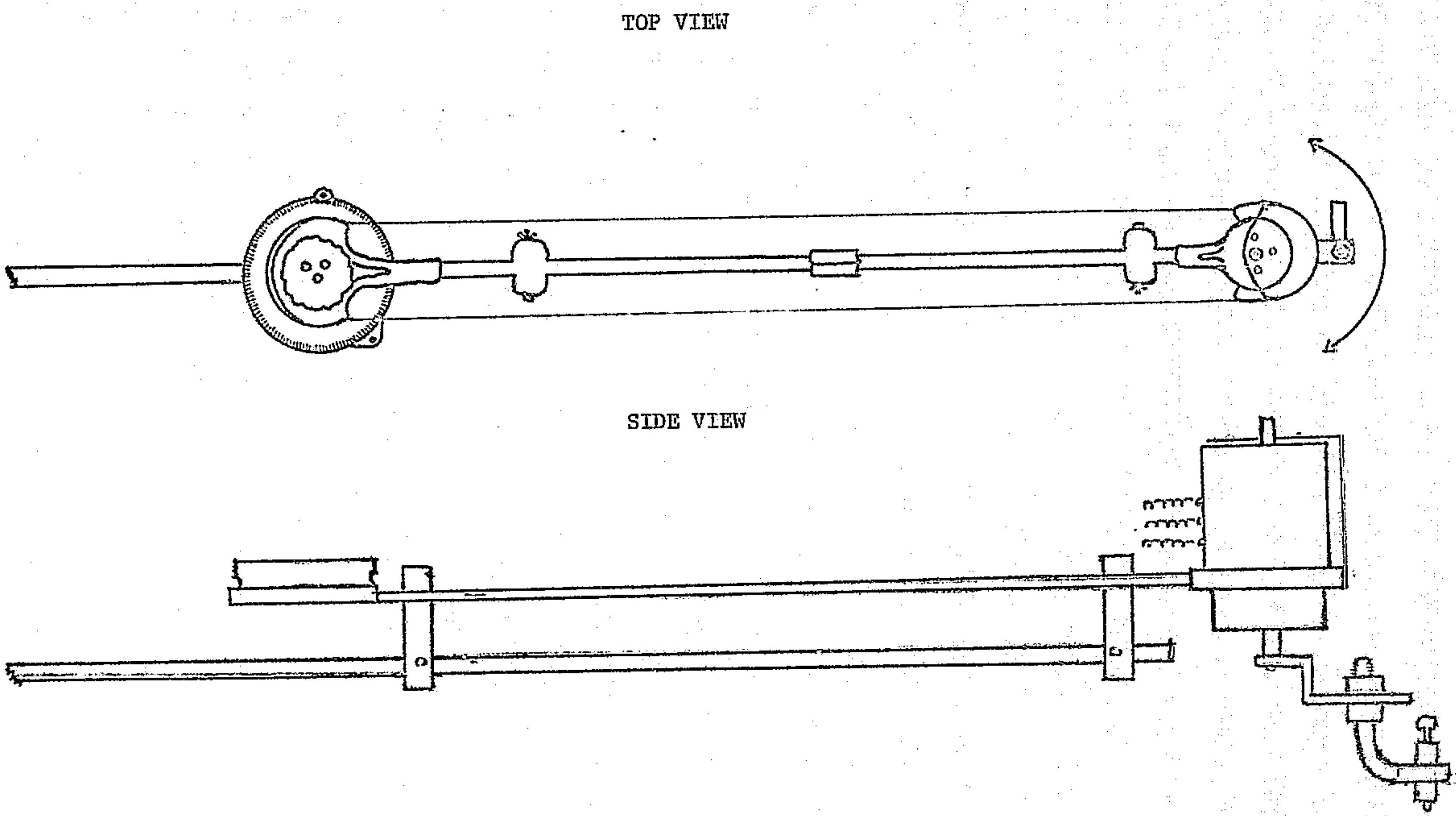


Figure 1. Azimuthal scan device made from a drafting machine. The position sensing mechanism is a 1 turn 1000Ω helipot connected to a 2.2 volt battery.

TABLE 1. Effect of exchanging sun and detector positions. Cotton measurements are shown for run prefix C and wheat is prefix W.

RUN	DATE	SUN POLAR & AZIMUTH	DETECTOR POLAR & AZIMUTH	WAVELENGTH (nm)	REFLECTANCE
C50	7/12/75	49°, 270°	45°, 90°	500	0.020±0.002
C7	7/21/75	55°, 90°	45°, 270°	500	0.020±0.002
C51	7/12/75	50°, 270°	45°, 90°	850	0.22±0.06
C8	7/21/75	54°, 90°	45°, 270°	850	0.27±0.06
W39	4/19/75	35°, 270°	30°, 90°	850	0.37±.015
W61	4/19/75	27°, 90°	30°, 270°	850	0.30±.014

The indicated errors are the combined effects of fluctuations on the radiometer signal from the cotton crop and from the reflectance panel and the estimated error in the absolute reflectance of the standard panel. The specular flux was 78% of the total at 850 nm and 75% at 500 nm on 7/21/75 at a solar polar angle of 45°; on 7/12/75 the specular flux was 89% at 850 nm and 83% at 500 nm at a solar polar angle of 11°. The fact that the solar polar angles were so much different for these measurements makes it impossible to compare sky conditions other than to say that both days were quite clear (based on similar data reported by Jones & Condit(6). Total solar irradiances for the two days in question were 233 watts/cm^2 for 7/12/75 and 225 on 7/21/75 at 850 nm; these differ by only 4%. This is justification for considering the sky conditions essentially the same. The 500 nm data has small error bars and shows exchange symmetry; the 850 nm data has large error bars and shows differences smaller than error limits. We thus conclude that the exchange of source and observer gives reflectance values that are essentially the same for cotton.

The exchange of source and observer for wheat is shown. There is not perfect exchange because the sun angle was 35° and 27° for the cases shown. The specular flux was variable because of the presence of high cirrus clouds; it varied from 61% to 68% specular. Because of these factors, the exchange property for wheat cannot be verified.

(b) The variation of reflectance of cotton with sun angle is shown in Fig. 2 and Fig. 3 for two angles of view at 500 nm and 850 nm, respectively. The general trends are toward moderate variations until the sun angle is more than 50° from vertical; the variation in reflectance is more pronounced in the near infrared compared to 500 nm. The data indicate that a minimum exists in the 850 nm data for a sun angle of about 40°-50° in cotton. The Suits model prediction is shown in Fig. 7 of the report of Chance and Cantu in the Appendix of this paper. The saddle-point behavior predicted by Suits shows a "plateau" and decreasing reflectance versus sun angle for a view angle of <35°. In the observer angle range >50°, there is an increasing reflectance versus sun angle; in the intermediate range the reflectance is nearly flat.

The only qualitative comparison one can make between the Suits prediction and experimental values is that there is a change in the general behavior of the reflectance in the vicinity of observer angle 40°. Since the cotton canopy is nearly uniform, it is clear that the effect is not a result of a layer structure at the tops of the plants reflecting most of the light at extreme solar angles and extreme observer angles. If one accepts the exchange symmetry, it follows that the isoreflectance surface must have diagonal symmetry and can have a maximum, a minimum, a saddle-point or completely flat. This all assumes the relative insensitivity to the azimuth angle between source and observer. We proceed to now look at reflectance versus observer angle at fixed sun angle, i.e., slicing the isoreflectance surface in the direction of fixed sun angle.

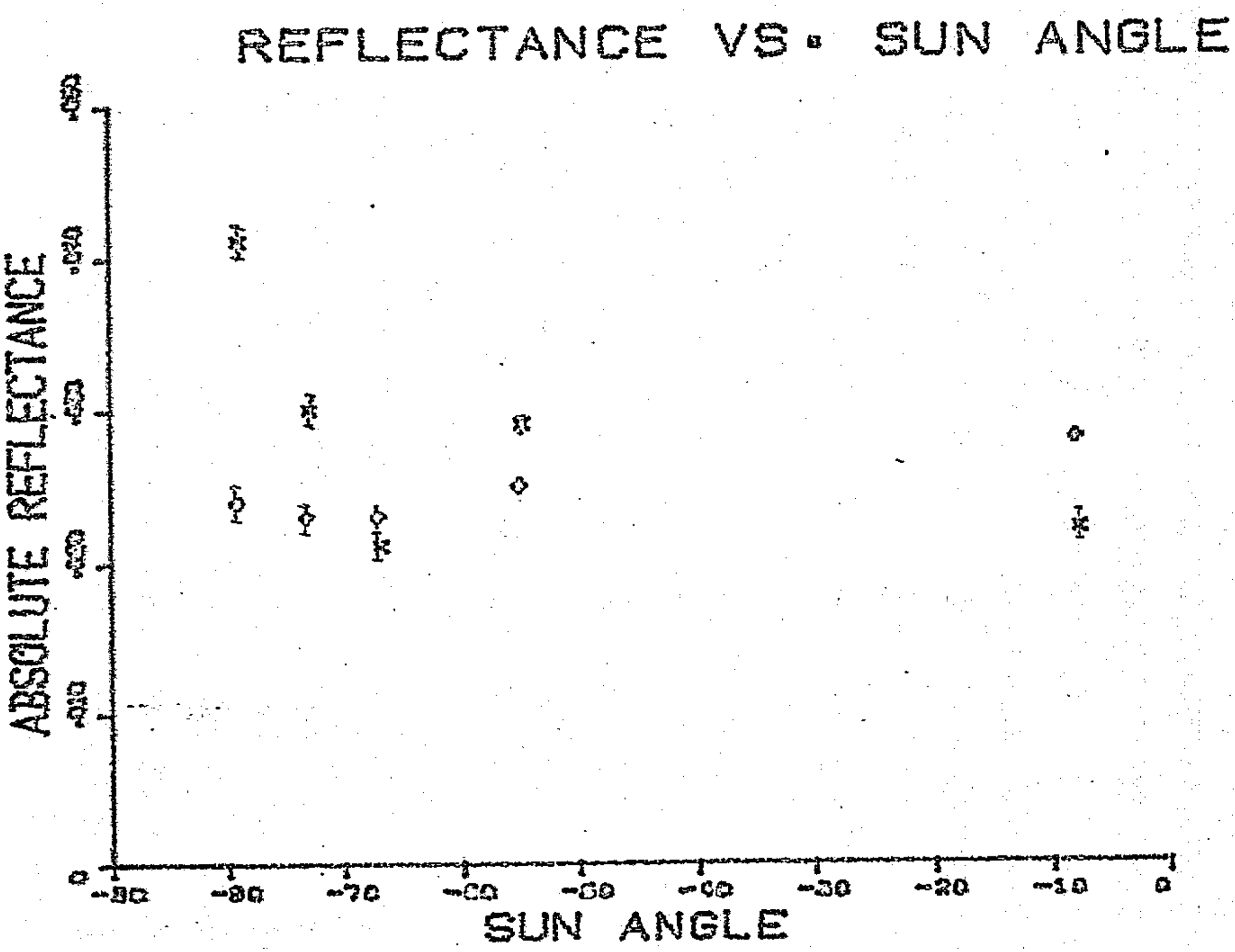


Figure 2. The reflectance of cotton at 500 nm for various sun polar angles. The detector polar angle was at ◇0° and x 30° looking at an azimuth angle 180° from the sun azimuth.

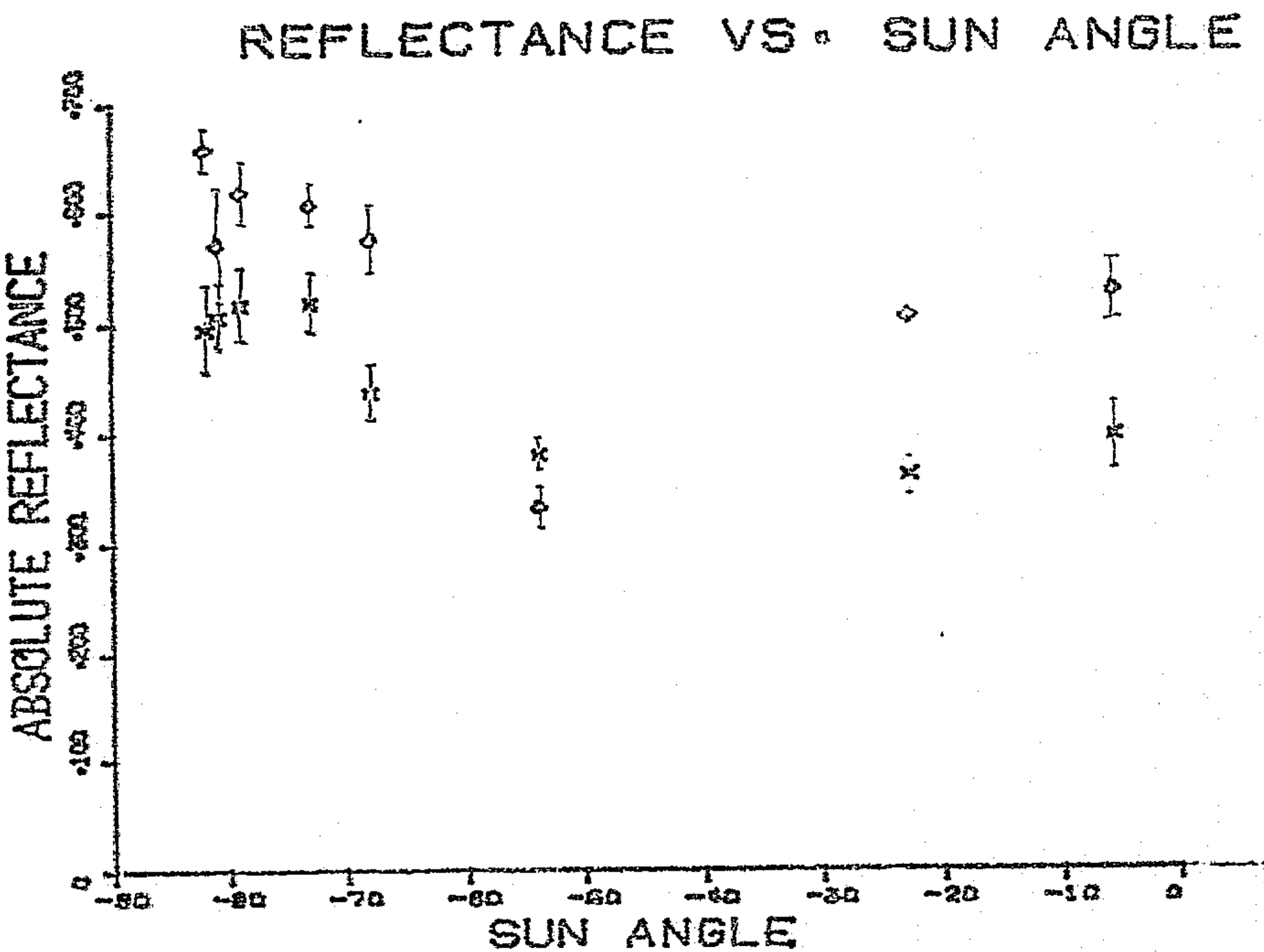


Figure 3. The reflectance of cotton at 850 nm for various sun polar angles. The detector polar angle was at ◊ 0° and x 30° looking at an azimuth angle 180° from the sun azimuth.

(c) The observer angle influence on the reflectance is observed in Figs. 4 and 5. The reflectance is shown here to be very sensitive to the polar angle of view at 850 nm, while the variation at 500 nm is only slight. The vertical plane through which the detector was scanned contained the sun in every case (the effect of choosing other planes of observation can be found from the azimuthal scans).

The combination of reflectance versus sun angle and reflectance versus observer angle makes possible an experimental plot of isoreflectance curves for purposes of comparing these results with the Suits model predictions. The general features of this plot for cotton do not agree with the saddle-point behavior predicted by the Suits model. The slices at constant sun angle and variable detector angle show maxima as seen in Figs. 4 and 5. This may be attributable to either top surface specular reflectance from the leaves or to heliotropic movement of the leaves. More experimental evidence will be presented to support the first effect; the second effect seems to explain the maximum that occurs in the data presented in Fig. 4 and Fig. 5. The maximum reflectance occurs when the observer has his back to the sun looking at about -5° in the IR and at -30° in the visible. This general behavior of the reflectance might be expected for a collection of leaves oriented toward the sun.

Shown in Figs. 6 and 7 is the reflectance of wheat at constant sun angle and variable detector polar angle. The negative sun angle indicates a morning sun. Note that there is no sign of row effects because the rows were only 0.16 meters apart. Since the L.A.I. was only 2.1, it seems that the data may have been greatly affected by the light, sandy-colored soil background. Looking vertically downward, the soil was readily visible; whereas at an angle of 30° or more only vegetation could be seen. The bare soil reflectance at 500 nm was 0.13 and at 850 nm it was 0.31.

The effect observed by Breece and Holmes on single-leaf non-Lambertian

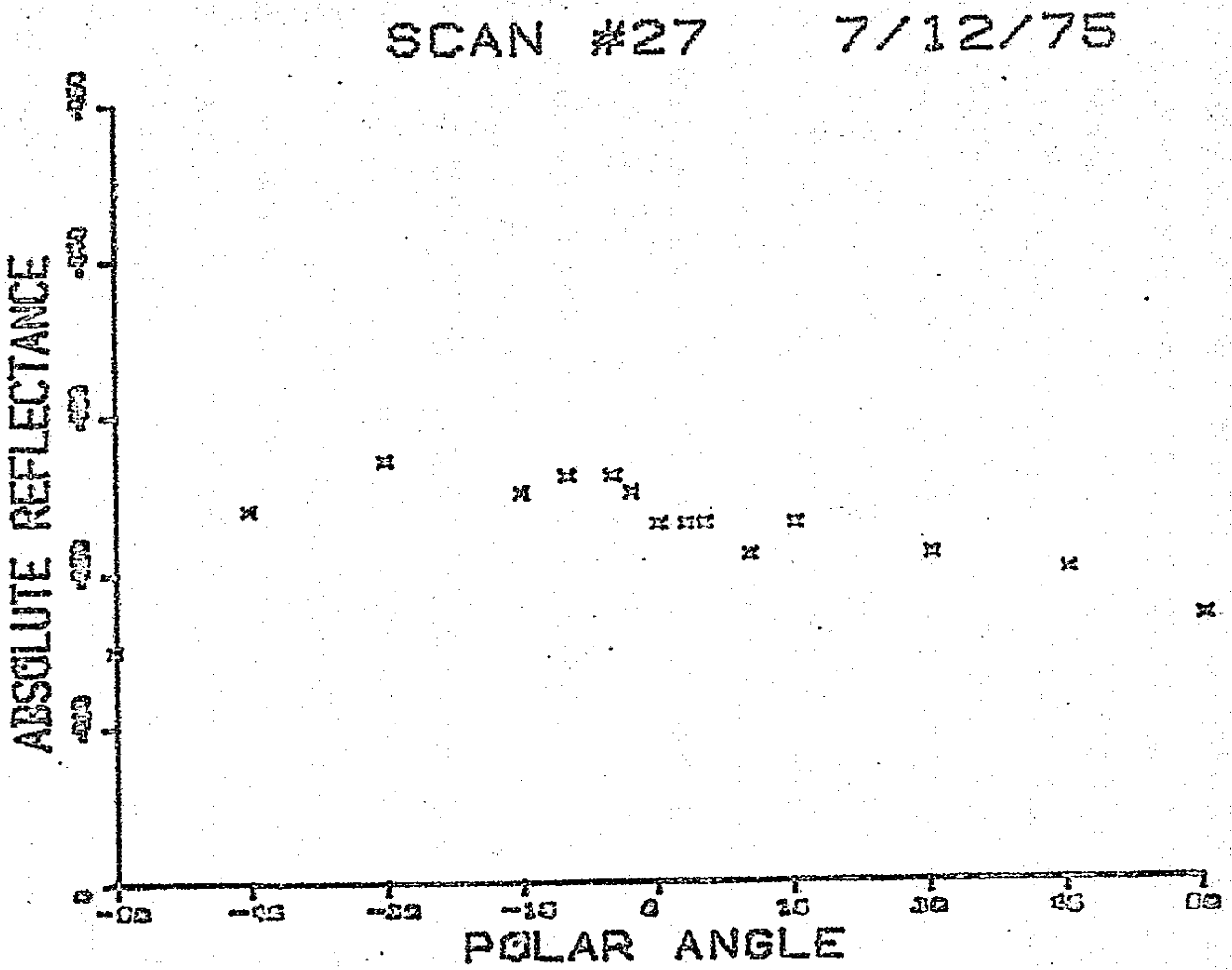


Figure 4. Cotton reflectance measurements at 500 nm at a sun polar angle of 31° from zenith in the west. The detector polar angle is positive when looking west in the vertical plane containing the sun.

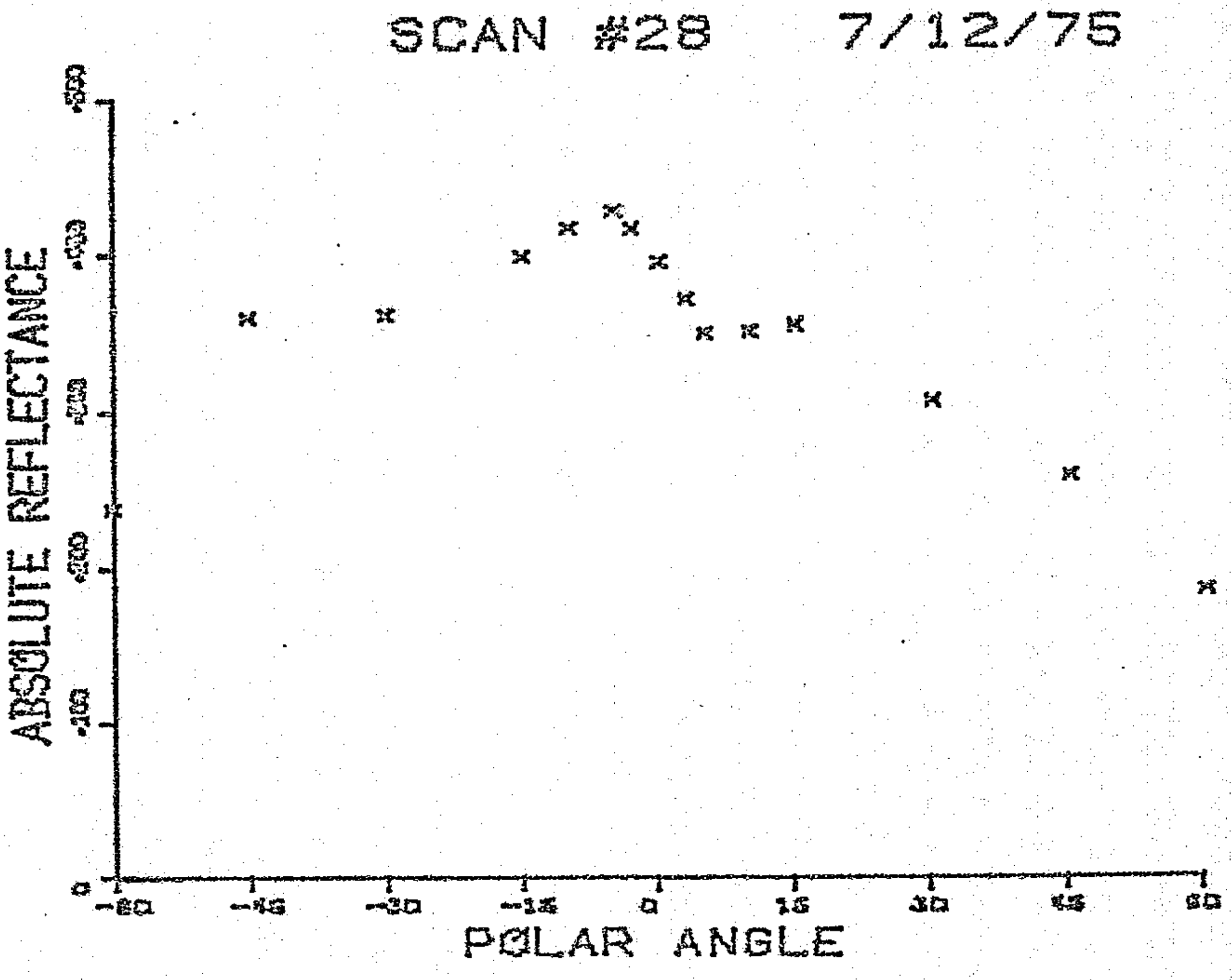


Figure 5. Cotton reflectance at 850 nm with a sun polar angle of 32° from zenith in the west. The positive direction is toward the west.

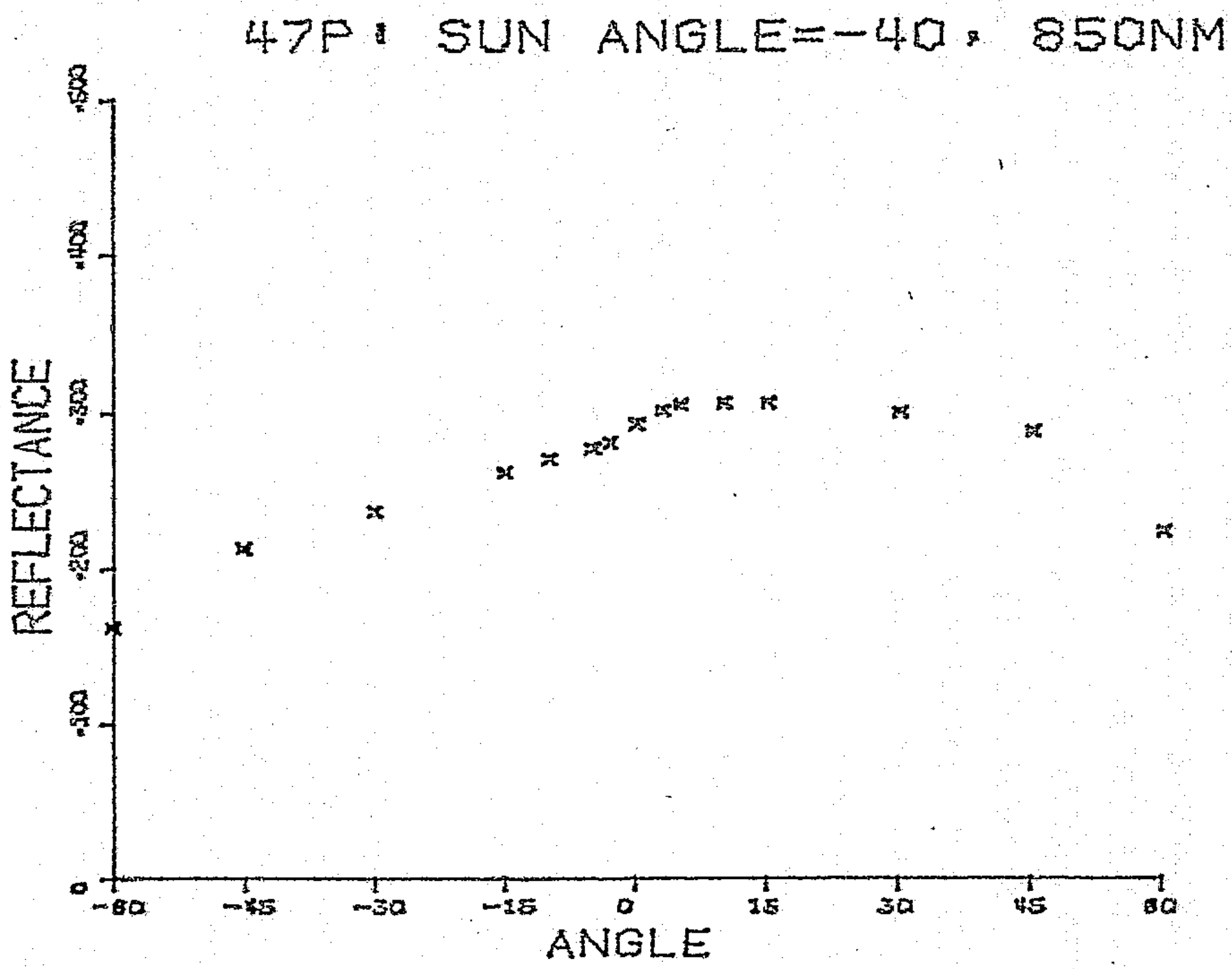


Figure 6. Wheat reflectance at 850 nm for a sun angle of 40° from zenith in the eastern sky. The positive polar angle indicated is the detector polar angle directed toward the north. The irrigation network in the field would not allow east-west polar scans.

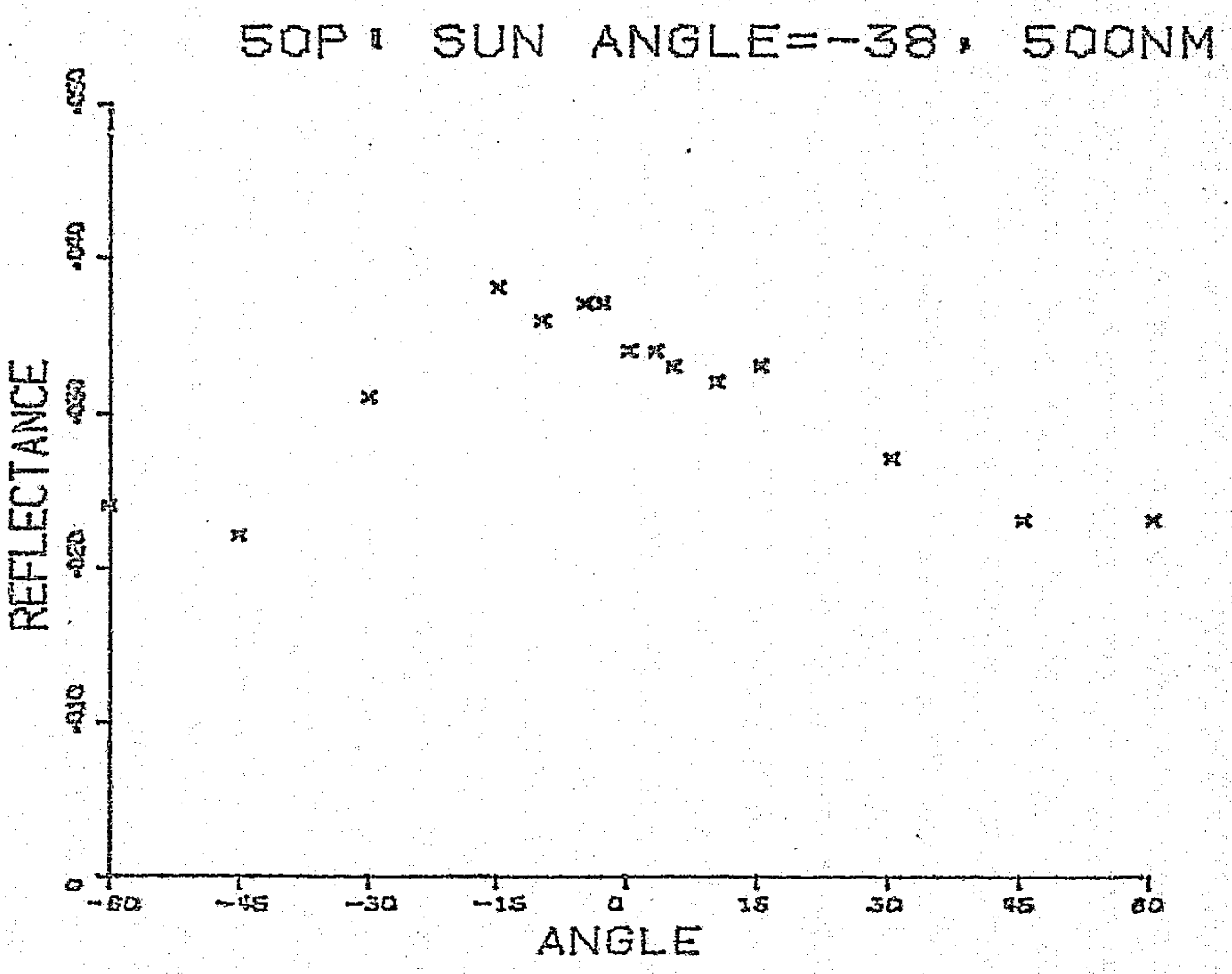


Figure 7. Wheat reflectance at 500 nm for a sun angle of 38° from zenith in the eastern sky.

reflectance was expected from the top surface of the canopy observed at extreme sun angles. Observer polar angle versus reflectance is shown in Figs. 8 and 9. The negative angles for the sun were morning positions; the negative angles for the observer polar angle were with the observer looking westward in the plane of the sun. The reflectance showed a high value when the observer was looking sunward in both morning and afternoon at 500 nm. The effect was not seen at 850 nm. This agrees with the observations of Breece and Holmes[3] on soybean leaves with a source polar angle of 60° (shown in their Fig. 6). Their observation was that there was a very strong tendency for specular reflectance in the visible wavelength ranges and the IR looked nearly Lambertian. One interesting feature of the 850 nm data was the marked effect of row structure at extreme sun angles compared to only a small effect at 500 nm (See Figs. 10 and 11). Compared to Figs. 8 and 9, the row effects are much smaller, while the sun has moved only 12°. These results can be accounted for by the following discussion. Cotton has very large, nearly flat leaves. These leaves are nearly symmetrically distributed about the central stalk of the plant as we determined by stripping 5 randomly-selected plants. The average leaf slope was 32°. The heliotropic effect was observed with time-lapse movie photography, but the magnitude of the change in leaf position was only a few degrees in the leaves in the upper part of the plant. The asymmetrical reflectance versus observer angle curves were probably due to non-Lambertian leaf reflectance as observed by Breece and Holmes on corn and soybeans. The row structure seen in the IR and not in the visible was most likely due to the fact that the IR penetration into the canopy is about 7 leaves deep and in the visible it is only about 2 leaves deep[7]. Therefore, the area between the rows which had a low L.A.I. of 2 to 5 would contrast with that in the rows of 10 or more if the light was penetrating to 7 leaves deep as in the infrared. To the human eye the canopy was nearly uniform.

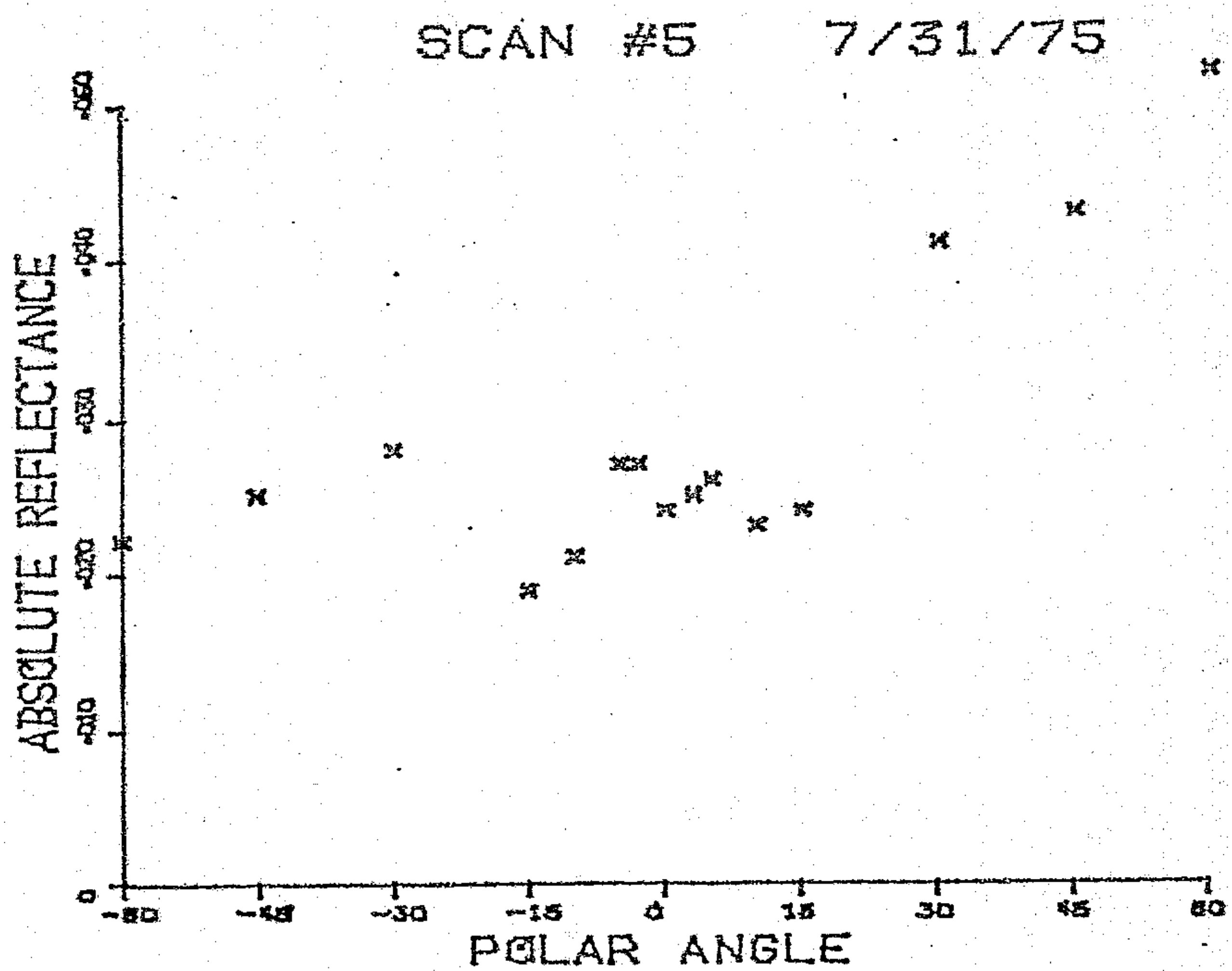


Figure 8. Cotton reflectance at an extreme sun angle of -79° at 500 nm. The scatter was not noise; data were repeatable and showed sinusoidal trends presumably from shadows between rows of plants. The positive angles shown are looking eastward toward the sun.

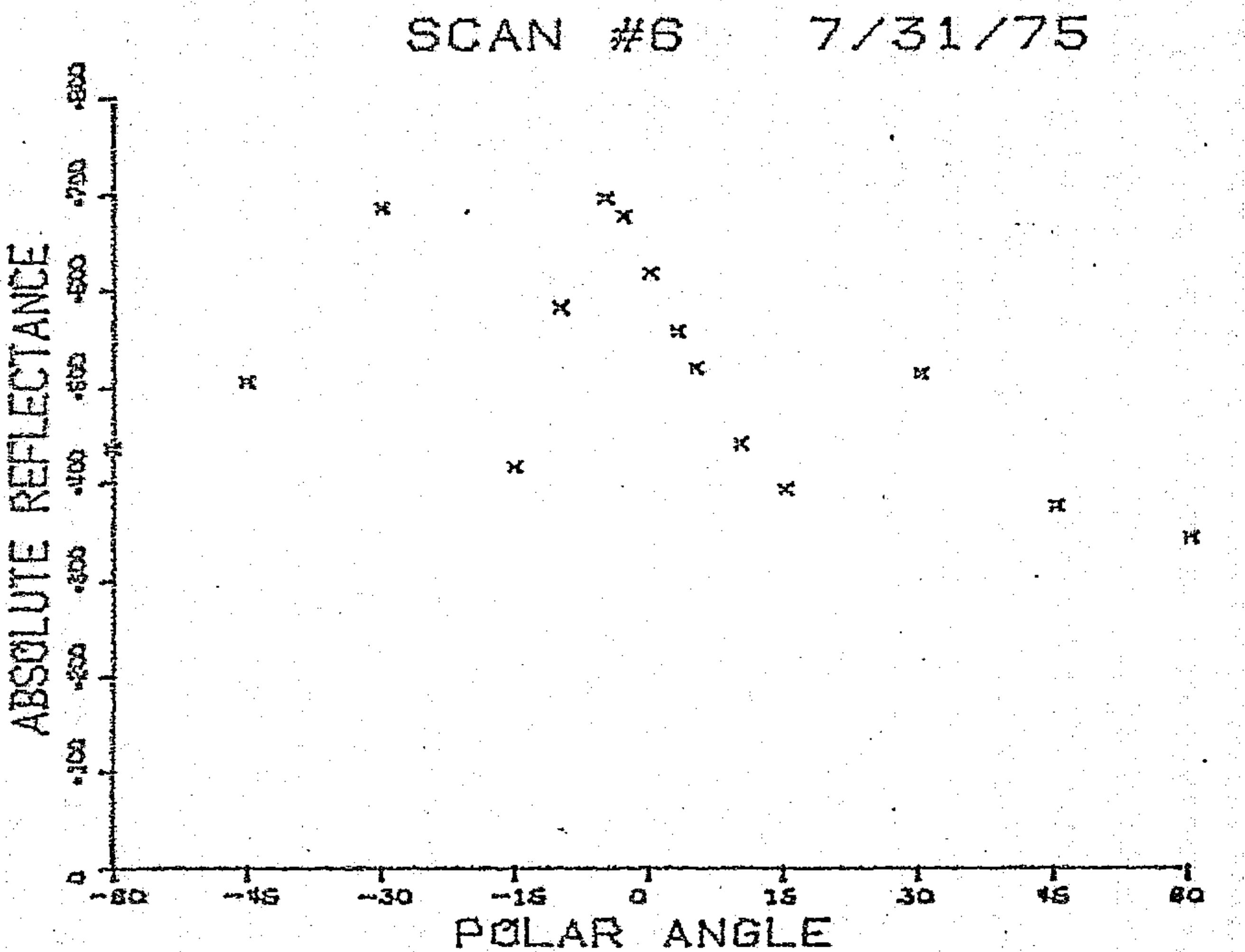


Figure 9. Cotton field reflectance at 850 nm. The spikes are row effects and were smooth, nearly sinusoidal curves in the original analog data. The sun angle was at −79° and the detector was looking eastward at positive polar angles.

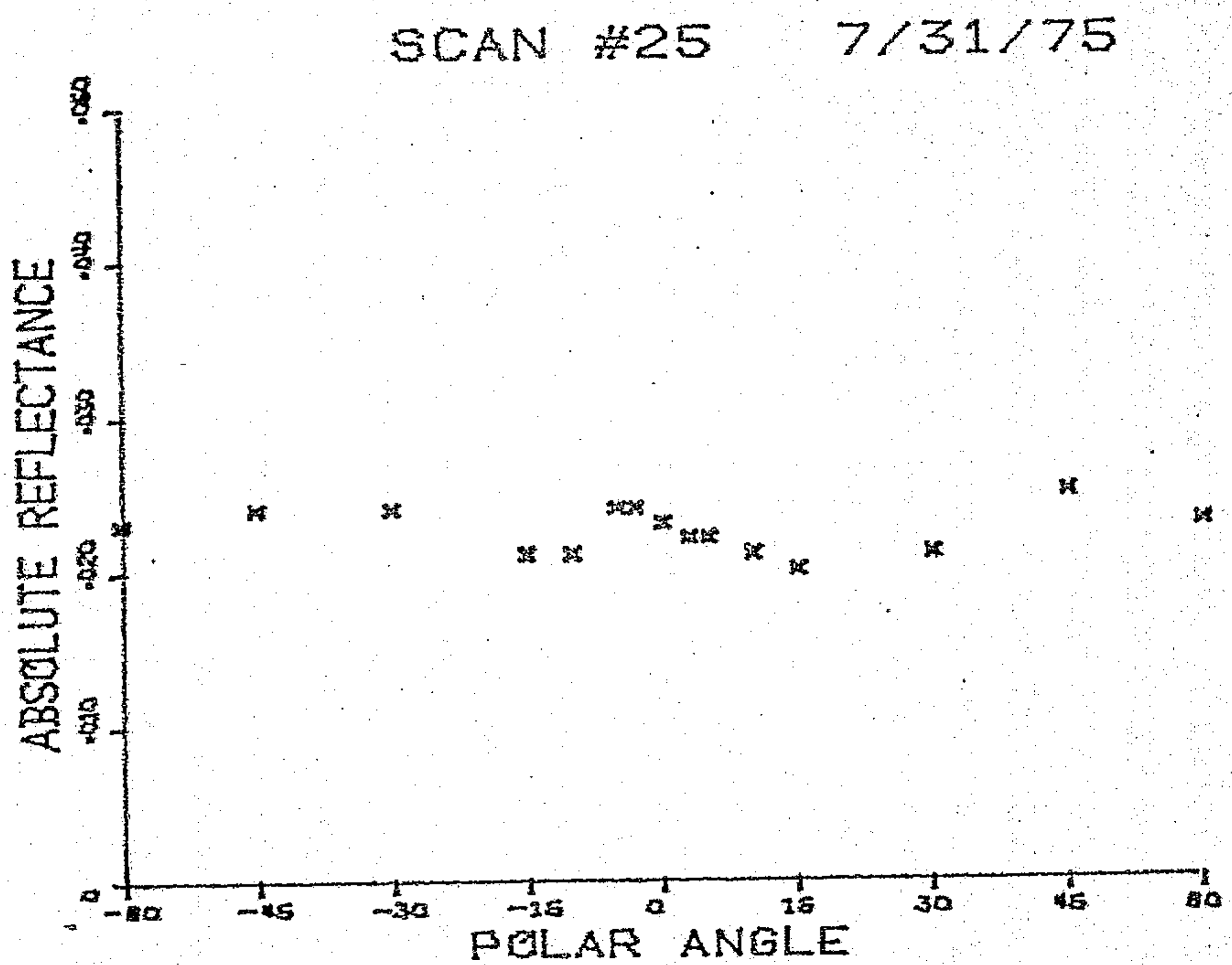


Figure 10. Cotton field reflectance at 500 nm for a sun polar angle of -67°. Observer positive angles are looking eastward.

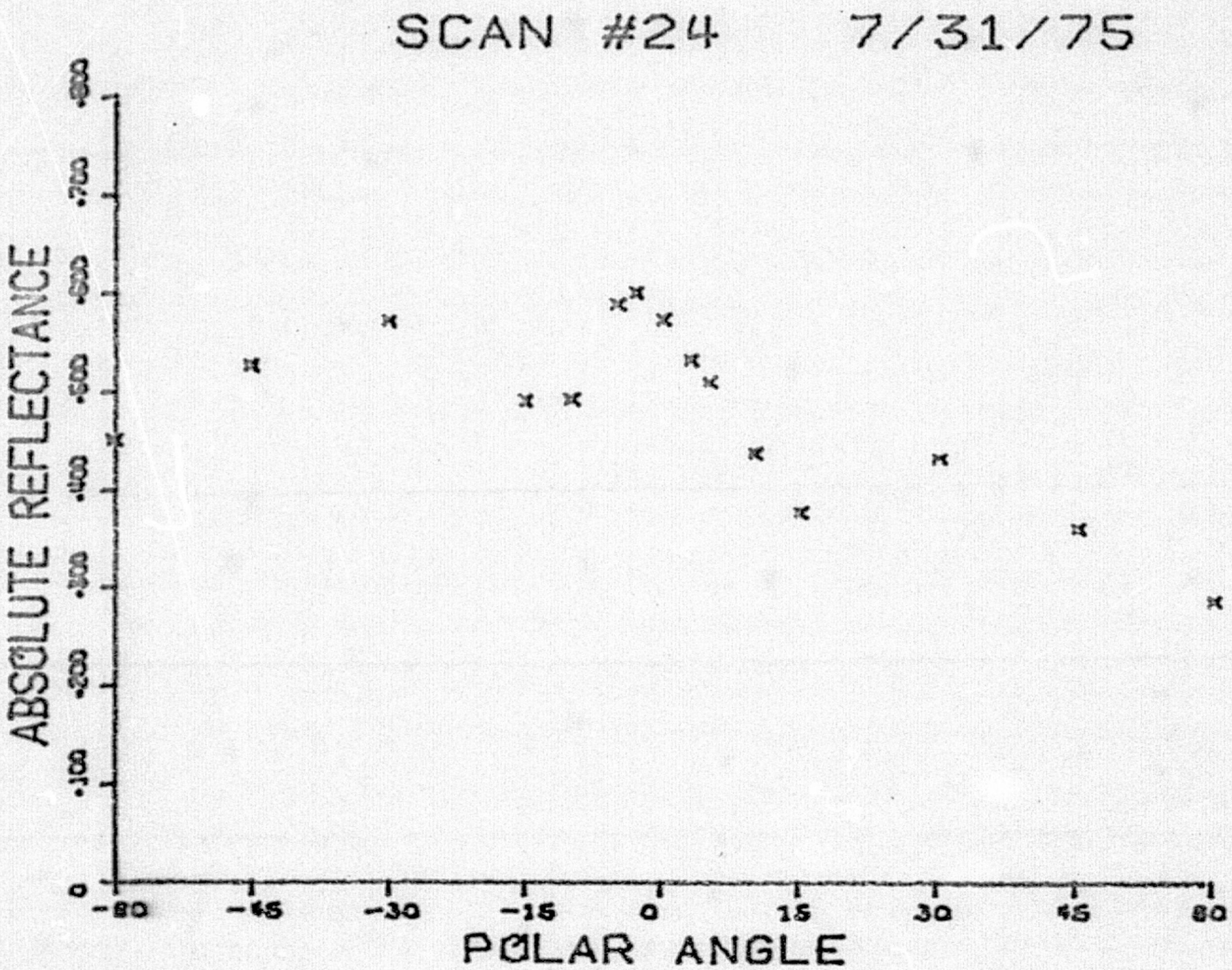


Figure 11. Cotton reflectance at 850 nm for a sun angle of -68°. Positive direction for the detector is eastward toward the sun.

(d) The qualitative agreement between the data gathered in the field and the Suits model is discussed beginning on page 39 of the report in the Appendix of this paper.

(e) The azimuthal variations in reflectance are shown for various sun angles in Figs. 12 and 13 for cotton and in Figs. 14 and 15 for wheat. The only features are row structures that show in the 850 nm data for cotton. Azimuthal positions of the detector yield different values of the measured reflectance by as much as 25% (Fig. 14), so one is forced to admit that azimuthal angles are important when modeling vegetation.

CONCLUSIONS:

1. Exchange symmetry predicted by the Suits model for interchange of source and detector is generally verified for pure specular irradiance.

2. The bidirectional reflectance function for cotton shows large variation in the IR at extreme sun angles (i.e., >50° from zenith) and moderate variation in the visible.

3. The general behavior of the bidirectional reflectance function with sun angle and observer angle predicted by the Suits model does not agree with the observed experimental data for cotton.

4. Azimuthal variation is of the order of 10% - 40° for both cotton and wheat.

5. The wheat reflectance data shows very few distinct trends. No effects of row structure are apparent. Insufficient data exist to make generalizations about trends of the reflectance with sun angle and observer angle.

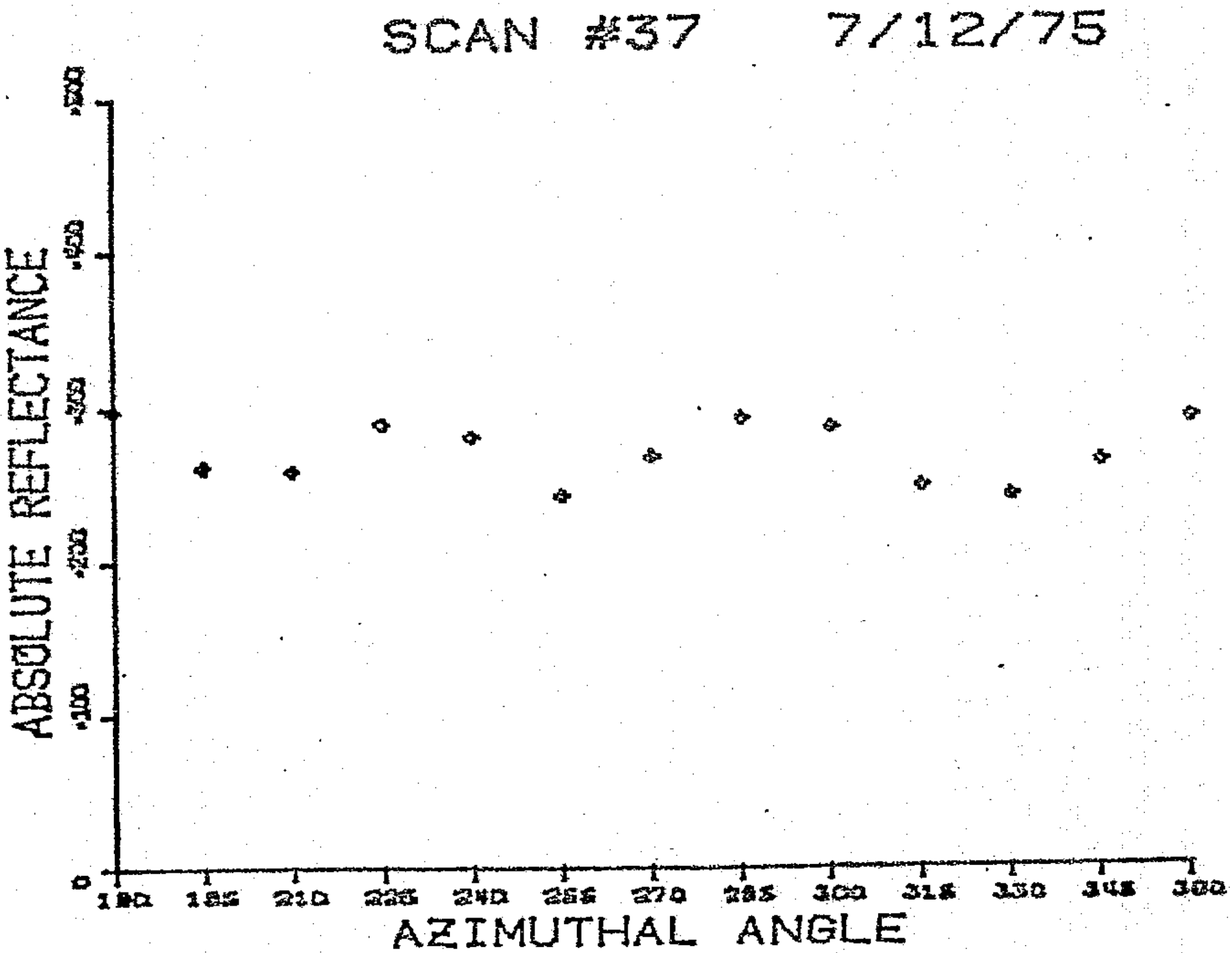


Figure 12. Cotton reflectance at 850 nm for a detector polar angle of 45° and a variable azimuth shown as the direction faced by the detector, i.e., from south through west to north. The sun was at 42° from zenith in the west.

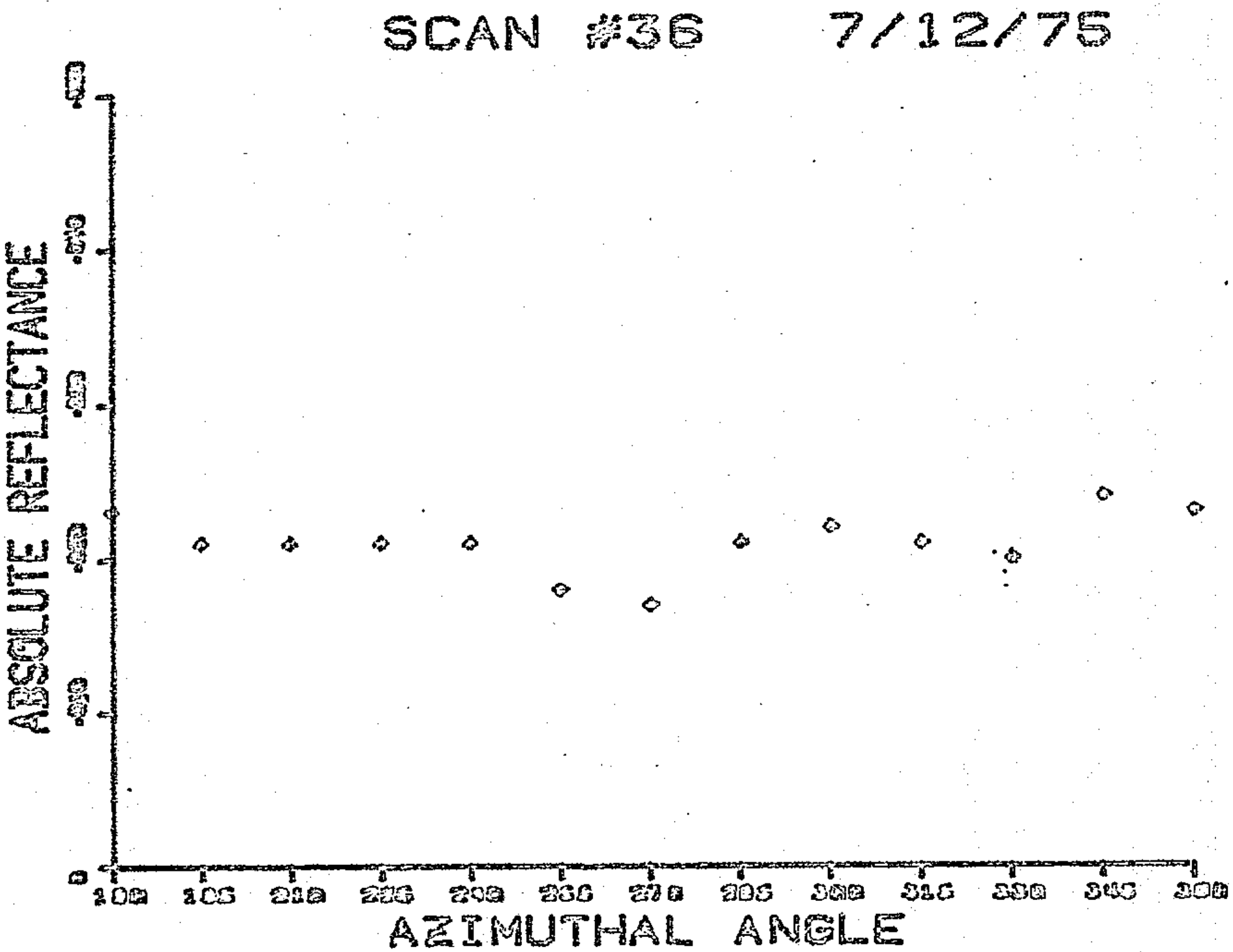


Figure 13. Cotton reflectance at 500 nm for detector polar angle of 45°, variable azimuth. The sun was at 42° from zenith in the west.

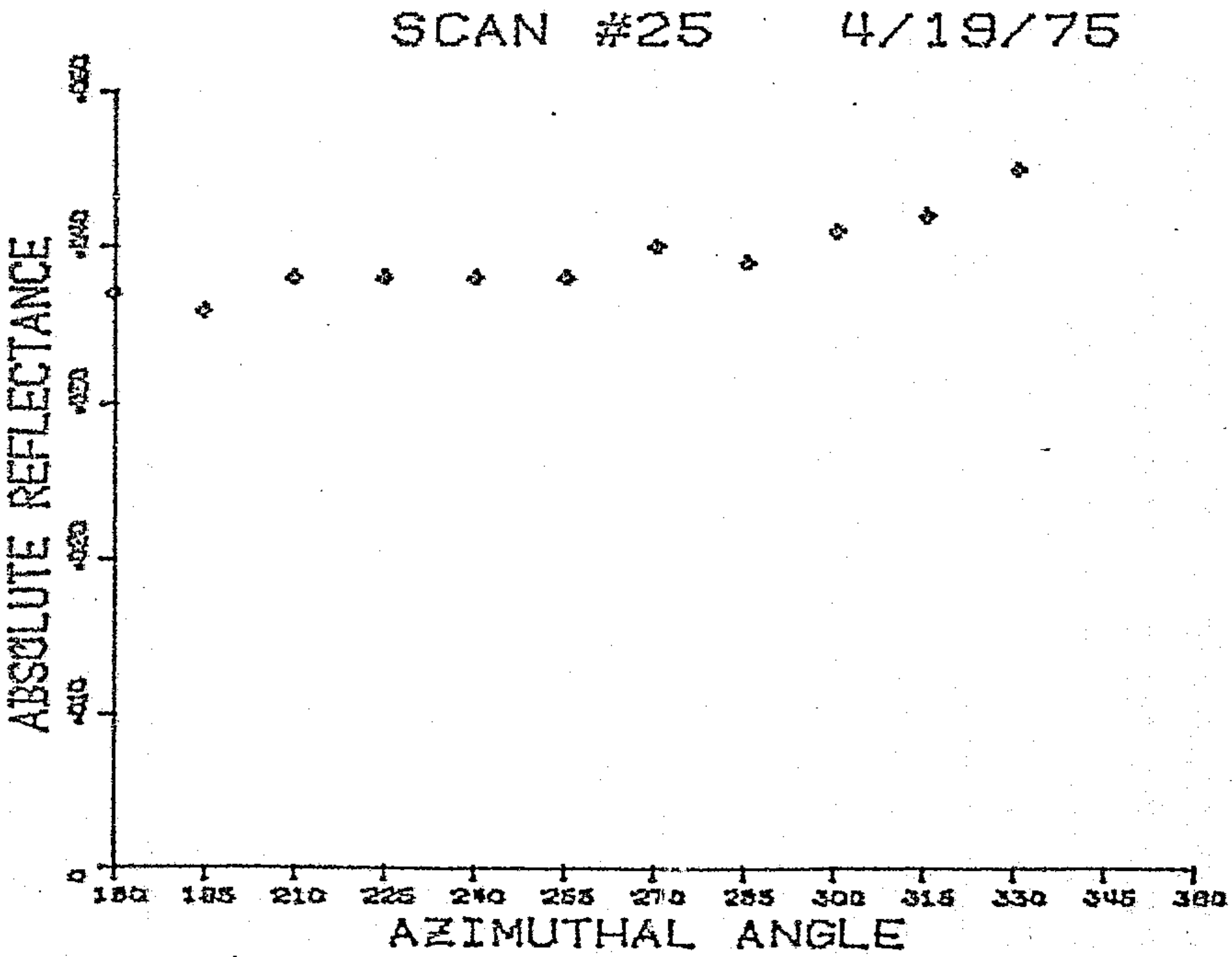


Figure 14. Wheat reflectance at 500 nm for a polar observer angle of 30°. An azimuth of 270° is with the detector pointing toward the west; the sun is 21° from zenith in the west.

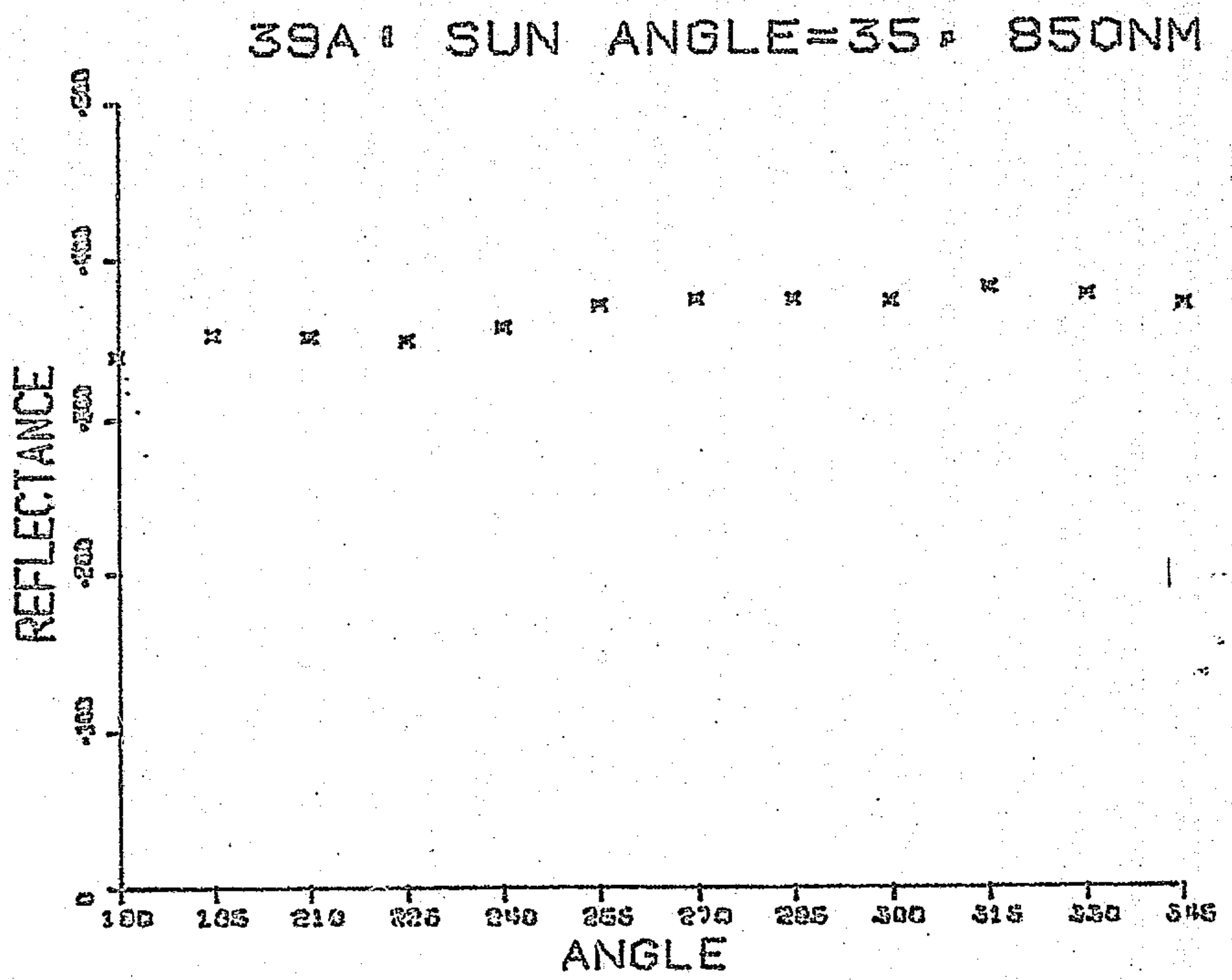


Figure 15. Wheat reflectance at 850 nm for a polar detector angle of 30°.

ACKNOWLEDGEMENTS

I wish to acknowledge the support of the National Aeronautics and Space Administration in this project and the technical help extended by the United States Department of Agriculture, Agricultural Research Service at Weslaco, Texas.

Thanks to the many students who have worked on the project - many times in physical discomfort: Doran Sprecher, Tomas Solis, James Guerra, Keith Jeske and David Egle.

Thanks to Mr. A. D. Bell for allowing us to view his cotton, and Mr. Alex Ritchie of Eagle Pass, Texas, for cooperating with our wheat data acquisition efforts.

REFERENCES

1. W. A. Malila, R. H. Hieber, and J. E. Sarno, "Analysis of Multispectral Signatures and Investigation of Multi-aspect Remote Sensing Techniques", Environmental Research Institute of Michigan, Report #NASA CR-140201, July, 1974.

2. G. H. Suits, Remote Sensing of Environment 2, 117 (1972).

   G. H. Suits, Gene Safir, and A. Ellingboe, Fifth Annual Earth Resources Program Review, pp 31.1-31.11 (1972). NASA, MSC. Houston, Texas.

3. H. T. Breece III and R. A. Holmes, Appl. Optics 10, 119 (1971).

4. M. V. Klein, Optics, John Wiley & Sons, Inc. New York (1970)

5. W. A. Allen and A. J. Richardson, Rev. Sci. Instruments 42, 1813 (1971).

6. L. Jones and H. Condit, J. Opt. Soc. Am., 38, 2 (1948).

7. See APPENDIX. J. E. Chance and J. M. Cantu, p. 30, also W. Allen and A. J. Richardson, J. Opt. Soc. Am. 58 (1968).

APPENDIX

A Study of Plant Canopy Reflectance Models

by

J. E. Chance
J. M. Cantu

This study was funded by the Graduate School of Pan American University for Summer 1975.

Foreword

For the interested reader who wishes to have exact values on Suits' Reflectance ($\pi L/E$) graphs contained in this paper, a description of each theoretical graph is included in Appendix 1. Also to be found in Appendix 1 are Suits' parameters for cotton and wheat.

Acknowledgements

The authors are indebted to Dr. E. W. LeMaster both for his helpful suggestions and criticisms and the data supplied for cotton and wheat plants necessary to calculate the parameters used in the mathematical models. The typing was done by Miss Ana Mendez, and help was supplied with the computer graphics by Mr. D. Egle.

In addition the authors wish to acknowledge the financial support supplied by the Department of Mathematics at Pan American University for publication and reproduction expenses.

TABLE OF CONTENTS

1. Introduction

Many of the problems relating to the growth of the world's population and the maintenance of basic subsistence levels for individuals within this population depend upon an accurate inventory of the world's crops. Current proposed solutions to the problem of crop inventory depend on data gathered by satellites-remote sensing. Using ground reflectance patterns at selected wavelengths as data a discrimination is to be made among the various crops. Typically, to discriminate between fields of cotton and wheat a mathematical model is employed that relates the remotely sensed reflectance patterns to their causes i.e. wheat canopies or cotton canopies. Many mathematical models are proposed by various investigators [2], [3], [5] with more or less unknown discrimination capabilities. These models are roughly divided into two categories-deterministic and statistical. The deterministic models attempt to develop a cause-effect relationship between the input parameters (sun angle, viewer angle, plant parameters, etc.) and the observed changes in the reflectance patterns, with the ability to explain observed changes in reflectance patterns. This type of model is desirable since the cause-effect relationship allows one to explain the alteration in reflectance patterns caused by crop diseases, pests, drought, and to optimize crop yields. The major weakness of this type of model is the inability to mathematically describe the subtle relationships that exists between the causes and effects. In fact this is such a serious problem that many researchers rely on statistical models using the philosophy that a crop can be identified by its reflectance properties by observing a large number of test plots of the same crop and measuring their variabilities. This type of mathematical model has shown good results in laboratory environments but fails to discriminate adequately with a moderate diurnal variation [1].

The purpose of this study is to examine a proposed family of deterministic mathematical models for vegetative canopy reflectance developed by G. H. Suits of the Environmental Research Institute of Michigan (ERIM) [2]. A solution for each of these models is given, along with a computer program for implementing these solutions. Properties of these solutions will be discussed, and the mathematical models will be compared with actual field data. Finally, suggestions will be given for improvements of these models and future areas for investigation will be discussed.

2. The Solution of the Suits' Model for Canopy Reflectance

The importance of recognizing plant canopies by using remote sensing techniques has continued to grow. Mathematical models have been developed to interpret data acquired by remote sensing devices. It is hoped that mathematical models can predict different reflectances for different crops. A study of the models should reveal the weaknesses and strengths of each model. Also, a study of the different models should expose the features which can best differentiate between different crops.

One of the models developed for identification of plant canopies was devised by Gwynn H. Suits. Suits' model marks the first attempt to account for directional reflectance as a function of view angle. It also attempts to trace changes in reflectance to spectral and geometric changes within the plant canopy. In Suits' model the plant canopy is divided into N layer which are infinitely extended. The last layer is always bounded by the soil. Each layer can have several components (leaves, stalks, flowers) which exhibit different optical and physical properties. The components are assumed to be randomly distributed and homogeneously mixed. The components in each layer are idealized as a combination of a vertical and a horizontal panel. The vertical and horizontal panels act as Lambertian surfaces; that is, they diffusely reflect and transmit the incident light. The area of the horizontal panel is obtained by projecting the area of the component on a horizontal plane. Similarly, the area of the vertical panel is obtained by projecting the area of the component on two orthogonal vertical planes.

The radiant flux that interacts with the canopy is divided into two types: specular flux which arrives directly from the sun and diffuse flux. The symbols $E_\lambda(s,i,x)$, $E_\lambda(+d,i,x)$, and $E_\lambda(-d,i,x)$ represent the specular flux, the upward directed diffuse flux, and the downward directed diffuse flux in the i^{th} layer and level x for a particular wavelength (λ). In Suits' model the level of a layer is measured downward from the top of the layer. Since downward in Suits' model is in the negative direction, then both the level and the total depth of a layer are negative numbers. To determine $E_\lambda(+d,i,x)$, $E_\lambda(-d,i,x)$, and $E_\lambda(s,i,x)$, the differential equations

$$\frac{dE_\lambda(+d,i,x)}{dx} = - a_i E_\lambda(+d,i,x) + b_i E_\lambda(-d,i,x) + c_i E_\lambda(s,i,x),$$

$$\frac{dE_\lambda(-d,i,x)}{dx} = a_i E_\lambda(-d,i,x) - b_i E_\lambda(+d,i,x) - c' E_\lambda(s,i,x),$$

and

$$\frac{dE_\lambda(s,i,x)}{dx} = k_i E_\lambda(s,i,x)$$

must be solved. The constants a_i, b_i, c_i, c_i', and k are derived from measurements of the canopy components. If only one type of component occupies the i^{th} layer, then

$$a_i = [\sigma_h n_h (1 - \tau) + \sigma_v n_v (1 - \frac{\rho + \tau}{2})],$$

$$b_i = [\sigma_h n_h \rho + \sigma_v n_v (\frac{\rho + \tau}{2})],$$

$$c_i = [\sigma_h n_h \rho + (\frac{2}{\pi}) \sigma_v n_v (\frac{\rho + \tau}{2}) \tan \theta],$$

$$c_i' = [\sigma_h n_h \tau + (\frac{2}{\pi}) \sigma_v n_v (\frac{\rho + \tau}{2}) \tan \theta],$$

and

$$k_i = [\sigma_h n_h + (\frac{2}{\pi}) \sigma_v n_v \tan \theta]$$

where σ_h is the average area of the projection of the component on a horizontal plane, σ_v is the average area of the projection of the canopy component on two orthogonal vertical planes, n_h is the number of horizontal projections per unit volume, n_v is the number of vertical projections per unit volume, θ is the polar angle for incident specular flux, ρ is the hemisperical reflectance of the component at this wavelength, and τ is the hemispherical transmittance of the component at this wavelength. If there are more than one type of components in layer i, then the values a, b, c, c', and k are obtained for each type separately and added together respectively to obtain a_i, b_i, c_i, c_i', and k_i. For example, if there are two types of components in layer i, then

$$a_i = a(\text{type 1}) + a(\text{type 2}).$$

The boundary conditions require that at the top of the first layer the only downward directed flux be specular flux. This can be stated as $E_\lambda(s,1,0) = 1$. Hence the downward directed diffuse flux is zero, or $E_\lambda(-d,1,0) = 0$. At the layer boundaries, the conditions require that the upward and downward directed flux be continuous. Finally, at the soil level, the boundary conditions require that all downward directed flux be reflected to produce upward directed diffuse flux. This last condition may be stated as

$$E_\lambda(+d,N,d_N) = \rho_s[E_\lambda(-d,N,d_N) + E_\lambda(s,N,d_N)]$$

where ρ_s is the soil reflectance, N is the last layer, and d_N is the depth of the last layer.

2.1 Solution to the Boundary-Value Problem Associated with the N Layer Model

The system of differential equations may be stated as

$\dot{E}_i(x) = N_i\, E_i(x)$ for $i = 1,2, \ldots , N$, where

$$\dot{E}_i(x) = \begin{bmatrix} \dfrac{dE_\lambda(+d,i,x)}{dx} \\ \dfrac{dE_\lambda(-d,i,x)}{dx} \\ \dfrac{dE_\lambda(s,i,x)}{dx} \end{bmatrix}, \quad N_i = \begin{bmatrix} -a_i & b_i & c_i \\ -b_i & a_i & -c_i{}' \\ 0 & 0 & k_i \end{bmatrix}, \text{ and}$$

$$E_i(x) = \begin{bmatrix} E_\lambda(+d,i,x) \\ E_\lambda(-d,i,x) \\ E_\lambda(s,i,x) \end{bmatrix}.$$ For a given x, N_i is the matrix representation

with respect to the natural basis of a linear transformation L of R^3 into R^3. If L has three distinct eigenvalues then the matrix representation of L with respect to an eigenvector basis will be a diagonal matrix. The elements of the diagonal matrix will be the eigenvalues. In this case, the eigenvalues are $g_i = (a_i^2 - b_i^2)^{1/2}$, $-g_i$, and k_i. If $\rho + \tau = 1$, then $g_i = 0$ and there are repeated eigenvalues. This is nearly the case in the infrared region which will be discussed in another section of this paper. On the other hand, if $\rho + \tau < 1$ then, with only one exception, there are three distinct eigenvalues. The exception occurs when

$\theta = \tan^{-1}\left[\frac{\pi}{2\sum_{m_i}\sigma_v n_v}(g_i - \sum_{m_i}\sigma_h n_h)\right]$ where m_i is the number of components in layer i. This angle of θ causes $g_i = k_i$.

An eigenvector associated with g_i is

$$e_{1i} = \begin{bmatrix} \frac{1}{a_i + g_i} \\ \frac{1}{\bar{b}_i} \\ 0 \end{bmatrix}.$$ An eigenvector associated with $-g_i$ is

$$e_{2i} = \begin{bmatrix} \frac{1}{a_i - g_i} \\ \frac{1}{\bar{b}_i} \\ 0 \end{bmatrix}.$$ Finally, an eigenvector associated with

$$e_{3i} = \begin{bmatrix} \xi_{1i} \\ \xi_{2i} \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{(a_i - k_i)\, c_i + c_i'\, b_i}{a_i^2 - k_i^2 - b_i^2} \\ \dfrac{(a_i + k_i)\, c_i' + c_i\, b_i}{a_i^2 - k_i^2 - b_i^2} \\ 1 \end{bmatrix}.$$ Hence the matrix

representation of L with respect to $S_i = \{e_{1i}, e_{2i}, e_{3i}\}$ is D_i where

$$D_i = \begin{bmatrix} g_i & 0 & 0 \\ 0 & -g_i & 0 \\ 0 & 0 & k_i \end{bmatrix}.$$ Now $\dot{E}_i(x) = N_i E_i(x)$ can be expressed as

$\dot{F}_i(x) = D_i F_i(x)$ where $\dot{F}_i(x) = M_i \dot{E}_i(x)$ and $F_i(x) = M_i E_i(x)$ where

$$M_i = \begin{bmatrix} \dfrac{1}{a_i + g_i} & \dfrac{1}{a_i - g_i} & \xi_{1i} \\ \dfrac{1}{b_i} & \dfrac{1}{b_i} & \xi_{2i} \\ 0 & 0 & 1 \end{bmatrix}.$$ The solution to $\dot{F}_i(x) = D_i F_i(x)$ is

$$F_i(x) = \begin{bmatrix} A_i\, e^{g_i x} \\ B_i\, e^{-g_i x} \\ C_i\, e^{k_i x} \end{bmatrix}$$ where A_i, B_i, and C_i are constants of integration.

Thus

$$E_i(x) = M_i F_i(x) \text{ or}$$

$$E_i(x) = A_i \, e^{g_i x} e_{1i} + B_i \, e^{g_i x} e_{2i} + C_i \, e^{k_i x} e_{3i}.$$

The constants of integration A_i, B_i, and C_i must now be determined such that they satisfy the boundary conditions.

At this point, a useful notation will be adopted to represent the boundary condition at the soil level.

Let $f_\rho \begin{bmatrix} a \\ b \\ c \end{bmatrix} = a - \rho(b + c)$. Thus $f_\rho \begin{bmatrix} E_\lambda(+d,N,d_N) \\ E_\lambda(-d,N,d_N) \\ E_\lambda(s,N,d_N) \end{bmatrix} = 0$

will satisfy the last boundary condition.

To determine A_i, B_i, and C_i, what will be done is to trade a boundary condition for an initial condition. Thus the boundary-value problem will reduce to an initial-value problem.

Let A_o represent the upward directed diffuse flux at the top of the first layer. Then

$$E_1(0) = \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix} = M_1 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}. \text{ Since}$$

$\det M_i = \left(\frac{1}{a_i + g_i} - \frac{1}{a_i - g_i}\right) \frac{1}{b_i} \neq 0$, then M_1 is nonsingular.

Thus

$$\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = M_1^{-1} \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix}.$$ The boundary conditions require that

$E_1(0) = E_1(d_1)$ where d_1 is the depth of the first layer.

Thus

$$M_2 \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = \overline{M}_1 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = \overline{M}_1 \, M_1^{-1} \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix}$$

where

$$\overline{M}_1 \begin{bmatrix} e^{g_1 d_1} \dfrac{1}{a_1 + g_1} & e^{-g_1 d_1} \dfrac{1}{a_1 - g_1} & e^{k_1 d_1} \xi_{11} \\ e^{g_1 d_1} \quad \dfrac{1}{\overline{b}_1} & e^{-g_1 d_1} \quad \dfrac{1}{\overline{b}_1} & e^{k_1 d_1} \xi_{21} \\ 0 & 0 & e^{k_1 d_1} \end{bmatrix}.$$

Again since M_2 is nonsingular

$$\begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = M_2^{-1} \, \overline{M}_1 \, M_1^{-1} \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix}.$$

In general $$\begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} = M_i^{-1} \, \overline{M}_{i-1} \, M_{i-1}^{-1} \cdots \overline{M}_1 \, M_1^{-1} \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix} \qquad (1).$$

At the soil boundary $E_N(d_N) = \overline{M}_N \begin{bmatrix} A_N \\ B_N \\ C_N \end{bmatrix} = \overline{M}_N M_N^{-1} \ldots M_1 M_1^{-1} \begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix}$.

Letting cl_1, cl_2, and cl_3 represent the columns of the 3 x 3 matrix $\overline{M}_N M_N^{-1} \ldots \overline{M}_1 M_1^{-1}$, then $E_N(d_N) = A_o\, cl_1 + cl_3$. The boundary condition requires that $f\rho(E_N(d_N)) = 0$.

Thus

$f\rho(E_N(d_N)) = f\rho(A_o\, cl_1 + cl_3) = A_o\, f\rho(cl_1) + f\rho(cl_3) = 0$, or

$A_o = -\dfrac{f\rho(cl_3)}{f\rho(cl_1)}$. It can be shown that for the one layer case $f\rho(cl_1) \neq 0$. An attempt was made to show $0 < A_o < 1$ for the one layer case, but the attempt proved unsuccessful. Beyond the one layer case, it comes difficult to show that $f\rho(cl_1) \neq 0$. Thus if $f\rho(cl_1) = 0$, then this problem has no solution. However, if $f\rho(cl_1) \neq 0$, then the constants of integration can be obtained from (1).

2.2 Solution to the Infinite Case for Suits' Model

Another case yet to be considered is one in which, due to dense foliage or due to the depth of the canopy, the soil reflectance is negligible. In this case the boundary conditions become

$E_\lambda(-d,1,0) = 0$,

$E_\lambda(s,1,0) = 1$, and

$E_\lambda(+d,1,x \to -\infty) = 0$.

From the previous section, the solution to the system of differential equations is $E_1(x) = A_1\ e^{g_i x}\ e_1 + B_1\ e^{g_i x}\ e_2 + C_1\ e^{k_i x}\ e_3$.

Again, let A_o be the upward directed flux at the top of the single layer. Thus the initial condition for the infinite one-layered case becomes

$$\begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a_1 + g_1} & \frac{1}{a_1 - g_1} & \xi_1 \\ \frac{1}{b_1} & \frac{1}{b_1} & \xi_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}. \text{ Hence}$$

$A_1 \frac{1}{b_1} + B_1 \frac{1}{b_1} + C_1\ \xi_2 = 0$ and $C_1 = 1$. The boundary condition requires that $f\rho(E_1(d_1)) = 0$. Thus $A_1\ e^{g_1 d_1} f\rho(e_1) + B_1\ e^{-g_1 d_1} f\rho(e_2) + C_1\ e^{k_1 d_1} f\rho(e_3) = 0$, or $f\rho(E_1(d_1)) = A_1\ e^{2g_1 d_1} f\rho(e_1) + B_1\ f\rho(e_2) + C_1\ e^{(g_1+k_1)d_1} f\rho(e_3) = 0$. The depth of the single layer (d_1) becomes large. Hence

$$\lim_{d_1 \to -\infty} [A_1\ e^{2g_1 d_1} f\rho(e_1) + B_1\ f\rho(e_2) + C_1\ e^{(g_1+k_1)d_1} f\rho(e_3)] = B_1\ f\rho(e_2) = 0.$$

For $0 < \rho < 1$, $f\rho(e_2) \neq 0$. Thus $B_1 = 0$. With $B_1 = 0$ and $C_1 = 1$, then using $A_1 \frac{1}{b_1} + B_1 \frac{1}{b_1} + C_1\ \xi_2 = 0$,

$$A_1 = \frac{(a_1 + k_1)\, b_1\ c_1' + b_1^2\ c_1}{b_1^2 + k_1^2 - a_1^2}.$$

2.3 Solution to the Combination Case for Suits' Model

The combination case was intended to describe the reflectance of a canopy primarily as initial surface reflectance from the top layer of the canopy. It is thought that the total reflectance from some crops such as wheat originate mainly from light interaction with the top of the canopy crop (the heads of the wheat). The combination case consists of two layers, the first having a finite depth and the second having an infinite depth. The flux densities of the first layer are described by

$E_1(x) = A_1\, e^{g_1 x}\, e_{11} + B_1\, e^{-g_1 x}\, e_{21} + C_1\, e^{k_1 x}\, e_{31}$ where A_1, B_1, and C_1 are to be determined. Similarly, the second layer of the combination case is described by

$$E_2(x) = A_2\, e^{g_2 x}\, e_{12} + B_2\, e^{-g_2 x}\, e_{22} + C_2\, e^{k_2 x}\, e_{32}.$$

The initial condition for the top layer is

$$\begin{bmatrix} A_0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{a_1 + g_1} & \frac{1}{a_1 - g_1} & \xi_{11} \\ \frac{1}{b_1} & \frac{1}{b_1} & \xi_{21} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}. \quad \text{Thus } C_1 = 1.$$

Also $A_1 \frac{1}{b_1} + B_1 \frac{1}{b_1} + C_1\, \xi_{21} = 0$ which implies that $B_1 = -(A_1 + \xi_{21}\, b_1)$.

In the second layer, the depth (d_2) becomes large thus

$$\lim_{d_2 \to -\infty} A_2\, e^{2 g_2 d_2} f\rho(e_{12}) + B_2\, f\rho(e_{22}) + C_2\, e^{(g_2 + k_2) d_2} f\rho(e_{22}) = B_2\, f\rho(e_{22}) = 0.$$

This implies that $B_2 = 0$ since $f\rho(e_{22}) \neq 0$ for $0 < \rho < 1$. The boundary conditions require that $E_2(0) = E_1(d_1)$, or

$$M_2 \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = \overline{M}_1 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} \text{ where } M_2 = \begin{bmatrix} \frac{1}{a_2 + g_2} & \frac{1}{a_2 - g_2} & \xi_{12} \\ \frac{1}{b_2} & \frac{1}{b_2} & \xi_{22} \\ 0 & 0 & 1 \end{bmatrix} \text{ and}$$

$$\overline{M}_1 = \begin{bmatrix} e^{g_1 d_1} \frac{1}{a_1 + g_1} & e^{-g_1 d_1} \frac{1}{a_1 - g_1} & e^{k_1 d_1} \xi_{11} \\ e^{g_1 d_1} \frac{1}{b_1} & e^{-g_1 d_1} \frac{1}{b_1} & e^{k_1 d_1} \xi_{21} \\ 0 & 0 & e^{k_1 d_1} \end{bmatrix}.$$

Again, M_2 is nonsingular since $\det M_2 = \left(\frac{1}{a_2 + g_2} \quad \frac{1}{a_2 - g_2}\right) \frac{1}{b_2} \neq 0.$

$$\text{Hence} \begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} = M_2^{-1} \overline{M}_1 \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} = M_2^{-1} \overline{M}_1 \begin{bmatrix} A_1 \\ -(A_1 + \xi_{21} b_1) \\ 1 \end{bmatrix}. \quad (2)$$

Let the elements of the second row of $M_2^{-1} M_1$ be h, i, and j. Then $A_1 h - (A_1 + \xi_{21} b_1)\, i + j = 0$, and

$A_1 = \dfrac{\xi_{21} b_1 i - j}{h - i}$. If h = i, then there is no solution to this problem. If $h \neq i$, then A_1, B_1, and C_1 are known and A_2, B_2, and C_2 are obtained from (2).

Once the constants of integration have been determined for each of the three cases, the radiance due to an infinitesimal strip in the i^{th} layer can be calculated from Suits' equation

$$\Delta L_\lambda = [u_i \frac{E_\lambda(+d,i,x)}{\pi} + v_i \frac{E_\lambda(-d,i,x)}{\pi} + w_i \frac{E_\lambda(s,i,x)}{\pi}] \quad \Delta x$$

where $u_i = \sum_{m_i} \sigma_h n_h \tau + \sigma_v n_v \frac{\tau + \rho}{2} (\frac{2}{\pi}) \tan \emptyset$

$$v_i = \sum_{m_i} \sigma_h n_h \rho + \sigma_v n_v \frac{\tau + \rho}{2} (\frac{2}{\pi}) \tan \emptyset$$

$$w_i = \sum_{m_i} \sigma_h n_h \rho + \sigma_v n_v \frac{\tau + \rho}{2} (\frac{2}{\pi})^2 \tan \emptyset \tan \theta$$

where $\emptyset$ is the polar viewer angle and m_i is the number of components in layer i. If i > 1, the fraction of radiance from this infinitesmal strip of layer i that is seen from the outside is

$\exp(\sum_{j=1}^{i-1} f_i d_j) \exp(f_i x)$ where $f_j = \sum_{m_j} \sigma_h n_h + (\frac{2}{\pi}) \sigma_v n_v \tan \emptyset$ and d_j is the depth of layer j. The radiance as seen from outside the plant canopy becomes

$\Delta L_\lambda(\text{outside}) = \exp(\sum_{j=1}^{i-1} f_j d_j) \exp(f_i x) \Delta L_\lambda$. If i = 1, the fraction simply becomes $\exp(f_i x)$ and

$\Delta L_\lambda(\text{outside}) = \exp(f_i x) \Delta L_\lambda$.

The radiance of the entire canopy as seen from the outside becomes the integral of all such contributions plus the contributions of the soil.

As an illustration, take the radiance of the infinite case. Now

$$E_\lambda(+d,1,x) = \frac{1}{a_1 + g_1} A_1 e^{g_1 x} + \xi_1 e^{k_1 x},$$

$$E_\lambda(-d,1,x) = \frac{1}{b_1} A_1 e^{g_1 x} + \xi_2 e^{k_1 x}, \text{ and}$$

$$E_\lambda(s,1,x) = e^{k_1 x}. \text{ Thus}$$

$$L_\lambda = \int_{-\infty}^{0} e^{f_1 x} [u_1 \frac{E_\lambda(+d,1,x)}{\pi} + v_1 \frac{E_\lambda(-d,1,x)}{\pi} + w_1 \frac{E_\lambda(s,1,x)}{\pi}] dx$$

$$= \frac{u_1}{\pi} \int_{-\infty}^{0} e^{f_1 x} [\frac{1}{a_1 + g_1} A \; e^{g_1 x} + \quad e^{k_1 x}] dx$$

$$+ \frac{v_1}{\pi} \int_{-\infty}^{0} e^{f_1 x} [\frac{1}{b_1} A_1 e^{g_1 x} + \xi_2 e^{k_1 x}] dx$$

$$+ \frac{w_1}{\pi} \int_{-\infty}^{0} e^{f_1 x} e^{k_1 x} dx$$

$$= \frac{u_1}{\pi} [A_1 \frac{1}{a_1 + g_1} \frac{1}{f_1 + g_1} + \xi_1 \frac{1}{f_1 + k_1}]$$

$$+ \frac{v_1}{\pi} [A_1 \frac{1}{b_1} \frac{1}{f_1 + g_1} + \xi_2 \frac{1}{f_1 + k_1}]$$

$$+ \frac{w_1}{\pi} [\frac{1}{f_1 + k_1}].$$

The reflectance $(\frac{\pi L}{E})$ from the plant canopy then becomes

$$\frac{\pi L}{E} = u_1 [A_1 \frac{1}{a_1 + g_1} \frac{1}{f_1 + g_1} + \xi_1 \frac{1}{f_1 + k_1}]$$

$$+ v_1 [A_1 \frac{1}{b_1} \frac{1}{f_1 + g_1} + \xi_2 \frac{1}{f_1 + k_1}]$$

$$+ w_1 [\frac{1}{f_1 + k_1}].$$

Note that in the infinite case and combination case there is no contribution to the reflectance by the soil. In the N layer case, the contribution to the reflectance by the soil can be expressed by

$$R(\text{soil}) = \exp[f_1 d_1 + f_2 d_2 + \ldots + f_N d_N]\, E_\lambda(+d,N,d_N).$$

2.4 Solution to the N layer Case for Suits' Model with Diffuse Light

In the three cases discussed above, it was assumed that the only downward directed flux at the top of the first layer was specular flux. In cloudy days, however, a significant part of the light falling on the top of the first layer is downward directed diffuse flux. Also, the amount of downward directed diffuse flux at the top of the first layer in the visible region of the spectrum is a function of the time of day (see Figure 1) as was shown by Jones and Condit [11]. Letting B_o be the amount of downward diffuse flux and C_o be the amount of specular flux at the top of the first layer, then the initial conditions become

$$E_\lambda(+d,1,o) = A_o$$
$$E_\lambda(-d,1,o) = B_o$$
$$E_\lambda(s,1,o) = C_o$$

where $B_o + C_o = 1$.

The following discussion shows how the N layer case is affected by changing the initial conditions from

$$\begin{bmatrix} A_o \\ o \\ 1 \end{bmatrix} \text{ to } \begin{bmatrix} A_o \\ B_o \\ C_o \end{bmatrix}$$ where B_o and C_o are known values.

A change in initial conditions will only change the constants of integration A_i, B_i, and C_i. Previously, the strategy used to solve for the

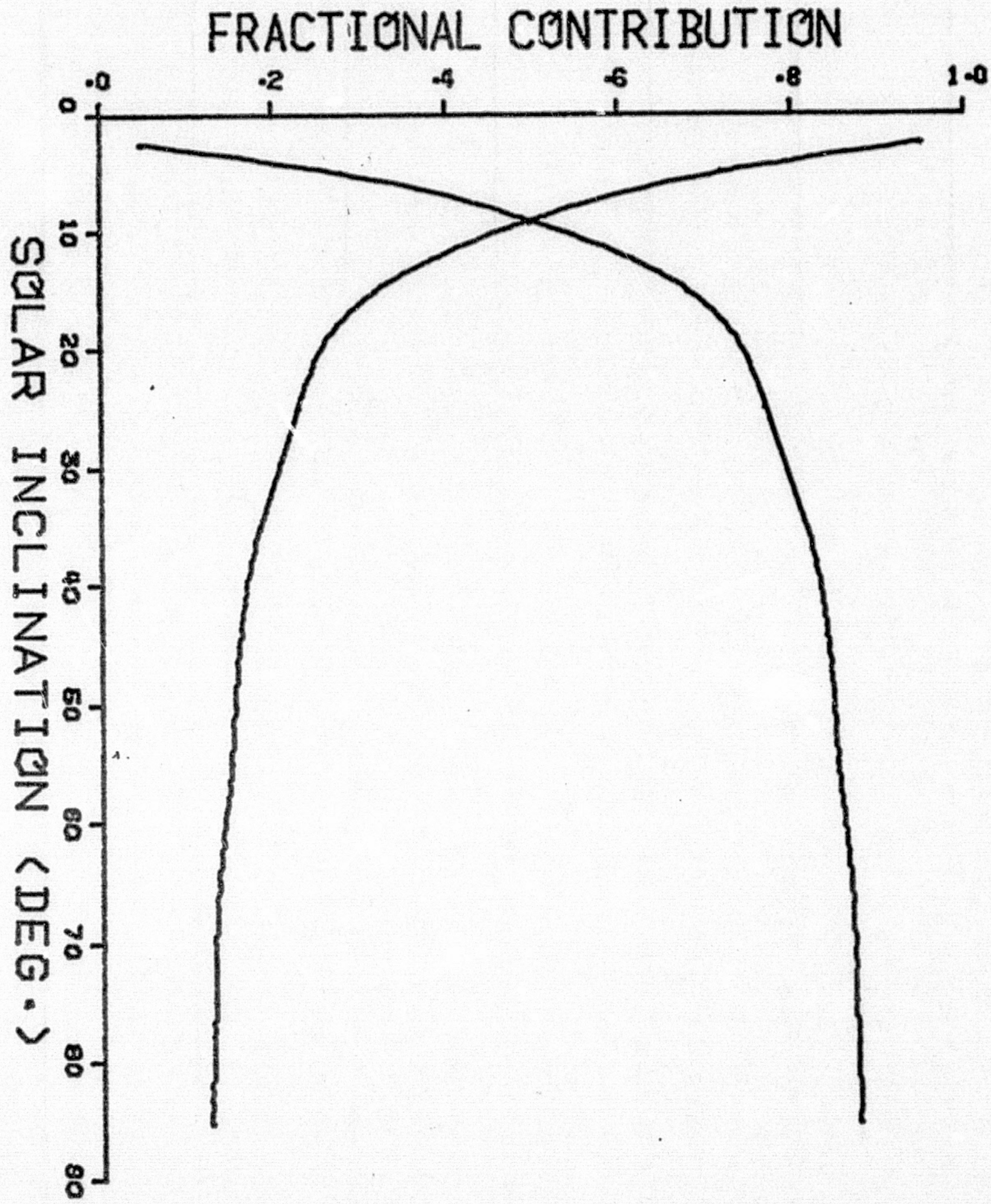


Figure 1

Direct Solar and Diffuse Skylight Fractions of Global Irradiation. These curves based upon data from Jones and Condit.

constants A_i, B_i, and C_i was to solve for A_o first. It was found that

$A_o = -\dfrac{f\rho(Cl_3)}{f\rho(Cl_1)}$. With A_o calculated, the constants A_i, B_i, and C_i

were obtained from

$$\begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} = M_i^{-1}\, \overline{M}_{i-1}\, M_{i-1} \ldots \overline{M}_1\, M_1^{-1} \begin{bmatrix} A_o \\ o \\ 1 \end{bmatrix} .$$

With a change in the initial conditions, the value of A_o changes. The value of A_o now becomes

$A_o = -\dfrac{B_o\, f\rho(Cl_2) - C_o\, f\rho(Cl_3)}{f\rho(Cl_1)}$. Again with A_o calculated, the constants

A_i, B_i, and C_i are now obtained from

$$\begin{bmatrix} A_i \\ B_i \\ C_i \end{bmatrix} = M^{-1}\, \overline{M}_{i-1}\, M_{i-1}^{-1} \ldots \overline{M}_1\, M_1^{-1} \begin{bmatrix} A_o \\ B_o \\ C_o \end{bmatrix} .$$

The following changes in NTH2 (a computer program designed to yield the radiance from Suits' N layer model) were made to take into account changes in the downward directed flux at the top of the first layer.

Lines

```
1304 Print "Enter fractional contribution of both the specular flux"
1305 Print "and Diffuse flux to the total downward directed flux.";
1306 Input L3, L2
```

were inserted. Lines 1920 - 2000 were changed to

```
1920 A2 = -1 * L2 * (G(1,2) -1 * S1 * (G(2,2) + G(3,2)))
1925 A3 = -1 * L3 * (G(1,3) -1 * S1 * (G(2,3) + G(3,3)))
1930 A4 = G(1,1) -1 * S1 * (G(2,1) + G(3,1))
1940 IF A4 = 0  THEN 2480
1950 AØ = (A2 + A3)/A4
1960 R5(1) = 0
1970 R6(0) = 0
1980 I(1,1) = AØ
1990 I(2,1) = L2
2000 I(3,1) = L3.
```

Similar changes were made on Inf 2 and Com 2 but are not included here since Inf 2 and Com 2 are special cases of NTH 2 (Inf 2 is the one layer NTH 2 with a large depth). The resulting programs with these changes were called NTH 3, Inf 3, and Com 3.

2.5 The Repeated Roots Case for Suits' Models

For incident light in the wave lengths from 760 to 1250 n.m. the single leaf absorption for a plant is small [4]. (See Figure 2) It can be shown that for small absorption, the Suits' differential equation coefficients a_i and b_i are almost equal. If one assumes that $a_i = b_i$ the eigenvalues of the matrix N_i as discussed in 2.1 are repeated, thereby yielding a different algebraic form for the solution to Suits' Model. Since this solution is relevant to the work done by Allen, Gale, and Richardson, (AGR Model) it is included to show the relationship between their vegetative reflectance canopy model and the Suits' Model.

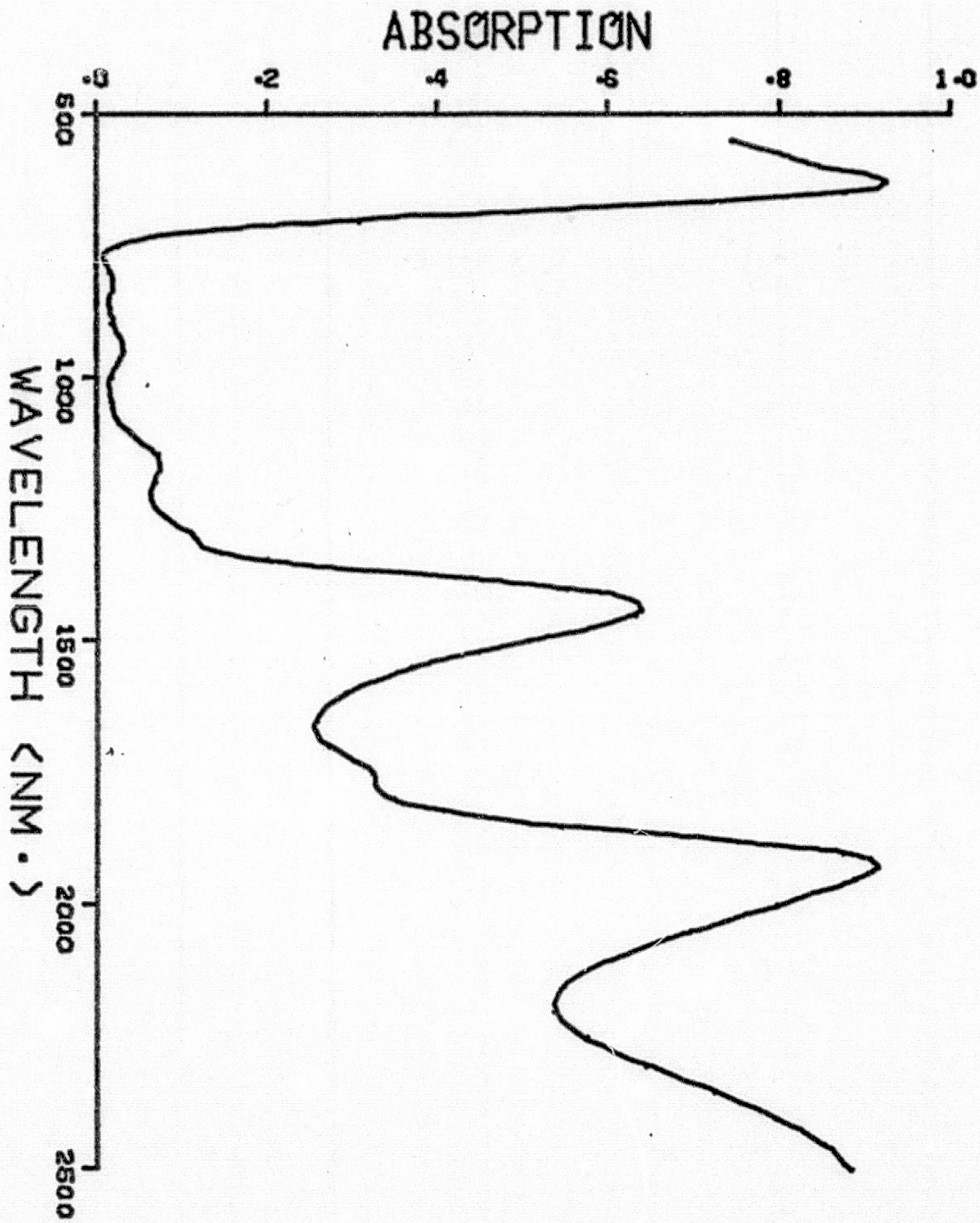


Figure 2

The Light Absorption for a Single Leaf of Cotton as a Function of the Wavelength.

The matrix $N_i = \begin{bmatrix} -a_i & b_i & c_i \\ -b_i & a_i & -c_i' \\ 0 & 0 & k_i \end{bmatrix}$ has characteristic polynomial $c(\lambda) = \lambda^2(k_i-\lambda)$ giving eigenvalues 0, 0, and k_i. An eigenvector corresponding to the eigenvalue k_i is

$$e_3 = \begin{bmatrix} \xi_1 \\ \xi_2 \\ 1 \end{bmatrix} \quad \text{where} \quad \xi_1 = -\frac{c_i(a_i - k_i) - b_i\, c_i'}{k_i^2} \tag{1}$$

$$\xi_2 = -\frac{b_i\, c_i + c_i'(a_i + k_i)}{k_i^2} \tag{2}$$

To find eigenvectors (if they exist) that form a basis for the subspace $N_i^2\, x = 0$ of three-space we must find non-zero vectors e_1 and e_2 such that

$N_i\, e_1 = 0$, $N_i\, e_2 = e_1$. Let

$$e_1 = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \text{ then}$$

$$-\, a_i\, \alpha_1 + b_i\, \alpha_2 + c_i\, \alpha_3 = 0$$

$$-\, b_i\, \alpha_1 + a_i\, \alpha_2 - c_i\, \alpha_3 = 0$$

$$k_i\, \alpha_3 = 0$$

α_3 must be zero, and a solution is $\alpha_1 = 1$, $\alpha_2 = 1$ so that

$$e_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

If $e_2 = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$ then the condition that

$N_i\ e_2 = e_1$ forces

$$- a_i\ \beta_1 + b_i\ \beta_2 + c_i\ \beta_3 = a_i$$

$$- b_i\ \beta_1 + a_i\ \beta_2 - c_i'\beta_3 = a_i$$

$$k_i\ \beta_3 = 0$$

A solution is $\beta_3 = 0$, $\beta_2 = 0$, $\beta_1 = -1$, so that

$$e_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.$$

The set $S = \{e_1, e_2, e_3\}$ is a basis for three-space. Denote the representation of N_i with respect to S as M_s. Since

$$M_s\ e_1 = 0,\ M_s\ e_2 = e_1,\ M_s\ e_3 = k_i\ e_3$$

$$M_s = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k_i \end{bmatrix}.$$ If we let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ the DE system

becomes

$$\dot{x}_1 = x_2,\ \dot{x}_2 = 0,\ \dot{x}_3 = k_i\ x_3,$$

yielding

$$x_3 = C_1\ e^{k_i x}\quad x_2 = B_1,\ x_1 = B_1 x + A_1$$

where A_1, B_1, C_1 are constants of integration.

The general solution to the original system is thus

$$E = x_1\, e_1 + x_2\, e_2 + x_3\, e_3$$

$$= (A_1 + B_1 x)\, e_1 + B_1\, e_2 + C_1\, e^{k_i x}\, e_3$$

so that

$$E(+d,i,x) = (A_1 - B_1) + B_1 x + C_1\, \xi_1\, e^{k_i x}$$

$$E(-d,i,x) = A_1 + B_1 x + C_1\, \xi_2\, e^{k_i x}$$

$$E(s,i,x) = C_1\, e^{k_i x}.$$

For the one layer case with the initial conditions $\begin{bmatrix} A_o \\ 0 \\ 1 \end{bmatrix}$ it is seen

that $C_1 = 1$, $A_1 = -\xi_2$, $B_1 = \xi_1 - \xi_2 - A_o$. Since $f\rho(e_1) = 1 - \rho$,

$f\rho(e_2) = -1$, $f\rho(e_3) = \xi_1 - \rho(\xi_2 + 1)$ then the boundary condition gives at $x = -d$ gives $0 = -\xi_2 + [\xi_1 - \xi_2 - A_o](-d)(1-\rho) + [\xi_1 - \xi_2 - A_o]\,(-1)$

$$+ e^{-k_i d}\, [\xi_1 - \rho(\xi_2 + 1)], \text{ or}$$

$$A_o = \frac{(\xi_1 - \xi_2)d(1 - \rho) + \xi_1 - e^{-k_i d}\,[\xi_1 - \rho(\xi_2 + 1)]}{1 + d(1 - \rho)}$$

yielding a solution to the one layer finite case. For the one layer infinite case, let $d \to -\infty$, which gives $A_o \to \xi_1 - \xi_2$ so that $C_1 = 1$, $B_1 = 0$, $A_1 = -\xi_2$.

The infinite solution is

$$E(+d,1,x) = -\xi_2 + \xi_1 \, e^{k_i x}$$

$$E(-d,1,x) = -\xi_2 + \xi_2 \, e^{k_i x}$$

$$E(s,1,x) = e^{k_i x}$$

The N-layer repeated problem is solved in a manner analogous to the N-layer non-repeated case discussed in 2.1.

Denote

$$M_i = \begin{bmatrix} 1 & -1 & \xi_1 \\ 1 & 0 & \xi_2 \\ 0 & 0 & 1 \end{bmatrix} \quad \overline{M}_i = \begin{bmatrix} 1 & -(1+d_i) & \xi_{1i} \; e^{-k_i d_i} \\ 1 & -d_i & \xi_{2i} \; e^{-k_i d_i} \\ 0 & 0 & e^{-k_i d_i} \end{bmatrix}$$

with ξ_{1i} and ξ_{2i} the eigenvector components using the parameters of layer i in equations (1) and (2). The unknown initial condition A_o solves the equation

$$A_o f\rho(Cl_1) + f\rho(Cl_3) = 0 \quad \text{where } [Cl_1, Cl_2, Cl_3] =$$

$M = (\overline{M}_N M_N^{-1}) \ldots (\overline{M}_1 M_1^{-1})$, ($Cl_1$, Cl_2, and Cl_3 are the three columns of M respectively).

If $f\rho(Cl_1) \neq 0$, the system has a solution with

$$A_o = -\frac{f\rho(Cl_3)}{f\rho(Cl_1)} .$$

To find a relationship between the Suits and AGR Models one uses the one layer Suits Model with repeated roots. Let $a_1 = b_1$, (which is equivalent to assuming that $\mu = 0$ for the AGR parameter for single leaf absorption). The following table relates the Suits parameters to the AGR parameters.

Suits Parameter	AGR Parameter
k_1	$\mu' + \beta' + F'$
a_1	$\mu + \beta$
b_1	β
c_1'	F'
c_1	β'
θ	ξ
$E(+d,i,x)$	$s(n)$
$E(-d,i,x)$	$t(n)$
$E(s,i,x)$	$I(n)$

Suits Conditions	AGR Conditions
$E(-d,1,0) = 0$	$t(0) = 1$
$E(s,1,0) = 1$	$I(0) = 1$
$E(+d,1,r) = \rho[E(-d,1,r) + E(s,1,r)]$	$s(r) = 0$
positive direction upward	positive direction downward

Table 1. Analogy Table for Suits and AGR Models

Using this table, it can be seen that the differential equations describing the two models are identical, but the boundary conditions are different.

As the AGR Model was not designed to measure light reflectance exterior to the canopy, the look angle $\emptyset$ in the Suits Model does not have an analogy. Experimental evidence [5], [6], indicates that these models agree with actual data taken within a plant canopy.

The AGR parameters were calculated by use of regression analysis on experimental data. With the AGR coefficients calculated for a cornfield in Ithica, New York [5] an attempt was made to calculate the Suits coefficients, but the conditions imposed on the Suits coefficients by Table 1, forced several of the Suits Parameters to be negative, a condition that is physically impossible.

3. Theoretical Light Penetration in a Plant Canopy Using the Suits Model

This section of the paper uses the Suits Model to discuss the importance of canopy depth in reflectance measurements. These determinations were based on the following definition:

> the penetration depth of a plant canopy at a given wavelength of light is that depth at which reflectance obtained from the one layer finite Suits Model is within 5% of the reflectance obtained from the infinite Suits Model.

Figure 3(a) illustrates a determination of the penetration depth for experimental data collected for a cotton canopy at 850 n.m. As the canopy depth increases, the reflectance increases due to the increased number of leaves, each leaf acting as a good reflector at this wave length (the absorption of a single cotton leaf is .018 at this wave length). The reflectance is within 5% of the infinite canopy reflectance (the horizontal asymptote) at a depth of 47 cm., indicating that vegetative structure below 47 cm. in the canopy contribute less than 5% to the total reflectance at 850 n. m. Thus, only 47 cm. of the cotton canopy is sampled by 850 n. m. light. That is, 850 n. m. light can be measured as a component of the total light transmitted to lower depths in the canopy, but would fail to exit the canopy upward into the

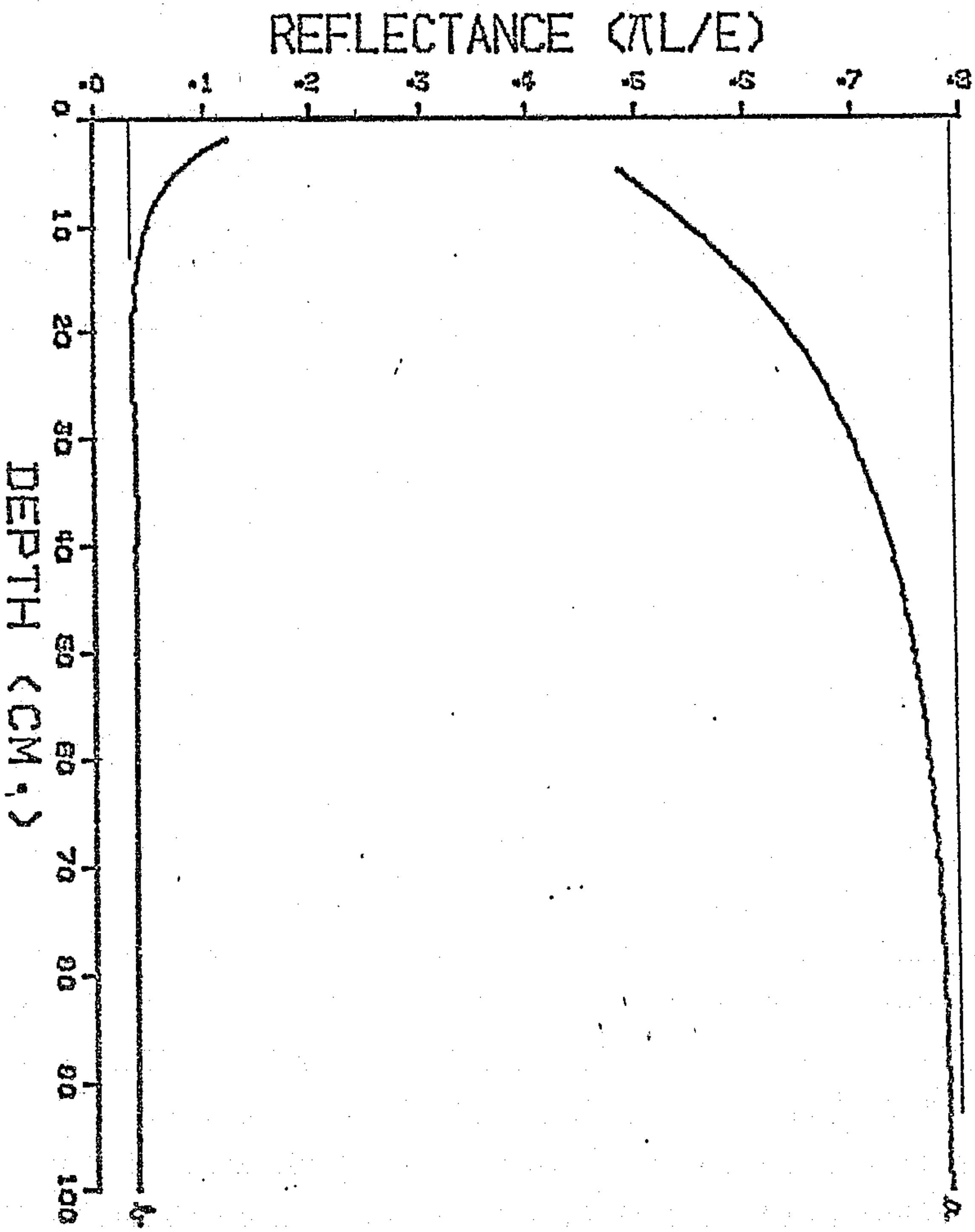


Figure 3

The relationship between canopy depth and reflectance $(\frac{\pi L}{E})$ at (a) 850 n.m. and (b) 650 n.m. for a cotton canopy. The horizontal asymptotes are the infinite case reflectance.

field of view of the observer. The behavior of 850 n.m. light is characteristic of most infrared radiation with respect to penetration depth.

Figure 3(b) indicates an interesting contrast that occurs for 650 n. m. light. As the canopy depth increases, the reflectance decreases, due to the increased number of leaves. Each leaf is now acting as a good absorber at this wave length (the absorption of a single cotton leaf is .877). Reflectance decreases to within 5% of the infinite canopy reflectance (the horizontal asymptote) at a canopy depth of 16.4 cm. This behavior is characteristic of most light in the visible spectrum.

The process of calculating the penetration depth for various wave lengths was continued, and the results are shown in Figure 4. To illustrate the difference between the visible and infrared penetration depth, the convention was adopted of assigning a negative sign to the penetration depth if the reflectance decreased to its limiting value and a positive sign if the reflectance increased to its limiting value. This graph indicates a greatest depth of 47 cm., so that only the first 47 cm. of the plant canopy are useful for reflection measurements at any wave length. As expected, infrared radiation penetrates to the greatest depths in the cotton canopy and the visible wave lengths penetrate the canopy to the least depth. Of great interest is the penetration depth for an estimated 695 n. m. radiation (no data was available at the wave length, and this result is based on interpolation). At this wave length the graph indicates that observed reflectance data fails to penetrate the canopy, and what

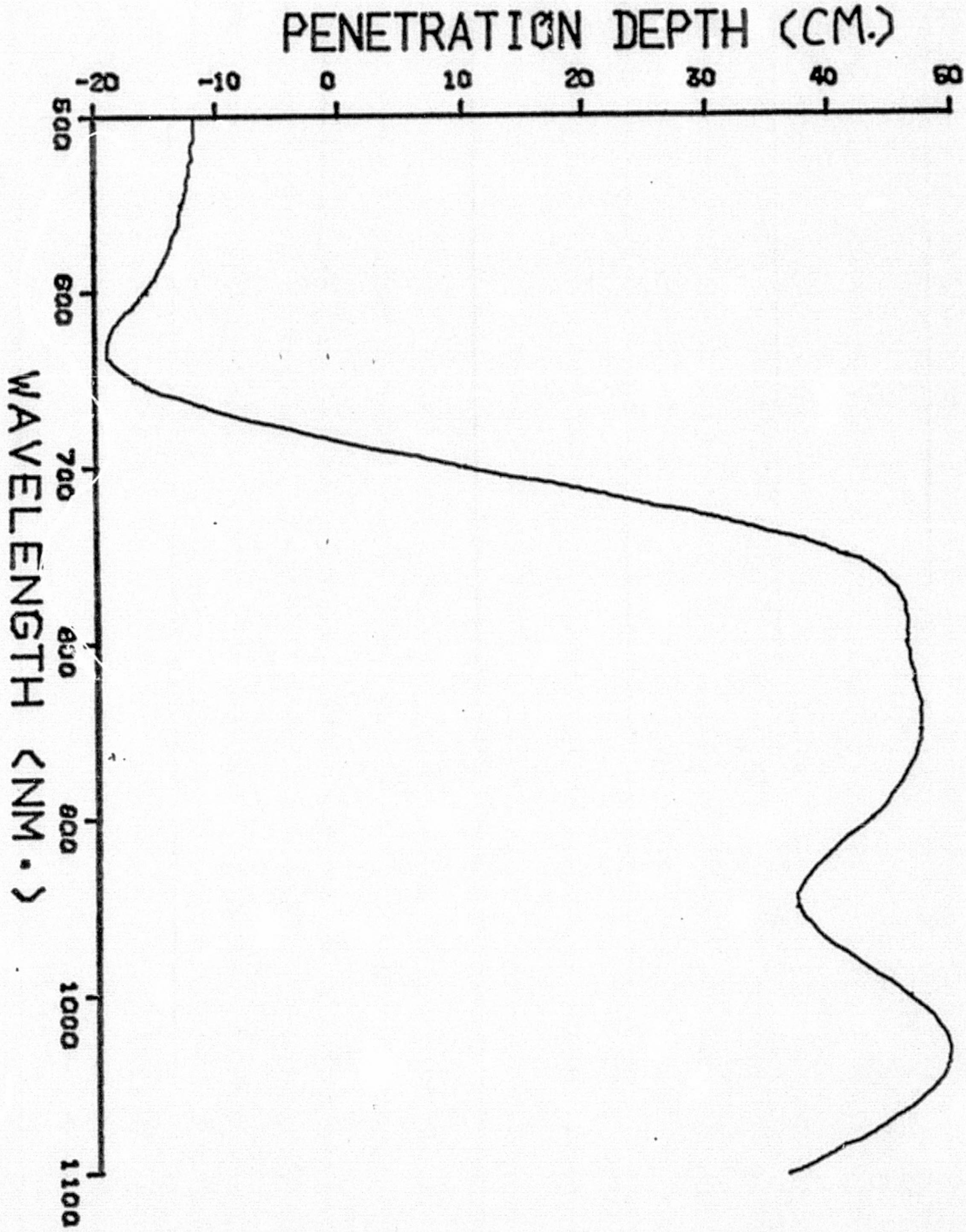


Figure 4

The Penetration Depth of Light in a Cotton Canopy as a Function of the Wavelength.

is recorded corresponds only to surface reflections. In summary, for the 189 cm. cotton canopy considered, incident light penetrated to a depth no greater than 47 cm. indicating that the soil reflectance data was not utilized in the model.

Allen and Richardson [7] have observed phenomena analogous to this with single leaf models. For the cotton crop considered $\sigma_h = 104$, $n_h = .00125$ and $\sigma_h n_h = .13$. For a maximum depth of 47 cm., the cumulative L.A.I. is .13 x 47 = 6.11, so that infrared light penetrating to this depth and exiting the canopy to the observer must pass through 12 leaves, and by an analogous calculation, light in the visible region must pass through 4 leaves, Allen and Richardson found that by stacking cotton leaves in a spectrophotometer, the reflectance ceased to vary with 8 cotton leaves in the infrared region and 2 cotton leaves in the visible spectrum, giving approximate agreement with the results found by use of the Suits Model.

On the basis of this limited data, one can conclude the following rules for use of the Suits Models:

(a) Since the maximum penetration occurs in the infrared, the effective depth d of a canopy is no larger than $d = 6/\sigma_h n_h$,

(b) If the canopy to be considered has depth greater than d, use the infinite model, and

(c) if the canopy considered has depth less than d, use the finite Suits Model and collect ground reflectance data.

These relationships are summarized in figure 5 for a typical plant from a canopy.

Penetration depth is a useful concept in explaining canopy reflectance for crops with layered structures such as wheat. As an

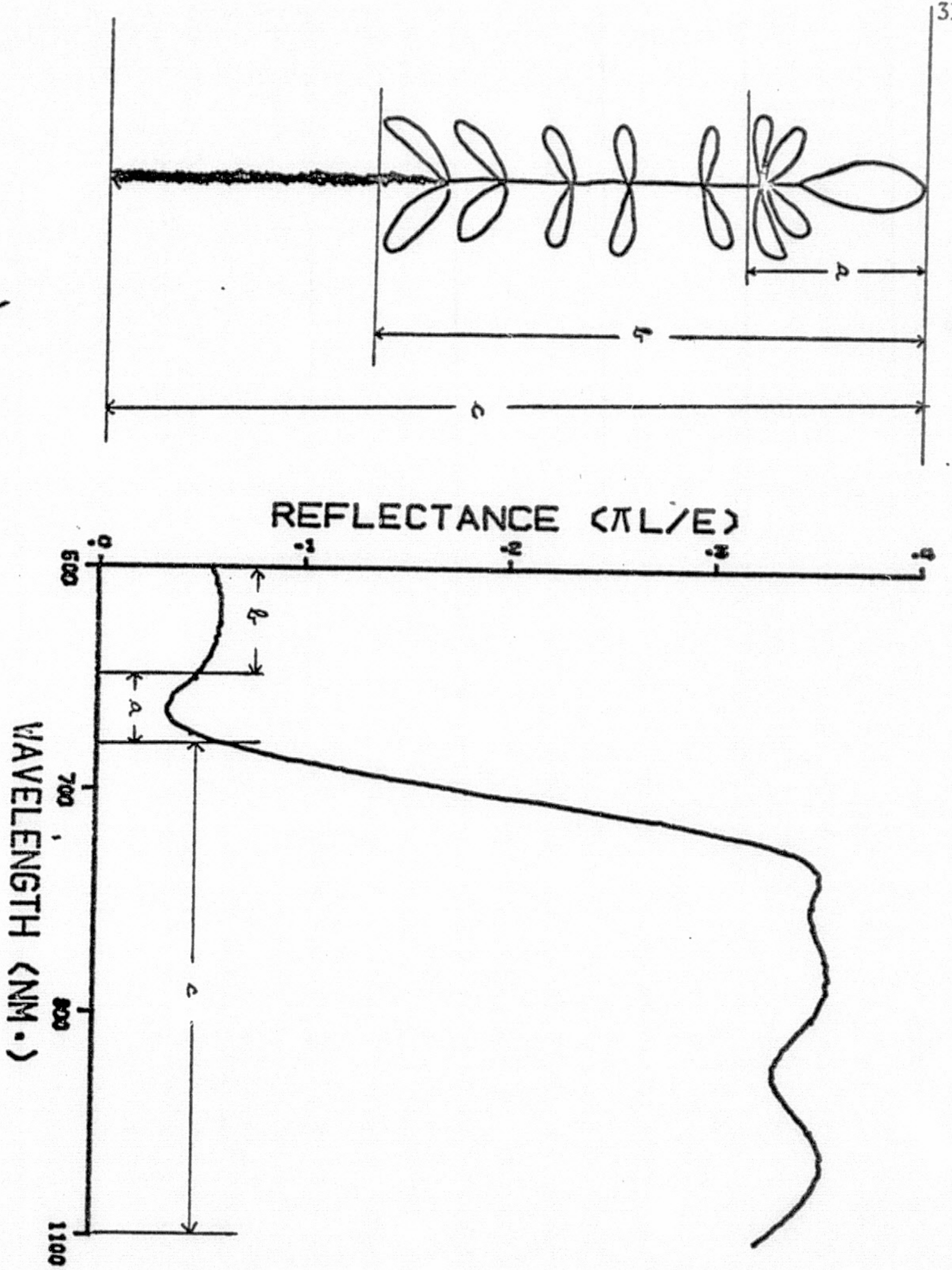


Figure 5

Relationship between Penetration Depth and Wavelength for a Typical Canopy.

a. 600-690 n.m. with small penetration into the first layer. The 1 layer infinite Suits' Model fits the data well.
b. 500-600 n.m. with moderate penetration into the second layer. The combination case for Suits' Model fits the data well.
c. 690 n.m. and above with deepest penetration. A multi-layered finite Suits' Model fits the data well.

example, consider Figure 6. In this case, two different Suits Models were employed on wheat data, the first used an infinite Suits Model, and the second a four layer finite Suits Model. The first model used only the parameters collected for the wheat heads and the second model used all parameters collected for each layer and the soil reflectance. As would be expected, the two graphs agree in the visible region, as the penetration depth is small and the second model is utilizing only the data from the wheat heads. The difference between the two graphs become noticeable around 700 n. m., where light has penetrated the layer of wheat heads in the second model and begins to sample the dissimilar reflectance properties of the green leaves. This difference continues to change until around 800 n. m., where the penetration depth has reached the soil level, and remains essentially constant thereafter. This example stresses the fact that reflectance data from around 650 n. m. to 850 n. m. contains a great deal of information on the internal reflective structure of a plant canopy.

4. The Interchange Property for Suits Models

This portion of the paper will establish a rather non-intuitive property of the Suits Model – the interchange property. In general, for an isotropic plant canopy, the reflectance is a function of the sun angle θ and the observer angle ϕ measured from the vertical, and can be written as

$$\text{canopy reflectance} = R(\theta,\phi).$$

The interchange property thus states that

$$R(\theta,\phi) = R(\phi,\theta),$$

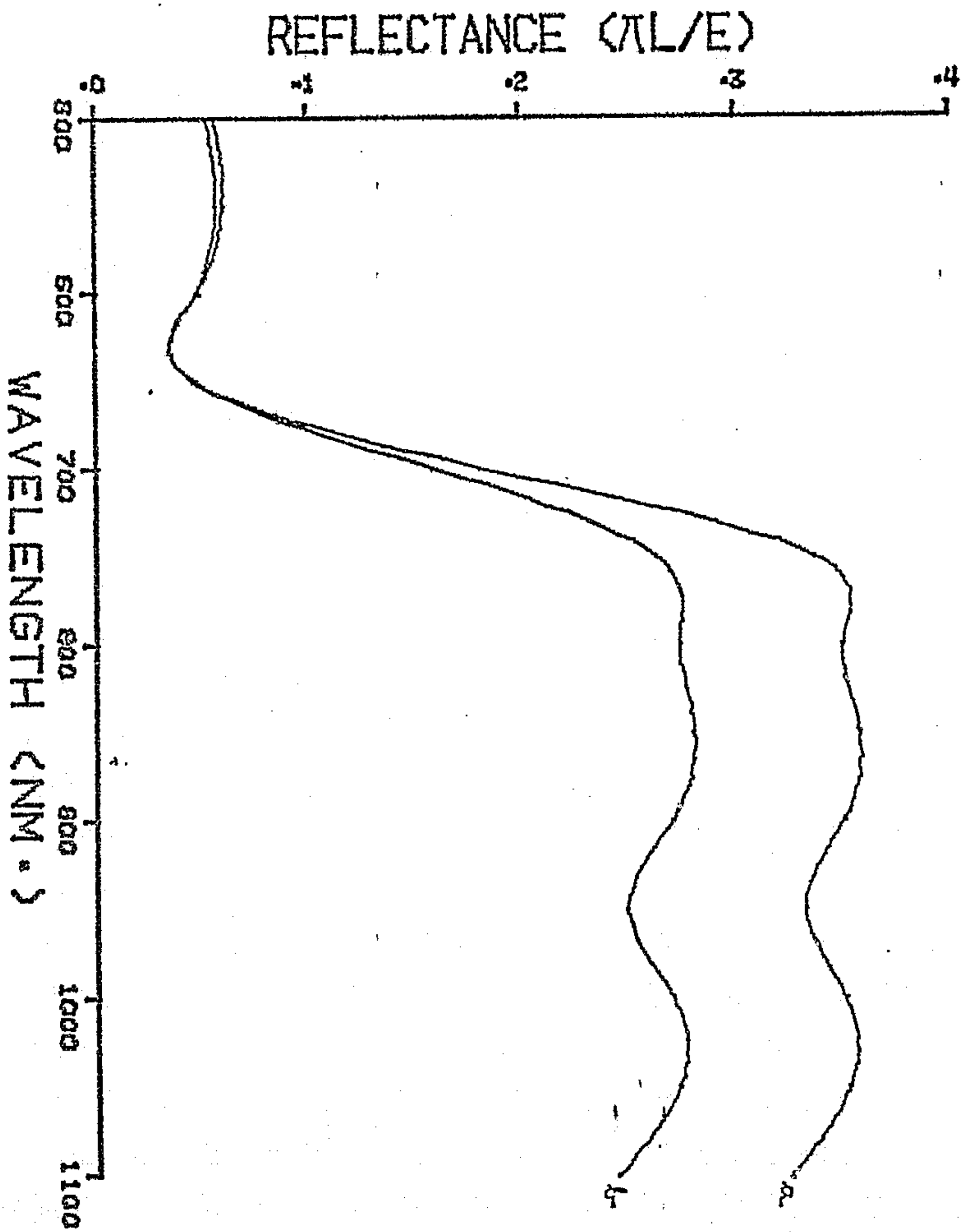


Figure 6

The Effect of the Reflectance from Lower Layers in a Wheat Canopy. (a) Is the 4 Layer Suits' Model and (b) is the Infinite Suits' Model using only Wheat head Parameters.

or that the same reflectance results whenever the position of the observer and the sun are interchanged.

The property will be established by use of the infinite case of Suits Model for non-diffuse light. From section 2.2 of the paper, using the infinite model,

$$R(\theta,\emptyset) = \left\{\frac{-b[bc + (a + k)c']}{(g^2 - k^2)(a + g)(g + k')} + \frac{c(a - k) + bc'}{(g^2 - k^2)(k + k')}\right\} u$$

$$+ \left\{\frac{-[bc + (a + k)c']}{(g^2 - k^2)(g + k')} + \frac{bc + (a + k)c'}{(g^2 - k^2)(k + k')}\right\} v \qquad (1)$$

$$+ \left\{\frac{1}{k + k'}\right\} w,$$

where these parameters are defined in section 2.1. One can observe a good deal of symmetry in these parameters, in fact, if one interchanges the roles of $\emptyset$ and θ, k becomes k', k' becomes k, u becomes c', c' becomes u, v becomes c, c becomes v, and w remains unchanged.

Expression (1) is formidable and the key step in proving this result is to re-write this expression. After considerable trial and error, one re-writes (1) as

$$R(\theta,\emptyset) = \frac{-b[bc + (a - g)c'] u - [bc + (a - g)c']v (a + g)}{2g(a + g)(g + k)(g + k')} +$$

$$\frac{\{[bc+(a+g)c'][bu+(a+g)v]\}(g+k)+(a+g)(g+k')\{[c(a+g)+bc]u+[bc+(a-g)c']v\}}{2g(a+g)(k+k')(g+k')(g+k)}$$

$$+ \left\{\frac{1}{k + k'}\right\} w.$$

While the first two terms are quite complicated, it can be seen by observation that both denominators are symmetric in k and k', thus one needs only show that the two numerators are symmetric in θ and ϕ. The third term is already symmetric in θ and ϕ.

Define the first numerator as

$$N_1(\theta,\phi) = -b[bc + (a-g)c']u - [bc + (a-g)c']v\ (a+g)$$

then

$$N_1(\phi,\theta) = -b[bv + (a-g)u]c' - [bv + (a-g)u]c\ (a+g)$$

or, rearranging

$$N_1(\phi,\theta) = [-b(a-g)c' - c(a+g)(a-g)]u + [-b^2c' - bc(a+g)]v.$$

Using the fact that $b^2 = a^2 - g^2$ and factoring gives

$$N_1(\phi,\theta) = -b[bc + (a-g)c']u - [bc + (a-g)c']v\ (a + g) = N_1(\theta,\phi).$$

In a similar manner, define the second numerator as

$$N_2(\theta,\phi) = \{[bc + (a+g)c'][bu + (a+g)v]\}(g+k) +$$

$$(a+g)(g+k')\{[c(a+g) + bc']u + [bc + (a-g)c']v\}.$$

$$N_2(\phi,\theta) = \{[bv + (a+g)u][bc' + (a+g)c]\}(g+k') +$$

$$(a+g)(g+k\{[v(a+g) + bu]c' + [bv + (a-g)u]c\}.$$

These terms can be re-arranged so that

$$N_2(\phi,\theta) = \{(a+g)u[bc' + (a+g)c] + bv[bc' + (a+g)c]\}(g+k') +$$

$$(g+k)\{[v(a+g)^2 + b(a+g)u]c' + [b(a+g)v + (a^2-g^2)u]c\}.$$

But $b^2 = a^2 - g^2$ and factoring gives

$$N_2(\emptyset,\theta) = \{u[bc' + (a+g)c] + [(a-g)c' + bc]v\}(a+g)(g+k') +$$

$$(g+k)\{[bc + (a+g)c'][bu + (a+g)v]\} = N_2(\theta,\emptyset).$$

Thus, it has been shown that

$$R(\theta,\emptyset) = R(\emptyset,\theta).$$

Considering that the Suits Model is a generalization of a math model used to describe light reflection from a stack of glass plates, and that in most optical models the interchange is valid, then this result should not appear too surprising.

It should be noted that the interchange is not valid in the presence of diffuse light i.e. $E_{-d}(0) > 0$, $E_s(0) > 0$, $E_{-d}(0) + E_s(0) = 1$. (See 2.4)

For example, in the case that $E_{-d}(0) = .6$, $E_s(0) = .4$ $R(10^o,50^o) = .752371$, but $R(50^o,10^o) = .736506$. (The infinite diffuse light Suits Model applied to the parameters for cotton in data set 1.)

Both the N-layered Suits Model and the Combined Suits Model exhibited the interchange property for non-diffuse light for all data run on the computer, but due to the complexity of the equations involved for reflectance, an analytic proof of the interchange property was not attempted. Similarly, both of these models did not exhibit the interchange property for diffuse light.

5. Directional Reflectance for the Suits Models

The general problem of calculating the canopy directional reflectance as a function of the sun angle, viewer angle, and viewer azimuth has been reduced by the Suits assumption of isotropic canopy conditions with respect to azimuth, so that the reflectance is a function of only the sun angle and viewer angle measured from the vertical. It is expected that such a simplifying assumption might lead to errors between experimental data and theoretical calculations of the bi-directional reflectance function.

This section of the paper will discuss the general shape of the surface $R(\theta,\emptyset)$ (the bi-directional reflectance function) and derive several useful formulas. Using the N layer Suits' Model, the radiance contribution from the i^{th} layer is given, from Suits equations as

$$R_i = P_i \int_{x_i}^{x_{i-1}} e^{k_i x} \, \Delta L_\lambda' dx$$

where the i^{th} layer has boundaries x_i and x_{i-1} respectively,

$P_i = 1$ if $i = 1$, and $P_i = \exp(\sum_{j=1}^{i-1} k_j x_j)$ if $i > 1$.

But

$$\Delta L_\lambda = uE_{+d} + vE_{-d} + wE_s \quad \text{where}$$

$$u = \sigma_h n_h \tau + \sigma_v n_v \frac{\rho + \tau}{2} \left(\frac{2}{\pi}\right) \tan \emptyset$$

$$v = \sigma_h n_h \rho + \sigma_v n_v \frac{\rho + \tau}{2} \left(\frac{2}{\pi}\right) \tan \emptyset$$

$$w = \sigma_h n_h \rho + \sigma_v n_v \frac{\rho + \tau}{2} \tan \emptyset \tan \theta \left(\frac{2}{\pi}\right)^2$$

and $E_{+d} = A_{1i} e^{-g_i x} + B_{1i} e^{g_i x} + C_{1i} e^{k_i x}$

$$E_{-d} = A_{2i} e^{-g_i x} + B_{2i} e^{g_i x} + C_{2i} e^{k_i x}$$

$$E_s = C_{3i} e^{k_i x}$$

Observe that A_{ij}, B_{ij}, C_{ij} for i = 1,2,3, j = 1,2,3 are independent of the view angle $\emptyset$. For i = 1, (the first layer) $x_0 = 0$. Integration and re-arrangement gives

$$R_1 = \frac{uA_{11} + vA_{21}}{k_1 - g_1} + \frac{uB_{11} + vB_{21}}{k_1 + g_1} + \frac{uC_{11} + vC_{21} + wC_{31}}{k_1 + k_1}$$

$$- \frac{uA_{11} + vA_{21}}{K_1 - g_1} \exp((k_1 - g_1)x_1) - \frac{uB_{11} + vB_{21}}{K_1 + g_1} \exp((k_1 + g_1)x_1)$$

$$- \frac{uC_{11} + vC_{21} + wC_{31}}{K_1 + k_1} \exp((K_1 + k_1)x_1).$$

As $\emptyset \to \frac{\pi}{2}$, each of the last three terms involving the exponential tend to zero, and the first three terms approach $\frac{\rho + \tau}{2}$ $[E(+d,1,0) + \frac{2}{\pi} \tan \theta]$.

For i > 1, as $\emptyset \to \frac{\pi}{2}$, each of the seven terms resulting from integration approaches zero. Thus $R_i \to 0$ as $\emptyset \to \frac{\pi}{2}$ for i > 1.

The ground radiance contribution after N layers is given by $P_N E(+d,N,x_N)$, and since $P_N \to 0$ as $\emptyset \to \frac{\pi}{2}$ and $E(+d,N,x_N)$ is independent of $\emptyset$, then the ground radiance contribution goes to zero.

Thus, we have shown, that as $\emptyset$ approaches $\frac{\pi}{2}$, the only contribution to the reflectance is from the top surface of the canopy. In fact,

$$\lim_{\emptyset \to \frac{\pi}{2}} R(\theta,\emptyset) = \frac{\rho + \tau}{2\pi} \; [E_{+d}(0) + \frac{2}{\pi} \tan \theta]. \qquad (1)$$

This result indicates for θ and $\emptyset$ sufficiently close to $\frac{\pi}{2}$, $R(\theta,\emptyset)$ becomes larger than one, a result that is physically impossible.

For non-diffuse light a typical surface for a single wave length of light, as calculated by the computer forms the shape of a saddle. Figure 7 is an example of such a surface for a cotton canopy with the iso-reflectance lines shown to represent the surface contours. One can observe that the upper right corner of the graph becomes larger than one as indicated by (1), and that the remainder of the graph exhibits only a small amount of variation.

For diffuse light of a single wave length with a proportion of 40% direct light and 60% diffuse light the reflectance surface for a cotton canopy is shown in Figure 8.

In this case one can observe that the interchange property fails to hold, and the contours of Figure 7 erode into the contours of Figure 8.

6. A Comparison of Experimental and Theoretical Results for Plant Canopies

A comparison of Suits' Reflectance ($^{\pi L}/E$) values and the experimental values for cotton at various wave lengths for the same sun and viewer angles (Figures 9a and 9b) shows that the qualitative trends exist. The maxima and minima do not occur at the same wave lengths due to a calibration problem in measuring single leaf reflectance values with the Beckman DK-2A spectrophotometer. The calibration problem caused a shift in wave length of about 30 nm. As can be noted, the theoretical values are significantly greater than the experimental values in the infrared region. Quantitative comparisons between the theoretical and experimental values are not possible at this time for several reasons. Among those reasons are normalization problems between the theoretical and experimental values. Also, there are varying

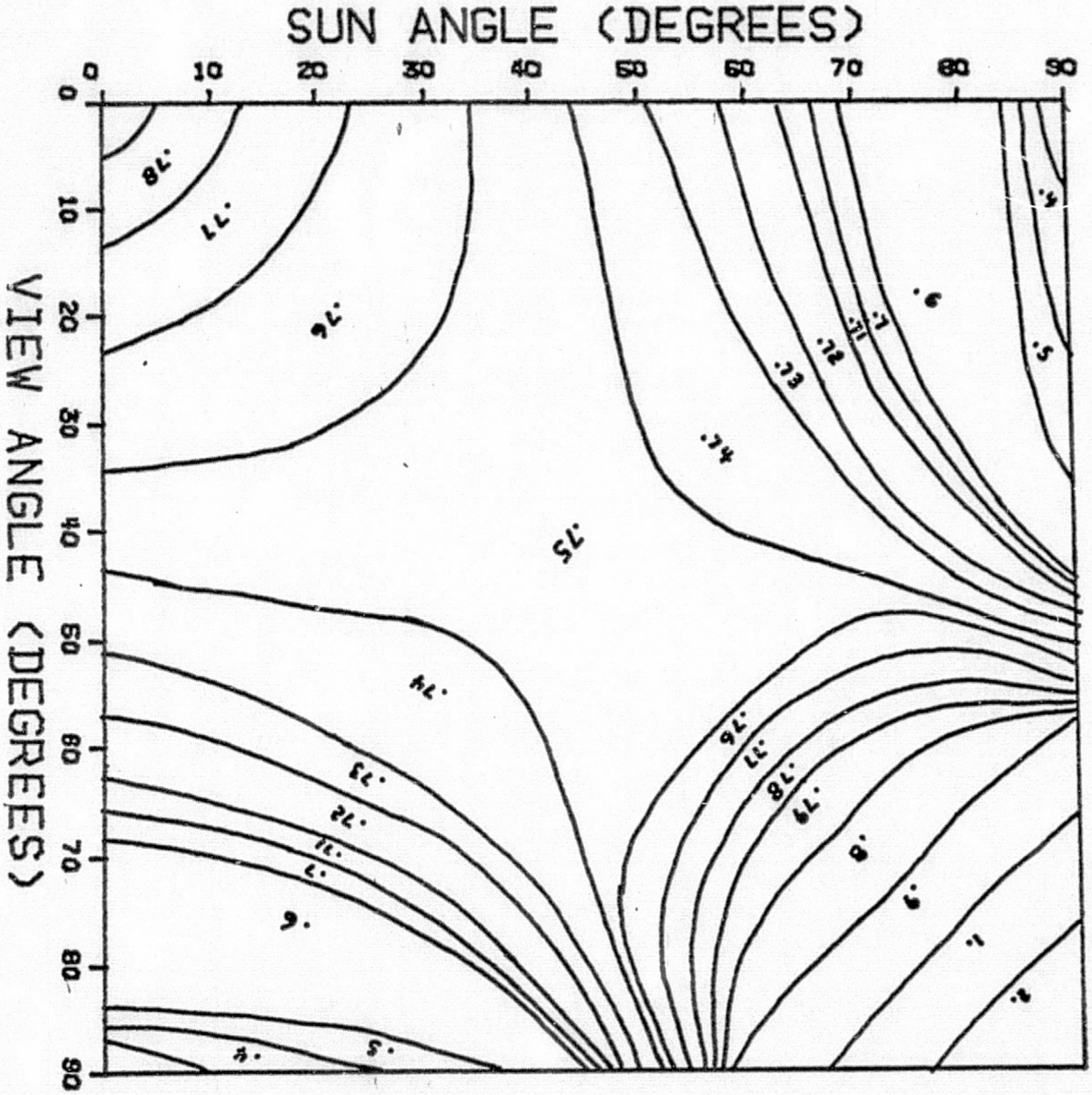


Figure 7

Iso-Reflectance Lines for the Reflectance Surface $R(\theta,\phi)$ of a Cotton Crop. θ is the Sun Angle and ϕ is the Viewer Angle.

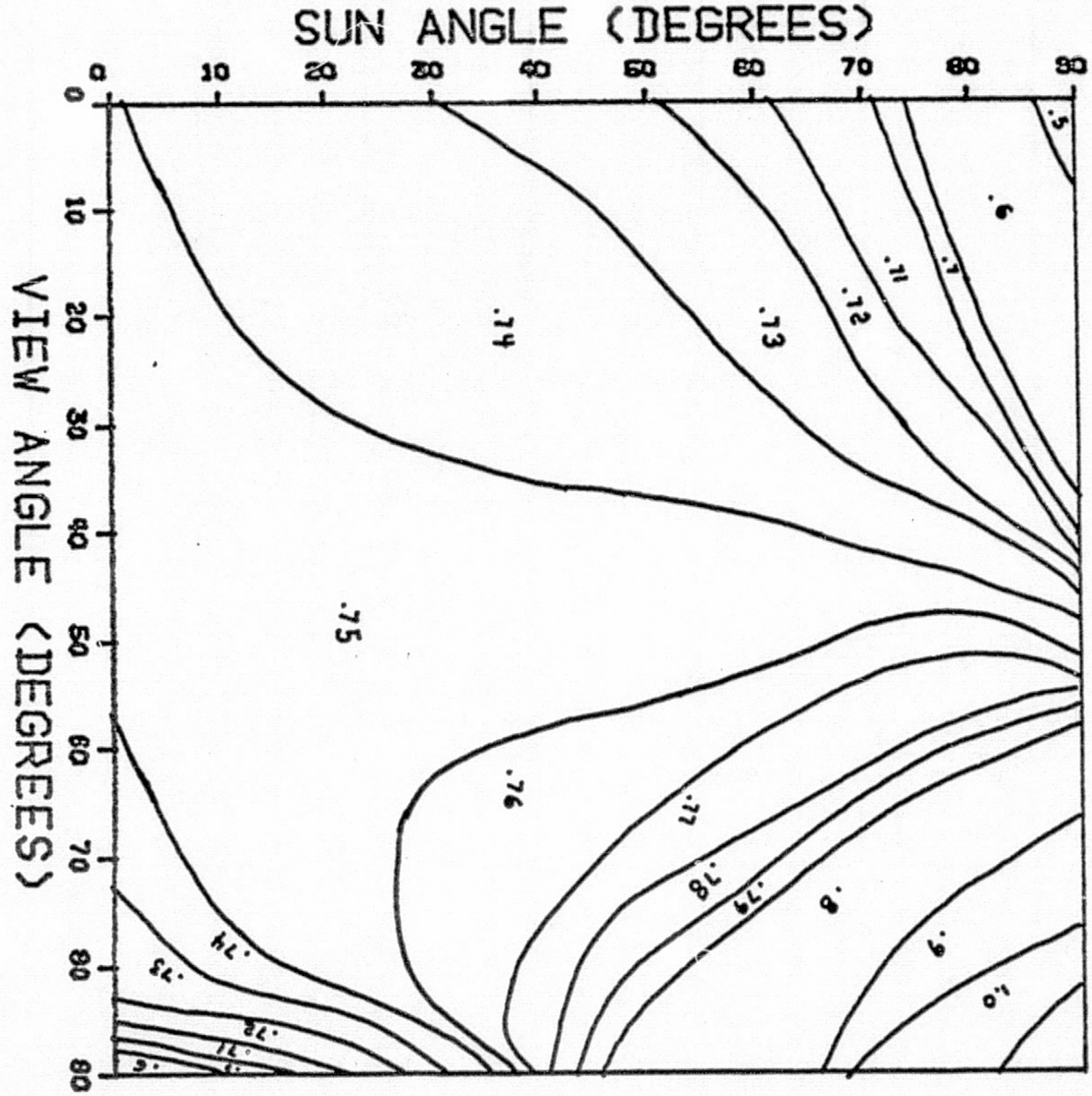


Figure 8

Iso-Reflectance Lines for the Reflectance Surface R(θ,∅) of a cotton crop using 40% Specular Light and 60% Diffuse Lights.

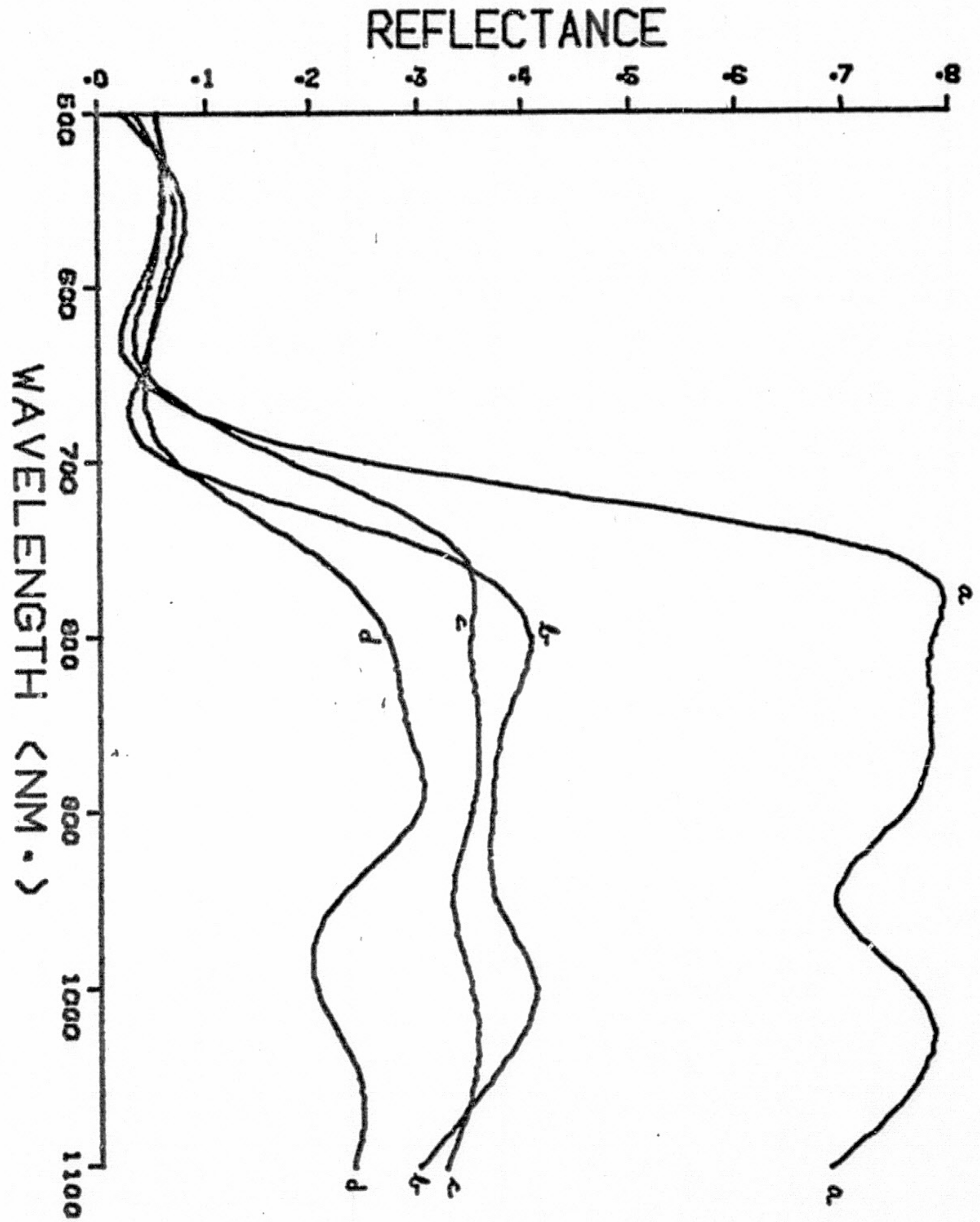


Figure 9

Theoretical and Experimental Reflectance Curves for Wheat and Cotton as a function of wavelength.

methods used by experimentalists to describe directional reflectance not all of which yield equivalent results. Another problem is the finite angular field of view that experimentalists must cope with to collect enough light for meter readings. In theoretical models, an aspect in question is the factor π that converts radiance to flux density. This is a well-established relationship for lambertian surfaces. It is not clear however that π converts from radiance to flux density for non-lambertian surfaces such as canopy surfaces.

Results similar to those of cotton were obtained in comparing Suits' Reflectance ($^{\pi L}/E$) values and the experimental values for wheat (Figure 9c and 9d). Again there is a shift of about 30 nm. between the experimental and theoretical results due to calibration problems. Similarly the theoretical values are greater than the experimental values in the infrared region. A comparison of Figures 9a and 9c shows that Suits' Reflectance ($^{\pi L}/E$) values for cotton are significantly greater than the Reflectance ($^{\pi L}/E$) values for wheat. This is primarily so because of the different absorption properties of the wheat components in the infrared region and also because of the soil reflectance values which play a significant role in canopy reflectance for wheat due to the low LAI of wheat.

7. Conclusions

Both the Suits and the AGR models for vegetative canopy reflectance are generalizations of the Kubelka-Munk theory for the transmission of diffuse light [8]. Both models assume incident specular light but assume that this light, upon being transmitted through a leaf is diffuse enough to satisfy the Kubelka-Munk hypothesis. In the words of [5] "Although a typical leaf such as corn has strong specular reflection, the structure of the leaf is such that the transmitted light is very diffuse". This article cites [9] as an example of the strong directional reflectance properties of leaves. Both of these models, as experiments verify, model light intensity interior to the canopy well, but fail to accurately model light reflected exterior to the canopy. [10] indicates experimental directional reflectance trends for a wheat field not found in the Suits Model. This deficiency in both Kubelka-Munk Models is due, in the opinion of the authors, to the failure in consideration of the strong specular reflectance from the top surface of the canopy. An initial attempt was made to alter the Suits Model by the development of the combined case, using a thin reflective top layer, and an infinite second layer, which resulted only in a partial success. On the top surface of the canopy, one must consider both the plant geometry, (leaf distributions, axial "droop", etc.) and the single leaf directional reflectance as measured by [9], rather than the single leaf hemispherical reflectance. The transmitted light through the top leaf surface is diffuse and a Kubelka-Munk type model such as the one proposed by Suits should be adequate to describe light reflected from within the canopy. Thus, it is felt that surface specular reflectance is a phenomena that must be modeled, and can be adjoined to the existing Suits Model as a refinement.

The existing Suits Models, on the other hand, showed qualitative agreement between the experimental results gathered for cotton and wheat and the theoretical calculated results for reflectance versus wave length.

Again, the authors felt that qualitative agreement could be converted to quantitative agreement either with a revised analysis of the conversion factors necessary to convert between radiant intensity and radiance for a non-Lambertian source or a re-evaluation of the experimental proceedures needed for normalization of field data.

The Suits Models agreed with work done by Allen and Richardson on the infinite reflectance of stacked leaf models, indicating that light will penetrate only through canopies with a cumulative LAI of less than 6, and that infrared light penetrates vegetation better than light in the visible region. The Suits Model gives a quantitative way of discussing light penetration through a canopy by means of the penetration depth. The model indicates that light incident on a plant canopy is reflected to an external observer from very near the canopy surface in the visible region and the reflections progress to vegetative deeper within the canopy as the light wave length progresses into the infrared reaching a maximum depth at about 850 nm. and leveling off thereafter. Thus, the authors believe that in the wave length region from 650 n.m. to 850 n.m. light scans a vertical profile of the plant canopy, and most of the information about the plant canopy is contained in this wave length region. This hypothesis is especially useful in explaining the differences between layered crops such as wheat and homogeneous crops such as cotton.

It is felt that continued work on the Suits Models should progress in the areas of data normalization, for both the experimental and the theoretical. Further, a "hybrid" model should be constructed that incorporates the specular directional reflectance of surface leaves and uses the Kubelka-Munk theory for the diffuse light that exists below the top surface. Both a lack of time and a lack of experimental data on both single leaf and canopy directional reflectance prevented the authors from completing such a model. Finally, the authors feel that work should continue on a study of the differential coefficients proposed by Suits. Some of the coefficients could be derived by the authors, but others were in direct conflict both with the author's results and established facts. But, since the purpose of this paper was to test the Suits Model, all coefficients used were those supplied by Suits [2]. This choice of coefficients explains, for example, why the reflectance is greater than one for large observer and sun angles on the directional surface $R(\theta,\phi)$ pictured in Figure 7, and could possibly explain the relative insensitivity of the model to changes in the parameters $\sigma_h n_h$ and $\sigma_v n_v$.

Appendix 1

A brief discussion on how to obtain the theoretical graphs of this paper follows. The experimental graphs were obtained with data supplied by Dr. Edwin LeMaster. Since the total LAI($\sigma_h n_h d_1$) of the cotton crop was 24.6, the one layer Infinite case was generally used on cotton. The four layer N layer case was used on wheat.

Figure 3. Graph a was obtained using Data set 1 (with $\theta = 0^o$, $\phi = 0^o$, and $\lambda = 850$ nm.) and using the Suits' one layer model, using the computer program NTH2. Graph b was obtained using Data set 1 (with $\theta = 0^o$, $\phi = 0^o$, and $\lambda = 650$ nm.) and using the Suits one layer model, using computer program NTH2. The horizontal asymptotes are the Infinite case reflectance value at the two wave lengths respectively.

Figure 4. The penetration depth at a particular wave length was obtained using Data set 1 (with $\theta = 0^o$) and using the computer program NTH2. The depth was varied until the Reflectance ($^{\pi L}/E$) using the computer program NTH2 was within 5% of the Reflectance ($^{\pi L}/E$) using the computer program Inf2 for a given wave length.

Figure 5. This graph was obtained using Data set 2 (with $\theta = 15^o$ and $\phi = 0^o$) and using Suits' four layer model, using the computer program NTH2.

Figure 6. Graph a was obtained using all of Data set 2 (with $\theta = 15^o$ and $\phi = 0^o$) and using Suits' four layer model, using the computer program NTH2. Graph b was obtained from the data on heads in Data set 2 (with $\theta = 15^o$ and $\phi = 0^o$) and using Suits' one layer model, using the computer program Inf2.

Figure 7. This graph was obtained using Data set 1 (with $\lambda = 850$ nm.) and using Suits' one layer model, using the computer program Inf2.

Figure 8. This graph was obtained using Data set 1 (with $\lambda = 850$ nm.) and with $E_\lambda(s,1,0) = .4$ and $E_\lambda(-d,1,0) = .6$ using Suits' one layer model, using the computer program Inf3 which is the altered Inf2 program.

Figure 9. For both the experimental and theoretical graphs in Figure 9, $0 = 15^o$ and $\emptyset = 0^o$. Graph a was obtained using Data set 1 and using Suits' one layer model, using the computer program Inf2. Graph c was obtained using Data set 2 and using Suits' four layer model, using the computer program NTH2.

Data Set 1
Cotton

Layer 1 1) green leaves

σ_h = 104 cm.^{2} n_h = .00125 1/cm.^{3} d_1 = 189 cm.

σ_v = 85 cm.^{2} n_v = .0007 1/cm.^{3}

λ(nm.)			
500	.099	.107	.100
550	.111	.145	.126
600	.076	.078	.160
650	.073	.050	.175
700	.308	.382	.190
750	.443	.531	.325
800	.444	.540	.417
850	.442	.542	.396
900	.438	.542	.398
950	.430	.539	.380
1000	.434	.547	.342
1050	.432	.551	.353
1100	.422	.546	.389

Data Set 2
Wheat

Layer 1 1) heads depth = .09 m

$\sigma_h = .000369\ m^2$ $n_h = 6170\ 1/m^3$

$\sigma_v = .001015\ m^2$ $n_v = 6170\ 1/m^3$

Layer 2 1) green leaves depth = .20 m

$\sigma_h = .00144\ m^2$ $n_h = 4360\ 1/m^3$

$\sigma_v = .00107\ m^2$ $n_v = 4360\ 1/m^3$

2) stems

$\sigma_h = 0\ m^2$ $n_h = 0\ 1/m^3$

$\sigma_v = .0006\ m^2$ $n_v = 2780\ 1/m^3$

Layer 3 1) green-brown leaves depth = .10 m

$\sigma_h = .00148\ m^2$ $n_h = 8330\ 1/m^3$

$\sigma_v = .000872\ m^2$ $n_v = 8330\ 1/m^3$

2) stems

$\sigma_h = 0\ m^2$ $n_h = 0\ 1/m^3$

$\sigma_v = .0003\ m^2$ $n_v = 5550\ 1/m^3$

Layer 4 1) brown leaves depth = .12 m

$\sigma_h = .000259\ m^2$ $n_h = 12350\ 1/m^3$

$\sigma_v = .0000646\ m^2$ $n_v = 12350\ 1/m^3$

2) stems

$\sigma_h = 0\ m^2$ $n_h = 0\ 1/m^3$

$\sigma_v = .00048\ m^2$ $n_v = 4360\ 1/m^3$

	Heads		Green leaves		Green Brown leaves		Brown leaves		Stems		
λ	ρ	τ	ρ	τ	ρ	τ	ρ	τ	ρ	τ	ρ_S
500	.126	.005	.115	.051	.154	.017	.203	.007	.115	.004	.129
550	.137	.004	.119	.066	.177	.021	.243	.008	.124	.004	.160
600	.115	.003	.095	.033	.146	.013	.287	.013	.093	.003	.176
650	.110	.003	.092	.027	.136	.021	.329	.021	.086	.003	.222
700	.371	.005	.308	.258	.366	.169	.370	.031	.367	.035	.203
750	.578	.011	.453	.381	.452	.266	.402	.043	.595	.102	.294
800	.591	.023	.453	.389	.457	.306	.433	.057	.605	.113	.293
850	.595	.038	.452	.392	.460	.333	.454	.069	.605	.114	.306
900	.577	.041	.446	.394	.456	.346	.472	.077	.589	.103	.329
950	.543	.036	.434	.394	.450	.355	.486	.086	.552	.080	.348
1000	.571	.053	.439	.400	.453	.362	.497	.094	.579	.102	.397
1050	.572	.061	.437	.402	.449	.367	.504	.101	.583	.109	.376
1100	.530	.047	.423	.398	.440	.368	.509	.104	.546	.081	.333

Appendix 2

This section contains a listing of the computer programs associated with the three cases of Suits' Model discussed in this paper. The computer program NTH2 was designed to calculate the radiance of the N layer model of Suits. Similarly, Inf2 and Com2 were designed to calculate the radiance of the Infinite case and Combination case respectively. Note that the computer programs yield the radiance and not the reflectance from a plant canopy. To convert from radiance to Reflectance ($^{\pi L}/E$), the radiance must be multiplied by π.

NTH2 10:18 27-MAY-75

```
1000 PRINT"DO YOU WISH TO READ INSTRUCTIONS (YES OR NO)";
1010 INPUT A$
1020 IF A$<>"YES" THEN 1230
1030 PRINT"    THIS PROGRAM DETERMINES THE RADIANCE, AS PROPOSED BY"
1040 PRINT"GWYNN SUITS, FROM A PLANT CANOPY CONSISTING OF N LAYERS."
1050 PRINT"THE PROGRAM WAS WRITTEN IN PARTIAL FULFILLMENT OF THE THESIS"
1060 PRINT"REQUIREMENTS OF JUAN M. CANTU, WHOSE GRADUATE ADVISOR WAS"
1070 PRINT"DR. JOE CHANCE."
1080 PRINT"    THROUGHOUT THE FOLLOWING CALCULATIONS, THE SAME UNITS OF"
1090 PRINT"MEASUREMENTS SHOULD BE USED.  THAT IS, IF THE DEPTH IS IN"
1100 PRINT"CM., THEN THE AREA IS IN CM.**2, AND THE VOLUME IS IN CM.**3."
1110 PRINT"    WITHIN A SINGLE LAYER THERE CAN EXIST SEVERAL COMPONENTS"
1120 PRINT"(LEAVES, STALKS, FRUITS, FLOWERS) EACH WITH ITS OWN SET OF"
1130 PRINT"PHYSICAL AND OPTICAL PROPERTIES.  THE AREA OF A COMPONENT"
1140 PRINT"IS DIVIDED INTO THE AREA PROJECTED ON A HORIZONTAL PLANE AND"
1150 PRINT"THE AREA PROJECTED ON TWO ORTHOGONAL VERTICAL PLANES.  IF"
1160 PRINT"THE AREA IS MEASURED IN CM.**2, THEN THE NUMBER OF HORIZONTAL"
1170 PRINT"AND VERTICAL COMPONENTS PER UNIT VOLUME SHOULD BE IN 1/CM.**3."
1180 PRINT"THE TRANSMITTANCE (T) AND REFLECTANCE (R) OF A COMPONENT ARE"
1190 PRINT"UNITLESS AND 0<T<1, 0<R<1, 0<T+R<1.  THE SUN ANGLE AND VIEW",
1200 PRINT"ANGLE ARE MEASURED IN DEGREES.  BECAUSE OF THE CHOICE OF CO-"
1210 PRINT"ORDINATE SYSTEM, THE DEPTH OF A LAYER SHOULD BE ENTERED AS"
1220 PRINT"A NEGATIVE NUMBER."
1230 DIM E(3,3),I(3,1)
1240 PRINT"ENTER WAVELENGTH IN NANOMETERS";
1250 INPUT J7
1260 PRINT"ENTER NUMBER OF LAYERS, SUN ANGLE, VIEW ANGLE, AND REFLECT-"
1270 PRINT"ANCE FOR SOIL";
1280 INPUT N1,S2,V2,S1
1290 S=S2*3.14159/180
1300 V=V2*3.14159/180
1310 MAT G=IDN(3,3)
1320 K3(0)=0
1330 FOR N=1 TO N1
1340 PRINT"ENTER NUMBER OF COMPONENTS IN, AND DEPTH OF LAYER";N;
1350 INPUT N2(N),D1(N)
1360 FOR M=1 TO N2(N)
1370 PRINT"ENTER AREA-HORIZONTAL, NUMBER-HORIZONTAL, AREA-VERTICAL,"
1380 PRINT"NUMBER-VERTICAL, REFLECTANCE, AND TRANSMITTANCE FOR COM-"
1390 PRINT"PONENT NUMBER";M
1400 INPUT H5(N,M),H6(N,M),V5(N,M),V6(N,M),R9(N,M),T(N,M)
1410 J=2/3.14159
1420 A(N,M)=H5(N,M)*H6(N,M)*(1-T(N,M))
1430 A(N,M)=A(N,M)+V5(N,M)*V6(N,M)*(1-(R9(N,M)+T(N,M))/2)
1440 B(N,M)=H5(N,M)*H6(N,M)*R9(N,M)+V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)
1450 C(N,M)=H5(N,M)*H6(N,M)*R9(N,M)
1460 C(N,M)=C(N,M)+J*V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)*TAN(S)
1470 C9(N,M)=J*V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)*TAN(S)
1480 C9(N,M)=C9(N,M)+H5(N,M)*H6(N,M)*T(N,M)
1490 K(N,M)=H5(N,M)*H6(N,M)+J*V5(N,M)*V6(N,M)*TAN(S)
1500 K2(N,M)=H5(N,M)*H6(N,M)+J*V5(N,M)*V6(N,M)*TAN(V)
1510 A1(N)=A1(N)+A(N,M)
1520 B1(N)=B1(N)+B(N,M)
1530 C1(N)=C1(N)+C(N,M)
1540 C2(N)=C2(N)+C9(N,M)
1550 K1(N)=K1(N)+K(N,M)
1560 K4(N)=K4(N)+K2(N,M)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
1570 NEXT M
1580 K3(N)=K3(N-1)+K4(N)*D1(N)
1590 K5(N)=EXP(K3(N-1))
1600 K6(N)=EXP(K3(N))
1610 G1(N)=SQR(A1(N)**2-B1(N)**2)
1620 Z1(N)=1/(A1(N)+G1(N))
1630 Z2(N)=1/(A1(N)-G1(N))
1640 Z7(N)=A1(N)**2-K1(N)**2-B1(N)**2
1650 IF Z7(N)=0 THEN 2500
1660 Z3(N)=((A1(N)-K1(N))*C1(N)+B1(N)*C2(N))/(A1(N)**2-K1(N)**2-B1(N)**2)
1670 Z4(N)=1/B1(N)
1680 Z5(N)=((A1(N)+K1(N))*C2(N)+B1(N)*C1(N))/(A1(N)**2-K1(N)**2-B1(N)**2)
1690 W1(N)=EXP(G1(N)*D1(N))
1700 W2(N)=EXP(-1*G1(N)*D1(N))
1710 W3(N)=EXP(K1(N)*D1(N))
1720 MAT E=ZER(3,3)
1730 E(1,1)=Z1(N)
1740 E(1,2)=Z2(N)
1750 E(1,3)=Z3(N)
1760 E(2,1)=Z4(N)
1770 E(2,2)=Z4(N)
1780 E(2,3)=Z5(N)
1790 E(3,1)=0
1800 E(3,2)=0
1810 E(3,3)=1
1820 MAT D=ZER(3,3)
1830 D(1,1)=W1(N)
1840 D(2,2)=W2(N)
1850 D(3,3)=W3(N)
1860 MAT P1=INV(E)
1870 MAT P2=E*D
1880 MAT P=P2*P1
1890 MAT H=G
1900 MAT G=P*H
1910 NEXT N
1920 A3=-1*(G(1,3)-1*S1*(G(2,3)+G(3,3)))
1930 A4=G(1,1)-1*S1*(G(2,1)+G(3,1))
1940 IF A4=0 THEN 2480
1950 A0=A3/A4
1960 R5(1)=0
1970 R6(0)=0
1980 I(1,1)=A0
1990 I(2,1)=0
2000 I(3,1)=1
2010 FOR N=1 TO N1
2020 E(1,1)=Z1(N)
2030 E(1,2)=Z2(N)
2040 E(1,3)=Z3(N)
2050 E(2,1)=Z4(N)
2060 E(2,2)=Z4(N)
2070 E(2,3)=Z5(N)
2080 E(3,1)=0
2090 E(3,2)=0
2100 E(3,3)=1
2110 D(1,1)=W1(N)
2120 D(2,2)=W2(N)
2130 D(3,3)=W3(N)
2140 MAT P1=INV(E)
2150 MAT P3=P1*I
2160 FOR M=1 TO N2(N)
2170 F1(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)*J*TAN(V)
2180 F1(N,M)=F1(N,M)+H5(N,M)*H6(N,M)*T(N,M)
2190 F2(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)*TAN(V)*J
2200 F2(N,M)=F2(N,M)+H5(N,M)*H6(N,M)*R9(N,M)
2210 F3(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)
2220 F3(N,M)=F3(N,M)*(J**2)*TAN(V)*TAN(S)
```

```
2230 F3(N,M)=F3(N,M)+H5(N,M)*H6(N,M)*R9(N,M)
2240 T1(N,M)=Z1(N)*P3(1,1)*(1/(K4(N)+G1(N)))*(1-EXP((K4(N)+G1(N))*D1(N)))
2250 T2(N,M)=Z2(N)*P3(2,1)*(1/(K4(N)-G1(N)))*(1-EXP((K4(N)-G1(N))*D1(N)))
2260 T3(N,M)=Z3(N)*P3(3,1)*(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1(N)))
2270 R1(N,M)=F1(N,M)*(T1(N,M)+T2(N,M)+T3(N,M))
2280 T4(N,M)=Z4(N)*P3(1,1)*(1/(K4(N)+G1(N)))*(1-EXP((K4(N)+G1(N))*D1(N)))
2290 T5(N,M)=Z4(N)*P3(2,1)*(1/(K4(N)-G1(N)))*(1-EXP((K4(N)-G1(N))*D1(N)))
2300 T6(N,M)=Z5(N)*P3(3,1)*(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1(N)))
2310 R2(N,M)=F2(N,M)*(T4(N,M)+T5(N,M)+T6(N,M))
2320 T7(N,M)=(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1(N)))
2330 R3(N,M)=F3(N,M)*P3(3,1)*T7(N,M)
2340 R4(N,M)=R1(N,M)+R2(N,M)+R3(N,M)
2350 R5(N)=R5(N)+R4(N,M)
2360 NEXT M
2370 R6(N)=R6(N-1)+K5(N)*R5(N)
2380 MAT P4=E*D
2390 MAT I=P4*P3
2400 NEXT N
2410 Q1=Z1(N1)*P3(1,1)*EXP(G1(N1)*D1(N1))
2420 Q2=Z2(N1)*P3(2,1)*EXP(-1*G1(N1)*D1(N1))
2430 Q3=Z3(N1)*P3(3,1)*EXP(K1(N1)*D1(N1))
2440 R7=K6(N1)*(Q1+Q2+Q3)
2450 R=R7+R6(N1)
2460 PRINT'THE RADIANCE IS';R/3.14159;'.'
2470 GO TO 2520
2480 print'this problem has no solution because frho(c11)=0.'
2490 GO TO 2520
2500 PRINT'THIS IS THE REPEATED ROOTS CASE WHERE K=G.  THIS SPECIAL'
2510 PRINT'CASE WAS NOT CONSIDERED.'
2520 END

READY
```

INF2 10:27 27-MAY-75

```
1000 PRINT'DO YOU WISH TO READ INSTRUCTIONS (YES OR NO)';
1010 INPUT A$
1020 IF A$<>'YES' THEN 1230
1030 PRINT'    THIS PROGRAM DETERMINES THE RADIANCE, AS PROPOSED BY'
1040 PRINT'GWYNN SUITS, FROM A SINGLE-LAYERED PLANT CANOPY WHOSE DEPTH'
1050 PRINT'IS CONSIDERED AS INFINITE.  THE PROGRAM WAS WRITTEN IN PARTIAL'
1060 PRINT'FULFILLMENT OF THE THESIS REQUIREMENTS OF JUAN M. CANTU, WHOSE'
1070 PRINT'GRADUATE ADVISOR WAS DR. JOE CHANCE.'
1080 PRINT'    THROUGHOUT THE FOLLOWING CALCULATIONS, THE SAME UNITS OF'
1090 PRINT'MEASUREMENTS SHOULD BE USED.  THAT IS, IF THE DEPTH IS IN CM.,'
1100 PRINT'THEN THE AREA IS IN CM.**2, AND THE VOLUME IS IN CM.**3.'
1110 PRINT'    WITHIN A SINGLE LAYER, THERE CAN EXIST SEVERAL COMPONENTS'
1120 PRINT'(LEAVES, STALKS, FRUITS, FLOWERS) EACH WITH ITS OWN SET OF'
1130 PRINT'PHYSICAL AND OPTICAL PROPERTIES.  THE AREA OF A COMPONENT IS'
1140 PRINT'DIVIDED INTO THE AREA PROJECTED ON A HORIZONTAL PLANE AND THE'
1150 PRINT'AREA PROJECTED ON TWO ORTHOGONAL VERTICAL PLANES.  IF THE AREA'
1160 PRINT'IS MEASURED IN CM.**2, THEN THE NUMBER OF HORIZONTAL AND VER-'
1170 PRINT'TICAL COMPONENTS PER UNIT VOLUME SHOULD BE IN 1/CM.**3.  THE'
1180 PRINT'TRANSMITTANCE (T) AND REFLECTANCE (R) OF A COMPONENT ARE UNIT-'
1190 PRINT'LESS AND 0<T<1, 0<R<1, 0<T+R<1.  THE SUN ANGLE AND VIEW ANGLE'
1200 PRINT'ARE MEASURED IN DEGREES.  BECAUSE OF THE CHOICE OF COORDINATE'
1210 PRINT'SYSTEM, THE DEPTH OF A LAYER SHOULD BE ENTERED AS A NEGATIVE'
1220 PRINT'NUMBER.'
1230 PRINT'ENTER THE WAVELENGTH IN NANOMETERS';
1240 INPUT J7
1250 PRINT'ENTER SUN ANGLE, VIEW ANGLE, AND NUMBER OF COMPONENTS';
1260 INPUT S2,V2,N2
1270 S=S2*3.14159/180
1280 V=V2*3.14159/180
1290 FOR M=1 TO N2
1300 PRINT 'ENTER AREA-HORIZONTAL, NUMBER-HORIZONTAL, AREA-VERTICAL,'
1310 PRINT'NUMBER-VERTICAL, REFLECTANCE, AND TRANSMITTANCE FOR COM-'
1320 PRINT'PONENT NUMBER';M
1330 INPUT H5(M),H6(M),V5(M),V6(M),R9(M),T(M)
1340 J=2/3.14159
1350 A(M)=H5(M)*H6(M)*(1-T(M))+V5(M)*V6(M)*(1-(R9(M)+T(M))/2)
1360 B(M)=H5(M)*H6(M)*R9(M)+V5(M)*V6(M)*((R9(M)+T(M))/2)
1370 C(M)=H5(M)*H6(M)*R9(M)+J*V5(M)*V6(M)*((R9(M)+T(M))/2)*TAN(S)
1380 C9(M)=H5(M)*H6(M)*T(M)+J*V5(M)*V6(M)*((R9(M)+T(M))/2)*TAN(S)
1390 K(M)=H5(M)*H6(M)+J*V5(M)*V6(M)*TAN(S)
1400 K2(M)=H5(M)*H6(M)+J*V5(M)*V6(M)*TAN(V)
1410 A1(M)=A1(M-1)+A(M)
1420 B1(M)=B1(M-1)+B(M)
1430 C1(M)=C1(M-1)+C(M)
1440 C2(M)=C2(M-1)+C9(M)
1450 K1(M)=K1(M-1)+K(M)
1460 K3(M)=K3(M-1)+K2(M)
1470 NEXT M
1480 Z1=((A1(N2)+K1(N2))*B1(N2)*C2(N2)+B1(N2)**2*C1(N2))
1490 Z2=B1(N2)**2+K1(N2)**2-A1(N2)**2
1500 IF Z2=0 THEN 1680
1510 Z=Z1/Z2
1520 G1=SQR(A1(N2)**2-B1(N2)**2)
1530 X1=1/(A1(N2)+G1)
1540 X2=((A1(N2)-K1(N2))*C1(N2)+C2(N2)*B1(N2))/(-1*Z2)
1550 X3=1/B1(N2)
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
1560 X4=((A1(N2)+K1(N2))*C2(N2)+B1(N2)*C1(N2))/(-1*Z2)
1570 FOR M=1 TO N2
1580 F1(M)=H5(M)*H6(M)*T(M)+V5(M)*V6(M)*((T(M)+R9(M))/2)*J*TAN(V)
1590 F2(M)=H5(M)*H6(M)*R9(M)+V5(M)*V6(M)*((T(M)+R9(M))/2)*J*TAN(V)
1600 F4(M)=V5(M)*V6(M)*((T(M)+R9(M))/2)*(J**2)*TAN(S)*TAN(V)
1610 F3(M)=F4(M)+H5(M)*H6(M)*R9(M)
1620 R1(M)=F1(M)*(X1*Z*(1/(G1+K3(N2)))+X2*(1/(K1(N2)+K3(N2))))
1630 R2(M)=F2(M)*(X3*Z*(1/(G1+K3(N2)))+X4*(1/(K1(N2)+K3(N2))))
1640 R3(M)=F3(M)*(1/(K1(N2)+K3(N2)))
1650 R4(M)=R4(M-1)+R1(M)+R2(M)+R3(M)
1660 NEXT M
1670 GO TO 1710
1680 PRINT"THIS IS THE REPEATED ROOTS CASE WHERE K=G.  THIS CASE WAS"
1690 PRINT"NOT CONSIDERED."
1700 GO TO 1720
1710 PRINT"THE RADIANCE IS";R4(N2)/3.14159;"."
1720 END

READY
```

```
COM2          10:23          27-MAY-75

1000 PRINT'DO YOU WISH TO READ INSTRUCTIONS (YES OR NO)';
1010 INPUT A$
1020 IF A$<>'YES' THEN 1240
1030 PRINT'     THIS PROGRAM DETERMINES THE RADIANCE FROM A PLANT CANOPY'
1040 PRINT'CONSISTING OF TWO LAYERS.  THE DEPTH OF THE FIRST LAYER IS'
1050 PRINT'FINITE WHILE THAT OF THE SECOND IS CONSIDERED AS INFINITE.'
1060 PRINT'THE PROGRAM WAS WRITTEN IN PARTIAL FULFULLMENT OF THE THESIS'
1070 PRINT'REQUIREMENTS OF JUAN M. CANTU, WHOSE GRADUATE ADVISOR WAS DR.'
1080 PRINT'JOE CHANCE.'
1090 PRINT'     THROUGHOUT THE FOLLOWING CALCULATIONS, THE SAME UNITS OF'
1100 PRINT'MEASUREMENTS SHOULD BE USED.  THAT IS, IF THE DEPTH IS IN CM.,'
1110 PRINT'THEN THE AREA IS IN CM.**2, AND THE VOLUME IS IN CM.**3.'
1120 PRINT'     WITHIN A SINGLE LAYER, THERE CAN EXIST SEVERAL COMPONENTS'
1130 PRINT'(LEAVES, STALKS, FRUITS, FLOWERS) EACH WITH ITS OWN SET OF'
1140 PRINT'PHYSICAL AND OPTICAL PROPERTIES.  THE AREA OF A COMPONENT IS '
1150 PRINT'DIVIDED INTO THE AREA PROJECTED ON A HORIZONTAL PLANE AND THE'
1160 PRINT'AREA PROJECTED ON TWO ORTHOGONAL VERTICAL PLANES.  IF THE AREA'
1170 PRINT'IS MEASURED IN CM.**2, THEN THE NUMBER OF HORIZONTAL AND VER-'
1180 PRINT'TICAL COMPONENTS PER UNIT VOLUME SHOULD BE IN 1/CM.**3.  THE'
1190 PRINT'TRANSMITTANCE (T) AND REFLECTANCE (R) OF A COMPONENT ARE UNIT-'
1200 PRINT'LESS AND 0<T<1, 0<R<1, 0<T+R<1.  THE SUN ANGLE AND VIEW ANGLE'
1210 PRINT'ARE MEASURED IN DEGREES.  BECAUSE OF THE CHOICE OF COORDINATE'
1220 PRINT'SYSTEM, THE DEPTH OF A LAYER SHOULD BE ENTERED AS A NEGATIVE'
1230 PRINT'NUMBER.'
1240 PRINT'ENTER WAVELENGTH IN NANOMETERS';
1250 INPUT J7
1260 PRINT'ENTER SUN ANGLE AND VIEW ANGLE';
1270 INPUT S2,V2
1280 S=S2*3.14159/180
1290 V=V2*3.14159/180
1300 FOR N=1 TO 2
1310 PRINT'FOR LAYER';N;'ENTER NUMBER OF COMPONENTS';
1320 INPUT N1(N)
1330 IF N=1 THEN 1350
1340 GO TO 1370
1350 PRINT'ENTER DEPTH OF FIRST LAYER';
1360 INPUT D1
1370 FOR M=1 TO N1(N)
1380 PRINT'ENTER AREA-HORIZONTAL, NUMBER-HORIZONTAL, AREA-VERTICAL'
1390 PRINT'NUMBER-VERTICAL, REFLECTANCE, AND TRANSMITTANCE FOR COMPONENT'
1400 PRINT'NUMBER';M
1410 INPUT H5(N,M),H6(N,M),V5(N,M),V6(N,M),R9(N,M),T(N,M)
1420 J=2/3.14159
1430 A(N,M)=V5(N,M)*V6(N,M)*(1-(R9(N,M)+T(N,M))/2)
1440 A(N,M)=A(N,M)+H5(N,M)*H6(N,M)*(1-T(N,M))
1450 B(N,M)=V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)
1460 B(N,M)=B(N,M)+H5(N,M)*H6(N,M)*R9(N,M)
1470 C(N,M)=J*V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)*TAN(S)
1480 C(N,M)=C(N,M)+H5(N,M)*H6(N,M)*R9(N,M)
1490 C9(N,M)=J*V5(N,M)*V6(N,M)*((R9(N,M)+T(N,M))/2)*TAN(S)
1500 C9(N,M)=C9(N,M)+H5(N,M)*H6(N,M)*T(N,M)
1510 K(N,M)=H5(N,M)*H6(N,M)+J*V5(N,M)*V6(N,M)*TAN(S)
1520 K2(N,M)=H5(N,M)*H6(N,M)+J*V5(N,M)*V6(N,M)*TAN(V)
1530 A1(N)=A1(N)+A(N,M)
1540 B1(N)=B1(N)+B(N,M)
1550 C1(N)=C1(N)+C(N,M)
1560 C2(N)=C2(N)+C9(N,M)
```

```
1570 K1(N)=K1(N)+K(N,M)
1580 K4(N)=K4(N)+K2(N,M)
1590 NEXT M
1600 K3(N)=EXP(K4(N-1)*D1)
1610 G1(N)=SQR(A1(N)**2-B1(N)**2)
1620 Z1(N)=1/(A1(N)+G1(N))
1630 Z2(N)=1/(A1(N)-G1(N))
1640 Z7(N)=A1(N)**2-K1(N)**2-B1(N)**2
1650 IF Z7(N)=0 THEN 2360
1660 Z3(N)=((A1(N)-K1(N))*C1(N)+B1(N)*C2(N))/(A1(N)**2-K1(N)**2-B1(N)**2)
1670 Z4(N)=1/B1(N)
1680 Z5(N)=((A1(N)+K1(N))*C2(N)+B1(N)*C1(N))/(A1(N)**2-K1(N)**2-B1(N)**2)
1690 MAT E=ZER(3,3)
1700 E(1,1)=Z1(N)
1710 E(1,2)=Z2(N)
1720 E(1,3)=Z3(N)
1730 E(2,1)=Z4(N)
1740 E(2,2)=Z4(N)
1750 E(2,3)=Z5(N)
1760 E(3,1)=0
1770 E(3,2)=0
1780 E(3,3)=1
1790 ON N GO TO 1800,1860
1800 MAT D=ZER(3,3)
1810 D(1,1)=EXP(G1(1)*D1)
1820 D(2,2)=EXP(-1*G1(1)*D1)
1830 D(3,3)=EXP(K1(1)*D1)
1840 MAT P1=E*D
1850 GO TO 1870
1860 MAT P2=INV(E)
1870 NEXT N
1880 MAT P=P2*P1
1890 IF P(2,1)-P(2,2)=0 THEN 2350
1900 X1(1)=(Z5(1)*B1(1)*P(2,2)-P(2,3))/(P(2,1)-P(2,2))
1910 X2(1)=-1*(X1(1)+Z5(1)*B1(1))
1920 X3(1)=1
1930 L1=X1(1)*EXP(G1(1)*D1)*(1/B1(1))+X2(1)*EXP(-1*G1(1)*D1)*(1/B1(1))
1940 L2=EXP(K1(1)*D1)*(Z5(1)-Z5(2))
1950 X1(2)=B1(2)*(L1+L2)
1960 X2(2)=0
1970 X3(2)=EXP(K1(1)*D1)
1980 FOR N=1 TO 2
1990 FOR M=1 TO N1(N)
2000 F1(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)*J*TAN(V)
2010 F1(N,M)=F1(N,M)+H5(N,M)*H6(N,M)*T(N,M)
2020 F2(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)*J*TAN(V)
2030 F2(N,M)=F2(N,M)+H5(N,M)*H6(N,M)*R9(N,M)
2040 T1(N,M)=V5(N,M)*V6(N,M)*((T(N,M)+R9(N,M))/2)
2050 T1(N,M)=T1(N,M)*(J**2)*TAN(V)*TAN(S)
2060 F3(N,M)=H5(N,M)*H6(N,M)*R9(N,M)+T1(N,M)
2070 IF N=1 THEN 2090
2080 GO TO 2210
2090 Q1(N)=Z1(N)*X1(N)*(1/(K4(N)+G1(N)))*(1-EXP((K4(N)+G1(N))*D1))
2100 Q2(N)=Z2(N)*X2(N)*(1/(K4(N)-G1(N)))*(1-EXP((K4(N)-G1(N))*D1))
2110 Q3(N)=Z3(N)*X3(N)*(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1))
2120 R1(N,M)=F1(N,M)*(Q1(N)+Q2(N)+Q3(N))
2130 Q4(N)=Z4(N)*X1(N)*(1/(K4(N)+G1(N)))*(1-EXP((K4(N)+G1(N))*D1))
2140 Q5(N)=Z4(N)*X2(N)*(1/(K4(N)-G1(N)))*(1-EXP((K4(N)-G1(N))*D1))
2150 Q6(N)=Z5(N)*X3(N)*(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1))
2160 R2(N,M)=F2(N,M)*(Q4(N)+Q5(N)+Q6(N))
2170 R3(N,M)=F3(N,M)*X3(N)*(1/(K1(N)+K4(N)))*(1-EXP((K1(N)+K4(N))*D1))
2180 R4(N,M)=R1(N,M)+R2(N,M)+R3(N,M)
2190 R5(N)=R5(N)+R4(N,M)
2200 GO TO 2300
2210 Q1(N)=Z1(N)*X1(N)*(1/(K4(N)+G1(N)))
2220 Q2(N)=Z3(N)*X3(N)*(1/(K1(N)+K4(N)))
```

```
2230 R1(N,M)=F1(N,M)*(Q1(N)+Q2(N))
2240 Q3(N)=Z4(N)*X1(N)*(1/(K4(N)+G1(N)))
2250 Q4(N)=Z5(N)*X3(N)*(1/(K1(N)+K4(N)))
2260 R2(N,M)=F2(N,M)*(Q3(N)+Q4(N))
2270 R3(N,M)=F3(N,M)*X3(N)*(1/(K1(N)+K4(N)))
2280 R4(N,M)=R1(N,M)+R2(N,M)+R3(N,M)
2290 R5(N)=R5(N)+R4(N,M)
2300 NEXT M
2310 R6(N)=R6(N-1)+K3(N)*R5(N)
2320 NEXT N
2330 PRINT'THE RADIANCE IS';R6(2)/3.14159;'.'
2340 GO TO 2380
2350 print'this problem has no solution because h=i.'
2360 PRINT'THIS IS THE REPEATED ROOTS CASE WHERE K=G. THIS SPECIAL'
2370 PRINT'CASE WAS NOT CONSIDERED.'
2380 END

READY
```

References

[1] W. A. Malila, R. H. Hieber, and J. E. Sarno, NASA Report, ERIM 190100-27-T. Contract No. NAS9-9784, TASK VI.

[2] G. H. Suits, "The Calculation of the Directional Reflectance of a Vegetative Crop", Remote Sensing of Environment 2 (1972).

[3] R. Oliver, J. Smith, "Vegetative Canopy Reflectance Models" Final Report, U. S. Army Research Office-Durham, DA-ARD-D-31-124-71-G165, June, 1973.

[4] A. Gausman, W. Allen, C. Wiegand, D. Escobar, R. Rodriguez, A. Richardson, "The Leaf Mesophylls of 20 Crops, Their Light Spectra, and Optical and Geometrical Parameters" U.S.D.A. NASA Contract R-09-038-002.

[5] W. Allen, T. Gayle, A. J. Richardson, "Plant-Canopy Irradiance Specified by the Duntley Equations", Journal of the Optical Society of America, 60, (1970).

[6] K. Jeske, "An Experimental Evaluation of a Crop Canopy Model Assumption", Senior Honors Research Project, Pan American University.

[7] W. Allen, A. Richardson, "Interaction of Light With a Plant Canopy", Journal of the Optical Society of America, 58, August, 1968.

[8] P. Kubelka, F. Munk, Z. Tech. Physik 12,593, (1931).

[9] H. Breece, R. Holmes, "Bi-directional Scattering Characteristics of Healthy Green Soybean and Corn Leaves in Vivo", Applied Optics, 10, Jan., 1971.

[10] M. Fuchs, G. Stanhill, A. Waanders, "Diurnal Variations of the Visible and Near Infrared Reflectance of a Wheat Crop", Israel J. Agric. Res. 22, July, 1972.

[11] L. Jones, H. Condit, "Sunlight and Skylight as Determinants of Photographic Exposure", J. Opt. Soc. Am., 38(2), 1948.