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TWINSAT EARTH GRAVITY FIELD MAPPING

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ABSTRACT

This paper describes the results of a sensitivity study on the proposed Lo-Lo (Twinsat) satellite-to-satellite tracking mission. The relative range-rate signal due to a local gravitational anomaly is investigated as a function of height and satellite separation. It is shown that the signal strength is weak and that an optimal combination of signal strength and resolution is achieved when the satellites are separated by 3° along-track. The signal does not resolve point masses closer than 5° apart when the satellites are at 300 km altitude. The influence of other factors on the system is evaluated, including the low frequency graviation field effect on the orbit and the dependence of the noise of the data type on (electronic) integration time.

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TWINSAT EARTH GRAVITY FIELD MAPPING

INTRODUCTION

The Lo-Lo, or Twinsat, system was proposed by Wolff (1969) to measure the high frequency components of the earth's gravitational field. The system consists of two satellites placed in identical circular, polar orbits at low altitude. One satellite leads the other in the ground track. The concept is that the satellites are affected similarly by all perturbations except those near the satellites on a scale comparable with the distance between the two satellites. The gravitational perturbations near the two satellites influences the range-rate on each satellite differently. If the range-rate between the pair is measured continuously, a global gravitational field may be obtained.

Wolff (1969) proposed using the relative range-rate as a direct measure of the gravitational potential to develop contour plots. This concept was extended by Comfort (1973). Schwarz (1970) tested the ability of the relative range-rate to obtain gravitational anomaly blocks using a least squares procedure for selected examples. This present paper is a sensitivity study to investigate the effect of a local gravitational anomaly on the relative range-rate signal for a variety of satellite configurations. A sensitivity study can yield a great deal of comparative information about the effects of varying the mission design parameters such as satellite altitude and separation distance between satellites. In addition, the direct visual presentation of the effects of gravity anomalies on a signal can promote insights which may be obscurred in the complexities of an error analysis study. Certainly the sensitivity analysis is simpler to prepare for the computer and uses less machine time. However, in the event that a sensitivity study demonstrates that a mission may be feasible, an error analysis is required to study the effects of unadjusted parameters (aliasing) or the correlations between adjusted parameters.

SIGNAL FROM ONE MASS POINT

The effects of local gravity variations on a Lo-Lo signal is best demonstrated by considering an idealized anomaly: a single mass point in an otherwise central force field. This allows a study of the effects of varying the design parameters of the configuration: the altitude of the satellites and the separation between satellites. The effects of more complex gravitational fields are sufficiently similar that the design parameters can be established in this manner.

The value of the mascon was equivalent to that of a surface density of 128 mgals spread in a 1/2° x 1/2° block. (Surface density in gm/cm^2 is converted into gals by multiplying by the universal gravitation constant $G = 6.67 \times 10^{-8}$ $cm^3/gm\ sec^2$. Surface densities are smaller than gravity anomalies, approximately by a factor of 2π). The point mass was simulated by using one integration point in this block. This value corresponds to a gravity anomaly of 50 mgals in a 2° x 2° block, or of 8 mgals in a 5° x 5° block, or about 10^{-8} earth masses.

The configuration used in the computation is shown in Figure 1. The satellites pass directly over the mascon. They are at a height h of 300 km and are separated by an angle α of 0.5° (equivalent to a linear separation of ~57 km). The Doppler signal is assumed to be transmitted by the first satellite and received

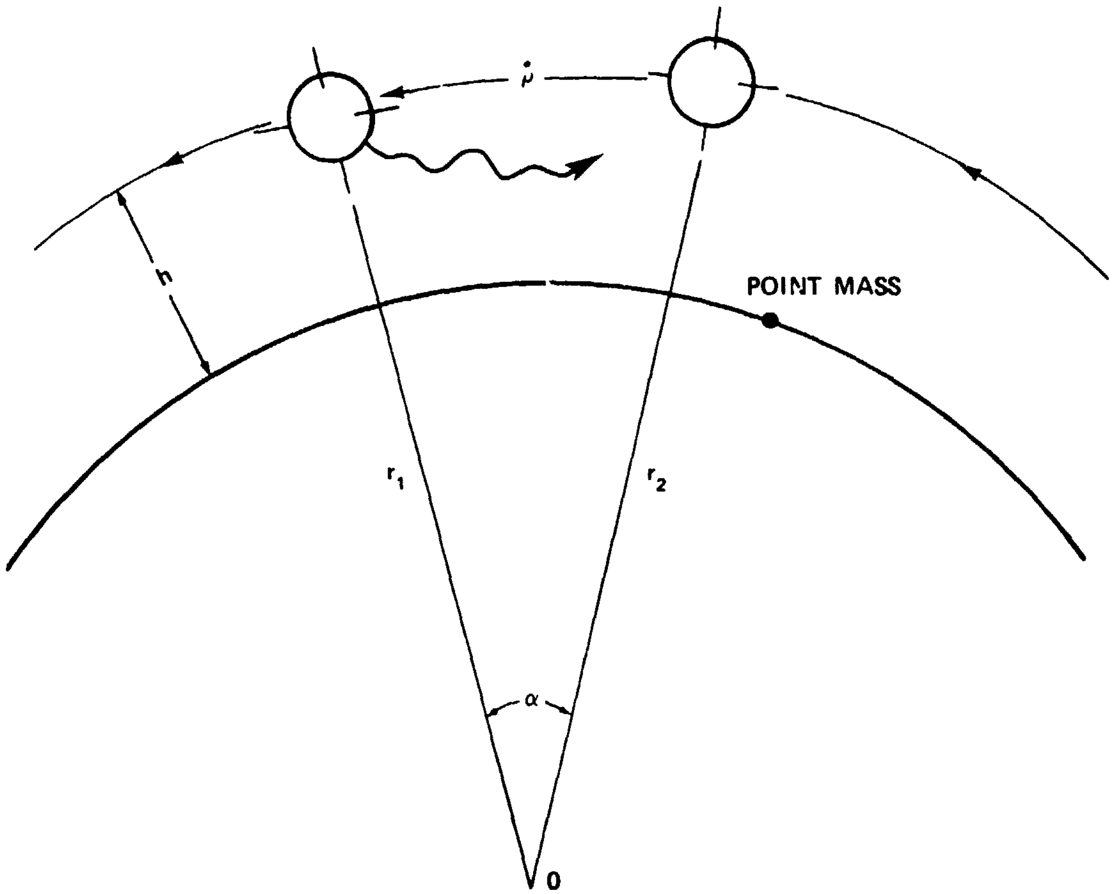

Figure 1. The Lo-Lo System

by the second satellite. Then the relative range-rate $\dot{\rho}$ is calculated by projecting the total range-rate difference $\dot{\vec{r}}_2 - \dot{\vec{r}}_1$ on the line between the satellites $\vec{\rho} = \vec{r}_2 - \vec{r}_1$

$$\dot{\rho} = \frac{(\dot{\vec{r}}_2 - \dot{\vec{r}}_1)}{|\rho|} \cdot \vec{\rho}$$

The values of $\vec{r}_1$, $\vec{r}_2$, $\dot{\vec{r}}_1$, $\dot{\vec{r}}_2$ are obtained from a trajectory integration using GEODYN (Martin, 1972). The relative range-rate $\dot{\rho}$ was computed at intervals of 5 seconds, an interval chosen to yield accurate values of the signal strengths while minimizing I/O machine time.

The signature for this configuration is shown in Figure 2 for a positive mascon and for a negative mascon (mass deficiency). As the satellites approach the mascon, their range-rates increase due to the attraction toward the (positive) mascon; when the first satellite has passed over the mascon, its range-rate is diminished due to an attraction opposed to its motion and a null relative range-rate results; after both satellites have passed the mascon, a negative peak occurs; this diminishes gradually as the satellites increase in distance from the mass point but there remains a small net change in relative range-rate. This signal is consistent with that obtained by Schwartz (1970).

It is to be noted that the relative range-rate does not return to zero (or the original value) after the point mass is passed. (In fact, the initial relative range-rate value of zero is an artifact of the run setup.) As Schwartz (1970) pointed out, the short term signal in Figure 2 is superimposed on a periodic variation in relative range-rate over the orbit. This is also induced by the point mass. This effect is shown in Figure 3. The orbits used to generate this signal were placed at an altitude of 1700 km: it may be that the periodic effect is larger relative to the short term effect as altitude increases.

A series of runs was made to investigate the signal strength as a function of the altitude of the satellite orbits and of the separation between the satellites. The satellite pairs were in identical circular orbits of heights of 200, 250, 300, 350, 400 km. The angular separations considered between the twin satellites were 0.5°, 1°, 2°, 3°, 4°, 5°, 6°. This is equivalent to linear separations ρ of 58, 120, 235, 352, 470, 587, 704 km (with slight variations according to satellite height).

The peak-to-peak signal strength was chosen as the parameter most convenient for summarizing the results of these runs. It is the difference in magnitude of $\dot{\rho}$ at the highest and lowest points of the signals. The peak-to-peak signal strengths as a function of linear separation between the satellites are shown in Figure 4 for several heights of the satellite pairs. The linear separation of 100 km

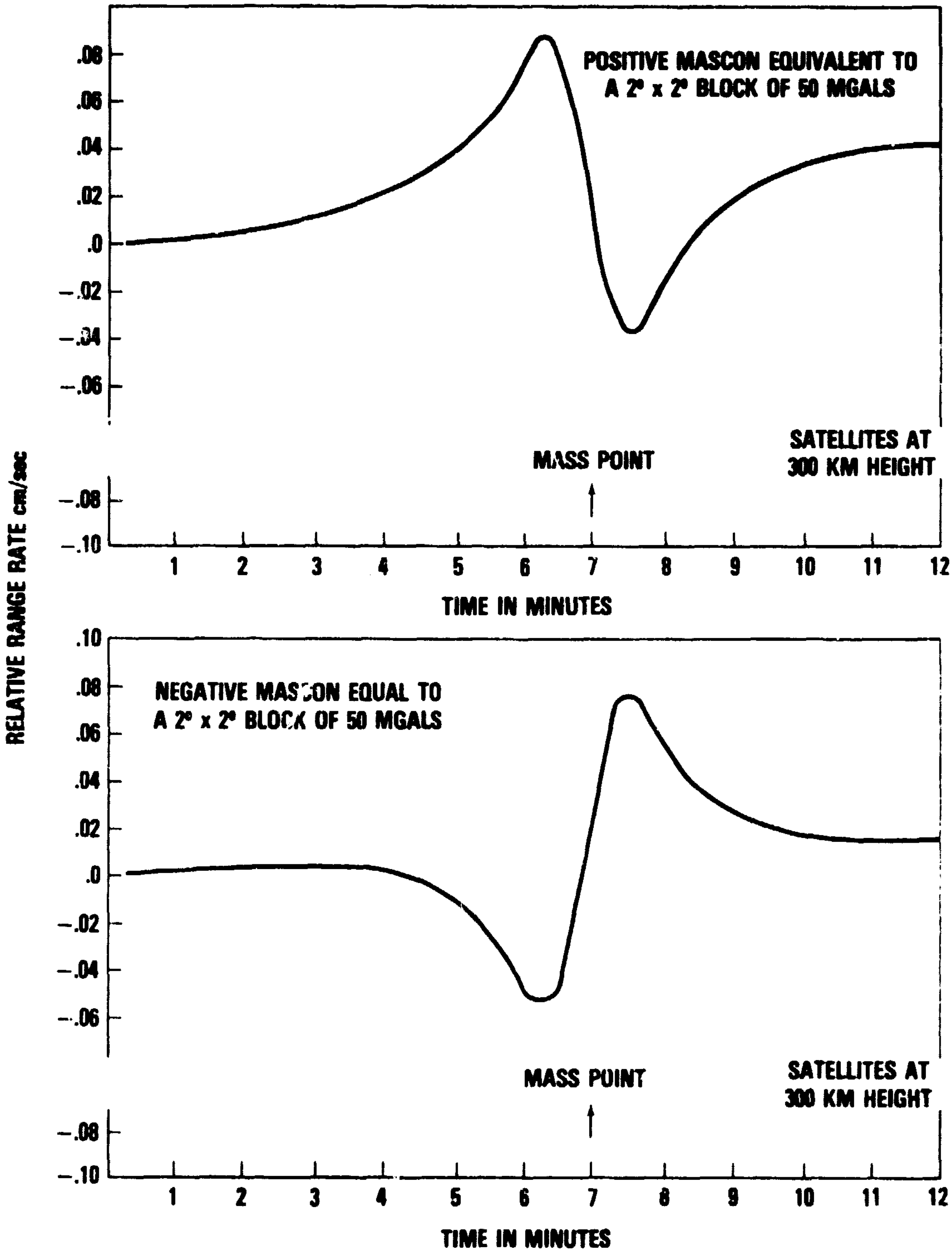

Figure 2. Range-Rate Signal From a Point Mass

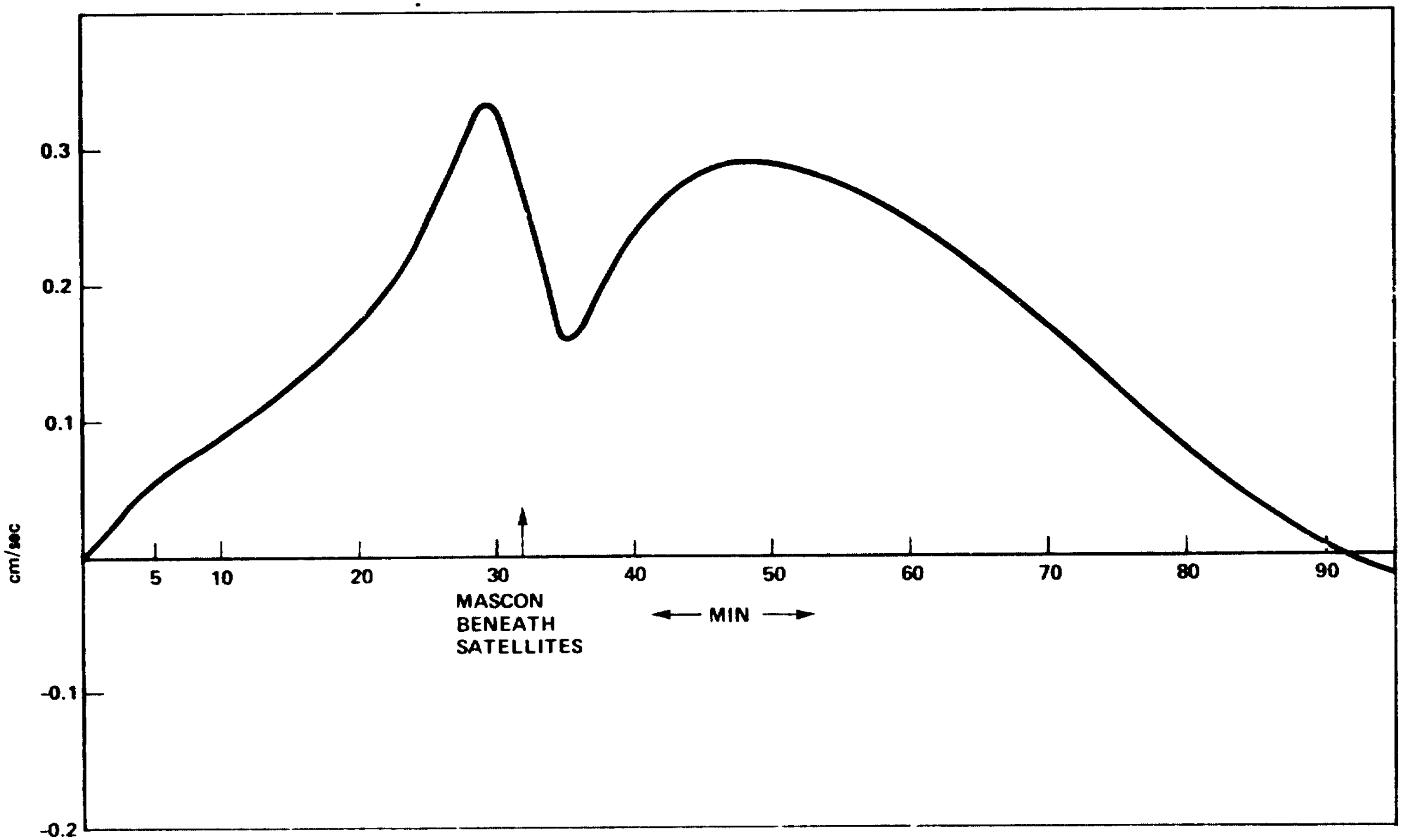

Figure 3. Effect of a Mascon (10^{-6} Earth Masses) on Relative Range-Rate of Two Satellites at 1700 km Height

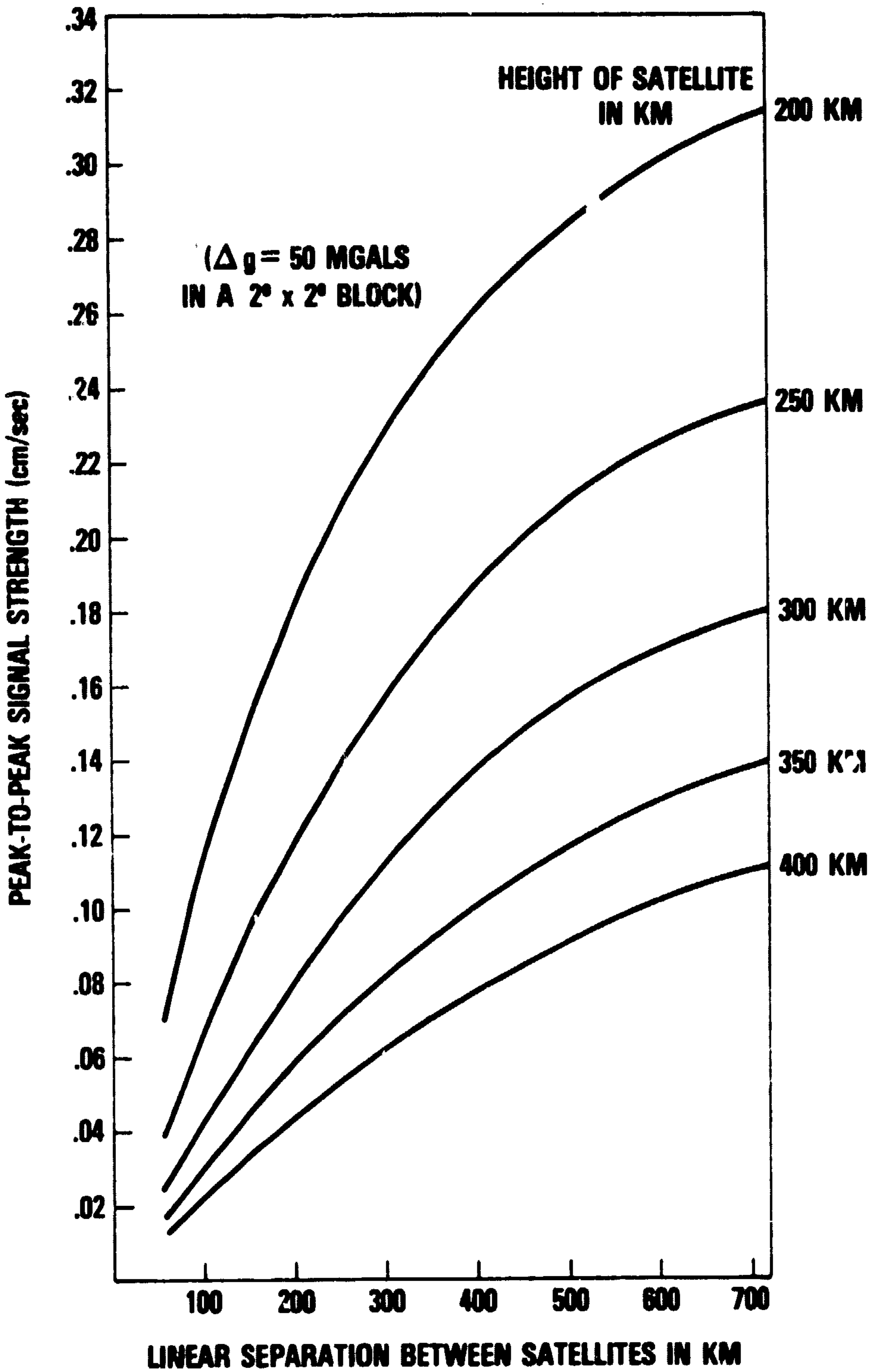

Figure 4. Peak-to-Peak Signal Strength vs. Satellite Separation

produces a small magnitude in the peak-to-peak signal, but as the linear separation increases, there is a considerable gain in magnitude. The increase is nearly linear until 300 or 400 km linear separation (3° or 4° angular separation). Then with increasing magnitude, the increase in magnitude begins to slow. This is shown more clearly in Figure 5 where the curves for a given angular separation are tending toward convergence at 5° or 6° angular separation. The impact of the satellite separation on resolution will be discussed below. It can be seen in both Figures 4 and 5 that a decrease in height produces an increase in magnitude. However the gain in magnitude is nearly linear with decreasing depth, while the increase in atmospheric drag is exponential. Thus the additional complexities introduced by increased atmospheric drag may outweigh any signal gain produced by lowering the satellite altitude.

RESOLVING POWER

The resolving power of the Lo-Lo range-rate data type was investigated by varying the angular separation of two mascons placed along the ground track of the satellite pairs.

The effect of varying the mascon separations is shown in Figure 6. The mascons used were equivalent to a surface density of 128 mgals in a 1/2° x 1/2° block (or 50 mgals gravity anomaly in a 2° x 2° area); they were placed on the ground track beneath two satellites at 300 km height with a linear separation of 300 km. For mascon separations of 3°, the signal produced was similar to one large point mass (double strength). As the mascon separation increased to 4°, a slight "knee" appeared in the middle of the signal; and at 5° mascon separation, a second hump developed. With increasing mascon separations, the humps grew in size until by 7° the two mascons produced two separate signals.

The resolving power of the Lo-Lo system proved to be insensitive to the satellite separations. This result was obtained from a set of runs using two very large mascons (512 mgals/1/2° x 1/2° block). Satellite separations above 1° showed no discernible difference in signals produced by a single mascon. Signals resulting from satellites separated by 0.5° and 1° showed a slight "knee;" however this effect was too small to be recoverable even with an instrument accuracy of 0.005 cm/sec. (Also, the signal strength is much weaker for this configuration, as discussed above.)

Therefore, it appears that the resolving power of a close Lo-Lo configuration at 300 km is at best 5°. This result is in apparent disagreement with Schwartz's "suggested" curve of resolution as a function of altitude (Schwartz, 1970, Fig. 6.1, p. 124).

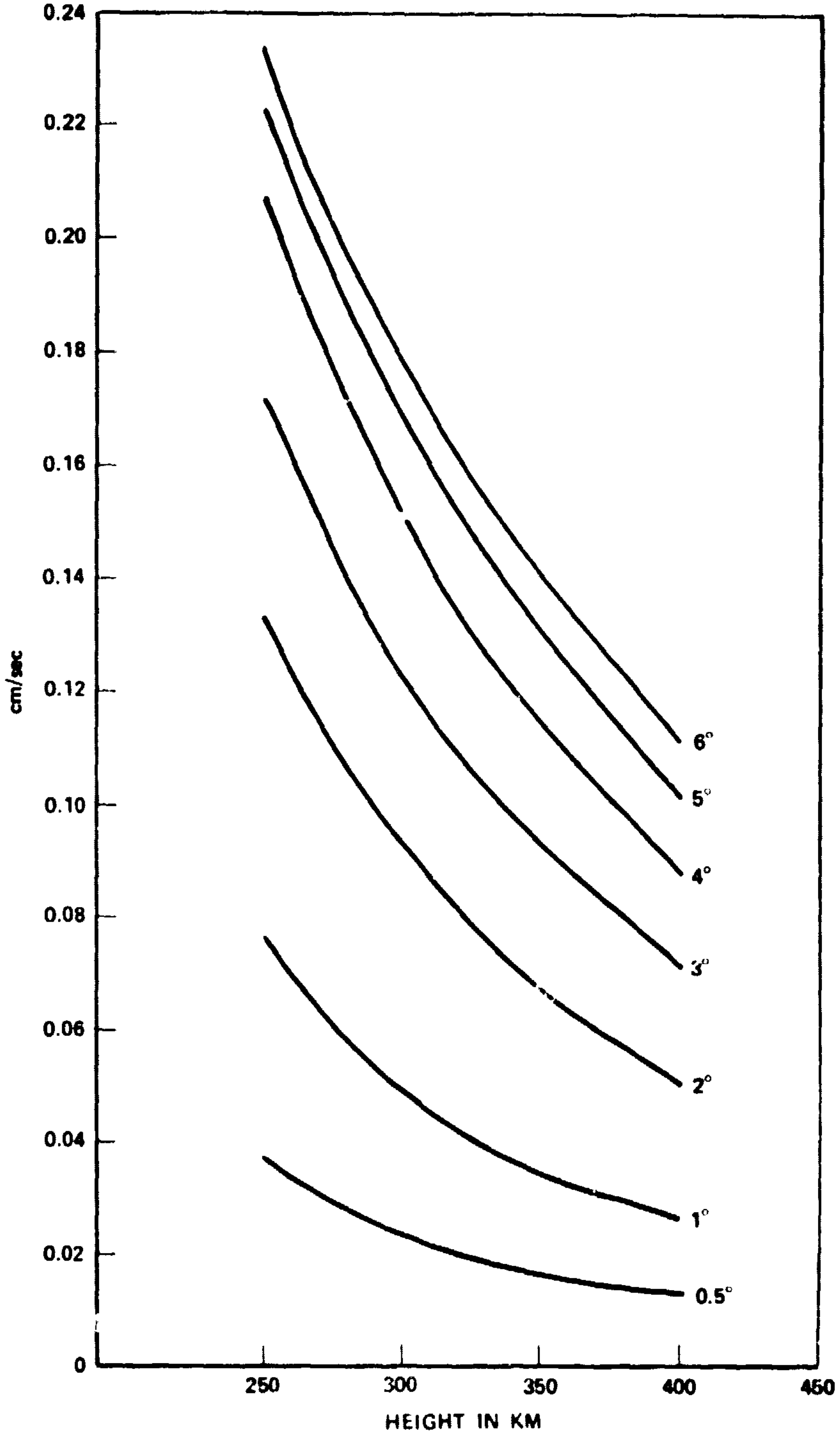

Figure 5. Peak-to-Peak Signal Strength vs. Satellite Altitude

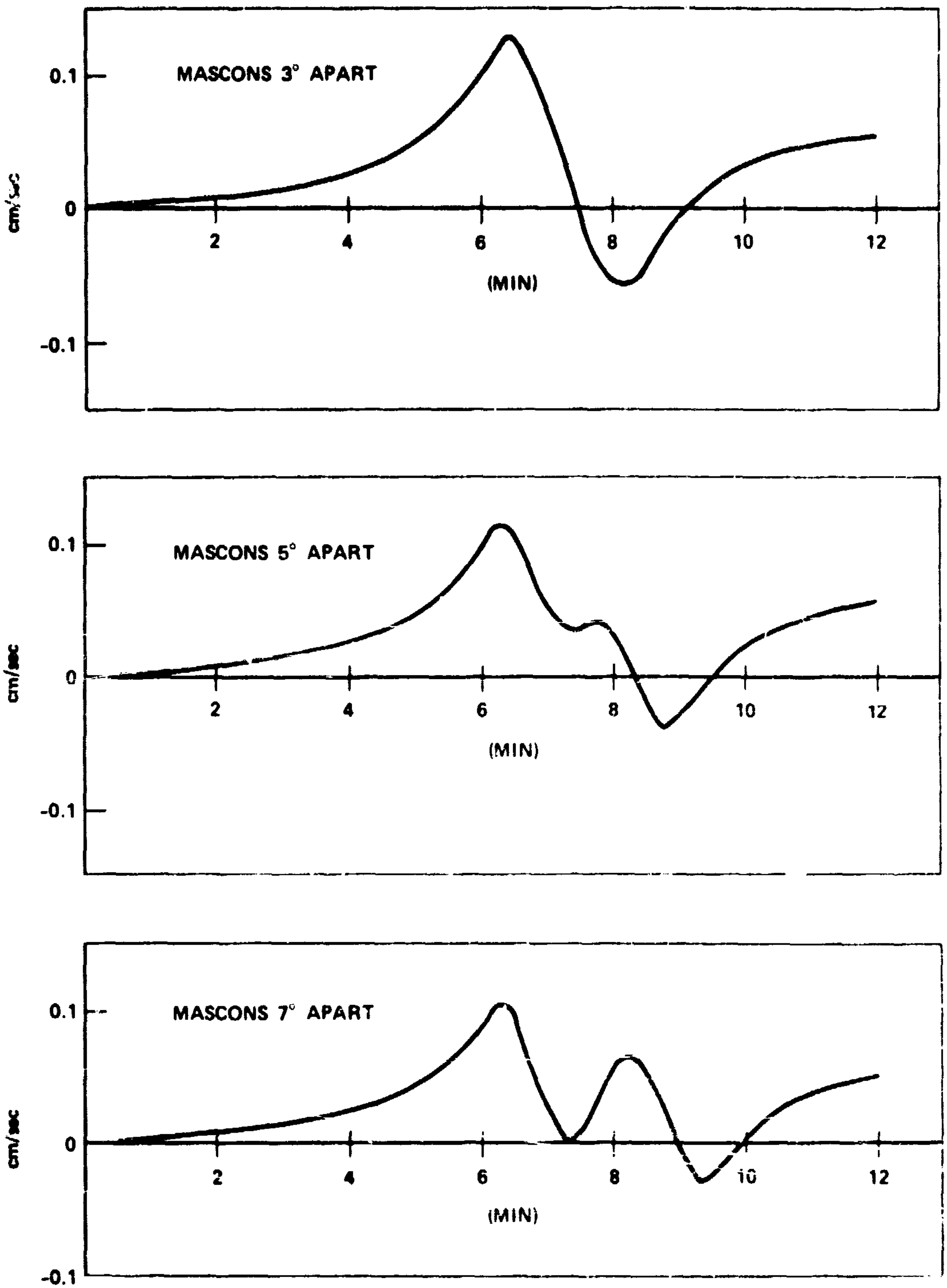

Figure 6. Resolution of Two Mascons Along Track

The increase in signal strength from a second mascon place perpendicular to the ground track from the first mascon was calculated. The decline in signal strength as the second mascon increases in distance from the first mascon is slow (Fig. 7). After the second mascon is more than 10° away from the ground track, the signal strength is affected by less than 2%.

SIGNALS FROM MULTI-ANOMALIES FIELDS

The general conclusions reached above were applicable when a more complex gravitational field was considered. The results discussed below were obtained by Business Technological Systems, Inc. under NASA Contract #NAS 5-20901.

The set of surface density blocks used in generating the signals was developed by Schwartz (1970) from a set of gravitational anomalies over the United States. The long wavelength features of the gravitational field were removed by removing the spherical harmonic portion of the gravitational field up to degree and order 12. A set of 36 5° x 5° surface density blocks was used in generating the signals.

The satellites followed a south-to-north trajectory over the blocks at an altitude of 250 km. The signals resulting from satellite separations of 1, 2, 5, 7° are shown in Figures 8, 9, 10, 11. There is a slight loss of resolution as the separation is increased from 1° to 5° but a large gain in magnitude of the signal strength. At 7° separation the signal strength increases little but the resolution is much poorer.

GRAVITY PERTURBATIONS ON A CLOSE EARTH CIRCULAR ORBIT

The overall gravity field considerably perturbs a near circular polar orbit to a degree that a "300 km" height orbit is only a fiction. For example, an initially circular polar ($e \sim 0.00005$) orbit at 250 km has a perigee of 249.7 km and an apogee of 250.3 (according to the two-body formulas). But when the orbit is computed in a standard gravitational field containing spherical harmonic coefficients up to degree 22 (GEM 1), the radial height of the satellite varies up to 265 km and down to 244 km in a 2 hr time period. These perturbations are reflected in an osculating value of the semimajor axis varying between 6608 km and 6628 km, the low value occurring over the earth's poles and the high value at the earth's equator. Thus, it appears that the J_2 term of the earth's gravitational field will not permit an orbit at a stationary height but induces a twice an orbit perturbation of 20 km on a 250 km polar orbit. In fact, it may be said that such an orbit has 2 perigees per revolution (and 2 apogees)!

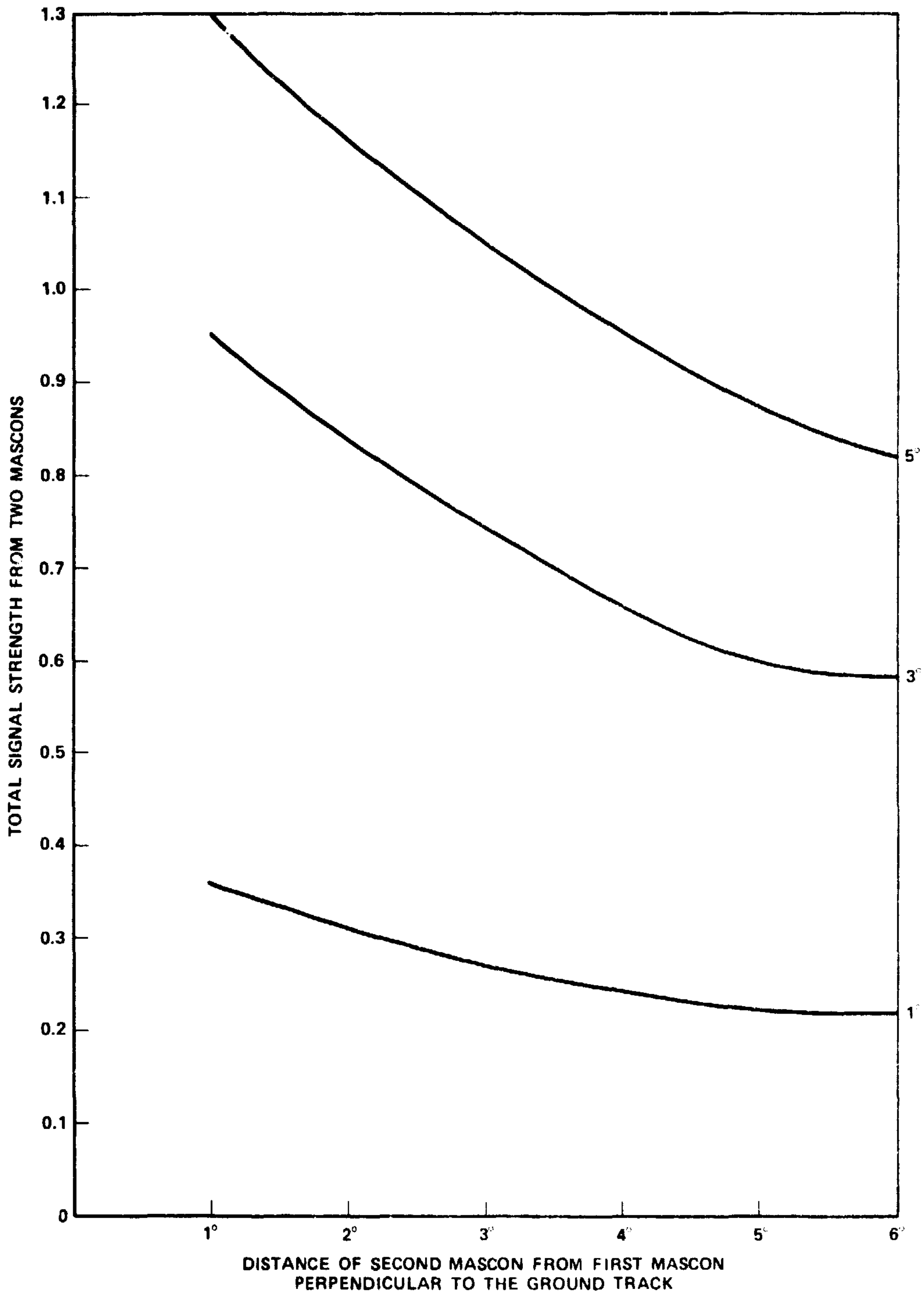

Figure 7. Effect of a Mascon Away From the Ground Track

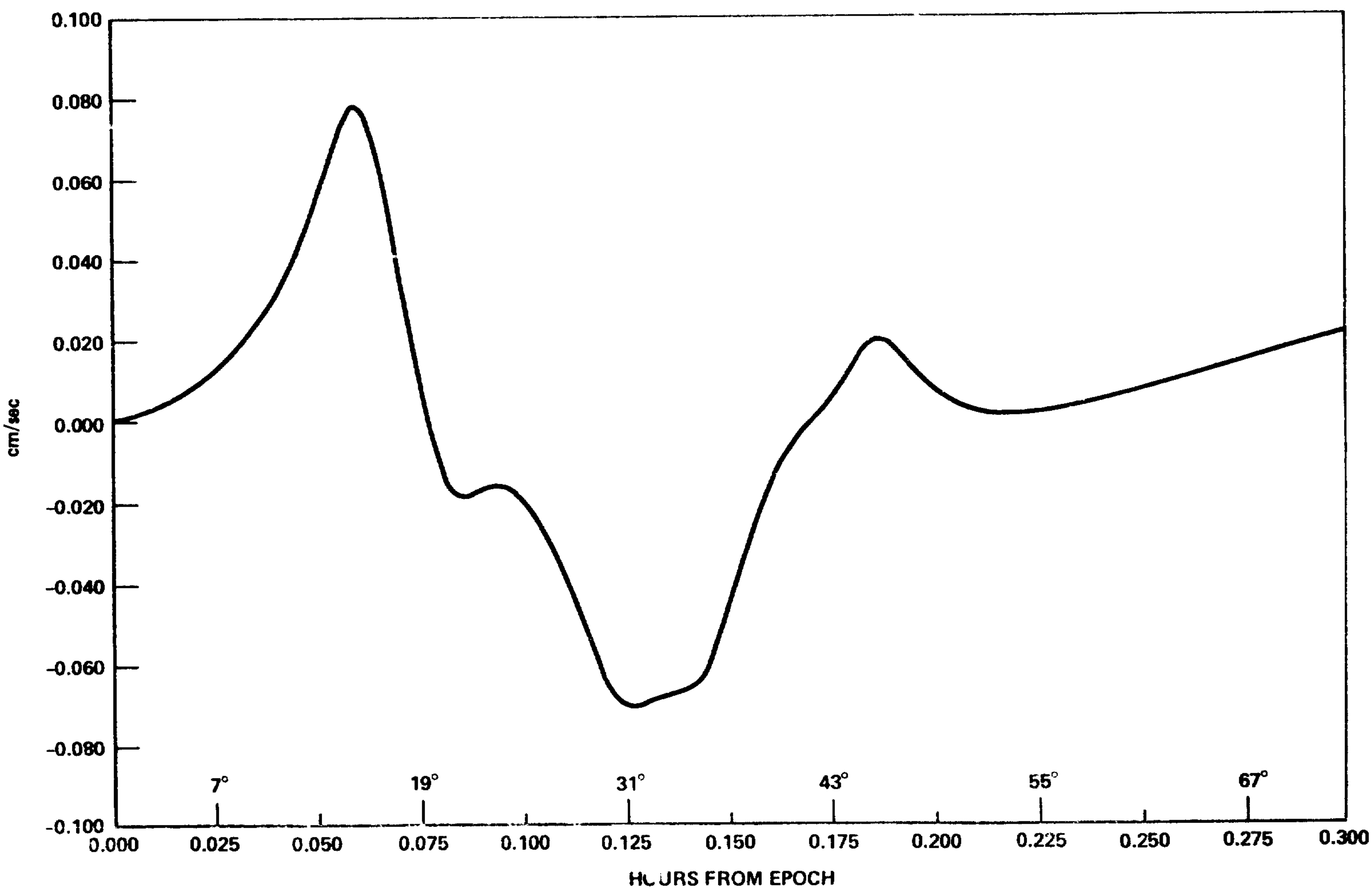

Figure 8. Signal in a Complex Field: 1° Satellite Separation

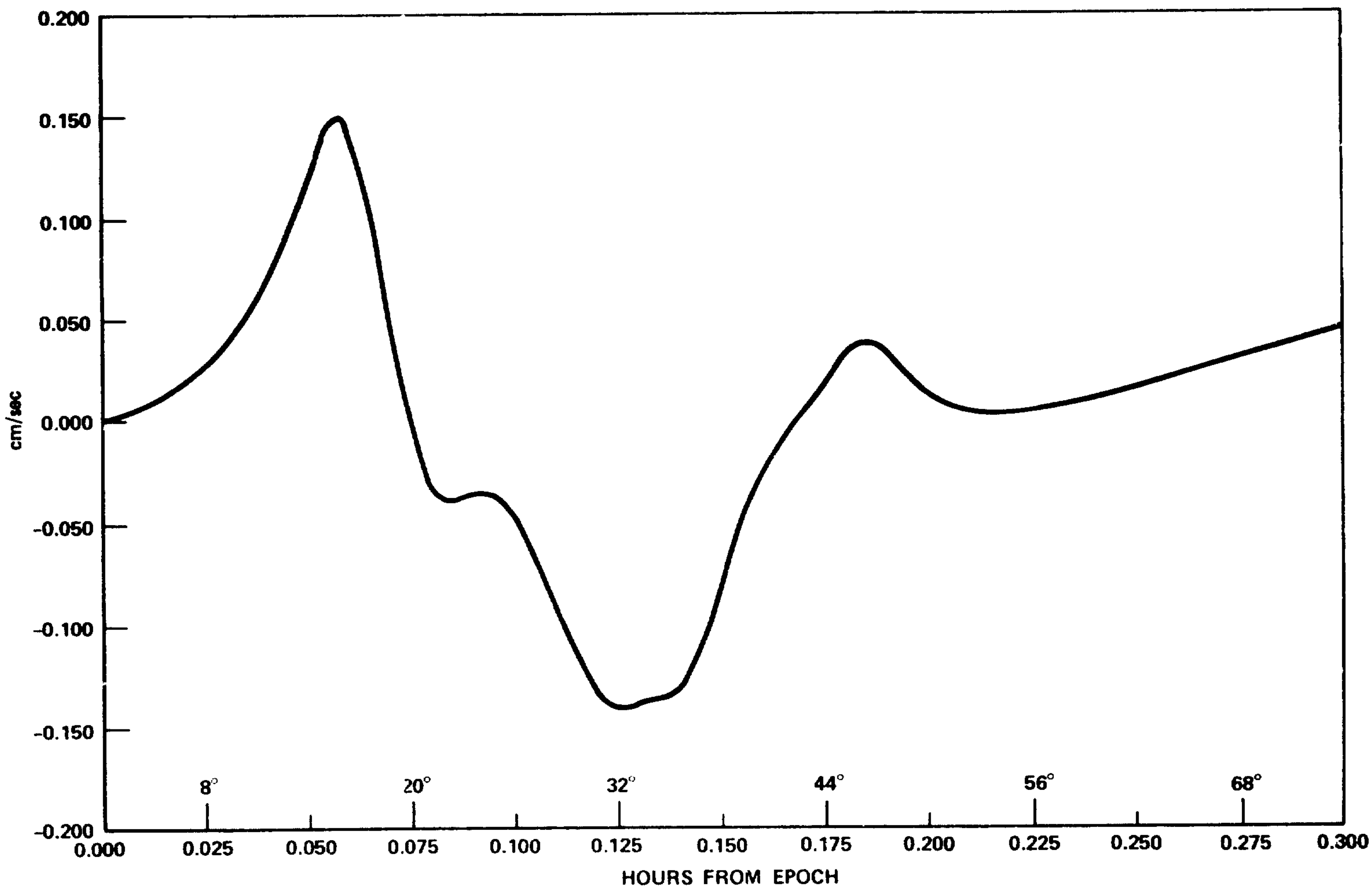

Figure 9. Signal in a Complex Field: 2° Satellite Separation

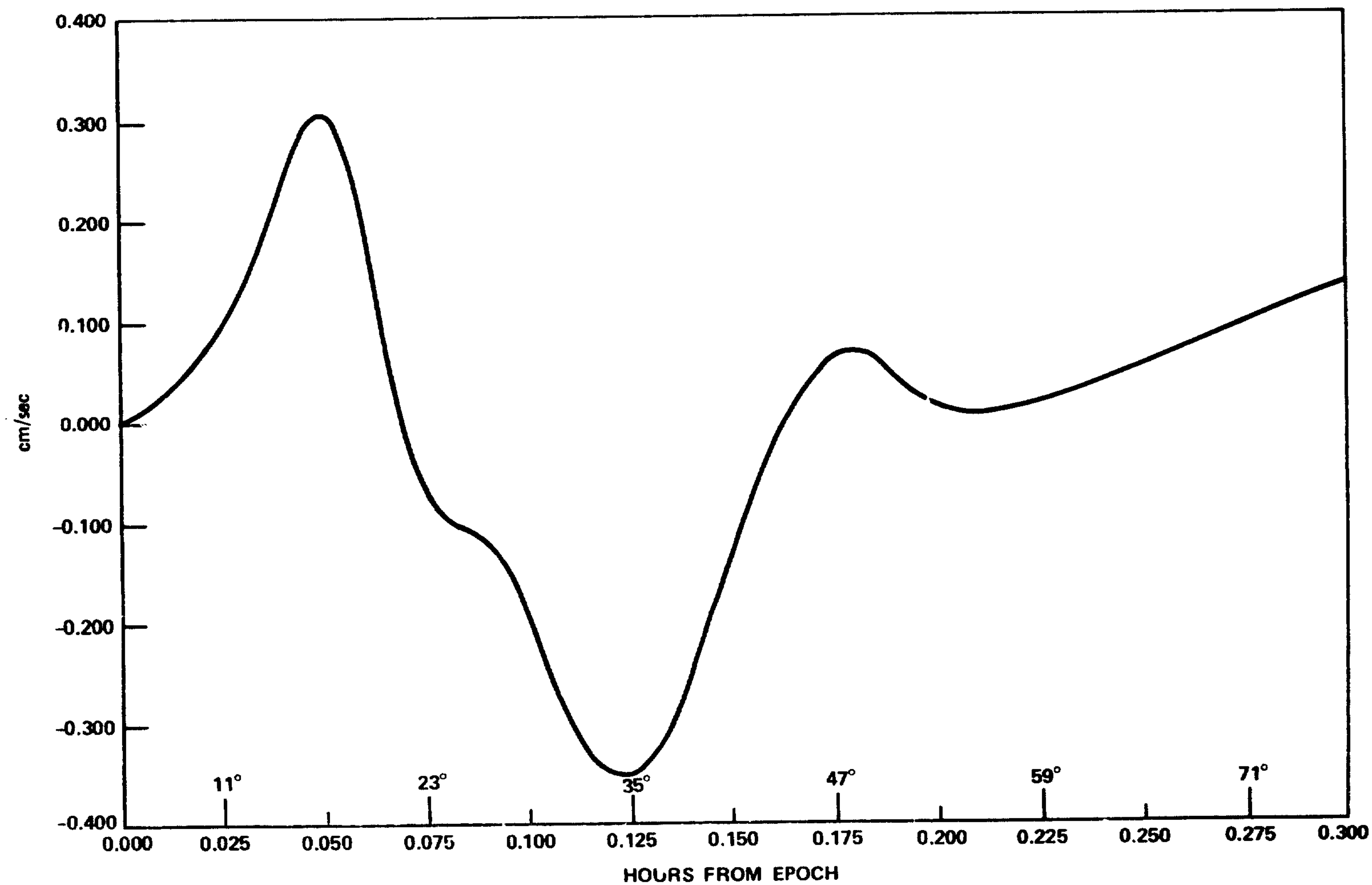

Figure 10. Signal in a Complex Field: 5° Satellite Separation

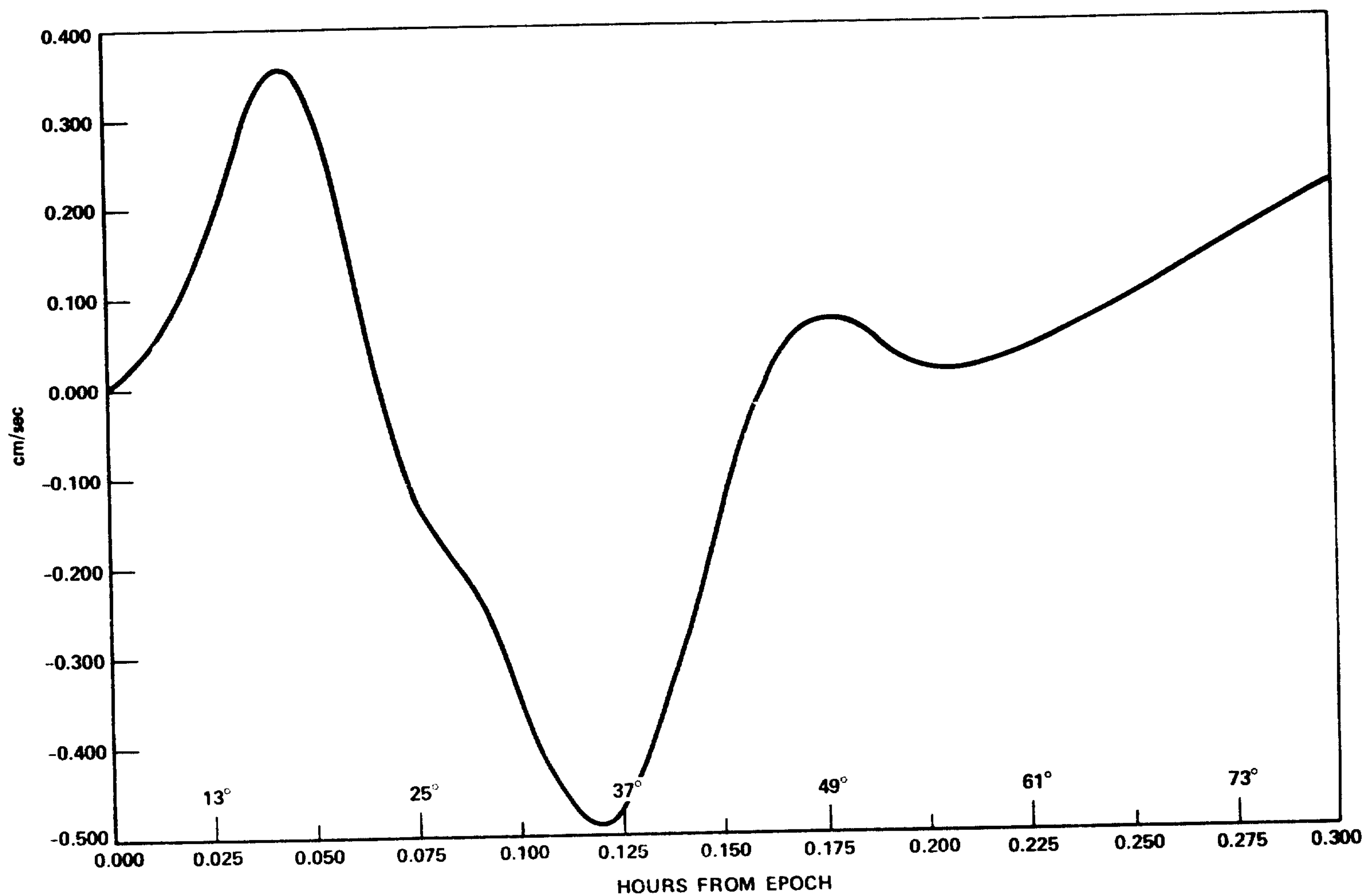

Figure 11. Signal in a Complex Field: 7° Satellite Separation

Table A

Time in Orbit Minutes	Latitude of Subsatellite Point	Height of Spacecraft km	Osculating Semimajor Axis
0	0°	250	6628
22.5	90°	265	6608
44.5	0°	244	6628
67	-90°	265	6608
99.5	0°	250	6628
112	90°	265	6608

The perturbations drive the eccentricity up to 0.002 within 2 hrs. This then is a lower limit for an attainable eccentricity of a near earth satellite.

This rapid variation of altitude would require consideration in the design of a surface drag compensation force. Such a system would have to respond rapidly to variations in atmospheric density.

Additionally, the J_2 term forms the major contribution to the relative range-rate of a satellite pair in polar orbits. Therefore, in the real situation, either the J_2 variation in range-rate would have to be subtracted before local gravity anomalies would be observable in the data, or else it would have to be accounted for in the mathematical model. Also, it may be desirable to employ a reference field containing higher order terms than J_2; a suitable reference field has not yet been established. But, in any event, the relative range-rate measured between the 2 satellites will be dominated by differential accelerations due to the J_2.

EFFECT OF NOISE ON THE LO-LO DATA TYPE

The strength of the signal in the relative range-rate data type is weak, on the order of 1 cm./sec for a mascon of 10^{-7} earth masses. Therefore the system noise is of importance in considering the feasibility of this data type. It is useful to compare the signal strength with the expected noise behavior of operational systems.

The Doppler measurement of velocity is fundamentally involved with the counting of the number of oscillations/second. Only ongoing positive crossings can be measured. Thus, the longer the integration time of the measurement, the smaller the error contributed by the quantization of the oscillation will be. For the ATS-6/Nimbus system the range-rate accuracy increases as 1/T for count times up to 5 seconds due to reduction of quantization error; for a longer integration time ($T > 5$ sec) the effect of transmitter reference phase jitter should also be considered (Schmidt, 1970). Thus the range-rate resolution

$$\sigma_R = \sqrt{\left(\frac{0.6}{T}\right)^2 + \left(\frac{0.1}{\sqrt{T}}\right)^2} \text{ cm/sec}$$

yielded

Integration Time sec	Range-Rate Resolution cm/sec
1	0.6
5	0.2
10	0.07

Therefore the noise in this data type (for current systems) is significant when compared with the signal strength of the Lo-Lo signal. Furthermore, the noise dependence on the integration time means that it is of little value to increase the number of data points along an arc. (Although improving the data density by combining data points from several arcs should improve the standard deviation by a factor $\sqrt{N}$, where N is the number of data points.) An integration time of 5 seconds implies that a satellite going 7 km/sec will take only 3 measurements per degree. Thus the data rate and the noise are inextricably correlated. The ability to resolve fine structure geopotential features must be compared with a system's noise properties as a function of data rate; increasing the data rate may increase the noise and show a geopotential feature no better than a slower rate with no noise.

CONCLUSIONS

The Lo-Lo range-rate data type produces a weak signal that is localized in the short term signal. Additionally, there is a periodic component acting over the entire orbit which may cause aliasing.

The recommended satellite configuration is two satellites separated by 3° along track placed at an altitude of 300 km. This will provide a signal strength on the order of 0.1 cm/sec for a gravity anomaly equivalent to 10 mgals in a 5° x 5° block. This system is capable of resolving two large mascons 5° apart, if the noise level on the data is sufficiently low, about 0.005 cm/sec, and if the aliasing is not too severe. The effects of aliasing require study by error analyses.

When the signal strength is compared with the noise properties of current systems, the outlook becomes more pessimistic. Current instrument noise levels of 0.05 cm/sec would tend to swamp a signal with peak-to-peak strength of 0.1 cm/sec. Because of the noise dependence of the system, increasing the data rate to along an arc to provide more density to improve the statistics would be offset by an increase in the noise level. The data density and thus the statistics could be improved by increasing the number of data arcs.

In designing a real mission for polar satellites at 300 km, the effects of the J_2 term of the earth's gravitational field must be accounted for. One effect is to produce a large range-rate signal, larger than other gravitational anomalies. Also, the J_2 will cause large changes in altitude, on the order of 20 km. This will result in an "orbit" with 2 perigees per revolution! Since the density of the earth's atmosphere varies exponentially with altitude, a drag-free polar satellite must be designed to expect wide variations in drag.

The weak signal strength in this data type can be improved somewhat but not substantially by designing an appropriate satellite configuration. In particular, the gain in magnitude of the signal strength and resolution achieved by lowering the satellite altitude must be weighed against the disadvantageous caused by increasing atmospheric drag.

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