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STOL AIRCRAFT TRANSIENT GROUND EFFECTS
PART I. FUNDAMENTAL ANALYTICAL STUDY

M. I. Goldhammer, J. P. Crowder, and D. N. Smyth

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NASA Project Engineer, D. G. Koenig, Large Scale Aerodynamics Branch, Ames Research Center, Mountain View, California

16. Abstract

The first phases of a fundamental analytical study of STOL ground effects are presented. Ground effects are studied in two dimensions to establish the importance of nonlinear effects, to examine transient aspects of ascent and descent near the ground, and to study the modelling of jet impingement on the ground. Powered lift system effects are treated using the jet-flap analogy. The development status of a three-dimensional jet-wing ground effect method is presented, including the description of a recently developed nonplanar, nonlinear lifting surface theory for the analysis of unblown wings in free air or in ground effect. Recommendations for future three-dimensional analytical developments are made.

The two-dimensional study has established the importance of nonlinear effects in ground proximity and has provided a simple means of modelling jet impingement. The study of transient phenomena in ground effect has shown for two-dimensional unblown airfoils that the transient effects are small and are primarily due to airfoil/freestream/ground orientation rather than to unsteady effects. Because of the limits of existing methodology, it is impossible at this time to fully assess analytically the importance of ground effect transients for STOL aircraft in relation to preformance, stability and control, and handling qualities.

The three-dimensional study of ground effects has shown phenomena similar to that shown in two dimensions. For unblown wings the wing/freestream/ground orientation effects have been shown to be of the same order of magnitude as for unblown airfoils, but no assessment of unsteady or jet effects can be made within the limits of existing methods. This study has provided the basis for the future development of a nonplanar, nonlinear jet-wing ground effect method.

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LIST OF SYMBOLS

$\vec{A}_{ij}$	Influence function
AR	Aspect ratio
b	Wing span
c	Airfoil or wing chord
c_f	Flap chord
c_p	Pressure coefficient
Δc_p	Pressure jump coefficient
c_μ	Sectional jet momentum coefficient
C_J	Total jet momentum coefficient
c_ℓ	Sectional lift coefficient
C_L	Total lift coefficient
C_{L_α}	Lift curve slope
C_D	Total drag coefficient
C_{D_i}	Induced drag coefficient
$c_{\ell_{p_{max}}}$	Maximum pressure lift coefficient
C_m	Pitching moment coefficient
g	Vortex distribution function
h	Ground clearance height (relative to leading edge of airfoil in two-dimensions and wing apex in three dimensions)
i,j	Dummy subscripts
$(\vec{i},\vec{j},\vec{k})$	Unit vectors in the (x,y,z) directions
$\vec{n}$	Unit normal vector
s	Arc length
t	Time
t/c	Thickness/chord ratio
$\vec{t}$	Unit tangent vector

u, v	Perturbation velocity components in (x,y) directions for two-dimensional analyses
u, v, w	Perturbation velocity components in (x,y,z) directions for three-dimensionsal analyses
u_∞, w_∞	x- and z- components of freestream flow
U_∞	Freestream velocity
$\vec{V}$	Total velocity vector
x, y, z	Cartesian coordinate variables
α	Angle of attack
ε	Camber angle
γ	Vortex strength
γ	Flight path angle
θ	Attitude angle
ξ, η, ξ	Integration dummy variables
$\Lambda_{1/4c}$	Sweepback of the 1/4-chord line
λ	Taper ratio
δ_f, δ_{flap}	Flap deflection angle
δ_J, δ_{Jet}	Jet deflection angle
∞	Infinity
∞	(Subscript) free air value
ϕ	Velocity potential

1.0 SUMMARY

A fundamental analytical study of STOL ground effects is presented. Ground effects are studied in two dimensions to establish the importance of nonlinear effects, to examine transient aspects of ascent and descent near the ground, and to study the modelling of jet impingement on the ground. Powered lift system effects are treated using the jet-flap analogy. The development status of a three-dimensional jet-wing ground effect method is presented, including the description of a recently developed nonplanar, nonlinear lifting surface theory for the analysis of unblown wings in free air or in ground effect. Recommendations for future three-dimensional analytical developments are made.

The two-dimensional study has established the importance of nonlinear effects in ground proximity and has provided a simple means of modelling jet impingement. The study of transient phenomena in ground effect has shown for two-dimensional unblown airfoils that the transient effects are small and are primarily due to airfoil/freestream/ground orientation rather than to unsteady effects. Because of the limits of existing methodology, it is impossible at this time to fully assess analytically the importance of ground effect transients for STOL aircraft in relation to performance, stability and control, and handling qualities.

The three-dimensional study of ground effects has shown phenomena similar to that shown in two dimensions. For unblown wings the wing/freestream/ground orientation effects have been shown to be of the same order of magnitude as for unblown airfoils, but no assessment of unsteady or jet effects can be made within the limits of existing methods. This study has provided the basis for the future development of a nonplanar, nonlinear jet-wing ground effect method.

2.0 INTRODUCTION

The analytic prediction of the effects of ground proximity on aircraft aerodynamics has been a subject of study for many years. While reasonably good solutions have been obtained for simple wings, the ground influence on wings with complex high lift systems has been difficult to predict accurately. For powered lift STOL aircraft the problem is even more difficult because of the very high lift coefficients required and the presence of a high velocity jet efflux which may impinge on the ground. In addition, there has been some concern that ascent or descent in ground proximity may introduce transients that can change the ground effect, especially for the high rates of ascent and descent typical of STOL aircraft.

Much of the previous analytical work on STOL ground effects has examined the problem in two dimensions. Lissaman (references 1 and 2) has approached the problem using linear theory while Huggett (reference 3) has used an experimental approach to establish prediction methods, particularly with regard to jet impingement. Halsey (reference 4) has solved the jet-flapped airfoil ground effect problem using a nonlinear finite element approach. Reference 5 has provided a linearized solution to the three-dimensional jet-wing ground effect problem. Although the method uses a correction term to approximately account for nonlinear effects, comparisons of its predictions with experimental data for STOL configurations show poor correlation.

The work presented here answers many fundamental questions regarding the nature of ground effects, both for conventional and STOL aircraft; and it establishes a basic framework from which prediction methods can be developed. Two-dimensional analytical methods have been used extensively to assess the importance of nonlinear effects, jet flap effects, and transients that may arise as a result of ascent or descent. Basic ground effect phenomena in three dimensions have also been studied. A nonplanar, nonlinear method for unblown wings has been developed, and recommendations for the development of a three-dimensional jet-wing ground effect method have been made. In the following sections each of these is discussed in detail.

3.0 TWO-DIMENSIONAL ANALYTICAL STUDY

An extensive analysis of the ground effect problem has been made using existing two-dimensional analytical techniques previously developed at the Douglas Aircraft Company. Although there are many valid questions concerning the extent of applicability of two-dimensional techniques to study a highly complex three-dimensional flow, experience has shown that basic phenomena and trends can be usefully studied two-dimensionally; however, absolute magnitudes must be obtained from the three-dimensional solution. The methods employed in the present study are potential flow techniques which have been in use at Douglas for some time. The Jet Flap Potential Flow Method (reference 4) solves the nonlinear airfoil/thin jet problem using an iterative technique to locate the jet sheet. The airfoil (either thin or thick) and the thin jet sheet are represented by distributed vorticity, and the problem is treated by solving the equations specifying no flow normal to the airfoil surfaces and a balance between jet centrifugal force and the pressure jump across the jet. The iteration continues until there is also no flow normal to the jet sheet. It should be noted that implicit in the use of this method (and other thin jet methods) is the assumption that the jet efflux can be adequately modelled by an infinitesimally thin jet sheet of infinite velocity but finite momentum. The jet-flap analogy, as it is known, has been shown to be a good model for both the internally ducted and the externally blown jet flap, but its applicability to upper surface blowing has not been established. Results computed by the Jet Flap Potential Flow Method have compared favorably with those obtained by other theoretical methods and with two-dimensional experimental data. Present capabilities of the method include the analysis of single- and multi-element airfoils with a thin jet, ground effect (modelled by an image airfoil technique), and non-uniform onset flows. Approximate techniques are employed to model boundary layers and jet entrainment. There presently is no method capable of analyzing a jet-flapped airfoil in an arbitrary transient motion, such as the flight into or out of ground effect. However, there does exist such a capability for unblown airfoils based on the Douglas Two-Dimensional Neumann Method (reference 6). These so-called unsteady Neumann techniques (references 7, 8, and 9) represent the airfoil by surface source distributions and internal vorticity, and they solve the unsteady problem by a time-step technique. Changes in lift with time require the shedding of vorticity into the wake to satisfy the Helmholtz conservation of vorticity law. The disposition of the vortex wake must be determined by an iterative technique. The ground plane is represented by an image airfoil technique.

Using the two methods described above as well as some simpler linearized methods, the two-dimensional ground effect problem has been studied, both for conventional and jet-flapped airfoils. Unsteady effects for conventional airfoils have been examined, and an approximate technique for modelling ascent or descent for airfoils with jet flaps has been developed. The details of this study are presented in the following sections.

3.1 Importance of Nonlinear Effects for Airfoils in Ground Proximity

Many practical airfoil analysis techniques have employed linearization assumptions to simplify the mathematical formulation and to reduce arithmetic labor. Fortunately for most problems such an approximation is quite valid, owing to the small angles and nearly planar nature of most airfoils (and wings in three-dimensions). However the ground effect problem does not fall into this category. The ground plane exerts a considerable influence on the flow field. Unlike the airfoil-alone problem, the induced flow due to the ground has a component tangential to the airfoil as well as normal to it. Expressed mathematically, the no-normal flow boundary condition,

$$\tan\ (\varepsilon+\alpha) = \frac{v}{U_\infty+u} \qquad (1)$$

cannot be linearized in the standard fashion because "u" induced by the ground plane is not necessarily small. To further aggravate the problem, most flight in ground proximity involves the use of high lift devices which makes linearization of the tangent function a poor approximation.

The importance of these nonlinearities was recognized at Douglas several years ago while studying the ground effect problem under AFFDL Contract F33615-71-C-1861. At that time a linearized three-dimensional jet-wing ground effect method based on the Douglas EVD method (reference 10) was being developed, but the importance of nonlinear effects was quickly recognized and a "u" perturbation correction term was included to partially account for the nonlinear ground influence (reference 5). In conjunction with that AFFDL contract, a study of the magnitude of nonlinear effects for conventional airfoils in ground proximity was made. Since at that time the jet iteration technique of the Jet Flap Potential Flow Method was not completely operational, a similar study could not be made for jet-flapped airfoils. This study has been made under the present contract, and the results of both are presented herein. This study assumes that the jet does not impinge on the ground. Jet impingement is discussed in Section 3.3.

A measure of the importance of nonlinear effects for conventional high lift airfoils represented by hinged flat plates in ground effect has been obtained using the linear and nonlinear mathematical models illustrated in figure 1. The linear theory represents the airfoil by a continuous distribution of vorticity [$\gamma(x)$] placed on a plate at a constant height (h) above the ground. The nonlinear theory uses a similar vortex distribution, but the vorticity [$\gamma(s)$] is placed on the actual plate whose leading edge is at the height h above the ground. Presented in figure 2 is the ratio of lift computed by the linear theory to that predicted by the nonlinear theory for various hinged flat plate airfoils. It is shown that linear theory can over-predict lift by as much as 50 percent of the nonlinear theory prediction for small values of h, and, as would be expected, nonlinear effects become more important with an increase in angle of attack and flap angle.

Typical effects of ground proximity on lift, calculated by nonlinear theory, for unblown hinged flat plate airfoils are presented in figure 3. These data show a lift increase due to the effect of ground proximity for the unflapped airfoil but a lift decrease for flapped airfoils. A detailed study of this problem indicates that three phenomena contribute to flow field changes as the ground is approached. Consider the ground to be simulated by placing an "image" airfoil at a distance $2h$ below the "real" airfoil. As shown schematically in figure 4, the image airfoil induces predominantly an up-wash on the real airfoil for forward (unflapped) loading but predominantly down-wash for aft (flapped) loading. Thus this first effect would tend to increase lift in ground effect for a flat plate airfoil but would tend to decrease lift

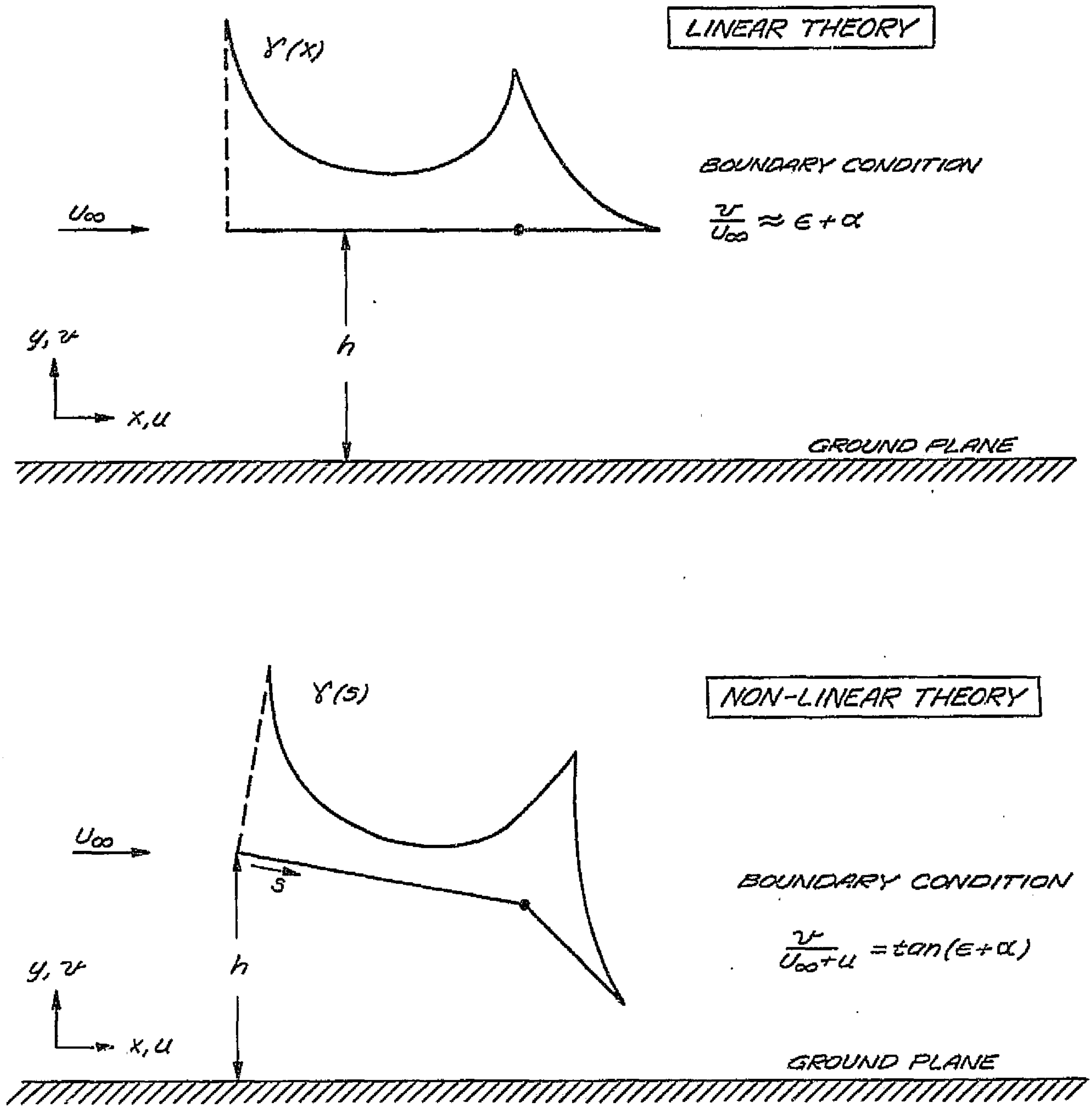

Figure 1. Illustration of Linear Theory and Nonlinear Theory Mathematical Models

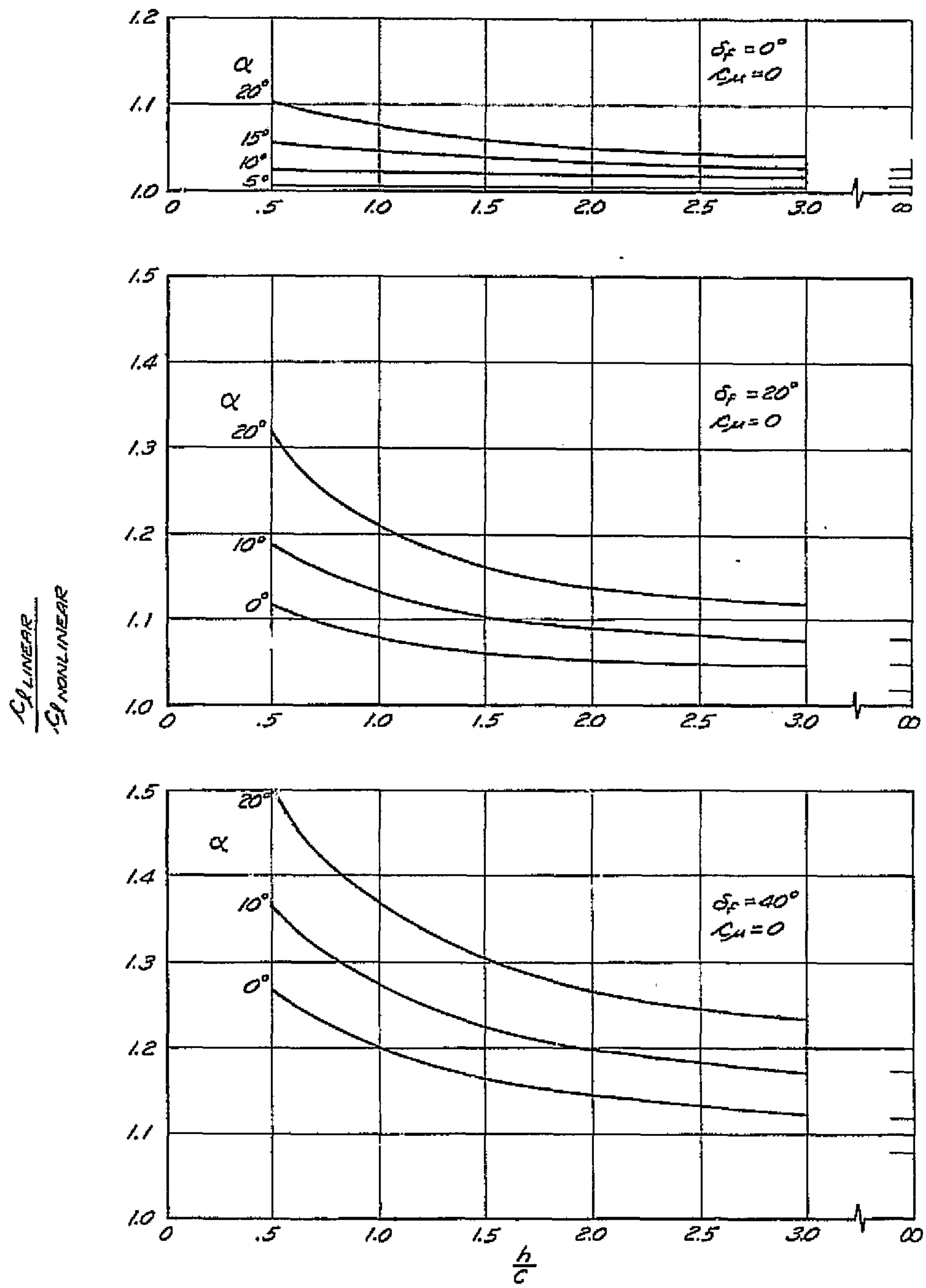

Figure 2. Comparison Between Linear and Nonlinear Theory for the Lift on a Two-Dimensional Hinged Flat Plate Airfoil in Ground Effect. ($c_f/c = 0.4$)

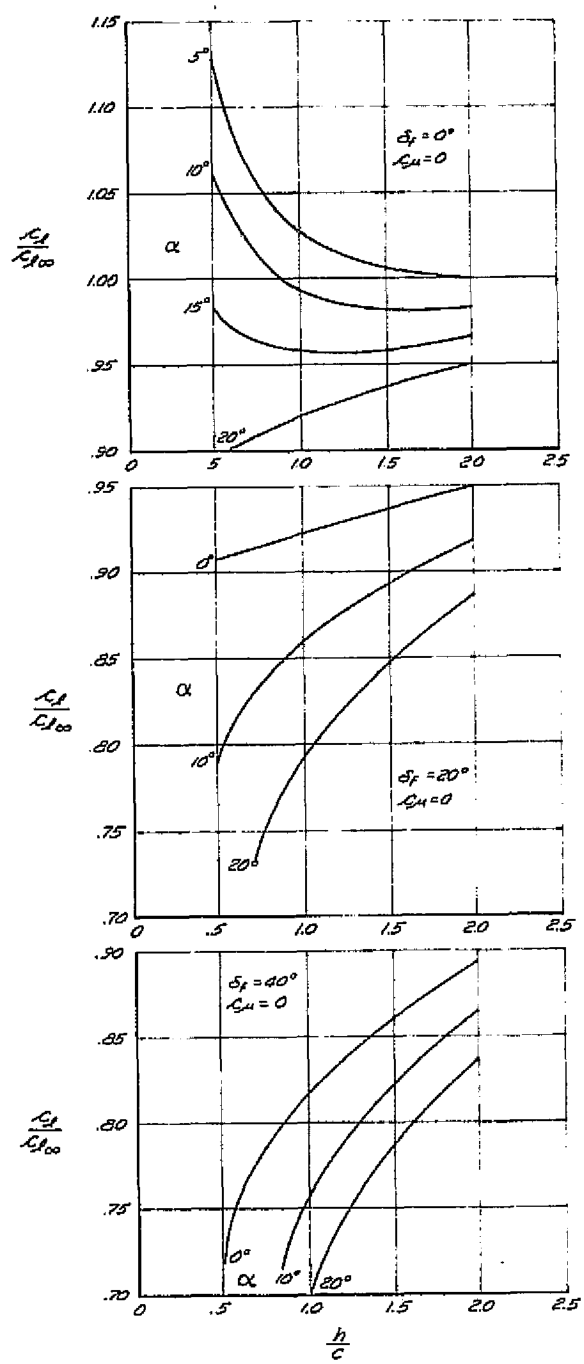

Figure 3. Ground Effect on a Two-Dimensional Hinged Flat Plate Airfoil Computed by the Method of Reference 4. ($c_f/c = 0.4$)

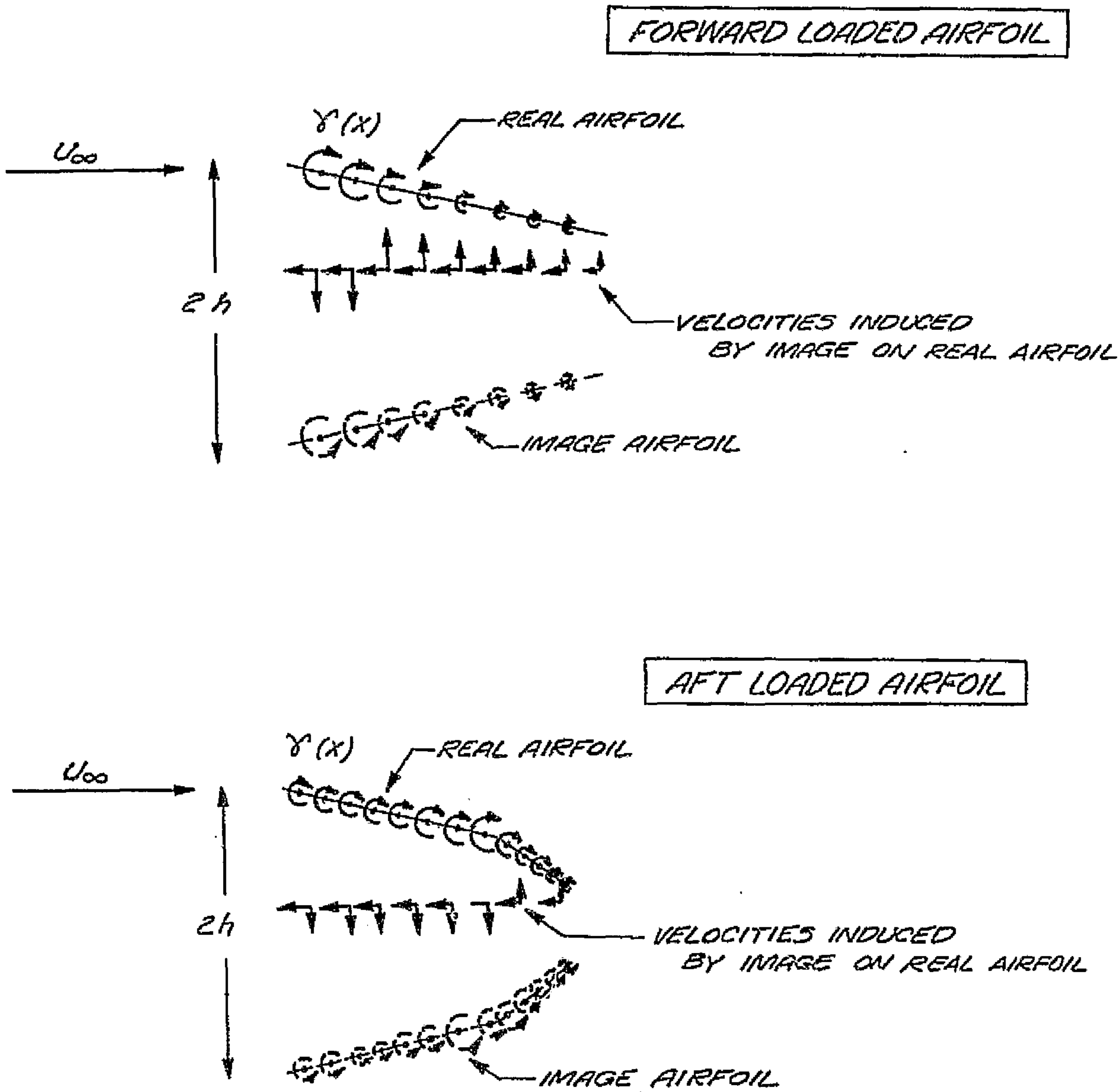

Figure 4. Schematic Illustration of Various Phenomena Contributing to Ground Effects

for a flapped airfoil, depending of course on the relative magnitude of the forward and aft loading. The second phenomena which contributes to ground effect is the perturbation velocity opposite to the freestream direction induced by the image airfoil. For positive lift, this effect of decreased dynamic pressure always reduces lift. The third effect is important only for large flap deflections. When a portion of the airfoil surface is highly deflected, downwash is no longer the predominant term in the boundary condition. Instead, the "u" perturbation velocity becomes of comparable magnitude. Since in ground proximity u induced by the image is generally opposite to the freestream, to satisfy equation 1 it follows that v must be smaller. For this condition to be satisfied lift must decrease.

The three phenomena discussed above cannot in reality be evaluated to assess the influence of ground proximity. The ultimate effect of ground proximity on lift is dependent on the relative importance of each phenomenon and can only be evaluated using a sophisticated analysis method which solves the complete ground effect problem. However, looking at the problem in this fundamental manner has led to further understanding of the ground effect problem.

A similar ground effect analysis has been conducted for hinged flat plate airfoils with jet flaps. Figure 5 presents a measure of the nonlinearities of the solution and shows that in terms of lift, nonlinear effects are still significant but are actually smaller than for unblown airfoils. Nonlinear effects tend to decrease with increasing jet momentum coefficient. This trend seems to contradict the data obtained for unblown airfoils where nonlinear effects became more important with increasing lift (see figure 2). However, for a jet-flapped airfoil in ground effect, the presence of the ground tends to restrict the downward trajectory of the jet, resulting in a flatter jet trajectory. Since linearized jet-flap theory assumes a flat jet, the effect of the ground in flattening out the jet may account for a portion of the reduction in nonlinear effects.

Typical effects on lift of ground proximity for jet-flapped airfoils is shown in figure 6. These data are very similar to the unblown airfoil data (figure 3) and show a generally adverse effect of ground proximity.

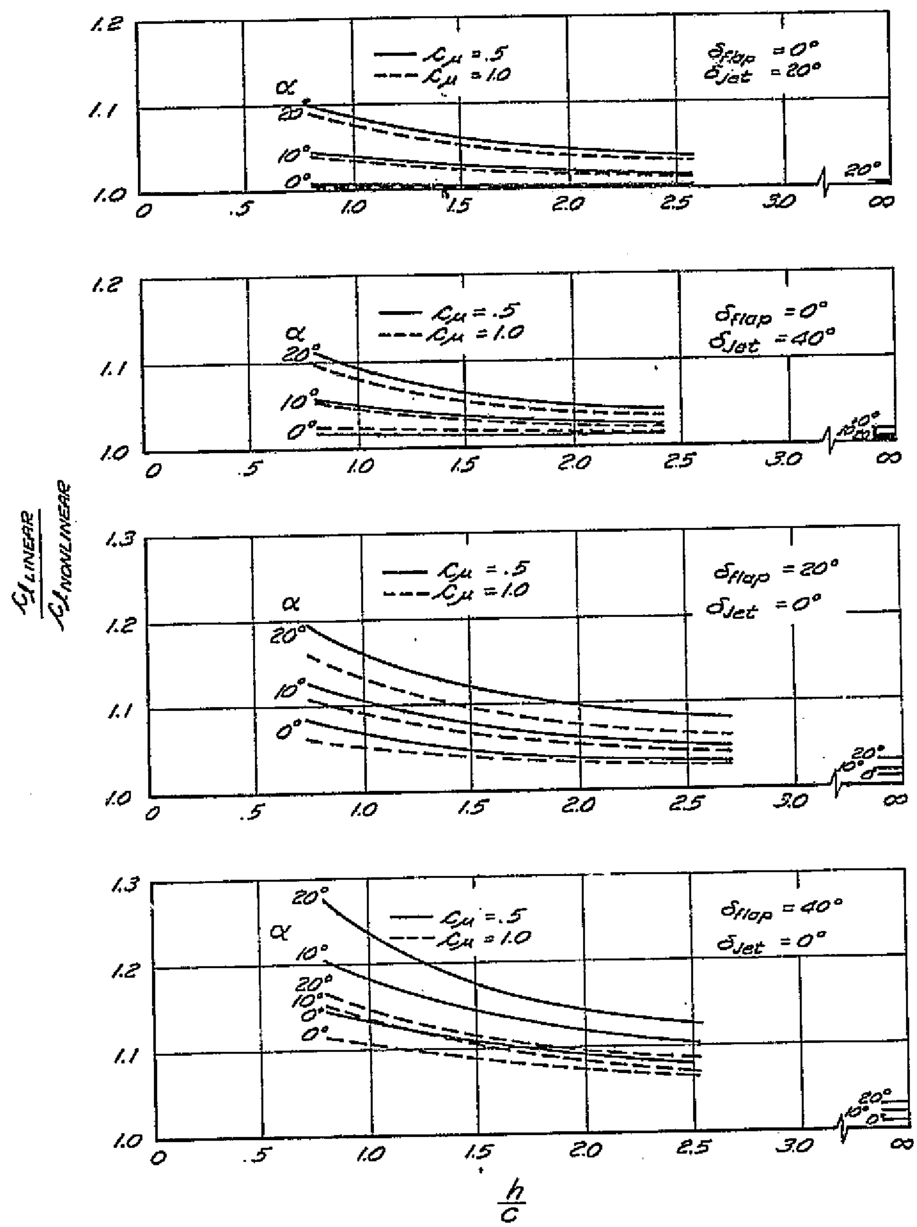

Figure 5. Comparison Between Linear and Nonlinear Theory for the Lift on a Two-Dimensional Hinged Flat Plate Jet-Flapped Airfoil in Ground Effect. (c_f/c = 0.4).

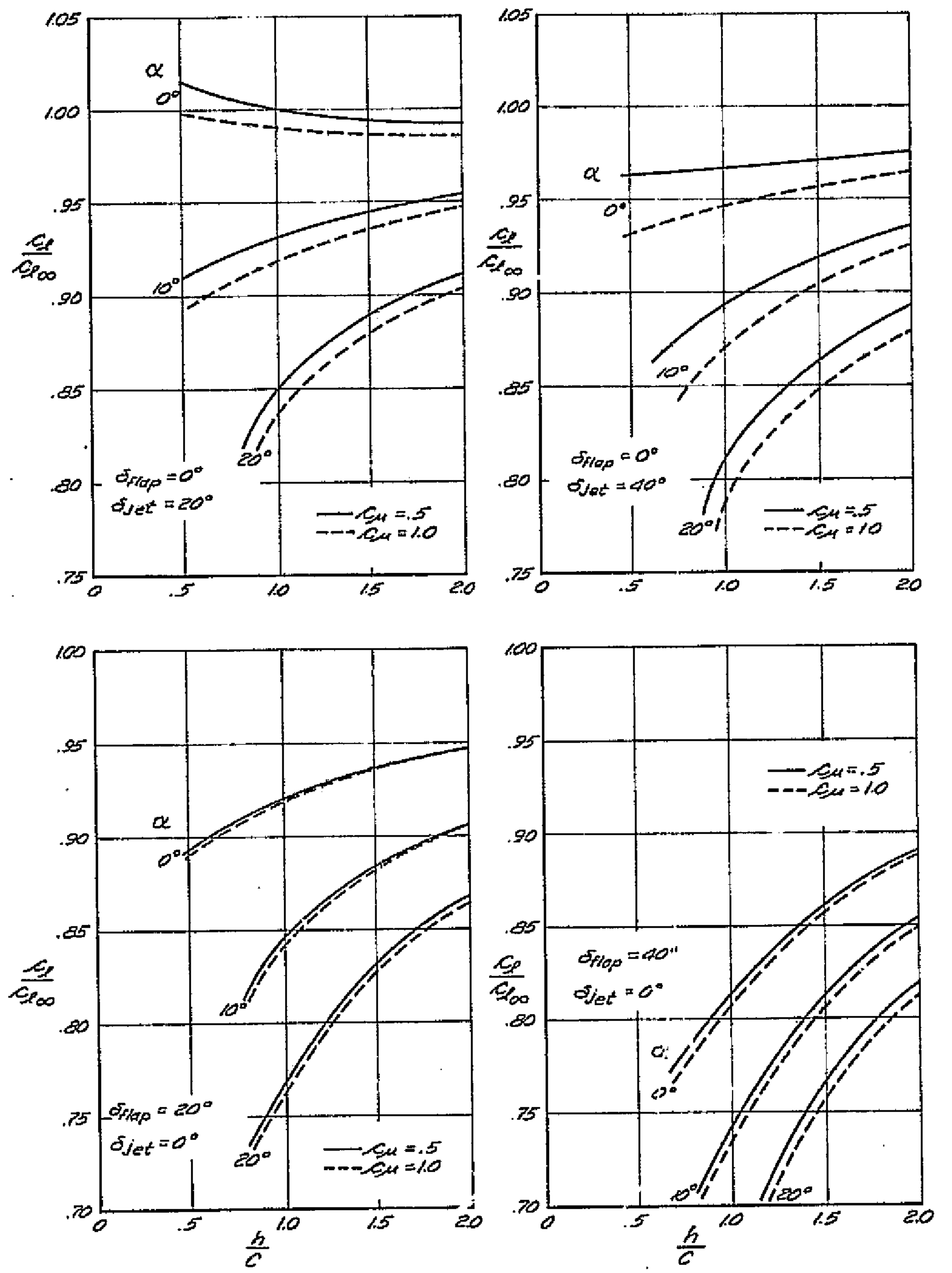

Figure 6. Ground Effect on a Two-Dimensional Hinged Flat Plate Jet-Flapped Airfoil Computed by the Method of Reference 4. ($c_f/c = 0.4$).

A jet flap tends to aft-load the airfoil, so these results are consistent with those of the aft-loaded unblown airfoils previously discussed. It should be noted that the data of figure 6 do not include the effects of jet impingement on the ground, which should further reduce lift. Jet impingement is discussed in Section 3.3.

In addition to nonlinear effects, the effects of airfoil thickness in relation to the ground effect problem have been examined. Effects of wing thickness are not considered in most lifiting surface theories because experience has shown the effects to be quite small. For an airfoil in free air, theoretical analyses show that thickness generally increases lift. The factor $(1 + t/c)$ is most often applied to approximate the effect. In practice, however, effects of the boundary layer tend to cancel thickness effects, so the thin wing result usually predicts lift very well, in the absence of any flow separation.

However, in ground proximity the effect of thickness is considerably different. Although the absolute magnitude of the lift on a thick airfoil in ground proximity may still be larger than for a corresponding thin airfoil, the ground effect (as a percentage of free air lift) is generally less favorable (or more adverse) for the thick airfoil than for the thin airfoil. This is shown in figure 7 for an NACA 0012 airfoil at various angles of attack and flap deflections. For an airfoil height of one chord the reduction in lift (i.e., c_ℓ/c_{ℓ_∞}) due to thickness is as large as three percent of the free air lift.

The thickness effect can be simply understood by considering the representation of the problem as that of two sources in a uniform flow (reference 18, page 210). The source (or a distribution of sources) can be used to model a thick body (with no circulation), and an image source can be used to model the ground. It is a well-known potential flow solution that two such sources will have a mutually attractive force between them, which can be interpreted as a reduction in lift.

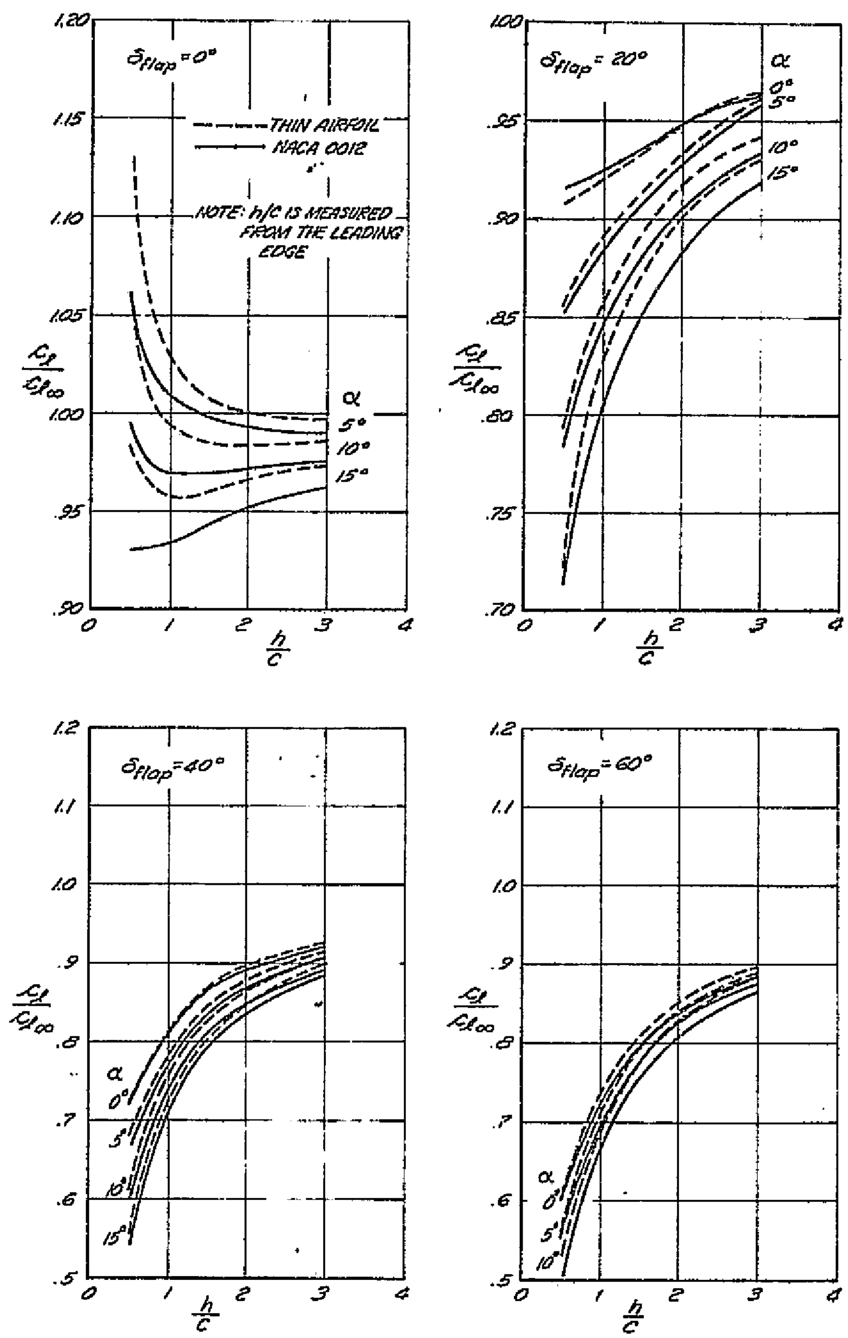

Figure 7. Comparison Between Theoretical Ground Effect Predictions for Lift on a Thin Hinged Flat Plate Airfoil and a NACA 0012 Airfoil with Flap. ($c_f/c = 0.4$).

3.2 Analytical Study of Ground Effect Transients

As an aircraft takes-off or lands, its attitude and distance relative to the ground are constantly changing. Because of the high rates of ascent and descent near the ground typical of STOL aircraft, there has been some concern that the effects of ground proximity will lag the motion. In other words, it is possible that aircraft disposition relative to the ground will change so rapidly that the ground effects will not quickly approach their steady state value. In addition, during ascent or descent the freestream flow is not parallel to the ground plane, as it is in the wind tunnel or in previous analytical methods; but rather the freestream is inclined to the ground by the flight path angle. Consequently the aircraft attitude angle is

$$\theta = \alpha + \gamma \qquad (2)$$

For the large flight path angles typical of STOL aircraft, these attitude effects may also effect the level of ground interference.

Using the previously discussed analytical methods, a two-dimensional assessment of transient ground effects has been made. The problem has been addressed from three levels of sophistication. The most approximate method neglects completely any aspects of ascent or descent and instead assumes flight at a constant height above the ground. This technique, called the "steady state method," is illustrated in figure 8a and is representative of current wind tunnel and analytical ground effect modelling. An improvement in modelling the ground effect problem is shown in figure 8b. Known as the "quasi-steady method," this technique models ascent or descent insofar as airfoil attitude relative to the ground is concerned but does not include any unsteady aspects of the flow. The complete transient modelling of the problem, known as the "unsteady" or "dynamic" method, uses essentially the same geometric model as shown in figure 8b but also takes into account the history of the motion. That is, as the airfoil ascends, descends, or changes attitude in ground proximity, changes in lift result in vorticity being shed in the wake. This affects the lift at all future times. Only unblown airfoils can presently be analyzed using the unsteady method.

An extensive analysis has been conducted to assess the three types of ground effect solutions described above. Simple NACA 0012 airfoils, with

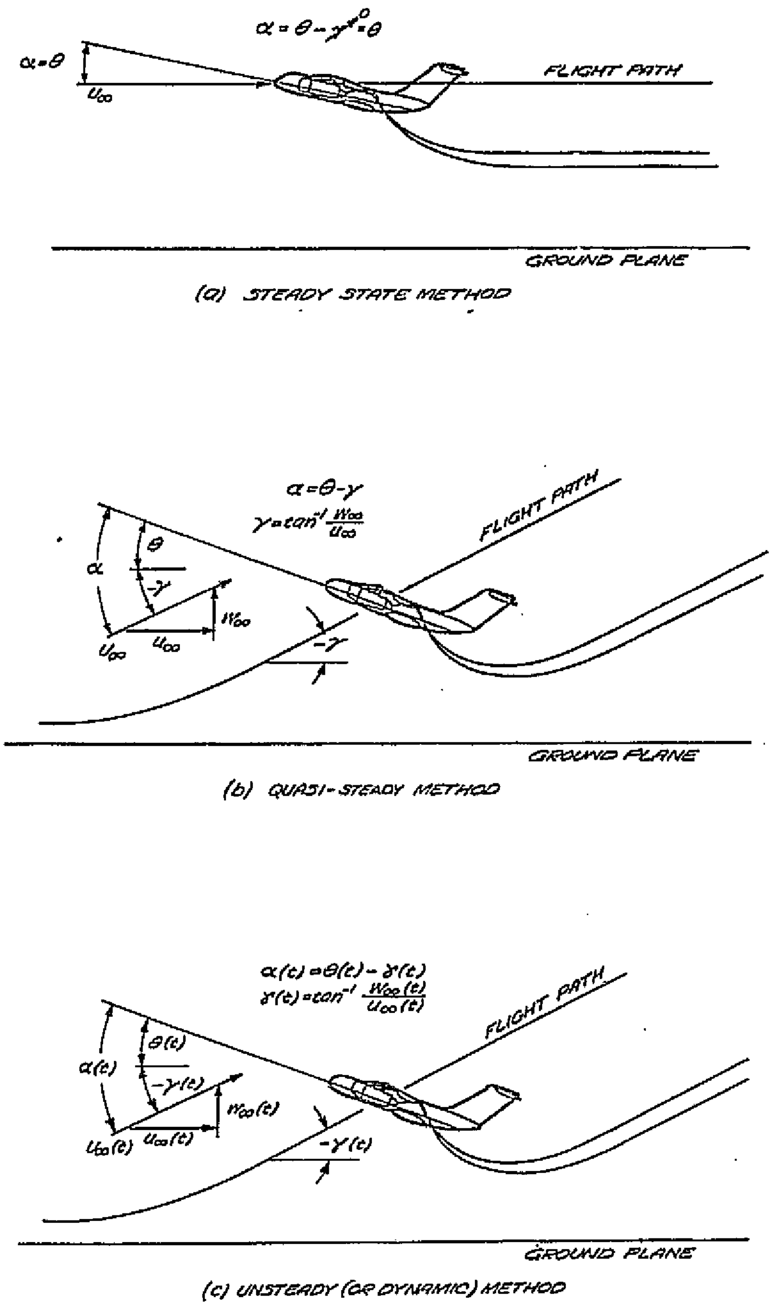

Figure 8. Illustration of Steady State, Quasi-Steady, and Dynamic Representations of the Ground Effect Problem.

and without flaps, have been analyzed to assess the effects of angle of attack, flap deflection, and descent (or ascent) angle on lift in ground effect. These data are presented in figures 9, 10, and 11, respectively. Flap deflections and descent angles typical of STOL transport aircraft have been used. From these figures, it can be seen that unsteady effects are rather small but that the effects of airfoil orientation relative to the ground are relatively large. According to these data the so-called lag in ground effects that has been observed experimentally (reference 11) may actually be due primarily to orientation effects. Figure 11 shows that this apparent lag increases with increasing descent angle. Similar effects have been calculated for pitching moment.

It is not possible at this time to evaluate fully the transient problem for jet-flapped airfoils, but a comparison of the steady-state and quasi-steady solutions for jet-flapped airfoils has been made to assess the importance of orientation effects. These results are presented in figure 12 for a flat plate airfoil with deflected jet and for a hinged flat plate airfoil with jet. These data show a considerably larger orientation effect than for unblown airfoils, consistent with the higher level of lift. It is unknown what effect transients would have, but it can be speculated that the flexibility of the jet sheet would lead to larger lags than calculated for unblown airfoils.

A two-dimensional analysis of a jet-flapped airfoil representative of the jet-wing used in the only known transient ground effect test (reference 11) has been made using the steady state and quasi-steady techniques in the Jet Flap Potential Flow Method. The wing tested in reference 11 was of rectangular planform of aspect ratio 6 and had a NACA 16-012 airfoil section with a ten percent chord flap deflected 60 degrees and a full span jet of strength $c_\mu = 3.5$. The test was made over a ramp inclined ten degrees and also over a flat ground. The two-dimensional analysis was made using a thin airfoil with a ten percent chord flap deflected 60 degrees and a jet of strength $c_\mu = 3.5$.

The analytical results and experimental data are presented in figure 13.

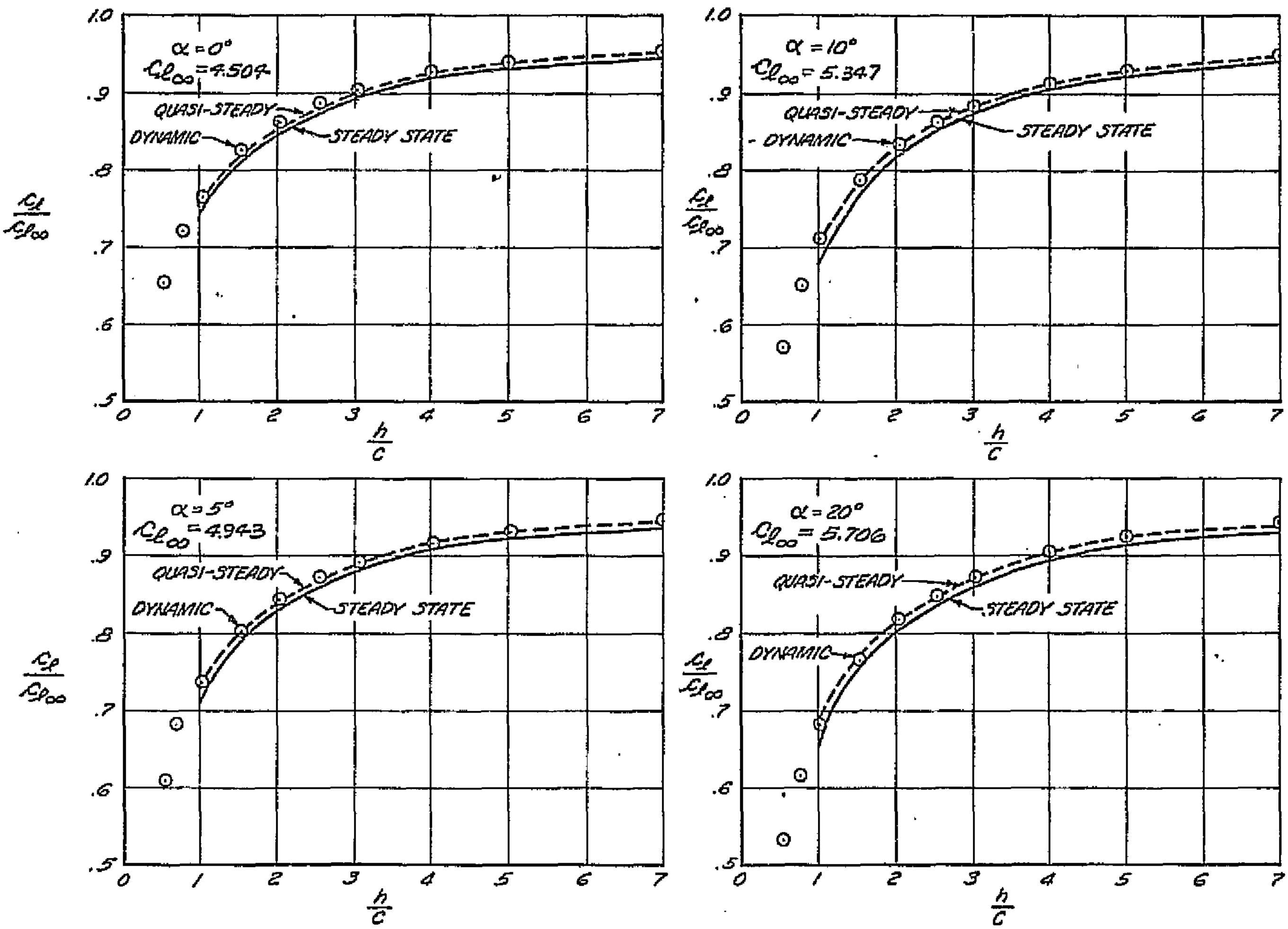

Figure 9. Comparison of Steady State, Quasi-Steady, and Dynamic Solutions for the Lift on a NACA 0012 Airfoil with Flap Descending Near the Ground. (c_f/c = 0.4, δ_f = 60°, γ = 5.71°(10%)).

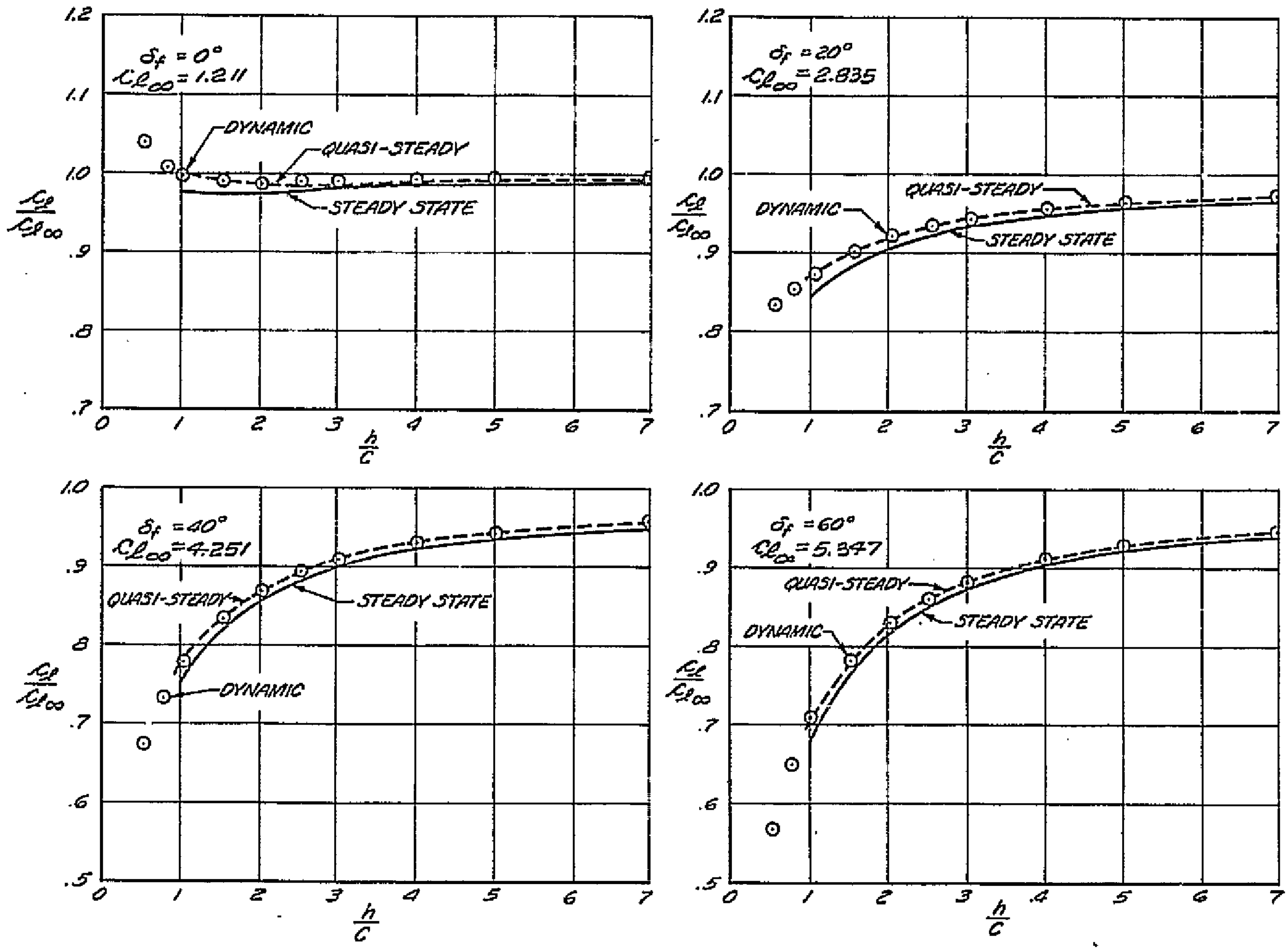

Figure 10. Comparison of Steady State, Quasi-Steady, and Dynamic Solutions for the Lift on a NACA 0012 Airfoil with Flap Descending Near the Ground. (c_f/c = 0.4, α = 10°, γ = 5.71° (10%)).

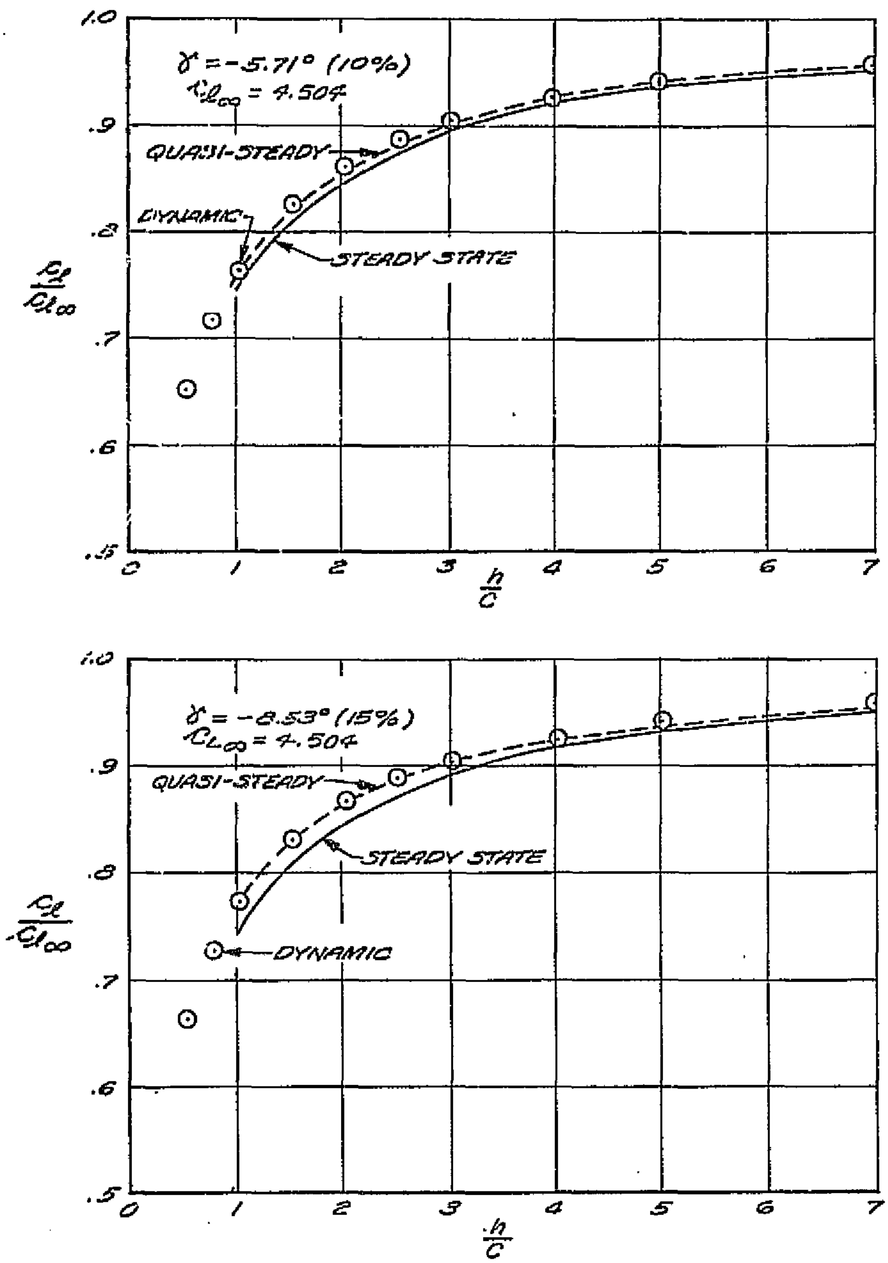

Figure 11. Comparison of Steady State, Quasi-Steady, and Dynamic Solutions for the Lift on a NACA 0012 Airfoil with Flap Descending Near the Ground. (c_f/c = 0.4, δ_f = 60°, α = 0°).

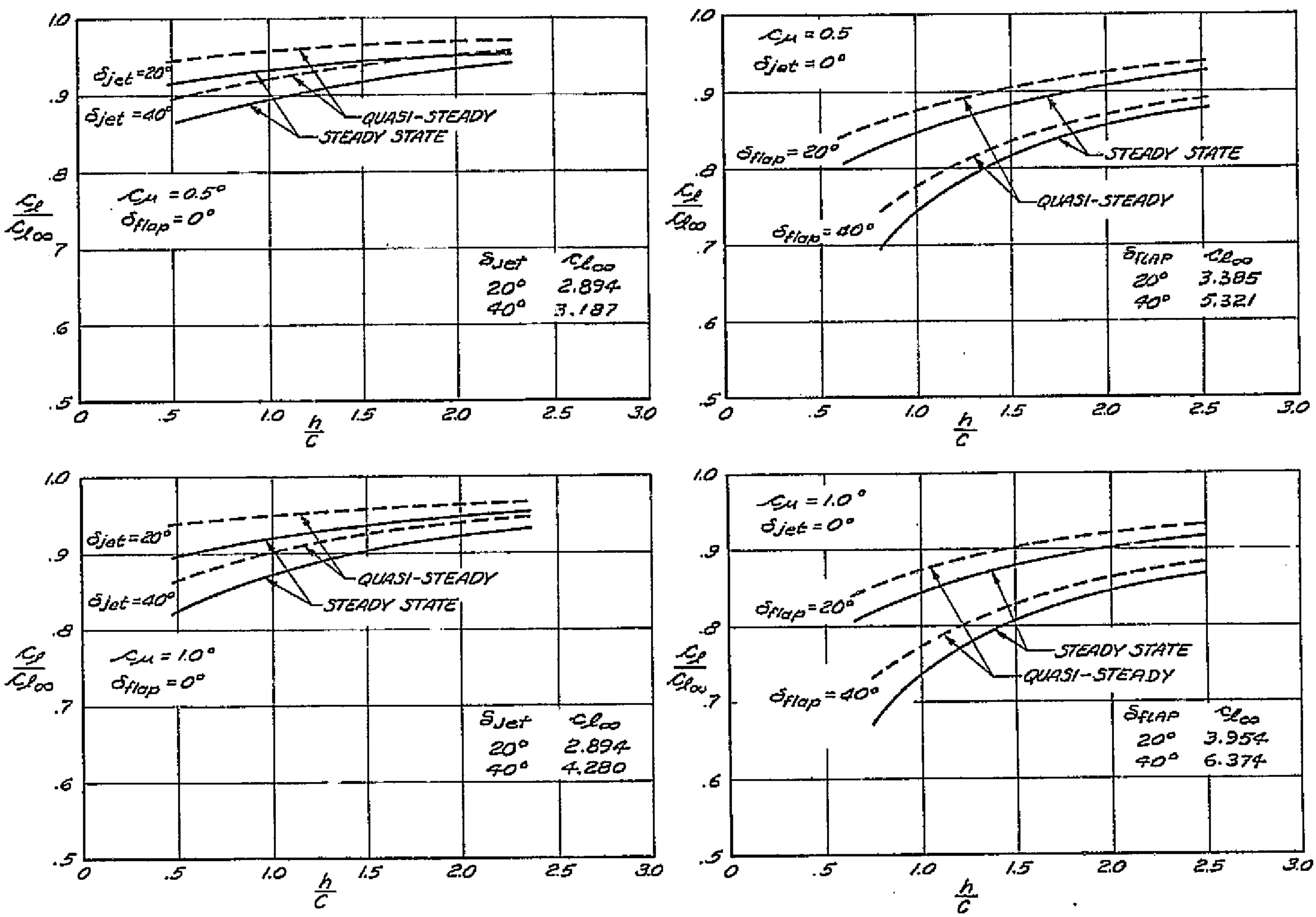

Figure 12. Comparison of Steady State and Quasi-Steady Solutions for the Lift on a Hinged Flat Plate Jet-Flapped Airfoil Descending Near the Ground. ($c_f/c = 0.4$, $\alpha = 10°$, $\gamma = -5.71°$ (10%)).

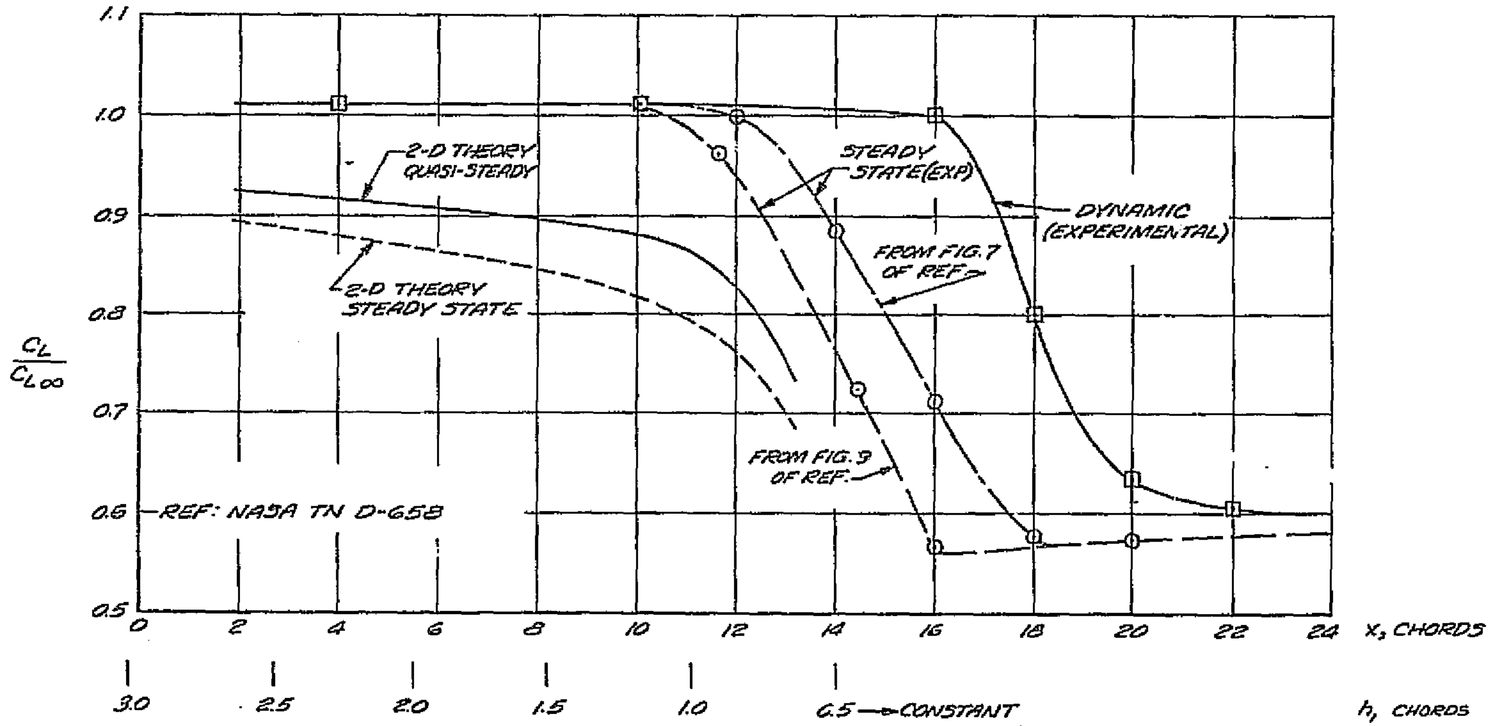

Figure 13. Comparison Between the Experimental Data of Reference 11 and Two-Dimensional Theory (Reference 4) for the Lift on a Jet-Flapped Wing Descending Near the Ground. (c_μ = 3.5, δ_f = 60°, α = 0, γ = -10°)

Before these results are compared, however, a number of comments concerning the experimental data must be made. First, the data of reference 11 are not consistent. The steady state data presented in figure 7 of that report do not agree with the steady state data presented in figure 9. These two sets of "steady state" data are shown in figure 13. In addition, the data obtained over the ramp ground board may be subject to large errors due to blockage effects of the tracked carriage, as indicated in the reference.

The results of the two-dimensional study are compared with these experimental data in figure 13. The analytical results are not complete because the solution would not converge at the closest ground height, 0.5 chords. Nevertheless, the solution shows the same general character as the test data, but the magnitudes of the C_L difference between the steady state and dynamic (or quasi-steady) cases do not agree. The analytical results show a fairly constant C_L difference (at a given h/c) on the order of 5% to 6%, while the experimental C_L difference is as much as approximately 40% of the free air value.

It is impossible to reach any firm conclusions based on this comparison because the theoretical analysis is limited by two-dimensional and quasi-steady assumptions. However, based on the two-dimensional unblown airfoil studies, it does not seem apparent that unsteady effects are causing the differences, although unsteady effects on the jet sheet are unknown. It is unlikely that three-dimensional unsteady effects are responsible for the discrepancy. A comparison of the work of Wagner (reference 12) with that of Jones (reference 13) shows that three-dimensional effects tend to decrease the time of response of lift to a sudden change in angle of attack relative to a two-dimensional airfoil. The large measured lift lag (reference 11) could be caused by a delay in the onset of jet impingement in the descending flight case. Or it is possible that carriage blockage correction errors are prejudicing the data. An additional item of concern about these experimental data is the large difference between wind tunnel results (steady state) and moving model results (figure 9 of reference 11). This difference may be due to boundary layer buildup on the wind tunnel ground board or it may be due to poor dynamic characteristics of the balance system on the moving model carriage. Additional transient ground effect test data would certainly help to clear up some of these questions. A test program of the type studied under this contract (see Part II) would resolve many of the problems that are beyond the capability of present and near-term future analytical methods.

3.3 Impingement of the Jet Sheet on the Ground

In addition to the formidable problems encountered in predicting ground effects for conventional high lift wings, powered lift systems have the problem of possible impingement of the jet sheet on the ground. A two-dimensional study of jet impingement has been made using existing anaytical methods in an attempt to simply model the problem. Questions which must be answered include: For a particular set of conditions (i.e., α, h/c, c_μ, δ_J) does impingement occur or not? Can the lift and pressure distribution be calculated when impingement does occur? And can the limiting lift (i.e., the maximum pressure lift which cannot be increased by more blowing) be predicted?

A number of investigators have previously studied the jet impingement problem. Huggett (reference 3) has studied the problem experimentally in two-dimensions and has used a simple mathematical model to predict the limiting lift. Lissaman has also studied the problem using a linearized analytical approach (reference 1).

The Douglas Jet Flap Potential Flow Method (reference 4) is the technique which has been used for impingement modelling in this study. Although impingment cannot be treated as a potential flow problem, it was felt that the relatively simple jet model in this method could be used at least to estimate the impingement point. Using the semi-infinite thin jet capability of the computer program, several attempts were made to force the jet through the ground so that it could be truncated at the impingement point. Unfortunately the solution would not converge, and in most cases the jet trajectory extended above and below the ground plane in successive iterations.

A reasonable means of predicting impingement was found, however, by using a finite length jet in place of an infinite jet for those cases where the infinite jet solution would not converge. The procedure requires some trial and error. The jet is truncated to some appropriate length, such as the straight line distance from the airfoil trailing edge to the ground along the initial jet direction. The problem is solved again, and if the resulting truncated jet trajectory curves nearly parallel to the ground, impingement is unlikely. However, if the resulting trajectory is "aimed" downward at a significant angle, then impingement is likely. There will be some cases where impingement may or may not occur. It may require several guesses of the truncated jet length until the jet trajectory comes close enough to the ground for a decision to be made.

An example of the procedure is shown in figure 14 for an elliptic airfoil at zero angle of attack with a 60 degree jet. Clearly there is no impingement for $c_\mu = 0.5$ and there is impingement for $c_\mu = 1.5$ and 2.6. It is uncertain what happens at $c_\mu = 1.0$. It is important to note that jet truncation does not seriously affect the solution. For the case of no impingement in figure 14, the lift is only three percent less than for the infinite jet solution.

A comparison of lift coefficients between the present jet impingement method calculation and experimental data for a two-dimensional jet-flapped airfoil (reference 3) is shown in figure 15 as a function of jet momentum coefficient. The theoretical results have been computed with and without jet entrainment. At low c_μ, where there is no impingement, the agreement is good. At the higher c_μ values, however, theory overpredicts lift. A possible explanation for this overprediction can be obtained by examining experimental and theoretical pressure distributions for non-impinged and impinged cases (figure 16). It is seen that a large loss in lift on the aft lower portion of the airfoil resulted from impingement but that this loss was not predicted theoretically. It is speculated that this loss in lift results from a trapped vortex in the cavity formed by the airfoil, jet, and ground. Flow visualization studies have confirmed the existence of such a vortex. Note that the experimental data show a leading edge bubble for both the non-impinged and impinged cases, which is not accounted for in the theoretical model.

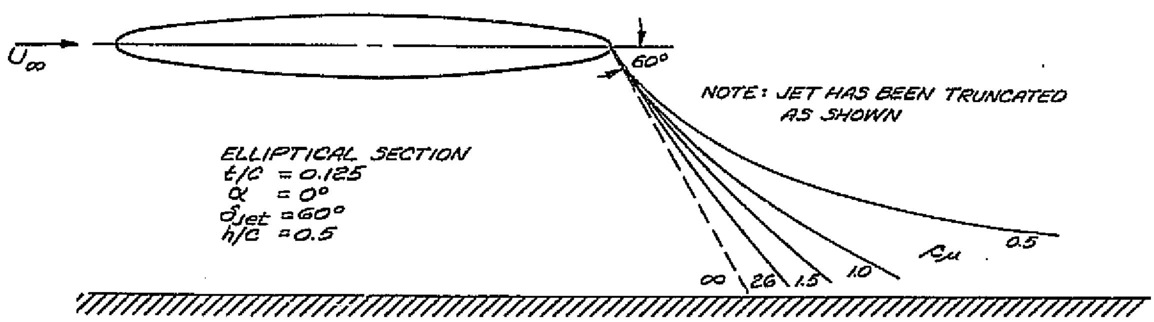

Figure 14. Illustration of Procedure used to Model Jet Impingement.

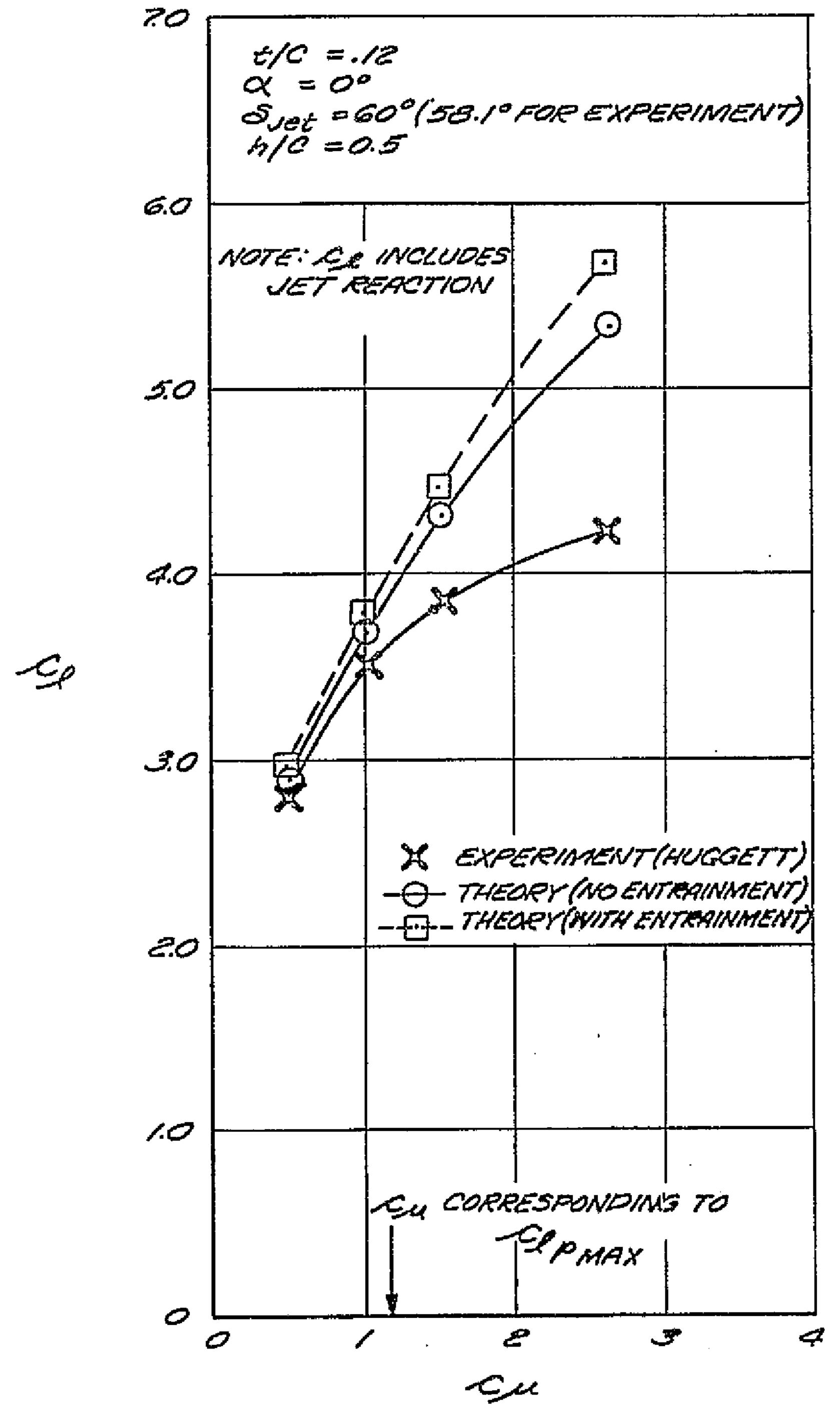

Figure 15. Comparison Between Theory and Experiment for the Lift on a Jet-Flapped Airfoil in Ground Effect for a Case Where Jet Impingement Occurs.

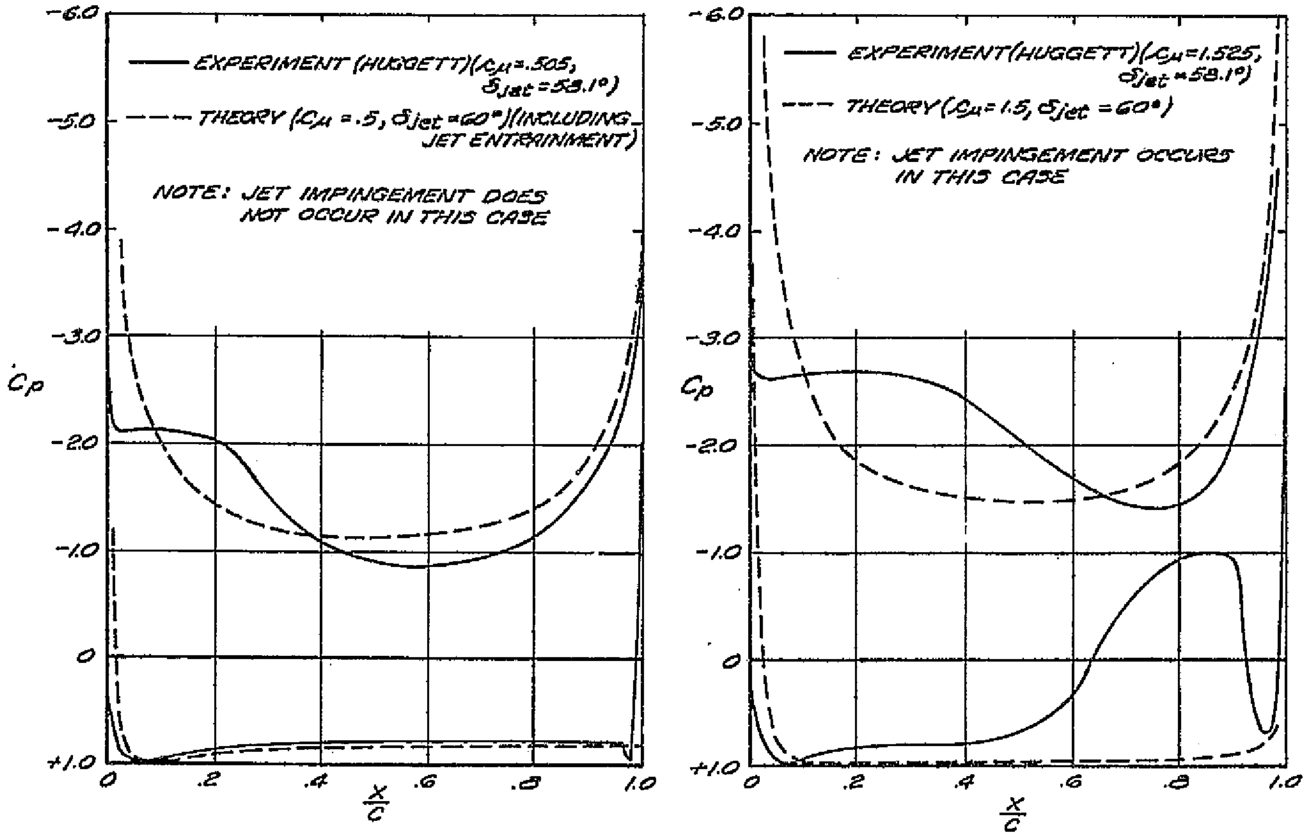

Figure 16. Comparison Between Theoretical and Experimental Pressure Distributions for a Jet-Flapped Airfoil in Ground Effect for Non-Impinged and Impinged Cases. (t/c = 0.125, α = 0°, h/c = 0.5).

The limiting lift is computed using the present technique by replacing the jet sheet by a rigid plate extending from the airfoil trailing edge to the ground at the initial jet deflection angle. This effectively represents a jet of infinite strength and hence should adequately predict the maximum attainable pressure lift. Figure 17 compares the results of this analysis with experiment (reference 3) and shows reasonably good correlation.

The technique used here to study impingement provides only a crude representation of the flow. In reality the jet is thick, and when impingement occurs the jet splits and flows both upstream and downstream along the ground. As evidenced by the pressure plots (figure 16), viscous effects are important underneath the airfoil. In three dimensions the problem is further complicated by the ability of the jet to spread spanwise and by "spanwise venting" of the flow beneath the wing.

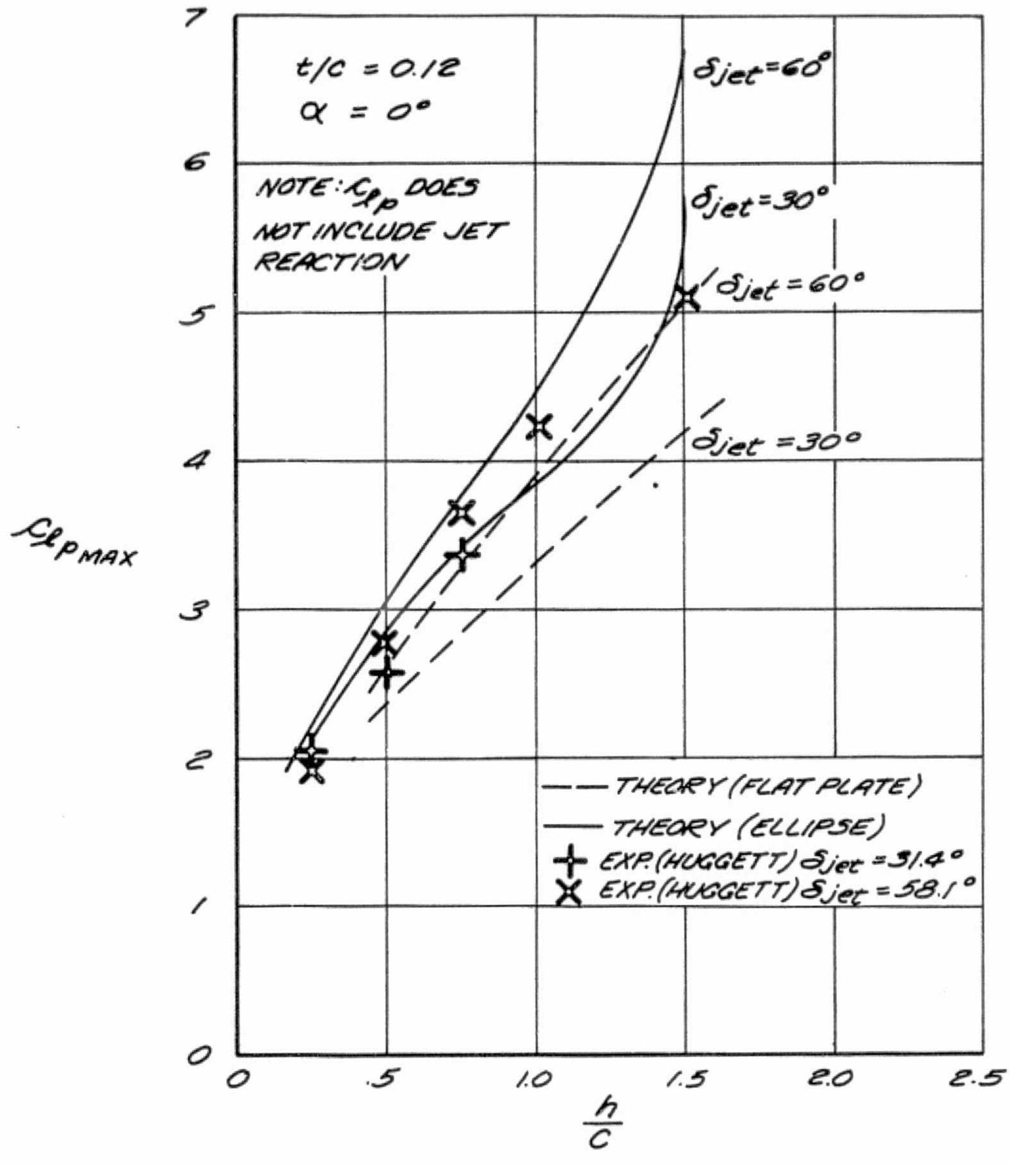

Figure 17. Comparison Between Theory and Experiment for the Limiting Lift on a Jet-Flapped Airfoil

4.0 THREE-DIMENSIONAL ANALYTICAL STUDY

There currently is no analytical method capable of accurately predicting the influence of ground proximity on the aerodynamic characteristics of jet-wings. Previous work at Douglas included the development of a linearized jet-wing lifting surface theory (reference 10) with a ground effect capability (reference 5), but it has been shown that such an approach is inadequate because of the extreme nonlinearity of the problem. Nonlinear effects may be even more significant in three dimensions than in two dimensions, as discussed in Section 3.1, because sweep, taper, and spanwise variations in geometry and blowing characteristics will add further nonplanar and nonlinear effects. Impingement of the jet sheet on the ground further complicates the problem. Transient effects in ground proximity, if important, would add further difficulty to obtaining a reliable three-dimensional solution.

In response to the lack of a reliable jet-wing ground effect method, Douglas, under its Independent Research and Development (IRAD) program, has been working on methods to provide this capability. Two approaches, in terms of the singularity distribution employed, are being pursued. The first is a nonplanar, nonlinear lifting surface theory using vortex distributions to represent the surfaces and a thin jet sheet model. The second approach uses a doublet singularity distribution to represent solid bodies and jet boundaries. When completed, this method would include a three-dimensional jet that correctly models the effects of finite mass flow, distortion and deflection, and inlet flow. While limited experience has been obtained with this jet model (reference 14), the method is still in the early stages of development and considerable additional effort is required before an operational program will be available.

The completion of the nonplanar, nonlinear lifting surface method, however, was felt to be much closer at hand, and its development has been accelerated to provide the needed capability. Several fundamental analytical stepping-stones have been developed in conjunction with the present contract work. In addition, considerable progress towards the desired analytical method has been made under a McDonnell Douglas Independent Research and

Development (IRAD) project. In fact, as a result of this work a fully nonplanar, nonlinear lifting surface theory for unblown wings, including ground effect, has been developed. The following sections describe the development of this method, demonstrate use of the present method for the analysis of three-dimensional ground effects, and define future tasks required to add the powered lift capability.

4.1 Development of a Three-Dimensional Ground Effect Method

Development of a fully nonlinear, nonplanar lifting surface theory was initiated as a 1974 McDonnell Douglas IRAD project. The term "lifting surface theory" is used to indicate that wing thickness effects are neglected. Considerable effort was devoted to developing a suitable singularity distribution to model the lifting surface. Factors such as accuracy, numerical behavior, and computing requirements were considered.

Nonplanar vortex distributions have been developed to model the wing, based on the planar Elementary Vortex Distributions (EVDs) developed at Douglas in 1970 (reference 10). Unlike their planar counterparts, however, the nonplanar singularities place appropriate vorticity distributions on the actual camber surface, which is required for the nonlinear solution, rather than on some mean plane. Three nonplanar vortex distributions have been developed, as shown in figure 18. To model the loading at the leading edge of a wing, an inverse-square-root distribution of vorticity is used $(\gamma \sim 1/\sqrt{x})$. For other portions of the wing surface (or the jet sheet), overlapping triangular vorticity distributions, which add to form simple linear distributions, have been derived. These triangular vortex distributions are placed on a piecewise broken camberline, as shown in figure 19. A third vortex distribution, which decays as $1/x^2$, has been derived for future use to model the loading on a semi-infinite jet sheet.

The nonplanar vortex distributions are implemented by dividing the camber surface into an array of finite elements. On a point within each element the no-normal flow boundary condition,

$$\vec{V}_i \cdot \vec{n}_i = 0 \qquad (3)$$

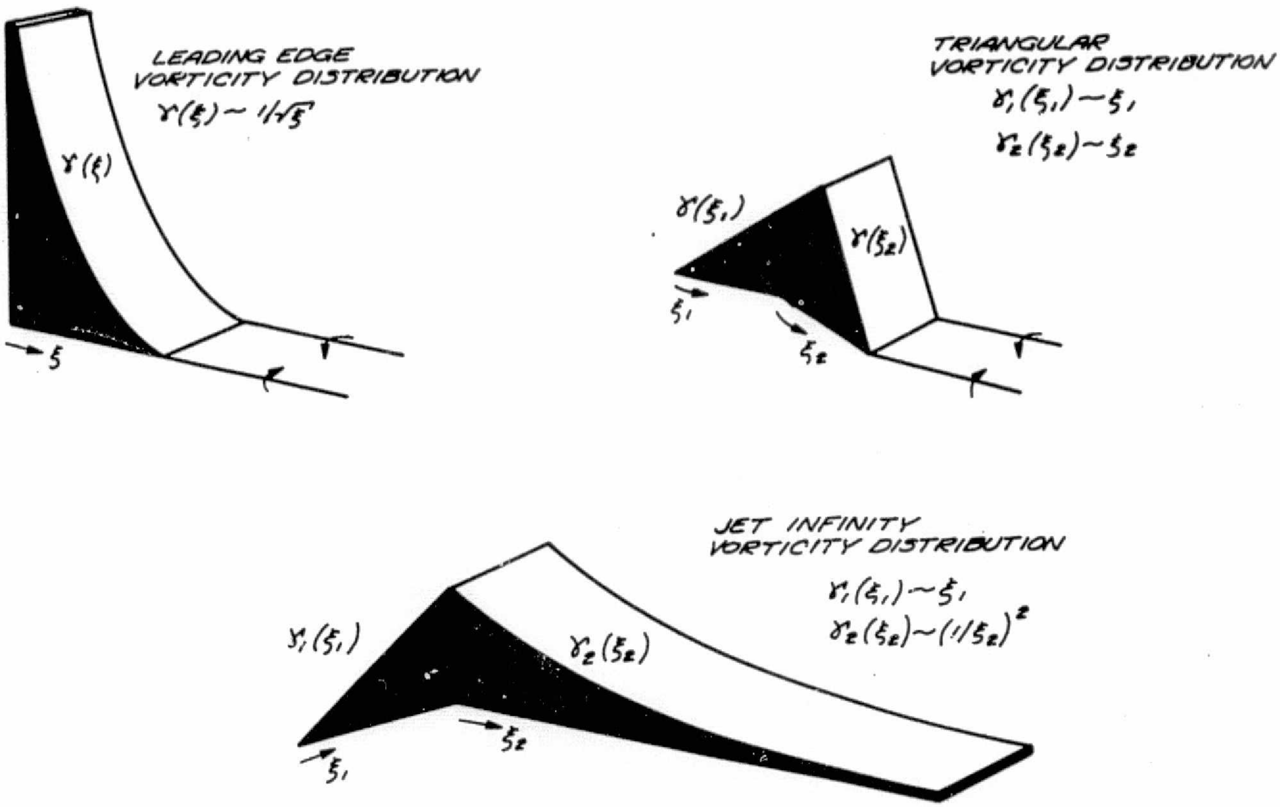

Figure 18. Illustration of Nonplanar Elementary Vortex Distributions.

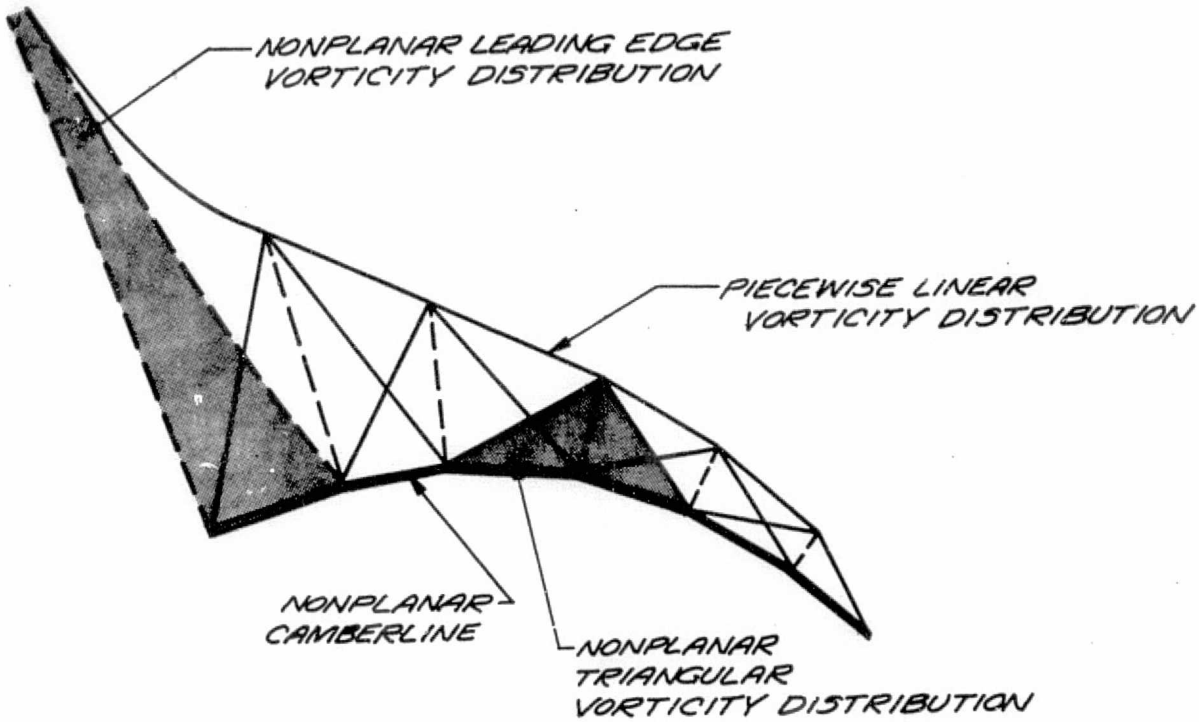

Figure 19. Illustration of Representation of Chordwise Loading by Nonplanar Elementary Vortex Distributions on a Nonplanar Camberline.

is imposed, where the subscript denotes the i^{th} element and $\vec{n}_i$ is the unit normal. The velocity $\vec{V}_i$ is composed of components of the freestream flow and of velocities induced by each vortex distribution on the wing surface. If γ_j is the strength of the vorticity on the j^{th} element, the velocity on the i^{th} element is then

$$\vec{V}_i = \vec{V}_\infty + \sum_j \vec{A}_{ij} \gamma_j \tag{4}$$

where the summation is taken over all the elements. Determination of $\vec{A}_{ij}$, known as the "normal-wash influence coefficient," is the crux of the problem. $\vec{A}_{ij}$ represents the velocity vector induced by the j^{th} element of unit vortex strength at the i^{th} boundary condition point. It is computed from the Biot-Savart Law. Using a local coordinate system (x,y,z) with the chosen vortex distribution in the xy-plane, the Biot-Savart Law can be expressed as

$$\phi(x,y,z) = -\frac{1}{4\pi} \int_x \int_y \int_{-\infty}^{x} \gamma(\xi,\eta) \frac{\partial}{\partial z} \left[\frac{1}{\sqrt{(\xi'-\xi)^2 + (y-\eta)^2 + z^2}} \right] d\xi' d\eta d\xi \tag{5}$$

where ξ, η, ξ' are integration dummy variables. The function $\gamma(\xi,\eta)$ is the chosen vorticity distribution on a small element of the camber surface.

In the present mathematical model the vorticity functions are assumed to be constant in the y-direction (spanwise) but have the desired behavior in the x-direction (chordwise). Thus the form of $\gamma(\xi,\eta)$ becomes

$$\gamma(\xi,\eta) = \gamma(\xi) = \bar{\gamma}\, g(\xi) \tag{6}$$

where $\bar{\gamma}$ is the mean vorticity strength and

$g(\xi) \sim 1/\sqrt{\xi}$	(leading edge elements)
$g(\xi) \sim \xi$	(other elements)
$g(\xi) \sim 1/\xi^2$	(jet semi-infinite elements)

It should be mentioned that, because of the assumption of constant vortex strength over the spanwise extent of an element, there is a pair of concentrated vortex legs extending downstream from each edge of an element. In the present formulation these vortex legs have been "broken" to follow the section camber line to the trailing edge and then extend downstream to infinity (see figure 20). Integration of equation (5) has been facilitated by evaluating velocity components rather than the velocity potential ϕ. The resulting integrals are quite lengthy and have the added complication of singular points, which must be handled using the Mangler Principal Value Theorem. For example, the velocity component normal to the local xy-plane of an arbitrary element is determined from

$$w(x,y,z) = \frac{1}{4\pi}\iint \gamma(\xi,\eta)\left\{\frac{(y-\eta)^2-z^2}{[(y-\eta)^2+z^2]^2}\right\}\left\{1+\frac{(x-\xi)}{\sqrt{(x-\xi)^2+(y-\eta)^2+z^2}}\right\}d\xi d\eta$$
$$-\frac{1}{4\pi}\iint \gamma(\xi,\eta)\left[\frac{z}{(y-\eta)^2+z^2}\right]\left\{\frac{(x-\xi)z}{[(x-\xi)^2+(y-\eta)^2+z^2]^{3/2}}\right\}d\xi d\eta \tag{7}$$

The numerical behavior and computer resource requirements of these nonplanar vortex distribution functions have been thoroughly analyzed and detailed comparisons with simple concentrated horseshoe vortices have been made, both on and off the plane of the singularity. One such comparison, for a triangular vortex distribution, is shown in figure 21. As would be expected, far from the element, where details of the vortex distribution are unimportant, the simple and complex functions agree well. However, close to the inducing element, in the region of most importance to the solution, there is a significantly different character to the induced velocity distribution.

The computing requirements of the nonplanar vortex distribution functions, while greater than for the simple concentrated vortex, are still within an acceptable limit for use within a frequently used design program. Simplifications to the functions for far field points have been included in the computer program to reduce computation requirements.

The finite element lifting surface theory problem is solved by combining equations 3 and 4 for the vortex strengths:

$$\sum_j \vec{A}_{ij} \cdot \vec{n}_i \, \gamma_j = -\vec{V}_\infty \cdot \vec{n}_i \tag{8}$$

Equation (8) is solved by matrix techniques on a digital computer. Once the γ_j values are known, the pressure jump coefficients $\Delta c_p(x,y,z)$ are computed from a form of the Kutta-Joukowski law,

$$\Delta c_p\,(x,y,z) = 2(\vec{V}\cdot\vec{t})\,\gamma(x,y,z) \tag{9}$$

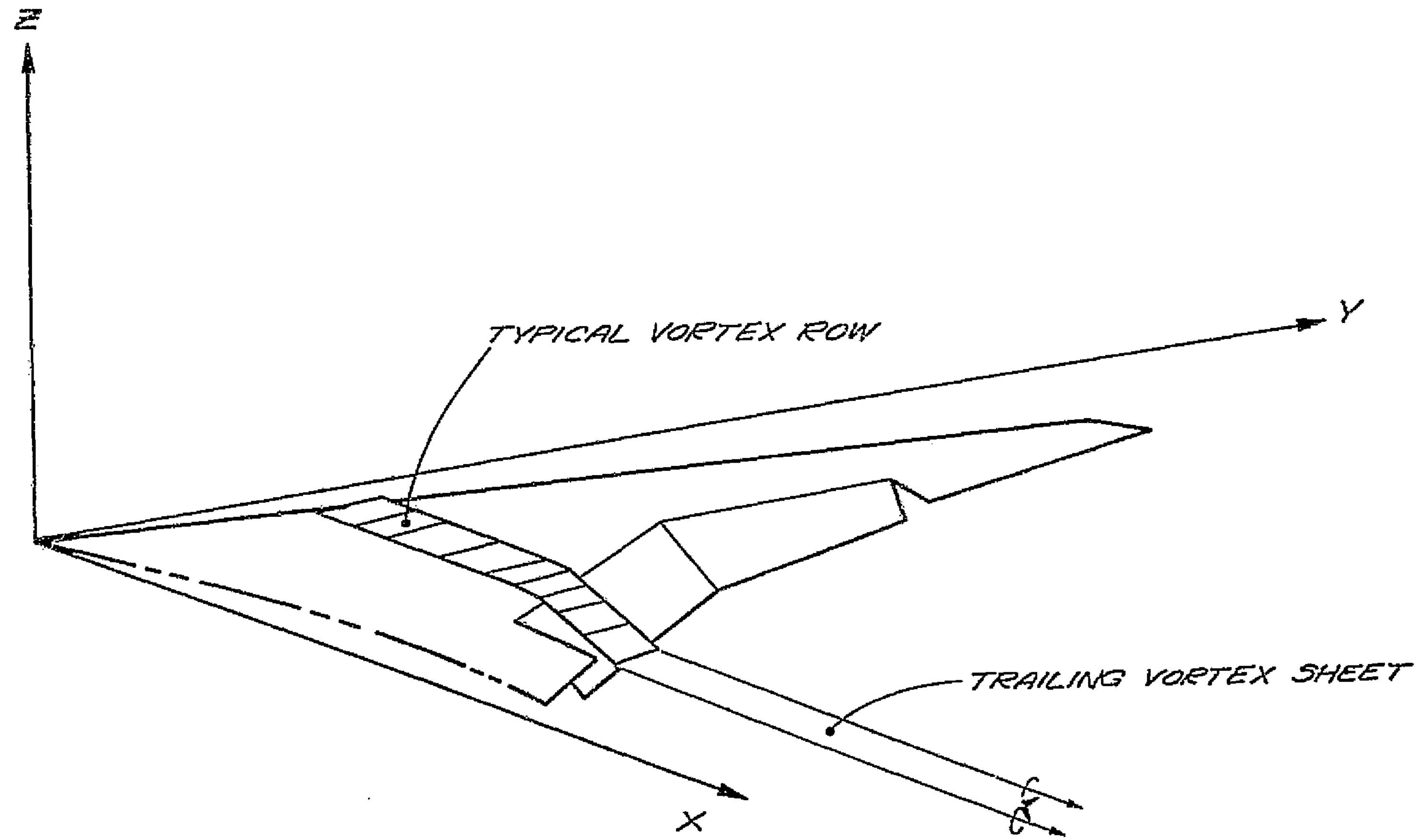

Figure 20. Illustration of Three-Dimensional Nonplanar Lifting Surface Theory Mathematical Model.

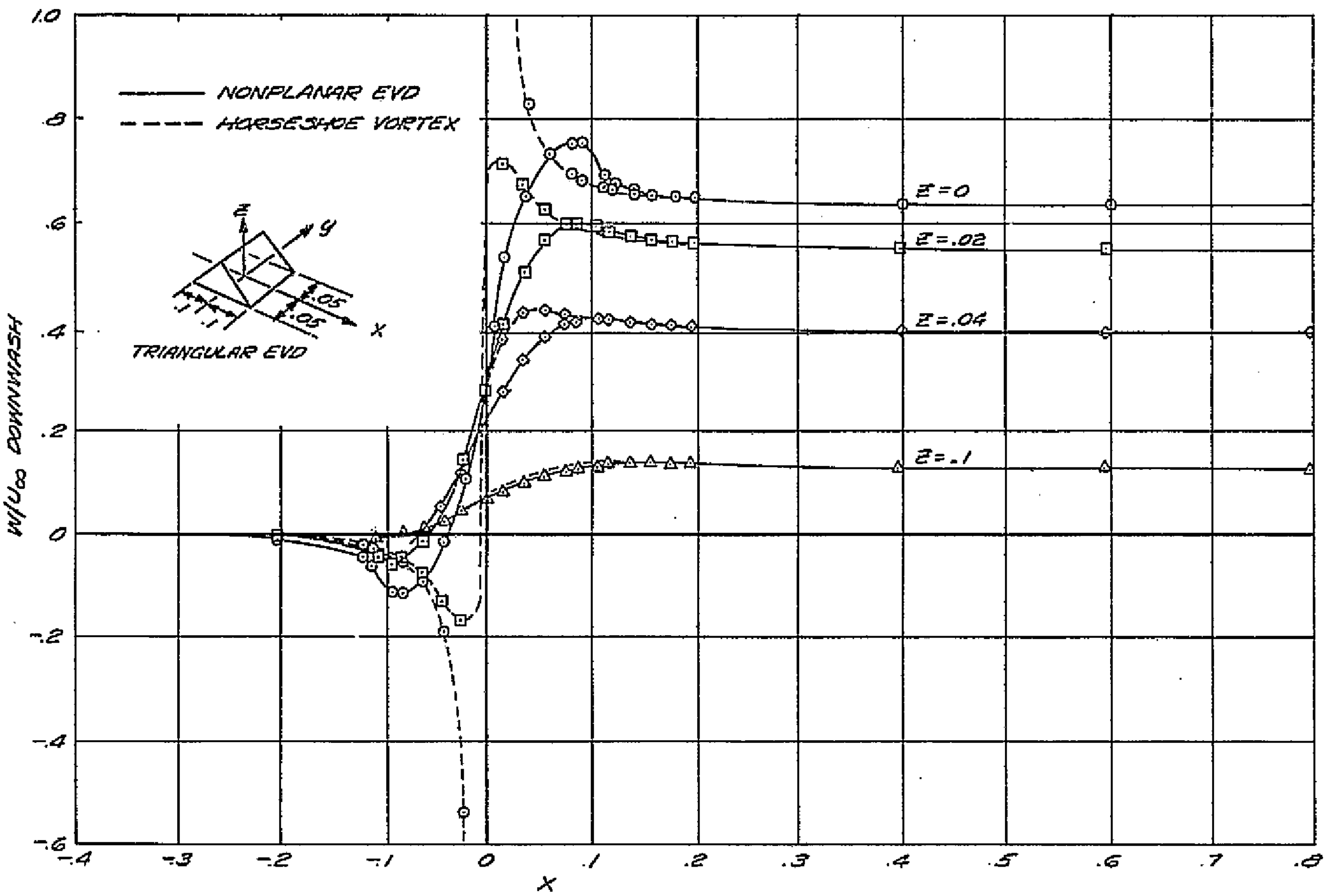

Figure 21. Comparison of Flow Induced by a Nonplanar Triangular Vortex Distribution and a Horseshoe Vortex of the Same Strength. (y = 0.0)

where $\vec{V}$ comes from equation (4) and $\vec{t}$ is the unit tangent vector. Aerodynamic forces and moments are simply calculated by suitable integrations of Δc_p over the surface. However, calculating induced drag requires not only a pressure integration but also a computation of leading edge suction. A leading edge suction term represents the chordwise force on the infinitesimally thin leading edge. The force is finite because the leading edge loading is singular. Because of the inverse-square-root singular vortex distribution employed in the present method, leading edge suction can be computed directly from the vortex strength solution.

The method described above, known as the Nonplanar Lifting Systems Program (NPLSP), has recently been expanded to include the effects of ground proximity. A standard image wing technique has been used, which effectively forms streamlines coincident with the ground plane. In order to consider effects of ascent or descent, a rotated freestream capability, identical to the "quasi-steady" technique discussed in Section 3.2, has been included. To facilitate its use, the computer program accepts inputs for flight path angle (γ) and attitude angle (θ) and computes angle of attack from equation (2). In the three-dimensional method the ground clearance is measured from the apex (leading edge at the centerline) of the wing.

An example of the validity of the Nonplanar Lifting Systems Program is shown in figure 22. Plotted are the lift, drag, and pitching moment coefficients for a simple aspect ratio 4 rectangular wing with an NACA 64_1A412 section, including experimental data from reference 15. Note that the predicted drag polar includes a zero lift friction drag estimate. Perhaps the most interesting feature of this comparison is the predicted nonlinearities of lift and pitching moment, which agree with experiment up to the onset of separated flow.

The validity of the ground effect capability has been ascertained using both experimental data and the Douglas Neumann Potential Flow Method (reference 16). The Neumann method not only considers the nonlinear aspects of the problem but also the effects of wing thickness. Figure 23 shows the ground effect on lift for an aspect ratio 6 rectangular wing computed by the

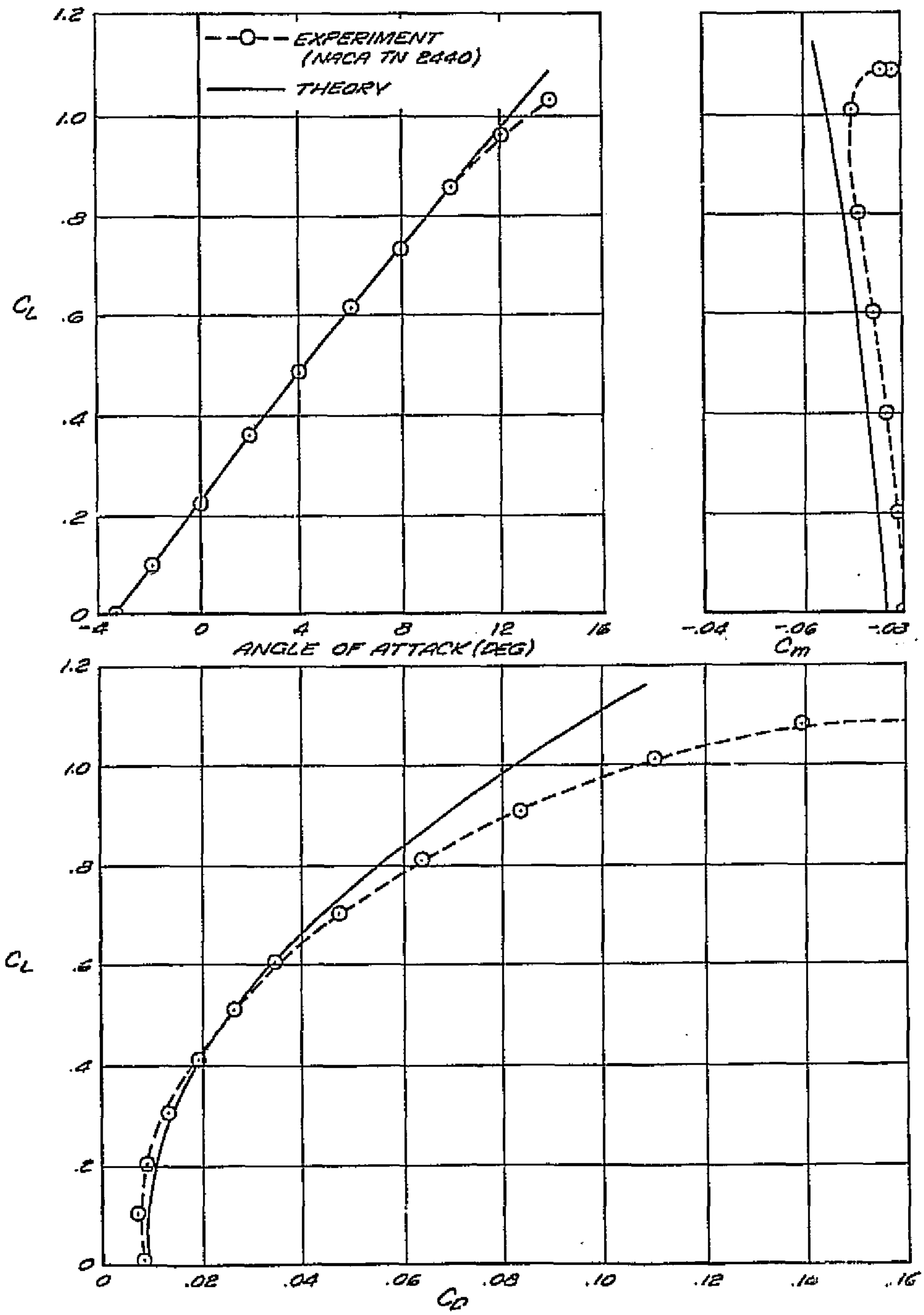

Figure 22. Example of Nonplanar Lifting Surface Theory Results for an Unblown Aspect Ratio 4 Rectangular Wing with a NACA 64_1A412 Section.

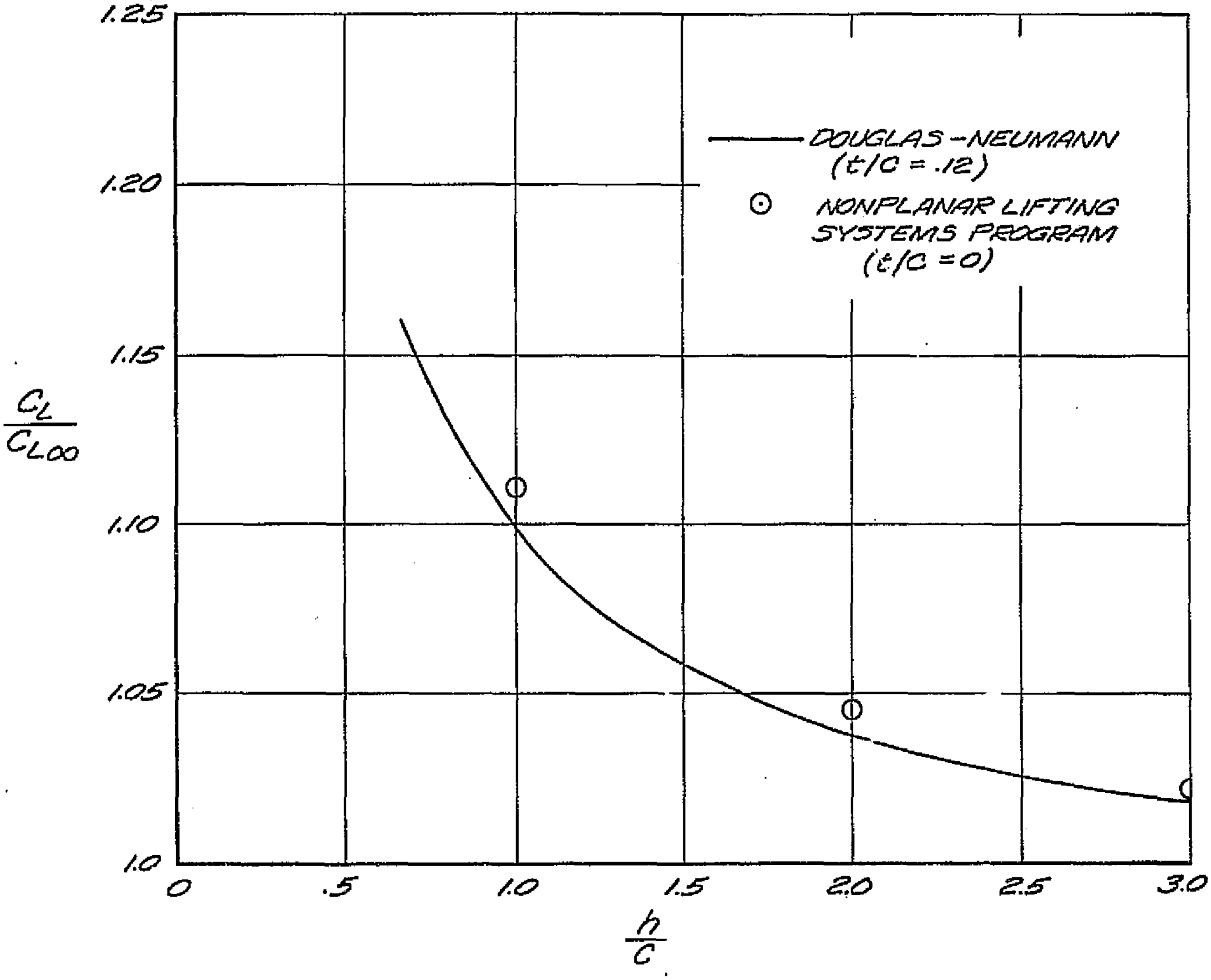

Figure 23. Comparison Between the Nonplanar Lifting Systems Program and the Douglas-Neumann Method for the Lift in Ground Proximity on an Unblown Aspect Ratio 6 Rectangular Wing.

Neumann method and by the present method. The small difference shown is undoubtedly due to the effects of wing thickness (t/c = .12) which tends to reduce lift in ground effect, as previously discussed.

Another verification of the ground effect capability of the present method is shown in figure 24, which includes experimental data from reference 17 for an aspect ratio 4 rectangular wing. The large thickness of the model (t/c = .22) again accounts for the small discrepancy shown.

Use of the present method to estimate the ground effect on a realistic transport aircraft configuration is shown in figure 25. The predicted reduction in angle of attack for a given lift coefficient agrees well with the plotted wind tunnel data.

4.2 Study of Three-Dimensional Ground Effects for Unblown Wings

The Douglas Nonplanar Lifting Systems Program, described in Section 4.1, has been used to study the nature of three-dimensional ground effect phenomena, particularly with regard to the influence of ascent and descent on lift and induced drag. Ascent and descent are modelled in the present method by suitable rotations of the freestream and wing. This procedure (i.e., quasi-steady technique) accounts for orientation effects but does not consider the unsteady aspects (i.e., the history) of the motion.

Figure 26 presents results of the ground effect analysis of an aspect ratio 7 rectangular wing, both clean and with a large full span flap. A larger than practical flight path angle (10 degrees) has been used to establish limits to the effects of ascent and descent. From these plots it can be seen that orientation effects can change the predicted ground effect by up to three percent of the free air lift and by up to ten percent of the free air induced drag. These data, it should be remembered, are for unblown wings, although the large flap provided lift coefficients typical of STOL aircraft.

4.3 Further Development of a Three-Dimensional Ground Effect Method

The method described in the preceding section provides the capability

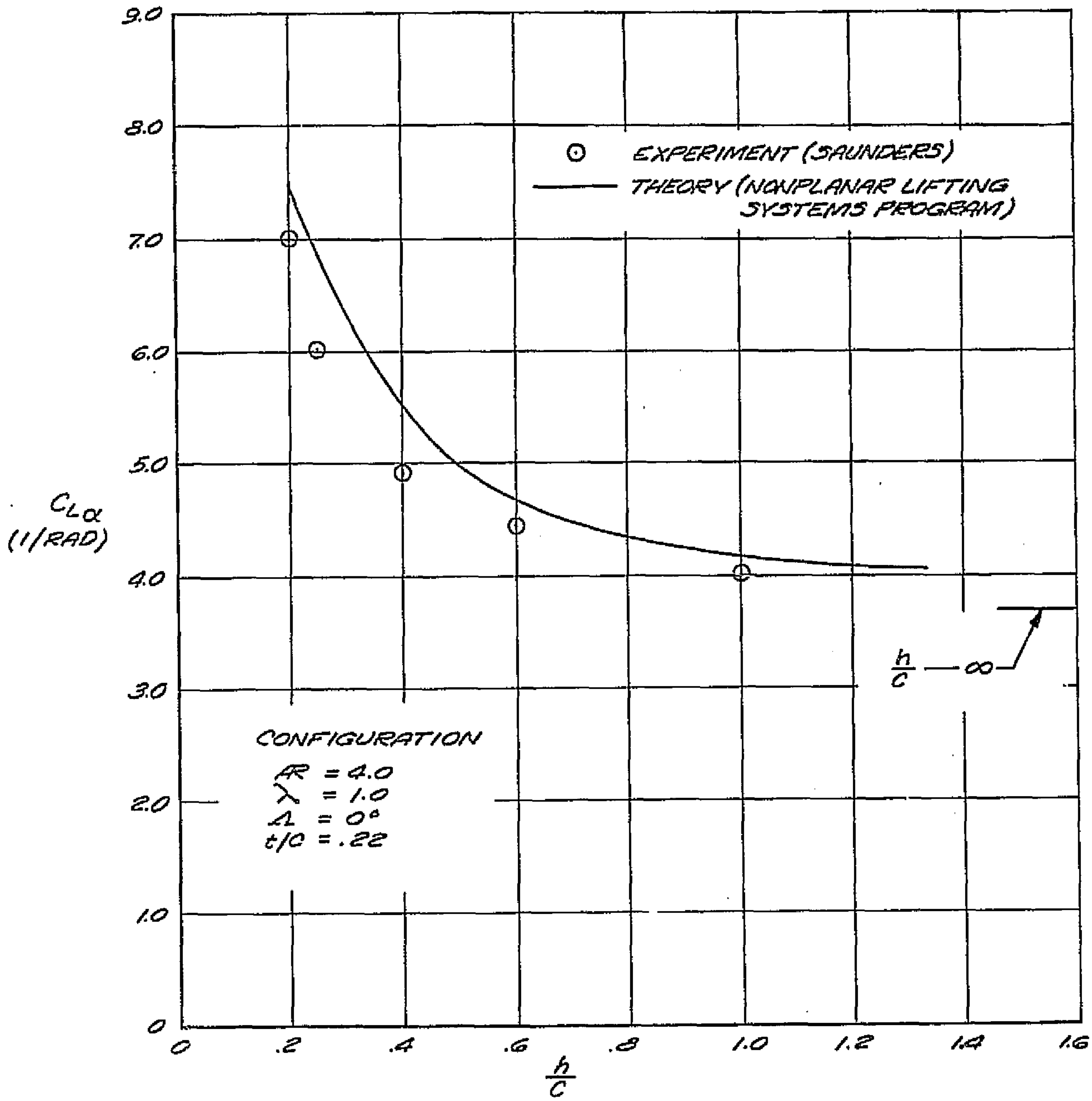

Figure 24. Comparison Between the Nonplanar Lifting Systems Program Results and Experiment (reference 17) for the Lift Curve Slope of a Rectangular Wing in Ground Effect.

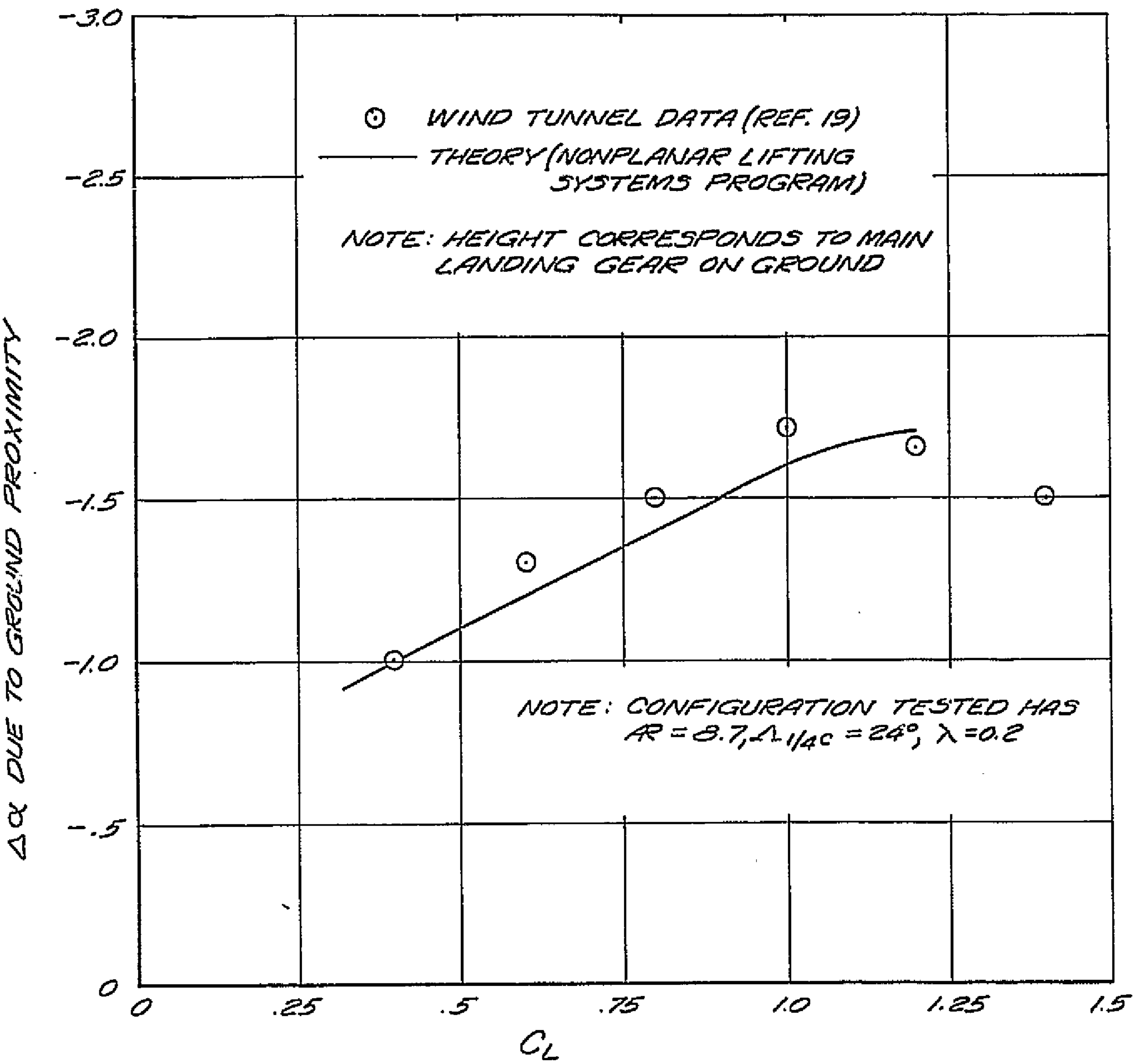

Figure 25. Comparison Between Theory and Wind Tunnel Data for the Ground Effect on Lift on a Subsonic Transport Aircraft with Flaps Deflected five degrees.

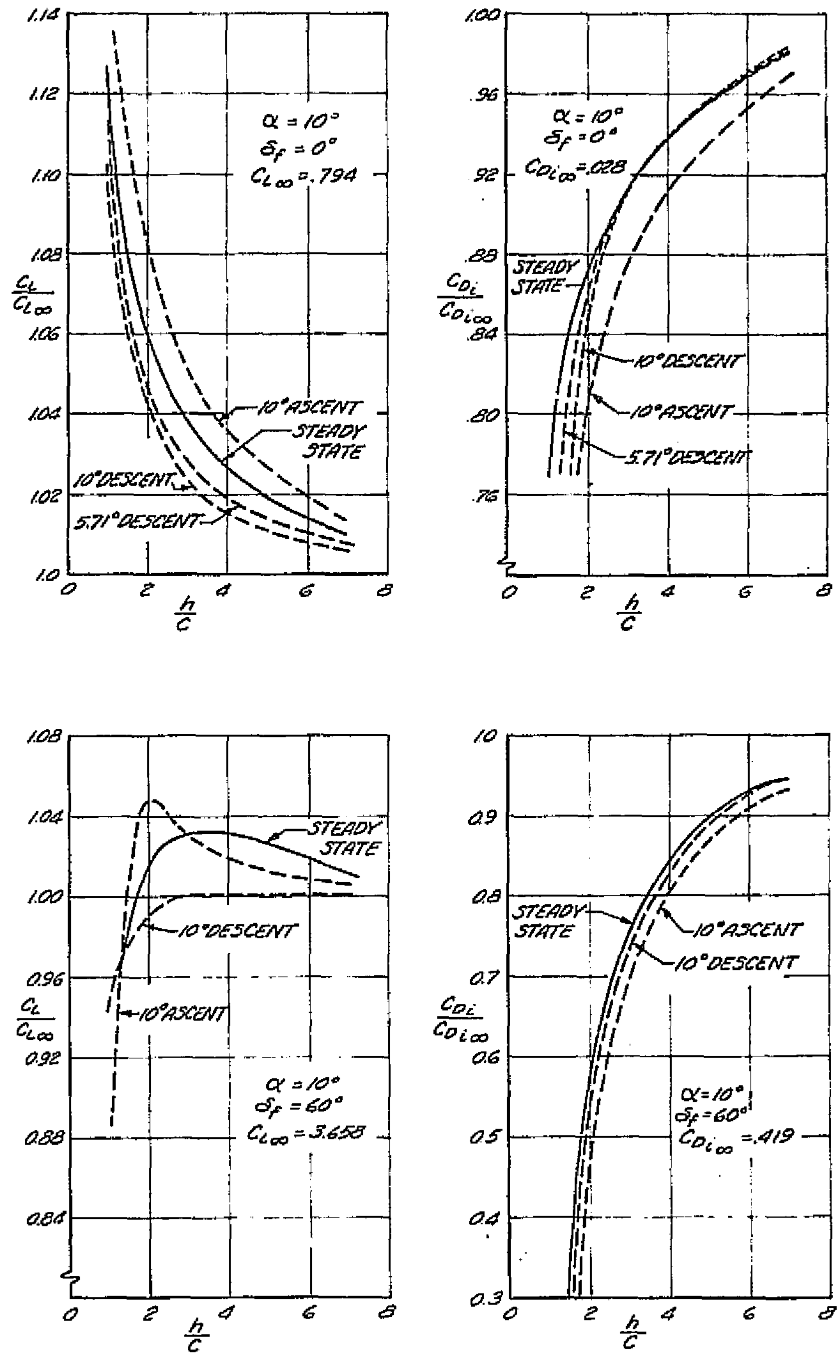

Figure 26. Comparison of Steady State and Quasi-Steady Solutions for Lift and Induced Drag on an Aspect Ratio 7 Rectangular Wing in Ground Effect (Nonplanar Lifting Systems Program Results).

to analyze conventional wings in ground proximity, including nonplanar and nonlinear effects. This work resulted in the development of the nonplanar vortex distribution influence functions which have been shown to be quite suitable in terms of numerical behavior and computer utilization. Use of these influence functions in a nonplanar lifting surface theory has been successfully demonstrated.

Further development of the present method to provide the needed jet-wing ground effect capability will require the addition of a thin jet sheet to the present mathematical model. The approach recommended is the same as that used in the method of reference 4 and described in Section 3.1. That approach models the jet sheet by a distribution of vorticity extending downstream to infinity. The jet shape will be determined through an iterative procedure by satisfying the dynamic boundary condition (pressure-curvature relationship) and modifying the jet trajectory until the no-normal-flow condition is also satisfied. It is unknown at this time whether the convergence characteristics of a three-dimensional solution will be as good as the two-dimensional method (typically 3 to 5 iterations required), but with the added complexity of spanwise variation of jet parameters it is likely that more iterations will be required. Because of the relatively large computer costs of a three-dimensional method, it will be desirable to explore simplications to the method to speed the iteration process. Possible simplications include changes to the normal-wash influence functions and to matrix solution techniques. Three-dimensional impingement modelling could be based on a scheme similar to that presented in Section 2.3, although it is likely that only a qualitative solution could be obtained with the approximate approach. Thick jet effects are considered to be a much more difficult analysis task.

Matrix partitioning had been considered to be a highly desirable timesaving technique because it allows the constant portion of a matrix to be solved only once while the changing portion is solved in each iteration. The advantage to be gained in using partitioning depends on the number of constant matrix elements relative to the number of changing elements and also on the number of iterations required. Initially it was thought that the wing-on-wing portion of the normal-wash matrix did not change with each iteration, but it has since been realized that this is true only for wing sections which have no jet sheet. Influence functions for elements on a jet-flapped wing section do change in the iteration process because of the change in position of trailing vorticity as the jet sheet moves. Thus the potential increase in computing efficiency by using a partitioning technique would not be as large as originally anticipated. However, partitioning still may prove useful, especially when a significant portion of the wing is unblown (such as upper surface blown configurations), since then a large portion of the matrix could be preserved. It is anticipated that matrix partitioning should be considered, but only after the jet-wing ground effect capability is developed.

5.0 CONCLUSIONS AND RECOMMENDATIONS

This analytical study of STOL ground effects has explored the problem two-dimensionally in considerable detail and has established the necessary theoretical basis from which a three-dimensional jet-wing ground effect method can be developed. It has been shown that the ground effect problem is highly nonlinear and that jet-flapped airfoils in ground effect generally show a larger loss in lift than conventional airfoils, especially when the effects of jet/ground impingement are considered. Ground effect transients have been studied to the limits of the existing methodology, and it has been shown for two-dimensional unblown airfoils that nearly all of the transient effect is a consequence of airfoil/freestream/ground orientation rather than of unsteady effects. Orientation effects for blown airfoils have been shown to be larger than for unblown airfoils. Study of unsteady effects for blown airfoils is beyond the capabilities of existing methodology, but it can be speculated that those effects may be larger than for unblown airfoils because of changes in the jet trajectory. For three-dimensional wings, orientation effects have been shown to be of the same order of magnitude as for unblown airfoils. Based on fundamental theoretical unsteady methods, it is likely that unsteady effects will be of less importance in three-dimensions than in two-dimensions.

Because of the limits of existing methodology, it is impossible at this time to assess analytically the importance of ground effect transients for STOL aircraft in relation to performance, stability and control, or handling qualities. The results of the two-dimensional study presented here, however, do show the effects to be small and to be primarily a result of airfoil orientation rather than of unsteady motion. Results obtained from this study do not indicate any profound difference, in terms of ground effect transients, between unblown and blown airfoils. However, because of the limits of existing methodology and serious concern about the validity of existing experimental data, no firm conclusions can be drawn.

This study has shown the importance of steady state STOL ground effects and has established the need for improved analytical methods. The significance of transient ground effect phenomena for powered lift systems cannot be assessed adequately within the scope of present analytical techniques, although the quasi-steady technique developed here does show promise of simply accounting for transients. Experiment and flight test could establish the significance of transients.

Work done in this study has established a firm theoretical framework for the development of a jet-wing ground effect method. A complete set of nonplanar vortex distribution influence functions has been developed, and the basic nonplanar lifting surface theory scheme has been successfully developed for unblown wings, including a ground effect capability. Some progress has also been made in modelling jet impingement. It is felt that sufficient progress has been made to continue, with confidence, the development of the nonplanar, nonlinear jet-wing ground effect method.

It is recommended that the following tasks be considered in any future research on STOL ground effects:

- Extension of the Douglas Nonplanar Lifting Systems Program to include a thin jet sheet, both in and out of ground proximity. Jet impingement modelling, based on the two-dimensional work done here, should be included.
- Extension of jet-flapped airfoil techniques to include unsteady effects.
- Thick jet/wing analysis in ground proximity. This could be done using a doublet approach or the Neumann method approach along with existing or new thick net techniques.
- An experimental program to establish the importance of ground effect transients.
- Study of viscous effects associated with a jet-wing in ground proximity.
- Further ground effect flight testing, both steady state and transient, of the Buffalo augmentor wing aircraft, the YC-15, and the YC-14 AMST prototypes.

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