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THE LAGEOS SYSTEM

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16. Abstract

The LAGEOS system is defined and its rationale is developed. This report was prepared in February 1974 and served as the basis for the LAGEOS Satellite Program development. Key features of the baseline system specified then included a circular orbit at 5900 km altitude and an inclination of 110°, and a satellite 60 cm in diameter weighing some 385 kg and mounting 440 retroreflectors, each having a diameter of 3.8 cm, leaving 30% of the spherical surface available for reflecting sunlight diffusely to facilitate tracking by Baker-Nunn cameras. The satellite weight was increased to 411 kg in the actual design through the addition of a 4th-stage apogee-kick motor. The number and diameter of the retroreflectors are now 426 and 3.81 cm, respectively. The net effect of these partially compensating changes is not significant. The diffusely reflecting area was increased to 47% of the satellite surface.

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FOREWORD

The LAGEOS system is defined and its rationale is developed. This report was prepared in February 1974 and served as the basis for the LAGEOS Satellite Program development. Baseline parameter values specified then and those actually selected for the design are as follows:

Parameter	Value: Baseline Feb.1974	Value: Actual Design
Altitude (km)	5900	5900
Inclination (deg)	110	110
Eccentricity	0	0
Diameter of Satellite (cm)	60	60
Weight of Satellite (kg)	385	411
Number of Retroreflectors	440	426
Retroreflector Diameter (cm)	3.8	3.81
Diffusely Reflecting Surface (%)	30	47
Dihedral Angle Offset (arc sec)	1.5	1.25
Recess Depth (mm)	1	1
Ratio of Moments of Inertia	1.1	1.03

The satellite weight was increased in the actual design as a result of launch vehicle modifications, which included the addition of a 4th-stage apogee-kick motor. The smaller number of retroreflectors is compensated for to a limited extent by the slightly larger retroreflector diameter. The net effect on performance is not significant. Covering the retroreflector mounting rings with aluminum, which detailed analysis of thermal and other factors showed to be feasible, increased by more than half the portion of the spherical surface area available for diffusely reflecting sunlight for Baker-Nunn camera tracking. Measurements of a number of retroreflection patterns showed that a dihedral angle of $90^{\circ} + 1''.25$ gave an energy maximum in the aberration annulus, hence this value was used instead of a theoretical estimate which was the basis for the earlier figure for the offset. The smaller moment of inertia ratio resulted in suitable stability characteristics for the satellite, as well as a considerable simplification in the fabrication process.

THE LAGEOS SYSTEM

I. INTRODUCTION

The LAGEOS program, centered around the first new spacecraft in the NASA Earth and Ocean Physics Applications Program (EOPAP), is entering an implementation phase as various aspects of the Phase B Definition Study get underway at the Marshall Space Flight Center. (1)

A review of the LAGEOS Program's objectives and scientific and technical features is in progress. The initial aims have been to review the study entitled "Use of a Passive Stable Satellite for Earth Physics Applications" which had been conducted by the Smithsonian Astrophysical Observatory (SAO), and to consider other views related to the orbit altitude and inclination and the satellite size and mass in order to provide a basis for the specification of the LAGEOS System as a basis for the Phase B Definition Study. (2)

These processes were begun at meetings on October 11, 1973, at which these latter points were discussed in considerable detail. Aspects of the SAO Study having to do with the retroreflectors themselves were considered at a meeting on October 29, 1973. (2) Both meetings revealed the need for further investigation of a number of specific points. Various studies have been conducted or initiated in response to the needs indicated during these meetings and in subsequent discussions. Tentative conclusions reached at these meetings and later on the basis of a number of the studies and additional discussions are described here, and the corresponding quantities are listed in Figure 1. These values

LAGEOS SYSTEM
NOMINAL BASELINE PARAMETERS

Altitude	5900 km
Inclination	110 deg
Eccentricity	0
Diameter of Satellite	60 cm
Weight of Satellite	385 kg
Number of Retroreflectors	440
Fraction of Surface Reflecting Diffusely	0.30
Retroreflector Diameter	3.8 cm
Dihedral Angle	$90^{\circ} + 1.5''$
Recess Depth	0.1 cm
Ratio of Moments of Inertia	1.1

FIGURE 1

represent judgements based on the available information. Their use enables the program to proceed.

Studies underway will be considered at appropriate times in order to provide the basis for reviews of these choices.

II. THE GENERAL LAGEOS REVIEW

A list of those who attended the General LAGEOS Review Meeting on October 11, 1973 is attached. (Cf. Fig. 2) Other organizations whose representatives were invited included the U.S. Geological Survey, the National Science Foundation, and the National Academy of Sciences.

The meeting opened with a review of the EOPAP objectives and the program as a whole which was presented by Mr. F. Williams, Director of the Special Programs Division of the NASA Office of Applications. The attached Figures 3 and 4 were part of his presentation. (1)

A discussion of the LAGEOS Program in the context of the overall EOPAP effort was presented by Dr. J. Siry. The attached Figures 5 through 13 were discussed. (1,2,4,5,25,26)

Dr. George Weiffenbach presented a review of the SAO Study. Copies of this report had been sent to the attendees before the meeting. His presentation included, in particular, the attached Figures 14 through 31. (2)

A brief review of other views concerning the orbital altitude and inclination and the satellite's size and mass was then presented by the Chairman. Dr. Siry included in this review the recommendations made in references 3 and 4, and in discussions with a number of those who had

THE GENERAL LAGEOS REVIEW
OCTOBER 11, 1973
ATTENDANCE

NAME	ORGANIZATION	PHONE NO.
R. Spencer	NASA-MSFC	205-453-2818
E.M. Gaposchkin	SAO	617-864-7910 x495
M. R. Pearlman	SAO	617-864-7910 x481
W. Lurie	SAO	617-864-7910 x485
George Weiffenbach	SAO	617-864-7910 x286
Bernard Chovitz	NOAA/NOS	301-496-8423
James Faller	JILA/NBS	303-499-1000 x3463
Michael Graber	NOAA/NOS	301-496-8556
Richard Anderle	NWL	703-663-8159
Donald Eckhardt	AFCRL	617-861-4550
Larry D. Beers	HQ DMA/PRA	703-254-4455
Wm. M. Kaula	UCLA	213-825-4363
F. L. Williams	NASA HQ	202-755-8458
J. P. Murphy	NASA HQ	202-755-3260
Jon Berger	UCSD	714-453-2000 x1798
J. W. Siry	NASA HQ	202-755-3837
D. E. Smith	GSFC	301-982-4555
T. S. Johnson	GSFC	301-982-4835
J. Whitcomb	JPL-Caltech	213-795-8806
I. J. Mueller	OSU	614-422-2269
D. Trask	JPL	213-354-4878
C. Scholz	LDGO	914-359-2900 x373
S. Yionoulis	APL	953-7100 x3057

FIGURE 2

EARTH AND OCEAN PHYSICS APPLICATIONS PROGRAM (EOPAP) OBJECTIVES

DEVELOPMENT AND VALIDATION OF METHODS OF OBSERVING THE EARTH'S DYNAMICAL MOTIONS USING SPACE TECHNIQUES TO MAKE UNIQUE CONTRIBUTIONS TO THE KNOWLEDGE OF EARTHQUAKE MECHANISMS AND THE DEVELOPMENT OF EARTHQUAKE PREDICTION APPROACHES.

DEVELOPMENT AND VALIDATION OF MEANS FOR PREDICTING THE GENERAL OCEAN CIRCULATION, SURFACE CURRENTS, AND THEIR TRANSPORT OF MASS AND HEAT.

DEVELOPMENT AND VALIDATION OF METHODS FOR SYNOPTIC MONITORING AND PREDICTING OF TRANSIENT SURFACE PHENOMENA, INCLUDING THE MAGNITUDES AND GEOGRAPHICAL DISTRIBUTIONS OF SEA STATE, STORM SURGES, SWELL, SURFACE WINDS, ETC., WITH EMPHASIS ON IDENTIFYING EXISTING AND POTENTIAL HAZARDS.

REFINEMENT OF THE GLOBAL GEOID, EXTENSION OF GEODETIC CONTROL TO INACCESSIBLE AREAS INCLUDING THE OCEAN FLOORS, AND IMPROVEMENT OF KNOWLEDGE OF THE GEOMAGNETIC FIELD FOR MAPPING AND GEOPHYSICAL APPLICATIONS.

FIGURE 3

EARTH AND OCEAN PHYSICS APPLICATION PROGRAM

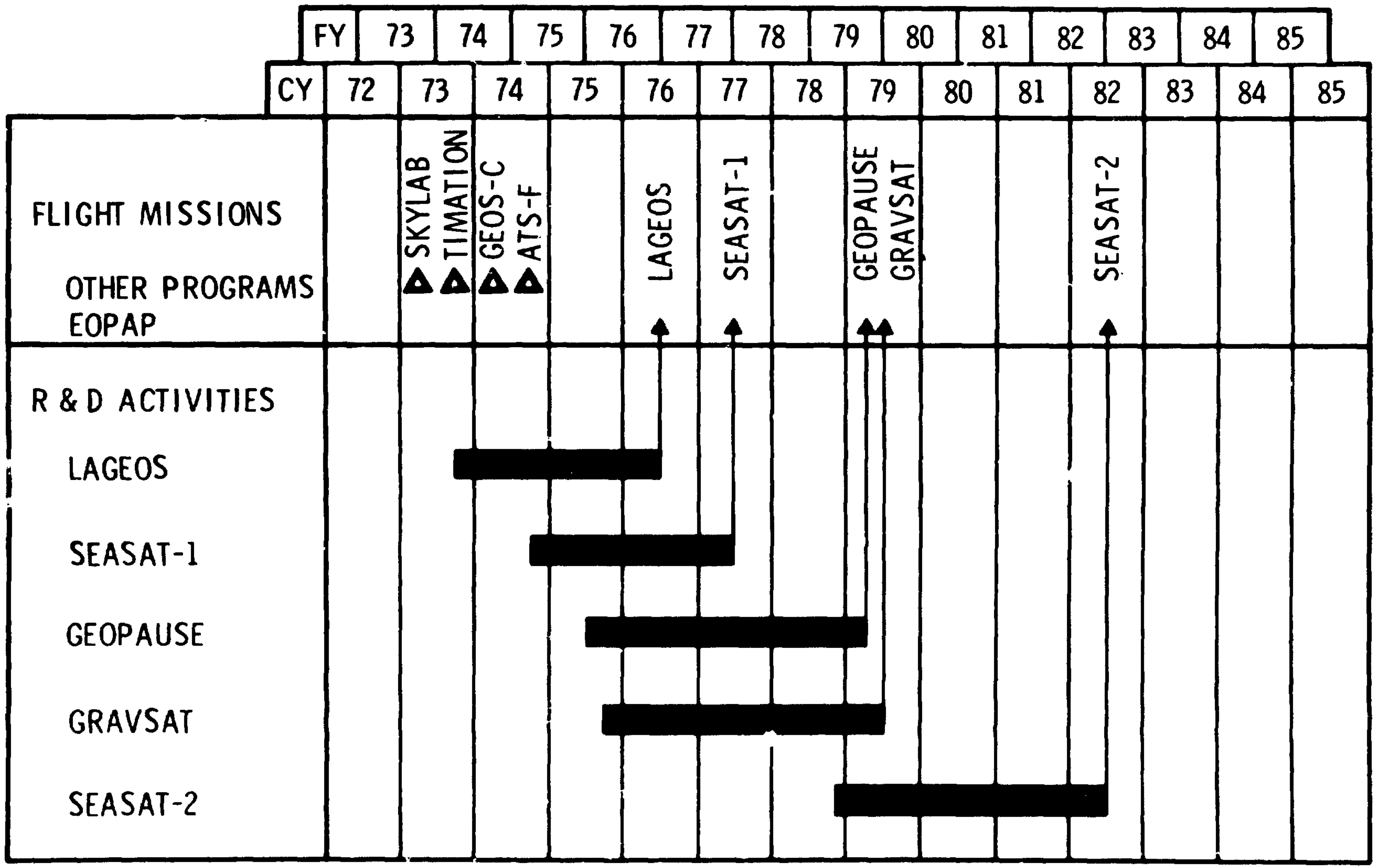


FIGURE 4

NASA ES73-1802
2-2-73

MEASUREMENT REQUIREMENTS SUMMARY

MEASUREMENT	ACCURACY
• CRUSTAL MOTION	1 cm / year
• POLAR MOTION, EARTH ROTATION	2 cm / 0.5 day
• SATELLITE ORBITS	10 cm
• GRAVITY FIELD / GEOID	10 cm
• SEA SURFACE TOPOGRAPHY	10 cm
• SEA STATE / WAVE HEIGHT	1–3 m
• SURFACE WINDS	2–5 m/s
• MAGNETIC FIELD	2 gamma, 0.5 arc min

FIGURE 5
(Cf. ref. 1)

EARTH DYNAMICS EXPERIMENTS

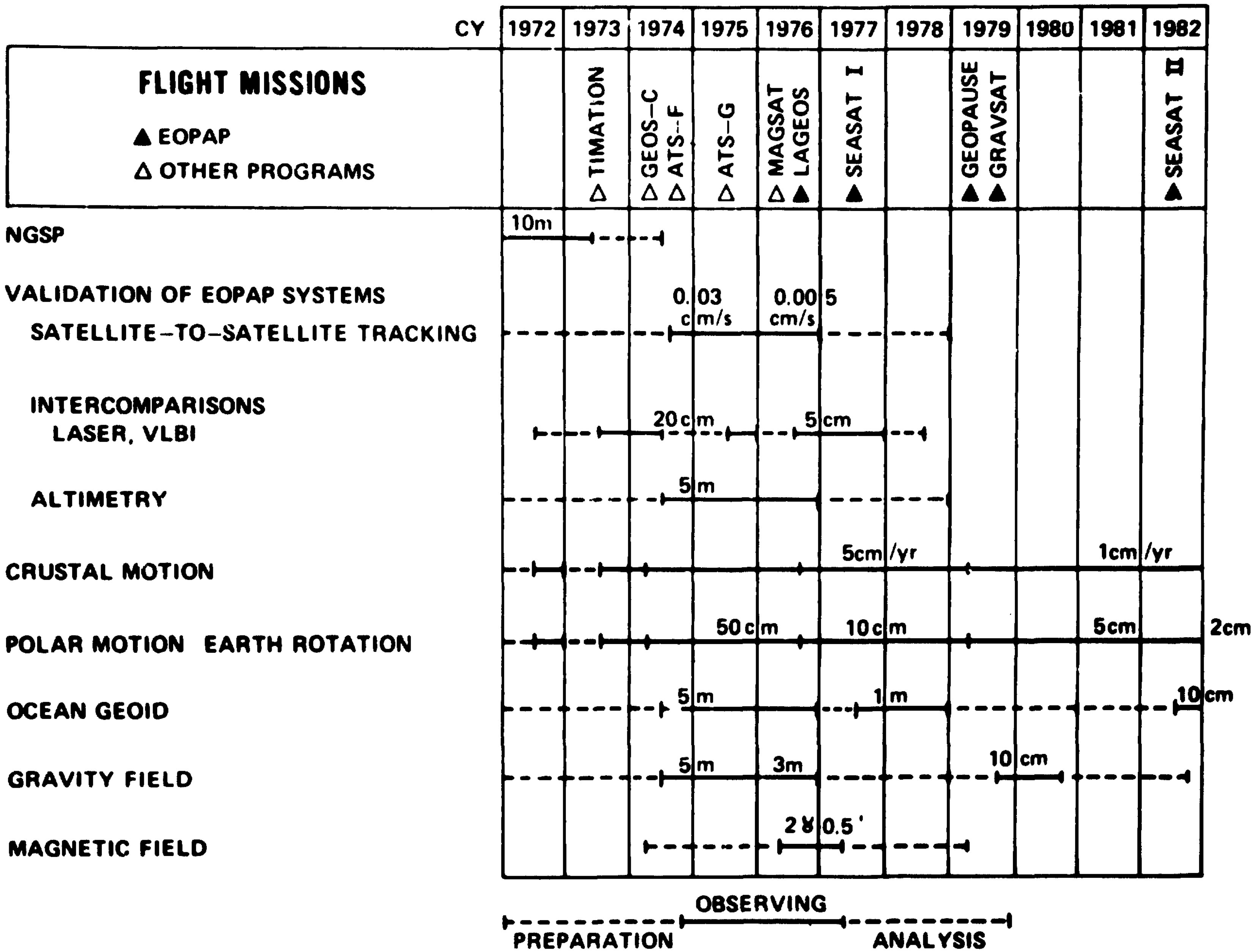


FIGURE 6
(Cf. ref. 1)

		Current Status	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	Final Goals
			CALENDAR YEAR											
SPACECRAFT	EARTH AND OCEAN DYNAMICS PROGRAM					▼ LAGEOS	▼ SEASAT 1		▼ GEOPAUSE ▼ GRAVSAT			▼ SEASAT 2		OPERATIONAL SPACECRAFT
	OTHER PROGRAMS	7 SATELLITES WITH REFLECTORS 4 LUNAR REFLECTORS	▽ SKYLAB ▽ TIMATION	▽ GEOS-C ▽ ATS-F	▽ ATS-G	▽ MAGSAT								
INSTRUMENTATION ACCURACY	LASERS, SATELLITE, FIXED	5 (50 cm)	5 (20 cm)	8 (10 cm)	8 (5 cm)			△ DECISION POINT ON FINAL EOPAP GROUND SYSTEM		△ AVAILABILITY OF 2 CM GROUND SYSTEM				OPERATIONAL NETWORK
	LASERS, SATELLITE, MOVABLE	2 (30 cm)	3 (20 cm)	4 (10 cm)	4 (5 cm)									
	LASERS, LUNAR, FIXED	1 (25 cm)	3 (15 cm)	3 (10 cm)	3 (5 cm)									
	LASERS, LUNAR, MOVABLE	–	–	1 (10 cm)	1 (5 cm)									
	VLBI TERMINALS, FIXED	6 (1 m)	6 (30 cm)	6 (10 cm)	6 (5 cm)									
	VLBI TERMINALS, MOVABLE	–	1 (30 cm)	2 (10 cm)	2 (5 cm)									
	C-BAND RADAR	7 (1–5 m)												
	USB/DSN	26 (10 m)												
	MODIFIED ATS/USB GROUND STATIONS			3 (10 cm) (0.01cm/sec)	3 (5 cm) (0.005cm/sec)									
	SATELLITE-TO-SATELLITE TRACKING			0.07 cm/sec 7 m					0.003 cm/sec 2–10 cm					
	SATELLITE ALTIMETRY SYSTEM			2 m			50 cm						10 cm	10 cm
EARTH PARAMETERS	POLE POSITION	1 m		50 cm			10 cm				5 cm			2 cm
	UT1, POINT ON EQUATOR	1 m		1 m			10 cm				5 cm			2 cm
	REGIONAL DISTANCE	50 cm		20 cm			5 cm				3 cm			2 cm
	GLOBAL DISTANCE	5 m		1 m			10 cm				5 cm			5 cm
	GEOID $\lambda/2 > 1500$ km	3–5 m			2 m			50 cm			10 cm			10 cm
	GEOID $1500 > \lambda/2 > 300$ km	10–25 m			3 m			1 m			10 cm			10 cm
	GEOID $300 > \lambda/2 > 10$ km	10–25 m			2 m			50 cm			50 cm			10 cm

FISCAL YEAR: 1973 | 1974 | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984

▼ EOPAP MISSIONS
▽ OTHER PROGRAMS

FIGURE 7. Systems capabilities and milestones.
(Cf. ref. 1)

LAGEOS PROGRAM OBJECTIVES

- DEMONSTRATE THE CAPABILITY FOR MAKING ACCURATE DETERMINATIONS OF THE EARTH'S CRUSTAL AND ROTATIONAL MOTIONS BY MEANS OF LASER SATELLITE TRACKING TECHNIQUES

- EMPLOY THIS CAPABILITY TO OBSERVE

  FAULT MOTION
  REGIONAL STRAIN FIELDS
  TECTONIC PLATE MOTION
  POLAR MOTION
  EARTH ROTATION
  SOLID EARTH TIDES
  STATION POSITIONS

- MAKE APPLICATION OF THE RESULTS TO THE SOLUTION OF PROBLEMS SUCH AS THOSE ASSOCIATED WITH EARTHQUAKE MECHANISMS AND ORBIT DETERMINATION FOR OCEAN DYNAMICS SPACECRAFT

FIGURE 8

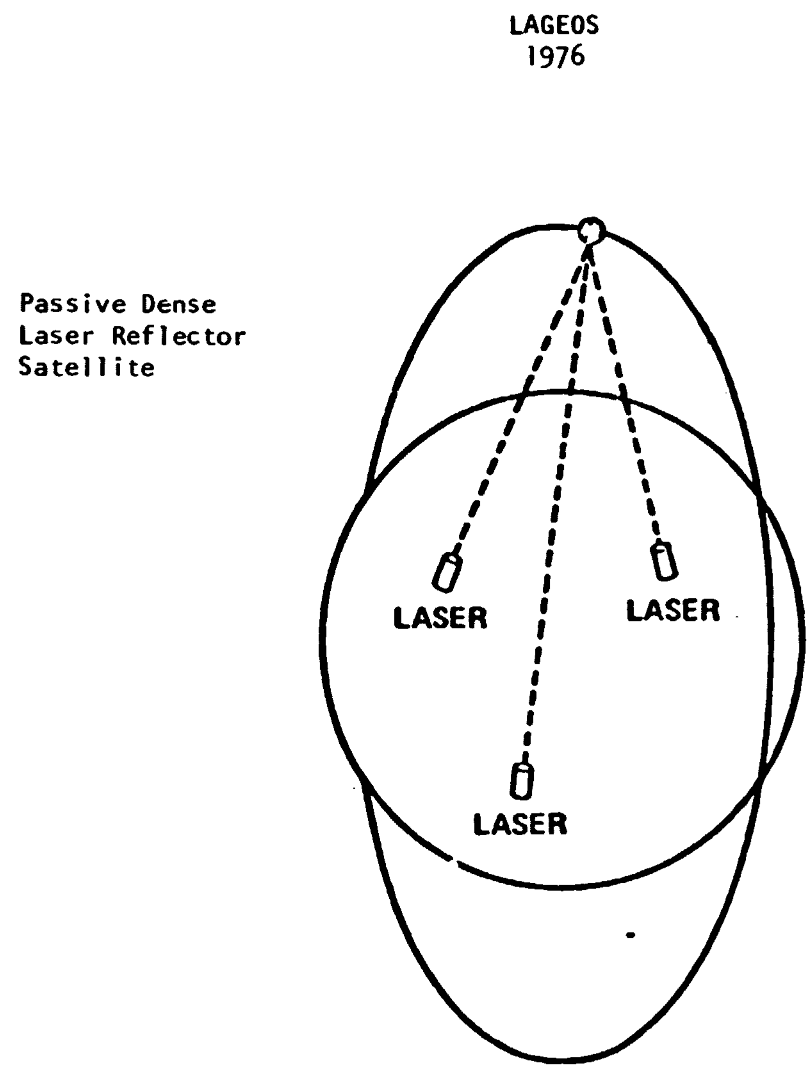


FIGURE 9
(Cf. ref. 1)

ORBIT SELECTION CONSIDERATIONS

ORBIT PROPERTY	a	i
PERTURBATION MINIMIZATION		
GRAVITATIONAL	LARGE	
RADIATION PRESSURE	SMALL	
TRACKABILITY		
RANGE	SMALL	
ANGULAR RATES	LARGE	
TRACKING COVERAGE		
AROUND ORBIT FOR PERTURBATION SENSING	LARGE	$60^0 - 90^0$
AT MAXIMUM LATITUDES FOR POLAR MOTION SENSING	LARGE	$70^0 - 90^0$
AROUND STATIONS FOR POSITIONING	LARGE	$70^0 - 90^0$

FIGURE 10

PREEXISTING SITES

FIXED

GSFC, MD
MT. HOPKINS, ARIZONA
NATAL, BRAZIL
AREQUIPA, PERU
JOHANNESBURG, SOUTH AFRICA

MOBILE

QUINCY, CALIF.
SAN DIEGO, CALIF.
WALLOPS STATION, VA
BERMUDA
GRAND TURK
CAPE KENNEDY
UTAH
MEXICO
CANAL ZONE

NEW LAGEOS SITES

FIXED

ORORRAL VALLEY, AUSTRALIA
FORT RESOLUTION, CANADA
SAO PAULO, BRAZIL
KASHIMA, JAPAN
MADRID, SPAIN

MOBILE

ADDIS ABABA, ETHIOPIA
NAINI TAL, INDIA
MAUI OR KAUAI, HAWAII
UPPSALA, SWEDEN
TAHITI
BANGKOK, THAILAND
FAIRBANKS, ALASKA

FIGURE 11
(Cf. ref. 27)

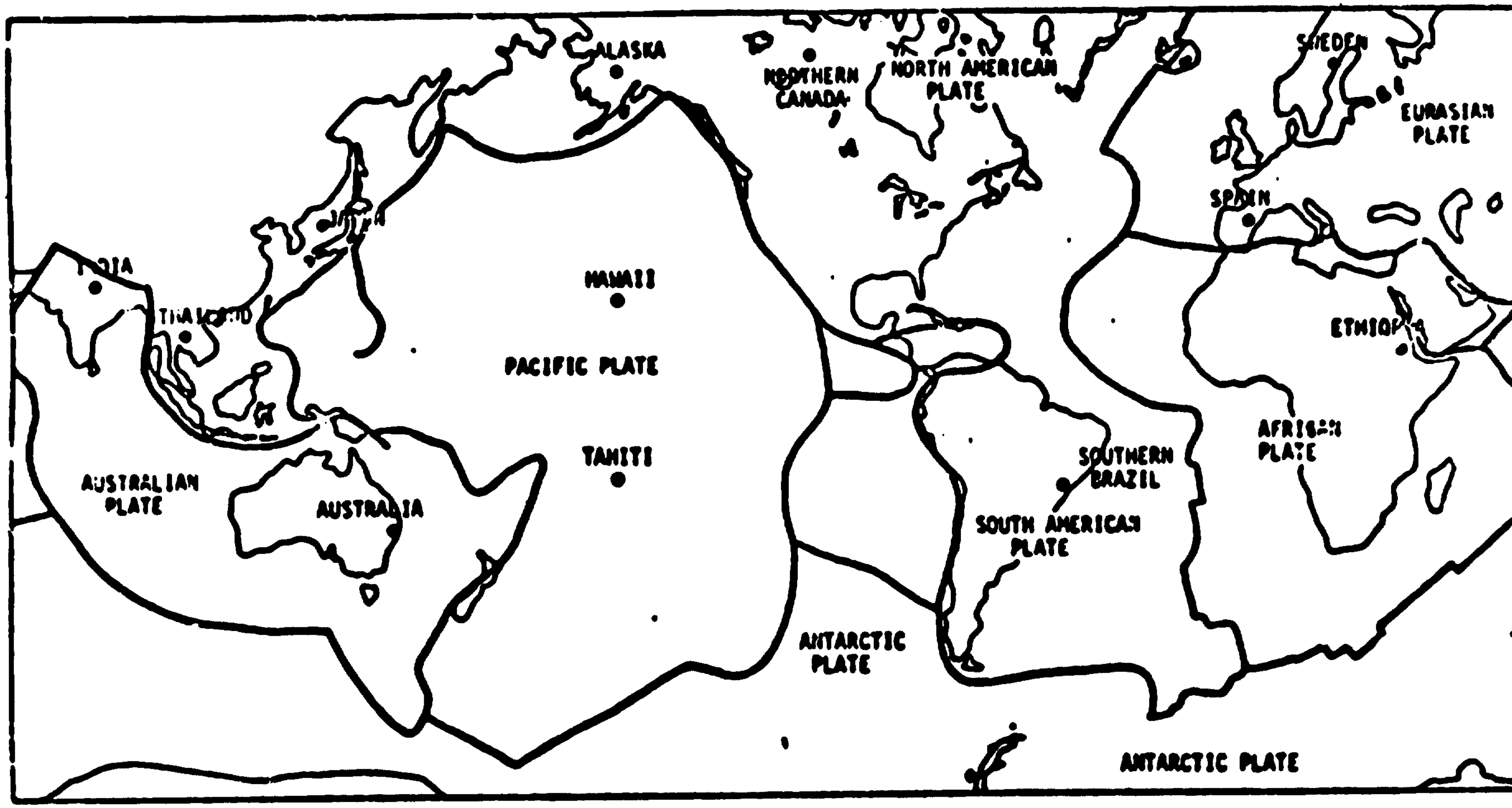


FIGURE 12
(Cf. ref. 27)

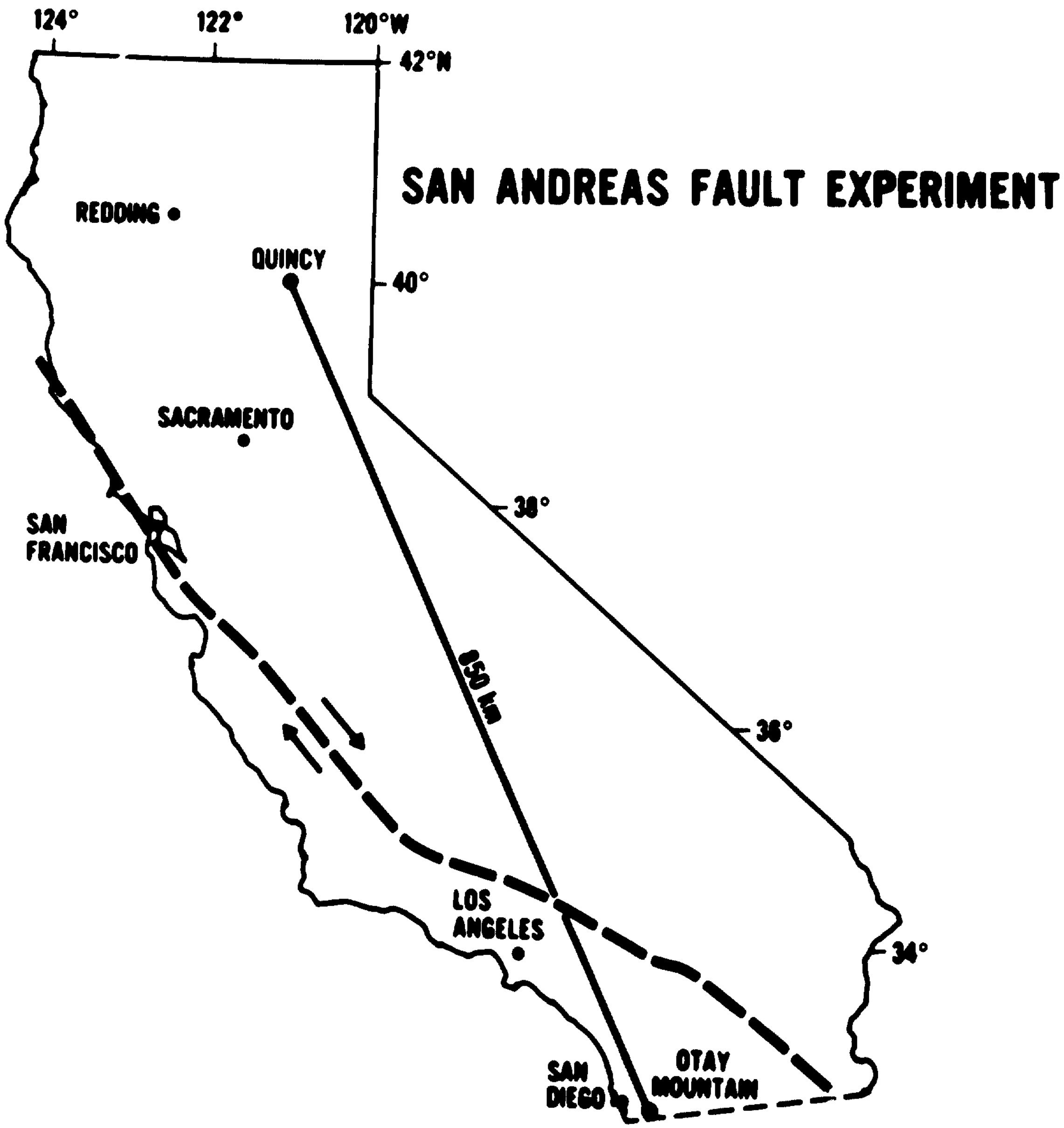


FIGURE 13
(Cf. ref. 5)

NASA HQ ES74-15603 (1)
2-20-74

LAGEOS MISSION OBJECTIVES

- USEFUL LIFE IN ORBIT . . . . 20 YEARS
- STATION POSITION ACCURACY. . . . . . 2 CM
- POLE POSITION ACCURACY . . . . 2 CM
- UT-1 (SHORT TERM). . . . . 2 CM

FIGURE 14
(Cf. ref. 2)

MISSION OBJECTIVES CAN BE SATISFIED WITH

- RANGE ACCURACY (PER PASS) . . . . 2 CM
- EPHEMERIS ACCURACY. . . . . 5 CM

FIGURE 15
(Cf. ref. 2)

FACTORS INFLUENCING EPHEMERIS ACCURACY

- ACCURACY OF RANGE MEASUREMENTS
- NUMBER OF TRACKING STATIONS AND GEOGRAPHIC DISTRIBUTION
- GEOPOTENTIAL MODEL ERRORS
- SURFACE FORCE MODEL ERRORS

  ATMOSPHERIC DRAG
  DIRECT SOLAR RADIATION PRESSURE
  EARTHSHINE RADIATION PRESSURE
  MICROMETEORITES

FIGURE 16
(Cf. ref. 2)

GEOPOTENTIAL MODEL ERRORS

- RESONANT HARMONICS

  CAN BE AVOIDED BY CHOICE OF PERIOD

- CURRENT GEOPOTENTIAL MODEL ERRORS

  FOR GEOS-2 . . . . ~5 M

- EXPECTED GEOPOTENTIAL MODEL ERRORS TO MEET EOPAP REQUIREMENTS:

  SEASAT REQUIREMENT . . . . 10 CM

FIGURE 17
(Cf. ref. 2)

SURFACE FORCE MODEL ERRORS

DRAG MICROMETEORITES	NEGLIGIBLE
DIRECT SOLAR RADIATION EARTHSHINE	MUST BE CORRECTED

FIGURE 18
(Cf. ref. 2)

RADIATION PERTURBATIONS VS ORBIT ALTITUDE

MEAN MOTION (REV/SID. DAY)	ORBIT ALTITUDE	PAYLOAD WEIGHT	RELATIVE PERTURBATION DIRECT SOLAR	EARTH-SHINE
8.55	3720 KM	680 KG	1.0	1.0
7.55	4600	600	2.9	1.9
6.55	5690	500	9.3	3.8
5.55	7100	440	33	7.9
4.55	9000	390	120	14.8
3.55	11,800	320	600	32

FIGURE 19
(Cf. ref. 2)

MAGNITUDES OF PERTURBING FORCES
$M/A = 4000\ KG/M^2$
ORBIT ALTITUDE = 3700 KM

	FORCE	ACCELERATION
DIRECT SOLAR	0.084 DYNE	120×10^{-9} CM/SEC
EARTHSHINE	VARIABLE UP TO ~0.02	UP TO 30
UNBALANCED SATELLITE THERMAL RADIATION	0.01	12
DRAG	10^{-5} TO 4×10^{-4}	0.01 TO 0.4
MICROMETEORITE IMPACTS	2.4×10^{-6}	0.004

FIGURE 20
(Cf. ref. 2)

ORBIT PARAMETERS

PERIOD	166 $\pm$ 2 MIN
INCLINATION	90° $\pm$ 1°
ECCENTRICITY	0.020 $\pm$ 0.015
NOMINAL ALTITUDE	3700 KM (2000 NM)

FIGURE 21
(Cf. ref. 2)

ERROR BUDGET FOR RANGE MEASUREMENTS

• TROPOSPHERE	15 MM
• LASER	
PULSE DETECTION	10 MM
RANGE COUNTER	5 MM
CALIBRATION, CABLES, MECHANICAL, ETC.	5 MM
• EPOCH SYNCH	5 MM
• SATELLITE	5 MM
RSS	.20 MM

FIGURE 22
(Cf. ref. 2)

SATELLITE PARAMETERS

RADIUS	22 CM
MASS	615 KG (1350 LB.)
M/A	4000 KG/M^2
SURFACE	DIFFUSE ALUMINUM

CUBE CORNERS

TOTAL NUMBER	240
CIRCULAR FRONT FACE	3.65 CM DIA.
DIHEDRAL ANGLE	$90° + 1.75 \pm 0.5$ ARCSEC

HIGH PURITY FUSED SILICA

NO REFLECTIVE COATINGS

NO ANTI-REFLECTION COATINGS

FIGURE 23
(Cf. ref. 2)

LAGEOS RECEIVED SIGNAL LEVELS FOR "GOOD" SEEING, 0.5 ARCMIN RADIUS, 1.5 J

EL ANGLE	S (PHOTONS)	N (ELECTRONS)
10°	60	6
15°	230	23
20°	520	52
25°	900	90
30°	1400	140
45°	3800	380
60°	6900	690
75°	9500	950
90°	10,000	1000

FIGURE 24
(Cf. ref. 2)

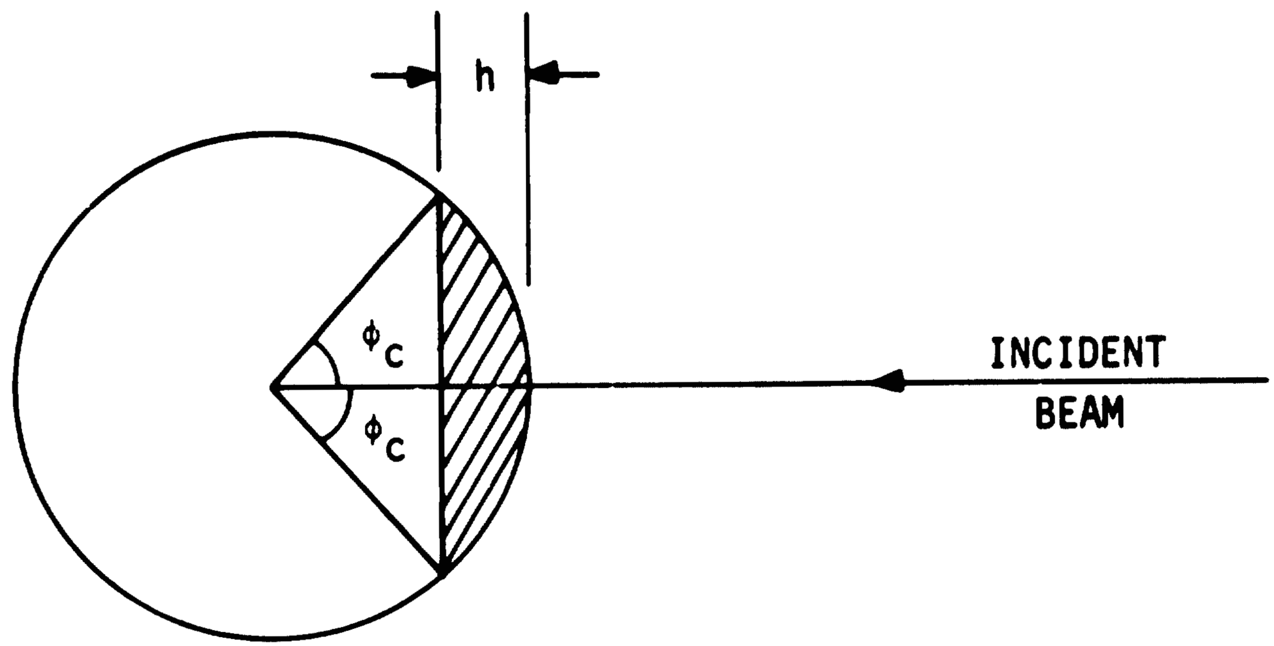
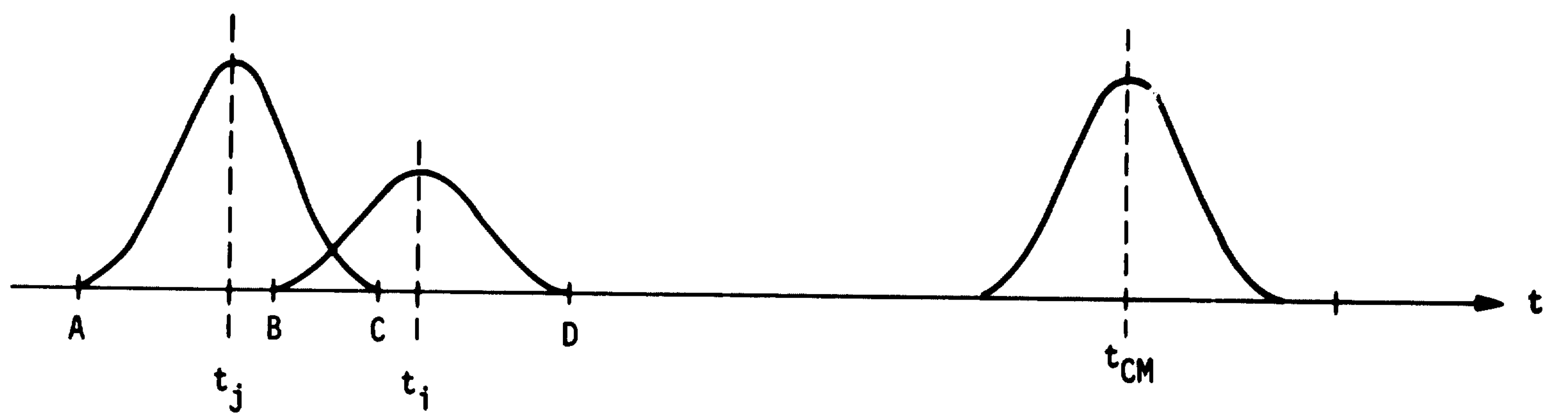
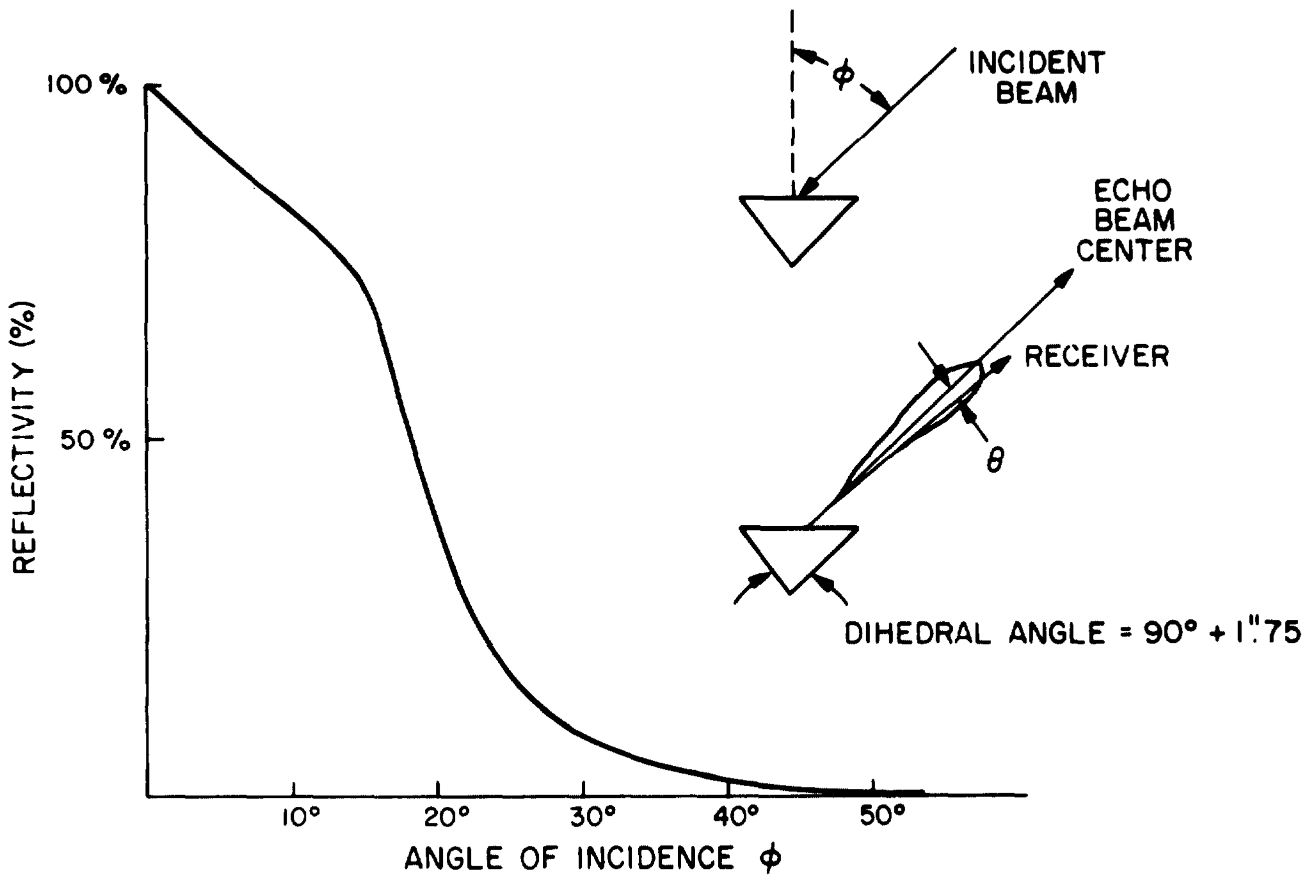


Figure 25
The shaded area is the spherical segment where the "active" cube corners are located when ϕ_c is the cube-corner cutoff angle. (Cf. ref. 2)



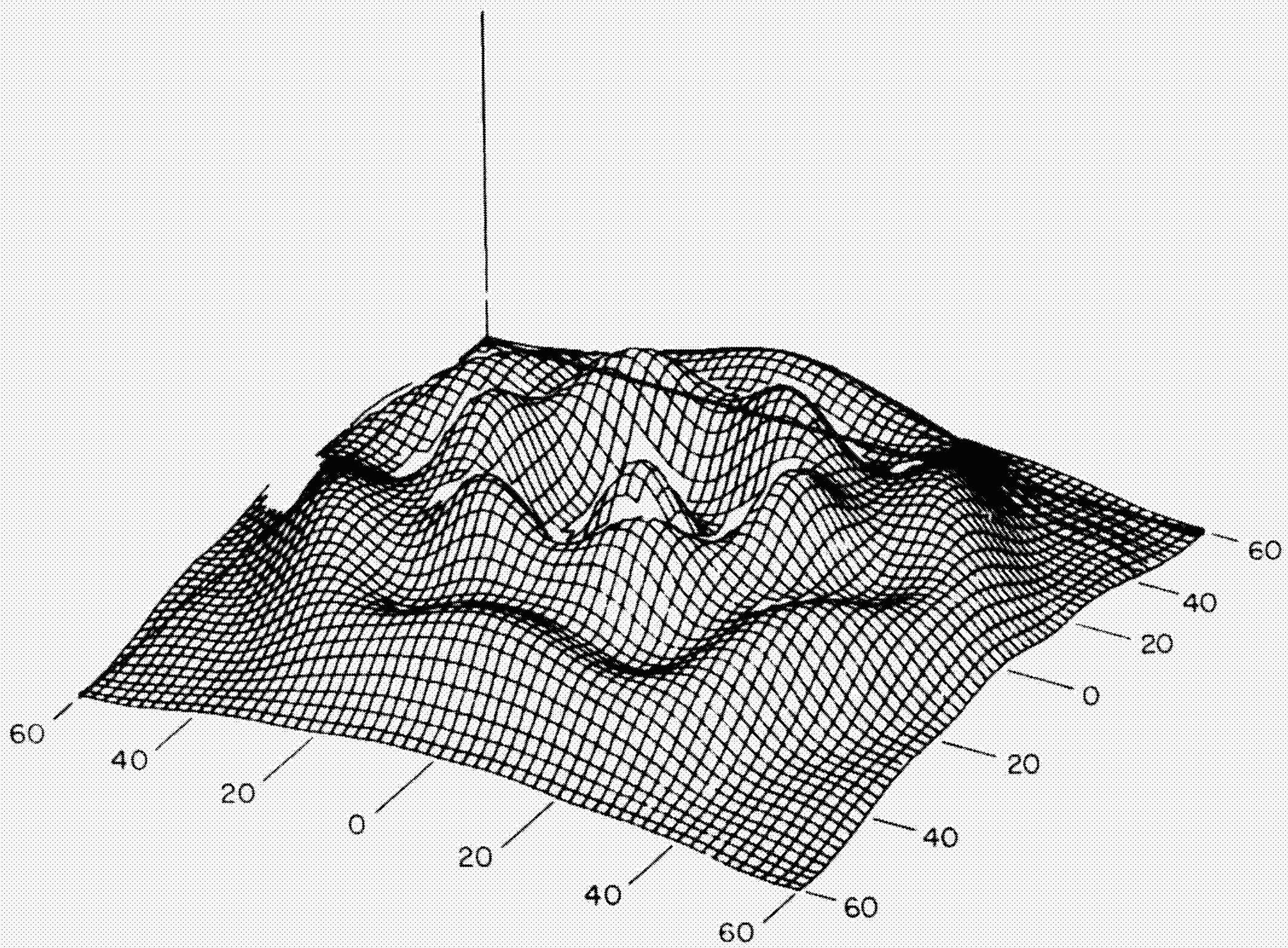
Time history of the echo signal.

FIGURE 26
(Cf. ref. 2)



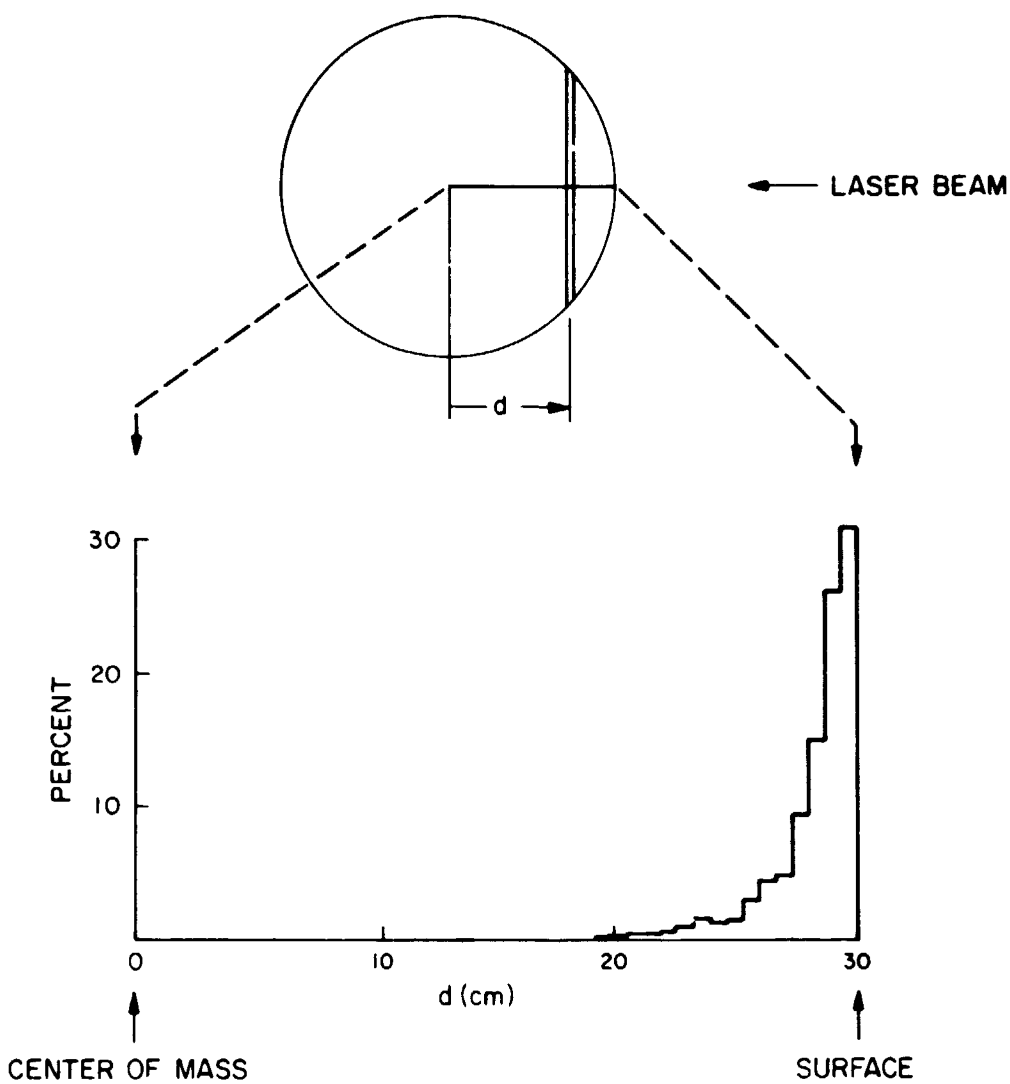
Reflectivity versus angle of incidence for an uncoated fused-silica cube corner with a circular aperture 3.65 cm in diameter and dihedral angles of 90° + 1.75 arcsec. The reflectivities are for a beam angle θ of 36-μrad, corresponding to a typical value of velocity aberration for the LAGEOS orbit. The reflectivity is the average for all azimuthal angles (taken around the normal to the front face).

FIGURE 27
(Cf. ref. 2)



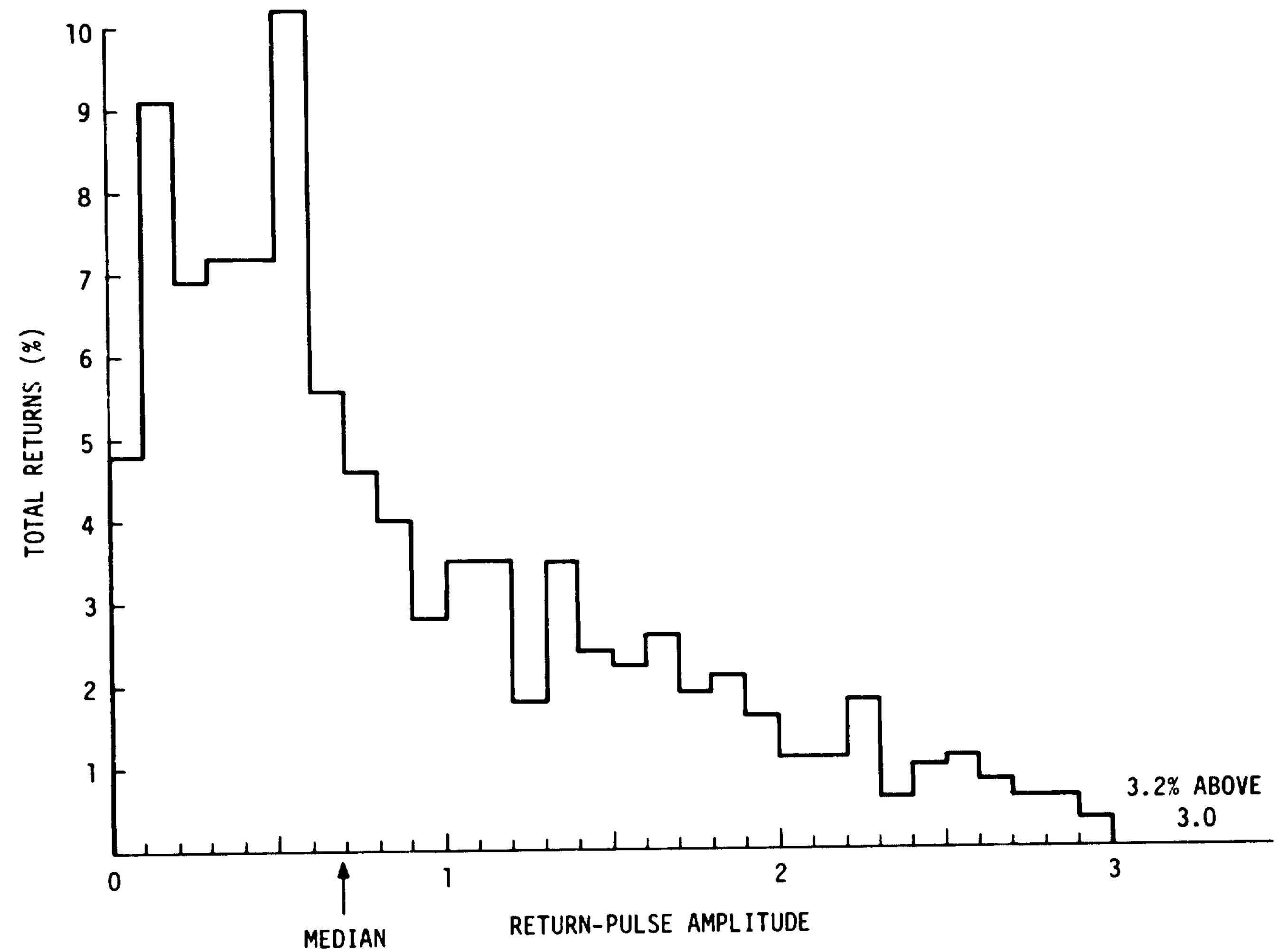
Beam pattern at normal incidence for an uncoated cube corner with a circular aperture 3.65 cm in diameter. Dihedral angle is 8.5 μrad (1.57 arcsec) from 90°. Beam angles are given in microradians. Wavelength is 6943 Å (ruby laser).

FIGURE 28
(Cf. ref. 2)



Effective reflectivity of segments of the LAGEOS retroreflector array versus the distance d from the center of mass. The histogram is the average over results computed for several aspect angles and for incoherent light; i.e., interference effects have been omitted. It is of interest to note that 57% of the total return is from the first centimeter, and 90% from the first 3 cm of the sphere.

FIGURE 29
(Cf. ref. 2)



Computed probability distribution of the intensity of LAGEOS retroreflected pulses. The histogram is normalized so that the intensity of an incoherent pulse is unity on the horizontal scale.

FIGURE 30
(Cf. ref. 2)

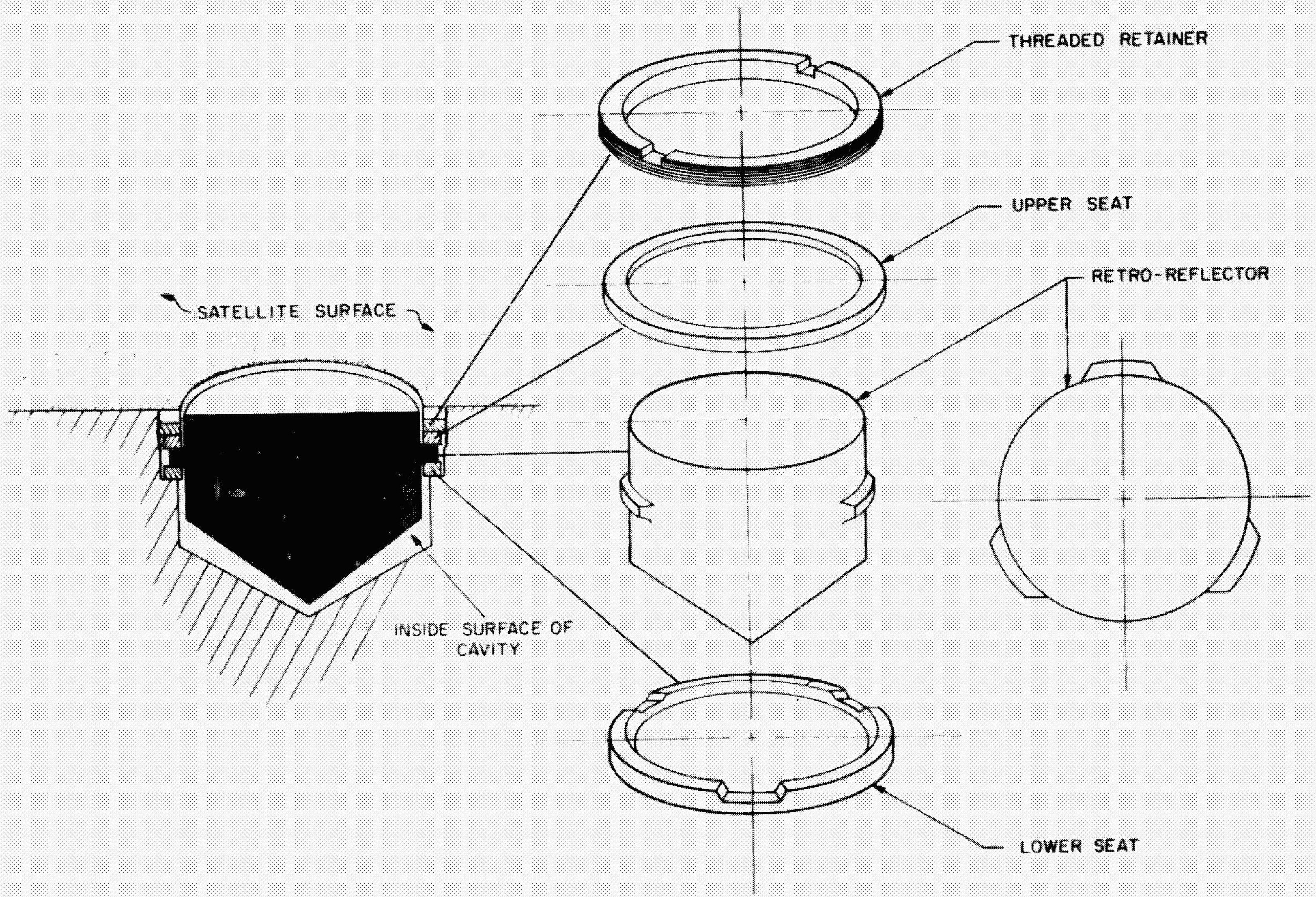


FIGURE 31
(Cf. ref. 2)

indicated an interest in these aspects of the program. Key points which were made by the various groups are summarized in the following paragraphs and indicated in Figure 10.

A. <u>The Orbital Altitude</u>

Dr. Weiffenbach, in his complete analysis of the LAGEOS system, pointed out that uncertainties associated with the geopotential and radiation pressure effects were the principal model errors influencing ephemeris accuracy. He listed a number of possible choices for the orbital altitude, each of which had good characteristics from the standpoint of gravitational resonance perturbations. The solar radiation pressure effects increased with increasing altitude since the area was increased and the mass decreased in accordance with the constraints described in the SAO Report. (2) It was pointed out in this study that the gravitational perturbation effects decrease with increasing altitude, and that the lowest altitude in the table of Figure 19 is a good choice when the accurate geopotential model to be provided by the EOPAP GRAVSAT mission is available.

De. J. Faller, Chairman of the Lunar Ranging Experiment (LURE) Team had suggested that a LAGEOS in a somewhat higher orbit of 6000 km altitude could be tracked more easily by certain lunar laser stations than one in a 3700 kilometer altitude orbit. He also indicated that the higher orbit would be advantageous due to its smaller gravitational perturbations.(3)

This latter thought had been echoed by Dr.F.Vonbun who also made the point that an orbit having a higher altitude (6000-10,000 km) and/or a lower inclination (60°-30°) would permit increased tracking coverage, and hence would improve the ability to model perturbations and determine polar motion. Such a choice would, for example, make possible the observation of LAGEOS at its maximum declination from middle latitude stations, and thus facilitate pole motion monitoring by means of the approach used by Dr. D. E. Smith. (4)

The discussion then procedded along the lines indicate in Figures 32 and 33, which are similar to tables which were developed at the board during the course of the meeting. Figure 32 deals with estimates of the relative effects of uncertainties associated with gravitational and radiation pressure perturbations as functions of the orbital altitude and the time. Figure 33 reflected an attempt to estimate the relative utility of several possible combinations of the orbital altitude and inclination from the standpoint of the principal LAGEOS Program objectives. Professor Kaula called attention to the importance of the determination of crustal motions over scales ranging up to a length of the order of a thousand kilometers.

In Figure 32, the first row and the first two entries in each of the first two columns reflect material in references 1 and 2. The satellite weights listed reflect data provided by the Delta Project Office and material elsewhere in the present discussion. (22) The radiation pressure perturbation estimates were based on the material in reference 2 corresponding to the first row. They were obtained from Table 2 of reference 2 and Figure 19 of this discussion by interpolation and by replacing

ESTIMATED ORBIT PERTURBATION UNCERTAINTIES
FOR CERTAIN LAGEOS ALTERNATIVES

Orbit Altitude (km)	Satellite		Estimated Orbit Perturbation Uncertainties (cm)			
			Radiation Pressure		Geopotential	
	Weight (kg)	Diameter (cm)	Direct Sunshine	Earth Shine	1976	1980
3720	680	44	1	5	50	5
4600	600	60	2	7		
5900	385	60	5	7		
6378	385	60	5	7	25	

FIGURE 32

Estimated Relative Effectiveness of Alternative
LAGEOS Orbits for Earth Dynamics Measurements

Orbit		Components of Intersite Vectors Having Lengths of:				
Altitude (km)	Inclination (deg)	200-1000 km	5000 km	Polar Motion	Earth Rotation	Reference Station Positions
3700	90	A	B	B	C	A
6378	90	A	A	A	B	A
3700	70	A	B	A	C	A
6378	70	A	A	A	B	A

FIGURE 33

the satellite weight and diameter assumptions of that reference with those listed in Columns 2 and 3.

The last two columns are rough estimates based partly on findings in reference 2. The gravitational perturbation uncertainty estimate of 5 centimeters in the 1980's is based on the EOPAP GRAVSAT goal, as that report points out. The 50 centimeter estimate reflects considerations presented there, as well as a further general discussion which took place during the meeting. The factor of two between the estimated gravitational perturbation uncertainties for the 3720 and 6378 km orbit altitudes was derived from results of calculations based on Kaula's theory. Among the quantities computed was the root mean square value of the amplitudes of non-resonant perturbation components corresponding to the terms in the l, m. p, and q, sequences which were calculated on the basis of the simple assumption that the uncertainties of the geopotential coefficients were constant through degree and order fifteen, and vanishingly small otherwise. The corresponding uncertainties for the 5900 km altitude orbit could be expected to be perhaps ten percent larger than those for the 6378 km altitude case.

These relative values for the ephemeris uncertainties are, to a certain extent, indicative of the corresponding relative values of the uncertainties in determining other derived quantities of interest such as components of station position and intersite vectors, and the polar motion. This tends to be the case when the dynamic method is used. Ephemeris uncertainties associated with gravitational perturbations have, in fact, been found to be among the principal contributors to the uncertainties in the determination of intersite vector components in the San Andreas Fault Experiment (SAFE) analysis which employs data from laser ground tracking sites. (5) Geometrical approaches can involve a larger number of lasers. (6,7) It was, accordingly, thought that these latter methods would be appropriate for consideration in a somewhat later phase of the program.

In Figure 33 the symbols A, B, and C, denote successively decreasing relative utilities. The differences among the various cases were not considered to be large enough to be conclusive.

The LAGEOS orbital altitude may be selected so as to attempt to minimize the perturbation uncertainties and maximize its usefulness in its early years when LAGEOS will be the key element in the satellite laser tracking system for measuring crustal motion, polar motion and earth rotation, and the critical laser - VLBI intercomparisons will occur. An intermediate altitude in Figure 19, which was presented in reference 2, is appropriate in this case. (Cf. also, for example, Figure 32.) Dr. Weiffenbach recommended, accordingly, that the 5690 km altitude be chosen, tentatively, and that more detailed calculations be made to confirm the initial estimates of the payload capability of the Delta launch vehicle.

B. The Orbital Inclination

Accurate station position determination is strongest when the satellite is observed in all directions around the site, hence the inclination should normally be somewhat greater than the highest latitude at which tracking systems are located.

It appeared that a 70° inclination would meet this requirement and, at the same time, permit tracking of the LAGEOS satellite over much of its orbit from middle latitude stations, thus enhancing the ability to model gravitational perturbation terms. Hence, it appeared that an inclination in the neighborhood of, say, 70°, would offer some advantages of the type indicated in Figure 10.

It was considered, tentatively, then, that an orbital altitude of 5690 km and an inclination of 70° would be useful candidates for consideration for LAGEOS. It was also concluded that detailed studies would be conducted to determine more accurately the payload capability of the Delta launch vehicle for this case, and for other inclinations between 60° and 90°. The altitudes of 4700 and 3720 km would be looked at, too, to provide contingency planning information in case, e.g., the payload capabilities for the 5690 km altitude proved to be inadequate. At the same time, interested groups, including those at UCLA, OSU, SAO, Goddard, and NWL, would give further consideration to one or more of the various factors affecting the orbit selection including those associated with the uses of the data to improve understanding of earthquake mechanisms, crustal motions and polar motions, as well as those having to do with the ability to determine these motions, such as geometrical coverage and uncertainties due to gravitational and radiation pressure perturbations, instrumental characteristics, etc.

The location of the laser tracking stations was not considered in detail. The possible new sites seen in Figures 11 and 12 are tentative and indicative of general concepts, but have not been finally selected.

It was pointed out, however, that a coverage gap exists in the southern part of the Western Hemisphere, and that the location of a laser at a place such as Comodoro Rivadavia, a former Baker-Nunn camera site, would strengthen the solutions. An Antarctic location would add even more, provided a site with reasonable weather conditions can be found.

III. THE LAGEOS RETROREFLECTOR REVIEW

A. The Retroreflector

Matters relating to the retroflectors including the number, diameter, shape, dihedral angle offset, recess, coatings, etc., were considered at a meeting on October 29, 1973, held at NASA Headquarters and attended by those listed in the attached Figure 34. The following approaches and rationale were developed at this meeting and in subsequent discussions. In a number of cases they reflect the views and results presented in the SAO Study, as the reference citations indicate. (2)

1. The Retroreflector Diameter

The optical antenna gain of the retroreflectors increases with the diameter hence, from this standpoint, it is advantageous to select the largest practical retroreflector face diameter. The Apollo retroreflectors had the largest face diameter used in space, i.e., 3.8 cm. Diameters significantly larger than this will probably begin to encounter problems associated with the manufacture of the raw material of suitable quality. Accordingly, it was concluded that the retroreflector diameter should be 3.8 cm.

THE LAGEOS RETROREFLECTOR REVIEW
OCTOBER 29, 1973
ATTENDANCE

NAME	ORGANIZATION	PHONE NO.
J. Siry	NASA Hq.	755-3837
T. Hoffman	SAO	617-864-7910 x492
Geo. Weiffenbach	SAO	617-864-7910 x286
J. L. Randall	NASA-MSFC	205-453-3770
P. O. Minott	NASA-GSFC	
Henry Plotkin	NASA-GSFC	301-982-6171
David Arnold	SAO	617-864-7910 x481
J. Faller	JILA-NBS	303-499-1000 x3463

FIGURE 34

2. Coatings

The use of retroreflectors without reflective coatings provides performance which is within about 20% of that which is obtainable through the use of such a coating. (2) This gain is considered to be insufficient to offset the risk associated with the possibility that the coating on some of the retroreflectors may deteriorate over the years. Such a partial deterioration would spread the pulse in unpredictible ways and decrease the accuracy. Accordingly it was tentatively concluded that no reflective coatings would be used.

For similar reasons, it was concluded that no anti-reflective coatings would be used. (Cf., again, reference 2.)

3. The Dihedral Angle

The selection of a dihedral angle offset was based on the data in Figure 35 which were supplied by the SAO. (8)

The offset of $1\overset{''}{.}5$ gives good performance over the entire range emcompassed by the uncertainity, i.e., $1\overset{''}{.}5 \pm 0\overset{''}{.}5$. It also has a markedly smaller gradient in the 30-40 microradian interval than the zero offset, for example. Hence it will probably also be less sensitive to degradation in performance due to non-nominal conditions associated with, say, material quality, thermal effects, etc.

The 1.5 arc second offset also gives better performance than the zero offset design, for example, for lasers operating at half the ruby wavelength of 6943 Å. The 1.5 arc second offset was, accordingly, tentatively chosen.

RETROREFLECTOR ARRAY GAIN
AS A FUNCTION OF
DIHEDRAL ANGLE OFFSET

DIHEDRAL Angle Offset Relative To 90° (ARC SEC)	Velocity Aberration (Microradians)	Average Gain Function (10^7)
	RUBY WAVELENGTH (6943 Å)	
2.00	30	5.05
	40	4.78
1.75	30	6.23
	40	5.39
1.50	30	7.33
	40	5.85
1.25	30	8.26
	40	6.11
1.00	30	8.97
	40	6.21
0.00	30	9.92
	40	6.04
	HALF RUBY WAVELENGTH ($\frac{1}{2}$x6943Å)	
1.50	30	12.11
	40	10.82
1.25	30	14.45
	40	8.82
1.0	30	14.20
	40	6.03
0.00	30	4.81
	40	1.97

Assumptions: 240 retroreflectors,

Diameter 1".437, uncoated, not recessed;

Satellite diameter 44 cm.

FIGURE 35
(Cf. ref. 8)

B. The Retroreflector Array

1. The Recess Depth

The depth of recess has an effect on both the amplitude and the shape of the return pulse. These effects are indicated in Figure 36, data for which were generated by the SAO. (11). It was concluded that, from these standpoints, a minimal depth is preferred.

A recess depth of 0.1 cm appears to be desirable from the standpoint of handling ease, etc. and will not significantly affect the return pulse strength or shape.

A greater depth may be advantageous from the standpoint of the thermal effects. The quantitative aspects of the thermal effects of varying the depth are not yet known but will be evaluated in the Phase B Definition Study. The depth of 0.1 cm is, accordingly, tentatively selected as the nominal value.

2. The Satellite Diameter and The Number of Retroreflectors

It was considered that the SAO and Goddard lasers could track effectively down to a threshhold value of four photoelectrons. It was estimated, tentatively, that a satellite having a diameter in the 50-60 cm range could contain an array having enough retroreflectors to permit laser tracking to an elevation angle in the neighborhood of 10° to 15°, and that the 30% or more of the spherical surface which would remain to reflect as diffuse aluminum would permit adequate tracking by the Baker-Nunn cameras.

Radiation pressure perturbations of the path of such a satellite in an orbit in the neighborhood of 5700 km altitude were also tentatively

EFFECT OF RECESSING CUBE CORNERS

DEPTH	ACTIVE REFLECTING AREA	RETURN PULSE SIGMA X 2 (CM)
0.	7.68	2.67
D/8	6.13	1.66
D/4	4.99	1.28
D/2	3.33	0.99

D = Diameter of Cube Corner

<u>Assumptions:</u>

Incident Pulse is much shorter than array size, i.e., 0.02 nanosec. half energy, full width.

Number of retroreflectors: 240

Diameter of Satellite: 44 cm

One cube corner at normal incidence has an active reflecting area of unity.

FIGURE 36
(Cf. ref. 11)

estimated to be acceptable, particularly during the critical early years of its lifetime when they are expected to be dominated by gravitational perturbations. (cf., e.g., Figure 32.)

It is seen from the Figure 37 and the data of ref. 12 that the four photoelectron threshold corresponds to an elevation angle between 10° and 15° for a 1.5 joule laser and a LAGEOS satellite at 5690 km altitude having an array containing 360 to 504 retroreflectors. Allowing a factor of two for effects of thermal distortion of the retroreflectors, a four photoelectron threshhold will allow tracking to about a 15° elevation angle for a satellite at 5900 km altitude having an array containing 440 cubes, which is near the center of the range covered by the second and third columns of Figure 37.

Tentative estimates indicated that a sphere diameter of about 60 cm would be consistent with this range for the number of retroreflectors. Studies conducted by the Marshall Space Flight Center have indicated that a sphere diameter of 60 cm would provide room for approximately 440 retroreflectors, where the diameter of the retroreflector, per se, is 3.8 cm and the diameter of the mounting apparatus is 4.76 cm. (13,24)

Some of the results of these studies are presented in Figure 38. Others appear in reference 13. It is seen that a 4.13 cm mounting diameter corresponds to 524 to 546 retroreflectors for a 60 cm diameter, the variation being a function of the array configuration.

3. Camera Tracking

All of these cases leave at least 30% of the spherical surface area available for diffuse reflection of sunlight to Baker-Nunn cameras. (13)

LAGEOS Signal Strengths
in Photoelectrons

Elevation Angle	No. of Retroreflectors		
(degrees)	360	504	672
10	2	3	4
15	7	9	13
20	13	19	25
25	22	31	41

Lageos Altitude: 5690 km

Laser output: 1.5 joules

Additional assumptions specified in ref. 12, which is the source of these values.

FIGURE 37

LAGEOS Retroreflector Array
Parameters

Sphere Diameter (cm)	Mounting Diameter (cm)	Placement Concept	Number of Retroreflectors	Surface Fraction Covered by Retroreflectors
55	4.76	Ring	370	0.70
60	4.76	Ring	440	0.69
60	4.13	Ring	524	0.62
60	4.13	Meshed	546	0.65

FIGURE 38
(Cf. Ref. 13, 24)

The ability of the Baker-Nunn cameras to track such a target is presented in the SAO discussion of reference 14.

4. Positioning

Experience indicates that normal manufacturing practice can achieve tolerances of about 0.015 cm. It was concluded that this level is adequate for the eccentricity of the center of mass relative to the center of figure. It was also concluded that the variation of the retroreflector apex from the nominal position should be no more than 0.025 cm in the radial direction and the radial direction and 0.0375 cm in the transverse direction, and that the variation of the actual surface from the best fit spherical surface should be no more than 0.025 cm.

C. Testing and Handling Procedures

It was also considered that criteria associated with thermal loading effects would be specified in terms of the far-field pattern. In particular, it was considered that, in terms of the far-field pattern, the return average signal intensity at the appropriate velocity aberration angle when the cube is thermally loaded to correspond to the worst case expected in orbit will not be reduced by more than a factor of two from the corresponding value obtained during the isothermal test.

It was concluded that edge sharpness or edge roll should be such that the energy return will be at least 80% of that from a retroreflector which has zero roll, but otherwise is a real object. This might be determined by means of a pin hole test or an interferometry test.

It was considered that the criteria and procedures for testing the GEOS-C and Timation retroreflectors which are set forth in references 9 and 10 will, in many cases, be applicable to the LAGEOS retroreflectors as well.

It was considered that the use of Twyman-Green interferograms and far-field diffraction patterns would be appropriate. In particular, it was concluded that each retroreflector should be tested in a Twyman-Green interferometer at 6328Å to demonstrate that:

1) The peak-to-valley wavefront deviation from the best fitting plane wave, in a least squares sense, is less than $\lambda/4$ over each of the six sectors of the aperture, and

2) that the dihedral angle meets the specifications.

The far field diffraction patterns will also be used to determine that they possess suitable symmetry, and that the dihedral angles meet the specifications in other respects.

It was considered that practices corresponding to those used for a class 100,000 clean room would be adequate.

IV. FURTHER CONSIDERATIONS

Further consideration was given to factors affecting the selection of the orbit and the satellite's mass, and the possibility of determining the satellite's attitude. The following sections deal with these topics.

A. The Orbit

1. The Eccentricity

A circular orbit has useful symmetry properties and appears to pose no particular difficulties. Accordingly, it is selected.

2. The Inclination

An orbit inclination of 70° is large enough to permit tracking on all sides of all stations, and it is small enough to be visible at its maximum northerly latitude from stations used for fault motion studies such as those at Quincy and San Diego, California. For example the corresponding elevation angles at these two sites for this case are about 32° & 22°, respectively. Lasers at these sites could thus observe with favorable geometry in both the fault motion and polar motion programs. Retrograde orbits afford roughly one more tracking pass each day than prograde orbits for the middle latitude locations were ground tracking stations are often placed for other reasons such as those associated with fault motion studies. (5,25) Better time resolution for polar motion and earth rotation studies can, accordingly, be obtained with such a retrograde orbit at 110° inclination. This factor outweights any disadvantage which may ultimately be associated with the fact that the long-period solar radiation pressure perturbation has a longer period, and hence a larger amplitude, for the retrograde orbit than it does for the prograde orbit, the two periods being about 580 and 290 days, respectively. This difference has no practical significance until orbits are determined for data spans exceeding about 290 days, however, this is not expected to pose a practical problem in the early years of the LAGEOS orbit. The longer period and corresponding larger amplitude of the solar radiation pressure perturbation associated with the retrograde orbit may actually turn out to be an advantage, since it may make the determination of the amplitude easier. Accordingly, the 110° inclination was considered to be the most suitable.

3. The Altitude

a. Resonance Effects Associated With a Specific Altitude Region

The best altitude in the neighborhood of 5690 km from the standpoint of resonance effects is 5900 km. It is seen from Figure 39 that this is a good choice for the case in which the uncertainty in the semi-major axis due to launch vehicle performance variations is no more than the 60 km or so expected from the Delta launch vehicle. The altitude of 5900 was, accordingly, selected as the appropriate choice in this altitude range. (21)

b. General Considerations

Two types of considerations arise in connection with the selection of the LAGEOS orbit. The first has to do with the program objectives, and the second with the ability to meet these objectives.

The measurement objectives of the LAGEOS program include the determination of fault motion, pole motion, plate motion, and reference station positions. The relative importance of certain of these in terms of their potential contribution to the meeting of the goal of achieving a better understanding of earthquake mechanisms is treated by Professor Kaula and Dr. Bender in references 16-18. The effect of different choices of the LAGEOS orbit altitude upon the ability to meet one or more of these objectives has been analyzed by R. J. Anderle, and Professor Mueller and Kaula in references 19, 20, and 26.

In references 16 and 17, Profe or Kaula treats a number of factors relating to the objectives, and points out the fundamental importance of measuring crustal motion at scales up to the order of a thousand kilometers. In reference 26 he gives results of a study of factors relating to the orbital altitude.

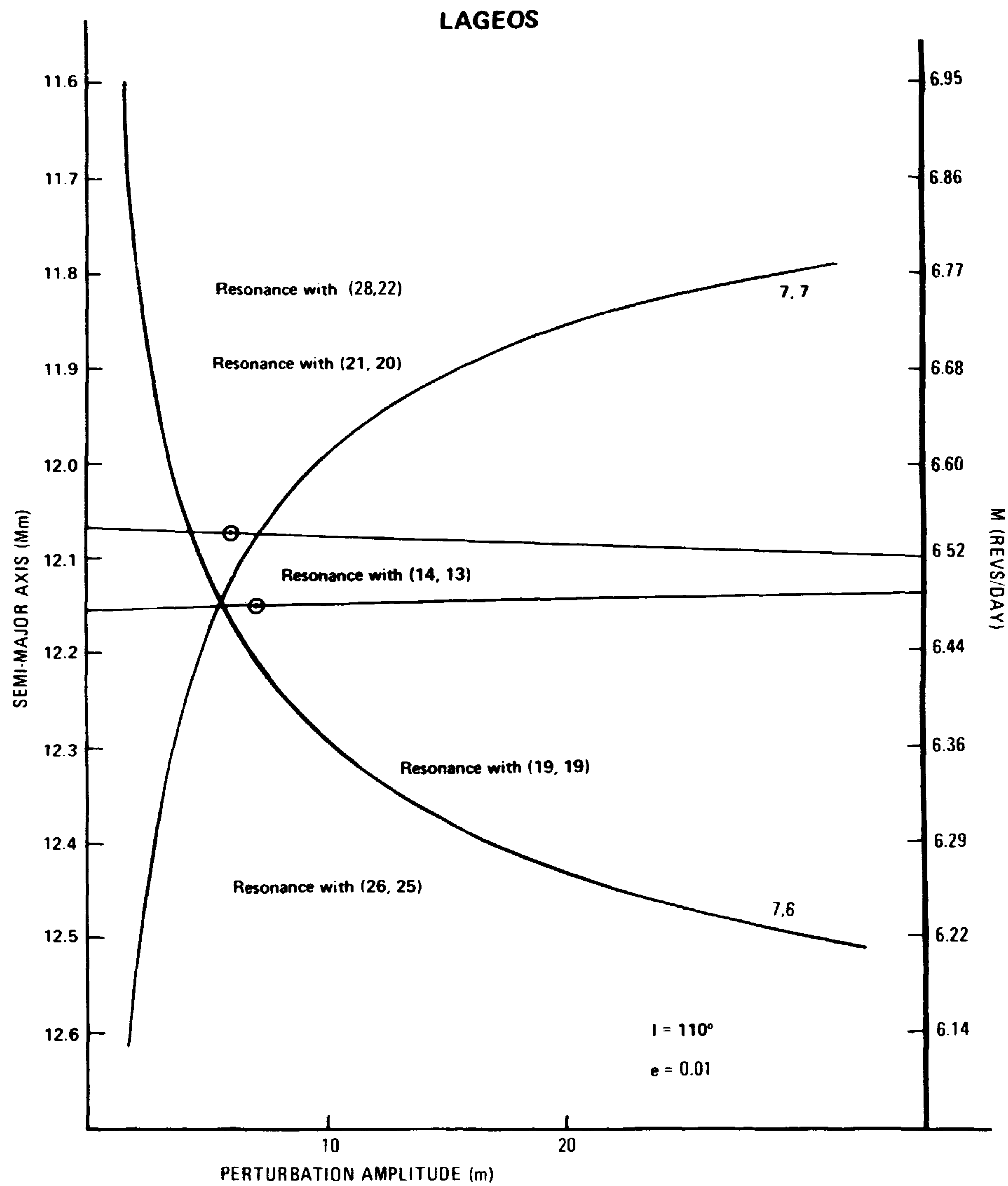


FIGURE 39
(Cf. ref. 21)

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Dr. Bender also considers the objectives and calls attention to the value of measuring the relative motions of the tectonic plates in the large, too. (18)

R. Anderle analyzed the effect of different choices of the orbital altitude and inclination on the ability to recover polar motion, and presented results in reference 19.

Professor Mueller and his collegues have studied the effect of the choice of the orbital altitude on the ability to determine positions of points on the earth's surface. The results are presented in reference 20.

None of the findings obtained up to now is inconsistent with the tentative selections of Sections II and IV and Figure 1. Studies of these types are continuing.

B. <u>The Satellite Mass</u>

The Delta launch vehicle has a gross payload capability of about 430 kg for this orbit.(22) It is tentatively estimated that ninety percent of this, i.e., about 385 kg., will be available for the LAGEOS satellite, per se. The radiation pressure perturbations associated with this combination of values for the orbital altitude and satellite diameter and mass, i.e., 5900 km, 60 cm, and 385 kg, respectively, were considered to be consistent with philosophy underlying the tentative selection of Section II above. (Cf., e.g., Figure 32) (Cf. Fig. 40 and ref. 22 provided by the Delta Project.)

C. <u>Attitude Determination</u>

The ability to determine the attitude of the LAGEOS satellite could be valuable in the case of any marked variation in the actual performance of

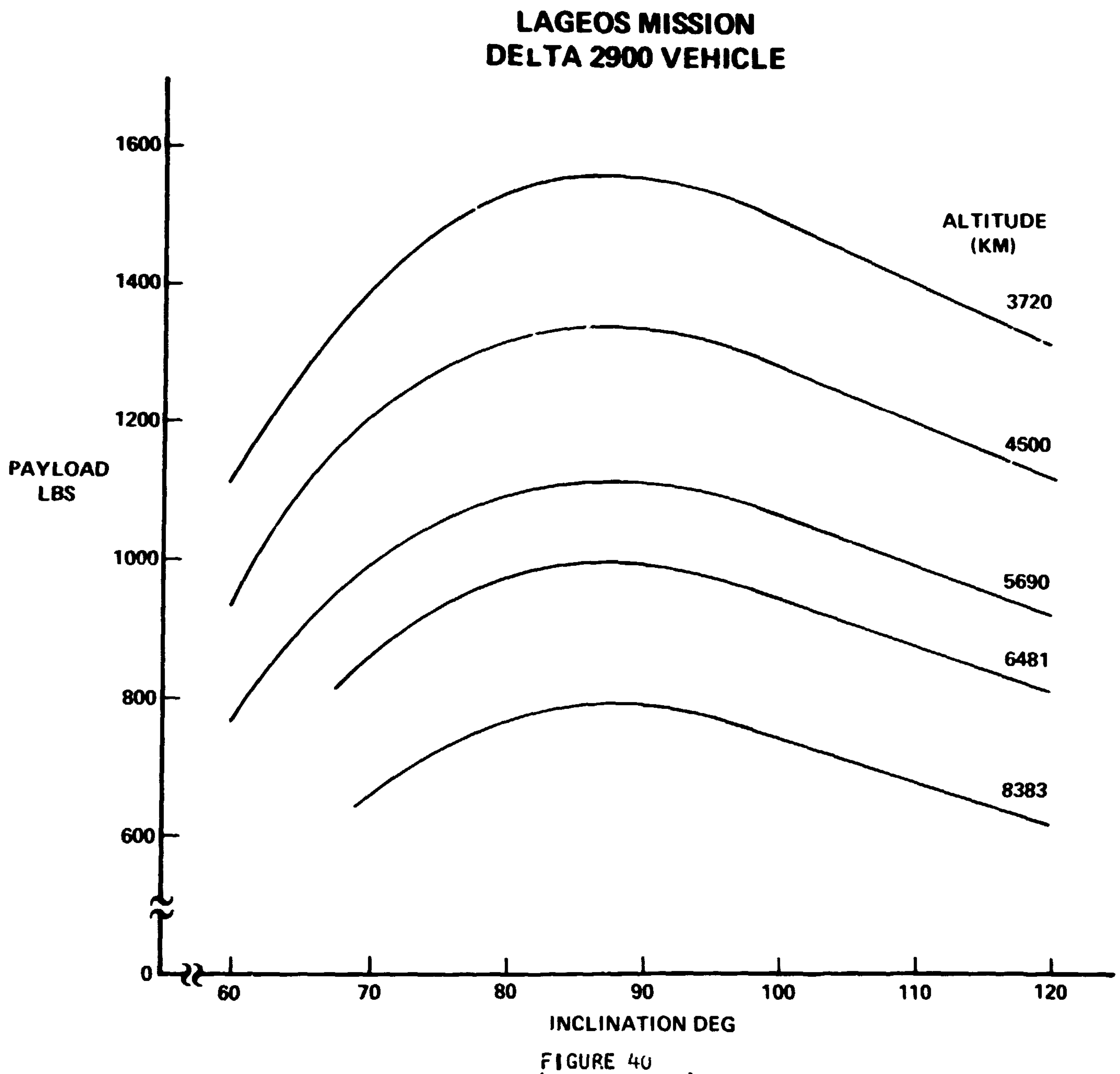

FIGURE 40
(Cf. ref. 22)

the different zonal regions of the retroreflector array in orbit, particularly in the early months of its orbital lifetime when its capabilities are being determined. Attitude determination ability can be achieved by two relatively simple design steps, neither of which should have any significant adverse effect on the basic capability of the satellite. The first is to choose a design for which the moment of inertia about the axis for which this moment is greatest is larger than the moments of inertia about the other two principal axes by a factor of at least 1.05. A ratio of about 1.1 appears to be suitable as a design goal. The moments about the other two axes would be designed to be equal. The axis about which the satellite has the maximum moment of inertia could be chosen as the spin axis at the point of injection into orbit. Attitude could be determined by means of reflection from two symmetrically placed rows of mirrors or flats, each at the same angular distance from the satellite's equator, i.e., the plane normal to the axis about which the moment of inertia is a maximum. These rows will be at an optimal angle from the equator, e. g., at an angle of the order of 30°, say. The "row" would consist of at least one flat, and as many more as would be practical. The flats would be located in regions where, for one reason or another, the space between adjacent retroreflectors is relatively large. The portion of the spherical surface in these regions would be replaced by inscribed plane circles made as large as practical. These flats would be specularly reflecting.

Variation of the spacing of the flats in each row would permit determination of the third component of the attitude, namely, the phase of rotation. This general approach has already actually been used in orbit in the case of the Telstar satellite to provide the capability for determining the spin axis direction.(23) Knowledge of the attitude will permit the determination of any variation in retroreflector array performance with position on the satellite.

V. ACKNOWLEDGEMENT

The foregoing sections reflect the efforts of many groups and individuals, as the references indicate. In particular, the key material provided by Dr. George Weiffenbach and his colleagues at the Smithsonian Astrophysical Observatory, including E. M. Gaposchkin M. Pearlman, and D. Arnold, has furnished the basis for a considerable amount of the discussion and the resulting body of conclusions which are reported upon here. Appreciation is also expressed for a number of additional valuable contributions including those made by W. M. Kaula, J. E. Faller, I. I. Mueller, R. J. Anderle, B. Chovitz, S. Yionoulis, D. Eckhardt, W. Melbourne, D. Trask, J. Whitcomb, C. Scholz, J. Berger, F. Vonbun, H. Plotkin, D. Smith, T. Johnson, P. Minott, D. Bowden, J. Randall, L. McNair, F. Williams, R. Spencer, and J. Murphy.

REFERENCES

1. NASA Earth and Ocean Physics Applications Program Plan, September, 1972

2. Weiffenbach, G.C., "Use of a Possible Stable Satellite For Earth-Physics Applications," Smithsonian Institution Astrophysical Observatory, April, 1973.

3. Faller, James E., Letter to Dr. John E. Naugle, Associate Administrator For Space Science, NASA, May 22, 1973.

4. Vonbun, F. O., "Laser Geodynamic Satellite (LAGEOS)", Memorandum to Mr. F. Williams, Director, Special Programs Division, Office of Applications, NASA, September 27, 1973.

5. Smith, D. E., Private Communication.

6. Escobal, P. R., Ong, K. M., Von Roos, O. H., Shumate, M. S., Jaffe, R. M., Fliegel, H. F., and Muller, P. M., "3-D Multilateration: A Precision Geodetic Measurement System," JPL Technical Memorandum 3[illegible] March 15, 1973.

7. Muller, . M., Private Communication.

8. Weiffenbach, G. C., and Arnold, D., Private Communication.

9. Plotkin, H., and Minott, P. O., furnished the Specifications for Fabrication of GEOS-C Cube Corners.

10. Minott, P. O., and Plotkin, H., furnished the Specifications for Fabrication of Timation III Cube Corners.

11. Weiffenbach, G. C., and Arnold, D., Private Communication.

12. Pearlman, M. R., and Arnold, D. A., "Lageos Signal Strengths," Memorandum For the Record, January 2, 1974.

13. Moore, J., "A Procedure for Placement of Corner Cube Retroreflectors (CCR) on the Laser Geodynamic Satellite (LAGEOS)", Marshall Space Flight Center, S&E-AERO-MX, January 1974.

14. Arnold, D., "Baker Nunn Tracking of LAGEOS", Memorandum For the Record, January 16, 1973, and Pearlman, M. R., Letter to R. Spencer, January 24, 1974.

15. Vonbun, F. O., Private Communication.

16. Kaula, William M., Letter to Joseph W. Siry, November 9, 1973.

17. Kaula, William M., Letter to Peter L. Bender, February 1, 1974.

18. Bender, Peter L., Letter to William M. Kaula, January 3, 1974.

19. Anderle, R. J. "LAGEOS Launch Conditions", January 10, 1974.

20. Mueller, Ivan I., Letter to Joseph W. Siry, January 25, 1974, enclosing "The Altitude of LAGEOS", preprinted from "Basic Research and Data Analysis For The National Geodetic Satellite Program and For the Earth and Ocean Physics Applications Program". Semi-annual Status Report, Department of Geodetic Science, The Ohio State University Research Foundation, January 1974.

21. Gaposchkin, E. M., "LAGEOS Orbit Specifications", Memorandum For The Record", December 10, 1973.

22. Schindler, W. R., "Delta Performance Estimates", Letter to Marshall Space Flight Center, December 14, 1973.

23. Courtney-Pratt, J. S., Hett, J. H., and McLaughlin, J. W., "Optical Measurements on Telstar Satellite to Determine The Orientation of the Spin Axis and the Spin Rate", Cf. the Bell System Technical Journal Vol. XLII, Number 4, Part 2, pp. 1477, 1478, and 1504, July 1963.

24. Moore, John E., "Placement of Corner Cube Retroreflectors on LAGEOS Satellite", Letter to Lewis McNair, George C. Marshall Space Flight Center, February 14, 1974.

25. Squires, R. K. and Cooley, J. L., "Interim Report on LAGEOS Mission Analyses" GSFC Report No. X-591-74-25, December 1973

26. Kaula, William M., Letter to Joseph W. Siry, March 15, 1974.

27. Milwitzky, B., "New Laser Tracking Sites for LAGEOS Support," Memorandum to R. Stephens, June 27, 1973.