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ELASTIC DEFORMATION OF THE GYRO ROTOR IN THE RELATIVITY GYRO EXPERIMENT

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16. ABSTRACT

The elastic deformation of a spinning sphere is calculated from first principles. Numerical results are quoted for a quartz sphere spinning at 200 Hz, the most likely configuration for the Relativity Gyro Experiment. The difference in principal moments of inertia is also calculated for the spinning gyro.

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LIST OF TERMS

e_{ij}	-	strain tensor
Θ	-	dilatation
τ_{ij}	-	stress tensor
λ, μ	-	Lamé constants
u_r	-	radial displacement
u_θ	-	angular displacement
ν	-	Poissons ratio
E	-	Young's modulus
Δ	-	equatorial minus polar radius
a	-	undeformed rotor radius
ω	-	angular rotation frequency
ρ	-	density of rotor
a_2	-	coefficient of second harmonic of rotor
ΔI	-	principal moment of inertia difference
I	-	moment of inertia of rotor

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ELASTIC DEFORMATION OF THE GYRO ROTOR IN THE RELATIVITY GYRO EXPERIMENT

INTRODUCTION

The elastic deformation of a spinning sphere is an old problem which was first solved by Chree in 1889 [1]. This solution is needed to evaluate the electrical torques on the rotor in the Stanford Gyro experiment. In this experiment, the elastic deformation due to spin will produce nonradial forces on the ball which will lead to drift-producing torques. These torques must be understood and calculated to estimate the ultimate accuracy of the experiment.

In the original derivation by Chree [1] and in subsequent treatments [2, 3], the results are exhibited without a derivation. A simplified derivation which points out the assumptions which are made in the derivation is presented here. Spherical coordinates are used so that the end result can readily be used as an input to the electrostatic torque calculation. Numerical results are presented for the most likely configuration in the gyro experiment, a quartz ball spinning at 200 Hz. The fractional change in difference of principal moments of inertia $\Delta I/I$ is then calculated including density changes in the ball. This result is needed for the gravity gradient torque calculations.

DEFORMATION CALCULATION

The problem is attacked by solving the Navier equations in elasticity theory with the appropriate boundary conditions. These equations must be written and solved in spherical coordinates because for our electrostatic torque calculations, the shape of the ball must be developed as a Fourier cosine series in the polar angle θ. To verify that the shape contains only an $a_2 \cos 2\theta$ term and to calculate a_2, we must work in polar coordinates. We start from first principles to bring out any approximations or assumptions made in the derivation. This presents a problem since most books give the equations only in cartesian coordinates. The simplest way to find the equations in spherical coordinates is to write the equations in their general form with covariant derivatives present and then find the form of the covariant derivatives in spherical coordinates. To begin, one calculates the strain tensor e_{ij} in terms of the displacements u_i using

$$e_{ij} = \frac{1}{2}\left[\frac{Du_i}{Dx^j} + \frac{Du_j}{Dx^i}\right] \tag{1}$$

where D/Dx^i denotes covariant derivatives. When this is done and the identification of the physical components of u_i and e_{ij} is made, one can obtain equation 48.17 of Sokolnikoff [4].

In spherical coordinates, the line element and Christoffel symbols are

$$ds^2 = dr^2 + r^2[d\theta^2 + \sin^2\theta d\phi^2]$$

$$\Gamma^2_{33} = -\sin\theta\cos\theta \quad \Gamma^1_{22} = -r \quad \Gamma^1_{33} = -r\sin^2\theta$$

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r} \quad \Gamma^3_{23} = \cot\theta \quad .$$

Identifying the "physical components" [5] of u_i as $u_r = u_1$, $u_\theta = u_2/r$, and $u_\phi = u_3/r\sin\theta$, one finds for the physical components of e_{ij} from equation (1):

$$e_{rr} = \frac{\partial u_r}{\partial r}$$

$$e_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_r}{r}$$

$$e_{\phi\phi} = \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial\phi} + \frac{\cot\theta}{r}u_\theta + \frac{u_r}{r} \tag{2}$$

$$e_{r\theta} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right]$$

$$e_{r\phi} = \frac{1}{2}\left[\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r}\right]$$

$$e_{\theta\phi} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial u_\phi}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi} - \frac{\cot\theta}{r}u_\phi\right] \tag{2}$$

(cont' d)

where the physical components of e_{ij} are $\sqrt{g^{ii}}\sqrt{g^{jj}}\,e_{ij}$. The dilatation Θ is given by

$$\Theta = g^{ij}e_{ij} = \frac{\partial u_r}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{2u_r}{r} + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{\cot\theta}{r}u_\theta \quad .$$

The equations of equilibrium in tensor notation are given by

$$g^{jk}\frac{D\tau_{ij}}{Dx^k} + F_i = 0 \quad , \tag{3}$$

where τ_{ij} is the stress tensor and F_i is the body force vector, in this case representing centrifugal force. The first term in this equation must be calculated in spherical coordinates. Using the definition of covariant derivative and the Christoffel symbols, the following equations are obtained:

$$\frac{\partial\tau_{rr}}{\partial r} + \frac{2}{r}\tau_{rr} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\tau_{r\phi}}{\partial\phi} - \frac{1}{r}(\tau_{\theta\theta} + \tau_{\phi\phi}) + \frac{1}{r}\cot\theta\,\tau_{r\theta} = -F_r \tag{4}$$

$$\frac{\partial\tau_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\tau_{\theta\phi}}{\partial\phi} + \frac{\cot\theta}{r}\tau_{\theta\theta} - \frac{\cot\theta}{r}\tau_{\phi\phi} + \frac{3\tau_{r\theta}}{r} = -F_\theta \tag{5}$$

$$\frac{\partial \tau_{r\phi}}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{1}{r} \frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{3\tau_{r\phi} + 2\tau_{\theta\phi} \cot \theta}{r} = -F_{\phi} \quad . \tag{6}$$

These equations contain the physical components of τ_{ij} and agree with equation 48.18 of Sokolnikoff [4].

To obtain equations containing only the u_i's (the Navier equations), the τ_{ij}'s must be expressed in terms of the u_i's. This is done by the generalized Hooke's Law valid for a homogeneous isotropic body

$$\tau_{ij} = \lambda \Theta g_{ij} + 2\mu e_{ij}$$

where λ and μ are the Lamé constants. Using equation (2), we obtain for the physical components of τ_{ij}:

$$\begin{aligned}
\tau_{rr} &= \lambda\Theta + 2\mu \frac{\partial u_r}{\partial r} \\
\tau_{\theta\theta} &= \lambda\Theta + 2\mu \left[\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right] \\
\tau_{\phi\phi} &= \lambda\Theta + 2\mu \left[\frac{u_r}{r} + \frac{\cot\theta}{r} u_\theta\right] \\
\tau_{r\theta} &= \mu \left[\frac{1}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r} + \frac{\partial u_\theta}{\partial r}\right]
\end{aligned} \tag{7}$$

where we have assumed that the u_i's are independent of ϕ (and $u_\phi = 0$). These equations can now be substituted into equation (4), from which we obtain

$$(\lambda + 2\mu) \frac{\partial \Theta}{\partial r} - \frac{\mu}{r^2 \sin \theta} \frac{\partial}{\partial \Theta} \sin \theta \frac{\partial}{\partial r} (r u_\theta) + \frac{\mu}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial u_r}{\partial \theta}\right) = -F_r \quad . \tag{8}$$

Substituting equation (7) into equation (5), we obtain

$$\frac{1}{r}\frac{\partial}{\partial\theta}(\lambda + 2\mu)\Theta + \frac{\mu}{r}\frac{\partial^2}{\partial r^2}(ru_\theta) - \frac{\mu}{r}\frac{\partial^2 u_r}{\partial r\partial\theta} = -F_\theta \quad . \tag{9}$$

Equations (8) and (9) are the Navier equations of elastic equilibrium written in spherical coordinates with no ϕ dependence. They agree with equation 110' of Honeywell's report No. 20831 FR [6].

They must be solved for the case of a body force given by the centrifugal force on the spinning ball. The solution must satisfy the boundary condition that the surface of the spinning sphere being considered is free from stress. That is, at $r = a$, we must have

$$\tau_{rr} = 0 \ , \ \tau_{r\theta} = 0 \quad . \tag{10}$$

The body force can be derived from a scalar potential given by

$$V = \frac{1}{2}\rho\omega^2 r^2 \sin^2\theta \quad ,$$

where ρ is the density of the ball and ω its angular velocity. Then, in spherical coordinates,

$$\begin{aligned} F_r &= \frac{\partial V}{\partial r} = \rho\omega^2 r \sin^2\theta \\ F_\theta &= \frac{1}{r}\frac{\partial V}{\partial\theta} = \rho\omega^2 r \sin\theta\cos\theta \quad . \end{aligned} \tag{11}$$

Now the solutions to equations (8) and (9) are written with the body force given by equation (11). This solution satisfies the boundary conditions of equation (10):

$$u_r = \rho\omega^2 a^2 \left\{ P_2(\cos\theta) \left[\alpha_1 \frac{r^3}{a^2} + \alpha_2 r \right] + \left[\alpha_3 \frac{r^3}{a^2} + \alpha_4 r \right] \right\}$$

$$u_\theta = \rho\omega^2 a^2 \sin\theta \cos\theta \left[\beta_1 \frac{r^3}{a^2} + \beta_2 r \right] \quad , \tag{12}$$

where

$$\alpha_1 = \frac{3\lambda + 2\mu}{3\mu(19\lambda + 14\mu)}$$

$$\alpha_2 = -\frac{2(4\lambda + 3\mu)}{3\mu(19\lambda + 14\mu)}$$

$$\alpha_3 = -\frac{1}{15(\lambda + 2\mu)}$$

$$\alpha_4 = \frac{(5\lambda + 6\mu)}{15(\lambda + 2\mu)(3\lambda + 2\mu)}$$

$$\beta_1 = -\frac{(5\lambda + 4\mu)}{2\mu(19\lambda + 14\mu)}$$

$$\beta_2 = \frac{4\lambda + 3\mu}{\mu(19\lambda + 14\mu)}$$

$$P_2(\cos\theta) = \frac{1}{2}[3\cos^2\theta - 1] \quad .$$

It can be verified that this is a solution of equation (8) as follows. First, if Θ is calculated from this solution,

$$(\lambda + 2\mu)\frac{\partial\Theta}{\partial r} = \rho\omega^2 a^2 \left\{ (\lambda + 2\mu) \left[P_2(\cos\theta)(5\alpha_1 + 2\beta_2)\frac{2r}{a^2} + 10\alpha_3 \frac{r}{a^2} \right] \right\} \quad .$$

The other terms in equation (8) give

$$-\frac{\mu}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\sin\theta\,\frac{\partial}{\partial r}(ru_\theta) = -\rho\omega^2 a^2\left\{-\mu\left(4\beta_1\frac{r}{a^2}+2\beta_2\frac{1}{r}\right)2P_2(\cos\theta)\right\}$$

$$\frac{\mu}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial u_r}{\partial\theta}\right) = \rho\omega^2 a^2\left\{-3\mu\left(\alpha_1\frac{r}{a^2}+\alpha_2\frac{1}{r}\right)2P_2(\cos\theta)\right\} \quad .$$

Adding these three terms and using the identities

$$\alpha_1(5\lambda+7\mu) + \beta_1(2\lambda) = \frac{1}{3}$$

$$10(\lambda+2\mu)\,\alpha_3 = -\frac{2}{3}$$

we find that the left side of equation (8) gives $-\rho\omega^2 r\sin^2\theta$ in agreement with equation (11). The three terms in equation (9) are

$$\frac{\partial^2}{\partial r^2}(ru_\theta) = \rho\omega^2 a^2\left\{\sin\theta\cos\theta\left[12\beta_1\frac{r^2}{a^2} + 2\beta_2\right]\right\}$$

$$\frac{\partial^2 u_r}{\partial r\partial\theta} = \rho\omega^2 a^2\left\{-3\cos\theta\sin\theta\left[3\alpha_1\frac{r^2}{a^2} + \alpha_2\right]\right\}$$

$$\frac{\partial\Theta}{\partial\theta} = \rho\omega^2 a^2\left\{-3\cos\theta\sin\theta\left[(5\alpha_1+2\beta_1)\frac{r^2}{a^2}\right]\right\} \quad .$$

Combining these terms, we verify that the left side of equation (9) gives $-\rho\omega^2 r^2\sin\theta\cos\theta$ in agreement with equation (11). Therefore, this verifies that our solution satisfies equations (8) and (9). It remains to show that the solution also satisfies the boundary conditions in equation (10). Using equation (7) with the differentiated form of equation (12), we find that at $r = a$

$$\tau_{rr} = \rho\omega^2 a^2 \{ P_2(\cos\theta) [\alpha_1(5\lambda + 6\mu) + 2\lambda\beta_1 + 2\mu\alpha_1]$$

$$+ [\alpha_3(5\lambda + 6\mu) + \alpha_4(3\lambda + 2\mu)]\} \quad .$$

The second square bracket is obviously zero and, using the expression for α_1, β_1 and α_2, the first square bracket is also zero. Again, using equation (7) and the differential form of equation (12), we find that at $r = a$

$$\tau_{r\theta} = \mu\rho^2\alpha^2 \sin\theta \cos\theta \{-3\alpha_1 - 3\alpha_2 + 2\beta_1\} \quad ,$$

and this is also zero by the definition of α_1, α_2, and β_1. This demonstrates that equation (12) satisfies both boundary conditions in equation (10).

Evaluating the solution at $r = a$ gives

$$u_r = \frac{2\rho\omega^2 a^3}{3E} \left\{ -P_2(\cos\theta) \frac{(1 + \nu)(2 + \nu)}{(7 + 5\nu)} + \frac{(1 - 2\nu)}{5} \right\} \quad , \tag{13}$$

where we have used the definitions of the Lamé constants in terms of Young's modulus and Poisson's ratio:

$$\nu = \frac{\lambda}{2(\lambda + \mu)}$$

$$\mu = \frac{E}{2(1 + \nu)} \quad .$$

From this we note that the fractional difference Δ between the equatorial and polar radii is

$$\Delta = \frac{\left[u_r\left(\theta = \frac{\pi}{2}\right) - u_r(\theta = 0) \right]}{a} = \frac{\rho\omega^2 a^2}{E} \left[\frac{(1 + \nu)(2 + \nu)}{7 + 5\nu} \right] \quad .$$

We also note that since

$$P_2(\cos\theta) = \frac{3}{2}\left(\frac{1+\cos 2\theta}{2}\right) - \frac{1}{2} \quad ,$$

the coefficient of the $\cos 2\theta$ term, which we will call a_2, is given by

$$a_2 = -\frac{\rho\omega^2 a^3}{2E}\left[\frac{(1+\nu)(2+\nu)}{7+5\nu}\right] = -\frac{1}{2}\Delta a \quad .$$

The coefficient a_2 is necessary to calculate electric torques on an electrostatic gyro since they are calculated in terms of harmonic coefficients of which a_2 is the lowest.

Now we can compute Δ and a_2 numerically for a quartz ball spinning at 200 Hz. We will use the following parameters:

$$\rho = 2.2\ \mathrm{g/cm^3}$$

$$\omega = 200\ \mathrm{cps}$$

$$a = 0.75\ \mathrm{in.} = 1.9\ \mathrm{cm}$$

$$E = 10.4 \times 10^6\ \mathrm{lb/in.^2} = 7.17 \times 10^{11}\ \mathrm{dynes/cm^2}$$

$$\nu = 0.16$$

$$\frac{\rho\omega^2 a^2}{E} = 1.76 \times 10^{-5}$$

$$\Delta = 5.62 \times 10^{-6}$$

$$a_2 = -2.1\ \mu\mathrm{in.} = -5.33 \times 10^{-6}\ \mathrm{cm}$$

Both these quantities scale as ω^2.

MOMENT OF INERTIA CALCULATION

Using the results of the last section, we can calculate the difference in the principal moments of inertia produced by the centrifugal expansion. This calculation is needed to evaluate the effects of gravity gradient torques on the gyro experiment. In the expression for gravity gradient torques, the quantity $\Delta I/I$ is needed, where

$$\Delta I = I_{33} - \frac{1}{2}(I_{11} + I_{22}) = \frac{1}{2}\int dv[r^2 - 3z^2]\rho(r) \tag{14}$$

and I_{11}, I_{22}, and I_{33} are the principal moments of inertia. There are two contributions to $\Delta I/I$: the first due to the oblateness of the ball, and the second due to density changes in the ball. The first effect is calculated as follows: the expansion of the ball given by $u_r(\theta)$ produces an extra infinitesimal element dm given by

$$dm = \rho u_r(\theta)(2\pi a \sin\theta)(a d\theta)$$

and one then needs to calculate

$$\Delta I = \frac{1}{2}\int a^2[1 - 3\cos^2\theta]\, dm = a^2 \int - P_2(\cos\theta)\, dm$$

where P_2 is the second Legendre polynomial. Using equation (13), this becomes

$$\Delta I = \frac{2\rho^2\omega^2 a^7}{3E}\left\{\frac{(1+\nu)(2+\nu)}{(7+5\nu)}\int_0^\pi [P_2(\cos\theta)]^2\, 2\pi\sin\theta\, d\theta - \frac{1-2\nu}{5}\int_0^\pi [P_2(\cos\theta)]\, 2\pi \sin\theta\, d\theta\right\}.$$

The second integral is zero because of the orthogonality of Legendre polynomials ($P_0 = 1$), and the first integral is equal to $4\pi/5$ by the normalization integrals for P_2. Hence,

$$\Delta I = \frac{8\pi\rho^2\omega^2 a^7}{15E}\left[\frac{(1+\nu)(2+\nu)}{(7+5\nu)}\right]$$

and, since $I = 8/15\ \pi\rho a^5$, we finally have

$$\frac{\Delta I}{I} = \frac{\rho\omega^2 a^2}{E}\ \frac{(1+\nu)(2+\nu)}{(7+5\nu)} = \frac{\rho\omega^2 a^2}{2\mu}\left(\frac{5\lambda+4\mu}{19\lambda+14\mu}\right)$$

which is equal to Δ. Therefore, we have the result that for the distorted sphere of uniform density, $\Delta I/I = \Delta$ [3]. However, in actuality, there will be a change in density when the gyro is spun up which will also produce a contribution to $\Delta I/I$. This is given by

$$\Delta I' = -2\pi \int_0^\pi \sin\theta\, d\theta \int r^2\, dr\ \frac{r^2}{2}\,[1 - 3\cos\theta]\rho\Theta \quad ,$$

where the integral is now over the entire volume of the rotor (assumed spherical) and $-\Theta$ is the fractional density change $\Delta\rho/\rho$. Using the expression for Θ computed from equation (12), this integral becomes

$$\Delta I' = \frac{2\pi\rho\omega^2 a^2}{E}\int_0^\pi r^2 dr \left\{\int_0^\pi [P_2(\cos\theta)]^2 \sin\theta\, d\theta\left[-\frac{2}{3}\,\frac{(1+\nu)(1-2\nu)}{(7+5\nu)}\,\frac{r^2}{a^2}\right]\right.$$

$$+ \int_0^\pi [P_2(\cos\theta)] \sin\theta\, d\theta\left[-\frac{1}{3}\,\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\,\frac{r^2}{a^2}\right.$$

$$\left.\left. + \frac{1}{5}\,\frac{(3-\nu)(1-2\nu)}{(1-\nu)}\right]\right\} .$$

As before, the second θ integral is zero and the first is 2/5; therefore, this expression gives

$$\Delta I' = \frac{-4\pi\rho^2\omega^2a^7}{21E} \frac{(1+\nu)(1-2\nu)}{(7+5\nu)} \frac{2}{5}$$

$$\frac{\Delta I'}{I'} = \frac{-\rho\omega^2a^2}{7E} \frac{(1+\nu)(1-2\nu)}{(7+5\nu)} = \frac{-\rho\omega^2a^2}{7} \frac{1}{19\lambda + 14\mu} \quad .$$

This gives for the total change in $\Delta I/I$

$$\frac{\Delta I}{I} = \frac{\rho\omega^2a^2}{14\mu} \frac{35\lambda + 26\mu}{19\lambda + 14\mu}$$

which is agreement with Reference 3. This gives a reduction in $\Delta I/I$ of

$$\frac{1-2\nu}{7(2+\nu)} = 4.5\% \quad .$$

Therefore, for the quartz sphere at 200 Hz, we have

$$\frac{\Delta I}{I} = 5.37 \times 10^{-6} \quad .$$

While this is a small number, the usual expression for gravity gradient torques involving $\Delta I/I$ indicates that they will contribute one of the largest torques on the gyro in the final gyro experiment. For this reason it is important to know the actual magnitude of the $\Delta I/I$ term accurately.

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APPROVAL

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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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