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"Sound Propagation from a Simple Source in a Wind Tunnel"

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A. Introduction and Overall Summary

The objective of the work performed under NASA Grant NGR 22-012-031 has been to examine both theoretically and experimentally the nature of the acoustic field of a simple source in a wind tunnel under flow conditions. The motivation of the study has been to seek to establish aspects of the theoretical framework for interpreting acoustic data taken in wind tunnels using "in wind" microphones.

There are three distinct investigations which have been performed under this grant. In the first which is described in Section B, the wind tunnel is modeled by a rectangular duct of infinite length. A point source is located inside the duct, and there is a uniform flow in the duct. The walls of the duct are characterized by an acoustic impedance. A steady-state sound field is assumed to exist in the duct. Owing to mathematical difficulties of this problem, the solution for the acoustic pressure field has not been previously calculated. An exact solution to this boundary-value problem however has been obtained in this work using a numerical procedure. In this solution there is no limitation placed on the value of the acoustic impedance of the walls. Hence, the effects of both acoustically hard and soft walls may be investigated. Owing to computer limitations, the maximum source frequency is approximately 250 Hz for a duct of dimensions 2 meters by 3 meters. As such this exact solution describes a "near-field" situation in a wind tunnel test.

The second investigation which is described in Section C, is also theoretical. The acoustic pressure, acoustic power, and Doppler shift in frequency of a moving point source in a moving acoustic medium are calculated. The subsonic source and medium speeds are arbitrary. This is a far-field calculation which neglects all wall effects. As such it pertains to high frequency measurements of noise using "in wind" microphones which are placed within the "hall" radius of the wind tunnel. This situation may be viewed as a simple model of jet noise measurements using "in wind" microphones. In part of this work, an existing expression for the acoustic intensity in a uniformly moving medium has been shown to be inappropriate for this free-field calculation.

Described in Section D is the third study which is an experimental investigation of pulse propagation in the 2.13 meter by 3.05 meter (7 feet by 10 feet) wind tunnel of the NASA Ames Research Center. Low frequency acoustic pulses are recorded by microphones in the presence of flow in the wind tunnel. These data are compared with the results of a theoretical model of pulse propagation in a long duct with acoustically hard walls. Both the experimental results and the theoretical results are in very good agreement with each other.

Detailed accounts of these investigations either currently appear or will appear at a later date in various forms. Some preliminary work on the material described in Section B appears in an article entitled "Acoustic Field of a Point Mass Source in an Infinitely Long Rectangular Duct with Flow" by J. E. Cole and I. I. Sarris published in the Proceedings of the Second Interagency Symposium on University Research in Transportation Noise (June, 1974). An article entitled "Theoretical Results Pertinent to Measurements of Jet Noise in Wind Tunnels" by J. E. Cole and I. I. Sarris which is based on the results of Section C and their relationship with jet noise measurements in wind tunnels appears in the Proceedings of the Third Interagency Symposium on

University Research in Transportation Noise (November 1975). A manuscript based on Section D and entitled "Acoustic Pulse Propagation in a Duct with Flow" by J. E. Cole was submitted in October 1975 to the Journal of Sound and Vibration for publication. A letter to the editor based on part of the work in Section C and entitled "Acoustic power of a moving source in a moving medium" by J. E. Cole and I. I. Sarris was submitted in October 1975 to the Journal of the Acoustical Society of America for publication. A detailed description of the work of Sections A and B will be included in a doctoral dissertation of I. I. Sarris who has been supported as a Research Assistant by this grant. It is also anticipated that a manuscript on the material of Section A will be prepared subsequently for publication. Finally, various aspects of this work were discussed in two semi-annual progress reports dated February 28, 1974 and October 15, 1974.

The NASA Technical officer for this grant is Warren F. Ahtye, Large-Scale Aerodynamics Branch, NASA Ames Research Center.

B. Low-to-mid Frequency Theoretical Solution

In this section the calculation is described of the acoustic field of a point source located in a long rectangular duct carrying a uniform flow. For this calculation the walls of the duct may be characterized by a constant acoustic impedance. In order to investigate the acoustic field of the sound source, the following assumptions are made:

(1) The medium is homogeneous, inviscid, non-heat conducting and in uniform motion.

(2) The source of sound is a fluctuating point mass.

(3) The duct is of rectangular cross section and of infinite length.

(4) The acoustic characteristics of the walls of the duct are represented by a constant impedance. The two pairs of opposite walls may have different acoustic impedances.

(5) The acoustic quantities have a harmonic time dependence.

The first two assumptions allow the linearized continuity and momentum equations to be combined to provide the following governing equation:

$$\frac{1}{a_o^2}\frac{D^2p}{Dt^2} - \nabla^2 p = \frac{Dq}{Dt} \tag{1}$$

where a_o is the speed of sound in the medium at rest, $p = p(x,y,z,t)$ is the acoustic pressure disturbance, ∇ is the three-dimensional gradient operator, $\frac{D}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x}$ is the "convective" time derivative, and U is the speed of the acoustic medium in the positive x-direction. For a harmonic point mass source located at (x_o,y_o,z_o) the source strength q is given by

$$q(x,y,z,t) = q_o e^{i\omega t}\,\delta(x-x_o)\delta(y-y_o)\delta(z-z_o) \tag{2}$$

where q_o is the mass rate, ω is the source driving frequency and δ is the Dirac delta function.

In accordance with the fourth assumption, the boundary conditions to be satisfied by the acoustic pressure at the walls of the duct are:

$$\frac{Dp}{Dt} \pm \frac{a_o}{\beta_y} \frac{\partial p}{\partial y} = 0 \quad @ \quad y = \pm \frac{b}{2}$$
$$\frac{Dp}{Dt} \pm \frac{a_o}{\beta_z} \frac{\partial p}{\partial z} = 0 \quad @ \quad z = \pm \frac{d}{2} \tag{3}$$

where b and d are the height and width of the duct and β_y, β_z are the specific acoustic admittances of the walls perpendicular to the y and z axes, respectively.

In a cartesian coordinate system the solution to equation (1) for the acoustic pressure can be written as

$$p(x,y,z,t) = \sum_{m=o}^{\infty} \sum_{n=o}^{\infty} A_{mn} \phi_{mn} (y,z) e^{-ik_x(x-x_o)} e^{i\omega t} \tag{4}$$

where $\phi_{mn}(y,z) = \begin{matrix}\cos\\ \sin\end{matrix} \left(\frac{\pi q_{ym}}{b} y\right) \begin{matrix}\cos\\ \sin\end{matrix} \left(\frac{\pi q_{zn}}{d} z\right)$, and m and n are integer indices for each mode. The cosine function is chosen for symmetric modes (even m,n) and the sine function for antisymmetric modes (odd m,n). A_{mn} is the modal amplitude coefficient and k_x, $\pi q_{ym}/b$ and $\pi q_{zn}/d$ are characteristic values, all to be determined subsequently. It is assumed that the characteristic functions (ϕ_{mn}'s) form a complete set. Only then can the acoustic pressure field be expressed in the series form given by equation (4). The characteristic functions however do not in general form an orthogonal set. The set of functions is orthogonal in the two special cases of either zero medium velocity or perfectly rigid duct walls. The nonorthogonality of the ϕ_{mn}'s impedes the use of analytical methods to obtain a simple closed-form solution for acoustic pressure. A numerical method of solution is therefore suggested.

The numerical scheme involves the calculation of three fundamental groups of quantities: a) the characteristic values, b) the modal amplitude coefficients and c) the total acoustic pressure. In order to determine the characteristic values for each mode, equation (4) is substituted into the boundary conditions (3) and into the homogeneous counterpart of the governing equation (1). The resulting nonlinear system of algebraic equations is solved numerically for k_x, $\pi q_{ym}/b$, and $\pi q_{zn}/d$ by means of the Newton-Raphson iteration technique. Analytic expressions derived for "slightly" soft walls are used as initial guesses in order to compute the exact modal characteristic values for any given values of β_y, β_z, and M.

The amplitude coefficients downstream (A^+_{mn}) and upstream (A^-_{mn}) of the source, are calculated by substituting equation (4) into the governing equation (1). This equation is then integrated across and along the duct. Application of pressure and pressure gradient discontinuity conditions at the source location in the resulting equation gives two sets of linear algebraic equations with an infinite number of unknowns $(A^+_{mn}$ and $A^-_{mn})$. This sytem of equations can be truncated at a number of terms (modes) which insures convergence of the solution. The truncated system of equations is then solved numerically for the amplitude coefficients.

Once the characteristic values and amplitude coefficients for each mode upstream and downstream are found, the modal pressure can immediately be determined. Finally, summation of the modal pressures according to equation (4) produces the total acoustic pressure at any location in the duct.

The input quantities needed for the above calculations are the duct dimensions, the wall specific acoustic admittances, the flow Mach number, the source location, and the driving frequency. The quantities printed out by the computer program may include the characteristic values and functions, modal amplitudes, modal pressures and total mean-square pressure at any point inside the duct. The computer program at its present stage is limited by core size to low and medium frequency calculations. A typical run using 35 modes requires less than 30 seconds of central processing time.

Several aspects of the acoustic field in a duct with flow have been investigated using this solution capability. Comparable solutions do not exist in the literature except for the two simple cases which yield orthogonal sets of characteristic functions. As a result, the investigations have been of an exploratory nature examining the variation of the mean-square pressure in the duct with frequency, source position, and wall impedance. The present capability of the computer allows approximately 30 modes to be summed. For a duct with a cross section of 2 meters by 3 meters this allows a maximum frequency of approximately 250 Hz to be investigated. Hence, any results would find applicability to measurements in the near-field of an acoustic source.

Based on results from a limited number of test cases, the following qualitative statements can be made:

1) For axial distances greater than 2-3 wavelengths from the source, medium motion reduces the acoustic pressure upstream and enhances it downstream when compared to the no flow case.

2) For hard walls the opposite is true, i.e., the acoustic pressure upstream is higher than that for the no flow case, while acoustic

pressure downstream is lower, as expected.

3) Acoustic pressure levels throughout a duct with hard walls are higher than levels in a duct whose walls have finite admittance. This holds for the particular wall admittance tested and for axial distances greater than one wavelength.

4) An increase in the source driving frequency initially increases the acoustic pressure levels across the duct.

5) The acoustic field in the duct is sensitive to source location.

6) In all cases, the acoustic pressure variations across planes perpendicular to the duct axis are more pronounced close to the source and damp out away from it.

The above conclusions are drawn on the basis of test runs for a 2 meter by 3 meter duct with $M = 0, .3$ and $\beta = 0, .1$. The source driven at 100 Hz and 200 Hz was located at $y_o = z_o = 0$ (center of the duct) and $y_o = .30m$, $z_o = .35m$.

C. High Frequency Theoretical Solution

In this section a simple theoretical model is described for jet noise measurements in a wind tunnel. For such measurements sound from moving acoustic sources (i.e., turbulent eddies) is measured by microphones located in a moving acoustic medium. A simple theoretical model for this situation is an acoustic point mass source which moves with constant speed through a uniform acoustic medium moving in the same direction with constant speed. This model is used to calculate explicitly the far-field acoustic pressure, the Doppler shift in frequency, and the acoustic power output of the source. Since the effects of the wind tunnel walls are absent from this model, the results are applicable to high frequencies where far-field measurements may be obtained within the "hall radius" of the wind tunnel.

The convective wave equation which describes the acoustic field of a mass source in a uniformly moving acoustic medium has been given previously by equation (1) in Section B. A point mass source which moves with constant speed V in the same direction as the medium is specified by expressing the source strength as follows:

$$q(x,y,z,t) = q(t)\delta(x-Vt)\delta(y)\delta(z) \ . \tag{5}$$

Since there are no boundaries in this model, the convective wave equation with the source strength specified above is solved for outgoing waves. The solution for the acoustic pressure is obtained by a standard method using a series of coordinate transformations. In order to correspond with "in wind" measurements of jet noise, the expression for the acoustic pressure is expressed in terms of the distance and angle between the "in wind" observer and the source at the time that the source emits the signal. The "Doppler shifted" frequency which is observed owing to the relative motion of the source and the observer is also calculated from the solution.

The relationship between far-field measurements of the acoustic pressure and the total power output of the source may be obtained by using the expression for the acoustic intensity. Two different expressions appear in the literature for the acoustic intensity for a uniformly moving acoustic medium. In order to ascertain the appropriateness of either of these expressions to the problem under consideration, both were used to obtain the corresponding results for the acoustic power.

The results calculated from this model indicate a relatively strong influence of the medium motion on the acoustic field of a moving point source. The directional dependence of the mean-square acoustic pressure in the far field is easily calculated from the solution. For a constant source Mach number, the mean-square pressure decreases for all observer locations as the value of the medium Mach number increases. This occurs even though the source strength remains unchanged. If the medium Mach number is different from zero, the mean-square pressure measured at an angle of ninety degrees to the source direction retains a dependence on both source and medium Mach numbers.

For a constant source Mach number, the ratio of observed to emitted frequency is found to decrease for all observer positions as the medium Mach number is increased. There is a Doppler shift in frequency found at an angle of ninety degrees to the source direction when the medium is in motion.

Of the two expressions for the acoustic power which correspond with the two definitions of the acoustic intensity, one expression contains results which are unreasonable. The second expression gives the acoustic power output as a function of both the relative speed between the source and medium and the medium speed. This suggests that only the latter expression for the acoustic intensity is acceptable for the model calculation and perhaps for any free-field calculation.

D. Experimental Investigation with Analytical Comparison

Aspects of sound propagation in the 2.13 meter by 3.05 meter wind tunnel of NASA Ames Research Center have been investigated experimentally. The primary objective of these experiments has been to explore the effects of a mean flow on sound propagation in the wind tunnel. To meet this objective, the propagation of acoustic pulses was investigated.

The acoustic source for these experiments is an AR-1 woofer. This speaker is mounted flush against a plywood window near the middle portion of the test section of the wind tunnel. A small hole is drilled through the plywood near the center of the speaker. A gated sinusoidal pulse is obtained by passing the output of an oscillator through a General Radio model 1396-B tone-burst generator. This device is set to provide a pulse of four cycles of the oscillator frequency. The pulse repeats itself typically every one-half second. The oscillator frequency for these tests ranged from 50 to 150 Hertz. The signal from the tone-burst generator is amplified by a power amplifier which drives the speaker.

The acoustic signal is received by two Brüel and Kjaer one-half inch condenser microphones. Both are flush-mounted in the ceiling of the wind tunnel. One is positioned approximately one meter upstream from the source and the second is positioned at an equal distance downstream.

When the wind tunnel is operating, there is a relatively large level of background noise received by the microphones. The primary sources of this noise are the wind tunnel drive fan and motor and the turbulent boundary-layer along the walls of the wind tunnel. The amplitude of the background noise at the higher tunnel speeds is far in excess of the maximum pulse amplitude which can be generated by the equipment used. The pulse signal however can be "recovered" from the noise by suitably averaging the signal over many realizations of the pulse. The microphone outputs are therefore connected to a SAICOR model SAI 43A correlation and probability analyzer which is used in the signal-enhancement mode. After many pulses are averaged, the resulting signal is plotted on paper by an X-Y recorder driven by the analyzer.

All of the pulse shapes recorded in this manner have certain similar characteristics. The initial four cycles of the signal are generally increasing in amplitude. The remaining portion of the signal (which will be referred to as the "tail") diminishes in amplitude with time. The specific shape of the tail is strongly dependent on the oscillator frequency. The effect of flow on sound propagation is examined by comparing the pulses received upstream and downstream of the source. The amplitude of the signal received upwind of the source is greater than that received downwind, as expected from theoretical considerations. The magnitude of the ratio of amplitudes received upwind to downwind depends on the oscillator frequency. There are also differences in the arrival time of the pulses upstream and downstream, as expected.

As a theoretical model of this experiment pulse propagation is considered from a point source located in a rectangular duct of infinite length. The duct has a constant cross section and contains a uniform flow with velocity V in the positive x-direction. The walls of the duct are perfectly rigid. Although not faithful in all respects to the experimental conditions, this model does contain many of the basic elements encountered during the wind tunnel tests. The governing equation which describes sound propagation from a point source in a uniformly moving acoustic medium is given by equation (1) in Section B. For a point mass source located at position (o,y_o,z_o),

$$q(o,y_o,z_o,t) = q(t)\delta(x)\delta(y-y_o)\delta(z-z_o) \ . \tag{6}$$

The boundary conditions imposed by the rigid walls are

$$\frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0, \text{ at the walls.} \tag{7}$$

The problem of pulse propagation in the duct is formulated by supplementing equations (1), (6) and (7) with suitable initial conditions. Before the pulse is initiated at time zero, we assume that there is no acoustic disturbance present in the duct. In order to compare with the experimental data, we take the source strength to be a four-cycle sinusoidal pulse defined as follows

$$q(t) = \begin{cases} 0 & t<0 \\ B\sin(2\pi f_o t) & 0\leq t\leq 4/f_o \\ 0 & t>4/f_o \end{cases} \tag{8}$$

The governing equation is solved using the Laplace transform. The acoustic pressure as a function of time is obtained from this solution by numerically evaluating the inverse Laplace transform. This procedure permits the theoretical results of pulse propagation in a duct with flow to be calculated in a straightforward manner for a realistic input pressure excitation. As perusual, pulses with significant high-frequency content require longer computation time than pulses of low frequency. The results of these calculations are found to be in very good qualitative agreement with the experimental data. Quantitative differences are found in the amplitude of the tail of the pulse. Such deviations from experimental results performed in a wind tunnel are to be expected from the use of a model based on a duct with uniform cross section. Nevertheless, pulse propagation in the wind tunnel appears to be well described by the theoretical model for the cases examined.