EFFECT OF POLARIZATION ON SUPERFLUIDITY IN LOW DENSITY NEUTRON MATTER

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Abstract

It is shown that, contrary to an earlier estimate, the polarizability of the neutron medium tends to suppress rather than enhance the isotropic energy gap in low-density neutron-star matter.

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The hydrodynamics of the superfluid interior of a neutron star--and associated relaxation phenomena presumably susceptible to observation--are quite sensitive to the isotropic energy gap Δ_{k_F} of the neutron matter of the crustal layers [1]. The most quantitative evaluation of Δ_{k_F} is the variational calculation of Yang and Clark [2] based on a wave function incorporating short-range Jastrow correlations as well as longer-range pairing or BCS correlations. For pure neutron matter and an adequately realistic two-neutron potential, these authors find that Δ_{k_F} peaks at $k_F = 0.72\ \mathrm{fm}^{-1}$, with a value $\Delta_{k_F} = 2.45\cdot\mathrm{MeV}$.

It has been argued by Pines [3] that this kind of evaluation is likely to yield a result for the energy gap which is on the low side, because the polarizability of the neutron medium has been essentially neglected. According to an estimate by Pines and Pethick, polarization of the medium tends to enhance a "bare" attractive ${}^1S_0$ interaction between two neutrons with wave vectors $\underset{\sim}{k}$, $-\underset{\sim}{k}$ near the Fermi surface. The enhancement factor was found to be $(1+F_0)^{-1}$, where F_0 is the leading Landau Fermi-liquid parameter appearing in the Legendre expansion of the spin-symmetric part of the interaction between two quasiparticles on the Fermi surface. With $F_0 \sim -0.7$, a substantial amplification of the pairing matrix elements would result, and the energy gap and condensation energy, which are extremely sensitive functions of these matrix elements, would increase dramatically. Indeed, these considerations raise the intriguing possibility that polarization and pairing may conspire to bring about a first-order phase transition in low-density neutron matter or even a bound state, metastable or stable.

In this note we shall take a closer look at the singlet-state quasiparticle interaction in neutron matter. Our considerations will be based on the results of a detailed evaluation of the Landau Fermi-liquid parameters for pure neutron matter [4] including polarization effects, i.e., including the interaction induced by exchange of density and spin-density excitations. It will be concluded that, owing to the spin-dependence of the quasiparticle interaction, and ultimately the balance of attraction, repulsion and spin-dependence in the fundamental two-neutron interaction, polarization actually works to suppress rather than to enhance the pairing matrix elements.

In the theory of Babu and Brown [5] the Landau quasiparticle interaction energy is approximated as

$$\mathcal{f}(\underset{\sim}{k}_1,\underset{\sim}{k}_2;\sigma_1,\sigma_2) = \mathcal{f}_d(\underset{\sim}{k}_1,\underset{\sim}{k}_2;\sigma_1,\sigma_2) + \mathcal{f}_i(\underset{\sim}{k}_1,\underset{\sim}{k}_2;\sigma_1,\sigma_2) \quad , \tag{1}$$

where the "direct" part $\mathcal{f}_d$ is obtained by functional differentiation of the lowest-order Brueckner approximation to the ground-state energy and the "induced" part of $\mathcal{f}_i$ arises from exchange of density and spin-density excitations. The induced interaction between quasiparticles with wave vectors $\underset{\sim}{k}_1,\underset{\sim}{k}_2$, in individual spin states s_1,s_2, is given by

$$\mathcal{f}_i^{s_1 s_2}(\underset{\sim}{k}_1,\underset{\sim}{k}_2) = -\sum_{\substack{\underset{\sim}{p},\underset{\sim}{p}'\\ \alpha\beta}} \mathcal{f}^{s_1\alpha}(\underset{\sim}{k}_1,\underset{\sim}{p}) \left(\frac{\delta n_{\underset{\sim}{p},\alpha}}{\delta u_{\underset{\sim}{p}',\beta}}\right)_{\underset{\sim}{q},0} \mathcal{f}^{\beta s_2}(\underset{\sim}{p}',\underset{\sim}{k}_2), \tag{2}$$

and represented graphically in fig. 1, where the blob stands for the response function $(\delta n_{\underset{\sim}{p},\alpha}/\delta u_{\underset{\sim}{p}',\beta})_{\underset{\sim}{q},0}$. Babu and Brown furnish a transport equation for $(\delta n_{\underset{\sim}{p},\alpha}/\delta u_{\underset{\sim}{p}',\beta})_{\underset{\sim}{q},\omega}$ in terms of the full interaction $\mathcal{f}^{\alpha\beta}(\underset{\sim}{p},\underset{\sim}{p}')$. (N.B. $\delta n_{\underset{\sim}{p},\alpha}$ is the change in the quasiparticle occupation number $n_{\underset{\sim}{p},\alpha}$ due to a weak external potential $\delta u_{\underset{\sim}{p}'\beta}(\underset{\sim}{q},\omega)$, where $\underset{\sim}{q},\omega$ is the four-vector of the induced disturbance.) Strictly, (2) applies to the long-wavelength limit $q \equiv |\underset{\sim}{k}_1-\underset{\sim}{k}_2| \rightarrow 0$; an extrapolation from this limit is performed as described in refs. [4,6], permitting (2) (with use of (1) and the transport equation) to be solved for the induced interaction $\mathcal{f}_i$. Certain diagrams must be omitted from the direct part of $\mathcal{f}$ so as to avoid double counting.

The spin dependence of the quasiparticle interaction takes the form

$$\mathcal{f}(\underset{\sim}{k}_1,\underset{\sim}{k}_2;\underset{\sim}{\sigma}_1,\underset{\sim}{\sigma}_2) = f(\underset{\sim}{k}_1,\underset{\sim}{k}_2) + g(\underset{\sim}{k}_1,\underset{\sim}{k}_2)\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2 \ , \tag{3}$$

and similarly for $\mathcal{f}_i$ and $\mathcal{f}_d$. The spin-symmetric part f and the spin-antisymmetric part g may be expanded in Legendre polynomials of $x = \cos(\underset{\sim}{k}_1,\underset{\sim}{k}_2)$:

$$f(\underset{\sim}{k}_1,\underset{\sim}{k}_2) = \sum_{\ell} f_\ell P_\ell(x) \ , \quad g(\underset{\sim}{k}_1,\underset{\sim}{k}_2) = \sum_{\ell} g_\ell P_\ell(x) \ . \tag{4}$$

With $k_1 = k_2 = k_F$, the Landau parameters f_ℓ, g_ℓ depend only on the density.

Keeping just the first two terms in these Legendre expansions, Bäckman <u>et</u> <u>al</u>. [4] derived a useful approximate expression for $\mathcal{f}_i$. Numerically it turns out (in their calculation for the Reid potential) that the parameters f_1 and g_1 are small compared to f_o and g_o in the density range of most interest for isotropic superfluidity ($k_F = 0.4$-$0.8\ \mathrm{fm}^{-1}$). It is therefore a good approximation to drop terms in f_1^2 and g_1^2, and work with this simplified version of the expression of ref. [4]:

$$F_i^{\uparrow\uparrow} \equiv N(0)\mathcal{f}_i^{\uparrow\uparrow} = \left[\frac{F_o^2}{1+F_oU(q)} + \frac{G_o^2}{1+G_oU(q)}\right]U(q) \ ,$$

$$F_i^{\uparrow\downarrow} \equiv N(0)\mathcal{f}_i^{\uparrow\downarrow} = 2\,\frac{G_o^2}{1+G_oU(q)}\,U(q) \ . \tag{5}$$

Here we use an arrow notation for the individual spin states s_1, s_2. The capital F's and G's are dimensionless, being obtained from the lower case f's and g's by multiplication with the density-of-states factor $N(0)=m^*k_F/\pi^2\hbar^2$ where m* is the quasiparticle effective mass. Finally, U(q) is the Lindhard function

$$U(q) = \frac{1}{2}\left[1 + \left(\frac{q}{4k_F} - \frac{k_F}{q}\right)\ln\left(\frac{k_F - q/2}{k_F + q/2}\right)\right]. \qquad (6)$$

The contribution of the induced interaction to the(dimensionless)singlet quasiparticle interaction is given by the spin-symmetric part of $N(0)f_i$ minus 3 times its spin-antisymmetric part(since $\underset{\sim}{\sigma}_1\cdot\underset{\sim}{\sigma}_2\to -3$), i. e., by

$$F_i({}^1s_0) = \frac{1}{2}\left[F_i^{\uparrow\uparrow} + F_i^{\uparrow\downarrow}\right] - \frac{3}{2}\left[F_i^{\uparrow\uparrow} - F_i^{\uparrow\downarrow}\right]$$
$$= -\left[\frac{F_o^2}{1+F_oU(q)} - 3\,\frac{G_o^2}{1+G_oU(q)}\right]U(q)\ . \qquad (7)$$

(We note that at these densities D and higher even waves are unimportant.) Taking numerical values for F_0 and G_0 from ref. 4 at $k_F\sim 0.5\ \mathrm{fm}^{-1}$, we find, to a very good approximation, $F_i({}^1s_o) \cong 1.2\ U(q)$. This is a smooth function of q on the individual $[0,\ 2k_F]$. Performing an angle average(a $q\,dq/2k_F^2$ integration over the stated interval), we arrive at $\bar{F}_i({}^1s_o) \cong 1.2\ U(1.5k_F)=0.9$. This result is almost independent of density in the aforementioned k_F range.

If the second term in square brackets in (7) were negligible compared to the first, our result would conform with the original estimate $-|VF_o|/(1+F_o)$ of Pines and Pethick, if U(q) were set unity(not far off) and the"bare" interaction V identified with F_o. However, with $G_o\sim 0.8$ and $F_o\sim -0.3$ according to [4] , the second term clearly dominates, reversing the sign of the polarization-induced interaction relative to the Pines-Pethick estimate.

We are indebted to C. Pethick for the following clarifying remarks: The main reason the present conclusion differs from that of ref. [7] is that the calculated equation of state of neutron matter has become less unstable against density fluctuations (F_0 has grown larger) since the time of the Pines-Pethick estimate The latter was made including only the density fluctuation channel, since it would be dominant over the spin-1 exchange channel for $F_0 \rightarrow -1$. (This is very similar to the paramagnon model for liquid ^{3}He, except that there $G_0 \rightarrow -1$.) Of course, if F_0 were close to -1, polarization effects would still enhance the singlet quasiparticle interaction even if the spin-dependence of the quasiparticle interaction were taken into account.

For the direct part of the quasiparticle interaction we may take the renormalized form [6]

$$\mathcal{F}_d^{s_1 s_2}(\underset{\sim}{k}_1,\underset{\sim}{k}_2) = (1-2\kappa)\langle \underset{\sim}{k}_1 s_1, \underset{\sim}{k}_2 s_2 | t | \underset{\sim}{k}_1 s_1, \underset{\sim}{k}_2 s_2 - ex \rangle \quad , \qquad (8)$$

where t is the Brueckner reaction operator defined in terms of the usual choice of single-particle spectrum having a jump at the Fermi surface [7] and κ is the wound parameter. At $k_F \sim 0.5\ fm^{-1}$, the direct part of the(dimensionless) singlet quasiparticle interaction is accordingly estimated to be $F_d(^1s_0) \cong -3.5$. The net effect of the polarization contribution is thus to suppress the singlet interaction by a factor $\beta \simeq 0.7-0.8$ relative to $F_d(^1s_0)$. The quantity $F_d(^1s_0)$ depends more strongly on density than $F_i(^1s_0)$; hence the suppression factor β will show some appreciable density dependence.

What effect will the associated supression of the pairing matrix elements have on the energy gap and condensation energy of neutron matter? A simple-minded answer may be based on the weak-coupling formula [8] $\Delta_{k_F} = (4\hbar^2 k_F^2/m^*)\exp(-1/N(0)V)$. This formula is used once, inserting the Δ_{k_F} result of Yang and Clark [2] and their m* value, to determine a "bare" V, then again, with $V \rightarrow \beta V$, to calculate the supressed gap. At k_F=0.6 fm^{-1}, we find, taking β =0.74, that the energy gap is cut down from 2.24 Mev to 0.69 MeV. Correspondingly, the weak-coupling approximation predicts that the condensation energy E_c is suppressed(from 0.301 MeV) by an order of magnitude. An elaborate evaluation of gap and condensation energy using the full method of Yang and Clark, with the pairing matrix elements $P_{\underset{\sim}{k}\underset{\sim}{\ell}}$ of that approach replaced by $\beta P_{\underset{\sim}{k}\underset{\sim}{\ell}}$ (but no other modifications), yields the suppressed values Δ_{k_F} = 0.64 MeV and $E_c \cong 0.03$ Mev. This evaluation rests on the unjustified but at first sight not implausible identification of the effective interaction $\langle 12|w_2|12-21\rangle$ of the Yang-Clark method with the direct quasiparticle interaction. It is important to remember, however, that the calculation of ref. [2] is variational in nature, dealing with the expectation value of the raw neutron-matter Hamiltonian. (The three-body and higher-order cluster contributions to the Hamiltonian expectation value, not treated, are almost certainly negligible at these low densities.) Therefore the gain of energy E_c which Yang and Clark obtain with their Jastrow-BCS trial wave function, over the simple Jastrow-Fermi gas energy evaluation, is surely genuine. However, it could be that the Jastrow wave function used for the normal ground state does not adequately incorporate the effects of low-lying virtual excitations(especially those corresponding to polarization of the medium). It could be that an improved superstate trial wave function, incorporating short-range correlations, polarization effects, and pairing correlations, would lead to essentially the same or a somewhat increased energy gain, but with a substantial reduction in the optimal energy gap Δ_{k_F}. We are currently looking into this possibility.

Evidently a workable first-principles theory of pairing in the presence of both short-range correlations and polarization is needed.

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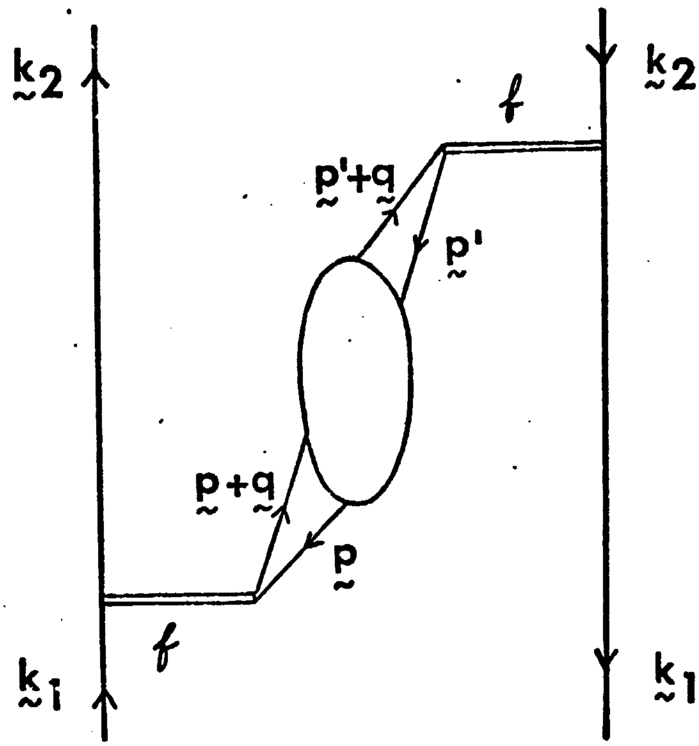
Figure Caption

Fig. 1. Induced interaction (spin labels omitted for simplicity).



k2
k2
f
p'+q
p'
p+q
p
f
k1
k1